Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.2-Quadratic-binomial/30-1.1.2.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**1304**]. This is test number [30].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (1304)	0.00 (0)
Mathematica	100.00 (1304)	0.00 (0)
Sympy	96.93 (1264)	3.07 (40)
Maple	61.73 (805)	38.27 (499)
Fricas	57.44 (749)	42.56 (555)
Mupad	56.29 (734)	43.71 (570)
Reduce	50.23 (655)	49.77 (649)
Maxima	48.54 (633)	51.46 (671)
Giac	46.93 (612)	53.07 (692)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

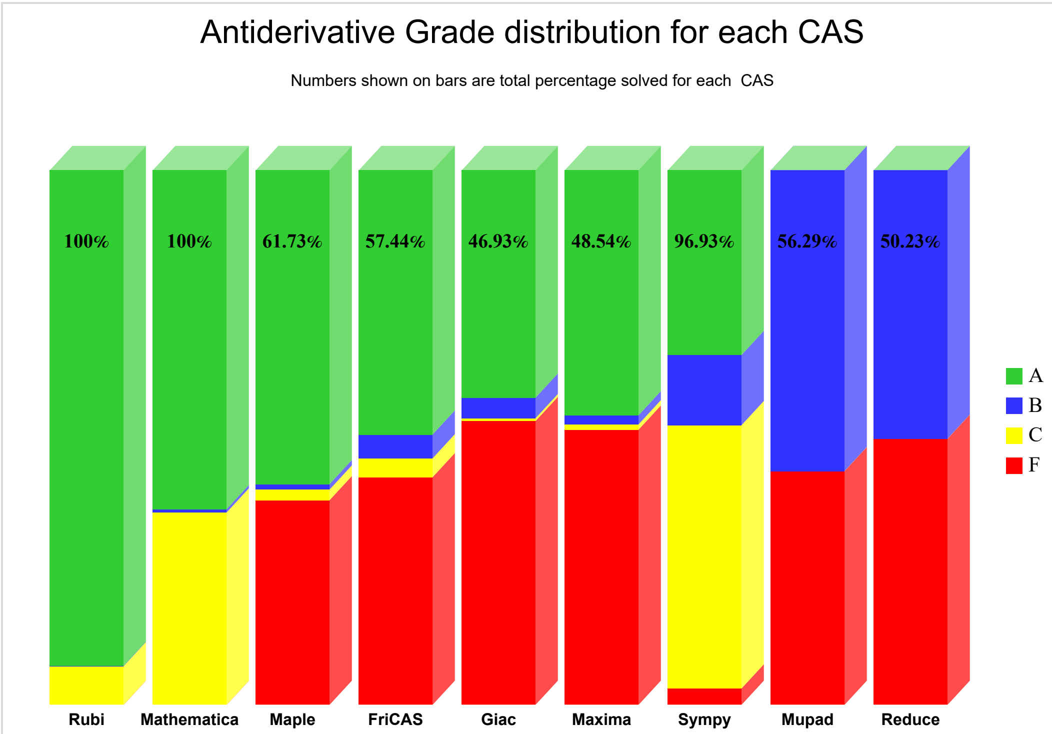
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

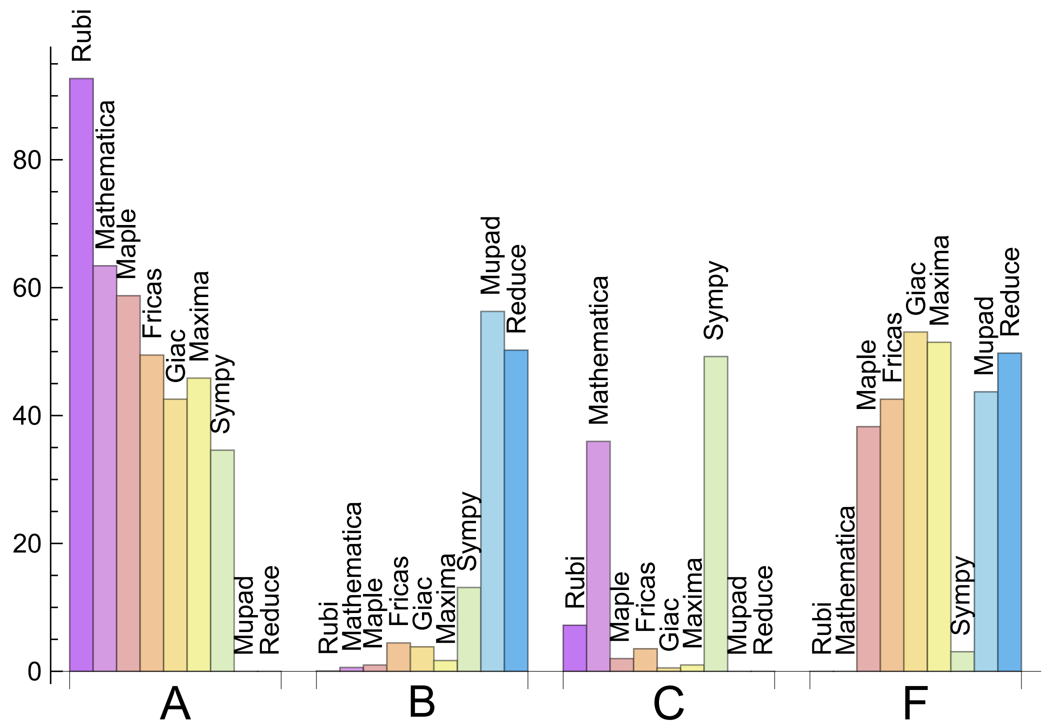
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.715	0.077	7.209	0.000
Mathematica	63.420	0.613	35.966	0.000
Maple	58.742	0.997	1.994	38.267
Fricas	49.463	4.448	3.528	42.561
Maxima	45.859	1.687	0.997	51.457
Giac	42.561	3.834	0.537	53.067
Sympy	34.586	13.113	49.233	3.067
Mupad	0.000	56.288	0.000	43.712
Reduce	0.000	50.230	0.000	49.770

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Sympy	40	2.50	97.50	0.00
Maple	499	100.00	0.00	0.00
Fricas	555	95.86	4.14	0.00
Mupad	570	0.00	100.00	0.00
Reduce	649	100.00	0.00	0.00
Maxima	671	100.00	0.00	0.00
Giac	692	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.06
Fricas	0.07
Giac	0.13
Rubi	0.21
Reduce	0.22
Mupad	0.31
Maple	0.54
Mathematica	3.55
Sympy	4.47

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	61.07	0.73	54.00	0.77
Mupad	61.72	0.88	41.00	0.81
Maple	65.50	0.81	47.00	0.78
Maxima	73.24	0.99	57.00	0.91
Giac	77.12	1.10	57.00	0.89
Reduce	101.23	1.39	62.00	1.06
Fricas	105.49	1.30	65.00	1.00
Sympy	115.15	1.48	44.00	0.79
Rubi	127.22	1.02	75.50	1.02

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

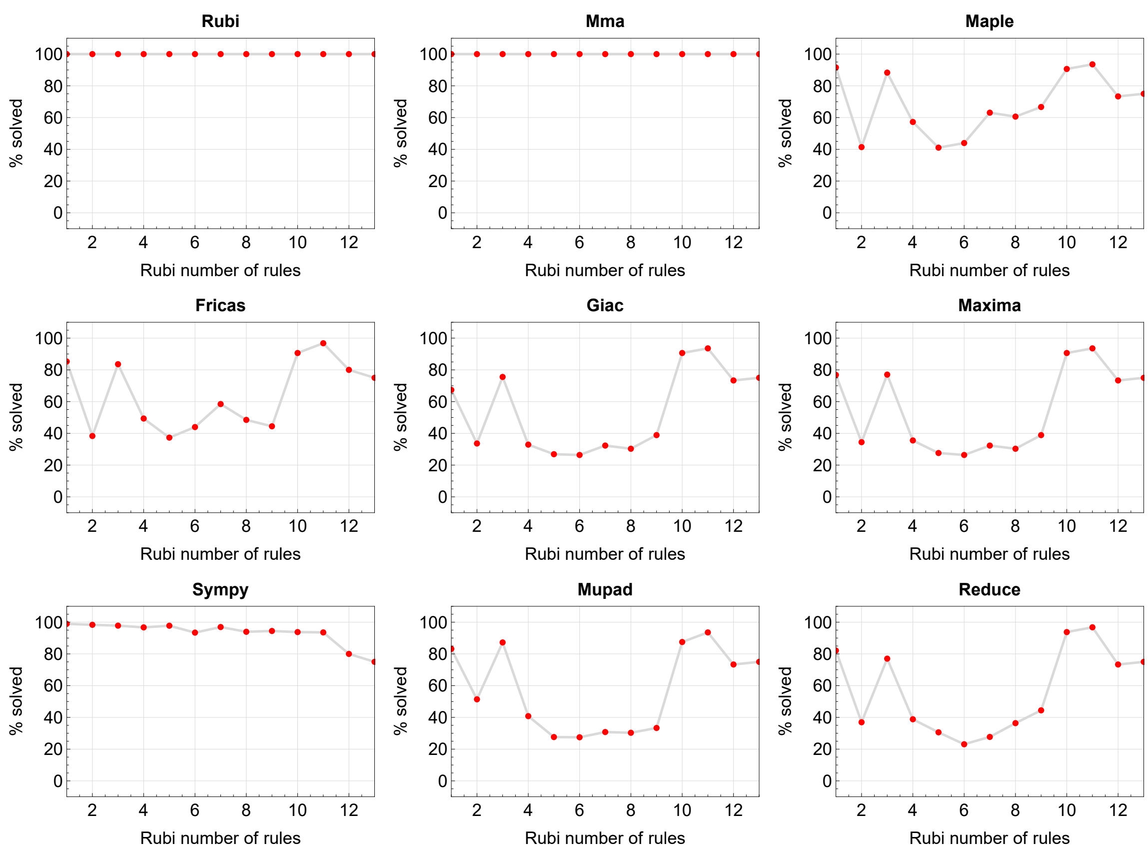


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

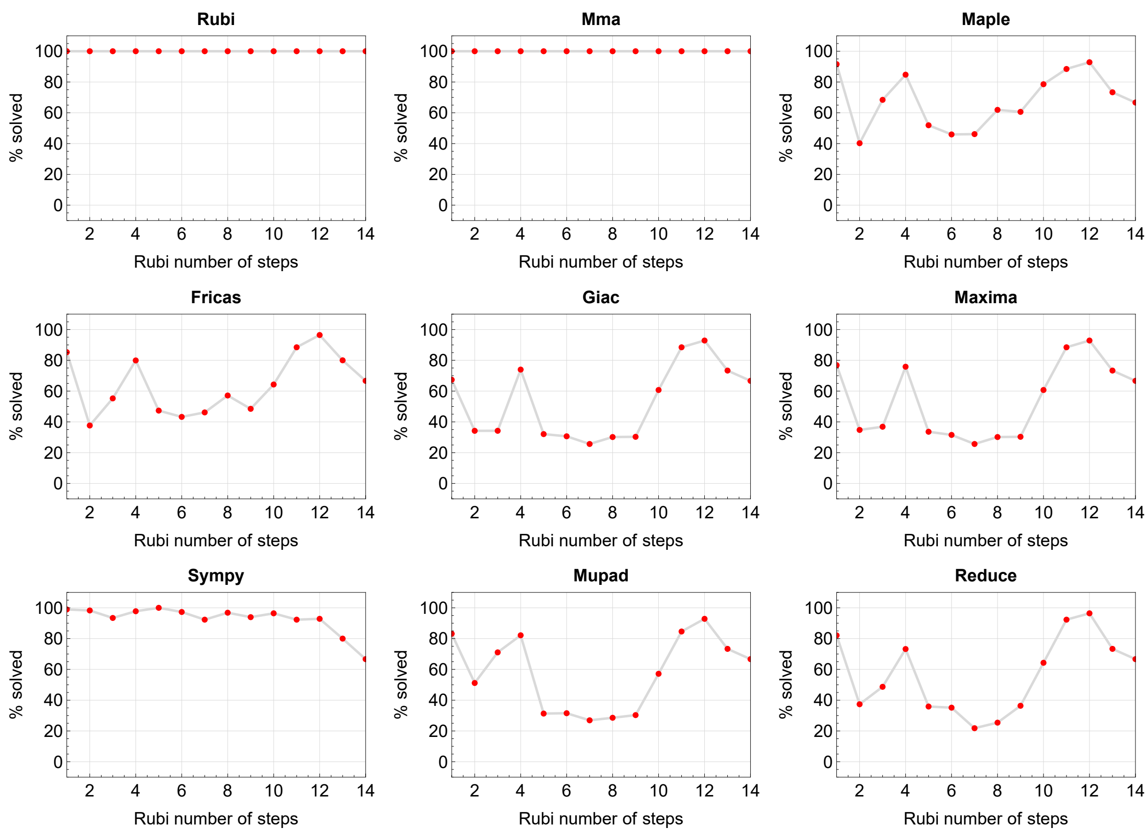


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

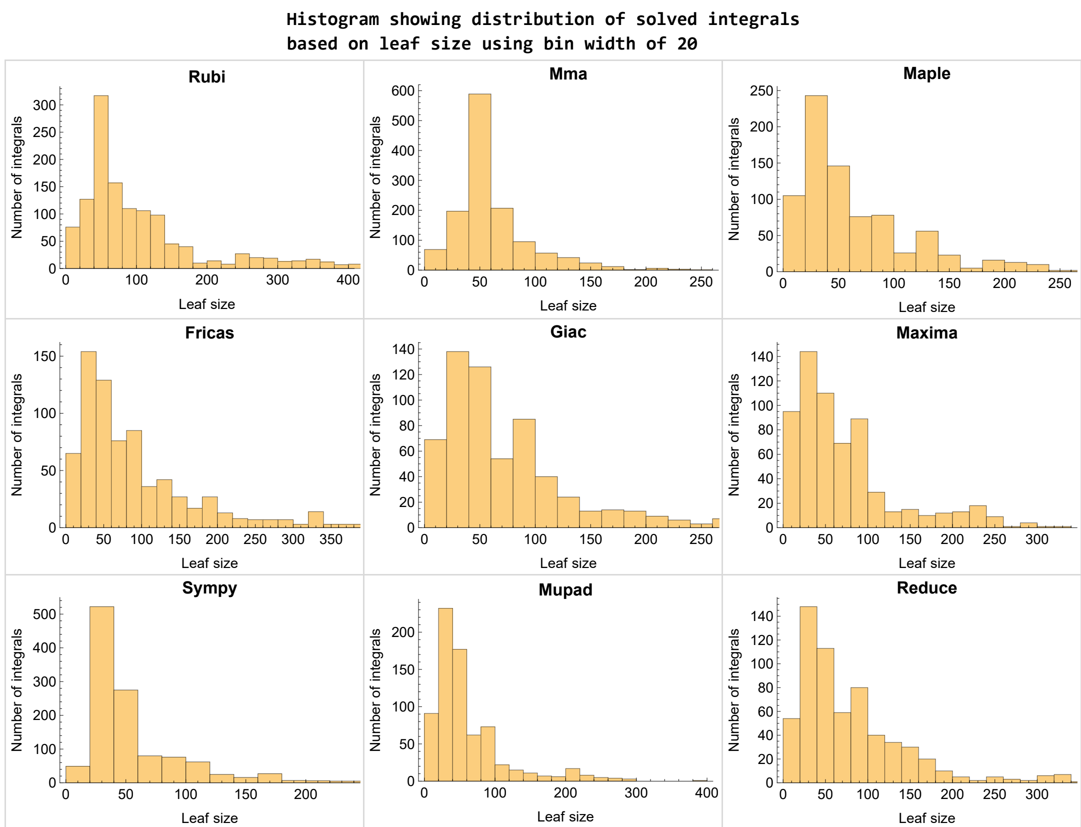


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

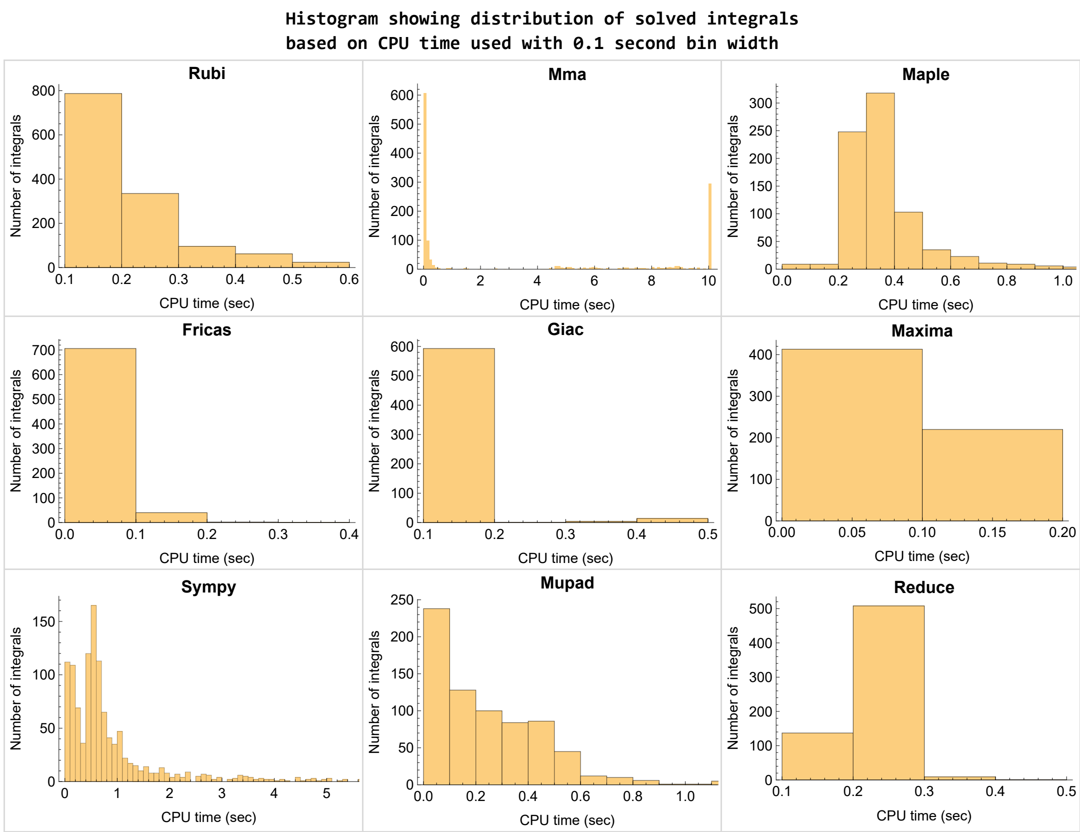


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

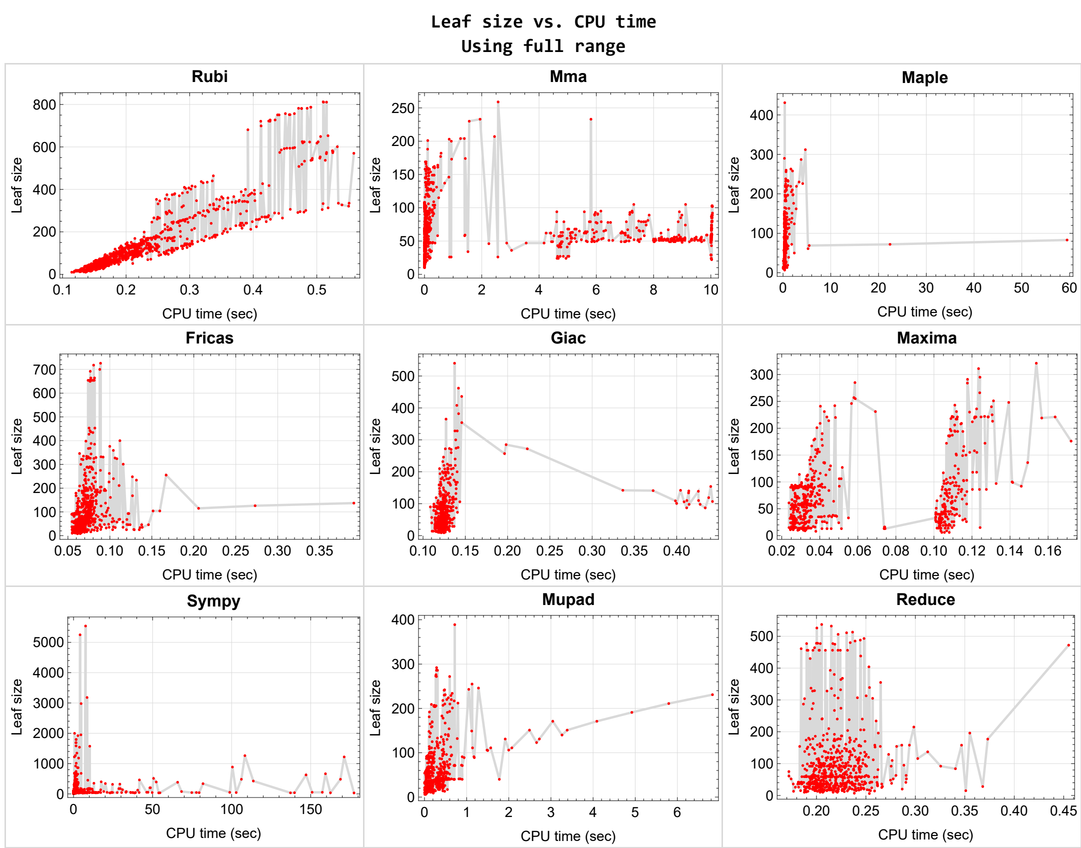


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {339, 341, 344, 346, 733, 734, 737, 757, 759, 760, 761, 781, 782, 783, 784, 785, 786, 787, 788, 789, 794, 795, 796, 797, 798, 802, 803, 804, 805, 806, 810, 811, 812, 813, 814, 815, 819, 820, 821, 822, 827, 828, 829, 830, 831, 983, 984, 985, 990, 991, 992, 993, 994, 995, 996, 1005, 1006, 1010, 1011, 1012, 1013, 1014, 1022, 1023, 1024, 1025, 1026, 1027, 1044, 1045, 1046, 1047, 1048, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118}

Mathematica {}

Maple {7, 955, 956, 957, 958, 959, 960, 961, 969, 970, 971, 972, 973, 974, 975, 1302, 1303}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
```

```
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
```

```
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

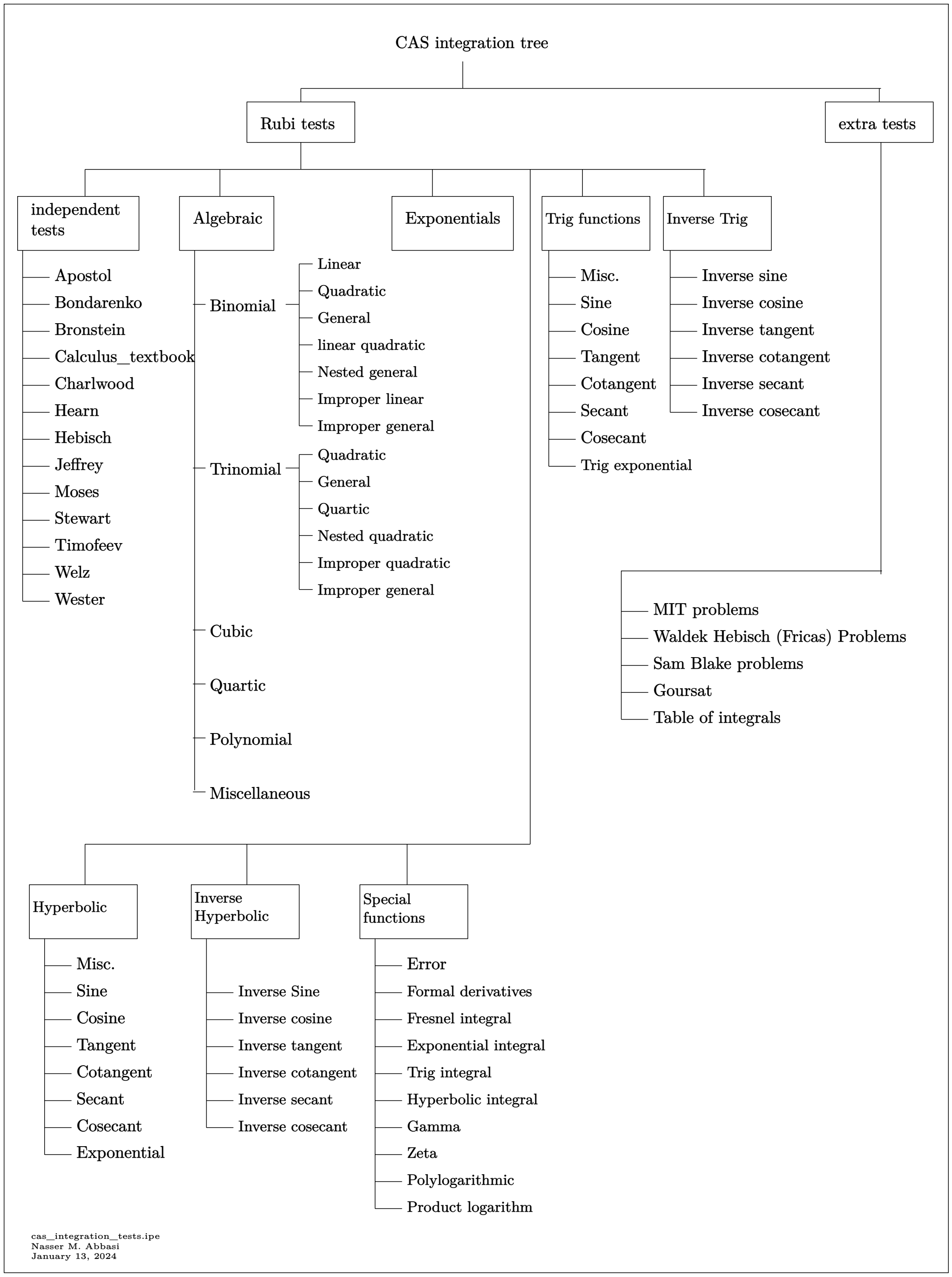
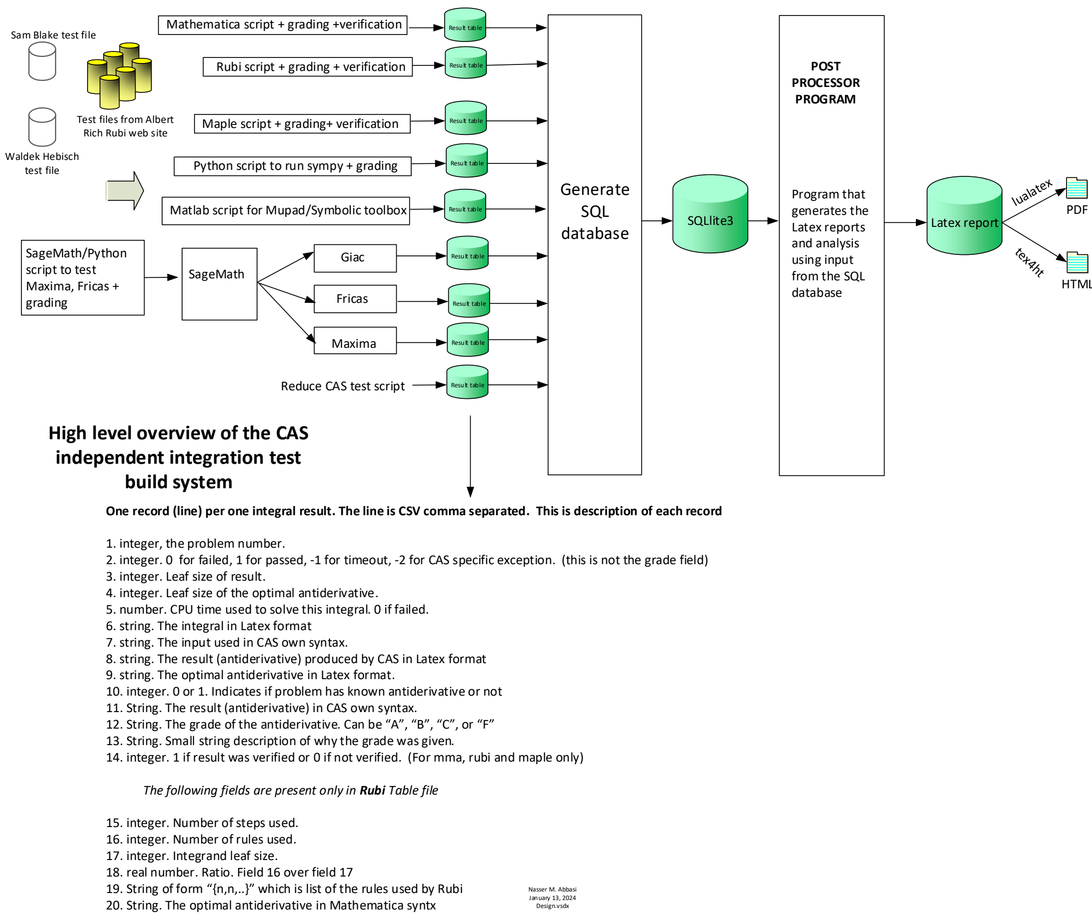


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26,
27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51,
52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76,
77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101,
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445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463,

464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 668, 669, 671, 674, 675, 677, 678, 679, 680, 687, 690, 691, 693, 696, 697, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 712, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

B grade { 235 }

C grade { 666, 667, 670, 672, 673, 676, 681, 682, 683, 684, 685, 686, 688, 689, 692, 694, 695, 698, 711, 713, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 741, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 762, 763, 764, 765, 766, 767, 768, 774, 775, 776, 777, 778, 779, 780, 786, 787, 788, 789, 790, 791, 792, 793, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 832, 833, 834, 948, 949, 950, 951, 983, 984, 985, 986, 987, 988, 989, 995, 996, 997, 998, 999, 1005, 1006, 1007, 1008, 1009, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1028, 1029, 1030, 1031, 1032, 1039, 1040, 1041, 1042, 1043, 1049, 1050, 1051, 1052, 1053, 1054, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1267, 1268, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

B grade { 38, 65, 90, 101, 102, 196, 197, 569 }

C grade { 348, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 710, 711, 712, 713, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 745, 746, 747, 748, 749, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 781, 782, 783, 784, 785, 794, 795, 796, 797, 798, 810, 811, 812, 813, 814, 815, 827, 828, 829, 830, 831, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 990, 991, 992, 993, 994, 1000, 1001, 1002, 1003, 1004, 1010, 1011, 1012, 1013, 1014, 1022, 1023, 1024, 1025, 1026, 1027, 1033, 1034, 1035, 1036, 1037, 1038, 1044, 1045, 1046, 1047, 1048, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1288, 1289 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 351, 352, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 488, 489, 490, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 664, 710, 712, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 741, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 762, 763, 764, 765, 766, 767, 768, 774, 775, 776, 777, 778, 779, 780, 790, 791, 792, 793, 807, 808, 809, 823, 824, 825, 826, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 986, 987, 988, 989, 997, 998, 999, 1007, 1008, 1009, 1018, 1019, 1020, 1021, 1030, 1031, 1032, 1041, 1042, 1043, 1051, 1052, 1053, 1054, 1267, 1268, 1269, 1270, 1288, 1289, 1290, 1299 }

B grade { 38, 65, 90, 101, 102, 196, 197, 337, 349, 350, 648, 711, 713 }

C grade { 362, 485, 487, 491, 576, 580, 661, 663, 699, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 1300, 1302, 1303 }

F normal fail { 354, 355, 356, 357, 358, 359, 360, 361, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 745, 746, 747, 748, 749, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 781, 782, 783, 784, 785, 786, 787, 788, 789, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 983, 984, 985, 990, 991, 992, 993, 994, 995, 996, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1301, 1304 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 260, 262, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 341, 344, 346, 348, 352, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 394, 395, 396, 397, 398, 399, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 488, 493, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 522, 524, 525, 526, 527, 528, 529, 530, 532, 535, 537, 539, 540, 541, 542, 543, 544, 545, 546, 549, 550, 551, 552, 553, 554, 555, 556, 557, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 579, 582, 584, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 663, 664, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 762, 763, 764, 765, 768, 774, 775, 776, 777, 778, 780, 790, 791, 792, 793, 807, 808, 809, 823, 824, 825, 826, 986, 987, 988, 989, 997, 998, 999, 1007, 1008, 1009, 1018, 1019, 1020, 1021, 1030, 1031, 1032, 1041, 1042, 1043, 1051, 1052, 1053, 1054, 1267, 1268, 1269, 1270, 1288, 1289, 1290 }

B grade { 33, 38, 58, 65, 90, 91, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 204, 205, 206, 207, 208, 221, 234, 235, 259, 261, 263, 337, 342, 343, 345, 347, 349, 350, 351, 382, 393, 400, 413, 425, 426, 444, 445, 520, 521, 523, 531, 533, 534, 536, 538, 547, 548, 558, 741, 766, 767, 779 }

C grade { 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 336, 485, 487, 489, 490, 491, 492, 494, 576, 578, 580, 581, 583, 585, 661, 662, 995, 996, 1028, 1029, 1049, 1050 }

F normal fail { 354, 355, 356, 357, 358, 359, 360, 361, 362, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 745, 746, 747, 748, 749, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 781, 782, 783, 784, 785, 794, 795, 796, 797, 798, 799, 800, 801, 810, 811, 812, 813, 814, 815, 816, 817, 818, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 990, 991, 992, 993, 994, 1000, 1001, 1002, 1003, 1004, 1010, 1011, 1012, 1013, 1014, 1022, 1023, 1024, 1025, 1026, 1027, 1033, 1034, 1035, 1036, 1037, 1038, 1044, 1045, 1046, 1047, 1048, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F(-1) timedout fail { 786, 787, 788, 789, 802, 803, 804, 805, 806, 819, 820, 821, 822, 983, 984, 985, 1005, 1006, 1015, 1016, 1017, 1039, 1040 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 486, 488, 493, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 528, 530, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 579, 582, 584, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 741, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 762, 763, 764, 765, 766, 767, 768, 774, 775, 776, 777, 778, 779, 780, 790, 793, 823, 826, 1267, 1268, 1269, 1270, 1288, 1289, 1290 }

B grade { 38, 65, 90, 101, 102, 174, 196, 197, 198, 199, 201, 202, 203, 235, 336, 337, 338, 340, 527, 529, 531, 533 }

C grade { 485, 487, 489, 490, 491, 492, 494, 576, 578, 580, 581, 583, 585 }

F normal fail { 354, 355, 356, 357, 358, 359, 360, 361, 362, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 721, 722, 723, 724, 725, 733, 734, 735, 736, 737, 745, 746, 747, 748, 749, 757, 758, 759, 760, 761, 769, 770, 771, 772, 773, 781, 782, 783, 784, 785, 786, 787, 788, 789, 791, 792, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 824, 825, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 451, 452, 453, 454, 455, 456, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 483, 490, 492, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 549, 550, 551, 552, 553, 554, 555, 556, 557, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 579, 581, 583, 714, 715, 716, 717, 718, 719, 720, 726, 727, 728, 729, 730, 731, 732, 738, 739, 740, 741, 742, 743, 744, 750, 751, 752, 753, 754, 755, 756, 762, 763, 764, 765, 766, 767, 768, 774, 775, 776, 777, 778, 779, 780, 1268, 1269, 1270 }

B grade { 38, 65, 90, 101, 102, 196, 197, 235, 336, 337, 349, 350, 351, 352, 375, 376, 377, 378, 392, 393, 394, 395, 396, 412, 413, 414, 415, 416, 417, 418, 442, 443, 444, 445, 446, 447, 448, 449, 450, 460, 471, 482, 515, 547, 548, 558, 559, 560, 562, 1267 }

C grade { 484, 486, 488, 494, 575, 577, 585 }

F normal fail { 354, 355, 356, 357, 358, 359, 360, 361, 362, 485, 487, 489, 491, 493, 576, 578, 580, 582, 584, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620,

621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639,
640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658,
659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677,
678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696,
697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 721, 722,
723, 724, 725, 733, 734, 735, 736, 737, 745, 746, 747, 748, 749, 757, 758, 759, 760, 761, 769,
770, 771, 772, 773, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795,
796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814,
815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833,
834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852,
853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871,
872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890,
891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909,
910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928,
929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947,
948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966,
967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985,
986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003,
1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018,
1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033,
1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048,
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1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078,
1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093,
1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108,
1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123,
1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138,
1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153,
1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168,
1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183,
1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198,
1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213,
1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228,
1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243,
1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258,
1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277,
1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292,
1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 363, 364, 365, 366, 367, 368, 369, 370, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 475, 477, 478, 479, 480, 481, 482, 483, 484, 486, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 508, 509, 510, 511, 512, 513, 514, 515, 517, 519, 520, 521, 522, 523, 524, 525, 526, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 583, 584, 585, 661, 710, 712, 714, 715, 716, 717, 718, 719, 720, 723, 724, 726, 727, 728, 729, 730, 731, 732, 735, 736, 738, 739, 740, 741, 742, 743, 744, 747, 748, 750, 751, 752, 753, 754, 755, 756, 759, 760, 762, 763, 764, 765, 766, 767, 768, 771, 772, 774, 775, 776, 777, 778, 779, 780, 783, 784, 837, 838, 843, 844, 849, 850, 855, 856, 861, 862, 867, 868, 874, 875, 881, 882, 888, 889, 895, 896, 902, 903, 909, 910, 916, 917, 923, 924, 930, 931, 937, 938, 944, 945, 951, 952, 958, 959, 965, 966, 972, 973, 979, 980, 986, 987, 988, 989, 997, 998, 999, 1007, 1008, 1009, 1018, 1019, 1020, 1021, 1030, 1031, 1032, 1041, 1042, 1043, 1051, 1052, 1053, 1054, 1077, 1078, 1085, 1086, 1093, 1094, 1101, 1102, 1108, 1109, 1115, 1116, 1122, 1123, 1130, 1131, 1138, 1139, 1146, 1147, 1154, 1155, 1161, 1162, 1168, 1169, 1175, 1176, 1182, 1183, 1189, 1190, 1196, 1197, 1204, 1205, 1212, 1213, 1220, 1221, 1228, 1229, 1235, 1236, 1242, 1243, 1249, 1250, 1256, 1257, 1263, 1264, 1267, 1268, 1269, 1270, 1276, 1277, 1288, 1289, 1290 }

C grade { }

F normal fail { }

F(-1) timedout fail { 354, 355, 356, 357, 358, 359, 360, 361, 362, 371, 372, 388, 389, 392, 407, 408, 411, 412, 435, 436, 437, 440, 441, 442, 443, 474, 476, 485, 487, 496, 507, 516, 518, 527, 529, 565, 576, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 711, 713, 721, 722, 725, 733, 734, 737, 745, 746, 749, 757, 758, 761, 769, 770, 773, 781, 782, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 839, 840, 841, 842, 845, 846, 847, 848, 851, 852, 853, 854, 857, 858, 859, 860, 863, 864, 865, 866, 869, 870, 871, 872, 873, 876, 877, 878, 879, 880, 883, 884, 885, 886, 887, 890, 891, 892, 893, 894, 897, 898, 899, 900, 901, 904, 905, 906, 907, 908, 911, 912, 913, 914, 915, 918, 919, 920, 921, 922, 925, 926, 927, 928, 929, 932, 933, 934, 935, 936, 939, 940, 941, 942, 943, 946, 947, 948, 949, 950, 953, 954, 955, 956, 957, 960, 961, 962, 963, 964, 967, 968, 969, 970, 971, 974, 975, 976, 977, 978, 981, 982, 983, 984, 985, 990, 991, 992, 993, 994, 995, 996, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1079, 1080, 1081, 1082, 1083, 1084, 1087, 1088, 1089, 1090, 1091, 1092, 1095, 1096, 1097, 1098, 1099, 1100, 1103, 1104, 1105, 1106, 1107, 1110, 1111, 1112, 1113, 1114, 1117, 1118, 1119, 1120, 1121, 1124, 1125, 1126, 1127, 1128, 1129, 1132, 1133, 1134, 1135, 1136, 1137, 1140, 1141, 1142, 1143, 1144, 1145, 1148, 1149, 1150, 1151, 1152, 1153, 1156, 1157, 1158, 1159, 1160, 1163, 1164, 1165, 1166, 1167, 1170, 1171, 1172, 1173, 1174, 1177, 1178, 1179, 1180, 1181, 1184, 1185, 1186, 1187, 1188, 1191, 1192, 1193, 1194, 1195, 1198, 1199, 1200, 1201, 1202, 1203, 1206, 1207, 1208, 1209, 1210, 1211, 1214, 1215, 1216, 1217, 1218, 1219, 1222, 1223, 1224, 1225, 1226, 1227, 1230, 1231, 1232, 1233, 1234, 1237, 1238, 1239, 1240, 1241, 1244, 1245, 1246, 1247, 1248, 1251, 1252, 1253, 1254, 1255, 1258, 1259, 1260, 1261, 1262, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 35, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 92, 93, 94, 95, 96, 97, 98, 99, 100, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 200, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 253, 254, 255, 256, 258, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 312, 313, 314, 315, 316, 317, 318, 319, 336, 348, 363, 364, 367, 368, 369, 370, 371, 372, 373, 374, 376, 379, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 427, 428, 429, 430, 431, 432, 433, 437, 438, 439, 440, 441, 442, 443, 451, 452, 453, 454, 456, 457, 458, 459, 461, 462, 464, 467, 469, 473, 475, 478, 480, 484, 486, 489, 491, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 513, 514, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 564, 565, 566, 567, 568, 569, 575, 576, 577, 578, 579, 580, 610, 611, 612, 613, 614, 615, 616, 617, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 721, 722, 723, 724, 725, 728, 733, 734, 735, 736, 737, 738, 745, 746, 747, 748, 749, 753, 757, 758, 759, 760, 761, 765, 769, 770, 771, 772, 773, 776, 777, 781, 782, 783, 784, 785, 823, 824, 1030, 1031, 1041, 1042, 1051, 1052, 1290 }

B grade { 17, 33, 38, 57, 58, 65, 89, 90, 91, 101, 102, 138, 139, 140, 141, 142, 143, 162, 163, 174, 186, 196, 197, 198, 199, 201, 202, 203, 204, 234, 235, 239, 241, 250, 251, 252, 257, 259, 296, 297, 298, 299, 300, 301, 302, 306, 307, 308, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 349, 350, 351, 352, 353, 365, 366, 375, 377, 378, 380, 381, 382, 393, 394, 395, 396, 397, 398, 399, 400, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 444, 445, 446, 447, 448, 449, 450, 455, 460, 466, 477, 488, 512, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 663, 714, 715, 716, 717, 726, 727, 729, 739, 740, 741, 750, 751, 752, 762, 763, 764, 774, 775, 790, 1018, 1019, 1032, 1043, 1053, 1267, 1268, 1269, 1270, 1288, 1289 }

C grade { 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 354, 355, 356, 357, 358, 359, 360, 361, 362, 463, 465, 468, 470, 471, 472, 474, 476, 479, 481, 482, 483, 485, 487, 490, 492, 493, 494, 559, 560, 561, 562, 563, 570, 571, 572, 573, 574, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 605, 606, 607, 608, 609, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 661, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 718, 719, 720, 730, 731, 732, 742, 743, 744, 754, 755, 756, 766, 767, 768, 778, 779, 780, 787, 788, 789, 794, 795, 796, 797, 799, 800, 801, 803, 804, 805, 806, 811, 812, 813, 814, 816, 817, 818, 821, 822, 827, 828, 829, 830, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1015, 1016, 1017, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1044, 1045, 1046, 1047, 1049, 1050, 1056, 1057, 1058, 1059, 1060, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1280, 1281, 1282, 1283, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F normal fail { 662 }

F(-1) timedout fail { 303, 304, 305, 309, 310, 311, 434, 435, 436, 604, 786, 791, 792, 793, 798, 802, 807, 808, 809, 810, 815, 819, 820, 825, 826, 831, 988, 989, 1014, 1020, 1021, 1048, 1054, 1055, 1061, 1278, 1279, 1284, 1285 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 710, 711, 712, 713, 714, 715, 716, 717, 726, 727, 728, 729, 738, 739, 740, 741, 765, 790, 791, 792, 793, 807, 808, 809, 986, 987, 988, 989, 996, 997, 998, 999, 1003, 1004, 1007, 1008, 1009, 1012, 1013, 1014, 1018, 1019, 1020, 1021, 1029, 1030, 1031, 1032, 1037, 1038, 1041, 1042, 1043, 1046, 1047, 1048, 1051, 1052, 1053, 1054, 1059, 1060, 1061, 1078, 1086, 1095, 1102, 1114, 1115, 1116, 1117, 1118, 1267, 1268, 1269, 1270, 1288, 1289, 1290 }

C grade { }

F normal fail { 354, 355, 356, 357, 358, 359, 360, 361, 362, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 718, 719, 720, 721, 722, 723, 724, 725, 730, 731, 732, 733, 734, 735, 736, 737, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 990, 991, 992, 993, 994, 995, 1000, 1001, 1002, 1005, 1006, 1010, 1011, 1015, 1016, 1017, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1033, 1034, 1035, 1036, 1039, 1040, 1044, 1045, 1049, 1050, 1055, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1096, 1097, 1098, 1099, 1100, 1101, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1112, 1113, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1218, 1219, 1220, 1221, 1222, 1223, 1224, 1225, 1226, 1227, 1228, 1229, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1291, 1292, 1293, 1294, 1295, 1296, 1297, 1298, 1299, 1300, 1301, 1302, 1303, 1304 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.132	0.001	0.148	0.041	0.057	0.016	0.116	0.226	0.023

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.137	0.001	0.135	0.033	0.058	0.018	0.119	0.184	0.023

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.88	0.76
time (sec)	N/A	0.135	0.001	0.149	0.026	0.057	0.017	0.122	0.351	0.021

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	13	12	13	14	13
N.S.	1	1.00	1.00	0.82	0.82	0.76	0.71	0.76	0.82	0.76
time (sec)	N/A	0.139	0.001	0.139	0.030	0.055	0.021	0.121	0.259	0.021

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	12	10
N.S.	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	1.00	0.83
time (sec)	N/A	0.134	0.000	0.033	0.031	0.055	0.016	0.124	0.190	0.002

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	11	10	14	11	11
N.S.	1	1.00	1.00	0.92	1.08	0.85	0.77	1.08	0.85	0.85
time (sec)	N/A	0.136	0.001	0.052	0.026	0.058	0.032	0.122	0.190	0.023

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	13	5	10	13	10
N.S.	1	1.00	1.00	1.10	1.00	1.30	0.50	1.00	1.30	1.00
time (sec)	N/A	0.136	0.001	0.050	0.028	0.056	0.029	0.119	0.201	0.026

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	14	17	10	20	17	11
N.S.	1	1.00	1.00	0.92	1.08	1.31	0.77	1.54	1.31	0.85
time (sec)	N/A	0.138	0.002	0.043	0.025	0.057	0.054	0.122	0.222	0.048

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.93	0.87	1.00	0.87
time (sec)	N/A	0.144	0.002	0.048	0.025	0.057	0.063	0.123	0.205	0.025

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	14	13	15	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.82	0.76	0.88	0.76
time (sec)	N/A	0.146	0.002	0.047	0.032	0.057	0.069	0.115	0.199	0.028

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.142	0.002	0.047	0.031	0.058	0.065	0.119	0.240	0.028

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	15	15	15	15	15	15
N.S.	1	1.00	1.00	0.82	0.88	0.88	0.88	0.88	0.88	0.88
time (sec)	N/A	0.142	0.002	0.046	0.028	0.058	0.080	0.125	0.210	0.027

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.163	0.001	0.286	0.025	0.055	0.020	0.121	0.192	0.035

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.153	0.000	0.276	0.026	0.055	0.019	0.122	0.179	0.034

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	24	24	24	24	26	24
N.S.	1	1.13	1.00	0.83	0.80	0.80	0.80	0.80	0.87	0.80
time (sec)	N/A	0.164	0.001	0.306	0.031	0.059	0.022	0.122	0.239	0.036

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	26	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.87	0.80
time (sec)	N/A	0.156	0.000	0.273	0.026	0.056	0.021	0.122	0.202	0.038

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	25	24
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	1.56	1.50
time (sec)	N/A	0.131	0.001	0.270	0.030	0.056	0.021	0.121	0.209	0.042

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	24	21
N.S.	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.96	0.84
time (sec)	N/A	0.150	0.000	0.237	0.032	0.058	0.018	0.124	0.201	0.003

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	22	24	21	20	24	21	21
N.S.	1	1.30	1.00	0.96	1.04	0.91	0.87	1.04	0.91	0.91
time (sec)	N/A	0.164	0.001	0.287	0.025	0.057	0.042	0.124	0.240	0.082

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	25	22
N.S.	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	1.04	0.92
time (sec)	N/A	0.155	0.000	0.234	0.029	0.057	0.041	0.123	0.198	0.035

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	28	27	24	24	27	24	32	27	23
N.S.	1	1.04	1.00	0.89	0.89	1.00	0.89	1.19	1.00	0.85
time (sec)	N/A	0.159	0.001	0.246	0.030	0.059	0.068	0.122	0.188	0.100

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	22	26	22	22	26	24
N.S.	1	1.00	1.00	0.96	0.96	1.13	0.96	0.96	1.13	1.04
time (sec)	N/A	0.154	0.001	0.242	0.027	0.057	0.063	0.119	0.203	0.030

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	30	24	23	26	28	24	34	28	24
N.S.	1	1.25	1.00	0.96	1.08	1.17	1.00	1.42	1.17	1.00
time (sec)	N/A	0.160	0.001	0.230	0.033	0.059	0.132	0.126	0.213	0.048

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	26	26	27	26	26	25
N.S.	1	1.00	1.00	0.89	0.93	0.93	0.96	0.93	0.93	0.89
time (sec)	N/A	0.154	0.001	0.254	0.027	0.056	0.104	0.122	0.192	0.037

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	30	25	24	24	26	24	26	26
N.S.	1	1.00	1.58	1.32	1.26	1.26	1.37	1.26	1.37	1.37
time (sec)	N/A	0.131	0.001	0.229	0.026	0.057	0.114	0.125	0.182	0.036

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.154	0.001	0.233	0.041	0.056	0.120	0.121	0.205	0.038

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	34	30	25	26	26	27	26	26	26
N.S.	1	1.13	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.160	0.001	0.230	0.034	0.061	0.113	0.127	0.217	0.039

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	26	27	26	26	26
N.S.	1	1.00	1.00	0.83	0.87	0.87	0.90	0.87	0.87	0.87
time (sec)	N/A	0.154	0.001	0.234	0.042	0.055	0.125	0.117	0.218	0.039

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	37	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.175	0.002	0.286	0.025	0.058	0.027	0.121	0.210	0.045

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	37	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.177	0.001	0.292	0.026	0.057	0.022	0.119	0.249	0.043

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	35	35	39	35	37	35
N.S.	1	1.09	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.175	0.001	0.289	0.026	0.055	0.022	0.122	0.207	0.044

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	43	36	35	35	37	35	37	35
N.S.	1	1.12	1.26	1.06	1.03	1.03	1.09	1.03	1.09	1.03
time (sec)	N/A	0.165	0.001	0.277	0.025	0.056	0.024	0.114	0.195	0.046

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	35	37	14	36	35
N.S.	1	1.00	1.00	0.94	0.88	2.19	2.31	0.88	2.25	2.19
time (sec)	N/A	0.130	0.001	0.279	0.027	0.057	0.023	0.125	0.226	0.043

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	36	33	37	36	33	33
N.S.	1	1.10	1.00	0.87	0.92	0.85	0.95	0.92	0.85	0.85
time (sec)	N/A	0.168	0.003	0.256	0.042	0.062	0.047	0.114	0.221	0.037

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	42	40	35	36	38	37	46	38	34
N.S.	1	1.05	1.00	0.88	0.90	0.95	0.92	1.15	0.95	0.85
time (sec)	N/A	0.167	0.005	0.306	0.026	0.059	0.069	0.119	0.201	0.038

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	40	35	37	39	37	46	39	37
N.S.	1	1.02	1.00	0.88	0.92	0.98	0.92	1.15	0.98	0.92
time (sec)	N/A	0.170	0.003	0.257	0.025	0.060	0.093	0.120	0.198	0.036

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	34	39	39	37	47	39	36
N.S.	1	1.10	1.00	0.87	1.00	1.00	0.95	1.21	1.00	0.92
time (sec)	N/A	0.170	0.003	0.238	0.025	0.060	0.128	0.121	0.227	0.103

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	36	35	35	37	35	37	37
N.S.	1	1.00	2.26	1.89	1.84	1.84	1.95	1.84	1.95	1.95
time (sec)	N/A	0.138	0.005	0.234	0.036	0.057	0.218	0.127	0.208	0.041

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	43	36	37	37	39	37	37	37
N.S.	1	1.10	1.08	0.90	0.92	0.92	0.98	0.92	0.92	0.92
time (sec)	N/A	0.152	0.003	0.237	0.032	0.055	0.168	0.115	0.212	0.032

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.169	0.003	0.232	0.030	0.056	0.182	0.121	0.219	0.093

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	43	36	37	37	39	37	37	37
N.S.	1	1.09	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.173	0.005	0.231	0.048	0.055	0.182	0.120	0.220	0.092

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.167	0.001	0.280	0.026	0.057	0.023	0.119	0.210	0.046

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.86	0.81
time (sec)	N/A	0.166	0.001	0.285	0.032	0.057	0.020	0.121	0.183	0.046

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	39	35	37	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.91	0.81	0.86	0.81
time (sec)	N/A	0.159	0.002	0.289	0.039	0.058	0.018	0.119	0.243	0.044

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	31	32	31	35	31
N.S.	1	1.00	1.00	0.91	0.89	0.89	0.91	0.89	1.00	0.89
time (sec)	N/A	0.159	0.000	0.230	0.026	0.056	0.018	0.122	0.208	0.003

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	32	36	29	32	36	32
N.S.	1	1.00	1.00	0.97	0.94	1.06	0.85	0.94	1.06	0.94
time (sec)	N/A	0.158	0.003	0.253	0.025	0.058	0.048	0.117	0.181	0.046

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	34	34	36	36	34	36	36
N.S.	1	1.00	1.00	0.92	0.92	0.97	0.97	0.92	0.97	0.97
time (sec)	N/A	0.159	0.003	0.239	0.030	0.057	0.069	0.120	0.197	0.039

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	37	34	33	37	34
N.S.	1	1.00	1.00	0.97	0.97	1.09	1.00	0.97	1.09	1.00
time (sec)	N/A	0.156	0.004	0.239	0.055	0.057	0.101	0.126	0.237	0.034

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	36	37	37	39	37	37	35
N.S.	1	1.00	1.00	0.92	0.95	0.95	1.00	0.95	0.95	0.90
time (sec)	N/A	0.157	0.003	0.240	0.045	0.056	0.133	0.123	0.205	0.031

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.161	0.003	0.314	0.026	0.061	0.146	0.119	0.201	0.034

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	37	37	39	37	37	37
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.160	0.006	0.236	0.032	0.056	0.142	0.123	0.221	0.037

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	69	58	57	57	65	57	59	57
N.S.	1	1.01	1.00	0.84	0.83	0.83	0.94	0.83	0.86	0.83
time (sec)	N/A	0.200	0.002	0.289	0.027	0.060	0.030	0.122	0.212	0.027

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	57	57	65	57	59	57
N.S.	1	1.06	1.00	0.84	0.83	0.83	0.94	0.83	0.86	0.83
time (sec)	N/A	0.194	0.002	0.289	0.025	0.060	0.022	0.121	0.190	0.027

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	57	57	66	57	59	57
N.S.	1	1.06	1.00	0.84	0.83	0.83	0.96	0.83	0.86	0.83
time (sec)	N/A	0.194	0.002	0.293	0.026	0.060	0.021	0.122	0.197	0.026

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	69	58	57	57	65	57	59	57
N.S.	1	1.06	0.96	0.81	0.79	0.79	0.90	0.79	0.82	0.79
time (sec)	N/A	0.198	0.002	0.299	0.034	0.057	0.024	0.123	0.225	0.026

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	66	57	56	56	63	56	59	56
N.S.	1	1.08	1.25	1.08	1.06	1.06	1.19	1.06	1.11	1.06
time (sec)	N/A	0.182	0.002	0.287	0.033	0.058	0.024	0.122	0.187	0.026

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	66	57	56	56	63	56	59	56
N.S.	1	1.12	1.94	1.68	1.65	1.65	1.85	1.65	1.74	1.65
time (sec)	N/A	0.163	0.002	0.280	0.027	0.058	0.033	0.120	0.201	0.026

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	57	65	14	58	57
N.S.	1	1.00	1.00	0.94	0.88	3.56	4.06	0.88	3.62	3.56
time (sec)	N/A	0.134	0.002	0.277	0.033	0.057	0.024	0.122	0.239	0.040

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	65	56	58	55	65	58	55	55
N.S.	1	1.03	1.00	0.86	0.89	0.85	1.00	0.89	0.85	0.85
time (sec)	N/A	0.189	0.003	0.252	0.035	0.059	0.064	0.119	0.229	0.030

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	64	57	58	61	63	68	61	56
N.S.	1	1.03	1.00	0.89	0.91	0.95	0.98	1.06	0.95	0.88
time (sec)	N/A	0.189	0.003	0.248	0.037	0.059	0.076	0.115	0.196	0.034

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	64	57	59	61	63	70	61	59
N.S.	1	1.06	1.00	0.89	0.92	0.95	0.98	1.09	0.95	0.92
time (sec)	N/A	0.186	0.005	0.251	0.029	0.059	0.106	0.124	0.195	0.031

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	68	64	57	61	61	65	72	61	59
N.S.	1	1.06	1.00	0.89	0.95	0.95	1.02	1.12	0.95	0.92
time (sec)	N/A	0.186	0.003	0.304	0.033	0.061	0.155	0.123	0.252	0.042

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	64	57	61	61	63	70	61	59
N.S.	1	1.02	1.00	0.89	0.95	0.95	0.98	1.09	0.95	0.92
time (sec)	N/A	0.182	0.003	0.243	0.033	0.060	0.174	0.124	0.209	0.045

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	67	65	56	61	61	61	69	61	58
N.S.	1	1.03	1.00	0.86	0.94	0.94	0.94	1.06	0.94	0.89
time (sec)	N/A	0.189	0.003	0.240	0.031	0.063	0.254	0.123	0.200	0.137

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	69	58	57	57	61	57	59	59
N.S.	1	1.00	3.63	3.05	3.00	3.00	3.21	3.00	3.11	3.11
time (sec)	N/A	0.133	0.003	0.240	0.029	0.057	0.280	0.118	0.222	0.043

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	67	58	59	59	63	59	59	58
N.S.	1	1.10	1.68	1.45	1.48	1.48	1.58	1.48	1.48	1.45
time (sec)	N/A	0.146	0.004	0.247	0.033	0.061	0.279	0.118	0.253	0.092

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	67	58	59	59	63	59	59	58
N.S.	1	1.16	1.08	0.94	0.95	0.95	1.02	0.95	0.95	0.94
time (sec)	N/A	0.161	0.003	0.246	0.040	0.057	0.307	0.127	0.185	0.043

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	69	58	59	59	63	59	59	59
N.S.	1	1.03	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.189	0.003	0.245	0.034	0.058	0.354	0.122	0.190	0.097

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	73	69	58	59	59	63	59	59	59
N.S.	1	1.06	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.190	0.003	0.243	0.026	0.061	0.343	0.121	0.225	0.097

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	59	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.86	0.83
time (sec)	N/A	0.185	0.002	0.296	0.026	0.058	0.020	0.125	0.221	0.026

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	65	57	59	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.94	0.83	0.86	0.83
time (sec)	N/A	0.185	0.002	0.306	0.037	0.059	0.025	0.121	0.215	0.028

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	57	66	57	59	57
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.96	0.83	0.86	0.83
time (sec)	N/A	0.182	0.002	0.292	0.033	0.059	0.027	0.121	0.205	0.025

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	66	57	56	56	63	56	59	56
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.95	0.85	0.89	0.85
time (sec)	N/A	0.182	0.002	0.309	0.035	0.061	0.021	0.127	0.228	0.025

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	55	54	54	61	54	57	54
N.S.	1	1.00	1.00	0.89	0.87	0.87	0.98	0.87	0.92	0.87
time (sec)	N/A	0.177	0.001	0.231	0.028	0.057	0.028	0.120	0.196	0.004

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	55	59	58	55	59	55
N.S.	1	1.00	1.00	0.92	0.90	0.97	0.95	0.90	0.97	0.90
time (sec)	N/A	0.182	0.003	0.255	0.026	0.058	0.055	0.119	0.210	0.026

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	55	59	60	55	59	57
N.S.	1	1.00	1.00	0.92	0.92	0.98	1.00	0.92	0.98	0.95
time (sec)	N/A	0.175	0.003	0.250	0.028	0.060	0.086	0.121	0.192	0.026

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	56	58	59	63	58	59	58
N.S.	1	1.00	1.00	0.89	0.92	0.94	1.00	0.92	0.94	0.92
time (sec)	N/A	0.183	0.003	0.263	0.029	0.058	0.129	0.122	0.253	0.050

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	56	58	59	61	58	59	59
N.S.	1	1.00	1.00	0.92	0.95	0.97	1.00	0.95	0.97	0.97
time (sec)	N/A	0.179	0.004	0.249	0.032	0.057	0.150	0.120	0.205	0.098

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	55	57	59	60	57	59	57
N.S.	1	1.00	1.00	0.92	0.95	0.98	1.00	0.95	0.98	0.95
time (sec)	N/A	0.183	0.005	0.253	0.031	0.059	0.179	0.119	0.188	0.042

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	58	59	59	63	59	59	58
N.S.	1	1.00	1.00	0.89	0.91	0.91	0.97	0.91	0.91	0.89
time (sec)	N/A	0.180	0.004	0.240	0.030	0.058	0.262	0.119	0.240	0.114

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	58	59	59	63	59	59	58
N.S.	1	1.00	1.00	0.87	0.88	0.88	0.94	0.88	0.88	0.87
time (sec)	N/A	0.186	0.004	0.246	0.034	0.055	0.264	0.123	0.246	0.095

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.183	0.003	0.257	0.033	0.055	0.322	0.122	0.208	0.094

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.183	0.004	0.280	0.035	0.057	0.284	0.123	0.194	0.095

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	59	59	63	59	59	59
N.S.	1	1.00	1.00	0.84	0.86	0.86	0.91	0.86	0.86	0.86
time (sec)	N/A	0.182	0.005	0.264	0.035	0.056	0.284	0.120	0.249	0.043

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	133	108	91	90	90	105	90	92	90
N.S.	1	1.03	0.84	0.71	0.70	0.70	0.81	0.70	0.71	0.70
time (sec)	N/A	0.251	0.002	0.300	0.026	0.057	0.038	0.122	0.211	0.192

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	114	108	91	90	90	107	90	92	90
N.S.	1	1.04	0.98	0.83	0.82	0.82	0.97	0.82	0.84	0.82
time (sec)	N/A	0.227	0.002	0.296	0.025	0.059	0.028	0.113	0.194	0.097

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	106	91	90	90	104	90	92	90
N.S.	1	1.04	1.16	1.00	0.99	0.99	1.14	0.99	1.01	0.99
time (sec)	N/A	0.215	0.002	0.302	0.033	0.058	0.034	0.121	0.220	0.143

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	106	91	90	90	105	90	92	90
N.S.	1	1.06	1.47	1.26	1.25	1.25	1.46	1.25	1.28	1.25
time (sec)	N/A	0.199	0.002	0.299	0.029	0.059	0.033	0.124	0.193	0.096

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	103	90	89	89	102	89	92	89
N.S.	1	1.08	1.94	1.70	1.68	1.68	1.92	1.68	1.74	1.68
time (sec)	N/A	0.183	0.002	0.302	0.029	0.055	0.029	0.119	0.204	0.162

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	106	91	90	90	105	90	92	90
N.S.	1	1.12	3.12	2.68	2.65	2.65	3.09	2.65	2.71	2.65
time (sec)	N/A	0.160	0.002	0.289	0.027	0.056	0.031	0.124	0.207	0.097

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	90	99	14	91	14
N.S.	1	1.00	1.00	0.94	0.88	5.62	6.19	0.88	5.69	0.88
time (sec)	N/A	0.129	0.002	0.289	0.031	0.056	0.033	0.119	0.226	0.126

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	106	100	89	91	88	102	91	88	88
N.S.	1	1.06	1.00	0.89	0.91	0.88	1.02	0.91	0.88	0.88
time (sec)	N/A	0.215	0.003	0.381	0.030	0.058	0.067	0.120	0.205	0.112

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	103	99	90	91	94	100	101	94	89
N.S.	1	1.04	1.00	0.91	0.92	0.95	1.01	1.02	0.95	0.90
time (sec)	N/A	0.218	0.004	0.264	0.032	0.058	0.089	0.118	0.208	0.128

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	103	101	90	92	94	104	103	94	92
N.S.	1	1.02	1.00	0.89	0.91	0.93	1.03	1.02	0.93	0.91
time (sec)	N/A	0.223	0.004	0.270	0.026	0.058	0.137	0.123	0.212	0.057

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	101	94	89	91	94	97	102	94	91
N.S.	1	1.07	1.00	0.95	0.97	1.00	1.03	1.09	1.00	0.97
time (sec)	N/A	0.221	0.004	0.266	0.026	0.058	0.154	0.120	0.217	0.056

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	103	97	90	94	94	100	105	94	92
N.S.	1	1.06	1.00	0.93	0.97	0.97	1.03	1.08	0.97	0.95
time (sec)	N/A	0.223	0.003	0.259	0.040	0.059	0.202	0.124	0.200	0.054

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	95	90	94	94	99	105	94	91
N.S.	1	1.06	1.00	0.95	0.99	0.99	1.04	1.11	0.99	0.96
time (sec)	N/A	0.219	0.004	0.265	0.034	0.058	0.258	0.121	0.186	0.107

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	103	101	90	94	94	99	105	94	92
N.S.	1	1.02	1.00	0.89	0.93	0.93	0.98	1.04	0.93	0.91
time (sec)	N/A	0.219	0.004	0.319	0.030	0.060	0.326	0.121	0.233	0.119

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	102	99	90	94	94	99	103	94	94
N.S.	1	1.03	1.00	0.91	0.95	0.95	1.00	1.04	0.95	0.95
time (sec)	N/A	0.214	0.003	0.267	0.025	0.062	0.432	0.131	0.207	0.081

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	106	100	89	94	94	97	102	94	91
N.S.	1	1.06	1.00	0.89	0.94	0.94	0.97	1.02	0.94	0.91
time (sec)	N/A	0.224	0.004	0.249	0.027	0.060	0.480	0.126	0.208	0.073

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	100	91	90	90	97	90	92	92
N.S.	1	1.00	5.26	4.79	4.74	4.74	5.11	4.74	4.84	4.84
time (sec)	N/A	0.131	0.003	0.250	0.036	0.055	0.534	0.116	0.205	0.133

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	106	91	92	92	99	92	92	92
N.S.	1	1.10	2.65	2.28	2.30	2.30	2.48	2.30	2.30	2.30
time (sec)	N/A	0.148	0.003	0.256	0.039	0.056	0.552	0.124	0.276	0.133

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	72	104	91	92	92	99	92	92	91
N.S.	1	1.16	1.68	1.47	1.48	1.48	1.60	1.48	1.48	1.47
time (sec)	N/A	0.160	0.003	0.248	0.028	0.060	0.590	0.116	0.204	0.080

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	100	106	91	92	92	99	92	92	92
N.S.	1	1.19	1.26	1.08	1.10	1.10	1.18	1.10	1.10	1.10
time (sec)	N/A	0.183	0.003	0.257	0.028	0.056	0.565	0.120	0.196	0.084

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	128	106	91	92	92	99	92	92	92
N.S.	1	1.21	1.00	0.86	0.87	0.87	0.93	0.87	0.87	0.87
time (sec)	N/A	0.191	0.003	0.250	0.031	0.058	0.652	0.122	0.208	0.153

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	108	91	92	92	99	92	92	92
N.S.	1	1.02	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.85
time (sec)	N/A	0.218	0.003	0.254	0.032	0.061	0.643	0.129	0.235	0.137

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	110	108	91	92	92	99	92	92	92
N.S.	1	1.02	1.00	0.84	0.85	0.85	0.92	0.85	0.85	0.85
time (sec)	N/A	0.219	0.003	0.267	0.028	0.061	0.695	0.129	0.228	0.148

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	110	106	91	92	92	99	92	92	91
N.S.	1	1.04	1.00	0.86	0.87	0.87	0.93	0.87	0.87	0.86
time (sec)	N/A	0.226	0.003	0.258	0.032	0.055	0.692	0.120	0.228	0.092

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	92	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.85	0.83
time (sec)	N/A	0.221	0.002	0.302	0.037	0.058	0.030	0.109	0.239	0.102

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	92	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.85	0.83
time (sec)	N/A	0.214	0.002	0.310	0.025	0.061	0.024	0.118	0.213	0.151

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	108	91	90	90	107	90	92	90
N.S.	1	1.00	1.00	0.84	0.83	0.83	0.99	0.83	0.85	0.83
time (sec)	N/A	0.218	0.002	0.305	0.041	0.056	0.024	0.119	0.217	0.096

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	90	90	105	90	92	90
N.S.	1	1.00	1.00	0.86	0.85	0.85	0.99	0.85	0.87	0.85
time (sec)	N/A	0.213	0.002	0.303	0.028	0.058	0.025	0.129	0.216	0.152

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	101	88	87	87	102	87	90	87
N.S.	1	1.00	1.00	0.87	0.86	0.86	1.01	0.86	0.89	0.86
time (sec)	N/A	0.216	0.001	0.256	0.037	0.056	0.029	0.117	0.227	0.045

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	88	92	99	88	92	88
N.S.	1	1.00	1.00	0.89	0.88	0.92	0.99	0.88	0.92	0.88
time (sec)	N/A	0.204	0.006	0.367	0.031	0.061	0.091	0.129	0.220	0.059

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	89	92	100	89	92	91
N.S.	1	1.00	1.00	0.91	0.91	0.94	1.02	0.91	0.94	0.93
time (sec)	N/A	0.209	0.006	0.269	0.028	0.056	0.095	0.123	0.203	0.056

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	102	91	92	91
N.S.	1	1.00	1.00	0.89	0.91	0.92	1.02	0.91	0.92	0.91
time (sec)	N/A	0.212	0.009	0.265	0.032	0.058	0.146	0.125	0.244	0.050

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	102	91	92	91
N.S.	1	1.00	1.00	0.87	0.89	0.90	1.00	0.89	0.90	0.89
time (sec)	N/A	0.209	0.005	0.326	0.033	0.056	0.153	0.126	0.251	0.052

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	102	89	91	92	100	91	92	91
N.S.	1	1.00	1.00	0.87	0.89	0.90	0.98	0.89	0.90	0.89
time (sec)	N/A	0.213	0.008	0.262	0.032	0.058	0.218	0.113	0.202	0.052

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	100	89	91	92	99	91	92	91
N.S.	1	1.00	1.00	0.89	0.91	0.92	0.99	0.91	0.92	0.91
time (sec)	N/A	0.210	0.008	0.266	0.026	0.060	0.277	0.118	0.238	0.131

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	98	89	91	92	97	91	92	92
N.S.	1	1.00	1.00	0.91	0.93	0.94	0.99	0.93	0.94	0.94
time (sec)	N/A	0.215	0.007	0.269	0.026	0.058	0.304	0.123	0.224	0.145

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	99	88	90	92	95	90	92	90
N.S.	1	1.00	1.00	0.89	0.91	0.93	0.96	0.91	0.93	0.91
time (sec)	N/A	0.210	0.005	0.259	0.043	0.056	0.384	0.113	0.208	0.074

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	91	92	92	99	92	92	91
N.S.	1	1.00	1.00	0.88	0.88	0.88	0.95	0.88	0.88	0.88
time (sec)	N/A	0.213	0.008	0.257	0.027	0.058	0.409	0.126	0.195	0.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	106	91	92	92	99	92	92	92
N.S.	1	1.00	1.00	0.86	0.87	0.87	0.93	0.87	0.87	0.87
time (sec)	N/A	0.214	0.007	0.251	0.029	0.060	0.463	0.122	0.224	0.080

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	78	79	68	68	67	68	69	67	67
N.S.	1	0.99	1.00	0.86	0.86	0.85	0.86	0.87	0.85	0.85
time (sec)	N/A	0.211	0.005	0.375	0.040	0.060	0.080	0.125	0.220	0.131

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	66	57	57	56	56	58	56	56
N.S.	1	0.98	1.00	0.86	0.86	0.85	0.85	0.88	0.85	0.85
time (sec)	N/A	0.189	0.005	0.452	0.030	0.061	0.095	0.125	0.198	0.077

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	53	46	46	45	44	47	45	45
N.S.	1	0.98	1.00	0.87	0.87	0.85	0.83	0.89	0.85	0.85
time (sec)	N/A	0.176	0.004	0.385	0.034	0.059	0.070	0.125	0.198	0.106

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	39	40	34	34	33	32	35	33	33
N.S.	1	0.98	1.00	0.85	0.85	0.82	0.80	0.88	0.82	0.82
time (sec)	N/A	0.166	0.004	0.418	0.025	0.058	0.067	0.121	0.207	0.051

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	23	22	20	24	22	22
N.S.	1	0.96	1.00	0.85	0.85	0.81	0.74	0.89	0.81	0.81
time (sec)	N/A	0.158	0.004	0.373	0.033	0.057	0.071	0.114	0.221	0.043

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87	0.87
time (sec)	N/A	0.133	0.002	0.348	0.025	0.058	0.076	0.122	0.191	0.092

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	22	21	23	18	15	24	20	18
N.S.	1	1.18	1.00	0.95	1.05	0.82	0.68	1.09	0.91	0.82
time (sec)	N/A	0.144	0.004	0.339	0.025	0.063	0.103	0.121	0.196	0.151

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	36	35	32	33	33	31	43	33	31
N.S.	1	1.03	1.00	0.91	0.94	0.94	0.89	1.23	0.94	0.89
time (sec)	N/A	0.168	0.006	0.361	0.026	0.060	0.142	0.123	0.236	0.081

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	50	49	44	47	45	42	57	47	46
N.S.	1	1.02	1.00	0.90	0.96	0.92	0.86	1.16	0.96	0.94
time (sec)	N/A	0.182	0.006	0.356	0.026	0.064	0.159	0.124	0.207	0.083

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	63	56	58	58	56	70	58	58
N.S.	1	1.02	1.00	0.89	0.92	0.92	0.89	1.11	0.92	0.92
time (sec)	N/A	0.190	0.006	0.351	0.027	0.064	0.186	0.120	0.216	0.086

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	75	66	69	69	68	81	69	68
N.S.	1	1.01	1.00	0.88	0.92	0.92	0.91	1.08	0.92	0.91
time (sec)	N/A	0.198	0.005	0.363	0.032	0.062	0.231	0.121	0.206	0.135

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	71	72	170	119	77	73	65
N.S.	1	1.00	1.00	0.88	0.89	2.10	1.47	0.95	0.90	0.80
time (sec)	N/A	0.190	0.018	0.383	0.112	0.069	0.121	0.122	0.244	0.061

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	60	60	148	107	65	62	54
N.S.	1	1.00	1.00	0.88	0.88	2.18	1.57	0.96	0.91	0.79
time (sec)	N/A	0.187	0.015	0.429	0.109	0.068	0.089	0.129	0.216	0.119

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	49	50	126	95	55	51	43
N.S.	1	1.00	1.00	0.89	0.91	2.29	1.73	1.00	0.93	0.78
time (sec)	N/A	0.176	0.015	0.369	0.110	0.067	0.093	0.126	0.200	0.131

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	37	99	80	40	37	32
N.S.	1	1.00	1.00	0.90	0.88	2.36	1.90	0.95	0.88	0.76
time (sec)	N/A	0.167	0.012	0.379	0.113	0.065	0.108	0.126	0.236	0.138

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	26	23
N.S.	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.84	0.74
time (sec)	N/A	0.145	0.006	0.393	0.109	0.067	0.075	0.121	0.230	0.039

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	23	16
N.S.	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.96	0.67
time (sec)	N/A	0.133	0.002	0.343	0.124	0.066	0.062	0.123	0.218	0.002

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	30	26
N.S.	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.88	0.76
time (sec)	N/A	0.143	0.009	0.387	0.115	0.066	0.080	0.135	0.219	0.060

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	51	43	39	40	106	87	40	43	37
N.S.	1	1.19	1.00	0.91	0.93	2.47	2.02	0.93	1.00	0.86
time (sec)	N/A	0.153	0.014	0.409	0.112	0.070	0.119	0.123	0.246	0.134

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	68	58	52	52	132	100	52	56	48
N.S.	1	1.17	1.00	0.90	0.90	2.28	1.72	0.90	0.97	0.83
time (sec)	N/A	0.169	0.015	0.356	0.109	0.068	0.128	0.120	0.245	0.066

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	85	69	61	62	154	112	62	67	59
N.S.	1	1.23	1.00	0.88	0.90	2.23	1.62	0.90	0.97	0.86
time (sec)	N/A	0.177	0.016	0.342	0.110	0.068	0.177	0.121	0.228	0.066

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	83	85	88	104	88	103	104	90
N.S.	1	0.97	0.88	0.90	0.94	1.11	0.94	1.10	1.11	0.96
time (sec)	N/A	0.223	0.020	0.313	0.027	0.062	0.152	0.117	0.232	0.155

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	72	74	77	93	80	92	93	79
N.S.	1	0.99	0.87	0.89	0.93	1.12	0.96	1.11	1.12	0.95
time (sec)	N/A	0.214	0.014	0.309	0.036	0.063	0.165	0.115	0.250	0.134

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	68	60	63	65	81	66	80	81	68
N.S.	1	0.97	0.86	0.90	0.93	1.16	0.94	1.14	1.16	0.97
time (sec)	N/A	0.200	0.015	0.309	0.033	0.057	0.144	0.127	0.226	0.074

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	56	49	54	54	70	53	67	70	57
N.S.	1	0.98	0.86	0.95	0.95	1.23	0.93	1.18	1.23	1.00
time (sec)	N/A	0.186	0.011	0.336	0.031	0.064	0.141	0.124	0.229	0.143

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	43	38	41	43	56	39	49	57	45
N.S.	1	0.98	0.86	0.93	0.98	1.27	0.89	1.11	1.30	1.02
time (sec)	N/A	0.173	0.011	0.299	0.033	0.061	0.123	0.120	0.214	0.110

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	31	27	30	32	35	29	48	44	29
N.S.	1	0.94	0.82	0.91	0.97	1.06	0.88	1.45	1.33	0.88
time (sec)	N/A	0.163	0.007	0.289	0.024	0.059	0.117	0.122	0.199	0.045

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	15	15	14	17	14
N.S.	1	1.00	1.00	0.94	0.88	0.94	0.94	0.88	1.06	0.88
time (sec)	N/A	0.130	0.002	0.287	0.032	0.058	0.070	0.112	0.211	0.095

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	39	33	35	37	47	34	47	59	34
N.S.	1	1.03	0.87	0.92	0.97	1.24	0.89	1.24	1.55	0.89
time (sec)	N/A	0.168	0.009	0.297	0.026	0.065	0.164	0.122	0.235	0.113

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	52	41	52	52	73	51	51	81	51
N.S.	1	1.06	0.84	1.06	1.06	1.49	1.04	1.04	1.65	1.04
time (sec)	N/A	0.177	0.024	0.286	0.026	0.063	0.209	0.119	0.243	0.070

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	68	57	65	70	90	68	86	94	67
N.S.	1	1.03	0.86	0.98	1.06	1.36	1.03	1.30	1.42	1.02
time (sec)	N/A	0.195	0.032	0.282	0.029	0.062	0.238	0.121	0.218	0.076

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	79	68	77	79	99	78	99	105	78
N.S.	1	0.99	0.85	0.96	0.99	1.24	0.98	1.24	1.31	0.98
time (sec)	N/A	0.210	0.033	0.286	0.027	0.063	0.257	0.113	0.219	0.180

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	94	79	88	92	112	94	110	116	89
N.S.	1	1.01	0.85	0.95	0.99	1.20	1.01	1.18	1.25	0.96
time (sec)	N/A	0.224	0.050	0.298	0.028	0.061	0.286	0.118	0.202	0.091

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	108	93	87	93	234	151	95	118	88
N.S.	1	1.04	0.89	0.84	0.89	2.25	1.45	0.91	1.13	0.85
time (sec)	N/A	0.210	0.040	0.315	0.111	0.070	0.173	0.120	0.216	0.072

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	82	76	82	212	134	84	107	77
N.S.	1	1.08	0.93	0.86	0.93	2.41	1.52	0.95	1.22	0.88
time (sec)	N/A	0.200	0.036	0.311	0.117	0.068	0.159	0.121	0.202	0.071

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	71	65	71	190	124	73	96	66
N.S.	1	1.05	0.91	0.83	0.91	2.44	1.59	0.94	1.23	0.85
time (sec)	N/A	0.194	0.042	0.290	0.108	0.070	0.158	0.121	0.223	0.128

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	69	60	54	59	164	107	61	83	56
N.S.	1	1.06	0.92	0.83	0.91	2.52	1.65	0.94	1.28	0.86
time (sec)	N/A	0.181	0.030	0.295	0.105	0.071	0.156	0.119	0.188	0.094

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	58	51	42	45	136	83	42	69	43
N.S.	1	1.14	1.00	0.82	0.88	2.67	1.63	0.82	1.35	0.84
time (sec)	N/A	0.160	0.021	0.279	0.107	0.066	0.141	0.120	0.208	0.101

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	36	120	78	35	62	33
N.S.	1	1.00	1.00	0.80	0.80	2.67	1.73	0.78	1.38	0.73
time (sec)	N/A	0.143	0.014	0.309	0.106	0.066	0.101	0.121	0.202	0.145

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	61	33
N.S.	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	1.36	0.73
time (sec)	N/A	0.141	0.003	0.263	0.107	0.067	0.099	0.123	0.249	0.003

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	54	45	49	136	92	47	72	44
N.S.	1	1.13	1.00	0.83	0.91	2.52	1.70	0.87	1.33	0.81
time (sec)	N/A	0.157	0.027	0.306	0.111	0.068	0.173	0.121	0.229	0.139

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	67	55	64	172	114	59	88	58
N.S.	1	1.16	1.00	0.82	0.96	2.57	1.70	0.88	1.31	0.87
time (sec)	N/A	0.169	0.025	0.329	0.110	0.071	0.189	0.126	0.217	0.154

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	95	80	67	75	198	126	70	101	70
N.S.	1	1.19	1.00	0.84	0.94	2.48	1.58	0.88	1.26	0.88
time (sec)	N/A	0.182	0.033	0.301	0.116	0.070	0.198	0.117	0.242	0.171

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	112	91	77	86	220	138	81	112	80
N.S.	1	1.23	1.00	0.85	0.95	2.42	1.52	0.89	1.23	0.88
time (sec)	N/A	0.193	0.030	0.314	0.115	0.072	0.240	0.124	0.247	0.230

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	111	97	97	111	137	119	114	141	111
N.S.	1	0.97	0.85	0.85	0.97	1.20	1.04	1.00	1.24	0.97
time (sec)	N/A	0.248	0.019	0.300	0.033	0.061	0.284	0.124	0.238	0.168

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	98	85	87	99	125	104	102	129	100
N.S.	1	0.98	0.85	0.87	0.99	1.25	1.04	1.02	1.29	1.00
time (sec)	N/A	0.232	0.019	0.290	0.030	0.062	0.231	0.122	0.192	0.157

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	75	76	89	115	92	92	119	90
N.S.	1	1.02	0.86	0.87	1.02	1.32	1.06	1.06	1.37	1.03
time (sec)	N/A	0.217	0.016	0.296	0.028	0.062	0.220	0.126	0.243	0.156

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	63	65	77	103	78	80	107	78
N.S.	1	1.03	0.85	0.88	1.04	1.39	1.05	1.08	1.45	1.05
time (sec)	N/A	0.203	0.015	0.329	0.034	0.060	0.233	0.122	0.242	0.174

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	62	48	54	66	91	68	62	95	68
N.S.	1	0.95	0.74	0.83	1.02	1.40	1.05	0.95	1.46	1.05
time (sec)	N/A	0.189	0.035	0.287	0.034	0.057	0.200	0.124	0.206	0.111

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	39	42	55	69	53	42	81	52
N.S.	1	1.04	0.80	0.86	1.12	1.41	1.08	0.86	1.65	1.06
time (sec)	N/A	0.182	0.011	0.277	0.026	0.068	0.166	0.126	0.262	0.065

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	23	36	36	36	22	28	37
N.S.	1	1.00	1.26	1.21	1.89	1.89	1.89	1.16	1.47	1.95
time (sec)	N/A	0.132	0.006	0.279	0.027	0.062	0.129	0.124	0.368	0.038

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	26	27	14	25	28
N.S.	1	1.00	1.00	0.94	0.88	1.62	1.69	0.88	1.56	1.75
time (sec)	N/A	0.135	0.002	0.282	0.025	0.059	0.129	0.125	0.266	0.037

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	55	43	46	60	90	56	59	109	56
N.S.	1	1.02	0.80	0.85	1.11	1.67	1.04	1.09	2.02	1.04
time (sec)	N/A	0.181	0.019	0.293	0.037	0.064	0.224	0.123	0.221	0.150

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	69	59	65	77	119	80	82	133	75
N.S.	1	1.03	0.88	0.97	1.15	1.78	1.19	1.22	1.99	1.12
time (sec)	N/A	0.200	0.034	0.293	0.031	0.066	0.260	0.124	0.224	0.171

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	88	74	77	92	134	90	80	148	88
N.S.	1	1.02	0.86	0.90	1.07	1.56	1.05	0.93	1.72	1.02
time (sec)	N/A	0.216	0.029	0.300	0.040	0.067	0.297	0.123	0.228	0.123

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	101	85	89	103	145	104	110	159	101
N.S.	1	1.06	0.89	0.94	1.08	1.53	1.09	1.16	1.67	1.06
time (sec)	N/A	0.233	0.043	0.301	0.037	0.064	0.324	0.132	0.201	0.131

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	109	96	100	114	156	116	119	170	111
N.S.	1	0.97	0.86	0.89	1.02	1.39	1.04	1.06	1.52	0.99
time (sec)	N/A	0.241	0.037	0.303	0.029	0.069	0.374	0.116	0.249	0.329

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	122	99	85	105	278	162	96	156	99
N.S.	1	1.12	0.91	0.78	0.96	2.55	1.49	0.88	1.43	0.91
time (sec)	N/A	0.224	0.039	0.324	0.107	0.070	0.258	0.120	0.217	0.163

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	109	88	74	93	256	144	84	145	87
N.S.	1	1.14	0.92	0.77	0.97	2.67	1.50	0.88	1.51	0.91
time (sec)	N/A	0.217	0.030	0.303	0.112	0.070	0.245	0.125	0.230	0.077

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	96	77	63	82	230	133	73	132	77
N.S.	1	1.13	0.91	0.74	0.96	2.71	1.56	0.86	1.55	0.91
time (sec)	N/A	0.211	0.029	0.296	0.114	0.071	0.254	0.126	0.198	0.173

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	85	66	51	68	202	107	54	118	64
N.S.	1	1.20	0.93	0.72	0.96	2.85	1.51	0.76	1.66	0.90
time (sec)	N/A	0.180	0.029	0.296	0.115	0.069	0.210	0.127	0.221	0.207

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	55	47	59	188	110	45	113	56
N.S.	1	1.12	0.86	0.73	0.92	2.94	1.72	0.70	1.77	0.88
time (sec)	N/A	0.165	0.024	0.322	0.117	0.067	0.154	0.126	0.214	0.181

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	58	49	62	190	110	50	110	55
N.S.	1	1.08	0.89	0.75	0.95	2.92	1.69	0.77	1.69	0.85
time (sec)	N/A	0.161	0.018	0.315	0.109	0.073	0.170	0.128	0.210	0.191

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	70	55	57	58	188	105	45	113	55
N.S.	1	1.13	0.89	0.92	0.94	3.03	1.69	0.73	1.82	0.89
time (sec)	N/A	0.158	0.003	0.253	0.106	0.071	0.167	0.117	0.233	0.074

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	88	68	54	71	202	116	57	121	66
N.S.	1	1.22	0.94	0.75	0.99	2.81	1.61	0.79	1.68	0.92
time (sec)	N/A	0.177	0.024	0.296	0.109	0.070	0.221	0.124	0.239	0.182

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	105	79	64	86	238	138	71	137	80
N.S.	1	1.21	0.91	0.74	0.99	2.74	1.59	0.82	1.57	0.92
time (sec)	N/A	0.200	0.026	0.305	0.115	0.071	0.267	0.128	0.209	0.191

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	122	90	75	97	264	150	80	150	92
N.S.	1	1.26	0.93	0.77	1.00	2.72	1.55	0.82	1.55	0.95
time (sec)	N/A	0.204	0.033	0.330	0.133	0.071	0.275	0.129	0.196	0.204

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	139	101	86	108	286	162	93	161	102
N.S.	1	1.25	0.91	0.77	0.97	2.58	1.46	0.84	1.45	0.92
time (sec)	N/A	0.215	0.031	0.319	0.109	0.075	0.298	0.127	0.230	0.235

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	212	169	151	242	346	260	168	404	242
N.S.	1	0.98	0.78	0.70	1.12	1.60	1.20	0.78	1.87	1.12
time (sec)	N/A	0.396	0.029	0.322	0.048	0.064	1.164	0.121	0.253	0.478

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	203	158	142	231	335	245	157	393	230
N.S.	1	0.99	0.77	0.69	1.13	1.63	1.20	0.77	1.92	1.12
time (sec)	N/A	0.354	0.018	0.360	0.042	0.068	1.084	0.121	0.214	0.467

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	185	145	129	220	322	233	139	380	220
N.S.	1	0.98	0.77	0.69	1.17	1.71	1.24	0.74	2.02	1.17
time (sec)	N/A	0.343	0.022	0.314	0.048	0.066	1.095	0.120	0.218	0.529

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	178	116	120	209	300	219	119	368	207
N.S.	1	0.99	0.65	0.67	1.17	1.68	1.22	0.66	2.06	1.16
time (sec)	N/A	0.325	0.021	0.299	0.040	0.065	0.854	0.124	0.226	0.438

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	101	100	190	190	202	99	105	192
N.S.	1	1.00	5.32	5.26	10.00	10.00	10.63	5.21	5.53	10.11
time (sec)	N/A	0.137	0.014	0.306	0.041	0.063	0.781	0.124	0.217	0.118

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	90	89	179	179	190	88	178	181
N.S.	1	1.10	2.31	2.28	4.59	4.59	4.87	2.26	4.56	4.64
time (sec)	N/A	0.152	0.012	0.294	0.041	0.060	0.732	0.123	0.196	0.111

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	70	79	78	168	168	178	77	167	170
N.S.	1	1.21	1.36	1.34	2.90	2.90	3.07	1.33	2.88	2.93
time (sec)	N/A	0.166	0.014	0.296	0.041	0.060	0.728	0.122	0.233	0.181

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	97	68	67	157	157	167	66	156	159
N.S.	1	1.26	0.88	0.87	2.04	2.04	2.17	0.86	2.03	2.06
time (sec)	N/A	0.175	0.013	0.298	0.039	0.062	0.667	0.126	0.223	0.178

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	57	56	146	146	155	55	145	148
N.S.	1	1.04	0.63	0.62	1.60	1.60	1.70	0.60	1.59	1.63
time (sec)	N/A	0.216	0.011	0.292	0.035	0.062	0.667	0.126	0.198	0.107

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	76	46	45	135	135	143	44	134	136
N.S.	1	1.06	0.64	0.62	1.88	1.88	1.99	0.61	1.86	1.89
time (sec)	N/A	0.202	0.009	0.306	0.043	0.062	0.667	0.121	0.212	0.121

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	57	35	34	124	124	131	33	123	125
N.S.	1	1.08	0.66	0.64	2.34	2.34	2.47	0.62	2.32	2.36
time (sec)	N/A	0.186	0.010	0.296	0.035	0.061	0.615	0.119	0.226	0.182

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	38	24	23	113	113	119	22	112	114
N.S.	1	1.12	0.71	0.68	3.32	3.32	3.50	0.65	3.29	3.35
time (sec)	N/A	0.166	0.007	0.302	0.043	0.062	0.645	0.125	0.243	0.113

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	103	110	14	102	14
N.S.	1	1.00	1.00	0.94	0.88	6.44	6.88	0.88	6.38	0.88
time (sec)	N/A	0.130	0.002	0.310	0.030	0.061	0.659	0.122	0.204	0.221

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	167	120	123	214	398	223	136	485	210
N.S.	1	1.01	0.72	0.74	1.29	2.40	1.34	0.82	2.92	1.27
time (sec)	N/A	0.296	0.053	0.310	0.045	0.071	0.916	0.120	0.239	0.602

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	136	142	231	427	245	159	511	229
N.S.	1	1.00	0.74	0.77	1.26	2.32	1.33	0.86	2.78	1.24
time (sec)	N/A	0.339	0.075	0.342	0.069	0.075	1.369	0.119	0.230	0.627

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	151	155	246	442	260	174	526	243
N.S.	1	1.00	0.70	0.71	1.13	2.04	1.20	0.80	2.42	1.12
time (sec)	N/A	0.382	0.058	0.344	0.057	0.076	1.010	0.125	0.200	1.051

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	224	162	165	257	453	270	187	537	255
N.S.	1	0.99	0.72	0.73	1.14	2.00	1.19	0.83	2.38	1.13
time (sec)	N/A	0.405	0.070	0.350	0.058	0.075	0.971	0.127	0.205	1.128

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	298	166	151	248	718	314	162	488	241
N.S.	1	1.25	0.70	0.63	1.04	3.02	1.32	0.68	2.05	1.01
time (sec)	N/A	0.390	0.053	0.349	0.139	0.081	1.087	0.121	0.244	0.465

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	285	155	140	236	692	299	150	475	231
N.S.	1	1.27	0.69	0.62	1.05	3.08	1.33	0.67	2.11	1.03
time (sec)	N/A	0.380	0.050	0.326	0.122	0.077	1.065	0.124	0.219	0.492

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	274	144	128	222	664	274	131	461	218
N.S.	1	1.30	0.68	0.61	1.05	3.15	1.30	0.62	2.18	1.03
time (sec)	N/A	0.324	0.052	0.336	0.127	0.077	0.937	0.120	0.185	0.436

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	261	134	124	213	650	277	122	456	210
N.S.	1	1.32	0.68	0.63	1.08	3.30	1.41	0.62	2.31	1.07
time (sec)	N/A	0.296	0.050	0.385	0.131	0.077	0.810	0.124	0.216	0.481

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	259	138	124	219	654	289	128	456	207
N.S.	1	1.31	0.70	0.63	1.11	3.30	1.46	0.65	2.30	1.05
time (sec)	N/A	0.293	0.046	0.368	0.121	0.077	0.757	0.129	0.222	0.230

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	257	138	122	221	654	291	128	456	205
N.S.	1	1.29	0.69	0.61	1.11	3.29	1.46	0.64	2.29	1.03
time (sec)	N/A	0.290	0.040	0.362	0.122	0.076	0.734	0.125	0.203	0.257

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	255	138	122	221	654	291	128	456	205
N.S.	1	1.28	0.69	0.61	1.10	3.27	1.46	0.64	2.28	1.02
time (sec)	N/A	0.290	0.045	0.378	0.121	0.077	0.692	0.129	0.191	0.222

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	253	138	122	221	654	291	128	456	205
N.S.	1	1.26	0.69	0.61	1.10	3.25	1.45	0.64	2.27	1.02
time (sec)	N/A	0.287	0.044	0.386	0.125	0.080	0.616	0.123	0.224	0.253

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	251	138	122	221	654	291	128	456	204
N.S.	1	1.24	0.68	0.60	1.09	3.24	1.44	0.63	2.26	1.01
time (sec)	N/A	0.286	0.037	0.397	0.128	0.080	0.628	0.125	0.190	0.236

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	249	138	122	221	654	291	128	456	204
N.S.	1	1.23	0.68	0.60	1.09	3.22	1.43	0.63	2.25	1.00
time (sec)	N/A	0.282	0.038	0.374	0.164	0.081	0.550	0.125	0.196	0.245

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	247	138	122	221	654	291	128	456	204
N.S.	1	1.21	0.68	0.60	1.08	3.21	1.43	0.63	2.24	1.00
time (sec)	N/A	0.274	0.040	0.369	0.129	0.078	0.533	0.123	0.203	0.251

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	245	138	124	219	654	286	128	456	206
N.S.	1	1.20	0.67	0.60	1.07	3.19	1.40	0.62	2.22	1.00
time (sec)	N/A	0.272	0.036	0.362	0.157	0.074	0.534	0.124	0.209	0.273

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	245	131	150	212	650	272	122	456	209
N.S.	1	1.35	0.72	0.83	1.17	3.59	1.50	0.67	2.52	1.15
time (sec)	N/A	0.274	0.057	0.356	0.120	0.076	0.562	0.127	0.231	0.165

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	277	147	131	225	664	282	134	464	220
N.S.	1	1.40	0.74	0.66	1.14	3.35	1.42	0.68	2.34	1.11
time (sec)	N/A	0.309	0.058	0.380	0.131	0.082	0.706	0.121	0.198	0.636

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	294	157	141	240	700	304	148	480	234
N.S.	1	1.30	0.69	0.62	1.06	3.08	1.34	0.65	2.11	1.03
time (sec)	N/A	0.330	0.052	0.391	0.130	0.088	0.756	0.119	0.234	0.657

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	311	169	153	251	726	316	159	493	246
N.S.	1	1.30	0.70	0.64	1.05	3.02	1.32	0.66	2.05	1.02
time (sec)	N/A	0.354	0.054	0.386	0.131	0.089	0.776	0.121	0.248	1.281

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	15	14	17	13	12	18	13	14
N.S.	1	1.27	1.00	0.93	1.13	0.87	0.80	1.20	0.87	0.93
time (sec)	N/A	0.133	0.004	0.337	0.031	0.060	0.077	0.123	0.177	0.142

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	20	18	16	17	15	12	18	25	16
N.S.	1	1.11	1.00	0.89	0.94	0.83	0.67	1.00	1.39	0.89
time (sec)	N/A	0.136	0.004	0.290	0.026	0.059	0.085	0.122	0.192	0.060

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	27	26	23	24	28	22	32	28	22
N.S.	1	1.04	1.00	0.88	0.92	1.08	0.85	1.23	1.08	0.85
time (sec)	N/A	0.159	0.005	0.307	0.028	0.061	0.123	0.119	0.228	0.064

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	26	27	23	24	28	22	32	41	22
N.S.	1	0.96	1.00	0.85	0.89	1.04	0.81	1.19	1.52	0.81
time (sec)	N/A	0.164	0.003	0.311	0.050	0.062	0.139	0.122	0.211	0.149

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	25	25	28	40	22	36	54	24
N.S.	1	1.00	0.89	0.89	1.00	1.43	0.79	1.29	1.93	0.86
time (sec)	N/A	0.157	0.008	0.312	0.034	0.061	0.115	0.119	0.203	0.151

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	28	40	22	36	74	26
N.S.	1	1.00	0.87	0.83	0.93	1.33	0.73	1.20	2.47	0.87
time (sec)	N/A	0.166	0.008	0.320	0.038	0.063	0.113	0.121	0.244	0.039

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	8	8	7	9	13	8
N.S.	1	1.00	0.83	0.75	0.67	0.67	0.58	0.75	1.08	0.67
time (sec)	N/A	0.127	0.001	0.290	0.038	0.061	0.028	0.125	0.207	0.048

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	21	12	15	22	17
N.S.	1	1.00	1.00	0.84	0.79	1.11	0.63	0.79	1.16	0.89
time (sec)	N/A	0.134	0.006	0.362	0.110	0.061	0.037	0.115	0.200	0.033

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	16	16	17	16	17	17
N.S.	1	1.00	1.00	0.77	0.73	0.73	0.77	0.73	0.77	0.77
time (sec)	N/A	0.143	0.001	0.318	0.025	0.057	0.015	0.122	0.203	0.038

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	11	10	9	24	22	9	24	9
N.S.	1	1.00	0.85	0.77	0.69	1.85	1.69	0.69	1.85	0.69
time (sec)	N/A	0.129	0.002	0.293	0.032	0.060	0.047	0.121	0.256	0.209

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	81	11	13	57	24	22	57	24	9
N.S.	1	6.23	0.85	1.00	4.38	1.85	1.69	4.38	1.85	0.69
time (sec)	N/A	0.184	0.001	0.830	0.031	0.060	0.156	0.121	0.214	0.029

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	25	23	22	26	41	23
N.S.	1	1.00	1.00	0.86	0.89	0.82	0.79	0.93	1.46	0.82
time (sec)	N/A	0.158	0.005	0.280	0.034	0.059	0.085	0.125	0.184	0.054

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	42	80	49	29	44	23
N.S.	1	1.00	1.00	0.87	1.35	2.58	1.58	0.94	1.42	0.74
time (sec)	N/A	0.140	0.006	0.295	0.105	0.069	0.078	0.118	0.210	0.097

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	15	12	16	33	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.75	1.00	2.06	0.94
time (sec)	N/A	0.128	0.003	0.271	0.024	0.060	0.062	0.127	0.205	0.038

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	31	68	46	18	38	16
N.S.	1	1.00	1.00	0.67	1.29	2.83	1.92	0.75	1.58	0.67
time (sec)	N/A	0.134	0.002	0.266	0.105	0.068	0.077	0.119	0.203	0.186

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	27	23	22	25	20	15	26	37	21
N.S.	1	1.17	1.00	0.96	1.09	0.87	0.65	1.13	1.61	0.91
time (sec)	N/A	0.138	0.005	0.279	0.026	0.064	0.124	0.120	0.196	0.191

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	44	82	58	31	48	25
N.S.	1	1.00	1.00	0.88	1.33	2.48	1.76	0.94	1.45	0.76
time (sec)	N/A	0.138	0.008	0.326	0.104	0.067	0.099	0.120	0.214	0.200

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	37	35	32	35	33	31	43	55	31
N.S.	1	1.06	1.00	0.91	1.00	0.94	0.89	1.23	1.57	0.89
time (sec)	N/A	0.162	0.007	0.286	0.025	0.063	0.140	0.116	0.220	0.182

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	33	29	32	35	42	29	53	83	32
N.S.	1	0.94	0.83	0.91	1.00	1.20	0.83	1.51	2.37	0.91
time (sec)	N/A	0.162	0.009	0.280	0.026	0.063	0.117	0.117	0.213	0.162

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	37	52	127	71	39	102	34
N.S.	1	1.00	1.02	0.80	1.13	2.76	1.54	0.85	2.22	0.74
time (sec)	N/A	0.145	0.019	0.295	0.106	0.067	0.113	0.123	0.226	0.115

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	16	15	16	18	15
N.S.	1	1.00	1.00	0.94	0.94	0.94	0.88	0.94	1.06	0.88
time (sec)	N/A	0.127	0.002	0.275	0.037	0.058	0.092	0.124	0.251	0.022

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	37	52	126	71	39	102	34
N.S.	1	1.00	1.02	0.80	1.13	2.74	1.54	0.85	2.22	0.74
time (sec)	N/A	0.144	0.003	0.274	0.106	0.067	0.119	0.127	0.218	0.155

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	41	35	37	41	53	34	51	96	36
N.S.	1	1.02	0.88	0.92	1.02	1.32	0.85	1.28	2.40	0.90
time (sec)	N/A	0.166	0.011	0.283	0.033	0.065	0.220	0.122	0.204	0.069

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	61	56	45	65	140	83	50	113	45
N.S.	1	1.11	1.02	0.82	1.18	2.55	1.51	0.91	2.05	0.82
time (sec)	N/A	0.161	0.024	0.309	0.107	0.069	0.158	0.114	0.225	0.092

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	53	44	53	57	80	49	56	127	55
N.S.	1	1.02	0.85	1.02	1.10	1.54	0.94	1.08	2.44	1.06
time (sec)	N/A	0.182	0.022	0.306	0.026	0.064	0.201	0.114	0.236	0.158

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	25	24	38	38	36	26	28	37
N.S.	1	1.00	1.25	1.20	1.90	1.90	1.80	1.30	1.40	1.85
time (sec)	N/A	0.136	0.007	0.286	0.047	0.058	0.125	0.126	0.224	0.045

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	56	50	76	188	105	53	176	54
N.S.	1	1.07	0.84	0.75	1.13	2.81	1.57	0.79	2.63	0.81
time (sec)	N/A	0.166	0.020	0.296	0.107	0.070	0.162	0.124	0.193	0.209

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	26	26	16	25	26
N.S.	1	1.00	1.00	0.94	0.94	1.53	1.53	0.94	1.47	1.53
time (sec)	N/A	0.129	0.002	0.283	0.026	0.055	0.124	0.125	0.219	0.109

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	72	56	59	73	188	99	49	178	55
N.S.	1	1.12	0.88	0.92	1.14	2.94	1.55	0.77	2.78	0.86
time (sec)	N/A	0.162	0.003	0.276	0.109	0.069	0.165	0.122	0.222	0.185

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	58	45	48	62	92	56	63	174	57
N.S.	1	1.02	0.79	0.84	1.09	1.61	0.98	1.11	3.05	1.00
time (sec)	N/A	0.186	0.022	0.290	0.038	0.062	0.217	0.127	0.190	0.162

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	89	69	54	86	202	107	61	187	66
N.S.	1	1.20	0.93	0.73	1.16	2.73	1.45	0.82	2.53	0.89
time (sec)	N/A	0.176	0.029	0.339	0.104	0.072	0.247	0.125	0.225	0.111

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	72	60	65	79	121	78	84	204	76
N.S.	1	1.04	0.87	0.94	1.14	1.75	1.13	1.22	2.96	1.10
time (sec)	N/A	0.201	0.034	0.300	0.039	0.063	0.252	0.119	0.232	0.183

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	25	24	60	60	60	39	57	59
N.S.	1	1.11	0.69	0.67	1.67	1.67	1.67	1.08	1.58	1.64
time (sec)	N/A	0.168	0.008	0.290	0.031	0.060	0.224	0.120	0.233	0.109

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	124	81	70	124	324	160	77	328	96
N.S.	1	1.14	0.74	0.64	1.14	2.97	1.47	0.71	3.01	0.88
time (sec)	N/A	0.198	0.033	0.326	0.113	0.072	0.289	0.123	0.196	0.231

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	16	48	49	16	47	48
N.S.	1	1.00	1.00	0.94	0.94	2.82	2.88	0.94	2.76	2.82
time (sec)	N/A	0.134	0.002	0.293	0.027	0.060	0.217	0.125	0.227	0.131

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	124	79	92	117	320	146	71	328	99
N.S.	1	1.24	0.79	0.92	1.17	3.20	1.46	0.71	3.28	0.99
time (sec)	N/A	0.189	0.027	0.327	0.105	0.068	0.276	0.123	0.211	0.204

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	92	67	70	106	180	104	85	332	101
N.S.	1	1.01	0.74	0.77	1.16	1.98	1.14	0.93	3.65	1.11
time (sec)	N/A	0.212	0.021	0.302	0.051	0.067	0.350	0.124	0.224	0.170

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	145	92	76	130	334	155	83	337	110
N.S.	1	1.29	0.82	0.68	1.16	2.98	1.38	0.74	3.01	0.98
time (sec)	N/A	0.220	0.033	0.345	0.111	0.072	0.415	0.124	0.194	0.249

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	108	83	87	123	209	126	106	364	120
N.S.	1	1.02	0.78	0.82	1.16	1.97	1.19	1.00	3.43	1.13
time (sec)	N/A	0.238	0.040	0.308	0.045	0.064	0.438	0.127	0.225	0.303

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.141	0.011	0.243	0.031	0.062	0.401	0.123	0.217	0.135

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	14	13	18	19	13	17	15
N.S.	1	1.00	0.90	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.142	0.010	0.236	0.051	0.062	0.261	0.123	0.206	0.031

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	18	19	13	17	15
N.S.	1	1.00	1.00	0.67	0.62	0.86	0.90	0.62	0.81	0.71
time (sec)	N/A	0.143	0.010	0.256	0.031	0.061	0.157	0.122	0.199	0.029

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	13	16	19	13	15	15
N.S.	1	1.00	1.00	0.67	0.62	0.76	0.90	0.62	0.71	0.71
time (sec)	N/A	0.143	0.010	0.109	0.044	0.060	0.369	0.123	0.255	0.035

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	20	14	13	14	17	13	13	14
N.S.	1	1.00	1.05	0.74	0.68	0.74	0.89	0.68	0.68	0.74
time (sec)	N/A	0.143	0.010	0.114	0.046	0.063	0.078	0.123	0.220	0.033

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	13	14	17	13	15	15
N.S.	1	1.00	1.00	0.74	0.68	0.74	0.89	0.68	0.79	0.79
time (sec)	N/A	0.142	0.012	0.105	0.025	0.066	0.162	0.121	0.198	0.031

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	15	17	13	19	15
N.S.	1	1.00	0.89	0.74	0.68	0.79	0.89	0.68	1.00	0.79
time (sec)	N/A	0.140	0.012	0.107	0.025	0.061	0.262	0.123	0.239	0.035

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	14	13	13	19	13	19	15
N.S.	1	1.00	0.89	0.74	0.68	0.68	1.00	0.68	1.00	0.79
time (sec)	N/A	0.141	0.014	0.125	0.032	0.064	0.287	0.118	0.198	0.029

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	25
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.69
time (sec)	N/A	0.147	0.014	0.320	0.027	0.059	0.630	0.120	0.188	0.165

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.72
time (sec)	N/A	0.149	0.013	0.306	0.044	0.060	0.392	0.117	0.207	0.041

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	29	34	24	28	26
N.S.	1	1.00	0.83	0.69	0.67	0.81	0.94	0.67	0.78	0.72
time (sec)	N/A	0.151	0.014	0.289	0.032	0.060	0.293	0.125	0.267	0.135

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	30	25	24	27	34	24	26	26
N.S.	1	1.00	0.83	0.69	0.67	0.75	0.94	0.67	0.72	0.72
time (sec)	N/A	0.147	0.013	0.280	0.027	0.061	0.411	0.120	0.204	0.151

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	25	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.74	0.76
time (sec)	N/A	0.147	0.013	0.429	0.025	0.061	0.149	0.117	0.190	0.040

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	27	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.79	0.76
time (sec)	N/A	0.145	0.016	0.277	0.038	0.062	0.257	0.122	0.196	0.040

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	24	26	32	24	30	26
N.S.	1	1.00	0.88	0.74	0.71	0.76	0.94	0.71	0.88	0.76
time (sec)	N/A	0.144	0.016	0.263	0.026	0.064	0.256	0.119	0.225	0.038

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	25	25	26	32	25	30	26
N.S.	1	1.00	0.88	0.74	0.74	0.76	0.94	0.74	0.88	0.76
time (sec)	N/A	0.143	0.018	0.270	0.032	0.063	0.327	0.125	0.217	0.035

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.153	0.016	0.325	0.035	0.063	0.931	0.122	0.211	0.062

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.150	0.016	0.332	0.036	0.063	0.636	0.122	0.250	0.046

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	40	49	35	39	35
N.S.	1	1.00	0.80	0.71	0.69	0.78	0.96	0.69	0.76	0.69
time (sec)	N/A	0.153	0.017	0.294	0.025	0.062	0.451	0.120	0.229	0.046

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	41	36	35	38	49	35	37	35
N.S.	1	1.00	0.80	0.71	0.69	0.75	0.96	0.69	0.73	0.69
time (sec)	N/A	0.154	0.015	0.292	0.046	0.061	0.552	0.122	0.191	0.046

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	36	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.73	0.71
time (sec)	N/A	0.153	0.015	0.263	0.026	0.064	0.236	0.119	0.192	0.048

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	35	37	46	35	38	35
N.S.	1	1.00	0.87	0.77	0.74	0.79	0.98	0.74	0.81	0.74
time (sec)	N/A	0.157	0.019	0.267	0.030	0.063	0.348	0.118	0.276	0.050

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	36	35	37	48	35	41	35
N.S.	1	1.00	0.84	0.73	0.71	0.76	0.98	0.71	0.84	0.71
time (sec)	N/A	0.154	0.019	0.267	0.030	0.063	0.376	0.119	0.187	0.049

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	36	36	37	46	36	41	37
N.S.	1	1.00	0.87	0.77	0.77	0.79	0.98	0.77	0.87	0.79
time (sec)	N/A	0.156	0.020	0.275	0.032	0.066	0.468	0.122	0.199	0.046

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	248	128	125	194	167	136	196	157	67
N.S.	1	1.56	0.81	0.79	1.22	1.05	0.86	1.23	0.99	0.42
time (sec)	N/A	0.420	0.146	0.325	0.112	0.074	17.173	0.124	0.204	0.213

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	231	119	116	186	158	124	178	147	54
N.S.	1	1.56	0.80	0.78	1.26	1.07	0.84	1.20	0.99	0.36
time (sec)	N/A	0.385	0.116	0.322	0.108	0.073	5.339	0.130	0.237	0.097

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	229	118	115	185	117	110	178	145	55
N.S.	1	1.56	0.80	0.78	1.26	0.80	0.75	1.21	0.99	0.37
time (sec)	N/A	0.381	0.115	0.312	0.108	0.074	2.053	0.128	0.214	0.094

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	214	91	106	172	132	104	182	112	38
N.S.	1	1.56	0.66	0.77	1.26	0.96	0.76	1.33	0.82	0.28
time (sec)	N/A	0.364	0.097	0.280	0.127	0.077	1.101	0.132	0.225	0.169

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	214	92	106	172	120	104	182	112	37
N.S.	1	1.57	0.68	0.78	1.26	0.88	0.76	1.34	0.82	0.27
time (sec)	N/A	0.356	0.098	0.290	0.107	0.073	1.846	0.123	0.241	0.092

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	229	117	115	186	140	114	190	155	54
N.S.	1	1.57	0.80	0.79	1.27	0.96	0.78	1.30	1.06	0.37
time (sec)	N/A	0.373	0.133	0.312	0.108	0.073	3.913	0.127	0.206	0.104

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	231	119	116	187	165	128	178	163	53
N.S.	1	1.55	0.80	0.78	1.26	1.11	0.86	1.19	1.09	0.36
time (sec)	N/A	0.379	0.136	0.303	0.110	0.074	9.168	0.121	0.203	0.096

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	248	127	124	198	191	139	200	180	66
N.S.	1	1.55	0.79	0.78	1.24	1.19	0.87	1.25	1.12	0.41
time (sec)	N/A	0.399	0.154	0.310	0.109	0.074	30.962	0.123	0.192	0.200

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	258	138	136	206	209	338	196	313	80
N.S.	1	1.46	0.78	0.77	1.16	1.18	1.91	1.11	1.77	0.45
time (sec)	N/A	0.408	0.258	0.349	0.111	0.080	81.878	0.130	0.250	0.118

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	124	195	200	393	199	307	64
N.S.	1	1.46	0.78	0.75	1.18	1.21	2.38	1.21	1.86	0.39
time (sec)	N/A	0.380	0.231	0.321	0.109	0.072	52.327	0.128	0.241	0.211

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	127	127	195	191	323	199	304	64
N.S.	1	1.46	0.77	0.77	1.18	1.16	1.96	1.21	1.84	0.39
time (sec)	N/A	0.388	0.223	0.287	0.111	0.078	29.299	0.127	0.188	0.185

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	127	194	204	400	199	305	64
N.S.	1	1.46	0.78	0.77	1.18	1.24	2.42	1.21	1.85	0.39
time (sec)	N/A	0.383	0.216	0.287	0.115	0.076	17.495	0.130	0.257	0.100

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	241	128	124	194	193	316	199	306	64
N.S.	1	1.46	0.78	0.75	1.18	1.17	1.92	1.21	1.85	0.39
time (sec)	N/A	0.390	0.211	0.239	0.111	0.073	24.666	0.126	0.222	0.188

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	258	138	136	208	218	512	210	329	77
N.S.	1	1.46	0.78	0.77	1.18	1.23	2.89	1.19	1.86	0.44
time (sec)	N/A	0.405	0.262	0.255	0.114	0.079	50.603	0.120	0.198	0.092

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	260	138	136	209	244	425	196	336	77
N.S.	1	1.47	0.78	0.77	1.18	1.38	2.40	1.11	1.90	0.44
time (sec)	N/A	0.410	0.261	0.368	0.114	0.080	113.714	0.127	0.215	0.216

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	277	149	145	221	263	0	220	355	87
N.S.	1	1.46	0.78	0.76	1.16	1.38	0.00	1.16	1.87	0.46
time (sec)	N/A	0.424	0.272	0.299	0.110	0.083	0.000	0.125	0.265	0.099

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	139	218	275	0	209	477	87
N.S.	1	1.45	0.74	0.75	1.17	1.48	0.00	1.12	2.56	0.47
time (sec)	N/A	0.418	0.312	0.328	0.109	0.089	0.000	0.133	0.190	0.228

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	270	136	138	222	289	0	212	478	85
N.S.	1	1.43	0.72	0.73	1.17	1.53	0.00	1.12	2.53	0.45
time (sec)	N/A	0.418	0.301	0.291	0.110	0.078	0.000	0.129	0.200	0.095

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	270	137	138	221	279	666	211	477	85
N.S.	1	1.43	0.72	0.73	1.17	1.48	3.52	1.12	2.52	0.45
time (sec)	N/A	0.414	0.292	0.308	0.112	0.080	159.351	0.136	0.217	0.200

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	150	217	281	887	209	478	86
N.S.	1	1.45	0.74	0.81	1.17	1.51	4.77	1.12	2.57	0.46
time (sec)	N/A	0.421	0.209	0.245	0.112	0.079	100.502	0.130	0.201	0.102

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	270	138	147	217	268	627	209	477	86
N.S.	1	1.45	0.74	0.79	1.17	1.44	3.37	1.12	2.56	0.46
time (sec)	N/A	0.423	0.203	0.256	0.112	0.076	147.056	0.126	0.204	0.200

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	287	149	145	230	284	0	220	506	99
N.S.	1	1.45	0.75	0.73	1.16	1.43	0.00	1.11	2.56	0.50
time (sec)	N/A	0.440	0.330	0.282	0.119	0.079	0.000	0.131	0.222	0.113

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	289	149	145	231	310	0	208	513	99
N.S.	1	1.46	0.75	0.73	1.17	1.57	0.00	1.05	2.59	0.50
time (sec)	N/A	0.434	0.334	0.279	0.112	0.081	0.000	0.129	0.236	0.128

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	306	160	155	243	329	0	232	532	109
N.S.	1	1.45	0.76	0.73	1.15	1.56	0.00	1.10	2.52	0.52
time (sec)	N/A	0.458	0.354	0.279	0.111	0.083	0.000	0.123	0.215	0.116

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	121	59	67	84	74	105	84	75	47
N.S.	1	1.48	0.72	0.82	1.02	0.90	1.28	1.02	0.91	0.57
time (sec)	N/A	0.289	0.071	0.823	0.107	0.067	0.445	0.117	0.172	0.203

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	114	55	62	79	69	99	79	69	42
N.S.	1	1.52	0.73	0.83	1.05	0.92	1.32	1.05	0.92	0.56
time (sec)	N/A	0.274	0.049	0.789	0.108	0.073	0.280	0.127	0.207	0.049

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	112	54	62	79	69	97	79	68	42
N.S.	1	1.51	0.73	0.84	1.07	0.93	1.31	1.07	0.92	0.57
time (sec)	N/A	0.275	0.048	0.826	0.106	0.068	0.208	0.113	0.228	0.047

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	104	41	56	74	64	90	74	58	37
N.S.	1	1.55	0.61	0.84	1.10	0.96	1.34	1.10	0.87	0.55
time (sec)	N/A	0.260	0.039	0.619	0.107	0.072	0.167	0.116	0.207	0.042

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	104	39	56	74	64	90	74	58	37
N.S.	1	1.58	0.59	0.85	1.12	0.97	1.36	1.12	0.88	0.56
time (sec)	N/A	0.251	0.040	0.574	0.104	0.065	0.188	0.123	0.209	0.031

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	112	53	62	79	77	97	79	78	42
N.S.	1	1.53	0.73	0.85	1.08	1.05	1.33	1.08	1.07	0.58
time (sec)	N/A	0.271	0.053	0.729	0.113	0.069	0.292	0.135	0.215	0.040

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	114	56	62	79	86	99	79	86	42
N.S.	1	1.50	0.74	0.82	1.04	1.13	1.30	1.04	1.13	0.55
time (sec)	N/A	0.267	0.057	0.505	0.107	0.079	0.437	0.129	0.222	0.042

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	121	62	67	86	93	105	86	99	48
N.S.	1	1.46	0.75	0.81	1.04	1.12	1.27	1.04	1.19	0.58
time (sec)	N/A	0.273	0.065	0.767	0.106	0.072	0.641	0.119	0.197	0.044

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	133	72	74	91	105	277	91	160	55
N.S.	1	1.34	0.73	0.75	0.92	1.06	2.80	0.92	1.62	0.56
time (sec)	N/A	0.287	0.125	0.530	0.105	0.070	1.104	0.127	0.194	0.094

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	65	69	86	98	264	86	154	51
N.S.	1	1.37	0.72	0.77	0.96	1.09	2.93	0.96	1.71	0.57
time (sec)	N/A	0.278	0.119	0.753	0.106	0.068	0.718	0.126	0.183	0.074

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	63	69	86	97	257	86	151	51
N.S.	1	1.37	0.70	0.77	0.96	1.08	2.86	0.96	1.68	0.57
time (sec)	N/A	0.272	0.113	0.764	0.128	0.077	0.592	0.123	0.218	0.081

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	64	69	86	97	257	86	152	50
N.S.	1	1.37	0.71	0.77	0.96	1.08	2.86	0.96	1.69	0.56
time (sec)	N/A	0.281	0.100	0.499	0.108	0.071	0.467	0.126	0.191	0.039

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	123	65	69	86	98	264	86	153	50
N.S.	1	1.37	0.72	0.77	0.96	1.09	2.93	0.96	1.70	0.56
time (sec)	N/A	0.273	0.097	0.530	0.108	0.071	0.524	0.124	0.204	0.049

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	133	72	74	92	105	366	92	174	55
N.S.	1	1.34	0.73	0.75	0.93	1.06	3.70	0.93	1.76	0.56
time (sec)	N/A	0.289	0.152	0.582	0.113	0.068	0.700	0.121	0.207	0.170

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	135	72	74	92	115	366	91	181	55
N.S.	1	1.36	0.73	0.75	0.93	1.16	3.70	0.92	1.83	0.56
time (sec)	N/A	0.291	0.140	0.632	0.146	0.071	1.067	0.127	0.234	0.089

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	142	77	79	97	120	384	98	194	59
N.S.	1	1.31	0.71	0.73	0.90	1.11	3.56	0.91	1.80	0.55
time (sec)	N/A	0.304	0.146	0.497	0.106	0.073	1.906	0.124	0.211	0.077

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	144	72	76	99	130	481	94	240	62
N.S.	1	1.36	0.68	0.72	0.93	1.23	4.54	0.89	2.26	0.58
time (sec)	N/A	0.294	0.176	0.606	0.141	0.073	2.227	0.126	0.206	0.201

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	144	72	76	99	132	481	94	241	62
N.S.	1	1.36	0.68	0.72	0.93	1.25	4.54	0.89	2.27	0.58
time (sec)	N/A	0.302	0.155	0.603	0.107	0.069	1.769	0.125	0.193	0.077

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	144	70	74	97	128	481	92	240	62
N.S.	1	1.36	0.66	0.70	0.92	1.21	4.54	0.87	2.26	0.58
time (sec)	N/A	0.296	0.142	0.531	0.105	0.074	1.304	0.121	0.246	0.054

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	144	72	76	99	132	481	94	241	61
N.S.	1	1.36	0.68	0.72	0.93	1.25	4.54	0.89	2.27	0.58
time (sec)	N/A	0.300	0.092	0.497	0.112	0.070	1.037	0.124	0.206	0.043

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	144	72	76	99	130	481	94	240	61
N.S.	1	1.36	0.68	0.72	0.93	1.23	4.54	0.89	2.26	0.58
time (sec)	N/A	0.292	0.098	0.519	0.110	0.072	1.288	0.122	0.221	0.089

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	154	77	81	102	135	653	99	267	65
N.S.	1	1.34	0.67	0.70	0.89	1.17	5.68	0.86	2.32	0.57
time (sec)	N/A	0.313	0.189	0.546	0.105	0.071	1.715	0.121	0.218	0.175

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	156	77	81	102	145	653	99	274	65
N.S.	1	1.36	0.67	0.70	0.89	1.26	5.68	0.86	2.38	0.57
time (sec)	N/A	0.304	0.190	0.575	0.116	0.070	2.698	0.120	0.225	0.091

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	163	82	86	107	150	678	106	287	69
N.S.	1	1.31	0.66	0.69	0.86	1.21	5.47	0.85	2.31	0.56
time (sec)	N/A	0.319	0.172	0.505	0.111	0.069	4.790	0.127	0.187	0.099

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	C	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	65	48	58	86	124	92	194	49	33
N.S.	1	1.12	0.83	1.00	1.48	2.14	1.59	3.34	0.84	0.57
time (sec)	N/A	0.166	0.034	0.253	0.113	0.074	0.935	0.120	0.194	0.181

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	23	15	24	23	23	26	24	20	11
N.S.	1	1.53	1.00	1.60	1.53	1.53	1.73	1.60	1.33	0.73
time (sec)	N/A	0.145	0.017	0.260	0.103	0.074	0.097	0.116	0.227	0.139

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	321	152	176	321	289	391	273	172	276
N.S.	1	1.37	0.65	0.75	1.37	1.23	1.66	1.16	0.73	1.17
time (sec)	N/A	0.550	0.151	2.216	0.154	0.073	1.545	0.131	0.219	0.274

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	175	159	115	149	143	175	158	135	158
N.S.	1	1.02	0.92	0.67	0.87	0.83	1.02	0.92	0.78	0.92
time (sec)	N/A	0.340	0.101	0.658	0.115	0.072	0.864	0.133	0.220	0.163

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	305	130	195	295	288	355	270	137	292
N.S.	1	1.38	0.59	0.88	1.33	1.30	1.61	1.22	0.62	1.32
time (sec)	N/A	0.508	0.105	0.514	0.124	0.073	1.191	0.133	0.233	0.287

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	199	201	141	150	156	204	178	174	165
N.S.	1	1.05	1.06	0.75	0.79	0.83	1.08	0.94	0.92	0.87
time (sec)	N/A	0.359	0.114	0.804	0.117	0.072	1.122	0.122	0.234	0.135

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	327	159	260	311	434	401	272	192	286
N.S.	1	1.37	0.67	1.09	1.30	1.82	1.68	1.14	0.80	1.20
time (sec)	N/A	0.540	0.139	0.645	0.123	0.080	1.769	0.126	0.218	0.284

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	335	153	225	291	368	391	272	177	272
N.S.	1	1.41	0.65	0.95	1.23	1.55	1.65	1.15	0.75	1.15
time (sec)	N/A	0.534	0.132	0.549	0.118	0.076	2.647	0.223	0.224	0.284

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	192	188	145	166	157	199	174	153	169
N.S.	1	1.03	1.01	0.78	0.89	0.84	1.06	0.93	0.82	0.90
time (sec)	N/A	0.352	0.109	0.290	0.116	0.073	1.448	0.137	0.217	0.184

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	313	128	193	266	332	366	257	137	273
N.S.	1	1.42	0.58	0.87	1.20	1.50	1.66	1.16	0.62	1.24
time (sec)	N/A	0.518	0.103	0.457	0.124	0.074	1.223	0.196	0.312	0.268

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	170	159	117	150	385	170	162	139	144
N.S.	1	0.99	0.92	0.68	0.87	2.24	0.99	0.94	0.81	0.84
time (sec)	N/A	0.331	0.090	0.277	0.112	0.082	0.842	0.132	0.234	0.140

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	335	157	238	284	388	391	285	187	286
N.S.	1	1.41	0.66	1.00	1.20	1.64	1.65	1.20	0.79	1.21
time (sec)	N/A	0.551	0.135	0.557	0.118	0.078	1.363	0.198	0.226	0.309

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	110	79	69	68	68	94	68	62	57
N.S.	1	1.51	1.08	0.95	0.93	0.93	1.29	0.93	0.85	0.78
time (sec)	N/A	0.264	0.095	5.483	0.105	0.072	0.353	0.124	0.213	0.110

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	88	431	97	367	1999	540	430	389
N.S.	1	1.00	0.91	4.44	1.00	3.78	20.61	5.57	4.43	4.01
time (sec)	N/A	0.221	0.095	0.363	0.030	0.074	0.647	0.137	0.194	0.718

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	290	79	251	1221	365	289	272
N.S.	1	1.00	0.91	3.67	1.00	3.18	15.46	4.62	3.66	3.44
time (sec)	N/A	0.196	0.053	0.292	0.036	0.071	0.519	0.127	0.197	0.596

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	72	61	157	683	224	176	167
N.S.	1	1.00	0.92	1.18	1.00	2.57	11.20	3.67	2.89	2.74
time (sec)	N/A	0.181	0.039	0.260	0.032	0.074	0.486	0.129	0.221	0.489

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	51	43	85	306	117	91	93
N.S.	1	1.00	0.93	1.19	1.00	1.98	7.12	2.72	2.12	2.16
time (sec)	N/A	0.169	0.030	0.240	0.026	0.077	0.293	0.126	0.229	0.430

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	30	25	33	94	43	33	34
N.S.	1	1.00	1.00	1.20	1.00	1.32	3.76	1.72	1.32	1.36
time (sec)	N/A	0.149	0.020	0.053	0.033	0.084	0.195	0.125	0.226	0.360

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	88	0	15	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	2.26	0.00	0.38	0.00
time (sec)	N/A	0.146	0.021	0.000	0.000	0.000	0.585	0.000	0.220	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	377	0	26	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	9.67	0.00	0.67	0.00
time (sec)	N/A	0.146	0.021	0.000	0.000	0.000	2.943	0.000	0.236	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	41	0	0	0	1574	0	37	0
N.S.	1	1.00	1.05	0.00	0.00	0.00	40.36	0.00	0.95	0.00
time (sec)	N/A	0.145	0.022	0.000	0.000	0.000	10.271	0.000	0.205	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	45	0	0	0	88	0	36	0
N.S.	1	1.00	1.02	0.00	0.00	0.00	2.00	0.00	0.82	0.00
time (sec)	N/A	0.150	0.025	0.000	0.000	0.000	0.597	0.000	0.197	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	0	0	0	95	0	19	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	2.16	0.00	0.43	0.00
time (sec)	N/A	0.145	0.005	0.000	0.000	0.000	0.601	0.000	0.227	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	0	0	0	51	0	24	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	1.34	0.00	0.63	0.00
time (sec)	N/A	0.142	0.023	0.000	0.000	0.000	0.604	0.000	0.234	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	0	0	0	99	0	26	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	2.11	0.00	0.55	0.00
time (sec)	N/A	0.152	0.026	0.000	0.000	0.000	0.771	0.000	0.213	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	85	0	26	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	1.89	0.00	0.58	0.00
time (sec)	N/A	0.151	0.027	0.000	0.000	0.000	0.640	0.000	0.188	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	92	0	0	347	0	30	0
N.S.	1	1.00	1.06	2.56	0.00	0.00	9.64	0.00	0.83	0.00
time (sec)	N/A	0.141	0.022	0.640	0.000	0.000	2.935	0.000	0.215	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	61	47	73	57	110	57	56	55
N.S.	1	1.05	0.76	0.59	0.91	0.71	1.38	0.71	0.70	0.69
time (sec)	N/A	0.185	0.025	0.275	0.028	0.068	0.267	0.120	0.232	0.342

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	50	36	53	46	87	43	45	44
N.S.	1	1.07	0.85	0.61	0.90	0.78	1.47	0.73	0.76	0.75
time (sec)	N/A	0.172	0.021	0.270	0.026	0.068	0.195	0.119	0.204	0.316

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	33	34	63	29	33	33
N.S.	1	1.11	1.00	0.66	0.87	0.89	1.66	0.76	0.87	0.87
time (sec)	N/A	0.160	0.020	0.270	0.026	0.068	0.160	0.120	0.204	0.302

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	39	14	20	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.17	0.78	1.11	0.78
time (sec)	N/A	0.127	0.002	0.289	0.034	0.070	0.076	0.125	0.248	0.173

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	37	30	27	80	56	33	60	29
N.S.	1	1.16	1.00	0.81	0.73	2.16	1.51	0.89	1.62	0.78
time (sec)	N/A	0.154	0.023	0.258	0.027	0.086	0.778	0.122	0.209	0.398

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	41	51	109	42	43	79	35
N.S.	1	1.00	1.00	0.87	1.09	2.32	0.89	0.91	1.68	0.74
time (sec)	N/A	0.152	0.051	0.356	0.026	0.074	0.917	0.127	0.190	0.447

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	73	62	56	73	134	92	72	100	54
N.S.	1	1.03	0.87	0.79	1.03	1.89	1.30	1.01	1.41	0.76
time (sec)	N/A	0.168	0.080	0.298	0.028	0.079	1.910	0.122	0.205	0.536

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	103	73	71	93	160	117	80	120	74
N.S.	1	1.08	0.77	0.75	0.98	1.68	1.23	0.84	1.26	0.78
time (sec)	N/A	0.186	0.096	0.305	0.035	0.079	4.637	0.127	0.253	0.602

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	103	82	62	69	146	117	64	80	0
N.S.	1	1.10	0.87	0.66	0.73	1.55	1.24	0.68	0.85	0.00
time (sec)	N/A	0.188	0.213	0.295	0.028	0.079	4.529	0.125	0.199	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	73	69	49	49	119	92	50	60	0
N.S.	1	1.04	0.99	0.70	0.70	1.70	1.31	0.71	0.86	0.00
time (sec)	N/A	0.167	0.144	0.290	0.027	0.082	1.863	0.134	0.211	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	48	36	28	94	41	37	40	35
N.S.	1	1.00	1.04	0.78	0.61	2.04	0.89	0.80	0.87	0.76
time (sec)	N/A	0.146	0.004	0.289	0.026	0.074	0.906	0.122	0.254	0.253

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	45	36	28	88	56	57	43	56
N.S.	1	1.00	1.07	0.86	0.67	2.10	1.33	1.36	1.02	1.33
time (sec)	N/A	0.146	0.036	0.286	0.027	0.073	0.775	0.129	0.196	0.534

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	17	42	59	42	17
N.S.	1	1.00	1.00	0.86	0.81	0.81	2.00	2.81	2.00	0.81
time (sec)	N/A	0.130	0.038	0.333	0.027	0.068	0.399	0.123	0.182	0.360

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	28	36	38	68	112	63	37
N.S.	1	1.00	0.95	0.64	0.82	0.86	1.55	2.55	1.43	0.84
time (sec)	N/A	0.146	0.047	0.298	0.045	0.070	0.495	0.132	0.196	0.429

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	53	39	56	49	359	138	82	73
N.S.	1	1.09	0.78	0.57	0.82	0.72	5.28	2.03	1.21	1.07
time (sec)	N/A	0.163	0.054	0.279	0.026	0.071	0.672	0.126	0.244	0.546

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	64	50	76	60	575	166	101	93
N.S.	1	1.13	0.70	0.54	0.83	0.65	6.25	1.80	1.10	1.01
time (sec)	N/A	0.181	0.061	0.291	0.027	0.077	0.934	0.130	0.203	0.632

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	73	68	133	57	67	64
N.S.	1	1.05	0.62	0.59	0.91	0.85	1.66	0.71	0.84	0.80
time (sec)	N/A	0.184	0.032	0.276	0.028	0.072	0.463	0.127	0.197	0.387

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	53	57	109	43	56	53
N.S.	1	1.07	0.66	0.61	0.90	0.97	1.85	0.73	0.95	0.90
time (sec)	N/A	0.176	0.027	0.264	0.032	0.069	0.317	0.122	0.220	0.328

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	33	45	85	29	44	42
N.S.	1	1.11	0.74	0.66	0.87	1.18	2.24	0.76	1.16	1.11
time (sec)	N/A	0.158	0.024	0.268	0.025	0.066	0.244	0.120	0.200	0.332

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	61	14	31	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.39	0.78	1.72	0.78
time (sec)	N/A	0.132	0.002	0.254	0.032	0.096	0.123	0.119	0.193	0.208

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	61	50	41	40	103	78	48	79	42
N.S.	1	1.13	0.93	0.76	0.74	1.91	1.44	0.89	1.46	0.78
time (sec)	N/A	0.169	0.033	0.271	0.032	0.077	1.163	0.125	0.203	0.341

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	65	57	52	63	122	88	57	91	47
N.S.	1	1.07	0.93	0.85	1.03	2.00	1.44	0.93	1.49	0.77
time (sec)	N/A	0.168	0.065	0.357	0.027	0.077	1.366	0.123	0.245	0.506

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	71	59	57	90	139	71	70	101	52
N.S.	1	1.03	0.86	0.83	1.30	2.01	1.03	1.01	1.46	0.75
time (sec)	N/A	0.165	0.088	0.296	0.036	0.078	1.532	0.117	0.207	0.596

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	97	73	71	110	160	119	80	120	72
N.S.	1	1.04	0.78	0.76	1.18	1.72	1.28	0.86	1.29	0.77
time (sec)	N/A	0.180	0.090	0.289	0.037	0.080	3.396	0.128	0.218	0.713

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	84	78	130	182	148	109	139	89
N.S.	1	1.09	0.72	0.67	1.11	1.56	1.26	0.93	1.19	0.76
time (sec)	N/A	0.194	0.119	0.348	0.046	0.088	11.821	0.128	0.222	0.941

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	127	93	73	87	168	148	76	99	0
N.S.	1	1.10	0.81	0.63	0.76	1.46	1.29	0.66	0.86	0.00
time (sec)	N/A	0.210	0.268	0.361	0.032	0.127	10.425	0.129	0.191	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	97	82	62	67	145	119	63	80	0
N.S.	1	1.07	0.90	0.68	0.74	1.59	1.31	0.69	0.88	0.00
time (sec)	N/A	0.181	0.225	0.290	0.025	0.080	3.337	0.125	0.198	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	68	60	48	43	124	70	49	61	37
N.S.	1	1.05	0.92	0.74	0.66	1.91	1.08	0.75	0.94	0.57
time (sec)	N/A	0.159	0.006	0.286	0.029	0.077	1.445	0.125	0.195	0.207

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	66	65	48	43	112	88	73	62	40
N.S.	1	1.03	1.02	0.75	0.67	1.75	1.38	1.14	0.97	0.62
time (sec)	N/A	0.164	0.114	0.293	0.033	0.075	1.318	0.141	0.226	0.708

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	63	57	44	66	112	78	114	58	0
N.S.	1	0.98	0.89	0.69	1.03	1.75	1.22	1.78	0.91	0.00
time (sec)	N/A	0.164	0.072	0.283	0.033	0.077	1.171	0.137	0.194	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	35	68	86	63	17
N.S.	1	1.00	1.00	0.86	0.81	1.67	3.24	4.10	3.00	0.81
time (sec)	N/A	0.128	0.052	0.319	0.031	0.068	0.505	0.133	0.199	0.545

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	49	94	166	82	71
N.S.	1	1.00	0.70	0.64	0.82	1.11	2.14	3.77	1.86	1.61
time (sec)	N/A	0.148	0.065	0.273	0.032	0.068	0.579	0.131	0.200	0.724

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	56	60	420	192	101	91
N.S.	1	1.09	0.62	0.57	0.82	0.88	6.18	2.82	1.49	1.34
time (sec)	N/A	0.170	0.073	0.286	0.035	0.082	0.828	0.135	0.247	0.895

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	76	71	648	220	120	111
N.S.	1	1.13	0.58	0.54	0.83	0.77	7.04	2.39	1.30	1.21
time (sec)	N/A	0.189	0.081	0.301	0.032	0.082	1.081	0.133	0.223	1.139

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	73	79	158	57	78	75
N.S.	1	1.05	0.62	0.59	0.91	0.99	1.98	0.71	0.98	0.94
time (sec)	N/A	0.185	0.031	0.316	0.032	0.070	0.588	0.126	0.207	0.375

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	53	68	133	43	67	64
N.S.	1	1.07	0.66	0.61	0.90	1.15	2.25	0.73	1.14	1.08
time (sec)	N/A	0.172	0.026	0.303	0.026	0.071	0.471	0.119	0.207	0.349

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	33	56	109	29	55	53
N.S.	1	1.11	0.74	0.66	0.87	1.47	2.87	0.76	1.45	1.39
time (sec)	N/A	0.159	0.026	0.259	0.027	0.071	0.379	0.121	0.207	0.324

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	43	85	14	42	14
N.S.	1	1.00	1.00	0.83	0.78	2.39	4.72	0.78	2.33	0.78
time (sec)	N/A	0.126	0.003	0.313	0.030	0.075	0.213	0.119	0.193	0.204

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	79	62	53	54	129	105	62	102	59
N.S.	1	1.10	0.86	0.74	0.75	1.79	1.46	0.86	1.42	0.82
time (sec)	N/A	0.174	0.039	0.312	0.027	0.075	2.270	0.129	0.195	0.399

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	83	68	68	76	145	112	73	112	66
N.S.	1	1.02	0.84	0.84	0.94	1.79	1.38	0.90	1.38	0.81
time (sec)	N/A	0.175	0.071	1.082	0.035	0.078	2.163	0.125	0.257	0.580

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	89	70	65	104	148	117	88	114	71
N.S.	1	1.02	0.80	0.75	1.20	1.70	1.34	1.01	1.31	0.82
time (sec)	N/A	0.176	0.084	0.296	0.034	0.083	2.360	0.125	0.217	0.700

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	70	68	127	161	99	75	120	72
N.S.	1	1.02	0.75	0.73	1.37	1.73	1.06	0.81	1.29	0.77
time (sec)	N/A	0.176	0.107	0.298	0.052	0.101	2.678	0.123	0.215	0.877

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	121	84	78	147	182	150	109	139	89
N.S.	1	1.03	0.72	0.67	1.26	1.56	1.28	0.93	1.19	0.76
time (sec)	N/A	0.193	0.109	0.438	0.038	0.084	6.991	0.125	0.208	1.182

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	151	95	88	167	204	175	108	158	106
N.S.	1	1.07	0.67	0.62	1.18	1.45	1.24	0.77	1.12	0.75
time (sec)	N/A	0.208	0.132	0.480	0.036	0.089	28.656	0.127	0.286	1.484

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	151	104	84	105	190	175	91	118	0
N.S.	1	1.11	0.76	0.62	0.77	1.40	1.29	0.67	0.87	0.00
time (sec)	N/A	0.224	0.359	0.416	0.033	0.093	29.562	0.135	0.210	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	121	93	73	85	167	150	77	99	0
N.S.	1	1.08	0.83	0.65	0.76	1.49	1.34	0.69	0.88	0.00
time (sec)	N/A	0.196	0.298	0.406	0.034	0.083	6.741	0.132	0.230	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	71	59	58	146	97	63	80	37
N.S.	1	1.07	0.85	0.70	0.69	1.74	1.15	0.75	0.95	0.44
time (sec)	N/A	0.167	0.012	0.358	0.029	0.080	2.563	0.131	0.237	0.216

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	88	73	61	59	140	117	87	85	40
N.S.	1	0.98	0.81	0.68	0.66	1.56	1.30	0.97	0.94	0.44
time (sec)	N/A	0.171	0.088	0.379	0.029	0.073	2.316	0.139	0.198	0.894

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	77	59	84	141	112	132	87	0
N.S.	1	1.00	0.86	0.66	0.93	1.57	1.24	1.47	0.97	0.00
time (sec)	N/A	0.178	0.139	0.408	0.040	0.075	2.102	0.139	0.229	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	84	68	57	104	140	105	168	89	0
N.S.	1	0.95	0.77	0.65	1.18	1.59	1.19	1.91	1.01	0.00
time (sec)	N/A	0.176	0.096	0.411	0.036	0.077	2.333	0.138	0.203	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	46	95	113	82	71
N.S.	1	1.00	1.00	0.86	0.81	2.19	4.52	5.38	3.90	3.38
time (sec)	N/A	0.128	0.068	0.394	0.034	0.069	0.614	0.135	0.232	0.898

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	60	121	220	101	91
N.S.	1	1.00	0.70	0.64	0.82	1.36	2.75	5.00	2.30	2.07
time (sec)	N/A	0.144	0.081	0.412	0.035	0.077	0.755	0.138	0.197	1.173

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	56	71	481	246	120	111
N.S.	1	1.09	0.62	0.57	0.82	1.04	7.07	3.62	1.76	1.63
time (sec)	N/A	0.166	0.091	0.486	0.037	0.083	1.245	0.124	0.211	1.572

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	76	82	721	274	139	131
N.S.	1	1.13	0.58	0.54	0.83	0.89	7.84	2.98	1.51	1.42
time (sec)	N/A	0.186	0.102	0.628	0.039	0.095	1.415	0.139	0.226	1.915

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	134	64	61	96	93	1012	300	158	151
N.S.	1	1.16	0.55	0.53	0.83	0.80	8.72	2.59	1.36	1.30
time (sec)	N/A	0.206	0.114	0.964	0.036	0.122	1.701	0.139	0.243	2.491

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	164	75	72	116	104	1346	328	177	171
N.S.	1	1.17	0.54	0.51	0.83	0.74	9.61	2.34	1.26	1.22
time (sec)	N/A	0.225	0.120	1.812	0.037	0.159	2.116	0.138	0.218	3.044

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	166	94	91	153	145	301	113	144	141
N.S.	1	1.03	0.58	0.57	0.95	0.90	1.87	0.70	0.89	0.88
time (sec)	N/A	0.256	0.056	0.942	0.038	0.084	2.638	0.127	0.221	0.562

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	145	83	80	133	134	277	99	133	130
N.S.	1	1.04	0.59	0.57	0.95	0.96	1.98	0.71	0.95	0.93
time (sec)	N/A	0.235	0.047	0.551	0.035	0.084	2.241	0.121	0.215	0.444

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	72	69	113	123	253	85	122	119
N.S.	1	1.03	0.59	0.57	0.93	1.01	2.07	0.70	1.00	0.98
time (sec)	N/A	0.217	0.037	0.452	0.034	0.079	1.931	0.121	0.206	0.459

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	61	58	93	112	230	71	111	108
N.S.	1	1.04	0.60	0.57	0.92	1.11	2.28	0.70	1.10	1.07
time (sec)	N/A	0.209	0.034	0.379	0.036	0.081	1.537	0.121	0.203	0.444

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	73	101	204	57	100	97
N.S.	1	1.05	0.62	0.59	0.91	1.26	2.55	0.71	1.25	1.21
time (sec)	N/A	0.190	0.033	0.359	0.035	0.073	1.239	0.121	0.207	0.431

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	53	90	180	43	89	86
N.S.	1	1.07	0.66	0.61	0.90	1.53	3.05	0.73	1.51	1.46
time (sec)	N/A	0.178	0.028	0.338	0.038	0.075	1.053	0.122	0.214	0.408

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	33	78	156	29	77	29
N.S.	1	1.11	0.74	0.66	0.87	2.05	4.11	0.76	2.03	0.76
time (sec)	N/A	0.159	0.028	0.341	0.032	0.070	0.869	0.118	0.209	0.369

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	65	133	14	64	14
N.S.	1	1.00	1.00	0.83	0.78	3.61	7.39	0.78	3.56	0.78
time (sec)	N/A	0.135	0.004	0.418	0.024	0.074	0.638	0.121	0.216	0.261

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	115	84	75	82	173	160	90	140	87
N.S.	1	1.06	0.78	0.69	0.76	1.60	1.48	0.83	1.30	0.81
time (sec)	N/A	0.198	0.047	0.359	0.036	0.084	10.347	0.122	0.230	0.363

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	119	90	90	106	191	167	101	154	95
N.S.	1	1.03	0.78	0.78	0.91	1.65	1.44	0.87	1.33	0.82
time (sec)	N/A	0.204	0.090	0.411	0.050	0.085	10.219	0.134	0.265	0.646

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	125	92	92	136	195	175	124	158	132
N.S.	1	0.98	0.72	0.72	1.07	1.54	1.38	0.98	1.24	1.04
time (sec)	N/A	0.200	0.100	0.418	0.042	0.094	9.534	0.121	0.259	0.777

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	131	92	92	156	193	175	106	154	149
N.S.	1	0.98	0.69	0.69	1.17	1.45	1.32	0.80	1.16	1.12
time (sec)	N/A	0.203	0.100	0.434	0.040	0.085	9.284	0.123	0.281	1.120

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	137	92	93	178	192	173	122	152	105
N.S.	1	1.01	0.68	0.69	1.32	1.42	1.28	0.90	1.13	0.78
time (sec)	N/A	0.203	0.101	0.431	0.037	0.083	10.067	0.131	0.259	1.500

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	143	92	90	201	205	153	103	158	106
N.S.	1	1.01	0.65	0.64	1.43	1.45	1.09	0.73	1.12	0.75
time (sec)	N/A	0.200	0.137	0.482	0.038	0.090	10.478	0.124	0.294	1.998

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	169	106	100	221	226	204	143	177	123
N.S.	1	1.02	0.64	0.61	1.34	1.37	1.24	0.87	1.07	0.75
time (sec)	N/A	0.214	0.138	0.594	0.044	0.099	48.643	0.125	0.373	2.659

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	199	117	111	241	248	0	136	196	140
N.S.	1	1.05	0.62	0.59	1.28	1.31	0.00	0.72	1.04	0.74
time (sec)	N/A	0.228	0.166	0.948	0.040	0.127	0.000	0.127	0.355	3.262

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	229	137	117	161	255	0	133	175	0
N.S.	1	1.13	0.68	0.58	0.80	1.26	0.00	0.66	0.87	0.00
time (sec)	N/A	0.284	0.704	0.488	0.039	0.167	0.000	0.130	0.211	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	199	126	106	141	234	0	119	156	0
N.S.	1	1.12	0.71	0.60	0.79	1.31	0.00	0.67	0.88	0.00
time (sec)	N/A	0.256	0.581	0.451	0.035	0.132	0.000	0.135	0.243	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	169	115	95	121	211	204	105	137	0
N.S.	1	1.10	0.75	0.62	0.79	1.37	1.32	0.68	0.89	0.00
time (sec)	N/A	0.232	0.495	0.407	0.036	0.111	48.288	0.136	0.220	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	134	93	81	88	190	151	91	118	37
N.S.	1	1.10	0.76	0.66	0.72	1.56	1.24	0.75	0.97	0.30
time (sec)	N/A	0.192	0.110	0.335	0.026	0.095	10.458	0.131	0.211	0.279

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	132	95	83	91	184	173	115	123	40
N.S.	1	0.96	0.69	0.60	0.66	1.33	1.25	0.83	0.89	0.29
time (sec)	N/A	0.201	0.114	0.398	0.035	0.085	9.976	0.139	0.249	1.775

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	132	95	83	120	189	175	160	129	0
N.S.	1	0.94	0.68	0.59	0.86	1.35	1.25	1.14	0.92	0.00
time (sec)	N/A	0.204	0.128	0.426	0.038	0.089	9.874	0.139	0.250	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	136	95	83	140	191	175	200	133	0
N.S.	1	0.97	0.68	0.59	1.00	1.36	1.25	1.43	0.95	0.00
time (sec)	N/A	0.204	0.144	0.412	0.036	0.084	10.413	0.140	0.245	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	99	81	160	187	167	240	129	0
N.S.	1	1.00	0.72	0.59	1.16	1.36	1.21	1.74	0.93	0.00
time (sec)	N/A	0.214	0.177	0.452	0.038	0.085	10.883	0.138	0.273	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	126	90	79	180	184	160	276	127	0
N.S.	1	0.93	0.66	0.58	1.32	1.35	1.18	2.03	0.93	0.00
time (sec)	N/A	0.216	0.148	0.474	0.037	0.082	11.787	0.141	0.235	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	17	68	150	167	120	111
N.S.	1	1.00	1.00	0.86	0.81	3.24	7.14	7.95	5.71	5.29
time (sec)	N/A	0.128	0.098	0.457	0.037	0.080	1.268	0.141	0.233	2.073

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	28	36	82	175	328	139	131
N.S.	1	1.00	0.70	0.64	0.82	1.86	3.98	7.45	3.16	2.98
time (sec)	N/A	0.146	0.118	0.653	0.049	0.093	1.502	0.140	0.222	2.725

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	42	39	56	93	604	354	158	151
N.S.	1	1.09	0.62	0.57	0.82	1.37	8.88	5.21	2.32	2.22
time (sec)	N/A	0.166	0.129	1.169	0.037	0.123	1.940	0.146	0.347	3.385

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	104	53	50	76	104	867	382	177	171
N.S.	1	1.13	0.58	0.54	0.83	1.13	9.42	4.15	1.92	1.86
time (sec)	N/A	0.191	0.142	2.365	0.040	0.151	2.508	0.142	0.240	4.090

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	134	64	61	96	115	1182	408	196	191
N.S.	1	1.16	0.55	0.53	0.83	0.99	10.19	3.52	1.69	1.65
time (sec)	N/A	0.208	0.157	5.240	0.041	0.206	3.216	0.140	0.263	4.920

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	164	75	72	116	126	1540	436	215	211
N.S.	1	1.17	0.54	0.51	0.83	0.90	11.00	3.11	1.54	1.51
time (sec)	N/A	0.233	0.168	22.404	0.040	0.273	3.764	0.146	0.298	5.795

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	194	86	83	136	137	1950	462	234	231
N.S.	1	1.18	0.52	0.51	0.83	0.84	11.89	2.82	1.43	1.41
time (sec)	N/A	0.255	0.186	59.480	0.043	0.391	4.665	0.142	0.260	6.832

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	24	40	28	61	34	27	25
N.S.	1	1.09	0.59	0.52	0.87	0.61	1.33	0.74	0.59	0.54
time (sec)	N/A	0.161	0.015	0.329	0.105	0.061	0.290	0.125	0.190	0.035

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	73	49	32	45	42	75	43	55	30
N.S.	1	1.16	0.78	0.51	0.71	0.67	1.19	0.68	0.87	0.48
time (sec)	N/A	0.169	0.032	0.396	0.106	0.062	4.916	0.127	0.227	0.033

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	19	26	23	44	23	22	20
N.S.	1	1.13	0.71	0.61	0.84	0.74	1.42	0.74	0.71	0.65
time (sec)	N/A	0.151	0.012	0.318	0.106	0.063	0.153	0.118	0.204	0.026

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	44	27	31	37	54	36	42	23
N.S.	1	1.11	0.98	0.60	0.69	0.82	1.20	0.80	0.93	0.51
time (sec)	N/A	0.149	0.022	0.390	0.103	0.069	1.893	0.128	0.219	0.039

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	17	16
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	1.13	1.07
time (sec)	N/A	0.129	0.001	0.315	0.031	0.060	0.080	0.118	0.226	0.024

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	20	19	29	22	29	29	16
N.S.	1	1.00	1.37	0.74	0.70	1.07	0.81	1.07	1.07	0.59
time (sec)	N/A	0.132	0.002	0.366	0.103	0.062	0.107	0.119	0.210	0.038

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	25	19	44	39	38	45	22
N.S.	1	1.20	1.00	0.83	0.63	1.47	1.30	1.27	1.50	0.73
time (sec)	N/A	0.151	0.017	0.366	0.107	0.067	0.763	0.127	0.204	0.030

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	22	21	35	19	40	36	19
N.S.	1	1.00	1.40	0.88	0.84	1.40	0.76	1.60	1.44	0.76
time (sec)	N/A	0.132	0.024	0.365	0.103	0.062	0.101	0.110	0.194	0.032

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	39	30	35	57	24	43	58	25
N.S.	1	1.05	1.00	0.77	0.90	1.46	0.62	1.10	1.49	0.64
time (sec)	N/A	0.146	0.027	0.434	0.107	0.064	1.007	0.115	0.219	0.037

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	20	34	42	34	27
N.S.	1	1.00	1.00	0.83	0.78	1.11	1.89	2.33	1.89	1.50
time (sec)	N/A	0.126	0.025	0.342	0.104	0.068	0.553	0.119	0.230	0.035

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	46	41	49	64	63	55	71	45
N.S.	1	1.07	0.81	0.72	0.86	1.12	1.11	0.96	1.25	0.79
time (sec)	N/A	0.151	0.038	0.399	0.112	0.068	2.192	0.129	0.209	0.038

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	32	24	40	28	61	52	27	28
N.S.	1	1.09	0.70	0.52	0.87	0.61	1.33	1.13	0.59	0.61
time (sec)	N/A	0.161	0.015	0.339	0.101	0.065	0.286	0.120	0.191	0.053

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	73	52	39	45	45	165	33	44	32
N.S.	1	1.16	0.83	0.62	0.71	0.71	2.62	0.52	0.70	0.51
time (sec)	N/A	0.158	0.098	0.443	0.105	0.067	4.909	0.117	0.232	0.033

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	19	26	23	44	32	22	23
N.S.	1	1.13	0.71	0.61	0.84	0.74	1.42	1.03	0.71	0.74
time (sec)	N/A	0.145	0.013	0.339	0.101	0.081	0.164	0.125	0.213	0.027

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	47	32	31	40	122	26	31	27
N.S.	1	1.11	1.04	0.71	0.69	0.89	2.71	0.58	0.69	0.60
time (sec)	N/A	0.145	0.063	0.400	0.104	0.064	1.983	0.129	0.201	0.036

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	27	11	17	18
N.S.	1	1.00	1.00	0.80	0.73	1.20	1.80	0.73	1.13	1.20
time (sec)	N/A	0.119	0.001	0.326	0.029	0.066	0.107	0.120	0.214	0.025

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	41	20	19	32	22	19	18	18
N.S.	1	1.00	1.52	0.74	0.70	1.19	0.81	0.70	0.67	0.67
time (sec)	N/A	0.130	0.003	0.372	0.103	0.067	0.110	0.123	0.216	0.027

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	25	35	28	75	40	20	32
N.S.	1	1.20	1.00	0.83	1.17	0.93	2.50	1.33	0.67	1.07
time (sec)	N/A	0.147	0.015	0.381	0.101	0.066	0.821	0.116	0.222	0.204

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	39	28	21	36	20	39	22	21
N.S.	1	1.00	1.56	1.12	0.84	1.44	0.80	1.56	0.88	0.84
time (sec)	N/A	0.132	0.031	0.400	0.106	0.064	0.102	0.125	0.197	0.032

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	39	34	51	38	97	45	29	35
N.S.	1	1.05	1.00	0.87	1.31	0.97	2.49	1.15	0.74	0.90
time (sec)	N/A	0.142	0.029	0.453	0.101	0.064	1.047	0.125	0.201	0.039

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	21	76	73	20	31
N.S.	1	1.00	1.00	0.83	0.78	1.17	4.22	4.06	1.11	1.72
time (sec)	N/A	0.124	0.022	0.326	0.107	0.063	0.559	0.124	0.226	0.038

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	46	41	65	45	139	57	42	49
N.S.	1	1.07	0.81	0.72	1.14	0.79	2.44	1.00	0.74	0.86
time (sec)	N/A	0.151	0.027	0.431	0.102	0.070	2.171	0.126	0.200	0.032

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	24	40	28	61	34	27	28
N.S.	1	1.09	0.59	0.52	0.87	0.61	1.33	0.74	0.59	0.61
time (sec)	N/A	0.153	0.014	0.343	0.110	0.065	0.294	0.120	0.204	0.328

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	82	49	42	57	42	165	44	55	0
N.S.	1	1.14	0.68	0.58	0.79	0.58	2.29	0.61	0.76	0.00
time (sec)	N/A	0.167	0.029	0.431	0.105	0.064	5.012	0.124	0.196	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	19	26	23	44	23	22	23
N.S.	1	1.13	0.71	0.61	0.84	0.74	1.42	0.74	0.71	0.74
time (sec)	N/A	0.147	0.012	0.365	0.117	0.068	0.148	0.119	0.215	0.289

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	44	37	43	37	122	37	42	0
N.S.	1	1.09	0.81	0.69	0.80	0.69	2.26	0.69	0.78	0.00
time (sec)	N/A	0.153	0.023	0.398	0.101	0.066	1.966	0.122	0.193	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	27	11	17	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	1.80	0.73	1.13	0.73
time (sec)	N/A	0.120	0.001	0.345	0.033	0.061	0.093	0.118	0.243	0.189

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	30	31	29	31	30	29	29
N.S.	1	1.00	1.03	0.83	0.86	0.81	0.86	0.83	0.81	0.81
time (sec)	N/A	0.135	0.002	0.393	0.104	0.068	0.095	0.123	0.229	0.199

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	25	19	28	80	24	26	24
N.S.	1	1.20	1.00	0.83	0.63	0.93	2.67	0.80	0.87	0.80
time (sec)	N/A	0.141	0.015	0.599	0.111	0.068	0.860	0.121	0.230	0.305

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	32	33	35	27	44	36	39
N.S.	1	1.00	1.03	0.94	0.97	1.03	0.79	1.29	1.06	1.15
time (sec)	N/A	0.141	0.024	0.407	0.101	0.071	0.109	0.126	0.239	0.463

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	39	30	35	38	97	29	36	29
N.S.	1	1.05	1.00	0.77	0.90	0.97	2.49	0.74	0.92	0.74
time (sec)	N/A	0.139	0.024	0.471	0.103	0.070	1.082	0.126	0.177	0.365

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	20	76	42	34	31
N.S.	1	1.00	1.00	0.83	0.78	1.11	4.22	2.33	1.89	1.72
time (sec)	N/A	0.127	0.023	0.363	0.102	0.064	0.555	0.129	0.247	0.306

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	46	42	49	45	139	41	49	43
N.S.	1	1.07	0.81	0.74	0.86	0.79	2.44	0.72	0.86	0.75
time (sec)	N/A	0.150	0.034	0.480	0.106	0.068	2.182	0.129	0.202	0.392

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	24	40	28	68	34	27	27
N.S.	1	1.09	0.59	0.52	0.87	0.61	1.48	0.74	0.59	0.59
time (sec)	N/A	0.155	0.016	0.339	0.108	0.062	0.348	0.121	0.210	0.336

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	82	48	44	45	72	83	0	45	0
N.S.	1	1.14	0.67	0.61	0.62	1.00	1.15	0.00	0.62	0.00
time (sec)	N/A	0.168	0.073	0.450	0.109	0.066	5.018	0.000	0.204	0.000

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	19	26	23	49	23	22	22
N.S.	1	1.13	0.71	0.61	0.84	0.74	1.58	0.74	0.71	0.71
time (sec)	N/A	0.146	0.013	0.326	0.110	0.065	0.168	0.121	0.248	0.310

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	43	39	31	67	61	0	32	0
N.S.	1	1.09	0.80	0.72	0.57	1.24	1.13	0.00	0.59	0.00
time (sec)	N/A	0.152	0.067	0.420	0.107	0.066	1.955	0.000	0.202	0.000

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	18	31	11	17	11
N.S.	1	1.00	1.00	0.80	0.73	1.20	2.07	0.73	1.13	0.73
time (sec)	N/A	0.120	0.001	0.346	0.025	0.061	0.105	0.126	0.199	0.074

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	19	59	34	0	19	28
N.S.	1	1.00	1.00	0.81	0.53	1.64	0.94	0.00	0.53	0.78
time (sec)	N/A	0.140	0.003	0.364	0.108	0.067	0.190	0.000	0.245	0.034

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	30	25	35	52	44	24	51	24
N.S.	1	1.20	1.00	0.83	1.17	1.73	1.47	0.80	1.70	0.80
time (sec)	N/A	0.143	0.014	0.579	0.106	0.065	0.805	0.125	0.220	0.297

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	34	21	64	32	0	25	41
N.S.	1	1.00	1.03	1.00	0.62	1.88	0.94	0.00	0.74	1.21
time (sec)	N/A	0.140	0.049	0.412	0.108	0.090	0.191	0.000	0.245	0.449

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	41	39	35	51	65	27	29	62	29
N.S.	1	1.05	1.00	0.90	1.31	1.67	0.69	0.74	1.59	0.74
time (sec)	N/A	0.143	0.021	0.442	0.110	0.065	1.198	0.121	0.210	0.355

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	37	0	35	31
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.06	0.00	1.94	1.72
time (sec)	N/A	0.122	0.034	0.369	0.103	0.066	0.538	0.000	0.238	0.329

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	61	46	42	65	72	68	43	75	43
N.S.	1	1.07	0.81	0.74	1.14	1.26	1.19	0.75	1.32	0.75
time (sec)	N/A	0.154	0.027	0.431	0.111	0.066	2.040	0.127	0.219	0.439

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	39	36	53	35	68	46	34	36
N.S.	1	1.09	0.70	0.64	0.95	0.62	1.21	0.82	0.61	0.64
time (sec)	N/A	0.172	0.021	0.337	0.026	0.067	0.205	0.120	0.214	0.386

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	79	71	51	51	124	95	54	61	0
N.S.	1	1.08	0.97	0.70	0.70	1.70	1.30	0.74	0.84	0.00
time (sec)	N/A	0.167	0.139	0.365	0.026	0.078	2.531	0.132	0.210	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	27	24	33	23	44	30	22	24
N.S.	1	1.11	0.75	0.67	0.92	0.64	1.22	0.83	0.61	0.67
time (sec)	N/A	0.156	0.019	0.320	0.026	0.066	0.181	0.131	0.234	0.340

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	57	39	31	93	42	40	41	56
N.S.	1	1.00	1.16	0.80	0.63	1.90	0.86	0.82	0.84	1.14
time (sec)	N/A	0.153	0.102	0.358	0.030	0.076	1.232	0.132	0.231	0.364

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	20	13	12	13
N.S.	1	1.00	1.00	0.93	0.87	0.87	1.33	0.87	0.80	0.87
time (sec)	N/A	0.126	0.001	0.335	0.030	0.066	0.089	0.118	0.228	0.211

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	13	59	17	37	25	20
N.S.	1	1.00	1.00	0.84	0.52	2.36	0.68	1.48	1.00	0.80
time (sec)	N/A	0.131	0.001	0.332	0.032	0.074	0.567	0.121	0.198	0.130

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	17	63	19	22	53	19
N.S.	1	1.00	1.00	0.80	0.68	2.52	0.76	0.88	2.12	0.76
time (sec)	N/A	0.143	0.018	0.333	0.026	0.069	0.546	0.126	0.241	0.431

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	19	30	23	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	1.00	1.58	1.21	0.89
time (sec)	N/A	0.130	0.027	0.329	0.025	0.062	0.376	0.125	0.242	0.174

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	49	50	43	36	108	42	48	79	38
N.S.	1	0.98	1.00	0.86	0.72	2.16	0.84	0.96	1.58	0.76
time (sec)	N/A	0.154	0.041	0.388	0.030	0.078	1.286	0.121	0.238	0.461

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	31	26	36	27	46	55	42	25
N.S.	1	1.00	0.70	0.59	0.82	0.61	1.05	1.25	0.95	0.57
time (sec)	N/A	0.148	0.037	0.339	0.035	0.066	0.523	0.131	0.255	0.329

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	62	60	56	138	97	75	101	57
N.S.	1	1.07	0.84	0.81	0.76	1.86	1.31	1.01	1.36	0.77
time (sec)	N/A	0.172	0.069	0.361	0.042	0.079	2.669	0.123	0.229	0.483

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	59	38	35	53	46	68	57	42	41
N.S.	1	1.07	0.69	0.64	0.96	0.84	1.24	1.04	0.76	0.75
time (sec)	N/A	0.173	0.025	0.384	0.026	0.066	0.231	0.126	0.185	0.489

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	74	66	54	49	159	71	51	114	0
N.S.	1	1.09	0.97	0.79	0.72	2.34	1.04	0.75	1.68	0.00
time (sec)	N/A	0.174	0.152	0.387	0.029	0.078	1.841	0.129	0.200	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	38	24	23	32	34	41	32	30	22
N.S.	1	1.19	0.75	0.72	1.00	1.06	1.28	1.00	0.94	0.69
time (sec)	N/A	0.162	0.021	0.365	0.027	0.068	0.185	0.126	0.262	0.368

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	54	37	29	130	37	39	88	36
N.S.	1	1.00	1.26	0.86	0.67	3.02	0.86	0.91	2.05	0.84
time (sec)	N/A	0.151	0.075	0.374	0.028	0.074	0.952	0.134	0.223	0.101

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	24	24	14	22	14
N.S.	1	1.00	1.00	0.94	0.88	1.50	1.50	0.88	1.38	0.88
time (sec)	N/A	0.129	0.002	0.330	0.026	0.075	0.102	0.121	0.219	0.042

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	14	23	17	14	39	14
N.S.	1	1.00	1.00	0.94	0.88	1.44	1.06	0.88	2.44	0.88
time (sec)	N/A	0.125	0.001	0.324	0.033	0.066	0.365	0.131	0.192	0.031

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	34	31	129	184	39	136	33
N.S.	1	1.12	1.00	0.83	0.76	3.15	4.49	0.95	3.32	0.80
time (sec)	N/A	0.157	0.031	0.358	0.027	0.079	0.983	0.126	0.249	0.445

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	28	26	34	35	46	50	56	35
N.S.	1	1.00	0.74	0.68	0.89	0.92	1.21	1.32	1.47	0.92
time (sec)	N/A	0.146	0.040	0.371	0.029	0.070	0.483	0.132	0.219	0.313

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	60	54	51	174	73	72	172	53
N.S.	1	1.07	0.87	0.78	0.74	2.52	1.06	1.04	2.49	0.77
time (sec)	N/A	0.173	0.061	0.402	0.029	0.079	1.879	0.127	0.218	0.533

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	68	42	37	54	50	233	106	81	38
N.S.	1	1.10	0.68	0.60	0.87	0.81	3.76	1.71	1.31	0.61
time (sec)	N/A	0.161	0.054	0.415	0.031	0.071	0.744	0.144	0.223	0.393

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	103	78	70	89	227	367	65	189	0
N.S.	1	1.13	0.86	0.77	0.98	2.49	4.03	0.71	2.08	0.00
time (sec)	N/A	0.187	0.216	0.490	0.034	0.083	2.856	0.122	0.207	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	39	36	52	58	138	49	54	38
N.S.	1	1.09	0.72	0.67	0.96	1.07	2.56	0.91	1.00	0.70
time (sec)	N/A	0.174	0.026	0.366	0.033	0.069	0.346	0.122	0.209	0.445

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	66	57	65	199	303	51	138	0
N.S.	1	1.08	1.03	0.89	1.02	3.11	4.73	0.80	2.16	0.00
time (sec)	N/A	0.168	0.148	0.367	0.036	0.077	1.626	0.135	0.188	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	28	25	33	47	92	24	43	24
N.S.	1	1.11	0.78	0.69	0.92	1.31	2.56	0.67	1.19	0.67
time (sec)	N/A	0.160	0.022	0.376	0.025	0.067	0.349	0.117	0.249	0.345

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	34	37	44	17	68	17
N.S.	1	1.00	1.00	0.86	1.62	1.76	2.10	0.81	3.24	0.81
time (sec)	N/A	0.127	0.043	0.421	0.035	0.104	0.444	0.121	0.227	0.330

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	35	46	14	33	14
N.S.	1	1.00	1.00	0.83	0.78	1.94	2.56	0.78	1.83	0.78
time (sec)	N/A	0.127	0.002	0.335	0.029	0.069	0.321	0.119	0.200	0.207

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	31	47	95	27	84	28
N.S.	1	1.00	0.74	0.67	0.79	1.21	2.44	0.69	2.15	0.72
time (sec)	N/A	0.136	0.003	0.335	0.032	0.067	0.521	0.127	0.341	0.190

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	69	54	54	45	200	740	50	238	47
N.S.	1	1.17	0.92	0.92	0.76	3.39	12.54	0.85	4.03	0.80
time (sec)	N/A	0.167	0.054	0.421	0.028	0.081	1.585	0.120	0.233	0.444

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	65	42	37	50	59	165	64	100	42
N.S.	1	1.02	0.66	0.58	0.78	0.92	2.58	1.00	1.56	0.66
time (sec)	N/A	0.159	0.057	0.404	0.025	0.071	0.763	0.130	0.208	0.349

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	71	72	66	244	864	73	272	73
N.S.	1	1.10	0.81	0.82	0.75	2.77	9.82	0.83	3.09	0.83
time (sec)	N/A	0.181	0.076	0.754	0.041	0.081	2.913	0.125	0.196	0.482

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	93	53	45	72	72	354	121	124	78
N.S.	1	1.11	0.63	0.54	0.86	0.86	4.21	1.44	1.48	0.93
time (sec)	N/A	0.176	0.063	0.431	0.027	0.074	1.014	0.136	0.250	0.442

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	161	100	94	285	359	3181	91	339	0
N.S.	1	1.23	0.76	0.72	2.18	2.74	24.28	0.69	2.59	0.00
time (sec)	N/A	0.226	0.348	0.556	0.058	0.104	8.550	0.135	0.254	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	99	61	58	92	102	454	77	98	80
N.S.	1	1.05	0.65	0.62	0.98	1.09	4.83	0.82	1.04	0.85
time (sec)	N/A	0.195	0.032	0.427	0.050	0.076	0.777	0.115	0.214	0.535

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	121	88	79	255	331	2980	78	314	0
N.S.	1	1.14	0.83	0.75	2.41	3.12	28.11	0.74	2.96	0.00
time (sec)	N/A	0.204	0.287	0.394	0.059	0.092	4.782	0.134	0.218	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	80	50	47	73	91	364	55	87	63
N.S.	1	1.07	0.67	0.63	0.97	1.21	4.85	0.73	1.16	0.84
time (sec)	N/A	0.181	0.031	0.352	0.038	0.075	0.771	0.122	0.243	0.466

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	103	59	95	17	116	68
N.S.	1	1.00	1.00	0.86	4.90	2.81	4.52	0.81	5.52	3.24
time (sec)	N/A	0.131	0.075	0.347	0.033	0.073	0.707	0.131	0.242	0.484

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	53	80	272	41	76	41
N.S.	1	1.07	0.66	0.61	0.90	1.36	4.61	0.69	1.29	0.69
time (sec)	N/A	0.168	0.028	0.341	0.031	0.076	0.760	0.121	0.225	0.451

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	31	28	85	71	199	29	136	68
N.S.	1	1.02	0.72	0.65	1.98	1.65	4.63	0.67	3.16	1.58
time (sec)	N/A	0.146	0.075	0.332	0.026	0.076	0.800	0.137	0.234	0.443

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	33	69	180	24	65	24
N.S.	1	1.11	0.74	0.66	0.87	1.82	4.74	0.63	1.71	0.63
time (sec)	N/A	0.160	0.024	0.320	0.027	0.074	0.763	0.126	0.215	0.418

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	74	42	39	70	82	517	43	155	70
N.S.	1	1.16	0.66	0.61	1.09	1.28	8.08	0.67	2.42	1.09
time (sec)	N/A	0.163	0.067	0.352	0.032	0.073	1.001	0.132	0.245	0.393

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	57	90	14	55	14
N.S.	1	1.00	1.00	0.83	0.78	3.17	5.00	0.78	3.06	0.78
time (sec)	N/A	0.128	0.003	0.341	0.031	0.072	0.736	0.121	0.193	0.225

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	93	51	48	61	91	1265	55	170	61
N.S.	1	1.21	0.66	0.62	0.79	1.18	16.43	0.71	2.21	0.79
time (sec)	N/A	0.167	0.005	0.341	0.024	0.079	1.178	0.137	0.238	0.232

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	115	76	76	73	332	5250	81	438	75
N.S.	1	1.21	0.80	0.80	0.77	3.49	55.26	0.85	4.61	0.79
time (sec)	N/A	0.193	0.063	0.353	0.033	0.089	4.126	0.123	0.245	0.465

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	119	64	59	82	103	400	90	186	76
N.S.	1	1.12	0.60	0.56	0.77	0.97	3.77	0.85	1.75	0.72
time (sec)	N/A	0.183	0.084	0.415	0.035	0.081	1.595	0.133	0.199	0.479

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	143	93	94	96	376	5540	104	472	113
N.S.	1	1.13	0.74	0.75	0.76	2.98	43.97	0.83	3.75	0.90
time (sec)	N/A	0.212	0.080	0.471	0.035	0.100	7.612	0.124	0.455	0.622

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	149	75	70	108	116	668	147	210	97
N.S.	1	1.16	0.59	0.55	0.84	0.91	5.22	1.15	1.64	0.76
time (sec)	N/A	0.202	0.093	0.411	0.036	0.098	1.928	0.139	0.212	0.458

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	23	40	23	44	34	22	21
N.S.	1	1.09	0.59	0.50	0.87	0.50	0.96	0.74	0.48	0.46
time (sec)	N/A	0.157	0.014	0.343	0.106	0.063	0.215	0.114	0.193	0.030

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	44	27	33	37	39	36	42	25
N.S.	1	1.11	0.98	0.60	0.73	0.82	0.87	0.80	0.93	0.56
time (sec)	N/A	0.150	0.034	0.383	0.103	0.063	0.158	0.122	0.256	0.050

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	18	26	18	27	23	17	15
N.S.	1	1.13	0.71	0.58	0.84	0.58	0.87	0.74	0.55	0.48
time (sec)	N/A	0.150	0.012	0.331	0.104	0.062	0.126	0.119	0.203	0.179

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	20	19	29	22	29	29	17
N.S.	1	1.00	1.37	0.74	0.70	1.07	0.81	1.07	1.07	0.63
time (sec)	N/A	0.131	0.024	0.386	0.106	0.062	0.090	0.119	0.223	0.036

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	10	9
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.67	0.60
time (sec)	N/A	0.127	0.001	0.324	0.029	0.061	0.073	0.118	0.204	0.017

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	7	6	16	7	29	17	6
N.S.	1	1.00	2.00	0.70	0.60	1.60	0.70	2.90	1.70	0.60
time (sec)	N/A	0.118	0.000	0.333	0.106	0.062	0.065	0.121	0.238	0.029

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	35	8	29	37	12
N.S.	1	1.00	1.00	0.75	0.45	1.75	0.40	1.45	1.85	0.60
time (sec)	N/A	0.135	0.014	0.341	0.104	0.064	0.547	0.114	0.207	0.188

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	15	23	19	12
N.S.	1	1.00	1.00	0.83	0.78	1.00	0.83	1.28	1.06	0.67
time (sec)	N/A	0.128	0.019	0.329	0.104	0.063	0.427	0.119	0.232	0.024

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	30	24	57	44	43	58	25
N.S.	1	1.10	1.00	0.77	0.62	1.46	1.13	1.10	1.49	0.64
time (sec)	N/A	0.145	0.028	0.432	0.104	0.063	1.322	0.121	0.233	0.032

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	28	32	42	34	19
N.S.	1	1.00	0.68	0.59	0.78	0.76	0.86	1.14	0.92	0.51
time (sec)	N/A	0.137	0.027	0.346	0.101	0.064	0.629	0.123	0.244	0.025

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	66	46	41	38	64	63	55	71	33
N.S.	1	1.16	0.81	0.72	0.67	1.12	1.11	0.96	1.25	0.58
time (sec)	N/A	0.154	0.035	0.425	0.103	0.067	2.613	0.116	0.208	0.175

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	23	40	23	46	43	22	23
N.S.	1	1.09	0.59	0.50	0.87	0.50	1.00	0.93	0.48	0.50
time (sec)	N/A	0.155	0.015	0.357	0.104	0.063	0.288	0.124	0.193	0.047

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	50	47	34	33	40	39	26	31	27
N.S.	1	1.11	1.04	0.76	0.73	0.89	0.87	0.58	0.69	0.60
time (sec)	N/A	0.146	0.062	0.407	0.103	0.065	0.177	0.128	0.238	0.040

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	18	26	18	29	23	17	18
N.S.	1	1.13	0.71	0.58	0.84	0.58	0.94	0.74	0.55	0.58
time (sec)	N/A	0.148	0.012	0.352	0.102	0.062	0.136	0.121	0.243	0.021

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	40	20	19	32	22	19	18	19
N.S.	1	1.00	1.48	0.74	0.70	1.19	0.81	0.70	0.67	0.70
time (sec)	N/A	0.132	0.039	0.401	0.109	0.065	0.121	0.129	0.221	0.025

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	12	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.80	0.73	0.67	0.73
time (sec)	N/A	0.123	0.001	0.332	0.026	0.064	0.079	0.120	0.217	0.211

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	7	6	19	7	19	6	6
N.S.	1	1.00	1.90	0.70	0.60	1.90	0.70	1.90	0.60	0.60
time (sec)	N/A	0.115	0.000	0.345	0.104	0.064	0.082	0.125	0.254	0.008

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	18	26	31	10	20
N.S.	1	1.00	1.00	0.75	1.25	0.90	1.30	1.55	0.50	1.00
time (sec)	N/A	0.137	0.012	0.487	0.103	0.063	0.591	0.123	0.221	0.118

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	36	33	13	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.00	1.83	0.72	0.78
time (sec)	N/A	0.129	0.021	0.342	0.106	0.059	0.514	0.123	0.222	0.025

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	30	40	38	99	45	29	35
N.S.	1	1.10	1.00	0.77	1.03	0.97	2.54	1.15	0.74	0.90
time (sec)	N/A	0.143	0.027	0.436	0.102	0.065	1.284	0.121	0.229	0.029

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	21	80	73	20	22
N.S.	1	1.00	0.68	0.59	0.78	0.57	2.16	1.97	0.54	0.59
time (sec)	N/A	0.139	0.028	0.277	0.103	0.069	0.678	0.124	0.236	0.024

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	66	46	41	54	45	136	57	42	49
N.S.	1	1.16	0.81	0.72	0.95	0.79	2.39	1.00	0.74	0.86
time (sec)	N/A	0.153	0.031	0.341	0.105	0.066	2.754	0.123	0.236	0.176

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	23	40	23	44	34	22	22
N.S.	1	1.09	0.59	0.50	0.87	0.50	0.96	0.74	0.48	0.48
time (sec)	N/A	0.156	0.014	0.283	0.106	0.061	0.213	0.119	0.189	0.381

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	44	39	45	37	48	37	42	0
N.S.	1	1.09	0.81	0.72	0.83	0.69	0.89	0.69	0.78	0.00
time (sec)	N/A	0.154	0.033	0.404	0.105	0.066	0.161	0.128	0.233	0.000

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	18	26	18	27	23	17	18
N.S.	1	1.13	0.71	0.58	0.84	0.58	0.87	0.74	0.55	0.58
time (sec)	N/A	0.151	0.012	0.339	0.102	0.063	0.125	0.124	0.211	0.361

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	30	31	29	31	30	29	29
N.S.	1	1.00	1.03	0.83	0.86	0.81	0.86	0.83	0.81	0.81
time (sec)	N/A	0.140	0.025	0.401	0.116	0.065	0.095	0.125	0.230	0.264

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	10	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.67	0.73	0.67	0.73
time (sec)	N/A	0.121	0.001	0.327	0.032	0.063	0.090	0.125	0.206	0.149

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	43	19	18	16	15	30	17	16
N.S.	1	1.00	2.26	1.00	0.95	0.84	0.79	1.58	0.89	0.84
time (sec)	N/A	0.127	0.000	0.340	0.103	0.066	0.081	0.128	0.253	0.094

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	9	18	26	14	17	20
N.S.	1	1.00	1.00	0.75	0.45	0.90	1.30	0.70	0.85	1.00
time (sec)	N/A	0.132	0.012	0.378	0.107	0.066	0.621	0.111	0.208	0.124

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	18	37	23	17	14
N.S.	1	1.00	1.00	0.83	0.78	1.00	2.06	1.28	0.94	0.78
time (sec)	N/A	0.121	0.020	0.533	0.103	0.064	0.466	0.130	0.189	0.180

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	30	24	38	99	29	36	29
N.S.	1	1.10	1.00	0.77	0.62	0.97	2.54	0.74	0.92	0.74
time (sec)	N/A	0.145	0.023	0.458	0.103	0.066	1.251	0.122	0.185	0.368

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	28	68	42	34	31
N.S.	1	1.00	0.68	0.59	0.78	0.76	1.84	1.14	0.92	0.84
time (sec)	N/A	0.135	0.025	0.394	0.103	0.067	0.686	0.130	0.245	0.296

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	66	46	42	38	45	136	41	49	57
N.S.	1	1.16	0.81	0.74	0.67	0.79	2.39	0.72	0.86	1.00
time (sec)	N/A	0.152	0.031	0.490	0.111	0.068	2.757	0.120	0.200	0.358

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	50	27	23	40	23	49	34	22	23
N.S.	1	1.09	0.59	0.50	0.87	0.50	1.07	0.74	0.48	0.50
time (sec)	N/A	0.151	0.014	0.334	0.103	0.064	0.239	0.119	0.222	0.378

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	43	39	33	67	53	0	32	0
N.S.	1	1.09	0.80	0.72	0.61	1.24	0.98	0.00	0.59	0.00
time (sec)	N/A	0.153	0.070	0.388	0.104	0.066	0.274	0.000	0.219	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	35	22	18	26	18	31	23	17	18
N.S.	1	1.13	0.71	0.58	0.84	0.58	1.00	0.74	0.55	0.58
time (sec)	N/A	0.146	0.013	0.328	0.102	0.064	0.136	0.126	0.215	0.331

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	29	19	59	36	0	19	31
N.S.	1	1.00	1.00	0.81	0.53	1.64	1.00	0.00	0.53	0.86
time (sec)	N/A	0.139	0.057	0.373	0.105	0.066	0.188	0.000	0.196	0.128

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	11	14	11	10	11
N.S.	1	1.00	1.00	0.80	0.73	0.73	0.93	0.73	0.67	0.73
time (sec)	N/A	0.123	0.001	0.346	0.032	0.062	0.085	0.118	0.188	0.077

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	C	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	8	6	47	17	0	7	15
N.S.	1	1.00	1.00	0.42	0.32	2.47	0.89	0.00	0.37	0.79
time (sec)	N/A	0.126	0.000	0.333	0.108	0.064	0.209	0.000	0.238	0.224

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	25	43	8	14	40	14
N.S.	1	1.00	1.00	0.75	1.25	2.15	0.40	0.70	2.00	0.70
time (sec)	N/A	0.137	0.021	0.351	0.105	0.064	0.724	0.121	0.194	0.436

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	15	0	20	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	0.83	0.00	1.11	0.78
time (sec)	N/A	0.124	0.040	0.345	0.102	0.060	0.461	0.000	0.206	0.214

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	43	39	30	40	65	46	29	62	29
N.S.	1	1.10	1.00	0.77	1.03	1.67	1.18	0.74	1.59	0.74
time (sec)	N/A	0.141	0.021	0.419	0.103	0.070	1.274	0.118	0.190	0.376

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	22	29	21	36	0	35	31
N.S.	1	1.00	0.68	0.59	0.78	0.57	0.97	0.00	0.95	0.84
time (sec)	N/A	0.136	0.044	0.332	0.104	0.059	0.667	0.000	0.243	0.353

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	C	C	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	66	46	42	54	72	65	43	75	60
N.S.	1	1.16	0.81	0.74	0.95	1.26	1.14	0.75	1.32	1.05
time (sec)	N/A	0.158	0.025	0.404	0.106	0.072	2.863	0.122	0.197	0.387

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	209	103	152	0	76	46	0	90	0
N.S.	1	1.14	0.56	0.83	0.00	0.41	0.25	0.00	0.49	0.00
time (sec)	N/A	0.266	10.044	0.635	0.000	0.077	14.085	0.000	0.227	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	171	85	138	0	57	46	0	67	0
N.S.	1	1.12	0.56	0.90	0.00	0.37	0.30	0.00	0.44	0.00
time (sec)	N/A	0.240	8.947	0.408	0.000	0.078	1.391	0.000	0.260	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	139	54	119	0	42	46	0	45	0
N.S.	1	1.10	0.43	0.94	0.00	0.33	0.37	0.00	0.36	0.00
time (sec)	N/A	0.211	6.240	0.391	0.000	0.075	0.582	0.000	0.234	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	139	56	120	0	44	49	0	54	0
N.S.	1	1.10	0.44	0.95	0.00	0.35	0.39	0.00	0.43	0.00
time (sec)	N/A	0.213	10.013	0.461	0.000	0.071	1.485	0.000	0.253	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	176	56	137	0	57	53	0	56	0
N.S.	1	1.14	0.36	0.88	0.00	0.37	0.34	0.00	0.36	0.00
time (sec)	N/A	0.244	10.012	0.585	0.000	0.068	15.744	0.000	0.295	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	216	56	151	0	70	53	0	56	0
N.S.	1	1.16	0.30	0.81	0.00	0.38	0.28	0.00	0.30	0.00
time (sec)	N/A	0.270	10.011	0.734	0.000	0.075	157.360	0.000	0.359	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	378	103	232	0	86	46	0	89	0
N.S.	1	1.14	0.31	0.70	0.00	0.26	0.14	0.00	0.27	0.00
time (sec)	N/A	0.402	10.043	0.546	0.000	0.100	41.500	0.000	0.241	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	340	85	221	0	71	46	0	68	0
N.S.	1	1.13	0.28	0.73	0.00	0.24	0.15	0.00	0.23	0.00
time (sec)	N/A	0.367	10.028	0.457	0.000	0.072	5.784	0.000	0.262	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	307	56	205	0	48	46	0	41	0
N.S.	1	1.14	0.21	0.76	0.00	0.18	0.17	0.00	0.15	0.00
time (sec)	N/A	0.342	7.109	0.419	0.000	0.071	0.662	0.000	0.251	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	303	54	194	0	49	49	0	50	0
N.S.	1	1.15	0.21	0.74	0.00	0.19	0.19	0.00	0.19	0.00
time (sec)	N/A	0.341	10.011	0.469	0.000	0.074	0.788	0.000	0.233	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	343	56	219	0	63	53	0	56	0
N.S.	1	1.13	0.18	0.72	0.00	0.21	0.17	0.00	0.18	0.00
time (sec)	N/A	0.373	10.013	0.561	0.000	0.070	5.307	0.000	0.298	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	383	56	234	0	78	53	0	56	0
N.S.	1	1.15	0.17	0.70	0.00	0.23	0.16	0.00	0.17	0.00
time (sec)	N/A	0.398	10.014	0.690	0.000	0.070	47.102	0.000	0.313	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	240	102	163	0	90	46	0	111	0
N.S.	1	1.13	0.48	0.77	0.00	0.42	0.22	0.00	0.52	0.00
time (sec)	N/A	0.274	10.053	1.239	0.000	0.091	24.730	0.000	0.246	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	202	89	150	0	69	46	0	88	0
N.S.	1	1.12	0.49	0.83	0.00	0.38	0.25	0.00	0.49	0.00
time (sec)	N/A	0.258	10.041	0.786	0.000	0.079	2.542	0.000	0.286	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	170	55	134	0	55	46	0	65	0
N.S.	1	1.12	0.36	0.88	0.00	0.36	0.30	0.00	0.43	0.00
time (sec)	N/A	0.235	9.094	0.648	0.000	0.075	1.090	0.000	0.231	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	171	57	125	0	53	49	0	73	0
N.S.	1	1.12	0.37	0.82	0.00	0.35	0.32	0.00	0.48	0.00
time (sec)	N/A	0.238	10.013	0.855	0.000	0.077	1.835	0.000	0.248	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	173	57	135	0	53	53	0	75	0
N.S.	1	1.13	0.37	0.88	0.00	0.35	0.35	0.00	0.49	0.00
time (sec)	N/A	0.243	10.011	1.332	0.000	0.073	15.784	0.000	0.293	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	210	57	151	0	70	53	0	76	0
N.S.	1	1.14	0.31	0.82	0.00	0.38	0.29	0.00	0.41	0.00
time (sec)	N/A	0.268	10.011	1.888	0.000	0.071	150.908	0.000	0.387	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	250	57	162	0	81	0	0	76	0
N.S.	1	1.16	0.27	0.75	0.00	0.38	0.00	0.00	0.35	0.00
time (sec)	N/A	0.293	10.028	2.747	0.000	0.073	0.000	0.000	0.518	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	371	89	232	0	85	46	0	89	0
N.S.	1	1.13	0.27	0.71	0.00	0.26	0.14	0.00	0.27	0.00
time (sec)	N/A	0.392	10.042	1.051	0.000	0.076	8.410	0.000	0.292	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	338	57	218	0	63	46	0	62	0
N.S.	1	1.14	0.19	0.73	0.00	0.21	0.15	0.00	0.21	0.00
time (sec)	N/A	0.372	10.010	0.470	0.000	0.081	1.349	0.000	0.236	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	337	55	208	0	60	49	0	69	0
N.S.	1	1.13	0.19	0.70	0.00	0.20	0.16	0.00	0.23	0.00
time (sec)	N/A	0.360	10.011	0.458	0.000	0.076	1.109	0.000	0.315	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	337	57	216	0	60	53	0	75	0
N.S.	1	1.13	0.19	0.72	0.00	0.20	0.18	0.00	0.25	0.00
time (sec)	N/A	0.359	10.012	0.659	0.000	0.080	5.096	0.000	0.296	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	377	57	234	0	78	53	0	76	0
N.S.	1	1.14	0.17	0.70	0.00	0.23	0.16	0.00	0.23	0.00
time (sec)	N/A	0.395	10.012	1.015	0.000	0.071	46.269	0.000	0.345	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	123	74	133	0	45	53	0	61	0
N.S.	1	1.04	0.63	1.13	0.00	0.38	0.45	0.00	0.52	0.00
time (sec)	N/A	0.235	7.740	0.367	0.000	0.065	1.687	0.000	0.293	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	51	124	0	40	53	0	47	0
N.S.	1	1.00	0.54	1.31	0.00	0.42	0.56	0.00	0.49	0.00
time (sec)	N/A	0.201	6.442	0.447	0.000	0.068	0.707	0.000	0.216	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	51	129	0	46	49	0	58	0
N.S.	1	1.00	0.53	1.33	0.00	0.47	0.51	0.00	0.60	0.00
time (sec)	N/A	0.201	10.013	0.450	0.000	0.066	1.490	0.000	0.256	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	136	53	137	0	52	60	0	60	0
N.S.	1	1.09	0.42	1.10	0.00	0.42	0.48	0.00	0.48	0.00
time (sec)	N/A	0.232	10.012	0.443	0.000	0.067	16.137	0.000	0.270	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	141	74	194	0	56	53	0	62	0
N.S.	1	0.78	0.41	1.07	0.00	0.31	0.29	0.00	0.34	0.00
time (sec)	N/A	0.235	9.109	0.484	0.000	0.070	4.442	0.000	0.224	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	99	51	183	0	39	53	0	43	0
N.S.	1	0.64	0.33	1.18	0.00	0.25	0.34	0.00	0.28	0.00
time (sec)	N/A	0.200	6.573	0.460	0.000	0.069	0.592	0.000	0.247	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	98	51	184	0	46	56	0	52	0
N.S.	1	0.66	0.34	1.23	0.00	0.31	0.38	0.00	0.35	0.00
time (sec)	N/A	0.204	10.011	0.459	0.000	0.066	0.762	0.000	0.281	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	144	53	192	0	55	49	0	60	0
N.S.	1	0.75	0.27	0.99	0.00	0.28	0.25	0.00	0.31	0.00
time (sec)	N/A	0.237	10.011	0.575	0.000	0.069	4.815	0.000	0.242	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	178	87	141	0	62	44	0	69	0
N.S.	1	1.14	0.56	0.90	0.00	0.40	0.28	0.00	0.44	0.00
time (sec)	N/A	0.250	10.025	0.825	0.000	0.076	10.155	0.000	0.239	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	140	69	125	0	41	44	0	48	0
N.S.	1	1.10	0.54	0.98	0.00	0.32	0.35	0.00	0.38	0.00
time (sec)	N/A	0.220	10.026	0.433	0.000	0.076	1.034	0.000	0.327	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	110	54	104	0	22	44	0	30	0
N.S.	1	1.13	0.56	1.07	0.00	0.23	0.45	0.00	0.31	0.00
time (sec)	N/A	0.194	10.015	0.711	0.000	0.073	0.639	0.000	0.256	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	142	56	123	0	45	48	0	32	0
N.S.	1	1.10	0.43	0.95	0.00	0.35	0.37	0.00	0.25	0.00
time (sec)	N/A	0.223	10.014	1.197	0.000	0.068	2.005	0.000	0.243	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	182	56	138	0	57	51	0	32	0
N.S.	1	1.15	0.35	0.87	0.00	0.36	0.32	0.00	0.20	0.00
time (sec)	N/A	0.247	10.016	1.869	0.000	0.073	21.186	0.000	0.250	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	347	87	221	0	72	44	0	68	0
N.S.	1	1.14	0.29	0.73	0.00	0.24	0.14	0.00	0.22	0.00
time (sec)	N/A	0.403	10.039	1.040	0.000	0.079	30.424	0.000	0.267	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	309	69	210	0	54	44	0	49	0
N.S.	1	1.13	0.25	0.77	0.00	0.20	0.16	0.00	0.18	0.00
time (sec)	N/A	0.338	10.026	0.735	0.000	0.072	3.148	0.000	0.250	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	278	56	132	0	27	44	0	25	0
N.S.	1	1.18	0.24	0.56	0.00	0.11	0.19	0.00	0.11	0.00
time (sec)	N/A	0.320	10.019	0.341	0.000	0.070	0.525	0.000	0.209	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	309	54	196	0	51	48	0	32	0
N.S.	1	1.15	0.20	0.73	0.00	0.19	0.18	0.00	0.12	0.00
time (sec)	N/A	0.353	10.013	0.951	0.000	0.066	0.826	0.000	0.236	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	349	56	219	0	65	51	0	32	0
N.S.	1	1.14	0.18	0.72	0.00	0.21	0.17	0.00	0.10	0.00
time (sec)	N/A	0.378	10.012	1.599	0.000	0.069	6.722	0.000	0.284	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	175	74	128	0	86	44	0	133	0
N.S.	1	1.14	0.48	0.84	0.00	0.56	0.29	0.00	0.87	0.00
time (sec)	N/A	0.247	10.022	1.727	0.000	0.084	12.711	0.000	0.269	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	138	59	115	0	61	44	0	109	0
N.S.	1	1.10	0.47	0.92	0.00	0.49	0.35	0.00	0.87	0.00
time (sec)	N/A	0.216	10.019	0.439	0.000	0.074	1.101	0.000	0.241	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	139	59	114	0	58	44	0	41	0
N.S.	1	1.10	0.47	0.90	0.00	0.46	0.35	0.00	0.33	0.00
time (sec)	N/A	0.220	9.867	1.257	0.000	0.071	0.943	0.000	0.207	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	175	59	124	0	79	48	0	43	0
N.S.	1	1.14	0.38	0.81	0.00	0.51	0.31	0.00	0.28	0.00
time (sec)	N/A	0.252	10.012	2.365	0.000	0.077	3.932	0.000	0.268	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	344	73	210	0	94	44	0	130	0
N.S.	1	1.15	0.24	0.70	0.00	0.31	0.15	0.00	0.43	0.00
time (sec)	N/A	0.386	10.029	1.940	0.000	0.081	37.622	0.000	0.293	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	304	60	197	0	76	44	0	110	0
N.S.	1	1.14	0.23	0.74	0.00	0.29	0.17	0.00	0.41	0.00
time (sec)	N/A	0.360	10.020	0.993	0.000	0.075	3.629	0.000	0.277	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	307	59	197	0	65	44	0	36	0
N.S.	1	1.15	0.22	0.74	0.00	0.24	0.17	0.00	0.14	0.00
time (sec)	N/A	0.344	10.013	0.414	0.000	0.072	0.656	0.000	0.250	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	342	57	197	0	83	48	0	43	0
N.S.	1	1.16	0.19	0.67	0.00	0.28	0.16	0.00	0.15	0.00
time (sec)	N/A	0.390	10.013	2.046	0.000	0.076	1.538	0.000	0.252	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	382	59	219	0	101	51	0	43	0
N.S.	1	1.15	0.18	0.66	0.00	0.31	0.15	0.00	0.13	0.00
time (sec)	N/A	0.412	10.014	2.810	0.000	0.074	14.480	0.000	0.308	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	212	92	230	0	125	44	0	244	0
N.S.	1	1.16	0.50	1.26	0.00	0.68	0.24	0.00	1.33	0.00
time (sec)	N/A	0.261	10.057	3.367	0.000	0.089	102.882	0.000	0.344	0.000

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	175	80	205	0	108	44	0	135	0
N.S.	1	1.13	0.52	1.32	0.00	0.70	0.28	0.00	0.87	0.00
time (sec)	N/A	0.246	10.036	1.753	0.000	0.077	12.396	0.000	0.352	0.000

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	176	79	208	0	98	44	0	200	0
N.S.	1	1.13	0.51	1.33	0.00	0.63	0.28	0.00	1.28	0.00
time (sec)	N/A	0.228	10.035	0.621	0.000	0.075	1.897	0.000	0.305	0.000

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	175	79	198	0	97	44	0	52	0
N.S.	1	1.11	0.50	1.26	0.00	0.62	0.28	0.00	0.33	0.00
time (sec)	N/A	0.249	10.028	2.108	0.000	0.077	2.359	0.000	0.261	0.000

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	211	59	226	0	115	48	0	54	0
N.S.	1	1.14	0.32	1.22	0.00	0.62	0.26	0.00	0.29	0.00
time (sec)	N/A	0.271	10.013	4.118	0.000	0.076	7.677	0.000	0.354	0.000

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	341	80	258	0	116	44	0	218	0
N.S.	1	1.14	0.27	0.86	0.00	0.39	0.15	0.00	0.73	0.00
time (sec)	N/A	0.391	10.044	1.984	0.000	0.081	39.180	0.000	0.353	0.000

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	344	74	263	0	118	44	0	199	0
N.S.	1	1.13	0.24	0.87	0.00	0.39	0.14	0.00	0.65	0.00
time (sec)	N/A	0.377	10.028	1.641	0.000	0.077	3.647	0.000	0.308	0.000

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	343	59	256	0	103	44	0	47	0
N.S.	1	1.14	0.20	0.85	0.00	0.34	0.15	0.00	0.16	0.00
time (sec)	N/A	0.383	10.012	0.622	0.000	0.073	1.411	0.000	0.235	0.000

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	378	57	287	0	119	48	0	54	0
N.S.	1	1.14	0.17	0.86	0.00	0.36	0.14	0.00	0.16	0.00
time (sec)	N/A	0.402	10.013	3.790	0.000	0.072	3.779	0.000	0.449	0.000

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	418	59	312	0	137	51	0	54	0
N.S.	1	1.15	0.16	0.86	0.00	0.38	0.14	0.00	0.15	0.00
time (sec)	N/A	0.425	10.021	4.681	0.000	0.095	24.103	0.000	0.406	0.000

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	61	131	0	42	51	0	49	0
N.S.	1	1.00	0.69	1.47	0.00	0.47	0.57	0.00	0.55	0.00
time (sec)	N/A	0.198	10.024	0.477	0.000	0.064	0.929	0.000	0.286	0.000

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	114	0	21	51	0	36	0
N.S.	1	1.00	0.88	1.78	0.00	0.33	0.80	0.00	0.56	0.00
time (sec)	N/A	0.177	10.029	0.477	0.000	0.085	0.580	0.000	0.225	0.000

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	53	132	0	47	54	0	38	0
N.S.	1	1.00	0.54	1.33	0.00	0.47	0.55	0.00	0.38	0.00
time (sec)	N/A	0.204	10.015	0.678	0.000	0.069	1.893	0.000	0.243	0.000

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	107	61	184	0	50	51	0	50	0
N.S.	1	0.65	0.37	1.12	0.00	0.30	0.31	0.00	0.30	0.00
time (sec)	N/A	0.206	10.028	0.520	0.000	0.068	3.206	0.000	0.268	0.000

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	67	53	152	0	20	51	0	31	0
N.S.	1	0.56	0.44	1.27	0.00	0.17	0.42	0.00	0.26	0.00
time (sec)	N/A	0.185	10.021	0.389	0.000	0.070	0.506	0.000	0.238	0.000

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	107	51	182	0	48	54	0	38	0
N.S.	1	0.66	0.31	1.12	0.00	0.30	0.33	0.00	0.23	0.00
time (sec)	N/A	0.212	10.020	0.621	0.000	0.067	0.817	0.000	0.220	0.000

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	129	67	134	0	81	51	0	114	0
N.S.	1	1.02	0.53	1.06	0.00	0.64	0.40	0.00	0.90	0.00
time (sec)	N/A	0.236	10.023	0.677	0.000	0.071	12.112	0.000	0.267	0.000

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	59	126	0	63	51	0	58	0
N.S.	1	1.00	0.62	1.33	0.00	0.66	0.54	0.00	0.61	0.00
time (sec)	N/A	0.201	9.848	0.463	0.000	0.067	1.057	0.000	0.240	0.000

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	59	122	0	61	51	0	40	0
N.S.	1	1.00	0.61	1.26	0.00	0.63	0.53	0.00	0.41	0.00
time (sec)	N/A	0.207	8.883	0.664	0.000	0.066	0.936	0.000	0.234	0.000

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	138	58	133	0	80	54	0	42	0
N.S.	1	1.04	0.44	1.00	0.00	0.60	0.41	0.00	0.32	0.00
time (sec)	N/A	0.234	10.017	0.806	0.000	0.071	3.886	0.000	0.327	0.000

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	147	67	210	0	85	51	0	111	0
N.S.	1	0.73	0.33	1.04	0.00	0.42	0.25	0.00	0.55	0.00
time (sec)	N/A	0.227	10.027	0.675	0.000	0.070	36.105	0.000	0.274	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	110	59	186	0	74	51	0	96	0
N.S.	1	0.65	0.35	1.09	0.00	0.44	0.30	0.00	0.56	0.00
time (sec)	N/A	0.210	10.017	0.535	0.000	0.071	3.446	0.000	0.236	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	101	58	182	0	63	51	0	35	0
N.S.	1	0.64	0.37	1.16	0.00	0.40	0.32	0.00	0.22	0.00
time (sec)	N/A	0.206	7.676	0.418	0.000	0.067	0.653	0.000	0.232	0.000

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	146	58	216	0	76	54	0	42	0
N.S.	1	0.74	0.29	1.09	0.00	0.38	0.27	0.00	0.21	0.00
time (sec)	N/A	0.235	10.020	0.721	0.000	0.072	1.564	0.000	0.267	0.000

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	15	0	9	32	0	22	16
N.S.	1	1.00	1.10	0.75	0.00	0.45	1.60	0.00	1.10	0.80
time (sec)	N/A	0.152	10.028	0.403	0.000	0.065	0.463	0.000	0.200	0.162

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	77	0	9	0	0	20	0
N.S.	1	1.00	0.84	1.51	0.00	0.18	0.00	0.00	0.39	0.00
time (sec)	N/A	0.210	10.022	0.335	0.000	0.064	0.000	0.000	0.197	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	0	20	36	0	31	0
N.S.	1	1.00	1.14	0.90	0.00	0.95	1.71	0.00	1.48	0.00
time (sec)	N/A	0.147	10.018	0.991	0.000	0.062	0.448	0.000	0.212	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	23	18	0	13	32	0	22	0
N.S.	1	1.00	0.34	0.27	0.00	0.19	0.48	0.00	0.33	0.00
time (sec)	N/A	0.172	10.016	0.605	0.000	0.066	0.434	0.000	0.211	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	537	570	85	0	0	0	46	0	66	0
N.S.	1	1.06	0.16	0.00	0.00	0.00	0.09	0.00	0.12	0.00
time (sec)	N/A	0.558	10.044	0.000	0.000	0.000	7.936	0.000	0.439	0.000

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1045	85	56	0	0	0	46	0	43	0
N.S.	1	0.08	0.05	0.00	0.00	0.00	0.04	0.00	0.04	0.00
time (sec)	N/A	0.225	10.012	0.000	0.000	0.000	2.395	0.000	0.592	0.000

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	998	85	56	0	0	0	46	0	43	0
N.S.	1	0.09	0.06	0.00	0.00	0.00	0.05	0.00	0.04	0.00
time (sec)	N/A	0.212	10.011	0.000	0.000	0.000	0.793	0.000	0.421	0.000

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	547	56	0	0	0	46	0	44	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.09	0.00	0.09	0.00
time (sec)	N/A	0.505	10.013	0.000	0.000	0.000	0.795	0.000	0.466	0.000

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	510	537	54	0	0	0	46	0	44	0
N.S.	1	1.05	0.11	0.00	0.00	0.00	0.09	0.00	0.09	0.00
time (sec)	N/A	0.491	10.009	0.000	0.000	0.000	0.721	0.000	0.517	0.000

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1018	83	54	0	0	0	49	0	46	0
N.S.	1	0.08	0.05	0.00	0.00	0.00	0.05	0.00	0.05	0.00
time (sec)	N/A	0.218	10.012	0.000	0.000	0.000	1.031	0.000	0.594	0.000

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	565	601	89	0	0	0	46	0	85	0
N.S.	1	1.06	0.16	0.00	0.00	0.00	0.08	0.00	0.15	0.00
time (sec)	N/A	0.532	10.053	0.000	0.000	0.000	13.738	0.000	0.480	0.000

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1073	116	57	0	0	0	46	0	62	0
N.S.	1	0.11	0.05	0.00	0.00	0.00	0.04	0.00	0.06	0.00
time (sec)	N/A	0.238	10.027	0.000	0.000	0.000	4.720	0.000	0.635	0.000

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1024	116	57	0	0	0	46	0	62	0
N.S.	1	0.11	0.06	0.00	0.00	0.00	0.04	0.00	0.06	0.00
time (sec)	N/A	0.236	10.011	0.000	0.000	0.000	1.352	0.000	0.453	0.000

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	578	57	0	0	0	46	0	63	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.09	0.00	0.12	0.00
time (sec)	N/A	0.525	10.010	0.000	0.000	0.000	1.036	0.000	0.483	0.000

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	540	568	55	0	0	0	46	0	63	0
N.S.	1	1.05	0.10	0.00	0.00	0.00	0.09	0.00	0.12	0.00
time (sec)	N/A	0.515	10.010	0.000	0.000	0.000	1.040	0.000	0.529	0.000

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1045	115	55	0	0	0	49	0	65	0
N.S.	1	0.11	0.05	0.00	0.00	0.00	0.05	0.00	0.06	0.00
time (sec)	N/A	0.246	10.014	0.000	0.000	0.000	1.896	0.000	0.643	0.000

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	507	539	69	0	0	0	44	0	49	0
N.S.	1	1.06	0.14	0.00	0.00	0.00	0.09	0.00	0.10	0.00
time (sec)	N/A	0.493	10.034	0.000	0.000	0.000	5.631	0.000	0.467	0.000

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	483	518	56	0	0	0	44	0	33	0
N.S.	1	1.07	0.12	0.00	0.00	0.00	0.09	0.00	0.07	0.00
time (sec)	N/A	0.476	10.014	0.000	0.000	0.000	0.695	0.000	0.468	0.000

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	508	54	0	0	0	44	0	35	0
N.S.	1	1.06	0.11	0.00	0.00	0.00	0.09	0.00	0.07	0.00
time (sec)	N/A	0.472	10.010	0.000	0.000	0.000	0.827	0.000	0.561	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	517	553	56	0	0	0	48	0	35	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.09	0.00	0.07	0.00
time (sec)	N/A	0.505	10.013	0.000	0.000	0.000	11.902	0.000	0.876	0.000

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1011	87	69	0	0	0	44	0	48	0
N.S.	1	0.09	0.07	0.00	0.00	0.00	0.04	0.00	0.05	0.00
time (sec)	N/A	0.218	10.032	0.000	0.000	0.000	49.754	0.000	0.475	0.000

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1019	58	56	0	0	0	44	0	27	0
N.S.	1	0.06	0.05	0.00	0.00	0.00	0.04	0.00	0.03	0.00
time (sec)	N/A	0.191	10.017	0.000	0.000	0.000	1.658	0.000	0.366	0.000

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	965	58	56	0	0	0	44	0	27	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.186	10.009	0.000	0.000	0.000	0.660	0.000	0.316	0.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	985	89	54	0	0	0	48	0	35	0
N.S.	1	0.09	0.05	0.00	0.00	0.00	0.05	0.00	0.04	0.00
time (sec)	N/A	0.217	10.010	0.000	0.000	0.000	1.370	0.000	0.524	0.000

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1013	91	56	0	0	0	48	0	35	0
N.S.	1	0.09	0.06	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.213	10.010	0.000	0.000	0.000	3.375	0.000	0.667	0.000

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1021	129	56	0	0	0	51	0	35	0
N.S.	1	0.13	0.05	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.242	10.020	0.000	0.000	0.000	98.794	0.000	1.177	0.000

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	503	533	59	0	0	0	44	0	111	0
N.S.	1	1.06	0.12	0.00	0.00	0.00	0.09	0.00	0.22	0.00
time (sec)	N/A	0.481	10.031	0.000	0.000	0.000	6.989	0.000	1.208	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1045	87	59	0	0	0	44	0	38	0
N.S.	1	0.08	0.06	0.00	0.00	0.00	0.04	0.00	0.04	0.00
time (sec)	N/A	0.219	10.013	0.000	0.000	0.000	2.248	0.000	0.501	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	998	87	59	0	0	0	44	0	38	0
N.S.	1	0.09	0.06	0.00	0.00	0.00	0.04	0.00	0.04	0.00
time (sec)	N/A	0.213	10.013	0.000	0.000	0.000	1.058	0.000	0.425	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	512	546	59	0	0	0	44	0	44	0
N.S.	1	1.07	0.12	0.00	0.00	0.00	0.09	0.00	0.09	0.00
time (sec)	N/A	0.493	10.016	0.000	0.000	0.000	1.187	0.000	1.039	0.000

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	514	537	60	0	0	0	44	0	46	0
N.S.	1	1.04	0.12	0.00	0.00	0.00	0.09	0.00	0.09	0.00
time (sec)	N/A	0.513	10.036	0.000	0.000	0.000	1.534	0.000	1.304	0.000

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1018	122	57	0	0	0	48	0	46	0
N.S.	1	0.12	0.06	0.00	0.00	0.00	0.05	0.00	0.05	0.00
time (sec)	N/A	0.245	10.024	0.000	0.000	0.000	2.795	0.000	1.559	0.000

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	540	574	80	0	0	0	44	0	201	0
N.S.	1	1.06	0.15	0.00	0.00	0.00	0.08	0.00	0.37	0.00
time (sec)	N/A	0.515	10.048	0.000	0.000	0.000	6.857	0.000	3.408	0.000

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1078	123	59	0	0	0	44	0	49	0
N.S.	1	0.11	0.05	0.00	0.00	0.00	0.04	0.00	0.05	0.00
time (sec)	N/A	0.249	10.034	0.000	0.000	0.000	3.560	0.000	0.938	0.000

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1029	123	59	0	0	0	44	0	49	0
N.S.	1	0.12	0.06	0.00	0.00	0.00	0.04	0.00	0.05	0.00
time (sec)	N/A	0.249	10.023	0.000	0.000	0.000	1.877	0.000	0.868	0.000

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	543	582	59	0	0	0	44	0	55	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.08	0.00	0.10	0.00
time (sec)	N/A	0.524	10.023	0.000	0.000	0.000	2.106	0.000	2.910	0.000

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	545	573	79	0	0	0	44	0	57	0
N.S.	1	1.05	0.14	0.00	0.00	0.00	0.08	0.00	0.10	0.00
time (sec)	N/A	0.525	10.044	0.000	0.000	0.000	3.870	0.000	3.335	0.000

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	158	57	0	0	0	48	0	57	0
N.S.	1	0.15	0.05	0.00	0.00	0.00	0.05	0.00	0.05	0.00
time (sec)	N/A	0.273	10.023	0.000	0.000	0.000	7.884	0.000	4.029	0.000

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	245	24	17	0	0	31	0	23	0
N.S.	1	1.03	0.10	0.07	0.00	0.00	0.13	0.00	0.10	0.00
time (sec)	N/A	0.297	10.010	0.415	0.000	0.000	0.594	0.000	0.247	0.000

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	67	0	0	0	58	0	40	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.84	0.00	0.58	0.00
time (sec)	N/A	0.176	0.210	0.000	0.000	0.000	1.419	0.000	0.283	0.000

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	58	0	18	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.85	0.00	0.26	0.00
time (sec)	N/A	0.178	0.111	0.000	0.000	0.000	0.663	0.000	0.248	0.000

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	66	0	0	0	56	0	20	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	0.29	0.00
time (sec)	N/A	0.167	0.135	0.000	0.000	0.000	0.589	0.000	0.243	0.000

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	36	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.51	0.00
time (sec)	N/A	0.169	0.204	0.000	0.000	0.000	0.796	0.000	0.256	0.000

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	69	0	0	0	56	0	55	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.77	0.00
time (sec)	N/A	0.168	0.222	0.000	0.000	0.000	1.417	0.000	0.250	0.000

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	53	0	19	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.84	0.00	0.30	0.00
time (sec)	N/A	0.168	0.144	0.000	0.000	0.000	0.582	0.000	0.220	0.000

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	46	0	17	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.73	0.00	0.27	0.00
time (sec)	N/A	0.170	0.144	0.000	0.000	0.000	0.574	0.000	0.243	0.000

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	53	0	16	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.84	0.00	0.25	0.00
time (sec)	N/A	0.171	0.012	0.000	0.000	0.000	0.563	0.000	0.259	0.000

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	51	0	19	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.89	0.00	0.33	0.00
time (sec)	N/A	0.163	0.141	0.000	0.000	0.000	0.593	0.000	0.208	0.000

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	65	0	0	0	53	0	19	0
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.80	0.00	0.29	0.00
time (sec)	N/A	0.169	0.143	0.000	0.000	0.000	0.602	0.000	0.240	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	104	16	16	16	196	0	63	24
N.S.	1	1.00	6.12	0.94	0.94	0.94	11.53	0.00	3.71	1.41
time (sec)	N/A	0.139	0.299	0.377	0.074	0.072	2.648	0.000	0.250	0.350

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	127	104	70	16	18	102	0	63	0
N.S.	1	7.47	6.12	4.12	0.94	1.06	6.00	0.00	3.71	0.00
time (sec)	N/A	0.236	0.003	0.519	0.074	0.070	0.926	0.000	0.224	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	103	14	13	16	107	0	206	13
N.S.	1	1.00	6.87	0.93	0.87	1.07	7.13	0.00	13.73	0.87
time (sec)	N/A	0.145	0.342	0.370	0.074	0.077	6.618	0.000	0.245	0.478

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	B	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	123	103	62	13	26	102	0	206	0
N.S.	1	8.20	6.87	4.13	0.87	1.73	6.80	0.00	13.73	0.00
time (sec)	N/A	0.232	0.004	0.462	0.074	0.073	1.090	0.000	0.248	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	57	1795	57	84	55
N.S.	1	1.05	0.62	0.59	0.80	0.71	22.44	0.71	1.05	0.69
time (sec)	N/A	0.198	0.031	0.353	0.032	0.066	1.600	0.124	0.223	0.392

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	700	43	73	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	11.86	0.73	1.24	0.75
time (sec)	N/A	0.179	0.025	0.353	0.031	0.064	1.100	0.123	0.203	0.305

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	38	25	30	34	223	29	61	33
N.S.	1	1.11	1.00	0.66	0.79	0.89	5.87	0.76	1.61	0.87
time (sec)	N/A	0.164	0.022	0.348	0.026	0.067	0.708	0.117	0.234	0.273

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	48	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	2.67	0.78
time (sec)	N/A	0.131	0.002	0.345	0.035	0.063	0.081	0.119	0.205	0.201

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	102	126	98	97	102	46	98	37	115
N.S.	1	1.01	1.25	0.97	0.96	1.01	0.46	0.97	0.37	1.14
time (sec)	N/A	0.213	0.109	1.330	0.109	0.072	0.694	0.415	0.235	0.299

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	108	135	111	103	151	42	106	46	125
N.S.	1	1.01	1.26	1.04	0.96	1.41	0.39	0.99	0.43	1.17
time (sec)	N/A	0.205	0.144	0.526	0.116	0.069	0.846	0.409	0.235	0.453

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	138	158	135	155	196	42	140	67	217
N.S.	1	1.02	1.17	1.00	1.15	1.45	0.31	1.04	0.50	1.61
time (sec)	N/A	0.224	0.146	0.550	0.123	0.074	1.181	0.414	0.234	0.704

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	323	94	0	0	0	29	0	70	0
N.S.	1	1.03	0.30	0.00	0.00	0.00	0.09	0.00	0.22	0.00
time (sec)	N/A	0.290	4.621	0.000	0.000	0.000	0.537	0.000	0.267	0.000

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	293	62	0	0	0	29	0	50	0
N.S.	1	1.01	0.21	0.00	0.00	0.00	0.10	0.00	0.17	0.00
time (sec)	N/A	0.263	4.395	0.000	0.000	0.000	0.483	0.000	0.249	0.000

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	46	0	0	0	26	0	27	37
N.S.	1	1.00	0.17	0.00	0.00	0.00	0.10	0.00	0.10	0.14
time (sec)	N/A	0.244	0.003	0.000	0.000	0.000	0.482	0.000	0.247	0.219

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	49	0	0	0	29	0	45	40
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.11	0.00	0.17	0.15
time (sec)	N/A	0.249	4.434	0.000	0.000	0.000	0.504	0.000	0.255	0.448

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	289	51	0	0	0	34	0	48	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	0.17	0.00
time (sec)	N/A	0.272	10.012	0.000	0.000	0.000	0.587	0.000	0.280	0.000

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	57	1795	57	92	55
N.S.	1	1.05	0.62	0.59	0.80	0.71	22.44	0.71	1.15	0.69
time (sec)	N/A	0.196	0.032	0.379	0.033	0.062	1.657	0.126	0.225	0.307

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	46	700	43	81	44
N.S.	1	1.07	0.66	0.61	0.80	0.78	11.86	0.73	1.37	0.75
time (sec)	N/A	0.181	0.027	0.373	0.032	0.064	1.270	0.115	0.208	0.279

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	39	25	30	35	66	29	70	33
N.S.	1	1.11	1.03	0.66	0.79	0.92	1.74	0.76	1.84	0.87
time (sec)	N/A	0.166	0.026	0.368	0.032	0.065	0.224	0.119	0.235	0.304

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	42	14	56	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	2.33	0.78	3.11	0.78
time (sec)	N/A	0.135	0.003	0.352	0.035	0.066	0.109	0.121	0.226	0.220

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	103	126	98	97	122	46	98	37	125
N.S.	1	1.02	1.25	0.97	0.96	1.21	0.46	0.97	0.37	1.24
time (sec)	N/A	0.209	0.077	0.494	0.109	0.072	0.773	0.428	0.246	0.289

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	136	112	103	290	42	107	46	136
N.S.	1	1.03	1.31	1.08	0.99	2.79	0.40	1.03	0.44	1.31
time (sec)	N/A	0.207	0.130	0.536	0.117	0.076	0.686	0.442	0.232	0.397

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	137	158	135	155	325	42	141	67	212
N.S.	1	1.01	1.17	1.00	1.15	2.41	0.31	1.04	0.50	1.57
time (sec)	N/A	0.224	0.175	0.513	0.119	0.081	1.190	0.372	0.275	0.794

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	601	649	94	0	0	0	29	0	70	0
N.S.	1	1.08	0.16	0.00	0.00	0.00	0.05	0.00	0.12	0.00
time (sec)	N/A	0.506	5.582	0.000	0.000	0.000	0.561	0.000	0.258	0.000

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	577	619	62	0	0	0	29	0	50	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.05	0.00	0.09	0.00
time (sec)	N/A	0.478	5.095	0.000	0.000	0.000	0.544	0.000	0.268	0.000

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	550	592	46	0	0	0	26	0	27	37
N.S.	1	1.08	0.08	0.00	0.00	0.00	0.05	0.00	0.05	0.07
time (sec)	N/A	0.449	0.003	0.000	0.000	0.000	0.517	0.000	0.277	0.230

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	538	586	49	0	0	0	29	0	45	40
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.08	0.07
time (sec)	N/A	0.444	5.306	0.000	0.000	0.000	0.531	0.000	0.276	0.514

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	575	618	51	0	0	0	34	0	48	0
N.S.	1	1.07	0.09	0.00	0.00	0.00	0.06	0.00	0.08	0.00
time (sec)	N/A	0.500	10.009	0.000	0.000	0.000	0.541	0.000	0.259	0.000

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	68	136	57	95	64
N.S.	1	1.05	0.62	0.59	0.80	0.85	1.70	0.71	1.19	0.80
time (sec)	N/A	0.201	0.035	0.350	0.027	0.065	0.495	0.122	0.288	0.340

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	57	112	43	84	53
N.S.	1	1.07	0.66	0.61	0.80	0.97	1.90	0.73	1.42	0.90
time (sec)	N/A	0.184	0.028	0.357	0.032	0.065	0.407	0.121	0.238	0.331

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	45	88	29	72	42
N.S.	1	1.11	0.74	0.66	0.79	1.18	2.32	0.76	1.89	1.11
time (sec)	N/A	0.171	0.025	0.336	0.031	0.066	0.335	0.119	0.209	0.277

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	32	65	14	59	14
N.S.	1	1.00	1.00	0.83	0.78	1.78	3.61	0.78	3.28	0.78
time (sec)	N/A	0.136	0.003	0.374	0.026	0.061	0.193	0.118	0.244	0.225

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	120	135	102	109	111	49	110	55	133
N.S.	1	1.03	1.15	0.87	0.93	0.95	0.42	0.94	0.47	1.14
time (sec)	N/A	0.218	0.093	0.416	0.110	0.070	1.128	0.426	0.250	0.307

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	124	140	111	116	129	46	119	63	141
N.S.	1	1.04	1.18	0.93	0.97	1.08	0.39	1.00	0.53	1.18
time (sec)	N/A	0.221	0.126	0.570	0.113	0.074	0.946	0.438	0.283	0.495

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	132	148	136	152	170	42	139	64	191
N.S.	1	0.99	1.11	1.02	1.14	1.28	0.32	1.05	0.48	1.44
time (sec)	N/A	0.220	0.146	0.547	0.112	0.073	1.006	0.426	0.305	0.544

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	347	79	0	0	0	29	0	90	0
N.S.	1	1.04	0.24	0.00	0.00	0.00	0.09	0.00	0.27	0.00
time (sec)	N/A	0.311	6.172	0.000	0.000	0.000	0.638	0.000	0.308	0.000

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	317	67	0	0	0	29	0	70	0
N.S.	1	1.02	0.22	0.00	0.00	0.00	0.09	0.00	0.23	0.00
time (sec)	N/A	0.288	5.871	0.000	0.000	0.000	0.598	0.000	0.264	0.000

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	288	47	0	0	0	26	0	45	37
N.S.	1	1.01	0.16	0.00	0.00	0.00	0.09	0.00	0.16	0.13
time (sec)	N/A	0.265	0.003	0.000	0.000	0.000	0.558	0.000	0.303	0.230

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	280	288	50	0	0	0	29	0	64	40
N.S.	1	1.03	0.18	0.00	0.00	0.00	0.10	0.00	0.23	0.14
time (sec)	N/A	0.263	5.914	0.000	0.000	0.000	0.592	0.000	0.268	0.690

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	52	0	0	0	34	0	66	0
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.12	0.00	0.23	0.00
time (sec)	N/A	0.264	10.011	0.000	0.000	0.000	0.570	0.000	0.275	0.000

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	84	50	47	64	46	1690	61	15	48
N.S.	1	1.05	0.62	0.59	0.80	0.58	21.12	0.76	0.19	0.60
time (sec)	N/A	0.196	0.035	0.355	0.028	0.065	1.513	0.119	0.264	0.358

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	63	39	36	47	35	631	47	15	36
N.S.	1	1.07	0.66	0.61	0.80	0.59	10.69	0.80	0.25	0.61
time (sec)	N/A	0.184	0.026	0.353	0.032	0.061	1.004	0.124	0.202	0.307

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	42	28	25	30	24	178	30	15	24
N.S.	1	1.11	0.74	0.66	0.79	0.63	4.68	0.79	0.39	0.63
time (sec)	N/A	0.169	0.023	0.329	0.027	0.063	0.654	0.117	0.224	0.297

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	13	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.72	0.78
time (sec)	N/A	0.137	0.002	0.309	0.033	0.062	0.086	0.119	0.263	0.241

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	85	103	83	86	235	41	87	15	106
N.S.	1	0.99	1.20	0.97	1.00	2.73	0.48	1.01	0.17	1.23
time (sec)	N/A	0.194	0.053	0.499	0.124	0.075	0.529	0.434	0.211	0.416

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	113	136	111	118	344	41	110	15	138
N.S.	1	1.03	1.24	1.01	1.07	3.13	0.37	1.00	0.14	1.25
time (sec)	N/A	0.216	0.105	0.509	0.122	0.078	0.780	0.412	0.198	0.568

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	143	143	136	158	326	41	142	15	201
N.S.	1	1.04	1.04	0.99	1.14	2.36	0.30	1.03	0.11	1.46
time (sec)	N/A	0.225	0.149	0.517	0.115	0.077	1.709	0.405	0.195	0.662

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	625	79	0	0	0	27	0	15	0
N.S.	1	1.08	0.14	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.477	4.615	0.000	0.000	0.000	0.584	0.000	0.249	0.000

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	556	595	62	0	0	0	27	0	15	0
N.S.	1	1.07	0.11	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.462	4.246	0.000	0.000	0.000	0.622	0.000	0.221	0.000

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	529	574	46	0	0	0	24	0	11	37
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.02	0.07
time (sec)	N/A	0.442	0.003	0.000	0.000	0.000	0.455	0.000	0.219	0.246

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	546	594	49	0	0	0	27	0	15	40
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.03	0.07
time (sec)	N/A	0.453	4.473	0.000	0.000	0.000	0.482	0.000	0.264	0.454

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	578	624	51	0	0	0	32	0	15	0
N.S.	1	1.08	0.09	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.502	10.010	0.000	0.000	0.000	0.541	0.000	0.222	0.000

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	50	47	64	46	1690	61	15	48
N.S.	1	1.02	0.62	0.59	0.80	0.58	21.12	0.76	0.19	0.60
time (sec)	N/A	0.198	0.038	0.334	0.041	0.062	1.567	0.123	0.210	0.328

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	61	39	36	47	35	631	47	15	36
N.S.	1	1.03	0.66	0.61	0.80	0.59	10.69	0.80	0.25	0.61
time (sec)	N/A	0.184	0.028	0.327	0.028	0.063	1.018	0.127	0.233	0.296

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	27	25	30	23	178	30	15	24
N.S.	1	1.05	0.71	0.66	0.79	0.61	4.68	0.79	0.39	0.63
time (sec)	N/A	0.173	0.026	0.309	0.027	0.064	0.656	0.122	0.234	0.329

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	14	24	14	14	14
N.S.	1	1.00	1.00	0.83	0.78	0.78	1.33	0.78	0.78	0.78
time (sec)	N/A	0.138	0.002	0.302	0.032	0.059	0.087	0.122	0.231	0.219

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	101	83	86	123	41	87	15	102
N.S.	1	1.00	1.17	0.97	1.00	1.43	0.48	1.01	0.17	1.19
time (sec)	N/A	0.197	0.060	0.368	0.120	0.068	0.574	0.412	0.242	0.458

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	135	111	118	179	41	109	15	130
N.S.	1	1.07	1.26	1.04	1.10	1.67	0.38	1.02	0.14	1.21
time (sec)	N/A	0.215	0.112	0.493	0.120	0.069	0.776	0.399	0.242	0.536

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	144	143	136	158	170	41	142	15	193
N.S.	1	1.04	1.04	0.99	1.14	1.23	0.30	1.03	0.11	1.40
time (sec)	N/A	0.228	0.132	0.517	0.121	0.078	1.595	0.336	0.215	0.596

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	299	79	0	0	0	27	0	15	0
N.S.	1	1.02	0.27	0.00	0.00	0.00	0.09	0.00	0.05	0.00
time (sec)	N/A	0.268	4.868	0.000	0.000	0.000	0.489	0.000	0.263	0.000

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	62	0	0	0	27	0	15	0
N.S.	1	1.00	0.23	0.00	0.00	0.00	0.10	0.00	0.06	0.00
time (sec)	N/A	0.250	4.411	0.000	0.000	0.000	0.457	0.000	0.215	0.000

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	46	0	0	0	24	0	11	37
N.S.	1	1.00	0.19	0.00	0.00	0.00	0.10	0.00	0.04	0.15
time (sec)	N/A	0.241	0.003	0.000	0.000	0.000	0.436	0.000	0.233	0.243

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	49	0	0	0	27	0	15	40
N.S.	1	1.00	0.18	0.00	0.00	0.00	0.10	0.00	0.06	0.15
time (sec)	N/A	0.252	4.812	0.000	0.000	0.000	0.538	0.000	0.215	0.472

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	295	51	0	0	0	32	0	15	0
N.S.	1	1.01	0.17	0.00	0.00	0.00	0.11	0.00	0.05	0.00
time (sec)	N/A	0.269	10.013	0.000	0.000	0.000	0.586	0.000	0.257	0.000

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	82	50	47	64	58	1584	70	34	55
N.S.	1	1.02	0.62	0.59	0.80	0.72	19.80	0.88	0.42	0.69
time (sec)	N/A	0.197	0.033	0.365	0.033	0.062	1.582	0.121	0.200	0.427

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	38	35	47	46	561	57	34	41
N.S.	1	1.00	0.64	0.59	0.80	0.78	9.51	0.97	0.58	0.69
time (sec)	N/A	0.180	0.031	0.351	0.028	0.060	1.019	0.110	0.226	0.369

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	27	24	30	35	46	34	34	24
N.S.	1	1.05	0.71	0.63	0.79	0.92	1.21	0.89	0.89	0.63
time (sec)	N/A	0.168	0.026	0.347	0.027	0.063	0.200	0.119	0.228	0.312

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	14	24	26	14	32	14
N.S.	1	1.00	1.00	0.83	0.78	1.33	1.44	0.78	1.78	0.78
time (sec)	N/A	0.137	0.002	0.325	0.025	0.062	0.111	0.120	0.243	0.194

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	106	121	122	100	327	41	101	31	123
N.S.	1	1.02	1.16	1.17	0.96	3.14	0.39	0.97	0.30	1.18
time (sec)	N/A	0.213	0.105	0.418	0.141	0.075	0.605	0.400	0.225	0.365

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	135	122	136	453	41	134	33	178
N.S.	1	1.09	1.10	0.99	1.11	3.68	0.33	1.09	0.27	1.45
time (sec)	N/A	0.221	0.156	0.631	0.149	0.082	0.935	0.414	0.216	0.441

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	164	154	163	176	437	41	154	33	224
N.S.	1	1.03	0.97	1.03	1.11	2.75	0.26	0.97	0.21	1.41
time (sec)	N/A	0.240	0.213	0.625	0.172	0.077	2.704	0.440	0.239	0.565

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	577	624	65	0	0	0	27	0	34	0
N.S.	1	1.08	0.11	0.00	0.00	0.00	0.05	0.00	0.06	0.00
time (sec)	N/A	0.480	5.408	0.000	0.000	0.000	0.539	0.000	0.220	0.000

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	553	594	55	0	0	0	27	0	34	0
N.S.	1	1.07	0.10	0.00	0.00	0.00	0.05	0.00	0.06	0.00
time (sec)	N/A	0.458	5.141	0.000	0.000	0.000	0.541	0.000	0.240	0.000

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	552	597	58	0	0	0	24	0	30	37
N.S.	1	1.08	0.11	0.00	0.00	0.00	0.04	0.00	0.05	0.07
time (sec)	N/A	0.465	0.007	0.000	0.000	0.000	0.507	0.000	0.224	0.286

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	571	623	52	0	0	0	27	0	33	40
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.06	0.07
time (sec)	N/A	0.489	5.479	0.000	0.000	0.000	0.618	0.000	0.236	0.562

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	A	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	599	653	54	0	0	0	32	0	33	0
N.S.	1	1.09	0.09	0.00	0.00	0.00	0.05	0.00	0.06	0.00
time (sec)	N/A	0.518	10.008	0.000	0.000	0.000	0.624	0.000	0.233	0.000

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	195	209	233	0	0	0	0	0	79	0
N.S.	1	1.07	1.19	0.00	0.00	0.00	0.00	0.00	0.41	0.00
time (sec)	N/A	0.303	1.941	0.000	0.000	0.000	0.000	0.000	0.312	0.000

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	171	204	0	0	0	46	0	59	0
N.S.	1	1.04	1.24	0.00	0.00	0.00	0.28	0.00	0.36	0.00
time (sec)	N/A	0.261	1.393	0.000	0.000	0.000	17.168	0.000	0.303	0.000

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	140	173	0	0	0	46	0	38	0
N.S.	1	1.05	1.30	0.00	0.00	0.00	0.35	0.00	0.29	0.00
time (sec)	N/A	0.236	0.947	0.000	0.000	0.000	0.833	0.000	0.282	0.000

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	182	0	0	0	49	0	19	0
N.S.	1	1.07	1.39	0.00	0.00	0.00	0.37	0.00	0.15	0.00
time (sec)	N/A	0.249	0.579	0.000	0.000	0.000	1.495	0.000	0.305	0.000

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	78	0	20	0
N.S.	1	1.00	0.93	0.75	1.25	0.89	2.79	0.00	0.71	0.00
time (sec)	N/A	0.148	0.931	0.335	0.041	0.079	33.361	0.000	0.287	0.000

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	46	31	0	46	0	0	41	0
N.S.	1	0.98	0.79	0.53	0.00	0.79	0.00	0.00	0.71	0.00
time (sec)	N/A	0.172	2.248	0.326	0.000	0.080	0.000	0.000	0.261	0.000

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	47	42	0	57	0	0	52	0
N.S.	1	1.04	0.53	0.47	0.00	0.64	0.00	0.00	0.58	0.00
time (sec)	N/A	0.189	2.871	0.338	0.000	0.082	0.000	0.000	0.291	0.000

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	129	58	53	64	68	0	0	63	0
N.S.	1	1.08	0.48	0.44	0.53	0.57	0.00	0.00	0.52	0.00
time (sec)	N/A	0.212	4.521	0.348	0.038	0.079	0.000	0.000	0.280	0.000

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	451	396	103	0	0	0	46	0	89	0
N.S.	1	0.88	0.23	0.00	0.00	0.00	0.10	0.00	0.20	0.00
time (sec)	N/A	0.402	10.045	0.000	0.000	0.000	77.678	0.000	0.350	0.000

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	418	358	85	0	0	0	46	0	69	0
N.S.	1	0.86	0.20	0.00	0.00	0.00	0.11	0.00	0.17	0.00
time (sec)	N/A	0.352	10.037	0.000	0.000	0.000	3.248	0.000	0.380	0.000

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	320	54	0	0	0	46	0	48	0
N.S.	1	0.84	0.14	0.00	0.00	0.00	0.12	0.00	0.13	0.00
time (sec)	N/A	0.347	10.009	0.000	0.000	0.000	0.743	0.000	0.356	0.000

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	391	326	56	0	0	0	32	0	51	0
N.S.	1	0.83	0.14	0.00	0.00	0.00	0.08	0.00	0.13	0.00
time (sec)	N/A	0.338	10.013	0.000	0.000	0.000	6.394	0.000	0.430	0.000

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	422	366	56	0	0	0	0	0	51	0
N.S.	1	0.87	0.13	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.369	10.016	0.000	0.000	0.000	0.000	0.000	0.488	0.000

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	38	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.66	0.00
time (sec)	N/A	0.168	10.011	0.000	0.000	0.000	0.980	0.000	0.415	0.000

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	48	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.83	0.00
time (sec)	N/A	0.169	10.011	0.000	0.000	0.000	0.628	0.000	0.319	0.000

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	49	0	51	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.88	0.00	0.91	0.00
time (sec)	N/A	0.169	10.014	0.000	0.000	0.000	1.083	0.000	0.392	0.000

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	223	240	259	0	0	0	0	0	99	0
N.S.	1	1.08	1.16	0.00	0.00	0.00	0.00	0.00	0.44	0.00
time (sec)	N/A	0.334	2.571	0.000	0.000	0.000	0.000	0.000	0.364	0.000

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	202	230	0	0	0	46	0	79	0
N.S.	1	1.05	1.20	0.00	0.00	0.00	0.24	0.00	0.41	0.00
time (sec)	N/A	0.285	1.562	0.000	0.000	0.000	37.361	0.000	0.323	0.000

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	163	171	204	0	0	0	46	0	56	0
N.S.	1	1.05	1.25	0.00	0.00	0.00	0.28	0.00	0.34	0.00
time (sec)	N/A	0.266	1.258	0.000	0.000	0.000	3.185	0.000	0.342	0.000

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	172	203	0	0	0	49	0	58	0
N.S.	1	1.12	1.33	0.00	0.00	0.00	0.32	0.00	0.38	0.00
time (sec)	N/A	0.268	0.863	0.000	0.000	0.000	3.549	0.000	0.371	0.000

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	171	200	0	0	0	53	0	57	0
N.S.	1	1.09	1.27	0.00	0.00	0.00	0.34	0.00	0.36	0.00
time (sec)	N/A	0.273	0.937	0.000	0.000	0.000	33.131	0.000	0.445	0.000

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	43	0	0	41	0
N.S.	1	1.00	0.93	0.75	0.00	1.54	0.00	0.00	1.46	0.00
time (sec)	N/A	0.146	2.563	0.323	0.000	0.088	0.000	0.000	0.290	0.000

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	36	31	0	57	0	0	52	0
N.S.	1	0.98	0.62	0.53	0.00	0.98	0.00	0.00	0.90	0.00
time (sec)	N/A	0.169	3.040	0.353	0.000	0.082	0.000	0.000	0.278	0.000

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	47	42	0	68	0	0	63	0
N.S.	1	1.04	0.53	0.47	0.00	0.76	0.00	0.00	0.71	0.00
time (sec)	N/A	0.189	4.176	0.359	0.000	0.120	0.000	0.000	0.291	0.000

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	479	427	102	0	0	0	0	0	109	0
N.S.	1	0.89	0.21	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.415	10.054	0.000	0.000	0.000	0.000	0.000	0.414	0.000

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	448	389	89	0	0	0	46	0	89	0
N.S.	1	0.87	0.20	0.00	0.00	0.00	0.10	0.00	0.20	0.00
time (sec)	N/A	0.381	10.043	0.000	0.000	0.000	7.132	0.000	0.374	0.000

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	414	351	55	0	0	0	46	0	66	0
N.S.	1	0.85	0.13	0.00	0.00	0.00	0.11	0.00	0.16	0.00
time (sec)	N/A	0.347	10.009	0.000	0.000	0.000	2.308	0.000	0.445	0.000

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	414	354	57	0	0	0	32	0	68	0
N.S.	1	0.86	0.14	0.00	0.00	0.00	0.08	0.00	0.16	0.00
time (sec)	N/A	0.342	10.016	0.000	0.000	0.000	8.358	0.000	0.453	0.000

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	419	360	57	0	0	0	32	0	68	0
N.S.	1	0.86	0.14	0.00	0.00	0.00	0.08	0.00	0.16	0.00
time (sec)	N/A	0.360	10.011	0.000	0.000	0.000	177.403	0.000	0.466	0.000

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	450	400	57	0	0	0	0	0	68	0
N.S.	1	0.89	0.13	0.00	0.00	0.00	0.00	0.00	0.15	0.00
time (sec)	N/A	0.388	10.014	0.000	0.000	0.000	0.000	0.000	0.521	0.000

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	56	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	0.95	0.00
time (sec)	N/A	0.173	10.012	0.000	0.000	0.000	4.001	0.000	0.506	0.000

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	46	0	66	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.78	0.00	1.12	0.00
time (sec)	N/A	0.172	10.008	0.000	0.000	0.000	2.348	0.000	0.352	0.000

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	0	49	0	69	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	1.21	0.00
time (sec)	N/A	0.172	10.010	0.000	0.000	0.000	2.318	0.000	0.365	0.000

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	198	216	233	0	0	0	0	0	19	0
N.S.	1	1.09	1.18	0.00	0.00	0.00	0.00	0.00	0.10	0.00
time (sec)	N/A	0.308	5.813	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	167	178	207	0	0	0	0	0	19	0
N.S.	1	1.07	1.24	0.00	0.00	0.00	0.00	0.00	0.11	0.00
time (sec)	N/A	0.286	2.445	0.000	0.000	0.000	0.000	0.000	0.234	0.000

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	174	0	0	0	44	0	19	0
N.S.	1	1.07	1.33	0.00	0.00	0.00	0.34	0.00	0.15	0.00
time (sec)	N/A	0.238	1.426	0.000	0.000	0.000	15.498	0.000	0.231	0.000

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	113	146	0	0	0	44	0	19	0
N.S.	1	1.07	1.38	0.00	0.00	0.00	0.42	0.00	0.18	0.00
time (sec)	N/A	0.213	0.851	0.000	0.000	0.000	0.695	0.000	0.235	0.000

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	35	25	36	0	19	0
N.S.	1	1.00	0.93	0.75	1.25	0.89	1.29	0.00	0.68	0.00
time (sec)	N/A	0.145	0.876	0.344	0.040	0.120	1.805	0.000	0.354	0.000

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	34	29	0	35	78	0	19	0
N.S.	1	0.98	0.59	0.50	0.00	0.60	1.34	0.00	0.33	0.00
time (sec)	N/A	0.171	1.518	0.346	0.000	0.139	68.302	0.000	0.283	0.000

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	91	47	42	0	46	0	0	19	0
N.S.	1	1.02	0.53	0.47	0.00	0.52	0.00	0.00	0.21	0.00
time (sec)	N/A	0.190	3.551	0.347	0.000	0.124	0.000	0.000	0.330	0.000

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	127	58	53	64	57	0	0	19	0
N.S.	1	1.06	0.48	0.44	0.53	0.48	0.00	0.00	0.16	0.00
time (sec)	N/A	0.209	5.095	0.352	0.047	0.116	0.000	0.000	0.330	0.000

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	421	365	87	0	0	0	44	0	19	0
N.S.	1	0.87	0.21	0.00	0.00	0.00	0.10	0.00	0.05	0.00
time (sec)	N/A	0.350	10.025	0.000	0.000	0.000	74.795	0.000	0.244	0.000

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	327	66	0	0	0	44	0	19	0
N.S.	1	0.84	0.17	0.00	0.00	0.00	0.11	0.00	0.05	0.00
time (sec)	N/A	0.332	10.018	0.000	0.000	0.000	2.393	0.000	0.267	0.000

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	364	299	54	0	0	0	31	0	19	0
N.S.	1	0.82	0.15	0.00	0.00	0.00	0.09	0.00	0.05	0.00
time (sec)	N/A	0.329	10.013	0.000	0.000	0.000	0.851	0.000	0.282	0.000

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	394	332	56	0	0	0	48	0	19	0
N.S.	1	0.84	0.14	0.00	0.00	0.00	0.12	0.00	0.05	0.00
time (sec)	N/A	0.337	10.011	0.000	0.000	0.000	13.842	0.000	0.272	0.000

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	425	372	56	0	0	0	0	0	19	0
N.S.	1	0.88	0.13	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	0.385	10.012	0.000	0.000	0.000	0.000	0.000	0.305	0.000

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.167	10.011	0.000	0.000	0.000	0.767	0.000	0.273	0.000

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	46	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.79	0.00	0.33	0.00
time (sec)	N/A	0.166	10.012	0.000	0.000	0.000	0.670	0.000	0.260	0.000

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	19	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	0.34	0.00
time (sec)	N/A	0.168	10.011	0.000	0.000	0.000	1.405	0.000	0.243	0.000

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	132	93	0	0	0	29	0	69	0
N.S.	1	1.09	0.77	0.00	0.00	0.00	0.24	0.00	0.57	0.00
time (sec)	N/A	0.214	5.971	0.000	0.000	0.000	0.500	0.000	0.243	0.000

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	102	62	0	0	0	29	0	49	0
N.S.	1	1.05	0.64	0.00	0.00	0.00	0.30	0.00	0.51	0.00
time (sec)	N/A	0.192	5.923	0.000	0.000	0.000	0.482	0.000	0.216	0.000

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	46	0	0	0	26	0	27	37
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.35	0.00	0.36	0.49
time (sec)	N/A	0.170	0.004	0.000	0.000	0.000	0.465	0.000	0.243	0.211

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	49	0	0	0	29	0	45	40
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.40	0.00	0.62	0.56
time (sec)	N/A	0.172	5.937	0.000	0.000	0.000	0.467	0.000	0.285	0.435

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	100	51	0	0	0	34	0	48	0
N.S.	1	1.01	0.52	0.00	0.00	0.00	0.34	0.00	0.48	0.00
time (sec)	N/A	0.192	10.008	0.000	0.000	0.000	0.515	0.000	0.263	0.000

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	130	51	0	0	0	34	0	48	0
N.S.	1	1.06	0.41	0.00	0.00	0.00	0.28	0.00	0.39	0.00
time (sec)	N/A	0.211	10.009	0.000	0.000	0.000	0.566	0.000	0.251	0.000

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	95	0	0	0	31	0	74	0
N.S.	1	1.09	0.75	0.00	0.00	0.00	0.25	0.00	0.59	0.00
time (sec)	N/A	0.218	6.160	0.000	0.000	0.000	0.503	0.000	0.264	0.000

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	64	0	0	0	31	0	52	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.31	0.00	0.51	0.00
time (sec)	N/A	0.197	6.006	0.000	0.000	0.000	0.476	0.000	0.253	0.000

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	29	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.37	0.49
time (sec)	N/A	0.169	0.003	0.000	0.000	0.000	0.471	0.000	0.248	0.236

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	48	41
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.63	0.54
time (sec)	N/A	0.175	6.022	0.000	0.000	0.000	0.487	0.000	0.263	0.448

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	52	0	0	0	36	0	51	0
N.S.	1	1.00	0.50	0.00	0.00	0.00	0.35	0.00	0.50	0.00
time (sec)	N/A	0.191	10.009	0.000	0.000	0.000	0.515	0.000	0.270	0.000

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	134	52	0	0	0	36	0	51	0
N.S.	1	1.05	0.41	0.00	0.00	0.00	0.28	0.00	0.40	0.00
time (sec)	N/A	0.208	10.011	0.000	0.000	0.000	0.592	0.000	0.244	0.000

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	143	156	93	0	0	0	29	0	70	0
N.S.	1	1.09	0.65	0.00	0.00	0.00	0.20	0.00	0.49	0.00
time (sec)	N/A	0.229	7.257	0.000	0.000	0.000	0.588	0.000	0.233	0.000

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	62	0	0	0	29	0	50	0
N.S.	1	1.06	0.52	0.00	0.00	0.00	0.24	0.00	0.42	0.00
time (sec)	N/A	0.203	6.873	0.000	0.000	0.000	0.543	0.000	0.281	0.000

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	46	0	0	0	26	0	27	37
N.S.	1	1.04	0.50	0.00	0.00	0.00	0.28	0.00	0.29	0.40
time (sec)	N/A	0.181	0.003	0.000	0.000	0.000	0.561	0.000	0.233	0.228

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	49	0	0	0	29	0	45	40
N.S.	1	1.09	0.56	0.00	0.00	0.00	0.33	0.00	0.51	0.45
time (sec)	N/A	0.186	6.982	0.000	0.000	0.000	0.533	0.000	0.247	0.528

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	51	0	0	0	34	0	48	0
N.S.	1	1.04	0.42	0.00	0.00	0.00	0.28	0.00	0.40	0.00
time (sec)	N/A	0.206	10.013	0.000	0.000	0.000	0.554	0.000	0.263	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	156	51	0	0	0	34	0	48	0
N.S.	1	1.08	0.35	0.00	0.00	0.00	0.23	0.00	0.33	0.00
time (sec)	N/A	0.230	10.010	0.000	0.000	0.000	0.658	0.000	0.267	0.000

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	95	0	0	0	31	0	74	0
N.S.	1	1.09	0.75	0.00	0.00	0.00	0.25	0.00	0.59	0.00
time (sec)	N/A	0.212	7.249	0.000	0.000	0.000	0.591	0.000	0.241	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	106	64	0	0	0	31	0	53	0
N.S.	1	1.05	0.63	0.00	0.00	0.00	0.31	0.00	0.52	0.00
time (sec)	N/A	0.190	6.893	0.000	0.000	0.000	0.539	0.000	0.269	0.000

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	47	0	0	0	27	0	29	38
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.35	0.00	0.37	0.49
time (sec)	N/A	0.169	0.003	0.000	0.000	0.000	0.506	0.000	0.232	0.224

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	50	0	0	0	31	0	48	41
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.41	0.00	0.63	0.54
time (sec)	N/A	0.175	7.043	0.000	0.000	0.000	0.544	0.000	0.243	0.553

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	104	52	0	0	0	36	0	51	0
N.S.	1	1.01	0.50	0.00	0.00	0.00	0.35	0.00	0.50	0.00
time (sec)	N/A	0.188	10.010	0.000	0.000	0.000	0.562	0.000	0.248	0.000

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	135	52	0	0	0	36	0	51	0
N.S.	1	1.05	0.41	0.00	0.00	0.00	0.28	0.00	0.40	0.00
time (sec)	N/A	0.212	10.010	0.000	0.000	0.000	0.628	0.000	0.278	0.000

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	156	79	0	0	0	29	0	90	0
N.S.	1	1.10	0.56	0.00	0.00	0.00	0.20	0.00	0.63	0.00
time (sec)	N/A	0.224	7.567	0.000	0.000	0.000	0.707	0.000	0.255	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	67	0	0	0	29	0	70	0
N.S.	1	1.07	0.57	0.00	0.00	0.00	0.25	0.00	0.59	0.00
time (sec)	N/A	0.203	7.096	0.000	0.000	0.000	0.632	0.000	0.237	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	47	0	0	0	26	0	45	37
N.S.	1	1.05	0.51	0.00	0.00	0.00	0.28	0.00	0.49	0.40
time (sec)	N/A	0.177	0.003	0.000	0.000	0.000	0.575	0.000	0.238	0.242

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	97	50	0	0	0	29	0	64	40
N.S.	1	1.05	0.54	0.00	0.00	0.00	0.32	0.00	0.70	0.43
time (sec)	N/A	0.179	7.142	0.000	0.000	0.000	0.637	0.000	0.250	0.702

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	52	0	0	0	34	0	65	0
N.S.	1	1.00	0.54	0.00	0.00	0.00	0.35	0.00	0.68	0.00
time (sec)	N/A	0.186	10.014	0.000	0.000	0.000	0.635	0.000	0.285	0.000

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	120	124	52	0	0	0	34	0	66	0
N.S.	1	1.03	0.43	0.00	0.00	0.00	0.28	0.00	0.55	0.00
time (sec)	N/A	0.206	10.008	0.000	0.000	0.000	0.693	0.000	0.278	0.000

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	162	81	0	0	0	31	0	95	0
N.S.	1	1.09	0.55	0.00	0.00	0.00	0.21	0.00	0.64	0.00
time (sec)	N/A	0.231	7.803	0.000	0.000	0.000	0.701	0.000	0.242	0.000

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	131	70	0	0	0	31	0	74	0
N.S.	1	1.07	0.57	0.00	0.00	0.00	0.25	0.00	0.60	0.00
time (sec)	N/A	0.210	7.410	0.000	0.000	0.000	0.641	0.000	0.241	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	101	48	0	0	0	27	0	48	38
N.S.	1	1.05	0.50	0.00	0.00	0.00	0.28	0.00	0.50	0.40
time (sec)	N/A	0.182	0.003	0.000	0.000	0.000	0.579	0.000	0.205	0.263

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	101	51	0	0	0	31	0	68	41
N.S.	1	1.05	0.53	0.00	0.00	0.00	0.32	0.00	0.71	0.43
time (sec)	N/A	0.187	7.449	0.000	0.000	0.000	0.665	0.000	0.253	0.816

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	101	53	0	0	0	36	0	69	0
N.S.	1	1.01	0.53	0.00	0.00	0.00	0.36	0.00	0.69	0.00
time (sec)	N/A	0.190	10.009	0.000	0.000	0.000	0.668	0.000	0.297	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	125	128	53	0	0	0	36	0	70	0
N.S.	1	1.02	0.42	0.00	0.00	0.00	0.29	0.00	0.56	0.00
time (sec)	N/A	0.207	10.009	0.000	0.000	0.000	0.733	0.000	0.321	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	162	90	0	0	0	27	0	15	0
N.S.	1	1.11	0.62	0.00	0.00	0.00	0.18	0.00	0.10	0.00
time (sec)	N/A	0.223	5.939	0.000	0.000	0.000	0.512	0.000	0.221	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	132	79	0	0	0	27	0	15	0
N.S.	1	1.08	0.65	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.206	5.756	0.000	0.000	0.000	0.495	0.000	0.228	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	102	62	0	0	0	27	0	15	0
N.S.	1	1.04	0.63	0.00	0.00	0.00	0.28	0.00	0.15	0.00
time (sec)	N/A	0.187	5.641	0.000	0.000	0.000	0.487	0.000	0.222	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	75	46	0	0	0	24	0	11	37
N.S.	1	1.06	0.65	0.00	0.00	0.00	0.34	0.00	0.15	0.52
time (sec)	N/A	0.170	0.003	0.000	0.000	0.000	0.464	0.000	0.227	0.243

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	102	49	0	0	0	27	0	15	40
N.S.	1	1.10	0.53	0.00	0.00	0.00	0.29	0.00	0.16	0.43
time (sec)	N/A	0.190	5.687	0.000	0.000	0.000	0.492	0.000	0.222	0.484

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	132	51	0	0	0	32	0	15	0
N.S.	1	1.06	0.41	0.00	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.208	10.009	0.000	0.000	0.000	0.541	0.000	0.208	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	162	51	0	0	0	32	0	15	0
N.S.	1	1.09	0.34	0.00	0.00	0.00	0.22	0.00	0.10	0.00
time (sec)	N/A	0.234	10.011	0.000	0.000	0.000	0.579	0.000	0.229	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	143	89	0	0	0	29	0	16	0
N.S.	1	1.11	0.69	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.220	5.942	0.000	0.000	0.000	0.541	0.000	0.246	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	79	0	0	0	29	0	16	0
N.S.	1	1.08	0.76	0.00	0.00	0.00	0.28	0.00	0.15	0.00
time (sec)	N/A	0.193	5.794	0.000	0.000	0.000	0.500	0.000	0.209	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	16	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	0.20	0.00
time (sec)	N/A	0.171	5.618	0.000	0.000	0.000	0.468	0.000	0.237	0.000

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	12	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.21	0.66
time (sec)	N/A	0.157	0.004	0.000	0.000	0.000	0.457	0.000	0.225	0.266

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	50	0	0	0	29	0	16	41
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.37	0.00	0.20	0.52
time (sec)	N/A	0.173	5.649	0.000	0.000	0.000	0.486	0.000	0.213	0.544

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	110	52	0	0	0	34	0	16	0
N.S.	1	1.04	0.49	0.00	0.00	0.00	0.32	0.00	0.15	0.00
time (sec)	N/A	0.195	10.010	0.000	0.000	0.000	0.555	0.000	0.225	0.000

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	141	52	0	0	0	34	0	16	0
N.S.	1	1.08	0.40	0.00	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.217	10.011	0.000	0.000	0.000	0.572	0.000	0.258	0.000

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	138	90	0	0	0	27	0	15	0
N.S.	1	1.11	0.73	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.217	6.049	0.000	0.000	0.000	0.495	0.000	0.263	0.000

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	108	78	0	0	0	27	0	15	0
N.S.	1	1.08	0.78	0.00	0.00	0.00	0.27	0.00	0.15	0.00
time (sec)	N/A	0.190	5.987	0.000	0.000	0.000	0.456	0.000	0.231	0.000

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	62	0	0	0	27	0	15	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.35	0.00	0.19	0.00
time (sec)	N/A	0.168	5.577	0.000	0.000	0.000	0.441	0.000	0.232	0.000

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	0	0	0	24	0	11	37
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.43	0.00	0.20	0.66
time (sec)	N/A	0.154	0.004	0.000	0.000	0.000	0.424	0.000	0.252	0.251

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	49	0	0	0	27	0	15	40
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.36	0.00	0.20	0.53
time (sec)	N/A	0.171	5.838	0.000	0.000	0.000	0.505	0.000	0.238	0.456

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	51	0	0	0	32	0	15	0
N.S.	1	1.04	0.50	0.00	0.00	0.00	0.31	0.00	0.15	0.00
time (sec)	N/A	0.192	10.008	0.000	0.000	0.000	0.556	0.000	0.211	0.000

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	136	51	0	0	0	32	0	15	0
N.S.	1	1.08	0.40	0.00	0.00	0.00	0.25	0.00	0.12	0.00
time (sec)	N/A	0.210	10.010	0.000	0.000	0.000	0.594	0.000	0.251	0.000

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	143	91	0	0	0	29	0	16	0
N.S.	1	1.11	0.71	0.00	0.00	0.00	0.22	0.00	0.12	0.00
time (sec)	N/A	0.213	6.051	0.000	0.000	0.000	0.504	0.000	0.219	0.000

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	112	77	0	0	0	29	0	16	0
N.S.	1	1.08	0.74	0.00	0.00	0.00	0.28	0.00	0.15	0.00
time (sec)	N/A	0.195	6.027	0.000	0.000	0.000	0.481	0.000	0.219	0.000

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	0	0	0	29	0	16	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.36	0.00	0.20	0.00
time (sec)	N/A	0.174	5.683	0.000	0.000	0.000	0.449	0.000	0.205	0.000

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	0	26	0	12	38
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.45	0.00	0.21	0.66
time (sec)	N/A	0.160	0.007	0.000	0.000	0.000	0.430	0.000	0.232	0.273

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	50	0	0	0	29	0	16	41
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.37	0.00	0.21	0.53
time (sec)	N/A	0.175	5.875	0.000	0.000	0.000	0.517	0.000	0.211	0.501

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	109	52	0	0	0	34	0	16	0
N.S.	1	1.03	0.49	0.00	0.00	0.00	0.32	0.00	0.15	0.00
time (sec)	N/A	0.196	10.009	0.000	0.000	0.000	0.588	0.000	0.205	0.000

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	140	52	0	0	0	34	0	16	0
N.S.	1	1.07	0.40	0.00	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.218	10.009	0.000	0.000	0.000	0.655	0.000	0.218	0.000

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	134	78	0	0	0	27	0	34	0
N.S.	1	1.08	0.63	0.00	0.00	0.00	0.22	0.00	0.27	0.00
time (sec)	N/A	0.215	6.407	0.000	0.000	0.000	0.543	0.000	0.211	0.000

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	104	65	0	0	0	27	0	34	0
N.S.	1	1.04	0.65	0.00	0.00	0.00	0.27	0.00	0.34	0.00
time (sec)	N/A	0.193	6.262	0.000	0.000	0.000	0.482	0.000	0.225	0.000

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	53	0	0	0	27	0	34	0
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.36	0.00	0.46	0.00
time (sec)	N/A	0.171	5.848	0.000	0.000	0.000	0.473	0.000	0.207	0.000

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	24	0	30	37
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.43	0.00	0.54	0.66
time (sec)	N/A	0.158	0.007	0.000	0.000	0.000	0.457	0.000	0.236	0.289

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	52	0	0	0	27	0	33	40
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.36	0.00	0.43	0.53
time (sec)	N/A	0.172	6.074	0.000	0.000	0.000	0.558	0.000	0.221	0.549

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	106	54	0	0	0	32	0	33	0
N.S.	1	1.04	0.53	0.00	0.00	0.00	0.31	0.00	0.32	0.00
time (sec)	N/A	0.195	10.010	0.000	0.000	0.000	0.597	0.000	0.211	0.000

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	136	54	0	0	0	32	0	33	0
N.S.	1	1.08	0.43	0.00	0.00	0.00	0.25	0.00	0.26	0.00
time (sec)	N/A	0.211	10.009	0.000	0.000	0.000	0.676	0.000	0.263	0.000

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	138	78	0	0	0	29	0	37	0
N.S.	1	1.11	0.63	0.00	0.00	0.00	0.23	0.00	0.30	0.00
time (sec)	N/A	0.214	6.497	0.000	0.000	0.000	0.521	0.000	0.224	0.000

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	107	66	0	0	0	29	0	37	0
N.S.	1	1.06	0.65	0.00	0.00	0.00	0.29	0.00	0.37	0.00
time (sec)	N/A	0.190	6.327	0.000	0.000	0.000	0.489	0.000	0.211	0.000

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	29	0	37	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.38	0.00	0.48	0.00
time (sec)	N/A	0.169	5.922	0.000	0.000	0.000	0.479	0.000	0.196	0.000

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	56	0	0	0	26	0	33	38
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.34	0.00	0.43	0.49
time (sec)	N/A	0.165	0.008	0.000	0.000	0.000	0.500	0.000	0.221	0.310

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	53	0	0	0	29	0	36	41
N.S.	1	1.06	0.54	0.00	0.00	0.00	0.29	0.00	0.36	0.41
time (sec)	N/A	0.198	6.182	0.000	0.000	0.000	0.578	0.000	0.215	0.564

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	136	55	0	0	0	34	0	36	0
N.S.	1	1.08	0.44	0.00	0.00	0.00	0.27	0.00	0.29	0.00
time (sec)	N/A	0.216	10.009	0.000	0.000	0.000	0.644	0.000	0.247	0.000

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	167	55	0	0	0	34	0	36	0
N.S.	1	1.11	0.36	0.00	0.00	0.00	0.23	0.00	0.24	0.00
time (sec)	N/A	0.246	10.009	0.000	0.000	0.000	0.686	0.000	0.269	0.000

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	137	79	0	0	0	27	0	34	0
N.S.	1	1.13	0.65	0.00	0.00	0.00	0.22	0.00	0.28	0.00
time (sec)	N/A	0.213	7.650	0.000	0.000	0.000	0.538	0.000	0.228	0.000

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	105	65	0	0	0	27	0	34	0
N.S.	1	1.06	0.66	0.00	0.00	0.00	0.27	0.00	0.34	0.00
time (sec)	N/A	0.190	7.517	0.000	0.000	0.000	0.534	0.000	0.209	0.000

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	27	0	34	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.35	0.00	0.44	0.00
time (sec)	N/A	0.169	7.193	0.000	0.000	0.000	0.531	0.000	0.215	0.000

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	0	0	0	24	0	30	37
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.31	0.00	0.38	0.47
time (sec)	N/A	0.166	0.008	0.000	0.000	0.000	0.572	0.000	0.230	0.330

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	105	52	0	0	0	27	0	33	40
N.S.	1	1.04	0.51	0.00	0.00	0.00	0.27	0.00	0.33	0.40
time (sec)	N/A	0.187	7.319	0.000	0.000	0.000	0.659	0.000	0.240	0.602

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	54	0	0	0	32	0	33	0
N.S.	1	1.10	0.45	0.00	0.00	0.00	0.26	0.00	0.27	0.00
time (sec)	N/A	0.210	10.009	0.000	0.000	0.000	0.725	0.000	0.228	0.000

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	165	54	0	0	0	32	0	33	0
N.S.	1	1.12	0.37	0.00	0.00	0.00	0.22	0.00	0.22	0.00
time (sec)	N/A	0.234	10.011	0.000	0.000	0.000	0.810	0.000	0.248	0.000

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	142	80	0	0	0	29	0	37	0
N.S.	1	1.13	0.63	0.00	0.00	0.00	0.23	0.00	0.29	0.00
time (sec)	N/A	0.210	7.479	0.000	0.000	0.000	0.551	0.000	0.240	0.000

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	109	66	0	0	0	29	0	37	0
N.S.	1	1.06	0.64	0.00	0.00	0.00	0.28	0.00	0.36	0.00
time (sec)	N/A	0.185	7.487	0.000	0.000	0.000	0.558	0.000	0.232	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	59	0	0	0	29	0	37	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.36	0.00	0.46	0.00
time (sec)	N/A	0.168	7.015	0.000	0.000	0.000	0.547	0.000	0.232	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	56	0	0	0	26	0	33	38
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.32	0.00	0.41	0.47
time (sec)	N/A	0.166	0.010	0.000	0.000	0.000	0.565	0.000	0.235	0.333

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	108	53	0	0	0	29	0	36	41
N.S.	1	1.03	0.50	0.00	0.00	0.00	0.28	0.00	0.34	0.39
time (sec)	N/A	0.189	7.548	0.000	0.000	0.000	0.639	0.000	0.223	0.639

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	137	55	0	0	0	34	0	36	0
N.S.	1	1.09	0.44	0.00	0.00	0.00	0.27	0.00	0.29	0.00
time (sec)	N/A	0.217	10.014	0.000	0.000	0.000	0.738	0.000	0.231	0.000

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	170	55	0	0	0	34	0	36	0
N.S.	1	1.11	0.36	0.00	0.00	0.00	0.22	0.00	0.24	0.00
time (sec)	N/A	0.234	10.010	0.000	0.000	0.000	0.890	0.000	0.248	0.000

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	110	54	20	0	0	27	0	15	0
N.S.	1	1.11	0.55	0.20	0.00	0.00	0.27	0.00	0.15	0.00
time (sec)	N/A	0.194	5.116	0.351	0.000	0.000	0.462	0.000	0.257	0.000

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	87	49	20	0	0	27	0	15	0
N.S.	1	1.07	0.60	0.25	0.00	0.00	0.33	0.00	0.19	0.00
time (sec)	N/A	0.176	5.111	0.341	0.000	0.000	0.441	0.000	0.248	0.000

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	64	41	20	0	0	27	0	15	0
N.S.	1	1.02	0.65	0.32	0.00	0.00	0.43	0.00	0.24	0.00
time (sec)	N/A	0.158	4.984	0.329	0.000	0.000	0.393	0.000	0.217	0.000

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	24	18	0	0	26	0	11	16
N.S.	1	1.00	0.56	0.42	0.00	0.00	0.60	0.00	0.26	0.37
time (sec)	N/A	0.148	4.942	0.246	0.000	0.000	0.377	0.000	0.238	0.100

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	66	27	20	0	0	29	0	15	36
N.S.	1	1.05	0.43	0.32	0.00	0.00	0.46	0.00	0.24	0.57
time (sec)	N/A	0.159	4.848	0.328	0.000	0.000	0.398	0.000	0.245	0.440

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	89	29	20	0	0	32	0	15	0
N.S.	1	1.07	0.35	0.24	0.00	0.00	0.39	0.00	0.18	0.00
time (sec)	N/A	0.178	10.005	0.345	0.000	0.000	0.477	0.000	0.252	0.000

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	112	29	20	0	0	32	0	15	0
N.S.	1	1.11	0.29	0.20	0.00	0.00	0.32	0.00	0.15	0.00
time (sec)	N/A	0.191	10.005	0.342	0.000	0.000	0.496	0.000	0.265	0.000

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	54	20	0	0	29	0	15	0
N.S.	1	1.12	0.65	0.24	0.00	0.00	0.35	0.00	0.18	0.00
time (sec)	N/A	0.184	4.980	0.366	0.000	0.000	0.460	0.000	0.230	0.000

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	49	20	0	0	29	0	15	0
N.S.	1	1.08	0.75	0.31	0.00	0.00	0.45	0.00	0.23	0.00
time (sec)	N/A	0.167	4.780	0.359	0.000	0.000	0.434	0.000	0.234	0.000

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	20	0	0	29	0	15	0
N.S.	1	1.00	0.87	0.43	0.00	0.00	0.62	0.00	0.32	0.00
time (sec)	N/A	0.152	4.690	0.342	0.000	0.000	0.413	0.000	0.219	0.000

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	24	18	0	0	27	0	11	16
N.S.	1	1.00	0.86	0.64	0.00	0.00	0.96	0.00	0.39	0.57
time (sec)	N/A	0.140	4.674	0.331	0.000	0.000	0.403	0.000	0.234	0.239

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	27	20	0	0	31	0	15	36
N.S.	1	1.00	0.57	0.43	0.00	0.00	0.66	0.00	0.32	0.77
time (sec)	N/A	0.153	4.738	0.353	0.000	0.000	0.434	0.000	0.237	0.437

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	70	29	20	0	0	34	0	15	0
N.S.	1	1.04	0.43	0.30	0.00	0.00	0.51	0.00	0.22	0.00
time (sec)	N/A	0.167	10.006	0.369	0.000	0.000	0.473	0.000	0.220	0.000

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	93	29	20	0	0	34	0	15	0
N.S.	1	1.09	0.34	0.24	0.00	0.00	0.40	0.00	0.18	0.00
time (sec)	N/A	0.182	10.006	0.362	0.000	0.000	0.524	0.000	0.260	0.000

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	54	20	0	0	27	0	15	0
N.S.	1	1.12	0.65	0.24	0.00	0.00	0.33	0.00	0.18	0.00
time (sec)	N/A	0.183	5.249	0.352	0.000	0.000	0.444	0.000	0.241	0.000

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	49	20	0	0	27	0	15	0
N.S.	1	1.08	0.75	0.31	0.00	0.00	0.42	0.00	0.23	0.00
time (sec)	N/A	0.166	5.120	0.324	0.000	0.000	0.402	0.000	0.225	0.000

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	20	0	0	27	0	15	0
N.S.	1	1.00	0.87	0.43	0.00	0.00	0.57	0.00	0.32	0.00
time (sec)	N/A	0.149	4.763	0.324	0.000	0.000	0.412	0.000	0.268	0.000

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	18	0	0	26	0	11	16
N.S.	1	1.00	0.89	0.67	0.00	0.00	0.96	0.00	0.41	0.59
time (sec)	N/A	0.138	4.629	0.246	0.000	0.000	0.430	0.000	0.239	0.221

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	20	0	0	29	0	15	36
N.S.	1	1.00	0.55	0.41	0.00	0.00	0.59	0.00	0.31	0.73
time (sec)	N/A	0.150	5.003	0.368	0.000	0.000	0.460	0.000	0.235	0.389

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	29	20	0	0	32	0	15	0
N.S.	1	1.07	0.43	0.30	0.00	0.00	0.48	0.00	0.22	0.00
time (sec)	N/A	0.166	10.005	0.373	0.000	0.000	0.491	0.000	0.226	0.000

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	29	20	0	0	32	0	15	0
N.S.	1	1.12	0.34	0.24	0.00	0.00	0.38	0.00	0.18	0.00
time (sec)	N/A	0.176	10.006	0.347	0.000	0.000	0.541	0.000	0.236	0.000

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	59	20	0	0	29	0	15	0
N.S.	1	1.12	0.71	0.24	0.00	0.00	0.35	0.00	0.18	0.00
time (sec)	N/A	0.177	5.277	0.332	0.000	0.000	0.448	0.000	0.240	0.000

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	70	54	20	0	0	29	0	15	0
N.S.	1	1.08	0.83	0.31	0.00	0.00	0.45	0.00	0.23	0.00
time (sec)	N/A	0.163	5.150	0.318	0.000	0.000	0.433	0.000	0.223	0.000

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	20	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.43	0.00	0.00	0.62	0.00	0.32	0.00
time (sec)	N/A	0.147	4.765	0.327	0.000	0.000	0.416	0.000	0.259	0.000

Problem 951	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	18	0	0	27	0	11	16
N.S.	1	1.00	1.00	0.67	0.00	0.00	1.00	0.00	0.41	0.59
time (sec)	N/A	0.133	4.647	0.284	0.000	0.000	0.392	0.000	0.230	0.086

Problem 952	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	27	20	0	0	31	0	15	36
N.S.	1	1.00	0.55	0.41	0.00	0.00	0.63	0.00	0.31	0.73
time (sec)	N/A	0.150	5.043	0.310	0.000	0.000	0.487	0.000	0.200	0.453

Problem 953	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	29	20	0	0	34	0	15	0
N.S.	1	1.07	0.43	0.30	0.00	0.00	0.51	0.00	0.22	0.00
time (sec)	N/A	0.162	10.005	0.298	0.000	0.000	0.501	0.000	0.262	0.000

Problem 954	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	29	20	0	0	34	0	15	0
N.S.	1	1.12	0.34	0.24	0.00	0.00	0.40	0.00	0.18	0.00
time (sec)	N/A	0.177	10.005	0.303	0.000	0.000	0.560	0.000	0.256	0.000

Problem 955	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	258	289	68	42	0	0	29	0	15	0
N.S.	1	1.12	0.26	0.16	0.00	0.00	0.11	0.00	0.06	0.00
time (sec)	N/A	0.316	5.001	0.362	0.000	0.000	0.472	0.000	0.240	0.000

Problem 956	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	240	266	63	42	0	0	29	0	15	0
N.S.	1	1.11	0.26	0.18	0.00	0.00	0.12	0.00	0.06	0.00
time (sec)	N/A	0.305	4.767	0.349	0.000	0.000	0.434	0.000	0.208	0.000

Problem 957	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	222	243	57	42	0	0	29	0	15	0
N.S.	1	1.09	0.26	0.19	0.00	0.00	0.13	0.00	0.07	0.00
time (sec)	N/A	0.282	4.691	0.335	0.000	0.000	0.396	0.000	0.231	0.000

Problem 958	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	199	224	43	40	0	0	27	0	11	34
N.S.	1	1.13	0.22	0.20	0.00	0.00	0.14	0.00	0.06	0.17
time (sec)	N/A	0.265	4.663	0.267	0.000	0.000	0.391	0.000	0.198	0.215

Problem 959	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	221	245	46	42	0	0	31	0	15	36
N.S.	1	1.11	0.21	0.19	0.00	0.00	0.14	0.00	0.07	0.16
time (sec)	N/A	0.280	4.732	0.362	0.000	0.000	0.453	0.000	0.241	0.336

Problem 960	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	242	268	48	42	0	0	34	0	15	0
N.S.	1	1.11	0.20	0.17	0.00	0.00	0.14	0.00	0.06	0.00
time (sec)	N/A	0.300	10.006	0.373	0.000	0.000	0.483	0.000	0.260	0.000

Problem 961	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	260	291	48	42	0	0	34	0	15	0
N.S.	1	1.12	0.18	0.16	0.00	0.00	0.13	0.00	0.06	0.00
time (sec)	N/A	0.314	10.006	0.389	0.000	0.000	0.511	0.000	0.212	0.000

Problem 962	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	299	68	23	0	0	34	0	15	0
N.S.	1	1.15	0.26	0.09	0.00	0.00	0.13	0.00	0.06	0.00
time (sec)	N/A	0.322	5.039	0.375	0.000	0.000	0.438	0.000	0.235	0.000

Problem 963	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	276	63	23	0	0	34	0	15	0
N.S.	1	1.14	0.26	0.10	0.00	0.00	0.14	0.00	0.06	0.00
time (sec)	N/A	0.308	4.824	0.375	0.000	0.000	0.419	0.000	0.251	0.000

Problem 964	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	253	58	23	0	0	34	0	15	0
N.S.	1	1.13	0.26	0.10	0.00	0.00	0.15	0.00	0.07	0.00
time (sec)	N/A	0.285	4.735	0.326	0.000	0.000	0.409	0.000	0.232	0.000

Problem 965	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	234	43	21	0	0	32	0	11	34
N.S.	1	1.16	0.21	0.10	0.00	0.00	0.16	0.00	0.05	0.17
time (sec)	N/A	0.261	4.727	0.263	0.000	0.000	0.387	0.000	0.224	0.082

Problem 966	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	255	46	23	0	0	36	0	15	36
N.S.	1	1.14	0.21	0.10	0.00	0.00	0.16	0.00	0.07	0.16
time (sec)	N/A	0.289	4.772	0.370	0.000	0.000	0.459	0.000	0.229	0.419

Problem 967	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	278	48	23	0	0	39	0	15	0
N.S.	1	1.14	0.20	0.09	0.00	0.00	0.16	0.00	0.06	0.00
time (sec)	N/A	0.308	10.006	0.325	0.000	0.000	0.496	0.000	0.252	0.000

Problem 968	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	301	48	23	0	0	39	0	15	0
N.S.	1	1.15	0.18	0.09	0.00	0.00	0.15	0.00	0.06	0.00
time (sec)	N/A	0.328	10.007	0.345	0.000	0.000	0.542	0.000	0.234	0.000

Problem 969	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	138	148	68	42	0	0	31	0	15	0
N.S.	1	1.07	0.49	0.30	0.00	0.00	0.22	0.00	0.11	0.00
time (sec)	N/A	0.233	5.248	0.363	0.000	0.000	0.464	0.000	0.231	0.000

Problem 970	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	120	125	63	42	0	0	31	0	15	0
N.S.	1	1.04	0.52	0.35	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.213	5.127	0.358	0.000	0.000	0.417	0.000	0.256	0.000

Problem 971	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	57	42	0	0	31	0	15	0
N.S.	1	1.00	0.56	0.41	0.00	0.00	0.30	0.00	0.15	0.00
time (sec)	N/A	0.194	4.756	0.376	0.000	0.000	0.421	0.000	0.251	0.000

Problem 972	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	43	40	0	0	29	0	11	34
N.S.	1	1.00	0.52	0.49	0.00	0.00	0.35	0.00	0.13	0.41
time (sec)	N/A	0.182	4.651	0.278	0.000	0.000	0.396	0.000	0.254	0.088

Problem 973	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	46	42	0	0	29	0	15	23
N.S.	1	1.00	0.44	0.40	0.00	0.00	0.28	0.00	0.14	0.22
time (sec)	N/A	0.196	5.042	0.357	0.000	0.000	0.476	0.000	0.261	0.423

Problem 974	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	48	42	0	0	32	0	15	0
N.S.	1	1.04	0.39	0.34	0.00	0.00	0.26	0.00	0.12	0.00
time (sec)	N/A	0.214	10.006	0.371	0.000	0.000	0.508	0.000	0.224	0.000

Problem 975	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	140	150	48	42	0	0	32	0	15	0
N.S.	1	1.07	0.34	0.30	0.00	0.00	0.23	0.00	0.11	0.00
time (sec)	N/A	0.225	10.006	0.380	0.000	0.000	0.550	0.000	0.221	0.000

Problem 976	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	149	68	23	0	0	36	0	15	0
N.S.	1	1.07	0.49	0.17	0.00	0.00	0.26	0.00	0.11	0.00
time (sec)	N/A	0.226	5.296	0.331	0.000	0.000	0.433	0.000	0.244	0.000

Problem 977	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	126	63	23	0	0	36	0	15	0
N.S.	1	1.04	0.52	0.19	0.00	0.00	0.30	0.00	0.12	0.00
time (sec)	N/A	0.208	5.170	0.293	0.000	0.000	0.462	0.000	0.254	0.000

Problem 978	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	58	23	0	0	36	0	15	0
N.S.	1	1.00	0.56	0.22	0.00	0.00	0.35	0.00	0.15	0.00
time (sec)	N/A	0.200	4.814	0.286	0.000	0.000	0.395	0.000	0.212	0.000

Problem 979	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	43	21	0	0	34	0	11	34
N.S.	1	1.00	0.51	0.25	0.00	0.00	0.40	0.00	0.13	0.40
time (sec)	N/A	0.181	4.685	0.275	0.000	0.000	0.378	0.000	0.219	0.082

Problem 980	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	46	23	0	0	34	0	15	36
N.S.	1	1.00	0.44	0.22	0.00	0.00	0.32	0.00	0.14	0.34
time (sec)	N/A	0.192	5.060	0.286	0.000	0.000	0.466	0.000	0.244	0.374

Problem 981	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	128	48	23	0	0	37	0	15	0
N.S.	1	1.04	0.39	0.19	0.00	0.00	0.30	0.00	0.12	0.00
time (sec)	N/A	0.213	10.007	0.287	0.000	0.000	0.501	0.000	0.235	0.000

Problem 982	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	151	48	23	0	0	37	0	15	0
N.S.	1	1.07	0.34	0.16	0.00	0.00	0.26	0.00	0.11	0.00
time (sec)	N/A	0.227	10.006	0.302	0.000	0.000	0.576	0.000	0.227	0.000

Problem 983	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	261	333	177	0	0	0	48	0	65	0
N.S.	1	1.28	0.68	0.00	0.00	0.00	0.18	0.00	0.25	0.00
time (sec)	N/A	0.510	0.563	0.000	0.000	0.000	4.489	0.000	0.286	0.000

Problem 984	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	227	295	162	0	0	0	48	0	37	0
N.S.	1	1.30	0.71	0.00	0.00	0.00	0.21	0.00	0.16	0.00
time (sec)	N/A	0.474	0.345	0.000	0.000	0.000	1.006	0.000	0.233	0.000

Problem 985	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	214	293	161	0	0	0	51	0	18	0
N.S.	1	1.37	0.75	0.00	0.00	0.00	0.24	0.00	0.08	0.00
time (sec)	N/A	0.480	0.310	0.000	0.000	0.000	1.189	0.000	200.031	0.000

Problem 986	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	35	178	0	36	38
N.S.	1	1.00	0.93	0.76	0.00	1.21	6.14	0.00	1.24	1.31
time (sec)	N/A	0.147	0.129	0.279	0.000	0.083	4.425	0.000	0.228	0.357

Problem 987	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	48	32	0	46	462	0	47	51
N.S.	1	0.98	0.80	0.53	0.00	0.77	7.70	0.00	0.78	0.85
time (sec)	N/A	0.171	0.146	0.276	0.000	0.122	41.652	0.000	0.207	0.392

Problem 988	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	48	43	0	58	0	0	59	65
N.S.	1	1.04	0.52	0.47	0.00	0.63	0.00	0.00	0.64	0.71
time (sec)	N/A	0.194	0.156	0.287	0.000	0.103	0.000	0.000	0.243	0.458

Problem 989	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	133	59	54	0	69	0	0	70	79
N.S.	1	1.07	0.48	0.44	0.00	0.56	0.00	0.00	0.56	0.64
time (sec)	N/A	0.221	1.415	0.289	0.000	0.112	0.000	0.000	0.215	0.467

Problem 990	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	126	88	0	0	0	48	0	73	0
N.S.	1	1.03	0.72	0.00	0.00	0.00	0.39	0.00	0.60	0.00
time (sec)	N/A	0.296	10.032	0.000	0.000	0.000	1.231	0.000	0.215	0.000

Problem 991	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	95	55	0	0	0	39	0	24	0
N.S.	1	1.03	0.60	0.00	0.00	0.00	0.42	0.00	0.26	0.00
time (sec)	N/A	0.262	10.010	0.000	0.000	0.000	0.676	0.000	0.196	0.000

Problem 992	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	57	0	0	0	36	0	62	0
N.S.	1	1.03	0.59	0.00	0.00	0.00	0.37	0.00	0.64	0.00
time (sec)	N/A	0.264	10.013	0.000	0.000	0.000	1.745	0.000	0.230	0.000

Problem 993	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	127	138	57	0	0	0	39	0	64	0
N.S.	1	1.09	0.45	0.00	0.00	0.00	0.31	0.00	0.50	0.00
time (sec)	N/A	0.295	10.013	0.000	0.000	0.000	14.056	0.000	0.278	0.000

Problem 994	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	159	179	57	0	0	0	39	0	64	0
N.S.	1	1.13	0.36	0.00	0.00	0.00	0.25	0.00	0.40	0.00
time (sec)	N/A	0.326	10.012	0.000	0.000	0.000	139.765	0.000	0.248	0.000

Problem 995	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	291	161	0	0	324	46	0	20	0
N.S.	1	1.28	0.71	0.00	0.00	1.42	0.20	0.00	0.09	0.00
time (sec)	N/A	0.470	0.466	0.000	0.000	0.108	1.201	0.000	0.207	0.000

Problem 996	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	259	125	0	0	262	46	0	39	0
N.S.	1	1.36	0.66	0.00	0.00	1.38	0.24	0.00	0.21	0.00
time (sec)	N/A	0.427	0.300	0.000	0.000	0.099	0.776	0.000	0.234	0.000

Problem 997	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	22	0	26	88	0	75	26
N.S.	1	1.00	0.93	0.76	0.00	0.90	3.03	0.00	2.59	0.90
time (sec)	N/A	0.148	0.161	0.301	0.000	0.089	1.775	0.000	0.230	0.485

Problem 998	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	59	37	32	0	36	343	0	97	41
N.S.	1	0.98	0.62	0.53	0.00	0.60	5.72	0.00	1.62	0.68
time (sec)	N/A	0.173	0.184	0.290	0.000	0.090	20.137	0.000	0.249	0.523

Problem 999	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	48	43	0	47	1221	0	116	55
N.S.	1	1.04	0.52	0.47	0.00	0.51	13.27	0.00	1.26	0.60
time (sec)	N/A	0.199	0.204	0.278	0.000	0.093	171.189	0.000	0.302	0.528

Problem 1000	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	128	131	71	0	0	0	46	0	24	0
N.S.	1	1.02	0.55	0.00	0.00	0.00	0.36	0.00	0.19	0.00
time (sec)	N/A	0.252	10.020	0.000	0.000	0.000	2.998	0.000	0.225	0.000

Problem 1001	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	57	0	0	0	46	0	18	0
N.S.	1	1.02	0.63	0.00	0.00	0.00	0.51	0.00	0.20	0.00
time (sec)	N/A	0.210	10.010	0.000	0.000	0.000	0.517	0.000	0.179	0.000

Problem 1002	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	70	55	0	0	0	32	0	35	0
N.S.	1	1.03	0.81	0.00	0.00	0.00	0.47	0.00	0.51	0.00
time (sec)	N/A	0.188	10.013	0.000	0.000	0.000	0.991	0.000	0.238	0.000

Problem 1003	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	102	57	0	0	0	39	0	47	0
N.S.	1	1.02	0.57	0.00	0.00	0.00	0.39	0.00	0.47	0.00
time (sec)	N/A	0.210	10.011	0.000	0.000	0.000	6.224	0.000	0.219	0.000

Problem 1004	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	143	57	0	0	0	36	0	58	0
N.S.	1	1.10	0.44	0.00	0.00	0.00	0.28	0.00	0.45	0.00
time (sec)	N/A	0.246	10.013	0.000	0.000	0.000	54.660	0.000	0.207	0.000

Problem 1005	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	301	164	0	0	0	46	0	24	0
N.S.	1	1.32	0.72	0.00	0.00	0.00	0.20	0.00	0.11	0.00
time (sec)	N/A	0.483	0.463	0.000	0.000	0.000	3.491	0.000	0.219	0.000

Problem 1006	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	191	265	127	0	0	0	46	0	18	0
N.S.	1	1.39	0.66	0.00	0.00	0.00	0.24	0.00	0.09	0.00
time (sec)	N/A	0.446	0.331	0.000	0.000	0.000	0.695	0.000	0.211	0.000

Problem 1007	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	22	0	26	90	0	24	23
N.S.	1	1.00	0.93	0.81	0.00	0.96	3.33	0.00	0.89	0.85
time (sec)	N/A	0.147	0.201	0.273	0.000	0.128	1.304	0.000	0.213	0.452

Problem 1008	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	57	35	30	0	34	352	0	37	41
N.S.	1	0.95	0.58	0.50	0.00	0.57	5.87	0.00	0.62	0.68
time (sec)	N/A	0.173	0.247	0.275	0.000	0.136	12.884	0.000	0.230	0.477

Problem 1009	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	C	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	48	43	0	47	1263	0	48	55
N.S.	1	1.00	0.52	0.47	0.00	0.51	13.73	0.00	0.52	0.60
time (sec)	N/A	0.206	0.292	0.281	0.000	0.120	108.388	0.000	0.275	0.552

Problem 1010	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	94	68	0	0	0	46	0	53	0
N.S.	1	1.03	0.75	0.00	0.00	0.00	0.51	0.00	0.58	0.00
time (sec)	N/A	0.266	10.020	0.000	0.000	0.000	0.974	0.000	0.219	0.000

Problem 1011	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	55	0	0	0	32	0	43	0
N.S.	1	1.04	0.81	0.00	0.00	0.00	0.47	0.00	0.63	0.00
time (sec)	N/A	0.239	10.011	0.000	0.000	0.000	0.865	0.000	0.229	0.000

Problem 1012	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	103	57	0	0	0	39	0	38	0
N.S.	1	1.03	0.57	0.00	0.00	0.00	0.39	0.00	0.38	0.00
time (sec)	N/A	0.272	10.011	0.000	0.000	0.000	3.527	0.000	0.224	0.000

Problem 1013	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	130	144	57	0	0	0	36	0	48	0
N.S.	1	1.11	0.44	0.00	0.00	0.00	0.28	0.00	0.37	0.00
time (sec)	N/A	0.304	10.013	0.000	0.000	0.000	38.820	0.000	0.206	0.000

Problem 1014	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	185	57	0	0	0	0	0	59	0
N.S.	1	1.14	0.35	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.340	10.011	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 1015	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	158	109	0	0	0	46	0	62	0
N.S.	1	1.07	0.74	0.00	0.00	0.00	0.31	0.00	0.42	0.00
time (sec)	N/A	0.276	0.323	0.000	0.000	0.000	4.430	0.000	0.234	0.000

Problem 1016	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	116	121	96	0	0	0	46	0	35	0
N.S.	1	1.04	0.83	0.00	0.00	0.00	0.40	0.00	0.30	0.00
time (sec)	N/A	0.237	0.208	0.000	0.000	0.000	0.910	0.000	0.236	0.000

Problem 1017	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	119	92	0	0	0	49	0	23	0
N.S.	1	1.11	0.86	0.00	0.00	0.00	0.46	0.00	0.21	0.00
time (sec)	N/A	0.239	0.175	0.000	0.000	0.000	1.082	0.000	0.215	0.000

Problem 1018	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	78	0	26	37
N.S.	1	1.00	0.93	0.75	0.00	0.89	2.79	0.00	0.93	1.32
time (sec)	N/A	0.146	0.125	0.270	0.000	0.135	4.270	0.000	0.199	0.355

Problem 1019	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	46	31	0	46	124	0	47	51
N.S.	1	0.98	0.79	0.53	0.00	0.79	2.14	0.00	0.81	0.88
time (sec)	N/A	0.170	0.141	0.263	0.000	0.138	40.577	0.000	0.233	0.403

Problem 1020	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	47	42	0	57	0	0	58	65
N.S.	1	1.04	0.53	0.47	0.00	0.64	0.00	0.00	0.65	0.73
time (sec)	N/A	0.193	0.153	0.296	0.000	0.108	0.000	0.000	0.220	0.455

Problem 1021	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	129	58	53	0	68	0	0	69	79
N.S.	1	1.08	0.48	0.44	0.00	0.57	0.00	0.00	0.58	0.66
time (sec)	N/A	0.214	1.403	0.276	0.000	0.111	0.000	0.000	0.214	0.455

Problem 1022	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	158	102	0	0	0	46	0	94	0
N.S.	1	1.04	0.67	0.00	0.00	0.00	0.30	0.00	0.62	0.00
time (sec)	N/A	0.322	10.041	0.000	0.000	0.000	11.838	0.000	0.277	0.000

Problem 1023	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	118	120	85	0	0	0	46	0	70	0
N.S.	1	1.02	0.72	0.00	0.00	0.00	0.39	0.00	0.59	0.00
time (sec)	N/A	0.280	10.030	0.000	0.000	0.000	1.211	0.000	0.234	0.000

Problem 1024	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	54	0	0	0	46	0	48	0
N.S.	1	1.03	0.61	0.00	0.00	0.00	0.52	0.00	0.54	0.00
time (sec)	N/A	0.254	10.009	0.000	0.000	0.000	0.662	0.000	0.346	0.000

Problem 1025	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	97	56	0	0	0	32	0	59	0
N.S.	1	1.03	0.60	0.00	0.00	0.00	0.34	0.00	0.63	0.00
time (sec)	N/A	0.261	10.012	0.000	0.000	0.000	1.673	0.000	0.272	0.000

Problem 1026	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	134	56	0	0	0	36	0	61	0
N.S.	1	1.09	0.46	0.00	0.00	0.00	0.29	0.00	0.50	0.00
time (sec)	N/A	0.300	10.013	0.000	0.000	0.000	13.781	0.000	0.270	0.000

Problem 1027	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	174	56	0	0	0	36	0	61	0
N.S.	1	1.13	0.36	0.00	0.00	0.00	0.23	0.00	0.40	0.00
time (sec)	N/A	0.335	10.010	0.000	0.000	0.000	137.208	0.000	0.256	0.000

Problem 1028	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	125	97	0	0	301	44	0	19	0
N.S.	1	1.07	0.83	0.00	0.00	2.57	0.38	0.00	0.16	0.00
time (sec)	N/A	0.231	0.258	0.000	0.000	0.116	1.143	0.000	0.228	0.000

Problem 1029	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	94	65	0	0	240	44	0	37	0
N.S.	1	1.13	0.78	0.00	0.00	2.89	0.53	0.00	0.45	0.00
time (sec)	N/A	0.206	0.178	0.000	0.000	0.104	0.692	0.000	0.224	0.000

Problem 1030	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	0	25	36	0	71	25
N.S.	1	1.00	0.93	0.75	0.00	0.89	1.29	0.00	2.54	0.89
time (sec)	N/A	0.147	0.141	0.263	0.000	0.129	1.614	0.000	0.240	0.449

Problem 1031	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	36	31	0	35	80	0	92	40
N.S.	1	0.98	0.62	0.53	0.00	0.60	1.38	0.00	1.59	0.69
time (sec)	N/A	0.169	0.165	0.284	0.000	0.116	19.128	0.000	0.230	0.471

Problem 1032	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	47	42	0	46	483	0	110	54
N.S.	1	1.04	0.53	0.47	0.00	0.52	5.43	0.00	1.24	0.61
time (sec)	N/A	0.194	0.186	0.262	0.000	0.146	168.841	0.000	0.277	0.521

Problem 1033	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	161	87	0	0	0	44	0	23	0
N.S.	1	1.04	0.56	0.00	0.00	0.00	0.28	0.00	0.15	0.00
time (sec)	N/A	0.265	10.023	0.000	0.000	0.000	27.615	0.000	0.221	0.000

Problem 1034	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	122	123	69	0	0	0	44	0	23	0
N.S.	1	1.01	0.57	0.00	0.00	0.00	0.36	0.00	0.19	0.00
time (sec)	N/A	0.239	10.017	0.000	0.000	0.000	2.784	0.000	0.204	0.000

Problem 1035	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	0	0	0	44	0	17	0
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.52	0.00	0.20	0.00
time (sec)	N/A	0.204	10.010	0.000	0.000	0.000	0.501	0.000	0.229	0.000

Problem 1036	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	54	0	0	0	31	0	33	0
N.S.	1	1.02	0.60	0.00	0.00	0.00	0.34	0.00	0.37	0.00
time (sec)	N/A	0.207	10.011	0.000	0.000	0.000	0.996	0.000	0.236	0.000

Problem 1037	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	132	56	0	0	0	34	0	36	0
N.S.	1	1.05	0.44	0.00	0.00	0.00	0.27	0.00	0.29	0.00
time (sec)	N/A	0.236	10.013	0.000	0.000	0.000	6.006	0.000	0.219	0.000

Problem 1038	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	172	56	0	0	0	34	0	57	0
N.S.	1	1.10	0.36	0.00	0.00	0.00	0.22	0.00	0.36	0.00
time (sec)	N/A	0.274	10.011	0.000	0.000	0.000	53.857	0.000	0.236	0.000

Problem 1039	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	127	97	0	0	0	44	0	23	0
N.S.	1	1.09	0.83	0.00	0.00	0.00	0.38	0.00	0.20	0.00
time (sec)	N/A	0.249	0.278	0.000	0.000	0.000	3.333	0.000	0.197	0.000

Problem 1040	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-1)	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	92	67	0	0	0	44	0	17	0
N.S.	1	1.10	0.80	0.00	0.00	0.00	0.52	0.00	0.20	0.00
time (sec)	N/A	0.217	0.203	0.000	0.000	0.000	0.673	0.000	0.203	0.000

Problem 1041	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	25	36	0	23	22
N.S.	1	1.00	0.92	0.81	0.00	0.96	1.38	0.00	0.88	0.85
time (sec)	N/A	0.147	0.180	0.266	0.000	0.103	1.292	0.000	0.247	0.450

Problem 1042	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	55	34	29	0	35	78	0	36	40
N.S.	1	0.95	0.59	0.50	0.00	0.60	1.34	0.00	0.62	0.69
time (sec)	N/A	0.170	0.226	0.268	0.000	0.099	12.348	0.000	0.266	0.473

Problem 1043	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	47	42	0	46	483	0	47	54
N.S.	1	1.00	0.53	0.47	0.00	0.52	5.43	0.00	0.53	0.61
time (sec)	N/A	0.193	0.275	0.279	0.000	0.088	106.178	0.000	0.240	0.536

Problem 1044	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	89	66	0	0	0	44	0	19	0
N.S.	1	1.03	0.77	0.00	0.00	0.00	0.51	0.00	0.22	0.00
time (sec)	N/A	0.252	10.019	0.000	0.000	0.000	0.960	0.000	0.195	0.000

Problem 1045	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	69	54	0	0	0	31	0	42	0
N.S.	1	1.05	0.82	0.00	0.00	0.00	0.47	0.00	0.64	0.00
time (sec)	N/A	0.234	10.011	0.000	0.000	0.000	0.847	0.000	0.244	0.000

Problem 1046	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	100	56	0	0	0	48	0	36	0
N.S.	1	1.03	0.58	0.00	0.00	0.00	0.49	0.00	0.37	0.00
time (sec)	N/A	0.260	10.011	0.000	0.000	0.000	3.437	0.000	0.205	0.000

Problem 1047	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	140	56	0	0	0	34	0	46	0
N.S.	1	1.11	0.44	0.00	0.00	0.00	0.27	0.00	0.37	0.00
time (sec)	N/A	0.299	10.011	0.000	0.000	0.000	38.538	0.000	0.200	0.000

Problem 1048	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	180	56	0	0	0	0	0	57	0
N.S.	1	1.15	0.36	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.338	10.010	0.000	0.000	0.000	0.000	0.000	0.259	0.000

Problem 1049	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	158	131	0	0	400	44	0	42	0
N.S.	1	1.08	0.90	0.00	0.00	2.74	0.30	0.00	0.29	0.00
time (sec)	N/A	0.265	0.422	0.000	0.000	0.112	11.971	0.000	0.219	0.000

Problem 1050	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	C	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	120	91	0	0	327	44	0	38	0
N.S.	1	1.12	0.85	0.00	0.00	3.06	0.41	0.00	0.36	0.00
time (sec)	N/A	0.229	0.324	0.000	0.000	0.106	1.869	0.000	0.200	0.000

Problem 1051	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	0	31	34	0	21	29
N.S.	1	1.00	0.92	0.81	0.00	1.19	1.31	0.00	0.81	1.12
time (sec)	N/A	0.147	0.179	0.254	0.000	0.085	1.341	0.000	0.229	0.440

Problem 1052	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	34	29	0	48	78	0	36	57
N.S.	1	1.00	0.62	0.53	0.00	0.87	1.42	0.00	0.65	1.04
time (sec)	N/A	0.167	0.216	0.275	0.000	0.098	7.191	0.000	0.262	0.504

Problem 1053	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	89	47	42	0	61	384	0	47	70
N.S.	1	1.06	0.56	0.50	0.00	0.73	4.57	0.00	0.56	0.83
time (sec)	N/A	0.195	0.366	0.274	0.000	0.105	65.831	0.000	0.242	0.560

Problem 1054	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	125	58	53	0	72	0	0	58	85
N.S.	1	1.09	0.50	0.46	0.00	0.63	0.00	0.00	0.50	0.74
time (sec)	N/A	0.213	0.458	0.281	0.000	0.103	0.000	0.000	0.261	0.595

Problem 1055	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	155	168	87	0	0	0	0	0	42	0
N.S.	1	1.08	0.56	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.270	10.029	0.000	0.000	0.000	0.000	0.000	0.254	0.000

Problem 1056	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	130	74	0	0	0	44	0	42	0
N.S.	1	1.05	0.60	0.00	0.00	0.00	0.35	0.00	0.34	0.00
time (sec)	N/A	0.240	10.022	0.000	0.000	0.000	36.213	0.000	0.218	0.000

Problem 1057	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	92	60	0	0	0	44	0	42	0
N.S.	1	1.02	0.67	0.00	0.00	0.00	0.49	0.00	0.47	0.00
time (sec)	N/A	0.209	10.021	0.000	0.000	0.000	3.419	0.000	0.211	0.000

Problem 1058	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	59	0	0	0	44	0	36	0
N.S.	1	1.03	0.94	0.00	0.00	0.00	0.70	0.00	0.57	0.00
time (sec)	N/A	0.187	10.010	0.000	0.000	0.000	1.001	0.000	0.216	0.000

Problem 1059	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	57	0	0	0	48	0	33	0
N.S.	1	1.02	0.61	0.00	0.00	0.00	0.52	0.00	0.35	0.00
time (sec)	N/A	0.210	10.011	0.000	0.000	0.000	2.589	0.000	0.213	0.000

Problem 1060	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	135	59	0	0	0	34	0	46	0
N.S.	1	1.07	0.47	0.00	0.00	0.00	0.27	0.00	0.37	0.00
time (sec)	N/A	0.247	10.013	0.000	0.000	0.000	22.968	0.000	0.205	0.000

Problem 1061	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	175	59	0	0	0	0	0	57	0
N.S.	1	1.11	0.38	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.276	10.012	0.000	0.000	0.000	0.000	0.000	0.239	0.000

Problem 1062	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.172	10.009	0.000	0.000	0.000	5.263	0.000	0.241	0.000

Problem 1063	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.169	10.009	0.000	0.000	0.000	1.556	0.000	0.502	0.000

Problem 1064	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.168	10.011	0.000	0.000	0.000	0.591	0.000	0.219	0.000

Problem 1065	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	0	0	0	44	0	19	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.76	0.00	0.33	0.00
time (sec)	N/A	0.167	10.011	0.000	0.000	0.000	0.625	0.000	0.237	0.000

Problem 1066	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	44	0	19	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.79	0.00	0.34	0.00
time (sec)	N/A	0.164	10.010	0.000	0.000	0.000	0.959	0.000	0.404	0.000

Problem 1067	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	0	0	0	48	0	19	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.86	0.00	0.34	0.00
time (sec)	N/A	0.168	10.011	0.000	0.000	0.000	1.690	0.000	0.278	0.000

Problem 1068	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	38	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	0.62	0.00
time (sec)	N/A	0.164	10.013	0.000	0.000	0.000	11.606	0.000	0.248	0.000

Problem 1069	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	38	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	0.62	0.00
time (sec)	N/A	0.165	10.013	0.000	0.000	0.000	6.618	0.000	0.535	0.000

Problem 1070	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	38	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	0.62	0.00
time (sec)	N/A	0.164	10.013	0.000	0.000	0.000	3.332	0.000	0.235	0.000

Problem 1071	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	44	0	37	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.72	0.00	0.61	0.00
time (sec)	N/A	0.163	10.014	0.000	0.000	0.000	3.708	0.000	0.227	0.000

Problem 1072	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	44	0	37	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.75	0.00	0.63	0.00
time (sec)	N/A	0.159	10.014	0.000	0.000	0.000	7.162	0.000	0.292	0.000

Problem 1073	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	57	0	0	0	48	0	37	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.81	0.00	0.63	0.00
time (sec)	N/A	0.162	10.013	0.000	0.000	0.000	12.980	0.000	0.229	0.000

Problem 1074	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	338	440	105	0	0	0	29	0	15	0
N.S.	1	1.30	0.31	0.00	0.00	0.00	0.09	0.00	0.04	0.00
time (sec)	N/A	0.336	7.309	0.000	0.000	0.000	0.603	0.000	0.222	0.000

Problem 1075	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	314	410	93	0	0	0	29	0	15	0
N.S.	1	1.31	0.30	0.00	0.00	0.00	0.09	0.00	0.05	0.00
time (sec)	N/A	0.292	7.173	0.000	0.000	0.000	0.546	0.000	0.222	0.000

Problem 1076	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	380	62	0	0	0	29	0	15	0
N.S.	1	1.31	0.21	0.00	0.00	0.00	0.10	0.00	0.05	0.00
time (sec)	N/A	0.277	7.011	0.000	0.000	0.000	0.499	0.000	0.242	0.000

Problem 1077	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	268	353	46	0	0	0	26	0	11	37
N.S.	1	1.32	0.17	0.00	0.00	0.00	0.10	0.00	0.04	0.14
time (sec)	N/A	0.248	0.003	0.000	0.000	0.000	0.483	0.000	0.208	0.237

Problem 1078	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	265	346	49	0	0	0	29	0	17	40
N.S.	1	1.31	0.18	0.00	0.00	0.00	0.11	0.00	0.06	0.15
time (sec)	N/A	0.253	7.009	0.000	0.000	0.000	0.520	0.000	0.188	0.452

Problem 1079	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	290	377	51	0	0	0	34	0	76	0
N.S.	1	1.30	0.18	0.00	0.00	0.00	0.12	0.00	0.26	0.00
time (sec)	N/A	0.276	10.009	0.000	0.000	0.000	0.592	0.000	0.237	0.000

Problem 1080	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	316	407	51	0	0	0	34	0	76	0
N.S.	1	1.29	0.16	0.00	0.00	0.00	0.11	0.00	0.24	0.00
time (sec)	N/A	0.299	10.008	0.000	0.000	0.000	0.678	0.000	0.243	0.000

Problem 1081	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	437	51	0	0	0	34	0	76	0
N.S.	1	1.29	0.15	0.00	0.00	0.00	0.10	0.00	0.22	0.00
time (sec)	N/A	0.318	10.010	0.000	0.000	0.000	0.713	0.000	0.264	0.000

Problem 1082	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	655	811	105	0	0	0	29	0	15	0
N.S.	1	1.24	0.16	0.00	0.00	0.00	0.04	0.00	0.02	0.00
time (sec)	N/A	0.511	9.117	0.000	0.000	0.000	0.959	0.000	0.215	0.000

Problem 1083	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	631	781	94	0	0	0	29	0	15	0
N.S.	1	1.24	0.15	0.00	0.00	0.00	0.05	0.00	0.02	0.00
time (sec)	N/A	0.476	8.901	0.000	0.000	0.000	0.836	0.000	0.242	0.000

Problem 1084	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	607	751	62	0	0	0	29	0	15	0
N.S.	1	1.24	0.10	0.00	0.00	0.00	0.05	0.00	0.02	0.00
time (sec)	N/A	0.438	8.258	0.000	0.000	0.000	0.713	0.000	0.210	0.000

Problem 1085	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	580	721	46	0	0	0	26	0	11	37
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.04	0.00	0.02	0.06
time (sec)	N/A	0.412	0.003	0.000	0.000	0.000	0.652	0.000	0.184	0.225

Problem 1086	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	574	721	49	0	0	0	29	0	45	40
N.S.	1	1.26	0.09	0.00	0.00	0.00	0.05	0.00	0.08	0.07
time (sec)	N/A	0.426	8.554	0.000	0.000	0.000	0.727	0.000	0.207	0.529

Problem 1087	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	605	751	51	0	0	0	34	0	79	0
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.06	0.00	0.13	0.00
time (sec)	N/A	0.441	10.008	0.000	0.000	0.000	0.699	0.000	0.257	0.000

Problem 1088	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	633	781	51	0	0	0	34	0	79	0
N.S.	1	1.23	0.08	0.00	0.00	0.00	0.05	0.00	0.12	0.00
time (sec)	N/A	0.476	10.010	0.000	0.000	0.000	0.815	0.000	0.255	0.000

Problem 1089	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	811	51	0	0	0	34	0	79	0
N.S.	1	1.23	0.08	0.00	0.00	0.00	0.05	0.00	0.12	0.00
time (sec)	N/A	0.515	10.008	0.000	0.000	0.000	0.920	0.000	0.264	0.000

Problem 1090	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	359	464	89	0	0	0	29	0	35	0
N.S.	1	1.29	0.25	0.00	0.00	0.00	0.08	0.00	0.10	0.00
time (sec)	N/A	0.337	8.989	0.000	0.000	0.000	1.135	0.000	0.262	0.000

Problem 1091	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	335	434	79	0	0	0	29	0	35	0
N.S.	1	1.30	0.24	0.00	0.00	0.00	0.09	0.00	0.10	0.00
time (sec)	N/A	0.328	8.859	0.000	0.000	0.000	1.001	0.000	0.246	0.000

Problem 1092	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	311	404	67	0	0	0	29	0	35	0
N.S.	1	1.30	0.22	0.00	0.00	0.00	0.09	0.00	0.11	0.00
time (sec)	N/A	0.294	8.512	0.000	0.000	0.000	0.888	0.000	0.269	0.000

Problem 1093	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	285	375	47	0	0	0	26	0	31	37
N.S.	1	1.32	0.16	0.00	0.00	0.00	0.09	0.00	0.11	0.13
time (sec)	N/A	0.264	0.003	0.000	0.000	0.000	0.781	0.000	0.220	0.242

Problem 1094	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	375	50	0	0	0	29	0	64	40
N.S.	1	1.33	0.18	0.00	0.00	0.00	0.10	0.00	0.23	0.14
time (sec)	N/A	0.273	8.617	0.000	0.000	0.000	0.943	0.000	0.216	0.710

Problem 1095	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	287	370	52	0	0	0	34	0	38	0
N.S.	1	1.29	0.18	0.00	0.00	0.00	0.12	0.00	0.13	0.00
time (sec)	N/A	0.284	10.008	0.000	0.000	0.000	0.913	0.000	0.211	0.000

Problem 1096	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	313	401	52	0	0	0	34	0	95	0
N.S.	1	1.28	0.17	0.00	0.00	0.00	0.11	0.00	0.30	0.00
time (sec)	N/A	0.302	10.009	0.000	0.000	0.000	0.961	0.000	0.277	0.000

Problem 1097	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	337	431	52	0	0	0	34	0	95	0
N.S.	1	1.28	0.15	0.00	0.00	0.00	0.10	0.00	0.28	0.00
time (sec)	N/A	0.321	10.008	0.000	0.000	0.000	1.043	0.000	0.280	0.000

Problem 1098	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	787	90	0	0	0	27	0	15	0
N.S.	1	1.24	0.14	0.00	0.00	0.00	0.04	0.00	0.02	0.00
time (sec)	N/A	0.491	7.167	0.000	0.000	0.000	0.524	0.000	0.215	0.000

Problem 1099	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	610	757	79	0	0	0	27	0	15	0
N.S.	1	1.24	0.13	0.00	0.00	0.00	0.04	0.00	0.02	0.00
time (sec)	N/A	0.465	7.049	0.000	0.000	0.000	0.481	0.000	0.197	0.000

Problem 1100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	586	727	62	0	0	0	27	0	15	0
N.S.	1	1.24	0.11	0.00	0.00	0.00	0.05	0.00	0.03	0.00
time (sec)	N/A	0.427	6.855	0.000	0.000	0.000	0.450	0.000	0.186	0.000

Problem 1101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	563	699	46	0	0	0	24	0	11	37
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.04	0.00	0.02	0.07
time (sec)	N/A	0.412	0.003	0.000	0.000	0.000	0.443	0.000	0.177	0.277

Problem 1102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	576	727	49	0	0	0	27	0	25	40
N.S.	1	1.26	0.09	0.00	0.00	0.00	0.05	0.00	0.04	0.07
time (sec)	N/A	0.434	6.868	0.000	0.000	0.000	0.486	0.000	0.214	0.432

Problem 1103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	608	757	51	0	0	0	32	0	75	0
N.S.	1	1.25	0.08	0.00	0.00	0.00	0.05	0.00	0.12	0.00
time (sec)	N/A	0.451	10.009	0.000	0.000	0.000	0.573	0.000	0.243	0.000

Problem 1104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	636	787	51	0	0	0	32	0	75	0
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.05	0.00	0.12	0.00
time (sec)	N/A	0.490	10.009	0.000	0.000	0.000	0.640	0.000	0.233	0.000

Problem 1105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	317	416	89	0	0	0	27	0	15	0
N.S.	1	1.31	0.28	0.00	0.00	0.00	0.09	0.00	0.05	0.00
time (sec)	N/A	0.304	7.845	0.000	0.000	0.000	0.502	0.000	0.223	0.000

Problem 1106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	386	79	0	0	0	27	0	15	0
N.S.	1	1.32	0.27	0.00	0.00	0.00	0.09	0.00	0.05	0.00
time (sec)	N/A	0.297	7.726	0.000	0.000	0.000	0.473	0.000	0.207	0.000

Problem 1107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	271	356	62	0	0	0	27	0	15	0
N.S.	1	1.31	0.23	0.00	0.00	0.00	0.10	0.00	0.06	0.00
time (sec)	N/A	0.262	6.952	0.000	0.000	0.000	0.476	0.000	0.186	0.000

Problem 1108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	332	46	0	0	0	24	0	38	37
N.S.	1	1.32	0.18	0.00	0.00	0.00	0.10	0.00	0.15	0.15
time (sec)	N/A	0.250	0.003	0.000	0.000	0.000	0.454	0.000	0.196	0.262

Problem 1109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	353	49	0	0	0	27	0	73	40
N.S.	1	1.33	0.18	0.00	0.00	0.00	0.10	0.00	0.27	0.15
time (sec)	N/A	0.267	7.499	0.000	0.000	0.000	0.548	0.000	0.249	0.494

Problem 1110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	293	383	51	0	0	0	32	0	80	0
N.S.	1	1.31	0.17	0.00	0.00	0.00	0.11	0.00	0.27	0.00
time (sec)	N/A	0.289	10.011	0.000	0.000	0.000	0.632	0.000	0.231	0.000

Problem 1111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	319	413	51	0	0	0	32	0	80	0
N.S.	1	1.29	0.16	0.00	0.00	0.00	0.10	0.00	0.25	0.00
time (sec)	N/A	0.309	10.008	0.000	0.000	0.000	0.706	0.000	0.227	0.000

Problem 1112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	629	782	79	0	0	0	27	0	114	0
N.S.	1	1.24	0.13	0.00	0.00	0.00	0.04	0.00	0.18	0.00
time (sec)	N/A	0.485	7.715	0.000	0.000	0.000	0.537	0.000	0.279	0.000

Problem 1113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	605	752	65	0	0	0	27	0	34	0
N.S.	1	1.24	0.11	0.00	0.00	0.00	0.04	0.00	0.06	0.00
time (sec)	N/A	0.457	7.632	0.000	0.000	0.000	0.537	0.000	0.206	0.000

Problem 1114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	583	722	58	0	0	0	27	0	35	0
N.S.	1	1.24	0.10	0.00	0.00	0.00	0.05	0.00	0.06	0.00
time (sec)	N/A	0.424	7.111	0.000	0.000	0.000	0.512	0.000	0.243	0.000

Problem 1115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	579	681	49	0	0	0	24	0	32	37
N.S.	1	1.18	0.08	0.00	0.00	0.00	0.04	0.00	0.06	0.06
time (sec)	N/A	0.392	0.004	0.000	0.000	0.000	0.509	0.000	0.262	0.304

Problem 1116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	593	752	52	0	0	0	27	0	39	40
N.S.	1	1.27	0.09	0.00	0.00	0.00	0.05	0.00	0.07	0.07
time (sec)	N/A	0.458	7.462	0.000	0.000	0.000	0.592	0.000	0.215	0.556

Problem 1117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	627	782	54	0	0	0	32	0	52	0
N.S.	1	1.25	0.09	0.00	0.00	0.00	0.05	0.00	0.08	0.00
time (sec)	N/A	0.471	10.008	0.000	0.000	0.000	0.663	0.000	0.196	0.000

Problem 1118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	C	F	B	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	655	812	54	0	0	0	32	0	65	0
N.S.	1	1.24	0.08	0.00	0.00	0.00	0.05	0.00	0.10	0.00
time (sec)	N/A	0.510	10.009	0.000	0.000	0.000	0.796	0.000	0.231	0.000

Problem 1119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.166	8.320	0.000	0.000	0.000	0.630	0.000	0.292	0.000

Problem 1120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.162	8.208	0.000	0.000	0.000	0.590	0.000	0.215	0.000

Problem 1121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.164	8.025	0.000	0.000	0.000	0.530	0.000	0.323	0.000

Problem 1122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.153	0.003	0.000	0.000	0.000	0.523	0.000	0.221	0.255

Problem 1123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	29	0	129	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	2.63	0.82
time (sec)	N/A	0.159	8.013	0.000	0.000	0.000	0.566	0.000	0.265	0.403

Problem 1124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	102	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	2.00	0.00
time (sec)	N/A	0.159	10.008	0.000	0.000	0.000	0.636	0.000	0.303	0.000

Problem 1125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	204	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	4.00	0.00
time (sec)	N/A	0.159	10.012	0.000	0.000	0.000	0.670	0.000	0.355	0.000

Problem 1126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	201	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	3.94	0.00
time (sec)	N/A	0.164	10.009	0.000	0.000	0.000	0.793	0.000	0.353	0.000

Problem 1127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.173	9.090	0.000	0.000	0.000	0.810	0.000	0.224	0.000

Problem 1128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.164	9.009	0.000	0.000	0.000	0.710	0.000	0.234	0.000

Problem 1129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.165	8.620	0.000	0.000	0.000	0.658	0.000	0.206	0.000

Problem 1130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.152	0.004	0.000	0.000	0.000	0.611	0.000	0.193	0.209

Problem 1131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	29	0	66	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	1.35	0.82
time (sec)	N/A	0.162	8.753	0.000	0.000	0.000	0.645	0.000	0.257	0.404

Problem 1132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	164	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	3.22	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	0.712	0.000	0.256	0.000

Problem 1133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	163	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	3.20	0.00
time (sec)	N/A	0.157	10.010	0.000	0.000	0.000	0.800	0.000	0.265	0.000

Problem 1134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	166	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	3.25	0.00
time (sec)	N/A	0.160	10.009	0.000	0.000	0.000	0.912	0.000	0.349	0.000

Problem 1135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.160	9.625	0.000	0.000	0.000	1.048	0.000	0.226	0.000

Problem 1136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.161	9.427	0.000	0.000	0.000	0.951	0.000	0.209	0.000

Problem 1137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.159	9.021	0.000	0.000	0.000	0.813	0.000	0.229	0.000

Problem 1138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.150	0.004	0.000	0.000	0.000	0.688	0.000	0.220	0.227

Problem 1139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	29	0	139	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	2.84	0.82
time (sec)	N/A	0.158	9.252	0.000	0.000	0.000	0.838	0.000	0.330	0.482

Problem 1140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	265	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	5.20	0.00
time (sec)	N/A	0.161	10.009	0.000	0.000	0.000	0.813	0.000	0.396	0.000

Problem 1141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	264	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	5.18	0.00
time (sec)	N/A	0.160	10.009	0.000	0.000	0.000	0.888	0.000	0.389	0.000

Problem 1142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	337	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	6.61	0.00
time (sec)	N/A	0.160	10.009	0.000	0.000	0.000	1.035	0.000	0.444	0.000

Problem 1143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.162	10.015	0.000	0.000	0.000	1.342	0.000	0.215	0.000

Problem 1144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.157	10.009	0.000	0.000	0.000	1.150	0.000	0.226	0.000

Problem 1145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	29	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.29	0.00
time (sec)	N/A	0.161	9.549	0.000	0.000	0.000	1.049	0.000	0.204	0.000

Problem 1146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	26	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.57	0.00	0.24	0.80
time (sec)	N/A	0.151	0.004	0.000	0.000	0.000	0.886	0.000	0.191	0.210

Problem 1147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	29	0	156	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	3.18	0.82
time (sec)	N/A	0.160	9.677	0.000	0.000	0.000	1.097	0.000	0.338	0.586

Problem 1148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	241	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	4.73	0.00
time (sec)	N/A	0.160	10.009	0.000	0.000	0.000	1.081	0.000	0.397	0.000

Problem 1149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	241	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	4.73	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	1.084	0.000	0.419	0.000

Problem 1150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	34	0	286	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.67	0.00	5.61	0.00
time (sec)	N/A	0.158	10.008	0.000	0.000	0.000	1.145	0.000	0.534	0.000

Problem 1151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.161	8.263	0.000	0.000	0.000	0.641	0.000	0.259	0.000

Problem 1152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.162	8.194	0.000	0.000	0.000	0.559	0.000	0.238	0.000

Problem 1153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.166	7.995	0.000	0.000	0.000	0.498	0.000	0.218	0.000

Problem 1154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.153	0.004	0.000	0.000	0.000	0.485	0.000	0.202	0.270

Problem 1155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	27	0	155	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	3.16	0.82
time (sec)	N/A	0.168	8.028	0.000	0.000	0.000	0.533	0.000	0.277	0.446

Problem 1156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	159	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	3.12	0.00
time (sec)	N/A	0.163	10.008	0.000	0.000	0.000	0.615	0.000	0.346	0.000

Problem 1157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	206	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	4.04	0.00
time (sec)	N/A	0.163	10.009	0.000	0.000	0.000	0.670	0.000	0.426	0.000

Problem 1158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.166	8.871	0.000	0.000	0.000	0.557	0.000	0.254	0.000

Problem 1159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.163	8.816	0.000	0.000	0.000	0.537	0.000	0.253	0.000

Problem 1160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.161	8.485	0.000	0.000	0.000	0.483	0.000	0.216	0.000

Problem 1161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.157	0.004	0.000	0.000	0.000	0.475	0.000	0.188	0.249

Problem 1162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	27	0	138	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	2.82	0.82
time (sec)	N/A	0.163	8.595	0.000	0.000	0.000	0.533	0.000	0.279	0.452

Problem 1163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	141	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	2.76	0.00
time (sec)	N/A	0.160	10.008	0.000	0.000	0.000	0.634	0.000	0.302	0.000

Problem 1164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	214	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	4.20	0.00
time (sec)	N/A	0.159	10.009	0.000	0.000	0.000	0.740	0.000	0.401	0.000

Problem 1165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.161	8.888	0.000	0.000	0.000	0.499	0.000	0.242	0.000

Problem 1166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.156	8.840	0.000	0.000	0.000	0.492	0.000	0.221	0.000

Problem 1167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.155	8.344	0.000	0.000	0.000	0.495	0.000	0.217	0.000

Problem 1168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	11	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.24	0.80
time (sec)	N/A	0.150	0.005	0.000	0.000	0.000	0.488	0.000	0.192	0.248

Problem 1169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	27	0	138	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	2.82	0.82
time (sec)	N/A	0.164	8.569	0.000	0.000	0.000	0.588	0.000	0.243	0.466

Problem 1170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	141	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	2.76	0.00
time (sec)	N/A	0.161	10.010	0.000	0.000	0.000	0.635	0.000	0.310	0.000

Problem 1171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	214	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	4.20	0.00
time (sec)	N/A	0.157	10.009	0.000	0.000	0.000	0.758	0.000	0.336	0.000

Problem 1172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.159	8.973	0.000	0.000	0.000	0.497	0.000	0.256	0.000

Problem 1173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.159	8.940	0.000	0.000	0.000	0.511	0.000	0.215	0.000

Problem 1174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	27	0	15	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	0.29	0.00
time (sec)	N/A	0.157	8.435	0.000	0.000	0.000	0.502	0.000	0.195	0.000

Problem 1175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	24	0	40	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.87	0.80
time (sec)	N/A	0.149	0.004	0.000	0.000	0.000	0.510	0.000	0.190	0.260

Problem 1176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	27	0	82	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	1.67	0.82
time (sec)	N/A	0.155	8.706	0.000	0.000	0.000	0.605	0.000	0.248	0.471

Problem 1177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	155	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	3.04	0.00
time (sec)	N/A	0.159	10.010	0.000	0.000	0.000	0.655	0.000	0.235	0.000

Problem 1178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	32	0	122	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.63	0.00	2.39	0.00
time (sec)	N/A	0.158	10.010	0.000	0.000	0.000	0.791	0.000	0.297	0.000

Problem 1179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	173	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	3.20	0.00
time (sec)	N/A	0.156	8.731	0.000	0.000	0.000	0.556	0.000	0.318	0.000

Problem 1180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	150	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	2.78	0.00
time (sec)	N/A	0.157	8.641	0.000	0.000	0.000	0.564	0.000	0.285	0.000

Problem 1181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	0.63	0.00
time (sec)	N/A	0.160	8.193	0.000	0.000	0.000	0.566	0.000	0.226	0.000

Problem 1182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.150	0.004	0.000	0.000	0.000	0.536	0.000	0.184	0.309

Problem 1183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	27	0	33	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.63	0.77
time (sec)	N/A	0.160	8.503	0.000	0.000	0.000	0.701	0.000	0.201	0.515

Problem 1184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	32	0	241	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	4.46	0.00
time (sec)	N/A	0.158	10.012	0.000	0.000	0.000	0.770	0.000	0.418	0.000

Problem 1185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	32	0	323	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	5.98	0.00
time (sec)	N/A	0.157	10.009	0.000	0.000	0.000	0.897	0.000	0.484	0.000

Problem 1186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	156	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	2.89	0.00
time (sec)	N/A	0.159	9.647	0.000	0.000	0.000	0.685	0.000	0.275	0.000

Problem 1187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	133	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	2.46	0.00
time (sec)	N/A	0.155	9.435	0.000	0.000	0.000	0.667	0.000	0.260	0.000

Problem 1188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	27	0	34	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	0.63	0.00
time (sec)	N/A	0.154	8.902	0.000	0.000	0.000	0.680	0.000	0.220	0.000

Problem 1189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	24	0	30	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.49	0.00	0.61	0.76
time (sec)	N/A	0.147	0.006	0.000	0.000	0.000	0.648	0.000	0.188	0.306

Problem 1190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	0	0	0	27	0	33	40
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.52	0.00	0.63	0.77
time (sec)	N/A	0.158	9.318	0.000	0.000	0.000	0.811	0.000	0.190	0.533

Problem 1191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	32	0	170	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	3.15	0.00
time (sec)	N/A	0.155	10.010	0.000	0.000	0.000	0.913	0.000	0.348	0.000

Problem 1192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	32	0	247	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.59	0.00	4.57	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	1.084	0.000	0.427	0.000

Problem 1193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	541	53	53	0	0	0	31	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.159	8.377	0.000	0.000	0.000	0.679	0.000	0.175	0.000

Problem 1194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	515	53	53	0	0	0	31	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.160	8.267	0.000	0.000	0.000	0.606	0.000	0.231	0.000

Problem 1195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	489	53	53	0	0	0	31	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.156	8.064	0.000	0.000	0.000	0.568	0.000	0.209	0.000

Problem 1196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	48	48	0	0	0	27	0	13	39
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.08
time (sec)	N/A	0.148	0.004	0.000	0.000	0.000	0.539	0.000	0.187	0.245

Problem 1197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	453	51	51	0	0	0	31	0	181	42
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.40	0.09
time (sec)	N/A	0.158	8.078	0.000	0.000	0.000	0.577	0.000	0.275	0.493

Problem 1198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	487	53	53	0	0	0	36	0	185	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.38	0.00
time (sec)	N/A	0.157	10.018	0.000	0.000	0.000	0.651	0.000	0.277	0.000

Problem 1199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	517	53	53	0	0	0	36	0	209	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.07	0.00	0.40	0.00
time (sec)	N/A	0.159	10.008	0.000	0.000	0.000	0.744	0.000	0.298	0.000

Problem 1200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	543	53	53	0	0	0	36	0	303	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.07	0.00	0.56	0.00
time (sec)	N/A	0.160	10.008	0.000	0.000	0.000	0.840	0.000	0.351	0.000

Problem 1201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	545	53	53	0	0	0	32	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.160	9.098	0.000	0.000	0.000	0.873	0.000	0.190	0.000

Problem 1202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	519	53	53	0	0	0	32	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.167	9.035	0.000	0.000	0.000	0.708	0.000	0.187	0.000

Problem 1203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	493	53	53	0	0	0	32	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.161	8.677	0.000	0.000	0.000	0.666	0.000	0.209	0.000

Problem 1204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	469	48	48	0	0	0	29	0	13	39
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.08
time (sec)	N/A	0.154	0.004	0.000	0.000	0.000	0.587	0.000	0.188	0.230

Problem 1205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	463	51	51	0	0	0	31	0	76	42
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.16	0.09
time (sec)	N/A	0.167	8.836	0.000	0.000	0.000	0.683	0.000	0.200	0.431

Problem 1206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	491	53	53	0	0	0	36	0	186	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.38	0.00
time (sec)	N/A	0.159	10.009	0.000	0.000	0.000	0.743	0.000	0.248	0.000

Problem 1207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	521	53	53	0	0	0	36	0	185	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.07	0.00	0.36	0.00
time (sec)	N/A	0.162	10.011	0.000	0.000	0.000	0.817	0.000	0.249	0.000

Problem 1208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	547	53	53	0	0	0	36	0	188	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.07	0.00	0.34	0.00
time (sec)	N/A	0.157	10.008	0.000	0.000	0.000	0.969	0.000	0.233	0.000

Problem 1209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	995	53	53	0	0	0	32	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.162	9.665	0.000	0.000	0.000	1.030	0.000	0.210	0.000

Problem 1210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	969	53	53	0	0	0	32	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.163	9.480	0.000	0.000	0.000	0.938	0.000	0.185	0.000

Problem 1211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	943	53	53	0	0	0	32	0	17	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.161	9.035	0.000	0.000	0.000	0.818	0.000	0.196	0.000

Problem 1212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	919	48	48	0	0	0	29	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.151	0.005	0.000	0.000	0.000	0.733	0.000	0.221	0.244

Problem 1213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	907	51	51	0	0	0	31	0	80	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.09	0.05
time (sec)	N/A	0.159	9.327	0.000	0.000	0.000	0.887	0.000	0.383	0.534

Problem 1214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	937	53	53	0	0	0	36	0	176	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.04	0.00	0.19	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	0.863	0.000	0.315	0.000

Problem 1215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	971	53	53	0	0	0	36	0	180	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.04	0.00	0.19	0.00
time (sec)	N/A	0.156	10.009	0.000	0.000	0.000	0.949	0.000	0.338	0.000

Problem 1216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	997	53	53	0	0	0	36	0	180	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.04	0.00	0.18	0.00
time (sec)	N/A	0.158	10.010	0.000	0.000	0.000	1.060	0.000	0.359	0.000

Problem 1217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1019	53	53	0	0	0	32	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.161	10.014	0.000	0.000	0.000	1.362	0.000	0.222	0.000

Problem 1218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	993	53	53	0	0	0	32	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.161	10.009	0.000	0.000	0.000	1.205	0.000	0.195	0.000

Problem 1219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	967	53	53	0	0	0	32	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.158	9.631	0.000	0.000	0.000	1.055	0.000	0.193	0.000

Problem 1220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	938	48	48	0	0	0	29	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.150	8.996	0.000	0.000	0.000	0.887	0.000	0.191	0.254

Problem 1221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	926	51	51	0	0	0	29	0	90	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.10	0.05
time (sec)	N/A	0.165	9.762	0.000	0.000	0.000	1.113	0.000	0.271	0.563

Problem 1222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	961	53	53	0	0	0	34	0	207	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.04	0.00	0.22	0.00
time (sec)	N/A	0.163	10.011	0.000	0.000	0.000	1.066	0.000	0.321	0.000

Problem 1223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	995	53	53	0	0	0	34	0	210	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.21	0.00
time (sec)	N/A	0.161	10.010	0.000	0.000	0.000	1.086	0.000	0.347	0.000

Problem 1224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1021	53	53	0	0	0	34	0	210	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.21	0.00
time (sec)	N/A	0.161	10.010	0.000	0.000	0.000	1.210	0.000	0.333	0.000

Problem 1225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	996	53	53	0	0	0	29	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.164	8.289	0.000	0.000	0.000	0.666	0.000	0.196	0.000

Problem 1226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	970	53	53	0	0	0	29	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.158	8.248	0.000	0.000	0.000	0.540	0.000	0.213	0.000

Problem 1227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	944	53	53	0	0	0	29	0	17	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.164	8.002	0.000	0.000	0.000	0.509	0.000	0.190	0.000

Problem 1228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	919	48	48	0	0	0	26	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.154	0.005	0.000	0.000	0.000	0.494	0.000	0.209	0.306

Problem 1229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	929	51	51	0	0	0	29	0	179	42
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.19	0.05
time (sec)	N/A	0.161	8.008	0.000	0.000	0.000	0.560	0.000	0.282	0.496

Problem 1230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	964	53	53	0	0	0	34	0	183	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.04	0.00	0.19	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	0.627	0.000	0.260	0.000

Problem 1231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	998	53	53	0	0	0	34	0	207	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.21	0.00
time (sec)	N/A	0.156	10.010	0.000	0.000	0.000	0.717	0.000	0.293	0.000

Problem 1232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	972	53	53	0	0	0	31	0	17	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.166	8.913	0.000	0.000	0.000	0.583	0.000	0.249	0.000

Problem 1233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	946	53	53	0	0	0	31	0	17	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.156	8.828	0.000	0.000	0.000	0.562	0.000	0.192	0.000

Problem 1234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	53	53	0	0	0	31	0	17	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.02	0.00
time (sec)	N/A	0.160	8.500	0.000	0.000	0.000	0.511	0.000	0.193	0.000

Problem 1235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	893	48	48	0	0	0	27	0	13	39
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.01	0.04
time (sec)	N/A	0.149	0.004	0.000	0.000	0.000	0.489	0.000	0.217	0.305

Problem 1236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	909	51	51	0	0	0	29	0	181	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.20	0.05
time (sec)	N/A	0.162	8.636	0.000	0.000	0.000	0.584	0.000	0.259	0.525

Problem 1237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	940	53	53	0	0	0	34	0	185	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.04	0.00	0.20	0.00
time (sec)	N/A	0.157	10.010	0.000	0.000	0.000	0.686	0.000	0.331	0.000

Problem 1238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	974	53	53	0	0	0	34	0	209	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.21	0.00
time (sec)	N/A	0.160	10.009	0.000	0.000	0.000	0.756	0.000	0.350	0.000

Problem 1239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	522	53	53	0	0	0	31	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.166	8.937	0.000	0.000	0.000	0.510	0.000	0.213	0.000

Problem 1240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	496	53	53	0	0	0	31	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.157	8.898	0.000	0.000	0.000	0.521	0.000	0.232	0.000

Problem 1241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	472	53	53	0	0	0	31	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.04	0.00
time (sec)	N/A	0.161	8.378	0.000	0.000	0.000	0.479	0.000	0.232	0.000

Problem 1242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	447	48	48	0	0	0	27	0	13	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.09
time (sec)	N/A	0.152	0.004	0.000	0.000	0.000	0.487	0.000	0.221	0.306

Problem 1243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	465	51	51	0	0	0	29	0	158	42
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.34	0.09
time (sec)	N/A	0.160	8.650	0.000	0.000	0.000	0.589	0.000	0.222	0.458

Problem 1244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	494	53	53	0	0	0	34	0	161	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.33	0.00
time (sec)	N/A	0.162	10.011	0.000	0.000	0.000	0.649	0.000	0.252	0.000

Problem 1245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	524	53	53	0	0	0	34	0	189	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.36	0.00
time (sec)	N/A	0.162	10.010	0.000	0.000	0.000	0.745	0.000	0.271	0.000

Problem 1246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	518	53	53	0	0	0	31	0	17	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.161	9.014	0.000	0.000	0.000	0.541	0.000	0.212	0.000

Problem 1247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	492	53	53	0	0	0	31	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.03	0.00
time (sec)	N/A	0.157	8.969	0.000	0.000	0.000	0.521	0.000	0.196	0.000

Problem 1248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	464	53	53	0	0	0	31	0	17	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.04	0.00
time (sec)	N/A	0.155	8.474	0.000	0.000	0.000	0.508	0.000	0.218	0.000

Problem 1249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	437	48	48	0	0	0	27	0	49	39
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.11	0.09
time (sec)	N/A	0.151	0.004	0.000	0.000	0.000	0.522	0.000	0.238	0.278

Problem 1250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	51	51	0	0	0	27	0	95	42
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.06	0.00	0.21	0.09
time (sec)	N/A	0.157	8.745	0.000	0.000	0.000	0.644	0.000	0.245	0.445

Problem 1251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	490	53	53	0	0	0	32	0	209	0
N.S.	1	0.11	0.11	0.00	0.00	0.00	0.07	0.00	0.43	0.00
time (sec)	N/A	0.157	10.011	0.000	0.000	0.000	0.684	0.000	0.305	0.000

Problem 1252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	520	53	53	0	0	0	32	0	209	0
N.S.	1	0.10	0.10	0.00	0.00	0.00	0.06	0.00	0.40	0.00
time (sec)	N/A	0.158	10.010	0.000	0.000	0.000	0.769	0.000	0.309	0.000

Problem 1253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	991	56	56	0	0	0	31	0	199	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.20	0.00
time (sec)	N/A	0.160	8.805	0.000	0.000	0.000	0.601	0.000	0.314	0.000

Problem 1254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	965	56	56	0	0	0	31	0	174	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.18	0.00
time (sec)	N/A	0.160	8.723	0.000	0.000	0.000	0.604	0.000	0.289	0.000

Problem 1255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	56	56	0	0	0	31	0	41	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.00
time (sec)	N/A	0.159	8.279	0.000	0.000	0.000	0.564	0.000	0.249	0.000

Problem 1256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	893	52	52	0	0	0	27	0	37	39
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.150	0.005	0.000	0.000	0.000	0.551	0.000	0.350	0.343

Problem 1257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	951	53	53	0	0	0	27	0	40	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.172	8.603	0.000	0.000	0.000	0.689	0.000	0.268	0.530

Problem 1258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	985	56	56	0	0	0	32	0	183	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.19	0.00
time (sec)	N/A	0.163	10.009	0.000	0.000	0.000	0.807	0.000	0.360	0.000

Problem 1259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1019	56	56	0	0	0	32	0	276	0
N.S.	1	0.05	0.05	0.00	0.00	0.00	0.03	0.00	0.27	0.00
time (sec)	N/A	0.162	10.009	0.000	0.000	0.000	0.897	0.000	0.438	0.000

Problem 1260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	969	56	56	0	0	0	31	0	201	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.21	0.00
time (sec)	N/A	0.161	9.672	0.000	0.000	0.000	0.690	0.000	0.291	0.000

Problem 1261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	945	56	56	0	0	0	31	0	176	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.19	0.00
time (sec)	N/A	0.165	9.489	0.000	0.000	0.000	0.696	0.000	0.315	0.000

Problem 1262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	56	56	0	0	0	31	0	41	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.00
time (sec)	N/A	0.158	8.940	0.000	0.000	0.000	0.685	0.000	0.236	0.000

Problem 1263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	922	52	52	0	0	0	27	0	37	39
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.152	0.005	0.000	0.000	0.000	0.660	0.000	0.203	0.324

Problem 1264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	935	53	53	0	0	0	29	0	40	42
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.04	0.04
time (sec)	N/A	0.160	9.335	0.000	0.000	0.000	0.843	0.000	0.225	0.575

Problem 1265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	963	56	56	0	0	0	34	0	185	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.04	0.00	0.19	0.00
time (sec)	N/A	0.158	10.009	0.000	0.000	0.000	0.941	0.000	0.396	0.000

Problem 1266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	C	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	997	56	56	0	0	0	34	0	279	0
N.S.	1	0.06	0.06	0.00	0.00	0.00	0.03	0.00	0.28	0.00
time (sec)	N/A	0.171	10.010	0.000	0.000	0.000	1.088	0.000	0.453	0.000

Problem 1267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	95	95	132	106	148	1923	260	147	183
N.S.	1	0.95	0.95	1.32	1.06	1.48	19.23	2.60	1.47	1.83
time (sec)	N/A	0.220	0.071	0.332	0.039	0.077	2.759	0.133	0.190	0.414

Problem 1268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	64	80	73	98	920	132	92	117
N.S.	1	0.97	0.89	1.11	1.01	1.36	12.78	1.83	1.28	1.62
time (sec)	N/A	0.209	0.046	0.296	0.035	0.096	1.303	0.130	0.326	0.320

Problem 1269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	47	40	42	47	58	333	51	52	68
N.S.	1	0.98	0.83	0.88	0.98	1.21	6.94	1.06	1.08	1.42
time (sec)	N/A	0.188	0.039	0.279	0.035	0.081	0.530	0.122	0.174	0.291

Problem 1270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	21	25	87	21	26	21
N.S.	1	1.00	0.96	0.96	0.91	1.09	3.78	0.91	1.13	0.91
time (sec)	N/A	0.152	0.002	0.270	0.026	0.072	0.281	0.114	0.233	0.309

Problem 1271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	0	0	0	39	0	42	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.95	0.00	1.02	0.00
time (sec)	N/A	0.164	0.029	0.000	0.000	0.000	1.018	0.000	0.229	0.000

Problem 1272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	41	0	47	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.98	0.00	1.12	0.00
time (sec)	N/A	0.165	0.028	0.000	0.000	0.000	2.055	0.000	0.212	0.000

Problem 1273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	26	0	657	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	13.41	0.00
time (sec)	N/A	0.169	0.037	0.000	0.000	0.000	5.734	0.000	0.229	0.000

Problem 1274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	26	0	419	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	8.55	0.00
time (sec)	N/A	0.171	0.034	0.000	0.000	0.000	3.330	0.000	0.198	0.000

Problem 1275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	26	0	233	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.53	0.00	4.76	0.00
time (sec)	N/A	0.168	0.028	0.000	0.000	0.000	1.850	0.000	0.190	0.000

Problem 1276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	22	0	94	41
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.50	0.00	2.14	0.93
time (sec)	N/A	0.159	0.024	0.000	0.000	0.000	1.064	0.000	0.201	0.851

Problem 1277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	0	0	0	26	0	115	58
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.55	0.00	2.45	1.23
time (sec)	N/A	0.164	0.034	0.000	0.000	0.000	1.718	0.000	0.213	0.398

Problem 1278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	455	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	8.92	0.00
time (sec)	N/A	0.161	0.087	0.000	0.000	0.000	0.000	0.000	0.233	0.000

Problem 1279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	247	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	4.84	0.00
time (sec)	N/A	0.161	0.116	0.000	0.000	0.000	0.000	0.000	0.216	0.000

Problem 1280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	254	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	4.98	0.00
time (sec)	N/A	0.162	0.102	0.000	0.000	0.000	162.501	0.000	0.200	0.000

Problem 1281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	37	0	107	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.73	0.00	2.10	0.00
time (sec)	N/A	0.170	0.082	0.000	0.000	0.000	23.446	0.000	0.223	0.000

Problem 1282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	37	0	106	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.76	0.00	2.16	0.00
time (sec)	N/A	0.161	0.110	0.000	0.000	0.000	14.988	0.000	0.192	0.000

Problem 1283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	0	0	0	41	0	123	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.84	0.00	2.51	0.00
time (sec)	N/A	0.166	0.129	0.000	0.000	0.000	79.453	0.000	0.197	0.000

Problem 1284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	128	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.51	0.00
time (sec)	N/A	0.161	0.131	0.000	0.000	0.000	0.000	0.000	0.247	0.000

Problem 1285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	132	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	2.59	0.00
time (sec)	N/A	0.161	0.106	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 1286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	63	0	0	0	51	0	171	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.84	0.00	2.80	0.00
time (sec)	N/A	0.175	0.022	0.000	0.000	0.000	9.732	0.000	0.199	0.000

Problem 1287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	174	0
N.S.	1	1.00	0.97	0.00	0.00	0.00	0.82	0.00	2.64	0.00
time (sec)	N/A	0.179	0.008	0.000	0.000	0.000	10.208	0.000	0.189	0.000

Problem 1288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	109	62	81	84	106	452	0	99	154
N.S.	1	1.04	0.59	0.77	0.80	1.01	4.30	0.00	0.94	1.47
time (sec)	N/A	0.223	0.027	0.440	0.045	0.097	9.039	0.000	0.215	0.584

Problem 1289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	62	45	59	67	162	0	61	96
N.S.	1	1.00	0.93	0.67	0.88	1.00	2.42	0.00	0.91	1.43
time (sec)	N/A	0.188	0.028	0.437	0.039	0.088	8.747	0.000	0.172	0.522

Problem 1290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	29	29	37	34	36	0	36	52
N.S.	1	1.00	0.97	0.97	1.23	1.13	1.20	0.00	1.20	1.73
time (sec)	N/A	0.150	0.024	0.431	0.049	0.106	8.688	0.000	0.192	0.467

Problem 1291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	37	0	22	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.66	0.00	0.39	0.00
time (sec)	N/A	0.177	0.022	0.000	0.000	0.000	5.762	0.000	0.193	0.000

Problem 1292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	0	0	0	39	0	65	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.61	0.00	1.02	0.00
time (sec)	N/A	0.184	0.028	0.000	0.000	0.000	8.520	0.000	0.216	0.000

Problem 1293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	61	0	0	0	39	0	132	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.61	0.00	2.06	0.00
time (sec)	N/A	0.185	0.029	0.000	0.000	0.000	10.000	0.000	0.201	0.000

Problem 1294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	0	0	54	0	71	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.77	0.00	1.01	0.00
time (sec)	N/A	0.188	0.030	0.000	0.000	0.000	8.636	0.000	0.196	0.000

Problem 1295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	0	0	54	0	71	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.77	0.00	1.01	0.00
time (sec)	N/A	0.185	0.031	0.000	0.000	0.000	8.585	0.000	0.200	0.000

Problem 1296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	0	0	51	0	68	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.73	0.00	0.97	0.00
time (sec)	N/A	0.187	0.030	0.000	0.000	0.000	8.351	0.000	0.204	0.000

Problem 1297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	65	0	0	0	32	0	61	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.46	0.00	0.88	0.00
time (sec)	N/A	0.187	0.027	0.000	0.000	0.000	6.837	0.000	0.230	0.000

Problem 1298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	65	0	0	0	46	0	129	0
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.67	0.00	1.87	0.00
time (sec)	N/A	0.184	0.029	0.000	0.000	0.000	8.910	0.000	0.219	0.000

Problem 1299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	0	0	42	0	54	0
N.S.	1	1.00	1.00	0.92	0.00	0.00	1.17	0.00	1.50	0.00
time (sec)	N/A	0.153	0.020	0.413	0.000	0.000	5.638	0.000	0.231	0.000

Problem 1300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	57	0	0	44	0	54	0
N.S.	1	1.00	1.00	1.04	0.00	0.00	0.80	0.00	0.98	0.00
time (sec)	N/A	0.173	0.021	0.404	0.000	0.000	5.728	0.000	0.244	0.000

Problem 1301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	0	0	0	42	0	56	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.74	0.00	0.98	0.00
time (sec)	N/A	0.170	0.023	0.000	0.000	0.000	7.777	0.000	0.215	0.000

Problem 1302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	87	0	0	42	0	132	0
N.S.	1	1.00	1.00	2.23	0.00	0.00	1.08	0.00	3.38	0.00
time (sec)	N/A	0.165	0.026	0.881	0.000	0.000	3.543	0.000	0.201	0.000

Problem 1303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	113	0	0	44	0	132	0
N.S.	1	1.00	1.00	1.92	0.00	0.00	0.75	0.00	2.24	0.00
time (sec)	N/A	0.187	0.028	0.812	0.000	0.000	3.448	0.000	0.230	0.000

Problem 1304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	0	0	0	51	0	135	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.85	0.00	2.25	0.00
time (sec)	N/A	0.190	0.032	0.000	0.000	0.000	4.159	0.000	0.229	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [221] had the largest ratio of [1.11111000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.00	11	0.182
3	A	2	2	1.00	11	0.182
4	A	2	2	1.00	9	0.222
5	A	1	1	1.00	7	0.143
6	A	2	2	1.00	11	0.182
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	11	0.182
9	A	2	2	1.00	11	0.182
10	A	2	2	1.00	11	0.182
11	A	2	2	1.00	11	0.182
12	A	2	2	1.00	11	0.182
13	A	4	3	1.13	13	0.231
14	A	2	2	1.00	13	0.154
15	A	4	3	1.13	13	0.231
16	A	2	2	1.00	13	0.154
17	A	1	1	1.00	11	0.091
18	A	2	2	1.00	9	0.222
19	A	4	3	1.30	13	0.231
20	A	2	2	1.00	13	0.154
21	A	4	3	1.04	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	13	0.154
23	A	4	3	1.25	13	0.231
24	A	2	2	1.00	13	0.154
25	A	1	1	1.00	13	0.077
26	A	2	2	1.00	13	0.154
27	A	4	3	1.13	13	0.231
28	A	2	2	1.00	13	0.154
29	A	4	3	1.09	13	0.231
30	A	4	3	1.09	13	0.231
31	A	4	3	1.09	13	0.231
32	A	4	3	1.12	13	0.231
33	A	1	1	1.00	11	0.091
34	A	4	3	1.10	13	0.231
35	A	4	3	1.05	13	0.231
36	A	4	3	1.02	13	0.231
37	A	4	3	1.10	13	0.231
38	A	1	1	1.00	13	0.077
39	A	4	3	1.10	13	0.231
40	A	4	3	1.09	13	0.231
41	A	4	3	1.09	13	0.231
42	A	2	2	1.00	13	0.154
43	A	2	2	1.00	13	0.154
44	A	2	2	1.00	13	0.154
45	A	2	2	1.00	9	0.222
46	A	2	2	1.00	13	0.154
47	A	2	2	1.00	13	0.154
48	A	2	2	1.00	13	0.154
49	A	2	2	1.00	13	0.154
50	A	2	2	1.00	13	0.154
51	A	2	2	1.00	13	0.154
52	A	4	3	1.01	13	0.231
53	A	4	3	1.06	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
54	A	4	3	1.06	13	0.231
55	A	4	3	1.06	13	0.231
56	A	4	3	1.08	13	0.231
57	A	4	3	1.12	13	0.231
58	A	1	1	1.00	11	0.091
59	A	4	3	1.03	13	0.231
60	A	4	3	1.03	13	0.231
61	A	4	3	1.06	13	0.231
62	A	4	3	1.06	13	0.231
63	A	4	3	1.02	13	0.231
64	A	4	3	1.03	13	0.231
65	A	1	1	1.00	13	0.077
66	A	4	3	1.10	13	0.231
67	A	5	4	1.16	13	0.308
68	A	4	3	1.03	13	0.231
69	A	4	3	1.06	13	0.231
70	A	2	2	1.00	13	0.154
71	A	2	2	1.00	13	0.154
72	A	2	2	1.00	13	0.154
73	A	2	2	1.00	13	0.154
74	A	2	2	1.00	9	0.222
75	A	2	2	1.00	13	0.154
76	A	2	2	1.00	13	0.154
77	A	2	2	1.00	13	0.154
78	A	2	2	1.00	13	0.154
79	A	2	2	1.00	13	0.154
80	A	2	2	1.00	13	0.154
81	A	2	2	1.00	13	0.154
82	A	2	2	1.00	13	0.154
83	A	2	2	1.00	13	0.154
84	A	2	2	1.00	13	0.154
85	A	4	3	1.03	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.04	13	0.231
87	A	4	3	1.04	13	0.231
88	A	4	3	1.06	13	0.231
89	A	4	3	1.08	13	0.231
90	A	4	3	1.12	13	0.231
91	A	1	1	1.00	11	0.091
92	A	4	3	1.06	13	0.231
93	A	4	3	1.04	13	0.231
94	A	4	3	1.02	13	0.231
95	A	4	3	1.07	13	0.231
96	A	4	3	1.06	13	0.231
97	A	4	3	1.06	13	0.231
98	A	4	3	1.02	13	0.231
99	A	4	3	1.03	13	0.231
100	A	4	3	1.06	13	0.231
101	A	1	1	1.00	13	0.077
102	A	4	3	1.10	13	0.231
103	A	5	4	1.16	13	0.308
104	A	6	5	1.19	13	0.385
105	A	7	6	1.21	13	0.462
106	A	4	3	1.02	13	0.231
107	A	4	3	1.02	13	0.231
108	A	4	3	1.04	13	0.231
109	A	2	2	1.00	13	0.154
110	A	2	2	1.00	13	0.154
111	A	2	2	1.00	13	0.154
112	A	2	2	1.00	13	0.154
113	A	2	2	1.00	9	0.222
114	A	2	2	1.00	13	0.154
115	A	2	2	1.00	13	0.154
116	A	2	2	1.00	13	0.154
117	A	2	2	1.00	13	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	2	2	1.00	13	0.154
119	A	2	2	1.00	13	0.154
120	A	2	2	1.00	13	0.154
121	A	2	2	1.00	13	0.154
122	A	2	2	1.00	13	0.154
123	A	2	2	1.00	13	0.154
124	A	4	3	0.99	13	0.231
125	A	4	3	0.98	13	0.231
126	A	4	3	0.98	13	0.231
127	A	4	3	0.98	13	0.231
128	A	4	3	0.96	13	0.231
129	A	1	1	1.00	11	0.091
130	A	5	4	1.18	13	0.308
131	A	4	3	1.03	13	0.231
132	A	4	3	1.02	13	0.231
133	A	4	3	1.02	13	0.231
134	A	4	3	1.01	13	0.231
135	A	2	2	1.00	13	0.154
136	A	2	2	1.00	13	0.154
137	A	2	2	1.00	13	0.154
138	A	2	2	1.00	13	0.154
139	A	2	2	1.00	13	0.154
140	A	1	1	1.00	9	0.111
141	A	2	2	1.00	13	0.154
142	A	3	3	1.19	13	0.231
143	A	4	4	1.17	13	0.308
144	A	5	5	1.23	13	0.385
145	A	4	3	0.97	13	0.231
146	A	4	3	0.99	13	0.231
147	A	4	3	0.97	13	0.231
148	A	4	3	0.98	13	0.231
149	A	4	3	0.98	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	3	0.94	13	0.231
151	A	1	1	1.00	11	0.091
152	A	4	3	1.03	13	0.231
153	A	4	3	1.06	13	0.231
154	A	4	3	1.03	13	0.231
155	A	4	3	0.99	13	0.231
156	A	4	3	1.01	13	0.231
157	A	3	3	1.04	13	0.231
158	A	3	3	1.08	13	0.231
159	A	3	3	1.05	13	0.231
160	A	3	3	1.06	13	0.231
161	A	3	3	1.14	13	0.231
162	A	2	2	1.00	13	0.154
163	A	2	2	1.00	9	0.222
164	A	3	3	1.13	13	0.231
165	A	4	4	1.16	13	0.308
166	A	5	5	1.19	13	0.385
167	A	6	6	1.23	13	0.462
168	A	4	3	0.97	13	0.231
169	A	4	3	0.98	13	0.231
170	A	4	3	1.02	13	0.231
171	A	4	3	1.03	13	0.231
172	A	4	3	0.95	13	0.231
173	A	4	3	1.04	13	0.231
174	A	1	1	1.00	13	0.077
175	A	1	1	1.00	11	0.091
176	A	4	3	1.02	13	0.231
177	A	4	3	1.03	13	0.231
178	A	4	3	1.02	13	0.231
179	A	4	3	1.06	13	0.231
180	A	4	3	0.97	13	0.231
181	A	4	4	1.12	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
182	A	4	4	1.14	13	0.308
183	A	4	4	1.13	13	0.308
184	A	4	4	1.20	13	0.308
185	A	3	3	1.12	13	0.231
186	A	3	3	1.08	13	0.231
187	A	3	3	1.13	9	0.333
188	A	4	4	1.22	13	0.308
189	A	5	5	1.21	13	0.385
190	A	6	6	1.26	13	0.462
191	A	7	7	1.25	13	0.538
192	A	4	3	0.98	13	0.231
193	A	4	3	0.99	13	0.231
194	A	4	3	0.98	13	0.231
195	A	4	3	0.99	13	0.231
196	A	1	1	1.00	13	0.077
197	A	4	3	1.10	13	0.231
198	A	5	4	1.21	13	0.308
199	A	6	5	1.26	13	0.385
200	A	4	3	1.04	13	0.231
201	A	4	3	1.06	13	0.231
202	A	4	3	1.08	13	0.231
203	A	4	3	1.12	13	0.231
204	A	1	1	1.00	11	0.091
205	A	4	3	1.01	13	0.231
206	A	4	3	1.00	13	0.231
207	A	4	3	1.00	13	0.231
208	A	4	3	0.99	13	0.231
209	A	11	11	1.25	13	0.846
210	A	11	11	1.27	13	0.846
211	A	11	11	1.30	13	0.846
212	A	10	10	1.32	13	0.769
213	A	10	10	1.31	13	0.769

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	10	10	1.29	13	0.769
215	A	10	10	1.28	13	0.769
216	A	10	10	1.26	13	0.769
217	A	10	10	1.24	13	0.769
218	A	10	10	1.23	13	0.769
219	A	10	10	1.21	13	0.769
220	A	10	10	1.20	13	0.769
221	A	10	10	1.35	9	1.111
222	A	11	11	1.40	13	0.846
223	A	12	12	1.30	13	0.923
224	A	13	13	1.30	13	1.000
225	A	5	4	1.27	13	0.308
226	A	6	5	1.11	13	0.385
227	A	4	3	1.04	13	0.231
228	A	5	4	0.96	13	0.308
229	A	4	3	1.00	13	0.231
230	A	4	3	1.00	13	0.231
231	A	1	1	1.00	9	0.111
232	A	2	2	1.00	11	0.182
233	A	2	2	1.00	13	0.154
234	A	1	1	1.00	11	0.091
235	B	1	1	6.23	73	0.014
236	A	4	3	1.00	14	0.214
237	A	2	2	1.00	14	0.143
238	A	1	1	1.00	12	0.083
239	A	1	1	1.00	10	0.100
240	A	5	4	1.17	14	0.286
241	A	2	2	1.00	14	0.143
242	A	4	3	1.06	14	0.214
243	A	4	3	0.94	14	0.214
244	A	2	2	1.00	14	0.143
245	A	1	1	1.00	12	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	2	2	1.00	10	0.200
247	A	4	3	1.02	14	0.214
248	A	3	3	1.11	14	0.214
249	A	4	3	1.02	14	0.214
250	A	1	1	1.00	14	0.071
251	A	3	3	1.07	14	0.214
252	A	1	1	1.00	12	0.083
253	A	3	3	1.12	10	0.300
254	A	4	3	1.02	14	0.214
255	A	4	4	1.20	14	0.286
256	A	4	3	1.04	14	0.214
257	A	4	3	1.11	14	0.214
258	A	5	5	1.14	14	0.357
259	A	1	1	1.00	12	0.083
260	A	5	5	1.24	10	0.500
261	A	4	3	1.01	14	0.214
262	A	6	6	1.29	14	0.429
263	A	4	3	1.02	14	0.214
264	A	2	2	1.00	13	0.154
265	A	2	2	1.00	13	0.154
266	A	2	2	1.00	13	0.154
267	A	2	2	1.00	13	0.154
268	A	2	2	1.00	13	0.154
269	A	2	2	1.00	13	0.154
270	A	2	2	1.00	13	0.154
271	A	2	2	1.00	13	0.154
272	A	2	2	1.00	15	0.133
273	A	2	2	1.00	15	0.133
274	A	2	2	1.00	15	0.133
275	A	2	2	1.00	15	0.133
276	A	2	2	1.00	15	0.133
277	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
278	A	2	2	1.00	15	0.133
279	A	2	2	1.00	15	0.133
280	A	2	2	1.00	15	0.133
281	A	2	2	1.00	15	0.133
282	A	2	2	1.00	15	0.133
283	A	2	2	1.00	15	0.133
284	A	2	2	1.00	15	0.133
285	A	2	2	1.00	15	0.133
286	A	2	2	1.00	15	0.133
287	A	2	2	1.00	15	0.133
288	A	12	11	1.56	15	0.733
289	A	11	10	1.56	15	0.667
290	A	11	10	1.56	15	0.667
291	A	10	9	1.56	15	0.600
292	A	10	9	1.57	15	0.600
293	A	11	10	1.57	15	0.667
294	A	11	10	1.55	15	0.667
295	A	12	11	1.55	15	0.733
296	A	12	11	1.46	15	0.733
297	A	11	10	1.46	15	0.667
298	A	11	10	1.46	15	0.667
299	A	11	10	1.46	15	0.667
300	A	11	10	1.46	15	0.667
301	A	12	11	1.46	15	0.733
302	A	12	11	1.47	15	0.733
303	A	13	12	1.46	15	0.800
304	A	12	11	1.45	15	0.733
305	A	12	11	1.43	15	0.733
306	A	12	11	1.43	15	0.733
307	A	12	11	1.45	15	0.733
308	A	12	11	1.45	15	0.733
309	A	13	12	1.45	15	0.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
310	A	13	12	1.46	15	0.800
311	A	14	13	1.45	15	0.867
312	A	12	11	1.48	13	0.846
313	A	11	10	1.52	13	0.769
314	A	11	10	1.51	13	0.769
315	A	10	9	1.55	13	0.692
316	A	10	9	1.58	13	0.692
317	A	11	10	1.53	13	0.769
318	A	11	10	1.50	13	0.769
319	A	12	11	1.46	13	0.846
320	A	12	11	1.34	13	0.846
321	A	11	10	1.37	13	0.769
322	A	11	10	1.37	13	0.769
323	A	11	10	1.37	13	0.769
324	A	11	10	1.37	13	0.769
325	A	12	11	1.34	13	0.846
326	A	12	11	1.36	13	0.846
327	A	13	12	1.31	13	0.923
328	A	12	11	1.36	13	0.846
329	A	12	11	1.36	13	0.846
330	A	12	11	1.36	13	0.846
331	A	12	11	1.36	13	0.846
332	A	12	11	1.36	13	0.846
333	A	13	12	1.34	13	0.923
334	A	13	12	1.36	13	0.923
335	A	14	13	1.31	13	1.000
336	A	5	4	1.12	16	0.250
337	A	5	4	1.53	15	0.267
338	A	12	11	1.37	17	0.647
339	A	12	11	1.02	17	0.647
340	A	11	10	1.38	17	0.588
341	A	13	12	1.05	17	0.706

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
342	A	12	11	1.37	17	0.647
343	A	13	12	1.41	17	0.706
344	A	13	12	1.03	17	0.706
345	A	12	11	1.42	17	0.647
346	A	12	11	0.99	17	0.647
347	A	13	12	1.41	17	0.706
348	A	10	9	1.51	13	0.692
349	A	2	2	1.00	13	0.154
350	A	2	2	1.00	13	0.154
351	A	2	2	1.00	13	0.154
352	A	2	2	1.00	13	0.154
353	A	2	2	1.00	11	0.182
354	A	1	1	1.00	13	0.077
355	A	1	1	1.00	13	0.077
356	A	1	1	1.00	13	0.077
357	A	1	1	1.00	17	0.059
358	A	1	1	1.00	15	0.067
359	A	1	1	1.00	17	0.059
360	A	1	1	1.00	17	0.059
361	A	1	1	1.00	17	0.059
362	A	1	1	1.00	16	0.062
363	A	4	3	1.05	15	0.200
364	A	4	3	1.07	15	0.200
365	A	4	3	1.11	15	0.200
366	A	1	1	1.00	13	0.077
367	A	5	4	1.16	15	0.267
368	A	5	4	1.00	15	0.267
369	A	6	5	1.03	15	0.333
370	A	7	6	1.08	15	0.400
371	A	6	5	1.10	15	0.333
372	A	5	4	1.04	15	0.267
373	A	4	3	1.00	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
374	A	4	3	1.00	15	0.200
375	A	1	1	1.00	15	0.067
376	A	2	2	1.00	15	0.133
377	A	3	3	1.09	15	0.200
378	A	4	4	1.13	15	0.267
379	A	4	3	1.05	15	0.200
380	A	4	3	1.07	15	0.200
381	A	4	3	1.11	15	0.200
382	A	1	1	1.00	13	0.077
383	A	6	5	1.13	15	0.333
384	A	6	5	1.07	15	0.333
385	A	6	5	1.03	15	0.333
386	A	7	6	1.04	15	0.400
387	A	8	7	1.09	15	0.467
388	A	7	6	1.10	15	0.400
389	A	6	5	1.07	15	0.333
390	A	5	4	1.05	11	0.364
391	A	5	4	1.03	15	0.267
392	A	5	4	0.98	15	0.267
393	A	1	1	1.00	15	0.067
394	A	2	2	1.00	15	0.133
395	A	3	3	1.09	15	0.200
396	A	4	4	1.13	15	0.267
397	A	4	3	1.05	15	0.200
398	A	4	3	1.07	15	0.200
399	A	4	3	1.11	15	0.200
400	A	1	1	1.00	13	0.077
401	A	7	6	1.10	15	0.400
402	A	7	6	1.02	15	0.400
403	A	7	6	1.02	15	0.400
404	A	7	6	1.02	15	0.400
405	A	8	7	1.03	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
406	A	9	8	1.07	15	0.533
407	A	8	7	1.11	15	0.467
408	A	7	6	1.08	15	0.400
409	A	6	5	1.07	11	0.455
410	A	6	5	0.98	15	0.333
411	A	6	5	1.00	15	0.333
412	A	6	5	0.95	15	0.333
413	A	1	1	1.00	15	0.067
414	A	2	2	1.00	15	0.133
415	A	3	3	1.09	15	0.200
416	A	4	4	1.13	15	0.267
417	A	5	5	1.16	15	0.333
418	A	6	6	1.17	15	0.400
419	A	4	3	1.03	15	0.200
420	A	4	3	1.04	15	0.200
421	A	4	3	1.03	15	0.200
422	A	4	3	1.04	15	0.200
423	A	4	3	1.05	15	0.200
424	A	4	3	1.07	15	0.200
425	A	4	3	1.11	15	0.200
426	A	1	1	1.00	13	0.077
427	A	9	8	1.06	15	0.533
428	A	9	8	1.03	15	0.533
429	A	9	8	0.98	15	0.533
430	A	9	8	0.98	15	0.533
431	A	9	8	1.01	15	0.533
432	A	9	8	1.01	15	0.533
433	A	10	9	1.02	15	0.600
434	A	11	10	1.05	15	0.667
435	A	11	10	1.13	15	0.667
436	A	10	9	1.12	15	0.600
437	A	9	8	1.10	15	0.533

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
438	A	8	7	1.10	11	0.636
439	A	8	7	0.96	15	0.467
440	A	8	7	0.94	15	0.467
441	A	8	7	0.97	15	0.467
442	A	8	7	1.00	15	0.467
443	A	8	7	0.93	15	0.467
444	A	1	1	1.00	15	0.067
445	A	2	2	1.00	15	0.133
446	A	3	3	1.09	15	0.200
447	A	4	4	1.13	15	0.267
448	A	5	5	1.16	15	0.333
449	A	6	6	1.17	15	0.400
450	A	7	7	1.18	15	0.467
451	A	4	3	1.09	15	0.200
452	A	4	4	1.16	15	0.267
453	A	4	3	1.13	15	0.200
454	A	3	3	1.11	15	0.200
455	A	1	1	1.00	13	0.077
456	A	2	2	1.00	11	0.182
457	A	5	4	1.20	15	0.267
458	A	2	2	1.00	15	0.133
459	A	5	4	1.05	15	0.267
460	A	1	1	1.00	15	0.067
461	A	6	5	1.07	15	0.333
462	A	4	3	1.09	15	0.200
463	A	4	4	1.16	15	0.267
464	A	4	3	1.13	15	0.200
465	A	3	3	1.11	15	0.200
466	A	1	1	1.00	13	0.077
467	A	2	2	1.00	11	0.182
468	A	5	4	1.20	15	0.267
469	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
470	A	5	4	1.05	15	0.267
471	A	1	1	1.00	15	0.067
472	A	6	5	1.07	15	0.333
473	A	4	3	1.09	15	0.200
474	A	6	5	1.14	15	0.333
475	A	4	3	1.13	15	0.200
476	A	5	4	1.09	15	0.267
477	A	1	1	1.00	13	0.077
478	A	4	3	1.00	11	0.273
479	A	5	4	1.20	15	0.267
480	A	4	3	1.00	15	0.200
481	A	5	4	1.05	15	0.267
482	A	1	1	1.00	15	0.067
483	A	6	5	1.07	15	0.333
484	A	4	3	1.09	15	0.200
485	A	6	5	1.14	15	0.333
486	A	4	3	1.13	15	0.200
487	A	5	4	1.09	15	0.267
488	A	1	1	1.00	13	0.077
489	A	4	3	1.00	11	0.273
490	A	5	4	1.20	15	0.267
491	A	4	3	1.00	15	0.200
492	A	5	4	1.05	15	0.267
493	A	1	1	1.00	15	0.067
494	A	6	5	1.07	15	0.333
495	A	4	3	1.09	15	0.200
496	A	5	4	1.08	15	0.267
497	A	4	3	1.11	15	0.200
498	A	4	3	1.00	15	0.200
499	A	1	1	1.00	13	0.077
500	A	3	2	1.00	11	0.182
501	A	4	3	1.00	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	1	1	1.00	15	0.067
503	A	5	4	0.98	15	0.267
504	A	2	2	1.00	15	0.133
505	A	6	5	1.07	15	0.333
506	A	4	3	1.07	15	0.200
507	A	5	4	1.09	15	0.267
508	A	4	3	1.19	15	0.200
509	A	4	3	1.00	15	0.200
510	A	1	1	1.00	13	0.077
511	A	1	1	1.00	11	0.091
512	A	5	4	1.12	15	0.267
513	A	2	2	1.00	15	0.133
514	A	6	5	1.07	15	0.333
515	A	3	3	1.10	15	0.200
516	A	6	5	1.13	15	0.333
517	A	4	3	1.09	15	0.200
518	A	5	4	1.08	15	0.267
519	A	4	3	1.11	15	0.200
520	A	1	1	1.00	15	0.067
521	A	1	1	1.00	13	0.077
522	A	2	2	1.00	11	0.182
523	A	6	5	1.17	15	0.333
524	A	3	3	1.02	15	0.200
525	A	7	6	1.10	15	0.400
526	A	4	4	1.11	15	0.267
527	A	8	7	1.23	15	0.467
528	A	4	3	1.05	15	0.200
529	A	7	6	1.14	15	0.400
530	A	4	3	1.07	15	0.200
531	A	1	1	1.00	15	0.067
532	A	4	3	1.07	15	0.200
533	A	2	2	1.02	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
534	A	4	3	1.11	15	0.200
535	A	3	3	1.16	15	0.200
536	A	1	1	1.00	13	0.077
537	A	4	4	1.21	11	0.364
538	A	8	7	1.21	15	0.467
539	A	5	5	1.12	15	0.333
540	A	9	8	1.13	15	0.533
541	A	6	6	1.16	15	0.400
542	A	4	3	1.09	15	0.200
543	A	3	3	1.11	15	0.200
544	A	4	3	1.13	15	0.200
545	A	2	2	1.00	15	0.133
546	A	1	1	1.00	13	0.077
547	A	1	1	1.00	11	0.091
548	A	4	3	1.00	15	0.200
549	A	1	1	1.00	15	0.067
550	A	5	4	1.10	15	0.267
551	A	2	2	1.00	15	0.133
552	A	6	5	1.16	15	0.333
553	A	4	3	1.09	15	0.200
554	A	3	3	1.11	15	0.200
555	A	4	3	1.13	15	0.200
556	A	2	2	1.00	15	0.133
557	A	1	1	1.00	13	0.077
558	A	1	1	1.00	11	0.091
559	A	4	3	1.00	15	0.200
560	A	1	1	1.00	15	0.067
561	A	5	4	1.10	15	0.267
562	A	2	2	1.00	15	0.133
563	A	6	5	1.16	15	0.333
564	A	4	3	1.09	15	0.200
565	A	5	4	1.09	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
566	A	4	3	1.13	15	0.200
567	A	4	3	1.00	15	0.200
568	A	1	1	1.00	13	0.077
569	A	3	2	1.00	11	0.182
570	A	4	3	1.00	15	0.200
571	A	1	1	1.00	15	0.067
572	A	5	4	1.10	15	0.267
573	A	2	2	1.00	15	0.133
574	A	6	5	1.16	15	0.333
575	A	4	3	1.09	15	0.200
576	A	5	4	1.09	15	0.267
577	A	4	3	1.13	15	0.200
578	A	4	3	1.00	15	0.200
579	A	1	1	1.00	13	0.077
580	A	3	2	1.00	11	0.182
581	A	4	3	1.00	15	0.200
582	A	1	1	1.00	15	0.067
583	A	5	4	1.10	15	0.267
584	A	2	2	1.00	15	0.133
585	A	6	5	1.16	15	0.333
586	A	6	5	1.14	19	0.263
587	A	5	4	1.12	19	0.211
588	A	4	3	1.10	19	0.158
589	A	4	3	1.10	19	0.158
590	A	5	4	1.14	19	0.211
591	A	6	5	1.16	19	0.263
592	A	9	8	1.14	19	0.421
593	A	8	7	1.13	19	0.368
594	A	7	6	1.14	19	0.316
595	A	7	6	1.15	19	0.316
596	A	8	7	1.13	19	0.368
597	A	9	8	1.15	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
598	A	7	6	1.13	19	0.316
599	A	6	5	1.12	19	0.263
600	A	5	4	1.12	19	0.211
601	A	5	4	1.12	19	0.211
602	A	5	4	1.13	19	0.211
603	A	6	5	1.14	19	0.263
604	A	7	6	1.16	19	0.316
605	A	9	8	1.13	19	0.421
606	A	8	7	1.14	19	0.368
607	A	8	7	1.13	19	0.368
608	A	8	7	1.13	19	0.368
609	A	9	8	1.14	19	0.421
610	A	6	5	1.04	22	0.227
611	A	5	4	1.00	22	0.182
612	A	5	4	1.00	22	0.182
613	A	6	5	1.09	22	0.227
614	A	8	7	0.78	22	0.318
615	A	7	6	0.64	22	0.273
616	A	7	6	0.66	22	0.273
617	A	8	7	0.75	22	0.318
618	A	5	4	1.14	19	0.211
619	A	4	3	1.10	19	0.158
620	A	3	2	1.13	19	0.105
621	A	4	3	1.10	19	0.158
622	A	5	4	1.15	19	0.211
623	A	8	7	1.14	19	0.368
624	A	7	6	1.13	19	0.316
625	A	6	5	1.18	19	0.263
626	A	7	6	1.15	19	0.316
627	A	8	7	1.14	19	0.368
628	A	5	4	1.14	19	0.211
629	A	4	3	1.10	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
630	A	4	3	1.10	19	0.158
631	A	5	4	1.14	19	0.211
632	A	8	7	1.15	19	0.368
633	A	7	6	1.14	19	0.316
634	A	7	6	1.15	19	0.316
635	A	8	7	1.16	19	0.368
636	A	9	8	1.15	19	0.421
637	A	6	5	1.16	19	0.263
638	A	5	4	1.13	19	0.211
639	A	5	4	1.13	19	0.211
640	A	5	4	1.11	19	0.211
641	A	6	5	1.14	19	0.263
642	A	8	7	1.14	19	0.368
643	A	8	7	1.13	19	0.368
644	A	8	7	1.14	19	0.368
645	A	9	8	1.14	19	0.421
646	A	10	9	1.15	19	0.474
647	A	5	4	1.00	22	0.182
648	A	4	3	1.00	22	0.136
649	A	5	4	1.00	22	0.182
650	A	7	6	0.65	22	0.273
651	A	6	5	0.56	22	0.227
652	A	7	6	0.66	22	0.273
653	A	6	5	1.02	22	0.227
654	A	5	4	1.00	22	0.182
655	A	5	4	1.00	22	0.182
656	A	6	5	1.04	22	0.227
657	A	8	7	0.73	22	0.318
658	A	7	6	0.65	22	0.273
659	A	7	6	0.64	22	0.273
660	A	8	7	0.74	22	0.318
661	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
662	A	4	3	1.00	15	0.200
663	A	3	2	1.00	20	0.100
664	A	3	2	1.00	17	0.118
665	A	7	6	1.06	19	0.316
666	C	5	4	0.08	19	0.211
667	C	5	4	0.09	19	0.211
668	A	6	5	1.07	19	0.263
669	A	6	5	1.05	19	0.263
670	C	5	4	0.08	19	0.211
671	A	8	7	1.06	19	0.368
672	C	6	5	0.11	19	0.263
673	C	6	5	0.11	19	0.263
674	A	7	6	1.07	19	0.316
675	A	7	6	1.05	19	0.316
676	C	6	5	0.11	19	0.263
677	A	6	5	1.06	19	0.263
678	A	5	4	1.07	19	0.211
679	A	5	4	1.06	19	0.211
680	A	6	5	1.07	19	0.263
681	C	5	4	0.09	19	0.211
682	C	4	3	0.06	19	0.158
683	C	4	3	0.06	19	0.158
684	C	5	4	0.09	19	0.211
685	C	5	4	0.09	19	0.211
686	C	6	5	0.13	19	0.263
687	A	6	5	1.06	19	0.263
688	C	5	4	0.08	19	0.211
689	C	5	4	0.09	19	0.211
690	A	6	5	1.07	19	0.263
691	A	6	5	1.04	19	0.263
692	C	6	5	0.12	19	0.263
693	A	7	6	1.06	19	0.316

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
694	C	6	5	0.11	19	0.263
695	C	6	5	0.12	19	0.263
696	A	7	6	1.07	19	0.316
697	A	7	6	1.05	19	0.316
698	C	7	6	0.15	19	0.316
699	A	4	3	1.03	15	0.200
700	A	2	2	1.00	17	0.118
701	A	2	2	1.00	17	0.118
702	A	2	2	1.00	17	0.118
703	A	2	2	1.00	17	0.118
704	A	2	2	1.00	17	0.118
705	A	2	2	1.00	17	0.118
706	A	2	2	1.00	17	0.118
707	A	2	2	1.00	15	0.133
708	A	2	2	1.00	17	0.118
709	A	2	2	1.00	17	0.118
710	A	1	1	1.00	31	0.032
711	C	1	1	7.47	43	0.023
712	A	1	1	1.00	29	0.034
713	C	1	1	8.20	38	0.026
714	A	4	3	1.05	15	0.200
715	A	4	3	1.07	15	0.200
716	A	4	3	1.11	15	0.200
717	A	1	1	1.00	13	0.077
718	A	7	6	1.01	15	0.400
719	A	7	6	1.01	15	0.400
720	A	8	7	1.02	15	0.467
721	A	6	5	1.03	15	0.333
722	A	5	4	1.01	15	0.267
723	A	4	3	1.00	11	0.273
724	A	4	3	1.00	15	0.200
725	A	5	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
726	A	4	3	1.05	15	0.200
727	A	4	3	1.07	15	0.200
728	A	4	3	1.11	15	0.200
729	A	1	1	1.00	13	0.077
730	A	7	6	1.02	15	0.400
731	A	7	6	1.03	15	0.400
732	A	8	7	1.01	15	0.467
733	A	8	7	1.08	15	0.467
734	A	7	6	1.07	15	0.400
735	A	6	5	1.08	11	0.455
736	A	6	5	1.09	15	0.333
737	A	7	6	1.07	15	0.400
738	A	4	3	1.05	15	0.200
739	A	4	3	1.07	15	0.200
740	A	4	3	1.11	15	0.200
741	A	1	1	1.00	13	0.077
742	A	8	7	1.03	15	0.467
743	A	8	7	1.04	15	0.467
744	A	8	7	0.99	15	0.467
745	A	7	6	1.04	15	0.400
746	A	6	5	1.02	15	0.333
747	A	5	4	1.01	11	0.364
748	A	5	4	1.03	15	0.267
749	A	5	4	1.00	15	0.267
750	A	4	3	1.05	15	0.200
751	A	4	3	1.07	15	0.200
752	A	4	3	1.11	15	0.200
753	A	1	1	1.00	13	0.077
754	A	6	5	0.99	15	0.333
755	A	7	6	1.03	15	0.400
756	A	8	7	1.04	15	0.467
757	A	7	6	1.08	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
758	A	6	5	1.07	15	0.333
759	A	5	4	1.09	11	0.364
760	A	6	5	1.09	15	0.333
761	A	7	6	1.08	15	0.400
762	A	4	3	1.02	15	0.200
763	A	4	3	1.03	15	0.200
764	A	4	3	1.05	15	0.200
765	A	1	1	1.00	13	0.077
766	A	6	5	1.00	15	0.333
767	A	7	6	1.07	15	0.400
768	A	8	7	1.04	15	0.467
769	A	5	4	1.02	15	0.267
770	A	4	3	1.00	15	0.200
771	A	3	2	1.00	11	0.182
772	A	4	3	1.00	15	0.200
773	A	5	4	1.01	15	0.267
774	A	4	3	1.02	15	0.200
775	A	4	3	1.00	15	0.200
776	A	4	3	1.05	15	0.200
777	A	1	1	1.00	13	0.077
778	A	7	6	1.02	15	0.400
779	A	8	7	1.09	15	0.467
780	A	9	8	1.03	15	0.533
781	A	7	6	1.08	15	0.400
782	A	6	5	1.07	15	0.333
783	A	6	5	1.08	11	0.455
784	A	7	6	1.09	15	0.400
785	A	8	7	1.09	15	0.467
786	A	7	6	1.07	19	0.316
787	A	6	5	1.04	19	0.263
788	A	5	4	1.05	19	0.211
789	A	5	4	1.07	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
790	A	1	1	1.00	19	0.053
791	A	2	2	0.98	19	0.105
792	A	3	3	1.04	19	0.158
793	A	4	4	1.08	19	0.211
794	A	7	6	0.88	19	0.316
795	A	6	5	0.86	19	0.263
796	A	5	4	0.84	19	0.211
797	A	5	4	0.83	19	0.211
798	A	6	5	0.87	19	0.263
799	A	2	2	1.00	19	0.105
800	A	2	2	1.00	19	0.105
801	A	2	2	1.00	19	0.105
802	A	8	7	1.08	19	0.368
803	A	7	6	1.05	19	0.316
804	A	6	5	1.05	19	0.263
805	A	6	5	1.12	19	0.263
806	A	6	5	1.09	19	0.263
807	A	1	1	1.00	19	0.053
808	A	2	2	0.98	19	0.105
809	A	3	3	1.04	19	0.158
810	A	8	7	0.89	19	0.368
811	A	7	6	0.87	19	0.316
812	A	6	5	0.85	19	0.263
813	A	6	5	0.86	19	0.263
814	A	6	5	0.86	19	0.263
815	A	7	6	0.89	19	0.316
816	A	2	2	1.00	19	0.105
817	A	2	2	1.00	19	0.105
818	A	2	2	1.00	19	0.105
819	A	7	6	1.09	19	0.316
820	A	6	5	1.07	19	0.263
821	A	5	4	1.07	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
822	A	4	3	1.07	19	0.158
823	A	1	1	1.00	19	0.053
824	A	2	2	0.98	19	0.105
825	A	3	3	1.02	19	0.158
826	A	4	4	1.06	19	0.211
827	A	6	5	0.87	19	0.263
828	A	5	4	0.84	19	0.211
829	A	4	3	0.82	19	0.158
830	A	5	4	0.84	19	0.211
831	A	6	5	0.88	19	0.263
832	A	2	2	1.00	19	0.105
833	A	2	2	1.00	19	0.105
834	A	2	2	1.00	19	0.105
835	A	5	5	1.09	15	0.333
836	A	4	4	1.05	15	0.267
837	A	3	3	1.00	11	0.273
838	A	3	3	1.00	15	0.200
839	A	4	4	1.01	15	0.267
840	A	5	5	1.06	15	0.333
841	A	5	5	1.09	16	0.312
842	A	4	4	1.05	16	0.250
843	A	3	3	1.00	12	0.250
844	A	3	3	1.00	16	0.188
845	A	4	4	1.00	16	0.250
846	A	5	5	1.05	16	0.312
847	A	6	6	1.09	15	0.400
848	A	5	5	1.06	15	0.333
849	A	4	4	1.04	11	0.364
850	A	4	4	1.09	15	0.267
851	A	5	5	1.04	15	0.333
852	A	6	6	1.08	15	0.400
853	A	5	5	1.09	16	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
854	A	4	4	1.05	16	0.250
855	A	3	3	1.00	12	0.250
856	A	3	3	1.00	16	0.188
857	A	4	4	1.01	16	0.250
858	A	5	5	1.05	16	0.312
859	A	6	6	1.10	15	0.400
860	A	5	5	1.07	15	0.333
861	A	4	4	1.05	11	0.364
862	A	4	4	1.05	15	0.267
863	A	4	4	1.00	15	0.267
864	A	5	5	1.03	15	0.333
865	A	6	6	1.09	16	0.375
866	A	5	5	1.07	16	0.312
867	A	4	4	1.05	12	0.333
868	A	4	4	1.05	16	0.250
869	A	4	4	1.01	16	0.250
870	A	5	5	1.02	16	0.312
871	A	6	6	1.11	15	0.400
872	A	5	5	1.08	15	0.333
873	A	4	4	1.04	15	0.267
874	A	3	3	1.06	11	0.273
875	A	4	4	1.10	15	0.267
876	A	5	5	1.06	15	0.333
877	A	6	6	1.09	15	0.400
878	A	5	5	1.11	16	0.312
879	A	4	4	1.08	16	0.250
880	A	3	3	1.00	16	0.188
881	A	2	2	1.00	12	0.167
882	A	3	3	1.00	16	0.188
883	A	4	4	1.04	16	0.250
884	A	5	5	1.08	16	0.312
885	A	5	5	1.11	15	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
886	A	4	4	1.08	15	0.267
887	A	3	3	1.00	15	0.200
888	A	2	2	1.00	11	0.182
889	A	3	3	1.00	15	0.200
890	A	4	4	1.04	15	0.267
891	A	5	5	1.08	15	0.333
892	A	5	5	1.11	16	0.312
893	A	4	4	1.08	16	0.250
894	A	3	3	1.00	16	0.188
895	A	2	2	1.00	12	0.167
896	A	3	3	1.00	16	0.188
897	A	4	4	1.03	16	0.250
898	A	5	5	1.07	16	0.312
899	A	5	5	1.08	15	0.333
900	A	4	4	1.04	15	0.267
901	A	3	3	1.00	15	0.200
902	A	2	2	1.00	11	0.182
903	A	3	3	1.00	15	0.200
904	A	4	4	1.04	15	0.267
905	A	5	5	1.08	15	0.333
906	A	5	5	1.11	16	0.312
907	A	4	4	1.06	16	0.250
908	A	3	3	1.00	16	0.188
909	A	3	3	1.00	12	0.250
910	A	4	4	1.06	16	0.250
911	A	5	5	1.08	16	0.312
912	A	6	6	1.11	16	0.375
913	A	5	5	1.13	15	0.333
914	A	4	4	1.06	15	0.267
915	A	3	3	1.00	15	0.200
916	A	3	3	1.00	11	0.273
917	A	4	4	1.04	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
918	A	5	5	1.10	15	0.333
919	A	6	6	1.12	15	0.400
920	A	5	5	1.13	16	0.312
921	A	4	4	1.06	16	0.250
922	A	3	3	1.00	16	0.188
923	A	3	3	1.00	12	0.250
924	A	4	4	1.03	16	0.250
925	A	5	5	1.09	16	0.312
926	A	6	6	1.11	16	0.375
927	A	5	5	1.11	15	0.333
928	A	4	4	1.07	15	0.267
929	A	3	3	1.02	15	0.200
930	A	2	2	1.00	11	0.182
931	A	3	3	1.05	15	0.200
932	A	4	4	1.07	15	0.267
933	A	5	5	1.11	15	0.333
934	A	4	4	1.12	15	0.267
935	A	3	3	1.08	15	0.200
936	A	2	2	1.00	15	0.133
937	A	1	1	1.00	11	0.091
938	A	2	2	1.00	15	0.133
939	A	3	3	1.04	15	0.200
940	A	4	4	1.09	15	0.267
941	A	4	4	1.12	15	0.267
942	A	3	3	1.08	15	0.200
943	A	2	2	1.00	15	0.133
944	A	1	1	1.00	11	0.091
945	A	2	2	1.00	15	0.133
946	A	3	3	1.07	15	0.200
947	A	4	4	1.12	15	0.267
948	A	4	4	1.12	15	0.267
949	A	3	3	1.08	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
950	A	2	2	1.00	15	0.133
951	A	1	1	1.00	11	0.091
952	A	2	2	1.00	15	0.133
953	A	3	3	1.07	15	0.200
954	A	4	4	1.12	15	0.267
955	A	10	9	1.12	15	0.600
956	A	9	8	1.11	15	0.533
957	A	8	7	1.09	15	0.467
958	A	7	6	1.13	11	0.545
959	A	8	7	1.11	15	0.467
960	A	9	8	1.11	15	0.533
961	A	10	9	1.12	15	0.600
962	A	10	9	1.15	15	0.600
963	A	9	8	1.14	15	0.533
964	A	8	7	1.13	15	0.467
965	A	7	6	1.16	11	0.545
966	A	8	7	1.14	15	0.467
967	A	9	8	1.14	15	0.533
968	A	10	9	1.15	15	0.600
969	A	6	5	1.07	15	0.333
970	A	5	4	1.04	15	0.267
971	A	4	3	1.00	15	0.200
972	A	3	2	1.00	11	0.182
973	A	4	3	1.00	15	0.200
974	A	5	4	1.04	15	0.267
975	A	6	5	1.07	15	0.333
976	A	6	5	1.07	15	0.333
977	A	5	4	1.04	15	0.267
978	A	4	3	1.00	15	0.200
979	A	3	2	1.00	11	0.182
980	A	4	3	1.00	15	0.200
981	A	5	4	1.04	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
982	A	6	5	1.07	15	0.333
983	A	14	13	1.28	20	0.650
984	A	13	12	1.30	20	0.600
985	A	13	12	1.37	20	0.600
986	A	1	1	1.00	20	0.050
987	A	2	2	0.98	20	0.100
988	A	3	3	1.04	20	0.150
989	A	4	4	1.07	20	0.200
990	A	8	7	1.03	20	0.350
991	A	7	6	1.03	20	0.300
992	A	7	6	1.03	20	0.300
993	A	8	7	1.09	20	0.350
994	A	9	8	1.13	20	0.400
995	A	13	12	1.28	20	0.600
996	A	12	11	1.36	20	0.550
997	A	1	1	1.00	20	0.050
998	A	2	2	0.98	20	0.100
999	A	3	3	1.04	20	0.150
1000	A	6	5	1.02	20	0.250
1001	A	5	4	1.02	20	0.200
1002	A	4	3	1.03	20	0.150
1003	A	5	4	1.02	20	0.200
1004	A	6	5	1.10	20	0.250
1005	A	13	12	1.32	20	0.600
1006	A	12	11	1.39	20	0.550
1007	A	1	1	1.00	20	0.050
1008	A	2	2	0.95	20	0.100
1009	A	3	3	1.00	20	0.150
1010	A	7	6	1.03	20	0.300
1011	A	6	5	1.04	20	0.250
1012	A	7	6	1.03	20	0.300
1013	A	8	7	1.11	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1014	A	9	8	1.14	20	0.400
1015	A	9	8	1.07	19	0.421
1016	A	8	7	1.04	19	0.368
1017	A	8	7	1.11	19	0.368
1018	A	1	1	1.00	19	0.053
1019	A	2	2	0.98	19	0.105
1020	A	3	3	1.04	19	0.158
1021	A	4	4	1.08	19	0.211
1022	A	9	8	1.04	19	0.421
1023	A	8	7	1.02	19	0.368
1024	A	7	6	1.03	19	0.316
1025	A	7	6	1.03	19	0.316
1026	A	8	7	1.09	19	0.368
1027	A	9	8	1.13	19	0.421
1028	A	7	6	1.07	19	0.316
1029	A	6	5	1.13	19	0.263
1030	A	1	1	1.00	19	0.053
1031	A	2	2	0.98	19	0.105
1032	A	3	3	1.04	19	0.158
1033	A	7	6	1.04	19	0.316
1034	A	6	5	1.01	19	0.263
1035	A	5	4	1.00	19	0.211
1036	A	5	4	1.02	19	0.211
1037	A	6	5	1.05	19	0.263
1038	A	7	6	1.10	19	0.316
1039	A	8	7	1.09	19	0.368
1040	A	7	6	1.10	19	0.316
1041	A	1	1	1.00	19	0.053
1042	A	2	2	0.95	19	0.105
1043	A	3	3	1.00	19	0.158
1044	A	7	6	1.03	19	0.316
1045	A	6	5	1.05	19	0.263

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1046	A	7	6	1.03	19	0.316
1047	A	8	7	1.11	19	0.368
1048	A	9	8	1.15	19	0.421
1049	A	8	7	1.08	19	0.368
1050	A	7	6	1.12	19	0.316
1051	A	1	1	1.00	19	0.053
1052	A	2	2	1.00	19	0.105
1053	A	3	3	1.06	19	0.158
1054	A	4	4	1.09	19	0.211
1055	A	7	6	1.08	19	0.316
1056	A	6	5	1.05	19	0.263
1057	A	5	4	1.02	19	0.211
1058	A	4	3	1.03	19	0.158
1059	A	5	4	1.02	19	0.211
1060	A	6	5	1.07	19	0.263
1061	A	7	6	1.11	19	0.316
1062	A	2	2	1.00	19	0.105
1063	A	2	2	1.00	19	0.105
1064	A	2	2	1.00	19	0.105
1065	A	2	2	1.00	19	0.105
1066	A	2	2	1.00	19	0.105
1067	A	2	2	1.00	19	0.105
1068	A	2	2	1.00	19	0.105
1069	A	2	2	1.00	19	0.105
1070	A	2	2	1.00	19	0.105
1071	A	2	2	1.00	19	0.105
1072	A	2	2	1.00	19	0.105
1073	A	2	2	1.00	19	0.105
1074	A	8	7	1.30	15	0.467
1075	A	7	6	1.31	15	0.400
1076	A	6	5	1.31	15	0.333
1077	A	5	4	1.32	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1078	A	5	4	1.31	15	0.267
1079	A	6	5	1.30	15	0.333
1080	A	7	6	1.29	15	0.400
1081	A	8	7	1.29	15	0.467
1082	A	11	10	1.24	15	0.667
1083	A	10	9	1.24	15	0.600
1084	A	9	8	1.24	15	0.533
1085	A	8	7	1.24	11	0.636
1086	A	8	7	1.26	15	0.467
1087	A	9	8	1.24	15	0.533
1088	A	10	9	1.23	15	0.600
1089	A	11	10	1.23	15	0.667
1090	A	9	8	1.29	15	0.533
1091	A	8	7	1.30	15	0.467
1092	A	7	6	1.30	15	0.400
1093	A	6	5	1.32	11	0.455
1094	A	6	5	1.33	15	0.333
1095	A	6	5	1.29	15	0.333
1096	A	7	6	1.28	15	0.400
1097	A	8	7	1.28	15	0.467
1098	A	10	9	1.24	15	0.600
1099	A	9	8	1.24	15	0.533
1100	A	8	7	1.24	15	0.467
1101	A	7	6	1.24	11	0.545
1102	A	8	7	1.26	15	0.467
1103	A	9	8	1.25	15	0.533
1104	A	10	9	1.24	15	0.600
1105	A	7	6	1.31	15	0.400
1106	A	6	5	1.32	15	0.333
1107	A	5	4	1.31	15	0.267
1108	A	4	3	1.32	11	0.273
1109	A	5	4	1.33	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1110	A	6	5	1.31	15	0.333
1111	A	7	6	1.29	15	0.400
1112	A	10	9	1.24	15	0.600
1113	A	9	8	1.24	15	0.533
1114	A	8	7	1.24	15	0.467
1115	A	6	5	1.18	11	0.455
1116	A	9	8	1.27	15	0.533
1117	A	10	9	1.25	15	0.600
1118	A	11	10	1.24	15	0.667
1119	A	2	2	1.00	15	0.133
1120	A	2	2	1.00	15	0.133
1121	A	2	2	1.00	15	0.133
1122	A	2	2	1.00	11	0.182
1123	A	2	2	1.00	15	0.133
1124	A	2	2	1.00	15	0.133
1125	A	2	2	1.00	15	0.133
1126	A	2	2	1.00	15	0.133
1127	A	2	2	1.00	15	0.133
1128	A	2	2	1.00	15	0.133
1129	A	2	2	1.00	15	0.133
1130	A	2	2	1.00	11	0.182
1131	A	2	2	1.00	15	0.133
1132	A	2	2	1.00	15	0.133
1133	A	2	2	1.00	15	0.133
1134	A	2	2	1.00	15	0.133
1135	A	2	2	1.00	15	0.133
1136	A	2	2	1.00	15	0.133
1137	A	2	2	1.00	15	0.133
1138	A	2	2	1.00	11	0.182
1139	A	2	2	1.00	15	0.133
1140	A	2	2	1.00	15	0.133
1141	A	2	2	1.00	15	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1142	A	2	2	1.00	15	0.133
1143	A	2	2	1.00	15	0.133
1144	A	2	2	1.00	15	0.133
1145	A	2	2	1.00	15	0.133
1146	A	2	2	1.00	11	0.182
1147	A	2	2	1.00	15	0.133
1148	A	2	2	1.00	15	0.133
1149	A	2	2	1.00	15	0.133
1150	A	2	2	1.00	15	0.133
1151	A	2	2	1.00	15	0.133
1152	A	2	2	1.00	15	0.133
1153	A	2	2	1.00	15	0.133
1154	A	2	2	1.00	11	0.182
1155	A	2	2	1.00	15	0.133
1156	A	2	2	1.00	15	0.133
1157	A	2	2	1.00	15	0.133
1158	A	2	2	1.00	15	0.133
1159	A	2	2	1.00	15	0.133
1160	A	2	2	1.00	15	0.133
1161	A	2	2	1.00	11	0.182
1162	A	2	2	1.00	15	0.133
1163	A	2	2	1.00	15	0.133
1164	A	2	2	1.00	15	0.133
1165	A	2	2	1.00	15	0.133
1166	A	2	2	1.00	15	0.133
1167	A	2	2	1.00	15	0.133
1168	A	2	2	1.00	11	0.182
1169	A	2	2	1.00	15	0.133
1170	A	2	2	1.00	15	0.133
1171	A	2	2	1.00	15	0.133
1172	A	2	2	1.00	15	0.133
1173	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1174	A	2	2	1.00	15	0.133
1175	A	2	2	1.00	11	0.182
1176	A	2	2	1.00	15	0.133
1177	A	2	2	1.00	15	0.133
1178	A	2	2	1.00	15	0.133
1179	A	2	2	1.00	15	0.133
1180	A	2	2	1.00	15	0.133
1181	A	2	2	1.00	15	0.133
1182	A	2	2	1.00	11	0.182
1183	A	2	2	1.00	15	0.133
1184	A	2	2	1.00	15	0.133
1185	A	2	2	1.00	15	0.133
1186	A	2	2	1.00	15	0.133
1187	A	2	2	1.00	15	0.133
1188	A	2	2	1.00	15	0.133
1189	A	2	2	1.00	11	0.182
1190	A	2	2	1.00	15	0.133
1191	A	2	2	1.00	15	0.133
1192	A	2	2	1.00	15	0.133
1193	C	2	2	0.10	17	0.118
1194	C	2	2	0.10	17	0.118
1195	C	2	2	0.11	17	0.118
1196	C	2	2	0.10	13	0.154
1197	C	2	2	0.11	17	0.118
1198	C	2	2	0.11	17	0.118
1199	C	2	2	0.10	17	0.118
1200	C	2	2	0.10	17	0.118
1201	C	2	2	0.10	17	0.118
1202	C	2	2	0.10	17	0.118
1203	C	2	2	0.11	17	0.118
1204	C	2	2	0.10	13	0.154
1205	C	2	2	0.11	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1206	C	2	2	0.11	17	0.118
1207	C	2	2	0.10	17	0.118
1208	C	2	2	0.10	17	0.118
1209	C	2	2	0.05	17	0.118
1210	C	2	2	0.05	17	0.118
1211	C	2	2	0.06	17	0.118
1212	C	2	2	0.05	13	0.154
1213	C	2	2	0.06	17	0.118
1214	C	2	2	0.06	17	0.118
1215	C	2	2	0.05	17	0.118
1216	C	2	2	0.05	17	0.118
1217	C	2	2	0.05	17	0.118
1218	C	2	2	0.05	17	0.118
1219	C	2	2	0.05	17	0.118
1220	C	2	2	0.05	13	0.154
1221	C	2	2	0.06	17	0.118
1222	C	2	2	0.06	17	0.118
1223	C	2	2	0.05	17	0.118
1224	C	2	2	0.05	17	0.118
1225	C	2	2	0.05	17	0.118
1226	C	2	2	0.05	17	0.118
1227	C	2	2	0.06	17	0.118
1228	C	2	2	0.05	13	0.154
1229	C	2	2	0.05	17	0.118
1230	C	2	2	0.05	17	0.118
1231	C	2	2	0.05	17	0.118
1232	C	2	2	0.05	17	0.118
1233	C	2	2	0.06	17	0.118
1234	C	2	2	0.06	17	0.118
1235	C	2	2	0.05	13	0.154
1236	C	2	2	0.06	17	0.118
1237	C	2	2	0.06	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
1238	C	2	2	0.05	17	0.118
1239	C	2	2	0.10	17	0.118
1240	C	2	2	0.11	17	0.118
1241	C	2	2	0.11	17	0.118
1242	C	2	2	0.11	13	0.154
1243	C	2	2	0.11	17	0.118
1244	C	2	2	0.11	17	0.118
1245	C	2	2	0.10	17	0.118
1246	C	2	2	0.10	17	0.118
1247	C	2	2	0.11	17	0.118
1248	C	2	2	0.11	17	0.118
1249	C	2	2	0.11	13	0.154
1250	C	2	2	0.11	17	0.118
1251	C	2	2	0.11	17	0.118
1252	C	2	2	0.10	17	0.118
1253	C	2	2	0.06	17	0.118
1254	C	2	2	0.06	17	0.118
1255	C	2	2	0.06	17	0.118
1256	C	2	2	0.06	13	0.154
1257	C	2	2	0.06	17	0.118
1258	C	2	2	0.06	17	0.118
1259	C	2	2	0.05	17	0.118
1260	C	2	2	0.06	17	0.118
1261	C	2	2	0.06	17	0.118
1262	C	2	2	0.06	17	0.118
1263	C	2	2	0.06	13	0.154
1264	C	2	2	0.06	17	0.118
1265	C	2	2	0.06	17	0.118
1266	C	2	2	0.06	17	0.118
1267	A	4	3	0.95	13	0.231
1268	A	4	3	0.97	13	0.231
1269	A	4	3	0.98	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1270	A	1	1	1.00	11	0.091
1271	A	3	2	1.00	13	0.154
1272	A	3	2	1.00	13	0.154
1273	A	2	2	1.00	13	0.154
1274	A	2	2	1.00	13	0.154
1275	A	2	2	1.00	13	0.154
1276	A	2	2	1.00	9	0.222
1277	A	2	2	1.00	13	0.154
1278	A	2	2	1.00	15	0.133
1279	A	2	2	1.00	15	0.133
1280	A	2	2	1.00	15	0.133
1281	A	2	2	1.00	15	0.133
1282	A	2	2	1.00	15	0.133
1283	A	2	2	1.00	15	0.133
1284	A	2	2	1.00	15	0.133
1285	A	2	2	1.00	15	0.133
1286	A	2	2	1.00	13	0.154
1287	A	2	2	1.00	15	0.133
1288	A	3	3	1.04	17	0.176
1289	A	2	2	1.00	17	0.118
1290	A	1	1	1.00	17	0.059
1291	A	2	2	1.00	17	0.118
1292	A	2	2	1.00	17	0.118
1293	A	2	2	1.00	17	0.118
1294	A	2	2	1.00	17	0.118
1295	A	2	2	1.00	17	0.118
1296	A	2	2	1.00	17	0.118
1297	A	2	2	1.00	15	0.133
1298	A	2	2	1.00	17	0.118
1299	A	1	1	1.00	17	0.059
1300	A	2	2	1.00	17	0.118
1301	A	2	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1302	A	1	1	1.00	17	0.059
1303	A	2	2	1.00	17	0.118
1304	A	2	2	1.00	17	0.118

CHAPTER 3

Listing of integrals

3.1 $\int x^4(a+bx^2)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^4(a+bx^2)\,dx = \frac{ax^5}{5}+\frac{bx^7}{7}$$

output
```
1/5*a*x^5+1/7*b*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)\,dx = \frac{ax^5}{5}+\frac{bx^7}{7}$$

input
```
Integrate[x^4*(a + b*x^2),x]
```

output
```
(a*x^5)/5 + (b*x^7)/7
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^2)\,dx$$
$$\Big\downarrow \text{ 244}$$
$$\int (ax^4+bx^6)\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{ax^5}{5}+\frac{bx^7}{7}$$

input
```
Int[x^4*(a + b*x^2),x]
```

output
```
(a*x^5)/5 + (b*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
default	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
norman	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
risch	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
parallelrisch	$\frac{1}{5}a\,x^5+\frac{1}{7}b\,x^7$	14
orering	$\frac{x^5(5b\,x^2+7a)}{35}$	16

input
```
int(x^4*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/7*b*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^2)\,dx=\frac{1}{7}\,bx^7+\frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^2+a),x, algorithm="fricas")
```

output
```
1/7*b*x^7 + 1/5*a*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^4(a+bx^2)\ dx = \frac{ax^5}{5} + \frac{bx^7}{7}$$

input
```
integrate(x**4*(b*x**2+a),x)
```

output
```
a*x**5/5 + b*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^2)\ dx = \frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^2+a),x, algorithm="maxima")
```

output
```
1/7*b*x^7 + 1/5*a*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^2)\ dx = \frac{1}{7}\,bx^7 + \frac{1}{5}\,ax^5$$

input
```
integrate(x^4*(b*x^2+a),x, algorithm="giac")
```

output
```
1/7*b*x^7 + 1/5*a*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^4 (a + bx^2) \, dx = \frac{b x^7}{7} + \frac{a x^5}{5}$$

input
```
int(x^4*(a + b*x^2),x)
```

output
```
(a*x^5)/5 + (b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^4 (a + bx^2) \, dx = \frac{x^5(5b x^2 + 7a)}{35}$$

input
```
int(x^4*(b*x^2+a),x)
```

output
```
(x**5*(7*a + 5*b*x**2))/35
```

3.2 $\int x^3(a+bx^2)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^3\left(a+bx^2\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

output
```
1/4*a*x^4+1/6*b*x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^2\right)\,dx=\frac{ax^4}{4}+\frac{bx^6}{6}$$

input
```
Integrate[x^3*(a + b*x^2),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a + bx^2) \, dx$$

$$\Big\downarrow \text{244}$$

$$\int (ax^3 + bx^5) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

input
```
Int[x^3*(a + b*x^2),x]
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
default	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
norman	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
risch	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
parallelrisch	$\frac{1}{4}a\,x^4+\frac{1}{6}b\,x^6$	14
orering	$\frac{x^4\left(2b\,x^2+3a\right)}{12}$	16

input
```
int(x^3*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/6*b*x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3\left(a+bx^2\right)\,dx=\frac{1}{6}\,bx^6+\frac{1}{4}\,ax^4$$

input
```
integrate(x^3*(b*x^2+a),x, algorithm="fricas")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^3(a+bx^2)\,dx = \frac{ax^4}{4} + \frac{bx^6}{6}$$

input
```
integrate(x**3*(b*x**2+a),x)
```

output
```
a*x**4/4 + b*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx^2)\,dx = \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

input
```
integrate(x^3*(b*x^2+a),x, algorithm="maxima")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3(a+bx^2)\,dx = \frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

input
```
integrate(x^3*(b*x^2+a),x, algorithm="giac")
```

output
```
1/6*b*x^6 + 1/4*a*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^3 (a + bx^2) \, dx = \frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

input
```
int(x^3*(a + b*x^2),x)
```

output
```
(a*x^4)/4 + (b*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^3 (a + bx^2) \, dx = \frac{x^4(2b\,x^2 + 3a)}{12}$$

input
```
int(x^3*(b*x^2+a),x)
```

output
```
(x**4*(3*a + 2*b*x**2))/12
```

3.3 $\int x^2(a+bx^2)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int x^2(a+bx^2)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

output
```
1/3*a*x^3+1/5*b*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)\,dx=\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Integrate[x^2*(a + b*x^2),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right) \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(ax^2+bx^4\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

input
```
Int[x^2*(a + b*x^2),x]
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{5}b\,x^5$	14
orering	$\frac{x^3(3b\,x^2+5a)}{15}$	16

input
```
int(x^2*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/5*b*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^2)\,dx=\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(b*x^2+a),x, algorithm="fricas")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x^2 (a + bx^2) \, dx = \frac{ax^3}{3} + \frac{bx^5}{5}$$

input
```
integrate(x**2*(b*x**2+a),x)
```

output
```
a*x**3/3 + b*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 (a + bx^2) \, dx = \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

input
```
integrate(x^2*(b*x^2+a),x, algorithm="maxima")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2 (a + bx^2) \, dx = \frac{1}{5} bx^5 + \frac{1}{3} ax^3$$

input
```
integrate(x^2*(b*x^2+a),x, algorithm="giac")
```

output
```
1/5*b*x^5 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x^2\left(a+bx^2\right)\,dx=\frac{b\,x^5}{5}+\frac{a\,x^3}{3}$$

input
```
int(x^2*(a + b*x^2),x)
```

output
```
(a*x^3)/3 + (b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2\left(a+bx^2\right)\,dx=\frac{x^3(3b\,x^2+5a)}{15}$$

input
```
int(x^2*(b*x^2+a),x)
```

output
```
(x**3*(5*a + 3*b*x**2))/15
```

3.4 $\int x(a+bx^2)\,dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int x(a+bx^2)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

output
```
1/2*a*x^2+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

input
```
Integrate[x*(a + b*x^2),x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)\,dx$$

$$\Big\downarrow \text{244}$$

$$\int (ax+bx^3)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

input
```
Int[x*(a + b*x^2),x]
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
norman	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
parallelrisch	$\frac{1}{2}a\,x^{2}+\frac{1}{4}b\,x^{4}$	14
default	$\frac{\left(b\,x^{2}+a\right)^{2}}{4b}$	15
orering	$\frac{x^{2}\left(b\,x^{2}+2a\right)}{4}$	15
risch	$\frac{b\,x^{4}}{4}+\frac{a\,x^{2}}{2}+\frac{a^{2}}{4b}$	22

input
```
int(x*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/4*b*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x\left(a+bx^2\right)\,dx=\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int x(a+bx^2)\, dx = \frac{ax^2}{2} + \frac{bx^4}{4}$$

input
```
integrate(x*(b*x**2+a),x)
```

output
```
a*x**2/2 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\, dx = \frac{(bx^2+a)^2}{4\,b}$$

input
```
integrate(x*(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^2/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^2)\, dx = \frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

input
```
integrate(x*(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/2*a*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int x(a+bx^2)\,dx=\frac{b\,x^4}{4}+\frac{a\,x^2}{2}$$

input
```
int(x*(a + b*x^2),x)
```

output
```
(a*x^2)/2 + (b*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\,dx=\frac{x^2(b\,x^2+2a)}{4}$$

input
```
int(x*(b*x^2+a),x)
```

output
```
(x**2*(2*a + b*x**2))/4
```

3.5 $\int (a + bx^2) \, dx$

Optimal result

Integrand size = 7, antiderivative size = 12

$$\int (a + bx^2) \, dx = ax + \frac{bx^3}{3}$$

output
```
a*x+1/3*b*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + bx^2) \, dx = ax + \frac{bx^3}{3}$$

input
```
Integrate[a + b*x^2,x]
```

output
```
a*x + (b*x^3)/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{bx^3}{3}$$

input
```
Int[a + b*x^2,x]
```

output
```
a*x + (b*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
gosper	$ax+\frac{1}{3}b\,x^3$	11
default	$ax+\frac{1}{3}b\,x^3$	11
norman	$ax+\frac{1}{3}b\,x^3$	11
risch	$ax+\frac{1}{3}b\,x^3$	11
parallelrisch	$ax+\frac{1}{3}b\,x^3$	11
parts	$ax+\frac{1}{3}b\,x^3$	11
orering	$\frac{x(b\,x^2+3a)}{3}$	13

input
```
int(b*x^2+a,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/3*b*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\,dx=\frac{1}{3}\,bx^3+ax$$

input
```
integrate(b*x^2+a,x, algorithm="fricas")
```

output
```
1/3*b*x^3 + a*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int (a+bx^2) \, dx = ax + \frac{bx^3}{3}$$

input
```
integrate(b*x**2+a,x)
```

output
```
a*x + b*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2) \, dx = \frac{1}{3}bx^3 + ax$$

input
```
integrate(b*x^2+a,x, algorithm="maxima")
```

output
```
1/3*b*x^3 + a*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a+bx^2) \, dx = \frac{1}{3}bx^3 + ax$$

input
```
integrate(b*x^2+a,x, algorithm="giac")
```

output
```
1/3*b*x^3 + a*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int (a + bx^2) \, dx = \frac{b\,x^3}{3} + a\,x$$

input
```
int(a + b*x^2,x)
```

output
```
a*x + (b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a + bx^2) \, dx = \frac{x(b\,x^2 + 3a)}{3}$$

input
```
int(b*x^2+a,x)
```

output
```
(x*(3*a + b*x**2))/3
```

3.6 $\int \frac{a+bx^2}{x} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^2}{x} \, dx = \frac{bx^2}{2} + a\log(x)$$

output
```
1/2*b*x^2+a*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x} \, dx = \frac{bx^2}{2} + a\log(x)$$

input
```
Integrate[(a + b*x^2)/x,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x}\,dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a}{x}+bx\right)dx$$

$$\Big\downarrow \; 2009$$

$$a\log(x)+\frac{bx^2}{2}$$

input
```
Int[(a + b*x^2)/x,x]
```

output
```
(b*x^2)/2 + a*Log[x]
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
norman	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
risch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12
parallelrisch	$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$	12

input
```
int((b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*x^2+a*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x}\,dx=\frac{1}{2}\,bx^2+a\log\left(x\right)$$

input
```
integrate((b*x^2+a)/x,x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^2}{x}\,dx=a\log\left(x\right)+\frac{bx^2}{2}$$

input
```
integrate((b*x**2+a)/x,x)
```

output
```
a*log(x) + b*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{x}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)/x,x, algorithm="maxima")
```

output
```
1/2*b*x^2 + 1/2*a*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{x}\,dx = \frac{1}{2}\,bx^2 + \frac{1}{2}\,a\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)/x,x, algorithm="giac")
```

output
```
1/2*b*x^2 + 1/2*a*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x}\,dx = \frac{b\,x^2}{2} + a\,\ln\left(x\right)$$

input
```
int((a + b*x^2)/x,x)
```

output
```
(b*x^2)/2 + a*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x}\,dx = \log(x)\,a + \frac{b\,x^2}{2}$$

input
```
int((b*x^2+a)/x,x)
```

output
```
(2*log(x)*a + b*x**2)/2
```

3.7 $\int \frac{a+bx^2}{x^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{a+bx^2}{x^2} dx = -\frac{a}{x} + bx$$

output
```
-a/x+b*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2} dx = -\frac{a}{x} + bx$$

input
```
Integrate[(a + b*x^2)/x^2,x]
```

output
```
-(a/x) + b*x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^2} \, dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a}{x^2}+b\right) dx$$

$$\Big\downarrow \ 2009$$

$$bx-\frac{a}{x}$$

input
```
Int[(a + b*x^2)/x^2,x]
```

output
```
-(a/x) + b*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{a}{x}+bx$	11
risch	$-\frac{a}{x}+bx$	11
gosper	$-\frac{-b\,x^{2}+a}{x}$	14
norman	$\frac{b\,x^{2}-a}{x}$	14
parallelrisch	$\frac{b\,x^{2}-a}{x}$	14
orering	$-\frac{-b\,x^{2}+a}{x}$	14

input
```
int((b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+b*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{a+bx^2}{x^2}\,dx=\frac{bx^2-a}{x}$$

input
```
integrate((b*x^2+a)/x^2,x, algorithm="fricas")
```

output
```
(b*x^2 - a)/x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.50

$$\int \frac{a+bx^2}{x^2}\,dx = -\frac{a}{x} + bx$$

input
```
integrate((b*x**2+a)/x**2,x)
```

output
```
-a/x + b*x
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2}\,dx = bx - \frac{a}{x}$$

input
```
integrate((b*x^2+a)/x^2,x, algorithm="maxima")
```

output
```
b*x - a/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2}\,dx = bx - \frac{a}{x}$$

input
```
integrate((b*x^2+a)/x^2,x, algorithm="giac")
```

output
```
b*x - a/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^2}\,dx = b\,x-\frac{a}{x}$$

input
```
int((a + b*x^2)/x^2,x)
```

output
```
b*x - a/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{a+bx^2}{x^2}\,dx = \frac{b\,x^2-a}{x}$$

input
```
int((b*x^2+a)/x^2,x)
```

output
```
( - a + b*x**2)/x
```

3.8 $\int \frac{a+bx^2}{x^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{a+bx^2}{x^3} \, dx = -\frac{a}{2x^2} + b\log(x)$$

output
```
-1/2*a/x^2+b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^3} \, dx = -\frac{a}{2x^2} + b\log(x)$$

input
```
Integrate[(a + b*x^2)/x^3,x]
```

output
```
-1/2*a/x^2 + b*Log[x]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^3}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a}{x^3}+\frac{b}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$b\log(x)-\frac{a}{2x^2}$$

input
```
Int[(a + b*x^2)/x^3,x]
```

output
```
-1/2*a/x^2 + b*Log[x]
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
norman	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
risch	$-\frac{a}{2x^2}+b\ln\left(x\right)$	12
parallelrisch	$\frac{2b\ln(x)x^2-a}{2x^2}$	18

input
```
int((b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^2}{x^3}\,dx = \frac{2\,bx^2\log\left(x\right)-a}{2\,x^2}$$

input
```
integrate((b*x^2+a)/x^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*x^2*log(x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{a+bx^2}{x^3}\,dx = -\frac{a}{2x^2}+b\log\left(x\right)$$

input
```
integrate((b*x**2+a)/x**3,x)
```

output
```
-a/(2*x**2) + b*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{a+bx^2}{x^3}\,dx = \frac{1}{2}\,b\log\left(x^2\right) - \frac{a}{2\,x^2}$$

input
```
integrate((b*x^2+a)/x^3,x, algorithm="maxima")
```

output
```
1/2*b*log(x^2) - 1/2*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \frac{a+bx^2}{x^3}\,dx = \frac{1}{2}\,b\log\left(x^2\right) - \frac{bx^2+a}{2\,x^2}$$

input
```
integrate((b*x^2+a)/x^3,x, algorithm="giac")
```

output
```
1/2*b*log(x^2) - 1/2*(b*x^2 + a)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{a+bx^2}{x^3}\,dx = b\,\ln\left(x\right) - \frac{a}{2\,x^2}$$

input
```
int((a + b*x^2)/x^3,x)
```

output
```
b*log(x) - a/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{a+bx^2}{x^3}\,dx = \frac{2\log(x)\,b\,x^2-a}{2x^2}$$

input
```
int((b*x^2+a)/x^3,x)
```

output
```
(2*log(x)*b*x**2 - a)/(2*x**2)
```

3.9 $\int \frac{a+bx^2}{x^4} dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{a+bx^2}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{x}$$

output
```
-1/3*a/x^3-b/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^4} dx = -\frac{a}{3x^3} - \frac{b}{x}$$

input
```
Integrate[(a + b*x^2)/x^4,x]
```

output
```
-1/3*a/x^3 - b/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^4}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a}{x^4}+\frac{b}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{3x^3}-\frac{b}{x}$$

input
```
Int[(a + b*x^2)/x^4,x]
```

output
```
-1/3*a/x^3 - b/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$-\frac{3b\,x^2+a}{3x^3}$	14
default	$-\frac{a}{3x^3}-\frac{b}{x}$	14
orering	$-\frac{3b\,x^2+a}{3x^3}$	14
norman	$\frac{-b\,x^2-\frac{a}{3}}{x^3}$	15
risch	$\frac{-b\,x^2-\frac{a}{3}}{x^3}$	15
parallelrisch	$\frac{-3b\,x^2-a}{3x^3}$	16

input
```
int((b*x^2+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b*x^2+a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4}\,dx=-\frac{3\,bx^2+a}{3\,x^3}$$

input
```
integrate((b*x^2+a)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{a + bx^2}{x^4} \, dx = \frac{-a - 3bx^2}{3x^3}$$

input
```
integrate((b*x**2+a)/x**4,x)
```

output
```
(-a - 3*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a + bx^2}{x^4} \, dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate((b*x^2+a)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a + bx^2}{x^4} \, dx = -\frac{3\,bx^2 + a}{3\,x^3}$$

input
```
integrate((b*x^2+a)/x^4,x, algorithm="giac")
```

output
```
-1/3*(3*b*x^2 + a)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+bx^2}{x^4}\,dx=-\frac{3\,b\,x^2+a}{3\,x^3}$$

input
```
int((a + b*x^2)/x^4,x)
```

output
```
-(a + 3*b*x^2)/(3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^4}\,dx=\frac{-3b\,x^2-a}{3x^3}$$

input
```
int((b*x^2+a)/x^4,x)
```

output
```
( - a - 3*b*x**2)/(3*x**3)
```

3.10 $\int \frac{a+bx^2}{x^5} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^2}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{2x^2}$$

output
```
-1/4*a/x^4-1/2*b/x^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^5} dx = -\frac{a}{4x^4} - \frac{b}{2x^2}$$

input
```
Integrate[(a + b*x^2)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(2*x^2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^5} \, dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a}{x^5}+\frac{b}{x^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a}{4x^4}-\frac{b}{2x^2}$$

input
```
Int[(a + b*x^2)/x^5,x]
```

output
```
-1/4*a/x^4 - b/(2*x^2)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$-\frac{2b\,x^2+a}{4x^4}$	14
default	$-\frac{a}{4x^4}-\frac{b}{2x^2}$	14
orering	$-\frac{2b\,x^2+a}{4x^4}$	14
norman	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}$	15
risch	$\frac{-\frac{b\,x^2}{2}-\frac{a}{4}}{x^4}$	15
parallelrisch	$\frac{-2b\,x^2-a}{4x^4}$	16

input
```
int((b*x^2+a)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*b*x^2+a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^5}\,dx=-\frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate((b*x^2+a)/x^5,x, algorithm="fricas")
```

output
```
-1/4*(2*b*x^2 + a)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{a+bx^2}{x^5}\,dx = \frac{-a-2bx^2}{4x^4}$$

input
```
integrate((b*x**2+a)/x**5,x)
```

output
```
(-a - 2*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^5}\,dx = -\frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate((b*x^2+a)/x^5,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 + a)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^5}\,dx = -\frac{2\,bx^2+a}{4\,x^4}$$

input
```
integrate((b*x^2+a)/x^5,x, algorithm="giac")
```

output
```
-1/4*(2*b*x^2 + a)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{a+bx^2}{x^5}\,dx=-\frac{2\,b\,x^2+a}{4\,x^4}$$

input
```
int((a + b*x^2)/x^5,x)
```

output
```
-(a + 2*b*x^2)/(4*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^5}\,dx=\frac{-2b\,x^2-a}{4x^4}$$

input
```
int((b*x^2+a)/x^5,x)
```

output
```
( - a - 2*b*x**2)/(4*x**4)
```

3.11 $\int \frac{a+bx^2}{x^6} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^2}{x^6} dx = -\frac{a}{5x^5} - \frac{b}{3x^3}$$

output
```
-1/5*a/x^5-1/3*b/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^6} dx = -\frac{a}{5x^5} - \frac{b}{3x^3}$$

input
```
Integrate[(a + b*x^2)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(3*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^6} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a}{x^6}+\frac{b}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{5x^5}-\frac{b}{3x^3}$$

input
```
Int[(a + b*x^2)/x^6,x]
```

output
```
-1/5*a/x^5 - b/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{5x^5}-\frac{b}{3x^3}$	14
norman	$\frac{-\frac{b\,x^2}{3}-\frac{a}{5}}{x^5}$	15
risch	$\frac{-\frac{b\,x^2}{3}-\frac{a}{5}}{x^5}$	15
gosper	$-\frac{5b\,x^2+3a}{15x^5}$	16
parallelrisch	$\frac{-5b\,x^2-3a}{15x^5}$	16
orering	$-\frac{5b\,x^2+3a}{15x^5}$	16

input
```
int((b*x^2+a)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/3*b/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6}\,dx=-\frac{5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((b*x^2+a)/x^6,x, algorithm="fricas")
```

output
```
-1/15*(5*b*x^2 + 3*a)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6}\,dx = \frac{-3a-5bx^2}{15x^5}$$

input
```
integrate((b*x**2+a)/x**6,x)
```

output
```
(-3*a - 5*b*x**2)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6}\,dx = -\frac{5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((b*x^2+a)/x^6,x, algorithm="maxima")
```

output
```
-1/15*(5*b*x^2 + 3*a)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6}\,dx = -\frac{5\,bx^2+3\,a}{15\,x^5}$$

input
```
integrate((b*x^2+a)/x^6,x, algorithm="giac")
```

output
```
-1/15*(5*b*x^2 + 3*a)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6} \, dx = -\frac{5\,b\,x^2+3\,a}{15\,x^5}$$

input
```
int((a + b*x^2)/x^6,x)
```

output
```
-(3*a + 5*b*x^2)/(15*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^6} \, dx = \frac{-5b\,x^2-3a}{15x^5}$$

input
```
int((b*x^2+a)/x^6,x)
```

output
```
( - 3*a - 5*b*x**2)/(15*x**5)
```

3.12 $\int \frac{a+bx^2}{x^7} dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int \frac{a+bx^2}{x^7} dx = -\frac{a}{6x^6} - \frac{b}{4x^4}$$

output
```
-1/6*a/x^6-1/4*b/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^7} dx = -\frac{a}{6x^6} - \frac{b}{4x^4}$$

input
```
Integrate[(a + b*x^2)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(4*x^4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^7} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a}{x^7}+\frac{b}{x^5}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a}{6x^6}-\frac{b}{4x^4}$$

input
```
Int[(a + b*x^2)/x^7,x]
```

output
```
-1/6*a/x^6 - b/(4*x^4)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{a}{6x^{6}}-\frac{b}{4x^{4}}$	14
norman	$\frac{-\frac{b\,x^{2}}{4}-\frac{a}{6}}{x^{6}}$	15
risch	$\frac{-\frac{b\,x^{2}}{4}-\frac{a}{6}}{x^{6}}$	15
gosper	$-\frac{3b\,x^{2}+2a}{12x^{6}}$	16
parallelrisch	$\frac{-3b\,x^{2}-2a}{12x^{6}}$	16
orering	$-\frac{3b\,x^{2}+2a}{12x^{6}}$	16

input
```
int((b*x^2+a)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/4*b/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx=-\frac{3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((b*x^2+a)/x^7,x, algorithm="fricas")
```

output
```
-1/12*(3*b*x^2 + 2*a)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx = \frac{-2a-3bx^2}{12x^6}$$

input
```
integrate((b*x**2+a)/x**7,x)
```

output
```
(-2*a - 3*b*x**2)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx = -\frac{3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((b*x^2+a)/x^7,x, algorithm="maxima")
```

output
```
-1/12*(3*b*x^2 + 2*a)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx = -\frac{3\,bx^2+2\,a}{12\,x^6}$$

input
```
integrate((b*x^2+a)/x^7,x, algorithm="giac")
```

output
```
-1/12*(3*b*x^2 + 2*a)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx = -\frac{3\,b\,x^2+2\,a}{12\,x^6}$$

input
```
int((a + b*x^2)/x^7,x)
```

output
```
-(2*a + 3*b*x^2)/(12*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{a+bx^2}{x^7}\,dx = \frac{-3b\,x^2-2a}{12x^6}$$

input
```
int((b*x^2+a)/x^7,x)
```

output
```
( - 2*a - 3*b*x**2)/(12*x**6)
```

3.13 $\int x^5(a+bx^2)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^5\left(a+bx^2\right)^2\,dx=\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

output
```
1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^5\left(a+bx^2\right)^2\,dx=\frac{a^2x^6}{6}+\frac{1}{4}abx^8+\frac{b^2x^{10}}{10}$$

input
```
Integrate[x^5*(a + b*x^2)^2,x]
```

output
```
(a^2*x^6)/6 + (a*b*x^8)/4 + (b^2*x^10)/10
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^2 dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(b^2x^8+2abx^6+a^2x^4\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^2x^6}{3}+\frac{1}{2}abx^8+\frac{b^2x^{10}}{5}\right)$$

input
```
Int[x^5*(a + b*x^2)^2,x]
```

output
```
((a^2*x^6)/3 + (a*b*x^8)/2 + (b^2*x^10)/5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
default	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
norman	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
risch	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
parallelrisch	$\frac{1}{6}a^2x^6+\frac{1}{4}ab\,x^8+\frac{1}{10}b^2x^{10}$	25
orering	$\frac{x^6(6b^2x^4+15ab\,x^2+10a^2)}{60}$	27

input
```
int(x^5*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*x^6+1/4*a*b*x^8+1/10*b^2*x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5(a+bx^2)^2\,dx=\frac{1}{10}\,b^2x^{10}+\frac{1}{4}\,abx^8+\frac{1}{6}\,a^2x^6$$

input
```
integrate(x^5*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 (a + bx^2)^2 \, dx = \frac{a^2 x^6}{6} + \frac{abx^8}{4} + \frac{b^2 x^{10}}{10}$$

input
```
integrate(x**5*(b*x**2+a)**2,x)
```

output
```
a**2*x**6/6 + a*b*x**8/4 + b**2*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 (a + bx^2)^2 \, dx = \frac{1}{10} b^2 x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2 x^6$$

input
```
integrate(x^5*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 (a + bx^2)^2 \, dx = \frac{1}{10} b^2 x^{10} + \frac{1}{4} abx^8 + \frac{1}{6} a^2 x^6$$

input
```
integrate(x^5*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/10*b^2*x^10 + 1/4*a*b*x^8 + 1/6*a^2*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^5 \left(a+bx^2\right)^2 \, dx = \frac{a^2 x^6}{6} + \frac{a\, b\, x^8}{4} + \frac{b^2 x^{10}}{10}$$

input
```
int(x^5*(a + b*x^2)^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*x^10)/10 + (a*b*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^5 \left(a+bx^2\right)^2 \, dx = \frac{x^6 (6b^2 x^4 + 15ab\, x^2 + 10a^2)}{60}$$

input
```
int(x^5*(b*x^2+a)^2,x)
```

output
```
(x**6*(10*a**2 + 15*a*b*x**2 + 6*b**2*x**4))/60
```

3.14 $\int x^4(a+bx^2)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^4\left(a+bx^2\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^2\,dx=\frac{a^2x^5}{5}+\frac{2}{7}abx^7+\frac{b^2x^9}{9}$$

input
```
Integrate[x^4*(a + b*x^2)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + bx^2)^2 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int (a^2x^4 + 2abx^6 + b^2x^8) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

input
```
Int[x^4*(a + b*x^2)^2,x]
```

output
```
(a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{5}a^2x^5 + \frac{2}{7}ab\,x^7 + \frac{1}{9}b^2x^9$	25
default	$\frac{1}{5}a^2x^5 + \frac{2}{7}ab\,x^7 + \frac{1}{9}b^2x^9$	25
norman	$\frac{1}{5}a^2x^5 + \frac{2}{7}ab\,x^7 + \frac{1}{9}b^2x^9$	25
risch	$\frac{1}{5}a^2x^5 + \frac{2}{7}ab\,x^7 + \frac{1}{9}b^2x^9$	25
parallelrisch	$\frac{1}{5}a^2x^5 + \frac{2}{7}ab\,x^7 + \frac{1}{9}b^2x^9$	25
orering	$\frac{x^5(35b^2x^4+90ab\,x^2+63a^2)}{315}$	27

input
```
int(x^4*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4(a+bx^2)^2\,dx = \frac{1}{9}\,b^2x^9 + \frac{2}{7}\,abx^7 + \frac{1}{5}\,a^2x^5$$

input
```
integrate(x^4*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4 (a + bx^2)^2 \, dx = \frac{a^2x^5}{5} + \frac{2abx^7}{7} + \frac{b^2x^9}{9}$$

input
```
integrate(x**4*(b*x**2+a)**2,x)
```

output
```
a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^2)^2 \, dx = \frac{1}{9} b^2x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2x^5$$

input
```
integrate(x^4*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^2)^2 \, dx = \frac{1}{9} b^2x^9 + \frac{2}{7} abx^7 + \frac{1}{5} a^2x^5$$

input
```
integrate(x^4*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^4 (a + bx^2)^2 \, dx = \frac{a^2 x^5}{5} + \frac{2 a b x^7}{7} + \frac{b^2 x^9}{9}$$

input
```
int(x^4*(a + b*x^2)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^4 (a + bx^2)^2 \, dx = \frac{x^5 (35b^2 x^4 + 90ab\, x^2 + 63a^2)}{315}$$

input
```
int(x^4*(b*x^2+a)^2,x)
```

output
```
(x**5*(63*a**2 + 90*a*b*x**2 + 35*b**2*x**4))/315
```

3.15 $\int x^3(a+bx^2)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^3\left(a+bx^2\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^2\right)^2\,dx=\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

input
```
Integrate[x^3*(a + b*x^2)^2,x]
```

output
```
(a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int x^2 \left(bx^2+a\right)^2 dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(b^2x^6+2abx^4+a^2x^2\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a^2x^4}{2}+\frac{2}{3}abx^6+\frac{b^2x^8}{4}\right)$$

input
```
Int[x^3*(a + b*x^2)^2,x]
```

output
```
((a^2*x^4)/2 + (2*a*b*x^6)/3 + (b^2*x^8)/4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
default	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
norman	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
risch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
parallelrisch	$\frac{1}{4}a^2x^4+\frac{1}{3}ab\,x^6+\frac{1}{8}b^2x^8$	25
orering	$\frac{x^4(3b^2x^4+8ab\,x^2+6a^2)}{24}$	27

input
```
int(x^3*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3(a+bx^2)^2\,dx=\frac{1}{8}\,b^2x^8+\frac{1}{3}\,abx^6+\frac{1}{4}\,a^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a + bx^2)^2 \, dx = \frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

input
```
integrate(x**3*(b*x**2+a)**2,x)
```

output
```
a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a + bx^2)^2 \, dx = \frac{1}{8} \, b^2x^8 + \frac{1}{3} \, abx^6 + \frac{1}{4} \, a^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a + bx^2)^2 \, dx = \frac{1}{8} \, b^2x^8 + \frac{1}{3} \, abx^6 + \frac{1}{4} \, a^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^3 (a+bx^2)^2 \, dx = \frac{a^2 \, x^4}{4} + \frac{a \, b \, x^6}{3} + \frac{b^2 \, x^8}{8}$$

input
```
int(x^3*(a + b*x^2)^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^3 (a+bx^2)^2 \, dx = \frac{x^4(3b^2x^4 + 8ab \, x^2 + 6a^2)}{24}$$

input
```
int(x^3*(b*x^2+a)^2,x)
```

output
```
(x**4*(6*a**2 + 8*a*b*x**2 + 3*b**2*x**4))/24
```

3.16 $\int x^2(a+bx^2)^2\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int x^2\left(a+bx^2\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^2\,dx=\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

input
```
Integrate[x^2*(a + b*x^2)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2)^2 \, dx$$

$$\Big\downarrow \ 244$$

$$\int (a^2x^2+2abx^4+b^2x^6) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

input
```
Int[x^2*(a + b*x^2)^2,x]
```

output
```
(a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
default	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
norman	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
risch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
parallelrisch	$\frac{1}{3}a^2x^3+\frac{2}{5}ab\,x^5+\frac{1}{7}b^2x^7$	25
orering	$\frac{x^3(15b^2x^4+42ab\,x^2+35a^2)}{105}$	27

input
```
int(x^2*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx^2)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{2}{5}\,abx^5+\frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2(a+bx^2)^2\,dx = \frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}$$

input
```
integrate(x**2*(b*x**2+a)**2,x)
```

output
```
a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx^2)^2\,dx = \frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2(a+bx^2)^2\,dx = \frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int x^2 (a + bx^2)^2 \, dx = \frac{a^2 x^3}{3} + \frac{2 a b x^5}{5} + \frac{b^2 x^7}{7}$$

input
```
int(x^2*(a + b*x^2)^2,x)
```

output
```
(a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int x^2 (a + bx^2)^2 \, dx = \frac{x^3(15b^2x^4 + 42ab x^2 + 35a^2)}{105}$$

input
```
int(x^2*(b*x^2+a)^2,x)
```

output
```
(x**3*(35*a**2 + 42*a*b*x**2 + 15*b**2*x**4))/105
```

3.17 $\int x(a+bx^2)^2\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x(a+bx^2)^2\,dx=\frac{(a+bx^2)^3}{6b}$$

output
```
1/6*(b*x^2+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)^2\,dx=\frac{(a+bx^2)^3}{6b}$$

input
```
Integrate[x*(a + b*x^2)^2,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2 \, dx$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx^2)^3}{6b}$$

input
```
Int[x*(a + b*x^2)^2,x]
```

output
```
(a + b*x^2)^3/(6*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^3}{6b}$	15
gosper	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
norman	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
parallelrisch	$\frac{1}{6}b^2x^6+\frac{1}{2}ab\,x^4+\frac{1}{2}a^2x^2$	25
orering	$\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$	26
risch	$\frac{b^2x^6}{6}+\frac{ab\,x^4}{2}+\frac{a^2x^2}{2}+\frac{a^3}{6b}$	33

input
```
int(x*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^2+a)^3/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x(a+bx^2)^2\,dx=\frac{1}{6}\,b^2x^6+\frac{1}{2}\,abx^4+\frac{1}{2}\,a^2x^2$$

input
```
integrate(x*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x\left(a+bx^2\right)^2\,dx=\frac{a^2x^2}{2}+\frac{abx^4}{2}+\frac{b^2x^6}{6}$$

input
```
integrate(x*(b*x**2+a)**2,x)
```

output
```
a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x\left(a+bx^2\right)^2\,dx=\frac{(bx^2+a)^3}{6\,b}$$

input
```
integrate(x*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^3/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x\left(a+bx^2\right)^2\,dx=\frac{(bx^2+a)^3}{6\,b}$$

input
```
integrate(x*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/6*(b*x^2 + a)^3/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int x(a+bx^2)^2\,dx=\frac{a^2\,x^2}{2}+\frac{a\,b\,x^4}{2}+\frac{b^2\,x^6}{6}$$

input
```
int(x*(a + b*x^2)^2,x)
```

output
```
(a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int x(a+bx^2)^2\,dx=\frac{x^2(b^2x^4+3ab\,x^2+3a^2)}{6}$$

input
```
int(x*(b*x^2+a)^2,x)
```

output
```
(x**2*(3*a**2 + 3*a*b*x**2 + b**2*x**4))/6
```

3.18 $\int (a+bx^2)^2\,dx$

Optimal result

Integrand size = 9, antiderivative size = 25

$$\int (a+bx^2)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^2\,dx = a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Integrate[(a + b*x^2)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 dx$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(a^2+2abx^2+b^2x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

input
```
Int[(a + b*x^2)^2,x]
```

output
```
a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
gosper	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
default	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
norman	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
risch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
parallelrisch	$a^2x+\frac{2}{3}ab\,x^3+\frac{1}{5}b^2x^5$	22
orering	$\frac{x(3b^2x^4+10ab\,x^2+15a^2)}{15}$	25

input
```
int((b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2\,dx=\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (a+bx^2)^2 \, dx = a^2x + \frac{2abx^3}{3} + \frac{b^2x^5}{5}$$

input
```
integrate((b*x**2+a)**2,x)
```

output
```
a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^2 \, dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a+bx^2)^2 \, dx = \frac{1}{5}\,b^2x^5 + \frac{2}{3}\,abx^3 + a^2x$$

input
```
integrate((b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int (a + bx^2)^2 \, dx = a^2 \, x + \frac{2 \, a \, b \, x^3}{3} + \frac{b^2 \, x^5}{5}$$

input
```
int((a + b*x^2)^2,x)
```

output
```
a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int (a + bx^2)^2 \, dx = \frac{x(3b^2x^4 + 10ab \, x^2 + 15a^2)}{15}$$

input
```
int((b*x^2+a)^2,x)
```

output
```
(x*(15*a**2 + 10*a*b*x**2 + 3*b**2*x**4))/15
```

3.19 $\int \frac{(a+bx^2)^2}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{(a+bx^2)^2}{x} \, dx = abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x} \, dx = abx^2 + \frac{b^2x^4}{4} + a^2\log(x)$$

input
```
Integrate[(a + b*x^2)^2/x,x]
```

output
```
a*b*x^2 + (b^2*x^4)/4 + a^2*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x}dx$$
$$\big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^2}dx^2$$
$$\big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(\frac{a^2}{x^2}+2ba+b^2x^2\right)dx^2$$
$$\big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(a^2\log\left(x^2\right)+2abx^2+\frac{b^2x^4}{2}\right)$$

input
```
Int[(a + b*x^2)^2/x,x]
```

output
```
(2*a*b*x^2 + (b^2*x^4)/2 + a^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$ab\,x^{2}+\frac{b^{2}x^{4}}{4}+a^{2}\ln\left(x\right)$	22
norman	$ab\,x^{2}+\frac{b^{2}x^{4}}{4}+a^{2}\ln\left(x\right)$	22
parallelrisch	$ab\,x^{2}+\frac{b^{2}x^{4}}{4}+a^{2}\ln\left(x\right)$	22
risch	$\frac{b^{2}x^{4}}{4}+ab\,x^{2}+a^{2}+a^{2}\ln\left(x\right)$	25

input
```
int((b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

output
```
a*b*x^2+1/4*b^2*x^4+a^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{x}\,dx=\frac{1}{4}\,b^2x^4+abx^2+a^2\log\left(x\right)$$

input
```
integrate((b*x^2+a)^2/x,x, algorithm="fricas")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + a^2*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x}\,dx = a^2\log(x) + abx^2 + \frac{b^2x^4}{4}$$

input
```
integrate((b*x**2+a)**2/x,x)
```

output
```
a**2*log(x) + a*b*x**2 + b**2*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x}\,dx = \frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

input
```
integrate((b*x^2+a)^2/x,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x}\,dx = \frac{1}{4}b^2x^4 + abx^2 + \frac{1}{2}a^2\log(x^2)$$

input
```
integrate((b*x^2+a)^2/x,x, algorithm="giac")
```

output
```
1/4*b^2*x^4 + a*b*x^2 + 1/2*a^2*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{x}\,dx = a^2\,\ln(x) + \frac{b^2\,x^4}{4} + a\,b\,x^2$$

input
```
int((a + b*x^2)^2/x,x)
```

output
```
a^2*log(x) + (b^2*x^4)/4 + a*b*x^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^2}{x}\,dx = \log(x)\,a^2 + ab\,x^2 + \frac{b^2x^4}{4}$$

input
```
int((b*x^2+a)^2/x,x)
```

output
```
(4*log(x)*a**2 + 4*a*b*x**2 + b**2*x**4)/4
```

3.20 $\int \frac{(a+bx^2)^2}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{(a+bx^2)^2}{x^2} dx = -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^2} dx = -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

input
```
Integrate[(a + b*x^2)^2/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^2} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^2}+2ab+b^2x^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

input
```
Int[(a + b*x^2)^2/x^2,x]
```

output
```
-(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
risch	$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$	23
norman	$\frac{\frac{1}{3}b^2x^4+2ab\,x^2-a^2}{x}$	26
parallelrisch	$\frac{b^2x^4+6ab\,x^2-3a^2}{3x}$	26
gosper	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27
orering	$-\frac{-b^2x^4-6ab\,x^2+3a^2}{3x}$	27

input
```
int((b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*a*b*x+1/3*b^2*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x^2}\,dx=\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

input
```
integrate((b*x^2+a)^2/x^2,x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{x^2}\,dx = -\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

input
```
integrate((b*x**2+a)**2/x**2,x)
```

output
```
-a**2/x + 2*a*b*x + b**2*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^2}\,dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((b*x^2+a)^2/x^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^2}\,dx = \frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

input
```
integrate((b*x^2+a)^2/x^2,x, algorithm="giac")
```

output
```
1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^2}{x^2}\,dx = \frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

input
```
int((a + b*x^2)^2/x^2,x)
```

output
```
(b^2*x^3)/3 - a^2/x + 2*a*b*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x^2}\,dx = \frac{b^2x^4 + 6ab\,x^2 - 3a^2}{3x}$$

input
```
int((b*x^2+a)^2/x^2,x)
```

output
```
( - 3*a**2 + 6*a*b*x**2 + b**2*x**4)/(3*x)
```

3.21 $\int \frac{(a+bx^2)^2}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{(a+bx^2)^2}{x^3} dx = -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)$$

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^3} dx = -\frac{a^2}{2x^2} + \frac{b^2x^2}{2} + 2ab\log(x)$$

input
```
Integrate[(a + b*x^2)^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + (b^2*x^2)/2 + 2*a*b*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^4}+\frac{2ba}{x^2}+b^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{x^2}+2ab\log\left(x^2\right)+b^2x^2\right)$$

input
```
Int[(a + b*x^2)^2/x^3,x]
```

output
```
(-(a^2/x^2) + b^2*x^2 + 2*a*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
risch	$-\frac{a^2}{2x^2}+\frac{b^2x^2}{2}+2ab\ln\left(x\right)$	24
norman	$\frac{-\frac{a^2}{2}+\frac{b^2x^4}{2}}{x^2}+2ab\ln\left(x\right)$	26
parallelrisch	$\frac{b^2x^4+4ab\ln(x)x^2-a^2}{2x^2}$	28

input
```
int((b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+1/2*b^2*x^2+2*a*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^3}\,dx=\frac{b^2x^4+4\,abx^2\log\left(x\right)-a^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2/x^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4 + 4*a*b*x^2*log(x) - a^2)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^3}\,dx = -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2}$$

input
```
integrate((b*x**2+a)**2/x**3,x)
```

output
```
-a**2/(2*x**2) + 2*a*b*log(x) + b**2*x**2/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^3}\,dx = \frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{a^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2/x^3,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2 + a*b*log(x^2) - 1/2*a^2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^2}{x^3}\,dx = \frac{1}{2}\,b^2x^2 + ab\log\left(x^2\right) - \frac{2\,abx^2 + a^2}{2\,x^2}$$

input
```
integrate((b*x^2+a)^2/x^3,x, algorithm="giac")
```

output
```
1/2*b^2*x^2 + a*b*log(x^2) - 1/2*(2*a*b*x^2 + a^2)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2}{x^3}\,dx = \frac{b^2\,x^2}{2} - \frac{a^2}{2\,x^2} + 2\,a\,b\,\ln{(x)}$$

input
```
int((a + b*x^2)^2/x^3,x)
```

output
```
(b^2*x^2)/2 - a^2/(2*x^2) + 2*a*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^3}\,dx = \frac{4\log(x)\,ab\,x^2 - a^2 + b^2x^4}{2x^2}$$

input
```
int((b*x^2+a)^2/x^3,x)
```

output
```
(4*log(x)*a*b*x**2 - a**2 + b**2*x**4)/(2*x**2)
```

3.22 $\int \frac{(a+bx^2)^2}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{(a+bx^2)^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^4} dx = -\frac{a^2}{3x^3} - \frac{2ab}{x} + b^2x$$

input
```
Integrate[(a + b*x^2)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^4} \, dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{a^2}{x^4}+\frac{2ab}{x^2}+b^2\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$$

input
```
Int[(a + b*x^2)^2/x^4,x]
```

output
```
-1/3*a^2/x^3 - (2*a*b)/x + b^2*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{3x^3}-\frac{2ab}{x}+b^2x$	22
risch	$b^2x+\frac{-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	24
gosper	$-\frac{-3b^2x^4+6ab\,x^2+a^2}{3x^3}$	25
norman	$\frac{b^2x^4-2ab\,x^2-\frac{1}{3}a^2}{x^3}$	25
orering	$-\frac{-3b^2x^4+6ab\,x^2+a^2}{3x^3}$	25
parallelrisch	$\frac{3b^2x^4-6ab\,x^2-a^2}{3x^3}$	27

input
```
int((b*x^2+a)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3-2*a*b/x+b^2*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^2}{x^4}\,dx=\frac{3\,b^2x^4-6\,abx^2-a^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2/x^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^2*x^4 - 6*a*b*x^2 - a^2)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^4}\,dx = b^2x + \frac{-a^2-6abx^2}{3x^3}$$

input
```
integrate((b*x**2+a)**2/x**4,x)
```

output
```
b**2*x + (-a**2 - 6*a*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^4}\,dx = b^2x - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2/x^4,x, algorithm="maxima")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^4}\,dx = b^2x - \frac{6\,abx^2+a^2}{3\,x^3}$$

input
```
integrate((b*x^2+a)^2/x^4,x, algorithm="giac")
```

output
```
b^2*x - 1/3*(6*a*b*x^2 + a^2)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2}{x^4}\,dx = b^2\,x - \frac{\frac{a^2}{3} + 2\,b\,a\,x^2}{x^3}$$

input
```
int((a + b*x^2)^2/x^4,x)
```

output
```
b^2*x - (a^2/3 + 2*a*b*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^2}{x^4}\,dx = \frac{3b^2x^4 - 6ab\,x^2 - a^2}{3x^3}$$

input
```
int((b*x^2+a)^2/x^4,x)
```

output
```
( - a**2 - 6*a*b*x**2 + 3*b**2*x**4)/(3*x**3)
```

3.23 $\int \frac{(a+bx^2)^2}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{(a+bx^2)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

output
```
-1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^5} dx = -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x)$$

input
```
Integrate[(a + b*x^2)^2/x^5,x]
```

output
```
-1/4*a^2/x^4 - (a*b)/x^2 + b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^5} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^6}+\frac{2ba}{x^4}+\frac{b^2}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{2x^4}-\frac{2ab}{x^2}+b^2\log\left(x^2\right)\right)$$

input
```
Int[(a + b*x^2)^2/x^5,x]
```

output
```
(-1/2*a^2/x^4 - (2*a*b)/x^2 + b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{a^2}{4x^4}-\frac{ab}{x^2}+b^2\ln\left(x\right)$	23
norman	$\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+b^2\ln\left(x\right)$	25
risch	$\frac{-\frac{1}{4}a^2-ab\,x^2}{x^4}+b^2\ln\left(x\right)$	25
parallelrisch	$\frac{4b^2\ln\left(x\right)x^4-4ab\,x^2-a^2}{4x^4}$	29

input
```
int((b*x^2+a)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2/x^4-a*b/x^2+b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int\frac{\left(a+bx^2\right)^2}{x^5}\,dx=\frac{4\,b^2x^4\log\left(x\right)-4\,abx^2-a^2}{4\,x^4}$$

input
```
integrate((b*x^2+a)^2/x^5,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*x^4*log(x) - 4*a*b*x^2 - a^2)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^5}\,dx = b^2\log(x) + \frac{-a^2-4abx^2}{4x^4}$$

input
```
integrate((b*x**2+a)**2/x**5,x)
```

output
```
b**2*log(x) + (-a**2 - 4*a*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2}{x^5}\,dx = \frac{1}{2}\,b^2\log\left(x^2\right) - \frac{4\,abx^2+a^2}{4\,x^4}$$

input
```
integrate((b*x^2+a)^2/x^5,x, algorithm="maxima")
```

output
```
1/2*b^2*log(x^2) - 1/4*(4*a*b*x^2 + a^2)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^2}{x^5}\,dx = \frac{1}{2}\,b^2\log\left(x^2\right) - \frac{3\,b^2x^4+4\,abx^2+a^2}{4\,x^4}$$

input
```
integrate((b*x^2+a)^2/x^5,x, algorithm="giac")
```

output
```
1/2*b^2*log(x^2) - 1/4*(3*b^2*x^4 + 4*a*b*x^2 + a^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^5}\,dx = b^2\,\ln(x) - \frac{\frac{a^2}{4} + b\,a\,x^2}{x^4}$$

input
```
int((a + b*x^2)^2/x^5,x)
```

output
```
b^2*log(x) - (a^2/4 + a*b*x^2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^2}{x^5}\,dx = \frac{4\log(x)\,b^2x^4 - a^2 - 4ab\,x^2}{4x^4}$$

input
```
int((b*x^2+a)^2/x^5,x)
```

output
```
(4*log(x)*b**2*x***4 - a**2 - 4*a*b*x**2)/(4*x***4)
```

3.24 $\int \frac{(a+bx^2)^2}{x^6}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{a^2}{5x^5} - \frac{2ab}{3x^3} - \frac{b^2}{x}$$

input
```
Integrate[(a + b*x^2)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^6} \, dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^2}{x^6}+\frac{2ab}{x^4}+\frac{b^2}{x^2}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$$

input
```
Int[(a + b*x^2)^2/x^6,x]
```

output
```
-1/5*a^2/x^5 - (2*a*b)/(3*x^3) - b^2/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^2}{5x^5}-\frac{2ab}{3x^3}-\frac{b^2}{x}$	25
norman	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
risch	$\frac{-b^2x^4-\frac{2}{3}ab\,x^2-\frac{1}{5}a^2}{x^5}$	26
gosper	$-\frac{15b^2x^4+10ab\,x^2+3a^2}{15x^5}$	27
parallelrisch	$\frac{-15b^2x^4-10ab\,x^2-3a^2}{15x^5}$	27
orering	$-\frac{15b^2x^4+10ab\,x^2+3a^2}{15x^5}$	27

input
```
int((b*x^2+a)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^2/x^5-2/3*a*b/x^3-b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2/x^6,x, algorithm="fricas")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = \frac{-3a^2-10abx^2-15b^2x^4}{15x^5}$$

input
```
integrate((b*x**2+a)**2/x**6,x)
```

output
```
(-3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2/x^6,x, algorithm="maxima")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{15\,b^2x^4+10\,abx^2+3\,a^2}{15\,x^5}$$

input
```
integrate((b*x^2+a)^2/x^6,x, algorithm="giac")
```

output
```
-1/15*(15*b^2*x^4 + 10*a*b*x^2 + 3*a^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = -\frac{\frac{a^2}{5}+\frac{2\,a\,b\,x^2}{3}+b^2\,x^4}{x^5}$$

input
```
int((a + b*x^2)^2/x^6,x)
```

output
```
-(a^2/5 + b^2*x^4 + (2*a*b*x^2)/3)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^2}{x^6}\,dx = \frac{-15b^2x^4-10ab\,x^2-3a^2}{15x^5}$$

input
```
int((b*x^2+a)^2/x^6,x)
```

output
```
( - 3*a**2 - 10*a*b*x**2 - 15*b**2*x**4)/(15*x**5)
```

3.25 $\int \frac{(a+bx^2)^2}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{(a+bx^2)^3}{6ax^6}$$

output
```
-1/6*(b*x^2+a)^3/a/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)^2}{x^7} dx = -\frac{a^2}{6x^6} - \frac{ab}{2x^4} - \frac{b^2}{2x^2}$$

input
```
Integrate[(a + b*x^2)^2/x^7,x]
```

output
```
-1/6*a^2/x^6 - (a*b)/(2*x^4) - b^2/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^7}\,dx$$

$$\Big\downarrow \; 242$$

$$-\frac{(a+bx^2)^3}{6ax^6}$$

input
```
Int[(a + b*x^2)^2/x^7,x]
```

output
```
-1/6*(a + b*x^2)^3/(a*x^6)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
gosper	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6x^6}$	25
default	$-\frac{b^2}{2x^2}-\frac{ab}{2x^4}-\frac{a^2}{6x^6}$	25
orering	$-\frac{3b^2x^4+3ab\,x^2+a^2}{6x^6}$	25
norman	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
risch	$\frac{-\frac{1}{2}b^2x^4-\frac{1}{2}ab\,x^2-\frac{1}{6}a^2}{x^6}$	26
parallelrisch	$\frac{-3b^2x^4-3ab\,x^2-a^2}{6x^6}$	27

input
```
int((b*x^2+a)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*b^2*x^4+3*a*b*x^2+a^2)/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = -\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

input
```
integrate((b*x^2+a)^2/x^7,x, algorithm="fricas")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = \frac{-a^2-3abx^2-3b^2x^4}{6x^6}$$

input
```
integrate((b*x**2+a)**2/x**7,x)
```

output
```
(-a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = -\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

input
```
integrate((b*x^2+a)^2/x^7,x, algorithm="maxima")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = -\frac{3\,b^2x^4+3\,abx^2+a^2}{6\,x^6}$$

input
```
integrate((b*x^2+a)^2/x^7,x, algorithm="giac")
```

output
```
-1/6*(3*b^2*x^4 + 3*a*b*x^2 + a^2)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = -\frac{\frac{a^2}{6}+\frac{a\,b\,x^2}{2}+\frac{b^2\,x^4}{2}}{x^6}$$

input
```
int((a + b*x^2)^2/x^7,x)
```

output
```
-(a^2/6 + (b^2*x^4)/2 + (a*b*x^2)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^2}{x^7}\,dx = \frac{-3b^2x^4 - 3ab\,x^2 - a^2}{6x^6}$$

input
```
int((b*x^2+a)^2/x^7,x)
```

output
```
( - a**2 - 3*a*b*x**2 - 3*b**2*x**4)/(6*x**6)
```

3.26 $\int \frac{(a+bx^2)^2}{x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^2)^2}{x^8}\,dx = -\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^8}\,dx = -\frac{a^2}{7x^7} - \frac{2ab}{5x^5} - \frac{b^2}{3x^3}$$

input
```
Integrate[(a + b*x^2)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^8}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^8}+\frac{2ab}{x^6}+\frac{b^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{7x^7}-\frac{2ab}{5x^5}-\frac{b^2}{3x^3}$$

input
```
Int[(a + b*x^2)^2/x^8,x]
```

output
```
-1/7*a^2/x^7 - (2*a*b)/(5*x^5) - b^2/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^{2}}{7x^{7}}-\frac{2ab}{5x^{5}}-\frac{b^{2}}{3x^{3}}$	25
norman	$\frac{-\frac{1}{3}b^{2}x^{4}-\frac{2}{5}ab\,x^{2}-\frac{1}{7}a^{2}}{x^{7}}$	26
risch	$\frac{-\frac{1}{3}b^{2}x^{4}-\frac{2}{5}ab\,x^{2}-\frac{1}{7}a^{2}}{x^{7}}$	26
gosper	$-\frac{35b^{2}x^{4}+42ab\,x^{2}+15a^{2}}{105x^{7}}$	27
parallelrisch	$\frac{-35b^{2}x^{4}-42ab\,x^{2}-15a^{2}}{105x^{7}}$	27
orering	$-\frac{35b^{2}x^{4}+42ab\,x^{2}+15a^{2}}{105x^{7}}$	27

input
```
int((b*x^2+a)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^2/x^7-2/5*a*b/x^5-1/3*b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^8}\,dx = -\frac{35\,b^2x^4+42\,abx^2+15\,a^2}{105\,x^7}$$

input
```
integrate((b*x^2+a)^2/x^8,x, algorithm="fricas")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2}{x^8} dx = \frac{-15a^2 - 42abx^2 - 35b^2x^4}{105x^7}$$

input
```
integrate((b*x**2+a)**2/x**8,x)
```

output
```
(-15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^8} dx = -\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

input
```
integrate((b*x^2+a)^2/x^8,x, algorithm="maxima")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^8} dx = -\frac{35\,b^2x^4 + 42\,abx^2 + 15\,a^2}{105\,x^7}$$

input
```
integrate((b*x^2+a)^2/x^8,x, algorithm="giac")
```

output
```
-1/105*(35*b^2*x^4 + 42*a*b*x^2 + 15*a^2)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^8}\,dx = -\frac{\frac{a^2}{7}+\frac{2\,a\,b\,x^2}{5}+\frac{b^2\,x^4}{3}}{x^7}$$

input
```
int((a + b*x^2)^2/x^8,x)
```

output
```
-(a^2/7 + (b^2*x^4)/3 + (2*a*b*x^2)/5)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^8}\,dx = \frac{-35b^2x^4-42ab\,x^2-15a^2}{105x^7}$$

input
```
int((b*x^2+a)^2/x^8,x)
```

output
```
( - 15*a**2 - 42*a*b*x**2 - 35*b**2*x**4)/(105*x**7)
```

3.27 $\int \frac{(a+bx^2)^2}{x^9} dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^2)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^9} dx = -\frac{a^2}{8x^8} - \frac{ab}{3x^6} - \frac{b^2}{4x^4}$$

input
```
Integrate[(a + b*x^2)^2/x^9,x]
```

output
```
-1/8*a^2/x^8 - (a*b)/(3*x^6) - b^2/(4*x^4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^9} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^2}{x^{10}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{x^{10}}+\frac{2ba}{x^8}+\frac{b^2}{x^6}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{4x^8}-\frac{2ab}{3x^6}-\frac{b^2}{2x^4}\right)$$

input
```
Int[(a + b*x^2)^2/x^9,x]
```

output
```
(-1/4*a^2/x^8 - (2*a*b)/(3*x^6) - b^2/(2*x^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{8x^8}-\frac{ab}{3x^6}-\frac{b^2}{4x^4}$	25
norman	$\frac{-\frac{1}{4}b^2x^4-\frac{1}{3}ab\,x^2-\frac{1}{8}a^2}{x^8}$	26
risch	$\frac{-\frac{1}{4}b^2x^4-\frac{1}{3}ab\,x^2-\frac{1}{8}a^2}{x^8}$	26
gosper	$-\frac{6b^2x^4+8ab\,x^2+3a^2}{24x^8}$	27
parallelrisch	$\frac{-6b^2x^4-8ab\,x^2-3a^2}{24x^8}$	27
orering	$-\frac{6b^2x^4+8ab\,x^2+3a^2}{24x^8}$	27

input
```
int((b*x^2+a)^2/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^2/x^8-1/3*a*b/x^6-1/4*b^2/x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = -\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2/x^9,x, algorithm="fricas")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = \frac{-3a^2-8abx^2-6b^2x^4}{24x^8}$$

input
```
integrate((b*x**2+a)**2/x**9,x)
```

output
```
(-3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = -\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2/x^9,x, algorithm="maxima")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = -\frac{6\,b^2x^4+8\,abx^2+3\,a^2}{24\,x^8}$$

input
```
integrate((b*x^2+a)^2/x^9,x, algorithm="giac")
```

output
```
-1/24*(6*b^2*x^4 + 8*a*b*x^2 + 3*a^2)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = -\frac{\frac{a^2}{8}+\frac{a\,b\,x^2}{3}+\frac{b^2\,x^4}{4}}{x^8}$$

input
```
int((a + b*x^2)^2/x^9,x)
```

output
```
-(a^2/8 + (b^2*x^4)/4 + (a*b*x^2)/3)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^9}\,dx = \frac{-6b^2x^4-8ab\,x^2-3a^2}{24x^8}$$

input
```
int((b*x^2+a)^2/x^9,x)
```

output
```
( - 3*a**2 - 8*a*b*x**2 - 6*b**2*x**4)/(24*x**8)
```

3.28 $\int \frac{(a+bx^2)^2}{x^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{(a+bx^2)^2}{x^{10}} \, dx = -\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

output
```
-1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2}{x^{10}} \, dx = -\frac{a^2}{9x^9} - \frac{2ab}{7x^7} - \frac{b^2}{5x^5}$$

input
```
Integrate[(a + b*x^2)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^{10}}+\frac{2ab}{x^8}+\frac{b^2}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$$

input
```
Int[(a + b*x^2)^2/x^10,x]
```

output
```
-1/9*a^2/x^9 - (2*a*b)/(7*x^7) - b^2/(5*x^5)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{a^2}{9x^9}-\frac{2ab}{7x^7}-\frac{b^2}{5x^5}$	25
norman	$\frac{-\frac{1}{5}b^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}a^2}{x^9}$	26
risch	$\frac{-\frac{1}{5}b^2x^4-\frac{2}{7}ab\,x^2-\frac{1}{9}a^2}{x^9}$	26
gosper	$-\frac{63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	27
parallelrisch	$\frac{-63b^2x^4-90ab\,x^2-35a^2}{315x^9}$	27
orering	$-\frac{63b^2x^4+90ab\,x^2+35a^2}{315x^9}$	27

input
```
int((b*x^2+a)^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^2/x^9-2/7*a*b/x^7-1/5*b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = -\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((b*x^2+a)^2/x^10,x, algorithm="fricas")
```

output
```
-1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = \frac{-35a^2-90abx^2-63b^2x^4}{315x^9}$$

input
```
integrate((b*x**2+a)**2/x**10,x)
```

output
```
(-35*a**2 - 90*a*b*x**2 - 63*b**2*x**4)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = -\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((b*x^2+a)^2/x^10,x, algorithm="maxima")
```

output
```
-1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = -\frac{63\,b^2x^4+90\,abx^2+35\,a^2}{315\,x^9}$$

input
```
integrate((b*x^2+a)^2/x^10,x, algorithm="giac")
```

output
```
-1/315*(63*b^2*x^4 + 90*a*b*x^2 + 35*a^2)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = -\frac{\frac{a^2}{9}+\frac{2\,a\,b\,x^2}{7}+\frac{b^2\,x^4}{5}}{x^9}$$

input
```
int((a + b*x^2)^2/x^10,x)
```

output
```
-(a^2/9 + (b^2*x^4)/5 + (2*a*b*x^2)/7)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^2}{x^{10}}\,dx = \frac{-63b^2x^4-90ab\,x^2-35a^2}{315x^9}$$

input
```
int((b*x^2+a)^2/x^10,x)
```

output
```
( - 35*a**2 - 90*a*b*x**2 - 63*b**2*x**4)/(315*x**9)
```

3.29 $\int x^9(a+bx^2)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

output
```
1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{a^3x^{10}}{10}+\frac{1}{4}a^2bx^{12}+\frac{3}{14}ab^2x^{14}+\frac{b^3x^{16}}{16}$$

input
```
Integrate[x^9*(a + b*x^2)^3,x]
```

output
```
(a^3*x^10)/10 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14 + (b^3*x^16)/16
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 (a+bx^2)^3 \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^8 (bx^2+a)^3 \, dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(b^3x^{14}+3ab^2x^{12}+3a^2bx^{10}+a^3x^8\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^3x^{10}}{5}+\frac{1}{2}a^2bx^{12}+\frac{3}{7}ab^2x^{14}+\frac{b^3x^{16}}{8}\right)$$

input
```
Int[x^9*(a + b*x^2)^3,x]
```

output
```
((a^3*x^10)/5 + (a^2*b*x^12)/2 + (3*a*b^2*x^14)/7 + (b^3*x^16)/8)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
default	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
norman	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
risch	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
parallelrisch	$\frac{1}{10}a^3x^{10}+\frac{1}{4}a^2b\,x^{12}+\frac{3}{14}a\,b^2x^{14}+\frac{1}{16}b^3x^{16}$	36
orering	$\frac{x^{10}\left(35b^3x^6+120a\,b^2x^4+140a^2b\,x^2+56a^3\right)}{560}$	38

input
```
int(x^9*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^3*x^10+1/4*a^2*b*x^12+3/14*a*b^2*x^14+1/16*b^3*x^16
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{a^3x^{10}}{10}+\frac{a^2bx^{12}}{4}+\frac{3ab^2x^{14}}{14}+\frac{b^3x^{16}}{16}$$

input
```
integrate(x**9*(b*x**2+a)**3,x)
```

output
```
a**3*x**10/10 + a**2*b*x**12/4 + 3*a*b**2*x**14/14 + b**3*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^9\left(a+bx^2\right)^3\,dx=\frac{1}{16}\,b^3x^{16}+\frac{3}{14}\,ab^2x^{14}+\frac{1}{4}\,a^2bx^{12}+\frac{1}{10}\,a^3x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/16*b^3*x^16 + 3/14*a*b^2*x^14 + 1/4*a^2*b*x^12 + 1/10*a^3*x^10
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^9 (a + bx^2)^3 \, dx = \frac{a^3 x^{10}}{10} + \frac{a^2 b x^{12}}{4} + \frac{3 a b^2 x^{14}}{14} + \frac{b^3 x^{16}}{16}$$

input
```
int(x^9*(a + b*x^2)^3,x)
```

output
```
(a^3*x^10)/10 + (b^3*x^16)/16 + (a^2*b*x^12)/4 + (3*a*b^2*x^14)/14
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^9 (a + bx^2)^3 \, dx = \frac{x^{10}(35b^3x^6 + 120a b^2x^4 + 140a^2b x^2 + 56a^3)}{560}$$

input
```
int(x^9*(b*x^2+a)^3,x)
```

output
```
(x**10*(56*a**3 + 140*a**2*b*x**2 + 120*a*b**2*x**4 + 35*b**3*x**6))/560
```

3.30 $\int x^7(a+bx^2)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^7\left(a+bx^2\right)^3\,dx=\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

output
```
1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^7\left(a+bx^2\right)^3\,dx=\frac{a^3x^8}{8}+\frac{3}{10}a^2bx^{10}+\frac{1}{4}ab^2x^{12}+\frac{b^3x^{14}}{14}$$

input
```
Integrate[x^7*(a + b*x^2)^3,x]
```

output
```
(a^3*x^8)/8 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4 + (b^3*x^14)/14
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + bx^2)^3 \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^6 (bx^2 + a)^3 \, dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int (b^3 x^{12} + 3ab^2 x^{10} + 3a^2 b x^8 + a^3 x^6) \, dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{a^3 x^8}{4} + \frac{3}{5} a^2 b x^{10} + \frac{1}{2} a b^2 x^{12} + \frac{b^3 x^{14}}{7} \right)$$

input
```
Int[x^7*(a + b*x^2)^3,x]
```

output
```
((a^3*x^8)/4 + (3*a^2*b*x^10)/5 + (a*b^2*x^12)/2 + (b^3*x^14)/7)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{8}a^3x^8 + \frac{3}{10}a^2b\,x^{10} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{14}b^3x^{14}$	36
default	$\frac{1}{8}a^3x^8 + \frac{3}{10}a^2b\,x^{10} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{14}b^3x^{14}$	36
norman	$\frac{1}{8}a^3x^8 + \frac{3}{10}a^2b\,x^{10} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{14}b^3x^{14}$	36
risch	$\frac{1}{8}a^3x^8 + \frac{3}{10}a^2b\,x^{10} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{14}b^3x^{14}$	36
parallelrisch	$\frac{1}{8}a^3x^8 + \frac{3}{10}a^2b\,x^{10} + \frac{1}{4}a\,b^2x^{12} + \frac{1}{14}b^3x^{14}$	36
orering	$\frac{x^8(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3)}{280}$	38

input
```
int(x^7*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^3*x^8+3/10*a^2*b*x^10+1/4*a*b^2*x^12+1/14*b^3*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^7(a+bx^2)^3\,dx = \frac{1}{14}\,b^3x^{14} + \frac{1}{4}\,ab^2x^{12} + \frac{3}{10}\,a^2bx^{10} + \frac{1}{8}\,a^3x^8$$

input
```
integrate(x^7*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^7 (a + bx^2)^3 \, dx = \frac{a^3x^8}{8} + \frac{3a^2bx^{10}}{10} + \frac{ab^2x^{12}}{4} + \frac{b^3x^{14}}{14}$$

input
```
integrate(x**7*(b*x**2+a)**3,x)
```

output
```
a**3*x**8/8 + 3*a**2*b*x**10/10 + a*b**2*x**12/4 + b**3*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^7 (a + bx^2)^3 \, dx = \frac{1}{14} b^3x^{14} + \frac{1}{4} ab^2x^{12} + \frac{3}{10} a^2bx^{10} + \frac{1}{8} a^3x^8$$

input
```
integrate(x^7*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^7 (a + bx^2)^3 \, dx = \frac{1}{14} b^3x^{14} + \frac{1}{4} ab^2x^{12} + \frac{3}{10} a^2bx^{10} + \frac{1}{8} a^3x^8$$

input
```
integrate(x^7*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/14*b^3*x^14 + 1/4*a*b^2*x^12 + 3/10*a^2*b*x^10 + 1/8*a^3*x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^7 \left(a+bx^2\right)^3 \, dx = \frac{a^3\,x^8}{8}+\frac{3\,a^2\,b\,x^{10}}{10}+\frac{a\,b^2\,x^{12}}{4}+\frac{b^3\,x^{14}}{14}$$

input
```
int(x^7*(a + b*x^2)^3,x)
```

output
```
(a^3*x^8)/8 + (b^3*x^14)/14 + (3*a^2*b*x^10)/10 + (a*b^2*x^12)/4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^7 \left(a+bx^2\right)^3 \, dx = \frac{x^8(20b^3x^6+70a\,b^2x^4+84a^2b\,x^2+35a^3)}{280}$$

input
```
int(x^7*(b*x^2+a)^3,x)
```

output
```
(x**8*(35*a**3 + 84*a**2*b*x**2 + 70*a*b**2*x**4 + 20*b**3*x**6))/280
```

3.31 $\int x^5(a+bx^2)^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^5\left(a+bx^2\right)^3 \, dx = \frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

output
```
1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^5\left(a+bx^2\right)^3 \, dx = \frac{a^3x^6}{6}+\frac{3}{8}a^2bx^8+\frac{3}{10}ab^2x^{10}+\frac{b^3x^{12}}{12}$$

input
```
Integrate[x^5*(a + b*x^2)^3,x]
```

output
```
(a^3*x^6)/6 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10 + (b^3*x^12)/12
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^4 \left(bx^2+a\right)^3 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(b^3x^{10}+3ab^2x^8+3a^2bx^6+a^3x^4\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^3x^6}{3}+\frac{3}{4}a^2bx^8+\frac{3}{5}ab^2x^{10}+\frac{b^3x^{12}}{6}\right)$$

input
```
Int[x^5*(a + b*x^2)^3,x]
```

output
```
((a^3*x^6)/3 + (3*a^2*b*x^8)/4 + (3*a*b^2*x^10)/5 + (b^3*x^12)/6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
default	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
norman	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
risch	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
parallelrisch	$\frac{1}{6}a^3x^6+\frac{3}{8}a^2b\,x^8+\frac{3}{10}a\,b^2x^{10}+\frac{1}{12}b^3x^{12}$	36
orering	$\frac{x^6(10b^3x^6+36a\,b^2x^4+45a^2b\,x^2+20a^3)}{120}$	38

input
```
int(x^5*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^3*x^6+3/8*a^2*b*x^8+3/10*a*b^2*x^10+1/12*b^3*x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^5(a+bx^2)^3\,dx=\frac{1}{12}\,b^3x^{12}+\frac{3}{10}\,ab^2x^{10}+\frac{3}{8}\,a^2bx^8+\frac{1}{6}\,a^3x^6$$

input
```
integrate(x^5*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^5 (a+bx^2)^3 \, dx = \frac{a^3x^6}{6} + \frac{3a^2bx^8}{8} + \frac{3ab^2x^{10}}{10} + \frac{b^3x^{12}}{12}$$

input
```
integrate(x**5*(b*x**2+a)**3,x)
```

output
```
a**3*x**6/6 + 3*a**2*b*x**8/8 + 3*a*b**2*x**10/10 + b**3*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^5 (a+bx^2)^3 \, dx = \frac{1}{12} b^3x^{12} + \frac{3}{10} ab^2x^{10} + \frac{3}{8} a^2bx^8 + \frac{1}{6} a^3x^6$$

input
```
integrate(x^5*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^5 (a+bx^2)^3 \, dx = \frac{1}{12} b^3x^{12} + \frac{3}{10} ab^2x^{10} + \frac{3}{8} a^2bx^8 + \frac{1}{6} a^3x^6$$

input
```
integrate(x^5*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/12*b^3*x^12 + 3/10*a*b^2*x^10 + 3/8*a^2*b*x^8 + 1/6*a^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^5 \left(a+bx^2\right)^3 \, dx = \frac{a^3 x^6}{6} + \frac{3 a^2 b x^8}{8} + \frac{3 a b^2 x^{10}}{10} + \frac{b^3 x^{12}}{12}$$

input
```
int(x^5*(a + b*x^2)^3,x)
```

output
```
(a^3*x^6)/6 + (b^3*x^12)/12 + (3*a^2*b*x^8)/8 + (3*a*b^2*x^10)/10
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^5 \left(a+bx^2\right)^3 \, dx = \frac{x^6 (10b^3x^6 + 36a b^2x^4 + 45a^2b x^2 + 20a^3)}{120}$$

input
```
int(x^5*(b*x^2+a)^3,x)
```

output
```
(x**6*(20*a**3 + 45*a**2*b*x**2 + 36*a*b**2*x**4 + 10*b**3*x**6))/120
```

3.32 $\int x^3(a+bx^2)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^3(a+bx^2)^3\,dx=-\frac{a(a+bx^2)^4}{8b^2}+\frac{(a+bx^2)^5}{10b^2}$$

output
```
-1/8*a*(b*x^2+a)^4/b^2+1/10*(b*x^2+a)^5/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int x^3(a+bx^2)^3\,dx=\frac{a^3x^4}{4}+\frac{1}{2}a^2bx^6+\frac{3}{8}ab^2x^8+\frac{b^3x^{10}}{10}$$

input
```
Integrate[x^3*(a + b*x^2)^3,x]
```

output
```
(a^3*x^4)/4 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8 + (b^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^3 dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2 \left(bx^2+a\right)^3 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^4}{b}-\frac{a\left(bx^2+a\right)^3}{b}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+bx^2\right)^5}{5b^2}-\frac{a\left(a+bx^2\right)^4}{4b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^3,x]
```

output
```
(-1/4*(a*(a + b*x^2)^4)/b^2 + (a + b*x^2)^5/(5*b^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
default	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
norman	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
risch	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
parallelrisch	$\frac{1}{10}b^3x^{10}+\frac{3}{8}a\,b^2x^8+\frac{1}{2}a^2b\,x^6+\frac{1}{4}a^3x^4$	36
orering	$\frac{x^4\left(4b^3x^6+15a\,b^2x^4+20a^2b\,x^2+10a^3\right)}{40}$	38

input
```
int(x^3*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*b^3*x^10+3/8*a*b^2*x^8+1/2*a^2*b*x^6+1/4*a^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^3\left(a+bx^2\right)^3\,dx=\frac{1}{10}\,b^3x^{10}+\frac{3}{8}\,ab^2x^8+\frac{1}{2}\,a^2bx^6+\frac{1}{4}\,a^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int x^3 (a + bx^2)^3 \, dx = \frac{a^3x^4}{4} + \frac{a^2bx^6}{2} + \frac{3ab^2x^8}{8} + \frac{b^3x^{10}}{10}$$

input
```
integrate(x**3*(b*x**2+a)**3,x)
```

output
```
a**3*x**4/4 + a**2*b*x**6/2 + 3*a*b**2*x**8/8 + b**3*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^3 (a + bx^2)^3 \, dx = \frac{1}{10} b^3x^{10} + \frac{3}{8} ab^2x^8 + \frac{1}{2} a^2bx^6 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^3 (a + bx^2)^3 \, dx = \frac{1}{10} b^3x^{10} + \frac{3}{8} ab^2x^8 + \frac{1}{2} a^2bx^6 + \frac{1}{4} a^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/10*b^3*x^10 + 3/8*a*b^2*x^8 + 1/2*a^2*b*x^6 + 1/4*a^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int x^3 (a + bx^2)^3 \, dx = \frac{a^3 x^4}{4} + \frac{a^2 b x^6}{2} + \frac{3 a b^2 x^8}{8} + \frac{b^3 x^{10}}{10}$$

input
```
int(x^3*(a + b*x^2)^3,x)
```

output
```
(a^3*x^4)/4 + (b^3*x^10)/10 + (a^2*b*x^6)/2 + (3*a*b^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int x^3 (a + bx^2)^3 \, dx = \frac{x^4(4b^3x^6 + 15a b^2x^4 + 20a^2b x^2 + 10a^3)}{40}$$

input
```
int(x^3*(b*x^2+a)^3,x)
```

output
```
(x**4*(10*a**3 + 20*a**2*b*x**2 + 15*a*b**2*x**4 + 4*b**3*x**6))/40
```

3.33 $\int x(a+bx^2)^3\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x\left(a+bx^2\right)^3\,dx=\frac{\left(a+bx^2\right)^4}{8b}$$

output
```
1/8*(b*x^2+a)^4/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)^3\,dx=\frac{\left(a+bx^2\right)^4}{8b}$$

input
```
Integrate[x*(a + b*x^2)^3,x]
```

output
```
(a + b*x^2)^4/(8*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^3 \, dx$$

$$\Big\downarrow \; 241$$

$$\frac{(a+bx^2)^4}{8b}$$

input
```
Int[x*(a + b*x^2)^3,x]
```

output
```
(a + b*x^2)^4/(8*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^4}{8b}$	15
gosper	$\frac{1}{8}b^3x^8+\frac{1}{2}a\,b^2x^6+\frac{3}{4}a^2b\,x^4+\frac{1}{2}a^3x^2$	36
norman	$\frac{1}{8}b^3x^8+\frac{1}{2}a\,b^2x^6+\frac{3}{4}a^2b\,x^4+\frac{1}{2}a^3x^2$	36
parallelrisch	$\frac{1}{8}b^3x^8+\frac{1}{2}a\,b^2x^6+\frac{3}{4}a^2b\,x^4+\frac{1}{2}a^3x^2$	36
orering	$\frac{x^2(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3)}{8}$	37
risch	$\frac{b^3x^8}{8}+\frac{a\,b^2x^6}{2}+\frac{3a^2b\,x^4}{4}+\frac{a^3x^2}{2}+\frac{a^4}{8b}$	44

input
```
int(x*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b*x^2+a)^4/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x(a+bx^2)^3\,dx = \frac{1}{8}\,b^3x^8+\frac{1}{2}\,ab^2x^6+\frac{3}{4}\,a^2bx^4+\frac{1}{2}\,a^3x^2$$

input
```
integrate(x*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*b^3*x^8 + 1/2*a*b^2*x^6 + 3/4*a^2*b*x^4 + 1/2*a^3*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.31

$$\int x(a+bx^2)^3\,dx = \frac{a^3x^2}{2} + \frac{3a^2bx^4}{4} + \frac{ab^2x^6}{2} + \frac{b^3x^8}{8}$$

input
```
integrate(x*(b*x**2+a)**3,x)
```

output
```
a**3*x**2/2 + 3*a**2*b*x**4/4 + a*b**2*x**6/2 + b**3*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^3\,dx = \frac{(bx^2+a)^4}{8\,b}$$

input
```
integrate(x*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^4/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^3\,dx = \frac{(bx^2+a)^4}{8\,b}$$

input
```
integrate(x*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(b*x^2 + a)^4/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int x(a+bx^2)^3\,dx=\frac{a^3\,x^2}{2}+\frac{3\,a^2\,b\,x^4}{4}+\frac{a\,b^2\,x^6}{2}+\frac{b^3\,x^8}{8}$$

input
```
int(x*(a + b*x^2)^3,x)
```

output
```
(a^3*x^2)/2 + (b^3*x^8)/8 + (3*a^2*b*x^4)/4 + (a*b^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int x(a+bx^2)^3\,dx=\frac{x^2(b^3x^6+4a\,b^2x^4+6a^2b\,x^2+4a^3)}{8}$$

input
```
int(x*(b*x^2+a)^3,x)
```

output
```
(x**2*(4*a**3 + 6*a**2*b*x**2 + 4*a*b**2*x**4 + b**3*x**6))/8
```

3.34 $\int \frac{(a+bx^2)^3}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^2)^3}{x} dx = \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3\log(x)$$

output
```
3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x} dx = \frac{3}{2}a^2bx^2 + \frac{3}{4}ab^2x^4 + \frac{b^3x^6}{6} + a^3\log(x)$$

input
```
Integrate[(a + b*x^2)^3/x,x]
```

output
```
(3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4 + (b^3*x^6)/6 + a^3*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^3x^4+3ab^2x^2+3a^2b+\frac{a^3}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^3\log\left(x^2\right)+3a^2bx^2+\frac{3}{2}ab^2x^4+\frac{b^3x^6}{3}\right)$$

input
```
Int[(a + b*x^2)^3/x,x]
```

output
```
(3*a^2*b*x^2 + (3*a*b^2*x^4)/2 + (b^3*x^6)/3 + a^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{3a^2b\,x^2}{2}+\frac{3a\,b^2x^4}{4}+\frac{b^3x^6}{6}+a^3\ln\left(x\right)$	34
norman	$\frac{3a^2b\,x^2}{2}+\frac{3a\,b^2x^4}{4}+\frac{b^3x^6}{6}+a^3\ln\left(x\right)$	34
risch	$\frac{3a^2b\,x^2}{2}+\frac{3a\,b^2x^4}{4}+\frac{b^3x^6}{6}+a^3\ln\left(x\right)$	34
parallelrisch	$\frac{3a^2b\,x^2}{2}+\frac{3a\,b^2x^4}{4}+\frac{b^3x^6}{6}+a^3\ln\left(x\right)$	34

input
```
int((b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

output
```
3/2*a^2*b*x^2+3/4*a*b^2*x^4+1/6*b^3*x^6+a^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int\frac{\left(a+bx^2\right)^3}{x}\,dx=\frac{1}{6}\,b^3x^6+\frac{3}{4}\,ab^2x^4+\frac{3}{2}\,a^2bx^2+a^3\log\left(x\right)$$

input
```
integrate((b*x^2+a)^3/x,x, algorithm="fricas")
```

output
```
1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + a^3*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x}\,dx = a^3\log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6}$$

input
```
integrate((b*x**2+a)**3/x,x)
```

output
```
a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x**4/4 + b**3*x**6/6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x}\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^3/x,x, algorithm="maxima")
```

output
```
1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x}\,dx = \frac{1}{6}\,b^3x^6 + \frac{3}{4}\,ab^2x^4 + \frac{3}{2}\,a^2bx^2 + \frac{1}{2}\,a^3\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^3/x,x, algorithm="giac")
```

output
```
1/6*b^3*x^6 + 3/4*a*b^2*x^4 + 3/2*a^2*b*x^2 + 1/2*a^3*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^3}{x}\,dx = a^3\,\ln(x) + \frac{b^3\,x^6}{6} + \frac{3\,a^2\,b\,x^2}{2} + \frac{3\,a\,b^2\,x^4}{4}$$

input
```
int((a + b*x^2)^3/x,x)
```

output
```
a^3*log(x) + (b^3*x^6)/6 + (3*a^2*b*x^2)/2 + (3*a*b^2*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^3}{x}\,dx = \log(x)\,a^3 + \frac{3a^2b\,x^2}{2} + \frac{3a\,b^2x^4}{4} + \frac{b^3x^6}{6}$$

input
```
int((b*x^2+a)^3/x,x)
```

output
```
(12*log(x)*a**3 + 18*a**2*b*x**2 + 9*a*b**2*x**4 + 2*b**3*x**6)/12
```

3.35 $\int \frac{(a+bx^2)^3}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4} + 3a^2b\log(x)$$

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + \frac{3}{2}ab^2x^2 + \frac{b^3x^4}{4} + 3a^2b\log(x)$$

input
```
Integrate[(a + b*x^2)^3/x^3,x]
```

output
```
-1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^4}+\frac{3ba^2}{x^2}+3b^2a+b^3x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{x^2}+3a^2b\log\left(x^2\right)+3ab^2x^2+\frac{b^3x^4}{2}\right)$$

input
```
Int[(a + b*x^2)^3/x^3,x]
```

output
```
(-(a^3/x^2) + 3*a*b^2*x^2 + (b^3*x^4)/2 + 3*a^2*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{2x^{2}}+\frac{3a\,b^{2}x^{2}}{2}+\frac{b^{3}x^{4}}{4}+3a^{2}b\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{2}a^{3}+\frac{1}{4}b^{3}x^{6}+\frac{3}{2}a\,b^{2}x^{4}}{x^{2}}+3a^{2}b\ln\left(x\right)$	37
parallelrisch	$\frac{b^{3}x^{6}+6a\,b^{2}x^{4}+12a^{2}b\ln(x)x^{2}-2a^{3}}{4x^{2}}$	39
risch	$\frac{b^{3}x^{4}}{4}+\frac{3a\,b^{2}x^{2}}{2}+\frac{9a^{2}b}{4}-\frac{a^{3}}{2x^{2}}+3a^{2}b\ln\left(x\right)$	41

input
```
int((b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int\frac{\left(a+bx^{2}\right)^{3}}{x^{3}}\,dx=\frac{b^{3}x^{6}+6\,ab^{2}x^{4}+12\,a^{2}bx^{2}\log\left(x\right)-2\,a^{3}}{4\,x^{2}}$$

input
```
integrate((b*x^2+a)^3/x^3,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = -\frac{a^3}{2x^2} + 3a^2b\log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

input
```
integrate((b*x**2+a)**3/x**3,x)
```

output
```
-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = \frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\log(x^2) - \frac{a^3}{2\,x^2}$$

input
```
integrate((b*x^2+a)^3/x^3,x, algorithm="maxima")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*a^3/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = \frac{1}{4}b^3x^4 + \frac{3}{2}ab^2x^2 + \frac{3}{2}a^2b\log(x^2) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

input
```
integrate((b*x^2+a)^3/x^3,x, algorithm="giac")
```

output
```
1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)
/x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = \frac{b^3\,x^4}{4} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,\ln(x)$$

input
```
int((a + b*x^2)^3/x^3,x)
```

output
```
(b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^3}\,dx = \frac{12\log(x)\,a^2b\,x^2 - 2a^3 + 6a\,b^2x^4 + b^3x^6}{4x^2}$$

input
```
int((b*x^2+a)^3/x^3,x)
```

output
```
(12*log(x)*a**2*b*x**2 - 2*a**3 + 6*a*b**2*x**4 + b**3*x**6)/(4*x**2)
```

3.36 $\int \frac{(a+bx^2)^3}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^2)^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3ab^2\log(x)$$

output
```
-1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^5} dx = -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + \frac{b^3x^2}{2} + 3ab^2\log(x)$$

input
```
Integrate[(a + b*x^2)^3/x^5,x]
```

output
```
-1/4*a^3/x^4 - (3*a^2*b)/(2*x^2) + (b^3*x^2)/2 + 3*a*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^5} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^6}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^6}+\frac{3ba^2}{x^4}+\frac{3b^2a}{x^2}+b^3\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{2x^4}-\frac{3a^2b}{x^2}+3ab^2\log\left(x^2\right)+b^3x^2\right)$$

input
```
Int[(a + b*x^2)^3/x^5,x]
```

output
```
(-1/2*a^3/x^4 - (3*a^2*b)/x^2 + b^3*x^2 + 3*a*b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{a^{3}}{4x^{4}}-\frac{3a^{2}b}{2x^{2}}+\frac{b^{3}x^{2}}{2}+3a\,b^{2}\ln\left(x\right)$	35
norman	$\frac{-\frac{1}{4}a^{3}+\frac{1}{2}b^{3}x^{6}-\frac{3}{2}a^{2}b\,x^{2}}{x^{4}}+3a\,b^{2}\ln\left(x\right)$	37
risch	$\frac{b^{3}x^{2}}{2}+\frac{-\frac{3}{2}a^{2}b\,x^{2}-\frac{1}{4}a^{3}}{x^{4}}+3a\,b^{2}\ln\left(x\right)$	37
parallelrisch	$\frac{2b^{3}x^{6}+12a\,b^{2}\ln\left(x\right)x^{4}-6a^{2}b\,x^{2}-a^{3}}{4x^{4}}$	40

input
```
int((b*x^2+a)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^3/x^4-3/2*a^2*b/x^2+1/2*b^3*x^2+3*a*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+bx^{2}\right)^{3}}{x^{5}}\,dx=\frac{2\,b^{3}x^{6}+12\,ab^{2}x^{4}\log\left(x\right)-6\,a^{2}bx^{2}-a^{3}}{4\,x^{4}}$$

input
```
integrate((b*x^2+a)^3/x^5,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*x^6 + 12*a*b^2*x^4*log(x) - 6*a^2*b*x^2 - a^3)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^5}\,dx = 3ab^2\log(x) + \frac{b^3x^2}{2} + \frac{-a^3-6a^2bx^2}{4x^4}$$

input
```
integrate((b*x**2+a)**3/x**5,x)
```

output
```
3*a*b**2*log(x) + b**3*x**2/2 + (-a**3 - 6*a**2*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^5}\,dx = \frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{6\,a^2bx^2+a^3}{4\,x^4}$$

input
```
integrate((b*x^2+a)^3/x^5,x, algorithm="maxima")
```

output
```
1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(6*a^2*b*x^2 + a^3)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3}{x^5}\,dx = \frac{1}{2}\,b^3x^2 + \frac{3}{2}\,ab^2\log\left(x^2\right) - \frac{9\,ab^2x^4+6\,a^2bx^2+a^3}{4\,x^4}$$

input
```
integrate((b*x^2+a)^3/x^5,x, algorithm="giac")
```

output
```
1/2*b^3*x^2 + 3/2*a*b^2*log(x^2) - 1/4*(9*a*b^2*x^4 + 6*a^2*b*x^2 + a^3)/x
^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^5}\,dx = \frac{b^3\,x^2}{2} - \frac{\frac{a^3}{4} + \frac{3\,b\,a^2\,x^2}{2}}{x^4} + 3\,a\,b^2\,\ln(x)$$

input
```
int((a + b*x^2)^3/x^5,x)
```

output
```
(b^3*x^2)/2 - (a^3/4 + (3*a^2*b*x^2)/2)/x^4 + 3*a*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{x^5}\,dx = \frac{12\log(x)\,a\,b^2x^4 - a^3 - 6a^2b\,x^2 + 2b^3x^6}{4x^4}$$

input
```
int((b*x^2+a)^3/x^5,x)
```

output
```
(12*log(x)*a*b**2*x**4 - a**3 - 6*a**2*b*x**2 + 2*b**3*x**6)/(4*x**4)
```

3.37 $\int \frac{(a+bx^2)^3}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3\log(x)$$

output
```
-1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3\log(x)$$

input
```
Integrate[(a + b*x^2)^3/x^7,x]
```

output
```
-1/6*a^3/x^6 - (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) + b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^7} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^3}{x^8} dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{a^3}{x^8} + \frac{3ba^2}{x^6} + \frac{3b^2a}{x^4} + \frac{b^3}{x^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3}{3x^6} - \frac{3a^2b}{2x^4} - \frac{3ab^2}{x^2} + b^3 \log\left(x^2\right)\right)$$

input
```
Int[(a + b*x^2)^3/x^7,x]
```

output
```
(-1/3*a^3/x^6 - (3*a^2*b)/(2*x^4) - (3*a*b^2)/x^2 + b^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^3}{6x^6}-\frac{3a^2b}{4x^4}-\frac{3a\,b^2}{2x^2}+b^3\ln\left(x\right)$	34
norman	$\frac{-\frac{1}{6}a^3-\frac{3}{2}a\,b^2x^4-\frac{3}{4}a^2b\,x^2}{x^6}+b^3\ln\left(x\right)$	36
risch	$\frac{-\frac{1}{6}a^3-\frac{3}{2}a\,b^2x^4-\frac{3}{4}a^2b\,x^2}{x^6}+b^3\ln\left(x\right)$	36
parallelrisch	$\frac{12b^3\ln(x)x^6-18a\,b^2x^4-9a^2b\,x^2-2a^3}{12x^6}$	40

input
```
int((b*x^2+a)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^3/x^6-3/4*a^2*b/x^4-3/2*a*b^2/x^2+b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int\frac{(a+bx^2)^3}{x^7}\,dx=\frac{12\,b^3x^6\log\left(x\right)-18\,ab^2x^4-9\,a^2bx^2-2\,a^3}{12\,x^6}$$

input
```
integrate((b*x^2+a)^3/x^7,x, algorithm="fricas")
```

output
```
1/12*(12*b^3*x^6*log(x) - 18*a*b^2*x^4 - 9*a^2*b*x^2 - 2*a^3)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = b^3\log(x) + \frac{-2a^3 - 9a^2bx^2 - 18ab^2x^4}{12x^6}$$

input
```
integrate((b*x**2+a)**3/x**7,x)
```

output
```
b**3*log(x) + (-2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = \frac{1}{2}b^3\log(x^2) - \frac{18\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

input
```
integrate((b*x^2+a)^3/x^7,x, algorithm="maxima")
```

output
```
1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = \frac{1}{2}b^3\log(x^2) - \frac{11\,b^3x^6 + 18\,ab^2x^4 + 9\,a^2bx^2 + 2\,a^3}{12\,x^6}$$

input
```
integrate((b*x^2+a)^3/x^7,x, algorithm="giac")
```

output
```
1/2*b^3*log(x^2) - 1/12*(11*b^3*x^6 + 18*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)/
x^6
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = b^3\,\ln(x) - \frac{\frac{a^3}{6} + \frac{3\,a^2\,b\,x^2}{4} + \frac{3\,a\,b^2\,x^4}{2}}{x^6}$$

input
```
int((a + b*x^2)^3/x^7,x)
```

output
```
b^3*log(x) - (a^3/6 + (3*a^2*b*x^2)/4 + (3*a*b^2*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^7}\,dx = \frac{12\log(x)\,b^3x^6 - 2a^3 - 9a^2b\,x^2 - 18a\,b^2x^4}{12x^6}$$

input
```
int((b*x^2+a)^3/x^7,x)
```

output
```
(12*log(x)*b**3*x**6 - 2*a**3 - 9*a**2*b*x**2 - 18*a*b**2*x**4)/(12*x**6)
```

3.38 $\int \frac{(a+bx^2)^3}{x^9} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^2)^3}{x^9} dx = -\frac{(a+bx^2)^4}{8ax^8}$$

output
```
-1/8*(b*x^2+a)^4/a/x^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{(a+bx^2)^3}{x^9} dx = -\frac{a^3}{8x^8} - \frac{a^2b}{2x^6} - \frac{3ab^2}{4x^4} - \frac{b^3}{2x^2}$$

input
```
Integrate[(a + b*x^2)^3/x^9,x]
```

output
```
-1/8*a^3/x^8 - (a^2*b)/(2*x^6) - (3*a*b^2)/(4*x^4) - b^3/(2*x^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^9}\,dx$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{(a+bx^2)^4}{8ax^8}$$

input
```
Int[(a + b*x^2)^3/x^9,x]
```

output
```
-1/8*(a + b*x^2)^4/(a*x^8)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

method	result	size
gosper	$-\frac{4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3}{8x^8}$	36
default	$-\frac{b^3}{2x^2}-\frac{3a\,b^2}{4x^4}-\frac{a^3}{8x^8}-\frac{a^2b}{2x^6}$	36
orering	$-\frac{4b^3x^6+6a\,b^2x^4+4a^2b\,x^2+a^3}{8x^8}$	36
norman	$\frac{-\frac{1}{2}b^3x^6-\frac{3}{4}a\,b^2x^4-\frac{1}{2}a^2b\,x^2-\frac{1}{8}a^3}{x^8}$	37
risch	$\frac{-\frac{1}{2}b^3x^6-\frac{3}{4}a\,b^2x^4-\frac{1}{2}a^2b\,x^2-\frac{1}{8}a^3}{x^8}$	37
parallelrisch	$\frac{-4b^3x^6-6a\,b^2x^4-4a^2b\,x^2-a^3}{8x^8}$	38

input
```
int((b*x^2+a)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(4*b^3*x^6+6*a*b^2*x^4+4*a^2*b*x^2+a^3)/x^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{4\,b^3x^6+6\,ab^2x^4+4\,a^2bx^2+a^3}{8\,x^8}$$

input
```
integrate((b*x^2+a)^3/x^9,x, algorithm="fricas")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(15) = 30$.

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = \frac{-a^3 - 4a^2bx^2 - 6ab^2x^4 - 4b^3x^6}{8x^8}$$

input
```
integrate((b*x**2+a)**3/x**9,x)
```

output
```
(-a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

input
```
integrate((b*x^2+a)^3/x^9,x, algorithm="maxima")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{4\,b^3x^6 + 6\,ab^2x^4 + 4\,a^2bx^2 + a^3}{8\,x^8}$$

input
```
integrate((b*x^2+a)^3/x^9,x, algorithm="giac")
```

output
```
-1/8*(4*b^3*x^6 + 6*a*b^2*x^4 + 4*a^2*b*x^2 + a^3)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = -\frac{\frac{a^3}{8}+\frac{a^2\,b\,x^2}{2}+\frac{3\,a\,b^2\,x^4}{4}+\frac{b^3\,x^6}{2}}{x^8}$$

input
```
int((a + b*x^2)^3/x^9,x)
```

output
```
-(a^3/8 + (b^3*x^6)/2 + (a^2*b*x^2)/2 + (3*a*b^2*x^4)/4)/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)^3}{x^9}\,dx = \frac{-4b^3x^6-6a\,b^2x^4-4a^2b\,x^2-a^3}{8x^8}$$

input
```
int((b*x^2+a)^3/x^9,x)
```

output
```
( - a**3 - 4*a**2*b*x**2 - 6*a*b**2*x**4 - 4*b**3*x**6)/(8*x**8)
```

3.39 $\int \frac{(a+bx^2)^3}{x^{11}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{(a+bx^2)^4}{10ax^{10}}+\frac{b(a+bx^2)^4}{40a^2x^8}$$

output
```
-1/10*(b*x^2+a)^4/a/x^10+1/40*b*(b*x^2+a)^4/a^2/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{a^3}{10x^{10}}-\frac{3a^2b}{8x^8}-\frac{ab^2}{2x^6}-\frac{b^3}{4x^4}$$

input
```
Integrate[(a + b*x^2)^3/x^11,x]
```

output
```
-1/10*a^3/x^10 - (3*a^2*b)/(8*x^8) - (a*b^2)/(2*x^6) - b^3/(4*x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^{12}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^3}{x^{10}}dx^2}{5a}-\frac{(a+bx^2)^4}{5ax^{10}}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{2}\left(\frac{b(a+bx^2)^4}{20a^2x^8}-\frac{(a+bx^2)^4}{5ax^{10}}\right)$$

input
```
Int[(a + b*x^2)^3/x^11,x]
```

output
```
(-1/5*(a + b*x^2)^4/(a*x^10) + (b*(a + b*x^2)^4)/(20*a^2*x^8))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{b^{3}}{4x^{4}}-\frac{3a^{2}b}{8x^{8}}-\frac{a^{3}}{10x^{10}}-\frac{a\,b^{2}}{2x^{6}}$	36
norman	$\frac{-\frac{1}{4}b^{3}x^{6}-\frac{1}{2}a\,b^{2}x^{4}-\frac{3}{8}a^{2}b\,x^{2}-\frac{1}{10}a^{3}}{x^{10}}$	37
risch	$\frac{-\frac{1}{4}b^{3}x^{6}-\frac{1}{2}a\,b^{2}x^{4}-\frac{3}{8}a^{2}b\,x^{2}-\frac{1}{10}a^{3}}{x^{10}}$	37
gosper	$-\frac{10b^{3}x^{6}+20a\,b^{2}x^{4}+15a^{2}b\,x^{2}+4a^{3}}{40x^{10}}$	38
parallelrisch	$\frac{-10b^{3}x^{6}-20a\,b^{2}x^{4}-15a^{2}b\,x^{2}-4a^{3}}{40x^{10}}$	38
orering	$-\frac{10b^{3}x^{6}+20a\,b^{2}x^{4}+15a^{2}b\,x^{2}+4a^{3}}{40x^{10}}$	38

input
```
int((b*x^2+a)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^3/x^4-3/8*a^2*b/x^8-1/10*a^3/x^10-1/2*a*b^2/x^6
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

input
```
integrate((b*x^2+a)^3/x^11,x, algorithm="fricas")
```

output
```
-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = \frac{-4a^3 - 15a^2bx^2 - 20ab^2x^4 - 10b^3x^6}{40x^{10}}$$

input
```
integrate((b*x**2+a)**3/x**11,x)
```

output
```
(-4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

input
```
integrate((b*x^2+a)^3/x^11,x, algorithm="maxima")
```

output
```
-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{10\,b^3x^6 + 20\,ab^2x^4 + 15\,a^2bx^2 + 4\,a^3}{40\,x^{10}}$$

input
```
integrate((b*x^2+a)^3/x^11,x, algorithm="giac")
```

output
```
-1/40*(10*b^3*x^6 + 20*a*b^2*x^4 + 15*a^2*b*x^2 + 4*a^3)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = -\frac{\frac{a^3}{10} + \frac{3\,a^2\,b\,x^2}{8} + \frac{a\,b^2\,x^4}{2} + \frac{b^3\,x^6}{4}}{x^{10}}$$

input
```
int((a + b*x^2)^3/x^11,x)
```

output
```
-(a^3/10 + (b^3*x^6)/4 + (3*a^2*b*x^2)/8 + (a*b^2*x^4)/2)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^{11}}\,dx = \frac{-10b^3x^6 - 20a\,b^2x^4 - 15a^2b\,x^2 - 4a^3}{40x^{10}}$$

input
```
int((b*x^2+a)^3/x^11,x)
```

output
```
( - 4*a**3 - 15*a**2*b*x**2 - 20*a*b**2*x**4 - 10*b**3*x**6)/(40*x**10)
```

3.40 $\int \frac{(a+bx^2)^3}{x^{13}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^2)^3}{x^{13}} \, dx = -\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

output
```
-1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^{13}} \, dx = -\frac{a^3}{12x^{12}} - \frac{3a^2b}{10x^{10}} - \frac{3ab^2}{8x^8} - \frac{b^3}{6x^6}$$

input
```
Integrate[(a + b*x^2)^3/x^13,x]
```

output
```
-1/12*a^3/x^12 - (3*a^2*b)/(10*x^10) - (3*a*b^2)/(8*x^8) - b^3/(6*x^6)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{13}}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^{14}}dx^2$$
$$\Big\downarrow \text{ 53}$$
$$\frac{1}{2}\int \left(\frac{a^3}{x^{14}}+\frac{3ba^2}{x^{12}}+\frac{3b^2a}{x^{10}}+\frac{b^3}{x^8}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{a^3}{6x^{12}}-\frac{3a^2b}{5x^{10}}-\frac{3ab^2}{4x^8}-\frac{b^3}{3x^6}\right)$$

input
```
Int[(a + b*x^2)^3/x^13,x]
```

output
```
(-1/6*a^3/x^12 - (3*a^2*b)/(5*x^10) - (3*a*b^2)/(4*x^8) - b^3/(3*x^6))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{3}}{12x^{12}}-\frac{3a^{2}b}{10x^{10}}-\frac{3a\,b^{2}}{8x^{8}}-\frac{b^{3}}{6x^{6}}$	36
norman	$\frac{-\frac{1}{6}b^{3}x^{6}-\frac{3}{8}a\,b^{2}x^{4}-\frac{3}{10}a^{2}b\,x^{2}-\frac{1}{12}a^{3}}{x^{12}}$	37
risch	$\frac{-\frac{1}{6}b^{3}x^{6}-\frac{3}{8}a\,b^{2}x^{4}-\frac{3}{10}a^{2}b\,x^{2}-\frac{1}{12}a^{3}}{x^{12}}$	37
gosper	$-\frac{20b^{3}x^{6}+45a\,b^{2}x^{4}+36a^{2}b\,x^{2}+10a^{3}}{120x^{12}}$	38
parallelrisch	$\frac{-20b^{3}x^{6}-45a\,b^{2}x^{4}-36a^{2}b\,x^{2}-10a^{3}}{120x^{12}}$	38
orering	$-\frac{20b^{3}x^{6}+45a\,b^{2}x^{4}+36a^{2}b\,x^{2}+10a^{3}}{120x^{12}}$	38

input
```
int((b*x^2+a)^3/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^3/x^12-3/10*a^2*b/x^10-3/8*a*b^2/x^8-1/6*b^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx=-\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

input
```
integrate((b*x^2+a)^3/x^13,x, algorithm="fricas")
```

output
```
-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx = \frac{-10a^3-36a^2bx^2-45ab^2x^4-20b^3x^6}{120x^{12}}$$

input
```
integrate((b*x**2+a)**3/x**13,x)
```

output
```
(-10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx = -\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

input
```
integrate((b*x^2+a)^3/x^13,x, algorithm="maxima")
```

output
```
-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx = -\frac{20\,b^3x^6+45\,ab^2x^4+36\,a^2bx^2+10\,a^3}{120\,x^{12}}$$

input
```
integrate((b*x^2+a)^3/x^13,x, algorithm="giac")
```

output
```
-1/120*(20*b^3*x^6 + 45*a*b^2*x^4 + 36*a^2*b*x^2 + 10*a^3)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx = -\frac{\frac{a^3}{12}+\frac{3\,a^2\,b\,x^2}{10}+\frac{3\,a\,b^2\,x^4}{8}+\frac{b^3\,x^6}{6}}{x^{12}}$$

input
```
int((a + b*x^2)^3/x^13,x)
```

output
```
-(a^3/12 + (b^3*x^6)/6 + (3*a^2*b*x^2)/10 + (3*a*b^2*x^4)/8)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{13}}\,dx = \frac{-20b^3x^6-45a\,b^2x^4-36a^2b\,x^2-10a^3}{120x^{12}}$$

input
```
int((b*x^2+a)^3/x^13,x)
```

output
```
( - 10*a**3 - 36*a**2*b*x**2 - 45*a*b**2*x**4 - 20*b**3*x**6)/(120*x**12)
```

3.41 $\int \frac{(a+bx^2)^3}{x^{15}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

output
```
-1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{a^3}{14x^{14}} - \frac{a^2b}{4x^{12}} - \frac{3ab^2}{10x^{10}} - \frac{b^3}{8x^8}$$

input
```
Integrate[(a + b*x^2)^3/x^15,x]
```

output
```
-1/14*a^3/x^14 - (a^2*b)/(4*x^12) - (3*a*b^2)/(10*x^10) - b^3/(8*x^8)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^3}{x^{16}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{a^3}{x^{16}}+\frac{3ba^2}{x^{14}}+\frac{3b^2a}{x^{12}}+\frac{b^3}{x^{10}}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{a^3}{7x^{14}}-\frac{a^2b}{2x^{12}}-\frac{3ab^2}{5x^{10}}-\frac{b^3}{4x^8}\right)$$

input
```
Int[(a + b*x^2)^3/x^15,x]
```

output
```
(-1/7*a^3/x^14 - (a^2*b)/(2*x^12) - (3*a*b^2)/(5*x^10) - b^3/(4*x^8))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{3}}{14 x^{14}}-\frac{a^{2} b}{4 x^{12}}-\frac{3 a \,b^{2}}{10 x^{10}}-\frac{b^{3}}{8 x^{8}}$	36
norman	$\frac{-\frac{1}{14} a^{3}-\frac{1}{4} a^{2} b \,x^{2}-\frac{3}{10} a \,b^{2} x^{4}-\frac{1}{8} b^{3} x^{6}}{x^{14}}$	37
risch	$\frac{-\frac{1}{14} a^{3}-\frac{1}{4} a^{2} b \,x^{2}-\frac{3}{10} a \,b^{2} x^{4}-\frac{1}{8} b^{3} x^{6}}{x^{14}}$	37
gosper	$-\frac{35 b^{3} x^{6}+84 a \,b^{2} x^{4}+70 a^{2} b \,x^{2}+20 a^{3}}{280 x^{14}}$	38
parallelrisch	$\frac{-35 b^{3} x^{6}-84 a \,b^{2} x^{4}-70 a^{2} b \,x^{2}-20 a^{3}}{280 x^{14}}$	38
orering	$-\frac{35 b^{3} x^{6}+84 a \,b^{2} x^{4}+70 a^{2} b \,x^{2}+20 a^{3}}{280 x^{14}}$	38

input
```
int((b*x^2+a)^3/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*a^3/x^14-1/4*a^2*b/x^12-3/10*a*b^2/x^10-1/8*b^3/x^8
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

input
```
integrate((b*x^2+a)^3/x^15,x, algorithm="fricas")
```

output
```
-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = \frac{-20a^3-70a^2bx^2-84ab^2x^4-35b^3x^6}{280x^{14}}$$

input
```
integrate((b*x**2+a)**3/x**15,x)
```

output
```
(-20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

input
```
integrate((b*x^2+a)^3/x^15,x, algorithm="maxima")
```

output
```
-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{35\,b^3x^6+84\,ab^2x^4+70\,a^2bx^2+20\,a^3}{280\,x^{14}}$$

input
```
integrate((b*x^2+a)^3/x^15,x, algorithm="giac")
```

output
```
-1/280*(35*b^3*x^6 + 84*a*b^2*x^4 + 70*a^2*b*x^2 + 20*a^3)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = -\frac{\frac{a^3}{14}+\frac{a^2\,b\,x^2}{4}+\frac{3\,a\,b^2\,x^4}{10}+\frac{b^3\,x^6}{8}}{x^{14}}$$

input
```
int((a + b*x^2)^3/x^15,x)
```

output
```
-(a^3/14 + (b^3*x^6)/8 + (a^2*b*x^2)/4 + (3*a*b^2*x^4)/10)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{15}}\,dx = \frac{-35b^3x^6-84a\,b^2x^4-70a^2b\,x^2-20a^3}{280x^{14}}$$

input
```
int((b*x^2+a)^3/x^15,x)
```

output
```
( - 20*a**3 - 70*a**2*b*x**2 - 84*a*b**2*x**4 - 35*b**3*x**6)/(280*x**14)
```

3.42 $\int x^6(a+bx^2)^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^6(a+bx^2)^3 \, dx = \frac{a^3x^7}{7} + \frac{1}{3}a^2bx^9 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{13}}{13}$$

output
```
1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^6(a+bx^2)^3 \, dx = \frac{a^3x^7}{7} + \frac{1}{3}a^2bx^9 + \frac{3}{11}ab^2x^{11} + \frac{b^3x^{13}}{13}$$

input
```
Integrate[x^6*(a + b*x^2)^3,x]
```

output
```
(a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3x^6+3a^2bx^8+3ab^2x^{10}+b^3x^{12}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^7}{7}+\frac{1}{3}a^2bx^9+\frac{3}{11}ab^2x^{11}+\frac{b^3x^{13}}{13}$$

input
```
Int[x^6*(a + b*x^2)^3,x]
```

output
```
(a^3*x^7)/7 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11 + (b^3*x^13)/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
default	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
norman	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
risch	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
parallelrisch	$\frac{1}{7}a^3x^7+\frac{1}{3}a^2b\,x^9+\frac{3}{11}a\,b^2x^{11}+\frac{1}{13}b^3x^{13}$	36
orering	$\frac{x^7(231b^3x^6+819a\,b^2x^4+1001a^2b\,x^2+429a^3)}{3003}$	38

input
```
int(x^6*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^3*x^7+1/3*a^2*b*x^9+3/11*a*b^2*x^11+1/13*b^3*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^6(a+bx^2)^3\,dx=\frac{1}{13}\,b^3x^{13}+\frac{3}{11}\,ab^2x^{11}+\frac{1}{3}\,a^2bx^9+\frac{1}{7}\,a^3x^7$$

input
```
integrate(x^6*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^6 (a+bx^2)^3 \, dx = \frac{a^3x^7}{7} + \frac{a^2bx^9}{3} + \frac{3ab^2x^{11}}{11} + \frac{b^3x^{13}}{13}$$

input
```
integrate(x**6*(b*x**2+a)**3,x)
```

output
```
a**3*x**7/7 + a**2*b*x**9/3 + 3*a*b**2*x**11/11 + b**3*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^6 (a+bx^2)^3 \, dx = \frac{1}{13}\,b^3x^{13} + \frac{3}{11}\,ab^2x^{11} + \frac{1}{3}\,a^2bx^9 + \frac{1}{7}\,a^3x^7$$

input
```
integrate(x^6*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^6 (a+bx^2)^3 \, dx = \frac{1}{13}\,b^3x^{13} + \frac{3}{11}\,ab^2x^{11} + \frac{1}{3}\,a^2bx^9 + \frac{1}{7}\,a^3x^7$$

input
```
integrate(x^6*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/13*b^3*x^13 + 3/11*a*b^2*x^11 + 1/3*a^2*b*x^9 + 1/7*a^3*x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^6 (a+bx^2)^3 \, dx = \frac{a^3 x^7}{7} + \frac{a^2 b x^9}{3} + \frac{3 a b^2 x^{11}}{11} + \frac{b^3 x^{13}}{13}$$

input
```
int(x^6*(a + b*x^2)^3,x)
```

output
```
(a^3*x^7)/7 + (b^3*x^13)/13 + (a^2*b*x^9)/3 + (3*a*b^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^6 (a+bx^2)^3 \, dx = \frac{x^7(231b^3x^6 + 819a b^2x^4 + 1001a^2b x^2 + 429a^3)}{3003}$$

input
```
int(x^6*(b*x^2+a)^3,x)
```

output
```
(x**7*(429*a**3 + 1001*a**2*b*x**2 + 819*a*b**2*x**4 + 231*b**3*x**6))/300
3
```

3.43 $\int x^4(a+bx^2)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^4(a+bx^2)^3\,dx=\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

output
```
1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^3\,dx=\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

input
```
Integrate[x^4*(a + b*x^2)^3,x]
```

output
```
(a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3x^4+3a^2bx^6+3ab^2x^8+b^3x^{10}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^5}{5}+\frac{3}{7}a^2bx^7+\frac{1}{3}ab^2x^9+\frac{b^3x^{11}}{11}$$

input
```
Int[x^4*(a + b*x^2)^3,x]
```

output
```
(a^3*x^5)/5 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3 + (b^3*x^11)/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^3x^5 + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a\,b^2x^9 + \frac{1}{11}b^3x^{11}$	36
default	$\frac{1}{5}a^3x^5 + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a\,b^2x^9 + \frac{1}{11}b^3x^{11}$	36
norman	$\frac{1}{5}a^3x^5 + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a\,b^2x^9 + \frac{1}{11}b^3x^{11}$	36
risch	$\frac{1}{5}a^3x^5 + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a\,b^2x^9 + \frac{1}{11}b^3x^{11}$	36
parallelrisch	$\frac{1}{5}a^3x^5 + \frac{3}{7}a^2b\,x^7 + \frac{1}{3}a\,b^2x^9 + \frac{1}{11}b^3x^{11}$	36
orering	$\frac{x^5\left(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3\right)}{1155}$	38

input
```
int(x^4*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^3*x^5+3/7*a^2*b*x^7+1/3*a*b^2*x^9+1/11*b^3*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4\left(a+bx^2\right)^3\,dx = \frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4 \left(a+bx^2\right)^3 dx = \frac{a^3x^5}{5} + \frac{3a^2bx^7}{7} + \frac{ab^2x^9}{3} + \frac{b^3x^{11}}{11}$$

input
```
integrate(x**4*(b*x**2+a)**3,x)
```

output
```
a**3*x**5/5 + 3*a**2*b*x**7/7 + a*b**2*x**9/3 + b**3*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 \left(a+bx^2\right)^3 dx = \frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4 \left(a+bx^2\right)^3 dx = \frac{1}{11}\,b^3x^{11} + \frac{1}{3}\,ab^2x^9 + \frac{3}{7}\,a^2bx^7 + \frac{1}{5}\,a^3x^5$$

input
```
integrate(x^4*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/11*b^3*x^11 + 1/3*a*b^2*x^9 + 3/7*a^2*b*x^7 + 1/5*a^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^4(a+bx^2)^3\,dx=\frac{a^3\,x^5}{5}+\frac{3\,a^2\,b\,x^7}{7}+\frac{a\,b^2\,x^9}{3}+\frac{b^3\,x^{11}}{11}$$

input
```
int(x^4*(a + b*x^2)^3,x)
```

output
```
(a^3*x^5)/5 + (b^3*x^11)/11 + (3*a^2*b*x^7)/7 + (a*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^2)^3\,dx=\frac{x^5(105b^3x^6+385a\,b^2x^4+495a^2b\,x^2+231a^3)}{1155}$$

input
```
int(x^4*(b*x^2+a)^3,x)
```

output
```
(x**5*(231*a**3 + 495*a**2*b*x**2 + 385*a*b**2*x**4 + 105*b**3*x**6))/1155
```

3.44 $\int x^2(a+bx^2)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^2\left(a+bx^2\right)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

output
```
1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^3\,dx=\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

input
```
Integrate[x^2*(a + b*x^2)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2)^3 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int (a^3x^2+3a^2bx^4+3ab^2x^6+b^3x^8) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^3}{3}+\frac{3}{5}a^2bx^5+\frac{3}{7}ab^2x^7+\frac{b^3x^9}{9}$$

input
```
Int[x^2*(a + b*x^2)^3,x]
```

output
```
(a^3*x^3)/3 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7 + (b^3*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
default	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
norman	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
risch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
parallelrisch	$\frac{1}{3}a^3x^3+\frac{3}{5}a^2b\,x^5+\frac{3}{7}a\,b^2x^7+\frac{1}{9}b^3x^9$	36
orering	$\frac{x^3(35b^3x^6+135a\,b^2x^4+189a^2b\,x^2+105a^3)}{315}$	38

input
```
int(x^2*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^3*x^3+3/5*a^2*b*x^5+3/7*a*b^2*x^7+1/9*b^3*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2\left(a+bx^2\right)^3\,dx=\frac{1}{9}\,b^3x^9+\frac{3}{7}\,ab^2x^7+\frac{3}{5}\,a^2bx^5+\frac{1}{3}\,a^3x^3$$

input
```
integrate(x^2*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int x^2 (a + bx^2)^3 \, dx = \frac{a^3x^3}{3} + \frac{3a^2bx^5}{5} + \frac{3ab^2x^7}{7} + \frac{b^3x^9}{9}$$

input
```
integrate(x**2*(b*x**2+a)**3,x)
```

output
```
a**3*x**3/3 + 3*a**2*b*x**5/5 + 3*a*b**2*x**7/7 + b**3*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^2)^3 \, dx = \frac{1}{9} b^3x^9 + \frac{3}{7} ab^2x^7 + \frac{3}{5} a^2bx^5 + \frac{1}{3} a^3x^3$$

input
```
integrate(x^2*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^2)^3 \, dx = \frac{1}{9} b^3x^9 + \frac{3}{7} ab^2x^7 + \frac{3}{5} a^2bx^5 + \frac{1}{3} a^3x^3$$

input
```
integrate(x^2*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/9*b^3*x^9 + 3/7*a*b^2*x^7 + 3/5*a^2*b*x^5 + 1/3*a^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int x^2 (a + bx^2)^3 \, dx = \frac{a^3 x^3}{3} + \frac{3 a^2 b x^5}{5} + \frac{3 a b^2 x^7}{7} + \frac{b^3 x^9}{9}$$

input
```
int(x^2*(a + b*x^2)^3,x)
```

output
```
(a^3*x^3)/3 + (b^3*x^9)/9 + (3*a^2*b*x^5)/5 + (3*a*b^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int x^2 (a + bx^2)^3 \, dx = \frac{x^3(35b^3x^6 + 135a b^2x^4 + 189a^2b x^2 + 105a^3)}{315}$$

input
```
int(x^2*(b*x^2+a)^3,x)
```

output
```
(x**3*(105*a**3 + 189*a**2*b*x**2 + 135*a*b**2*x**4 + 35*b**3*x**6))/315
```

3.45 $\int (a+bx^2)^3 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 35

$$\int (a+bx^2)^3 \, dx = a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

output
```
a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^3 \, dx = a^3x + a^2bx^3 + \frac{3}{5}ab^2x^5 + \frac{b^3x^7}{7}$$

input
```
Integrate[(a + b*x^2)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 210}$$

$$\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^3x+a^2bx^3+\frac{3}{5}ab^2x^5+\frac{b^3x^7}{7}$$

input
```
Int[(a + b*x^2)^3,x]
```

output
```
a^3*x + a^2*b*x^3 + (3*a*b^2*x^5)/5 + (b^3*x^7)/7
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
gosper	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
default	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
norman	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
risch	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
parallelrisch	$a^3x + a^2b\,x^3 + \frac{3}{5}a\,b^2x^5 + \frac{1}{7}b^3x^7$	32
orering	$\frac{x(5b^3x^6+21a\,b^2x^4+35a^2b\,x^2+35a^3)}{35}$	36

input
```
int((b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+a^2*b*x^3+3/5*a*b^2*x^5+1/7*b^3*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3\,dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(a+bx^2\right)^3 \, dx = a^3x + a^2bx^3 + \frac{3ab^2x^5}{5} + \frac{b^3x^7}{7}$$

input
```
integrate((b*x**2+a)**3,x)
```

output
```
a**3*x + a**2*b*x**3 + 3*a*b**2*x**5/5 + b**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3 \, dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a+bx^2\right)^3 \, dx = \frac{1}{7}\,b^3x^7 + \frac{3}{5}\,ab^2x^5 + a^2bx^3 + a^3x$$

input
```
integrate((b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/7*b^3*x^7 + 3/5*a*b^2*x^5 + a^2*b*x^3 + a^3*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \left(a + bx^2\right)^3 \, dx = a^3 \, x + a^2 \, b \, x^3 + \frac{3 \, a \, b^2 \, x^5}{5} + \frac{b^3 \, x^7}{7}$$

input
```
int((a + b*x^2)^3,x)
```

output
```
a^3*x + (b^3*x^7)/7 + a^2*b*x^3 + (3*a*b^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \left(a + bx^2\right)^3 \, dx = \frac{x(5b^3x^6 + 21a \, b^2x^4 + 35a^2b \, x^2 + 35a^3)}{35}$$

input
```
int((b*x^2+a)^3,x)
```

output
```
(x*(35*a**3 + 35*a**2*b*x**2 + 21*a*b**2*x**4 + 5*b**3*x**6))/35
```

3.46 $\int \frac{(a+bx^2)^3}{x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(a+bx^2)^3}{x^2} \, dx = -\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

output
```
-a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^2} \, dx = -\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

input
```
Integrate[(a + b*x^2)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^2} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^3}{x^2}+3a^2b+3ab^2x^2+b^3x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{x}+3a^2bx+ab^2x^3+\frac{b^3x^5}{5}$$

input
```
Int[(a + b*x^2)^3/x^2,x]
```

output
```
-(a^3/x) + 3*a^2*b*x + a*b^2*x^3 + (b^3*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{a^{3}}{x}+3a^{2}bx+a\,b^{2}x^{3}+\frac{b^{3}x^{5}}{5}$	33
risch	$-\frac{a^{3}}{x}+3a^{2}bx+a\,b^{2}x^{3}+\frac{b^{3}x^{5}}{5}$	33
norman	$\frac{\frac{1}{5}b^{3}x^{6}+a\,b^{2}x^{4}+3a^{2}b\,x^{2}-a^{3}}{x}$	36
parallelrisch	$\frac{b^{3}x^{6}+5a\,b^{2}x^{4}+15a^{2}b\,x^{2}-5a^{3}}{5x}$	37
gosper	$-\frac{-b^{3}x^{6}-5a\,b^{2}x^{4}-15a^{2}b\,x^{2}+5a^{3}}{5x}$	38
orering	$-\frac{-b^{3}x^{6}-5a\,b^{2}x^{4}-15a^{2}b\,x^{2}+5a^{3}}{5x}$	38

input
```
int((b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^3/x+3*a^2*b*x+a*b^2*x^3+1/5*b^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3}{x^2}\,dx=\frac{b^3x^6+5\,ab^2x^4+15\,a^2bx^2-5\,a^3}{5\,x}$$

input
```
integrate((b*x^2+a)^3/x^2,x, algorithm="fricas")
```

output
```
1/5*(b^3*x^6 + 5*a*b^2*x^4 + 15*a^2*b*x^2 - 5*a^3)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^3}{x^2} dx = -\frac{a^3}{x} + 3a^2bx + ab^2x^3 + \frac{b^3x^5}{5}$$

input
```
integrate((b*x**2+a)**3/x**2,x)
```

output
```
-a**3/x + 3*a**2*b*x + a*b**2*x**3 + b**3*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3}{x^2} dx = \frac{1}{5}b^3x^5 + ab^2x^3 + 3a^2bx - \frac{a^3}{x}$$

input
```
integrate((b*x^2+a)^3/x^2,x, algorithm="maxima")
```

output
```
1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3}{x^2} dx = \frac{1}{5}b^3x^5 + ab^2x^3 + 3a^2bx - \frac{a^3}{x}$$

input
```
integrate((b*x^2+a)^3/x^2,x, algorithm="giac")
```

output
```
1/5*b^3*x^5 + a*b^2*x^3 + 3*a^2*b*x - a^3/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3}{x^2}\,dx = \frac{b^3\,x^5}{5} - \frac{a^3}{x} + a\,b^2\,x^3 + 3\,a^2\,b\,x$$

input
```
int((a + b*x^2)^3/x^2,x)
```

output
```
(b^3*x^5)/5 - a^3/x + a*b^2*x^3 + 3*a^2*b*x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3}{x^2}\,dx = \frac{b^3x^6 + 5a\,b^2x^4 + 15a^2b\,x^2 - 5a^3}{5x}$$

input
```
int((b*x^2+a)^3/x^2,x)
```

output
```
( - 5*a**3 + 15*a**2*b*x**2 + 5*a*b**2*x**4 + b**3*x**6)/(5*x)
```

3.47 $\int \frac{(a+bx^2)^3}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{(a+bx^2)^3}{x^4} dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

output
```
-1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^4} dx = -\frac{a^3}{3x^3} - \frac{3a^2b}{x} + 3ab^2x + \frac{b^3x^3}{3}$$

input
```
Integrate[(a + b*x^2)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^4} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^4}+\frac{3a^2b}{x^2}+3ab^2+b^3x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3ab^2x+\frac{b^3x^3}{3}$$

input
```
Int[(a + b*x^2)^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (3*a^2*b)/x + 3*a*b^2*x + (b^3*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^3}{3x^3}-\frac{3a^2b}{x}+3a\,b^2x+\frac{b^3x^3}{3}$	34
gosper	$-\frac{-b^3x^6-9a\,b^2x^4+9a^2b\,x^2+a^3}{3x^3}$	36
risch	$\frac{b^3x^3}{3}+3a\,b^2x+\frac{-3a^2b\,x^2-\frac{1}{3}a^3}{x^3}$	36
orering	$-\frac{-b^3x^6-9a\,b^2x^4+9a^2b\,x^2+a^3}{3x^3}$	36
norman	$\frac{\frac{1}{3}b^3x^6+3a\,b^2x^4-3a^2b\,x^2-\frac{1}{3}a^3}{x^3}$	37
parallelrisch	$\frac{b^3x^6+9a\,b^2x^4-9a^2b\,x^2-a^3}{3x^3}$	37

input
```
int((b*x^2+a)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^3/x^3-3*a^2*b/x+3*a*b^2*x+1/3*b^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = \frac{b^3x^6+9\,ab^2x^4-9\,a^2bx^2-a^3}{3\,x^3}$$

input
```
integrate((b*x^2+a)^3/x^4,x, algorithm="fricas")
```

output
```
1/3*(b^3*x^6 + 9*a*b^2*x^4 - 9*a^2*b*x^2 - a^3)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = 3ab^2x + \frac{b^3x^3}{3} + \frac{-a^3-9a^2bx^2}{3x^3}$$

input
```
integrate((b*x**2+a)**3/x**4,x)
```

output
```
3*a*b**2*x + b**3*x**3/3 + (-a**3 - 9*a**2*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = \frac{1}{3}\,b^3x^3 + 3\,ab^2x - \frac{9\,a^2bx^2+a^3}{3\,x^3}$$

input
```
integrate((b*x^2+a)^3/x^4,x, algorithm="maxima")
```

output
```
1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = \frac{1}{3}\,b^3x^3 + 3\,ab^2x - \frac{9\,a^2bx^2+a^3}{3\,x^3}$$

input
```
integrate((b*x^2+a)^3/x^4,x, algorithm="giac")
```

output
```
1/3*b^3*x^3 + 3*a*b^2*x - 1/3*(9*a^2*b*x^2 + a^3)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = \frac{b^3\,x^3}{3} - \frac{\frac{a^3}{3} + 3\,b\,a^2\,x^2}{x^3} + 3\,a\,b^2\,x$$

input
```
int((a + b*x^2)^3/x^4,x)
```

output
```
(b^3*x^3)/3 - (a^3/3 + 3*a^2*b*x^2)/x^3 + 3*a*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^4}\,dx = \frac{b^3x^6 + 9a\,b^2x^4 - 9a^2b\,x^2 - a^3}{3x^3}$$

input
```
int((b*x^2+a)^3/x^4,x)
```

output
```
( - a**3 - 9*a**2*b*x**2 + 9*a*b**2*x**4 + b**3*x**6)/(3*x**3)
```

3.48 $\int \frac{(a+bx^2)^3}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{(a+bx^2)^3}{x^6} dx = -\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + b^3x$$

output
```
-1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^6} dx = -\frac{a^3}{5x^5} - \frac{a^2b}{x^3} - \frac{3ab^2}{x} + b^3x$$

input
```
Integrate[(a + b*x^2)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^6} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^6}+\frac{3a^2b}{x^4}+\frac{3ab^2}{x^2}+b^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3ab^2}{x}+b^3x$$

input
```
Int[(a + b*x^2)^3/x^6,x]
```

output
```
-1/5*a^3/x^5 - (a^2*b)/x^3 - (3*a*b^2)/x + b^3*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{a^3}{5x^5}-\frac{a^2b}{x^3}-\frac{3a\,b^2}{x}+b^3x$	33
risch	$b^3x+\frac{-3a\,b^2x^4-a^2b\,x^2-\frac{1}{5}a^3}{x^5}$	35
gosper	$-\frac{-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3}{5x^5}$	36
norman	$\frac{b^3x^6-3a\,b^2x^4-a^2b\,x^2-\frac{1}{5}a^3}{x^5}$	36
orering	$-\frac{-5b^3x^6+15a\,b^2x^4+5a^2b\,x^2+a^3}{5x^5}$	36
parallelrisch	$\frac{5b^3x^6-15a\,b^2x^4-5a^2b\,x^2-a^3}{5x^5}$	38

input
```
int((b*x^2+a)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^3/x^5-a^2*b/x^3-3*a*b^2/x+b^3*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3}{x^6}\,dx=\frac{5\,b^3x^6-15\,ab^2x^4-5\,a^2bx^2-a^3}{5\,x^5}$$

input
```
integrate((b*x^2+a)^3/x^6,x, algorithm="fricas")
```

output
```
1/5*(5*b^3*x^6 - 15*a*b^2*x^4 - 5*a^2*b*x^2 - a^3)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^6}\,dx = b^3x + \frac{-a^3 - 5a^2bx^2 - 15ab^2x^4}{5x^5}$$

input
```
integrate((b*x**2+a)**3/x**6,x)
```

output
```
b**3*x + (-a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4)/(5*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^6}\,dx = b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

input
```
integrate((b*x^2+a)^3/x^6,x, algorithm="maxima")
```

output
```
b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^3}{x^6}\,dx = b^3x - \frac{15\,ab^2x^4 + 5\,a^2bx^2 + a^3}{5\,x^5}$$

input
```
integrate((b*x^2+a)^3/x^6,x, algorithm="giac")
```

output
```
b^3*x - 1/5*(15*a*b^2*x^4 + 5*a^2*b*x^2 + a^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^6}\,dx = b^3\,x - \frac{\frac{a^3}{5} + a^2\,b\,x^2 + 3\,a\,b^2\,x^4}{x^5}$$

input
```
int((a + b*x^2)^3/x^6,x)
```

output
```
b^3*x - (a^3/5 + a^2*b*x^2 + 3*a*b^2*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3}{x^6}\,dx = \frac{5b^3x^6 - 15a\,b^2x^4 - 5a^2b\,x^2 - a^3}{5x^5}$$

input
```
int((b*x^2+a)^3/x^6,x)
```

output
```
( - a**3 - 5*a**2*b*x**2 - 15*a*b**2*x**4 + 5*b**3*x**6)/(5*x**5)
```

3.49 $\int \frac{(a+bx^2)^3}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{(a+bx^2)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

output
```
-1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^8} dx = -\frac{a^3}{7x^7} - \frac{3a^2b}{5x^5} - \frac{ab^2}{x^3} - \frac{b^3}{x}$$

input
```
Integrate[(a + b*x^2)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^8} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^8}+\frac{3a^2b}{x^6}+\frac{3ab^2}{x^4}+\frac{b^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{7x^7}-\frac{3a^2b}{5x^5}-\frac{ab^2}{x^3}-\frac{b^3}{x}$$

input
```
Int[(a + b*x^2)^3/x^8,x]
```

output
```
-1/7*a^3/x^7 - (3*a^2*b)/(5*x^5) - (a*b^2)/x^3 - b^3/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^{3}}{7x^{7}}-\frac{3a^{2}b}{5x^{5}}-\frac{a\,b^{2}}{x^{3}}-\frac{b^{3}}{x}$	36
norman	$\frac{-b^{3}x^{6}-a\,b^{2}x^{4}-\frac{3}{5}a^{2}b\,x^{2}-\frac{1}{7}a^{3}}{x^{7}}$	37
risch	$\frac{-b^{3}x^{6}-a\,b^{2}x^{4}-\frac{3}{5}a^{2}b\,x^{2}-\frac{1}{7}a^{3}}{x^{7}}$	37
gosper	$-\frac{35b^{3}x^{6}+35a\,b^{2}x^{4}+21a^{2}b\,x^{2}+5a^{3}}{35x^{7}}$	38
parallelrisch	$\frac{-35b^{3}x^{6}-35a\,b^{2}x^{4}-21a^{2}b\,x^{2}-5a^{3}}{35x^{7}}$	38
orering	$-\frac{35b^{3}x^{6}+35a\,b^{2}x^{4}+21a^{2}b\,x^{2}+5a^{3}}{35x^{7}}$	38

input
```
int((b*x^2+a)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^3/x^7-3/5*a^2*b/x^5-a*b^2/x^3-b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^8}\,dx=-\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

input
```
integrate((b*x^2+a)^3/x^8,x, algorithm="fricas")
```

output
```
-1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^8}\,dx = \frac{-5a^3-21a^2bx^2-35ab^2x^4-35b^3x^6}{35x^7}$$

input
```
integrate((b*x**2+a)**3/x**8,x)
```

output
```
(-5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^8}\,dx = -\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

input
```
integrate((b*x^2+a)^3/x^8,x, algorithm="maxima")
```

output
```
-1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^8}\,dx = -\frac{35\,b^3x^6+35\,ab^2x^4+21\,a^2bx^2+5\,a^3}{35\,x^7}$$

input
```
integrate((b*x^2+a)^3/x^8,x, algorithm="giac")
```

output
```
-1/35*(35*b^3*x^6 + 35*a*b^2*x^4 + 21*a^2*b*x^2 + 5*a^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^3}{x^8}\,dx = -\frac{\frac{a^3}{7}+\frac{3\,a^2\,b\,x^2}{5}+a\,b^2\,x^4+b^3\,x^6}{x^7}$$

input
```
int((a + b*x^2)^3/x^8,x)
```

output
```
-(a^3/7 + b^3*x^6 + (3*a^2*b*x^2)/5 + a*b^2*x^4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^3}{x^8}\,dx = \frac{-35b^3x^6-35a\,b^2x^4-21a^2b\,x^2-5a^3}{35x^7}$$

input
```
int((b*x^2+a)^3/x^8,x)
```

output
```
( - 5*a**3 - 21*a**2*b*x**2 - 35*a*b**2*x**4 - 35*b**3*x**6)/(35*x**7)
```

3.50 $\int \frac{(a+bx^2)^3}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

output
```
-1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{a^3}{9x^9} - \frac{3a^2b}{7x^7} - \frac{3ab^2}{5x^5} - \frac{b^3}{3x^3}$$

input
```
Integrate[(a + b*x^2)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{10}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^3}{x^{10}}+\frac{3a^2b}{x^8}+\frac{3ab^2}{x^6}+\frac{b^3}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3}{9x^9}-\frac{3a^2b}{7x^7}-\frac{3ab^2}{5x^5}-\frac{b^3}{3x^3}$$

input
```
Int[(a + b*x^2)^3/x^10,x]
```

output
```
-1/9*a^3/x^9 - (3*a^2*b)/(7*x^7) - (3*a*b^2)/(5*x^5) - b^3/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{3}}{9 x^{9}}-\frac{3 a^{2} b}{7 x^{7}}-\frac{3 a \,b^{2}}{5 x^{5}}-\frac{b^{3}}{3 x^{3}}$	36
norman	$\frac{-\frac{1}{3} b^{3} x^{6}-\frac{3}{5} a \,b^{2} x^{4}-\frac{3}{7} a^{2} b \,x^{2}-\frac{1}{9} a^{3}}{x^{9}}$	37
risch	$\frac{-\frac{1}{3} b^{3} x^{6}-\frac{3}{5} a \,b^{2} x^{4}-\frac{3}{7} a^{2} b \,x^{2}-\frac{1}{9} a^{3}}{x^{9}}$	37
gosper	$-\frac{105 b^{3} x^{6}+189 a \,b^{2} x^{4}+135 a^{2} b \,x^{2}+35 a^{3}}{315 x^{9}}$	38
parallelrisch	$\frac{-105 b^{3} x^{6}-189 a \,b^{2} x^{4}-135 a^{2} b \,x^{2}-35 a^{3}}{315 x^{9}}$	38
orering	$-\frac{105 b^{3} x^{6}+189 a \,b^{2} x^{4}+135 a^{2} b \,x^{2}+35 a^{3}}{315 x^{9}}$	38

input
```
int((b*x^2+a)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^3/x^9-3/7*a^2*b/x^7-3/5*a*b^2/x^5-1/3*b^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{105\,b^3x^6+189\,ab^2x^4+135\,a^2bx^2+35\,a^3}{315\,x^9}$$

input
```
integrate((b*x^2+a)^3/x^10,x, algorithm="fricas")
```

output
```
-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = \frac{-35a^3 - 135a^2bx^2 - 189ab^2x^4 - 105b^3x^6}{315x^9}$$

input
```
integrate((b*x**2+a)**3/x**10,x)
```

output
```
(-35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

input
```
integrate((b*x^2+a)^3/x^10,x, algorithm="maxima")
```

output
```
-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{105\,b^3x^6 + 189\,ab^2x^4 + 135\,a^2bx^2 + 35\,a^3}{315\,x^9}$$

input
```
integrate((b*x^2+a)^3/x^10,x, algorithm="giac")
```

output
```
-1/315*(105*b^3*x^6 + 189*a*b^2*x^4 + 135*a^2*b*x^2 + 35*a^3)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = -\frac{\frac{a^3}{9}+\frac{3\,a^2\,b\,x^2}{7}+\frac{3\,a\,b^2\,x^4}{5}+\frac{b^3\,x^6}{3}}{x^9}$$

input
```
int((a + b*x^2)^3/x^10,x)
```

output
```
-(a^3/9 + (b^3*x^6)/3 + (3*a^2*b*x^2)/7 + (3*a*b^2*x^4)/5)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{10}}\,dx = \frac{-105b^3x^6-189a\,b^2x^4-135a^2b\,x^2-35a^3}{315x^9}$$

input
```
int((b*x^2+a)^3/x^10,x)
```

output
```
( - 35*a**3 - 135*a**2*b*x**2 - 189*a*b**2*x**4 - 105*b**3*x**6)/(315*x**9
)
```

3.51 $\int \frac{(a+bx^2)^3}{x^{12}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

output
```
-1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{a^3}{11x^{11}} - \frac{a^2b}{3x^9} - \frac{3ab^2}{7x^7} - \frac{b^3}{5x^5}$$

input
```
Integrate[(a + b*x^2)^3/x^12,x]
```

output
```
-1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^3}{x^{12}}+\frac{3a^2b}{x^{10}}+\frac{3ab^2}{x^8}+\frac{b^3}{x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3}{11x^{11}}-\frac{a^2b}{3x^9}-\frac{3ab^2}{7x^7}-\frac{b^3}{5x^5}$$

input
```
Int[(a + b*x^2)^3/x^12,x]
```

output
```
-1/11*a^3/x^11 - (a^2*b)/(3*x^9) - (3*a*b^2)/(7*x^7) - b^3/(5*x^5)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{3}}{11x^{11}}-\frac{a^{2}b}{3x^{9}}-\frac{3a\,b^{2}}{7x^{7}}-\frac{b^{3}}{5x^{5}}$	36
norman	$\frac{-\frac{1}{5}b^{3}x^{6}-\frac{3}{7}a\,b^{2}x^{4}-\frac{1}{3}a^{2}b\,x^{2}-\frac{1}{11}a^{3}}{x^{11}}$	37
risch	$\frac{-\frac{1}{5}b^{3}x^{6}-\frac{3}{7}a\,b^{2}x^{4}-\frac{1}{3}a^{2}b\,x^{2}-\frac{1}{11}a^{3}}{x^{11}}$	37
gosper	$-\frac{231b^{3}x^{6}+495a\,b^{2}x^{4}+385a^{2}b\,x^{2}+105a^{3}}{1155x^{11}}$	38
parallelrisch	$\frac{-231b^{3}x^{6}-495a\,b^{2}x^{4}-385a^{2}b\,x^{2}-105a^{3}}{1155x^{11}}$	38
orering	$-\frac{231b^{3}x^{6}+495a\,b^{2}x^{4}+385a^{2}b\,x^{2}+105a^{3}}{1155x^{11}}$	38

input
```
int((b*x^2+a)^3/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^3/x^11-1/3*a^2*b/x^9-3/7*a*b^2/x^7-1/5*b^3/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

input
```
integrate((b*x^2+a)^3/x^12,x, algorithm="fricas")
```

output
```
-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = \frac{-105a^3-385a^2bx^2-495ab^2x^4-231b^3x^6}{1155x^{11}}$$

input
```
integrate((b*x**2+a)**3/x**12,x)
```

output
```
(-105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x**1
1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

input
```
integrate((b*x^2+a)^3/x^12,x, algorithm="maxima")
```

output
```
-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{231\,b^3x^6+495\,ab^2x^4+385\,a^2bx^2+105\,a^3}{1155\,x^{11}}$$

input
```
integrate((b*x^2+a)^3/x^12,x, algorithm="giac")
```

output
```
-1/1155*(231*b^3*x^6 + 495*a*b^2*x^4 + 385*a^2*b*x^2 + 105*a^3)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = -\frac{\frac{a^3}{11}+\frac{a^2\,b\,x^2}{3}+\frac{3\,a\,b^2\,x^4}{7}+\frac{b^3\,x^6}{5}}{x^{11}}$$

input
```
int((a + b*x^2)^3/x^12,x)
```

output
```
-(a^3/11 + (b^3*x^6)/5 + (a^2*b*x^2)/3 + (3*a*b^2*x^4)/7)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^3}{x^{12}}\,dx = \frac{-231b^3x^6-495a\,b^2x^4-385a^2b\,x^2-105a^3}{1155x^{11}}$$

input
```
int((b*x^2+a)^3/x^12,x)
```

output
```
( - 105*a**3 - 385*a**2*b*x**2 - 495*a*b**2*x**4 - 231*b**3*x**6)/(1155*x*
*11)
```

3.52 $\int x^{13}(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^{13}(a+bx^2)^5\,dx=\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

output
```
1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4
*x^22+1/24*b^5*x^24
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^{13}(a+bx^2)^5\,dx=\frac{a^5x^{14}}{14}+\frac{5}{16}a^4bx^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}ab^4x^{22}+\frac{b^5x^{24}}{24}$$

input
```
Integrate[x^13*(a + b*x^2)^5,x]
```

output
```
(a^5*x^14)/14 + (5*a^4*b*x^16)/16 + (5*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2
+ (5*a*b^4*x^22)/22 + (b^5*x^24)/24
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a+bx^2\right)^5 \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{12}\left(bx^2+a\right)^5 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^5x^{22}+5ab^4x^{20}+10a^2b^3x^{18}+10a^3b^2x^{16}+5a^4bx^{14}+a^5x^{12}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^5x^{14}}{7}+\frac{5}{8}a^4bx^{16}+\frac{10}{9}a^3b^2x^{18}+a^2b^3x^{20}+\frac{5}{11}ab^4x^{22}+\frac{b^5x^{24}}{12}\right)$$

input
```
Int[x^13*(a + b*x^2)^5,x]
```

output
```
((a^5*x^14)/7 + (5*a^4*b*x^16)/8 + (10*a^3*b^2*x^18)/9 + a^2*b^3*x^20 + (5
*a*b^4*x^22)/11 + (b^5*x^24)/12)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
default	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
norman	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
risch	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
parallelrisch	$\frac{1}{14}a^5x^{14}+\frac{5}{16}a^4b\,x^{16}+\frac{5}{9}a^3b^2x^{18}+\frac{1}{2}a^2b^3x^{20}+\frac{5}{22}a\,b^4x^{22}+\frac{1}{24}b^5x^{24}$	58
orering	$\frac{x^{14}\left(462b^5x^{10}+2520a\,b^4x^8+5544a^2b^3x^6+6160a^3b^2x^4+3465a^4b\,x^2+792a^5\right)}{11088}$	60

input
```
int(x^13*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/14*a^5*x^14+5/16*a^4*b*x^16+5/9*a^3*b^2*x^18+1/2*a^2*b^3*x^20+5/22*a*b^4
*x^22+1/24*b^5*x^24
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{13}\left(a+bx^2\right)^5\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/
16*a^4*b*x^16 + 1/14*a^5*x^14
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^{13}\left(a+bx^2\right)^5\,dx=\frac{a^5x^{14}}{14}+\frac{5a^4bx^{16}}{16}+\frac{5a^3b^2x^{18}}{9}+\frac{a^2b^3x^{20}}{2}+\frac{5ab^4x^{22}}{22}+\frac{b^5x^{24}}{24}$$

input
```
integrate(x**13*(b*x**2+a)**5,x)
```

output
```
a**5*x**14/14 + 5*a**4*b*x**16/16 + 5*a**3*b**2*x**18/9 + a**2*b**3*x**20/
2 + 5*a*b**4*x**22/22 + b**5*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{13}\left(a+bx^2\right)^5\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/
16*a^4*b*x^16 + 1/14*a^5*x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{13}(a+bx^2)^5\,dx=\frac{1}{24}\,b^5x^{24}+\frac{5}{22}\,ab^4x^{22}+\frac{1}{2}\,a^2b^3x^{20}+\frac{5}{9}\,a^3b^2x^{18}+\frac{5}{16}\,a^4bx^{16}+\frac{1}{14}\,a^5x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/24*b^5*x^24 + 5/22*a*b^4*x^22 + 1/2*a^2*b^3*x^20 + 5/9*a^3*b^2*x^18 + 5/
16*a^4*b*x^16 + 1/14*a^5*x^14
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{13}(a+bx^2)^5\,dx=\frac{a^5\,x^{14}}{14}+\frac{5\,a^4\,b\,x^{16}}{16}+\frac{5\,a^3\,b^2\,x^{18}}{9}+\frac{a^2\,b^3\,x^{20}}{2}+\frac{5\,a\,b^4\,x^{22}}{22}+\frac{b^5\,x^{24}}{24}$$

input
```
int(x^13*(a + b*x^2)^5,x)
```

output
```
(a^5*x^14)/14 + (b^5*x^24)/24 + (5*a^4*b*x^16)/16 + (5*a*b^4*x^22)/22 + (5
*a^3*b^2*x^18)/9 + (a^2*b^3*x^20)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^{13}(a+bx^2)^5\,dx=\frac{x^{14}(462b^5x^{10}+2520a\,b^4x^8+5544a^2b^3x^6+6160a^3b^2x^4+3465a^4b\,x^2+792a^5)}{11088}$$

input
```
int(x^13*(b*x^2+a)^5,x)
```

output
```
(x**14*(792*a**5 + 3465*a**4*b*x**2 + 6160*a**3*b**2*x**4 + 5544*a**2*b**3
*x**6 + 2520*a*b**4*x**8 + 462*b**5*x**10))/11088
```

3.53 $\int x^{11}(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^{11}\left(a+bx^2\right)^5\,dx=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

output
```
1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*
x^20+1/22*b^5*x^22
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^{11}\left(a+bx^2\right)^5\,dx=\frac{a^5x^{12}}{12}+\frac{5}{14}a^4bx^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}ab^4x^{20}+\frac{b^5x^{22}}{22}$$

input
```
Integrate[x^11*(a + b*x^2)^5,x]
```

output
```
(a^5*x^12)/12 + (5*a^4*b*x^14)/14 + (5*a^3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/
9 + (a*b^4*x^20)/4 + (b^5*x^22)/22
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^2\right)^5\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{10}\left(bx^2+a\right)^5\,dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^5x^{20}+5ab^4x^{18}+10a^2b^3x^{16}+10a^3b^2x^{14}+5a^4bx^{12}+a^5x^{10}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^5x^{12}}{6}+\frac{5}{7}a^4bx^{14}+\frac{5}{4}a^3b^2x^{16}+\frac{10}{9}a^2b^3x^{18}+\frac{1}{2}ab^4x^{20}+\frac{b^5x^{22}}{11}\right)$$

input
```
Int[x^11*(a + b*x^2)^5,x]
```

output
```
((a^5*x^12)/6 + (5*a^4*b*x^14)/7 + (5*a^3*b^2*x^16)/4 + (10*a^2*b^3*x^18)/
9 + (a*b^4*x^20)/2 + (b^5*x^22)/11)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
default	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
norman	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
risch	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
parallelrisch	$\frac{1}{12}a^5x^{12}+\frac{5}{14}a^4b\,x^{14}+\frac{5}{8}a^3b^2x^{16}+\frac{5}{9}a^2b^3x^{18}+\frac{1}{4}a\,b^4x^{20}+\frac{1}{22}b^5x^{22}$	58
orering	$\frac{x^{12}\left(252b^5x^{10}+1386a\,b^4x^8+3080a^2b^3x^6+3465a^3b^2x^4+1980a^4b\,x^2+462a^5\right)}{5544}$	60

input
```
int(x^11*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^5*x^12+5/14*a^4*b*x^14+5/8*a^3*b^2*x^16+5/9*a^2*b^3*x^18+1/4*a*b^4*
x^20+1/22*b^5*x^22
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a+bx^2\right)^5\,dx=\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/1
4*a^4*b*x^14 + 1/12*a^5*x^12
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^{11}\left(a+bx^2\right)^5\,dx=\frac{a^5x^{12}}{12}+\frac{5a^4bx^{14}}{14}+\frac{5a^3b^2x^{16}}{8}+\frac{5a^2b^3x^{18}}{9}+\frac{ab^4x^{20}}{4}+\frac{b^5x^{22}}{22}$$

input
```
integrate(x**11*(b*x**2+a)**5,x)
```

output
```
a**5*x**12/12 + 5*a**4*b*x**14/14 + 5*a**3*b**2*x**16/8 + 5*a**2*b**3*x**1
8/9 + a*b**4*x**20/4 + b**5*x**22/22
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{11}\left(a+bx^2\right)^5\,dx=\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/1
4*a^4*b*x^14 + 1/12*a^5*x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{11}(a+bx^2)^5\,dx=\frac{1}{22}\,b^5x^{22}+\frac{1}{4}\,ab^4x^{20}+\frac{5}{9}\,a^2b^3x^{18}+\frac{5}{8}\,a^3b^2x^{16}+\frac{5}{14}\,a^4bx^{14}+\frac{1}{12}\,a^5x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/22*b^5*x^22 + 1/4*a*b^4*x^20 + 5/9*a^2*b^3*x^18 + 5/8*a^3*b^2*x^16 + 5/1
4*a^4*b*x^14 + 1/12*a^5*x^12
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^{11}(a+bx^2)^5\,dx=\frac{a^5\,x^{12}}{12}+\frac{5\,a^4\,b\,x^{14}}{14}+\frac{5\,a^3\,b^2\,x^{16}}{8}+\frac{5\,a^2\,b^3\,x^{18}}{9}+\frac{a\,b^4\,x^{20}}{4}+\frac{b^5\,x^{22}}{22}$$

input
```
int(x^11*(a + b*x^2)^5,x)
```

output
```
(a^5*x^12)/12 + (b^5*x^22)/22 + (5*a^4*b*x^14)/14 + (a*b^4*x^20)/4 + (5*a^
3*b^2*x^16)/8 + (5*a^2*b^3*x^18)/9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^{11}(a+bx^2)^5\,dx=\frac{x^{12}(252b^5x^{10}+1386a\,b^4x^8+3080a^2b^3x^6+3465a^3b^2x^4+1980a^4b\,x^2+462a^5)}{5544}$$

input
```
int(x^11*(b*x^2+a)^5,x)
```

output
```
(x**12*(462*a**5 + 1980*a**4*b*x**2 + 3465*a**3*b**2*x**4 + 3080*a**2*b**3
*x**6 + 1386*a*b**4*x**8 + 252*b**5*x**10))/5544
```

3.54 $\int x^9(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^9\left(a+bx^2\right)^5\,dx=\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

output
```
1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4
*x^18+1/20*b^5*x^20
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^9\left(a+bx^2\right)^5\,dx=\frac{a^5x^{10}}{10}+\frac{5}{12}a^4bx^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}ab^4x^{18}+\frac{b^5x^{20}}{20}$$

input
```
Integrate[x^9*(a + b*x^2)^5,x]
```

output
```
(a^5*x^10)/10 + (5*a^4*b*x^12)/12 + (5*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/
8 + (5*a*b^4*x^18)/18 + (b^5*x^20)/20
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^2\right)^5 \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int x^8 \left(bx^2+a\right)^5 dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2} \int \left(b^5x^{18}+5ab^4x^{16}+10a^2b^3x^{14}+10a^3b^2x^{12}+5a^4bx^{10}+a^5x^8\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a^5x^{10}}{5}+\frac{5}{6}a^4bx^{12}+\frac{10}{7}a^3b^2x^{14}+\frac{5}{4}a^2b^3x^{16}+\frac{5}{9}ab^4x^{18}+\frac{b^5x^{20}}{10}\right)$$

input
```
Int[x^9*(a + b*x^2)^5,x]
```

output
```
((a^5*x^10)/5 + (5*a^4*b*x^12)/6 + (10*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/
4 + (5*a*b^4*x^18)/9 + (b^5*x^20)/10)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
default	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
norman	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
risch	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
parallelrisch	$\frac{1}{10}a^5x^{10}+\frac{5}{12}a^4b\,x^{12}+\frac{5}{7}a^3b^2x^{14}+\frac{5}{8}a^2b^3x^{16}+\frac{5}{18}a\,b^4x^{18}+\frac{1}{20}b^5x^{20}$	58
orering	$\frac{x^{10}\left(126b^5x^{10}+700a\,b^4x^8+1575a^2b^3x^6+1800a^3b^2x^4+1050a^4b\,x^2+252a^5\right)}{2520}$	60

input
```
int(x^9*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/10*a^5*x^10+5/12*a^4*b*x^12+5/7*a^3*b^2*x^14+5/8*a^2*b^3*x^16+5/18*a*b^4
*x^18+1/20*b^5*x^20
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^9\left(a+bx^2\right)^5\,dx=\frac{1}{20}\,b^5x^{20}+\frac{5}{18}\,ab^4x^{18}+\frac{5}{8}\,a^2b^3x^{16}+\frac{5}{7}\,a^3b^2x^{14}+\frac{5}{12}\,a^4bx^{12}+\frac{1}{10}\,a^5x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/
12*a^4*b*x^12 + 1/10*a^5*x^10
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^9 \left(a+bx^2\right)^5 \,dx = \frac{a^5x^{10}}{10} + \frac{5a^4bx^{12}}{12} + \frac{5a^3b^2x^{14}}{7} + \frac{5a^2b^3x^{16}}{8} + \frac{5ab^4x^{18}}{18} + \frac{b^5x^{20}}{20}$$

input
```
integrate(x**9*(b*x**2+a)**5,x)
```

output
```
a**5*x**10/10 + 5*a**4*b*x**12/12 + 5*a**3*b**2*x**14/7 + 5*a**2*b**3*x**1
6/8 + 5*a*b**4*x**18/18 + b**5*x**20/20
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^9 \left(a+bx^2\right)^5 \,dx = \frac{1}{20}\,b^5x^{20} + \frac{5}{18}\,ab^4x^{18} + \frac{5}{8}\,a^2b^3x^{16} + \frac{5}{7}\,a^3b^2x^{14} + \frac{5}{12}\,a^4bx^{12} + \frac{1}{10}\,a^5x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/
12*a^4*b*x^12 + 1/10*a^5*x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^9 (a+bx^2)^5 \, dx = \frac{1}{20} b^5 x^{20} + \frac{5}{18} a b^4 x^{18} + \frac{5}{8} a^2 b^3 x^{16} + \frac{5}{7} a^3 b^2 x^{14} + \frac{5}{12} a^4 b x^{12} + \frac{1}{10} a^5 x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/20*b^5*x^20 + 5/18*a*b^4*x^18 + 5/8*a^2*b^3*x^16 + 5/7*a^3*b^2*x^14 + 5/
12*a^4*b*x^12 + 1/10*a^5*x^10
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^9 (a+bx^2)^5 \, dx = \frac{a^5 x^{10}}{10} + \frac{5 a^4 b x^{12}}{12} + \frac{5 a^3 b^2 x^{14}}{7} + \frac{5 a^2 b^3 x^{16}}{8} + \frac{5 a b^4 x^{18}}{18} + \frac{b^5 x^{20}}{20}$$

input
```
int(x^9*(a + b*x^2)^5,x)
```

output
```
(a^5*x^10)/10 + (b^5*x^20)/20 + (5*a^4*b*x^12)/12 + (5*a*b^4*x^18)/18 + (5
*a^3*b^2*x^14)/7 + (5*a^2*b^3*x^16)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^9 (a+bx^2)^5 \, dx = \frac{x^{10} (126 b^5 x^{10} + 700 a b^4 x^8 + 1575 a^2 b^3 x^6 + 1800 a^3 b^2 x^4 + 1050 a^4 b x^2 + 252 a^5)}{2520}$$

input
```
int(x^9*(b*x^2+a)^5,x)
```

output
```
(x**10*(252*a**5 + 1050*a**4*b*x**2 + 1800*a**3*b**2*x**4 + 1575*a**2*b**3
*x**6 + 700*a*b**4*x**8 + 126*b**5*x**10))/2520
```

3.55 $\int x^7(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^7\left(a+bx^2\right)^5\,dx=-\frac{a^3(a+bx^2)^6}{12b^4}+\frac{3a^2(a+bx^2)^7}{14b^4}-\frac{3a(a+bx^2)^8}{16b^4}+\frac{(a+bx^2)^9}{18b^4}$$

output
```
-1/12*a^3*(b*x^2+a)^6/b^4+3/14*a^2*(b*x^2+a)^7/b^4-3/16*a*(b*x^2+a)^8/b^4+
1/18*(b*x^2+a)^9/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int x^7\left(a+bx^2\right)^5\,dx=\frac{a^5x^8}{8}+\frac{1}{2}a^4bx^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}ab^4x^{16}+\frac{b^5x^{18}}{18}$$

input
```
Integrate[x^7*(a + b*x^2)^5,x]
```

output
```
(a^5*x^8)/8 + (a^4*b*x^10)/2 + (5*a^3*b^2*x^12)/6 + (5*a^2*b^3*x^14)/7 + (
5*a*b^4*x^16)/16 + (b^5*x^18)/18
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^5 \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^5 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^8}{b^3} - \frac{3a\left(bx^2+a\right)^7}{b^3} + \frac{3a^2\left(bx^2+a\right)^6}{b^3} - \frac{a^3\left(bx^2+a\right)^5}{b^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^3\left(a+bx^2\right)^6}{6b^4} + \frac{3a^2\left(a+bx^2\right)^7}{7b^4} + \frac{\left(a+bx^2\right)^9}{9b^4} - \frac{3a\left(a+bx^2\right)^8}{8b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^5,x]
```

output
```
(-1/6*(a^3*(a + b*x^2)^6)/b^4 + (3*a^2*(a + b*x^2)^7)/(7*b^4) - (3*a*(a +
b*x^2)^8)/(8*b^4) + (a + b*x^2)^9/(9*b^4))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
default	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
norman	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
risch	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
parallelrisch	$\frac{1}{8}a^5x^8+\frac{1}{2}a^4b\,x^{10}+\frac{5}{6}a^3b^2x^{12}+\frac{5}{7}a^2b^3x^{14}+\frac{5}{16}a\,b^4x^{16}+\frac{1}{18}b^5x^{18}$	58
orering	$\frac{x^8(56b^5x^{10}+315a\,b^4x^8+720a^2b^3x^6+840a^3b^2x^4+504a^4b\,x^2+126a^5)}{1008}$	60

input
```
int(x^7*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*a^5*x^8+1/2*a^4*b*x^10+5/6*a^3*b^2*x^12+5/7*a^2*b^3*x^14+5/16*a*b^4*x^
16+1/18*b^5*x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^7(a+bx^2)^5\,dx=\frac{1}{18}\,b^5x^{18}+\frac{5}{16}\,ab^4x^{16}+\frac{5}{7}\,a^2b^3x^{14}+\frac{5}{6}\,a^3b^2x^{12}+\frac{1}{2}\,a^4bx^{10}+\frac{1}{8}\,a^5x^8$$

input
```
integrate(x^7*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/
2*a^4*b*x^10 + 1/8*a^5*x^8
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int x^7 (a + bx^2)^5 \, dx = \frac{a^5x^8}{8} + \frac{a^4bx^{10}}{2} + \frac{5a^3b^2x^{12}}{6} + \frac{5a^2b^3x^{14}}{7} + \frac{5ab^4x^{16}}{16} + \frac{b^5x^{18}}{18}$$

input
```
integrate(x**7*(b*x**2+a)**5,x)
```

output
```
a**5*x**8/8 + a**4*b*x**10/2 + 5*a**3*b**2*x**12/6 + 5*a**2*b**3*x**14/7 +
 5*a*b**4*x**16/16 + b**5*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^7 (a + bx^2)^5 \, dx = \frac{1}{18} b^5x^{18} + \frac{5}{16} ab^4x^{16} + \frac{5}{7} a^2b^3x^{14} + \frac{5}{6} a^3b^2x^{12} + \frac{1}{2} a^4bx^{10} + \frac{1}{8} a^5x^8$$

input
```
integrate(x^7*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/
2*a^4*b*x^10 + 1/8*a^5*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^7 \left(a+bx^2\right)^5 \, dx = \frac{1}{18} b^5 x^{18} + \frac{5}{16} a b^4 x^{16} + \frac{5}{7} a^2 b^3 x^{14} + \frac{5}{6} a^3 b^2 x^{12} + \frac{1}{2} a^4 b x^{10} + \frac{1}{8} a^5 x^8$$

input
```
integrate(x^7*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/18*b^5*x^18 + 5/16*a*b^4*x^16 + 5/7*a^2*b^3*x^14 + 5/6*a^3*b^2*x^12 + 1/
2*a^4*b*x^10 + 1/8*a^5*x^8
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^7 \left(a+bx^2\right)^5 \, dx = \frac{a^5\,x^8}{8} + \frac{a^4\,b\,x^{10}}{2} + \frac{5\,a^3\,b^2\,x^{12}}{6} + \frac{5\,a^2\,b^3\,x^{14}}{7} + \frac{5\,a\,b^4\,x^{16}}{16} + \frac{b^5\,x^{18}}{18}$$

input
```
int(x^7*(a + b*x^2)^5,x)
```

output
```
(a^5*x^8)/8 + (b^5*x^18)/18 + (a^4*b*x^10)/2 + (5*a*b^4*x^16)/16 + (5*a^3*
b^2*x^12)/6 + (5*a^2*b^3*x^14)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int x^7 \left(a+bx^2\right)^5 \, dx = \frac{x^8 (56 b^5 x^{10} + 315 a\, b^4 x^8 + 720 a^2 b^3 x^6 + 840 a^3 b^2 x^4 + 504 a^4 b\, x^2 + 126 a^5)}{1008}$$

input
```
int(x^7*(b*x^2+a)^5,x)
```

output
```
(x**8*(126*a**5 + 504*a**4*b*x**2 + 840*a**3*b**2*x**4 + 720*a**2*b**3*x**
6 + 315*a*b**4*x**8 + 56*b**5*x**10))/1008
```

3.56 $\int x^5(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^5\left(a+bx^2\right)^5\,dx=\frac{a^2(a+bx^2)^6}{12b^3}-\frac{a(a+bx^2)^7}{7b^3}+\frac{(a+bx^2)^8}{16b^3}$$

output
```
1/12*a^2*(b*x^2+a)^6/b^3-1/7*a*(b*x^2+a)^7/b^3+1/16*(b*x^2+a)^8/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int x^5\left(a+bx^2\right)^5\,dx=\frac{a^5x^6}{6}+\frac{5}{8}a^4bx^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}ab^4x^{14}+\frac{b^5x^{16}}{16}$$

input
```
Integrate[x^5*(a + b*x^2)^5,x]
```

output
```
(a^5*x^6)/6 + (5*a^4*b*x^8)/8 + a^3*b^2*x^10 + (5*a^2*b^3*x^12)/6 + (5*a*b
^4*x^14)/14 + (b^5*x^16)/16
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^5 \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int x^4 \left(bx^2+a\right)^5 dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^7}{b^2} - \frac{2a\left(bx^2+a\right)^6}{b^2} + \frac{a^2\left(bx^2+a\right)^5}{b^2}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^2\left(a+bx^2\right)^6}{6b^3} + \frac{\left(a+bx^2\right)^8}{8b^3} - \frac{2a\left(a+bx^2\right)^7}{7b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^5,x]
```

output
```
((a^2*(a + b*x^2)^6)/(6*b^3) - (2*a*(a + b*x^2)^7)/(7*b^3) + (a + b*x^2)^8
/(8*b^3))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

method	result	size
gosper	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
default	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
norman	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
risch	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
parallelrisch	$\frac{1}{6}a^5x^6+\frac{5}{8}a^4b\,x^8+a^3b^2x^{10}+\frac{5}{6}a^2b^3x^{12}+\frac{5}{14}a\,b^4x^{14}+\frac{1}{16}b^5x^{16}$	57
orering	$\frac{x^6\left(21b^5x^{10}+120a\,b^4x^8+280a^2b^3x^6+336a^3b^2x^4+210a^4b\,x^2+56a^5\right)}{336}$	60

input
```
int(x^5*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^5*x^6+5/8*a^4*b*x^8+a^3*b^2*x^10+5/6*a^2*b^3*x^12+5/14*a*b^4*x^14+1/
16*b^5*x^16
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^5\left(a+bx^2\right)^5\,dx=\frac{1}{16}\,b^5x^{16}+\frac{5}{14}\,ab^4x^{14}+\frac{5}{6}\,a^2b^3x^{12}+a^3b^2x^{10}+\frac{5}{8}\,a^4bx^8+\frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^
4*b*x^8 + 1/6*a^5*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int x^5 \left(a+bx^2\right)^5 \, dx = \frac{a^5x^6}{6} + \frac{5a^4bx^8}{8} + a^3b^2x^{10} + \frac{5a^2b^3x^{12}}{6} + \frac{5ab^4x^{14}}{14} + \frac{b^5x^{16}}{16}$$

input
```
integrate(x**5*(b*x**2+a)**5,x)
```

output
```
a**5*x**6/6 + 5*a**4*b*x**8/8 + a**3*b**2*x**10 + 5*a**2*b**3*x**12/6 + 5*
a*b**4*x**14/14 + b**5*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^5 \left(a+bx^2\right)^5 \, dx = \frac{1}{16}\,b^5x^{16} + \frac{5}{14}\,ab^4x^{14} + \frac{5}{6}\,a^2b^3x^{12} + a^3b^2x^{10} + \frac{5}{8}\,a^4bx^8 + \frac{1}{6}\,a^5x^6$$

input
```
integrate(x^5*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^
4*b*x^8 + 1/6*a^5*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^5 (a+bx^2)^5 \, dx = \frac{1}{16} b^5 x^{16} + \frac{5}{14} a b^4 x^{14} + \frac{5}{6} a^2 b^3 x^{12} + a^3 b^2 x^{10} + \frac{5}{8} a^4 b x^8 + \frac{1}{6} a^5 x^6$$

input
```
integrate(x^5*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/16*b^5*x^16 + 5/14*a*b^4*x^14 + 5/6*a^2*b^3*x^12 + a^3*b^2*x^10 + 5/8*a^
4*b*x^8 + 1/6*a^5*x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x^5 (a+bx^2)^5 \, dx = \frac{a^5 x^6}{6} + \frac{5 a^4 b x^8}{8} + a^3 b^2 x^{10} + \frac{5 a^2 b^3 x^{12}}{6} + \frac{5 a b^4 x^{14}}{14} + \frac{b^5 x^{16}}{16}$$

input
```
int(x^5*(a + b*x^2)^5,x)
```

output
```
(a^5*x^6)/6 + (b^5*x^16)/16 + (5*a^4*b*x^8)/8 + (5*a*b^4*x^14)/14 + a^3*b^
2*x^10 + (5*a^2*b^3*x^12)/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int x^5 (a+bx^2)^5 \, dx = \frac{x^6 (21 b^5 x^{10} + 120 a b^4 x^8 + 280 a^2 b^3 x^6 + 336 a^3 b^2 x^4 + 210 a^4 b x^2 + 56 a^5)}{336}$$

input
```
int(x^5*(b*x^2+a)^5,x)
```

output
```
(x**6*(56*a**5 + 210*a**4*b*x**2 + 336*a**3*b**2*x**4 + 280*a**2*b**3*x**6
 + 120*a*b**4*x**8 + 21*b**5*x**10))/336
```

3.57 $\int x^3(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^3\left(a+bx^2\right)^5\,dx=-\frac{a\left(a+bx^2\right)^6}{12b^2}+\frac{\left(a+bx^2\right)^7}{14b^2}$$

output
```
-1/12*a*(b*x^2+a)^6/b^2+1/14*(b*x^2+a)^7/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int x^3\left(a+bx^2\right)^5\,dx=\frac{a^5x^4}{4}+\frac{5}{6}a^4bx^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}ab^4x^{12}+\frac{b^5x^{14}}{14}$$

input
```
Integrate[x^3*(a + b*x^2)^5,x]
```

output
```
(a^5*x^4)/4 + (5*a^4*b*x^6)/6 + (5*a^3*b^2*x^8)/4 + a^2*b^3*x^10 + (5*a*b^
4*x^12)/12 + (b^5*x^14)/14
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^5 \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int x^2 (bx^2+a)^5 \, dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2} \int \left(\frac{(bx^2+a)^6}{b} - \frac{a(bx^2+a)^5}{b} \right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(\frac{(a+bx^2)^7}{7b^2} - \frac{a(a+bx^2)^6}{6b^2} \right)$$

input
```
Int[x^3*(a + b*x^2)^5,x]
```

output
```
(-1/6*(a*(a + b*x^2)^6)/b^2 + (a + b*x^2)^7/(7*b^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

method	result	size
gosper	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
default	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
norman	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
risch	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
parallelrisch	$\frac{1}{4}a^5x^4+\frac{5}{6}a^4b\,x^6+\frac{5}{4}a^3b^2x^8+a^2b^3x^{10}+\frac{5}{12}a\,b^4x^{12}+\frac{1}{14}b^5x^{14}$	57
orering	$\frac{x^4(6b^5x^{10}+35a\,b^4x^8+84a^2b^3x^6+105a^3b^2x^4+70a^4b\,x^2+21a^5)}{84}$	60

input
```
int(x^3*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^5*x^4+5/6*a^4*b*x^6+5/4*a^3*b^2*x^8+a^2*b^3*x^10+5/12*a*b^4*x^12+1/1
4*b^5*x^14
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^3(a+bx^2)^5\,dx=\frac{1}{14}\,b^5x^{14}+\frac{5}{12}\,ab^4x^{12}+a^2b^3x^{10}+\frac{5}{4}\,a^3b^2x^8+\frac{5}{6}\,a^4bx^6+\frac{1}{4}\,a^5x^4$$

input
```
integrate(x^3*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4
*b*x^6 + 1/4*a^5*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.85

$$\int x^3 (a + bx^2)^5 \, dx = \frac{a^5x^4}{4} + \frac{5a^4bx^6}{6} + \frac{5a^3b^2x^8}{4} + a^2b^3x^{10} + \frac{5ab^4x^{12}}{12} + \frac{b^5x^{14}}{14}$$

input
```
integrate(x**3*(b*x**2+a)**5,x)
```

output
```
a**5*x**4/4 + 5*a**4*b*x**6/6 + 5*a**3*b**2*x**8/4 + a**2*b**3*x**10 + 5*a
*b**4*x**12/12 + b**5*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^3 (a + bx^2)^5 \, dx = \frac{1}{14} b^5x^{14} + \frac{5}{12} ab^4x^{12} + a^2b^3x^{10} + \frac{5}{4} a^3b^2x^8 + \frac{5}{6} a^4bx^6 + \frac{1}{4} a^5x^4$$

input
```
integrate(x^3*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4
*b*x^6 + 1/4*a^5*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^3 (a + bx^2)^5 \, dx = \frac{1}{14} b^5 x^{14} + \frac{5}{12} ab^4 x^{12} + a^2 b^3 x^{10} + \frac{5}{4} a^3 b^2 x^8 + \frac{5}{6} a^4 b x^6 + \frac{1}{4} a^5 x^4$$

input
```
integrate(x^3*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/14*b^5*x^14 + 5/12*a*b^4*x^12 + a^2*b^3*x^10 + 5/4*a^3*b^2*x^8 + 5/6*a^4
*b*x^6 + 1/4*a^5*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.65

$$\int x^3 (a + bx^2)^5 \, dx = \frac{a^5 x^4}{4} + \frac{5 a^4 b x^6}{6} + \frac{5 a^3 b^2 x^8}{4} + a^2 b^3 x^{10} + \frac{5 a b^4 x^{12}}{12} + \frac{b^5 x^{14}}{14}$$

input
```
int(x^3*(a + b*x^2)^5,x)
```

output
```
(a^5*x^4)/4 + (b^5*x^14)/14 + (5*a^4*b*x^6)/6 + (5*a*b^4*x^12)/12 + (5*a^3
*b^2*x^8)/4 + a^2*b^3*x^10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.74

$$\int x^3 (a + bx^2)^5 \, dx = \frac{x^4 (6b^5 x^{10} + 35a b^4 x^8 + 84a^2 b^3 x^6 + 105a^3 b^2 x^4 + 70a^4 b x^2 + 21a^5)}{84}$$

input
```
int(x^3*(b*x^2+a)^5,x)
```

output
```
(x**4*(21*a**5 + 70*a**4*b*x**2 + 105*a**3*b**2*x**4 + 84*a**2*b**3*x**6 +
 35*a*b**4*x**8 + 6*b**5*x**10))/84
```

3.58 $\int x(a+bx^2)^5\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x\left(a+bx^2\right)^5\,dx=\frac{\left(a+bx^2\right)^6}{12b}$$

output
```
1/12*(b*x^2+a)^6/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)^5\,dx=\frac{\left(a+bx^2\right)^6}{12b}$$

input
```
Integrate[x*(a + b*x^2)^5,x]
```

output
```
(a + b*x^2)^6/(12*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^5\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx^2)^6}{12b}$$

input
```
Int[x*(a + b*x^2)^5,x]
```

output
```
(a + b*x^2)^6/(12*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{(b\,x^2+a)^6}{12b}$	15
gosper	$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$	58
norman	$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$	58
parallelrisch	$\frac{1}{12}b^5x^{12}+\frac{1}{2}a\,b^4x^{10}+\frac{5}{4}a^2b^3x^8+\frac{5}{3}a^3b^2x^6+\frac{5}{4}a^4b\,x^4+\frac{1}{2}a^5x^2$	58
orering	$\frac{x^2(b^5x^{10}+6a\,b^4x^8+15a^2b^3x^6+20a^3b^2x^4+15a^4b\,x^2+6a^5)}{12}$	59
risch	$\frac{b^5x^{12}}{12}+\frac{a\,b^4x^{10}}{2}+\frac{5a^2b^3x^8}{4}+\frac{5a^3b^2x^6}{3}+\frac{5a^4b\,x^4}{4}+\frac{a^5x^2}{2}+\frac{a^6}{12b}$	66

input
```
int(x*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(b*x^2+a)^6/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int x\left(a+bx^2\right)^5\,dx=\frac{1}{12}\,b^5x^{12}+\frac{1}{2}\,ab^4x^{10}+\frac{5}{4}\,a^2b^3x^8+\frac{5}{3}\,a^3b^2x^6+\frac{5}{4}\,a^4bx^4+\frac{1}{2}\,a^5x^2$$

input
```
integrate(x*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/12*b^5*x^12 + 1/2*a*b^4*x^10 + 5/4*a^2*b^3*x^8 + 5/3*a^3*b^2*x^6 + 5/4*a
^4*b*x^4 + 1/2*a^5*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(10) = 20$.

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 4.06

$$\int x(a+bx^2)^5\,dx = \frac{a^5x^2}{2} + \frac{5a^4bx^4}{4} + \frac{5a^3b^2x^6}{3} + \frac{5a^2b^3x^8}{4} + \frac{ab^4x^{10}}{2} + \frac{b^5x^{12}}{12}$$

input
```
integrate(x*(b*x**2+a)**5,x)
```

output
```
a**5*x**2/2 + 5*a**4*b*x**4/4 + 5*a**3*b**2*x**6/3 + 5*a**2*b**3*x**8/4 +
a*b**4*x**10/2 + b**5*x**12/12
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^5\,dx = \frac{(bx^2+a)^6}{12\,b}$$

input
```
integrate(x*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^6/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^5\,dx = \frac{(bx^2+a)^6}{12\,b}$$

input
```
integrate(x*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/12*(b*x^2 + a)^6/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.56

$$\int x(a+bx^2)^5 \, dx = \frac{a^5 x^2}{2} + \frac{5 a^4 b x^4}{4} + \frac{5 a^3 b^2 x^6}{3} + \frac{5 a^2 b^3 x^8}{4} + \frac{a b^4 x^{10}}{2} + \frac{b^5 x^{12}}{12}$$

input
```
int(x*(a + b*x^2)^5,x)
```

output
```
(a^5*x^2)/2 + (b^5*x^12)/12 + (5*a^4*b*x^4)/4 + (a*b^4*x^10)/2 + (5*a^3*b^
2*x^6)/3 + (5*a^2*b^3*x^8)/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.62

$$\int x(a+bx^2)^5 \, dx = \frac{x^2(b^5x^{10} + 6a b^4x^8 + 15a^2b^3x^6 + 20a^3b^2x^4 + 15a^4b x^2 + 6a^5)}{12}$$

input
```
int(x*(b*x^2+a)^5,x)
```

output
```
(x**2*(6*a**5 + 15*a**4*b*x**2 + 20*a**3*b**2*x**4 + 15*a**2*b**3*x**6 + 6
*a*b**4*x**8 + b**5*x**10))/12
```

3.59 $\int \frac{(a+bx^2)^5}{x} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^2)^5}{x} \, dx = \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10} + a^5\log(x)$$

output
```
5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+
a^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x} \, dx = \frac{5}{2}a^4bx^2 + \frac{5}{2}a^3b^2x^4 + \frac{5}{3}a^2b^3x^6 + \frac{5}{8}ab^4x^8 + \frac{b^5x^{10}}{10} + a^5\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x,x]
```

output
```
(5*a^4*b*x^2)/2 + (5*a^3*b^2*x^4)/2 + (5*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/8
+ (b^5*x^10)/10 + a^5*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^5}{x^2} dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(b^5x^8 + 5ab^4x^6 + 10a^2b^3x^4 + 10a^3b^2x^2 + 5a^4b + \frac{a^5}{x^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^5 \log\left(x^2\right) + 5a^4bx^2 + 5a^3b^2x^4 + \frac{10}{3}a^2b^3x^6 + \frac{5}{4}ab^4x^8 + \frac{b^5x^{10}}{5}\right)$$

input
```
Int[(a + b*x^2)^5/x,x]
```

output
```
(5*a^4*b*x^2 + 5*a^3*b^2*x^4 + (10*a^2*b^3*x^6)/3 + (5*a*b^4*x^8)/4 + (b^5
*x^10)/5 + a^5*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56
norman	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56
risch	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56
parallelrisch	$\frac{5a^4b\,x^2}{2}+\frac{5a^3b^2x^4}{2}+\frac{5a^2b^3x^6}{3}+\frac{5a\,b^4x^8}{8}+\frac{b^5x^{10}}{10}+a^5\ln\left(x\right)$	56

input
```
int((b*x^2+a)^5/x,x,method=_RETURNVERBOSE)
```

output
```
5/2*a^4*b*x^2+5/2*a^3*b^2*x^4+5/3*a^2*b^3*x^6+5/8*a*b^4*x^8+1/10*b^5*x^10+
a^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int\frac{\left(a+bx^2\right)^5}{x}\,dx=\frac{1}{10}\,b^5x^{10}+\frac{5}{8}\,ab^4x^8+\frac{5}{3}\,a^2b^3x^6+\frac{5}{2}\,a^3b^2x^4+\frac{5}{2}\,a^4bx^2+a^5\log\left(x\right)$$

input
```
integrate((b*x^2+a)^5/x,x, algorithm="fricas")
```

output
```
1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^
4*b*x^2 + a^5*log(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x}\,dx = a^5\log(x) + \frac{5a^4bx^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5ab^4x^8}{8} + \frac{b^5x^{10}}{10}$$

input
```
integrate((b*x**2+a)**5/x,x)
```

output
```
a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 +
5*a*b**4*x**8/8 + b**5*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^5}{x}\,dx = \frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + \frac{1}{2}\,a^5\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^5/x,x, algorithm="maxima")
```

output
```
1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^
4*b*x^2 + 1/2*a^5*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^5}{x}\,dx = \frac{1}{10}\,b^5x^{10} + \frac{5}{8}\,ab^4x^8 + \frac{5}{3}\,a^2b^3x^6 + \frac{5}{2}\,a^3b^2x^4 + \frac{5}{2}\,a^4bx^2 + \frac{1}{2}\,a^5\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^5/x,x, algorithm="giac")
```

output
```
1/10*b^5*x^10 + 5/8*a*b^4*x^8 + 5/3*a^2*b^3*x^6 + 5/2*a^3*b^2*x^4 + 5/2*a^
4*b*x^2 + 1/2*a^5*log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^5}{x}\,dx = a^5\,\ln(x) + \frac{b^5\,x^{10}}{10} + \frac{5\,a^4\,b\,x^2}{2} + \frac{5\,a\,b^4\,x^8}{8} + \frac{5\,a^3\,b^2\,x^4}{2} + \frac{5\,a^2\,b^3\,x^6}{3}$$

input
```
int((a + b*x^2)^5/x,x)
```

output
```
a^5*log(x) + (b^5*x^10)/10 + (5*a^4*b*x^2)/2 + (5*a*b^4*x^8)/8 + (5*a^3*b^
2*x^4)/2 + (5*a^2*b^3*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^5}{x}\,dx = \log(x)\,a^5 + \frac{5a^4b\,x^2}{2} + \frac{5a^3b^2x^4}{2} + \frac{5a^2b^3x^6}{3} + \frac{5a\,b^4x^8}{8} + \frac{b^5x^{10}}{10}$$

input
```
int((b*x^2+a)^5/x,x)
```

output
```
(120*log(x)*a**5 + 300*a**4*b*x**2 + 300*a**3*b**2*x**4 + 200*a**2*b**3*x*
*6 + 75*a*b**4*x**8 + 12*b**5*x**10)/120
```

3.60 $\int \frac{(a+bx^2)^5}{x^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = -\frac{a^5}{2x^2} + 5a^3b^2x^2 + \frac{5}{2}a^2b^3x^4 + \frac{5}{6}ab^4x^6 + \frac{b^5x^8}{8} + 5a^4b\log(x)$$

output
```
-1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4
*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = -\frac{a^5}{2x^2} + 5a^3b^2x^2 + \frac{5}{2}a^2b^3x^4 + \frac{5}{6}ab^4x^6 + \frac{b^5x^8}{8} + 5a^4b\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x^3,x]
```

output
```
-1/2*a^5/x^2 + 5*a^3*b^2*x^2 + (5*a^2*b^3*x^4)/2 + (5*a*b^4*x^6)/6 + (b^5*
x^8)/8 + 5*a^4*b*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^5x^6+5ab^4x^4+10a^2b^3x^2+10a^3b^2+\frac{5a^4b}{x^2}+\frac{a^5}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{x^2}+5a^4b\log\left(x^2\right)+10a^3b^2x^2+5a^2b^3x^4+\frac{5}{3}ab^4x^6+\frac{b^5x^8}{4}\right)$$

input
```
Int[(a + b*x^2)^5/x^3,x]
```

output
```
(-(a^5/x^2) + 10*a^3*b^2*x^2 + 5*a^2*b^3*x^4 + (5*a*b^4*x^6)/3 + (b^5*x^8)
/4 + 5*a^4*b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^{5}}{2x^{2}}+5a^{3}b^{2}x^{2}+\frac{5a^{2}b^{3}x^{4}}{2}+\frac{5a\,b^{4}x^{6}}{6}+\frac{b^{5}x^{8}}{8}+5a^{4}b\ln\left(x\right)$	57
risch	$-\frac{a^{5}}{2x^{2}}+5a^{3}b^{2}x^{2}+\frac{5a^{2}b^{3}x^{4}}{2}+\frac{5a\,b^{4}x^{6}}{6}+\frac{b^{5}x^{8}}{8}+5a^{4}b\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{2}a^{5}+\frac{1}{8}b^{5}x^{10}+\frac{5}{6}a\,b^{4}x^{8}+\frac{5}{2}a^{2}b^{3}x^{6}+5a^{3}b^{2}x^{4}}{x^{2}}+5a^{4}b\ln\left(x\right)$	59
parallelrisch	$\frac{3b^{5}x^{10}+20a\,b^{4}x^{8}+60a^{2}b^{3}x^{6}+120a^{3}b^{2}x^{4}+120a^{4}b\ln\left(x\right)x^{2}-12a^{5}}{24x^{2}}$	62

input
```
int((b*x^2+a)^5/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^5/x^2+5*a^3*b^2*x^2+5/2*a^2*b^3*x^4+5/6*a*b^4*x^6+1/8*b^5*x^8+5*a^4
*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx^2)^5}{x^3}\,dx=\frac{3\,b^5x^{10}+20\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4+120\,a^4bx^2\log\left(x\right)-12\,a^5}{24\,x^2}$$

input
```
integrate((b*x^2+a)^5/x^3,x, algorithm="fricas")
```

output
```
1/24*(3*b^5*x^10 + 20*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4 + 120*a
^4*b*x^2*log(x) - 12*a^5)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = -\frac{a^5}{2x^2} + 5a^4b\log(x) + 5a^3b^2x^2 + \frac{5a^2b^3x^4}{2} + \frac{5ab^4x^6}{6} + \frac{b^5x^8}{8}$$

input
```
integrate((b*x**2+a)**5/x**3,x)
```

output
```
-a**5/(2*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 +
 5*a*b**4*x**6/6 + b**5*x**8/8
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = \frac{1}{8}\,b^5x^8 + \frac{5}{6}\,ab^4x^6 + \frac{5}{2}\,a^2b^3x^4 + 5\,a^3b^2x^2 + \frac{5}{2}\,a^4b\log\left(x^2\right) - \frac{a^5}{2\,x^2}$$

input
```
integrate((b*x^2+a)^5/x^3,x, algorithm="maxima")
```

output
```
1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*
log(x^2) - 1/2*a^5/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = \frac{1}{8}b^5x^8 + \frac{5}{6}ab^4x^6 + \frac{5}{2}a^2b^3x^4 + 5a^3b^2x^2 + \frac{5}{2}a^4b\log\left(x^2\right) - \frac{5a^4bx^2 + a^5}{2x^2}$$

input
```
integrate((b*x^2+a)^5/x^3,x, algorithm="giac")
```

output
```
1/8*b^5*x^8 + 5/6*a*b^4*x^6 + 5/2*a^2*b^3*x^4 + 5*a^3*b^2*x^2 + 5/2*a^4*b*
log(x^2) - 1/2*(5*a^4*b*x^2 + a^5)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = \frac{b^5\,x^8}{8} - \frac{a^5}{2\,x^2} + \frac{5\,a\,b^4\,x^6}{6} + 5\,a^4\,b\,\ln(x) + 5\,a^3\,b^2\,x^2 + \frac{5\,a^2\,b^3\,x^4}{2}$$

input
```
int((a + b*x^2)^5/x^3,x)
```

output
```
(b^5*x^8)/8 - a^5/(2*x^2) + (5*a*b^4*x^6)/6 + 5*a^4*b*log(x) + 5*a^3*b^2*x
^2 + (5*a^2*b^3*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^3}\,dx = \frac{120\log(x)\,a^4b\,x^2 - 12a^5 + 120a^3b^2x^4 + 60a^2b^3x^6 + 20a\,b^4x^8 + 3b^5x^{10}}{24x^2}$$

input
```
int((b*x^2+a)^5/x^3,x)
```

output
```
(120*log(x)*a**4*b*x**2 - 12*a**5 + 120*a**3*b**2*x**4 + 60*a**2*b**3*x**6
 + 20*a*b**4*x**8 + 3*b**5*x**10)/(24*x**2)
```

3.61 $\int \frac{(a+bx^2)^5}{x^5}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6} + 10a^3b^2\log(x)$$

output
```
-1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*
b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = -\frac{a^5}{4x^4} - \frac{5a^4b}{2x^2} + 5a^2b^3x^2 + \frac{5}{4}ab^4x^4 + \frac{b^5x^6}{6} + 10a^3b^2\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x^5,x]
```

output
```
-1/4*a^5/x^4 - (5*a^4*b)/(2*x^2) + 5*a^2*b^3*x^2 + (5*a*b^4*x^4)/4 + (b^5*
x^6)/6 + 10*a^3*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^5} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^6}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^5}{x^6}+\frac{5ba^4}{x^4}+\frac{10b^2a^3}{x^2}+10b^3a^2+5b^4x^2a+b^5x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{2x^4}-\frac{5a^4b}{x^2}+10a^3b^2\log\left(x^2\right)+10a^2b^3x^2+\frac{5}{2}ab^4x^4+\frac{b^5x^6}{3}\right)$$

input
```
Int[(a + b*x^2)^5/x^5,x]
```

output
```
(-1/2*a^5/x^4 - (5*a^4*b)/x^2 + 10*a^2*b^3*x^2 + (5*a*b^4*x^4)/2 + (b^5*x^
6)/3 + 10*a^3*b^2*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^5}{4x^4}-\frac{5a^4b}{2x^2}+5a^2b^3x^2+\frac{5a\,b^4x^4}{4}+\frac{b^5x^6}{6}+10a^3b^2\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{4}a^5+\frac{1}{6}b^5x^{10}+\frac{5}{4}a\,b^4x^8+5a^2b^3x^6-\frac{5}{2}a^4b\,x^2}{x^4}+10a^3b^2\ln\left(x\right)$	59
risch	$\frac{b^5x^6}{6}+\frac{5a\,b^4x^4}{4}+5a^2b^3x^2+\frac{-\frac{5}{2}a^4b\,x^2-\frac{1}{4}a^5}{x^4}+10a^3b^2\ln\left(x\right)$	59
parallelrisch	$\frac{2b^5x^{10}+15a\,b^4x^8+60a^2b^3x^6+120a^3b^2\ln(x)x^4-30a^4b\,x^2-3a^5}{12x^4}$	62

input
```
int((b*x^2+a)^5/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^5/x^4-5/2*a^4*b/x^2+5*a^2*b^3*x^2+5/4*a*b^4*x^4+1/6*b^5*x^6+10*a^3*
b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx^2)^5}{x^5}\,dx=\frac{2\,b^5x^{10}+15\,ab^4x^8+60\,a^2b^3x^6+120\,a^3b^2x^4\log\left(x\right)-30\,a^4bx^2-3\,a^5}{12\,x^4}$$

input
```
integrate((b*x^2+a)^5/x^5,x, algorithm="fricas")
```

output
```
1/12*(2*b^5*x^10 + 15*a*b^4*x^8 + 60*a^2*b^3*x^6 + 120*a^3*b^2*x^4*log(x)
- 30*a^4*b*x^2 - 3*a^5)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = 10a^3b^2\log{(x)} + 5a^2b^3x^2 + \frac{5ab^4x^4}{4} + \frac{b^5x^6}{6} + \frac{-a^5 - 10a^4bx^2}{4x^4}$$

input
```
integrate((b*x**2+a)**5/x**5,x)
```

output
```
10*a**3*b**2*log(x) + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6 + (
-a**5 - 10*a**4*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = \frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 5\,a^3b^2\log\left(x^2\right) - \frac{10\,a^4bx^2 + a^5}{4\,x^4}$$

input
```
integrate((b*x^2+a)^5/x^5,x, algorithm="maxima")
```

output
```
1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(10
*a^4*b*x^2 + a^5)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = \frac{1}{6}\,b^5x^6 + \frac{5}{4}\,ab^4x^4 + 5\,a^2b^3x^2 + 5\,a^3b^2\log\left(x^2\right) - \frac{30\,a^3b^2x^4 + 10\,a^4bx^2 + a^5}{4\,x^4}$$

input
```
integrate((b*x^2+a)^5/x^5,x, algorithm="giac")
```

output
```
1/6*b^5*x^6 + 5/4*a*b^4*x^4 + 5*a^2*b^3*x^2 + 5*a^3*b^2*log(x^2) - 1/4*(30
*a^3*b^2*x^4 + 10*a^4*b*x^2 + a^5)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = \frac{b^5\,x^6}{6} - \frac{\frac{a^5}{4}+\frac{5\,b\,a^4\,x^2}{2}}{x^4} + \frac{5\,a\,b^4\,x^4}{4} + 5\,a^2\,b^3\,x^2 + 10\,a^3\,b^2\,\ln(x)$$

input
```
int((a + b*x^2)^5/x^5,x)
```

output
```
(b^5*x^6)/6 - (a^5/4 + (5*a^4*b*x^2)/2)/x^4 + (5*a*b^4*x^4)/4 + 5*a^2*b^3*
x^2 + 10*a^3*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^5}\,dx = \frac{120\log(x)\,a^3b^2x^4 - 3a^5 - 30a^4b\,x^2 + 60a^2b^3x^6 + 15a\,b^4x^8 + 2b^5x^{10}}{12x^4}$$

input
```
int((b*x^2+a)^5/x^5,x)
```

output
```
(120*log(x)*a**3*b**2*x**4 - 3*a**5 - 30*a**4*b*x**2 + 60*a**2*b**3*x**6 +
 15*a*b**4*x**8 + 2*b**5*x**10)/(12*x**4)
```

3.62 $\int \frac{(a+bx^2)^5}{x^7}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)$$

output
```
-1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*
b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = -\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5}{2}ab^4x^2+\frac{b^5x^4}{4}+10a^2b^3\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x^7,x]
```

output
```
-1/6*a^5/x^6 - (5*a^4*b)/(4*x^4) - (5*a^3*b^2)/x^2 + (5*a*b^4*x^2)/2 + (b^
5*x^4)/4 + 10*a^2*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^8}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^5}{x^8}+\frac{5ba^4}{x^6}+\frac{10b^2a^3}{x^4}+\frac{10b^3a^2}{x^2}+5b^4a+b^5x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{3x^6}-\frac{5a^4b}{2x^4}-\frac{10a^3b^2}{x^2}+10a^2b^3\log\left(x^2\right)+5ab^4x^2+\frac{b^5x^4}{2}\right)$$

input
```
Int[(a + b*x^2)^5/x^7,x]
```

output
```
(-1/3*a^5/x^6 - (5*a^4*b)/(2*x^4) - (10*a^3*b^2)/x^2 + 5*a*b^4*x^2 + (b^5*
x^4)/2 + 10*a^2*b^3*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^5}{6x^6}-\frac{5a^4b}{4x^4}-\frac{5a^3b^2}{x^2}+\frac{5a\,b^4x^2}{2}+\frac{b^5x^4}{4}+10a^2b^3\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{6}a^5+\frac{1}{4}b^5x^{10}+\frac{5}{2}a\,b^4x^8-5a^3b^2x^4-\frac{5}{4}a^4b\,x^2}{x^6}+10a^2b^3\ln\left(x\right)$	59
parallelrisch	$\frac{3b^5x^{10}+30a\,b^4x^8+120a^2b^3\ln(x)x^6-60a^3b^2x^4-15a^4b\,x^2-2a^5}{12x^6}$	62
risch	$\frac{b^5x^4}{4}+\frac{5a\,b^4x^2}{2}+\frac{25a^2b^3}{4}+\frac{-5a^3b^2x^4-\frac{5}{4}a^4b\,x^2-\frac{1}{6}a^5}{x^6}+10a^2b^3\ln\left(x\right)$	67

input
```
int((b*x^2+a)^5/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^5/x^6-5/4*a^4*b/x^4-5*a^3*b^2/x^2+5/2*a*b^4*x^2+1/4*b^5*x^4+10*a^2*
b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx^2)^5}{x^7}\,dx=\frac{3\,b^5x^{10}+30\,ab^4x^8+120\,a^2b^3x^6\log\left(x\right)-60\,a^3b^2x^4-15\,a^4bx^2-2\,a^5}{12\,x^6}$$

input
```
integrate((b*x^2+a)^5/x^7,x, algorithm="fricas")
```

output
```
1/12*(3*b^5*x^10 + 30*a*b^4*x^8 + 120*a^2*b^3*x^6*log(x) - 60*a^3*b^2*x^4
- 15*a^4*b*x^2 - 2*a^5)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = 10a^2b^3\log(x) + \frac{5ab^4x^2}{2} + \frac{b^5x^4}{4} + \frac{-2a^5-15a^4bx^2-60a^3b^2x^4}{12x^6}$$

input
```
integrate((b*x**2+a)**5/x**7,x)
```

output
```
10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**4/4 + (-2*a**5 - 15*a**4*b
*x**2 - 60*a**3*b**2*x**4)/(12*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = \frac{1}{4}\,b^5x^4 + \frac{5}{2}\,ab^4x^2 + 5\,a^2b^3\log\left(x^2\right) - \frac{60\,a^3b^2x^4 + 15\,a^4bx^2 + 2\,a^5}{12\,x^6}$$

input
```
integrate((b*x^2+a)^5/x^7,x, algorithm="maxima")
```

output
```
1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(60*a^3*b^2*x^4 +
15*a^4*b*x^2 + 2*a^5)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = \frac{1}{4}\,b^5x^4 + \frac{5}{2}\,ab^4x^2 + 5\,a^2b^3\log\left(x^2\right) - \frac{110\,a^2b^3x^6 + 60\,a^3b^2x^4 + 15\,a^4bx^2 + 2\,a^5}{12\,x^6}$$

input
```
integrate((b*x^2+a)^5/x^7,x, algorithm="giac")
```

output
```
1/4*b^5*x^4 + 5/2*a*b^4*x^2 + 5*a^2*b^3*log(x^2) - 1/12*(110*a^2*b^3*x^6 +
 60*a^3*b^2*x^4 + 15*a^4*b*x^2 + 2*a^5)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = \frac{b^5\,x^4}{4} - \frac{\frac{a^5}{6} + \frac{5\,a^4\,b\,x^2}{4} + 5\,a^3\,b^2\,x^4}{x^6} + \frac{5\,a\,b^4\,x^2}{2} + 10\,a^2\,b^3\,\ln(x)$$

input
```
int((a + b*x^2)^5/x^7,x)
```

output
```
(b^5*x^4)/4 - (a^5/6 + (5*a^4*b*x^2)/4 + 5*a^3*b^2*x^4)/x^6 + (5*a*b^4*x^2
)/2 + 10*a^2*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^7}\,dx = \frac{120\log(x)\,a^2b^3x^6 - 2a^5 - 15a^4b\,x^2 - 60a^3b^2x^4 + 30a\,b^4x^8 + 3b^5x^{10}}{12x^6}$$

input
```
int((b*x^2+a)^5/x^7,x)
```

output
```
(120*log(x)*a**2*b**3*x**6 - 2*a**5 - 15*a**4*b*x**2 - 60*a**3*b**2*x**4 +
 30*a*b**4*x**8 + 3*b**5*x**10)/(12*x**6)
```

3.63 $\int \frac{(a+bx^2)^5}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = -\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + \frac{b^5x^2}{2} + 5ab^4\log(x)$$

output
```
-1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b
^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = -\frac{a^5}{8x^8} - \frac{5a^4b}{6x^6} - \frac{5a^3b^2}{2x^4} - \frac{5a^2b^3}{x^2} + \frac{b^5x^2}{2} + 5ab^4\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x^9,x]
```

output
```
-1/8*a^5/x^8 - (5*a^4*b)/(6*x^6) - (5*a^3*b^2)/(2*x^4) - (5*a^2*b^3)/x^2 +
 (b^5*x^2)/2 + 5*a*b^4*Log[x]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^5}{x^{10}}+\frac{5ba^4}{x^8}+\frac{10b^2a^3}{x^6}+\frac{10b^3a^2}{x^4}+\frac{5b^4a}{x^2}+b^5\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{4x^8}-\frac{5a^4b}{3x^6}-\frac{5a^3b^2}{x^4}-\frac{10a^2b^3}{x^2}+5ab^4\log\left(x^2\right)+b^5x^2\right)$$

input
```
Int[(a + b*x^2)^5/x^9,x]
```

output
```
(-1/4*a^5/x^8 - (5*a^4*b)/(3*x^6) - (5*a^3*b^2)/x^4 - (10*a^2*b^3)/x^2 + b
^5*x^2 + 5*a*b^4*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^5}{8x^8}-\frac{5a^4b}{6x^6}-\frac{5a^3b^2}{2x^4}-\frac{5a^2b^3}{x^2}+\frac{b^5x^2}{2}+5a\,b^4\ln\left(x\right)$	57
norman	$\frac{-\frac{1}{8}a^5+\frac{1}{2}b^5x^{10}-5a^2b^3x^6-\frac{5}{2}a^3b^2x^4-\frac{5}{6}a^4b\,x^2}{x^8}+5a\,b^4\ln\left(x\right)$	59
risch	$\frac{b^5x^2}{2}+\frac{-5a^2b^3x^6-\frac{5}{2}a^3b^2x^4-\frac{5}{6}a^4b\,x^2-\frac{1}{8}a^5}{x^8}+5a\,b^4\ln\left(x\right)$	59
parallelrisch	$\frac{12b^5x^{10}+120a\,b^4\ln(x)x^8-120a^2b^3x^6-60a^3b^2x^4-20a^4b\,x^2-3a^5}{24x^8}$	62

input
```
int((b*x^2+a)^5/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^5/x^8-5/6*a^4*b/x^6-5/2*a^3*b^2/x^4-5*a^2*b^3/x^2+1/2*b^5*x^2+5*a*b
^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int\frac{(a+bx^2)^5}{x^9}\,dx=\frac{12\,b^5x^{10}+120\,ab^4x^8\log\left(x\right)-120\,a^2b^3x^6-60\,a^3b^2x^4-20\,a^4bx^2-3\,a^5}{24\,x^8}$$

input
```
integrate((b*x^2+a)^5/x^9,x, algorithm="fricas")
```

output
```
1/24*(12*b^5*x^10 + 120*a*b^4*x^8*log(x) - 120*a^2*b^3*x^6 - 60*a^3*b^2*x^
4 - 20*a^4*b*x^2 - 3*a^5)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = 5ab^4\log(x) + \frac{b^5x^2}{2} + \frac{-3a^5 - 20a^4bx^2 - 60a^3b^2x^4 - 120a^2b^3x^6}{24x^8}$$

input
```
integrate((b*x**2+a)**5/x**9,x)
```

output
```
5*a*b**4*log(x) + b**5*x**2/2 + (-3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x
**4 - 120*a**2*b**3*x**6)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = \frac{1}{2}\,b^5x^2 + \frac{5}{2}\,ab^4\log(x^2) - \frac{120\,a^2b^3x^6 + 60\,a^3b^2x^4 + 20\,a^4bx^2 + 3\,a^5}{24\,x^8}$$

input
```
integrate((b*x^2+a)^5/x^9,x, algorithm="maxima")
```

output
```
1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(120*a^2*b^3*x^6 + 60*a^3*b^2*x^4
+ 20*a^4*b*x^2 + 3*a^5)/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = \frac{1}{2}\,b^5x^2+\frac{5}{2}\,ab^4\log\left(x^2\right) - \frac{125\,ab^4x^8+120\,a^2b^3x^6+60\,a^3b^2x^4+20\,a^4bx^2+3\,a^5}{24\,x^8}$$

input
```
integrate((b*x^2+a)^5/x^9,x, algorithm="giac")
```

output
```
1/2*b^5*x^2 + 5/2*a*b^4*log(x^2) - 1/24*(125*a*b^4*x^8 + 120*a^2*b^3*x^6 +
 60*a^3*b^2*x^4 + 20*a^4*b*x^2 + 3*a^5)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = \frac{b^5\,x^2}{2}-\frac{\frac{a^5}{8}+\frac{5\,a^4\,b\,x^2}{6}+\frac{5\,a^3\,b^2\,x^4}{2}+5\,a^2\,b^3\,x^6}{x^8}+5\,a\,b^4\,\ln\left(x\right)$$

input
```
int((a + b*x^2)^5/x^9,x)
```

output
```
(b^5*x^2)/2 - (a^5/8 + (5*a^4*b*x^2)/6 + (5*a^3*b^2*x^4)/2 + 5*a^2*b^3*x^6
)/x^8 + 5*a*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^9}\,dx = \frac{120\log(x)\,a\,b^4x^8-3a^5-20a^4b\,x^2-60a^3b^2x^4-120a^2b^3x^6+12b^5x^{10}}{24x^8}$$

input
```
int((b*x^2+a)^5/x^9,x)
```

output
```
(120*log(x)*a*b**4*x**8 - 3*a**5 - 20*a**4*b*x**2 - 60*a**3*b**2*x**4 - 12
0*a**2*b**3*x**6 + 12*b**5*x**10)/(24*x**8)
```

3.64 $\int \frac{(a+bx^2)^5}{x^{11}} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^2)^5}{x^{11}} dx = -\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5\log(x)$$

output
```
-1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2
+b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{11}} dx = -\frac{a^5}{10x^{10}} - \frac{5a^4b}{8x^8} - \frac{5a^3b^2}{3x^6} - \frac{5a^2b^3}{2x^4} - \frac{5ab^4}{2x^2} + b^5\log(x)$$

input
```
Integrate[(a + b*x^2)^5/x^11,x]
```

output
```
-1/10*a^5/x^10 - (5*a^4*b)/(8*x^8) - (5*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/(2*
x^4) - (5*a*b^4)/(2*x^2) + b^5*Log[x]
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{12}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^5}{x^{12}}+\frac{5ba^4}{x^{10}}+\frac{10b^2a^3}{x^8}+\frac{10b^3a^2}{x^6}+\frac{5b^4a}{x^4}+\frac{b^5}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{5x^{10}}-\frac{5a^4b}{4x^8}-\frac{10a^3b^2}{3x^6}-\frac{5a^2b^3}{x^4}-\frac{5ab^4}{x^2}+b^5\log\left(x^2\right)\right)$$

input
```
Int[(a + b*x^2)^5/x^11,x]
```

output
```
(-1/5*a^5/x^10 - (5*a^4*b)/(4*x^8) - (10*a^3*b^2)/(3*x^6) - (5*a^2*b^3)/x^
4 - (5*a*b^4)/x^2 + b^5*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{a^{5}}{10 x^{10}}-\frac{5 a^{4} b}{8 x^{8}}-\frac{5 a^{3} b^{2}}{3 x^{6}}-\frac{5 a^{2} b^{3}}{2 x^{4}}-\frac{5 a \,b^{4}}{2 x^{2}}+b^{5} \ln \left(x \right)$	56
norman	$\frac{-\frac{1}{10} a^{5}-\frac{5}{2} a \,b^{4} x^{8}-\frac{5}{2} a^{2} b^{3} x^{6}-\frac{5}{3} a^{3} b^{2} x^{4}-\frac{5}{8} a^{4} b \,x^{2}}{x^{10}}+b^{5} \ln \left(x \right)$	58
risch	$\frac{-\frac{1}{10} a^{5}-\frac{5}{2} a \,b^{4} x^{8}-\frac{5}{2} a^{2} b^{3} x^{6}-\frac{5}{3} a^{3} b^{2} x^{4}-\frac{5}{8} a^{4} b \,x^{2}}{x^{10}}+b^{5} \ln \left(x \right)$	58
parallelrisch	$\frac{120 b^{5} \ln \left(x \right) x^{10}-300 a \,b^{4} x^{8}-300 a^{2} b^{3} x^{6}-200 a^{3} b^{2} x^{4}-75 a^{4} b \,x^{2}-12 a^{5}}{120 x^{10}}$	62

input
```
int((b*x^2+a)^5/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^5/x^10-5/8*a^4*b/x^8-5/3*a^3*b^2/x^6-5/2*a^2*b^3/x^4-5/2*a*b^4/x^2
+b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = \frac{120\,b^5x^{10}\log(x)-300\,ab^4x^8-300\,a^2b^3x^6-200\,a^3b^2x^4-75\,a^4bx^2-12\,a^5}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5/x^11,x, algorithm="fricas")
```

output
```
1/120*(120*b^5*x^10*log(x) - 300*a*b^4*x^8 - 300*a^2*b^3*x^6 - 200*a^3*b^2
*x^4 - 75*a^4*b*x^2 - 12*a^5)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = b^5\log(x) + \frac{-12a^5-75a^4bx^2-200a^3b^2x^4-300a^2b^3x^6-300ab^4x^8}{120x^{10}}$$

input
```
integrate((b*x**2+a)**5/x**11,x)
```

output
```
b**5*log(x) + (-12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 300*a**2*b
**3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = \frac{1}{2}\,b^5\log(x^2) - \frac{300\,ab^4x^8+300\,a^2b^3x^6+200\,a^3b^2x^4+75\,a^4bx^2+12\,a^5}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5/x^11,x, algorithm="maxima")
```

output
```
1/2*b^5*log(x^2) - 1/120*(300*a*b^4*x^8 + 300*a^2*b^3*x^6 + 200*a^3*b^2*x^
4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = \frac{1}{2}\,b^5\log\left(x^2\right) - \frac{137\,b^5x^{10} + 300\,ab^4x^8 + 300\,a^2b^3x^6 + 200\,a^3b^2x^4 + 75\,a^4bx^2 + 12\,a^5}{120\,x^{10}}$$

input
```
integrate((b*x^2+a)^5/x^11,x, algorithm="giac")
```

output
```
1/2*b^5*log(x^2) - 1/120*(137*b^5*x^10 + 300*a*b^4*x^8 + 300*a^2*b^3*x^6 +
 200*a^3*b^2*x^4 + 75*a^4*b*x^2 + 12*a^5)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = b^5\,\ln(x) - \frac{\frac{a^5}{10} + \frac{5\,a^4\,b\,x^2}{8} + \frac{5\,a^3\,b^2\,x^4}{3} + \frac{5\,a^2\,b^3\,x^6}{2} + \frac{5\,a\,b^4\,x^8}{2}}{x^{10}}$$

input
```
int((a + b*x^2)^5/x^11,x)
```

output
```
b^5*log(x) - (a^5/10 + (5*a^4*b*x^2)/8 + (5*a*b^4*x^8)/2 + (5*a^3*b^2*x^4)
/3 + (5*a^2*b^3*x^6)/2)/x^10
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{11}}\,dx = \frac{120\log(x)\,b^5x^{10} - 12a^5 - 75a^4b\,x^2 - 200a^3b^2x^4 - 300a^2b^3x^6 - 300a\,b^4x^8}{120x^{10}}$$

input
```
int((b*x^2+a)^5/x^11,x)
```

output
```
(120*log(x)*b**5*x**10 - 12*a**5 - 75*a**4*b*x**2 - 200*a**3*b**2*x**4 - 3
00*a**2*b**3*x**6 - 300*a*b**4*x**8)/(120*x**10)
```

3.65 $\int \frac{(a+bx^2)^5}{x^{13}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{(a+bx^2)^6}{12ax^{12}}$$

output
```
-1/12*(b*x^2+a)^6/a/x^12
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.63

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{a^5}{12x^{12}} - \frac{a^4b}{2x^{10}} - \frac{5a^3b^2}{4x^8} - \frac{5a^2b^3}{3x^6} - \frac{5ab^4}{4x^4} - \frac{b^5}{2x^2}$$

input
```
Integrate[(a + b*x^2)^5/x^13,x]
```

output
```
-1/12*a^5/x^12 - (a^4*b)/(2*x^10) - (5*a^3*b^2)/(4*x^8) - (5*a^2*b^3)/(3*x
^6) - (5*a*b^4)/(4*x^4) - b^5/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx$$

$$\Big\downarrow \; 242$$

$$-\frac{(a+bx^2)^6}{12ax^{12}}$$

input
```
Int[(a + b*x^2)^5/x^13,x]
```

output
```
-1/12*(a + b*x^2)^6/(a*x^12)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 3.05

method	result	size
gosper	$-\frac{6b^5x^{10}+15a\,b^4x^8+20a^2b^3x^6+15a^3b^2x^4+6a^4b\,x^2+a^5}{12x^{12}}$	58
default	$-\frac{b^5}{2x^2}-\frac{5a\,b^4}{4x^4}-\frac{5a^3b^2}{4x^8}-\frac{a^4b}{2x^{10}}-\frac{5a^2b^3}{3x^6}-\frac{a^5}{12x^{12}}$	58
orering	$-\frac{6b^5x^{10}+15a\,b^4x^8+20a^2b^3x^6+15a^3b^2x^4+6a^4b\,x^2+a^5}{12x^{12}}$	58
norman	$\frac{-\frac{1}{2}b^5x^{10}-\frac{5}{4}a\,b^4x^8-\frac{5}{3}a^2b^3x^6-\frac{5}{4}a^3b^2x^4-\frac{1}{2}a^4b\,x^2-\frac{1}{12}a^5}{x^{12}}$	59
risch	$\frac{-\frac{1}{2}b^5x^{10}-\frac{5}{4}a\,b^4x^8-\frac{5}{3}a^2b^3x^6-\frac{5}{4}a^3b^2x^4-\frac{1}{2}a^4b\,x^2-\frac{1}{12}a^5}{x^{12}}$	59
parallelrisch	$\frac{-6b^5x^{10}-15a\,b^4x^8-20a^2b^3x^6-15a^3b^2x^4-6a^4b\,x^2-a^5}{12x^{12}}$	60

input
```
int((b*x^2+a)^5/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(6*b^5*x^10+15*a*b^4*x^8+20*a^2*b^3*x^6+15*a^3*b^2*x^4+6*a^4*b*x^2+a
^5)/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{6\,b^5x^{10}+15\,ab^4x^8+20\,a^2b^3x^6+15\,a^3b^2x^4+6\,a^4bx^2+a^5}{12\,x^{12}}$$

input
```
integrate((b*x^2+a)^5/x^13,x, algorithm="fricas")
```

output
```
-1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4
*b*x^2 + a^5)/x^12
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = \frac{-a^5 - 6a^4bx^2 - 15a^3b^2x^4 - 20a^2b^3x^6 - 15ab^4x^8 - 6b^5x^{10}}{12x^{12}}$$

input
```
integrate((b*x**2+a)**5/x**13,x)
```

output
```
(-a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b**4
*x**8 - 6*b**5*x**10)/(12*x**12)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{6\,b^5x^{10} + 15\,ab^4x^8 + 20\,a^2b^3x^6 + 15\,a^3b^2x^4 + 6\,a^4bx^2 + a^5}{12\,x^{12}}$$

input
```
integrate((b*x^2+a)^5/x^13,x, algorithm="maxima")
```

output
```
-1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4
*b*x^2 + a^5)/x^12
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{6\,b^5x^{10} + 15\,ab^4x^8 + 20\,a^2b^3x^6 + 15\,a^3b^2x^4 + 6\,a^4bx^2 + a^5}{12\,x^{12}}$$

input
```
integrate((b*x^2+a)^5/x^13,x, algorithm="giac")
```

output
```
-1/12*(6*b^5*x^10 + 15*a*b^4*x^8 + 20*a^2*b^3*x^6 + 15*a^3*b^2*x^4 + 6*a^4
*b*x^2 + a^5)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = -\frac{\frac{a^5}{12}+\frac{a^4\,b\,x^2}{2}+\frac{5\,a^3\,b^2\,x^4}{4}+\frac{5\,a^2\,b^3\,x^6}{3}+\frac{5\,a\,b^4\,x^8}{4}+\frac{b^5\,x^{10}}{2}}{x^{12}}$$

input
```
int((a + b*x^2)^5/x^13,x)
```

output
```
-(a^5/12 + (b^5*x^10)/2 + (a^4*b*x^2)/2 + (5*a*b^4*x^8)/4 + (5*a^3*b^2*x^4
)/4 + (5*a^2*b^3*x^6)/3)/x^12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^2)^5}{x^{13}}\,dx = \frac{-6b^5x^{10}-15a\,b^4x^8-20a^2b^3x^6-15a^3b^2x^4-6a^4b\,x^2-a^5}{12x^{12}}$$

input
```
int((b*x^2+a)^5/x^13,x)
```

output
```
( - a**5 - 6*a**4*b*x**2 - 15*a**3*b**2*x**4 - 20*a**2*b**3*x**6 - 15*a*b*
*4*x**8 - 6*b**5*x**10)/(12*x**12)
```

3.66 $\int \frac{(a+bx^2)^5}{x^{15}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{(a+bx^2)^6}{14ax^{14}} + \frac{b(a+bx^2)^6}{84a^2x^{12}}$$

output
```
-1/14*(b*x^2+a)^6/a/x^14+1/84*b*(b*x^2+a)^6/a^2/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{a^5}{14x^{14}} - \frac{5a^4b}{12x^{12}} - \frac{a^3b^2}{x^{10}} - \frac{5a^2b^3}{4x^8} - \frac{5ab^4}{6x^6} - \frac{b^5}{4x^4}$$

input
```
Integrate[(a + b*x^2)^5/x^15,x]
```

output
```
-1/14*a^5/x^14 - (5*a^4*b)/(12*x^12) - (a^3*b^2)/x^10 - (5*a^2*b^3)/(4*x^8
) - (5*a*b^4)/(6*x^6) - b^5/(4*x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{16}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^5}{x^{14}}dx^2}{7a}-\frac{(a+bx^2)^6}{7ax^{14}}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{2}\left(\frac{b(a+bx^2)^6}{42a^2x^{12}}-\frac{(a+bx^2)^6}{7ax^{14}}\right)$$

input
```
Int[(a + b*x^2)^5/x^15,x]
```

output
```
(-1/7*(a + b*x^2)^6/(a*x^14) + (b*(a + b*x^2)^6)/(42*a^2*x^12))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

method	result	size
default	$-\frac{b^{5}}{4x^{4}}-\frac{5a^{2}b^{3}}{4x^{8}}-\frac{a^{3}b^{2}}{x^{10}}-\frac{a^{5}}{14x^{14}}-\frac{5a\,b^{4}}{6x^{6}}-\frac{5a^{4}b}{12x^{12}}$	58
norman	$\frac{-\frac{1}{14}a^{5}-\frac{5}{12}a^{4}b\,x^{2}-a^{3}b^{2}x^{4}-\frac{5}{4}a^{2}b^{3}x^{6}-\frac{5}{6}a\,b^{4}x^{8}-\frac{1}{4}b^{5}x^{10}}{x^{14}}$	59
risch	$\frac{-\frac{1}{14}a^{5}-\frac{5}{12}a^{4}b\,x^{2}-a^{3}b^{2}x^{4}-\frac{5}{4}a^{2}b^{3}x^{6}-\frac{5}{6}a\,b^{4}x^{8}-\frac{1}{4}b^{5}x^{10}}{x^{14}}$	59
gosper	$-\frac{21b^{5}x^{10}+70a\,b^{4}x^{8}+105a^{2}b^{3}x^{6}+84a^{3}b^{2}x^{4}+35a^{4}b\,x^{2}+6a^{5}}{84x^{14}}$	60
parallelrisch	$\frac{-21b^{5}x^{10}-70a\,b^{4}x^{8}-105a^{2}b^{3}x^{6}-84a^{3}b^{2}x^{4}-35a^{4}b\,x^{2}-6a^{5}}{84x^{14}}$	60
orering	$-\frac{21b^{5}x^{10}+70a\,b^{4}x^{8}+105a^{2}b^{3}x^{6}+84a^{3}b^{2}x^{4}+35a^{4}b\,x^{2}+6a^{5}}{84x^{14}}$	60

input
```
int((b*x^2+a)^5/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^5/x^4-5/4*a^2*b^3/x^8-a^3*b^2/x^10-1/14*a^5/x^14-5/6*a*b^4/x^6-5/12
*a^4*b/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5/x^15,x, algorithm="fricas")
```

output
```
-1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*
a^4*b*x^2 + 6*a^5)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.58

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = \frac{-6a^5-35a^4bx^2-84a^3b^2x^4-105a^2b^3x^6-70ab^4x^8-21b^5x^{10}}{84x^{14}}$$

input
```
integrate((b*x**2+a)**5/x**15,x)
```

output
```
(-6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*a*
b**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5/x^15,x, algorithm="maxima")
```

output
```
-1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*
a^4*b*x^2 + 6*a^5)/x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{21\,b^5x^{10}+70\,ab^4x^8+105\,a^2b^3x^6+84\,a^3b^2x^4+35\,a^4bx^2+6\,a^5}{84\,x^{14}}$$

input
```
integrate((b*x^2+a)^5/x^15,x, algorithm="giac")
```

output
```
-1/84*(21*b^5*x^10 + 70*a*b^4*x^8 + 105*a^2*b^3*x^6 + 84*a^3*b^2*x^4 + 35*
a^4*b*x^2 + 6*a^5)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = -\frac{\frac{a^5}{14}+\frac{5\,a^4\,b\,x^2}{12}+a^3\,b^2\,x^4+\frac{5\,a^2\,b^3\,x^6}{4}+\frac{5\,a\,b^4\,x^8}{6}+\frac{b^5\,x^{10}}{4}}{x^{14}}$$

input
```
int((a + b*x^2)^5/x^15,x)
```

output
```
-(a^5/14 + (b^5*x^10)/4 + (5*a^4*b*x^2)/12 + (5*a*b^4*x^8)/6 + a^3*b^2*x^4
 + (5*a^2*b^3*x^6)/4)/x^14
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^5}{x^{15}}\,dx = \frac{-21b^5x^{10}-70a\,b^4x^8-105a^2b^3x^6-84a^3b^2x^4-35a^4b\,x^2-6a^5}{84x^{14}}$$

input
```
int((b*x^2+a)^5/x^15,x)
```

output
```
( - 6*a**5 - 35*a**4*b*x**2 - 84*a**3*b**2*x**4 - 105*a**2*b**3*x**6 - 70*
a*b**4*x**8 - 21*b**5*x**10)/(84*x**14)
```

3.67 $\int \frac{(a+bx^2)^5}{x^{17}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{(a+bx^2)^6}{16ax^{16}} + \frac{b(a+bx^2)^6}{56a^2x^{14}} - \frac{b^2(a+bx^2)^6}{336a^3x^{12}}$$

output
```
-1/16*(b*x^2+a)^6/a/x^16+1/56*b*(b*x^2+a)^6/a^2/x^14-1/336*b^2*(b*x^2+a)^6
/a^3/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{a^5}{16x^{16}} - \frac{5a^4b}{14x^{14}} - \frac{5a^3b^2}{6x^{12}} - \frac{a^2b^3}{x^{10}} - \frac{5ab^4}{8x^8} - \frac{b^5}{6x^6}$$

input
```
Integrate[(a + b*x^2)^5/x^17,x]
```

output
```
-1/16*a^5/x^16 - (5*a^4*b)/(14*x^14) - (5*a^3*b^2)/(6*x^12) - (a^2*b^3)/x^
10 - (5*a*b^4)/(8*x^8) - b^5/(6*x^6)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {243, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{17}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{18}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^5}{x^{16}}dx^2}{4a}-\frac{(a+bx^2)^6}{8ax^{16}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{b\int \frac{(bx^2+a)^5}{x^{14}}dx^2}{7a}-\frac{(a+bx^2)^6}{7ax^{14}}\right)}{4a}-\frac{(a+bx^2)^6}{8ax^{16}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{b(a+bx^2)^6}{42a^2x^{12}}-\frac{(a+bx^2)^6}{7ax^{14}}\right)}{4a}-\frac{(a+bx^2)^6}{8ax^{16}}\right)$$

input
```
Int[(a + b*x^2)^5/x^17,x]
```

output
```
(-1/8*(a + b*x^2)^6/(a*x^16) - (b*(-1/7*(a + b*x^2)^6/(a*x^14) + (b*(a + b
*x^2)^6)/(42*a^2*x^12)))/(4*a))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{5 a\, b^{4}}{8 x^{8}}-\frac{a^{2} b^{3}}{x^{10}}-\frac{5 a^{4} b}{14 x^{14}}-\frac{b^{5}}{6 x^{6}}-\frac{a^{5}}{16 x^{16}}-\frac{5 a^{3} b^{2}}{6 x^{12}}$	58
norman	$\frac{-\frac{1}{16} a^{5}-\frac{5}{14} a^{4} b\, x^{2}-\frac{5}{6} a^{3} b^{2} x^{4}-a^{2} b^{3} x^{6}-\frac{5}{8} a\, b^{4} x^{8}-\frac{1}{6} b^{5} x^{10}}{x^{16}}$	59
risch	$\frac{-\frac{1}{16} a^{5}-\frac{5}{14} a^{4} b\, x^{2}-\frac{5}{6} a^{3} b^{2} x^{4}-a^{2} b^{3} x^{6}-\frac{5}{8} a\, b^{4} x^{8}-\frac{1}{6} b^{5} x^{10}}{x^{16}}$	59
gosper	$-\frac{56 b^{5} x^{10}+210 a\, b^{4} x^{8}+336 a^{2} b^{3} x^{6}+280 a^{3} b^{2} x^{4}+120 a^{4} b\, x^{2}+21 a^{5}}{336 x^{16}}$	60
parallelrisch	$\frac{-56 b^{5} x^{10}-210 a\, b^{4} x^{8}-336 a^{2} b^{3} x^{6}-280 a^{3} b^{2} x^{4}-120 a^{4} b\, x^{2}-21 a^{5}}{336 x^{16}}$	60
orering	$-\frac{56 b^{5} x^{10}+210 a\, b^{4} x^{8}+336 a^{2} b^{3} x^{6}+280 a^{3} b^{2} x^{4}+120 a^{4} b\, x^{2}+21 a^{5}}{336 x^{16}}$	60

input
```
int((b*x^2+a)^5/x^17,x,method=_RETURNVERBOSE)
```

output
```
-5/8*a*b^4/x^8-a^2*b^3/x^10-5/14*a^4*b/x^14-1/6*b^5/x^6-1/16*a^5/x^16-5/6*
a^3*b^2/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5/x^17,x, algorithm="fricas")
```

output
```
-1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 +
120*a^4*b*x^2 + 21*a^5)/x^16
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = \frac{-21a^5-120a^4bx^2-280a^3b^2x^4-336a^2b^3x^6-210ab^4x^8-56b^5x^{10}}{336x^{16}}$$

input
```
integrate((b*x**2+a)**5/x**17,x)
```

output
```
(-21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 - 21
0*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5/x^17,x, algorithm="maxima")
```

output
```
-1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 +
120*a^4*b*x^2 + 21*a^5)/x^16
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{56\,b^5x^{10}+210\,ab^4x^8+336\,a^2b^3x^6+280\,a^3b^2x^4+120\,a^4bx^2+21\,a^5}{336\,x^{16}}$$

input
```
integrate((b*x^2+a)^5/x^17,x, algorithm="giac")
```

output
```
-1/336*(56*b^5*x^10 + 210*a*b^4*x^8 + 336*a^2*b^3*x^6 + 280*a^3*b^2*x^4 +
120*a^4*b*x^2 + 21*a^5)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = -\frac{\frac{a^5}{16}+\frac{5\,a^4\,b\,x^2}{14}+\frac{5\,a^3\,b^2\,x^4}{6}+a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{8}+\frac{b^5\,x^{10}}{6}}{x^{16}}$$

input
```
int((a + b*x^2)^5/x^17,x)
```

output
```
-(a^5/16 + (b^5*x^10)/6 + (5*a^4*b*x^2)/14 + (5*a*b^4*x^8)/8 + (5*a^3*b^2*
x^4)/6 + a^2*b^3*x^6)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{17}}\,dx = \frac{-56b^5x^{10}-210a\,b^4x^8-336a^2b^3x^6-280a^3b^2x^4-120a^4b\,x^2-21a^5}{336x^{16}}$$

input
```
int((b*x^2+a)^5/x^17,x)
```

output
```
( - 21*a**5 - 120*a**4*b*x**2 - 280*a**3*b**2*x**4 - 336*a**2*b**3*x**6 -
210*a*b**4*x**8 - 56*b**5*x**10)/(336*x**16)
```

3.68 $\int \frac{(a+bx^2)^5}{x^{19}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

output
```
-1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4
/x^10-1/8*b^5/x^8
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{ab^4}{2x^{10}}-\frac{b^5}{8x^8}$$

input
```
Integrate[(a + b*x^2)^5/x^19,x]
```

output
```
-1/18*a^5/x^18 - (5*a^4*b)/(16*x^16) - (5*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/
(6*x^12) - (a*b^4)/(2*x^10) - b^5/(8*x^8)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{19}}dx$$
$$\Big\downarrow \ 243$$
$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{20}}dx^2$$
$$\Big\downarrow \ 53$$
$$\frac{1}{2}\int \left(\frac{a^5}{x^{20}}+\frac{5ba^4}{x^{18}}+\frac{10b^2a^3}{x^{16}}+\frac{10b^3a^2}{x^{14}}+\frac{5b^4a}{x^{12}}+\frac{b^5}{x^{10}}\right)dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\left(-\frac{a^5}{9x^{18}}-\frac{5a^4b}{8x^{16}}-\frac{10a^3b^2}{7x^{14}}-\frac{5a^2b^3}{3x^{12}}-\frac{ab^4}{x^{10}}-\frac{b^5}{4x^8}\right)$$

input
```
Int[(a + b*x^2)^5/x^19,x]
```

output
```
(-1/9*a^5/x^18 - (5*a^4*b)/(8*x^16) - (10*a^3*b^2)/(7*x^14) - (5*a^2*b^3)/
(3*x^12) - (a*b^4)/x^10 - b^5/(4*x^8))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^5}{18x^{18}}-\frac{5a^4b}{16x^{16}}-\frac{5a^3b^2}{7x^{14}}-\frac{5a^2b^3}{6x^{12}}-\frac{a\,b^4}{2x^{10}}-\frac{b^5}{8x^8}$	58
norman	$\frac{-\frac{1}{18}a^5-\frac{5}{16}a^4b\,x^2-\frac{5}{7}a^3b^2x^4-\frac{5}{6}a^2b^3x^6-\frac{1}{2}a\,b^4x^8-\frac{1}{8}b^5x^{10}}{x^{18}}$	59
risch	$\frac{-\frac{1}{18}a^5-\frac{5}{16}a^4b\,x^2-\frac{5}{7}a^3b^2x^4-\frac{5}{6}a^2b^3x^6-\frac{1}{2}a\,b^4x^8-\frac{1}{8}b^5x^{10}}{x^{18}}$	59
gosper	$-\frac{126b^5x^{10}+504a\,b^4x^8+840a^2b^3x^6+720a^3b^2x^4+315a^4b\,x^2+56a^5}{1008x^{18}}$	60
parallelrisch	$\frac{-126b^5x^{10}-504a\,b^4x^8-840a^2b^3x^6-720a^3b^2x^4-315a^4b\,x^2-56a^5}{1008x^{18}}$	60
orering	$-\frac{126b^5x^{10}+504a\,b^4x^8+840a^2b^3x^6+720a^3b^2x^4+315a^4b\,x^2+56a^5}{1008x^{18}}$	60

input
```
int((b*x^2+a)^5/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*a^5/x^18-5/16*a^4*b/x^16-5/7*a^3*b^2/x^14-5/6*a^2*b^3/x^12-1/2*a*b^4
/x^10-1/8*b^5/x^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

input
```
integrate((b*x^2+a)^5/x^19,x, algorithm="fricas")
```

output
```
-1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4
+ 315*a^4*b*x^2 + 56*a^5)/x^18
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = \frac{-56a^5 - 315a^4bx^2 - 720a^3b^2x^4 - 840a^2b^3x^6 - 504ab^4x^8 - 126b^5x^{10}}{1008x^{18}}$$

input
```
integrate((b*x**2+a)**5/x**19,x)
```

output
```
(-56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 - 50
4*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{126\,b^5x^{10} + 504\,ab^4x^8 + 840\,a^2b^3x^6 + 720\,a^3b^2x^4 + 315\,a^4bx^2 + 56\,a^5}{1008\,x^{18}}$$

input
```
integrate((b*x^2+a)^5/x^19,x, algorithm="maxima")
```

output
```
-1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4
+ 315*a^4*b*x^2 + 56*a^5)/x^18
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{126\,b^5x^{10}+504\,ab^4x^8+840\,a^2b^3x^6+720\,a^3b^2x^4+315\,a^4bx^2+56\,a^5}{1008\,x^{18}}$$

input
```
integrate((b*x^2+a)^5/x^19,x, algorithm="giac")
```

output
```
-1/1008*(126*b^5*x^10 + 504*a*b^4*x^8 + 840*a^2*b^3*x^6 + 720*a^3*b^2*x^4
+ 315*a^4*b*x^2 + 56*a^5)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = -\frac{\frac{a^5}{18}+\frac{5\,a^4\,b\,x^2}{16}+\frac{5\,a^3\,b^2\,x^4}{7}+\frac{5\,a^2\,b^3\,x^6}{6}+\frac{a\,b^4\,x^8}{2}+\frac{b^5\,x^{10}}{8}}{x^{18}}$$

input
```
int((a + b*x^2)^5/x^19,x)
```

output
```
-(a^5/18 + (b^5*x^10)/8 + (5*a^4*b*x^2)/16 + (a*b^4*x^8)/2 + (5*a^3*b^2*x^
4)/7 + (5*a^2*b^3*x^6)/6)/x^18
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{19}}\,dx = \frac{-126b^5x^{10}-504a\,b^4x^8-840a^2b^3x^6-720a^3b^2x^4-315a^4b\,x^2-56a^5}{1008x^{18}}$$

input
```
int((b*x^2+a)^5/x^19,x)
```

output
```
( - 56*a**5 - 315*a**4*b*x**2 - 720*a**3*b**2*x**4 - 840*a**2*b**3*x**6 -
504*a*b**4*x**8 - 126*b**5*x**10)/(1008*x**18)
```

3.69 $\int \frac{(a+bx^2)^5}{x^{21}} dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^2)^5}{x^{21}} dx = -\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

output
```
-1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^
4/x^12-1/10*b^5/x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{21}} dx = -\frac{a^5}{20x^{20}} - \frac{5a^4b}{18x^{18}} - \frac{5a^3b^2}{8x^{16}} - \frac{5a^2b^3}{7x^{14}} - \frac{5ab^4}{12x^{12}} - \frac{b^5}{10x^{10}}$$

input
```
Integrate[(a + b*x^2)^5/x^21,x]
```

output
```
-1/20*a^5/x^20 - (5*a^4*b)/(18*x^18) - (5*a^3*b^2)/(8*x^16) - (5*a^2*b^3)/
(7*x^14) - (5*a*b^4)/(12*x^12) - b^5/(10*x^10)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{21}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^5}{x^{22}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^5}{x^{22}}+\frac{5ba^4}{x^{20}}+\frac{10b^2a^3}{x^{18}}+\frac{10b^3a^2}{x^{16}}+\frac{5b^4a}{x^{14}}+\frac{b^5}{x^{12}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5}{10x^{20}}-\frac{5a^4b}{9x^{18}}-\frac{5a^3b^2}{4x^{16}}-\frac{10a^2b^3}{7x^{14}}-\frac{5ab^4}{6x^{12}}-\frac{b^5}{5x^{10}}\right)$$

input
```
Int[(a + b*x^2)^5/x^21,x]
```

output
```
(-1/10*a^5/x^20 - (5*a^4*b)/(9*x^18) - (5*a^3*b^2)/(4*x^16) - (10*a^2*b^3)
/(7*x^14) - (5*a*b^4)/(6*x^12) - b^5/(5*x^10))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^5}{20x^{20}}-\frac{5a^4b}{18x^{18}}-\frac{5a^3b^2}{8x^{16}}-\frac{5a^2b^3}{7x^{14}}-\frac{5a\,b^4}{12x^{12}}-\frac{b^5}{10x^{10}}$	58
norman	$\frac{-\frac{1}{20}a^5-\frac{5}{18}a^4b\,x^2-\frac{5}{8}a^3b^2x^4-\frac{5}{7}a^2b^3x^6-\frac{5}{12}a\,b^4x^8-\frac{1}{10}b^5x^{10}}{x^{20}}$	59
risch	$\frac{-\frac{1}{20}a^5-\frac{5}{18}a^4b\,x^2-\frac{5}{8}a^3b^2x^4-\frac{5}{7}a^2b^3x^6-\frac{5}{12}a\,b^4x^8-\frac{1}{10}b^5x^{10}}{x^{20}}$	59
gosper	$-\frac{252b^5x^{10}+1050a\,b^4x^8+1800a^2b^3x^6+1575a^3b^2x^4+700a^4b\,x^2+126a^5}{2520x^{20}}$	60
parallelrisch	$\frac{-252b^5x^{10}-1050a\,b^4x^8-1800a^2b^3x^6-1575a^3b^2x^4-700a^4b\,x^2-126a^5}{2520x^{20}}$	60
orering	$-\frac{252b^5x^{10}+1050a\,b^4x^8+1800a^2b^3x^6+1575a^3b^2x^4+700a^4b\,x^2+126a^5}{2520x^{20}}$	60

input
```
int((b*x^2+a)^5/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/20*a^5/x^20-5/18*a^4*b/x^18-5/8*a^3*b^2/x^16-5/7*a^2*b^3/x^14-5/12*a*b^
4/x^12-1/10*b^5/x^10
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = -\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

input
```
integrate((b*x^2+a)^5/x^21,x, algorithm="fricas")
```

output
```
-1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x
^4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = \frac{-126a^5-700a^4bx^2-1575a^3b^2x^4-1800a^2b^3x^6-1050ab^4x^8-252b^5x^{10}}{2520x^{20}}$$

input
```
integrate((b*x**2+a)**5/x**21,x)
```

output
```
(-126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6 -
 1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = -\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

input
```
integrate((b*x^2+a)^5/x^21,x, algorithm="maxima")
```

output
```
-1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x
^4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = -\frac{252\,b^5x^{10}+1050\,ab^4x^8+1800\,a^2b^3x^6+1575\,a^3b^2x^4+700\,a^4bx^2+126\,a^5}{2520\,x^{20}}$$

input
```
integrate((b*x^2+a)^5/x^21,x, algorithm="giac")
```

output
```
-1/2520*(252*b^5*x^10 + 1050*a*b^4*x^8 + 1800*a^2*b^3*x^6 + 1575*a^3*b^2*x
^4 + 700*a^4*b*x^2 + 126*a^5)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = -\frac{\frac{a^5}{20}+\frac{5\,a^4\,b\,x^2}{18}+\frac{5\,a^3\,b^2\,x^4}{8}+\frac{5\,a^2\,b^3\,x^6}{7}+\frac{5\,a\,b^4\,x^8}{12}+\frac{b^5\,x^{10}}{10}}{x^{20}}$$

input
```
int((a + b*x^2)^5/x^21,x)
```

output
```
-(a^5/20 + (b^5*x^10)/10 + (5*a^4*b*x^2)/18 + (5*a*b^4*x^8)/12 + (5*a^3*b^
2*x^4)/8 + (5*a^2*b^3*x^6)/7)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{21}}\,dx = \frac{-252b^5x^{10}-1050a\,b^4x^8-1800a^2b^3x^6-1575a^3b^2x^4-700a^4b\,x^2-126a^5}{2520x^{20}}$$

input
```
int((b*x^2+a)^5/x^21,x)
```

output
```
( - 126*a**5 - 700*a**4*b*x**2 - 1575*a**3*b**2*x**4 - 1800*a**2*b**3*x**6
 - 1050*a*b**4*x**8 - 252*b**5*x**10)/(2520*x**20)
```

3.70 $\int x^8(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^8\left(a+bx^2\right)^5\,dx=\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

output
```
1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4
*x^17+1/19*b^5*x^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^8\left(a+bx^2\right)^5\,dx=\frac{a^5x^9}{9}+\frac{5}{11}a^4bx^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}ab^4x^{17}+\frac{b^5x^{19}}{19}$$

input
```
Integrate[x^8*(a + b*x^2)^5,x]
```

output
```
(a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/
3 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b x^2\right)^5 \, dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^5 x^8+5 a^4 b x^{10}+10 a^3 b^2 x^{12}+10 a^2 b^3 x^{14}+5 a b^4 x^{16}+b^5 x^{18}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^5 x^9}{9}+\frac{5}{11} a^4 b x^{11}+\frac{10}{13} a^3 b^2 x^{13}+\frac{2}{3} a^2 b^3 x^{15}+\frac{5}{17} a b^4 x^{17}+\frac{b^5 x^{19}}{19}$$

input
```
Int[x^8*(a + b*x^2)^5,x]
```

output
```
(a^5*x^9)/9 + (5*a^4*b*x^11)/11 + (10*a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/
3 + (5*a*b^4*x^17)/17 + (b^5*x^19)/19
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
default	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
norman	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
risch	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
parallelrisch	$\frac{1}{9}a^5x^9+\frac{5}{11}a^4b\,x^{11}+\frac{10}{13}a^3b^2x^{13}+\frac{2}{3}a^2b^3x^{15}+\frac{5}{17}a\,b^4x^{17}+\frac{1}{19}b^5x^{19}$	58
orering	$\frac{x^9(21879b^5x^{10}+122265a\,b^4x^8+277134a^2b^3x^6+319770a^3b^2x^4+188955a^4b\,x^2+46189a^5)}{415701}$	60

input
```
int(x^8*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^5*x^9+5/11*a^4*b*x^11+10/13*a^3*b^2*x^13+2/3*a^2*b^3*x^15+5/17*a*b^4
*x^17+1/19*b^5*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^8(a+bx^2)^5\,dx=\frac{1}{19}\,b^5x^{19}+\frac{5}{17}\,ab^4x^{17}+\frac{2}{3}\,a^2b^3x^{15}+\frac{10}{13}\,a^3b^2x^{13}+\frac{5}{11}\,a^4bx^{11}+\frac{1}{9}\,a^5x^9$$

input
```
integrate(x^8*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 +
5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^8 (a + bx^2)^5 \, dx = \frac{a^5x^9}{9} + \frac{5a^4bx^{11}}{11} + \frac{10a^3b^2x^{13}}{13} + \frac{2a^2b^3x^{15}}{3} + \frac{5ab^4x^{17}}{17} + \frac{b^5x^{19}}{19}$$

input
```
integrate(x**8*(b*x**2+a)**5,x)
```

output
```
a**5*x**9/9 + 5*a**4*b*x**11/11 + 10*a**3*b**2*x**13/13 + 2*a**2*b**3*x**1
5/3 + 5*a*b**4*x**17/17 + b**5*x**19/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^8 (a + bx^2)^5 \, dx = \frac{1}{19} b^5x^{19} + \frac{5}{17} ab^4x^{17} + \frac{2}{3} a^2b^3x^{15} + \frac{10}{13} a^3b^2x^{13} + \frac{5}{11} a^4bx^{11} + \frac{1}{9} a^5x^9$$

input
```
integrate(x^8*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 +
5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^8 (a + bx^2)^5 \, dx = \frac{1}{19} b^5x^{19} + \frac{5}{17} ab^4x^{17} + \frac{2}{3} a^2b^3x^{15} + \frac{10}{13} a^3b^2x^{13} + \frac{5}{11} a^4bx^{11} + \frac{1}{9} a^5x^9$$

input
```
integrate(x^8*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/19*b^5*x^19 + 5/17*a*b^4*x^17 + 2/3*a^2*b^3*x^15 + 10/13*a^3*b^2*x^13 +
5/11*a^4*b*x^11 + 1/9*a^5*x^9
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^8 (a+bx^2)^5 \, dx = \frac{a^5 x^9}{9} + \frac{5 a^4 b x^{11}}{11} + \frac{10 a^3 b^2 x^{13}}{13} + \frac{2 a^2 b^3 x^{15}}{3} + \frac{5 a b^4 x^{17}}{17} + \frac{b^5 x^{19}}{19}$$

input
```
int(x^8*(a + b*x^2)^5,x)
```

output
```
(a^5*x^9)/9 + (b^5*x^19)/19 + (5*a^4*b*x^11)/11 + (5*a*b^4*x^17)/17 + (10*
a^3*b^2*x^13)/13 + (2*a^2*b^3*x^15)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^8 (a+bx^2)^5 \, dx = \frac{x^9 (21879 b^5 x^{10} + 122265 a b^4 x^8 + 277134 a^2 b^3 x^6 + 319770 a^3 b^2 x^4 + 188955 a^4 b x^2 + 46189 a^5)}{415701}$$

input
```
int(x^8*(b*x^2+a)^5,x)
```

output
```
(x**9*(46189*a**5 + 188955*a**4*b*x**2 + 319770*a**3*b**2*x**4 + 277134*a*
*2*b**3*x**6 + 122265*a*b**4*x**8 + 21879*b**5*x**10))/415701
```

3.71 $\int x^6(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^6\left(a+bx^2\right)^5\,dx=\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

output
```
1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*
x^15+1/17*b^5*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^6\left(a+bx^2\right)^5\,dx=\frac{a^5x^7}{7}+\frac{5}{9}a^4bx^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}ab^4x^{15}+\frac{b^5x^{17}}{17}$$

input
```
Integrate[x^6*(a + b*x^2)^5,x]
```

output
```
(a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/1
3 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (a + bx^2)^5 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int (a^5x^6 + 5a^4bx^8 + 10a^3b^2x^{10} + 10a^2b^3x^{12} + 5ab^4x^{14} + b^5x^{16}) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5x^7}{7} + \frac{5}{9}a^4bx^9 + \frac{10}{11}a^3b^2x^{11} + \frac{10}{13}a^2b^3x^{13} + \frac{1}{3}ab^4x^{15} + \frac{b^5x^{17}}{17}$$

input
```
Int[x^6*(a + b*x^2)^5,x]
```

output
```
(a^5*x^7)/7 + (5*a^4*b*x^9)/9 + (10*a^3*b^2*x^11)/11 + (10*a^2*b^3*x^13)/1
3 + (a*b^4*x^15)/3 + (b^5*x^17)/17
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
default	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
norman	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
risch	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
parallelrisch	$\frac{1}{7}a^5x^7+\frac{5}{9}a^4b\,x^9+\frac{10}{11}a^3b^2x^{11}+\frac{10}{13}a^2b^3x^{13}+\frac{1}{3}a\,b^4x^{15}+\frac{1}{17}b^5x^{17}$	58
orering	$\frac{x^7\left(9009b^5x^{10}+51051a\,b^4x^8+117810a^2b^3x^6+139230a^3b^2x^4+85085a^4b\,x^2+21879a^5\right)}{153153}$	60

input
```
int(x^6*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^5*x^7+5/9*a^4*b*x^9+10/11*a^3*b^2*x^11+10/13*a^2*b^3*x^13+1/3*a*b^4*
x^15+1/17*b^5*x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^6\left(a+bx^2\right)^5\,dx=\frac{1}{17}\,b^5x^{17}+\frac{1}{3}\,ab^4x^{15}+\frac{10}{13}\,a^2b^3x^{13}+\frac{10}{11}\,a^3b^2x^{11}+\frac{5}{9}\,a^4bx^9+\frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^6 \left(a+bx^2\right)^5 \, dx = \frac{a^5x^7}{7} + \frac{5a^4bx^9}{9} + \frac{10a^3b^2x^{11}}{11} + \frac{10a^2b^3x^{13}}{13} + \frac{ab^4x^{15}}{3} + \frac{b^5x^{17}}{17}$$

input
```
integrate(x**6*(b*x**2+a)**5,x)
```

output
```
a**5*x**7/7 + 5*a**4*b*x**9/9 + 10*a**3*b**2*x**11/11 + 10*a**2*b**3*x**13
/13 + a*b**4*x**15/3 + b**5*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^6 \left(a+bx^2\right)^5 \, dx = \frac{1}{17}\,b^5x^{17} + \frac{1}{3}\,ab^4x^{15} + \frac{10}{13}\,a^2b^3x^{13} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{9}\,a^4bx^9 + \frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^6 \left(a+bx^2\right)^5 \, dx = \frac{1}{17}\,b^5x^{17} + \frac{1}{3}\,ab^4x^{15} + \frac{10}{13}\,a^2b^3x^{13} + \frac{10}{11}\,a^3b^2x^{11} + \frac{5}{9}\,a^4bx^9 + \frac{1}{7}\,a^5x^7$$

input
```
integrate(x^6*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/17*b^5*x^17 + 1/3*a*b^4*x^15 + 10/13*a^2*b^3*x^13 + 10/11*a^3*b^2*x^11 +
 5/9*a^4*b*x^9 + 1/7*a^5*x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^6 \left(a+bx^2\right)^5 \, dx = \frac{a^5\,x^7}{7}+\frac{5\,a^4\,b\,x^9}{9}+\frac{10\,a^3\,b^2\,x^{11}}{11}+\frac{10\,a^2\,b^3\,x^{13}}{13}+\frac{a\,b^4\,x^{15}}{3}+\frac{b^5\,x^{17}}{17}$$

input
```
int(x^6*(a + b*x^2)^5,x)
```

output
```
(a^5*x^7)/7 + (b^5*x^17)/17 + (5*a^4*b*x^9)/9 + (a*b^4*x^15)/3 + (10*a^3*b
^2*x^11)/11 + (10*a^2*b^3*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^6 \left(a+bx^2\right)^5 \, dx = \frac{x^7 (9009 b^5 x^{10}+51051 a\, b^4 x^8+117810 a^2 b^3 x^6+139230 a^3 b^2 x^4+85085 a^4 b\, x^2+21879 a^5)}{153153}$$

input
```
int(x^6*(b*x^2+a)^5,x)
```

output
```
(x**7*(21879*a**5 + 85085*a**4*b*x**2 + 139230*a**3*b**2*x**4 + 117810*a**
2*b**3*x**6 + 51051*a*b**4*x**8 + 9009*b**5*x**10))/153153
```

3.72 $\int x^4(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int x^4\left(a+bx^2\right)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

output
```
1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x
^13+1/15*b^5*x^15
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^5\,dx=\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

input
```
Integrate[x^4*(a + b*x^2)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
+ (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^5 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^5x^4+5a^4bx^6+10a^3b^2x^8+10a^2b^3x^{10}+5ab^4x^{12}+b^5x^{14}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5x^5}{5}+\frac{5}{7}a^4bx^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}ab^4x^{13}+\frac{b^5x^{15}}{15}$$

input
```
Int[x^4*(a + b*x^2)^5,x]
```

output
```
(a^5*x^5)/5 + (5*a^4*b*x^7)/7 + (10*a^3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
+ (5*a*b^4*x^13)/13 + (b^5*x^15)/15
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
default	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
norman	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
risch	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
parallelrisch	$\frac{1}{5}a^5x^5+\frac{5}{7}a^4b\,x^7+\frac{10}{9}a^3b^2x^9+\frac{10}{11}a^2b^3x^{11}+\frac{5}{13}a\,b^4x^{13}+\frac{1}{15}b^5x^{15}$	58
orering	$\frac{x^5(3003b^5x^{10}+17325a\,b^4x^8+40950a^2b^3x^6+50050a^3b^2x^4+32175a^4b\,x^2+9009a^5)}{45045}$	60

input
```
int(x^4*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5*x^5+5/7*a^4*b*x^7+10/9*a^3*b^2*x^9+10/11*a^2*b^3*x^11+5/13*a*b^4*x
^13+1/15*b^5*x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4(a+bx^2)^5\,dx=\frac{1}{15}\,b^5x^{15}+\frac{5}{13}\,ab^4x^{13}+\frac{10}{11}\,a^2b^3x^{11}+\frac{10}{9}\,a^3b^2x^9+\frac{5}{7}\,a^4bx^7+\frac{1}{5}\,a^5x^5$$

input
```
integrate(x^4*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 +
5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^4 (a+bx^2)^5 \, dx = \frac{a^5x^5}{5} + \frac{5a^4bx^7}{7} + \frac{10a^3b^2x^9}{9} + \frac{10a^2b^3x^{11}}{11} + \frac{5ab^4x^{13}}{13} + \frac{b^5x^{15}}{15}$$

input
```
integrate(x**4*(b*x**2+a)**5,x)
```

output
```
a**5*x**5/5 + 5*a**4*b*x**7/7 + 10*a**3*b**2*x**9/9 + 10*a**2*b**3*x**11/1
1 + 5*a*b**4*x**13/13 + b**5*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a+bx^2)^5 \, dx = \frac{1}{15}b^5x^{15} + \frac{5}{13}ab^4x^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$$

input
```
integrate(x^4*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 +
5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a+bx^2)^5 \, dx = \frac{1}{15}b^5x^{15} + \frac{5}{13}ab^4x^{13} + \frac{10}{11}a^2b^3x^{11} + \frac{10}{9}a^3b^2x^9 + \frac{5}{7}a^4bx^7 + \frac{1}{5}a^5x^5$$

input
```
integrate(x^4*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/15*b^5*x^15 + 5/13*a*b^4*x^13 + 10/11*a^2*b^3*x^11 + 10/9*a^3*b^2*x^9 +
5/7*a^4*b*x^7 + 1/5*a^5*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^4 (a+bx^2)^5 \, dx = \frac{a^5 x^5}{5} + \frac{5 a^4 b x^7}{7} + \frac{10 a^3 b^2 x^9}{9} + \frac{10 a^2 b^3 x^{11}}{11} + \frac{5 a b^4 x^{13}}{13} + \frac{b^5 x^{15}}{15}$$

input
```
int(x^4*(a + b*x^2)^5,x)
```

output
```
(a^5*x^5)/5 + (b^5*x^15)/15 + (5*a^4*b*x^7)/7 + (5*a*b^4*x^13)/13 + (10*a^
3*b^2*x^9)/9 + (10*a^2*b^3*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int x^4 (a+bx^2)^5 \, dx = \frac{x^5(3003b^5x^{10} + 17325a\, b^4x^8 + 40950a^2b^3x^6 + 50050a^3b^2x^4 + 32175a^4b\, x^2 + 9009a^5)}{45045}$$

input
```
int(x^4*(b*x^2+a)^5,x)
```

output
```
(x**5*(9009*a**5 + 32175*a**4*b*x**2 + 50050*a**3*b**2*x**4 + 40950*a**2*b
**3*x**6 + 17325*a*b**4*x**8 + 3003*b**5*x**10))/45045
```

3.73 $\int x^2(a+bx^2)^5\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int x^2\left(a+bx^2\right)^5\,dx=\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

output
```
1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/
13*b^5*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^5\,dx=\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

input
```
Integrate[x^2*(a + b*x^2)^5,x]
```

output
```
(a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b
^4*x^11)/11 + (b^5*x^13)/13
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^5 \, dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^5x^2+5a^4bx^4+10a^3b^2x^6+10a^2b^3x^8+5ab^4x^{10}+b^5x^{12}\right) dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^5x^3}{3}+a^4bx^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}ab^4x^{11}+\frac{b^5x^{13}}{13}$$

input
```
Int[x^2*(a + b*x^2)^5,x]
```

output
```
(a^5*x^3)/3 + a^4*b*x^5 + (10*a^3*b^2*x^7)/7 + (10*a^2*b^3*x^9)/9 + (5*a*b
^4*x^11)/11 + (b^5*x^13)/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
default	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
norman	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
risch	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
parallelrisch	$\frac{1}{3}a^5x^3+a^4b\,x^5+\frac{10}{7}a^3b^2x^7+\frac{10}{9}a^2b^3x^9+\frac{5}{11}a\,b^4x^{11}+\frac{1}{13}b^5x^{13}$	57
orering	$\frac{x^3\left(693b^5x^{10}+4095a\,b^4x^8+10010a^2b^3x^6+12870a^3b^2x^4+9009a^4b\,x^2+3003a^5\right)}{9009}$	60

input
```
int(x^2*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^5*x^3+a^4*b*x^5+10/7*a^3*b^2*x^7+10/9*a^2*b^3*x^9+5/11*a*b^4*x^11+1/
13*b^5*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2\left(a+bx^2\right)^5\,dx=\frac{1}{13}\,b^5x^{13}+\frac{5}{11}\,ab^4x^{11}+\frac{10}{9}\,a^2b^3x^9+\frac{10}{7}\,a^3b^2x^7+a^4bx^5+\frac{1}{3}\,a^5x^3$$

input
```
integrate(x^2*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^
4*b*x^5 + 1/3*a^5*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^2 (a + bx^2)^5 \, dx = \frac{a^5x^3}{3} + a^4bx^5 + \frac{10a^3b^2x^7}{7} + \frac{10a^2b^3x^9}{9} + \frac{5ab^4x^{11}}{11} + \frac{b^5x^{13}}{13}$$

input
```
integrate(x**2*(b*x**2+a)**5,x)
```

output
```
a**5*x**3/3 + a**4*b*x**5 + 10*a**3*b**2*x**7/7 + 10*a**2*b**3*x**9/9 + 5*
a*b**4*x**11/11 + b**5*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx^2)^5 \, dx = \frac{1}{13} b^5x^{13} + \frac{5}{11} ab^4x^{11} + \frac{10}{9} a^2b^3x^9 + \frac{10}{7} a^3b^2x^7 + a^4bx^5 + \frac{1}{3} a^5x^3$$

input
```
integrate(x^2*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^
4*b*x^5 + 1/3*a^5*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx^2)^5 \, dx = \frac{1}{13} b^5x^{13} + \frac{5}{11} ab^4x^{11} + \frac{10}{9} a^2b^3x^9 + \frac{10}{7} a^3b^2x^7 + a^4bx^5 + \frac{1}{3} a^5x^3$$

input
```
integrate(x^2*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/13*b^5*x^13 + 5/11*a*b^4*x^11 + 10/9*a^2*b^3*x^9 + 10/7*a^3*b^2*x^7 + a^
4*b*x^5 + 1/3*a^5*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int x^2\left(a+bx^2\right)^5\,dx=\frac{a^5\,x^3}{3}+a^4\,b\,x^5+\frac{10\,a^3\,b^2\,x^7}{7}+\frac{10\,a^2\,b^3\,x^9}{9}+\frac{5\,a\,b^4\,x^{11}}{11}+\frac{b^5\,x^{13}}{13}$$

input
```
int(x^2*(a + b*x^2)^5,x)
```

output
```
(a^5*x^3)/3 + (b^5*x^13)/13 + a^4*b*x^5 + (5*a*b^4*x^11)/11 + (10*a^3*b^2*
x^7)/7 + (10*a^2*b^3*x^9)/9
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2\left(a+bx^2\right)^5\,dx =\frac{x^3(693b^5x^{10}+4095a\,b^4x^8+10010a^2b^3x^6+12870a^3b^2x^4+9009a^4b\,x^2+3003a^5)}{9009}$$

input
```
int(x^2*(b*x^2+a)^5,x)
```

output
```
(x**3*(3003*a**5 + 9009*a**4*b*x**2 + 12870*a**3*b**2*x**4 + 10010*a**2*b*
*3*x**6 + 4095*a*b**4*x**8 + 693*b**5*x**10))/9009
```

3.74 $\int (a+bx^2)^5 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 62

$$\int (a+bx^2)^5 \, dx = a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

output
```
a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*
x^11
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^5 \, dx = a^5x + \frac{5}{3}a^4bx^3 + 2a^3b^2x^5 + \frac{10}{7}a^2b^3x^7 + \frac{5}{9}ab^4x^9 + \frac{b^5x^{11}}{11}$$

input
```
Integrate[(a + b*x^2)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^
9)/9 + (b^5*x^11)/11
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^5 \, dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^5+5a^4bx^2+10a^3b^2x^4+10a^2b^3x^6+5ab^4x^8+b^5x^{10}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^5x+\frac{5}{3}a^4bx^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}ab^4x^9+\frac{b^5x^{11}}{11}$$

input
```
Int[(a + b*x^2)^5,x]
```

output
```
a^5*x + (5*a^4*b*x^3)/3 + 2*a^3*b^2*x^5 + (10*a^2*b^3*x^7)/7 + (5*a*b^4*x^
9)/9 + (b^5*x^11)/11
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

method	result	size
gosper	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
default	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
norman	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
risch	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
parallelrisch	$a^5x+\frac{5}{3}a^4b\,x^3+2a^3b^2x^5+\frac{10}{7}a^2b^3x^7+\frac{5}{9}a\,b^4x^9+\frac{1}{11}b^5x^{11}$	55
orering	$\frac{x(63b^5x^{10}+385a\,b^4x^8+990a^2b^3x^6+1386a^3b^2x^4+1155a^4b\,x^2+693a^5)}{693}$	58

input
```
int((b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*x+5/3*a^4*b*x^3+2*a^3*b^2*x^5+10/7*a^2*b^3*x^7+5/9*a*b^4*x^9+1/11*b^5*
x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \left(a+bx^2\right)^5\,dx=\frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

$$\int (a+bx^2)^5\,dx = a^5x+\frac{5a^4bx^3}{3}+2a^3b^2x^5+\frac{10a^2b^3x^7}{7}+\frac{5ab^4x^9}{9}+\frac{b^5x^{11}}{11}$$

input
```
integrate((b*x**2+a)**5,x)
```

output
```
a**5*x + 5*a**4*b*x**3/3 + 2*a**3*b**2*x**5 + 10*a**2*b**3*x**7/7 + 5*a*b*
*4*x**9/9 + b**5*x**11/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^5\,dx = \frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^5\,dx = \frac{1}{11}\,b^5x^{11}+\frac{5}{9}\,ab^4x^9+\frac{10}{7}\,a^2b^3x^7+2\,a^3b^2x^5+\frac{5}{3}\,a^4bx^3+a^5x$$

input
```
integrate((b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/11*b^5*x^11 + 5/9*a*b^4*x^9 + 10/7*a^2*b^3*x^7 + 2*a^3*b^2*x^5 + 5/3*a^4
*b*x^3 + a^5*x
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int (a+bx^2)^5\, dx = a^5\,x + \frac{5\,a^4\,b\,x^3}{3} + 2\,a^3\,b^2\,x^5 + \frac{10\,a^2\,b^3\,x^7}{7} + \frac{5\,a\,b^4\,x^9}{9} + \frac{b^5\,x^{11}}{11}$$

input
```
int((a + b*x^2)^5,x)
```

output
```
a^5*x + (b^5*x^11)/11 + (5*a^4*b*x^3)/3 + (5*a*b^4*x^9)/9 + 2*a^3*b^2*x^5
+ (10*a^2*b^3*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^5\, dx = \frac{x(63b^5x^{10} + 385a\,b^4x^8 + 990a^2b^3x^6 + 1386a^3b^2x^4 + 1155a^4b\,x^2 + 693a^5)}{693}$$

input
```
int((b*x^2+a)^5,x)
```

output
```
(x*(693*a**5 + 1155*a**4*b*x**2 + 1386*a**3*b**2*x**4 + 990*a**2*b**3*x**6
 + 385*a*b**4*x**8 + 63*b**5*x**10))/693
```

3.75 $\int \frac{(a+bx^2)^5}{x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = -\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

output
```
-a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = -\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

input
```
Integrate[(a + b*x^2)^5/x^2,x]
```

output
```
-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/
7 + (b^5*x^9)/9
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^2}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^5}{x^2}+5a^4b+10a^3b^2x^2+10a^2b^3x^4+5ab^4x^6+b^5x^8\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5}{x}+5a^4bx+\frac{10}{3}a^3b^2x^3+2a^2b^3x^5+\frac{5}{7}ab^4x^7+\frac{b^5x^9}{9}$$

input
```
Int[(a + b*x^2)^5/x^2,x]
```

output
```
-(a^5/x) + 5*a^4*b*x + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5 + (5*a*b^4*x^7)/
7 + (b^5*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{x}+5a^4bx+\frac{10a^3b^2x^3}{3}+2a^2b^3x^5+\frac{5a\,b^4x^7}{7}+\frac{b^5x^9}{9}$	56
risch	$-\frac{a^5}{x}+5a^4bx+\frac{10a^3b^2x^3}{3}+2a^2b^3x^5+\frac{5a\,b^4x^7}{7}+\frac{b^5x^9}{9}$	56
norman	$\frac{\frac{1}{9}b^5x^{10}+\frac{5}{7}a\,b^4x^8+2a^2b^3x^6+\frac{10}{3}a^3b^2x^4+5a^4b\,x^2-a^5}{x}$	59
gosper	$-\frac{-7b^5x^{10}-45a\,b^4x^8-126a^2b^3x^6-210a^3b^2x^4-315a^4b\,x^2+63a^5}{63x}$	60
parallelrisch	$\frac{7b^5x^{10}+45a\,b^4x^8+126a^2b^3x^6+210a^3b^2x^4+315a^4b\,x^2-63a^5}{63x}$	60
orering	$-\frac{-7b^5x^{10}-45a\,b^4x^8-126a^2b^3x^6-210a^3b^2x^4-315a^4b\,x^2+63a^5}{63x}$	60

input
```
int((b*x^2+a)^5/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^5/x+5*a^4*b*x+10/3*a^3*b^2*x^3+2*a^2*b^3*x^5+5/7*a*b^4*x^7+1/9*b^5*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^2}\,dx=\frac{7\,b^5x^{10}+45\,ab^4x^8+126\,a^2b^3x^6+210\,a^3b^2x^4+315\,a^4bx^2-63\,a^5}{63\,x}$$

input
```
integrate((b*x^2+a)^5/x^2,x, algorithm="fricas")
```

output
```
1/63*(7*b^5*x^10 + 45*a*b^4*x^8 + 126*a^2*b^3*x^6 + 210*a^3*b^2*x^4 + 315*
a^4*b*x^2 - 63*a^5)/x
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = -\frac{a^5}{x} + 5a^4bx + \frac{10a^3b^2x^3}{3} + 2a^2b^3x^5 + \frac{5ab^4x^7}{7} + \frac{b^5x^9}{9}$$

input
```
integrate((b*x**2+a)**5/x**2,x)
```

output
```
-a**5/x + 5*a**4*b*x + 10*a**3*b**2*x**3/3 + 2*a**2*b**3*x**5 + 5*a*b**4*x
**7/7 + b**5*x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{7}\,ab^4x^7 + 2\,a^2b^3x^5 + \frac{10}{3}\,a^3b^2x^3 + 5\,a^4bx - \frac{a^5}{x}$$

input
```
integrate((b*x^2+a)^5/x^2,x, algorithm="maxima")
```

output
```
1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
 - a^5/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = \frac{1}{9}\,b^5x^9 + \frac{5}{7}\,ab^4x^7 + 2\,a^2b^3x^5 + \frac{10}{3}\,a^3b^2x^3 + 5\,a^4bx - \frac{a^5}{x}$$

input
```
integrate((b*x^2+a)^5/x^2,x, algorithm="giac")
```

output
```
1/9*b^5*x^9 + 5/7*a*b^4*x^7 + 2*a^2*b^3*x^5 + 10/3*a^3*b^2*x^3 + 5*a^4*b*x
 - a^5/x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = \frac{b^5\,x^9}{9} - \frac{a^5}{x} + \frac{5\,a\,b^4\,x^7}{7} + \frac{10\,a^3\,b^2\,x^3}{3} + 2\,a^2\,b^3\,x^5 + 5\,a^4\,b\,x$$

input
```
int((a + b*x^2)^5/x^2,x)
```

output
```
(b^5*x^9)/9 - a^5/x + (5*a*b^4*x^7)/7 + (10*a^3*b^2*x^3)/3 + 2*a^2*b^3*x^5
 + 5*a^4*b*x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^2}\,dx = \frac{7b^5x^{10} + 45a\,b^4x^8 + 126a^2b^3x^6 + 210a^3b^2x^4 + 315a^4b\,x^2 - 63a^5}{63x}$$

input
```
int((b*x^2+a)^5/x^2,x)
```

output
```
( - 63*a**5 + 315*a**4*b*x**2 + 210*a**3*b**2*x**4 + 126*a**2*b**3*x**6 +
45*a*b**4*x**8 + 7*b**5*x**10)/(63*x)
```

3.76 $\int \frac{(a+bx^2)^5}{x^4} dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{(a+bx^2)^5}{x^4} dx = -\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

output
```
-1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^4} dx = -\frac{a^5}{3x^3} - \frac{5a^4b}{x} + 10a^3b^2x + \frac{10}{3}a^2b^3x^3 + ab^4x^5 + \frac{b^5x^7}{7}$$

input
```
Integrate[(a + b*x^2)^5/x^4,x]
```

output
```
-1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
 + (b^5*x^7)/7
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^4} \, dx$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\frac{a^5}{x^4}+\frac{5a^4b}{x^2}+10a^3b^2+10a^2b^3x^2+5ab^4x^4+b^5x^6\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10}{3}a^2b^3x^3+ab^4x^5+\frac{b^5x^7}{7}$$

input
```
Int[(a + b*x^2)^5/x^4,x]
```

output
```
-1/3*a^5/x^3 - (5*a^4*b)/x + 10*a^3*b^2*x + (10*a^2*b^3*x^3)/3 + a*b^4*x^5
 + (b^5*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{3x^3}-\frac{5a^4b}{x}+10a^3b^2x+\frac{10a^2b^3x^3}{3}+a\,b^4x^5+\frac{b^5x^7}{7}$	55
risch	$\frac{b^5x^7}{7}+a\,b^4x^5+\frac{10a^2b^3x^3}{3}+10a^3b^2x+\frac{-5a^4b\,x^2-\frac{1}{3}a^5}{x^3}$	57
norman	$\frac{\frac{1}{7}b^5x^{10}+a\,b^4x^8+\frac{10}{3}a^2b^3x^6+10a^3b^2x^4-5a^4b\,x^2-\frac{1}{3}a^5}{x^3}$	58
gosper	$-\frac{-3b^5x^{10}-21a\,b^4x^8-70a^2b^3x^6-210a^3b^2x^4+105a^4b\,x^2+7a^5}{21x^3}$	60
parallelrisch	$\frac{3b^5x^{10}+21a\,b^4x^8+70a^2b^3x^6+210a^3b^2x^4-105a^4b\,x^2-7a^5}{21x^3}$	60
orering	$-\frac{-3b^5x^{10}-21a\,b^4x^8-70a^2b^3x^6-210a^3b^2x^4+105a^4b\,x^2+7a^5}{21x^3}$	60

input
```
int((b*x^2+a)^5/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^5/x^3-5*a^4*b/x+10*a^3*b^2*x+10/3*a^2*b^3*x^3+a*b^4*x^5+1/7*b^5*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = \frac{3\,b^5x^{10}+21\,ab^4x^8+70\,a^2b^3x^6+210\,a^3b^2x^4-105\,a^4bx^2-7\,a^5}{21\,x^3}$$

input
```
integrate((b*x^2+a)^5/x^4,x, algorithm="fricas")
```

output
```
1/21*(3*b^5*x^10 + 21*a*b^4*x^8 + 70*a^2*b^3*x^6 + 210*a^3*b^2*x^4 - 105*a
^4*b*x^2 - 7*a^5)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = 10a^3b^2x + \frac{10a^2b^3x^3}{3} + ab^4x^5 + \frac{b^5x^7}{7} + \frac{-a^5 - 15a^4bx^2}{3x^3}$$

input
```
integrate((b*x**2+a)**5/x**4,x)
```

output
```
10*a**3*b**2*x + 10*a**2*b**3*x**3/3 + a*b**4*x**5 + b**5*x**7/7 + (-a**5
- 15*a**4*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = \frac{1}{7}\,b^5x^7 + ab^4x^5 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2x - \frac{15\,a^4bx^2 + a^5}{3\,x^3}$$

input
```
integrate((b*x^2+a)^5/x^4,x, algorithm="maxima")
```

output
```
1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*
x^2 + a^5)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = \frac{1}{7}\,b^5x^7 + ab^4x^5 + \frac{10}{3}\,a^2b^3x^3 + 10\,a^3b^2x - \frac{15\,a^4bx^2 + a^5}{3\,x^3}$$

input
```
integrate((b*x^2+a)^5/x^4,x, algorithm="giac")
```

output
```
1/7*b^5*x^7 + a*b^4*x^5 + 10/3*a^2*b^3*x^3 + 10*a^3*b^2*x - 1/3*(15*a^4*b*
x^2 + a^5)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = \frac{b^5\,x^7}{7} - \frac{\frac{a^5}{3}+5\,b\,a^4\,x^2}{x^3} + 10\,a^3\,b^2\,x + a\,b^4\,x^5 + \frac{10\,a^2\,b^3\,x^3}{3}$$

input
```
int((a + b*x^2)^5/x^4,x)
```

output
```
(b^5*x^7)/7 - (a^5/3 + 5*a^4*b*x^2)/x^3 + 10*a^3*b^2*x + a*b^4*x^5 + (10*a
^2*b^3*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^4}\,dx = \frac{3b^5x^{10}+21a\,b^4x^8+70a^2b^3x^6+210a^3b^2x^4-105a^4b\,x^2-7a^5}{21x^3}$$

input
```
int((b*x^2+a)^5/x^4,x)
```

output
```
( - 7*a**5 - 105*a**4*b*x**2 + 210*a**3*b**2*x**4 + 70*a**2*b**3*x**6 + 21
*a*b**4*x**8 + 3*b**5*x**10)/(21*x**3)
```

3.77 $\int \frac{(a+bx^2)^5}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{(a+bx^2)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5}{3}ab^4x^3 + \frac{b^5x^5}{5}$$

output
```
-1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5
*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^6} dx = -\frac{a^5}{5x^5} - \frac{5a^4b}{3x^3} - \frac{10a^3b^2}{x} + 10a^2b^3x + \frac{5}{3}ab^4x^3 + \frac{b^5x^5}{5}$$

input
```
Integrate[(a + b*x^2)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^
4*x^3)/3 + (b^5*x^5)/5
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^6} \, dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^5}{x^6}+\frac{5a^4b}{x^4}+\frac{10a^3b^2}{x^2}+10a^2b^3+5ab^4x^2+b^5x^4\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5}{3}ab^4x^3+\frac{b^5x^5}{5}$$

input
```
Int[(a + b*x^2)^5/x^6,x]
```

output
```
-1/5*a^5/x^5 - (5*a^4*b)/(3*x^3) - (10*a^3*b^2)/x + 10*a^2*b^3*x + (5*a*b^
4*x^3)/3 + (b^5*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^5}{5x^5}-\frac{5a^4b}{3x^3}-\frac{10a^3b^2}{x}+10a^2b^3x+\frac{5a\,b^4x^3}{3}+\frac{b^5x^5}{5}$	56
risch	$\frac{b^5x^5}{5}+\frac{5a\,b^4x^3}{3}+10a^2b^3x+\frac{-10a^3b^2x^4-\frac{5}{3}a^4b\,x^2-\frac{1}{5}a^5}{x^5}$	58
norman	$\frac{\frac{1}{5}b^5x^{10}+\frac{5}{3}a\,b^4x^8+10a^2b^3x^6-10a^3b^2x^4-\frac{5}{3}a^4b\,x^2-\frac{1}{5}a^5}{x^5}$	59
gosper	$-\frac{-3b^5x^{10}-25a\,b^4x^8-150a^2b^3x^6+150a^3b^2x^4+25a^4b\,x^2+3a^5}{15x^5}$	60
parallelrisch	$\frac{3b^5x^{10}+25a\,b^4x^8+150a^2b^3x^6-150a^3b^2x^4-25a^4b\,x^2-3a^5}{15x^5}$	60
orering	$-\frac{-3b^5x^{10}-25a\,b^4x^8-150a^2b^3x^6+150a^3b^2x^4+25a^4b\,x^2+3a^5}{15x^5}$	60

input
```
int((b*x^2+a)^5/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^5/x^5-5/3*a^4*b/x^3-10*a^3*b^2/x+10*a^2*b^3*x+5/3*a*b^4*x^3+1/5*b^5
*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = \frac{3\,b^5x^{10}+25\,ab^4x^8+150\,a^2b^3x^6-150\,a^3b^2x^4-25\,a^4bx^2-3\,a^5}{15\,x^5}$$

input
```
integrate((b*x^2+a)^5/x^6,x, algorithm="fricas")
```

output
```
1/15*(3*b^5*x^10 + 25*a*b^4*x^8 + 150*a^2*b^3*x^6 - 150*a^3*b^2*x^4 - 25*a
^4*b*x^2 - 3*a^5)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = 10a^2b^3x + \frac{5ab^4x^3}{3} + \frac{b^5x^5}{5} + \frac{-3a^5 - 25a^4bx^2 - 150a^3b^2x^4}{15x^5}$$

input
```
integrate((b*x**2+a)**5/x**6,x)
```

output
```
10*a**2*b**3*x + 5*a*b**4*x**3/3 + b**5*x**5/5 + (-3*a**5 - 25*a**4*b*x**2
 - 150*a**3*b**2*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{150\,a^3b^2x^4 + 25\,a^4bx^2 + 3\,a^5}{15\,x^5}$$

input
```
integrate((b*x^2+a)^5/x^6,x, algorithm="maxima")
```

output
```
1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^
4*b*x^2 + 3*a^5)/x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = \frac{1}{5}\,b^5x^5 + \frac{5}{3}\,ab^4x^3 + 10\,a^2b^3x - \frac{150\,a^3b^2x^4 + 25\,a^4bx^2 + 3\,a^5}{15\,x^5}$$

input
```
integrate((b*x^2+a)^5/x^6,x, algorithm="giac")
```

output
```
1/5*b^5*x^5 + 5/3*a*b^4*x^3 + 10*a^2*b^3*x - 1/15*(150*a^3*b^2*x^4 + 25*a^
4*b*x^2 + 3*a^5)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = \frac{b^5\,x^5}{5} - \frac{\frac{a^5}{5} + \frac{5\,a^4\,b\,x^2}{3} + 10\,a^3\,b^2\,x^4}{x^5} + 10\,a^2\,b^3\,x + \frac{5\,a\,b^4\,x^3}{3}$$

input
```
int((a + b*x^2)^5/x^6,x)
```

output
```
(b^5*x^5)/5 - (a^5/5 + (5*a^4*b*x^2)/3 + 10*a^3*b^2*x^4)/x^5 + 10*a^2*b^3*
x + (5*a*b^4*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^6}\,dx = \frac{3b^5x^{10} + 25a\,b^4x^8 + 150a^2b^3x^6 - 150a^3b^2x^4 - 25a^4b\,x^2 - 3a^5}{15x^5}$$

input
```
int((b*x^2+a)^5/x^6,x)
```

output
```
( - 3*a**5 - 25*a**4*b*x**2 - 150*a**3*b**2*x**4 + 150*a**2*b**3*x**6 + 25
*a*b**4*x**8 + 3*b**5*x**10)/(15*x**5)
```

3.78 $\int \frac{(a+bx^2)^5}{x^8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = -\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

output
```
-1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = -\frac{a^5}{7x^7} - \frac{a^4b}{x^5} - \frac{10a^3b^2}{3x^3} - \frac{10a^2b^3}{x} + 5ab^4x + \frac{b^5x^3}{3}$$

input
```
Integrate[(a + b*x^2)^5/x^8,x]
```

output
```
-1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b
^4*x + (b^5*x^3)/3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^8} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^5}{x^8}+\frac{5a^4b}{x^6}+\frac{10a^3b^2}{x^4}+\frac{10a^2b^3}{x^2}+5ab^4+b^5x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{7x^7}-\frac{a^4b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5ab^4x+\frac{b^5x^3}{3}$$

input
```
Int[(a + b*x^2)^5/x^8,x]
```

output
```
-1/7*a^5/x^7 - (a^4*b)/x^5 - (10*a^3*b^2)/(3*x^3) - (10*a^2*b^3)/x + 5*a*b
^4*x + (b^5*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{7x^7}-\frac{a^4 b}{x^5}-\frac{10a^3b^2}{3x^3}-\frac{10a^2b^3}{x}+5a\,b^4x+\frac{b^5x^3}{3}$	56
risch	$\frac{b^5x^3}{3}+5a\,b^4x+\frac{-10a^2b^3x^6-\frac{10}{3}a^3b^2x^4-a^4b\,x^2-\frac{1}{7}a^5}{x^7}$	58
norman	$\frac{\frac{1}{3}b^5x^{10}+5a\,b^4x^8-10a^2b^3x^6-\frac{10}{3}a^3b^2x^4-a^4b\,x^2-\frac{1}{7}a^5}{x^7}$	59
gosper	$-\frac{-7b^5x^{10}-105a\,b^4x^8+210a^2b^3x^6+70a^3b^2x^4+21a^4b\,x^2+3a^5}{21x^7}$	60
parallelrisch	$\frac{7b^5x^{10}+105a\,b^4x^8-210a^2b^3x^6-70a^3b^2x^4-21a^4b\,x^2-3a^5}{21x^7}$	60
orering	$-\frac{-7b^5x^{10}-105a\,b^4x^8+210a^2b^3x^6+70a^3b^2x^4+21a^4b\,x^2+3a^5}{21x^7}$	60

input
```
int((b*x^2+a)^5/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^5/x^7-a^4*b/x^5-10/3*a^3*b^2/x^3-10*a^2*b^3/x+5*a*b^4*x+1/3*b^5*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = \frac{7\,b^5x^{10}+105\,ab^4x^8-210\,a^2b^3x^6-70\,a^3b^2x^4-21\,a^4bx^2-3\,a^5}{21\,x^7}$$

input
```
integrate((b*x^2+a)^5/x^8,x, algorithm="fricas")
```

output
```
1/21*(7*b^5*x^10 + 105*a*b^4*x^8 - 210*a^2*b^3*x^6 - 70*a^3*b^2*x^4 - 21*a
^4*b*x^2 - 3*a^5)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = 5ab^4x + \frac{b^5x^3}{3} + \frac{-3a^5 - 21a^4bx^2 - 70a^3b^2x^4 - 210a^2b^3x^6}{21x^7}$$

input
```
integrate((b*x**2+a)**5/x**8,x)
```

output
```
5*a*b**4*x + b**5*x**3/3 + (-3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 -
 210*a**2*b**3*x**6)/(21*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = \frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

input
```
integrate((b*x^2+a)^5/x^8,x, algorithm="maxima")
```

output
```
1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*
b*x^2 + 3*a^5)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = \frac{1}{3}\,b^5x^3 + 5\,ab^4x - \frac{210\,a^2b^3x^6 + 70\,a^3b^2x^4 + 21\,a^4bx^2 + 3\,a^5}{21\,x^7}$$

input
```
integrate((b*x^2+a)^5/x^8,x, algorithm="giac")
```

output
```
1/3*b^5*x^3 + 5*a*b^4*x - 1/21*(210*a^2*b^3*x^6 + 70*a^3*b^2*x^4 + 21*a^4*
b*x^2 + 3*a^5)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = -\frac{3\,a^5+21\,a^4\,b\,x^2+70\,a^3\,b^2\,x^4+210\,a^2\,b^3\,x^6-105\,a\,b^4\,x^8-7\,b^5\,x^{10}}{21\,x^7}$$

input
```
int((a + b*x^2)^5/x^8,x)
```

output
```
-(3*a^5 - 7*b^5*x^10 + 21*a^4*b*x^2 - 105*a*b^4*x^8 + 70*a^3*b^2*x^4 + 210
*a^2*b^3*x^6)/(21*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^8}\,dx = \frac{7b^5x^{10}+105a\,b^4x^8-210a^2b^3x^6-70a^3b^2x^4-21a^4b\,x^2-3a^5}{21x^7}$$

input
```
int((b*x^2+a)^5/x^8,x)
```

output
```
( - 3*a**5 - 21*a**4*b*x**2 - 70*a**3*b**2*x**4 - 210*a**2*b**3*x**6 + 105
*a*b**4*x**8 + 7*b**5*x**10)/(21*x**7)
```

3.79 $\int \frac{(a+bx^2)^5}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 60

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = -\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

output
```
-1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = -\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

input
```
Integrate[(a + b*x^2)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3)
- (5*a*b^4)/x + b^5*x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^5}{x^{10}}+\frac{5a^4b}{x^8}+\frac{10a^3b^2}{x^6}+\frac{10a^2b^3}{x^4}+\frac{5ab^4}{x^2}+b^5\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5ab^4}{x}+b^5x$$

input
```
Int[(a + b*x^2)^5/x^10,x]
```

output
```
-1/9*a^5/x^9 - (5*a^4*b)/(7*x^7) - (2*a^3*b^2)/x^5 - (10*a^2*b^3)/(3*x^3)
- (5*a*b^4)/x + b^5*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
default	$-\frac{a^5}{9x^9}-\frac{5a^4b}{7x^7}-\frac{2a^3b^2}{x^5}-\frac{10a^2b^3}{3x^3}-\frac{5a\,b^4}{x}+b^5x$	55
risch	$b^5x+\frac{-5a\,b^4x^8-\frac{10}{3}a^2b^3x^6-2a^3b^2x^4-\frac{5}{7}a^4b\,x^2-\frac{1}{9}a^5}{x^9}$	57
norman	$\frac{b^5x^{10}-5a\,b^4x^8-\frac{10}{3}a^2b^3x^6-2a^3b^2x^4-\frac{5}{7}a^4b\,x^2-\frac{1}{9}a^5}{x^9}$	58
gosper	$-\frac{-63b^5x^{10}+315a\,b^4x^8+210a^2b^3x^6+126a^3b^2x^4+45a^4b\,x^2+7a^5}{63x^9}$	60
parallelrisch	$\frac{63b^5x^{10}-315a\,b^4x^8-210a^2b^3x^6-126a^3b^2x^4-45a^4b\,x^2-7a^5}{63x^9}$	60
orering	$-\frac{-63b^5x^{10}+315a\,b^4x^8+210a^2b^3x^6+126a^3b^2x^4+45a^4b\,x^2+7a^5}{63x^9}$	60

input
```
int((b*x^2+a)^5/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^5/x^9-5/7*a^4*b/x^7-2*a^3*b^2/x^5-10/3*a^2*b^3/x^3-5*a*b^4/x+b^5*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx=\frac{63\,b^5x^{10}-315\,ab^4x^8-210\,a^2b^3x^6-126\,a^3b^2x^4-45\,a^4bx^2-7\,a^5}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5/x^10,x, algorithm="fricas")
```

output
```
1/63*(63*b^5*x^10 - 315*a*b^4*x^8 - 210*a^2*b^3*x^6 - 126*a^3*b^2*x^4 - 45
*a^4*b*x^2 - 7*a^5)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = b^5x + \frac{-7a^5 - 45a^4bx^2 - 126a^3b^2x^4 - 210a^2b^3x^6 - 315ab^4x^8}{63x^9}$$

input
```
integrate((b*x**2+a)**5/x**10,x)
```

output
```
b**5*x + (-7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x*
*6 - 315*a*b**4*x**8)/(63*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5/x^10,x, algorithm="maxima")
```

output
```
b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b
*x^2 + 7*a^5)/x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = b^5x - \frac{315\,ab^4x^8 + 210\,a^2b^3x^6 + 126\,a^3b^2x^4 + 45\,a^4bx^2 + 7\,a^5}{63\,x^9}$$

input
```
integrate((b*x^2+a)^5/x^10,x, algorithm="giac")
```

output
```
b^5*x - 1/63*(315*a*b^4*x^8 + 210*a^2*b^3*x^6 + 126*a^3*b^2*x^4 + 45*a^4*b
*x^2 + 7*a^5)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = b^5\,x - \frac{\frac{a^5}{9} + \frac{5\,a^4\,b\,x^2}{7} + 2\,a^3\,b^2\,x^4 + \frac{10\,a^2\,b^3\,x^6}{3} + 5\,a\,b^4\,x^8}{x^9}$$

input
```
int((a + b*x^2)^5/x^10,x)
```

output
```
b^5*x - (a^5/9 + (5*a^4*b*x^2)/7 + 5*a*b^4*x^8 + 2*a^3*b^2*x^4 + (10*a^2*b
^3*x^6)/3)/x^9
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^5}{x^{10}}\,dx = \frac{63b^5x^{10} - 315a\,b^4x^8 - 210a^2b^3x^6 - 126a^3b^2x^4 - 45a^4b\,x^2 - 7a^5}{63x^9}$$

input
```
int((b*x^2+a)^5/x^10,x)
```

output
```
( - 7*a**5 - 45*a**4*b*x**2 - 126*a**3*b**2*x**4 - 210*a**2*b**3*x**6 - 31
5*a*b**4*x**8 + 63*b**5*x**10)/(63*x**9)
```

3.80 $\int \frac{(a+bx^2)^5}{x^{12}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{(a+bx^2)^5}{x^{12}} \, dx = -\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

output
```
-1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-
b^5/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{12}} \, dx = -\frac{a^5}{11x^{11}} - \frac{5a^4b}{9x^9} - \frac{10a^3b^2}{7x^7} - \frac{2a^2b^3}{x^5} - \frac{5ab^4}{3x^3} - \frac{b^5}{x}$$

input
```
Integrate[(a + b*x^2)^5/x^12,x]
```

output
```
-1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^
5 - (5*a*b^4)/(3*x^3) - b^5/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^5}{x^{12}}+\frac{5a^4b}{x^{10}}+\frac{10a^3b^2}{x^8}+\frac{10a^2b^3}{x^6}+\frac{5ab^4}{x^4}+\frac{b^5}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5ab^4}{3x^3}-\frac{b^5}{x}$$

input
```
Int[(a + b*x^2)^5/x^12,x]
```

output
```
-1/11*a^5/x^11 - (5*a^4*b)/(9*x^9) - (10*a^3*b^2)/(7*x^7) - (2*a^2*b^3)/x^
5 - (5*a*b^4)/(3*x^3) - b^5/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{a^5}{11x^{11}}-\frac{5a^4b}{9x^9}-\frac{10a^3b^2}{7x^7}-\frac{2a^2b^3}{x^5}-\frac{5a\,b^4}{3x^3}-\frac{b^5}{x}$	58
norman	$\frac{-b^5x^{10}-\frac{5}{3}a\,b^4x^8-2a^2b^3x^6-\frac{10}{7}a^3b^2x^4-\frac{5}{9}a^4b\,x^2-\frac{1}{11}a^5}{x^{11}}$	59
risch	$\frac{-b^5x^{10}-\frac{5}{3}a\,b^4x^8-2a^2b^3x^6-\frac{10}{7}a^3b^2x^4-\frac{5}{9}a^4b\,x^2-\frac{1}{11}a^5}{x^{11}}$	59
gosper	$-\frac{693b^5x^{10}+1155a\,b^4x^8+1386a^2b^3x^6+990a^3b^2x^4+385a^4b\,x^2+63a^5}{693x^{11}}$	60
parallelrisch	$\frac{-693b^5x^{10}-1155a\,b^4x^8-1386a^2b^3x^6-990a^3b^2x^4-385a^4b\,x^2-63a^5}{693x^{11}}$	60
orering	$-\frac{693b^5x^{10}+1155a\,b^4x^8+1386a^2b^3x^6+990a^3b^2x^4+385a^4b\,x^2+63a^5}{693x^{11}}$	60

input
```
int((b*x^2+a)^5/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^5/x^11-5/9*a^4*b/x^9-10/7*a^3*b^2/x^7-2*a^2*b^3/x^5-5/3*a*b^4/x^3-
b^5/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = -\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5/x^12,x, algorithm="fricas")
```

output
```
-1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = \frac{-63a^5-385a^4bx^2-990a^3b^2x^4-1386a^2b^3x^6-1155ab^4x^8-693b^5x^{10}}{693x^{11}}$$

input
```
integrate((b*x**2+a)**5/x**12,x)
```

output
```
(-63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 - 1
155*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = -\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5/x^12,x, algorithm="maxima")
```

output
```
-1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = -\frac{693\,b^5x^{10}+1155\,ab^4x^8+1386\,a^2b^3x^6+990\,a^3b^2x^4+385\,a^4bx^2+63\,a^5}{693\,x^{11}}$$

input
```
integrate((b*x^2+a)^5/x^12,x, algorithm="giac")
```

output
```
-1/693*(693*b^5*x^10 + 1155*a*b^4*x^8 + 1386*a^2*b^3*x^6 + 990*a^3*b^2*x^4
 + 385*a^4*b*x^2 + 63*a^5)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = -\frac{\frac{a^5}{11}+\frac{5\,a^4\,b\,x^2}{9}+\frac{10\,a^3\,b^2\,x^4}{7}+2\,a^2\,b^3\,x^6+\frac{5\,a\,b^4\,x^8}{3}+b^5\,x^{10}}{x^{11}}$$

input
```
int((a + b*x^2)^5/x^12,x)
```

output
```
-(a^5/11 + b^5*x^10 + (5*a^4*b*x^2)/9 + (5*a*b^4*x^8)/3 + (10*a^3*b^2*x^4)
/7 + 2*a^2*b^3*x^6)/x^11
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{12}}\,dx = \frac{-693b^5x^{10}-1155a\,b^4x^8-1386a^2b^3x^6-990a^3b^2x^4-385a^4b\,x^2-63a^5}{693x^{11}}$$

input
```
int((b*x^2+a)^5/x^12,x)
```

output
```
( - 63*a**5 - 385*a**4*b*x**2 - 990*a**3*b**2*x**4 - 1386*a**2*b**3*x**6 -
 1155*a*b**4*x**8 - 693*b**5*x**10)/(693*x**11)
```

3.81 $\int \frac{(a+bx^2)^5}{x^{14}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

output
```
-1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5
-1/3*b^5/x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{a^5}{13x^{13}} - \frac{5a^4b}{11x^{11}} - \frac{10a^3b^2}{9x^9} - \frac{10a^2b^3}{7x^7} - \frac{ab^4}{x^5} - \frac{b^5}{3x^3}$$

input
```
Integrate[(a + b*x^2)^5/x^14,x]
```

output
```
-1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)
/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{14}} \, dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^5}{x^{14}}+\frac{5a^4b}{x^{12}}+\frac{10a^3b^2}{x^{10}}+\frac{10a^2b^3}{x^8}+\frac{5ab^4}{x^6}+\frac{b^5}{x^4}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^5}{13x^{13}}-\frac{5a^4b}{11x^{11}}-\frac{10a^3b^2}{9x^9}-\frac{10a^2b^3}{7x^7}-\frac{ab^4}{x^5}-\frac{b^5}{3x^3}$$

input
```
Int[(a + b*x^2)^5/x^14,x]
```

output
```
-1/13*a^5/x^13 - (5*a^4*b)/(11*x^11) - (10*a^3*b^2)/(9*x^9) - (10*a^2*b^3)
/(7*x^7) - (a*b^4)/x^5 - b^5/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{5}}{13x^{13}}-\frac{5a^{4}b}{11x^{11}}-\frac{10a^{3}b^{2}}{9x^{9}}-\frac{10a^{2}b^{3}}{7x^{7}}-\frac{a\,b^{4}}{x^{5}}-\frac{b^{5}}{3x^{3}}$	58
norman	$\frac{-\frac{1}{3}b^{5}x^{10}-a\,b^{4}x^{8}-\frac{10}{7}a^{2}b^{3}x^{6}-\frac{10}{9}a^{3}b^{2}x^{4}-\frac{5}{11}a^{4}b\,x^{2}-\frac{1}{13}a^{5}}{x^{13}}$	59
risch	$\frac{-\frac{1}{3}b^{5}x^{10}-a\,b^{4}x^{8}-\frac{10}{7}a^{2}b^{3}x^{6}-\frac{10}{9}a^{3}b^{2}x^{4}-\frac{5}{11}a^{4}b\,x^{2}-\frac{1}{13}a^{5}}{x^{13}}$	59
gosper	$-\frac{3003b^{5}x^{10}+9009a\,b^{4}x^{8}+12870a^{2}b^{3}x^{6}+10010a^{3}b^{2}x^{4}+4095a^{4}b\,x^{2}+693a^{5}}{9009x^{13}}$	60
parallelrisch	$\frac{-3003b^{5}x^{10}-9009a\,b^{4}x^{8}-12870a^{2}b^{3}x^{6}-10010a^{3}b^{2}x^{4}-4095a^{4}b\,x^{2}-693a^{5}}{9009x^{13}}$	60
orering	$-\frac{3003b^{5}x^{10}+9009a\,b^{4}x^{8}+12870a^{2}b^{3}x^{6}+10010a^{3}b^{2}x^{4}+4095a^{4}b\,x^{2}+693a^{5}}{9009x^{13}}$	60

input
```
int((b*x^2+a)^5/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^5/x^13-5/11*a^4*b/x^11-10/9*a^3*b^2/x^9-10/7*a^2*b^3/x^7-a*b^4/x^5
-1/3*b^5/x^3
```

Fricas [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5/x^14,x, algorithm="fricas")
```

output
```
-1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^
2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = \frac{-693a^5 - 4095a^4bx^2 - 10010a^3b^2x^4 - 12870a^2b^3x^6 - 9009ab^4x^8 - 3003b^5x^{10}}{9009x^{13}}$$

input
```
integrate((b*x**2+a)**5/x**14,x)
```

output
```
(-693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x**
6 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{3003\,b^5x^{10} + 9009\,ab^4x^8 + 12870\,a^2b^3x^6 + 10010\,a^3b^2x^4 + 4095\,a^4bx^2 + 693\,a^5}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5/x^14,x, algorithm="maxima")
```

output
```
-1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^
2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{3003\,b^5x^{10}+9009\,ab^4x^8+12870\,a^2b^3x^6+10010\,a^3b^2x^4+4095\,a^4bx^2+693\,a^5}{9009\,x^{13}}$$

input
```
integrate((b*x^2+a)^5/x^14,x, algorithm="giac")
```

output
```
-1/9009*(3003*b^5*x^10 + 9009*a*b^4*x^8 + 12870*a^2*b^3*x^6 + 10010*a^3*b^
2*x^4 + 4095*a^4*b*x^2 + 693*a^5)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = -\frac{\frac{a^5}{13}+\frac{5\,a^4\,b\,x^2}{11}+\frac{10\,a^3\,b^2\,x^4}{9}+\frac{10\,a^2\,b^3\,x^6}{7}+a\,b^4\,x^8+\frac{b^5\,x^{10}}{3}}{x^{13}}$$

input
```
int((a + b*x^2)^5/x^14,x)
```

output
```
-(a^5/13 + (b^5*x^10)/3 + (5*a^4*b*x^2)/11 + a*b^4*x^8 + (10*a^3*b^2*x^4)/
9 + (10*a^2*b^3*x^6)/7)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^5}{x^{14}}\,dx = \frac{-3003b^5x^{10}-9009a\,b^4x^8-12870a^2b^3x^6-10010a^3b^2x^4-4095a^4b\,x^2-693a^5}{9009x^{13}}$$

input
```
int((b*x^2+a)^5/x^14,x)
```

output
```
( - 693*a**5 - 4095*a**4*b*x**2 - 10010*a**3*b**2*x**4 - 12870*a**2*b**3*x
**6 - 9009*a*b**4*x**8 - 3003*b**5*x**10)/(9009*x**13)
```

3.82 $\int \frac{(a+bx^2)^5}{x^{16}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

output
```
-1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b
^4/x^7-1/5*b^5/x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{a^5}{15x^{15}} - \frac{5a^4b}{13x^{13}} - \frac{10a^3b^2}{11x^{11}} - \frac{10a^2b^3}{9x^9} - \frac{5ab^4}{7x^7} - \frac{b^5}{5x^5}$$

input
```
Integrate[(a + b*x^2)^5/x^16,x]
```

output
```
-1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^
3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{16}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^5}{x^{16}}+\frac{5a^4b}{x^{14}}+\frac{10a^3b^2}{x^{12}}+\frac{10a^2b^3}{x^{10}}+\frac{5ab^4}{x^8}+\frac{b^5}{x^6}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5}{15x^{15}}-\frac{5a^4b}{13x^{13}}-\frac{10a^3b^2}{11x^{11}}-\frac{10a^2b^3}{9x^9}-\frac{5ab^4}{7x^7}-\frac{b^5}{5x^5}$$

input
```
Int[(a + b*x^2)^5/x^16,x]
```

output
```
-1/15*a^5/x^15 - (5*a^4*b)/(13*x^13) - (10*a^3*b^2)/(11*x^11) - (10*a^2*b^
3)/(9*x^9) - (5*a*b^4)/(7*x^7) - b^5/(5*x^5)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{5}}{15x^{15}}-\frac{5a^{4}b}{13x^{13}}-\frac{10a^{3}b^{2}}{11x^{11}}-\frac{10a^{2}b^{3}}{9x^{9}}-\frac{5a\,b^{4}}{7x^{7}}-\frac{b^{5}}{5x^{5}}$	58
norman	$\frac{-\frac{1}{15}a^{5}-\frac{5}{13}a^{4}b\,x^{2}-\frac{10}{11}a^{3}b^{2}x^{4}-\frac{10}{9}a^{2}b^{3}x^{6}-\frac{5}{7}a\,b^{4}x^{8}-\frac{1}{5}b^{5}x^{10}}{x^{15}}$	59
risch	$\frac{-\frac{1}{15}a^{5}-\frac{5}{13}a^{4}b\,x^{2}-\frac{10}{11}a^{3}b^{2}x^{4}-\frac{10}{9}a^{2}b^{3}x^{6}-\frac{5}{7}a\,b^{4}x^{8}-\frac{1}{5}b^{5}x^{10}}{x^{15}}$	59
gosper	$-\frac{9009b^{5}x^{10}+32175a\,b^{4}x^{8}+50050a^{2}b^{3}x^{6}+40950a^{3}b^{2}x^{4}+17325a^{4}b\,x^{2}+3003a^{5}}{45045x^{15}}$	60
parallelrisch	$\frac{-9009b^{5}x^{10}-32175a\,b^{4}x^{8}-50050a^{2}b^{3}x^{6}-40950a^{3}b^{2}x^{4}-17325a^{4}b\,x^{2}-3003a^{5}}{45045x^{15}}$	60
orering	$-\frac{9009b^{5}x^{10}+32175a\,b^{4}x^{8}+50050a^{2}b^{3}x^{6}+40950a^{3}b^{2}x^{4}+17325a^{4}b\,x^{2}+3003a^{5}}{45045x^{15}}$	60

input
```
int((b*x^2+a)^5/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^5/x^15-5/13*a^4*b/x^13-10/11*a^3*b^2/x^11-10/9*a^2*b^3/x^9-5/7*a*b
^4/x^7-1/5*b^5/x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5/x^16,x, algorithm="fricas")
```

output
```
-1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*
b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = \frac{-3003a^5-17325a^4bx^2-40950a^3b^2x^4-50050a^2b^3x^6-32175ab^4x^8-9009b^5x^{10}}{45045x^{15}}$$

input
```
integrate((b*x**2+a)**5/x**16,x)
```

output
```
(-3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3*x
**6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5/x^16,x, algorithm="maxima")
```

output
```
-1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*
b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{9009\,b^5x^{10}+32175\,ab^4x^8+50050\,a^2b^3x^6+40950\,a^3b^2x^4+17325\,a^4bx^2+3003\,a^5}{45045\,x^{15}}$$

input
```
integrate((b*x^2+a)^5/x^16,x, algorithm="giac")
```

output
```
-1/45045*(9009*b^5*x^10 + 32175*a*b^4*x^8 + 50050*a^2*b^3*x^6 + 40950*a^3*
b^2*x^4 + 17325*a^4*b*x^2 + 3003*a^5)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = -\frac{\frac{a^5}{15}+\frac{5\,a^4\,b\,x^2}{13}+\frac{10\,a^3\,b^2\,x^4}{11}+\frac{10\,a^2\,b^3\,x^6}{9}+\frac{5\,a\,b^4\,x^8}{7}+\frac{b^5\,x^{10}}{5}}{x^{15}}$$

input
```
int((a + b*x^2)^5/x^16,x)
```

output
```
-(a^5/15 + (b^5*x^10)/5 + (5*a^4*b*x^2)/13 + (5*a*b^4*x^8)/7 + (10*a^3*b^2
*x^4)/11 + (10*a^2*b^3*x^6)/9)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{16}}\,dx = \frac{-9009b^5x^{10}-32175a\,b^4x^8-50050a^2b^3x^6-40950a^3b^2x^4-17325a^4b\,x^2-3003a^5}{45045x^{15}}$$

input
```
int((b*x^2+a)^5/x^16,x)
```

output
```
( - 3003*a**5 - 17325*a**4*b*x**2 - 40950*a**3*b**2*x**4 - 50050*a**2*b**3
*x**6 - 32175*a*b**4*x**8 - 9009*b**5*x**10)/(45045*x**15)
```

3.83 $\int \frac{(a+bx^2)^5}{x^{18}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

output
```
-1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*
b^4/x^9-1/7*b^5/x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{a^5}{17x^{17}} - \frac{a^4b}{3x^{15}} - \frac{10a^3b^2}{13x^{13}} - \frac{10a^2b^3}{11x^{11}} - \frac{5ab^4}{9x^9} - \frac{b^5}{7x^7}$$

input
```
Integrate[(a + b*x^2)^5/x^18,x]
```

output
```
-1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/
(11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{18}} \, dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a^5}{x^{18}}+\frac{5a^4b}{x^{16}}+\frac{10a^3b^2}{x^{14}}+\frac{10a^2b^3}{x^{12}}+\frac{5ab^4}{x^{10}}+\frac{b^5}{x^8}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^5}{17x^{17}}-\frac{a^4b}{3x^{15}}-\frac{10a^3b^2}{13x^{13}}-\frac{10a^2b^3}{11x^{11}}-\frac{5ab^4}{9x^9}-\frac{b^5}{7x^7}$$

input
```
Int[(a + b*x^2)^5/x^18,x]
```

output
```
-1/17*a^5/x^17 - (a^4*b)/(3*x^15) - (10*a^3*b^2)/(13*x^13) - (10*a^2*b^3)/
(11*x^11) - (5*a*b^4)/(9*x^9) - b^5/(7*x^7)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^{5}}{17x^{17}}-\frac{a^{4}b}{3x^{15}}-\frac{10a^{3}b^{2}}{13x^{13}}-\frac{10a^{2}b^{3}}{11x^{11}}-\frac{5a\,b^{4}}{9x^{9}}-\frac{b^{5}}{7x^{7}}$	58
norman	$\frac{-\frac{1}{17}a^{5}-\frac{1}{3}a^{4}b\,x^{2}-\frac{10}{13}a^{3}b^{2}x^{4}-\frac{10}{11}a^{2}b^{3}x^{6}-\frac{5}{9}a\,b^{4}x^{8}-\frac{1}{7}b^{5}x^{10}}{x^{17}}$	59
risch	$\frac{-\frac{1}{17}a^{5}-\frac{1}{3}a^{4}b\,x^{2}-\frac{10}{13}a^{3}b^{2}x^{4}-\frac{10}{11}a^{2}b^{3}x^{6}-\frac{5}{9}a\,b^{4}x^{8}-\frac{1}{7}b^{5}x^{10}}{x^{17}}$	59
gosper	$-\frac{21879b^{5}x^{10}+85085a\,b^{4}x^{8}+139230a^{2}b^{3}x^{6}+117810a^{3}b^{2}x^{4}+51051a^{4}b\,x^{2}+9009a^{5}}{153153x^{17}}$	60
parallelrisch	$\frac{-21879b^{5}x^{10}-85085a\,b^{4}x^{8}-139230a^{2}b^{3}x^{6}-117810a^{3}b^{2}x^{4}-51051a^{4}b\,x^{2}-9009a^{5}}{153153x^{17}}$	60
orering	$-\frac{21879b^{5}x^{10}+85085a\,b^{4}x^{8}+139230a^{2}b^{3}x^{6}+117810a^{3}b^{2}x^{4}+51051a^{4}b\,x^{2}+9009a^{5}}{153153x^{17}}$	60

input
```
int((b*x^2+a)^5/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/17*a^5/x^17-1/3*a^4*b/x^15-10/13*a^3*b^2/x^13-10/11*a^2*b^3/x^11-5/9*a*
b^4/x^9-1/7*b^5/x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

input
```
integrate((b*x^2+a)^5/x^18,x, algorithm="fricas")
```

output
```
-1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*
a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = \frac{-9009a^5 - 51051a^4bx^2 - 117810a^3b^2x^4 - 139230a^2b^3x^6 - 85085ab^4x^8 - 21879b^5x^{10}}{153153x^{17}}$$

input
```
integrate((b*x**2+a)**5/x**18,x)
```

output
```
(-9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b**3
*x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{21879\,b^5x^{10} + 85085\,ab^4x^8 + 139230\,a^2b^3x^6 + 117810\,a^3b^2x^4 + 51051\,a^4bx^2 + 9009\,a^5}{153153\,x^{17}}$$

input
```
integrate((b*x^2+a)^5/x^18,x, algorithm="maxima")
```

output
```
-1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*
a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{21879\,b^5x^{10}+85085\,ab^4x^8+139230\,a^2b^3x^6+117810\,a^3b^2x^4+51051\,a^4bx^2+9009\,a^5}{153153\,x^{17}}$$

input
```
integrate((b*x^2+a)^5/x^18,x, algorithm="giac")
```

output
```
-1/153153*(21879*b^5*x^10 + 85085*a*b^4*x^8 + 139230*a^2*b^3*x^6 + 117810*
a^3*b^2*x^4 + 51051*a^4*b*x^2 + 9009*a^5)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = -\frac{\frac{a^5}{17}+\frac{a^4\,b\,x^2}{3}+\frac{10\,a^3\,b^2\,x^4}{13}+\frac{10\,a^2\,b^3\,x^6}{11}+\frac{5\,a\,b^4\,x^8}{9}+\frac{b^5\,x^{10}}{7}}{x^{17}}$$

input
```
int((a + b*x^2)^5/x^18,x)
```

output
```
-(a^5/17 + (b^5*x^10)/7 + (a^4*b*x^2)/3 + (5*a*b^4*x^8)/9 + (10*a^3*b^2*x^
4)/13 + (10*a^2*b^3*x^6)/11)/x^17
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{18}}\,dx = \frac{-21879b^5x^{10}-85085a\,b^4x^8-139230a^2b^3x^6-117810a^3b^2x^4-51051a^4b\,x^2-9009a^5}{153153x^{17}}$$

input
```
int((b*x^2+a)^5/x^18,x)
```

output
```
( - 9009*a**5 - 51051*a**4*b*x**2 - 117810*a**3*b**2*x**4 - 139230*a**2*b*
*3*x**6 - 85085*a*b**4*x**8 - 21879*b**5*x**10)/(153153*x**17)
```

3.84 $\int \frac{(a+bx^2)^5}{x^{20}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

output
```
-1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*
b^4/x^11-1/9*b^5/x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

input
```
Integrate[(a + b*x^2)^5/x^20,x]
```

output
```
-1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)
/(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^5}{x^{20}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^5}{x^{20}}+\frac{5a^4b}{x^{18}}+\frac{10a^3b^2}{x^{16}}+\frac{10a^2b^3}{x^{14}}+\frac{5ab^4}{x^{12}}+\frac{b^5}{x^{10}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5ab^4}{11x^{11}}-\frac{b^5}{9x^9}$$

input
```
Int[(a + b*x^2)^5/x^20,x]
```

output
```
-1/19*a^5/x^19 - (5*a^4*b)/(17*x^17) - (2*a^3*b^2)/(3*x^15) - (10*a^2*b^3)
/(13*x^13) - (5*a*b^4)/(11*x^11) - b^5/(9*x^9)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{a^5}{19x^{19}}-\frac{5a^4b}{17x^{17}}-\frac{2a^3b^2}{3x^{15}}-\frac{10a^2b^3}{13x^{13}}-\frac{5a\,b^4}{11x^{11}}-\frac{b^5}{9x^9}$	58
norman	$\frac{-\frac{1}{19}a^5-\frac{5}{17}a^4b\,x^2-\frac{2}{3}a^3b^2x^4-\frac{10}{13}a^2b^3x^6-\frac{5}{11}a\,b^4x^8-\frac{1}{9}b^5x^{10}}{x^{19}}$	59
risch	$\frac{-\frac{1}{19}a^5-\frac{5}{17}a^4b\,x^2-\frac{2}{3}a^3b^2x^4-\frac{10}{13}a^2b^3x^6-\frac{5}{11}a\,b^4x^8-\frac{1}{9}b^5x^{10}}{x^{19}}$	59
gosper	$-\frac{46189b^5x^{10}+188955a\,b^4x^8+319770a^2b^3x^6+277134a^3b^2x^4+122265a^4b\,x^2+21879a^5}{415701x^{19}}$	60
parallelrisch	$\frac{-46189b^5x^{10}-188955a\,b^4x^8-319770a^2b^3x^6-277134a^3b^2x^4-122265a^4b\,x^2-21879a^5}{415701x^{19}}$	60
orering	$-\frac{46189b^5x^{10}+188955a\,b^4x^8+319770a^2b^3x^6+277134a^3b^2x^4+122265a^4b\,x^2+21879a^5}{415701x^{19}}$	60

input
```
int((b*x^2+a)^5/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^5/x^19-5/17*a^4*b/x^17-2/3*a^3*b^2/x^15-10/13*a^2*b^3/x^13-5/11*a*
b^4/x^11-1/9*b^5/x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

input
```
integrate((b*x^2+a)^5/x^20,x, algorithm="fricas")
```

output
```
-1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134
*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = \frac{-21879a^5-122265a^4bx^2-277134a^3b^2x^4-319770a^2b^3x^6-188955ab^4x^8-46189b^5x^{10}}{415701x^{19}}$$

input
```
integrate((b*x**2+a)**5/x**20,x)
```

output
```
(-21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*b*
*3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

input
```
integrate((b*x^2+a)^5/x^20,x, algorithm="maxima")
```

output
```
-1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134
*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{46189\,b^5x^{10}+188955\,ab^4x^8+319770\,a^2b^3x^6+277134\,a^3b^2x^4+122265\,a^4bx^2+21879\,a^5}{415701\,x^{19}}$$

input
```
integrate((b*x^2+a)^5/x^20,x, algorithm="giac")
```

output
```
-1/415701*(46189*b^5*x^10 + 188955*a*b^4*x^8 + 319770*a^2*b^3*x^6 + 277134
*a^3*b^2*x^4 + 122265*a^4*b*x^2 + 21879*a^5)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = -\frac{\frac{a^5}{19}+\frac{5\,a^4\,b\,x^2}{17}+\frac{2\,a^3\,b^2\,x^4}{3}+\frac{10\,a^2\,b^3\,x^6}{13}+\frac{5\,a\,b^4\,x^8}{11}+\frac{b^5\,x^{10}}{9}}{x^{19}}$$

input
```
int((a + b*x^2)^5/x^20,x)
```

output
```
-(a^5/19 + (b^5*x^10)/9 + (5*a^4*b*x^2)/17 + (5*a*b^4*x^8)/11 + (2*a^3*b^2
*x^4)/3 + (10*a^2*b^3*x^6)/13)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^5}{x^{20}}\,dx = \frac{-46189b^5x^{10}-188955a\,b^4x^8-319770a^2b^3x^6-277134a^3b^2x^4-122265a^4b\,x^2-21879a^5}{415701x^{19}}$$

input

```
int((b*x^2+a)^5/x^20,x)
```

output

```
( - 21879*a**5 - 122265*a**4*b*x**2 - 277134*a**3*b**2*x**4 - 319770*a**2*
b**3*x**6 - 188955*a*b**4*x**8 - 46189*b**5*x**10)/(415701*x**19)
```

3.85 $\int x^{13}(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 129

$$\int x^{13}\left(a+bx^2\right)^8\,dx=\frac{a^6(a+bx^2)^9}{18b^7}-\frac{3a^5(a+bx^2)^{10}}{10b^7}+\frac{15a^4(a+bx^2)^{11}}{22b^7}-\frac{5a^3(a+bx^2)^{12}}{6b^7}+\frac{15a^2(a+bx^2)^{13}}{26b^7}-\frac{3a(a+bx^2)^{14}}{14b^7}+\frac{(a+bx^2)^{15}}{30b^7}$$

output
```
1/18*a^6*(b*x^2+a)^9/b^7-3/10*a^5*(b*x^2+a)^10/b^7+15/22*a^4*(b*x^2+a)^11/
b^7-5/6*a^3*(b*x^2+a)^12/b^7+15/26*a^2*(b*x^2+a)^13/b^7-3/14*a*(b*x^2+a)^1
4/b^7+1/30*(b*x^2+a)^15/b^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int x^{13}\left(a+bx^2\right)^8\,dx=\frac{a^8x^{14}}{14}+\frac{1}{2}a^7bx^{16}+\frac{14}{9}a^6b^2x^{18}+\frac{14}{5}a^5b^3x^{20}+\frac{35}{11}a^4b^4x^{22}+\frac{7}{3}a^3b^5x^{24}+\frac{14}{13}a^2b^6x^{26}+\frac{2}{7}ab^7x^{28}+\frac{b^8x^{30}}{30}$$

input
```
Integrate[x^13*(a + b*x^2)^8,x]
```

output
```
(a^8*x^14)/14 + (a^7*b*x^16)/2 + (14*a^6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5
 + (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^24)/3 + (14*a^2*b^6*x^26)/13 + (2*a
*b^7*x^28)/7 + (b^8*x^30)/30
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a+bx^2\right)^8\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{12}\left(bx^2+a\right)^8\,dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{14}}{b^6}-\frac{6a\left(bx^2+a\right)^{13}}{b^6}+\frac{15a^2\left(bx^2+a\right)^{12}}{b^6}-\frac{20a^3\left(bx^2+a\right)^{11}}{b^6}+\frac{15a^4\left(bx^2+a\right)^{10}}{b^6}-\frac{6a^5\left(bx^2+a\right)^{9}}{b^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^6\left(a+bx^2\right)^9}{9b^7}-\frac{3a^5\left(a+bx^2\right)^{10}}{5b^7}+\frac{15a^4\left(a+bx^2\right)^{11}}{11b^7}-\frac{5a^3\left(a+bx^2\right)^{12}}{3b^7}+\frac{15a^2\left(a+bx^2\right)^{13}}{13b^7}+\frac{\left(a+bx^2\right)^{15}}{15b^7}-\right.$$

input
```
Int[x^13*(a + b*x^2)^8,x]
```

output
```
((a^6*(a + b*x^2)^9)/(9*b^7) - (3*a^5*(a + b*x^2)^10)/(5*b^7) + (15*a^4*(a
 + b*x^2)^11)/(11*b^7) - (5*a^3*(a + b*x^2)^12)/(3*b^7) + (15*a^2*(a + b*x
^2)^13)/(13*b^7) - (3*a*(a + b*x^2)^14)/(7*b^7) + (a + b*x^2)^15/(15*b^7))
/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

method	result
gosper	$\frac{2}{7}a\,b^7x^{28}+\frac{1}{14}a^8x^{14}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{30}b^8x^{30}+\frac{1}{2}a^7b\,x^{16}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{13}a^2b^6x^{26}+\frac{14}{5}a^5b^3x^{20}$
default	$\frac{2}{7}a\,b^7x^{28}+\frac{1}{14}a^8x^{14}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{30}b^8x^{30}+\frac{1}{2}a^7b\,x^{16}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{13}a^2b^6x^{26}+\frac{14}{5}a^5b^3x^{20}$
norman	$\frac{2}{7}a\,b^7x^{28}+\frac{1}{14}a^8x^{14}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{30}b^8x^{30}+\frac{1}{2}a^7b\,x^{16}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{13}a^2b^6x^{26}+\frac{14}{5}a^5b^3x^{20}$
risch	$\frac{2}{7}a\,b^7x^{28}+\frac{1}{14}a^8x^{14}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{30}b^8x^{30}+\frac{1}{2}a^7b\,x^{16}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{13}a^2b^6x^{26}+\frac{14}{5}a^5b^3x^{20}$
parallelrisch	$\frac{2}{7}a\,b^7x^{28}+\frac{1}{14}a^8x^{14}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{30}b^8x^{30}+\frac{1}{2}a^7b\,x^{16}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{13}a^2b^6x^{26}+\frac{14}{5}a^5b^3x^{20}$
orering	$\frac{x^{14}(3003b^8x^{16}+25740a\,b^7x^{14}+97020a^2b^6x^{12}+210210a^3b^5x^{10}+286650a^4b^4x^8+252252a^5b^3x^6+140140a^6b^2x^4+45045a^7b\,x}{90090}$

input
```
int(x^13*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
2/7*a*b^7*x^28+1/14*a^8*x^14+14/9*a^6*b^2*x^18+1/30*b^8*x^30+1/2*a^7*b*x^1
6+35/11*a^4*b^4*x^22+14/13*a^2*b^6*x^26+14/5*a^5*b^3*x^20+7/3*a^3*b^5*x^24
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{13}\left(a+bx^2\right)^8 dx = \frac{1}{30}b^8x^{30} + \frac{2}{7}ab^7x^{28} + \frac{14}{13}a^2b^6x^{26} + \frac{7}{3}a^3b^5x^{24} + \frac{35}{11}a^4b^4x^{22} + \frac{14}{5}a^5b^3x^{20} + \frac{14}{9}a^6b^2x^{18} + \frac{1}{2}a^7bx^{16} + \frac{1}{14}a^8x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 3
5/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16
 + 1/14*a^8*x^14
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int x^{13}\left(a+bx^2\right)^8 dx = \frac{a^8x^{14}}{14} + \frac{a^7bx^{16}}{2} + \frac{14a^6b^2x^{18}}{9} + \frac{14a^5b^3x^{20}}{5} + \frac{35a^4b^4x^{22}}{11} + \frac{7a^3b^5x^{24}}{3} + \frac{14a^2b^6x^{26}}{13} + \frac{2ab^7x^{28}}{7} + \frac{b^8x^{30}}{30}$$

input
```
integrate(x**13*(b*x**2+a)**8,x)
```

output
```
a**8*x**14/14 + a**7*b*x**16/2 + 14*a**6*b**2*x**18/9 + 14*a**5*b**3*x**20
/5 + 35*a**4*b**4*x**22/11 + 7*a**3*b**5*x**24/3 + 14*a**2*b**6*x**26/13 +
 2*a*b**7*x**28/7 + b**8*x**30/30
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{13}(a+bx^2)^8\,dx = \frac{1}{30}b^8x^{30}+\frac{2}{7}ab^7x^{28}+\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7bx^{16}+\frac{1}{14}a^8x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 3
5/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16
 + 1/14*a^8*x^14
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{13}(a+bx^2)^8\,dx = \frac{1}{30}b^8x^{30}+\frac{2}{7}ab^7x^{28}+\frac{14}{13}a^2b^6x^{26}+\frac{7}{3}a^3b^5x^{24}+\frac{35}{11}a^4b^4x^{22}+\frac{14}{5}a^5b^3x^{20}+\frac{14}{9}a^6b^2x^{18}+\frac{1}{2}a^7bx^{16}+\frac{1}{14}a^8x^{14}$$

input
```
integrate(x^13*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/30*b^8*x^30 + 2/7*a*b^7*x^28 + 14/13*a^2*b^6*x^26 + 7/3*a^3*b^5*x^24 + 3
5/11*a^4*b^4*x^22 + 14/5*a^5*b^3*x^20 + 14/9*a^6*b^2*x^18 + 1/2*a^7*b*x^16
 + 1/14*a^8*x^14
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.70

$$\int x^{13}(a+bx^2)^8\,dx = \frac{a^8\,x^{14}}{14}+\frac{a^7\,b\,x^{16}}{2}+\frac{14\,a^6\,b^2\,x^{18}}{9}+\frac{14\,a^5\,b^3\,x^{20}}{5}+\frac{35\,a^4\,b^4\,x^{22}}{11}+\frac{7\,a^3\,b^5\,x^{24}}{3}+\frac{14\,a^2\,b^6\,x^{26}}{13}+\frac{2\,a\,b^7\,x^{28}}{7}+\frac{b^8\,x^{30}}{30}$$

input
```
int(x^13*(a + b*x^2)^8,x)
```

output
```
(a^8*x^14)/14 + (b^8*x^30)/30 + (a^7*b*x^16)/2 + (2*a*b^7*x^28)/7 + (14*a^
6*b^2*x^18)/9 + (14*a^5*b^3*x^20)/5 + (35*a^4*b^4*x^22)/11 + (7*a^3*b^5*x^
24)/3 + (14*a^2*b^6*x^26)/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.71

$$\int x^{13}(a+bx^2)^8\,dx$$
$$= \frac{x^{14}(3003b^8x^{16}+25740a\,b^7x^{14}+97020a^2b^6x^{12}+210210a^3b^5x^{10}+286650a^4b^4x^8+252252a^5b^3x^6+1401}{90090}$$

input
```
int(x^13*(b*x^2+a)^8,x)
```

output
```
(x**14*(6435*a**8 + 45045*a**7*b*x**2 + 140140*a**6*b**2*x**4 + 252252*a**
5*b**3*x**6 + 286650*a**4*b**4*x**8 + 210210*a**3*b**5*x**10 + 97020*a**2*
b**6*x**12 + 25740*a*b**7*x**14 + 3003*b**8*x**16))/90090
```

3.86 $\int x^{11}(a+bx^2)^8 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int x^{11}(a+bx^2)^8 \, dx = -\frac{a^5(a+bx^2)^9}{18b^6} + \frac{a^4(a+bx^2)^{10}}{4b^6} - \frac{5a^3(a+bx^2)^{11}}{11b^6} + \frac{5a^2(a+bx^2)^{12}}{12b^6} - \frac{5a(a+bx^2)^{13}}{26b^6} + \frac{(a+bx^2)^{14}}{28b^6}$$

output
```
-1/18*a^5*(b*x^2+a)^9/b^6+1/4*a^4*(b*x^2+a)^10/b^6-5/11*a^3*(b*x^2+a)^11/b
^6+5/12*a^2*(b*x^2+a)^12/b^6-5/26*a*(b*x^2+a)^13/b^6+1/28*(b*x^2+a)^14/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int x^{11}(a+bx^2)^8 \, dx = \frac{a^8x^{12}}{12} + \frac{4}{7}a^7bx^{14} + \frac{7}{4}a^6b^2x^{16} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{6}a^2b^6x^{24} + \frac{4}{13}ab^7x^{26} + \frac{b^8x^{28}}{28}$$

input
```
Integrate[x^11*(a + b*x^2)^8,x]
```

output
```
(a^8*x^12)/12 + (4*a^7*b*x^14)/7 + (7*a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/
9 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x^22)/11 + (7*a^2*b^6*x^24)/6 + (4*a*
b^7*x^26)/13 + (b^8*x^28)/28
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^2\right)^8\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{10}\left(bx^2+a\right)^8dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{13}}{b^5}-\frac{5a\left(bx^2+a\right)^{12}}{b^5}+\frac{10a^2\left(bx^2+a\right)^{11}}{b^5}-\frac{10a^3\left(bx^2+a\right)^{10}}{b^5}+\frac{5a^4\left(bx^2+a\right)^9}{b^5}-\frac{a^5\left(bx^2+a\right)^8}{b^5}\right)d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^5\left(a+bx^2\right)^9}{9b^6}+\frac{a^4\left(a+bx^2\right)^{10}}{2b^6}-\frac{10a^3\left(a+bx^2\right)^{11}}{11b^6}+\frac{5a^2\left(a+bx^2\right)^{12}}{6b^6}+\frac{\left(a+bx^2\right)^{14}}{14b^6}-\frac{5a\left(a+bx^2\right)^{13}}{13b^6}\right)$$

input
```
Int[x^11*(a + b*x^2)^8,x]
```

output
```
(-1/9*(a^5*(a + b*x^2)^9)/b^6 + (a^4*(a + b*x^2)^10)/(2*b^6) - (10*a^3*(a
+ b*x^2)^11)/(11*b^6) + (5*a^2*(a + b*x^2)^12)/(6*b^6) - (5*a*(a + b*x^2)^
13)/(13*b^6) + (a + b*x^2)^14/(14*b^6))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

method	result
gosper	$\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{11}a^3b^5x^{22}+\frac{1}{28}b^8x^{28}+\frac{7}{2}a^4b^4x^{20}+\frac{1}{12}a^8x^{12}+\frac{28}{9}a^5b^3x^{18}$
default	$\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{11}a^3b^5x^{22}+\frac{1}{28}b^8x^{28}+\frac{7}{2}a^4b^4x^{20}+\frac{1}{12}a^8x^{12}+\frac{28}{9}a^5b^3x^{18}$
norman	$\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{11}a^3b^5x^{22}+\frac{1}{28}b^8x^{28}+\frac{7}{2}a^4b^4x^{20}+\frac{1}{12}a^8x^{12}+\frac{28}{9}a^5b^3x^{18}$
risch	$\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{11}a^3b^5x^{22}+\frac{1}{28}b^8x^{28}+\frac{7}{2}a^4b^4x^{20}+\frac{1}{12}a^8x^{12}+\frac{28}{9}a^5b^3x^{18}$
parallelrisch	$\frac{4}{7}a^7b\,x^{14}+\frac{7}{4}a^6b^2x^{16}+\frac{4}{13}a\,b^7x^{26}+\frac{28}{11}a^3b^5x^{22}+\frac{1}{28}b^8x^{28}+\frac{7}{2}a^4b^4x^{20}+\frac{1}{12}a^8x^{12}+\frac{28}{9}a^5b^3x^{18}$
orering	$\frac{x^{12}(1287b^8x^{16}+11088a\,b^7x^{14}+42042a^2b^6x^{12}+91728a^3b^5x^{10}+126126a^4b^4x^8+112112a^5b^3x^6+63063a^6b^2x^4+20592a^7b\,x^2+}{36036}$

input
```
int(x^11*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
4/7*a^7*b*x^14+7/4*a^6*b^2*x^16+4/13*a*b^7*x^26+28/11*a^3*b^5*x^22+1/28*b^
8*x^28+7/2*a^4*b^4*x^20+1/12*a^8*x^12+28/9*a^5*b^3*x^18+7/6*a^2*b^6*x^24
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{11}\left(a+bx^2\right)^8\,dx=\frac{1}{28}\,b^8x^{28}+\frac{4}{13}\,ab^7x^{26}+\frac{7}{6}\,a^2b^6x^{24}+\frac{28}{11}\,a^3b^5x^{22}+\frac{7}{2}\,a^4b^4x^{20}+\frac{28}{9}\,a^5b^3x^{18}+\frac{7}{4}\,a^6b^2x^{16}+\frac{4}{7}\,a^7bx^{14}+\frac{1}{12}\,a^8x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 +
7/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 +
 1/12*a^8*x^12
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97

$$\int x^{11}\left(a+bx^2\right)^8\,dx=\frac{a^8x^{12}}{12}+\frac{4a^7bx^{14}}{7}+\frac{7a^6b^2x^{16}}{4}+\frac{28a^5b^3x^{18}}{9}+\frac{7a^4b^4x^{20}}{2}+\frac{28a^3b^5x^{22}}{11}+\frac{7a^2b^6x^{24}}{6}+\frac{4ab^7x^{26}}{13}+\frac{b^8x^{28}}{28}$$

input
```
integrate(x**11*(b*x**2+a)**8,x)
```

output
```
a**8*x**12/12 + 4*a**7*b*x**14/7 + 7*a**6*b**2*x**16/4 + 28*a**5*b**3*x**1
8/9 + 7*a**4*b**4*x**20/2 + 28*a**3*b**5*x**22/11 + 7*a**2*b**6*x**24/6 +
4*a*b**7*x**26/13 + b**8*x**28/28
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{11}(a+bx^2)^8\,dx = \frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 +
7/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 +
 1/12*a^8*x^12
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{11}(a+bx^2)^8\,dx = \frac{1}{28}b^8x^{28} + \frac{4}{13}ab^7x^{26} + \frac{7}{6}a^2b^6x^{24} + \frac{28}{11}a^3b^5x^{22} + \frac{7}{2}a^4b^4x^{20} + \frac{28}{9}a^5b^3x^{18} + \frac{7}{4}a^6b^2x^{16} + \frac{4}{7}a^7bx^{14} + \frac{1}{12}a^8x^{12}$$

input
```
integrate(x^11*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/28*b^8*x^28 + 4/13*a*b^7*x^26 + 7/6*a^2*b^6*x^24 + 28/11*a^3*b^5*x^22 +
7/2*a^4*b^4*x^20 + 28/9*a^5*b^3*x^18 + 7/4*a^6*b^2*x^16 + 4/7*a^7*b*x^14 +
 1/12*a^8*x^12
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int x^{11}\left(a+bx^2\right)^8\,dx=\frac{a^8\,x^{12}}{12}+\frac{4\,a^7\,b\,x^{14}}{7}+\frac{7\,a^6\,b^2\,x^{16}}{4}+\frac{28\,a^5\,b^3\,x^{18}}{9}+\frac{7\,a^4\,b^4\,x^{20}}{2}+\frac{28\,a^3\,b^5\,x^{22}}{11}+\frac{7\,a^2\,b^6\,x^{24}}{6}+\frac{4\,a\,b^7\,x^{26}}{13}+\frac{b^8\,x^{28}}{28}$$

input
```
int(x^11*(a + b*x^2)^8,x)
```

output
```
(a^8*x^12)/12 + (b^8*x^28)/28 + (4*a^7*b*x^14)/7 + (4*a*b^7*x^26)/13 + (7*
a^6*b^2*x^16)/4 + (28*a^5*b^3*x^18)/9 + (7*a^4*b^4*x^20)/2 + (28*a^3*b^5*x
^22)/11 + (7*a^2*b^6*x^24)/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int x^{11}\left(a+bx^2\right)^8\,dx=\frac{x^{12}(1287b^8x^{16}+11088a\,b^7x^{14}+42042a^2b^6x^{12}+91728a^3b^5x^{10}+126126a^4b^4x^8+112112a^5b^3x^6+63063}{36036}$$

input
```
int(x^11*(b*x^2+a)^8,x)
```

output
```
(x**12*(3003*a**8 + 20592*a**7*b*x**2 + 63063*a**6*b**2*x**4 + 112112*a**5
*b**3*x**6 + 126126*a**4*b**4*x**8 + 91728*a**3*b**5*x**10 + 42042*a**2*b*
*6*x**12 + 11088*a*b**7*x**14 + 1287*b**8*x**16))/36036
```

3.87 $\int x^9(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int x^9(a+bx^2)^8\,dx=\frac{a^4(a+bx^2)^9}{18b^5}-\frac{a^3(a+bx^2)^{10}}{5b^5}+\frac{3a^2(a+bx^2)^{11}}{11b^5}-\frac{a(a+bx^2)^{12}}{6b^5}+\frac{(a+bx^2)^{13}}{26b^5}$$

output
```
1/18*a^4*(b*x^2+a)^9/b^5-1/5*a^3*(b*x^2+a)^10/b^5+3/11*a^2*(b*x^2+a)^11/b^
5-1/6*a*(b*x^2+a)^12/b^5+1/26*(b*x^2+a)^13/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int x^9(a+bx^2)^8\,dx=\frac{a^8x^{10}}{10}+\frac{2}{3}a^7bx^{12}+2a^6b^2x^{14}+\frac{7}{2}a^5b^3x^{16}+\frac{35}{9}a^4b^4x^{18}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+\frac{1}{3}ab^7x^{24}+\frac{b^8x^{26}}{26}$$

input
```
Integrate[x^9*(a + b*x^2)^8,x]
```

output
```
(a^8*x^10)/10 + (2*a^7*b*x^12)/3 + 2*a^6*b^2*x^14 + (7*a^5*b^3*x^16)/2 + (
35*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5 + (14*a^2*b^6*x^22)/11 + (a*b^7*x
^24)/3 + (b^8*x^26)/26
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^8 \left(bx^2+a\right)^8 dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{12}}{b^4} - \frac{4a\left(bx^2+a\right)^{11}}{b^4} + \frac{6a^2\left(bx^2+a\right)^{10}}{b^4} - \frac{4a^3\left(bx^2+a\right)^9}{b^4} + \frac{a^4\left(bx^2+a\right)^8}{b^4}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^4\left(a+bx^2\right)^9}{9b^5} - \frac{2a^3\left(a+bx^2\right)^{10}}{5b^5} + \frac{6a^2\left(a+bx^2\right)^{11}}{11b^5} + \frac{\left(a+bx^2\right)^{13}}{13b^5} - \frac{a\left(a+bx^2\right)^{12}}{3b^5}\right)$$

input
```
Int[x^9*(a + b*x^2)^8,x]
```

output
```
((a^4*(a + b*x^2)^9)/(9*b^5) - (2*a^3*(a + b*x^2)^10)/(5*b^5) + (6*a^2*(a
+ b*x^2)^11)/(11*b^5) - (a*(a + b*x^2)^12)/(3*b^5) + (a + b*x^2)^13/(13*b^
5))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{7}{2}a^5b^3x^{16}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{3}a\,b^7x^{24}+$
default	$\frac{7}{2}a^5b^3x^{16}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{3}a\,b^7x^{24}+$
norman	$\frac{7}{2}a^5b^3x^{16}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{3}a\,b^7x^{24}+$
risch	$\frac{7}{2}a^5b^3x^{16}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{3}a\,b^7x^{24}+$
parallelrisch	$\frac{7}{2}a^5b^3x^{16}+\frac{1}{26}b^8x^{26}+\frac{1}{10}a^8x^{10}+\frac{14}{5}a^3b^5x^{20}+\frac{14}{11}a^2b^6x^{22}+2a^6b^2x^{14}+\frac{2}{3}a^7b\,x^{12}+\frac{1}{3}a\,b^7x^{24}+$
orering	$\frac{x^{10}(495b^8x^{16}+4290a\,b^7x^{14}+16380a^2b^6x^{12}+36036a^3b^5x^{10}+50050a^4b^4x^8+45045a^5b^3x^6+25740a^6b^2x^4+8580a^7b\,x^2+1287a}{12870}$

input
```
int(x^9*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
7/2*a^5*b^3*x^16+1/26*b^8*x^26+1/10*a^8*x^10+14/5*a^3*b^5*x^20+14/11*a^2*b
^6*x^22+2*a^6*b^2*x^14+2/3*a^7*b*x^12+1/3*a*b^7*x^24+35/9*a^4*b^4*x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^9 (a + bx^2)^8 \, dx = \frac{1}{26} b^8 x^{26} + \frac{1}{3} a b^7 x^{24} + \frac{14}{11} a^2 b^6 x^{22} + \frac{14}{5} a^3 b^5 x^{20} + \frac{35}{9} a^4 b^4 x^{18} + \frac{7}{2} a^5 b^3 x^{16} + 2 a^6 b^2 x^{14} + \frac{2}{3} a^7 b x^{12} + \frac{1}{10} a^8 x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 +
35/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1
/10*a^8*x^10
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int x^9 (a + bx^2)^8 \, dx = \frac{a^8 x^{10}}{10} + \frac{2 a^7 b x^{12}}{3} + 2 a^6 b^2 x^{14} + \frac{7 a^5 b^3 x^{16}}{2} + \frac{35 a^4 b^4 x^{18}}{9} + \frac{14 a^3 b^5 x^{20}}{5} + \frac{14 a^2 b^6 x^{22}}{11} + \frac{a b^7 x^{24}}{3} + \frac{b^8 x^{26}}{26}$$

input
```
integrate(x**9*(b*x**2+a)**8,x)
```

output
```
a**8*x**10/10 + 2*a**7*b*x**12/3 + 2*a**6*b**2*x**14 + 7*a**5*b**3*x**16/2
 + 35*a**4*b**4*x**18/9 + 14*a**3*b**5*x**20/5 + 14*a**2*b**6*x**22/11 + a
*b**7*x**24/3 + b**8*x**26/26
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^9 \left(a+bx^2\right)^8 \,dx = \frac{1}{26}\,b^8x^{26} + \frac{1}{3}\,ab^7x^{24} + \frac{14}{11}\,a^2b^6x^{22} + \frac{14}{5}\,a^3b^5x^{20} + \frac{35}{9}\,a^4b^4x^{18} + \frac{7}{2}\,a^5b^3x^{16} + 2\,a^6b^2x^{14} + \frac{2}{3}\,a^7bx^{12} + \frac{1}{10}\,a^8x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 +
35/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1
/10*a^8*x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^9 \left(a+bx^2\right)^8 \,dx = \frac{1}{26}\,b^8x^{26} + \frac{1}{3}\,ab^7x^{24} + \frac{14}{11}\,a^2b^6x^{22} + \frac{14}{5}\,a^3b^5x^{20} + \frac{35}{9}\,a^4b^4x^{18} + \frac{7}{2}\,a^5b^3x^{16} + 2\,a^6b^2x^{14} + \frac{2}{3}\,a^7bx^{12} + \frac{1}{10}\,a^8x^{10}$$

input
```
integrate(x^9*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/26*b^8*x^26 + 1/3*a*b^7*x^24 + 14/11*a^2*b^6*x^22 + 14/5*a^3*b^5*x^20 +
35/9*a^4*b^4*x^18 + 7/2*a^5*b^3*x^16 + 2*a^6*b^2*x^14 + 2/3*a^7*b*x^12 + 1
/10*a^8*x^10
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.99

$$\int x^9 (a+bx^2)^8 \, dx = \frac{a^8 x^{10}}{10} + \frac{2 a^7 b x^{12}}{3} + 2 a^6 b^2 x^{14} + \frac{7 a^5 b^3 x^{16}}{2} + \frac{35 a^4 b^4 x^{18}}{9} + \frac{14 a^3 b^5 x^{20}}{5} + \frac{14 a^2 b^6 x^{22}}{11} + \frac{a b^7 x^{24}}{3} + \frac{b^8 x^{26}}{26}$$

input
```
int(x^9*(a + b*x^2)^8,x)
```

output
```
(a^8*x^10)/10 + (b^8*x^26)/26 + (2*a^7*b*x^12)/3 + (a*b^7*x^24)/3 + 2*a^6*
b^2*x^14 + (7*a^5*b^3*x^16)/2 + (35*a^4*b^4*x^18)/9 + (14*a^3*b^5*x^20)/5
+ (14*a^2*b^6*x^22)/11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int x^9 (a+bx^2)^8 \, dx = \frac{x^{10}(495b^8x^{16} + 4290a b^7x^{14} + 16380a^2b^6x^{12} + 36036a^3b^5x^{10} + 50050a^4b^4x^8 + 45045a^5b^3x^6 + 25740a^6b^2x}{12870}$$

input
```
int(x^9*(b*x^2+a)^8,x)
```

output
```
(x**10*(1287*a**8 + 8580*a**7*b*x**2 + 25740*a**6*b**2*x**4 + 45045*a**5*b
**3*x**6 + 50050*a**4*b**4*x**8 + 36036*a**3*b**5*x**10 + 16380*a**2*b**6*
x**12 + 4290*a*b**7*x**14 + 495*b**8*x**16))/12870
```

3.88 $\int x^7(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^7(a+bx^2)^8\,dx=-\frac{a^3(a+bx^2)^9}{18b^4}+\frac{3a^2(a+bx^2)^{10}}{20b^4}-\frac{3a(a+bx^2)^{11}}{22b^4}+\frac{(a+bx^2)^{12}}{24b^4}$$

output
```
-1/18*a^3*(b*x^2+a)^9/b^4+3/20*a^2*(b*x^2+a)^10/b^4-3/22*a*(b*x^2+a)^11/b^
4+1/24*(b*x^2+a)^12/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.47

$$\int x^7(a+bx^2)^8\,dx=\frac{a^8x^8}{8}+\frac{4}{5}a^7bx^{10}+\frac{7}{3}a^6b^2x^{12}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{28}{9}a^3b^5x^{18}+\frac{7}{5}a^2b^6x^{20}+\frac{4}{11}ab^7x^{22}+\frac{b^8x^{24}}{24}$$

input
```
Integrate[x^7*(a + b*x^2)^8,x]
```

output
```
(a^8*x^8)/8 + (4*a^7*b*x^10)/5 + (7*a^6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35
*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9 + (7*a^2*b^6*x^20)/5 + (4*a*b^7*x^2
2)/11 + (b^8*x^24)/24
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^8 dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{11}}{b^3} - \frac{3a\left(bx^2+a\right)^{10}}{b^3} + \frac{3a^2\left(bx^2+a\right)^9}{b^3} - \frac{a^3\left(bx^2+a\right)^8}{b^3}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{a^3\left(a+bx^2\right)^9}{9b^4} + \frac{3a^2\left(a+bx^2\right)^{10}}{10b^4} + \frac{\left(a+bx^2\right)^{12}}{12b^4} - \frac{3a\left(a+bx^2\right)^{11}}{11b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^8,x]
```

output
```
(-1/9*(a^3*(a + b*x^2)^9)/b^4 + (3*a^2*(a + b*x^2)^10)/(10*b^4) - (3*a*(a
+ b*x^2)^11)/(11*b^4) + (a + b*x^2)^12/(12*b^4))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.26

method	result
gosper	$\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{1}{24}b^8x^{24}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{1}{8}a^8x^8+\frac{2}{9}$
default	$\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{1}{24}b^8x^{24}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{1}{8}a^8x^8+\frac{2}{9}$
norman	$\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{1}{24}b^8x^{24}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{1}{8}a^8x^8+\frac{2}{9}$
risch	$\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{1}{24}b^8x^{24}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{1}{8}a^8x^8+\frac{2}{9}$
parallelrisch	$\frac{4}{11}a\,b^7x^{22}+\frac{7}{5}a^2b^6x^{20}+\frac{1}{24}b^8x^{24}+\frac{7}{3}a^6b^2x^{12}+\frac{4}{5}a^7b\,x^{10}+4a^5b^3x^{14}+\frac{35}{8}a^4b^4x^{16}+\frac{1}{8}a^8x^8+\frac{2}{9}$
orering	$\frac{x^8\left(165b^8x^{16}+1440a\,b^7x^{14}+5544a^2b^6x^{12}+12320a^3b^5x^{10}+17325a^4b^4x^8+15840a^5b^3x^6+9240a^6b^2x^4+3168a^7b\,x^2+495a^8\right)}{3960}$

input
```
int(x^7*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
4/11*a*b^7*x^22+7/5*a^2*b^6*x^20+1/24*b^8*x^24+7/3*a^6*b^2*x^12+4/5*a^7*b*
x^10+4*a^5*b^3*x^14+35/8*a^4*b^4*x^16+1/8*a^8*x^8+28/9*a^3*b^5*x^18
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^7(a+bx^2)^8\,dx = \frac{1}{24}\,b^8x^{24} + \frac{4}{11}\,ab^7x^{22} + \frac{7}{5}\,a^2b^6x^{20} + \frac{28}{9}\,a^3b^5x^{18} + \frac{35}{8}\,a^4b^4x^{16} + 4\,a^5b^3x^{14} + \frac{7}{3}\,a^6b^2x^{12} + \frac{4}{5}\,a^7bx^{10} + \frac{1}{8}\,a^8x^8$$

input
```
integrate(x^7*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 3
5/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/
8*a^8*x^8
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.46

$$\int x^7(a+bx^2)^8\,dx = \frac{a^8x^8}{8} + \frac{4a^7bx^{10}}{5} + \frac{7a^6b^2x^{12}}{3} + 4a^5b^3x^{14} + \frac{35a^4b^4x^{16}}{8} + \frac{28a^3b^5x^{18}}{9} + \frac{7a^2b^6x^{20}}{5} + \frac{4ab^7x^{22}}{11} + \frac{b^8x^{24}}{24}$$

input
```
integrate(x**7*(b*x**2+a)**8,x)
```

output
```
a**8*x**8/8 + 4*a**7*b*x**10/5 + 7*a**6*b**2*x**12/3 + 4*a**5*b**3*x**14 +
 35*a**4*b**4*x**16/8 + 28*a**3*b**5*x**18/9 + 7*a**2*b**6*x**20/5 + 4*a*b
**7*x**22/11 + b**8*x**24/24
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^7 (a+bx^2)^8 \, dx = \frac{1}{24} b^8 x^{24} + \frac{4}{11} a b^7 x^{22} + \frac{7}{5} a^2 b^6 x^{20} + \frac{28}{9} a^3 b^5 x^{18} + \frac{35}{8} a^4 b^4 x^{16} + 4 a^5 b^3 x^{14} + \frac{7}{3} a^6 b^2 x^{12} + \frac{4}{5} a^7 b x^{10} + \frac{1}{8} a^8 x^8$$

input
```
integrate(x^7*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 3
5/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/
8*a^8*x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^7 (a+bx^2)^8 \, dx = \frac{1}{24} b^8 x^{24} + \frac{4}{11} a b^7 x^{22} + \frac{7}{5} a^2 b^6 x^{20} + \frac{28}{9} a^3 b^5 x^{18} + \frac{35}{8} a^4 b^4 x^{16} + 4 a^5 b^3 x^{14} + \frac{7}{3} a^6 b^2 x^{12} + \frac{4}{5} a^7 b x^{10} + \frac{1}{8} a^8 x^8$$

input
```
integrate(x^7*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/24*b^8*x^24 + 4/11*a*b^7*x^22 + 7/5*a^2*b^6*x^20 + 28/9*a^3*b^5*x^18 + 3
5/8*a^4*b^4*x^16 + 4*a^5*b^3*x^14 + 7/3*a^6*b^2*x^12 + 4/5*a^7*b*x^10 + 1/
8*a^8*x^8
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int x^7 (a + bx^2)^8 \, dx = \frac{a^8 x^8}{8} + \frac{4 a^7 b x^{10}}{5} + \frac{7 a^6 b^2 x^{12}}{3} + 4 a^5 b^3 x^{14} + \frac{35 a^4 b^4 x^{16}}{8} + \frac{28 a^3 b^5 x^{18}}{9} + \frac{7 a^2 b^6 x^{20}}{5} + \frac{4 a b^7 x^{22}}{11} + \frac{b^8 x^{24}}{24}$$

input
```
int(x^7*(a + b*x^2)^8,x)
```

output
```
(a^8*x^8)/8 + (b^8*x^24)/24 + (4*a^7*b*x^10)/5 + (4*a*b^7*x^22)/11 + (7*a^
6*b^2*x^12)/3 + 4*a^5*b^3*x^14 + (35*a^4*b^4*x^16)/8 + (28*a^3*b^5*x^18)/9
 + (7*a^2*b^6*x^20)/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int x^7 (a + bx^2)^8 \, dx = \frac{x^8(165b^8x^{16} + 1440a b^7x^{14} + 5544a^2b^6x^{12} + 12320a^3b^5x^{10} + 17325a^4b^4x^8 + 15840a^5b^3x^6 + 9240a^6b^2x^4 +}{3960}$$

input
```
int(x^7*(b*x^2+a)^8,x)
```

output
```
(x**8*(495*a**8 + 3168*a**7*b*x**2 + 9240*a**6*b**2*x**4 + 15840*a**5*b**3
*x**6 + 17325*a**4*b**4*x**8 + 12320*a**3*b**5*x**10 + 5544*a**2*b**6*x**1
2 + 1440*a*b**7*x**14 + 165*b**8*x**16))/3960
```

3.89 $\int x^5(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int x^5\left(a+bx^2\right)^8\,dx=\frac{a^2(a+bx^2)^9}{18b^3}-\frac{a(a+bx^2)^{10}}{10b^3}+\frac{(a+bx^2)^{11}}{22b^3}$$

output
```
1/18*a^2*(b*x^2+a)^9/b^3-1/10*a*(b*x^2+a)^10/b^3+1/22*(b*x^2+a)^11/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.94

$$\int x^5\left(a+bx^2\right)^8\,dx=\frac{a^8x^6}{6}+a^7bx^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}ab^7x^{20}+\frac{b^8x^{22}}{22}$$

input
```
Integrate[x^5*(a + b*x^2)^8,x]
```

output
```
(a^8*x^6)/6 + a^7*b*x^8 + (14*a^6*b^2*x^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^
4*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a^2*b^6*x^18)/9 + (2*a*b^7*x^20)/5 +
 (b^8*x^22)/22
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^4 \left(bx^2+a\right)^8 dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{10}}{b^2}-\frac{2a\left(bx^2+a\right)^9}{b^2}+\frac{a^2\left(bx^2+a\right)^8}{b^2}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^2\left(a+bx^2\right)^9}{9b^3}+\frac{\left(a+bx^2\right)^{11}}{11b^3}-\frac{a\left(a+bx^2\right)^{10}}{5b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^8,x]
```

output
```
((a^2*(a + b*x^2)^9)/(9*b^3) - (a*(a + b*x^2)^10)/(5*b^3) + (a + b*x^2)^11
/(11*b^3))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.70

method	result
gosper	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{2}$
default	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{2}$
norman	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{2}$
risch	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{2}$
parallelrisch	$\frac{1}{6}a^8x^6+a^7b\,x^8+\frac{14}{5}a^6b^2x^{10}+\frac{14}{3}a^5b^3x^{12}+5a^4b^4x^{14}+\frac{7}{2}a^3b^5x^{16}+\frac{14}{9}a^2b^6x^{18}+\frac{2}{5}a\,b^7x^{20}+\frac{1}{2}$
orering	$\frac{x^6\left(45b^8x^{16}+396a\,b^7x^{14}+1540a^2b^6x^{12}+3465a^3b^5x^{10}+4950a^4b^4x^8+4620a^5b^3x^6+2772a^6b^2x^4+990a^7b\,x^2+165a^8\right)}{990}$

input
```
int(x^5*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^8*x^6+a^7*b*x^8+14/5*a^6*b^2*x^10+14/3*a^5*b^3*x^12+5*a^4*b^4*x^14+7
/2*a^3*b^5*x^16+14/9*a^2*b^6*x^18+2/5*a*b^7*x^20+1/22*b^8*x^22
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int x^5\left(a+bx^2\right)^8\,dx=\frac{1}{22}\,b^8x^{22}+\frac{2}{5}\,ab^7x^{20}+\frac{14}{9}\,a^2b^6x^{18}+\frac{7}{2}\,a^3b^5x^{16}+5\,a^4b^4x^{14}+\frac{14}{3}\,a^5b^3x^{12}+\frac{14}{5}\,a^6b^2x^{10}+a^7bx^8+\frac{1}{6}\,a^8x^6$$

input
```
integrate(x^5*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*
a^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8
*x^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(44) = 88$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.92

$$\int x^5 (a + bx^2)^8 \, dx = \frac{a^8x^6}{6} + a^7bx^8 + \frac{14a^6b^2x^{10}}{5} + \frac{14a^5b^3x^{12}}{3} + 5a^4b^4x^{14} + \frac{7a^3b^5x^{16}}{2} + \frac{14a^2b^6x^{18}}{9} + \frac{2ab^7x^{20}}{5} + \frac{b^8x^{22}}{22}$$

input
```
integrate(x**5*(b*x**2+a)**8,x)
```

output
```
a**8*x**6/6 + a**7*b*x**8 + 14*a**6*b**2*x**10/5 + 14*a**5*b**3*x**12/3 +
5*a**4*b**4*x**14 + 7*a**3*b**5*x**16/2 + 14*a**2*b**6*x**18/9 + 2*a*b**7*
x**20/5 + b**8*x**22/22
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^2)^8 \, dx = \frac{1}{22} b^8x^{22} + \frac{2}{5} ab^7x^{20} + \frac{14}{9} a^2b^6x^{18} + \frac{7}{2} a^3b^5x^{16} + 5 a^4b^4x^{14} + \frac{14}{3} a^5b^3x^{12} + \frac{14}{5} a^6b^2x^{10} + a^7bx^8 + \frac{1}{6} a^8x^6$$

input
```
integrate(x^5*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*
a^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8
*x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^2)^8 \, dx = \frac{1}{22} b^8 x^{22} + \frac{2}{5} a b^7 x^{20} + \frac{14}{9} a^2 b^6 x^{18} + \frac{7}{2} a^3 b^5 x^{16} + 5 a^4 b^4 x^{14} + \frac{14}{3} a^5 b^3 x^{12} + \frac{14}{5} a^6 b^2 x^{10} + a^7 b x^8 + \frac{1}{6} a^8 x^6$$

input
```
integrate(x^5*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/22*b^8*x^22 + 2/5*a*b^7*x^20 + 14/9*a^2*b^6*x^18 + 7/2*a^3*b^5*x^16 + 5*
a^4*b^4*x^14 + 14/3*a^5*b^3*x^12 + 14/5*a^6*b^2*x^10 + a^7*b*x^8 + 1/6*a^8
*x^6
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int x^5 (a + bx^2)^8 \, dx = \frac{a^8 x^6}{6} + a^7 b x^8 + \frac{14 a^6 b^2 x^{10}}{5} + \frac{14 a^5 b^3 x^{12}}{3} + 5 a^4 b^4 x^{14} + \frac{7 a^3 b^5 x^{16}}{2} + \frac{14 a^2 b^6 x^{18}}{9} + \frac{2 a b^7 x^{20}}{5} + \frac{b^8 x^{22}}{22}$$

input
```
int(x^5*(a + b*x^2)^8,x)
```

output
```
(a^8*x^6)/6 + (b^8*x^22)/22 + a^7*b*x^8 + (2*a*b^7*x^20)/5 + (14*a^6*b^2*x
^10)/5 + (14*a^5*b^3*x^12)/3 + 5*a^4*b^4*x^14 + (7*a^3*b^5*x^16)/2 + (14*a
^2*b^6*x^18)/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int x^5(a+bx^2)^8\,dx$$
$$= \frac{x^6(45b^8x^{16}+396a\,b^7x^{14}+1540a^2b^6x^{12}+3465a^3b^5x^{10}+4950a^4b^4x^8+4620a^5b^3x^6+2772a^6b^2x^4+990a}{990}$$

input
```
int(x^5*(b*x^2+a)^8,x)
```

output
```
(x**6*(165*a**8 + 990*a**7*b*x**2 + 2772*a**6*b**2*x**4 + 4620*a**5*b**3*x
**6 + 4950*a**4*b**4*x**8 + 3465*a**3*b**5*x**10 + 1540*a**2*b**6*x**12 +
396*a*b**7*x**14 + 45*b**8*x**16))/990
```

3.90 $\int x^3(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int x^3(a+bx^2)^8\,dx=-\frac{a(a+bx^2)^9}{18b^2}+\frac{(a+bx^2)^{10}}{20b^2}$$

output
```
-1/18*a*(b*x^2+a)^9/b^2+1/20*(b*x^2+a)^10/b^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(34) = 68.

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.12

$$\int x^3(a+bx^2)^8\,dx=\frac{a^8x^4}{4}+\frac{4}{3}a^7bx^6+\frac{7}{2}a^6b^2x^8+\frac{28}{5}a^5b^3x^{10}+\frac{35}{6}a^4b^4x^{12}+4a^3b^5x^{14}+\frac{7}{4}a^2b^6x^{16}+\frac{4}{9}ab^7x^{18}+\frac{b^8x^{20}}{20}$$

input
```
Integrate[x^3*(a + b*x^2)^8,x]
```

output
```
(a^8*x^4)/4 + (4*a^7*b*x^6)/3 + (7*a^6*b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 +
(35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 + (7*a^2*b^6*x^16)/4 + (4*a*b^7*x^18)
/9 + (b^8*x^20)/20
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^8 \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2 (bx^2+a)^8 \, dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^9}{b} - \frac{a(bx^2+a)^8}{b}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{10}}{10b^2} - \frac{a(a+bx^2)^9}{9b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^8,x]
```

output
```
(-1/9*(a*(a + b*x^2)^9)/b^2 + (a + b*x^2)^10/(10*b^2))/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.68

method	result
gosper	$\frac{1}{4}a^8x^4 + \frac{4}{3}a^7b\,x^6 + \frac{7}{2}a^6b^2x^8 + \frac{28}{5}a^5b^3x^{10} + \frac{35}{6}a^4b^4x^{12} + 4a^3b^5x^{14} + \frac{7}{4}a^2b^6x^{16} + \frac{4}{9}a\,b^7x^{18} + \frac{1}{20}$
default	$\frac{1}{4}a^8x^4 + \frac{4}{3}a^7b\,x^6 + \frac{7}{2}a^6b^2x^8 + \frac{28}{5}a^5b^3x^{10} + \frac{35}{6}a^4b^4x^{12} + 4a^3b^5x^{14} + \frac{7}{4}a^2b^6x^{16} + \frac{4}{9}a\,b^7x^{18} + \frac{1}{20}$
norman	$\frac{1}{4}a^8x^4 + \frac{4}{3}a^7b\,x^6 + \frac{7}{2}a^6b^2x^8 + \frac{28}{5}a^5b^3x^{10} + \frac{35}{6}a^4b^4x^{12} + 4a^3b^5x^{14} + \frac{7}{4}a^2b^6x^{16} + \frac{4}{9}a\,b^7x^{18} + \frac{1}{20}$
risch	$\frac{1}{4}a^8x^4 + \frac{4}{3}a^7b\,x^6 + \frac{7}{2}a^6b^2x^8 + \frac{28}{5}a^5b^3x^{10} + \frac{35}{6}a^4b^4x^{12} + 4a^3b^5x^{14} + \frac{7}{4}a^2b^6x^{16} + \frac{4}{9}a\,b^7x^{18} + \frac{1}{20}$
parallelrisch	$\frac{1}{4}a^8x^4 + \frac{4}{3}a^7b\,x^6 + \frac{7}{2}a^6b^2x^8 + \frac{28}{5}a^5b^3x^{10} + \frac{35}{6}a^4b^4x^{12} + 4a^3b^5x^{14} + \frac{7}{4}a^2b^6x^{16} + \frac{4}{9}a\,b^7x^{18} + \frac{1}{20}$
orering	$\frac{x^4(9b^8x^{16}+80a\,b^7x^{14}+315a^2b^6x^{12}+720a^3b^5x^{10}+1050a^4b^4x^8+1008a^5b^3x^6+630a^6b^2x^4+240a^7b\,x^2+45a^8)}{180}$

input
```
int(x^3*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^8*x^4+4/3*a^7*b*x^6+7/2*a^6*b^2*x^8+28/5*a^5*b^3*x^10+35/6*a^4*b^4*x
^12+4*a^3*b^5*x^14+7/4*a^2*b^6*x^16+4/9*a*b^7*x^18+1/20*b^8*x^20
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(30) = 60.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^3 (a+bx^2)^8 \, dx = \frac{1}{20} b^8 x^{20} + \frac{4}{9} a b^7 x^{18} + \frac{7}{4} a^2 b^6 x^{16} + 4 a^3 b^5 x^{14} + \frac{35}{6} a^4 b^4 x^{12} + \frac{28}{5} a^5 b^3 x^{10} + \frac{7}{2} a^6 b^2 x^8 + \frac{4}{3} a^7 b x^6 + \frac{1}{4} a^8 x^4$$

input
```
integrate(x^3*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*
a^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a
^8*x^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 3.09

$$\int x^3 (a+bx^2)^8 \, dx = \frac{a^8 x^4}{4} + \frac{4a^7 b x^6}{3} + \frac{7a^6 b^2 x^8}{2} + \frac{28a^5 b^3 x^{10}}{5} + \frac{35a^4 b^4 x^{12}}{6} + 4a^3 b^5 x^{14} + \frac{7a^2 b^6 x^{16}}{4} + \frac{4ab^7 x^{18}}{9} + \frac{b^8 x^{20}}{20}$$

input
```
integrate(x**3*(b*x**2+a)**8,x)
```

output
```
a**8*x**4/4 + 4*a**7*b*x**6/3 + 7*a**6*b**2*x**8/2 + 28*a**5*b**3*x**10/5
+ 35*a**4*b**4*x**12/6 + 4*a**3*b**5*x**14 + 7*a**2*b**6*x**16/4 + 4*a*b**
7*x**18/9 + b**8*x**20/20
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^3 (a + bx^2)^8 \, dx = \frac{1}{20} b^8 x^{20} + \frac{4}{9} a b^7 x^{18} + \frac{7}{4} a^2 b^6 x^{16} + 4 a^3 b^5 x^{14} + \frac{35}{6} a^4 b^4 x^{12} + \frac{28}{5} a^5 b^3 x^{10} + \frac{7}{2} a^6 b^2 x^8 + \frac{4}{3} a^7 b x^6 + \frac{1}{4} a^8 x^4$$

input
```
integrate(x^3*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*
a^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a
^8*x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(30) = 60$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^3 (a + bx^2)^8 \, dx = \frac{1}{20} b^8 x^{20} + \frac{4}{9} a b^7 x^{18} + \frac{7}{4} a^2 b^6 x^{16} + 4 a^3 b^5 x^{14} + \frac{35}{6} a^4 b^4 x^{12} + \frac{28}{5} a^5 b^3 x^{10} + \frac{7}{2} a^6 b^2 x^8 + \frac{4}{3} a^7 b x^6 + \frac{1}{4} a^8 x^4$$

input
```
integrate(x^3*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/20*b^8*x^20 + 4/9*a*b^7*x^18 + 7/4*a^2*b^6*x^16 + 4*a^3*b^5*x^14 + 35/6*
a^4*b^4*x^12 + 28/5*a^5*b^3*x^10 + 7/2*a^6*b^2*x^8 + 4/3*a^7*b*x^6 + 1/4*a
^8*x^4
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.65

$$\int x^3 \left(a+bx^2\right)^8 \, dx = \frac{a^8\,x^4}{4}+\frac{4\,a^7\,b\,x^6}{3}+\frac{7\,a^6\,b^2\,x^8}{2}+\frac{28\,a^5\,b^3\,x^{10}}{5}+\frac{35\,a^4\,b^4\,x^{12}}{6}+4\,a^3\,b^5\,x^{14}+\frac{7\,a^2\,b^6\,x^{16}}{4}+\frac{4\,a\,b^7\,x^{18}}{9}+\frac{b^8\,x^{20}}{20}$$

input
```
int(x^3*(a + b*x^2)^8,x)
```

output
```
(a^8*x^4)/4 + (b^8*x^20)/20 + (4*a^7*b*x^6)/3 + (4*a*b^7*x^18)/9 + (7*a^6*
b^2*x^8)/2 + (28*a^5*b^3*x^10)/5 + (35*a^4*b^4*x^12)/6 + 4*a^3*b^5*x^14 +
(7*a^2*b^6*x^16)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.71

$$\int x^3 \left(a+bx^2\right)^8 \, dx = \frac{x^4(9b^8x^{16}+80a\,b^7x^{14}+315a^2b^6x^{12}+720a^3b^5x^{10}+1050a^4b^4x^8+1008a^5b^3x^6+630a^6b^2x^4+240a^7b\,x^2}{180}$$

input
```
int(x^3*(b*x^2+a)^8,x)
```

output
```
(x**4*(45*a**8 + 240*a**7*b*x**2 + 630*a**6*b**2*x**4 + 1008*a**5*b**3*x**
6 + 1050*a**4*b**4*x**8 + 720*a**3*b**5*x**10 + 315*a**2*b**6*x**12 + 80*a
*b**7*x**14 + 9*b**8*x**16))/180
```

3.91 $\int x(a+bx^2)^8 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int x(a+bx^2)^8 \, dx = \frac{(a+bx^2)^9}{18b}$$

output
```
1/18*(b*x^2+a)^9/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)^8 \, dx = \frac{(a+bx^2)^9}{18b}$$

input
```
Integrate[x*(a + b*x^2)^8,x]
```

output
```
(a + b*x^2)^9/(18*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^8\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{(a+bx^2)^9}{18b}$$

input
```
Int[x*(a + b*x^2)^8,x]
```

output
```
(a + b*x^2)^9/(18*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
default	$\frac{(b\,x^2+a)^9}{18b}$
gosper	$\frac{1}{2}a^8x^2+2a^7b\,x^4+\frac{14}{3}a^6b^2x^6+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14}{3}a^3b^5x^{12}+2a^2b^6x^{14}+\frac{1}{2}a\,b^7x^{16}+\frac{1}{18}b$
norman	$\frac{1}{2}a^8x^2+2a^7b\,x^4+\frac{14}{3}a^6b^2x^6+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14}{3}a^3b^5x^{12}+2a^2b^6x^{14}+\frac{1}{2}a\,b^7x^{16}+\frac{1}{18}b$
parallelrisch	$\frac{1}{2}a^8x^2+2a^7b\,x^4+\frac{14}{3}a^6b^2x^6+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14}{3}a^3b^5x^{12}+2a^2b^6x^{14}+\frac{1}{2}a\,b^7x^{16}+\frac{1}{18}b$
orering	$\frac{x^2(b^8x^{16}+9a\,b^7x^{14}+36a^2b^6x^{12}+84a^3b^5x^{10}+126a^4b^4x^8+126a^5b^3x^6+84a^6b^2x^4+36a^7b\,x^2+9a^8)}{18}$
risch	$\frac{b^8x^{18}}{18}+\frac{a\,b^7x^{16}}{2}+2a^2b^6x^{14}+\frac{14a^3b^5x^{12}}{3}+7a^4b^4x^{10}+7a^5b^3x^8+\frac{14a^6b^2x^6}{3}+2a^7b\,x^4+\frac{a^8x^2}{2}+\frac{a^9}{18b}$

input
```
int(x*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/18*(b*x^2+a)^9/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.62

$$\int x(a+bx^2)^8\,dx=\frac{1}{18}b^8x^{18}+\frac{1}{2}ab^7x^{16}+2a^2b^6x^{14}+\frac{14}{3}a^3b^5x^{12}+7a^4b^4x^{10}+7a^5b^3x^8+\frac{14}{3}a^6b^2x^6+2a^7bx^4+\frac{1}{2}a^8x^2$$

input
```
integrate(x*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/18*b^8*x^18 + 1/2*a*b^7*x^16 + 2*a^2*b^6*x^14 + 14/3*a^3*b^5*x^12 + 7*a^
4*b^4*x^10 + 7*a^5*b^3*x^8 + 14/3*a^6*b^2*x^6 + 2*a^7*b*x^4 + 1/2*a^8*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 6.19

$$\int x\left(a+bx^2\right)^8\,dx=\frac{a^8x^2}{2}+2a^7bx^4+\frac{14a^6b^2x^6}{3}+7a^5b^3x^8+7a^4b^4x^{10}+\frac{14a^3b^5x^{12}}{3}+2a^2b^6x^{14}+\frac{ab^7x^{16}}{2}+\frac{b^8x^{18}}{18}$$

input
```
integrate(x*(b*x**2+a)**8,x)
```

output
```
a**8*x**2/2 + 2*a**7*b*x**4 + 14*a**6*b**2*x**6/3 + 7*a**5*b**3*x**8 + 7*a
**4*b**4*x**10 + 14*a**3*b**5*x**12/3 + 2*a**2*b**6*x**14 + a*b**7*x**16/2
 + b**8*x**18/18
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x\left(a+bx^2\right)^8\,dx=\frac{\left(bx^2+a\right)^9}{18\,b}$$

input
```
integrate(x*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/18*(b*x^2 + a)^9/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^8\,dx = \frac{(bx^2+a)^9}{18\,b}$$

input
```
integrate(x*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/18*(b*x^2 + a)^9/b
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^8\,dx = \frac{(b\,x^2+a)^9}{18\,b}$$

input
```
int(x*(a + b*x^2)^8,x)
```

output
```
(a + b*x^2)^9/(18*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 5.69

$$\int x(a+bx^2)^8\,dx = \frac{x^2(b^8x^{16}+9a\,b^7x^{14}+36a^2b^6x^{12}+84a^3b^5x^{10}+126a^4b^4x^8+126a^5b^3x^6+84a^6b^2x^4+36a^7b\,x^2+9a^8)}{18}$$

input
```
int(x*(b*x^2+a)^8,x)
```

output
```
(x**2*(9*a**8 + 36*a**7*b*x**2 + 84*a**6*b**2*x**4 + 126*a**5*b**3*x**6 +
126*a**4*b**4*x**8 + 84*a**3*b**5*x**10 + 36*a**2*b**6*x**12 + 9*a*b**7*x*
*14 + b**8*x**16))/18
```

3.92 $\int \frac{(a+bx^2)^8}{x}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^2)^8}{x}\,dx = 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\log(x)$$

output
```
4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x
^10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x}\,dx = 4a^7bx^2+7a^6b^2x^4+\frac{28}{3}a^5b^3x^6+\frac{35}{4}a^4b^4x^8+\frac{28}{5}a^3b^5x^{10}+\frac{7}{3}a^2b^6x^{12}+\frac{4}{7}ab^7x^{14}+\frac{b^8x^{16}}{16}+a^8\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x,x]
```

output
```
4*a^7*b*x^2 + 7*a^6*b^2*x^4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (2
8*a^3*b^5*x^10)/5 + (7*a^2*b^6*x^12)/3 + (4*a*b^7*x^14)/7 + (b^8*x^16)/16
+ a^8*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^8x^{14}+8ab^7x^{12}+28a^2b^6x^{10}+56a^3b^5x^8+70a^4b^4x^6+56a^5b^3x^4+28a^6b^2x^2+8a^7b+\frac{a^8}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(a^8\log\left(x^2\right)+8a^7bx^2+14a^6b^2x^4+\frac{56}{3}a^5b^3x^6+\frac{35}{2}a^4b^4x^8+\frac{56}{5}a^3b^5x^{10}+\frac{14}{3}a^2b^6x^{12}+\frac{8}{7}ab^7x^{14}+\frac{b^8x^{16}}{8}\right)$$

input
```
Int[(a + b*x^2)^8/x,x]
```

output
```
(8*a^7*b*x^2 + 14*a^6*b^2*x^4 + (56*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/2 +
(56*a^3*b^5*x^10)/5 + (14*a^2*b^6*x^12)/3 + (8*a*b^7*x^14)/7 + (b^8*x^16)/
8 + a^8*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
default	$4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{16}}{16}+a^8\ln\left(x\right)$
norman	$4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{16}}{16}+a^8\ln\left(x\right)$
parallelrisch	$4a^7b\,x^2+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+\frac{4a\,b^7x^{14}}{7}+\frac{b^8x^{16}}{16}+a^8\ln\left(x\right)$
risch	$4a^7b\,x^2+\frac{176a^8}{105}+a^8\ln\left(x\right)+\frac{b^8x^{16}}{16}+7a^6b^2x^4+\frac{28a^5b^3x^6}{3}+\frac{35a^4b^4x^8}{4}+\frac{28a^3b^5x^{10}}{5}+\frac{7a^2b^6x^{12}}{3}+4a$

input
```
int((b*x^2+a)^8/x,x,method=_RETURNVERBOSE)
```

output
```
4*a^7*b*x^2+7*a^6*b^2*x^4+28/3*a^5*b^3*x^6+35/4*a^4*b^4*x^8+28/5*a^3*b^5*x
^10+7/3*a^2*b^6*x^12+4/7*a*b^7*x^14+1/16*b^8*x^16+a^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x}\,dx = \frac{1}{16}\,b^8x^{16} + \frac{4}{7}\,ab^7x^{14} + \frac{7}{3}\,a^2b^6x^{12} + \frac{28}{5}\,a^3b^5x^{10} + \frac{35}{4}\,a^4b^4x^8 + \frac{28}{3}\,a^5b^3x^6 + 7\,a^6b^2x^4 + 4\,a^7bx^2 + a^8\log(x)$$

input
```
integrate((b*x^2+a)^8/x,x, algorithm="fricas")
```

output
```
1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35
/4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8*log(
x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x}\,dx = a^8\log(x) + 4a^7bx^2 + 7a^6b^2x^4 + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{5} + \frac{7a^2b^6x^{12}}{3} + \frac{4ab^7x^{14}}{7} + \frac{b^8x^{16}}{16}$$

input
```
integrate((b*x**2+a)**8/x,x)
```

output
```
a**8*log(x) + 4*a**7*b*x**2 + 7*a**6*b**2*x**4 + 28*a**5*b**3*x**6/3 + 35*
a**4*b**4*x**8/4 + 28*a**3*b**5*x**10/5 + 7*a**2*b**6*x**12/3 + 4*a*b**7*x
**14/7 + b**8*x**16/16
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x}\,dx = \frac{1}{16}\,b^8x^{16} + \frac{4}{7}\,ab^7x^{14} + \frac{7}{3}\,a^2b^6x^{12} + \frac{28}{5}\,a^3b^5x^{10} + \frac{35}{4}\,a^4b^4x^8 + \frac{28}{3}\,a^5b^3x^6 + 7\,a^6b^2x^4 + 4\,a^7bx^2 + \frac{1}{2}\,a^8\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^8/x,x, algorithm="maxima")
```

output
```
1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35
/4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*
log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x}\,dx = \frac{1}{16}\,b^8x^{16} + \frac{4}{7}\,ab^7x^{14} + \frac{7}{3}\,a^2b^6x^{12} + \frac{28}{5}\,a^3b^5x^{10} + \frac{35}{4}\,a^4b^4x^8 + \frac{28}{3}\,a^5b^3x^6 + 7\,a^6b^2x^4 + 4\,a^7bx^2 + \frac{1}{2}\,a^8\log\left(x^2\right)$$

input
```
integrate((b*x^2+a)^8/x,x, algorithm="giac")
```

output
```
1/16*b^8*x^16 + 4/7*a*b^7*x^14 + 7/3*a^2*b^6*x^12 + 28/5*a^3*b^5*x^10 + 35
/4*a^4*b^4*x^8 + 28/3*a^5*b^3*x^6 + 7*a^6*b^2*x^4 + 4*a^7*b*x^2 + 1/2*a^8*
log(x^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x}\,dx = a^8\ \ln(x) + \frac{b^8\,x^{16}}{16} + 4\,a^7\,b\,x^2 + \frac{4\,a\,b^7\,x^{14}}{7} + 7\,a^6\,b^2\,x^4 + \frac{28\,a^5\,b^3\,x^6}{3} + \frac{35\,a^4\,b^4\,x^8}{4} + \frac{28\,a^3\,b^5\,x^{10}}{5} + \frac{7\,a^2\,b^6\,x^{12}}{3}$$

input
```
int((a + b*x^2)^8/x,x)
```

output
```
a^8*log(x) + (b^8*x^16)/16 + 4*a^7*b*x^2 + (4*a*b^7*x^14)/7 + 7*a^6*b^2*x^
4 + (28*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/5 + (7*a^2
*b^6*x^12)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x}\,dx = \log(x)\,a^8 + 4a^7b\,x^2 + 7a^6b^2x^4 + \frac{28a^5b^3x^6}{3} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{5} + \frac{7a^2b^6x^{12}}{3} + \frac{4a\,b^7x^{14}}{7} + \frac{b^8x^{16}}{16}$$

input
```
int((b*x^2+a)^8/x,x)
```

output
```
(1680*log(x)*a**8 + 6720*a**7*b*x**2 + 11760*a**6*b**2*x**4 + 15680*a**5*b
**3*x**6 + 14700*a**4*b**4*x**8 + 9408*a**3*b**5*x**10 + 3920*a**2*b**6*x*
*12 + 960*a*b**7*x**14 + 105*b**8*x**16)/1680
```

3.93 $\int \frac{(a+bx^2)^8}{x^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{(a+bx^2)^8}{x^3} \, dx = -\frac{a^8}{2x^2} + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35}{3}a^4b^4x^6 + 7a^3b^5x^8 + \frac{14}{5}a^2b^6x^{10} + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{14}}{14} + 8a^7b\log(x)$$

output
```
-1/2*a^8/x^2+14*a^6*b^2*x^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+
14/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^3} \, dx = -\frac{a^8}{2x^2} + 14a^6b^2x^2 + 14a^5b^3x^4 + \frac{35}{3}a^4b^4x^6 + 7a^3b^5x^8 + \frac{14}{5}a^2b^6x^{10} + \frac{2}{3}ab^7x^{12} + \frac{b^8x^{14}}{14} + 8a^7b\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^3,x]
```

output
```
-1/2*a^8/x^2 + 14*a^6*b^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^
3*b^5*x^8 + (14*a^2*b^6*x^10)/5 + (2*a*b^7*x^12)/3 + (b^8*x^14)/14 + 8*a^7
*b*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^3}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^4}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(b^8x^{12}+8ab^7x^{10}+28a^2b^6x^8+56a^3b^5x^6+70a^4b^4x^4+56a^5b^3x^2+28a^6b^2+\frac{8a^7b}{x^2}+\frac{a^8}{x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^8}{x^2}+8a^7b\log\left(x^2\right)+28a^6b^2x^2+28a^5b^3x^4+\frac{70}{3}a^4b^4x^6+14a^3b^5x^8+\frac{28}{5}a^2b^6x^{10}+\frac{4}{3}ab^7x^{12}+\frac{b^8x^{14}}{7}\right)$$

input
```
Int[(a + b*x^2)^8/x^3,x]
```

output
```
(-(a^8/x^2) + 28*a^6*b^2*x^2 + 28*a^5*b^3*x^4 + (70*a^4*b^4*x^6)/3 + 14*a^
3*b^5*x^8 + (28*a^2*b^6*x^10)/5 + (4*a*b^7*x^12)/3 + (b^8*x^14)/7 + 8*a^7*
b*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^8}{2x^2}+14a^6x^2b^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2a\,b^7x^{12}}{3}+\frac{b^8x^{14}}{14}+8a^7b\ln(x)$
risch	$-\frac{a^8}{2x^2}+14a^6x^2b^2+14a^5b^3x^4+\frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2a\,b^7x^{12}}{3}+\frac{b^8x^{14}}{14}+8a^7b\ln(x)$
norman	$\frac{-\frac{1}{2}a^8+\frac{1}{14}b^8x^{16}+\frac{2}{3}a\,b^7x^{14}+\frac{14}{5}a^2b^6x^{12}+7a^3b^5x^{10}+\frac{35}{3}a^4b^4x^8+14a^5b^3x^6+14a^6b^2x^4}{x^2}+8a^7b\ln(x)$
parallelrisch	$\frac{15b^8x^{16}+140a\,b^7x^{14}+588a^2b^6x^{12}+1470a^3b^5x^{10}+2450a^4b^4x^8+2940a^5b^3x^6+2940a^6b^2x^4+1680a^7b\ln(x)x^2-105a^8}{210x^2}$

input
```
int((b*x^2+a)^8/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^8/x^2+14*a^6*x^2*b^2+14*a^5*b^3*x^4+35/3*a^4*b^4*x^6+7*a^3*b^5*x^8+
14/5*a^2*b^6*x^10+2/3*a*b^7*x^12+1/14*b^8*x^14+8*a^7*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^3}\,dx$$
$$= \frac{15\,b^8x^{16}+140\,ab^7x^{14}+588\,a^2b^6x^{12}+1470\,a^3b^5x^{10}+2450\,a^4b^4x^8+2940\,a^5b^3x^6+2940\,a^6b^2x^4+1680\,a^7}{210\,x^2}$$

input
```
integrate((b*x^2+a)^8/x^3,x, algorithm="fricas")
```

output
```
1/210*(15*b^8*x^16 + 140*a*b^7*x^14 + 588*a^2*b^6*x^12 + 1470*a^3*b^5*x^10
 + 2450*a^4*b^4*x^8 + 2940*a^5*b^3*x^6 + 2940*a^6*b^2*x^4 + 1680*a^7*b*x^2
*log(x) - 105*a^8)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^8}{x^3}\,dx = -\frac{a^8}{2x^2}+8a^7b\log{(x)}+14a^6b^2x^2+14a^5b^3x^4 + \frac{35a^4b^4x^6}{3}+7a^3b^5x^8+\frac{14a^2b^6x^{10}}{5}+\frac{2ab^7x^{12}}{3}+\frac{b^8x^{14}}{14}$$

input
```
integrate((b*x**2+a)**8/x**3,x)
```

output
```
-a**8/(2*x**2) + 8*a**7*b*log(x) + 14*a**6*b**2*x**2 + 14*a**5*b**3*x**4 +
 35*a**4*b**4*x**6/3 + 7*a**3*b**5*x**8 + 14*a**2*b**6*x**10/5 + 2*a*b**7*
x**12/3 + b**8*x**14/14
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^3}\,dx = \frac{1}{14}\,b^8x^{14} + \frac{2}{3}\,ab^7x^{12} + \frac{14}{5}\,a^2b^6x^{10} + 7\,a^3b^5x^8 + \frac{35}{3}\,a^4b^4x^6 + 14\,a^5b^3x^4 + 14\,a^6b^2x^2 + 4\,a^7b\log\left(x^2\right) - \frac{a^8}{2\,x^2}$$

input
```
integrate((b*x^2+a)^8/x^3,x, algorithm="maxima")
```

output
```
1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*
a^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*a^8
/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x^3}\,dx = \frac{1}{14}\,b^8x^{14} + \frac{2}{3}\,ab^7x^{12} + \frac{14}{5}\,a^2b^6x^{10} + 7\,a^3b^5x^8 + \frac{35}{3}\,a^4b^4x^6 + 14\,a^5b^3x^4 + 14\,a^6b^2x^2 + 4\,a^7b\log\left(x^2\right) - \frac{8\,a^7bx^2 + a^8}{2\,x^2}$$

input
```
integrate((b*x^2+a)^8/x^3,x, algorithm="giac")
```

output
```
1/14*b^8*x^14 + 2/3*a*b^7*x^12 + 14/5*a^2*b^6*x^10 + 7*a^3*b^5*x^8 + 35/3*
a^4*b^4*x^6 + 14*a^5*b^3*x^4 + 14*a^6*b^2*x^2 + 4*a^7*b*log(x^2) - 1/2*(8*
a^7*b*x^2 + a^8)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^8}{x^3}\,dx = \frac{b^8\,x^{14}}{14} - \frac{a^8}{2\,x^2} + \frac{2\,a\,b^7\,x^{12}}{3} + 8\,a^7\,b\,\ln(x) + 14\,a^6\,b^2\,x^2 + 14\,a^5\,b^3\,x^4 + \frac{35\,a^4\,b^4\,x^6}{3} + 7\,a^3\,b^5\,x^8 + \frac{14\,a^2\,b^6\,x^{10}}{5}$$

input
```
int((a + b*x^2)^8/x^3,x)
```

output
```
(b^8*x^14)/14 - a^8/(2*x^2) + (2*a*b^7*x^12)/3 + 8*a^7*b*log(x) + 14*a^6*b
^2*x^2 + 14*a^5*b^3*x^4 + (35*a^4*b^4*x^6)/3 + 7*a^3*b^5*x^8 + (14*a^2*b^6
*x^10)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^3}\,dx = \frac{1680\log(x)\,a^7b\,x^2 - 105a^8 + 2940a^6b^2x^4 + 2940a^5b^3x^6 + 2450a^4b^4x^8 + 1470a^3b^5x^{10} + 588a^2b^6x^{12} + 140}{210x^2}$$

input
```
int((b*x^2+a)^8/x^3,x)
```

output
```
(1680*log(x)*a**7*b*x**2 - 105*a**8 + 2940*a**6*b**2*x**4 + 2940*a**5*b**3
*x**6 + 2450*a**4*b**4*x**8 + 1470*a**3*b**5*x**10 + 588*a**2*b**6*x**12 +
 140*a*b**7*x**14 + 15*b**8*x**16)/(210*x**2)
```

3.94 $\int \frac{(a+bx^2)^8}{x^5} dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{(a+bx^2)^8}{x^5} dx = -\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 28a^5b^3x^2 + \frac{35}{2}a^4b^4x^4 + \frac{28}{3}a^3b^5x^6 + \frac{7}{2}a^2b^6x^8 + \frac{4}{5}ab^7x^{10} + \frac{b^8x^{12}}{12} + 28a^6b^2\log(x)$$

output
```
-1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*b^4*x^4+28/3*a^3*b^5*x^6+
7/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^5} dx = -\frac{a^8}{4x^4} - \frac{4a^7b}{x^2} + 28a^5b^3x^2 + \frac{35}{2}a^4b^4x^4 + \frac{28}{3}a^3b^5x^6 + \frac{7}{2}a^2b^6x^8 + \frac{4}{5}ab^7x^{10} + \frac{b^8x^{12}}{12} + 28a^6b^2\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^5,x]
```

output
```
-1/4*a^8/x^4 - (4*a^7*b)/x^2 + 28*a^5*b^3*x^2 + (35*a^4*b^4*x^4)/2 + (28*a
^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + (4*a*b^7*x^10)/5 + (b^8*x^12)/12 + 28*
a^6*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^5}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^6}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(b^8x^{10}+8ab^7x^8+28a^2b^6x^6+56a^3b^5x^4+70a^4b^4x^2+56a^5b^3+\frac{28a^6b^2}{x^2}+\frac{8a^7b}{x^4}+\frac{a^8}{x^6}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{2x^4}-\frac{8a^7b}{x^2}+28a^6b^2\log\left(x^2\right)+56a^5b^3x^2+35a^4b^4x^4+\frac{56}{3}a^3b^5x^6+7a^2b^6x^8+\frac{8}{5}ab^7x^{10}+\frac{b^8x^{12}}{6}\right)$$

input
```
Int[(a + b*x^2)^8/x^5,x]
```

output
```
(-1/2*a^8/x^4 - (8*a^7*b)/x^2 + 56*a^5*b^3*x^2 + 35*a^4*b^4*x^4 + (56*a^3*
b^5*x^6)/3 + 7*a^2*b^6*x^8 + (8*a*b^7*x^10)/5 + (b^8*x^12)/6 + 28*a^6*b^2*
Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result	si
default	$-\frac{a^{8}}{4 x^{4}}-\frac{4 a^{7} b}{x^{2}}+28 a^{5} b^{3} x^{2}+\frac{35 a^{4} x^{4} b^{4}}{2}+\frac{28 a^{3} b^{5} x^{6}}{3}+\frac{7 a^{2} b^{6} x^{8}}{2}+\frac{4 a\, b^{7} x^{10}}{5}+\frac{b^{8} x^{12}}{12}+28 a^{6} b^{2} \ln \left(x \right)$	90
norman	$\frac{-\frac{1}{4} a^{8}+\frac{1}{12} b^{8} x^{16}+\frac{4}{5} a\, b^{7} x^{14}+\frac{7}{2} a^{2} b^{6} x^{12}+\frac{28}{3} a^{3} b^{5} x^{10}+\frac{35}{2} a^{4} b^{4} x^{8}+28 a^{5} b^{3} x^{6}-4 a^{7} b\, x^{2}}{x^{4}}+28 a^{6} b^{2} \ln \left(x \right)$	92
risch	$\frac{b^{8} x^{12}}{12}+\frac{4 a\, b^{7} x^{10}}{5}+\frac{7 a^{2} b^{6} x^{8}}{2}+\frac{28 a^{3} b^{5} x^{6}}{3}+\frac{35 a^{4} x^{4} b^{4}}{2}+28 a^{5} b^{3} x^{2}+\frac{-4 a^{7} b\, x^{2}-\frac{1}{4} a^{8}}{x^{4}}+28 a^{6} b^{2} \ln \left(x \right)$	92
parallelrisch	$\frac{5 b^{8} x^{16}+48 a\, b^{7} x^{14}+210 a^{2} b^{6} x^{12}+560 a^{3} b^{5} x^{10}+1050 a^{4} b^{4} x^{8}+1680 a^{5} b^{3} x^{6}+1680 a^{6} b^{2} \ln \left(x \right) x^{4}-240 a^{7} b\, x^{2}-15 a^{8}}{60 x^{4}}$	95

input
```
int((b*x^2+a)^8/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^8/x^4-4*a^7*b/x^2+28*a^5*b^3*x^2+35/2*a^4*x^4*b^4+28/3*a^3*b^5*x^6+
7/2*a^2*b^6*x^8+4/5*a*b^7*x^10+1/12*b^8*x^12+28*a^6*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^5}\,dx$$
$$= \frac{5\,b^8x^{16}+48\,ab^7x^{14}+210\,a^2b^6x^{12}+560\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6+1680\,a^6b^2x^4\log(x)-240}{60\,x^4}$$

input
```
integrate((b*x^2+a)^8/x^5,x, algorithm="fricas")
```

output
```
1/60*(5*b^8*x^16 + 48*a*b^7*x^14 + 210*a^2*b^6*x^12 + 560*a^3*b^5*x^10 + 1
050*a^4*b^4*x^8 + 1680*a^5*b^3*x^6 + 1680*a^6*b^2*x^4*log(x) - 240*a^7*b*x
^2 - 15*a^8)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = 28a^6b^2\log(x)+28a^5b^3x^2+\frac{35a^4b^4x^4}{2}+\frac{28a^3b^5x^6}{3}+\frac{7a^2b^6x^8}{2}+\frac{4ab^7x^{10}}{5}+\frac{b^8x^{12}}{12}+\frac{-a^8-16a^7bx^2}{4x^4}$$

input
```
integrate((b*x**2+a)**8/x**5,x)
```

output
```
28*a**6*b**2*log(x) + 28*a**5*b**3*x**2 + 35*a**4*b**4*x**4/2 + 28*a**3*b*
*5*x**6/3 + 7*a**2*b**6*x**8/2 + 4*a*b**7*x**10/5 + b**8*x**12/12 + (-a**8
 - 16*a**7*b*x**2)/(4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = \frac{1}{12}\,b^8x^{12} + \frac{4}{5}\,ab^7x^{10} + \frac{7}{2}\,a^2b^6x^8 + \frac{28}{3}\,a^3b^5x^6 + \frac{35}{2}\,a^4b^4x^4 + 28\,a^5b^3x^2 + 14\,a^6b^2\log\left(x^2\right) - \frac{16\,a^7bx^2 + a^8}{4\,x^4}$$

input
```
integrate((b*x^2+a)^8/x^5,x, algorithm="maxima")
```

output
```
1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2
*a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(16*a^7*b*x^2 +
a^8)/x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = \frac{1}{12}\,b^8x^{12} + \frac{4}{5}\,ab^7x^{10} + \frac{7}{2}\,a^2b^6x^8 + \frac{28}{3}\,a^3b^5x^6 + \frac{35}{2}\,a^4b^4x^4 + 28\,a^5b^3x^2 + 14\,a^6b^2\log\left(x^2\right) - \frac{84\,a^6b^2x^4 + 16\,a^7bx^2 + a^8}{4\,x^4}$$

input
```
integrate((b*x^2+a)^8/x^5,x, algorithm="giac")
```

output
```
1/12*b^8*x^12 + 4/5*a*b^7*x^10 + 7/2*a^2*b^6*x^8 + 28/3*a^3*b^5*x^6 + 35/2
*a^4*b^4*x^4 + 28*a^5*b^3*x^2 + 14*a^6*b^2*log(x^2) - 1/4*(84*a^6*b^2*x^4
+ 16*a^7*b*x^2 + a^8)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = \frac{b^8\,x^{12}}{12} - \frac{\frac{a^8}{4}+4\,b\,a^7\,x^2}{x^4} + \frac{4\,a\,b^7\,x^{10}}{5} + 28\,a^5\,b^3\,x^2 + \frac{35\,a^4\,b^4\,x^4}{2} + \frac{28\,a^3\,b^5\,x^6}{3} + \frac{7\,a^2\,b^6\,x^8}{2} + 28\,a^6\,b^2\,\ln(x)$$

input
```
int((a + b*x^2)^8/x^5,x)
```

output
```
(b^8*x^12)/12 - (a^8/4 + 4*a^7*b*x^2)/x^4 + (4*a*b^7*x^10)/5 + 28*a^5*b^3*
x^2 + (35*a^4*b^4*x^4)/2 + (28*a^3*b^5*x^6)/3 + (7*a^2*b^6*x^8)/2 + 28*a^6
*b^2*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^5}\,dx = \frac{1680\log(x)\,a^6b^2x^4 - 15a^8 - 240a^7b\,x^2 + 1680a^5b^3x^6 + 1050a^4b^4x^8 + 560a^3b^5x^{10} + 210a^2b^6x^{12} + 48a\,b^7x}{60x^4}$$

input
```
int((b*x^2+a)^8/x^5,x)
```

output
```
(1680*log(x)*a**6*b**2*x**4 - 15*a**8 - 240*a**7*b*x**2 + 1680*a**5*b**3*x
**6 + 1050*a**4*b**4*x**8 + 560*a**3*b**5*x**10 + 210*a**2*b**6*x**12 + 48
*a*b**7*x**14 + 5*b**8*x**16)/(60*x**4)
```

3.95 $\int \frac{(a+bx^2)^8}{x^7} dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{(a+bx^2)^8}{x^7} dx = -\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14}{3}a^2b^6x^6 + ab^7x^8 + \frac{b^8x^{10}}{10} + 56a^5b^3\log(x)$$

output
```
-1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3
*a^2*b^6*x^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^7} dx = -\frac{a^8}{6x^6} - \frac{2a^7b}{x^4} - \frac{14a^6b^2}{x^2} + 35a^4b^4x^2 + 14a^3b^5x^4 + \frac{14}{3}a^2b^6x^6 + ab^7x^8 + \frac{b^8x^{10}}{10} + 56a^5b^3\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^7,x]
```

output
```
-1/6*a^8/x^6 - (2*a^7*b)/x^4 - (14*a^6*b^2)/x^2 + 35*a^4*b^4*x^2 + 14*a^3*
b^5*x^4 + (14*a^2*b^6*x^6)/3 + a*b^7*x^8 + (b^8*x^10)/10 + 56*a^5*b^3*Log[
x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^7}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^8}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^8}+\frac{8ba^7}{x^6}+\frac{28b^2a^6}{x^4}+\frac{56b^3a^5}{x^2}+70b^4a^4+56b^5x^2a^3+28b^6x^4a^2+8b^7x^6a+b^8x^8\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{3x^6}-\frac{4a^7b}{x^4}-\frac{28a^6b^2}{x^2}+56a^5b^3\log\left(x^2\right)+70a^4b^4x^2+28a^3b^5x^4+\frac{28}{3}a^2b^6x^6+2ab^7x^8+\frac{b^8x^{10}}{5}\right)$$

input
```
Int[(a + b*x^2)^8/x^7,x]
```

output
```
(-1/3*a^8/x^6 - (4*a^7*b)/x^4 - (28*a^6*b^2)/x^2 + 70*a^4*b^4*x^2 + 28*a^3
*b^5*x^4 + (28*a^2*b^6*x^6)/3 + 2*a*b^7*x^8 + (b^8*x^10)/5 + 56*a^5*b^3*Lo
g[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

method	result	si
default	$-\frac{a^{8}}{6x^{6}}-\frac{2a^{7}b}{x^{4}}-\frac{14a^{6}b^{2}}{x^{2}}+35a^{4}b^{4}x^{2}+14a^{3}b^{5}x^{4}+\frac{14a^{2}x^{6}b^{6}}{3}+a\,b^{7}x^{8}+\frac{b^{8}x^{10}}{10}+56a^{5}b^{3}\ln\left(x\right)$	89
norman	$\frac{a\,b^{7}x^{14}-\frac{1}{6}a^{8}+\frac{1}{10}b^{8}x^{16}+\frac{14}{3}a^{2}b^{6}x^{12}+14a^{3}b^{5}x^{10}+35a^{4}b^{4}x^{8}-14a^{6}b^{2}x^{4}-2a^{7}b\,x^{2}}{x^{6}}+56a^{5}b^{3}\ln\left(x\right)$	91
risch	$\frac{b^{8}x^{10}}{10}+a\,b^{7}x^{8}+\frac{14a^{2}x^{6}b^{6}}{3}+14a^{3}b^{5}x^{4}+35a^{4}b^{4}x^{2}+\frac{-14a^{6}b^{2}x^{4}-2a^{7}b\,x^{2}-\frac{1}{6}a^{8}}{x^{6}}+56a^{5}b^{3}\ln\left(x\right)$	91
parallelrisch	$\frac{3b^{8}x^{16}+30a\,b^{7}x^{14}+140a^{2}b^{6}x^{12}+420a^{3}b^{5}x^{10}+1050a^{4}b^{4}x^{8}+1680a^{5}b^{3}\ln\left(x\right)x^{6}-420a^{6}b^{2}x^{4}-60a^{7}b\,x^{2}-5a^{8}}{30x^{6}}$	95

input
```
int((b*x^2+a)^8/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a^8/x^6-2*a^7*b/x^4-14*a^6*b^2/x^2+35*a^4*b^4*x^2+14*a^3*b^5*x^4+14/3
*a^2*x^6*b^6+a*b^7*x^8+1/10*b^8*x^10+56*a^5*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = \frac{3\,b^8x^{16}+30\,ab^7x^{14}+140\,a^2b^6x^{12}+420\,a^3b^5x^{10}+1050\,a^4b^4x^8+1680\,a^5b^3x^6\log(x)-420\,a^6b^2x^4-60\,a}{30\,x^6}$$

input
```
integrate((b*x^2+a)^8/x^7,x, algorithm="fricas")
```

output
```
1/30*(3*b^8*x^16 + 30*a*b^7*x^14 + 140*a^2*b^6*x^12 + 420*a^3*b^5*x^10 + 1
050*a^4*b^4*x^8 + 1680*a^5*b^3*x^6*log(x) - 420*a^6*b^2*x^4 - 60*a^7*b*x^2
 - 5*a^8)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = 56a^5b^3\log(x)+35a^4b^4x^2+14a^3b^5x^4+\frac{14a^2b^6x^6}{3}+ab^7x^8+\frac{b^8x^{10}}{10}+\frac{-a^8-12a^7bx^2-84a^6b^2x^4}{6x^6}$$

input
```
integrate((b*x**2+a)**8/x**7,x)
```

output
```
56*a**5*b**3*log(x) + 35*a**4*b**4*x**2 + 14*a**3*b**5*x**4 + 14*a**2*b**6
*x**6/3 + a*b**7*x**8 + b**8*x**10/10 + (-a**8 - 12*a**7*b*x**2 - 84*a**6*
b**2*x**4)/(6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = \frac{1}{10}\,b^8x^{10} + ab^7x^8 + \frac{14}{3}\,a^2b^6x^6 + 14\,a^3b^5x^4 + 35\,a^4b^4x^2 + 28\,a^5b^3\log\left(x^2\right) - \frac{84\,a^6b^2x^4 + 12\,a^7bx^2 + a^8}{6\,x^6}$$

input
```
integrate((b*x^2+a)^8/x^7,x, algorithm="maxima")
```

output
```
1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4
*x^2 + 28*a^5*b^3*log(x^2) - 1/6*(84*a^6*b^2*x^4 + 12*a^7*b*x^2 + a^8)/x^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = \frac{1}{10}\,b^8x^{10} + ab^7x^8 + \frac{14}{3}\,a^2b^6x^6 + 14\,a^3b^5x^4 + 35\,a^4b^4x^2 + 28\,a^5b^3\log\left(x^2\right) - \frac{308\,a^5b^3x^6 + 84\,a^6b^2x^4 + 12\,a^7bx^2 + a^8}{6\,x^6}$$

input
```
integrate((b*x^2+a)^8/x^7,x, algorithm="giac")
```

output
```
1/10*b^8*x^10 + a*b^7*x^8 + 14/3*a^2*b^6*x^6 + 14*a^3*b^5*x^4 + 35*a^4*b^4
*x^2 + 28*a^5*b^3*log(x^2) - 1/6*(308*a^5*b^3*x^6 + 84*a^6*b^2*x^4 + 12*a^
7*b*x^2 + a^8)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = \frac{b^8\,x^{10}}{10} - \frac{\frac{a^8}{6} + 2\,a^7\,b\,x^2 + 14\,a^6\,b^2\,x^4}{x^6} + a\,b^7\,x^8 + 35\,a^4\,b^4\,x^2 + 14\,a^3\,b^5\,x^4 + \frac{14\,a^2\,b^6\,x^6}{3} + 56\,a^5\,b^3\,\ln(x)$$

input
```
int((a + b*x^2)^8/x^7,x)
```

output
```
(b^8*x^10)/10 - (a^8/6 + 2*a^7*b*x^2 + 14*a^6*b^2*x^4)/x^6 + a*b^7*x^8 + 3
5*a^4*b^4*x^2 + 14*a^3*b^5*x^4 + (14*a^2*b^6*x^6)/3 + 56*a^5*b^3*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^7}\,dx = \frac{1680\log(x)\,a^5b^3x^6 - 5a^8 - 60a^7b\,x^2 - 420a^6b^2x^4 + 1050a^4b^4x^8 + 420a^3b^5x^{10} + 140a^2b^6x^{12} + 30a\,b^7x^{14} +}{30x^6}$$

input
```
int((b*x^2+a)^8/x^7,x)
```

output
```
(1680*log(x)*a**5*b**3*x**6 - 5*a**8 - 60*a**7*b*x**2 - 420*a**6*b**2*x**4
 + 1050*a**4*b**4*x**8 + 420*a**3*b**5*x**10 + 140*a**2*b**6*x**12 + 30*a*
b**7*x**14 + 3*b**8*x**16)/(30*x**6)
```

3.96 $\int \frac{(a+bx^2)^8}{x^9}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)$$

output
```
-1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a
^2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = -\frac{a^8}{8x^8}-\frac{4a^7b}{3x^6}-\frac{7a^6b^2}{x^4}-\frac{28a^5b^3}{x^2}+28a^3b^5x^2+7a^2b^6x^4+\frac{4}{3}ab^7x^6+\frac{b^8x^8}{8}+70a^4b^4\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^9,x]
```

output
```
-1/8*a^8/x^8 - (4*a^7*b)/(3*x^6) - (7*a^6*b^2)/x^4 - (28*a^5*b^3)/x^2 + 28
*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + (4*a*b^7*x^6)/3 + (b^8*x^8)/8 + 70*a^4*b^4*
Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^9}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{10}}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{10}}+\frac{8ba^7}{x^8}+\frac{28b^2a^6}{x^6}+\frac{56b^3a^5}{x^4}+\frac{70b^4a^4}{x^2}+56b^5a^3+28b^6x^2a^2+8b^7x^4a+b^8x^6\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{4x^8}-\frac{8a^7b}{3x^6}-\frac{14a^6b^2}{x^4}-\frac{56a^5b^3}{x^2}+70a^4b^4\log\left(x^2\right)+56a^3b^5x^2+14a^2b^6x^4+\frac{8}{3}ab^7x^6+\frac{b^8x^8}{4}\right)$$

input
```
Int[(a + b*x^2)^8/x^9,x]
```

output
```
(-1/4*a^8/x^8 - (8*a^7*b)/(3*x^6) - (14*a^6*b^2)/x^4 - (56*a^5*b^3)/x^2 +
56*a^3*b^5*x^2 + 14*a^2*b^6*x^4 + (8*a*b^7*x^6)/3 + (b^8*x^8)/4 + 70*a^4*b
^4*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{a^{8}}{8x^{8}}-\frac{4a^{7}b}{3x^{6}}-\frac{7a^{6}b^{2}}{x^{4}}-\frac{28a^{5}b^{3}}{x^{2}}+28a^{3}b^{5}x^{2}+7a^{2}b^{6}x^{4}+\frac{4a\,b^{7}x^{6}}{3}+\frac{b^{8}x^{8}}{8}+70a^{4}b^{4}\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{8}a^{8}+\frac{1}{8}b^{8}x^{16}+\frac{4}{3}a\,b^{7}x^{14}+7a^{2}b^{6}x^{12}+28a^{3}b^{5}x^{10}-28a^{5}b^{3}x^{6}-7a^{6}b^{2}x^{4}-\frac{4}{3}a^{7}b\,x^{2}}{x^{8}}+70a^{4}b^{4}\ln\left(x\right)$	92
risch	$\frac{b^{8}x^{8}}{8}+\frac{4a\,b^{7}x^{6}}{3}+7a^{2}b^{6}x^{4}+28a^{3}b^{5}x^{2}+\frac{-28a^{5}b^{3}x^{6}-7a^{6}b^{2}x^{4}-\frac{4}{3}a^{7}b\,x^{2}-\frac{1}{8}a^{8}}{x^{8}}+70a^{4}b^{4}\ln\left(x\right)$	92
parallelrisch	$\frac{3b^{8}x^{16}+32a\,b^{7}x^{14}+168a^{2}b^{6}x^{12}+672a^{3}b^{5}x^{10}+1680a^{4}b^{4}\ln\left(x\right)x^{8}-672a^{5}b^{3}x^{6}-168a^{6}b^{2}x^{4}-32a^{7}b\,x^{2}-3a^{8}}{24x^{8}}$	95

input
```
int((b*x^2+a)^8/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*a^8/x^8-4/3*a^7*b/x^6-7*a^6*b^2/x^4-28*a^5*b^3/x^2+28*a^3*b^5*x^2+7*a
^2*b^6*x^4+4/3*a*b^7*x^6+1/8*b^8*x^8+70*a^4*b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = \frac{3\,b^8x^{16}+32\,ab^7x^{14}+168\,a^2b^6x^{12}+672\,a^3b^5x^{10}+1680\,a^4b^4x^8\log(x)-672\,a^5b^3x^6-168\,a^6b^2x^4-32\,a^7b}{24\,x^8}$$

input
```
integrate((b*x^2+a)^8/x^9,x, algorithm="fricas")
```

output
```
1/24*(3*b^8*x^16 + 32*a*b^7*x^14 + 168*a^2*b^6*x^12 + 672*a^3*b^5*x^10 + 1
680*a^4*b^4*x^8*log(x) - 672*a^5*b^3*x^6 - 168*a^6*b^2*x^4 - 32*a^7*b*x^2
- 3*a^8)/x^8
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = 70a^4b^4\log(x)+28a^3b^5x^2+7a^2b^6x^4+\frac{4ab^7x^6}{3} + \frac{b^8x^8}{8}+\frac{-3a^8-32a^7bx^2-168a^6b^2x^4-672a^5b^3x^6}{24x^8}$$

input
```
integrate((b*x**2+a)**8/x**9,x)
```

output
```
70*a**4*b**4*log(x) + 28*a**3*b**5*x**2 + 7*a**2*b**6*x**4 + 4*a*b**7*x**6
/3 + b**8*x**8/8 + (-3*a**8 - 32*a**7*b*x**2 - 168*a**6*b**2*x**4 - 672*a*
*5*b**3*x**6)/(24*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = \frac{1}{8}\,b^8x^8 + \frac{4}{3}\,ab^7x^6 + 7\,a^2b^6x^4 + 28\,a^3b^5x^2 + 35\,a^4b^4\log\left(x^2\right) - \frac{672\,a^5b^3x^6 + 168\,a^6b^2x^4 + 32\,a^7bx^2 + 3\,a^8}{24\,x^8}$$

input
```
integrate((b*x^2+a)^8/x^9,x, algorithm="maxima")
```

output
```
1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*
log(x^2) - 1/24*(672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32*a^7*b*x^2 + 3*a^8)
/x^8
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = \frac{1}{8}\,b^8x^8 + \frac{4}{3}\,ab^7x^6 + 7\,a^2b^6x^4 + 28\,a^3b^5x^2 + 35\,a^4b^4\log\left(x^2\right) - \frac{1750\,a^4b^4x^8 + 672\,a^5b^3x^6 + 168\,a^6b^2x^4 + 32\,a^7bx^2 + 3\,a^8}{24\,x^8}$$

input
```
integrate((b*x^2+a)^8/x^9,x, algorithm="giac")
```

output
```
1/8*b^8*x^8 + 4/3*a*b^7*x^6 + 7*a^2*b^6*x^4 + 28*a^3*b^5*x^2 + 35*a^4*b^4*
log(x^2) - 1/24*(1750*a^4*b^4*x^8 + 672*a^5*b^3*x^6 + 168*a^6*b^2*x^4 + 32
*a^7*b*x^2 + 3*a^8)/x^8
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = \frac{b^8\,x^8}{8} - \frac{\frac{a^8}{8}+\frac{4\,a^7\,b\,x^2}{3}+7\,a^6\,b^2\,x^4+28\,a^5\,b^3\,x^6}{x^8} + \frac{4\,a\,b^7\,x^6}{3} + 28\,a^3\,b^5\,x^2 + 7\,a^2\,b^6\,x^4 + 70\,a^4\,b^4\,\ln(x)$$

input
```
int((a + b*x^2)^8/x^9,x)
```

output
```
(b^8*x^8)/8 - (a^8/8 + (4*a^7*b*x^2)/3 + 7*a^6*b^2*x^4 + 28*a^5*b^3*x^6)/x
^8 + (4*a*b^7*x^6)/3 + 28*a^3*b^5*x^2 + 7*a^2*b^6*x^4 + 70*a^4*b^4*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^9}\,dx = \frac{1680\log(x)\,a^4b^4x^8 - 3a^8 - 32a^7b\,x^2 - 168a^6b^2x^4 - 672a^5b^3x^6 + 672a^3b^5x^{10} + 168a^2b^6x^{12} + 32a\,b^7x^{14} +}{24x^8}$$

input
```
int((b*x^2+a)^8/x^9,x)
```

output
```
(1680*log(x)*a**4*b**4*x**8 - 3*a**8 - 32*a**7*b*x**2 - 168*a**6*b**2*x**4
 - 672*a**5*b**3*x**6 + 672*a**3*b**5*x**10 + 168*a**2*b**6*x**12 + 32*a*b
**7*x**14 + 3*b**8*x**16)/(24*x**8)
```

3.97 $\int \frac{(a+bx^2)^8}{x^{11}} dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \frac{(a+bx^2)^8}{x^{11}} dx = -\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + 56a^3b^5\log(x)$$

output
```
-1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14
*a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{11}} dx = -\frac{a^8}{10x^{10}} - \frac{a^7b}{x^8} - \frac{14a^6b^2}{3x^6} - \frac{14a^5b^3}{x^4} - \frac{35a^4b^4}{x^2} + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + 56a^3b^5\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^11,x]
```

output
```
-1/10*a^8/x^10 - (a^7*b)/x^8 - (14*a^6*b^2)/(3*x^6) - (14*a^5*b^3)/x^4 - (
35*a^4*b^4)/x^2 + 14*a^2*b^6*x^2 + 2*a*b^7*x^4 + (b^8*x^6)/6 + 56*a^3*b^5*
Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{11}} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{12}}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{12}}+\frac{8ba^7}{x^{10}}+\frac{28b^2a^6}{x^8}+\frac{56b^3a^5}{x^6}+\frac{70b^4a^4}{x^4}+\frac{56b^5a^3}{x^2}+28b^6a^2+8b^7x^2a+b^8x^4\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{5x^{10}}-\frac{2a^7b}{x^8}-\frac{28a^6b^2}{3x^6}-\frac{28a^5b^3}{x^4}-\frac{70a^4b^4}{x^2}+56a^3b^5\log\left(x^2\right)+28a^2b^6x^2+4ab^7x^4+\frac{b^8x^6}{3}\right)$$

input
```
Int[(a + b*x^2)^8/x^11,x]
```

output
```
(-1/5*a^8/x^10 - (2*a^7*b)/x^8 - (28*a^6*b^2)/(3*x^6) - (28*a^5*b^3)/x^4 -
 (70*a^4*b^4)/x^2 + 28*a^2*b^6*x^2 + 4*a*b^7*x^4 + (b^8*x^6)/3 + 56*a^3*b^
5*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{a^{8}}{10x^{10}}-\frac{a^{7}b}{x^{8}}-\frac{14a^{6}b^{2}}{3x^{6}}-\frac{14a^{5}b^{3}}{x^{4}}-\frac{35a^{4}b^{4}}{x^{2}}+14a^{2}b^{6}x^{2}+2a\,b^{7}x^{4}+\frac{b^{8}x^{6}}{6}+56a^{3}b^{5}\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{10}a^{8}+\frac{1}{6}b^{8}x^{16}+2a\,b^{7}x^{14}+14a^{2}b^{6}x^{12}-35a^{4}b^{4}x^{8}-14a^{5}b^{3}x^{6}-\frac{14}{3}a^{6}b^{2}x^{4}-a^{7}b\,x^{2}}{x^{10}}+56a^{3}b^{5}\ln\left(x\right)$	92
risch	$\frac{b^{8}x^{6}}{6}+2a\,b^{7}x^{4}+14a^{2}b^{6}x^{2}+\frac{-35a^{4}b^{4}x^{8}-14a^{5}b^{3}x^{6}-\frac{14}{3}a^{6}b^{2}x^{4}-a^{7}b\,x^{2}-\frac{1}{10}a^{8}}{x^{10}}+56a^{3}b^{5}\ln\left(x\right)$	92
parallelrisch	$\frac{5b^{8}x^{16}+60a\,b^{7}x^{14}+420a^{2}b^{6}x^{12}+1680a^{3}b^{5}\ln\left(x\right)x^{10}-1050a^{4}b^{4}x^{8}-420a^{5}b^{3}x^{6}-140a^{6}b^{2}x^{4}-30a^{7}b\,x^{2}-3a^{8}}{30x^{10}}$	95

input
```
int((b*x^2+a)^8/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*a^8/x^10-a^7*b/x^8-14/3*a^6*b^2/x^6-14*a^5*b^3/x^4-35*a^4*b^4/x^2+14
*a^2*b^6*x^2+2*a*b^7*x^4+1/6*b^8*x^6+56*a^3*b^5*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx$$
$$= \frac{5\,b^8x^{16} + 60\,ab^7x^{14} + 420\,a^2b^6x^{12} + 1680\,a^3b^5x^{10}\log(x) - 1050\,a^4b^4x^8 - 420\,a^5b^3x^6 - 140\,a^6b^2x^4 - 30\,a}{30\,x^{10}}$$

input
```
integrate((b*x^2+a)^8/x^11,x, algorithm="fricas")
```

output
```
1/30*(5*b^8*x^16 + 60*a*b^7*x^14 + 420*a^2*b^6*x^12 + 1680*a^3*b^5*x^10*lo
g(x) - 1050*a^4*b^4*x^8 - 420*a^5*b^3*x^6 - 140*a^6*b^2*x^4 - 30*a^7*b*x^2
 - 3*a^8)/x^10
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx = 56a^3b^5\log(x) + 14a^2b^6x^2 + 2ab^7x^4 + \frac{b^8x^6}{6} + \frac{-3a^8 - 30a^7bx^2 - 140a^6b^2x^4 - 420a^5b^3x^6 - 1050a^4b^4x^8}{30x^{10}}$$

input
```
integrate((b*x**2+a)**8/x**11,x)
```

output
```
56*a**3*b**5*log(x) + 14*a**2*b**6*x**2 + 2*a*b**7*x**4 + b**8*x**6/6 + (-
3*a**8 - 30*a**7*b*x**2 - 140*a**6*b**2*x**4 - 420*a**5*b**3*x**6 - 1050*a
**4*b**4*x**8)/(30*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx = \frac{1}{6}\,b^8x^6 + 2\,ab^7x^4 + 14\,a^2b^6x^2 + 28\,a^3b^5\log\left(x^2\right) - \frac{1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 + 3\,a^8}{30\,x^{10}}$$

input
```
integrate((b*x^2+a)^8/x^11,x, algorithm="maxima")
```

output
```
1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(1
050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 + 30*a^7*b*x^2 + 3*a^8
)/x^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx = \frac{1}{6}\,b^8x^6 + 2\,ab^7x^4 + 14\,a^2b^6x^2 + 28\,a^3b^5\log\left(x^2\right) - \frac{1918\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 420\,a^5b^3x^6 + 140\,a^6b^2x^4 + 30\,a^7bx^2 + 3\,a^8}{30\,x^{10}}$$

input
```
integrate((b*x^2+a)^8/x^11,x, algorithm="giac")
```

output
```
1/6*b^8*x^6 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 28*a^3*b^5*log(x^2) - 1/30*(1
918*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 420*a^5*b^3*x^6 + 140*a^6*b^2*x^4 +
30*a^7*b*x^2 + 3*a^8)/x^10
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx = \frac{b^8\,x^6}{6} - \frac{\frac{a^8}{10} + a^7\,b\,x^2 + \frac{14\,a^6\,b^2\,x^4}{3} + 14\,a^5\,b^3\,x^6 + 35\,a^4\,b^4\,x^8}{x^{10}} + 2\,a\,b^7\,x^4 + 14\,a^2\,b^6\,x^2 + 56\,a^3\,b^5\,\ln(x)$$

input
```
int((a + b*x^2)^8/x^11,x)
```

output
```
(b^8*x^6)/6 - (a^8/10 + a^7*b*x^2 + (14*a^6*b^2*x^4)/3 + 14*a^5*b^3*x^6 +
35*a^4*b^4*x^8)/x^10 + 2*a*b^7*x^4 + 14*a^2*b^6*x^2 + 56*a^3*b^5*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^{11}}\,dx = \frac{1680\log(x)\,a^3b^5x^{10} - 3a^8 - 30a^7b\,x^2 - 140a^6b^2x^4 - 420a^5b^3x^6 - 1050a^4b^4x^8 + 420a^2b^6x^{12} + 60a\,b^7x^{14}}{30x^{10}}$$

input
```
int((b*x^2+a)^8/x^11,x)
```

output
```
(1680*log(x)*a**3*b**5*x**10 - 3*a**8 - 30*a**7*b*x**2 - 140*a**6*b**2*x**
4 - 420*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 + 420*a**2*b**6*x**12 + 60*a*
b**7*x**14 + 5*b**8*x**16)/(30*x**10)
```

3.98 $\int \frac{(a+bx^2)^8}{x^{13}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = -\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 4ab^7x^2 + \frac{b^8x^4}{4} + 28a^2b^6\log(x)$$

output
```
-1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4*b^
4/x^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = -\frac{a^8}{12x^{12}} - \frac{4a^7b}{5x^{10}} - \frac{7a^6b^2}{2x^8} - \frac{28a^5b^3}{3x^6} - \frac{35a^4b^4}{2x^4} - \frac{28a^3b^5}{x^2} + 4ab^7x^2 + \frac{b^8x^4}{4} + 28a^2b^6\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^13,x]
```

output
```
-1/12*a^8/x^12 - (4*a^7*b)/(5*x^10) - (7*a^6*b^2)/(2*x^8) - (28*a^5*b^3)/(
3*x^6) - (35*a^4*b^4)/(2*x^4) - (28*a^3*b^5)/x^2 + 4*a*b^7*x^2 + (b^8*x^4)
/4 + 28*a^2*b^6*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{14}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{14}}+\frac{8ba^7}{x^{12}}+\frac{28b^2a^6}{x^{10}}+\frac{56b^3a^5}{x^8}+\frac{70b^4a^4}{x^6}+\frac{56b^5a^3}{x^4}+\frac{28b^6a^2}{x^2}+8b^7a+b^8x^2\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^8}{6x^{12}}-\frac{8a^7b}{5x^{10}}-\frac{7a^6b^2}{x^8}-\frac{56a^5b^3}{3x^6}-\frac{35a^4b^4}{x^4}-\frac{56a^3b^5}{x^2}+28a^2b^6\log\left(x^2\right)+8ab^7x^2+\frac{b^8x^4}{2}\right)$$

input
```
Int[(a + b*x^2)^8/x^13,x]
```

output
```
(-1/6*a^8/x^12 - (8*a^7*b)/(5*x^10) - (7*a^6*b^2)/x^8 - (56*a^5*b^3)/(3*x^
6) - (35*a^4*b^4)/x^4 - (56*a^3*b^5)/x^2 + 8*a*b^7*x^2 + (b^8*x^4)/2 + 28*
a^2*b^6*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{8}}{12 x^{12}}-\frac{4 a^{7} b}{5 x^{10}}-\frac{7 a^{6} b^{2}}{2 x^{8}}-\frac{28 a^{5} b^{3}}{3 x^{6}}-\frac{35 a^{4} b^{4}}{2 x^{4}}-\frac{28 a^{3} b^{5}}{x^{2}}+4 a\, b^{7} x^{2}+\frac{b^{8} x^{4}}{4}+28 a^{2} b^{6} \ln \left(x \right)$
norman	$\frac{-\frac{1}{12} a^{8}+\frac{1}{4} b^{8} x^{16}+4 a\, b^{7} x^{14}-28 a^{3} b^{5} x^{10}-\frac{35}{2} a^{4} b^{4} x^{8}-\frac{28}{3} a^{5} b^{3} x^{6}-\frac{7}{2} a^{6} b^{2} x^{4}-\frac{4}{5} a^{7} b\, x^{2}}{x^{12}}+28 a^{2} b^{6} \ln \left(x \right)$
parallelrisch	$\frac{15 b^{8} x^{16}+240 a\, b^{7} x^{14}+1680 a^{2} b^{6} \ln \left(x \right) x^{12}-1680 a^{3} b^{5} x^{10}-1050 a^{4} b^{4} x^{8}-560 a^{5} b^{3} x^{6}-210 a^{6} b^{2} x^{4}-48 a^{7} b\, x^{2}-5 a^{8}}{60 x^{12}}$
risch	$\frac{b^{8} x^{4}}{4}+4 a\, b^{7} x^{2}+16 a^{2} b^{6}+\frac{-28 a^{3} b^{5} x^{10}-\frac{35}{2} a^{4} b^{4} x^{8}-\frac{28}{3} a^{5} b^{3} x^{6}-\frac{7}{2} a^{6} b^{2} x^{4}-\frac{4}{5} a^{7} b\, x^{2}-\frac{1}{12} a^{8}}{x^{12}}+28 a^{2} b^{6} \ln \left(x \right)$

input
```
int((b*x^2+a)^8/x^13,x,method=_RETURNVERBOSE)
```

output
```
-1/12*a^8/x^12-4/5*a^7*b/x^10-7/2*a^6*b^2/x^8-28/3*a^5*b^3/x^6-35/2*a^4/x^
4*b^4-28*a^3*b^5/x^2+4*a*b^7*x^2+1/4*b^8*x^4+28*a^2*b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = \frac{15\,b^8x^{16}+240\,ab^7x^{14}+1680\,a^2b^6x^{12}\log(x)-1680\,a^3b^5x^{10}-1050\,a^4b^4x^8-560\,a^5b^3x^6-210\,a^6b^2x^4-4}{60\,x^{12}}$$

input
```
integrate((b*x^2+a)^8/x^13,x, algorithm="fricas")
```

output
```
1/60*(15*b^8*x^16 + 240*a*b^7*x^14 + 1680*a^2*b^6*x^12*log(x) - 1680*a^3*b
^5*x^10 - 1050*a^4*b^4*x^8 - 560*a^5*b^3*x^6 - 210*a^6*b^2*x^4 - 48*a^7*b*
x^2 - 5*a^8)/x^12
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = 28a^2b^6\log(x)+4ab^7x^2+\frac{b^8x^4}{4} + \frac{-5a^8-48a^7bx^2-210a^6b^2x^4-560a^5b^3x^6-1050a^4b^4x^8-1680a^3b^5x^{10}}{60x^{12}}$$

input
```
integrate((b*x**2+a)**8/x**13,x)
```

output
```
28*a**2*b**6*log(x) + 4*a*b**7*x**2 + b**8*x**4/4 + (-5*a**8 - 48*a**7*b*x
**2 - 210*a**6*b**2*x**4 - 560*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 - 1680
*a**3*b**5*x**10)/(60*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = \frac{1}{4}\,b^8x^4 + 4\,ab^7x^2 + 14\,a^2b^6\log\left(x^2\right) - \frac{1680\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 560\,a^5b^3x^6 + 210\,a^6b^2x^4 + 48\,a^7bx^2 + 5\,a^8}{60\,x^{12}}$$

input
```
integrate((b*x^2+a)^8/x^13,x, algorithm="maxima")
```

output
```
1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(1680*a^3*b^5*x^10
+ 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4 + 48*a^7*b*x^2 + 5*
a^8)/x^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^8}{x^{13}}\,dx = \frac{1}{4}\,b^8x^4 + 4\,ab^7x^2 + 14\,a^2b^6\log\left(x^2\right) - \frac{2058\,a^2b^6x^{12} + 1680\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 560\,a^5b^3x^6 + 210\,a^6b^2x^4 + 48\,a^7bx^2 + 5\,a^8}{60\,x^{12}}$$

input
```
integrate((b*x^2+a)^8/x^13,x, algorithm="giac")
```

output
```
1/4*b^8*x^4 + 4*a*b^7*x^2 + 14*a^2*b^6*log(x^2) - 1/60*(2058*a^2*b^6*x^12
+ 1680*a^3*b^5*x^10 + 1050*a^4*b^4*x^8 + 560*a^5*b^3*x^6 + 210*a^6*b^2*x^4
 + 48*a^7*b*x^2 + 5*a^8)/x^12
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+bx^2\right)^8}{x^{13}}\,dx = \frac{b^8\,x^4}{4} - \frac{\frac{a^8}{12} + \frac{4\,a^7\,b\,x^2}{5} + \frac{7\,a^6\,b^2\,x^4}{2} + \frac{28\,a^5\,b^3\,x^6}{3} + \frac{35\,a^4\,b^4\,x^8}{2} + 28\,a^3\,b^5\,x^{10}}{x^{12}} + 4\,a\,b^7\,x^2 + 28\,a^2\,b^6\,\ln\left(x\right)$$

input
```
int((a + b*x^2)^8/x^13,x)
```

output
```
(b^8*x^4)/4 - (a^8/12 + (4*a^7*b*x^2)/5 + (7*a^6*b^2*x^4)/2 + (28*a^5*b^3*
x^6)/3 + (35*a^4*b^4*x^8)/2 + 28*a^3*b^5*x^10)/x^12 + 4*a*b^7*x^2 + 28*a^2
*b^6*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{\left(a+bx^2\right)^8}{x^{13}}\,dx = \frac{1680\log(x)\,a^2b^6x^{12} - 5a^8 - 48a^7b\,x^2 - 210a^6b^2x^4 - 560a^5b^3x^6 - 1050a^4b^4x^8 - 1680a^3b^5x^{10} + 240a\,b^7x^1}{60x^{12}}$$

input
```
int((b*x^2+a)^8/x^13,x)
```

output
```
(1680*log(x)*a**2*b**6*x**12 - 5*a**8 - 48*a**7*b*x**2 - 210*a**6*b**2*x**
4 - 560*a**5*b**3*x**6 - 1050*a**4*b**4*x**8 - 1680*a**3*b**5*x**10 + 240*
a*b**7*x**14 + 15*b**8*x**16)/(60*x**12)
```

3.99 $\int \frac{(a+bx^2)^8}{x^{15}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)$$

output
```
-1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4
/x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = -\frac{a^8}{14x^{14}}-\frac{2a^7b}{3x^{12}}-\frac{14a^6b^2}{5x^{10}}-\frac{7a^5b^3}{x^8}-\frac{35a^4b^4}{3x^6}-\frac{14a^3b^5}{x^4}-\frac{14a^2b^6}{x^2}+\frac{b^8x^2}{2}+8ab^7\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^15,x]
```

output
```
-1/14*a^8/x^14 - (2*a^7*b)/(3*x^12) - (14*a^6*b^2)/(5*x^10) - (7*a^5*b^3)/
x^8 - (35*a^4*b^4)/(3*x^6) - (14*a^3*b^5)/x^4 - (14*a^2*b^6)/x^2 + (b^8*x^
2)/2 + 8*a*b^7*Log[x]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{16}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{16}}+\frac{8ba^7}{x^{14}}+\frac{28b^2a^6}{x^{12}}+\frac{56b^3a^5}{x^{10}}+\frac{70b^4a^4}{x^8}+\frac{56b^5a^3}{x^6}+\frac{28b^6a^2}{x^4}+\frac{8b^7a}{x^2}+b^8\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^8}{7x^{14}}-\frac{4a^7b}{3x^{12}}-\frac{28a^6b^2}{5x^{10}}-\frac{14a^5b^3}{x^8}-\frac{70a^4b^4}{3x^6}-\frac{28a^3b^5}{x^4}-\frac{28a^2b^6}{x^2}+8ab^7\log\left(x^2\right)+b^8x^2\right)$$

input
```
Int[(a + b*x^2)^8/x^15,x]
```

output
```
(-1/7*a^8/x^14 - (4*a^7*b)/(3*x^12) - (28*a^6*b^2)/(5*x^10) - (14*a^5*b^3)
/x^8 - (70*a^4*b^4)/(3*x^6) - (28*a^3*b^5)/x^4 - (28*a^2*b^6)/x^2 + b^8*x^
2 + 8*a*b^7*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a^{8}}{14x^{14}}-\frac{2a^{7}b}{3x^{12}}-\frac{14a^{6}b^{2}}{5x^{10}}-\frac{7a^{5}b^{3}}{x^{8}}-\frac{35a^{4}b^{4}}{3x^{6}}-\frac{14a^{3}b^{5}}{x^{4}}-\frac{14a^{2}b^{6}}{x^{2}}+\frac{b^{8}x^{2}}{2}+8a\,b^{7}\ln\left(x\right)$	90
norman	$\frac{-\frac{1}{14}a^{8}+\frac{1}{2}b^{8}x^{16}-14a^{2}b^{6}x^{12}-14a^{3}b^{5}x^{10}-\frac{35}{3}a^{4}b^{4}x^{8}-7a^{5}b^{3}x^{6}-\frac{14}{5}a^{6}b^{2}x^{4}-\frac{2}{3}a^{7}b\,x^{2}}{x^{14}}+8a\,b^{7}\ln\left(x\right)$	92
risch	$\frac{b^{8}x^{2}}{2}+\frac{-14a^{2}b^{6}x^{12}-14a^{3}b^{5}x^{10}-\frac{35}{3}a^{4}b^{4}x^{8}-7a^{5}b^{3}x^{6}-\frac{14}{5}a^{6}b^{2}x^{4}-\frac{2}{3}a^{7}b\,x^{2}-\frac{1}{14}a^{8}}{x^{14}}+8a\,b^{7}\ln\left(x\right)$	92
parallelrisch	$\frac{105b^{8}x^{16}+1680a\,b^{7}\ln\left(x\right)x^{14}-2940a^{2}b^{6}x^{12}-2940a^{3}b^{5}x^{10}-2450a^{4}b^{4}x^{8}-1470a^{5}b^{3}x^{6}-588a^{6}b^{2}x^{4}-140a^{7}b\,x^{2}-15a^{8}}{210x^{14}}$	95

input
```
int((b*x^2+a)^8/x^15,x,method=_RETURNVERBOSE)
```

output
```
-1/14*a^8/x^14-2/3*a^7*b/x^12-14/5*a^6*b^2/x^10-7*a^5*b^3/x^8-35/3*a^4*b^4
/x^6-14*a^3*b^5/x^4-14*a^2*b^6/x^2+1/2*b^8*x^2+8*a*b^7*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = \frac{105\,b^8x^{16} + 1680\,ab^7x^{14}\log(x) - 2940\,a^2b^6x^{12} - 2940\,a^3b^5x^{10} - 2450\,a^4b^4x^8 - 1470\,a^5b^3x^6 - 588\,a^6b^2x^4}{210\,x^{14}}$$

input
```
integrate((b*x^2+a)^8/x^15,x, algorithm="fricas")
```

output
```
1/210*(105*b^8*x^16 + 1680*a*b^7*x^14*log(x) - 2940*a^2*b^6*x^12 - 2940*a^
3*b^5*x^10 - 2450*a^4*b^4*x^8 - 1470*a^5*b^3*x^6 - 588*a^6*b^2*x^4 - 140*a
^7*b*x^2 - 15*a^8)/x^14
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = 8ab^7\log(x) + \frac{b^8x^2}{2} + \frac{-15a^8 - 140a^7bx^2 - 588a^6b^2x^4 - 1470a^5b^3x^6 - 2450a^4b^4x^8 - 2940a^3b^5x^{10} - 2940a^2b^6x^{12}}{210x^{14}}$$

input
```
integrate((b*x**2+a)**8/x**15,x)
```

output
```
8*a*b**7*log(x) + b**8*x**2/2 + (-15*a**8 - 140*a**7*b*x**2 - 588*a**6*b**
2*x**4 - 1470*a**5*b**3*x**6 - 2450*a**4*b**4*x**8 - 2940*a**3*b**5*x**10
- 2940*a**2*b**6*x**12)/(210*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = \frac{1}{2}\,b^8x^2 + 4\,ab^7\log\left(x^2\right) - \frac{2940\,a^2b^6x^{12} + 2940\,a^3b^5x^{10} + 2450\,a^4b^4x^8 + 1470\,a^5b^3x^6 + 588\,a^6b^2x^4 + 140\,a^7bx^2 + 15\,a^8}{210\,x^{14}}$$

input
```
integrate((b*x^2+a)^8/x^15,x, algorithm="maxima")
```

output
```
1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2940*a^2*b^6*x^12 + 2940*a^3*b^5*x
^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*x^4 + 140*a^7*b*x^
2 + 15*a^8)/x^14
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = \frac{1}{2}\,b^8x^2 + 4\,ab^7\log\left(x^2\right) - \frac{2178\,ab^7x^{14} + 2940\,a^2b^6x^{12} + 2940\,a^3b^5x^{10} + 2450\,a^4b^4x^8 + 1470\,a^5b^3x^6 + 588\,a^6b^2x^4 + 140\,a^7bx^2 + 15}{210\,x^{14}}$$

input
```
integrate((b*x^2+a)^8/x^15,x, algorithm="giac")
```

output
```
1/2*b^8*x^2 + 4*a*b^7*log(x^2) - 1/210*(2178*a*b^7*x^14 + 2940*a^2*b^6*x^1
2 + 2940*a^3*b^5*x^10 + 2450*a^4*b^4*x^8 + 1470*a^5*b^3*x^6 + 588*a^6*b^2*
x^4 + 140*a^7*b*x^2 + 15*a^8)/x^14
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = -\frac{\frac{a^8}{14}-\frac{b^8\,x^{16}}{2}+\frac{2\,a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{5}+7\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{3}+14\,a^3\,b^5\,x^{10}+14\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}\,\ln\left(x\right)}{x^{14}}$$

input
```
int((a + b*x^2)^8/x^15,x)
```

output
```
-(a^8/14 - (b^8*x^16)/2 + (2*a^7*b*x^2)/3 + (14*a^6*b^2*x^4)/5 + 7*a^5*b^3
*x^6 + (35*a^4*b^4*x^8)/3 + 14*a^3*b^5*x^10 + 14*a^2*b^6*x^12 - 8*a*b^7*x^
14*log(x))/x^14
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^{15}}\,dx = \frac{1680\log(x)\,a\,b^7x^{14}-15a^8-140a^7b\,x^2-588a^6b^2x^4-1470a^5b^3x^6-2450a^4b^4x^8-2940a^3b^5x^{10}-2940a}{210x^{14}}$$

input
```
int((b*x^2+a)^8/x^15,x)
```

output
```
(1680*log(x)*a*b**7*x**14 - 15*a**8 - 140*a**7*b*x**2 - 588*a**6*b**2*x**4
 - 1470*a**5*b**3*x**6 - 2450*a**4*b**4*x**8 - 2940*a**3*b**5*x**10 - 2940
*a**2*b**6*x**12 + 105*b**8*x**16)/(210*x**14)
```

3.100 $\int \frac{(a+bx^2)^8}{x^{17}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = -\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + b^8\log(x)$$

output
```
-1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*
b^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = -\frac{a^8}{16x^{16}} - \frac{4a^7b}{7x^{14}} - \frac{7a^6b^2}{3x^{12}} - \frac{28a^5b^3}{5x^{10}} - \frac{35a^4b^4}{4x^8} - \frac{28a^3b^5}{3x^6} - \frac{7a^2b^6}{x^4} - \frac{4ab^7}{x^2} + b^8\log(x)$$

input
```
Integrate[(a + b*x^2)^8/x^17,x]
```

output
```
-1/16*a^8/x^16 - (4*a^7*b)/(7*x^14) - (7*a^6*b^2)/(3*x^12) - (28*a^5*b^3)/
(5*x^10) - (35*a^4*b^4)/(4*x^8) - (28*a^3*b^5)/(3*x^6) - (7*a^2*b^6)/x^4 -
 (4*a*b^7)/x^2 + b^8*Log[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{17}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{18}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{18}}+\frac{8ba^7}{x^{16}}+\frac{28b^2a^6}{x^{14}}+\frac{56b^3a^5}{x^{12}}+\frac{70b^4a^4}{x^{10}}+\frac{56b^5a^3}{x^8}+\frac{28b^6a^2}{x^6}+\frac{8b^7a}{x^4}+\frac{b^8}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^8}{8x^{16}}-\frac{8a^7b}{7x^{14}}-\frac{14a^6b^2}{3x^{12}}-\frac{56a^5b^3}{5x^{10}}-\frac{35a^4b^4}{2x^8}-\frac{56a^3b^5}{3x^6}-\frac{14a^2b^6}{x^4}-\frac{8ab^7}{x^2}+b^8\log\left(x^2\right)\right)$$

input
```
Int[(a + b*x^2)^8/x^17,x]
```

output
```
(-1/8*a^8/x^16 - (8*a^7*b)/(7*x^14) - (14*a^6*b^2)/(3*x^12) - (56*a^5*b^3)
/(5*x^10) - (35*a^4*b^4)/(2*x^8) - (56*a^3*b^5)/(3*x^6) - (14*a^2*b^6)/x^4
 - (8*a*b^7)/x^2 + b^8*Log[x^2])/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^{8}}{16 x^{16}}-\frac{4 a^{7} b}{7 x^{14}}-\frac{7 a^{6} b^{2}}{3 x^{12}}-\frac{28 a^{5} b^{3}}{5 x^{10}}-\frac{35 a^{4} b^{4}}{4 x^{8}}-\frac{28 a^{3} b^{5}}{3 x^{6}}-\frac{7 a^{2} b^{6}}{x^{4}}-\frac{4 a\, b^{7}}{x^{2}}+b^{8} \ln \left(x \right)$
norman	$\frac{-\frac{1}{16} a^{8}-4 a\, b^{7} x^{14}-7 a^{2} b^{6} x^{12}-\frac{28}{3} a^{3} b^{5} x^{10}-\frac{35}{4} a^{4} b^{4} x^{8}-\frac{28}{5} a^{5} b^{3} x^{6}-\frac{7}{3} a^{6} b^{2} x^{4}-\frac{4}{7} a^{7} b\, x^{2}}{x^{16}}+b^{8} \ln \left(x \right)$
risch	$\frac{-\frac{1}{16} a^{8}-4 a\, b^{7} x^{14}-7 a^{2} b^{6} x^{12}-\frac{28}{3} a^{3} b^{5} x^{10}-\frac{35}{4} a^{4} b^{4} x^{8}-\frac{28}{5} a^{5} b^{3} x^{6}-\frac{7}{3} a^{6} b^{2} x^{4}-\frac{4}{7} a^{7} b\, x^{2}}{x^{16}}+b^{8} \ln \left(x \right)$
parallelrisch	$\frac{1680 b^{8} \ln \left(x \right) x^{16}-6720 a\, b^{7} x^{14}-11760 a^{2} b^{6} x^{12}-15680 a^{3} b^{5} x^{10}-14700 a^{4} b^{4} x^{8}-9408 a^{5} b^{3} x^{6}-3920 a^{6} b^{2} x^{4}-960 a^{7} b\, x^{2}-105 a^{8}}{1680 x^{16}}$

input
```
int((b*x^2+a)^8/x^17,x,method=_RETURNVERBOSE)
```

output
```
-1/16*a^8/x^16-4/7*a^7*b/x^14-7/3*a^6*b^2/x^12-28/5*a^5*b^3/x^10-35/4*a^4*
b^4/x^8-28/3*a^3*b^5/x^6-7*a^2*b^6/x^4-4*a*b^7/x^2+b^8*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = \frac{1680\,b^8x^{16}\log(x) - 6720\,ab^7x^{14} - 11760\,a^2b^6x^{12} - 15680\,a^3b^5x^{10} - 14700\,a^4b^4x^8 - 9408\,a^5b^3x^6 - 3920\,a}{1680\,x^{16}}$$

input
```
integrate((b*x^2+a)^8/x^17,x, algorithm="fricas")
```

output
```
1/1680*(1680*b^8*x^16*log(x) - 6720*a*b^7*x^14 - 11760*a^2*b^6*x^12 - 1568
0*a^3*b^5*x^10 - 14700*a^4*b^4*x^8 - 9408*a^5*b^3*x^6 - 3920*a^6*b^2*x^4 -
 960*a^7*b*x^2 - 105*a^8)/x^16
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = b^8\log(x) + \frac{-105a^8 - 960a^7bx^2 - 3920a^6b^2x^4 - 9408a^5b^3x^6 - 14700a^4b^4x^8 - 15680a^3b^5x^{10} - 11760a^2b^6x^{12} - 672}{1680x^{16}}$$

input
```
integrate((b*x**2+a)**8/x**17,x)
```

output
```
b**8*log(x) + (-105*a**8 - 960*a**7*b*x**2 - 3920*a**6*b**2*x**4 - 9408*a*
*5*b**3*x**6 - 14700*a**4*b**4*x**8 - 15680*a**3*b**5*x**10 - 11760*a**2*b
**6*x**12 - 6720*a*b**7*x**14)/(1680*x**16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = \frac{1}{2}\,b^8\log\left(x^2\right) - \frac{6720\,ab^7x^{14} + 11760\,a^2b^6x^{12} + 15680\,a^3b^5x^{10} + 14700\,a^4b^4x^8 + 9408\,a^5b^3x^6 + 3920\,a^6b^2x^4 + 960\,a^7bx^2}{1680\,x^{16}}$$

input
```
integrate((b*x^2+a)^8/x^17,x, algorithm="maxima")
```

output
```
1/2*b^8*log(x^2) - 1/1680*(6720*a*b^7*x^14 + 11760*a^2*b^6*x^12 + 15680*a^
3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a^6*b^2*x^4 + 960
*a^7*b*x^2 + 105*a^8)/x^16
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx = \frac{1}{2}\,b^8\log\left(x^2\right) - \frac{2283\,b^8x^{16} + 6720\,ab^7x^{14} + 11760\,a^2b^6x^{12} + 15680\,a^3b^5x^{10} + 14700\,a^4b^4x^8 + 9408\,a^5b^3x^6 + 3920\,a^6b^2x}{1680\,x^{16}}$$

input
```
integrate((b*x^2+a)^8/x^17,x, algorithm="giac")
```

output
```
1/2*b^8*log(x^2) - 1/1680*(2283*b^8*x^16 + 6720*a*b^7*x^14 + 11760*a^2*b^6
*x^12 + 15680*a^3*b^5*x^10 + 14700*a^4*b^4*x^8 + 9408*a^5*b^3*x^6 + 3920*a
^6*b^2*x^4 + 960*a^7*b*x^2 + 105*a^8)/x^16
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx$$
$$= b^8 \ln(x) - \frac{\frac{a^8}{16} + \frac{4a^7bx^2}{7} + \frac{7a^6b^2x^4}{3} + \frac{28a^5b^3x^6}{5} + \frac{35a^4b^4x^8}{4} + \frac{28a^3b^5x^{10}}{3} + 7a^2b^6x^{12} + 4ab^7x^{14}}{x^{16}}$$

input
```
int((a + b*x^2)^8/x^17,x)
```

output
```
b^8*log(x) - (a^8/16 + (4*a^7*b*x^2)/7 + 4*a*b^7*x^14 + (7*a^6*b^2*x^4)/3
+ (28*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/4 + (28*a^3*b^5*x^10)/3 + 7*a^2*b^
6*x^12)/x^16
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{17}}\,dx$$
$$= \frac{1680\log(x)\,b^8x^{16} - 105a^8 - 960a^7b\,x^2 - 3920a^6b^2x^4 - 9408a^5b^3x^6 - 14700a^4b^4x^8 - 15680a^3b^5x^{10} - 117}{1680x^{16}}$$

input
```
int((b*x^2+a)^8/x^17,x)
```

output
```
(1680*log(x)*b**8*x**16 - 105*a**8 - 960*a**7*b*x**2 - 3920*a**6*b**2*x**4
 - 9408*a**5*b**3*x**6 - 14700*a**4*b**4*x**8 - 15680*a**3*b**5*x**10 - 11
760*a**2*b**6*x**12 - 6720*a*b**7*x**14)/(1680*x**16)
```

3.101 $\int \frac{(a+bx^2)^8}{x^{19}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{(a+bx^2)^8}{x^{19}} \, dx = -\frac{(a+bx^2)^9}{18ax^{18}}$$

output
```
-1/18*(b*x^2+a)^9/a/x^18
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.26

$$\int \frac{(a+bx^2)^8}{x^{19}} \, dx = -\frac{a^8}{18x^{18}} - \frac{a^7b}{2x^{16}} - \frac{2a^6b^2}{x^{14}} - \frac{14a^5b^3}{3x^{12}} - \frac{7a^4b^4}{x^{10}} - \frac{7a^3b^5}{x^8} - \frac{14a^2b^6}{3x^6} - \frac{2ab^7}{x^4} - \frac{b^8}{2x^2}$$

input
```
Integrate[(a + b*x^2)^8/x^19,x]
```

output
```
-1/18*a^8/x^18 - (a^7*b)/(2*x^16) - (2*a^6*b^2)/x^14 - (14*a^5*b^3)/(3*x^1
2) - (7*a^4*b^4)/x^10 - (7*a^3*b^5)/x^8 - (14*a^2*b^6)/(3*x^6) - (2*a*b^7)
/x^4 - b^8/(2*x^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{242\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{19}} \, dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{(a+bx^2)^9}{18ax^{18}}$$

input
```
Int[(a + b*x^2)^8/x^19,x]
```

output
```
-1/18*(a + b*x^2)^9/(a*x^18)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.79

method	result	size
gosper	$-\frac{9b^8x^{16}+36a\,b^7x^{14}+84a^2b^6x^{12}+126a^3b^5x^{10}+126a^4b^4x^8+84a^5b^3x^6+36a^6b^2x^4+9a^7b\,x^2+a^8}{18x^{18}}$	91
default	$-\frac{b^8}{2x^2}-\frac{2a\,b^7}{x^4}-\frac{7a^3b^5}{x^8}-\frac{7a^4b^4}{x^{10}}-\frac{2a^6b^2}{x^{14}}-\frac{14a^2b^6}{3x^6}-\frac{a^8}{18x^{18}}-\frac{a^7b}{2x^{16}}-\frac{14a^5b^3}{3x^{12}}$	91
orering	$-\frac{9b^8x^{16}+36a\,b^7x^{14}+84a^2b^6x^{12}+126a^3b^5x^{10}+126a^4b^4x^8+84a^5b^3x^6+36a^6b^2x^4+9a^7b\,x^2+a^8}{18x^{18}}$	91
norman	$\frac{-\frac{1}{18}a^8-\frac{1}{2}a^7b\,x^2-2a^6b^2x^4-\frac{14}{3}a^5b^3x^6-7a^4b^4x^8-7a^3b^5x^{10}-\frac{14}{3}a^2b^6x^{12}-2a\,b^7x^{14}-\frac{1}{2}b^8x^{16}}{x^{18}}$	92
risch	$\frac{-\frac{1}{18}a^8-\frac{1}{2}a^7b\,x^2-2a^6b^2x^4-\frac{14}{3}a^5b^3x^6-7a^4b^4x^8-7a^3b^5x^{10}-\frac{14}{3}a^2b^6x^{12}-2a\,b^7x^{14}-\frac{1}{2}b^8x^{16}}{x^{18}}$	92
parallelrisch	$\frac{-9b^8x^{16}-36a\,b^7x^{14}-84a^2b^6x^{12}-126a^3b^5x^{10}-126a^4b^4x^8-84a^5b^3x^6-36a^6b^2x^4-9a^7b\,x^2-a^8}{18x^{18}}$	93

input
```
int((b*x^2+a)^8/x^19,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(9*b^8*x^16+36*a*b^7*x^14+84*a^2*b^6*x^12+126*a^3*b^5*x^10+126*a^4*b
^4*x^8+84*a^5*b^3*x^6+36*a^6*b^2*x^4+9*a^7*b*x^2+a^8)/x^18
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

input
```
integrate((b*x^2+a)^8/x^19,x, algorithm="fricas")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(15) = 30$.

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.11

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = \frac{-a^8 - 9a^7bx^2 - 36a^6b^2x^4 - 84a^5b^3x^6 - 126a^4b^4x^8 - 126a^3b^5x^{10} - 84a^2b^6x^{12} - 36ab^7x^{14} - 9b^8x^{16}}{18x^{18}}$$

input
```
integrate((b*x**2+a)**8/x**19,x)
```

output
```
(-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*
b**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9
*b**8*x**16)/(18*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a^7bx^2 + a^8}{18\,x^{18}}$$

input
```
integrate((b*x^2+a)^8/x^19,x, algorithm="maxima")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.74

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,x^{18}}$$

input
```
integrate((b*x^2+a)^8/x^19,x, algorithm="giac")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/x^18
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = -\frac{\frac{a^8}{18}+\frac{a^7\,b\,x^2}{2}+2\,a^6\,b^2\,x^4+\frac{14\,a^5\,b^3\,x^6}{3}+7\,a^4\,b^4\,x^8+7\,a^3\,b^5\,x^{10}+\frac{14\,a^2\,b^6\,x^{12}}{3}+2\,a\,b^7\,x^{14}+\frac{b^8\,x^{16}}{2}}{x^{18}}$$

input
```
int((a + b*x^2)^8/x^19,x)
```

output
```
-(a^8/18 + (b^8*x^16)/2 + (a^7*b*x^2)/2 + 2*a*b^7*x^14 + 2*a^6*b^2*x^4 + (
14*a^5*b^3*x^6)/3 + 7*a^4*b^4*x^8 + 7*a^3*b^5*x^10 + (14*a^2*b^6*x^12)/3)/
x^18
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(a+bx^2)^8}{x^{19}}\,dx = \frac{-9b^8x^{16}-36a\,b^7x^{14}-84a^2b^6x^{12}-126a^3b^5x^{10}-126a^4b^4x^8-84a^5b^3x^6-36a^6b^2x^4-9a^7b\,x^2-a^8}{18x^{18}}$$

input
```
int((b*x^2+a)^8/x^19,x)
```

output
```
( - a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**
4*b**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 -
 9*b**8*x**16)/(18*x**18)
```

3.102 $\int \frac{(a+bx^2)^8}{x^{21}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{(a+bx^2)^9}{20ax^{20}} + \frac{b(a+bx^2)^9}{180a^2x^{18}}$$

output
```
-1/20*(b*x^2+a)^9/a/x^20+1/180*b*(b*x^2+a)^9/a^2/x^18
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(40) = 80.

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.65

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{a^8}{20x^{20}} - \frac{4a^7b}{9x^{18}} - \frac{7a^6b^2}{4x^{16}} - \frac{4a^5b^3}{x^{14}} - \frac{35a^4b^4}{6x^{12}} - \frac{28a^3b^5}{5x^{10}} - \frac{7a^2b^6}{2x^8} - \frac{4ab^7}{3x^6} - \frac{b^8}{4x^4}$$

input
```
Integrate[(a + b*x^2)^8/x^21,x]
```

output
```
-1/20*a^8/x^20 - (4*a^7*b)/(9*x^18) - (7*a^6*b^2)/(4*x^16) - (4*a^5*b^3)/x
^14 - (35*a^4*b^4)/(6*x^12) - (28*a^3*b^5)/(5*x^10) - (7*a^2*b^6)/(2*x^8)
- (4*a*b^7)/(3*x^6) - b^8/(4*x^4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{22}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^8}{x^{20}}dx^2}{10a}-\frac{(a+bx^2)^9}{10ax^{20}}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{2}\left(\frac{b(a+bx^2)^9}{90a^2x^{18}}-\frac{(a+bx^2)^9}{10ax^{20}}\right)$$

input
```
Int[(a + b*x^2)^8/x^21,x]
```

output
```
(-1/10*(a + b*x^2)^9/(a*x^20) + (b*(a + b*x^2)^9)/(90*a^2*x^18))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

method	result	size
default	$-\frac{b^8}{4x^4}-\frac{7a^2b^6}{2x^8}-\frac{28a^3b^5}{5x^{10}}-\frac{4a^5b^3}{x^{14}}-\frac{a^8}{20x^{20}}-\frac{4a\,b^7}{3x^6}-\frac{4a^7b}{9x^{18}}-\frac{7a^6b^2}{4x^{16}}-\frac{35a^4b^4}{6x^{12}}$	91
norman	$\frac{-\frac{1}{20}a^8-\frac{4}{9}a^7b\,x^2-\frac{7}{4}a^6b^2x^4-4a^5b^3x^6-\frac{35}{6}a^4b^4x^8-\frac{28}{5}a^3b^5x^{10}-\frac{7}{2}a^2b^6x^{12}-\frac{4}{3}a\,b^7x^{14}-\frac{1}{4}b^8x^{16}}{x^{20}}$	92
risch	$\frac{-\frac{1}{20}a^8-\frac{4}{9}a^7b\,x^2-\frac{7}{4}a^6b^2x^4-4a^5b^3x^6-\frac{35}{6}a^4b^4x^8-\frac{28}{5}a^3b^5x^{10}-\frac{7}{2}a^2b^6x^{12}-\frac{4}{3}a\,b^7x^{14}-\frac{1}{4}b^8x^{16}}{x^{20}}$	92
gosper	$-\frac{45b^8x^{16}+240a\,b^7x^{14}+630a^2b^6x^{12}+1008a^3b^5x^{10}+1050a^4b^4x^8+720a^5b^3x^6+315a^6b^2x^4+80a^7b\,x^2+9a^8}{180x^{20}}$	93
parallelrisch	$\frac{-45b^8x^{16}-240a\,b^7x^{14}-630a^2b^6x^{12}-1008a^3b^5x^{10}-1050a^4b^4x^8-720a^5b^3x^6-315a^6b^2x^4-80a^7b\,x^2-9a^8}{180x^{20}}$	93
orering	$-\frac{45b^8x^{16}+240a\,b^7x^{14}+630a^2b^6x^{12}+1008a^3b^5x^{10}+1050a^4b^4x^8+720a^5b^3x^6+315a^6b^2x^4+80a^7b\,x^2+9a^8}{180x^{20}}$	93

input
```
int((b*x^2+a)^8/x^21,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^8/x^4-7/2*a^2*b^6/x^8-28/5*a^3*b^5/x^10-4*a^5*b^3/x^14-1/20*a^8/x^2
0-4/3*a*b^7/x^6-4/9*a^7*b/x^18-7/4*a^6*b^2/x^16-35/6*a^4*b^4/x^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{45\,b^8x^{16}+240\,ab^7x^{14}+630\,a^2b^6x^{12}+1008\,a^3b^5x^{10}+1050\,a^4b^4x^8+720\,a^5b^3x^6+315\,a^6b^2x^4+80\,a^7bx}{180\,x^{20}}$$

input
```
integrate((b*x^2+a)^8/x^21,x, algorithm="fricas")
```

output
```
-1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^1
0 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 +
9*a^8)/x^20
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(32) = 64$.

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.48

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = \frac{-9a^8-80a^7bx^2-315a^6b^2x^4-720a^5b^3x^6-1050a^4b^4x^8-1008a^3b^5x^{10}-630a^2b^6x^{12}-240ab^7x^{14}-45b}{180x^{20}}$$

input
```
integrate((b*x**2+a)**8/x**21,x)
```

output
```
(-9*a**8 - 80*a**7*b*x**2 - 315*a**6*b**2*x**4 - 720*a**5*b**3*x**6 - 1050
*a**4*b**4*x**8 - 1008*a**3*b**5*x**10 - 630*a**2*b**6*x**12 - 240*a*b**7*
x**14 - 45*b**8*x**16)/(180*x**20)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{45\,b^8x^{16} + 240\,ab^7x^{14} + 630\,a^2b^6x^{12} + 1008\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 720\,a^5b^3x^6 + 315\,a^6b^2x^4 + 80\,a^7bx}{180\,x^{20}}$$

input
```
integrate((b*x^2+a)^8/x^21,x, algorithm="maxima")
```

output
```
-1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^1
0 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 +
9*a^8)/x^20
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(36) = 72$.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{45\,b^8x^{16} + 240\,ab^7x^{14} + 630\,a^2b^6x^{12} + 1008\,a^3b^5x^{10} + 1050\,a^4b^4x^8 + 720\,a^5b^3x^6 + 315\,a^6b^2x^4 + 80\,a^7bx}{180\,x^{20}}$$

input
```
integrate((b*x^2+a)^8/x^21,x, algorithm="giac")
```

output
```
-1/180*(45*b^8*x^16 + 240*a*b^7*x^14 + 630*a^2*b^6*x^12 + 1008*a^3*b^5*x^1
0 + 1050*a^4*b^4*x^8 + 720*a^5*b^3*x^6 + 315*a^6*b^2*x^4 + 80*a^7*b*x^2 +
9*a^8)/x^20
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = -\frac{\frac{a^8}{20}+\frac{4\,a^7\,b\,x^2}{9}+\frac{7\,a^6\,b^2\,x^4}{4}+4\,a^5\,b^3\,x^6+\frac{35\,a^4\,b^4\,x^8}{6}+\frac{28\,a^3\,b^5\,x^{10}}{5}+\frac{7\,a^2\,b^6\,x^{12}}{2}+\frac{4\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{4}}{x^{20}}$$

input
```
int((a + b*x^2)^8/x^21,x)
```

output
```
-(a^8/20 + (b^8*x^16)/4 + (4*a^7*b*x^2)/9 + (4*a*b^7*x^14)/3 + (7*a^6*b^2*
x^4)/4 + 4*a^5*b^3*x^6 + (35*a^4*b^4*x^8)/6 + (28*a^3*b^5*x^10)/5 + (7*a^2
*b^6*x^12)/2)/x^20
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^8}{x^{21}}\,dx = \frac{-45b^8x^{16}-240a\,b^7x^{14}-630a^2b^6x^{12}-1008a^3b^5x^{10}-1050a^4b^4x^8-720a^5b^3x^6-315a^6b^2x^4-80a^7b\,x^2}{180x^{20}}$$

input
```
int((b*x^2+a)^8/x^21,x)
```

output
```
( - 9*a**8 - 80*a**7*b*x**2 - 315*a**6*b**2*x**4 - 720*a**5*b**3*x**6 - 10
50*a**4*b**4*x**8 - 1008*a**3*b**5*x**10 - 630*a**2*b**6*x**12 - 240*a*b**
7*x**14 - 45*b**8*x**16)/(180*x**20)
```

3.103 $\int \frac{(a+bx^2)^8}{x^{23}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = -\frac{(a+bx^2)^9}{22ax^{22}}+\frac{b(a+bx^2)^9}{110a^2x^{20}}-\frac{b^2(a+bx^2)^9}{990a^3x^{18}}$$

output
```
-1/22*(b*x^2+a)^9/a/x^22+1/110*b*(b*x^2+a)^9/a^2/x^20-1/990*b^2*(b*x^2+a)^
9/a^3/x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = -\frac{a^8}{22x^{22}}-\frac{2a^7b}{5x^{20}}-\frac{14a^6b^2}{9x^{18}}-\frac{7a^5b^3}{2x^{16}}-\frac{5a^4b^4}{x^{14}}-\frac{14a^3b^5}{3x^{12}}-\frac{14a^2b^6}{5x^{10}}-\frac{ab^7}{x^8}-\frac{b^8}{6x^6}$$

input
```
Integrate[(a + b*x^2)^8/x^23,x]
```

output
```
-1/22*a^8/x^22 - (2*a^7*b)/(5*x^20) - (14*a^6*b^2)/(9*x^18) - (7*a^5*b^3)/
(2*x^16) - (5*a^4*b^4)/x^14 - (14*a^3*b^5)/(3*x^12) - (14*a^2*b^6)/(5*x^10
) - (a*b^7)/x^8 - b^8/(6*x^6)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {243, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{24}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{(bx^2+a)^8}{x^{22}}dx^2}{11a}-\frac{(a+bx^2)^9}{11ax^{22}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\int \frac{(bx^2+a)^8}{x^{20}}dx^2}{10a}-\frac{(a+bx^2)^9}{10ax^{20}}\right)}{11a}-\frac{(a+bx^2)^9}{11ax^{22}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{2b\left(\frac{b(a+bx^2)^9}{90a^2x^{18}}-\frac{(a+bx^2)^9}{10ax^{20}}\right)}{11a}-\frac{(a+bx^2)^9}{11ax^{22}}\right)$$

input
```
Int[(a + b*x^2)^8/x^23,x]
```

output
```
(-1/11*(a + b*x^2)^9/(a*x^22) - (2*b*(-1/10*(a + b*x^2)^9/(a*x^20) + (b*(a
 + b*x^2)^9)/(90*a^2*x^18)))/(11*a))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{a^{8}}{22x^{22}}-\frac{a\,b^{7}}{x^{8}}-\frac{14a^{2}b^{6}}{5x^{10}}-\frac{5a^{4}b^{4}}{x^{14}}-\frac{2a^{7}b}{5x^{20}}-\frac{b^{8}}{6x^{6}}-\frac{14a^{6}b^{2}}{9x^{18}}-\frac{7a^{5}b^{3}}{2x^{16}}-\frac{14a^{3}b^{5}}{3x^{12}}$	91
norman	$\frac{-\frac{1}{22}a^{8}-\frac{2}{5}a^{7}b\,x^{2}-\frac{14}{9}a^{6}b^{2}x^{4}-\frac{7}{2}a^{5}b^{3}x^{6}-5a^{4}b^{4}x^{8}-\frac{14}{3}a^{3}b^{5}x^{10}-\frac{14}{5}a^{2}b^{6}x^{12}-a\,b^{7}x^{14}-\frac{1}{6}b^{8}x^{16}}{x^{22}}$	92
risch	$\frac{-\frac{1}{22}a^{8}-\frac{2}{5}a^{7}b\,x^{2}-\frac{14}{9}a^{6}b^{2}x^{4}-\frac{7}{2}a^{5}b^{3}x^{6}-5a^{4}b^{4}x^{8}-\frac{14}{3}a^{3}b^{5}x^{10}-\frac{14}{5}a^{2}b^{6}x^{12}-a\,b^{7}x^{14}-\frac{1}{6}b^{8}x^{16}}{x^{22}}$	92
gosper	$-\frac{165b^{8}x^{16}+990a\,b^{7}x^{14}+2772a^{2}b^{6}x^{12}+4620a^{3}b^{5}x^{10}+4950a^{4}b^{4}x^{8}+3465a^{5}b^{3}x^{6}+1540a^{6}b^{2}x^{4}+396a^{7}b\,x^{2}+45a^{8}}{990x^{22}}$	93
parallelrisch	$\frac{-165b^{8}x^{16}-990a\,b^{7}x^{14}-2772a^{2}b^{6}x^{12}-4620a^{3}b^{5}x^{10}-4950a^{4}b^{4}x^{8}-3465a^{5}b^{3}x^{6}-1540a^{6}b^{2}x^{4}-396a^{7}b\,x^{2}-45a^{8}}{990x^{22}}$	93
orering	$-\frac{165b^{8}x^{16}+990a\,b^{7}x^{14}+2772a^{2}b^{6}x^{12}+4620a^{3}b^{5}x^{10}+4950a^{4}b^{4}x^{8}+3465a^{5}b^{3}x^{6}+1540a^{6}b^{2}x^{4}+396a^{7}b\,x^{2}+45a^{8}}{990x^{22}}$	93

input
```
int((b*x^2+a)^8/x^23,x,method=_RETURNVERBOSE)
```

output
```
-1/22*a^8/x^22-a*b^7/x^8-14/5*a^2*b^6/x^10-5*a^4*b^4/x^14-2/5*a^7*b/x^20-1
/6*b^8/x^6-14/9*a^6*b^2/x^18-7/2*a^5*b^3/x^16-14/3*a^3*b^5/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = -\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+39}{990\,x^{22}}$$

input
```
integrate((b*x^2+a)^8/x^23,x, algorithm="fricas")
```

output
```
-1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x
^10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x
^2 + 45*a^8)/x^22
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx$$
$$= \frac{-45a^8 - 396a^7bx^2 - 1540a^6b^2x^4 - 3465a^5b^3x^6 - 4950a^4b^4x^8 - 4620a^3b^5x^{10} - 2772a^2b^6x^{12} - 990ab^7x^{14}}{990x^{22}}$$

input
```
integrate((b*x**2+a)**8/x**23,x)
```

output
```
(-45*a**8 - 396*a**7*b*x**2 - 1540*a**6*b**2*x**4 - 3465*a**5*b**3*x**6 -
4950*a**4*b**4*x**8 - 4620*a**3*b**5*x**10 - 2772*a**2*b**6*x**12 - 990*a*
b**7*x**14 - 165*b**8*x**16)/(990*x**22)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx =$$
$$-\frac{165\,b^8x^{16} + 990\,ab^7x^{14} + 2772\,a^2b^6x^{12} + 4620\,a^3b^5x^{10} + 4950\,a^4b^4x^8 + 3465\,a^5b^3x^6 + 1540\,a^6b^2x^4 + 39}{990\,x^{22}}$$

input
```
integrate((b*x^2+a)^8/x^23,x, algorithm="maxima")
```

output
```
-1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x
^10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x
^2 + 45*a^8)/x^22
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = -\frac{165\,b^8x^{16}+990\,ab^7x^{14}+2772\,a^2b^6x^{12}+4620\,a^3b^5x^{10}+4950\,a^4b^4x^8+3465\,a^5b^3x^6+1540\,a^6b^2x^4+39}{990\,x^{22}}$$

input
```
integrate((b*x^2+a)^8/x^23,x, algorithm="giac")
```

output
```
-1/990*(165*b^8*x^16 + 990*a*b^7*x^14 + 2772*a^2*b^6*x^12 + 4620*a^3*b^5*x
^10 + 4950*a^4*b^4*x^8 + 3465*a^5*b^3*x^6 + 1540*a^6*b^2*x^4 + 396*a^7*b*x
^2 + 45*a^8)/x^22
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx = -\frac{\frac{a^8}{22}+\frac{2\,a^7\,b\,x^2}{5}+\frac{14\,a^6\,b^2\,x^4}{9}+\frac{7\,a^5\,b^3\,x^6}{2}+5\,a^4\,b^4\,x^8+\frac{14\,a^3\,b^5\,x^{10}}{3}+\frac{14\,a^2\,b^6\,x^{12}}{5}+a\,b^7\,x^{14}+\frac{b^8\,x^{16}}{6}}{x^{22}}$$

input
```
int((a + b*x^2)^8/x^23,x)
```

output
```
-(a^8/22 + (b^8*x^16)/6 + (2*a^7*b*x^2)/5 + a*b^7*x^14 + (14*a^6*b^2*x^4)/
9 + (7*a^5*b^3*x^6)/2 + 5*a^4*b^4*x^8 + (14*a^3*b^5*x^10)/3 + (14*a^2*b^6*
x^12)/5)/x^22
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^8}{x^{23}}\,dx$$
$$= \frac{-165b^8x^{16} - 990a\,b^7x^{14} - 2772a^2b^6x^{12} - 4620a^3b^5x^{10} - 4950a^4b^4x^8 - 3465a^5b^3x^6 - 1540a^6b^2x^4 - 396a}{990x^{22}}$$

input
```
int((b*x^2+a)^8/x^23,x)
```

output
```
( - 45*a**8 - 396*a**7*b*x**2 - 1540*a**6*b**2*x**4 - 3465*a**5*b**3*x**6
- 4950*a**4*b**4*x**8 - 4620*a**3*b**5*x**10 - 2772*a**2*b**6*x**12 - 990*
a*b**7*x**14 - 165*b**8*x**16)/(990*x**22)
```

3.104 $\int \frac{(a+bx^2)^8}{x^{25}} dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{(a+bx^2)^9}{24ax^{24}}+\frac{b(a+bx^2)^9}{88a^2x^{22}}-\frac{b^2(a+bx^2)^9}{440a^3x^{20}}+\frac{b^3(a+bx^2)^9}{3960a^4x^{18}}$$

output
```
-1/24*(b*x^2+a)^9/a/x^24+1/88*b*(b*x^2+a)^9/a^2/x^22-1/440*b^2*(b*x^2+a)^9
/a^3/x^20+1/3960*b^3*(b*x^2+a)^9/a^4/x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{a^8}{24x^{24}}-\frac{4a^7b}{11x^{22}}-\frac{7a^6b^2}{5x^{20}}-\frac{28a^5b^3}{9x^{18}} -\frac{35a^4b^4}{8x^{16}}-\frac{4a^3b^5}{x^{14}}-\frac{7a^2b^6}{3x^{12}}-\frac{4ab^7}{5x^{10}}-\frac{b^8}{8x^8}$$

input
```
Integrate[(a + b*x^2)^8/x^25,x]
```

output
```
-1/24*a^8/x^24 - (4*a^7*b)/(11*x^22) - (7*a^6*b^2)/(5*x^20) - (28*a^5*b^3)
/(9*x^18) - (35*a^4*b^4)/(8*x^16) - (4*a^3*b^5)/x^14 - (7*a^2*b^6)/(3*x^12
) - (4*a*b^7)/(5*x^10) - b^8/(8*x^8)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {243, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{26}}dx^2$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{(bx^2+a)^8}{x^{24}}dx^2}{4a}-\frac{(a+bx^2)^9}{12ax^{24}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{2b\int \frac{(bx^2+a)^8}{x^{22}}dx^2}{11a}-\frac{(a+bx^2)^9}{11ax^{22}}\right)}{4a}-\frac{(a+bx^2)^9}{12ax^{24}}\right)$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{2b\left(-\frac{b\int\frac{\left(bx^2+a\right)^8}{x^{20}}dx^2}{10a}-\frac{\left(a+bx^2\right)^9}{10ax^{20}}\right)}{11a}-\frac{\left(a+bx^2\right)^9}{11ax^{22}}\right)}{4a}-\frac{\left(a+bx^2\right)^9}{12ax^{24}}\right)$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{2b\left(\frac{b\left(a+bx^2\right)^9}{90a^2x^{18}}-\frac{\left(a+bx^2\right)^9}{10ax^{20}}\right)}{11a}-\frac{\left(a+bx^2\right)^9}{11ax^{22}}\right)}{4a}-\frac{\left(a+bx^2\right)^9}{12ax^{24}}\right)$$

input

```
Int[(a + b*x^2)^8/x^25,x]
```

output

```
(-1/12*(a + b*x^2)^9/(a*x^24) - (b*(-1/11*(a + b*x^2)^9/(a*x^22) - (2*b*(-
1/10*(a + b*x^2)^9/(a*x^20) + (b*(a + b*x^2)^9)/(90*a^2*x^18)))/(11*a)))/(
4*a))/2
```

Defintions of rubi rules used

rule 48

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.08

method	result
default	$-\frac{4a^7b}{11x^{22}}-\frac{a^8}{24x^{24}}-\frac{b^8}{8x^8}-\frac{4a\,b^7}{5x^{10}}-\frac{4a^3b^5}{x^{14}}-\frac{7a^6b^2}{5x^{20}}-\frac{28a^5b^3}{9x^{18}}-\frac{35a^4b^4}{8x^{16}}-\frac{7a^2b^6}{3x^{12}}$
norman	$\frac{-\frac{1}{24}a^8-\frac{4}{5}a\,b^7x^{14}-\frac{7}{5}a^6b^2x^4-\frac{28}{9}a^5b^3x^6-\frac{4}{11}a^7b\,x^2-4a^3b^5x^{10}-\frac{7}{3}a^2b^6x^{12}-\frac{1}{8}b^8x^{16}-\frac{35}{8}a^4b^4x^8}{x^{24}}$
risch	$\frac{-\frac{1}{24}a^8-\frac{4}{5}a\,b^7x^{14}-\frac{7}{5}a^6b^2x^4-\frac{28}{9}a^5b^3x^6-\frac{4}{11}a^7b\,x^2-4a^3b^5x^{10}-\frac{7}{3}a^2b^6x^{12}-\frac{1}{8}b^8x^{16}-\frac{35}{8}a^4b^4x^8}{x^{24}}$
gosper	$-\frac{495b^8x^{16}+3168a\,b^7x^{14}+9240a^2b^6x^{12}+15840a^3b^5x^{10}+17325a^4b^4x^8+12320a^5b^3x^6+5544a^6b^2x^4+1440a^7b\,x^2+165a^8}{3960x^{24}}$
parallelrisch	$\frac{-495b^8x^{16}-3168a\,b^7x^{14}-9240a^2b^6x^{12}-15840a^3b^5x^{10}-17325a^4b^4x^8-12320a^5b^3x^6-5544a^6b^2x^4-1440a^7b\,x^2-165a^8}{3960x^{24}}$
orering	$-\frac{495b^8x^{16}+3168a\,b^7x^{14}+9240a^2b^6x^{12}+15840a^3b^5x^{10}+17325a^4b^4x^8+12320a^5b^3x^6+5544a^6b^2x^4+1440a^7b\,x^2+165a^8}{3960x^{24}}$

input
```
int((b*x^2+a)^8/x^25,x,method=_RETURNVERBOSE)
```

output
```
-4/11*a^7*b/x^22-1/24*a^8/x^24-1/8*b^8/x^8-4/5*a*b^7/x^10-4*a^3*b^5/x^14-7
/5*a^6*b^2/x^20-28/9*a^5*b^3/x^18-35/8*a^4*b^4/x^16-7/3*a^2*b^6/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4}{3960\,x^{24}}$$

input
```
integrate((b*x^2+a)^8/x^25,x, algorithm="fricas")
```

output
```
-1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^
5*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a
^7*b*x^2 + 165*a^8)/x^24
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = \frac{-165a^8-1440a^7bx^2-5544a^6b^2x^4-12320a^5b^3x^6-17325a^4b^4x^8-15840a^3b^5x^{10}-9240a^2b^6x^{12}-3168}{3960x^{24}}$$

input
```
integrate((b*x**2+a)**8/x**25,x)
```

output
```
(-165*a**8 - 1440*a**7*b*x**2 - 5544*a**6*b**2*x**4 - 12320*a**5*b**3*x**6
 - 17325*a**4*b**4*x**8 - 15840*a**3*b**5*x**10 - 9240*a**2*b**6*x**12 - 3
168*a*b**7*x**14 - 495*b**8*x**16)/(3960*x**24)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4}{3960\,x^{24}}$$

input
```
integrate((b*x^2+a)^8/x^25,x, algorithm="maxima")
```

output
```
-1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^
5*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a
^7*b*x^2 + 165*a^8)/x^24
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{495\,b^8x^{16}+3168\,ab^7x^{14}+9240\,a^2b^6x^{12}+15840\,a^3b^5x^{10}+17325\,a^4b^4x^8+12320\,a^5b^3x^6+5544\,a^6b^2x^4}{3960\,x^{24}}$$

input
```
integrate((b*x^2+a)^8/x^25,x, algorithm="giac")
```

output
```
-1/3960*(495*b^8*x^16 + 3168*a*b^7*x^14 + 9240*a^2*b^6*x^12 + 15840*a^3*b^
5*x^10 + 17325*a^4*b^4*x^8 + 12320*a^5*b^3*x^6 + 5544*a^6*b^2*x^4 + 1440*a
^7*b*x^2 + 165*a^8)/x^24
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = -\frac{\frac{a^8}{24}+\frac{4\,a^7\,b\,x^2}{11}+\frac{7\,a^6\,b^2\,x^4}{5}+\frac{28\,a^5\,b^3\,x^6}{9}+\frac{35\,a^4\,b^4\,x^8}{8}+4\,a^3\,b^5\,x^{10}+\frac{7\,a^2\,b^6\,x^{12}}{3}+\frac{4\,a\,b^7\,x^{14}}{5}+\frac{b^8\,x^{16}}{8}}{x^{24}}$$

input
```
int((a + b*x^2)^8/x^25,x)
```

output
```
-(a^8/24 + (b^8*x^16)/8 + (4*a^7*b*x^2)/11 + (4*a*b^7*x^14)/5 + (7*a^6*b^2
*x^4)/5 + (28*a^5*b^3*x^6)/9 + (35*a^4*b^4*x^8)/8 + 4*a^3*b^5*x^10 + (7*a^
2*b^6*x^12)/3)/x^24
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^8}{x^{25}}\,dx = \frac{-495b^8x^{16}-3168a\,b^7x^{14}-9240a^2b^6x^{12}-15840a^3b^5x^{10}-17325a^4b^4x^8-12320a^5b^3x^6-5544a^6b^2x^4-}{3960x^{24}}$$

input
```
int((b*x^2+a)^8/x^25,x)
```

output
```
( - 165*a**8 - 1440*a**7*b*x**2 - 5544*a**6*b**2*x**4 - 12320*a**5*b**3*x**
*6 - 17325*a**4*b**4*x**8 - 15840*a**3*b**5*x**10 - 9240*a**2*b**6*x**12 -
 3168*a*b**7*x**14 - 495*b**8*x**16)/(3960*x**24)
```

3.105 $\int \frac{(a+bx^2)^8}{x^{27}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{(a+bx^2)^9}{26ax^{26}}+\frac{b(a+bx^2)^9}{78a^2x^{24}}-\frac{b^2(a+bx^2)^9}{286a^3x^{22}}+\frac{b^3(a+bx^2)^9}{1430a^4x^{20}}-\frac{b^4(a+bx^2)^9}{12870a^5x^{18}}$$

output
```
-1/26*(b*x^2+a)^9/a/x^26+1/78*b*(b*x^2+a)^9/a^2/x^24-1/286*b^2*(b*x^2+a)^9
/a^3/x^22+1/1430*b^3*(b*x^2+a)^9/a^4/x^20-1/12870*b^4*(b*x^2+a)^9/a^5/x^18
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{a^8}{26x^{26}}-\frac{a^7b}{3x^{24}}-\frac{14a^6b^2}{11x^{22}}-\frac{14a^5b^3}{5x^{20}}-\frac{35a^4b^4}{9x^{18}}-\frac{7a^3b^5}{2x^{16}}-\frac{2a^2b^6}{x^{14}}-\frac{2ab^7}{3x^{12}}-\frac{b^8}{10x^{10}}$$

input
```
Integrate[(a + b*x^2)^8/x^27,x]
```

output
```
-1/26*a^8/x^26 - (a^7*b)/(3*x^24) - (14*a^6*b^2)/(11*x^22) - (14*a^5*b^3)/
(5*x^20) - (35*a^4*b^4)/(9*x^18) - (7*a^3*b^5)/(2*x^16) - (2*a^2*b^6)/x^14
 - (2*a*b^7)/(3*x^12) - b^8/(10*x^10)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {243, 55, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{28}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{4b\int \frac{(bx^2+a)^8}{x^{26}}dx^2}{13a}-\frac{(a+bx^2)^9}{13ax^{26}}\right)$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{4b\left(-\frac{b\int \frac{(bx^2+a)^8}{x^{24}}dx^2}{4a}-\frac{(a+bx^2)^9}{12ax^{24}}\right)}{13a}-\frac{(a+bx^2)^9}{13ax^{26}}\right)$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\int \frac{\left(bx^2+a\right)^8}{x^{22}}dx^2}{11a}-\frac{\left(a+bx^2\right)^9}{11ax^{22}}\right)}{4a}-\frac{\left(a+bx^2\right)^9}{12ax^{24}}\right)}{13a}-\frac{\left(a+bx^2\right)^9}{13ax^{26}}\right)$$

$$\Big\downarrow \; 55$$

$$\frac{1}{2}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\left(-\frac{b\int \frac{\left(bx^2+a\right)^8}{x^{20}}dx^2}{10a}-\frac{\left(a+bx^2\right)^9}{10ax^{20}}\right)}{11a}-\frac{\left(a+bx^2\right)^9}{11ax^{22}}\right)}{4a}-\frac{\left(a+bx^2\right)^9}{12ax^{24}}\right)}{13a}-\frac{\left(a+bx^2\right)^9}{13ax^{26}}\right)$$

$$\Big\downarrow \; 48$$

$$\frac{1}{2}\left(-\frac{4b\left(-\frac{b\left(-\frac{2b\left(\frac{b(a+bx^2)^9}{90a^2x^{18}}-\frac{(a+bx^2)^9}{10ax^{20}}\right)}{11a}-\frac{(a+bx^2)^9}{11ax^{22}}\right)}{4a}-\frac{(a+bx^2)^9}{12ax^{24}}\right)}{13a}-\frac{(a+bx^2)^9}{13ax^{26}}\right)$$

input
```
Int[(a + b*x^2)^8/x^27,x]
```

output
```
(-1/13*(a + b*x^2)^9/(a*x^26) - (4*b*(-1/12*(a + b*x^2)^9/(a*x^24) - (b*(-
1/11*(a + b*x^2)^9/(a*x^22) - (2*b*(-1/10*(a + b*x^2)^9/(a*x^20) + (b*(a +
 b*x^2)^9)/(90*a^2*x^18)))/(11*a)))/(4*a)))/(13*a))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
default	$-\frac{14a^6b^2}{11x^{22}}-\frac{a^7b}{3x^{24}}-\frac{b^8}{10x^{10}}-\frac{a^8}{26x^{26}}-\frac{2a^2b^6}{x^{14}}-\frac{14a^5b^3}{5x^{20}}-\frac{35a^4b^4}{9x^{18}}-\frac{7a^3b^5}{2x^{16}}-\frac{2a\,b^7}{3x^{12}}$
norman	$\frac{-\frac{1}{26}a^8-2a^2b^6x^{12}-\frac{2}{3}a\,b^7x^{14}-\frac{1}{3}a^7b\,x^2-\frac{1}{10}b^8x^{16}-\frac{7}{2}a^3b^5x^{10}-\frac{35}{9}a^4b^4x^8-\frac{14}{11}a^6b^2x^4-\frac{14}{5}a^5b^3x^6}{x^{26}}$
risch	$\frac{-\frac{1}{26}a^8-2a^2b^6x^{12}-\frac{2}{3}a\,b^7x^{14}-\frac{1}{3}a^7b\,x^2-\frac{1}{10}b^8x^{16}-\frac{7}{2}a^3b^5x^{10}-\frac{35}{9}a^4b^4x^8-\frac{14}{11}a^6b^2x^4-\frac{14}{5}a^5b^3x^6}{x^{26}}$
gosper	$-\frac{1287b^8x^{16}+8580a\,b^7x^{14}+25740a^2b^6x^{12}+45045a^3b^5x^{10}+50050a^4b^4x^8+36036a^5b^3x^6+16380a^6b^2x^4+4290a^7b\,x^2+495a^8}{12870x^{26}}$
parallelrisch	$\frac{-1287b^8x^{16}-8580a\,b^7x^{14}-25740a^2b^6x^{12}-45045a^3b^5x^{10}-50050a^4b^4x^8-36036a^5b^3x^6-16380a^6b^2x^4-4290a^7b\,x^2-495a^8}{12870x^{26}}$
orering	$-\frac{1287b^8x^{16}+8580a\,b^7x^{14}+25740a^2b^6x^{12}+45045a^3b^5x^{10}+50050a^4b^4x^8+36036a^5b^3x^6+16380a^6b^2x^4+4290a^7b\,x^2+495a^8}{12870x^{26}}$

input
```
int((b*x^2+a)^8/x^27,x,method=_RETURNVERBOSE)
```

output
```
-14/11*a^6*b^2/x^22-1/3*a^7*b/x^24-1/10*b^8/x^10-1/26*a^8/x^26-2*a^2*b^6/x
^14-14/5*a^5*b^3/x^20-35/9*a^4*b^4/x^18-7/2*a^3*b^5/x^16-2/3*a*b^7/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b}{12870\,x^{26}}$$

input
```
integrate((b*x^2+a)^8/x^27,x, algorithm="fricas")
```

output
```
-1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3
*b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 42
90*a^7*b*x^2 + 495*a^8)/x^26
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = \frac{-495a^8-4290a^7bx^2-16380a^6b^2x^4-36036a^5b^3x^6-50050a^4b^4x^8-45045a^3b^5x^{10}-25740a^2b^6x^{12}-85}{12870x^{26}}$$

input
```
integrate((b*x**2+a)**8/x**27,x)
```

output
```
(-495*a**8 - 4290*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 36036*a**5*b**3*x**
6 - 50050*a**4*b**4*x**8 - 45045*a**3*b**5*x**10 - 25740*a**2*b**6*x**12 -
 8580*a*b**7*x**14 - 1287*b**8*x**16)/(12870*x**26)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b}{12870\,x^{26}}$$

input
```
integrate((b*x^2+a)^8/x^27,x, algorithm="maxima")
```

output
```
-1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3
*b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 42
90*a^7*b*x^2 + 495*a^8)/x^26
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{1287\,b^8x^{16}+8580\,ab^7x^{14}+25740\,a^2b^6x^{12}+45045\,a^3b^5x^{10}+50050\,a^4b^4x^8+36036\,a^5b^3x^6+16380\,a^6b}{12870\,x^{26}}$$

input
```
integrate((b*x^2+a)^8/x^27,x, algorithm="giac")
```

output
```
-1/12870*(1287*b^8*x^16 + 8580*a*b^7*x^14 + 25740*a^2*b^6*x^12 + 45045*a^3
*b^5*x^10 + 50050*a^4*b^4*x^8 + 36036*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 42
90*a^7*b*x^2 + 495*a^8)/x^26
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx = -\frac{\frac{a^8}{26}+\frac{a^7\,b\,x^2}{3}+\frac{14\,a^6\,b^2\,x^4}{11}+\frac{14\,a^5\,b^3\,x^6}{5}+\frac{35\,a^4\,b^4\,x^8}{9}+\frac{7\,a^3\,b^5\,x^{10}}{2}+2\,a^2\,b^6\,x^{12}+\frac{2\,a\,b^7\,x^{14}}{3}+\frac{b^8\,x^{16}}{10}}{x^{26}}$$

input
```
int((a + b*x^2)^8/x^27,x)
```

output
```
-(a^8/26 + (b^8*x^16)/10 + (a^7*b*x^2)/3 + (2*a*b^7*x^14)/3 + (14*a^6*b^2*
x^4)/11 + (14*a^5*b^3*x^6)/5 + (35*a^4*b^4*x^8)/9 + (7*a^3*b^5*x^10)/2 + 2
*a^2*b^6*x^12)/x^26
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{27}}\,dx$$
$$= \frac{-1287b^8x^{16} - 8580a\,b^7x^{14} - 25740a^2b^6x^{12} - 45045a^3b^5x^{10} - 50050a^4b^4x^8 - 36036a^5b^3x^6 - 16380a^6b^2x}{12870x^{26}}$$

input
```
int((b*x^2+a)^8/x^27,x)
```

output
```
( - 495*a**8 - 4290*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 36036*a**5*b**3*x
**6 - 50050*a**4*b**4*x**8 - 45045*a**3*b**5*x**10 - 25740*a**2*b**6*x**12
 - 8580*a*b**7*x**14 - 1287*b**8*x**16)/(12870*x**26)
```

3.106 $\int \frac{(a+bx^2)^8}{x^{29}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

output
```
-1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4
*b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{a^8}{28x^{28}} - \frac{4a^7b}{13x^{26}} - \frac{7a^6b^2}{6x^{24}} - \frac{28a^5b^3}{11x^{22}} - \frac{7a^4b^4}{2x^{20}} - \frac{28a^3b^5}{9x^{18}} - \frac{7a^2b^6}{4x^{16}} - \frac{4ab^7}{7x^{14}} - \frac{b^8}{12x^{12}}$$

input
```
Integrate[(a + b*x^2)^8/x^29,x]
```

output
```
-1/28*a^8/x^28 - (4*a^7*b)/(13*x^26) - (7*a^6*b^2)/(6*x^24) - (28*a^5*b^3)
/(11*x^22) - (7*a^4*b^4)/(2*x^20) - (28*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(4
*x^16) - (4*a*b^7)/(7*x^14) - b^8/(12*x^12)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{30}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{30}}+\frac{8ba^7}{x^{28}}+\frac{28b^2a^6}{x^{26}}+\frac{56b^3a^5}{x^{24}}+\frac{70b^4a^4}{x^{22}}+\frac{56b^5a^3}{x^{20}}+\frac{28b^6a^2}{x^{18}}+\frac{8b^7a}{x^{16}}+\frac{b^8}{x^{14}}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{14x^{28}}-\frac{8a^7b}{13x^{26}}-\frac{7a^6b^2}{3x^{24}}-\frac{56a^5b^3}{11x^{22}}-\frac{7a^4b^4}{x^{20}}-\frac{56a^3b^5}{9x^{18}}-\frac{7a^2b^6}{2x^{16}}-\frac{8ab^7}{7x^{14}}-\frac{b^8}{6x^{12}}\right)$$

input
```
Int[(a + b*x^2)^8/x^29,x]
```

output
```
(-1/14*a^8/x^28 - (8*a^7*b)/(13*x^26) - (7*a^6*b^2)/(3*x^24) - (56*a^5*b^3
)/(11*x^22) - (7*a^4*b^4)/x^20 - (56*a^3*b^5)/(9*x^18) - (7*a^2*b^6)/(2*x^
16) - (8*a*b^7)/(7*x^14) - b^8/(6*x^12))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^8}{28x^{28}}-\frac{4a^7b}{13x^{26}}-\frac{7a^6b^2}{6x^{24}}-\frac{28a^5b^3}{11x^{22}}-\frac{7a^4b^4}{2x^{20}}-\frac{28a^3b^5}{9x^{18}}-\frac{7a^2b^6}{4x^{16}}-\frac{4a\,b^7}{7x^{14}}-\frac{b^8}{12x^{12}}$
norman	$\frac{-\frac{1}{28}a^8-\frac{7}{4}a^2b^6x^{12}-\frac{28}{11}a^5b^3x^6-\frac{7}{2}a^4b^4x^8-\frac{4}{7}a\,b^7x^{14}-\frac{28}{9}a^3b^5x^{10}-\frac{7}{6}a^6b^2x^4-\frac{1}{12}b^8x^{16}-\frac{4}{13}a^7b\,x^2}{x^{28}}$
risch	$\frac{-\frac{1}{28}a^8-\frac{7}{4}a^2b^6x^{12}-\frac{28}{11}a^5b^3x^6-\frac{7}{2}a^4b^4x^8-\frac{4}{7}a\,b^7x^{14}-\frac{28}{9}a^3b^5x^{10}-\frac{7}{6}a^6b^2x^4-\frac{1}{12}b^8x^{16}-\frac{4}{13}a^7b\,x^2}{x^{28}}$
gosper	$-\frac{3003b^8x^{16}+20592a\,b^7x^{14}+63063a^2b^6x^{12}+112112a^3b^5x^{10}+126126a^4b^4x^8+91728a^5b^3x^6+42042a^6b^2x^4+11088a^7b\,x^2+1\ldots}{36036x^{28}}$
parallelrisch	$\frac{-3003b^8x^{16}-20592a\,b^7x^{14}-63063a^2b^6x^{12}-112112a^3b^5x^{10}-126126a^4b^4x^8-91728a^5b^3x^6-42042a^6b^2x^4-11088a^7b\,x^2-12\ldots}{36036x^{28}}$
orering	$-\frac{3003b^8x^{16}+20592a\,b^7x^{14}+63063a^2b^6x^{12}+112112a^3b^5x^{10}+126126a^4b^4x^8+91728a^5b^3x^6+42042a^6b^2x^4+11088a^7b\,x^2+1\ldots}{36036x^{28}}$

input
```
int((b*x^2+a)^8/x^29,x,method=_RETURNVERBOSE)
```

output
```
-1/28*a^8/x^28-4/13*a^7*b/x^26-7/6*a^6*b^2/x^24-28/11*a^5*b^3/x^22-7/2*a^4
*b^4/x^20-28/9*a^3*b^5/x^18-7/4*a^2*b^6/x^16-4/7*a*b^7/x^14-1/12*b^8/x^12
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{3003\,b^8x^{16} + 20592\,ab^7x^{14} + 63063\,a^2b^6x^{12} + 112112\,a^3b^5x^{10} + 126126\,a^4b^4x^8 + 91728\,a^5b^3x^6 + 42042}{36036\,x^{28}}$$

input
```
integrate((b*x^2+a)^8/x^29,x, algorithm="fricas")
```

output
```
-1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a
^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 +
 11088*a^7*b*x^2 + 1287*a^8)/x^28
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = \frac{-1287a^8 - 11088a^7bx^2 - 42042a^6b^2x^4 - 91728a^5b^3x^6 - 126126a^4b^4x^8 - 112112a^3b^5x^{10} - 63063a^2b^6x^{12}}{36036x^{28}}$$

input
```
integrate((b*x**2+a)**8/x**29,x)
```

output
```
(-1287*a**8 - 11088*a**7*b*x**2 - 42042*a**6*b**2*x**4 - 91728*a**5*b**3*x
**6 - 126126*a**4*b**4*x**8 - 112112*a**3*b**5*x**10 - 63063*a**2*b**6*x**
12 - 20592*a*b**7*x**14 - 3003*b**8*x**16)/(36036*x**28)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042}{36036\,x^{28}}$$

input
```
integrate((b*x^2+a)^8/x^29,x, algorithm="maxima")
```

output
```
-1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a
^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 +
 11088*a^7*b*x^2 + 1287*a^8)/x^28
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{3003\,b^8x^{16}+20592\,ab^7x^{14}+63063\,a^2b^6x^{12}+112112\,a^3b^5x^{10}+126126\,a^4b^4x^8+91728\,a^5b^3x^6+42042}{36036\,x^{28}}$$

input
```
integrate((b*x^2+a)^8/x^29,x, algorithm="giac")
```

output
```
-1/36036*(3003*b^8*x^16 + 20592*a*b^7*x^14 + 63063*a^2*b^6*x^12 + 112112*a
^3*b^5*x^10 + 126126*a^4*b^4*x^8 + 91728*a^5*b^3*x^6 + 42042*a^6*b^2*x^4 +
 11088*a^7*b*x^2 + 1287*a^8)/x^28
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = -\frac{\frac{a^8}{28}+\frac{4\,a^7\,b\,x^2}{13}+\frac{7\,a^6\,b^2\,x^4}{6}+\frac{28\,a^5\,b^3\,x^6}{11}+\frac{7\,a^4\,b^4\,x^8}{2}+\frac{28\,a^3\,b^5\,x^{10}}{9}+\frac{7\,a^2\,b^6\,x^{12}}{4}+\frac{4\,a\,b^7\,x^{14}}{7}+\frac{b^8\,x^{16}}{12}}{x^{28}}$$

input
```
int((a + b*x^2)^8/x^29,x)
```

output
```
-(a^8/28 + (b^8*x^16)/12 + (4*a^7*b*x^2)/13 + (4*a*b^7*x^14)/7 + (7*a^6*b^
2*x^4)/6 + (28*a^5*b^3*x^6)/11 + (7*a^4*b^4*x^8)/2 + (28*a^3*b^5*x^10)/9 +
 (7*a^2*b^6*x^12)/4)/x^28
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{29}}\,dx = \frac{-3003b^8x^{16}-20592a\,b^7x^{14}-63063a^2b^6x^{12}-112112a^3b^5x^{10}-126126a^4b^4x^8-91728a^5b^3x^6-42042a^6}{36036x^{28}}$$

input
```
int((b*x^2+a)^8/x^29,x)
```

output
```
( - 1287*a**8 - 11088*a**7*b*x**2 - 42042*a**6*b**2*x**4 - 91728*a**5*b**3
*x**6 - 126126*a**4*b**4*x**8 - 112112*a**3*b**5*x**10 - 63063*a**2*b**6*x
**12 - 20592*a*b**7*x**14 - 3003*b**8*x**16)/(36036*x**28)
```

3.107 $\int \frac{(a+bx^2)^8}{x^{31}} dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{(a+bx^2)^8}{x^{31}} dx = -\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

output
```
-1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^
4*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^1
4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{31}} dx = -\frac{a^8}{30x^{30}} - \frac{2a^7b}{7x^{28}} - \frac{14a^6b^2}{13x^{26}} - \frac{7a^5b^3}{3x^{24}} - \frac{35a^4b^4}{11x^{22}} - \frac{14a^3b^5}{5x^{20}} - \frac{14a^2b^6}{9x^{18}} - \frac{ab^7}{2x^{16}} - \frac{b^8}{14x^{14}}$$

input
```
Integrate[(a + b*x^2)^8/x^31,x]
```

output
```
-1/30*a^8/x^30 - (2*a^7*b)/(7*x^28) - (14*a^6*b^2)/(13*x^26) - (7*a^5*b^3)
/(3*x^24) - (35*a^4*b^4)/(11*x^22) - (14*a^3*b^5)/(5*x^20) - (14*a^2*b^6)/
(9*x^18) - (a*b^7)/(2*x^16) - b^8/(14*x^14)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{31}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{32}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{32}}+\frac{8ba^7}{x^{30}}+\frac{28b^2a^6}{x^{28}}+\frac{56b^3a^5}{x^{26}}+\frac{70b^4a^4}{x^{24}}+\frac{56b^5a^3}{x^{22}}+\frac{28b^6a^2}{x^{20}}+\frac{8b^7a}{x^{18}}+\frac{b^8}{x^{16}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^8}{15x^{30}}-\frac{4a^7b}{7x^{28}}-\frac{28a^6b^2}{13x^{26}}-\frac{14a^5b^3}{3x^{24}}-\frac{70a^4b^4}{11x^{22}}-\frac{28a^3b^5}{5x^{20}}-\frac{28a^2b^6}{9x^{18}}-\frac{ab^7}{x^{16}}-\frac{b^8}{7x^{14}}\right)$$

input
```
Int[(a + b*x^2)^8/x^31,x]
```

output
```
(-1/15*a^8/x^30 - (4*a^7*b)/(7*x^28) - (28*a^6*b^2)/(13*x^26) - (14*a^5*b^
3)/(3*x^24) - (70*a^4*b^4)/(11*x^22) - (28*a^3*b^5)/(5*x^20) - (28*a^2*b^6
)/(9*x^18) - (a*b^7)/x^16 - b^8/(7*x^14))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
default	$-\frac{a^8}{30x^{30}}-\frac{2a^7b}{7x^{28}}-\frac{14a^6b^2}{13x^{26}}-\frac{7a^5b^3}{3x^{24}}-\frac{35a^4b^4}{11x^{22}}-\frac{14a^3b^5}{5x^{20}}-\frac{14a^2b^6}{9x^{18}}-\frac{a\,b^7}{2x^{16}}-\frac{b^8}{14x^{14}}$
norman	$\frac{-\frac{1}{30}a^8-\frac{14}{9}a^2b^6x^{12}-\frac{7}{3}a^5b^3x^6-\frac{35}{11}a^4b^4x^8-\frac{1}{2}a\,b^7x^{14}-\frac{14}{5}a^3b^5x^{10}-\frac{14}{13}a^6b^2x^4-\frac{1}{14}b^8x^{16}-\frac{2}{7}a^7b\,x^2}{x^{30}}$
risch	$\frac{-\frac{1}{30}a^8-\frac{14}{9}a^2b^6x^{12}-\frac{7}{3}a^5b^3x^6-\frac{35}{11}a^4b^4x^8-\frac{1}{2}a\,b^7x^{14}-\frac{14}{5}a^3b^5x^{10}-\frac{14}{13}a^6b^2x^4-\frac{1}{14}b^8x^{16}-\frac{2}{7}a^7b\,x^2}{x^{30}}$
gosper	$-\frac{6435b^8x^{16}+45045a\,b^7x^{14}+140140a^2b^6x^{12}+252252a^3b^5x^{10}+286650a^4b^4x^8+210210a^5b^3x^6+97020a^6b^2x^4+25740a^7b\,x^2+}{90090x^{30}}$
parallelrisch	$\frac{-6435b^8x^{16}-45045a\,b^7x^{14}-140140a^2b^6x^{12}-252252a^3b^5x^{10}-286650a^4b^4x^8-210210a^5b^3x^6-97020a^6b^2x^4-25740a^7b\,x^2-}{90090x^{30}}$
orering	$-\frac{6435b^8x^{16}+45045a\,b^7x^{14}+140140a^2b^6x^{12}+252252a^3b^5x^{10}+286650a^4b^4x^8+210210a^5b^3x^6+97020a^6b^2x^4+25740a^7b\,x^2+}{90090x^{30}}$

input
```
int((b*x^2+a)^8/x^31,x,method=_RETURNVERBOSE)
```

output
```
-1/30*a^8/x^30-2/7*a^7*b/x^28-14/13*a^6*b^2/x^26-7/3*a^5*b^3/x^24-35/11*a^
4*b^4/x^22-14/5*a^3*b^5/x^20-14/9*a^2*b^6/x^18-1/2*a*b^7/x^16-1/14*b^8/x^1
4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx =$$
$$-\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 970}{90090\,x^{30}}$$

input
```
integrate((b*x^2+a)^8/x^31,x, algorithm="fricas")
```

output
```
-1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*
a^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4
 + 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx$$
$$= \frac{-3003a^8 - 25740a^7bx^2 - 97020a^6b^2x^4 - 210210a^5b^3x^6 - 286650a^4b^4x^8 - 252252a^3b^5x^{10} - 140140a^2b^6x}{90090x^{30}}$$

input
```
integrate((b*x**2+a)**8/x**31,x)
```

output
```
(-3003*a**8 - 25740*a**7*b*x**2 - 97020*a**6*b**2*x**4 - 210210*a**5*b**3*
x**6 - 286650*a**4*b**4*x**8 - 252252*a**3*b**5*x**10 - 140140*a**2*b**6*x
**12 - 45045*a*b**7*x**14 - 6435*b**8*x**16)/(90090*x**30)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx = -\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 970}{90090\,x^{30}}$$

input
```
integrate((b*x^2+a)^8/x^31,x, algorithm="maxima")
```

output
```
-1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*
a^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4
 + 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx = -\frac{6435\,b^8x^{16} + 45045\,ab^7x^{14} + 140140\,a^2b^6x^{12} + 252252\,a^3b^5x^{10} + 286650\,a^4b^4x^8 + 210210\,a^5b^3x^6 + 970}{90090\,x^{30}}$$

input
```
integrate((b*x^2+a)^8/x^31,x, algorithm="giac")
```

output
```
-1/90090*(6435*b^8*x^16 + 45045*a*b^7*x^14 + 140140*a^2*b^6*x^12 + 252252*
a^3*b^5*x^10 + 286650*a^4*b^4*x^8 + 210210*a^5*b^3*x^6 + 97020*a^6*b^2*x^4
 + 25740*a^7*b*x^2 + 3003*a^8)/x^30
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx = -\frac{\frac{a^8}{30}+\frac{2\,a^7\,b\,x^2}{7}+\frac{14\,a^6\,b^2\,x^4}{13}+\frac{7\,a^5\,b^3\,x^6}{3}+\frac{35\,a^4\,b^4\,x^8}{11}+\frac{14\,a^3\,b^5\,x^{10}}{5}+\frac{14\,a^2\,b^6\,x^{12}}{9}+\frac{a\,b^7\,x^{14}}{2}+\frac{b^8\,x^{16}}{14}}{x^{30}}$$

input
```
int((a + b*x^2)^8/x^31,x)
```

output
```
-(a^8/30 + (b^8*x^16)/14 + (2*a^7*b*x^2)/7 + (a*b^7*x^14)/2 + (14*a^6*b^2*
x^4)/13 + (7*a^5*b^3*x^6)/3 + (35*a^4*b^4*x^8)/11 + (14*a^3*b^5*x^10)/5 +
(14*a^2*b^6*x^12)/9)/x^30
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^8}{x^{31}}\,dx = \frac{-6435b^8x^{16}-45045a\,b^7x^{14}-140140a^2b^6x^{12}-252252a^3b^5x^{10}-286650a^4b^4x^8-210210a^5b^3x^6-97020}{90090x^{30}}$$

input
```
int((b*x^2+a)^8/x^31,x)
```

output
```
( - 3003*a**8 - 25740*a**7*b*x**2 - 97020*a**6*b**2*x**4 - 210210*a**5*b**
3*x**6 - 286650*a**4*b**4*x**8 - 252252*a**3*b**5*x**10 - 140140*a**2*b**6
*x**12 - 45045*a*b**7*x**14 - 6435*b**8*x**16)/(90090*x**30)
```

3.108 $\int \frac{(a+bx^2)^8}{x^{33}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx=-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

output
```
-1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b
^4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx=-\frac{a^8}{32x^{32}}-\frac{4a^7b}{15x^{30}}-\frac{a^6b^2}{x^{28}}-\frac{28a^5b^3}{13x^{26}}-\frac{35a^4b^4}{12x^{24}}-\frac{28a^3b^5}{11x^{22}}-\frac{7a^2b^6}{5x^{20}}-\frac{4ab^7}{9x^{18}}-\frac{b^8}{16x^{16}}$$

input
```
Integrate[(a + b*x^2)^8/x^33,x]
```

output
```
-1/32*a^8/x^32 - (4*a^7*b)/(15*x^30) - (a^6*b^2)/x^28 - (28*a^5*b^3)/(13*x
^26) - (35*a^4*b^4)/(12*x^24) - (28*a^3*b^5)/(11*x^22) - (7*a^2*b^6)/(5*x^
20) - (4*a*b^7)/(9*x^18) - b^8/(16*x^16)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^8}{x^{34}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{a^8}{x^{34}}+\frac{8ba^7}{x^{32}}+\frac{28b^2a^6}{x^{30}}+\frac{56b^3a^5}{x^{28}}+\frac{70b^4a^4}{x^{26}}+\frac{56b^5a^3}{x^{24}}+\frac{28b^6a^2}{x^{22}}+\frac{8b^7a}{x^{20}}+\frac{b^8}{x^{18}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^8}{16x^{32}}-\frac{8a^7b}{15x^{30}}-\frac{2a^6b^2}{x^{28}}-\frac{56a^5b^3}{13x^{26}}-\frac{35a^4b^4}{6x^{24}}-\frac{56a^3b^5}{11x^{22}}-\frac{14a^2b^6}{5x^{20}}-\frac{8ab^7}{9x^{18}}-\frac{b^8}{8x^{16}}\right)$$

input
```
Int[(a + b*x^2)^8/x^33,x]
```

output
```
(-1/16*a^8/x^32 - (8*a^7*b)/(15*x^30) - (2*a^6*b^2)/x^28 - (56*a^5*b^3)/(1
3*x^26) - (35*a^4*b^4)/(6*x^24) - (56*a^3*b^5)/(11*x^22) - (14*a^2*b^6)/(5
*x^20) - (8*a*b^7)/(9*x^18) - b^8/(8*x^16))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
default	$-\frac{a^{8}}{32 x^{32}}-\frac{4 a^{7} b}{15 x^{30}}-\frac{a^{6} b^{2}}{x^{28}}-\frac{28 a^{5} b^{3}}{13 x^{26}}-\frac{35 a^{4} b^{4}}{12 x^{24}}-\frac{28 a^{3} b^{5}}{11 x^{22}}-\frac{7 a^{2} b^{6}}{5 x^{20}}-\frac{4 a \,b^{7}}{9 x^{18}}-\frac{b^{8}}{16 x^{16}}$
norman	$\frac{-\frac{1}{32} a^{8}-a^{6} b^{2} x^{4}-\frac{35}{12} a^{4} b^{4} x^{8}-\frac{4}{9} a \,b^{7} x^{14}-\frac{7}{5} a^{2} b^{6} x^{12}-\frac{4}{15} a^{7} b \,x^{2}-\frac{28}{11} a^{3} b^{5} x^{10}-\frac{1}{16} b^{8} x^{16}-\frac{28}{13} a^{5} b^{3} x^{6}}{x^{32}}$
risch	$\frac{-\frac{1}{32} a^{8}-a^{6} b^{2} x^{4}-\frac{35}{12} a^{4} b^{4} x^{8}-\frac{4}{9} a \,b^{7} x^{14}-\frac{7}{5} a^{2} b^{6} x^{12}-\frac{4}{15} a^{7} b \,x^{2}-\frac{28}{11} a^{3} b^{5} x^{10}-\frac{1}{16} b^{8} x^{16}-\frac{28}{13} a^{5} b^{3} x^{6}}{x^{32}}$
gosper	$-\frac{12870 b^{8} x^{16}+91520 a \,b^{7} x^{14}+288288 a^{2} b^{6} x^{12}+524160 a^{3} b^{5} x^{10}+600600 a^{4} b^{4} x^{8}+443520 a^{5} b^{3} x^{6}+205920 a^{6} b^{2} x^{4}+54912 a^{7} b \,x}{205920 x^{32}}$
parallelrisch	$\frac{-12870 b^{8} x^{16}-91520 a \,b^{7} x^{14}-288288 a^{2} b^{6} x^{12}-524160 a^{3} b^{5} x^{10}-600600 a^{4} b^{4} x^{8}-443520 a^{5} b^{3} x^{6}-205920 a^{6} b^{2} x^{4}-54912 a^{7} b \,x}{205920 x^{32}}$
orering	$-\frac{12870 b^{8} x^{16}+91520 a \,b^{7} x^{14}+288288 a^{2} b^{6} x^{12}+524160 a^{3} b^{5} x^{10}+600600 a^{4} b^{4} x^{8}+443520 a^{5} b^{3} x^{6}+205920 a^{6} b^{2} x^{4}+54912 a^{7} b \,x}{205920 x^{32}}$

input
```
int((b*x^2+a)^8/x^33,x,method=_RETURNVERBOSE)
```

output
```
-1/32*a^8/x^32-4/15*a^7*b/x^30-a^6*b^2/x^28-28/13*a^5*b^3/x^26-35/12*a^4*b
^4/x^24-28/11*a^3*b^5/x^22-7/5*a^2*b^6/x^20-4/9*a*b^7/x^18-1/16*b^8/x^16
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = -\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+20}{205920\,x^{32}}$$

input
```
integrate((b*x^2+a)^8/x^33,x, algorithm="fricas")
```

output
```
-1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 52416
0*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*
x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = \frac{-6435a^8-54912a^7bx^2-205920a^6b^2x^4-443520a^5b^3x^6-600600a^4b^4x^8-524160a^3b^5x^{10}-288288a^2b^6}{205920x^{32}}$$

input
```
integrate((b*x**2+a)**8/x**33,x)
```

output
```
(-6435*a**8 - 54912*a**7*b*x**2 - 205920*a**6*b**2*x**4 - 443520*a**5*b**3
*x**6 - 600600*a**4*b**4*x**8 - 524160*a**3*b**5*x**10 - 288288*a**2*b**6*
x**12 - 91520*a*b**7*x**14 - 12870*b**8*x**16)/(205920*x**32)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = -\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+20}{205920\,x^{32}}$$

input
```
integrate((b*x^2+a)^8/x^33,x, algorithm="maxima")
```

output
```
-1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 52416
0*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*
x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = -\frac{12870\,b^8x^{16}+91520\,ab^7x^{14}+288288\,a^2b^6x^{12}+524160\,a^3b^5x^{10}+600600\,a^4b^4x^8+443520\,a^5b^3x^6+20}{205920\,x^{32}}$$

input
```
integrate((b*x^2+a)^8/x^33,x, algorithm="giac")
```

output
```
-1/205920*(12870*b^8*x^16 + 91520*a*b^7*x^14 + 288288*a^2*b^6*x^12 + 52416
0*a^3*b^5*x^10 + 600600*a^4*b^4*x^8 + 443520*a^5*b^3*x^6 + 205920*a^6*b^2*
x^4 + 54912*a^7*b*x^2 + 6435*a^8)/x^32
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = -\frac{\frac{a^8}{32}+\frac{4\,a^7\,b\,x^2}{15}+a^6\,b^2\,x^4+\frac{28\,a^5\,b^3\,x^6}{13}+\frac{35\,a^4\,b^4\,x^8}{12}+\frac{28\,a^3\,b^5\,x^{10}}{11}+\frac{7\,a^2\,b^6\,x^{12}}{5}+\frac{4\,a\,b^7\,x^{14}}{9}+\frac{b^8\,x^{16}}{16}}{x^{32}}$$

input
```
int((a + b*x^2)^8/x^33,x)
```

output
```
-(a^8/32 + (b^8*x^16)/16 + (4*a^7*b*x^2)/15 + (4*a*b^7*x^14)/9 + a^6*b^2*x
^4 + (28*a^5*b^3*x^6)/13 + (35*a^4*b^4*x^8)/12 + (28*a^3*b^5*x^10)/11 + (7
*a^2*b^6*x^12)/5)/x^32
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{33}}\,dx = \frac{-12870b^8x^{16}-91520a\,b^7x^{14}-288288a^2b^6x^{12}-524160a^3b^5x^{10}-600600a^4b^4x^8-443520a^5b^3x^6-2059}{205920x^{32}}$$

input
```
int((b*x^2+a)^8/x^33,x)
```

output
```
( - 6435*a**8 - 54912*a**7*b*x**2 - 205920*a**6*b**2*x**4 - 443520*a**5*b*
*3*x**6 - 600600*a**4*b**4*x**8 - 524160*a**3*b**5*x**10 - 288288*a**2*b**
6*x**12 - 91520*a*b**7*x**14 - 12870*b**8*x**16)/(205920*x**32)
```

3.109 $\int x^8(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int x^8(a+bx^2)^8\,dx=\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

output
```
1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^
4*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^
25
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int x^8(a+bx^2)^8\,dx=\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

input
```
Integrate[x^8*(a + b*x^2)^8,x]
```

output
```
(a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)
/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (
8*a*b^7*x^23)/23 + (b^8*x^25)/25
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^8x^8+8a^7bx^{10}+28a^6b^2x^{12}+56a^5b^3x^{14}+70a^4b^4x^{16}+56a^3b^5x^{18}+28a^2b^6x^{20}+8ab^7x^{22}+b^8x^{24}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^8x^9}{9}+\frac{8}{11}a^7bx^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}ab^7x^{23}+\frac{b^8x^{25}}{25}$$

input
```
Int[x^8*(a + b*x^2)^8,x]
```

output
```
(a^8*x^9)/9 + (8*a^7*b*x^11)/11 + (28*a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)
/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b^5*x^19)/19 + (4*a^2*b^6*x^21)/3 + (
8*a*b^7*x^23)/23 + (b^8*x^25)/25
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{2}$
default	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{2}$
norman	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{2}$
risch	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{2}$
parallelrisch	$\frac{1}{9}a^8x^9+\frac{8}{11}a^7b\,x^{11}+\frac{28}{13}a^6b^2x^{13}+\frac{56}{15}a^5b^3x^{15}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{19}a^3b^5x^{19}+\frac{4}{3}a^2b^6x^{21}+\frac{8}{23}a\,b^7x^{2}$
orering	$\frac{x^9(9561123b^8x^{16}+83140200a\,b^7x^{14}+318704100a^2b^6x^{12}+704503800a^3b^5x^{10}+984233250a^4b^4x^8+892371480a^5b^3x^6+51482}{239028075}$

input
```
int(x^8*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^8*x^9+8/11*a^7*b*x^11+28/13*a^6*b^2*x^13+56/15*a^5*b^3*x^15+70/17*a^
4*b^4*x^17+56/19*a^3*b^5*x^19+4/3*a^2*b^6*x^21+8/23*a*b^7*x^23+1/25*b^8*x^
25
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^8 (a + bx^2)^8 \, dx = \frac{1}{25} b^8 x^{25} + \frac{8}{23} a b^7 x^{23} + \frac{4}{3} a^2 b^6 x^{21} + \frac{56}{19} a^3 b^5 x^{19} + \frac{70}{17} a^4 b^4 x^{17} + \frac{56}{15} a^5 b^3 x^{15} + \frac{28}{13} a^6 b^2 x^{13} + \frac{8}{11} a^7 b x^{11} + \frac{1}{9} a^8 x^9$$

input
```
integrate(x^8*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 +
70/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*
x^11 + 1/9*a^8*x^9
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int x^8 (a + bx^2)^8 \, dx = \frac{a^8 x^9}{9} + \frac{8a^7 b x^{11}}{11} + \frac{28a^6 b^2 x^{13}}{13} + \frac{56a^5 b^3 x^{15}}{15} + \frac{70a^4 b^4 x^{17}}{17} + \frac{56a^3 b^5 x^{19}}{19} + \frac{4a^2 b^6 x^{21}}{3} + \frac{8ab^7 x^{23}}{23} + \frac{b^8 x^{25}}{25}$$

input
```
integrate(x**8*(b*x**2+a)**8,x)
```

output
```
a**8*x**9/9 + 8*a**7*b*x**11/11 + 28*a**6*b**2*x**13/13 + 56*a**5*b**3*x**
15/15 + 70*a**4*b**4*x**17/17 + 56*a**3*b**5*x**19/19 + 4*a**2*b**6*x**21/
3 + 8*a*b**7*x**23/23 + b**8*x**25/25
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^8(a+bx^2)^8\,dx = \frac{1}{25}b^8x^{25}+\frac{8}{23}ab^7x^{23}+\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{1}{9}a^8x^9$$

input
```
integrate(x^8*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 +
70/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*
x^11 + 1/9*a^8*x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^8(a+bx^2)^8\,dx = \frac{1}{25}b^8x^{25}+\frac{8}{23}ab^7x^{23}+\frac{4}{3}a^2b^6x^{21}+\frac{56}{19}a^3b^5x^{19}+\frac{70}{17}a^4b^4x^{17}+\frac{56}{15}a^5b^3x^{15}+\frac{28}{13}a^6b^2x^{13}+\frac{8}{11}a^7bx^{11}+\frac{1}{9}a^8x^9$$

input
```
integrate(x^8*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/25*b^8*x^25 + 8/23*a*b^7*x^23 + 4/3*a^2*b^6*x^21 + 56/19*a^3*b^5*x^19 +
70/17*a^4*b^4*x^17 + 56/15*a^5*b^3*x^15 + 28/13*a^6*b^2*x^13 + 8/11*a^7*b*
x^11 + 1/9*a^8*x^9
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^8 (a+bx^2)^8 \, dx = \frac{a^8 x^9}{9} + \frac{8 a^7 b x^{11}}{11} + \frac{28 a^6 b^2 x^{13}}{13} + \frac{56 a^5 b^3 x^{15}}{15} + \frac{70 a^4 b^4 x^{17}}{17} + \frac{56 a^3 b^5 x^{19}}{19} + \frac{4 a^2 b^6 x^{21}}{3} + \frac{8 a b^7 x^{23}}{23} + \frac{b^8 x^{25}}{25}$$

input
```
int(x^8*(a + b*x^2)^8,x)
```

output
```
(a^8*x^9)/9 + (b^8*x^25)/25 + (8*a^7*b*x^11)/11 + (8*a*b^7*x^23)/23 + (28*
a^6*b^2*x^13)/13 + (56*a^5*b^3*x^15)/15 + (70*a^4*b^4*x^17)/17 + (56*a^3*b
^5*x^19)/19 + (4*a^2*b^6*x^21)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int x^8 (a+bx^2)^8 \, dx = \frac{x^9(9561123b^8x^{16} + 83140200a b^7x^{14} + 318704100a^2b^6x^{12} + 704503800a^3b^5x^{10} + 984233250a^4b^4x^8 + 892}{239028075}$$

input
```
int(x^8*(b*x^2+a)^8,x)
```

output
```
(x**9*(26558675*a**8 + 173838600*a**7*b*x**2 + 514829700*a**6*b**2*x**4 +
892371480*a**5*b**3*x**6 + 984233250*a**4*b**4*x**8 + 704503800*a**3*b**5*
x**10 + 318704100*a**2*b**6*x**12 + 83140200*a*b**7*x**14 + 9561123*b**8*x
**16))/239028075
```

3.110 $\int x^6(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int x^6\left(a+bx^2\right)^8\,dx=\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

output
```
1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b
^4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^2
3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int x^6\left(a+bx^2\right)^8\,dx=\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

input
```
Integrate[x^6*(a + b*x^2)^8,x]
```

output
```
(a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/1
3 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8
*a*b^7*x^21)/21 + (b^8*x^23)/23
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (a+bx^2)^8 \, dx$$

$$\Big\downarrow \ 244$$

$$\int \left(a^8x^6+8a^7bx^8+28a^6b^2x^{10}+56a^5b^3x^{12}+70a^4b^4x^{14}+56a^3b^5x^{16}+28a^2b^6x^{18}+8ab^7x^{20}+b^8x^{22}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^8x^7}{7}+\frac{8}{9}a^7bx^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}ab^7x^{21}+\frac{b^8x^{23}}{23}$$

input
```
Int[x^6*(a + b*x^2)^8,x]
```

output
```
(a^8*x^7)/7 + (8*a^7*b*x^9)/9 + (28*a^6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/1
3 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*x^17)/17 + (28*a^2*b^6*x^19)/19 + (8
*a*b^7*x^21)/21 + (b^8*x^23)/23
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}$
default	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}$
norman	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}$
risch	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}$
parallelrisch	$\frac{1}{7}a^8x^7+\frac{8}{9}a^7b\,x^9+\frac{28}{11}a^6b^2x^{11}+\frac{56}{13}a^5b^3x^{13}+\frac{14}{3}a^4b^4x^{15}+\frac{56}{17}a^3b^5x^{17}+\frac{28}{19}a^2b^6x^{19}+\frac{8}{21}a\,b^7x^{21}$
orering	$\frac{x^7(2909907b^8x^{16}+25496328a\,b^7x^{14}+98630532a^2b^6x^{12}+220468248a^3b^5x^{10}+312330018a^4b^4x^8+288304632a^5b^3x^6+170361}{66927861}$

input
```
int(x^6*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^8*x^7+8/9*a^7*b*x^9+28/11*a^6*b^2*x^11+56/13*a^5*b^3*x^13+14/3*a^4*b
^4*x^15+56/17*a^3*b^5*x^17+28/19*a^2*b^6*x^19+8/21*a*b^7*x^21+1/23*b^8*x^2
3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^6 (a+bx^2)^8 \, dx = \frac{1}{23} b^8 x^{23} + \frac{8}{21} a b^7 x^{21} + \frac{28}{19} a^2 b^6 x^{19} + \frac{56}{17} a^3 b^5 x^{17} + \frac{14}{3} a^4 b^4 x^{15} + \frac{56}{13} a^5 b^3 x^{13} + \frac{28}{11} a^6 b^2 x^{11} + \frac{8}{9} a^7 b x^9 + \frac{1}{7} a^8 x^7$$

input
```
integrate(x^6*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17
+ 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*
x^9 + 1/7*a^8*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int x^6 (a+bx^2)^8 \, dx = \frac{a^8 x^7}{7} + \frac{8a^7 b x^9}{9} + \frac{28a^6 b^2 x^{11}}{11} + \frac{56a^5 b^3 x^{13}}{13} + \frac{14a^4 b^4 x^{15}}{3} + \frac{56a^3 b^5 x^{17}}{17} + \frac{28a^2 b^6 x^{19}}{19} + \frac{8ab^7 x^{21}}{21} + \frac{b^8 x^{23}}{23}$$

input
```
integrate(x**6*(b*x**2+a)**8,x)
```

output
```
a**8*x**7/7 + 8*a**7*b*x**9/9 + 28*a**6*b**2*x**11/11 + 56*a**5*b**3*x**13
/13 + 14*a**4*b**4*x**15/3 + 56*a**3*b**5*x**17/17 + 28*a**2*b**6*x**19/19
 + 8*a*b**7*x**21/21 + b**8*x**23/23
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^6(a+bx^2)^8\,dx = \frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$$

input
```
integrate(x^6*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17
+ 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*
x^9 + 1/7*a^8*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^6(a+bx^2)^8\,dx = \frac{1}{23}b^8x^{23} + \frac{8}{21}ab^7x^{21} + \frac{28}{19}a^2b^6x^{19} + \frac{56}{17}a^3b^5x^{17} + \frac{14}{3}a^4b^4x^{15} + \frac{56}{13}a^5b^3x^{13} + \frac{28}{11}a^6b^2x^{11} + \frac{8}{9}a^7bx^9 + \frac{1}{7}a^8x^7$$

input
```
integrate(x^6*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/23*b^8*x^23 + 8/21*a*b^7*x^21 + 28/19*a^2*b^6*x^19 + 56/17*a^3*b^5*x^17
+ 14/3*a^4*b^4*x^15 + 56/13*a^5*b^3*x^13 + 28/11*a^6*b^2*x^11 + 8/9*a^7*b*
x^9 + 1/7*a^8*x^7
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^6 \left(a+bx^2\right)^8 \, dx = \frac{a^8\,x^7}{7} + \frac{8\,a^7\,b\,x^9}{9} + \frac{28\,a^6\,b^2\,x^{11}}{11} + \frac{56\,a^5\,b^3\,x^{13}}{13} + \frac{14\,a^4\,b^4\,x^{15}}{3} + \frac{56\,a^3\,b^5\,x^{17}}{17} + \frac{28\,a^2\,b^6\,x^{19}}{19} + \frac{8\,a\,b^7\,x^{21}}{21} + \frac{b^8\,x^{23}}{23}$$

input
```
int(x^6*(a + b*x^2)^8,x)
```

output
```
(a^8*x^7)/7 + (b^8*x^23)/23 + (8*a^7*b*x^9)/9 + (8*a*b^7*x^21)/21 + (28*a^
6*b^2*x^11)/11 + (56*a^5*b^3*x^13)/13 + (14*a^4*b^4*x^15)/3 + (56*a^3*b^5*
x^17)/17 + (28*a^2*b^6*x^19)/19
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int x^6 \left(a+bx^2\right)^8 \, dx = \frac{x^7 (2909907 b^8 x^{16} + 25496328 a\, b^7 x^{14} + 98630532 a^2 b^6 x^{12} + 220468248 a^3 b^5 x^{10} + 312330018 a^4 b^4 x^8 + 2883}{66927861}$$

input
```
int(x^6*(b*x^2+a)^8,x)
```

output
```
(x**7*(9561123*a**8 + 59491432*a**7*b*x**2 + 170361828*a**6*b**2*x**4 + 28
8304632*a**5*b**3*x**6 + 312330018*a**4*b**4*x**8 + 220468248*a**3*b**5*x*
*10 + 98630532*a**2*b**6*x**12 + 25496328*a*b**7*x**14 + 2909907*b**8*x**1
6))/66927861
```

3.111 $\int x^4(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int x^4(a+bx^2)^8\,dx=\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

output
```
1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^
4*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^8\,dx=\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

input
```
Integrate[x^4*(a + b*x^2)^8,x]
```

output
```
(a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11
+ (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*
a*b^7*x^19)/19 + (b^8*x^21)/21
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^8x^4+8a^7bx^6+28a^6b^2x^8+56a^5b^3x^{10}+70a^4b^4x^{12}+56a^3b^5x^{14}+28a^2b^6x^{16}+8ab^7x^{18}+b^8x^{20}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^8x^5}{5}+\frac{8}{7}a^7bx^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}ab^7x^{19}+\frac{b^8x^{21}}{21}$$

input
```
Int[x^4*(a + b*x^2)^8,x]
```

output

```
(a^8*x^5)/5 + (8*a^7*b*x^7)/7 + (28*a^6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11
+ (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x^15)/15 + (28*a^2*b^6*x^17)/17 + (8*
a*b^7*x^19)/19 + (b^8*x^21)/21
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84

method	result
gosper	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}$
default	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}$
norman	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}$
risch	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}$
parallelrisch	$\frac{1}{5}a^8x^5+\frac{8}{7}a^7b\,x^7+\frac{28}{9}a^6b^2x^9+\frac{56}{11}a^5b^3x^{11}+\frac{70}{13}a^4b^4x^{13}+\frac{56}{15}a^3b^5x^{15}+\frac{28}{17}a^2b^6x^{17}+\frac{8}{19}a\,b^7x^{19}$
orering	$\frac{x^5(692835b^8x^{16}+6126120a\,b^7x^{14}+23963940a^2b^6x^{12}+54318264a^3b^5x^{10}+78343650a^4b^4x^8+74070360a^5b^3x^6+45265220a^6b}{14549535}$

input
```
int(x^4*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^8*x^5+8/7*a^7*b*x^7+28/9*a^6*b^2*x^9+56/11*a^5*b^3*x^11+70/13*a^4*b^
4*x^13+56/15*a^3*b^5*x^15+28/17*a^2*b^6*x^17+8/19*a*b^7*x^19+1/21*b^8*x^21
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^4(a+bx^2)^8\,dx=\frac{1}{21}\,b^8x^{21}+\frac{8}{19}\,ab^7x^{19}+\frac{28}{17}\,a^2b^6x^{17}+\frac{56}{15}\,a^3b^5x^{15}+\frac{70}{13}\,a^4b^4x^{13}+\frac{56}{11}\,a^5b^3x^{11}+\frac{28}{9}\,a^6b^2x^9+\frac{8}{7}\,a^7bx^7+\frac{1}{5}\,a^8x^5$$

input
```
integrate(x^4*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15
+ 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x
^7 + 1/5*a^8*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int x^4 \left(a+bx^2\right)^8 \, dx = \frac{a^8x^5}{5} + \frac{8a^7bx^7}{7} + \frac{28a^6b^2x^9}{9} + \frac{56a^5b^3x^{11}}{11} + \frac{70a^4b^4x^{13}}{13} + \frac{56a^3b^5x^{15}}{15} + \frac{28a^2b^6x^{17}}{17} + \frac{8ab^7x^{19}}{19} + \frac{b^8x^{21}}{21}$$

input
```
integrate(x**4*(b*x**2+a)**8,x)
```

output
```
a**8*x**5/5 + 8*a**7*b*x**7/7 + 28*a**6*b**2*x**9/9 + 56*a**5*b**3*x**11/1
1 + 70*a**4*b**4*x**13/13 + 56*a**3*b**5*x**15/15 + 28*a**2*b**6*x**17/17
+ 8*a*b**7*x**19/19 + b**8*x**21/21
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^4 \left(a+bx^2\right)^8 \, dx = \frac{1}{21}\,b^8x^{21} + \frac{8}{19}\,ab^7x^{19} + \frac{28}{17}\,a^2b^6x^{17} + \frac{56}{15}\,a^3b^5x^{15} + \frac{70}{13}\,a^4b^4x^{13} + \frac{56}{11}\,a^5b^3x^{11} + \frac{28}{9}\,a^6b^2x^9 + \frac{8}{7}\,a^7bx^7 + \frac{1}{5}\,a^8x^5$$

input
```
integrate(x^4*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15
+ 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x
^7 + 1/5*a^8*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^4 (a+bx^2)^8 \, dx = \frac{1}{21} b^8 x^{21} + \frac{8}{19} a b^7 x^{19} + \frac{28}{17} a^2 b^6 x^{17} + \frac{56}{15} a^3 b^5 x^{15} + \frac{70}{13} a^4 b^4 x^{13} + \frac{56}{11} a^5 b^3 x^{11} + \frac{28}{9} a^6 b^2 x^9 + \frac{8}{7} a^7 b x^7 + \frac{1}{5} a^8 x^5$$

input
```
integrate(x^4*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/21*b^8*x^21 + 8/19*a*b^7*x^19 + 28/17*a^2*b^6*x^17 + 56/15*a^3*b^5*x^15
+ 70/13*a^4*b^4*x^13 + 56/11*a^5*b^3*x^11 + 28/9*a^6*b^2*x^9 + 8/7*a^7*b*x
^7 + 1/5*a^8*x^5
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int x^4 (a+bx^2)^8 \, dx = \frac{a^8 x^5}{5} + \frac{8 a^7 b x^7}{7} + \frac{28 a^6 b^2 x^9}{9} + \frac{56 a^5 b^3 x^{11}}{11} + \frac{70 a^4 b^4 x^{13}}{13} + \frac{56 a^3 b^5 x^{15}}{15} + \frac{28 a^2 b^6 x^{17}}{17} + \frac{8 a b^7 x^{19}}{19} + \frac{b^8 x^{21}}{21}$$

input
```
int(x^4*(a + b*x^2)^8,x)
```

output
```
(a^8*x^5)/5 + (b^8*x^21)/21 + (8*a^7*b*x^7)/7 + (8*a*b^7*x^19)/19 + (28*a^
6*b^2*x^9)/9 + (56*a^5*b^3*x^11)/11 + (70*a^4*b^4*x^13)/13 + (56*a^3*b^5*x
^15)/15 + (28*a^2*b^6*x^17)/17
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int x^4 (a + bx^2)^8 \, dx$$
$$= \frac{x^5(692835b^8x^{16} + 6126120a\,b^7x^{14} + 23963940a^2b^6x^{12} + 54318264a^3b^5x^{10} + 78343650a^4b^4x^8 + 74070360}{14549535}$$

input
```
int(x^4*(b*x^2+a)^8,x)
```

output
```
(x**5*(2909907*a**8 + 16628040*a**7*b*x**2 + 45265220*a**6*b**2*x**4 + 740
70360*a**5*b**3*x**6 + 78343650*a**4*b**4*x**8 + 54318264*a**3*b**5*x**10
+ 23963940*a**2*b**6*x**12 + 6126120*a*b**7*x**14 + 692835*b**8*x**16))/14
549535
```

3.112 $\int x^2(a+bx^2)^8\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int x^2\left(a+bx^2\right)^8\,dx=\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

output
```
1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^1
1+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^8\,dx=\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

input
```
Integrate[x^2*(a + b*x^2)^8,x]
```

output
```
(a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a
^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x
^17)/17 + (b^8*x^19)/19
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^8 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^8x^2+8a^7bx^4+28a^6b^2x^6+56a^5b^3x^8+70a^4b^4x^{10}+56a^3b^5x^{12}+28a^2b^6x^{14}+8ab^7x^{16}+b^8x^{18}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^8x^3}{3}+\frac{8}{5}a^7bx^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}ab^7x^{17}+\frac{b^8x^{19}}{19}$$

input
```
Int[x^2*(a + b*x^2)^8,x]
```

output
```
(a^8*x^3)/3 + (8*a^7*b*x^5)/5 + 4*a^6*b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a
^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13 + (28*a^2*b^6*x^15)/15 + (8*a*b^7*x
^17)/17 + (b^8*x^19)/19
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+$
default	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+$
norman	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+$
risch	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+$
parallelrisch	$\frac{1}{3}a^8x^3+\frac{8}{5}a^7b\,x^5+4a^6b^2x^7+\frac{56}{9}a^5b^3x^9+\frac{70}{11}a^4b^4x^{11}+\frac{56}{13}a^3b^5x^{13}+\frac{28}{15}a^2b^6x^{15}+\frac{8}{17}a\,b^7x^{17}+$
orering	$\frac{x^3(109395b^8x^{16}+978120a\,b^7x^{14}+3879876a^2b^6x^{12}+8953560a^3b^5x^{10}+13226850a^4b^4x^8+12932920a^5b^3x^6+8314020a^6b^2x^4+}{2078505}$

input
```
int(x^2*(b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^8*x^3+8/5*a^7*b*x^5+4*a^6*b^2*x^7+56/9*a^5*b^3*x^9+70/11*a^4*b^4*x^1
1+56/13*a^3*b^5*x^13+28/15*a^2*b^6*x^15+8/17*a*b^7*x^17+1/19*b^8*x^19
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^2)^8\,dx=\frac{1}{19}\,b^8x^{19}+\frac{8}{17}\,ab^7x^{17}+\frac{28}{15}\,a^2b^6x^{15}+\frac{56}{13}\,a^3b^5x^{13}+\frac{70}{11}\,a^4b^4x^{11}+\frac{56}{9}\,a^5b^3x^9+4\,a^6b^2x^7+\frac{8}{5}\,a^7bx^5+\frac{1}{3}\,a^8x^3$$

input
```
integrate(x^2*(b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13
+ 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 +
1/3*a^8*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int x^2 (a + bx^2)^8 \, dx = \frac{a^8x^3}{3} + \frac{8a^7bx^5}{5} + 4a^6b^2x^7 + \frac{56a^5b^3x^9}{9} + \frac{70a^4b^4x^{11}}{11} + \frac{56a^3b^5x^{13}}{13} + \frac{28a^2b^6x^{15}}{15} + \frac{8ab^7x^{17}}{17} + \frac{b^8x^{19}}{19}$$

input
```
integrate(x**2*(b*x**2+a)**8,x)
```

output
```
a**8*x**3/3 + 8*a**7*b*x**5/5 + 4*a**6*b**2*x**7 + 56*a**5*b**3*x**9/9 + 7
0*a**4*b**4*x**11/11 + 56*a**3*b**5*x**13/13 + 28*a**2*b**6*x**15/15 + 8*a
*b**7*x**17/17 + b**8*x**19/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx^2)^8 \, dx = \frac{1}{19} b^8x^{19} + \frac{8}{17} ab^7x^{17} + \frac{28}{15} a^2b^6x^{15} + \frac{56}{13} a^3b^5x^{13} + \frac{70}{11} a^4b^4x^{11} + \frac{56}{9} a^5b^3x^9 + 4 a^6b^2x^7 + \frac{8}{5} a^7bx^5 + \frac{1}{3} a^8x^3$$

input
```
integrate(x^2*(b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13
+ 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 +
1/3*a^8*x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx^2)^8 \, dx = \frac{1}{19} b^8 x^{19} + \frac{8}{17} ab^7 x^{17} + \frac{28}{15} a^2 b^6 x^{15} + \frac{56}{13} a^3 b^5 x^{13} + \frac{70}{11} a^4 b^4 x^{11} + \frac{56}{9} a^5 b^3 x^9 + 4 a^6 b^2 x^7 + \frac{8}{5} a^7 b x^5 + \frac{1}{3} a^8 x^3$$

input
```
integrate(x^2*(b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/19*b^8*x^19 + 8/17*a*b^7*x^17 + 28/15*a^2*b^6*x^15 + 56/13*a^3*b^5*x^13
+ 70/11*a^4*b^4*x^11 + 56/9*a^5*b^3*x^9 + 4*a^6*b^2*x^7 + 8/5*a^7*b*x^5 +
1/3*a^8*x^3
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int x^2 (a + bx^2)^8 \, dx = \frac{a^8 x^3}{3} + \frac{8 a^7 b x^5}{5} + 4 a^6 b^2 x^7 + \frac{56 a^5 b^3 x^9}{9} + \frac{70 a^4 b^4 x^{11}}{11} + \frac{56 a^3 b^5 x^{13}}{13} + \frac{28 a^2 b^6 x^{15}}{15} + \frac{8 a b^7 x^{17}}{17} + \frac{b^8 x^{19}}{19}$$

input
```
int(x^2*(a + b*x^2)^8,x)
```

output
```
(a^8*x^3)/3 + (b^8*x^19)/19 + (8*a^7*b*x^5)/5 + (8*a*b^7*x^17)/17 + 4*a^6*
b^2*x^7 + (56*a^5*b^3*x^9)/9 + (70*a^4*b^4*x^11)/11 + (56*a^3*b^5*x^13)/13
 + (28*a^2*b^6*x^15)/15
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int x^2(a+bx^2)^8\,dx$$
$$= \frac{x^3(109395b^8x^{16}+978120a\,b^7x^{14}+3879876a^2b^6x^{12}+8953560a^3b^5x^{10}+13226850a^4b^4x^8+12932920a^5b}{2078505}$$

input
```
int(x^2*(b*x^2+a)^8,x)
```

output
```
(x**3*(692835*a**8 + 3325608*a**7*b*x**2 + 8314020*a**6*b**2*x**4 + 129329
20*a**5*b**3*x**6 + 13226850*a**4*b**4*x**8 + 8953560*a**3*b**5*x**10 + 38
79876*a**2*b**6*x**12 + 978120*a*b**7*x**14 + 109395*b**8*x**16))/2078505
```

3.113 $\int (a+bx^2)^8 \, dx$

Optimal result

Integrand size = 9, antiderivative size = 101

$$\int (a+bx^2)^8 \, dx = a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{17}}{17}$$

output
```
a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*
a^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^8 \, dx = a^8x + \frac{8}{3}a^7bx^3 + \frac{28}{5}a^6b^2x^5 + 8a^5b^3x^7 + \frac{70}{9}a^4b^4x^9 + \frac{56}{11}a^3b^5x^{11} + \frac{28}{13}a^2b^6x^{13} + \frac{8}{15}ab^7x^{15} + \frac{b^8x^{17}}{17}$$

input
```
Integrate[(a + b*x^2)^8,x]
```

output
```
a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4
*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15
+ (b^8*x^17)/17
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^8 dx$$

$$\Big\downarrow \text{210}$$

$$\int \left(a^8+8a^7bx^2+28a^6b^2x^4+56a^5b^3x^6+70a^4b^4x^8+56a^3b^5x^{10}+28a^2b^6x^{12}+8ab^7x^{14}+b^8x^{16}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^8x+\frac{8}{3}a^7bx^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}ab^7x^{15}+\frac{b^8x^{17}}{17}$$

input
```
Int[(a + b*x^2)^8,x]
```

output

```
a^8*x + (8*a^7*b*x^3)/3 + (28*a^6*b^2*x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4
*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a^2*b^6*x^13)/13 + (8*a*b^7*x^15)/15
+ (b^8*x^17)/17
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

method	result
gosper	$a^8x+\frac{8}{3}a^7b\,x^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}a\,b^7x^{15}+\frac{1}{17}b$
default	$a^8x+\frac{8}{3}a^7b\,x^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}a\,b^7x^{15}+\frac{1}{17}b$
norman	$a^8x+\frac{8}{3}a^7b\,x^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}a\,b^7x^{15}+\frac{1}{17}b$
risch	$a^8x+\frac{8}{3}a^7b\,x^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}a\,b^7x^{15}+\frac{1}{17}b$
parallelrisch	$a^8x+\frac{8}{3}a^7b\,x^3+\frac{28}{5}a^6b^2x^5+8a^5b^3x^7+\frac{70}{9}a^4b^4x^9+\frac{56}{11}a^3b^5x^{11}+\frac{28}{13}a^2b^6x^{13}+\frac{8}{15}a\,b^7x^{15}+\frac{1}{17}b$
orering	$\frac{x\left(6435b^8x^{16}+58344a\,b^7x^{14}+235620a^2b^6x^{12}+556920a^3b^5x^{10}+850850a^4b^4x^8+875160a^5b^3x^6+612612a^6b^2x^4+291720a^7b\,x\right.}{109395}$

input
```
int((b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

output
```
a^8*x+8/3*a^7*b*x^3+28/5*a^6*b^2*x^5+8*a^5*b^3*x^7+70/9*a^4*b^4*x^9+56/11*
a^3*b^5*x^11+28/13*a^2*b^6*x^13+8/15*a*b^7*x^15+1/17*b^8*x^17
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)^8\,dx=\frac{1}{17}\,b^8x^{17}+\frac{8}{15}\,ab^7x^{15}+\frac{28}{13}\,a^2b^6x^{13}+\frac{56}{11}\,a^3b^5x^{11}+\frac{70}{9}\,a^4b^4x^9+8\,a^5b^3x^7+\frac{28}{5}\,a^6b^2x^5+\frac{8}{3}\,a^7bx^3+a^8x$$

input
```
integrate((b*x^2+a)^8,x, algorithm="fricas")
```

output
```
1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11
+ 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^
8*x
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \left(a+bx^2\right)^8 \,dx = a^8x + \frac{8a^7bx^3}{3} + \frac{28a^6b^2x^5}{5} + 8a^5b^3x^7 + \frac{70a^4b^4x^9}{9} + \frac{56a^3b^5x^{11}}{11} + \frac{28a^2b^6x^{13}}{13} + \frac{8ab^7x^{15}}{15} + \frac{b^8x^{17}}{17}$$

input
```
integrate((b*x**2+a)**8,x)
```

output
```
a**8*x + 8*a**7*b*x**3/3 + 28*a**6*b**2*x**5/5 + 8*a**5*b**3*x**7 + 70*a**
4*b**4*x**9/9 + 56*a**3*b**5*x**11/11 + 28*a**2*b**6*x**13/13 + 8*a*b**7*x
**15/15 + b**8*x**17/17
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)^8 \,dx = \frac{1}{17}\,b^8x^{17} + \frac{8}{15}\,ab^7x^{15} + \frac{28}{13}\,a^2b^6x^{13} + \frac{56}{11}\,a^3b^5x^{11} + \frac{70}{9}\,a^4b^4x^9 + 8\,a^5b^3x^7 + \frac{28}{5}\,a^6b^2x^5 + \frac{8}{3}\,a^7bx^3 + a^8x$$

input
```
integrate((b*x^2+a)^8,x, algorithm="maxima")
```

output
```
1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11
+ 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^
8*x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)^8 \, dx = \frac{1}{17}b^8x^{17} + \frac{8}{15}ab^7x^{15} + \frac{28}{13}a^2b^6x^{13} + \frac{56}{11}a^3b^5x^{11} + \frac{70}{9}a^4b^4x^9 + 8\,a^5b^3x^7 + \frac{28}{5}a^6b^2x^5 + \frac{8}{3}a^7bx^3 + a^8x$$

input
```
integrate((b*x^2+a)^8,x, algorithm="giac")
```

output
```
1/17*b^8*x^17 + 8/15*a*b^7*x^15 + 28/13*a^2*b^6*x^13 + 56/11*a^3*b^5*x^11
+ 70/9*a^4*b^4*x^9 + 8*a^5*b^3*x^7 + 28/5*a^6*b^2*x^5 + 8/3*a^7*b*x^3 + a^
8*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int \left(a+bx^2\right)^8 \, dx = a^8\,x + \frac{8\,a^7\,b\,x^3}{3} + \frac{28\,a^6\,b^2\,x^5}{5} + 8\,a^5\,b^3\,x^7 + \frac{70\,a^4\,b^4\,x^9}{9} + \frac{56\,a^3\,b^5\,x^{11}}{11} + \frac{28\,a^2\,b^6\,x^{13}}{13} + \frac{8\,a\,b^7\,x^{15}}{15} + \frac{b^8\,x^{17}}{17}$$

input
```
int((a + b*x^2)^8,x)
```

output
```
a^8*x + (b^8*x^17)/17 + (8*a^7*b*x^3)/3 + (8*a*b^7*x^15)/15 + (28*a^6*b^2*
x^5)/5 + 8*a^5*b^3*x^7 + (70*a^4*b^4*x^9)/9 + (56*a^3*b^5*x^11)/11 + (28*a
^2*b^6*x^13)/13
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int (a + bx^2)^8 \, dx$$
$$= \frac{x(6435b^8x^{16} + 58344a\,b^7x^{14} + 235620a^2b^6x^{12} + 556920a^3b^5x^{10} + 850850a^4b^4x^8 + 875160a^5b^3x^6 + 61261}{109395}$$

input
```
int((b*x^2+a)^8,x)
```

output
```
(x*(109395*a**8 + 291720*a**7*b*x**2 + 612612*a**6*b**2*x**4 + 875160*a**5
*b**3*x**6 + 850850*a**4*b**4*x**8 + 556920*a**3*b**5*x**10 + 235620*a**2*
b**6*x**12 + 58344*a*b**7*x**14 + 6435*b**8*x**16))/109395
```

3.114 $\int \frac{(a+bx^2)^8}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^2)^8}{x^2} dx = -\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

output
```
-a^8/x+8*a^7*b*x+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3
*b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^2} dx = -\frac{a^8}{x} + 8a^7bx + \frac{28}{3}a^6b^2x^3 + \frac{56}{5}a^5b^3x^5 + 10a^4b^4x^7 + \frac{56}{9}a^3b^5x^9 + \frac{28}{11}a^2b^6x^{11} + \frac{8}{13}ab^7x^{13} + \frac{b^8x^{15}}{15}$$

input
```
Integrate[(a + b*x^2)^8/x^2,x]
```

output
```
-(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^
4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b
^8*x^15)/15
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^2}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^8}{x^2}+8a^7b+28a^6b^2x^2+56a^5b^3x^4+70a^4b^4x^6+56a^3b^5x^8+28a^2b^6x^{10}+8ab^7x^{12}+b^8x^{14}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{x}+8a^7bx+\frac{28}{3}a^6b^2x^3+\frac{56}{5}a^5b^3x^5+10a^4b^4x^7+\frac{56}{9}a^3b^5x^9+\frac{28}{11}a^2b^6x^{11}+\frac{8}{13}ab^7x^{13}+\frac{b^8x^{15}}{15}$$

input
```
Int[(a + b*x^2)^8/x^2,x]
```

output

```
-(a^8/x) + 8*a^7*b*x + (28*a^6*b^2*x^3)/3 + (56*a^5*b^3*x^5)/5 + 10*a^4*b^
4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + (8*a*b^7*x^13)/13 + (b
^8*x^15)/15
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^8}{x}+8a^7xb+\frac{28a^6b^2x^3}{3}+\frac{56a^5b^3x^5}{5}+10a^4b^4x^7+\frac{56a^3b^5x^9}{9}+\frac{28a^2b^6x^{11}}{11}+\frac{8a\,b^7x^{13}}{13}+\frac{b^8x^{15}}{15}$
risch	$-\frac{a^8}{x}+8a^7xb+\frac{28a^6b^2x^3}{3}+\frac{56a^5b^3x^5}{5}+10a^4b^4x^7+\frac{56a^3b^5x^9}{9}+\frac{28a^2b^6x^{11}}{11}+\frac{8a\,b^7x^{13}}{13}+\frac{b^8x^{15}}{15}$
norman	$\frac{-a^8+8a^7b\,x^2+\frac{28}{3}a^6b^2x^4+\frac{56}{5}a^5b^3x^6+10a^4b^4x^8+\frac{56}{9}a^3b^5x^{10}+\frac{28}{11}a^2b^6x^{12}+\frac{8}{13}a\,b^7x^{14}+\frac{1}{15}b^8x^{16}}{x}$
gosper	$-\frac{-429b^8x^{16}-3960a\,b^7x^{14}-16380a^2b^6x^{12}-40040a^3b^5x^{10}-64350a^4b^4x^8-72072a^5b^3x^6-60060a^6b^2x^4-51480a^7b\,x^2+6435}{6435x}$
parallelrisch	$\frac{429b^8x^{16}+3960a\,b^7x^{14}+16380a^2b^6x^{12}+40040a^3b^5x^{10}+64350a^4b^4x^8+72072a^5b^3x^6+60060a^6b^2x^4+51480a^7b\,x^2-6435a^8}{6435x}$
orering	$-\frac{-429b^8x^{16}-3960a\,b^7x^{14}-16380a^2b^6x^{12}-40040a^3b^5x^{10}-64350a^4b^4x^8-72072a^5b^3x^6-60060a^6b^2x^4-51480a^7b\,x^2+6435}{6435x}$

input
```
int((b*x^2+a)^8/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^8/x+8*a^7*x*b+28/3*a^6*b^2*x^3+56/5*a^5*b^3*x^5+10*a^4*b^4*x^7+56/9*a^3
*b^5*x^9+28/11*a^2*b^6*x^11+8/13*a*b^7*x^13+1/15*b^8*x^15
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^2}\,dx$$
$$=\frac{429\,b^8x^{16}+3960\,ab^7x^{14}+16380\,a^2b^6x^{12}+40040\,a^3b^5x^{10}+64350\,a^4b^4x^8+72072\,a^5b^3x^6+60060\,a^6b^2x^4}{6435\,x}$$

input
```
integrate((b*x^2+a)^8/x^2,x, algorithm="fricas")
```

output
```
1/6435*(429*b^8*x^16 + 3960*a*b^7*x^14 + 16380*a^2*b^6*x^12 + 40040*a^3*b^
5*x^10 + 64350*a^4*b^4*x^8 + 72072*a^5*b^3*x^6 + 60060*a^6*b^2*x^4 + 51480
*a^7*b*x^2 - 6435*a^8)/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^2}\,dx = -\frac{a^8}{x} + 8a^7bx + \frac{28a^6b^2x^3}{3} + \frac{56a^5b^3x^5}{5} + 10a^4b^4x^7 + \frac{56a^3b^5x^9}{9} + \frac{28a^2b^6x^{11}}{11} + \frac{8ab^7x^{13}}{13} + \frac{b^8x^{15}}{15}$$

input
```
integrate((b*x**2+a)**8/x**2,x)
```

output
```
-a**8/x + 8*a**7*b*x + 28*a**6*b**2*x**3/3 + 56*a**5*b**3*x**5/5 + 10*a**4
*b**4*x**7 + 56*a**3*b**5*x**9/9 + 28*a**2*b**6*x**11/11 + 8*a*b**7*x**13/
13 + b**8*x**15/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^2}\,dx = \frac{1}{15}\,b^8x^{15} + \frac{8}{13}\,ab^7x^{13} + \frac{28}{11}\,a^2b^6x^{11} + \frac{56}{9}\,a^3b^5x^9 + 10\,a^4b^4x^7 + \frac{56}{5}\,a^5b^3x^5 + \frac{28}{3}\,a^6b^2x^3 + 8\,a^7bx - \frac{a^8}{x}$$

input
```
integrate((b*x^2+a)^8/x^2,x, algorithm="maxima")
```

output
```
1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 +
10*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^2} \, dx = \frac{1}{15} b^8 x^{15} + \frac{8}{13} a b^7 x^{13} + \frac{28}{11} a^2 b^6 x^{11} + \frac{56}{9} a^3 b^5 x^9 + 10 a^4 b^4 x^7 + \frac{56}{5} a^5 b^3 x^5 + \frac{28}{3} a^6 b^2 x^3 + 8 a^7 b x - \frac{a^8}{x}$$

input
```
integrate((b*x^2+a)^8/x^2,x, algorithm="giac")
```

output
```
1/15*b^8*x^15 + 8/13*a*b^7*x^13 + 28/11*a^2*b^6*x^11 + 56/9*a^3*b^5*x^9 +
10*a^4*b^4*x^7 + 56/5*a^5*b^3*x^5 + 28/3*a^6*b^2*x^3 + 8*a^7*b*x - a^8/x
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^2} \, dx = \frac{b^8 x^{15}}{15} - \frac{a^8}{x} + \frac{8 a b^7 x^{13}}{13} + \frac{28 a^6 b^2 x^3}{3} + \frac{56 a^5 b^3 x^5}{5} + 10 a^4 b^4 x^7 + \frac{56 a^3 b^5 x^9}{9} + \frac{28 a^2 b^6 x^{11}}{11} + 8 a^7 b x$$

input
```
int((a + b*x^2)^8/x^2,x)
```

output
```
(b^8*x^15)/15 - a^8/x + (8*a*b^7*x^13)/13 + (28*a^6*b^2*x^3)/3 + (56*a^5*b
^3*x^5)/5 + 10*a^4*b^4*x^7 + (56*a^3*b^5*x^9)/9 + (28*a^2*b^6*x^11)/11 + 8
*a^7*b*x
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^2}\,dx$$
$$= \frac{429b^8x^{16} + 3960a\,b^7x^{14} + 16380a^2b^6x^{12} + 40040a^3b^5x^{10} + 64350a^4b^4x^8 + 72072a^5b^3x^6 + 60060a^6b^2x^4 +}{6435x}$$

input
```
int((b*x^2+a)^8/x^2,x)
```

output
```
( - 6435*a**8 + 51480*a**7*b*x**2 + 60060*a**6*b**2*x**4 + 72072*a**5*b**3
*x**6 + 64350*a**4*b**4*x**8 + 40040*a**3*b**5*x**10 + 16380*a**2*b**6*x**
12 + 3960*a*b**7*x**14 + 429*b**8*x**16)/(6435*x)
```

3.115 $\int \frac{(a+bx^2)^8}{x^4} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{(a+bx^2)^8}{x^4} \, dx = -\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56}{3}a^5b^3x^3 + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28}{9}a^2b^6x^9 + \frac{8}{11}ab^7x^{11} + \frac{b^8x^{13}}{13}$$

output
```
-1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*
b^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^4} \, dx = -\frac{a^8}{3x^3} - \frac{8a^7b}{x} + 28a^6b^2x + \frac{56}{3}a^5b^3x^3 + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28}{9}a^2b^6x^9 + \frac{8}{11}ab^7x^{11} + \frac{b^8x^{13}}{13}$$

input
```
Integrate[(a + b*x^2)^8/x^4,x]
```

output
```
-1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^
4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13
)/13
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^4} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^8}{x^4}+\frac{8a^7b}{x^2}+28a^6b^2+56a^5b^3x^2+70a^4b^4x^4+56a^3b^5x^6+28a^2b^6x^8+8ab^7x^{10}+b^8x^{12}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^8}{3x^3}-\frac{8a^7b}{x}+28a^6b^2x+\frac{56}{3}a^5b^3x^3+14a^4b^4x^5+8a^3b^5x^7+\frac{28}{9}a^2b^6x^9+\frac{8}{11}ab^7x^{11}+\frac{b^8x^{13}}{13}$$

input
```
Int[(a + b*x^2)^8/x^4,x]
```

output

```
-1/3*a^8/x^3 - (8*a^7*b)/x + 28*a^6*b^2*x + (56*a^5*b^3*x^3)/3 + 14*a^4*b^
4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9)/9 + (8*a*b^7*x^11)/11 + (b^8*x^13
)/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^{8}}{3x^{3}}-\frac{8a^{7}b}{x}+28a^{6}b^{2}x+\frac{56a^{5}b^{3}x^{3}}{3}+14a^{4}b^{4}x^{5}+8a^{3}b^{5}x^{7}+\frac{28a^{2}b^{6}x^{9}}{9}+\frac{8a\,b^{7}x^{11}}{11}+\frac{b^{8}x^{13}}{13}$
risch	$\frac{b^{8}x^{13}}{13}+\frac{8a\,b^{7}x^{11}}{11}+\frac{28a^{2}b^{6}x^{9}}{9}+8a^{3}b^{5}x^{7}+14a^{4}b^{4}x^{5}+\frac{56a^{5}b^{3}x^{3}}{3}+28a^{6}b^{2}x+\frac{-8a^{7}b\,x^{2}-\frac{1}{3}a^{8}}{x^{3}}$
norman	$\frac{-\frac{1}{3}a^{8}-8a^{7}b\,x^{2}+28a^{6}b^{2}x^{4}+\frac{56}{3}a^{5}b^{3}x^{6}+14a^{4}b^{4}x^{8}+8a^{3}b^{5}x^{10}+\frac{28}{9}a^{2}b^{6}x^{12}+\frac{8}{11}a\,b^{7}x^{14}+\frac{1}{13}b^{8}x^{16}}{x^{3}}$
gosper	$-\frac{-99b^{8}x^{16}-936a\,b^{7}x^{14}-4004a^{2}b^{6}x^{12}-10296a^{3}b^{5}x^{10}-18018a^{4}b^{4}x^{8}-24024a^{5}b^{3}x^{6}-36036a^{6}b^{2}x^{4}+10296a^{7}b\,x^{2}+429a^{8}}{1287x^{3}}$
parallelrisch	$\frac{99b^{8}x^{16}+936a\,b^{7}x^{14}+4004a^{2}b^{6}x^{12}+10296a^{3}b^{5}x^{10}+18018a^{4}b^{4}x^{8}+24024a^{5}b^{3}x^{6}+36036a^{6}b^{2}x^{4}-10296a^{7}b\,x^{2}-429a^{8}}{1287x^{3}}$
orering	$-\frac{-99b^{8}x^{16}-936a\,b^{7}x^{14}-4004a^{2}b^{6}x^{12}-10296a^{3}b^{5}x^{10}-18018a^{4}b^{4}x^{8}-24024a^{5}b^{3}x^{6}-36036a^{6}b^{2}x^{4}+10296a^{7}b\,x^{2}+429a^{8}}{1287x^{3}}$

input
```
int((b*x^2+a)^8/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^8/x^3-8*a^7*b/x+28*a^6*b^2*x+56/3*a^5*b^3*x^3+14*a^4*b^4*x^5+8*a^3*
b^5*x^7+28/9*a^2*b^6*x^9+8/11*a*b^7*x^11+1/13*b^8*x^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \frac{99\,b^8x^{16} + 936\,ab^7x^{14} + 4004\,a^2b^6x^{12} + 10296\,a^3b^5x^{10} + 18018\,a^4b^4x^8 + 24024\,a^5b^3x^6 + 36036\,a^6b^2x^4 - 1}{1287\,x^3}$$

input
```
integrate((b*x^2+a)^8/x^4,x, algorithm="fricas")
```

output
```
1/1287*(99*b^8*x^16 + 936*a*b^7*x^14 + 4004*a^2*b^6*x^12 + 10296*a^3*b^5*x
^10 + 18018*a^4*b^4*x^8 + 24024*a^5*b^3*x^6 + 36036*a^6*b^2*x^4 - 10296*a^
7*b*x^2 - 429*a^8)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = 28a^6b^2x + \frac{56a^5b^3x^3}{3} + 14a^4b^4x^5 + 8a^3b^5x^7 + \frac{28a^2b^6x^9}{9} + \frac{8ab^7x^{11}}{11} + \frac{b^8x^{13}}{13} + \frac{-a^8 - 24a^7bx^2}{3x^3}$$

input
```
integrate((b*x**2+a)**8/x**4,x)
```

output
```
28*a**6*b**2*x + 56*a**5*b**3*x**3/3 + 14*a**4*b**4*x**5 + 8*a**3*b**5*x**
7 + 28*a**2*b**6*x**9/9 + 8*a*b**7*x**11/11 + b**8*x**13/13 + (-a**8 - 24*
a**7*b*x**2)/(3*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \frac{1}{13}\,b^8x^{13} + \frac{8}{11}\,ab^7x^{11} + \frac{28}{9}\,a^2b^6x^9 + 8\,a^3b^5x^7 + 14\,a^4b^4x^5 + \frac{56}{3}\,a^5b^3x^3 + 28\,a^6b^2x - \frac{24\,a^7bx^2 + a^8}{3\,x^3}$$

input
```
integrate((b*x^2+a)^8/x^4,x, algorithm="maxima")
```

output
```
1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^
4*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \frac{1}{13}\,b^8x^{13} + \frac{8}{11}\,ab^7x^{11} + \frac{28}{9}\,a^2b^6x^9 + 8\,a^3b^5x^7 + 14\,a^4b^4x^5 + \frac{56}{3}\,a^5b^3x^3 + 28\,a^6b^2x - \frac{24\,a^7bx^2 + a^8}{3\,x^3}$$

input
```
integrate((b*x^2+a)^8/x^4,x, algorithm="giac")
```

output
```
1/13*b^8*x^13 + 8/11*a*b^7*x^11 + 28/9*a^2*b^6*x^9 + 8*a^3*b^5*x^7 + 14*a^
4*b^4*x^5 + 56/3*a^5*b^3*x^3 + 28*a^6*b^2*x - 1/3*(24*a^7*b*x^2 + a^8)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \frac{b^8\,x^{13}}{13} - \frac{\frac{a^8}{3} + 8\,b\,a^7\,x^2}{x^3} + 28\,a^6\,b^2\,x + \frac{8\,a\,b^7\,x^{11}}{11} + \frac{56\,a^5\,b^3\,x^3}{3} + 14\,a^4\,b^4\,x^5 + 8\,a^3\,b^5\,x^7 + \frac{28\,a^2\,b^6\,x^9}{9}$$

input
```
int((a + b*x^2)^8/x^4,x)
```

output
```
(b^8*x^13)/13 - (a^8/3 + 8*a^7*b*x^2)/x^3 + 28*a^6*b^2*x + (8*a*b^7*x^11)/
11 + (56*a^5*b^3*x^3)/3 + 14*a^4*b^4*x^5 + 8*a^3*b^5*x^7 + (28*a^2*b^6*x^9
)/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^4}\,dx = \frac{99b^8x^{16} + 936a\,b^7x^{14} + 4004a^2b^6x^{12} + 10296a^3b^5x^{10} + 18018a^4b^4x^8 + 24024a^5b^3x^6 + 36036a^6b^2x^4 - 102}{1287x^3}$$

input
```
int((b*x^2+a)^8/x^4,x)
```

output
```
( - 429*a**8 - 10296*a**7*b*x**2 + 36036*a**6*b**2*x**4 + 24024*a**5*b**3*
x**6 + 18018*a**4*b**4*x**8 + 10296*a**3*b**5*x**10 + 4004*a**2*b**6*x**12
 + 936*a*b**7*x**14 + 99*b**8*x**16)/(1287*x**3)
```

3.116 $\int \frac{(a+bx^2)^8}{x^6} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^2)^8}{x^6} dx = -\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70}{3}a^4b^4x^3 + \frac{56}{5}a^3b^5x^5 + 4a^2b^6x^7 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{11}}{11}$$

output
```
-1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5
*a^3*b^5*x^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^6} dx = -\frac{a^8}{5x^5} - \frac{8a^7b}{3x^3} - \frac{28a^6b^2}{x} + 56a^5b^3x + \frac{70}{3}a^4b^4x^3 + \frac{56}{5}a^3b^5x^5 + 4a^2b^6x^7 + \frac{8}{9}ab^7x^9 + \frac{b^8x^{11}}{11}$$

input
```
Integrate[(a + b*x^2)^8/x^6,x]
```

output
```
-1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4
*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*
x^11)/11
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^6}\,dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a^8}{x^6}+\frac{8a^7b}{x^4}+\frac{28a^6b^2}{x^2}+56a^5b^3+70a^4b^4x^2+56a^3b^5x^4+28a^2b^6x^6+8ab^7x^8+b^8x^{10}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70}{3}a^4b^4x^3+\frac{56}{5}a^3b^5x^5+4a^2b^6x^7+\frac{8}{9}ab^7x^9+\frac{b^8x^{11}}{11}$$

input
```
Int[(a + b*x^2)^8/x^6,x]
```

output
```
-1/5*a^8/x^5 - (8*a^7*b)/(3*x^3) - (28*a^6*b^2)/x + 56*a^5*b^3*x + (70*a^4
*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^6*x^7 + (8*a*b^7*x^9)/9 + (b^8*
x^11)/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result	si
default	$-\frac{a^8}{5x^5}-\frac{8a^7b}{3x^3}-\frac{28a^6b^2}{x}+56a^5b^3x+\frac{70a^4b^4x^3}{3}+\frac{56a^3x^5b^5}{5}+4a^2b^6x^7+\frac{8a\,b^7x^9}{9}+\frac{b^8x^{11}}{11}$	8
risch	$\frac{b^8x^{11}}{11}+\frac{8a\,b^7x^9}{9}+4a^2b^6x^7+\frac{56a^3x^5b^5}{5}+\frac{70a^4b^4x^3}{3}+56a^5b^3x+\frac{-28a^6b^2x^4-\frac{8}{3}a^7b\,x^2-\frac{1}{5}a^8}{x^5}$	9
norman	$\frac{-\frac{1}{5}a^8-\frac{8}{3}a^7b\,x^2-28a^6b^2x^4+56a^5b^3x^6+\frac{70}{3}a^4b^4x^8+\frac{56}{5}a^3b^5x^{10}+4a^2b^6x^{12}+\frac{8}{9}a\,b^7x^{14}+\frac{1}{11}b^8x^{16}}{x^5}$	9
gosper	$-\frac{-45b^8x^{16}-440a\,b^7x^{14}-1980a^2b^6x^{12}-5544a^3b^5x^{10}-11550a^4b^4x^8-27720a^5b^3x^6+13860a^6b^2x^4+1320a^7b\,x^2+99a^8}{495x^5}$	9
parallelrisch	$\frac{45b^8x^{16}+440a\,b^7x^{14}+1980a^2b^6x^{12}+5544a^3b^5x^{10}+11550a^4b^4x^8+27720a^5b^3x^6-13860a^6b^2x^4-1320a^7b\,x^2-99a^8}{495x^5}$	9
orering	$-\frac{-45b^8x^{16}-440a\,b^7x^{14}-1980a^2b^6x^{12}-5544a^3b^5x^{10}-11550a^4b^4x^8-27720a^5b^3x^6+13860a^6b^2x^4+1320a^7b\,x^2+99a^8}{495x^5}$	9

input
```
int((b*x^2+a)^8/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a^8/x^5-8/3*a^7*b/x^3-28*a^6*b^2/x+56*a^5*b^3*x+70/3*a^4*b^4*x^3+56/5
*a^3*x^5*b^5+4*a^2*b^6*x^7+8/9*a*b^7*x^9+1/11*b^8*x^11
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = \frac{45\,b^8x^{16}+440\,ab^7x^{14}+1980\,a^2b^6x^{12}+5544\,a^3b^5x^{10}+11550\,a^4b^4x^8+27720\,a^5b^3x^6-13860\,a^6b^2x^4-13}{495\,x^5}$$

input
```
integrate((b*x^2+a)^8/x^6,x, algorithm="fricas")
```

output
```
1/495*(45*b^8*x^16 + 440*a*b^7*x^14 + 1980*a^2*b^6*x^12 + 5544*a^3*b^5*x^1
0 + 11550*a^4*b^4*x^8 + 27720*a^5*b^3*x^6 - 13860*a^6*b^2*x^4 - 1320*a^7*b
*x^2 - 99*a^8)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = 56a^5b^3x + \frac{70a^4b^4x^3}{3} + \frac{56a^3b^5x^5}{5} + 4a^2b^6x^7 + \frac{8ab^7x^9}{9} + \frac{b^8x^{11}}{11} + \frac{-3a^8-40a^7bx^2-420a^6b^2x^4}{15x^5}$$

input
```
integrate((b*x**2+a)**8/x**6,x)
```

output
```
56*a**5*b**3*x + 70*a**4*b**4*x**3/3 + 56*a**3*b**5*x**5/5 + 4*a**2*b**6*x
**7 + 8*a*b**7*x**9/9 + b**8*x**11/11 + (-3*a**8 - 40*a**7*b*x**2 - 420*a*
*6*b**2*x**4)/(15*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = \frac{1}{11}\,b^8x^{11} + \frac{8}{9}\,ab^7x^9 + 4\,a^2b^6x^7 + \frac{56}{5}\,a^3b^5x^5 + \frac{70}{3}\,a^4b^4x^3 + 56\,a^5b^3x - \frac{420\,a^6b^2x^4 + 40\,a^7bx^2 + 3\,a^8}{15\,x^5}$$

input
```
integrate((b*x^2+a)^8/x^6,x, algorithm="maxima")
```

output
```
1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^
4*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x
^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = \frac{1}{11}\,b^8x^{11} + \frac{8}{9}\,ab^7x^9 + 4\,a^2b^6x^7 + \frac{56}{5}\,a^3b^5x^5 + \frac{70}{3}\,a^4b^4x^3 + 56\,a^5b^3x - \frac{420\,a^6b^2x^4 + 40\,a^7bx^2 + 3\,a^8}{15\,x^5}$$

input
```
integrate((b*x^2+a)^8/x^6,x, algorithm="giac")
```

output
```
1/11*b^8*x^11 + 8/9*a*b^7*x^9 + 4*a^2*b^6*x^7 + 56/5*a^3*b^5*x^5 + 70/3*a^
4*b^4*x^3 + 56*a^5*b^3*x - 1/15*(420*a^6*b^2*x^4 + 40*a^7*b*x^2 + 3*a^8)/x
^5
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = \frac{b^8\,x^{11}}{11} - \frac{\frac{a^8}{5} + \frac{8\,a^7\,b\,x^2}{3} + 28\,a^6\,b^2\,x^4}{x^5} + 56\,a^5\,b^3\,x + \frac{8\,a\,b^7\,x^9}{9} + \frac{70\,a^4\,b^4\,x^3}{3} + \frac{56\,a^3\,b^5\,x^5}{5} + 4\,a^2\,b^6\,x^7$$

input
```
int((a + b*x^2)^8/x^6,x)
```

output
```
(b^8*x^11)/11 - (a^8/5 + (8*a^7*b*x^2)/3 + 28*a^6*b^2*x^4)/x^5 + 56*a^5*b^
3*x + (8*a*b^7*x^9)/9 + (70*a^4*b^4*x^3)/3 + (56*a^3*b^5*x^5)/5 + 4*a^2*b^
6*x^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^6}\,dx = \frac{45b^8x^{16} + 440a\,b^7x^{14} + 1980a^2b^6x^{12} + 5544a^3b^5x^{10} + 11550a^4b^4x^8 + 27720a^5b^3x^6 - 13860a^6b^2x^4 - 1320}{495x^5}$$

input
```
int((b*x^2+a)^8/x^6,x)
```

output
```
( - 99*a**8 - 1320*a**7*b*x**2 - 13860*a**6*b**2*x**4 + 27720*a**5*b**3*x*
*6 + 11550*a**4*b**4*x**8 + 5544*a**3*b**5*x**10 + 1980*a**2*b**6*x**12 +
440*a*b**7*x**14 + 45*b**8*x**16)/(495*x**5)
```

3.117 $\int \frac{(a+bx^2)^8}{x^8} dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{(a+bx^2)^8}{x^8} dx = -\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56}{3}a^3b^5x^3 + \frac{28}{5}a^2b^6x^5 + \frac{8}{7}ab^7x^7 + \frac{b^8x^9}{9}$$

output
```
-1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3
*a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*b^7*x^7+1/9*b^8*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^8} dx = -\frac{a^8}{7x^7} - \frac{8a^7b}{5x^5} - \frac{28a^6b^2}{3x^3} - \frac{56a^5b^3}{x} + 70a^4b^4x + \frac{56}{3}a^3b^5x^3 + \frac{28}{5}a^2b^6x^5 + \frac{8}{7}ab^7x^7 + \frac{b^8x^9}{9}$$

input
```
Integrate[(a + b*x^2)^8/x^8,x]
```

output
```
-1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
 70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7
+ (b^8*x^9)/9
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^8}\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^8}{x^8}+\frac{8a^7b}{x^6}+\frac{28a^6b^2}{x^4}+\frac{56a^5b^3}{x^2}+70a^4b^4+56a^3b^5x^2+28a^2b^6x^4+8ab^7x^6+b^8x^8\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^8}{7x^7}-\frac{8a^7b}{5x^5}-\frac{28a^6b^2}{3x^3}-\frac{56a^5b^3}{x}+70a^4b^4x+\frac{56}{3}a^3b^5x^3+\frac{28}{5}a^2b^6x^5+\frac{8}{7}ab^7x^7+\frac{b^8x^9}{9}$$

input
```
Int[(a + b*x^2)^8/x^8,x]
```

output

```
-1/7*a^8/x^7 - (8*a^7*b)/(5*x^5) - (28*a^6*b^2)/(3*x^3) - (56*a^5*b^3)/x +
 70*a^4*b^4*x + (56*a^3*b^5*x^3)/3 + (28*a^2*b^6*x^5)/5 + (8*a*b^7*x^7)/7
+ (b^8*x^9)/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^{8}}{7x^{7}}-\frac{8a^{7}b}{5x^{5}}-\frac{28a^{6}b^{2}}{3x^{3}}-\frac{56a^{5}b^{3}}{x}+70a^{4}b^{4}x+\frac{56a^{3}b^{5}x^{3}}{3}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{8a\,x^{7}b^{7}}{7}+\frac{b^{8}x^{9}}{9}$	89
risch	$\frac{b^{8}x^{9}}{9}+\frac{8a\,x^{7}b^{7}}{7}+\frac{28a^{2}b^{6}x^{5}}{5}+\frac{56a^{3}b^{5}x^{3}}{3}+70a^{4}b^{4}x+\frac{-56a^{5}b^{3}x^{6}-\frac{28}{3}a^{6}b^{2}x^{4}-\frac{8}{5}a^{7}b\,x^{2}-\frac{1}{7}a^{8}}{x^{7}}$	91
norman	$\frac{-\frac{1}{7}a^{8}-\frac{8}{5}a^{7}b\,x^{2}-\frac{28}{3}a^{6}b^{2}x^{4}-56a^{5}b^{3}x^{6}+70a^{4}b^{4}x^{8}+\frac{56}{3}a^{3}b^{5}x^{10}+\frac{28}{5}a^{2}b^{6}x^{12}+\frac{8}{7}a\,b^{7}x^{14}+\frac{1}{9}b^{8}x^{16}}{x^{7}}$	92
gosper	$-\frac{-35b^{8}x^{16}-360a\,b^{7}x^{14}-1764a^{2}b^{6}x^{12}-5880a^{3}b^{5}x^{10}-22050a^{4}b^{4}x^{8}+17640a^{5}b^{3}x^{6}+2940a^{6}b^{2}x^{4}+504a^{7}b\,x^{2}+45a^{8}}{315x^{7}}$	93
parallelrisch	$\frac{35b^{8}x^{16}+360a\,b^{7}x^{14}+1764a^{2}b^{6}x^{12}+5880a^{3}b^{5}x^{10}+22050a^{4}b^{4}x^{8}-17640a^{5}b^{3}x^{6}-2940a^{6}b^{2}x^{4}-504a^{7}b\,x^{2}-45a^{8}}{315x^{7}}$	93
orering	$-\frac{-35b^{8}x^{16}-360a\,b^{7}x^{14}-1764a^{2}b^{6}x^{12}-5880a^{3}b^{5}x^{10}-22050a^{4}b^{4}x^{8}+17640a^{5}b^{3}x^{6}+2940a^{6}b^{2}x^{4}+504a^{7}b\,x^{2}+45a^{8}}{315x^{7}}$	93

input
```
int((b*x^2+a)^8/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*a^8/x^7-8/5*a^7*b/x^5-28/3*a^6*b^2/x^3-56*a^5*b^3/x+70*a^4*b^4*x+56/3
*a^3*b^5*x^3+28/5*a^2*b^6*x^5+8/7*a*x^7*b^7+1/9*b^8*x^9
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = \frac{35\,b^8x^{16}+360\,ab^7x^{14}+1764\,a^2b^6x^{12}+5880\,a^3b^5x^{10}+22050\,a^4b^4x^8-17640\,a^5b^3x^6-2940\,a^6b^2x^4-504}{315\,x^7}$$

input
```
integrate((b*x^2+a)^8/x^8,x, algorithm="fricas")
```

output
```
1/315*(35*b^8*x^16 + 360*a*b^7*x^14 + 1764*a^2*b^6*x^12 + 5880*a^3*b^5*x^1
0 + 22050*a^4*b^4*x^8 - 17640*a^5*b^3*x^6 - 2940*a^6*b^2*x^4 - 504*a^7*b*x
^2 - 45*a^8)/x^7
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = 70a^4b^4x+\frac{56a^3b^5x^3}{3}+\frac{28a^2b^6x^5}{5}+\frac{8ab^7x^7}{7}+\frac{b^8x^9}{9} + \frac{-15a^8-168a^7bx^2-980a^6b^2x^4-5880a^5b^3x^6}{105x^7}$$

input
```
integrate((b*x**2+a)**8/x**8,x)
```

output
```
70*a**4*b**4*x + 56*a**3*b**5*x**3/3 + 28*a**2*b**6*x**5/5 + 8*a*b**7*x**7
/7 + b**8*x**9/9 + (-15*a**8 - 168*a**7*b*x**2 - 980*a**6*b**2*x**4 - 5880
*a**5*b**3*x**6)/(105*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = \frac{1}{9}\,b^8x^9 + \frac{8}{7}\,ab^7x^7 + \frac{28}{5}\,a^2b^6x^5 + \frac{56}{3}\,a^3b^5x^3 + 70\,a^4b^4x - \frac{5880\,a^5b^3x^6 + 980\,a^6b^2x^4 + 168\,a^7bx^2 + 15\,a^8}{105\,x^7}$$

input
```
integrate((b*x^2+a)^8/x^8,x, algorithm="maxima")
```

output
```
1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4
*b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^
8)/x^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = \frac{1}{9}\,b^8x^9 + \frac{8}{7}\,ab^7x^7 + \frac{28}{5}\,a^2b^6x^5 + \frac{56}{3}\,a^3b^5x^3 + 70\,a^4b^4x - \frac{5880\,a^5b^3x^6 + 980\,a^6b^2x^4 + 168\,a^7bx^2 + 15\,a^8}{105\,x^7}$$

input
```
integrate((b*x^2+a)^8/x^8,x, algorithm="giac")
```

output
```
1/9*b^8*x^9 + 8/7*a*b^7*x^7 + 28/5*a^2*b^6*x^5 + 56/3*a^3*b^5*x^3 + 70*a^4
*b^4*x - 1/105*(5880*a^5*b^3*x^6 + 980*a^6*b^2*x^4 + 168*a^7*b*x^2 + 15*a^
8)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = \frac{b^8\,x^9}{9} - \frac{\frac{a^8}{7} + \frac{8\,a^7\,b\,x^2}{5} + \frac{28\,a^6\,b^2\,x^4}{3} + 56\,a^5\,b^3\,x^6}{x^7} + 70\,a^4\,b^4\,x + \frac{8\,a\,b^7\,x^7}{7} + \frac{56\,a^3\,b^5\,x^3}{3} + \frac{28\,a^2\,b^6\,x^5}{5}$$

input
```
int((a + b*x^2)^8/x^8,x)
```

output
```
(b^8*x^9)/9 - (a^8/7 + (8*a^7*b*x^2)/5 + (28*a^6*b^2*x^4)/3 + 56*a^5*b^3*x
^6)/x^7 + 70*a^4*b^4*x + (8*a*b^7*x^7)/7 + (56*a^3*b^5*x^3)/3 + (28*a^2*b^
6*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^8}{x^8}\,dx = \frac{35b^8x^{16} + 360a\,b^7x^{14} + 1764a^2b^6x^{12} + 5880a^3b^5x^{10} + 22050a^4b^4x^8 - 17640a^5b^3x^6 - 2940a^6b^2x^4 - 504a^7}{315x^7}$$

input
```
int((b*x^2+a)^8/x^8,x)
```

output
```
( - 45*a**8 - 504*a**7*b*x**2 - 2940*a**6*b**2*x**4 - 17640*a**5*b**3*x**6
 + 22050*a**4*b**4*x**8 + 5880*a**3*b**5*x**10 + 1764*a**2*b**6*x**12 + 36
0*a*b**7*x**14 + 35*b**8*x**16)/(315*x**7)
```

3.118 $\int \frac{(a+bx^2)^8}{x^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 102

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

output
```
-1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+
56*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = -\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

input
```
Integrate[(a + b*x^2)^8/x^10,x]
```

output
```
-1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*
x^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/
5 + (b^8*x^7)/7
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{10}} \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^8}{x^{10}}+\frac{8a^7b}{x^8}+\frac{28a^6b^2}{x^6}+\frac{56a^5b^3}{x^4}+\frac{70a^4b^4}{x^2}+56a^3b^5+28a^2b^6x^2+8ab^7x^4+b^8x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28}{3}a^2b^6x^3+\frac{8}{5}ab^7x^5+\frac{b^8x^7}{7}$$

input
```
Int[(a + b*x^2)^8/x^10,x]
```

output
```
-1/9*a^8/x^9 - (8*a^7*b)/(7*x^7) - (28*a^6*b^2)/(5*x^5) - (56*a^5*b^3)/(3*
x^3) - (70*a^4*b^4)/x + 56*a^3*b^5*x + (28*a^2*b^6*x^3)/3 + (8*a*b^7*x^5)/
5 + (b^8*x^7)/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{a^8}{9x^9}-\frac{8a^7b}{7x^7}-\frac{28a^6b^2}{5x^5}-\frac{56a^5b^3}{3x^3}-\frac{70a^4b^4}{x}+56a^3b^5x+\frac{28a^2b^6x^3}{3}+\frac{8a\,b^7x^5}{5}+\frac{b^8x^7}{7}$	89
risch	$\frac{b^8x^7}{7}+\frac{8a\,b^7x^5}{5}+\frac{28a^2b^6x^3}{3}+56a^3b^5x+\frac{-70a^4b^4x^8-\frac{56}{3}a^5b^3x^6-\frac{28}{5}a^6b^2x^4-\frac{8}{7}a^7b\,x^2-\frac{1}{9}a^8}{x^9}$	91
norman	$\frac{-\frac{1}{9}a^8-\frac{8}{7}a^7b\,x^2-\frac{28}{5}a^6b^2x^4-\frac{56}{3}a^5b^3x^6-70a^4b^4x^8+56a^3b^5x^{10}+\frac{28}{3}a^2b^6x^{12}+\frac{8}{5}a\,b^7x^{14}+\frac{1}{7}b^8x^{16}}{x^9}$	92
gosper	$-\frac{-45b^8x^{16}-504a\,b^7x^{14}-2940a^2b^6x^{12}-17640a^3b^5x^{10}+22050a^4b^4x^8+5880a^5b^3x^6+1764a^6b^2x^4+360a^7b\,x^2+35a^8}{315x^9}$	93
parallelrisch	$\frac{45b^8x^{16}+504a\,b^7x^{14}+2940a^2b^6x^{12}+17640a^3b^5x^{10}-22050a^4b^4x^8-5880a^5b^3x^6-1764a^6b^2x^4-360a^7b\,x^2-35a^8}{315x^9}$	93
orering	$-\frac{-45b^8x^{16}-504a\,b^7x^{14}-2940a^2b^6x^{12}-17640a^3b^5x^{10}+22050a^4b^4x^8+5880a^5b^3x^6+1764a^6b^2x^4+360a^7b\,x^2+35a^8}{315x^9}$	93

input
```
int((b*x^2+a)^8/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^8/x^9-8/7*a^7*b/x^7-28/5*a^6*b^2/x^5-56/3*a^5*b^3/x^3-70*a^4*b^4/x+
56*a^3*b^5*x+28/3*a^2*b^6*x^3+8/5*a*b^7*x^5+1/7*b^8*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = \frac{45\,b^8x^{16} + 504\,ab^7x^{14} + 2940\,a^2b^6x^{12} + 17640\,a^3b^5x^{10} - 22050\,a^4b^4x^8 - 5880\,a^5b^3x^6 - 1764\,a^6b^2x^4 - 360}{315\,x^9}$$

input
```
integrate((b*x^2+a)^8/x^10,x, algorithm="fricas")
```

output
```
1/315*(45*b^8*x^16 + 504*a*b^7*x^14 + 2940*a^2*b^6*x^12 + 17640*a^3*b^5*x^
10 - 22050*a^4*b^4*x^8 - 5880*a^5*b^3*x^6 - 1764*a^6*b^2*x^4 - 360*a^7*b*x
^2 - 35*a^8)/x^9
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = 56a^3b^5x + \frac{28a^2b^6x^3}{3} + \frac{8ab^7x^5}{5} + \frac{b^8x^7}{7} + \frac{-35a^8 - 360a^7bx^2 - 1764a^6b^2x^4 - 5880a^5b^3x^6 - 22050a^4b^4x^8}{315x^9}$$

input
```
integrate((b*x**2+a)**8/x**10,x)
```

output
```
56*a**3*b**5*x + 28*a**2*b**6*x**3/3 + 8*a*b**7*x**5/5 + b**8*x**7/7 + (-3
5*a**8 - 360*a**7*b*x**2 - 1764*a**6*b**2*x**4 - 5880*a**5*b**3*x**6 - 220
50*a**4*b**4*x**8)/(315*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = \frac{1}{7}\,b^8x^7 + \frac{8}{5}\,ab^7x^5 + \frac{28}{3}\,a^2b^6x^3 + 56\,a^3b^5x - \frac{22050\,a^4b^4x^8 + 5880\,a^5b^3x^6 + 1764\,a^6b^2x^4 + 360\,a^7bx^2 + 35\,a^8}{315\,x^9}$$

input
```
integrate((b*x^2+a)^8/x^10,x, algorithm="maxima")
```

output
```
1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(220
50*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*
a^8)/x^9
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = \frac{1}{7}\,b^8x^7 + \frac{8}{5}\,ab^7x^5 + \frac{28}{3}\,a^2b^6x^3 + 56\,a^3b^5x - \frac{22050\,a^4b^4x^8 + 5880\,a^5b^3x^6 + 1764\,a^6b^2x^4 + 360\,a^7bx^2 + 35\,a^8}{315\,x^9}$$

input
```
integrate((b*x^2+a)^8/x^10,x, algorithm="giac")
```

output
```
1/7*b^8*x^7 + 8/5*a*b^7*x^5 + 28/3*a^2*b^6*x^3 + 56*a^3*b^5*x - 1/315*(220
50*a^4*b^4*x^8 + 5880*a^5*b^3*x^6 + 1764*a^6*b^2*x^4 + 360*a^7*b*x^2 + 35*
a^8)/x^9
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = \frac{b^8\,x^7}{7} - \frac{\frac{a^8}{9} + \frac{8\,a^7\,b\,x^2}{7} + \frac{28\,a^6\,b^2\,x^4}{5} + \frac{56\,a^5\,b^3\,x^6}{3} + 70\,a^4\,b^4\,x^8}{x^9} + 56\,a^3\,b^5\,x + \frac{8\,a\,b^7\,x^5}{5} + \frac{28\,a^2\,b^6\,x^3}{3}$$

input

```
int((a + b*x^2)^8/x^10,x)
```

output

```
(b^8*x^7)/7 - (a^8/9 + (8*a^7*b*x^2)/7 + (28*a^6*b^2*x^4)/5 + (56*a^5*b^3*
x^6)/3 + 70*a^4*b^4*x^8)/x^9 + 56*a^3*b^5*x + (8*a*b^7*x^5)/5 + (28*a^2*b^
6*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^8}{x^{10}}\,dx = \frac{45b^8x^{16} + 504a\,b^7x^{14} + 2940a^2b^6x^{12} + 17640a^3b^5x^{10} - 22050a^4b^4x^8 - 5880a^5b^3x^6 - 1764a^6b^2x^4 - 360a^7}{315x^9}$$

input

```
int((b*x^2+a)^8/x^10,x)
```

output

```
( - 35*a**8 - 360*a**7*b*x**2 - 1764*a**6*b**2*x**4 - 5880*a**5*b**3*x**6
- 22050*a**4*b**4*x**8 + 17640*a**3*b**5*x**10 + 2940*a**2*b**6*x**12 + 50
4*a*b**7*x**14 + 45*b**8*x**16)/(315*x**9)
```

3.119 $\int \frac{(a+bx^2)^8}{x^{12}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = -\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8}{3}ab^7x^3 + \frac{b^8x^5}{5}$$

output
```
-1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x
^3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = -\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8}{3}ab^7x^3 + \frac{b^8x^5}{5}$$

input
```
Integrate[(a + b*x^2)^8/x^12,x]
```

output
```
-1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5
) - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3
 + (b^8*x^5)/5
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^8}{x^{12}}+\frac{8a^7b}{x^{10}}+\frac{28a^6b^2}{x^8}+\frac{56a^5b^3}{x^6}+\frac{70a^4b^4}{x^4}+\frac{56a^3b^5}{x^2}+28a^2b^6+8ab^7x^2+b^8x^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{11x^{11}}-\frac{8a^7b}{9x^9}-\frac{4a^6b^2}{x^7}-\frac{56a^5b^3}{5x^5}-\frac{70a^4b^4}{3x^3}-\frac{56a^3b^5}{x}+28a^2b^6x+\frac{8}{3}ab^7x^3+\frac{b^8x^5}{5}$$

input
```
Int[(a + b*x^2)^8/x^12,x]
```

output
```
-1/11*a^8/x^11 - (8*a^7*b)/(9*x^9) - (4*a^6*b^2)/x^7 - (56*a^5*b^3)/(5*x^5
) - (70*a^4*b^4)/(3*x^3) - (56*a^3*b^5)/x + 28*a^2*b^6*x + (8*a*b^7*x^3)/3
 + (b^8*x^5)/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

method	result	si
default	$-\frac{a^8}{11x^{11}} - \frac{8a^7b}{9x^9} - \frac{4a^6b^2}{x^7} - \frac{56a^5b^3}{5x^5} - \frac{70a^4b^4}{3x^3} - \frac{56a^3b^5}{x} + 28a^2b^6x + \frac{8a\,b^7x^3}{3} + \frac{b^8x^5}{5}$	8
risch	$\frac{b^8x^5}{5} + \frac{8a\,b^7x^3}{3} + 28a^2b^6x + \frac{-56a^3b^5x^{10} - \frac{70}{3}a^4b^4x^8 - \frac{56}{5}a^5b^3x^6 - 4a^6b^2x^4 - \frac{8}{9}a^7b\,x^2 - \frac{1}{11}a^8}{x^{11}}$	9
norman	$\frac{-\frac{1}{11}a^8 - \frac{8}{9}a^7b\,x^2 - 4a^6b^2x^4 - \frac{56}{5}a^5b^3x^6 - \frac{70}{3}a^4b^4x^8 - 56a^3b^5x^{10} + 28a^2b^6x^{12} + \frac{8}{3}a\,b^7x^{14} + \frac{1}{5}b^8x^{16}}{x^{11}}$	9
gosper	$-\frac{-99b^8x^{16} - 1320a\,b^7x^{14} - 13860a^2b^6x^{12} + 27720a^3b^5x^{10} + 11550a^4b^4x^8 + 5544a^5b^3x^6 + 1980a^6b^2x^4 + 440a^7b\,x^2 + 45a^8}{495x^{11}}$	9
parallelrisch	$\frac{99b^8x^{16} + 1320a\,b^7x^{14} + 13860a^2b^6x^{12} - 27720a^3b^5x^{10} - 11550a^4b^4x^8 - 5544a^5b^3x^6 - 1980a^6b^2x^4 - 440a^7b\,x^2 - 45a^8}{495x^{11}}$	9
orering	$-\frac{-99b^8x^{16} - 1320a\,b^7x^{14} - 13860a^2b^6x^{12} + 27720a^3b^5x^{10} + 11550a^4b^4x^8 + 5544a^5b^3x^6 + 1980a^6b^2x^4 + 440a^7b\,x^2 + 45a^8}{495x^{11}}$	9

input
```
int((b*x^2+a)^8/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*a^8/x^11-8/9*a^7*b/x^9-4*a^6*b^2/x^7-56/5*a^5*b^3/x^5-70/3*a^4*b^4/x
^3-56*a^3*b^5/x+28*a^2*b^6*x+8/3*a*b^7*x^3+1/5*b^8*x^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = \frac{99\,b^8x^{16}+1320\,ab^7x^{14}+13860\,a^2b^6x^{12}-27720\,a^3b^5x^{10}-11550\,a^4b^4x^8-5544\,a^5b^3x^6-1980\,a^6b^2x^4-4}{495\,x^{11}}$$

input
```
integrate((b*x^2+a)^8/x^12,x, algorithm="fricas")
```

output
```
1/495*(99*b^8*x^16 + 1320*a*b^7*x^14 + 13860*a^2*b^6*x^12 - 27720*a^3*b^5*
x^10 - 11550*a^4*b^4*x^8 - 5544*a^5*b^3*x^6 - 1980*a^6*b^2*x^4 - 440*a^7*b
*x^2 - 45*a^8)/x^11
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = 28a^2b^6x+\frac{8ab^7x^3}{3}+\frac{b^8x^5}{5} + \frac{-45a^8-440a^7bx^2-1980a^6b^2x^4-5544a^5b^3x^6-11550a^4b^4x^8-27720a^3b^5x^{10}}{495x^{11}}$$

input
```
integrate((b*x**2+a)**8/x**12,x)
```

output
```
28*a**2*b**6*x + 8*a*b**7*x**3/3 + b**8*x**5/5 + (-45*a**8 - 440*a**7*b*x*
*2 - 1980*a**6*b**2*x**4 - 5544*a**5*b**3*x**6 - 11550*a**4*b**4*x**8 - 27
720*a**3*b**5*x**10)/(495*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = \frac{1}{5}\,b^8x^5 + \frac{8}{3}\,ab^7x^3 + 28\,a^2b^6x - \frac{27720\,a^3b^5x^{10} + 11550\,a^4b^4x^8 + 5544\,a^5b^3x^6 + 1980\,a^6b^2x^4 + 440\,a^7bx^2 + 45\,a^8}{495\,x^{11}}$$

input
```
integrate((b*x^2+a)^8/x^12,x, algorithm="maxima")
```

output
```
1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 1
1550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 4
5*a^8)/x^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = \frac{1}{5}\,b^8x^5 + \frac{8}{3}\,ab^7x^3 + 28\,a^2b^6x - \frac{27720\,a^3b^5x^{10} + 11550\,a^4b^4x^8 + 5544\,a^5b^3x^6 + 1980\,a^6b^2x^4 + 440\,a^7bx^2 + 45\,a^8}{495\,x^{11}}$$

input
```
integrate((b*x^2+a)^8/x^12,x, algorithm="giac")
```

output
```
1/5*b^8*x^5 + 8/3*a*b^7*x^3 + 28*a^2*b^6*x - 1/495*(27720*a^3*b^5*x^10 + 1
1550*a^4*b^4*x^8 + 5544*a^5*b^3*x^6 + 1980*a^6*b^2*x^4 + 440*a^7*b*x^2 + 4
5*a^8)/x^11
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = \frac{b^8\,x^5}{5} - \frac{\frac{a^8}{11} + \frac{8\,a^7\,b\,x^2}{9} + 4\,a^6\,b^2\,x^4 + \frac{56\,a^5\,b^3\,x^6}{5} + \frac{70\,a^4\,b^4\,x^8}{3} + 56\,a^3\,b^5\,x^{10}}{x^{11}} + 28\,a^2\,b^6\,x + \frac{8\,a\,b^7\,x^3}{3}$$

input
```
int((a + b*x^2)^8/x^12,x)
```

output
```
(b^8*x^5)/5 - (a^8/11 + (8*a^7*b*x^2)/9 + 4*a^6*b^2*x^4 + (56*a^5*b^3*x^6)
/5 + (70*a^4*b^4*x^8)/3 + 56*a^3*b^5*x^10)/x^11 + 28*a^2*b^6*x + (8*a*b^7*
x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^8}{x^{12}}\,dx = \frac{99b^8x^{16} + 1320a\,b^7x^{14} + 13860a^2b^6x^{12} - 27720a^3b^5x^{10} - 11550a^4b^4x^8 - 5544a^5b^3x^6 - 1980a^6b^2x^4 - 440}{495x^{11}}$$

input
```
int((b*x^2+a)^8/x^12,x)
```

output
```
( - 45*a**8 - 440*a**7*b*x**2 - 1980*a**6*b**2*x**4 - 5544*a**5*b**3*x**6
- 11550*a**4*b**4*x**8 - 27720*a**3*b**5*x**10 + 13860*a**2*b**6*x**12 + 1
320*a*b**7*x**14 + 99*b**8*x**16)/(495*x**11)
```

3.120 $\int \frac{(a+bx^2)^8}{x^{14}} dx$

Optimal result

Integrand size = 13, antiderivative size = 98

$$\int \frac{(a+bx^2)^8}{x^{14}} dx = -\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

output
```
-1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x
^5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{14}} dx = -\frac{a^8}{13x^{13}} - \frac{8a^7b}{11x^{11}} - \frac{28a^6b^2}{9x^9} - \frac{8a^5b^3}{x^7} - \frac{14a^4b^4}{x^5} - \frac{56a^3b^5}{3x^3} - \frac{28a^2b^6}{x} + 8ab^7x + \frac{b^8x^3}{3}$$

input
```
Integrate[(a + b*x^2)^8/x^14,x]
```

output
```
-1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/
x^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x
 + (b^8*x^3)/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^8}{x^{14}}+\frac{8a^7b}{x^{12}}+\frac{28a^6b^2}{x^{10}}+\frac{56a^5b^3}{x^8}+\frac{70a^4b^4}{x^6}+\frac{56a^3b^5}{x^4}+\frac{28a^2b^6}{x^2}+8ab^7+b^8x^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8ab^7x+\frac{b^8x^3}{3}$$

input
```
Int[(a + b*x^2)^8/x^14,x]
```

output

```
-1/13*a^8/x^13 - (8*a^7*b)/(11*x^11) - (28*a^6*b^2)/(9*x^9) - (8*a^5*b^3)/
x^7 - (14*a^4*b^4)/x^5 - (56*a^3*b^5)/(3*x^3) - (28*a^2*b^6)/x + 8*a*b^7*x
 + (b^8*x^3)/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.91

method	result
default	$-\frac{a^8}{13x^{13}}-\frac{8a^7b}{11x^{11}}-\frac{28a^6b^2}{9x^9}-\frac{8a^5b^3}{x^7}-\frac{14a^4b^4}{x^5}-\frac{56a^3b^5}{3x^3}-\frac{28a^2b^6}{x}+8a\,b^7x+\frac{b^8x^3}{3}$
risch	$\frac{b^8x^3}{3}+8a\,b^7x+\frac{-28a^2b^6x^{12}-\frac{56}{3}a^3b^5x^{10}-14a^4b^4x^8-8a^5b^3x^6-\frac{28}{9}a^6b^2x^4-\frac{8}{11}a^7b\,x^2-\frac{1}{13}a^8}{x^{13}}$
norman	$\frac{-\frac{1}{13}a^8-\frac{8}{11}a^7b\,x^2-\frac{28}{9}a^6b^2x^4-8a^5b^3x^6-14a^4b^4x^8-\frac{56}{3}a^3b^5x^{10}-28a^2b^6x^{12}+8a\,b^7x^{14}+\frac{1}{3}b^8x^{16}}{x^{13}}$
gosper	$-\frac{-429b^8x^{16}-10296a\,b^7x^{14}+36036a^2b^6x^{12}+24024a^3b^5x^{10}+18018a^4b^4x^8+10296a^5b^3x^6+4004a^6b^2x^4+936a^7b\,x^2+99a^8}{1287x^{13}}$
parallelrisch	$\frac{429b^8x^{16}+10296a\,b^7x^{14}-36036a^2b^6x^{12}-24024a^3b^5x^{10}-18018a^4b^4x^8-10296a^5b^3x^6-4004a^6b^2x^4-936a^7b\,x^2-99a^8}{1287x^{13}}$
orering	$-\frac{-429b^8x^{16}-10296a\,b^7x^{14}+36036a^2b^6x^{12}+24024a^3b^5x^{10}+18018a^4b^4x^8+10296a^5b^3x^6+4004a^6b^2x^4+936a^7b\,x^2+99a^8}{1287x^{13}}$

input
```
int((b*x^2+a)^8/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/13*a^8/x^13-8/11*a^7*b/x^11-28/9*a^6*b^2/x^9-8*a^5*b^3/x^7-14*a^4*b^4/x
^5-56/3*a^3*b^5/x^3-28*a^2*b^6/x+8*a*b^7*x+1/3*b^8*x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = \frac{429\,b^8x^{16}+10296\,ab^7x^{14}-36036\,a^2b^6x^{12}-24024\,a^3b^5x^{10}-18018\,a^4b^4x^8-10296\,a^5b^3x^6-4004\,a^6b^2x^4}{1287\,x^{13}}$$

input
```
integrate((b*x^2+a)^8/x^14,x, algorithm="fricas")
```

output
```
1/1287*(429*b^8*x^16 + 10296*a*b^7*x^14 - 36036*a^2*b^6*x^12 - 24024*a^3*b
^5*x^10 - 18018*a^4*b^4*x^8 - 10296*a^5*b^3*x^6 - 4004*a^6*b^2*x^4 - 936*a
^7*b*x^2 - 99*a^8)/x^13
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = 8ab^7x+\frac{b^8x^3}{3} + \frac{-99a^8-936a^7bx^2-4004a^6b^2x^4-10296a^5b^3x^6-18018a^4b^4x^8-24024a^3b^5x^{10}-36036a^2b^6x^{12}}{1287x^{13}}$$

input
```
integrate((b*x**2+a)**8/x**14,x)
```

output
```
8*a*b**7*x + b**8*x**3/3 + (-99*a**8 - 936*a**7*b*x**2 - 4004*a**6*b**2*x*
*4 - 10296*a**5*b**3*x**6 - 18018*a**4*b**4*x**8 - 24024*a**3*b**5*x**10 -
 36036*a**2*b**6*x**12)/(1287*x**13)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = \frac{1}{3}\,b^8x^3 + 8\,ab^7x - \frac{36036\,a^2b^6x^{12} + 24024\,a^3b^5x^{10} + 18018\,a^4b^4x^8 + 10296\,a^5b^3x^6 + 4004\,a^6b^2x^4 + 936\,a^7bx^2 + 99\,a^8}{1287\,x^{13}}$$

input
```
integrate((b*x^2+a)^8/x^14,x, algorithm="maxima")
```

output
```
1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10
+ 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2
 + 99*a^8)/x^13
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = \frac{1}{3}\,b^8x^3 + 8\,ab^7x - \frac{36036\,a^2b^6x^{12} + 24024\,a^3b^5x^{10} + 18018\,a^4b^4x^8 + 10296\,a^5b^3x^6 + 4004\,a^6b^2x^4 + 936\,a^7bx^2 + 99\,a^8}{1287\,x^{13}}$$

input
```
integrate((b*x^2+a)^8/x^14,x, algorithm="giac")
```

output
```
1/3*b^8*x^3 + 8*a*b^7*x - 1/1287*(36036*a^2*b^6*x^12 + 24024*a^3*b^5*x^10
+ 18018*a^4*b^4*x^8 + 10296*a^5*b^3*x^6 + 4004*a^6*b^2*x^4 + 936*a^7*b*x^2
 + 99*a^8)/x^13
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = -\frac{\frac{a^8}{13}+\frac{8\,a^7\,b\,x^2}{11}+\frac{28\,a^6\,b^2\,x^4}{9}+8\,a^5\,b^3\,x^6+14\,a^4\,b^4\,x^8+\frac{56\,a^3\,b^5\,x^{10}}{3}+28\,a^2\,b^6\,x^{12}-8\,a\,b^7\,x^{14}-\frac{b^8\,x^{16}}{3}}{x^{13}}$$

input
```
int((a + b*x^2)^8/x^14,x)
```

output
```
-(a^8/13 - (b^8*x^16)/3 + (8*a^7*b*x^2)/11 - 8*a*b^7*x^14 + (28*a^6*b^2*x^
4)/9 + 8*a^5*b^3*x^6 + 14*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/3 + 28*a^2*b^6*x
^12)/x^13
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^8}{x^{14}}\,dx = \frac{429b^8x^{16}+10296a\,b^7x^{14}-36036a^2b^6x^{12}-24024a^3b^5x^{10}-18018a^4b^4x^8-10296a^5b^3x^6-4004a^6b^2x^4-}{1287x^{13}}$$

input
```
int((b*x^2+a)^8/x^14,x)
```

output
```
( - 99*a**8 - 936*a**7*b*x**2 - 4004*a**6*b**2*x**4 - 10296*a**5*b**3*x**6
 - 18018*a**4*b**4*x**8 - 24024*a**3*b**5*x**10 - 36036*a**2*b**6*x**12 +
10296*a*b**7*x**14 + 429*b**8*x**16)/(1287*x**13)
```

3.121 $\int \frac{(a+bx^2)^8}{x^{16}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = -\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + b^8x$$

output
```
-1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*
b^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = -\frac{a^8}{15x^{15}} - \frac{8a^7b}{13x^{13}} - \frac{28a^6b^2}{11x^{11}} - \frac{56a^5b^3}{9x^9} - \frac{10a^4b^4}{x^7} - \frac{56a^3b^5}{5x^5} - \frac{28a^2b^6}{3x^3} - \frac{8ab^7}{x} + b^8x$$

input
```
Integrate[(a + b*x^2)^8/x^16,x]
```

output
```
-1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^
3)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3
) - (8*a*b^7)/x + b^8*x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{244, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^8}{x^{16}}+\frac{8a^7b}{x^{14}}+\frac{28a^6b^2}{x^{12}}+\frac{56a^5b^3}{x^{10}}+\frac{70a^4b^4}{x^8}+\frac{56a^3b^5}{x^6}+\frac{28a^2b^6}{x^4}+\frac{8ab^7}{x^2}+b^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8ab^7}{x}+b^8x$$

input
```
Int[(a + b*x^2)^8/x^16,x]
```

output
```
-1/15*a^8/x^15 - (8*a^7*b)/(13*x^13) - (28*a^6*b^2)/(11*x^11) - (56*a^5*b^
3)/(9*x^9) - (10*a^4*b^4)/x^7 - (56*a^3*b^5)/(5*x^5) - (28*a^2*b^6)/(3*x^3
) - (8*a*b^7)/x + b^8*x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

method	result
default	$-\frac{a^8}{15x^{15}}-\frac{8a^7b}{13x^{13}}-\frac{28a^6b^2}{11x^{11}}-\frac{56a^5b^3}{9x^9}-\frac{10a^4b^4}{x^7}-\frac{56a^3b^5}{5x^5}-\frac{28a^2b^6}{3x^3}-\frac{8a\,b^7}{x}+b^8x$
risch	$b^8x+\frac{-\frac{1}{15}a^8-\frac{8}{13}a^7b\,x^2-\frac{28}{11}a^6b^2x^4-\frac{56}{9}a^5b^3x^6-10a^4b^4x^8-\frac{56}{5}a^3b^5x^{10}-\frac{28}{3}a^2b^6x^{12}-8a\,b^7x^{14}}{x^{15}}$
norman	$\frac{-\frac{1}{15}a^8-\frac{8}{13}a^7b\,x^2-\frac{28}{11}a^6b^2x^4-\frac{56}{9}a^5b^3x^6-10a^4b^4x^8-\frac{56}{5}a^3b^5x^{10}-\frac{28}{3}a^2b^6x^{12}-8a\,b^7x^{14}+b^8x^{16}}{x^{15}}$
gosper	$-\frac{-6435b^8x^{16}+51480a\,b^7x^{14}+60060a^2b^6x^{12}+72072a^3b^5x^{10}+64350a^4b^4x^8+40040a^5b^3x^6+16380a^6b^2x^4+3960a^7b\,x^2+429}{6435x^{15}}$
parallelrisch	$\frac{6435b^8x^{16}-51480a\,b^7x^{14}-60060a^2b^6x^{12}-72072a^3b^5x^{10}-64350a^4b^4x^8-40040a^5b^3x^6-16380a^6b^2x^4-3960a^7b\,x^2-429a^8}{6435x^{15}}$
orering	$-\frac{-6435b^8x^{16}+51480a\,b^7x^{14}+60060a^2b^6x^{12}+72072a^3b^5x^{10}+64350a^4b^4x^8+40040a^5b^3x^6+16380a^6b^2x^4+3960a^7b\,x^2+429}{6435x^{15}}$

input
```
int((b*x^2+a)^8/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/15*a^8/x^15-8/13*a^7*b/x^13-28/11*a^6*b^2/x^11-56/9*a^5*b^3/x^9-10*a^4*
b^4/x^7-56/5*a^3*b^5/x^5-28/3*a^2*b^6/x^3-8*a*b^7/x+b^8*x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = \frac{6435\,b^8x^{16} - 51480\,ab^7x^{14} - 60060\,a^2b^6x^{12} - 72072\,a^3b^5x^{10} - 64350\,a^4b^4x^8 - 40040\,a^5b^3x^6 - 16380\,a^6b^2}{6435\,x^{15}}$$

input
```
integrate((b*x^2+a)^8/x^16,x, algorithm="fricas")
```

output
```
1/6435*(6435*b^8*x^16 - 51480*a*b^7*x^14 - 60060*a^2*b^6*x^12 - 72072*a^3*
b^5*x^10 - 64350*a^4*b^4*x^8 - 40040*a^5*b^3*x^6 - 16380*a^6*b^2*x^4 - 396
0*a^7*b*x^2 - 429*a^8)/x^15
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = b^8x + \frac{-429a^8 - 3960a^7bx^2 - 16380a^6b^2x^4 - 40040a^5b^3x^6 - 64350a^4b^4x^8 - 72072a^3b^5x^{10} - 60060a^2b^6x^{12} -}{6435x^{15}}$$

input
```
integrate((b*x**2+a)**8/x**16,x)
```

output
```
b**8*x + (-429*a**8 - 3960*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 40040*a**5
*b**3*x**6 - 64350*a**4*b**4*x**8 - 72072*a**3*b**5*x**10 - 60060*a**2*b**
6*x**12 - 51480*a*b**7*x**14)/(6435*x**15)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = b^8x - \frac{51480\,ab^7x^{14} + 60060\,a^2b^6x^{12} + 72072\,a^3b^5x^{10} + 64350\,a^4b^4x^8 + 40040\,a^5b^3x^6 + 16380\,a^6b^2x^4 + 3960\,a^7bx^2 + 429\,a^8}{6435\,x^{15}}$$

input
```
integrate((b*x^2+a)^8/x^16,x, algorithm="maxima")
```

output
```
b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
 + 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*
x^2 + 429*a^8)/x^15
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx = b^8x - \frac{51480\,ab^7x^{14} + 60060\,a^2b^6x^{12} + 72072\,a^3b^5x^{10} + 64350\,a^4b^4x^8 + 40040\,a^5b^3x^6 + 16380\,a^6b^2x^4 + 3960\,a^7bx^2 + 429\,a^8}{6435\,x^{15}}$$

input
```
integrate((b*x^2+a)^8/x^16,x, algorithm="giac")
```

output
```
b^8*x - 1/6435*(51480*a*b^7*x^14 + 60060*a^2*b^6*x^12 + 72072*a^3*b^5*x^10
 + 64350*a^4*b^4*x^8 + 40040*a^5*b^3*x^6 + 16380*a^6*b^2*x^4 + 3960*a^7*b*
x^2 + 429*a^8)/x^15
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx$$
$$= b^8\,x - \frac{\frac{a^8}{15}+\frac{8\,a^7\,b\,x^2}{13}+\frac{28\,a^6\,b^2\,x^4}{11}+\frac{56\,a^5\,b^3\,x^6}{9}+10\,a^4\,b^4\,x^8+\frac{56\,a^3\,b^5\,x^{10}}{5}+\frac{28\,a^2\,b^6\,x^{12}}{3}+8\,a\,b^7\,x^{14}}{x^{15}}$$

input
```
int((a + b*x^2)^8/x^16,x)
```

output
```
b^8*x - (a^8/15 + (8*a^7*b*x^2)/13 + 8*a*b^7*x^14 + (28*a^6*b^2*x^4)/11 +
(56*a^5*b^3*x^6)/9 + 10*a^4*b^4*x^8 + (56*a^3*b^5*x^10)/5 + (28*a^2*b^6*x^
12)/3)/x^15
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{16}}\,dx$$
$$= \frac{6435b^8x^{16}-51480a\,b^7x^{14}-60060a^2b^6x^{12}-72072a^3b^5x^{10}-64350a^4b^4x^8-40040a^5b^3x^6-16380a^6b^2x^4}{6435x^{15}}$$

input
```
int((b*x^2+a)^8/x^16,x)
```

output
```
( - 429*a**8 - 3960*a**7*b*x**2 - 16380*a**6*b**2*x**4 - 40040*a**5*b**3*x
**6 - 64350*a**4*b**4*x**8 - 72072*a**3*b**5*x**10 - 60060*a**2*b**6*x**12
 - 51480*a*b**7*x**14 + 6435*b**8*x**16)/(6435*x**15)
```

3.122 $\int \frac{(a+bx^2)^8}{x^{18}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = -\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

output
```
-1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*
a^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = -\frac{a^8}{17x^{17}} - \frac{8a^7b}{15x^{15}} - \frac{28a^6b^2}{13x^{13}} - \frac{56a^5b^3}{11x^{11}} - \frac{70a^4b^4}{9x^9} - \frac{8a^3b^5}{x^7} - \frac{28a^2b^6}{5x^5} - \frac{8ab^7}{3x^3} - \frac{b^8}{x}$$

input
```
Integrate[(a + b*x^2)^8/x^18,x]
```

output
```
-1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^
3)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^
5) - (8*a*b^7)/(3*x^3) - b^8/x
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^8}{x^{18}}+\frac{8a^7b}{x^{16}}+\frac{28a^6b^2}{x^{14}}+\frac{56a^5b^3}{x^{12}}+\frac{70a^4b^4}{x^{10}}+\frac{56a^3b^5}{x^8}+\frac{28a^2b^6}{x^6}+\frac{8ab^7}{x^4}+\frac{b^8}{x^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^8}{17x^{17}}-\frac{8a^7b}{15x^{15}}-\frac{28a^6b^2}{13x^{13}}-\frac{56a^5b^3}{11x^{11}}-\frac{70a^4b^4}{9x^9}-\frac{8a^3b^5}{x^7}-\frac{28a^2b^6}{5x^5}-\frac{8ab^7}{3x^3}-\frac{b^8}{x}$$

input
```
Int[(a + b*x^2)^8/x^18,x]
```

output

```
-1/17*a^8/x^17 - (8*a^7*b)/(15*x^15) - (28*a^6*b^2)/(13*x^13) - (56*a^5*b^
3)/(11*x^11) - (70*a^4*b^4)/(9*x^9) - (8*a^3*b^5)/x^7 - (28*a^2*b^6)/(5*x^
5) - (8*a*b^7)/(3*x^3) - b^8/x
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

method	result
default	$-\frac{a^{8}}{17 x^{17}}-\frac{8 a^{7} b}{15 x^{15}}-\frac{28 a^{6} b^{2}}{13 x^{13}}-\frac{56 a^{5} b^{3}}{11 x^{11}}-\frac{70 a^{4} b^{4}}{9 x^{9}}-\frac{8 a^{3} b^{5}}{x^{7}}-\frac{28 a^{2} b^{6}}{5 x^{5}}-\frac{8 a\,b^{7}}{3 x^{3}}-\frac{b^{8}}{x}$
norman	$\frac{-\frac{1}{17} a^{8}-\frac{8}{15} a^{7} b\,x^{2}-\frac{28}{13} a^{6} b^{2} x^{4}-\frac{56}{11} a^{5} b^{3} x^{6}-\frac{70}{9} a^{4} b^{4} x^{8}-8 a^{3} b^{5} x^{10}-\frac{28}{5} a^{2} b^{6} x^{12}-\frac{8}{3} a\,b^{7} x^{14}-b^{8} x^{16}}{x^{17}}$
risch	$\frac{-\frac{1}{17} a^{8}-\frac{8}{15} a^{7} b\,x^{2}-\frac{28}{13} a^{6} b^{2} x^{4}-\frac{56}{11} a^{5} b^{3} x^{6}-\frac{70}{9} a^{4} b^{4} x^{8}-8 a^{3} b^{5} x^{10}-\frac{28}{5} a^{2} b^{6} x^{12}-\frac{8}{3} a\,b^{7} x^{14}-b^{8} x^{16}}{x^{17}}$
gosper	$-\frac{109395 b^{8} x^{16}+291720 a\,b^{7} x^{14}+612612 a^{2} b^{6} x^{12}+875160 a^{3} b^{5} x^{10}+850850 a^{4} b^{4} x^{8}+556920 a^{5} b^{3} x^{6}+235620 a^{6} b^{2} x^{4}+58344 a^{7}}{109395 x^{17}}$
parallelrisch	$\frac{-109395 b^{8} x^{16}-291720 a\,b^{7} x^{14}-612612 a^{2} b^{6} x^{12}-875160 a^{3} b^{5} x^{10}-850850 a^{4} b^{4} x^{8}-556920 a^{5} b^{3} x^{6}-235620 a^{6} b^{2} x^{4}-58344 a^{7} b}{109395 x^{17}}$
orering	$-\frac{109395 b^{8} x^{16}+291720 a\,b^{7} x^{14}+612612 a^{2} b^{6} x^{12}+875160 a^{3} b^{5} x^{10}+850850 a^{4} b^{4} x^{8}+556920 a^{5} b^{3} x^{6}+235620 a^{6} b^{2} x^{4}+58344 a^{7}}{109395 x^{17}}$

input
```
int((b*x^2+a)^8/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/17*a^8/x^17-8/15*a^7*b/x^15-28/13*a^6*b^2/x^13-56/11*a^5*b^3/x^11-70/9*
a^4*b^4/x^9-8*a^3*b^5/x^7-28/5*a^2*b^6/x^5-8/3*a*b^7/x^3-b^8/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx =$$
$$-\frac{109395\,b^8x^{16}+291720\,ab^7x^{14}+612612\,a^2b^6x^{12}+875160\,a^3b^5x^{10}+850850\,a^4b^4x^8+556920\,a^5b^3x^6+}{109395\,x^{17}}$$

input
```
integrate((b*x^2+a)^8/x^18,x, algorithm="fricas")
```

output
```
-1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 875
160*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^
2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx$$
$$=\frac{-6435a^8-58344a^7bx^2-235620a^6b^2x^4-556920a^5b^3x^6-850850a^4b^4x^8-875160a^3b^5x^{10}-612612a^2b^6}{109395x^{17}}$$

input
```
integrate((b*x**2+a)**8/x**18,x)
```

output
```
(-6435*a**8 - 58344*a**7*b*x**2 - 235620*a**6*b**2*x**4 - 556920*a**5*b**3
*x**6 - 850850*a**4*b**4*x**8 - 875160*a**3*b**5*x**10 - 612612*a**2*b**6*
x**12 - 291720*a*b**7*x**14 - 109395*b**8*x**16)/(109395*x**17)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = -\frac{109395\,b^8x^{16} + 291720\,ab^7x^{14} + 612612\,a^2b^6x^{12} + 875160\,a^3b^5x^{10} + 850850\,a^4b^4x^8 + 556920\,a^5b^3x^6 + \cdots}{109395\,x^{17}}$$

input
```
integrate((b*x^2+a)^8/x^18,x, algorithm="maxima")
```

output
```
-1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 875
160*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^
2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = -\frac{109395\,b^8x^{16} + 291720\,ab^7x^{14} + 612612\,a^2b^6x^{12} + 875160\,a^3b^5x^{10} + 850850\,a^4b^4x^8 + 556920\,a^5b^3x^6 + \cdots}{109395\,x^{17}}$$

input
```
integrate((b*x^2+a)^8/x^18,x, algorithm="giac")
```

output
```
-1/109395*(109395*b^8*x^16 + 291720*a*b^7*x^14 + 612612*a^2*b^6*x^12 + 875
160*a^3*b^5*x^10 + 850850*a^4*b^4*x^8 + 556920*a^5*b^3*x^6 + 235620*a^6*b^
2*x^4 + 58344*a^7*b*x^2 + 6435*a^8)/x^17
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = -\frac{\frac{a^8}{17}+\frac{8\,a^7\,b\,x^2}{15}+\frac{28\,a^6\,b^2\,x^4}{13}+\frac{56\,a^5\,b^3\,x^6}{11}+\frac{70\,a^4\,b^4\,x^8}{9}+8\,a^3\,b^5\,x^{10}+\frac{28\,a^2\,b^6\,x^{12}}{5}+\frac{8\,a\,b^7\,x^{14}}{3}+b^8\,x^{16}}{x^{17}}$$

input
```
int((a + b*x^2)^8/x^18,x)
```

output
```
-(a^8/17 + b^8*x^16 + (8*a^7*b*x^2)/15 + (8*a*b^7*x^14)/3 + (28*a^6*b^2*x^
4)/13 + (56*a^5*b^3*x^6)/11 + (70*a^4*b^4*x^8)/9 + 8*a^3*b^5*x^10 + (28*a^
2*b^6*x^12)/5)/x^17
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^8}{x^{18}}\,dx = \frac{-109395b^8x^{16}-291720a\,b^7x^{14}-612612a^2b^6x^{12}-875160a^3b^5x^{10}-850850a^4b^4x^8-556920a^5b^3x^6-23}{109395x^{17}}$$

input
```
int((b*x^2+a)^8/x^18,x)
```

output
```
( - 6435*a**8 - 58344*a**7*b*x**2 - 235620*a**6*b**2*x**4 - 556920*a**5*b*
*3*x**6 - 850850*a**4*b**4*x**8 - 875160*a**3*b**5*x**10 - 612612*a**2*b**
6*x**12 - 291720*a*b**7*x**14 - 109395*b**8*x**16)/(109395*x**17)
```

3.123 $\int \frac{(a+bx^2)^8}{x^{20}} dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{(a+bx^2)^8}{x^{20}} dx = -\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

output
```
-1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11
*a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^8}{x^{20}} dx = -\frac{a^8}{19x^{19}} - \frac{8a^7b}{17x^{17}} - \frac{28a^6b^2}{15x^{15}} - \frac{56a^5b^3}{13x^{13}} - \frac{70a^4b^4}{11x^{11}} - \frac{56a^3b^5}{9x^9} - \frac{4a^2b^6}{x^7} - \frac{8ab^7}{5x^5} - \frac{b^8}{3x^3}$$

input
```
Integrate[(a + b*x^2)^8/x^20,x]
```

output
```
-1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^
3)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)
/x^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{244, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^8}{x^{20}}\,dx$$

$$\Big\downarrow \; 244$$

$$\int \left(\frac{a^8}{x^{20}}+\frac{8a^7b}{x^{18}}+\frac{28a^6b^2}{x^{16}}+\frac{56a^5b^3}{x^{14}}+\frac{70a^4b^4}{x^{12}}+\frac{56a^3b^5}{x^{10}}+\frac{28a^2b^6}{x^8}+\frac{8ab^7}{x^6}+\frac{b^8}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8ab^7}{5x^5}-\frac{b^8}{3x^3}$$

input
```
Int[(a + b*x^2)^8/x^20,x]
```

output

```
-1/19*a^8/x^19 - (8*a^7*b)/(17*x^17) - (28*a^6*b^2)/(15*x^15) - (56*a^5*b^
3)/(13*x^13) - (70*a^4*b^4)/(11*x^11) - (56*a^3*b^5)/(9*x^9) - (4*a^2*b^6)
/x^7 - (8*a*b^7)/(5*x^5) - b^8/(3*x^3)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
default	$-\frac{a^8}{19x^{19}}-\frac{8a^7b}{17x^{17}}-\frac{28a^6b^2}{15x^{15}}-\frac{56a^5b^3}{13x^{13}}-\frac{70a^4b^4}{11x^{11}}-\frac{56a^3b^5}{9x^9}-\frac{4a^2b^6}{x^7}-\frac{8a\,b^7}{5x^5}-\frac{b^8}{3x^3}$
norman	$\frac{-\frac{1}{19}a^8-\frac{8}{17}a^7b\,x^2-\frac{28}{15}a^6b^2x^4-\frac{56}{13}a^5b^3x^6-\frac{70}{11}a^4b^4x^8-\frac{56}{9}a^3b^5x^{10}-4a^2b^6x^{12}-\frac{8}{5}a\,b^7x^{14}-\frac{1}{3}b^8x^{16}}{x^{19}}$
risch	$\frac{-\frac{1}{19}a^8-\frac{8}{17}a^7b\,x^2-\frac{28}{15}a^6b^2x^4-\frac{56}{13}a^5b^3x^6-\frac{70}{11}a^4b^4x^8-\frac{56}{9}a^3b^5x^{10}-4a^2b^6x^{12}-\frac{8}{5}a\,b^7x^{14}-\frac{1}{3}b^8x^{16}}{x^{19}}$
gosper	$-\frac{692835b^8x^{16}+3325608a\,b^7x^{14}+8314020a^2b^6x^{12}+12932920a^3b^5x^{10}+13226850a^4b^4x^8+8953560a^5b^3x^6+3879876a^6b^2x^4}{2078505x^{19}}$
parallelrisch	$\frac{-692835b^8x^{16}-3325608a\,b^7x^{14}-8314020a^2b^6x^{12}-12932920a^3b^5x^{10}-13226850a^4b^4x^8-8953560a^5b^3x^6-3879876a^6b^2x^4}{2078505x^{19}}$
orering	$-\frac{692835b^8x^{16}+3325608a\,b^7x^{14}+8314020a^2b^6x^{12}+12932920a^3b^5x^{10}+13226850a^4b^4x^8+8953560a^5b^3x^6+3879876a^6b^2x^4}{2078505x^{19}}$

input
```
int((b*x^2+a)^8/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/19*a^8/x^19-8/17*a^7*b/x^17-28/15*a^6*b^2/x^15-56/13*a^5*b^3/x^13-70/11
*a^4*b^4/x^11-56/9*a^3*b^5/x^9-4*a^2*b^6/x^7-8/5*a*b^7/x^5-1/3*b^8/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx =$$
$$-\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a}{2078505\,x^{19}}$$

input
```
integrate((b*x^2+a)^8/x^20,x, algorithm="fricas")
```

output
```
-1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 +
12932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 38798
76*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx$$
$$=\frac{-109395a^8-978120a^7bx^2-3879876a^6b^2x^4-8953560a^5b^3x^6-13226850a^4b^4x^8-12932920a^3b^5x^{10}-}{2078505x^{19}}$$

input
```
integrate((b*x**2+a)**8/x**20,x)
```

output
```
(-109395*a**8 - 978120*a**7*b*x**2 - 3879876*a**6*b**2*x**4 - 8953560*a**5
*b**3*x**6 - 13226850*a**4*b**4*x**8 - 12932920*a**3*b**5*x**10 - 8314020*
a**2*b**6*x**12 - 3325608*a*b**7*x**14 - 692835*b**8*x**16)/(2078505*x**19
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx = -\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a}{2078505\,x^{19}}$$

input
```
integrate((b*x^2+a)^8/x^20,x, algorithm="maxima")
```

output
```
-1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 +
12932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 38798
76*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx = -\frac{692835\,b^8x^{16}+3325608\,ab^7x^{14}+8314020\,a^2b^6x^{12}+12932920\,a^3b^5x^{10}+13226850\,a^4b^4x^8+8953560\,a}{2078505\,x^{19}}$$

input
```
integrate((b*x^2+a)^8/x^20,x, algorithm="giac")
```

output
```
-1/2078505*(692835*b^8*x^16 + 3325608*a*b^7*x^14 + 8314020*a^2*b^6*x^12 +
12932920*a^3*b^5*x^10 + 13226850*a^4*b^4*x^8 + 8953560*a^5*b^3*x^6 + 38798
76*a^6*b^2*x^4 + 978120*a^7*b*x^2 + 109395*a^8)/x^19
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx = -\frac{\frac{a^8}{19}+\frac{8\,a^7\,b\,x^2}{17}+\frac{28\,a^6\,b^2\,x^4}{15}+\frac{56\,a^5\,b^3\,x^6}{13}+\frac{70\,a^4\,b^4\,x^8}{11}+\frac{56\,a^3\,b^5\,x^{10}}{9}+4\,a^2\,b^6\,x^{12}+\frac{8\,a\,b^7\,x^{14}}{5}+\frac{b^8\,x^{16}}{3}}{x^{19}}$$

input
```
int((a + b*x^2)^8/x^20,x)
```

output
```
-(a^8/19 + (b^8*x^16)/3 + (8*a^7*b*x^2)/17 + (8*a*b^7*x^14)/5 + (28*a^6*b^
2*x^4)/15 + (56*a^5*b^3*x^6)/13 + (70*a^4*b^4*x^8)/11 + (56*a^3*b^5*x^10)/
9 + 4*a^2*b^6*x^12)/x^19
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^8}{x^{20}}\,dx = \frac{-692835b^8x^{16}-3325608a\,b^7x^{14}-8314020a^2b^6x^{12}-12932920a^3b^5x^{10}-13226850a^4b^4x^8-8953560a^5b}{2078505x^{19}}$$

input
```
int((b*x^2+a)^8/x^20,x)
```

output
```
( - 109395*a**8 - 978120*a**7*b*x**2 - 3879876*a**6*b**2*x**4 - 8953560*a*
*5*b**3*x**6 - 13226850*a**4*b**4*x**8 - 12932920*a**3*b**5*x**10 - 831402
0*a**2*b**6*x**12 - 3325608*a*b**7*x**14 - 692835*b**8*x**16)/(2078505*x**
19)
```

3.124 $\int \frac{x^{11}}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int \frac{x^{11}}{a+bx^2} dx = \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b} - \frac{a^5 \log(a+bx^2)}{2b^6}$$

output
```
1/2*a^4*x^2/b^5-1/4*a^3*x^4/b^4+1/6*a^2*x^6/b^3-1/8*a*x^8/b^2+1/10*x^10/b-
1/2*a^5*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int \frac{x^{11}}{a+bx^2} dx = \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b} - \frac{a^5 \log(a+bx^2)}{2b^6}$$

input
```
Integrate[x^11/(a + b*x^2),x]
```

output
```
(a^4*x^2)/(2*b^5) - (a^3*x^4)/(4*b^4) + (a^2*x^6)/(6*b^3) - (a*x^8)/(8*b^2
) + x^10/(10*b) - (a^5*Log[a + b*x^2])/(2*b^6)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^{10}}{bx^2+a}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{x^8}{b}-\frac{ax^6}{b^2}+\frac{a^2x^4}{b^3}-\frac{a^3x^2}{b^4}-\frac{a^5}{b^5\left(bx^2+a\right)}+\frac{a^4}{b^5}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^5\log\left(a+bx^2\right)}{b^6}+\frac{a^4x^2}{b^5}-\frac{a^3x^4}{2b^4}+\frac{a^2x^6}{3b^3}-\frac{ax^8}{4b^2}+\frac{x^{10}}{5b}\right)$$

input
```
Int[x^11/(a + b*x^2),x]
```

output
```
((a^4*x^2)/b^5 - (a^3*x^4)/(2*b^4) + (a^2*x^6)/(3*b^3) - (a*x^8)/(4*b^2) +
 x^10/(5*b) - (a^5*Log[a + b*x^2])/b^6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\frac{1}{5} b^{4} x^{10}-\frac{1}{4} a \,b^{3} x^{8}+\frac{1}{3} a^{2} b^{2} x^{6}-\frac{1}{2} a^{3} b \,x^{4}+a^{4} x^{2}}{2 b^{5}}-\frac{a^{5} \ln \left(b \,x^{2}+a \right)}{2 b^{6}}$	68
norman	$\frac{a^{4} x^{2}}{2 b^{5}}-\frac{a^{3} x^{4}}{4 b^{4}}+\frac{a^{2} x^{6}}{6 b^{3}}-\frac{a \,x^{8}}{8 b^{2}}+\frac{x^{10}}{10 b}-\frac{a^{5} \ln \left(b \,x^{2}+a \right)}{2 b^{6}}$	68
risch	$\frac{a^{4} x^{2}}{2 b^{5}}-\frac{a^{3} x^{4}}{4 b^{4}}+\frac{a^{2} x^{6}}{6 b^{3}}-\frac{a \,x^{8}}{8 b^{2}}+\frac{x^{10}}{10 b}-\frac{a^{5} \ln \left(b \,x^{2}+a \right)}{2 b^{6}}$	68
parallelrisch	$-\frac{-12 b^{5} x^{10}+15 a \,b^{4} x^{8}-20 a^{2} b^{3} x^{6}+30 a^{3} b^{2} x^{4}-60 a^{4} b \,x^{2}+60 a^{5} \ln \left(b \,x^{2}+a \right)}{120 b^{6}}$	68

input
```
int(x^11/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^5*(1/5*b^4*x^10-1/4*a*b^3*x^8+1/3*a^2*b^2*x^6-1/2*a^3*b*x^4+a^4*x^2)
-1/2*a^5*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{a+bx^2}\,dx = \frac{12\,b^{5}x^{10}-15\,ab^{4}x^{8}+20\,a^{2}b^{3}x^{6}-30\,a^{3}b^{2}x^{4}+60\,a^{4}bx^{2}-60\,a^{5}\log\left(bx^{2}+a\right)}{120\,b^{6}}$$

input
```
integrate(x^11/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/120*(12*b^5*x^10 - 15*a*b^4*x^8 + 20*a^2*b^3*x^6 - 30*a^3*b^2*x^4 + 60*a
^4*b*x^2 - 60*a^5*log(b*x^2 + a))/b^6
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{a+bx^2}\,dx = -\frac{a^5\log(a+bx^2)}{2b^6} + \frac{a^4x^2}{2b^5} - \frac{a^3x^4}{4b^4} + \frac{a^2x^6}{6b^3} - \frac{ax^8}{8b^2} + \frac{x^{10}}{10b}$$

input
```
integrate(x**11/(b*x**2+a),x)
```

output
```
-a**5*log(a + b*x**2)/(2*b**6) + a**4*x**2/(2*b**5) - a**3*x**4/(4*b**4) +
 a**2*x**6/(6*b**3) - a*x**8/(8*b**2) + x**10/(10*b)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{a+bx^2}\,dx = -\frac{a^5\log(bx^2+a)}{2\,b^6} + \frac{12\,b^4x^{10} - 15\,ab^3x^8 + 20\,a^2b^2x^6 - 30\,a^3bx^4 + 60\,a^4x^2}{120\,b^5}$$

input
```
integrate(x^11/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*a^5*log(b*x^2 + a)/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*a^2*b
^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}}{a+bx^2}\,dx = -\frac{a^5\log\left(|bx^2+a|\right)}{2\,b^6} + \frac{12\,b^4x^{10}-15\,ab^3x^8+20\,a^2b^2x^6-30\,a^3bx^4+60\,a^4x^2}{120\,b^5}$$

input
```
integrate(x^11/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*a^5*log(abs(b*x^2 + a))/b^6 + 1/120*(12*b^4*x^10 - 15*a*b^3*x^8 + 20*
a^2*b^2*x^6 - 30*a^3*b*x^4 + 60*a^4*x^2)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{a+bx^2}\,dx = \frac{x^{10}}{10\,b} - \frac{a\,x^8}{8\,b^2} - \frac{a^5\,\ln\left(b\,x^2+a\right)}{2\,b^6} + \frac{a^2\,x^6}{6\,b^3} - \frac{a^3\,x^4}{4\,b^4} + \frac{a^4\,x^2}{2\,b^5}$$

input
```
int(x^11/(a + b*x^2),x)
```

output
```
x^10/(10*b) - (a*x^8)/(8*b^2) - (a^5*log(a + b*x^2))/(2*b^6) + (a^2*x^6)/(
6*b^3) - (a^3*x^4)/(4*b^4) + (a^4*x^2)/(2*b^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{x^{11}}{a+bx^2}\,dx = \frac{-60\log(b\,x^2+a)\,a^5+60a^4b\,x^2-30a^3b^2x^4+20a^2b^3x^6-15a\,b^4x^8+12b^5x^{10}}{120b^6}$$

input
```
int(x^11/(b*x^2+a),x)
```

output
```
( - 60*log(a + b*x**2)*a**5 + 60*a**4*b*x**2 - 30*a**3*b**2*x**4 + 20*a**2
*b**3*x**6 - 15*a*b**4*x**8 + 12*b**5*x**10)/(120*b**6)
```

3.125 $\int \frac{x^9}{a+bx^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{x^9}{a+bx^2} \, dx = -\frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b} + \frac{a^4 \log (a+bx^2)}{2b^5}$$

output
```
-1/2*a^3*x^2/b^4+1/4*a^2*x^4/b^3-1/6*a*x^6/b^2+1/8*x^8/b+1/2*a^4*ln(b*x^2+
a)/b^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int \frac{x^9}{a+bx^2} \, dx = -\frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b} + \frac{a^4 \log (a+bx^2)}{2b^5}$$

input
```
Integrate[x^9/(a + b*x^2),x]
```

output
```
-1/2*(a^3*x^2)/b^4 + (a^2*x^4)/(4*b^3) - (a*x^6)/(6*b^2) + x^8/(8*b) + (a^
4*Log[a + b*x^2])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{a+bx^2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{x^8}{bx^2+a}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{x^6}{b} - \frac{ax^4}{b^2} + \frac{a^2x^2}{b^3} + \frac{a^4}{b^4\left(bx^2+a\right)} - \frac{a^3}{b^4}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^4\log\left(a+bx^2\right)}{b^5} - \frac{a^3x^2}{b^4} + \frac{a^2x^4}{2b^3} - \frac{ax^6}{3b^2} + \frac{x^8}{4b}\right)$$

input
```
Int[x^9/(a + b*x^2),x]
```

output
```
(-((a^3*x^2)/b^4) + (a^2*x^4)/(2*b^3) - (a*x^6)/(3*b^2) + x^8/(4*b) + (a^4
*Log[a + b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-\frac{1}{4}b^3x^8+\frac{1}{3}a\,b^2x^6-\frac{1}{2}a^2b\,x^4+a^3x^2}{2b^4}+\frac{a^4\ln\left(b\,x^2+a\right)}{2b^5}$	57
norman	$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{a\,x^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\ln\left(b\,x^2+a\right)}{2b^5}$	57
risch	$-\frac{a^3x^2}{2b^4}+\frac{a^2x^4}{4b^3}-\frac{a\,x^6}{6b^2}+\frac{x^8}{8b}+\frac{a^4\ln\left(b\,x^2+a\right)}{2b^5}$	57
parallelrisch	$\frac{3b^4x^8-4a\,b^3x^6+6a^2b^2x^4-12a^3b\,x^2+12a^4\ln\left(b\,x^2+a\right)}{24b^5}$	57

input
```
int(x^9/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^4*(-1/4*b^3*x^8+1/3*a*b^2*x^6-1/2*a^2*b*x^4+a^3*x^2)+1/2*a^4*ln(b*x
^2+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{a+bx^2}\,dx=\frac{3\,b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-12\,a^3bx^2+12\,a^4\log\left(bx^2+a\right)}{24\,b^5}$$

input
```
integrate(x^9/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/24*(3*b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 12*a^3*b*x^2 + 12*a^4*log(
b*x^2 + a))/b^5
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{a+bx^2}\,dx = \frac{a^4\log\left(a+bx^2\right)}{2b^5} - \frac{a^3x^2}{2b^4} + \frac{a^2x^4}{4b^3} - \frac{ax^6}{6b^2} + \frac{x^8}{8b}$$

input
```
integrate(x**9/(b*x**2+a),x)
```

output
```
a**4*log(a + b*x**2)/(2*b**5) - a**3*x**2/(2*b**4) + a**2*x**4/(4*b**3) -
a*x**6/(6*b**2) + x**8/(8*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{a+bx^2}\,dx = \frac{a^4\log\left(bx^2+a\right)}{2\,b^5} + \frac{3\,b^3x^8 - 4\,ab^2x^6 + 6\,a^2bx^4 - 12\,a^3x^2}{24\,b^4}$$

input
```
integrate(x^9/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*a^4*log(b*x^2 + a)/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*x^4 -
 12*a^3*x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{x^9}{a+bx^2}\,dx = \frac{a^4\log\left(|bx^2+a|\right)}{2\,b^5} + \frac{3\,b^3x^8 - 4\,ab^2x^6 + 6\,a^2bx^4 - 12\,a^3x^2}{24\,b^4}$$

input
```
integrate(x^9/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*a^4*log(abs(b*x^2 + a))/b^5 + 1/24*(3*b^3*x^8 - 4*a*b^2*x^6 + 6*a^2*b*
x^4 - 12*a^3*x^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{a+bx^2}\,dx = \frac{x^8}{8\,b} - \frac{a\,x^6}{6\,b^2} + \frac{a^4\,\ln\left(b\,x^2+a\right)}{2\,b^5} + \frac{a^2\,x^4}{4\,b^3} - \frac{a^3\,x^2}{2\,b^4}$$

input
```
int(x^9/(a + b*x^2),x)
```

output
```
x^8/(8*b) - (a*x^6)/(6*b^2) + (a^4*log(a + b*x^2))/(2*b^5) + (a^2*x^4)/(4*
b^3) - (a^3*x^2)/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{a+bx^2}\,dx = \frac{12\log(b\,x^2+a)\,a^4-12a^3b\,x^2+6a^2b^2x^4-4a\,b^3x^6+3b^4x^8}{24b^5}$$

input
```
int(x^9/(b*x^2+a),x)
```

output
```
(12*log(a + b*x**2)*a**4 - 12*a**3*b*x**2 + 6*a**2*b**2*x**4 - 4*a*b**3*x*
*6 + 3*b**4*x**8)/(24*b**5)
```

3.126 $\int \frac{x^7}{a+bx^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^7}{a+bx^2} \, dx = \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b} - \frac{a^3\log(a+bx^2)}{2b^4}$$

output
```
1/2*a^2*x^2/b^3-1/4*a*x^4/b^2+1/6*x^6/b-1/2*a^3*ln(b*x^2+a)/b^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{a+bx^2} \, dx = \frac{a^2x^2}{2b^3} - \frac{ax^4}{4b^2} + \frac{x^6}{6b} - \frac{a^3\log(a+bx^2)}{2b^4}$$

input
```
Integrate[x^7/(a + b*x^2),x]
```

output
```
(a^2*x^2)/(2*b^3) - (a*x^4)/(4*b^2) + x^6/(6*b) - (a^3*Log[a + b*x^2])/(2*
b^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{a+bx^2} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{x^6}{bx^2+a}dx^2$$
$$\Big\downarrow \text{49}$$
$$\frac{1}{2} \int \left(\frac{x^4}{b} - \frac{ax^2}{b^2} - \frac{a^3}{b^3 (bx^2+a)} + \frac{a^2}{b^3}\right) dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(-\frac{a^3 \log (a+bx^2)}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^4}{2b^2} + \frac{x^6}{3b}\right)$$

input
```
Int[x^7/(a + b*x^2),x]
```

output
```
((a^2*x^2)/b^3 - (a*x^4)/(2*b^2) + x^6/(3*b) - (a^3*Log[a + b*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\frac{1}{3}b^2x^6-\frac{1}{2}ab\,x^4+a^2x^2}{2b^3}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46
norman	$\frac{a^2x^2}{2b^3}-\frac{a\,x^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46
risch	$\frac{a^2x^2}{2b^3}-\frac{a\,x^4}{4b^2}+\frac{x^6}{6b}-\frac{a^3\ln\left(b\,x^2+a\right)}{2b^4}$	46
parallelrisch	$-\frac{-2b^3x^6+3a\,b^2x^4-6a^2b\,x^2+6a^3\ln\left(b\,x^2+a\right)}{12b^4}$	46

input
```
int(x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3*(1/3*b^2*x^6-1/2*a*b*x^4+a^2*x^2)-1/2*a^3*ln(b*x^2+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{a+bx^2}\,dx=\frac{2\,b^3x^6-3\,ab^2x^4+6\,a^2bx^2-6\,a^3\log\left(bx^2+a\right)}{12\,b^4}$$

input
```
integrate(x^7/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^6 - 3*a*b^2*x^4 + 6*a^2*b*x^2 - 6*a^3*log(b*x^2 + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^7}{a+bx^2}\,dx = -\frac{a^3\log\left(a+bx^2\right)}{2b^4}+\frac{a^2x^2}{2b^3}-\frac{ax^4}{4b^2}+\frac{x^6}{6b}$$

input
```
integrate(x**7/(b*x**2+a),x)
```

output
```
-a**3*log(a + b*x**2)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x*
*6/(6*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \frac{x^7}{a+bx^2}\,dx = -\frac{a^3\log\left(bx^2+a\right)}{2\,b^4}+\frac{2\,b^2x^6-3\,abx^4+6\,a^2x^2}{12\,b^3}$$

input
```
integrate(x^7/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*a^3*log(b*x^2 + a)/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{x^7}{a+bx^2}\,dx = -\frac{a^3\log\left(|bx^2+a|\right)}{2\,b^4}+\frac{2\,b^2x^6-3\,abx^4+6\,a^2x^2}{12\,b^3}$$

input
```
integrate(x^7/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*a^3*log(abs(b*x^2 + a))/b^4 + 1/12*(2*b^2*x^6 - 3*a*b*x^4 + 6*a^2*x^2
)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{a+bx^2}\,dx = \frac{x^6}{6\,b} - \frac{a\,x^4}{4\,b^2} - \frac{a^3\,\ln\left(b\,x^2+a\right)}{2\,b^4} + \frac{a^2\,x^2}{2\,b^3}$$

input
```
int(x^7/(a + b*x^2),x)
```

output
```
x^6/(6*b) - (a*x^4)/(4*b^2) - (a^3*log(a + b*x^2))/(2*b^4) + (a^2*x^2)/(2*
b^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \frac{x^7}{a+bx^2}\,dx = \frac{-6\log(b\,x^2+a)\,a^3+6a^2b\,x^2-3a\,b^2x^4+2b^3x^6}{12b^4}$$

input
```
int(x^7/(b*x^2+a),x)
```

output
```
( - 6*log(a + b*x**2)*a**3 + 6*a**2*b*x**2 - 3*a*b**2*x**4 + 2*b**3*x**6)/
(12*b**4)
```

3.127 $\int \frac{x^5}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{x^5}{a+bx^2} dx = -\frac{ax^2}{2b^2} + \frac{x^4}{4b} + \frac{a^2 \log(a+bx^2)}{2b^3}$$

output
```
-1/2*a*x^2/b^2+1/4*x^4/b+1/2*a^2*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{a+bx^2} dx = -\frac{ax^2}{2b^2} + \frac{x^4}{4b} + \frac{a^2 \log(a+bx^2)}{2b^3}$$

input
```
Integrate[x^5/(a + b*x^2),x]
```

output
```
-1/2*(a*x^2)/b^2 + x^4/(4*b) + (a^2*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^4}{bx^2+a}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2\,(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^2\log\left(a+bx^2\right)}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{2b}\right)$$

input
```
Int[x^5/(a + b*x^2),x]
```

output
```
(-((a*x^2)/b^2) + x^4/(2*b) + (a^2*Log[a + b*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{b^{2} x^{4}-2 a b \,x^{2}+2 a^{2} \ln \left(b \,x^{2}+a \right)}{4 b^{3}}$	34
default	$-\frac{-\frac{1}{2} b \,x^{4}+a \,x^{2}}{2 b^{2}}+\frac{a^{2} \ln \left(b \,x^{2}+a \right)}{2 b^{3}}$	35
norman	$-\frac{a \,x^{2}}{2 b^{2}}+\frac{x^{4}}{4 b}+\frac{a^{2} \ln \left(b \,x^{2}+a \right)}{2 b^{3}}$	35
risch	$\frac{x^{4}}{4 b}-\frac{a \,x^{2}}{2 b^{2}}+\frac{a^{2}}{4 b^{3}}+\frac{a^{2} \ln \left(b \,x^{2}+a \right)}{2 b^{3}}$	43

input
```
int(x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*x^4-2*a*b*x^2+2*a^2*ln(b*x^2+a))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a+bx^2}\,dx=\frac{b^2x^4-2\,abx^2+2\,a^2\log\left(bx^2+a\right)}{4\,b^3}$$

input
```
integrate(x^5/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*x^4 - 2*a*b*x^2 + 2*a^2*log(b*x^2 + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{a+bx^2}\,dx = \frac{a^2\log\left(a+bx^2\right)}{2b^3} - \frac{ax^2}{2b^2} + \frac{x^4}{4b}$$

input
```
integrate(x**5/(b*x**2+a),x)
```

output
```
a**2*log(a + b*x**2)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{x^5}{a+bx^2}\,dx = \frac{a^2\log\left(bx^2+a\right)}{2\,b^3} + \frac{bx^4-2\,ax^2}{4\,b^2}$$

input
```
integrate(x^5/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*a^2*log(b*x^2 + a)/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{x^5}{a+bx^2}\,dx = \frac{a^2\log\left(|bx^2+a|\right)}{2\,b^3} + \frac{bx^4-2\,ax^2}{4\,b^2}$$

input
```
integrate(x^5/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*a^2*log(abs(b*x^2 + a))/b^3 + 1/4*(b*x^4 - 2*a*x^2)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a+bx^2}\,dx = \frac{2\,a^2\,\ln\left(b\,x^2+a\right)+b^2\,x^4-2\,a\,b\,x^2}{4\,b^3}$$

input
```
int(x^5/(a + b*x^2),x)
```

output
```
(2*a^2*log(a + b*x^2) + b^2*x^4 - 2*a*b*x^2)/(4*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{a+bx^2}\,dx = \frac{2\log(b\,x^2+a)\,a^2-2ab\,x^2+b^2x^4}{4b^3}$$

input
```
int(x^5/(b*x^2+a),x)
```

output
```
(2*log(a + b*x**2)*a**2 - 2*a*b*x**2 + b**2*x**4)/(4*b**3)
```

3.128 $\int \frac{x^3}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{x^3}{a+bx^2} dx = \frac{x^2}{2b} - \frac{a \log(a+bx^2)}{2b^2}$$

output
```
1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a+bx^2} dx = \frac{x^2}{2b} - \frac{a \log(a+bx^2)}{2b^2}$$

input
```
Integrate[x^3/(a + b*x^2),x]
```

output
```
x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{bx^2+a}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{1}{b}-\frac{a}{b\left(bx^2+a\right)}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{x^2}{b}-\frac{a\log\left(a+bx^2\right)}{b^2}\right)$$

input
```
Int[x^3/(a + b*x^2),x]
```

output
```
(x^2/b - (a*Log[a + b*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$-\frac{-b\,x^{2}+a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	23
default	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24
norman	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24
risch	$\frac{x^{2}}{2b}-\frac{a\ln\left(b\,x^{2}+a\right)}{2b^{2}}$	24

input
```
int(x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-b*x^2+a*ln(b*x^2+a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{a+bx^2}\,dx=\frac{bx^2-a\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{a+bx^2}\,dx = -\frac{a\log(a+bx^2)}{2b^2} + \frac{x^2}{2b}$$

input
```
integrate(x**3/(b*x**2+a),x)
```

output
```
-a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{a+bx^2}\,dx = \frac{x^2}{2\,b} - \frac{a\log(bx^2+a)}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{a+bx^2}\,dx = \frac{x^2}{2\,b} - \frac{a\log(|bx^2+a|)}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{a+bx^2}\,dx=-\frac{a\,\ln\left(b\,x^2+a\right)-b\,x^2}{2\,b^2}$$

input
```
int(x^3/(a + b*x^2),x)
```

output
```
-(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^3}{a+bx^2}\,dx=\frac{-\log(b\,x^2+a)\,a+b\,x^2}{2b^2}$$

input
```
int(x^3/(b*x^2+a),x)
```

output
```
( - log(a + b*x**2)*a + b*x**2)/(2*b**2)
```

3.129 $\int \frac{x}{a+bx^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 15

$$\int \frac{x}{a+bx^2} dx = \frac{\log(a+bx^2)}{2b}$$

output
```
1/2*ln(b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{a+bx^2} dx = \frac{\log(a+bx^2)}{2b}$$

input
```
Integrate[x/(a + b*x^2),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+bx^2} \, dx$$
$$\Big\downarrow \; 240$$
$$\frac{\log \left(a+bx^2\right)}{2b}$$

input
```
Int[x/(a + b*x^2),x]
```

output
```
Log[a + b*x^2]/(2*b)
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\ln(b\,x^2+a)}{2b}$	14
default	$\frac{\ln(b\,x^2+a)}{2b}$	14
norman	$\frac{\ln(b\,x^2+a)}{2b}$	14
risch	$\frac{\ln(b\,x^2+a)}{2b}$	14
parallelrisch	$\frac{\ln(b\,x^2+a)}{2b}$	14

input
```
int(x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a+bx^2}\,dx=\frac{\log\left(bx^2+a\right)}{2\,b}$$

input
```
integrate(x/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{a+bx^2}\,dx = \frac{\log(a+bx^2)}{2b}$$

input
```
integrate(x/(b*x**2+a),x)
```

output
```
log(a + b*x**2)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a+bx^2}\,dx = \frac{\log(bx^2+a)}{2\,b}$$

input
```
integrate(x/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x}{a+bx^2}\,dx = \frac{\log(|bx^2+a|)}{2\,b}$$

input
```
integrate(x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a+bx^2}\,dx=\frac{\ln\left(b\,x^2+a\right)}{2\,b}$$

input
```
int(x/(a + b*x^2),x)
```

output
```
log(a + b*x^2)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{a+bx^2}\,dx=\frac{\log(b\,x^2+a)}{2b}$$

input
```
int(x/(b*x^2+a),x)
```

output
```
log(a + b*x**2)/(2*b)
```

3.130 $\int \frac{1}{x(a+bx^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{1}{x(a+bx^2)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^2)} dx = \frac{\log(x)}{a} - \frac{\log(a+bx^2)}{2a}$$

input
```
Integrate[1/(x*(a + b*x^2)),x]
```

output
```
Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2+a)}dx^2$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{x^2}dx^2}{a} - \frac{b \int \frac{1}{bx^2+a}dx^2}{a} \right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a} - \frac{b \int \frac{1}{bx^2+a}dx^2}{a} \right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2} \left(\frac{\log (x^2)}{a} - \frac{\log (a+bx^2)}{a} \right)$$

input
```
Int[1/(x*(a + b*x^2)),x]
```

output
```
(Log[x^2]/a - Log[a + b*x^2]/a)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
norman	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
risch	$\frac{\ln(x)}{a}-\frac{\ln(b\,x^2+a)}{2a}$	21
parallelrisch	$\frac{2\ln(x)-\ln(b\,x^2+a)}{2a}$	21

input
```
int(1/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2*ln(b*x^2+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a+bx^2)}\,dx = -\frac{\log(bx^2+a)-2\log(x)}{2a}$$

input
```
integrate(1/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(log(b*x^2 + a) - 2*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{1}{x(a+bx^2)}\,dx = \frac{\log(x)}{a} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a}$$

input
```
integrate(1/x/(b*x**2+a),x)
```

output
```
log(x)/a - log(a/b + x**2)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(a+bx^2)}\,dx = -\frac{\log(bx^2+a)}{2a} + \frac{\log(x^2)}{2a}$$

input
```
integrate(1/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + a)/a + 1/2*log(x^2)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(a+bx^2)}\,dx = \frac{\log(x^2)}{2a} - \frac{\log(|bx^2+a|)}{2a}$$

input
```
integrate(1/x/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x(a+bx^2)}\,dx = -\frac{\ln(b\,x^2+a) - 2\,\ln(x)}{2\,a}$$

input
```
int(1/(x*(a + b*x^2)),x)
```

output
```
-(log(a + b*x^2) - 2*log(x))/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(a+bx^2)}\,dx = \frac{-\log(b\,x^2+a) + 2\log(x)}{2a}$$

input
```
int(1/x/(b*x^2+a),x)
```

output
```
( - log(a + b*x**2) + 2*log(x))/(2*a)
```

3.131 $\int \frac{1}{x^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{x^3(a+bx^2)}\,dx = -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a+bx^2)}\,dx = -\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log(a+bx^2)}{2a^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)} \, dx$$
$$\Big\downarrow \ 243$$
$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)} dx^2$$
$$\Big\downarrow \ 54$$
$$\frac{1}{2} \int \left(\frac{b^2}{a^2 (bx^2+a)} - \frac{b}{a^2 x^2} + \frac{1}{a x^4}\right) dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2} \left(-\frac{b \log (x^2)}{a^2} + \frac{b \log (a+bx^2)}{a^2} - \frac{1}{a x^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)),x]
```

output
```
(-(1/(a*x^2)) - (b*Log[x^2])/a^2 + (b*Log[a + b*x^2])/a^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}}$	32
norman	$-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(b\,x^{2}+a\right)}{2a^{2}}$	32
parallelrisch	$-\frac{2b\ln\left(x\right)x^{2}-b\ln\left(b\,x^{2}+a\right)x^{2}+a}{2a^{2}x^{2}}$	33
risch	$-\frac{1}{2a\,x^{2}}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(-b\,x^{2}-a\right)}{2a^{2}}$	35

input
```
int(1/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\,(a+bx^2)}\,dx=\frac{bx^2\log\left(bx^2+a\right)-2\,bx^2\log\left(x\right)-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\left(a+bx^2\right)}\,dx=-\frac{1}{2ax^2}-\frac{b\log\left(x\right)}{a^2}+\frac{b\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(1/x**3/(b*x**2+a),x)
```

output
```
-1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\left(a+bx^2\right)}\,dx=\frac{b\log\left(bx^2+a\right)}{2\,a^2}-\frac{b\log\left(x^2\right)}{2\,a^2}-\frac{1}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + a)/a^2 - 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3\left(a+bx^2\right)}\,dx=-\frac{b\log\left(x^2\right)}{2\,a^2}+\frac{b\log\left(\left|bx^2+a\right|\right)}{2\,a^2}+\frac{bx^2-a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2
*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3\,(a+bx^2)}\,dx = \frac{b\,\ln\left(b\,x^2+a\right)}{2\,a^2} - \frac{1}{2\,a\,x^2} - \frac{b\,\ln\left(x\right)}{a^2}$$

input
```
int(1/(x^3*(a + b*x^2)),x)
```

output
```
(b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\,(a+bx^2)}\,dx = \frac{\log(b\,x^2+a)\,b\,x^2 - 2\log(x)\,b\,x^2 - a}{2a^2x^2}$$

input
```
int(1/x^3/(b*x^2+a),x)
```

output
```
(log(a + b*x**2)*b*x**2 - 2*log(x)*b*x**2 - a)/(2*a**2*x**2)
```

3.132 $\int \frac{1}{x^5(a+bx^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^5 (a+bx^2)} dx = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx^2)}{2a^3}$$

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5 (a+bx^2)} dx = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2 \log(x)}{a^3} - \frac{b^2 \log(a+bx^2)}{2a^3}$$

input
```
Integrate[1/(x^5*(a + b*x^2)),x]
```

output
```
-1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(
2*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2)} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^2+a)} dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(-\frac{b^3}{a^3 (bx^2+a)} + \frac{b^2}{a^3 x^2} - \frac{b}{a^2 x^4} + \frac{1}{a x^6}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{b^2 \log(x^2)}{a^3} - \frac{b^2 \log(a+bx^2)}{a^3} + \frac{b}{a^2 x^2} - \frac{1}{2 a x^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)),x]
```

output
```
(-1/2*1/(a*x^4) + b/(a^2*x^2) + (b^2*Log[x^2])/a^3 - (b^2*Log[a + b*x^2])/
a^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{1}{4 a \,x^{4}}+\frac{b}{2 a^{2} x^{2}}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(b \,x^{2}+a \right)}{2 a^{3}}$	44
norman	$\frac{-\frac{1}{4 a}+\frac{b \,x^{2}}{2 a^{2}}}{x^{4}}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(b \,x^{2}+a \right)}{2 a^{3}}$	46
risch	$\frac{-\frac{1}{4 a}+\frac{b \,x^{2}}{2 a^{2}}}{x^{4}}+\frac{b^{2} \ln \left(x \right)}{a^{3}}-\frac{b^{2} \ln \left(b \,x^{2}+a \right)}{2 a^{3}}$	46
parallelrisch	$\frac{4 b^{2} \ln \left(x \right) x^{4}-2 b^{2} \ln \left(b \,x^{2}+a \right) x^{4}+2 a b \,x^{2}-a^{2}}{4 a^{3} x^{4}}$	48

input
```
int(1/x^5/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^5 (a+bx^2)} \, dx=-\frac{2 \, b^{2} x^{4} \log \left(b x^{2} + a\right) - 4 \, b^{2} x^{4} \log \left(x\right) - 2 \, a b x^{2} + a^{2}}{4 \, a^{3} x^{4}}$$

input
```
integrate(1/x^5/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*
x^4)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 (a+bx^2)}\,dx = \frac{-a+2bx^2}{4a^2x^4} + \frac{b^2\log(x)}{a^3} - \frac{b^2\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

input
```
integrate(1/x**5/(b*x**2+a),x)
```

output
```
(-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2
*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5 (a+bx^2)}\,dx = -\frac{b^2\log(bx^2+a)}{2\,a^3} + \frac{b^2\log(x^2)}{2\,a^3} + \frac{2\,bx^2-a}{4\,a^2x^4}$$

input
```
integrate(1/x^5/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*b^2*log(b*x^2 + a)/a^3 + 1/2*b^2*log(x^2)/a^3 + 1/4*(2*b*x^2 - a)/(a^
2*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^5 (a+bx^2)}\,dx = \frac{b^2\log(x^2)}{2\,a^3} - \frac{b^2\log(|bx^2+a|)}{2\,a^3} - \frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

input
```
integrate(1/x^5/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 -
2*a*b*x^2 + a^2)/(a^3*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^5 (a+bx^2)} \, dx = \frac{b^2 \ln(x)}{a^3} - \frac{b^2 \ln(b\,x^2+a)}{2\,a^3} - \frac{\frac{1}{4\,a} - \frac{b\,x^2}{2\,a^2}}{x^4}$$

input
```
int(1/(x^5*(a + b*x^2)),x)
```

output
```
(b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^
2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5 (a+bx^2)} \, dx = \frac{-2\log(b\,x^2+a)\,b^2x^4 + 4\log(x)\,b^2x^4 - a^2 + 2ab\,x^2}{4a^3x^4}$$

input
```
int(1/x^5/(b*x^2+a),x)
```

output
```
( - 2*log(a + b*x**2)*b**2*x**4 + 4*log(x)*b**2*x**4 - a**2 + 2*a*b*x**2)/
(4*a**3*x**4)
```

3.133 $\int \frac{1}{x^7(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 63

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=-\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}$$

output
```
-1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)
/a^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=-\frac{1}{6ax^6}+\frac{b}{4a^2x^4}-\frac{b^2}{2a^3x^2}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log\left(a+bx^2\right)}{2a^4}$$

input
```
Integrate[1/(x^7*(a + b*x^2)),x]
```

output
```
-1/6*1/(a*x^6) + b/(4*a^2*x^4) - b^2/(2*a^3*x^2) - (b^3*Log[x])/a^4 + (b^3
*Log[a + b*x^2])/(2*a^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^2+a)} dx^2$$

$$\Big\downarrow \ 54$$

$$\frac{1}{2} \int \left(\frac{b^4}{a^4 (bx^2+a)} - \frac{b^3}{a^4 x^2} + \frac{b^2}{a^3 x^4} - \frac{b}{a^2 x^6} + \frac{1}{a x^8}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(-\frac{b^3 \log(x^2)}{a^4} + \frac{b^3 \log(a+bx^2)}{a^4} - \frac{b^2}{a^3 x^2} + \frac{b}{2a^2 x^4} - \frac{1}{3a x^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^2)),x]
```

output
```
(-1/3*1/(a*x^6) + b/(2*a^2*x^4) - b^2/(a^3*x^2) - (b^3*Log[x^2])/a^4 + (b^
3*Log[a + b*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{1}{6a\,x^{6}}+\frac{b}{4a^{2}x^{4}}-\frac{b^{2}}{2a^{3}x^{2}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{4}}$	56
norman	$\frac{-\frac{1}{6a}+\frac{b\,x^{2}}{4a^{2}}-\frac{b^{2}x^{4}}{2a^{3}}}{x^{6}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(b\,x^{2}+a\right)}{2a^{4}}$	58
parallelrisch	$-\frac{12b^{3}\ln\left(x\right)x^{6}-6b^{3}\ln\left(b\,x^{2}+a\right)x^{6}+6a\,b^{2}x^{4}-3a^{2}b\,x^{2}+2a^{3}}{12a^{4}x^{6}}$	59
risch	$\frac{-\frac{1}{6a}+\frac{b\,x^{2}}{4a^{2}}-\frac{b^{2}x^{4}}{2a^{3}}}{x^{6}}-\frac{b^{3}\ln\left(x\right)}{a^{4}}+\frac{b^{3}\ln\left(-b\,x^{2}-a\right)}{2a^{4}}$	61

input
```
int(1/x^7/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/a/x^6+1/4*b/a^2/x^4-1/2*b^2/a^3/x^2-b^3*ln(x)/a^4+1/2*b^3*ln(b*x^2+a)
/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=\frac{6\,b^3x^6\log\left(bx^2+a\right)-12\,b^3x^6\log\left(x\right)-6\,ab^2x^4+3\,a^2bx^2-2\,a^3}{12\,a^4x^6}$$

input
```
integrate(1/x^7/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/12*(6*b^3*x^6*log(b*x^2 + a) - 12*b^3*x^6*log(x) - 6*a*b^2*x^4 + 3*a^2*b
*x^2 - 2*a^3)/(a^4*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7 (a + bx^2)} \, dx = \frac{-2a^2 + 3abx^2 - 6b^2x^4}{12a^3x^6} - \frac{b^3 \log(x)}{a^4} + \frac{b^3 \log\left(\frac{a}{b} + x^2\right)}{2a^4}$$

input
```
integrate(1/x**7/(b*x**2+a),x)
```

output
```
(-2*a**2 + 3*a*b*x**2 - 6*b**2*x**4)/(12*a**3*x**6) - b**3*log(x)/a**4 + b
**3*log(a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7 (a + bx^2)} \, dx = \frac{b^3 \log(bx^2 + a)}{2 \, a^4} - \frac{b^3 \log(x^2)}{2 \, a^4} - \frac{6 \, b^2x^4 - 3 \, abx^2 + 2 \, a^2}{12 \, a^3x^6}$$

input
```
integrate(1/x^7/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*b^3*log(b*x^2 + a)/a^4 - 1/2*b^3*log(x^2)/a^4 - 1/12*(6*b^2*x^4 - 3*a*
b*x^2 + 2*a^2)/(a^3*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=-\frac{b^3\log\left(x^2\right)}{2\,a^4}+\frac{b^3\log\left(\left|bx^2+a\right|\right)}{2\,a^4}+\frac{11\,b^3x^6-6\,ab^2x^4+3\,a^2bx^2-2\,a^3}{12\,a^4x^6}$$

input
```
integrate(1/x^7/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*b^3*log(x^2)/a^4 + 1/2*b^3*log(abs(b*x^2 + a))/a^4 + 1/12*(11*b^3*x^6
 - 6*a*b^2*x^4 + 3*a^2*b*x^2 - 2*a^3)/(a^4*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=\frac{b^3\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{\frac{1}{6\,a}-\frac{b\,x^2}{4\,a^2}+\frac{b^2\,x^4}{2\,a^3}}{x^6}-\frac{b^3\,\ln\left(x\right)}{a^4}$$

input
```
int(1/(x^7*(a + b*x^2)),x)
```

output
```
(b^3*log(a + b*x^2))/(2*a^4) - (1/(6*a) - (b*x^2)/(4*a^2) + (b^2*x^4)/(2*a
^3))/x^6 - (b^3*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^7\left(a+bx^2\right)}\,dx=\frac{6\log\left(b\,x^2+a\right)b^3x^6-12\log\left(x\right)b^3x^6-2a^3+3a^2b\,x^2-6a\,b^2x^4}{12a^4x^6}$$

input
```
int(1/x^7/(b*x^2+a),x)
```

output
```
(6*log(a + b*x**2)*b**3*x**6 - 12*log(x)*b**3*x**6 - 2*a**3 + 3*a**2*b*x**
2 - 6*a*b**2*x**4)/(12*a**4*x**6)
```

3.134 $\int \frac{1}{x^9(a+bx^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{1}{x^9(a+bx^2)} dx = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx^2)}{2a^5}$$

output
```
-1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2
*b^4*ln(b*x^2+a)/a^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^9(a+bx^2)} dx = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4\log(x)}{a^5} - \frac{b^4\log(a+bx^2)}{2a^5}$$

input
```
Integrate[1/(x^9*(a + b*x^2)),x]
```

output
```
-1/8*1/(a*x^8) + b/(6*a^2*x^6) - b^2/(4*a^3*x^4) + b^3/(2*a^4*x^2) + (b^4*
Log[x])/a^5 - (b^4*Log[a + b*x^2])/(2*a^5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^2)} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int \frac{1}{x^{10} (bx^2+a)} dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2} \int \left(-\frac{b^5}{a^5 (bx^2+a)} + \frac{b^4}{a^5 x^2} - \frac{b^3}{a^4 x^4} + \frac{b^2}{a^3 x^6} - \frac{b}{a^2 x^8} + \frac{1}{a x^{10}}\right) dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{b^4 \log(x^2)}{a^5} - \frac{b^4 \log(a+bx^2)}{a^5} + \frac{b^3}{a^4 x^2} - \frac{b^2}{2a^3 x^4} + \frac{b}{3a^2 x^6} - \frac{1}{4a x^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^2)),x]
```

output
```
(-1/4*1/(a*x^8) + b/(3*a^2*x^6) - b^2/(2*a^3*x^4) + b^3/(a^4*x^2) + (b^4*L
og[x^2])/a^5 - (b^4*Log[a + b*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{8a\,x^{8}}+\frac{b}{6a^{2}x^{6}}-\frac{b^{2}}{4a^{3}x^{4}}+\frac{b^{3}}{2a^{4}x^{2}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{5}}$	66
norman	$\frac{-\frac{1}{8a}+\frac{b\,x^{2}}{6a^{2}}-\frac{b^{2}x^{4}}{4a^{3}}+\frac{b^{3}x^{6}}{2a^{4}}}{x^{8}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{5}}$	68
risch	$\frac{-\frac{1}{8a}+\frac{b\,x^{2}}{6a^{2}}-\frac{b^{2}x^{4}}{4a^{3}}+\frac{b^{3}x^{6}}{2a^{4}}}{x^{8}}+\frac{b^{4}\ln\left(x\right)}{a^{5}}-\frac{b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{5}}$	68
parallelrisch	$\frac{24b^{4}\ln\left(x\right)x^{8}-12b^{4}\ln\left(b\,x^{2}+a\right)x^{8}+12a\,b^{3}x^{6}-6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}-3a^{4}}{24a^{5}x^{8}}$	70

input
```
int(1/x^9/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8/a/x^8+1/6*b/a^2/x^6-1/4*b^2/a^3/x^4+1/2*b^3/a^4/x^2+b^4*ln(x)/a^5-1/2
*b^4*ln(b*x^2+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^9\,(a+bx^2)}\,dx = -\frac{12\,b^4x^8\log\left(bx^2+a\right)-24\,b^4x^8\log\left(x\right)-12\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+3\,a^4}{24\,a^5x^8}$$

input
```
integrate(1/x^9/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/24*(12*b^4*x^8*log(b*x^2 + a) - 24*b^4*x^8*log(x) - 12*a*b^3*x^6 + 6*a^
2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^9 (a+bx^2)} dx = \frac{-3a^3 + 4a^2bx^2 - 6ab^2x^4 + 12b^3x^6}{24a^4x^8} + \frac{b^4 \log(x)}{a^5} - \frac{b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^5}$$

input
```
integrate(1/x**9/(b*x**2+a),x)
```

output
```
(-3*a**3 + 4*a**2*b*x**2 - 6*a*b**2*x**4 + 12*b**3*x**6)/(24*a**4*x**8) +
b**4*log(x)/a**5 - b**4*log(a/b + x**2)/(2*a**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^9 (a+bx^2)} dx = -\frac{b^4 \log(bx^2 + a)}{2a^5} + \frac{b^4 \log(x^2)}{2a^5} + \frac{12 b^3x^6 - 6 ab^2x^4 + 4 a^2bx^2 - 3 a^3}{24 a^4x^8}$$

input
```
integrate(1/x^9/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*b^4*log(b*x^2 + a)/a^5 + 1/2*b^4*log(x^2)/a^5 + 1/24*(12*b^3*x^6 - 6*
a*b^2*x^4 + 4*a^2*b*x^2 - 3*a^3)/(a^4*x^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^9 (a + bx^2)} \, dx = \frac{b^4 \log (x^2)}{2 a^5} - \frac{b^4 \log (|bx^2 + a|)}{2 a^5} - \frac{25 b^4 x^8 - 12 ab^3 x^6 + 6 a^2 b^2 x^4 - 4 a^3 b x^2 + 3 a^4}{24 a^5 x^8}$$

input
```
integrate(1/x^9/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*b^4*log(x^2)/a^5 - 1/2*b^4*log(abs(b*x^2 + a))/a^5 - 1/24*(25*b^4*x^8
- 12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + 3*a^4)/(a^5*x^8)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^9 (a + bx^2)} \, dx = \frac{b^4 \ln (x)}{a^5} - \frac{b^4 \ln (b x^2 + a)}{2 a^5} - \frac{\frac{1}{8 a} - \frac{b x^2}{6 a^2} + \frac{b^2 x^4}{4 a^3} - \frac{b^3 x^6}{2 a^4}}{x^8}$$

input
```
int(1/(x^9*(a + b*x^2)),x)
```

output
```
(b^4*log(x))/a^5 - (b^4*log(a + b*x^2))/(2*a^5) - (1/(8*a) - (b*x^2)/(6*a^
2) + (b^2*x^4)/(4*a^3) - (b^3*x^6)/(2*a^4))/x^8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^9 (a + bx^2)} \, dx = \frac{-12 \log(b x^2 + a) b^4 x^8 + 24 \log(x) b^4 x^8 - 3a^4 + 4a^3 b x^2 - 6a^2 b^2 x^4 + 12a b^3 x^6}{24a^5 x^8}$$

input
```
int(1/x^9/(b*x^2+a),x)
```

output
```
( - 12*log(a + b*x**2)*b**4*x**8 + 24*log(x)*b**4*x**8 - 3*a**4 + 4*a**3*b
*x**2 - 6*a**2*b**2*x**4 + 12*a*b**3*x**6)/(24*a**5*x**8)
```

3.135 $\int \frac{x^{10}}{a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 81

$$\int \frac{x^{10}}{a+bx^2}\,dx=\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

output
```
a^4*x/b^5-1/3*a^3*x^3/b^4+1/5*a^2*x^5/b^3-1/7*a*x^7/b^2+1/9*x^9/b-a^(9/2)*
arctan(b^(1/2)*x/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{x^{10}}{a+bx^2}\,dx=\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}-\frac{a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[x^10/(a + b*x^2),x]
```

output
```
(a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^
9/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{a+bx^2}\,dx$$
$$\Big\downarrow \ 254$$
$$\int \left(-\frac{a^5}{b^5\,(a+bx^2)}+\frac{a^4}{b^5}-\frac{a^3x^2}{b^4}+\frac{a^2x^4}{b^3}-\frac{ax^6}{b^2}+\frac{x^8}{b}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}$$

input
```
Int[x^10/(a + b*x^2),x]
```

output
```
(a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^5)/(5*b^3) - (a*x^7)/(7*b^2) + x^
9/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\frac{1}{9}b^4x^9-\frac{1}{7}a\,b^3x^7+\frac{1}{5}a^2b^2x^5-\frac{1}{3}a^3b\,x^3+a^4x}{b^5}-\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^5\sqrt{ab}}$	71
risch	$\frac{x^9}{9b}-\frac{a\,x^7}{7b^2}+\frac{a^2x^5}{5b^3}-\frac{a^3x^3}{3b^4}+\frac{a^4x}{b^5}+\frac{\sqrt{-ab}\,a^4\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^6}-\frac{\sqrt{-ab}\,a^4\ln\left(\sqrt{-ab}\,x-a\right)}{2b^6}$	104

input
```
int(x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/9*b^4*x^9-1/7*a*b^3*x^7+1/5*a^2*b^2*x^5-1/3*a^3*b*x^3+a^4*x)-a^5/
b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int \frac{x^{10}}{a+bx^2}\,dx$$
$$=\left[\frac{70\,b^4x^9-90\,ab^3x^7+126\,a^2b^2x^5-210\,a^3bx^3+315\,a^4\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+630\,a^4x}{630\,b^5},\frac{35\,b^4x^9-4}{}\right.$$

input
```
integrate(x^10/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/630*(70*b^4*x^9 - 90*a*b^3*x^7 + 126*a^2*b^2*x^5 - 210*a^3*b*x^3 + 315*
a^4*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 630*a^4*x
)/b^5, 1/315*(35*b^4*x^9 - 45*a*b^3*x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 -
 315*a^4*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 315*a^4*x)/b^5]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.47

$$\int \frac{x^{10}}{a+bx^2}\,dx = \frac{a^4x}{b^5} - \frac{a^3x^3}{3b^4} + \frac{a^2x^5}{5b^3} - \frac{ax^7}{7b^2} + \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} - \frac{\sqrt{-\frac{a^9}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^9}{b^{11}}}}{a^4}\right)}{2} + \frac{x^9}{9b}$$

input
```
integrate(x**10/(b*x**2+a),x)
```

output
```
a**4*x/b**5 - a**3*x**3/(3*b**4) + a**2*x**5/(5*b**3) - a*x**7/(7*b**2) +
sqrt(-a**9/b**11)*log(x - b**5*sqrt(-a**9/b**11)/a**4)/2 - sqrt(-a**9/b**1
1)*log(x + b**5*sqrt(-a**9/b**11)/a**4)/2 + x**9/(9*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{x^{10}}{a+bx^2}\,dx = -\frac{a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^4x^9 - 45\,ab^3x^7 + 63\,a^2b^2x^5 - 105\,a^3bx^3 + 315\,a^4x}{315\,b^5}$$

input
```
integrate(x^10/(b*x^2+a),x, algorithm="maxima")
```

output
```
-a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^4*x^9 - 45*a*b^3*
x^7 + 63*a^2*b^2*x^5 - 105*a^3*b*x^3 + 315*a^4*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x^{10}}{a+bx^2}\,dx = -\frac{a^5 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8x^9 - 45\,ab^7x^7 + 63\,a^2b^6x^5 - 105\,a^3b^5x^3 + 315\,a^4b^4x}{315\,b^9}$$

input
```
integrate(x^10/(b*x^2+a),x, algorithm="giac")
```

output
```
-a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/315*(35*b^8*x^9 - 45*a*b^7*
x^7 + 63*a^2*b^6*x^5 - 105*a^3*b^5*x^3 + 315*a^4*b^4*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{x^{10}}{a+bx^2}\,dx = \frac{x^9}{9\,b} - \frac{a\,x^7}{7\,b^2} + \frac{a^4\,x}{b^5} - \frac{a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}} + \frac{a^2\,x^5}{5\,b^3} - \frac{a^3\,x^3}{3\,b^4}$$

input
```
int(x^10/(a + b*x^2),x)
```

output
```
x^9/(9*b) - (a*x^7)/(7*b^2) + (a^4*x)/b^5 - (a^(9/2)*atan((b^(1/2)*x)/a^(1
/2)))/b^(11/2) + (a^2*x^5)/(5*b^3) - (a^3*x^3)/(3*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{x^{10}}{a+bx^2}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4 + 315a^4bx - 105a^3b^2x^3 + 63a^2b^3x^5 - 45a\,b^4x^7 + 35b^5x^9}{315b^6}$$

input
```
int(x^10/(b*x^2+a),x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 315*a**4*b*x
- 105*a**3*b**2*x**3 + 63*a**2*b**3*x**5 - 45*a*b**4*x**7 + 35*b**5*x**9)/
(315*b**6)
```

3.136 $\int \frac{x^8}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{x^8}{a+bx^2} dx = -\frac{a^3x}{b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b} + \frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
-a^3*x/b^4+1/3*a^2*x^3/b^3-1/5*a*x^5/b^2+1/7*x^7/b+a^(7/2)*arctan(b^(1/2)*
x/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{x^8}{a+bx^2} dx = -\frac{a^3x}{b^4} + \frac{a^2x^3}{3b^3} - \frac{ax^5}{5b^2} + \frac{x^7}{7b} + \frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[x^8/(a + b*x^2),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{a+bx^2}dx$$

$$\Big\downarrow \text{ 254}$$

$$\int \left(\frac{a^4}{b^4 (a+bx^2)}-\frac{a^3}{b^4}+\frac{a^2x^2}{b^3}-\frac{ax^4}{b^2}+\frac{x^6}{b}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}$$

input
```
Int[x^8/(a + b*x^2),x]
```

output
```
-((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{-\frac{1}{7}b^3x^7+\frac{1}{5}a\,b^2x^5-\frac{1}{3}a^2b\,x^3+a^3x}{b^4}+\frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^4\sqrt{ab}}$	60
risch	$\frac{x^7}{7b}-\frac{a\,x^5}{5b^2}+\frac{a^2x^3}{3b^3}-\frac{a^3x}{b^4}+\frac{\sqrt{-ab}\,a^3\ln\left(-\sqrt{-ab}\,x+a\right)}{2b^5}-\frac{\sqrt{-ab}\,a^3\ln\left(\sqrt{-ab}\,x+a\right)}{2b^5}$	90

input
```
int(x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/7*b^3*x^7+1/5*a*b^2*x^5-1/3*a^2*b*x^3+a^3*x)+a^4/b^4/(a*b)^(1/2
)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.18

$$\int \frac{x^8}{a+bx^2}\,dx = \left[\frac{30\,b^3x^7-42\,ab^2x^5+70\,a^2bx^3+105\,a^3\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-210\,a^3x}{210\,b^4},\frac{15\,b^3x^7-21\,ab^2x^5+35\,a\right.$$

input
```
integrate(x^8/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*b^3*x^7 - 42*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*sqrt(-a/b)*log(
(b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 210*a^3*x)/b^4, 1/105*(15*b^
3*x^7 - 21*a*b^2*x^5 + 35*a^2*b*x^3 + 105*a^3*sqrt(a/b)*arctan(b*x*sqrt(a/
b)/a) - 105*a^3*x)/b^4]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.57

$$\int \frac{x^8}{a+bx^2}\,dx = -\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}-\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x-\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{\sqrt{-\frac{a^7}{b^9}}\log\left(x+\frac{b^4\sqrt{-\frac{a^7}{b^9}}}{a^3}\right)}{2}+\frac{x^7}{7b}$$

input
```
integrate(x**8/(b*x**2+a),x)
```

output
```
-a**3*x/b**4 + a**2*x**3/(3*b**3) - a*x**5/(5*b**2) - sqrt(-a**7/b**9)*log
(x - b**4*sqrt(-a**7/b**9)/a**3)/2 + sqrt(-a**7/b**9)*log(x + b**4*sqrt(-a
**7/b**9)/a**3)/2 + x**7/(7*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^8}{a+bx^2}\,dx = \frac{a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4}+\frac{15\,b^3x^7-21\,ab^2x^5+35\,a^2bx^3-105\,a^3x}{105\,b^4}$$

input
```
integrate(x^8/(b*x^2+a),x, algorithm="maxima")
```

output
```
a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^3*x^7 - 21*a*b^2*x
^5 + 35*a^2*b*x^3 - 105*a^3*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{a+bx^2}\,dx = \frac{a^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6x^7 - 21\,ab^5x^5 + 35\,a^2b^4x^3 - 105\,a^3b^3x}{105\,b^7}$$

input
```
integrate(x^8/(b*x^2+a),x, algorithm="giac")
```

output
```
a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/105*(15*b^6*x^7 - 21*a*b^5*x
^5 + 35*a^2*b^4*x^3 - 105*a^3*b^3*x)/b^7
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \frac{x^8}{a+bx^2}\,dx = \frac{x^7}{7\,b} - \frac{a\,x^5}{5\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}} + \frac{a^2\,x^3}{3\,b^3}$$

input
```
int(x^8/(a + b*x^2),x)
```

output
```
x^7/(7*b) - (a*x^5)/(5*b^2) - (a^3*x)/b^4 + (a^(7/2)*atan((b^(1/2)*x)/a^(1
/2)))/b^(9/2) + (a^2*x^3)/(3*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{a+bx^2}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3 - 105a^3bx + 35a^2b^2x^3 - 21a\,b^3x^5 + 15b^4x^7}{105b^5}$$

input
```
int(x^8/(b*x^2+a),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 - 105*a**3*b*x + 3
5*a**2*b**2*x**3 - 21*a*b**3*x**5 + 15*b**4*x**7)/(105*b**5)
```

3.137 $\int \frac{x^6}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{x^6}{a+bx^2} dx = \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

output
```
a^2*x/b^3-1/3*a*x^3/b^2+1/5*x^5/b-a^(5/2)*arctan(b^(1/2)*x/a^(1/2))/b^(7/2
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{a+bx^2} dx = \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{x^5}{5b} - \frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2),x]
```

output
```
(a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{a+bx^2} \, dx$$

$$\Big\downarrow \text{254}$$

$$\int \left(-\frac{a^3}{b^3 (a+bx^2)}+\frac{a^2}{b^3}-\frac{ax^2}{b^2}+\frac{x^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{5/2} \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2 x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}$$

input
```
Int[x^6/(a + b*x^2),x]
```

output
```
(a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/b^(7/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-\frac{1}{3}ab\,x^3+a^2x}{b^3}-\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$	49
risch	$\frac{x^5}{5b}-\frac{a\,x^3}{3b^2}+\frac{a^2x}{b^3}+\frac{\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^4}-\frac{\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{2b^4}$	82

input
```
int(x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*b^2*x^5-1/3*a*b*x^3+a^2*x)-a^3/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int\frac{x^6}{a+bx^2}\,dx = \left[\frac{6\,b^2x^5-10\,abx^3+15\,a^2\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+30\,a^2x}{30\,b^3},\ \frac{3\,b^2x^5-5\,abx^3-15\,a^2\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right.}{15\,b^3}\right.$$

input
```
integrate(x^6/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*b^2*x^5 - 10*a*b*x^3 + 15*a^2*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(
-a/b) - a)/(b*x^2 + a)) + 30*a^2*x)/b^3, 1/15*(3*b^2*x^5 - 5*a*b*x^3 - 15*
a^2*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 15*a^2*x)/b^3]
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int \frac{x^6}{a+bx^2}\,dx = \frac{a^2x}{b^3} - \frac{ax^3}{3b^2} + \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x - \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} - \frac{\sqrt{-\frac{a^5}{b^7}}\log\left(x + \frac{b^3\sqrt{-\frac{a^5}{b^7}}}{a^2}\right)}{2} + \frac{x^5}{5b}$$

input
```
integrate(x**6/(b*x**2+a),x)
```

output
```
a**2*x/b**3 - a*x**3/(3*b**2) + sqrt(-a**5/b**7)*log(x - b**3*sqrt(-a**5/b
**7)/a**2)/2 - sqrt(-a**5/b**7)*log(x + b**3*sqrt(-a**5/b**7)/a**2)/2 + x*
*5/(5*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{a+bx^2}\,dx = -\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2x^5 - 5\,abx^3 + 15\,a^2x}{15\,b^3}$$

input
```
integrate(x^6/(b*x^2+a),x, algorithm="maxima")
```

output
```
-a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^2*x^5 - 5*a*b*x^3 +
 15*a^2*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{a+bx^2}\,dx = -\frac{a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^4x^5 - 5\,ab^3x^3 + 15\,a^2b^2x}{15\,b^5}$$

input
```
integrate(x^6/(b*x^2+a),x, algorithm="giac")
```

output
```
-a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/15*(3*b^4*x^5 - 5*a*b^3*x^3
 + 15*a^2*b^2*x)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int \frac{x^6}{a+bx^2}\,dx = \frac{x^5}{5\,b} - \frac{a\,x^3}{3\,b^2} + \frac{a^2\,x}{b^3} - \frac{a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}$$

input
```
int(x^6/(a + b*x^2),x)
```

output
```
x^5/(5*b) - (a*x^3)/(3*b^2) + (a^2*x)/b^3 - (a^(5/2)*atan((b^(1/2)*x)/a^(1
/2)))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{a+bx^2}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2 + 15a^2bx - 5a\,b^2x^3 + 3b^3x^5}{15b^4}$$

input
```
int(x^6/(b*x^2+a),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 15*a**2*b*x -
5*a*b**2*x**3 + 3*b**3*x**5)/(15*b**4)
```

3.138 $\int \frac{x^4}{a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{x^4}{a+bx^2}\,dx=-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

output
```
-a*x/b^2+1/3*x^3/b+a^(3/2)*arctan(b^(1/2)*x/a^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+bx^2}\,dx=-\frac{ax}{b^2}+\frac{x^3}{3b}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2),x]
```

output
```
-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{a+bx^2}\,dx$$
$$\Big\downarrow \ 254$$
$$\int \left(\frac{a^2}{b^2\left(a+bx^2\right)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}$$

input
```
Int[x^4/(a + b*x^2),x]
```

output
```
-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)
```

Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{a x-\frac{1}{3} b \,x^{3}}{b^{2}}+\frac{a^{2} \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b^{2} \sqrt{a b}}$	38
risch	$\frac{x^{3}}{3 b}-\frac{a x}{b^{2}}+\frac{\sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x+a\right)}{2 b^{3}}-\frac{\sqrt{-a b}\, a \ln \left(\sqrt{-a b}\, x+a\right)}{2 b^{3}}$	64

input
```
int(x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(a*x-1/3*b*x^3)+a^2/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.36

$$\int \frac{x^4}{a+bx^2}\,dx = \left[\frac{2\,bx^{3}+3\,a\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,ax}{6\,b^{2}}, \frac{bx^{3}+3\,a\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,ax}{3\,b^{2}}\right]$$

input
```
integrate(x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*b*x^3 + 3*a*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 +
 a)) - 6*a*x)/b^2, 1/3*(b*x^3 + 3*a*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*
a*x)/b^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{x^4}{a+bx^2}\,dx = -\frac{ax}{b^2} - \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x - \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{\sqrt{-\frac{a^3}{b^5}}\log\left(x + \frac{b^2\sqrt{-\frac{a^3}{b^5}}}{a}\right)}{2} + \frac{x^3}{3b}$$

input
```
integrate(x**4/(b*x**2+a),x)
```

output
```
-a*x/b**2 - sqrt(-a**3/b**5)*log(x - b**2*sqrt(-a**3/b**5)/a)/2 + sqrt(-a*
*3/b**5)*log(x + b**2*sqrt(-a**3/b**5)/a)/2 + x**3/(3*b)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{a+bx^2}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{bx^3 - 3\,ax}{3\,b^2}$$

input
```
integrate(x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3 - 3*a*x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x^4}{a+bx^2}\,dx = \frac{a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{b^2x^3 - 3\,abx}{3\,b^3}$$

input
```
integrate(x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*x^3 - 3*a*b*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{a+bx^2}\,dx=\frac{x^3}{3\,b}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a\,x}{b^2}$$

input
```
int(x^4/(a + b*x^2),x)
```

output
```
x^3/(3*b) + (a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(5/2) - (a*x)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{a+bx^2}\,dx=\frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a-3abx+b^2x^3}{3b^3}$$

input
```
int(x^4/(b*x^2+a),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a - 3*a*b*x + b**2*x**3)/
(3*b**3)
```

3.139 $\int \frac{x^2}{a+bx^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{x^2}{a+bx^2} dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
x/b-a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+bx^2} dx = \frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+bx^2} \, dx$$
$$\Big\downarrow \; 262$$
$$\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}$$
$$\Big\downarrow \; 218$$
$$\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Int[x^2/(a + b*x^2),x]
```

output
```
x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{x}{b}-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$\frac{x}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{2b^2}-\frac{\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{2b^2}$	56

input
```
int(x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
x/b-a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.65

$$\int \frac{x^2}{a+bx^2}\,dx=\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

input
```
integrate(x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
 -(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.81

$$\int \frac{x^2}{a+bx^2}\,dx = \frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2} + \frac{x}{b}$$

input
```
integrate(x**2/(b*x**2+a),x)
```

output
```
sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b*
*3) + x)/2 + x/b
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx^2}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx^2}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{x}{b}$$

input
```
integrate(x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{a+bx^2} \, dx = \frac{x}{b} - \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int(x^2/(a + b*x^2),x)
```

output
```
x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{a+bx^2} \, dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)+bx}{b^2}$$

input
```
int(x^2/(b*x^2+a),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))) + b*x)/b**2
```

3.140 $\int \frac{1}{a+bx^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 24

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+bx^2} dx = \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+bx^2} \, dx$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a + b*x^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$-\frac{\ln\left(bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{2\sqrt{-ab}}$	41

input
```
int(1/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{1}{a+bx^2}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt
(a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{1}{a+bx^2}\,dx = -\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-
1/(a*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="maxima")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{1}{a+bx^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

input
```
integrate(1/(b*x^2+a),x, algorithm="giac")
```

output
```
arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a+bx^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2),x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{1}{a+bx^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{ab}$$

input
```
int(1/(b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b)
```

3.141 $\int \frac{1}{x^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{1}{x^2(a+bx^2)}\,dx=-\frac{1}{ax}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-1/a/x-b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^2)}\,dx=-\frac{1}{ax}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a+bx^2)}\,dx$$
$$\Big\downarrow \text{264}$$
$$-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}$$
$$\Big\downarrow \text{218}$$
$$-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)),x]
```

output
```
-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	30
risch	$-\frac{1}{ax}+\frac{\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{2a^2}-\frac{\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{2a^2}$	58

input
```
int(1/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-b/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^2\,(a+bx^2)}\,dx = \left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

input
```
integrate(1/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a
*x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.91

$$\int \frac{1}{x^2(a+bx^2)}\,dx = \frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2} - \frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2} - \frac{1}{ax}$$

input
```
integrate(1/x**2/(b*x**2+a),x)
```

output
```
sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sq
rt(-b/a**3)/b + x)/2 - 1/(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(a+bx^2)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2(a+bx^2)}\,dx = -\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2 (a + bx^2)} dx = -\frac{1}{a\,x} - \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x^2*(a + b*x^2)),x)
```

output
```
- 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2 (a + bx^2)} dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)x - a}{a^2 x}$$

input
```
int(1/x^2/(b*x^2+a),x)
```

output
```
( - (sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*x + a))/(a**2*x)
```

3.142 $\int \frac{1}{x^4(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{1}{x^4(a+bx^2)}\,dx = -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
-1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx^2)}\,dx = -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a + bx^2)} \, dx$$

$$\Big\downarrow \ 264$$

$$-\frac{b \int \frac{1}{x^2 (bx^2+a)} dx}{a} - \frac{1}{3ax^3}$$

$$\Big\downarrow \ 264$$

$$-\frac{b \left(-\frac{b \int \frac{1}{bx^2+a} dx}{a} - \frac{1}{ax}\right)}{a} - \frac{1}{3ax^3}$$

$$\Big\downarrow \ 218$$

$$-\frac{b \left(-\frac{\sqrt{b} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}\right)}{a} - \frac{1}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^
(3/2)))/a
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{b^{2} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a^{2} \sqrt{a b}}-\frac{1}{3 a \,x^{3}}+\frac{b}{a^{2} x}$	39
risch	$\frac{\frac{b \,x^{2}}{a^{2}}-\frac{1}{3 a}}{x^{3}}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(a^{5} \_Z^{2}+b^{3}\right)}\_R \ln\left(\left(3 a^{5} \_R^{2}+2 b^{3}\right) x -a^{3} b \_R \right)\right)}{2}$	64

input
```
int(1/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b^2/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/3/a/x^3+b/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.47

$$\int \frac{1}{x^4 (a+bx^2)}\, dx = \left[\frac{3\, b x^{3} \sqrt{-\frac{b}{a}} \log\left(\frac{b x^{2} + 2\, a x \sqrt{-\frac{b}{a}} - a}{b x^{2} + a}\right) + 6\, b x^{2} - 2\, a}{6\, a^{2} x^{3}}, \frac{3\, b x^{3} \sqrt{\frac{b}{a}} \arctan\left(x \sqrt{\frac{b}{a}}\right) + 3\, b x^{2} - a}{3\, a^{2} x^{3}}\right]$$

input
```
integrate(1/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
 6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*
b*x^2 - a)/(a^2*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.02

$$\int \frac{1}{x^4 (a+bx^2)} \, dx = -\frac{\sqrt{-\frac{b^3}{a^5}} \log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2} + \frac{\sqrt{-\frac{b^3}{a^5}} \log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2} + \frac{-a+3bx^2}{3a^2x^3}$$

input
```
integrate(1/x**4/(b*x**2+a),x)
```

output
```
-sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5
)*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (a+bx^2)} \, dx = \frac{b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4(a+bx^2)}\,dx = \frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{3\,bx^2-a}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4(a+bx^2)}\,dx = \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

input
```
int(1/(x^4*(a + b*x^2)),x)
```

output
```
(b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4(a+bx^2)}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^3-a^2+3ab\,x^2}{3a^3x^3}$$

input
```
int(1/x^4/(b*x^2+a),x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - a**2 + 3*a*b*x**
2)/(3*a**3*x**3)
```

3.143 $\int \frac{1}{x^6(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{1}{x^6\left(a+bx^2\right)}\,dx=-\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}$$

output
```
-1/5/a/x^5+1/3*b/a^2/x^3-b^2/a^3/x-b^(5/2)*arctan(b^(1/2)*x/a^(1/2))/a^(7/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6\left(a+bx^2\right)}\,dx=-\frac{1}{5ax^5}+\frac{b}{3a^2x^3}-\frac{b^2}{a^3x}-\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*1/(a*x^5) + b/(3*a^2*x^3) - b^2/(a^3*x) - (b^(5/2)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/a^(7/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)} \, dx$$

$$\Big\downarrow \; 264$$

$$-\frac{b \int \frac{1}{x^4 (bx^2+a)}dx}{a}-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 264$$

$$-\frac{b \left(-\frac{b \int \frac{1}{x^2 \left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 264$$

$$-\frac{b \left(-\frac{b \left(-\frac{b \int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}$$

$$\Big\downarrow \; 218$$

$$-\frac{b \left(-\frac{b \left(-\frac{\sqrt{b} \arctan \left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
default	$-\frac{b^{3} \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a^{3} \sqrt{a b}}-\frac{1}{5 a \,x^{5}}-\frac{b^{2}}{a^{3} x}+\frac{b}{3 a^{2} x^{3}}$	52
risch	$\frac{-\frac{b^{2} x^{4}}{a^{3}}+\frac{b \,x^{2}}{3 a^{2}}-\frac{1}{5 a}}{x^{5}}+\frac{\sqrt{-a b}\, b^{2} \ln\left(-b x+\sqrt{-a b}\right)}{2 a^{4}}-\frac{\sqrt{-a b}\, b^{2} \ln\left(-b x-\sqrt{-a b}\right)}{2 a^{4}}$	86

input
```
int(1/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-b^3/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/5/a/x^5-b^2/a^3/x+1/3*b/a^2
/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.28

$$\int \frac{1}{x^6 (a+bx^2)} dx = \left[\frac{15 b^2 x^5 \sqrt{-\frac{b}{a}} \log \left(\frac{bx^2 - 2 ax\sqrt{-\frac{b}{a}} - a}{bx^2+a} \right) - 30 b^2 x^4 + 10 abx^2 - 6 a^2}{30 a^3 x^5}, - \frac{15 b^2 x^5 \sqrt{\frac{b}{a}} \arctan \left(x\sqrt{\frac{b}{a}} \right) + 15 b^2 x^4 - 5 abx^2 + 3 a^2}{15 a^3 x^5} \right]$$

input
```
integrate(1/x^6/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(15*b^2*x^5*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a
)) - 30*b^2*x^4 + 10*a*b*x^2 - 6*a^2)/(a^3*x^5), -1/15*(15*b^2*x^5*sqrt(b/
a)*arctan(x*sqrt(b/a)) + 15*b^2*x^4 - 5*a*b*x^2 + 3*a^2)/(a^3*x^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^6 (a+bx^2)} dx = \frac{\sqrt{-\frac{b^5}{a^7}} \log \left(-\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3} + x \right)}{2} - \frac{\sqrt{-\frac{b^5}{a^7}} \log \left(\frac{a^4\sqrt{-\frac{b^5}{a^7}}}{b^3} + x \right)}{2} + \frac{-3a^2 + 5abx^2 - 15b^2x^4}{15a^3x^5}$$

input
```
integrate(1/x**6/(b*x**2+a),x)
```

output
```
sqrt(-b**5/a**7)*log(-a**4*sqrt(-b**5/a**7)/b**3 + x)/2 - sqrt(-b**5/a**7)
*log(a**4*sqrt(-b**5/a**7)/b**3 + x)/2 + (-3*a**2 + 5*a*b*x**2 - 15*b**2*x
**4)/(15*a**3*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^6 (a + bx^2)}\, dx = -\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\, b^2x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3x^5}$$

input
```
integrate(1/x^6/(b*x^2+a),x, algorithm="maxima")
```

output
```
-b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2
+ 3*a^2)/(a^3*x^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^6 (a + bx^2)}\, dx = -\frac{b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\, b^2x^4 - 5\, abx^2 + 3\, a^2}{15\, a^3x^5}$$

input
```
integrate(1/x^6/(b*x^2+a),x, algorithm="giac")
```

output
```
-b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/15*(15*b^2*x^4 - 5*a*b*x^2
+ 3*a^2)/(a^3*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^6 (a + bx^2)}\, dx = -\frac{\frac{1}{5\,a} - \frac{b\,x^2}{3\,a^2} + \frac{b^2\,x^4}{a^3}}{x^5} - \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
int(1/(x^6*(a + b*x^2)),x)
```

output
```
- (1/(5*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3)/x^5 - (b^(5/2)*atan((b^(1/2)
*x)/a^(1/2)))/a^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^6 (a+bx^2)} dx = \frac{-15\sqrt{b}\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right) b^2x^5 - 3a^3 + 5a^2b\,x^2 - 15a\,b^2x^4}{15a^4x^5}$$

input
```
int(1/x^6/(b*x^2+a),x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 3*a**3 +
5*a**2*b*x**2 - 15*a*b**2*x**4)/(15*a**4*x**5)
```

3.144 $\int \frac{1}{x^8(a+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{x^8(a+bx^2)}\,dx=-\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-1/7/a/x^7+1/5*b/a^2/x^5-1/3*b^2/a^3/x^3+b^3/a^4/x+b^(7/2)*arctan(b^(1/2)*
x/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8(a+bx^2)}\,dx=-\frac{1}{7ax^7}+\frac{b}{5a^2x^5}-\frac{b^2}{3a^3x^3}+\frac{b^3}{a^4x}+\frac{b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[1/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*1/(a*x^7) + b/(5*a^2*x^5) - b^2/(3*a^3*x^3) + b^3/(a^4*x) + (b^(7/2)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^2)}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{1}{x^6 (bx^2+a)}dx}{a}-\frac{1}{7ax^7}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(-\frac{b\int \frac{1}{x^4 \left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2 \left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}$$

$$\Big\downarrow \text{218}$$

$$-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}$$

input
```
Int[1/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*1/(a*x^7) - (b*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x))
- (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/a
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{1}{7a\,x^7}-\frac{b^2}{3a^3x^3}+\frac{b}{5a^2x^5}+\frac{b^3}{a^4x}$	61
risch	$\frac{\frac{b^3x^6}{a^4}-\frac{b^2x^4}{3a^3}+\frac{b\,x^2}{5a^2}-\frac{1}{7a}}{x^7}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^9\_Z^2+b^7\right)}\_R\ln\left(\left(3\_R^2a^9+2b^7\right)x-a^5b^3\_R\right)\right)}{2}$	88

input
```
int(1/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b^4/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/7/a/x^7-1/3*b^2/a^3/x^3+1/5*
b/a^2/x^5+b^3/a^4/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.23

$$\int \frac{1}{x^8\,(a+bx^2)}\,dx = \left[\frac{105\,b^3x^7\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+210\,b^3x^6-70\,ab^2x^4+42\,a^2bx^2-30\,a^3}{210\,a^4x^7}, \frac{105\,b^3x^7\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\right.}\right.}{}\right.$$

input
```
integrate(1/x^8/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(105*b^3*x^7*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 +
 a)) + 210*b^3*x^6 - 70*a*b^2*x^4 + 42*a^2*b*x^2 - 30*a^3)/(a^4*x^7), 1/10
5*(105*b^3*x^7*sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*b^3*x^6 - 35*a*b^2*x^4
+ 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)]
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.62

$$\int \frac{1}{x^8\,(a+bx^2)}\,dx = -\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2}+\frac{\sqrt{-\frac{b^7}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^7}{a^9}}}{b^4}+x\right)}{2} + \frac{-15a^3+21a^2bx^2-35ab^2x^4+105b^3x^6}{105a^4x^7}$$

input
```
integrate(1/x**8/(b*x**2+a),x)
```

output
```
-sqrt(-b**7/a**9)*log(-a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + sqrt(-b**7/a**9
)*log(a**5*sqrt(-b**7/a**9)/b**4 + x)/2 + (-15*a**3 + 21*a**2*b*x**2 - 35*
a*b**2*x**4 + 105*b**3*x**6)/(105*a**4*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^8 (a + bx^2)} \, dx = \frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105 \, b^3 x^6 - 35 \, ab^2 x^4 + 21 \, a^2 b x^2 - 15 \, a^3}{105 \, a^4 x^7}$$

input
```
integrate(1/x^8/(b*x^2+a),x, algorithm="maxima")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*
x^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^8 (a + bx^2)} \, dx = \frac{b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105 \, b^3 x^6 - 35 \, ab^2 x^4 + 21 \, a^2 b x^2 - 15 \, a^3}{105 \, a^4 x^7}$$

input
```
integrate(1/x^8/(b*x^2+a),x, algorithm="giac")
```

output
```
b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/105*(105*b^3*x^6 - 35*a*b^2*
x^4 + 21*a^2*b*x^2 - 15*a^3)/(a^4*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^8 (a+bx^2)} \, dx = \frac{b^{7/2} \operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}} - \frac{\frac{1}{7\,a} - \frac{b\,x^2}{5\,a^2} + \frac{b^2\,x^4}{3\,a^3} - \frac{b^3\,x^6}{a^4}}{x^7}$$

input
```
int(1/(x^8*(a + b*x^2)),x)
```

output
```
(b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(9/2) - (1/(7*a) - (b*x^2)/(5*a^2) +
 (b^2*x^4)/(3*a^3) - (b^3*x^6)/a^4)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^8 (a+bx^2)} \, dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^3x^7 - 15a^4 + 21a^3b\,x^2 - 35a^2b^2x^4 + 105a\,b^3x^6}{105a^5x^7}$$

input
```
int(1/x^8/(b*x^2+a),x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**7 - 15*a**4 + 2
1*a**3*b*x**2 - 35*a**2*b**2*x**4 + 105*a*b**3*x**6)/(105*a**5*x**7)
```

3.145 $\int \frac{x^{13}}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{x^{13}}{(a+bx^2)^2} dx = \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2} - \frac{a^6}{2b^7(a+bx^2)} - \frac{3a^5\log(a+bx^2)}{b^7}$$

output
```
5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/
2*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x^{13}}{(a+bx^2)^2} dx = \frac{50a^4bx^2 - 20a^3b^2x^4 + 10a^2b^3x^6 - 5ab^4x^8 + 2b^5x^{10} - \frac{10a^6}{a+bx^2} - 60a^5\log(a+bx^2)}{20b^7}$$

input
```
Integrate[x^13/(a + b*x^2)^2,x]
```

output
```
(50*a^4*b*x^2 - 20*a^3*b^2*x^4 + 10*a^2*b^3*x^6 - 5*a*b^4*x^8 + 2*b^5*x^10
 - (10*a^6)/(a + b*x^2) - 60*a^5*Log[a + b*x^2])/(20*b^7)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{x^8}{b^2}-\frac{2ax^6}{b^3}+\frac{3a^2x^4}{b^4}-\frac{4a^3x^2}{b^5}-\frac{6a^5}{b^6(bx^2+a)}+\frac{a^6}{b^6(bx^2+a)^2}+\frac{5a^4}{b^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{b^7(a+bx^2)}-\frac{6a^5\log(a+bx^2)}{b^7}+\frac{5a^4x^2}{b^6}-\frac{2a^3x^4}{b^5}+\frac{a^2x^6}{b^4}-\frac{ax^8}{2b^3}+\frac{x^{10}}{5b^2}\right)$$

input
```
Int[x^13/(a + b*x^2)^2,x]
```

output
```
((5*a^4*x^2)/b^6 - (2*a^3*x^4)/b^5 + (a^2*x^6)/b^4 - (a*x^8)/(2*b^3) + x^1
0/(5*b^2) - a^6/(b^7*(a + b*x^2)) - (6*a^5*Log[a + b*x^2])/b^7)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{5a^4x^2}{2b^6}-\frac{a^3x^4}{b^5}+\frac{a^2x^6}{2b^4}-\frac{a\,x^8}{4b^3}+\frac{x^{10}}{10b^2}-\frac{a^6}{2b^7(b\,x^2+a)}-\frac{3a^5\ln\left(b\,x^2+a\right)}{b^7}$	85
norman	$\frac{\frac{x^{12}}{10b}-\frac{3a\,x^{10}}{20b^2}-\frac{3a^6}{b^7}+\frac{a^2x^8}{4b^3}-\frac{a^3x^6}{2b^4}+\frac{3a^4x^4}{2b^5}}{b\,x^2+a}-\frac{3a^5\ln\left(b\,x^2+a\right)}{b^7}$	87
default	$\frac{\frac{1}{10}b^4x^{10}-\frac{1}{4}a\,b^3x^8+\frac{1}{2}a^2b^2x^6-a^3b\,x^4+\frac{5}{2}a^4x^2}{b^6}-\frac{a^5\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{6\ln\left(b\,x^2+a\right)}{b}\right)}{2b^6}$	88
parallelrisch	$-\frac{-2b^6x^{12}+3a\,b^5x^{10}-5a^2b^4x^8+10a^3x^6b^3-30a^4b^2x^4+60\ln\left(b\,x^2+a\right)x^2a^5b+60\ln\left(b\,x^2+a\right)a^6+60a^6}{20b^7\left(b\,x^2+a\right)}$	101

input
```
int(x^13/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
5/2*a^4*x^2/b^6-a^3*x^4/b^5+1/2*a^2*x^6/b^4-1/4*a*x^8/b^3+1/10*x^10/b^2-1/
2*a^6/b^7/(b*x^2+a)-3*a^5*ln(b*x^2+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = \frac{2\,b^6x^{12} - 3\,ab^5x^{10} + 5\,a^2b^4x^8 - 10\,a^3b^3x^6 + 30\,a^4b^2x^4 + 50\,a^5bx^2 - 10\,a^6 - 60\,(a^5bx^2 + a^6)\log(bx^2 + a)}{20\,(b^8x^2 + ab^7)}$$

input
```
integrate(x^13/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/20*(2*b^6*x^12 - 3*a*b^5*x^10 + 5*a^2*b^4*x^8 - 10*a^3*b^3*x^6 + 30*a^4*
b^2*x^4 + 50*a^5*b*x^2 - 10*a^6 - 60*(a^5*b*x^2 + a^6)*log(b*x^2 + a))/(b^
8*x^2 + a*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = -\frac{a^6}{2ab^7 + 2b^8x^2} - \frac{3a^5\log(a+bx^2)}{b^7} + \frac{5a^4x^2}{2b^6} - \frac{a^3x^4}{b^5} + \frac{a^2x^6}{2b^4} - \frac{ax^8}{4b^3} + \frac{x^{10}}{10b^2}$$

input
```
integrate(x**13/(b*x**2+a)**2,x)
```

output
```
-a**6/(2*a*b**7 + 2*b**8*x**2) - 3*a**5*log(a + b*x**2)/b**7 + 5*a**4*x**2
/(2*b**6) - a**3*x**4/b**5 + a**2*x**6/(2*b**4) - a*x**8/(4*b**3) + x**10/
(10*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = -\frac{a^6}{2\,(b^8x^2+ab^7)} - \frac{3\,a^5\log\left(bx^2+a\right)}{b^7} + \frac{2\,b^4x^{10} - 5\,ab^3x^8 + 10\,a^2b^2x^6 - 20\,a^3bx^4 + 50\,a^4x^2}{20\,b^6}$$

input
```
integrate(x^13/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a^6/(b^8*x^2 + a*b^7) - 3*a^5*log(b*x^2 + a)/b^7 + 1/20*(2*b^4*x^10 -
 5*a*b^3*x^8 + 10*a^2*b^2*x^6 - 20*a^3*b*x^4 + 50*a^4*x^2)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = -\frac{3\,a^5\log\left(|bx^2+a|\right)}{b^7} + \frac{6\,a^5bx^2+5\,a^6}{2\,(bx^2+a)b^7} + \frac{2\,b^8x^{10} - 5\,ab^7x^8 + 10\,a^2b^6x^6 - 20\,a^3b^5x^4 + 50\,a^4b^4x^2}{20\,b^{10}}$$

input
```
integrate(x^13/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-3*a^5*log(abs(b*x^2 + a))/b^7 + 1/2*(6*a^5*b*x^2 + 5*a^6)/((b*x^2 + a)*b^
7) + 1/20*(2*b^8*x^10 - 5*a*b^7*x^8 + 10*a^2*b^6*x^6 - 20*a^3*b^5*x^4 + 50
*a^4*b^4*x^2)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = \frac{x^{10}}{10\,b^2} - \frac{a^6}{2\,b\,\left(b^7\,x^2+a\,b^6\right)} - \frac{a\,x^8}{4\,b^3} - \frac{3\,a^5\,\ln\left(b\,x^2+a\right)}{b^7} + \frac{a^2\,x^6}{2\,b^4} - \frac{a^3\,x^4}{b^5} + \frac{5\,a^4\,x^2}{2\,b^6}$$

input
```
int(x^13/(a + b*x^2)^2,x)
```

output
```
x^10/(10*b^2) - a^6/(2*b*(a*b^6 + b^7*x^2)) - (a*x^8)/(4*b^3) - (3*a^5*log
(a + b*x^2))/b^7 + (a^2*x^6)/(2*b^4) - (a^3*x^4)/b^5 + (5*a^4*x^2)/(2*b^6)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int \frac{x^{13}}{(a+bx^2)^2}\,dx = \frac{-60\log(b\,x^2+a)\,a^6-60\log(b\,x^2+a)\,a^5b\,x^2+60a^5b\,x^2+30a^4b^2x^4-10a^3b^3x^6+5a^2b^4x^8-3a\,b^5x^{10}+}{20b^7\,(b\,x^2+a)}$$

input
```
int(x^13/(b*x^2+a)^2,x)
```

output
```
( - 60*log(a + b*x**2)*a**6 - 60*log(a + b*x**2)*a**5*b*x**2 + 60*a**5*b*x
**2 + 30*a**4*b**2*x**4 - 10*a**3*b**3*x**6 + 5*a**2*b**4*x**8 - 3*a*b**5*
x**10 + 2*b**6*x**12)/(20*b**7*(a + b*x**2))
```

3.146 $\int \frac{x^{11}}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{x^{11}}{(a+bx^2)^2} dx = -\frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2} + \frac{a^5}{2b^6(a+bx^2)} + \frac{5a^4\log(a+bx^2)}{2b^6}$$

output
```
-2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^
2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int \frac{x^{11}}{(a+bx^2)^2} dx = \frac{-48a^3bx^2 + 18a^2b^2x^4 - 8ab^3x^6 + 3b^4x^8 + \frac{12a^5}{a+bx^2} + 60a^4\log(a+bx^2)}{24b^6}$$

input
```
Integrate[x^11/(a + b*x^2)^2,x]
```

output
```
(-48*a^3*b*x^2 + 18*a^2*b^2*x^4 - 8*a*b^3*x^6 + 3*b^4*x^8 + (12*a^5)/(a +
b*x^2) + 60*a^4*Log[a + b*x^2])/(24*b^6)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{x^{10}}{(bx^2+a)^2} dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2} \int \left(\frac{x^6}{b^2} - \frac{2ax^4}{b^3} + \frac{3a^2x^2}{b^4} + \frac{5a^4}{b^5(bx^2+a)} - \frac{a^5}{b^5(bx^2+a)^2} - \frac{4a^3}{b^5}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^5}{b^6(a+bx^2)} + \frac{5a^4 \log(a+bx^2)}{b^6} - \frac{4a^3x^2}{b^5} + \frac{3a^2x^4}{2b^4} - \frac{2ax^6}{3b^3} + \frac{x^8}{4b^2}\right)$$

input
```
Int[x^11/(a + b*x^2)^2,x]
```

output
```
((-4*a^3*x^2)/b^5 + (3*a^2*x^4)/(2*b^4) - (2*a*x^6)/(3*b^3) + x^8/(4*b^2)
+ a^5/(b^6*(a + b*x^2)) + (5*a^4*Log[a + b*x^2])/b^6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

method	result	size
risch	$-\frac{2a^3x^2}{b^5}+\frac{3a^2x^4}{4b^4}-\frac{a\,x^6}{3b^3}+\frac{x^8}{8b^2}+\frac{a^5}{2b^6(b\,x^2+a)}+\frac{5a^4\ln\left(b\,x^2+a\right)}{2b^6}$	74
norman	$\frac{\frac{x^{10}}{8b}-\frac{5a\,x^8}{24b^2}+\frac{5a^5}{2b^6}+\frac{5a^2x^6}{12b^3}-\frac{5a^3x^4}{4b^4}}{b\,x^2+a}+\frac{5a^4\ln\left(b\,x^2+a\right)}{2b^6}$	76
default	$-\frac{-\frac{1}{8}b^3x^8+\frac{1}{3}a\,b^2x^6-\frac{3}{4}a^2b\,x^4+2a^3x^2}{b^5}+\frac{a^4\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{5\ln\left(b\,x^2+a\right)}{b}\right)}{2b^5}$	78
parallelrisch	$\frac{3b^5x^{10}-5a\,b^4x^8+10a^2b^3x^6-30a^3b^2x^4+60\ln\left(b\,x^2+a\right)x^2a^4b+60\ln\left(b\,x^2+a\right)a^5+60a^5}{24b^6\left(b\,x^2+a\right)}$	90

input
```
int(x^11/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*a^3*x^2/b^5+3/4*a^2*x^4/b^4-1/3*a*x^6/b^3+1/8*x^8/b^2+1/2*a^5/b^6/(b*x^
2+a)+5/2*a^4*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{3\,b^5x^{10}-5\,ab^4x^8+10\,a^2b^3x^6-30\,a^3b^2x^4-48\,a^4bx^2+12\,a^5+60\,(a^4bx^2+a^5)\log\left(bx^2+a\right)}{24\,(b^7x^2+ab^6)}$$

input
```
integrate(x^11/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*b^5*x^10 - 5*a*b^4*x^8 + 10*a^2*b^3*x^6 - 30*a^3*b^2*x^4 - 48*a^4*
b*x^2 + 12*a^5 + 60*(a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^7*x^2 + a*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{a^5}{2ab^6+2b^7x^2} + \frac{5a^4\log{(a+bx^2)}}{2b^6} - \frac{2a^3x^2}{b^5} + \frac{3a^2x^4}{4b^4} - \frac{ax^6}{3b^3} + \frac{x^8}{8b^2}$$

input
```
integrate(x**11/(b*x**2+a)**2,x)
```

output
```
a**5/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*log(a + b*x**2)/(2*b**6) - 2*a**3*x
**2/b**5 + 3*a**2*x**4/(4*b**4) - a*x**6/(3*b**3) + x**8/(8*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{a^5}{2\,(b^7x^2+ab^6)} + \frac{5\,a^4\log{(bx^2+a)}}{2\,b^6} + \frac{3\,b^3x^8-8\,ab^2x^6+18\,a^2bx^4-48\,a^3x^2}{24\,b^5}$$

input
```
integrate(x^11/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a^5/(b^7*x^2 + a*b^6) + 5/2*a^4*log(b*x^2 + a)/b^6 + 1/24*(3*b^3*x^8 -
 8*a*b^2*x^6 + 18*a^2*b*x^4 - 48*a^3*x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{5\,a^4\log\left(|bx^2+a|\right)}{2\,b^6} - \frac{5\,a^4bx^2+4\,a^5}{2\,(bx^2+a)b^6} + \frac{3\,b^6x^8-8\,ab^5x^6+18\,a^2b^4x^4-48\,a^3b^3x^2}{24\,b^8}$$

input
```
integrate(x^11/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
5/2*a^4*log(abs(b*x^2 + a))/b^6 - 1/2*(5*a^4*b*x^2 + 4*a^5)/((b*x^2 + a)*b
^6) + 1/24*(3*b^6*x^8 - 8*a*b^5*x^6 + 18*a^2*b^4*x^4 - 48*a^3*b^3*x^2)/b^8
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{x^8}{8\,b^2} + \frac{a^5}{2\,b\,\left(b^6\,x^2+a\,b^5\right)} - \frac{a\,x^6}{3\,b^3} + \frac{5\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^6} + \frac{3\,a^2\,x^4}{4\,b^4} - \frac{2\,a^3\,x^2}{b^5}$$

input
```
int(x^11/(a + b*x^2)^2,x)
```

output
```
x^8/(8*b^2) + a^5/(2*b*(a*b^5 + b^6*x^2)) - (a*x^6)/(3*b^3) + (5*a^4*log(a
 + b*x^2))/(2*b^6) + (3*a^2*x^4)/(4*b^4) - (2*a^3*x^2)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{x^{11}}{(a+bx^2)^2}\,dx = \frac{60\log(b\,x^2+a)\,a^5+60\log(b\,x^2+a)\,a^4b\,x^2-60a^4b\,x^2-30a^3b^2x^4+10a^2b^3x^6-5a\,b^4x^8+3b^5x^{10}}{24b^6\left(b\,x^2+a\right)}$$

input

```
int(x^11/(b*x^2+a)^2,x)
```

output

```
(60*log(a + b*x**2)*a**5 + 60*log(a + b*x**2)*a**4*b*x**2 - 60*a**4*b*x**2
 - 30*a**3*b**2*x**4 + 10*a**2*b**3*x**6 - 5*a*b**4*x**8 + 3*b**5*x**10)/(
24*b**6*(a + b*x**2))
```

3.147 $\int \frac{x^9}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 70

$$\int \frac{x^9}{(a+bx^2)^2} dx = \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2} - \frac{a^4}{2b^5(a+bx^2)} - \frac{2a^3\log(a+bx^2)}{b^5}$$

output
```
3/2*a^2*x^2/b^4-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b
*x^2+a)/b^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int \frac{x^9}{(a+bx^2)^2} dx = \frac{9a^2bx^2 - 3ab^2x^4 + b^3x^6 - \frac{3a^4}{a+bx^2} - 12a^3\log(a+bx^2)}{6b^5}$$

input
```
Integrate[x^9/(a + b*x^2)^2,x]
```

output
```
(9*a^2*b*x^2 - 3*a*b^2*x^4 + b^3*x^6 - (3*a^4)/(a + b*x^2) - 12*a^3*Log[a
+ b*x^2])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^8}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4(bx^2+a)^2}-\frac{4a^3}{b^4(bx^2+a)}+\frac{3a^2}{b^4}-\frac{2x^2a}{b^3}+\frac{x^4}{b^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{a^4}{b^5(a+bx^2)}-\frac{4a^3\log(a+bx^2)}{b^5}+\frac{3a^2x^2}{b^4}-\frac{ax^4}{b^3}+\frac{x^6}{3b^2}\right)$$

input
```
Int[x^9/(a + b*x^2)^2,x]
```

output
```
((3*a^2*x^2)/b^4 - (a*x^4)/b^3 + x^6/(3*b^2) - a^4/(b^5*(a + b*x^2)) - (4*
a^3*Log[a + b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

method	result	size
risch	$\frac{3a^2x^2}{2b^4}-\frac{a\,x^4}{2b^3}+\frac{x^6}{6b^2}-\frac{a^4}{2b^5(b\,x^2+a)}-\frac{2a^3\ln\left(b\,x^2+a\right)}{b^5}$	63
norman	$\frac{\frac{a^2x^4}{b^3}+\frac{x^8}{6b}-\frac{a\,x^6}{3b^2}-\frac{2a^4}{b^5}}{b\,x^2+a}-\frac{2a^3\ln\left(b\,x^2+a\right)}{b^5}$	64
default	$\frac{\frac{1}{6}b^2x^6-\frac{1}{2}ab\,x^4+\frac{3}{2}a^2x^2}{b^4}-\frac{a^3\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{4\ln\left(b\,x^2+a\right)}{b}\right)}{2b^4}$	66
parallelrisch	$-\frac{-b^4x^8+2a\,b^3x^6-6a^2b^2x^4+12\ln\left(b\,x^2+a\right)x^2a^3b+12a^4\ln\left(b\,x^2+a\right)+12a^4}{6b^5\left(b\,x^2+a\right)}$	79

input
```
int(x^9/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*a^2/b^4*x^2-1/2*a*x^4/b^3+1/6*x^6/b^2-1/2*a^4/b^5/(b*x^2+a)-2*a^3*ln(b
*x^2+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = \frac{b^4x^8-2\,ab^3x^6+6\,a^2b^2x^4+9\,a^3bx^2-3\,a^4-12\,(a^3bx^2+a^4)\log\left(bx^2+a\right)}{6\,(b^6x^2+ab^5)}$$

input
```
integrate(x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/6*(b^4*x^8 - 2*a*b^3*x^6 + 6*a^2*b^2*x^4 + 9*a^3*b*x^2 - 3*a^4 - 12*(a^3
*b*x^2 + a^4)*log(b*x^2 + a))/(b^6*x^2 + a*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = -\frac{a^4}{2ab^5+2b^6x^2} - \frac{2a^3\log(a+bx^2)}{b^5} + \frac{3a^2x^2}{2b^4} - \frac{ax^4}{2b^3} + \frac{x^6}{6b^2}$$

input
```
integrate(x**9/(b*x**2+a)**2,x)
```

output
```
-a**4/(2*a*b**5 + 2*b**6*x**2) - 2*a**3*log(a + b*x**2)/b**5 + 3*a**2*x**2
/(2*b**4) - a*x**4/(2*b**3) + x**6/(6*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = -\frac{a^4}{2\,(b^6x^2+ab^5)} - \frac{2\,a^3\log(bx^2+a)}{b^5} + \frac{b^2x^6 - 3\,abx^4 + 9\,a^2x^2}{6\,b^4}$$

input
```
integrate(x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a^4/(b^6*x^2 + a*b^5) - 2*a^3*log(b*x^2 + a)/b^5 + 1/6*(b^2*x^6 - 3*a
*b*x^4 + 9*a^2*x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = -\frac{2\,a^3\log\left(|bx^2+a|\right)}{b^5} + \frac{b^4x^6 - 3\,ab^3x^4 + 9\,a^2b^2x^2}{6\,b^6} + \frac{4\,a^3bx^2 + 3\,a^4}{2\,(bx^2+a)b^5}$$

input
```
integrate(x^9/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-2*a^3*log(abs(b*x^2 + a))/b^5 + 1/6*(b^4*x^6 - 3*a*b^3*x^4 + 9*a^2*b^2*x^
2)/b^6 + 1/2*(4*a^3*b*x^2 + 3*a^4)/((b*x^2 + a)*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = \frac{x^6}{6\,b^2} - \frac{a^4}{2\,b\,\left(b^5\,x^2 + a\,b^4\right)} - \frac{a\,x^4}{2\,b^3} - \frac{2\,a^3\,\ln\left(b\,x^2+a\right)}{b^5} + \frac{3\,a^2\,x^2}{2\,b^4}$$

input
```
int(x^9/(a + b*x^2)^2,x)
```

output
```
x^6/(6*b^2) - a^4/(2*b*(a*b^4 + b^5*x^2)) - (a*x^4)/(2*b^3) - (2*a^3*log(a
 + b*x^2))/b^5 + (3*a^2*x^2)/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.16

$$\int \frac{x^9}{(a+bx^2)^2}\,dx = \frac{-12\log(b\,x^2+a)\,a^4 - 12\log(b\,x^2+a)\,a^3b\,x^2 + 12a^3b\,x^2 + 6a^2b^2x^4 - 2a\,b^3x^6 + b^4x^8}{6b^5\,(b\,x^2+a)}$$

input
```
int(x^9/(b*x^2+a)^2,x)
```

output

```
( - 12*log(a + b*x**2)*a**4 - 12*log(a + b*x**2)*a**3*b*x**2 + 12*a**3*b*x
**2 + 6*a**2*b**2*x**4 - 2*a*b**3*x**6 + b**4*x**8)/(6*b**5*(a + b*x**2))
```

3.148 $\int \frac{x^7}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{x^7}{(a+bx^2)^2} dx = -\frac{ax^2}{b^3} + \frac{x^4}{4b^2} + \frac{a^3}{2b^4(a+bx^2)} + \frac{3a^2 \log(a+bx^2)}{2b^4}$$

output
```
-a*x^2/b^3+1/4*x^4/b^2+1/2*a^3/b^4/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{x^7}{(a+bx^2)^2} dx = \frac{-4abx^2 + b^2x^4 + \frac{2a^3}{a+bx^2} + 6a^2 \log(a+bx^2)}{4b^4}$$

input
```
Integrate[x^7/(a + b*x^2)^2,x]
```

output
```
(-4*a*b*x^2 + b^2*x^4 + (2*a^3)/(a + b*x^2) + 6*a^2*Log[a + b*x^2])/(4*b^4
)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^2}+\frac{3a^2}{b^3(bx^2+a)}-\frac{2a}{b^3}+\frac{x^2}{b^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a^3}{b^4(a+bx^2)}+\frac{3a^2\log(a+bx^2)}{b^4}-\frac{2ax^2}{b^3}+\frac{x^4}{2b^2}\right)$$

input
```
Int[x^7/(a + b*x^2)^2,x]
```

output
```
((-2*a*x^2)/b^3 + x^4/(2*b^2) + a^3/(b^4*(a + b*x^2)) + (3*a^2*Log[a + b*x
^2])/b^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{\frac{x^{6}}{4b}-\frac{3a\,x^{4}}{4b^{2}}+\frac{3a^{3}}{2b^{4}}}{b\,x^{2}+a}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	54
default	$\frac{\left(-b\,x^{2}+2a\right)^{2}}{4b^{4}}+\frac{a^{2}\left(\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{3\ln\left(b\,x^{2}+a\right)}{b}\right)}{2b^{3}}$	55
risch	$\frac{x^{4}}{4b^{2}}-\frac{a\,x^{2}}{b^{3}}+\frac{a^{2}}{b^{4}}+\frac{a^{3}}{2b^{4}\left(b\,x^{2}+a\right)}+\frac{3a^{2}\ln\left(b\,x^{2}+a\right)}{2b^{4}}$	59
parallelrisch	$\frac{b^{3}x^{6}-3a\,b^{2}x^{4}+6\ln\left(b\,x^{2}+a\right)x^{2}a^{2}b+6a^{3}\ln\left(b\,x^{2}+a\right)+6a^{3}}{4b^{4}\left(b\,x^{2}+a\right)}$	67

input
```
int(x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/4/b*x^6-3/4*a*x^4/b^2+3/2*a^3/b^4)/(b*x^2+a)+3/2*a^2*ln(b*x^2+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{(a+bx^2)^2}\,dx=\frac{b^3x^6-3\,ab^2x^4-4\,a^2bx^2+2\,a^3+6\,(a^2bx^2+a^3)\log\left(bx^2+a\right)}{4\,(b^5x^2+ab^4)}$$

input
```
integrate(x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(b^3*x^6 - 3*a*b^2*x^4 - 4*a^2*b*x^2 + 2*a^3 + 6*(a^2*b*x^2 + a^3)*log
(b*x^2 + a))/(b^5*x^2 + a*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^7}{(a+bx^2)^2}\,dx = \frac{a^3}{2ab^4+2b^5x^2} + \frac{3a^2\log(a+bx^2)}{2b^4} - \frac{ax^2}{b^3} + \frac{x^4}{4b^2}$$

input
```
integrate(x**7/(b*x**2+a)**2,x)
```

output
```
a**3/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*log(a + b*x**2)/(2*b**4) - a*x**2/b
**3 + x**4/(4*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(a+bx^2)^2}\,dx = \frac{a^3}{2\,(b^5x^2+ab^4)} + \frac{3\,a^2\log(bx^2+a)}{2\,b^4} + \frac{bx^4-4\,ax^2}{4\,b^3}$$

input
```
integrate(x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a^3/(b^5*x^2 + a*b^4) + 3/2*a^2*log(b*x^2 + a)/b^4 + 1/4*(b*x^4 - 4*a*
x^2)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{x^7}{(a+bx^2)^2}\,dx = \frac{3\,a^2\log\left(|bx^2+a|\right)}{2\,b^4} + \frac{b^2x^4-4\,abx^2}{4\,b^4} - \frac{3\,a^2bx^2+2\,a^3}{2\,(bx^2+a)b^4}$$

input
```
integrate(x^7/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
3/2*a^2*log(abs(b*x^2 + a))/b^4 + 1/4*(b^2*x^4 - 4*a*b*x^2)/b^4 - 1/2*(3*a
^2*b*x^2 + 2*a^3)/((b*x^2 + a)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^7}{(a+bx^2)^2}\,dx = \frac{x^4}{4\,b^2} + \frac{a^3}{2\,b\,\left(b^4\,x^2+a\,b^3\right)} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

input
```
int(x^7/(a + b*x^2)^2,x)
```

output
```
x^4/(4*b^2) + a^3/(2*b*(a*b^3 + b^4*x^2)) - (a*x^2)/b^3 + (3*a^2*log(a + b
*x^2))/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.23

$$\int \frac{x^7}{(a+bx^2)^2}\,dx = \frac{6\log(b\,x^2+a)\,a^3+6\log(b\,x^2+a)\,a^2b\,x^2-6a^2b\,x^2-3a\,b^2x^4+b^3x^6}{4b^4\,(b\,x^2+a)}$$

input
```
int(x^7/(b*x^2+a)^2,x)
```

output
```
(6*log(a + b*x**2)*a**3 + 6*log(a + b*x**2)*a**2*b*x**2 - 6*a**2*b*x**2 -
3*a*b**2*x**4 + b**3*x**6)/(4*b**4*(a + b*x**2))
```

3.149 $\int \frac{x^5}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{x^5}{(a+bx^2)^2} \, dx = \frac{x^2}{2b^2} - \frac{a^2}{2b^3(a+bx^2)} - \frac{a\log(a+bx^2)}{b^3}$$

output
```
1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^2)^2} \, dx = \frac{bx^2 - \frac{a^2}{a+bx^2} - 2a\log(a+bx^2)}{2b^3}$$

input
```
Integrate[x^5/(a + b*x^2)^2,x]
```

output
```
(b*x^2 - a^2/(a + b*x^2) - 2*a*Log[a + b*x^2])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^2}-\frac{2a}{b^2(bx^2+a)}+\frac{1}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{b^3(a+bx^2)}-\frac{2a\log(a+bx^2)}{b^3}+\frac{x^2}{b^2}\right)$$

input
```
Int[x^5/(a + b*x^2)^2,x]
```

output
```
(x^2/b^2 - a^2/(b^3*(a + b*x^2)) - (2*a*Log[a + b*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{x^2}{2b^2}-\frac{a^2}{2b^3(b\,x^2+a)}-\frac{a\ln\left(b\,x^2+a\right)}{b^3}$	41
norman	$\frac{\frac{x^4}{2b}-\frac{a^2}{b^3}}{b\,x^2+a}-\frac{a\ln\left(b\,x^2+a\right)}{b^3}$	43
default	$\frac{x^2}{2b^2}-\frac{a\left(\frac{a}{b\left(b\,x^2+a\right)}+\frac{2\ln\left(b\,x^2+a\right)}{b}\right)}{2b^2}$	44
parallelrisch	$-\frac{-b^2x^4+2\ln\left(b\,x^2+a\right)x^2ab+2\ln\left(b\,x^2+a\right)a^2+2a^2}{2b^3\left(b\,x^2+a\right)}$	57

input
```
int(x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^2-1/2*a^2/b^3/(b*x^2+a)-a*ln(b*x^2+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27

$$\int \frac{x^5}{(a+bx^2)^2}\,dx=\frac{b^2x^4+abx^2-a^2-2\,(abx^2+a^2)\log\left(bx^2+a\right)}{2\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^4 + a*b*x^2 - a^2 - 2*(a*b*x^2 + a^2)*log(b*x^2 + a))/(b^4*x^2
+ a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{(a+bx^2)^2}\,dx = -\frac{a^2}{2ab^3+2b^4x^2} - \frac{a\log(a+bx^2)}{b^3} + \frac{x^2}{2b^2}$$

input
```
integrate(x**5/(b*x**2+a)**2,x)
```

output
```
-a**2/(2*a*b**3 + 2*b**4*x**2) - a*log(a + b*x**2)/b**3 + x**2/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{x^5}{(a+bx^2)^2}\,dx = -\frac{a^2}{2\,(b^4x^2+ab^3)} + \frac{x^2}{2\,b^2} - \frac{a\log(bx^2+a)}{b^3}$$

input
```
integrate(x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2/(b^4*x^2 + a*b^3) + 1/2*x^2/b^2 - a*log(b*x^2 + a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{x^5}{(a+bx^2)^2}\,dx = \frac{x^2}{2\,b^2} - \frac{a\log(|bx^2+a|)}{b^3} + \frac{2\,abx^2+a^2}{2\,(bx^2+a)b^3}$$

input
```
integrate(x^5/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*x^2/b^2 - a*log(abs(b*x^2 + a))/b^3 + 1/2*(2*a*b*x^2 + a^2)/((b*x^2 +
a)*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{x^5}{(a+bx^2)^2}\,dx = \frac{x^2}{2\,b^2} - \frac{a^2}{2\,(b^4\,x^2+a\,b^3)} - \frac{a\,\ln(b\,x^2+a)}{b^3}$$

input
```
int(x^5/(a + b*x^2)^2,x)
```

output
```
x^2/(2*b^2) - a^2/(2*(a*b^3 + b^4*x^2)) - (a*log(a + b*x^2))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{x^5}{(a+bx^2)^2}\,dx = \frac{-2\log(b\,x^2+a)\,a^2 - 2\log(b\,x^2+a)\,ab\,x^2 + 2ab\,x^2 + b^2x^4}{2b^3\,(b\,x^2+a)}$$

input
```
int(x^5/(b*x^2+a)^2,x)
```

output
```
( - 2*log(a + b*x**2)*a**2 - 2*log(a + b*x**2)*a*b*x**2 + 2*a*b*x**2 + b**
2*x**4)/(2*b**3*(a + b*x**2))
```

3.150 $\int \frac{x^3}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{a}{2b^2(a+bx^2)} + \frac{\log(a+bx^2)}{2b^2}$$

output
```
1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{\frac{a}{a+bx^2} + \log(a+bx^2)}{2b^2}$$

input
```
Integrate[x^3/(a + b*x^2)^2,x]
```

output
```
(a/(a + b*x^2) + Log[a + b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{1}{b(bx^2+a)}-\frac{a}{b(bx^2+a)^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a}{b^2(a+bx^2)}+\frac{\log(a+bx^2)}{b^2}\right)$$

input
```
Int[x^3/(a + b*x^2)^2,x]
```

output
```
(a/(b^2*(a + b*x^2)) + Log[a + b*x^2]/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
norman	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
risch	$\frac{a}{2b^2(b\,x^2+a)}+\frac{\ln(b\,x^2+a)}{2b^2}$	30
parallelrisch	$\frac{b\ln(b\,x^2+a)x^2+a\ln(b\,x^2+a)+a}{2b^2(b\,x^2+a)}$	40

input
```
int(x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2/(b*x^2+a)+1/2*ln(b*x^2+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{(bx^2+a)\log(bx^2+a)+a}{2\,(b^3x^2+ab^2)}$$

input
```
integrate(x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x^2 + a)*log(b*x^2 + a) + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{a}{2ab^2+2b^3x^2} + \frac{\log(a+bx^2)}{2b^2}$$

input
```
integrate(x**3/(b*x**2+a)**2,x)
```

output
```
a/(2*a*b**2 + 2*b**3*x**2) + log(a + b*x**2)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{a}{2\,(b^3x^2+ab^2)} + \frac{\log(bx^2+a)}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a/(b^3*x^2 + a*b^2) + 1/2*log(b*x^2 + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = -\frac{\frac{\log\left(\frac{|bx^2+a|}{(bx^2+a)^2|b|}\right)}{b} - \frac{a}{(bx^2+a)b}}{2\,b}$$

input
```
integrate(x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(log(abs(b*x^2 + a)/((b*x^2 + a)^2*abs(b)))/b - a/((b*x^2 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{a}{2\,b^2\,\left(b\,x^2+a\right)}$$

input
```
int(x^3/(a + b*x^2)^2,x)
```

output
```
log(a + b*x^2)/(2*b^2) + a/(2*b^2*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{x^3}{(a+bx^2)^2}\,dx = \frac{\log(b\,x^2+a)\,a+\log(b\,x^2+a)\,b\,x^2-b\,x^2}{2b^2\,(b\,x^2+a)}$$

input
```
int(x^3/(b*x^2+a)^2,x)
```

output
```
(log(a + b*x**2)*a + log(a + b*x**2)*b*x**2 - b*x**2)/(2*b**2*(a + b*x**2)
)
```

3.151 $\int \frac{x}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2b(a+bx^2)}$$

output
```
-1/2/b/(b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2b(a+bx^2)}$$

input
```
Integrate[x/(a + b*x^2)^2,x]
```

output
```
-1/2*1/(b*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^2} \, dx$$
$$\Big\downarrow \; 241$$
$$-\frac{1}{2b\,(a+bx^2)}$$

input
```
Int[x/(a + b*x^2)^2,x]
```

output
```
-1/2*1/(b*(a + b*x^2))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{2b(b\,x^{2}+a)}$	15
derivativedivides	$-\frac{1}{2b(b\,x^{2}+a)}$	15
default	$-\frac{1}{2b(b\,x^{2}+a)}$	15
norman	$-\frac{1}{2b(b\,x^{2}+a)}$	15
risch	$-\frac{1}{2b(b\,x^{2}+a)}$	15
parallelrisch	$-\frac{1}{2b(b\,x^{2}+a)}$	15
orering	$-\frac{1}{2b(b\,x^{2}+a)}$	15

input
```
int(x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2\,(b^2x^2+ab)}$$

input
```
integrate(x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2/(b^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2ab+2b^2x^2}$$

input
```
integrate(x/(b*x**2+a)**2,x)
```

output
```
-1/(2*a*b + 2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2\,(bx^2+a)b}$$

input
```
integrate(x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2/((b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2\,(bx^2+a)b}$$

input
```
integrate(x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^2}\,dx = -\frac{1}{2\,b\,(b\,x^2+a)}$$

input
```
int(x/(a + b*x^2)^2,x)
```

output
```
-1/(2*b*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a+bx^2)^2}\,dx = \frac{x^2}{2a\,(b\,x^2+a)}$$

input
```
int(x/(b*x^2+a)^2,x)
```

output
```
x**2/(2*a*(a + b*x**2))
```

3.152 $\int \frac{1}{x(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx=\frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}$$

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx=\frac{\frac{a}{a+bx^2}+2\log(x)-\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[1/(x*(a + b*x^2)^2),x]
```

output
```
(a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^2}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^2}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$\frac{1}{2}\int \left(-\frac{b}{a^2\left(bx^2+a\right)}-\frac{b}{a\left(bx^2+a\right)^2}+\frac{1}{a^2x^2}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(-\frac{\log\left(a+bx^2\right)}{a^2}+\frac{\log\left(x^2\right)}{a^2}+\frac{1}{a\left(a+bx^2\right)}\right)$$

input
```
Int[1/(x*(a + b*x^2)^2),x]
```

output
```
(1/(a*(a + b*x^2)) + Log[x^2]/a^2 - Log[a + b*x^2]/a^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	35
norman	$-\frac{b\,x^2}{2a^2(b\,x^2+a)}+\frac{\ln(x)}{a^2}-\frac{\ln(b\,x^2+a)}{2a^2}$	39
default	$-\frac{b\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{\ln\left(b\,x^2+a\right)}{b}\right)}{2a^2}+\frac{\ln(x)}{a^2}$	42
parallelrisch	$\frac{2b\ln(x)x^2-b\ln(b\,x^2+a)x^2-b\,x^2+2a\ln(x)-a\ln(b\,x^2+a)}{2a^2(b\,x^2+a)}$	60

input
```
int(1/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx=-\frac{(bx^2+a)\log\left(bx^2+a\right)-2\left(bx^2+a\right)\log\left(x\right)-a}{2\left(a^2bx^2+a^3\right)}$$

input
```
integrate(1/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 +
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2a^2+2abx^2} + \frac{\log\left(x\right)}{a^2} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(1/x/(b*x**2+a)**2,x)
```

output
```
1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{1}{2\left(abx^2+a^2\right)} - \frac{\log\left(bx^2+a\right)}{2\,a^2} + \frac{\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate(1/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + 1/2*log(x^2)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(|bx^2+a|\right)}{2\,a^2} + \frac{bx^2+2\,a}{2\,(bx^2+a)a^2}$$

input
```
integrate(1/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
 + a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2+a\right)} - \frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

input
```
int(1/(x*(a + b*x^2)^2),x)
```

output
```
log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\left(a+bx^2\right)^2}\,dx = \frac{-\log(b\,x^2+a)\,a - \log(b\,x^2+a)\,b\,x^2 + 2\log(x)\,a + 2\log(x)\,b\,x^2 - b\,x^2}{2a^2\left(b\,x^2+a\right)}$$

input
```
int(1/x/(b*x^2+a)^2,x)
```

output
```
( - log(a + b*x**2)*a - log(a + b*x**2)*b*x**2 + 2*log(x)*a + 2*log(x)*b*x
**2 - b*x**2)/(2*a**2*(a + b*x**2))
```

3.153 $\int \frac{1}{x^3(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{1}{x^3(a+bx^2)^2} dx = -\frac{1}{2a^2x^2} - \frac{b}{2a^2(a+bx^2)} - \frac{2b\log(x)}{a^3} + \frac{b\log(a+bx^2)}{a^3}$$

output
```
-1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3(a+bx^2)^2} dx = -\frac{a\left(\frac{1}{x^2} + \frac{b}{a+bx^2}\right) + 4b\log(x) - 2b\log(a+bx^2)}{2a^3}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^2),x]
```

output
```
-1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^2} dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(\frac{2b^2}{a^3 (bx^2+a)} + \frac{b^2}{a^2 (bx^2+a)^2} - \frac{2b}{a^3x^2} + \frac{1}{a^2x^4}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2b\log(x^2)}{a^3} + \frac{2b\log(a+bx^2)}{a^3} - \frac{b}{a^2(a+bx^2)} - \frac{1}{a^2x^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^2),x]
```

output
```
(-(1/(a^2*x^2)) - b/(a^2*(a + b*x^2)) - (2*b*Log[x^2])/a^3 + (2*b*Log[a +
b*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

method	result	size
norman	$\frac{\frac{b^2 x^4}{a^3}-\frac{1}{2a}}{x^2(b\,x^2+a)}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{2b\ln\left(x\right)}{a^3}$	52
risch	$\frac{-\frac{b\,x^2}{a^2}-\frac{1}{2a}}{x^2(b\,x^2+a)}-\frac{2b\ln\left(x\right)}{a^3}+\frac{b\ln\left(-b\,x^2-a\right)}{a^3}$	54
default	$\frac{b^2\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{2\ln\left(b\,x^2+a\right)}{b}\right)}{2a^3}-\frac{1}{2a^2x^2}-\frac{2b\ln\left(x\right)}{a^3}$	55
parallelrisch	$-\frac{4b^2\ln\left(x\right)x^4-2b^2\ln\left(b\,x^2+a\right)x^4-2b^2x^4+4ab\ln\left(x\right)x^2-2\ln\left(b\,x^2+a\right)x^2ab+a^2}{2a^3x^2\left(b\,x^2+a\right)}$	80

input
```
int(1/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2/a^3*x^4-1/2/a)/x^2/(b*x^2+a)+b*ln(b*x^2+a)/a^3-2*b*ln(x)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx = -\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log\left(x\right)}{2\left(a^3bx^4+a^4x^2\right)}$$

input
```
integrate(1/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4
+ a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = \frac{-a-2bx^2}{2a^3x^2+2a^2bx^4} - \frac{2b\log(x)}{a^3} + \frac{b\log\left(\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate(1/x**3/(b*x**2+a)**2,x)
```

output
```
(-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/
b + x**2)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = -\frac{2\,bx^2+a}{2\,(a^2bx^4+a^3x^2)} + \frac{b\log(bx^2+a)}{a^3} - \frac{b\log(x^2)}{a^3}$$

input
```
integrate(1/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - b*log(x^
2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = -\frac{b \log(x^2)}{a^3} + \frac{b \log(|bx^2+a|)}{a^3} - \frac{2bx^2+a}{2(bx^4+ax^2)a^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 +
a*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = \frac{b \ln(b x^2 + a)}{a^3} - \frac{\frac{1}{2a} + \frac{b x^2}{a^2}}{b x^4 + a x^2} - \frac{2 b \ln(x)}{a^3}$$

input
```
int(1/(x^3*(a + b*x^2)^2),x)
```

output
```
(b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*lo
g(x))/a^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{1}{x^3 (a+bx^2)^2} \, dx = \frac{2 \log(b x^2 + a) ab x^2 + 2 \log(b x^2 + a) b^2 x^4 - 4 \log(x) ab x^2 - 4 \log(x) b^2 x^4 - a^2 + 2b^2 x^4}{2a^3 x^2 (b x^2 + a)}$$

input
```
int(1/x^3/(b*x^2+a)^2,x)
```

output
```
(2*log(a + b*x**2)*a*b*x**2 + 2*log(a + b*x**2)*b**2*x**4 - 4*log(x)*a*b*x
**2 - 4*log(x)*b**2*x**4 - a**2 + 2*b**2*x**4)/(2*a**3*x**2*(a + b*x**2))
```

3.154 $\int \frac{1}{x^5(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{1}{x^5 \left(a+bx^2\right)^2}\,dx=-\frac{1}{4a^2x^4}+\frac{b}{a^3x^2}+\frac{b^2}{2a^3\left(a+bx^2\right)}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(a+bx^2\right)}{2a^4}$$

output
```
-1/4/a^2/x^4+b/a^3/x^2+1/2*b^2/a^3/(b*x^2+a)+3*b^2*ln(x)/a^4-3/2*b^2*ln(b*
x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 \left(a+bx^2\right)^2}\,dx=\frac{a\left(-\frac{a}{x^4}+\frac{4b}{x^2}+\frac{2b^2}{a+bx^2}\right)+12b^2\log(x)-6b^2\log\left(a+bx^2\right)}{4a^4}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^2),x]
```

output
```
(a*(-(a/x^4) + (4*b)/x^2 + (2*b^2)/(a + b*x^2)) + 12*b^2*Log[x] - 6*b^2*Lo
g[a + b*x^2])/(4*a^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2} \int \frac{1}{x^6 \left(bx^2+a\right)^2}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2} \int \left(-\frac{3b^3}{a^4 \left(bx^2+a\right)}-\frac{b^3}{a^3 \left(bx^2+a\right)^2}+\frac{3b^2}{a^4 x^2}-\frac{2b}{a^3 x^4}+\frac{1}{a^2 x^6}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2} \left(\frac{3b^2 \log \left(x^2\right)}{a^4}-\frac{3b^2 \log \left(a+bx^2\right)}{a^4}+\frac{b^2}{a^3 \left(a+bx^2\right)}+\frac{2b}{a^3 x^2}-\frac{1}{2a^2 x^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)^2),x]
```

output
```
(-1/2*1/(a^2*x^4) + (2*b)/(a^3*x^2) + b^2/(a^3*(a + b*x^2)) + (3*b^2*Log[x
^2])/a^4 - (3*b^2*Log[a + b*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{b^{3}\left(-\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{3\ln\left(b\,x^{2}+a\right)}{b}\right)}{2a^{4}}-\frac{1}{4a^{2}x^{4}}+\frac{b}{a^{3}x^{2}}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}$	65
norman	$\frac{-\frac{1}{4a}+\frac{3b\,x^{2}}{4a^{2}}-\frac{3b^{3}x^{6}}{2a^{4}}}{x^{4}\left(b\,x^{2}+a\right)}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}-\frac{3b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{4}}$	67
risch	$\frac{\frac{3b^{2}x^{4}}{2a^{3}}+\frac{3b\,x^{2}}{4a^{2}}-\frac{1}{4a}}{x^{4}\left(b\,x^{2}+a\right)}+\frac{3b^{2}\ln\left(x\right)}{a^{4}}-\frac{3b^{2}\ln\left(b\,x^{2}+a\right)}{2a^{4}}$	67
parallelrisch	$\frac{12b^{3}\ln\left(x\right)x^{6}-6b^{3}\ln\left(b\,x^{2}+a\right)x^{6}-6b^{3}x^{6}+12a\,b^{2}\ln\left(x\right)x^{4}-6\ln\left(b\,x^{2}+a\right)x^{4}a\,b^{2}+3a^{2}b\,x^{2}-a^{3}}{4a^{4}x^{4}\left(b\,x^{2}+a\right)}$	95

input
```
int(1/x^5/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^3/a^4*(-a/b/(b*x^2+a)+3*ln(b*x^2+a)/b)-1/4/a^2/x^4+b/a^3/x^2+3*b^2*
ln(x)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5\left(a+bx^2\right)^2}\,dx = \frac{6\,ab^2x^4+3\,a^2bx^2-a^3-6\left(b^3x^6+ab^2x^4\right)\log\left(bx^2+a\right)+12\left(b^3x^6+ab^2x^4\right)\log\left(x\right)}{4\left(a^4bx^6+a^5x^4\right)}$$

input
```
integrate(1/x^5/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*a*b^2*x^4 + 3*a^2*b*x^2 - a^3 - 6*(b^3*x^6 + a*b^2*x^4)*log(b*x^2 +
 a) + 12*(b^3*x^6 + a*b^2*x^4)*log(x))/(a^4*b*x^6 + a^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (a+bx^2)^2}\,dx = \frac{-a^2+3abx^2+6b^2x^4}{4a^4x^4+4a^3bx^6}+\frac{3b^2\log(x)}{a^4}-\frac{3b^2\log\left(\frac{a}{b}+x^2\right)}{2a^4}$$

input
```
integrate(1/x**5/(b*x**2+a)**2,x)
```

output
```
(-a**2 + 3*a*b*x**2 + 6*b**2*x**4)/(4*a**4*x**4 + 4*a**3*b*x**6) + 3*b**2*
log(x)/a**4 - 3*b**2*log(a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^5 (a+bx^2)^2}\,dx = \frac{6\,b^2x^4+3\,abx^2-a^2}{4\,(a^3bx^6+a^4x^4)}-\frac{3\,b^2\log(bx^2+a)}{2\,a^4}+\frac{3\,b^2\log(x^2)}{2\,a^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/4*(6*b^2*x^4 + 3*a*b*x^2 - a^2)/(a^3*b*x^6 + a^4*x^4) - 3/2*b^2*log(b*x^
2 + a)/a^4 + 3/2*b^2*log(x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^5 (a+bx^2)^2} \, dx = \frac{3 b^2 \log(x^2)}{2 a^4} - \frac{3 b^2 \log(|bx^2+a|)}{2 a^4} + \frac{3 b^3 x^2 + 4 a b^2}{2 (bx^2+a) a^4} - \frac{9 b^2 x^4 - 4 a b x^2 + a^2}{4 a^4 x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
3/2*b^2*log(x^2)/a^4 - 3/2*b^2*log(abs(b*x^2 + a))/a^4 + 1/2*(3*b^3*x^2 +
4*a*b^2)/((b*x^2 + a)*a^4) - 1/4*(9*b^2*x^4 - 4*a*b*x^2 + a^2)/(a^4*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5 (a+bx^2)^2} \, dx = \frac{\frac{3 b x^2}{4 a^2} - \frac{1}{4 a} + \frac{3 b^2 x^4}{2 a^3}}{b x^6 + a x^4} - \frac{3 b^2 \ln(b x^2 + a)}{2 a^4} + \frac{3 b^2 \ln(x)}{a^4}$$

input
```
int(1/(x^5*(a + b*x^2)^2),x)
```

output
```
((3*b*x^2)/(4*a^2) - 1/(4*a) + (3*b^2*x^4)/(2*a^3))/(a*x^4 + b*x^6) - (3*b
^2*log(a + b*x^2))/(2*a^4) + (3*b^2*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^5 (a+bx^2)^2} \, dx = \frac{-6 \log(b x^2+a) a b^2 x^4 - 6 \log(b x^2+a) b^3 x^6 + 12 \log(x) a b^2 x^4 + 12 \log(x) b^3 x^6 - a^3 + 3a^2 b x^2 - 6 b^3 x^6}{4 a^4 x^4 (b x^2 + a)}$$

input
```
int(1/x^5/(b*x^2+a)^2,x)
```

output
```
( - 6*log(a + b*x**2)*a*b**2*x**4 - 6*log(a + b*x**2)*b**3*x**6 + 12*log(x
)*a*b**2*x**4 + 12*log(x)*b**3*x**6 - a**3 + 3*a**2*b*x**2 - 6*b**3*x**6)/
(4*a**4*x**4*(a + b*x**2))
```

3.155 $\int \frac{1}{x^7(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx=-\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{b^3}{2a^4\left(a+bx^2\right)}-\frac{4b^3\log(x)}{a^5}+\frac{2b^3\log\left(a+bx^2\right)}{a^5}$$

output
```
-1/6/a^2/x^6+1/2*b/a^3/x^4-3/2*b^2/a^4/x^2-1/2*b^3/a^4/(b*x^2+a)-4*b^3*ln(
x)/a^5+2*b^3*ln(b*x^2+a)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^7\left(a+bx^2\right)^2}\,dx=\frac{a\left(-\frac{a^2}{x^6}+\frac{3ab}{x^4}-\frac{9b^2}{x^2}-\frac{3b^3}{a+bx^2}\right)-24b^3\log(x)+12b^3\log\left(a+bx^2\right)}{6a^5}$$

input
```
Integrate[1/(x^7*(a + b*x^2)^2),x]
```

output
```
(a*(-(a^2/x^6) + (3*a*b)/x^4 - (9*b^2)/x^2 - (3*b^3)/(a + b*x^2)) - 24*b^3
*Log[x] + 12*b^3*Log[a + b*x^2])/(6*a^5)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^8 (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{4b^4}{a^5 (bx^2+a)}+\frac{b^4}{a^4 (bx^2+a)^2}-\frac{4b^3}{a^5x^2}+\frac{3b^2}{a^4x^4}-\frac{2b}{a^3x^6}+\frac{1}{a^2x^8}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{4b^3\log(x^2)}{a^5}+\frac{4b^3\log(a+bx^2)}{a^5}-\frac{b^3}{a^4 (a+bx^2)}-\frac{3b^2}{a^4x^2}+\frac{b}{a^3x^4}-\frac{1}{3a^2x^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^2)^2),x]
```

output
```
(-1/3*1/(a^2*x^6) + b/(a^3*x^4) - (3*b^2)/(a^4*x^2) - b^3/(a^4*(a + b*x^2)
) - (4*b^3*Log[x^2])/a^5 + (4*b^3*Log[a + b*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{b^4\left(-\frac{a}{b\left(b\,x^2+a\right)}+\frac{4\ln\left(b\,x^2+a\right)}{b}\right)}{2a^5}-\frac{1}{6a^2x^6}+\frac{b}{2a^3x^4}-\frac{3b^2}{2a^4x^2}-\frac{4b^3\ln\left(x\right)}{a^5}$	77
norman	$\frac{\frac{2b^4x^8}{a^5}-\frac{1}{6a}+\frac{b\,x^2}{3a^2}-\frac{b^2x^4}{a^3}}{x^6\left(b\,x^2+a\right)}-\frac{4b^3\ln\left(x\right)}{a^5}+\frac{2b^3\ln\left(b\,x^2+a\right)}{a^5}$	78
risch	$\frac{-\frac{2b^3x^6}{a^4}-\frac{b^2x^4}{a^3}+\frac{b\,x^2}{3a^2}-\frac{1}{6a}}{x^6\left(b\,x^2+a\right)}-\frac{4b^3\ln\left(x\right)}{a^5}+\frac{2b^3\ln\left(-b\,x^2-a\right)}{a^5}$	81
parallelrisch	$-\frac{24b^4\ln\left(x\right)x^8-12b^4\ln\left(b\,x^2+a\right)x^8-12b^4x^8+24\ln\left(x\right)x^6a\,b^3-12\ln\left(b\,x^2+a\right)x^6a\,b^3+6a^2b^2x^4-2a^3b\,x^2+a^4}{6a^5x^6\left(b\,x^2+a\right)}$	104

input
```
int(1/x^7/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*b^4/a^5*(-a/b/(b*x^2+a)+4*ln(b*x^2+a)/b)-1/6/a^2/x^6+1/2*b/a^3/x^4-3/2
*b^2/a^4/x^2-4*b^3*ln(x)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^7 (a+bx^2)^2} \, dx = -\frac{12\, ab^3x^6 + 6\, a^2b^2x^4 - 2\, a^3bx^2 + a^4 - 12\, (b^4x^8 + ab^3x^6) \log(bx^2+a) + 24\, (b^4x^8 + ab^3x^6) \log(x)}{6\, (a^5bx^8 + a^6x^6)}$$

input
```
integrate(1/x^7/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(12*a*b^3*x^6 + 6*a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4 - 12*(b^4*x^8 + a*b
^3*x^6)*log(b*x^2 + a) + 24*(b^4*x^8 + a*b^3*x^6)*log(x))/(a^5*b*x^8 + a^6
*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (a+bx^2)^2} \, dx = \frac{-a^3 + 2a^2bx^2 - 6ab^2x^4 - 12b^3x^6}{6a^5x^6 + 6a^4bx^8} - \frac{4b^3 \log(x)}{a^5} + \frac{2b^3 \log\left(\frac{a}{b} + x^2\right)}{a^5}$$

input
```
integrate(1/x**7/(b*x**2+a)**2,x)
```

output
```
(-a**3 + 2*a**2*b*x**2 - 6*a*b**2*x**4 - 12*b**3*x**6)/(6*a**5*x**6 + 6*a*
*4*b*x**8) - 4*b**3*log(x)/a**5 + 2*b**3*log(a/b + x**2)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^7 (a+bx^2)^2} \, dx = -\frac{12\, b^3x^6 + 6\, ab^2x^4 - 2\, a^2bx^2 + a^3}{6\, (a^4bx^8 + a^5x^6)} + \frac{2\, b^3 \log(bx^2+a)}{a^5} - \frac{2\, b^3 \log(x^2)}{a^5}$$

input
```
integrate(1/x^7/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(12*b^3*x^6 + 6*a*b^2*x^4 - 2*a^2*b*x^2 + a^3)/(a^4*b*x^8 + a^5*x^6)
+ 2*b^3*log(b*x^2 + a)/a^5 - 2*b^3*log(x^2)/a^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^7 (a+bx^2)^2} \, dx = -\frac{2 b^3 \log(x^2)}{a^5} + \frac{2 b^3 \log(|bx^2+a|)}{a^5} - \frac{4 b^4 x^2 + 5 ab^3}{2 (bx^2+a) a^5} + \frac{22 b^3 x^6 - 9 ab^2 x^4 + 3 a^2 b x^2 - a^3}{6 a^5 x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-2*b^3*log(x^2)/a^5 + 2*b^3*log(abs(b*x^2 + a))/a^5 - 1/2*(4*b^4*x^2 + 5*a
*b^3)/((b*x^2 + a)*a^5) + 1/6*(22*b^3*x^6 - 9*a*b^2*x^4 + 3*a^2*b*x^2 - a^
3)/(a^5*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^7 (a+bx^2)^2} \, dx = \frac{2 b^3 \ln(b x^2 + a)}{a^5} - \frac{\frac{1}{6 a} - \frac{b x^2}{3 a^2} + \frac{b^2 x^4}{a^3} + \frac{2 b^3 x^6}{a^4}}{b x^8 + a x^6} - \frac{4 b^3 \ln(x)}{a^5}$$

input
```
int(1/(x^7*(a + b*x^2)^2),x)
```

output
```
(2*b^3*log(a + b*x^2))/a^5 - (1/(6*a) - (b*x^2)/(3*a^2) + (b^2*x^4)/a^3 +
(2*b^3*x^6)/a^4)/(a*x^6 + b*x^8) - (4*b^3*log(x))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^7 (a+bx^2)^2}\,dx$$
$$= \frac{12\log(b\,x^2+a)\,a\,b^3x^6+12\log(b\,x^2+a)\,b^4x^8-24\log(x)\,a\,b^3x^6-24\log(x)\,b^4x^8-a^4+2a^3b\,x^2-6a^2b^2}{6a^5x^6\,(b\,x^2+a)}$$

input
```
int(1/x^7/(b*x^2+a)^2,x)
```

output
```
(12*log(a + b*x**2)*a*b**3*x**6 + 12*log(a + b*x**2)*b**4*x**8 - 24*log(x)
*a*b**3*x**6 - 24*log(x)*b**4*x**8 - a**4 + 2*a**3*b*x**2 - 6*a**2*b**2*x*
*4 + 12*b**4*x**8)/(6*a**5*x**6*(a + b*x**2))
```

3.156 $\int \frac{1}{x^9(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 93

$$\int \frac{1}{x^9\left(a+bx^2\right)^2}\,dx = -\frac{1}{8a^2x^8}+\frac{b}{3a^3x^6}-\frac{3b^2}{4a^4x^4}+\frac{2b^3}{a^5x^2}+\frac{b^4}{2a^5\left(a+bx^2\right)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log\left(a+bx^2\right)}{2a^6}$$

output
```
-1/8/a^2/x^8+1/3*b/a^3/x^6-3/4*b^2/a^4/x^4+2*b^3/a^5/x^2+1/2*b^4/a^5/(b*x^
2+a)+5*b^4*ln(x)/a^6-5/2*b^4*ln(b*x^2+a)/a^6
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^9\left(a+bx^2\right)^2}\,dx = \frac{a\left(-\frac{3a^3}{x^8}+\frac{8a^2b}{x^6}-\frac{18ab^2}{x^4}+12b^3\left(\frac{4}{x^2}+\frac{b}{a+bx^2}\right)\right)+120b^4\log(x)-60b^4\log\left(a+bx^2\right)}{24a^6}$$

input
```
Integrate[1/(x^9*(a + b*x^2)^2),x]
```

output
```
(a*((-3*a^3)/x^8 + (8*a^2*b)/x^6 - (18*a*b^2)/x^4 + 12*b^3*(4/x^2 + b/(a +
 b*x^2))) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2])/(24*a^6)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{243, 54, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^{10} (bx^2+a)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{5b^5}{a^6 (bx^2+a)}-\frac{b^5}{a^5 (bx^2+a)^2}+\frac{5b^4}{a^6x^2}-\frac{4b^3}{a^5x^4}+\frac{3b^2}{a^4x^6}-\frac{2b}{a^3x^8}+\frac{1}{a^2x^{10}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{5b^4\log(x^2)}{a^6}-\frac{5b^4\log(a+bx^2)}{a^6}+\frac{b^4}{a^5 (a+bx^2)}+\frac{4b^3}{a^5x^2}-\frac{3b^2}{2a^4x^4}+\frac{2b}{3a^3x^6}-\frac{1}{4a^2x^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^2)^2),x]
```

output
```
(-1/4*1/(a^2*x^8) + (2*b)/(3*a^3*x^6) - (3*b^2)/(2*a^4*x^4) + (4*b^3)/(a^5
*x^2) + b^4/(a^5*(a + b*x^2)) + (5*b^4*Log[x^2])/a^6 - (5*b^4*Log[a + b*x^
2])/a^6)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result
default	$-\frac{b^{5}\left(-\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{5\ln\left(b\,x^{2}+a\right)}{b}\right)}{2a^{6}}-\frac{1}{8a^{2}x^{8}}+\frac{5b^{4}\ln\left(x\right)}{a^{6}}+\frac{2b^{3}}{a^{5}x^{2}}-\frac{3b^{2}}{4a^{4}x^{4}}+\frac{b}{3a^{3}x^{6}}$
norman	$\frac{-\frac{1}{8a}+\frac{5b\,x^{2}}{24a^{2}}-\frac{5b^{2}x^{4}}{12a^{3}}+\frac{5b^{3}x^{6}}{4a^{4}}-\frac{5b^{5}x^{10}}{2a^{6}}}{x^{8}\left(b\,x^{2}+a\right)}+\frac{5b^{4}\ln\left(x\right)}{a^{6}}-\frac{5b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{6}}$
risch	$\frac{\frac{5b^{4}x^{8}}{2a^{5}}+\frac{5b^{3}x^{6}}{4a^{4}}-\frac{5b^{2}x^{4}}{12a^{3}}+\frac{5b\,x^{2}}{24a^{2}}-\frac{1}{8a}}{x^{8}\left(b\,x^{2}+a\right)}+\frac{5b^{4}\ln\left(x\right)}{a^{6}}-\frac{5b^{4}\ln\left(b\,x^{2}+a\right)}{2a^{6}}$
parallelrisch	$\frac{120\ln\left(x\right)x^{10}b^{5}-60\ln\left(b\,x^{2}+a\right)x^{10}b^{5}-60b^{5}x^{10}+120a\,b^{4}\ln\left(x\right)x^{8}-60\ln\left(b\,x^{2}+a\right)x^{8}a\,b^{4}+30a^{2}b^{3}x^{6}-10a^{3}b^{2}x^{4}+5a^{4}b\,x^{2}-3a^{5}}{24a^{6}x^{8}\left(b\,x^{2}+a\right)}$

input
```
int(1/x^9/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b^5/a^6*(-a/b/(b*x^2+a)+5*ln(b*x^2+a)/b)-1/8/a^2/x^8+5*b^4*ln(x)/a^6+
2*b^3/a^5/x^2-3/4*b^2/a^4/x^4+1/3*b/a^3/x^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^9 (a+bx^2)^2}\,dx$$
$$= \frac{60\,ab^4x^8 + 30\,a^2b^3x^6 - 10\,a^3b^2x^4 + 5\,a^4bx^2 - 3\,a^5 - 60\,(b^5x^{10} + ab^4x^8)\log(bx^2 + a) + 120\,(b^5x^{10} + ab^4x}{24\,(a^6bx^{10} + a^7x^8)}$$

input
```
integrate(1/x^9/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/24*(60*a*b^4*x^8 + 30*a^2*b^3*x^6 - 10*a^3*b^2*x^4 + 5*a^4*b*x^2 - 3*a^5
 - 60*(b^5*x^10 + a*b^4*x^8)*log(b*x^2 + a) + 120*(b^5*x^10 + a*b^4*x^8)*l
og(x))/(a^6*b*x^10 + a^7*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^9 (a+bx^2)^2}\,dx = \frac{-3a^4 + 5a^3bx^2 - 10a^2b^2x^4 + 30ab^3x^6 + 60b^4x^8}{24a^6x^8 + 24a^5bx^{10}} + \frac{5b^4\log(x)}{a^6} - \frac{5b^4\log\left(\frac{a}{b} + x^2\right)}{2a^6}$$

input
```
integrate(1/x**9/(b*x**2+a)**2,x)
```

output
```
(-3*a**4 + 5*a**3*b*x**2 - 10*a**2*b**2*x**4 + 30*a*b**3*x**6 + 60*b**4*x*
*8)/(24*a**6*x**8 + 24*a**5*b*x**10) + 5*b**4*log(x)/a**6 - 5*b**4*log(a/b
 + x**2)/(2*a**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^9 (a + bx^2)^2} \, dx = \frac{60 \, b^4 x^8 + 30 \, a b^3 x^6 - 10 \, a^2 b^2 x^4 + 5 \, a^3 b x^2 - 3 \, a^4}{24 \, (a^5 b x^{10} + a^6 x^8)} - \frac{5 \, b^4 \log \left(b x^2 + a\right)}{2 \, a^6} + \frac{5 \, b^4 \log \left(x^2\right)}{2 \, a^6}$$

input
```
integrate(1/x^9/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/24*(60*b^4*x^8 + 30*a*b^3*x^6 - 10*a^2*b^2*x^4 + 5*a^3*b*x^2 - 3*a^4)/(a
^5*b*x^10 + a^6*x^8) - 5/2*b^4*log(b*x^2 + a)/a^6 + 5/2*b^4*log(x^2)/a^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^9 (a + bx^2)^2} \, dx = \frac{5 \, b^4 \log \left(x^2\right)}{2 \, a^6} - \frac{5 \, b^4 \log \left(|b x^2 + a|\right)}{2 \, a^6} + \frac{5 \, b^5 x^2 + 6 \, a b^4}{2 \, (b x^2 + a) a^6} - \frac{125 \, b^4 x^8 - 48 \, a b^3 x^6 + 18 \, a^2 b^2 x^4 - 8 \, a^3 b x^2 + 3 \, a^4}{24 \, a^6 x^8}$$

input
```
integrate(1/x^9/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
5/2*b^4*log(x^2)/a^6 - 5/2*b^4*log(abs(b*x^2 + a))/a^6 + 1/2*(5*b^5*x^2 +
6*a*b^4)/((b*x^2 + a)*a^6) - 1/24*(125*b^4*x^8 - 48*a*b^3*x^6 + 18*a^2*b^2
*x^4 - 8*a^3*b*x^2 + 3*a^4)/(a^6*x^8)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^9 (a+bx^2)^2}\, dx = \frac{\frac{5\,b\,x^2}{24\,a^2} - \frac{1}{8\,a} - \frac{5\,b^2\,x^4}{12\,a^3} + \frac{5\,b^3\,x^6}{4\,a^4} + \frac{5\,b^4\,x^8}{2\,a^5}}{b\,x^{10} + a\,x^8} - \frac{5\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^6} + \frac{5\,b^4\,\ln\left(x\right)}{a^6}$$

input
```
int(1/(x^9*(a + b*x^2)^2),x)
```

output
```
((5*b*x^2)/(24*a^2) - 1/(8*a) - (5*b^2*x^4)/(12*a^3) + (5*b^3*x^6)/(4*a^4)
 + (5*b^4*x^8)/(2*a^5))/(a*x^8 + b*x^10) - (5*b^4*log(a + b*x^2))/(2*a^6)
+ (5*b^4*log(x))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^9 (a+bx^2)^2}\, dx = \frac{-60\log(b\,x^2+a)\,a\,b^4x^8 - 60\log(b\,x^2+a)\,b^5x^{10} + 120\log(x)\,a\,b^4x^8 + 120\log(x)\,b^5x^{10} - 3a^5 + 5a^4b\,x^2 -}{24a^6x^8\left(b\,x^2+a\right)}$$

input
```
int(1/x^9/(b*x^2+a)^2,x)
```

output
```
( - 60*log(a + b*x**2)*a*b**4*x**8 - 60*log(a + b*x**2)*b**5*x**10 + 120*l
og(x)*a*b**4*x**8 + 120*log(x)*b**5*x**10 - 3*a**5 + 5*a**4*b*x**2 - 10*a*
*3*b**2*x**4 + 30*a**2*b**3*x**6 - 60*b**5*x**10)/(24*a**6*x**8*(a + b*x**
2))
```

3.157 $\int \frac{x^{12}}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{x^{12}}{(a+bx^2)^2} dx = \frac{5a^4x}{b^6} - \frac{4a^3x^3}{3b^5} + \frac{3a^2x^5}{5b^4} - \frac{2ax^7}{7b^3} + \frac{x^9}{9b^2} + \frac{a^5x}{2b^6(a+bx^2)} - \frac{11a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

output
```
5*a^4*x/b^6-4/3*a^3*x^3/b^5+3/5*a^2*x^5/b^4-2/7*a*x^7/b^3+1/9*x^9/b^2+1/2*
a^5*x/b^6/(b*x^2+a)-11/2*a^(9/2)*arctan(b^(1/2)*x/a^(1/2))/b^(13/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^{12}}{(a+bx^2)^2} dx = \frac{x\left(3150a^4 - 840a^3bx^2 + 378a^2b^2x^4 - 180ab^3x^6 + 70b^4x^8 + \frac{315a^5}{a+bx^2}\right)}{630b^6} - \frac{11a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

input
```
Integrate[x^12/(a + b*x^2)^2,x]
```

output
```
(x*(3150*a^4 - 840*a^3*b*x^2 + 378*a^2*b^2*x^4 - 180*a*b^3*x^6 + 70*b^4*x^
8 + (315*a^5)/(a + b*x^2)))/(630*b^6) - (11*a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{11\int \frac{x^{10}}{bx^2+a}dx}{2b}-\frac{x^{11}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{254}$$

$$\frac{11\int \left(\frac{x^8}{b}-\frac{ax^6}{b^2}+\frac{a^2x^4}{b^3}-\frac{a^3x^2}{b^4}-\frac{a^5}{b^5(bx^2+a)}+\frac{a^4}{b^5}\right)dx}{2b}-\frac{x^{11}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{11\left(-\frac{a^{9/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}+\frac{a^4x}{b^5}-\frac{a^3x^3}{3b^4}+\frac{a^2x^5}{5b^3}-\frac{ax^7}{7b^2}+\frac{x^9}{9b}\right)}{2b}-\frac{x^{11}}{2b(a+bx^2)}$$

input
```
Int[x^12/(a + b*x^2)^2,x]
```

output
```
-1/2*x^11/(b*(a + b*x^2)) + (11*((a^4*x)/b^5 - (a^3*x^3)/(3*b^4) + (a^2*x^
5)/(5*b^3) - (a*x^7)/(7*b^2) + x^9/(9*b) - (a^(9/2)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(11/2)))/(2*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

method	result
default	$\frac{\frac{1}{9}b^4x^9-\frac{2}{7}a\,b^3x^7+\frac{3}{5}a^2b^2x^5-\frac{4}{3}a^3b\,x^3+5a^4x}{b^6}-\frac{a^5\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{11\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^6}$
risch	$\frac{x^9}{9b^2}-\frac{2a\,x^7}{7b^3}+\frac{3a^2x^5}{5b^4}-\frac{4a^3x^3}{3b^5}+\frac{5a^4x}{b^6}+\frac{a^5x}{2b^6(b\,x^2+a)}+\frac{11\sqrt{-ab}\,a^4\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^7}-\frac{11\sqrt{-ab}\,a^4\ln\left(\sqrt{-ab}\,x-a\right)}{4b^7}$

input
```
int(x^12/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/9*b^4*x^9-2/7*a*b^3*x^7+3/5*a^2*b^2*x^5-4/3*a^3*b*x^3+5*a^4*x)-a^
5/b^6*(-1/2*x/(b*x^2+a)+11/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.25

$$\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx = \left[\frac{140\,b^5x^{11} - 220\,ab^4x^9 + 396\,a^2b^3x^7 - 924\,a^3b^2x^5 + 4620\,a^4bx^3 + 6930\,a^5x + 3465\,\left(a^4bx^2 + a^5\right)\sqrt{-\frac{a}{b}}\log\right.}{1260\,\left(b^7x^2 + ab^6\right)}$$

input
```
integrate(x^12/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(140*b^5*x^11 - 220*a*b^4*x^9 + 396*a^2*b^3*x^7 - 924*a^3*b^2*x^5
+ 4620*a^4*b*x^3 + 6930*a^5*x + 3465*(a^4*b*x^2 + a^5)*sqrt(-a/b)*log((b*x
^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^2 + a*b^6), 1/630*(70*b^5*
x^11 - 110*a*b^4*x^9 + 198*a^2*b^3*x^7 - 462*a^3*b^2*x^5 + 2310*a^4*b*x^3
+ 3465*a^5*x - 3465*(a^4*b*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(
b^7*x^2 + a*b^6)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.45

$$\int \frac{x^{12}}{\left(a+bx^2\right)^2}\,dx = \frac{a^5x}{2ab^6+2b^7x^2} + \frac{5a^4x}{b^6} - \frac{4a^3x^3}{3b^5} + \frac{3a^2x^5}{5b^4} - \frac{2ax^7}{7b^3} + \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} - \frac{11\sqrt{-\frac{a^9}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^9}{b^{13}}}}{a^4}\right)}{4} + \frac{x^9}{9b^2}$$

input
```
integrate(x**12/(b*x**2+a)**2,x)
```

output
```
a**5*x/(2*a*b**6 + 2*b**7*x**2) + 5*a**4*x/b**6 - 4*a**3*x**3/(3*b**5) + 3
*a**2*x**5/(5*b**4) - 2*a*x**7/(7*b**3) + 11*sqrt(-a**9/b**13)*log(x - b**
6*sqrt(-a**9/b**13)/a**4)/4 - 11*sqrt(-a**9/b**13)*log(x + b**6*sqrt(-a**9
/b**13)/a**4)/4 + x**9/(9*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{x^{12}}{(a+bx^2)^2}\,dx = \frac{a^5x}{2\,(b^7x^2+ab^6)} - \frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} + \frac{35\,b^4x^9 - 90\,ab^3x^7 + 189\,a^2b^2x^5 - 420\,a^3bx^3 + 1575\,a^4x}{315\,b^6}$$

input
```
integrate(x^12/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a^5*x/(b^7*x^2 + a*b^6) - 11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
6) + 1/315*(35*b^4*x^9 - 90*a*b^3*x^7 + 189*a^2*b^2*x^5 - 420*a^3*b*x^3 +
1575*a^4*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{x^{12}}{(a+bx^2)^2}\,dx = -\frac{11\,a^5\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} + \frac{a^5x}{2\,(bx^2+a)b^6} + \frac{35\,b^{16}x^9 - 90\,ab^{15}x^7 + 189\,a^2b^{14}x^5 - 420\,a^3b^{13}x^3 + 1575\,a^4b^{12}x}{315\,b^{18}}$$

input
```
integrate(x^12/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-11/2*a^5*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/2*a^5*x/((b*x^2 + a)*b
^6) + 1/315*(35*b^16*x^9 - 90*a*b^15*x^7 + 189*a^2*b^14*x^5 - 420*a^3*b^13
*x^3 + 1575*a^4*b^12*x)/b^18
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^{12}}{(a+bx^2)^2}\,dx = \frac{x^9}{9\,b^2} - \frac{2\,a\,x^7}{7\,b^3} + \frac{5\,a^4\,x}{b^6} - \frac{11\,a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{13/2}} + \frac{3\,a^2\,x^5}{5\,b^4} - \frac{4\,a^3\,x^3}{3\,b^5} + \frac{a^5\,x}{2\,(b^7\,x^2+a\,b^6)}$$

input
```
int(x^12/(a + b*x^2)^2,x)
```

output
```
x^9/(9*b^2) - (2*a*x^7)/(7*b^3) + (5*a^4*x)/b^6 - (11*a^(9/2)*atan((b^(1/2
)*x)/a^(1/2)))/(2*b^(13/2)) + (3*a^2*x^5)/(5*b^4) - (4*a^3*x^3)/(3*b^5) +
(a^5*x)/(2*(a*b^6 + b^7*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int \frac{x^{12}}{(a+bx^2)^2}\,dx = \frac{-3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5 - 3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2 + 3465a^5bx + 2310a^4b^2x^3 - 462a^3b^3x^5 +}{630b^7\,(b\,x^2+a)}$$

input
```
int(x^12/(b*x^2+a)^2,x)
```

output
```
( - 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 - 3465*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 3465*a**5*b*x + 2310*
a**4*b**2*x**3 - 462*a**3*b**3*x**5 + 198*a**2*b**4*x**7 - 110*a*b**5*x**9
 + 70*b**6*x**11)/(630*b**7*(a + b*x**2))
```

3.158 $\int \frac{x^{10}}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{x^{10}}{(a+bx^2)^2} dx = -\frac{4a^3x}{b^5} + \frac{a^2x^3}{b^4} - \frac{2ax^5}{5b^3} + \frac{x^7}{7b^2} - \frac{a^4x}{2b^5(a+bx^2)} + \frac{9a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

output
```
-4*a^3*x/b^5+a^2*x^3/b^4-2/5*a*x^5/b^3+1/7*x^7/b^2-1/2*a^4*x/b^5/(b*x^2+a)
+9/2*a^(7/2)*arctan(b^(1/2)*x/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}}{(a+bx^2)^2} dx = \frac{x\left(-280a^3 + 70a^2bx^2 - 28ab^2x^4 + 10b^3x^6 - \frac{35a^4}{a+bx^2}\right)}{70b^5} + \frac{9a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

input
```
Integrate[x^10/(a + b*x^2)^2,x]
```

output
```
(x*(-280*a^3 + 70*a^2*b*x^2 - 28*a*b^2*x^4 + 10*b^3*x^6 - (35*a^4)/(a + b*
x^2)))/(70*b^5) + (9*a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(11/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{9\int \frac{x^8}{bx^2+a}dx}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{254}$$

$$\frac{9\int \left(\frac{x^6}{b}-\frac{ax^4}{b^2}+\frac{a^2x^2}{b^3}+\frac{a^4}{b^4(bx^2+a)}-\frac{a^3}{b^4}\right)dx}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{9\left(\frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}\right)}{2b}-\frac{x^9}{2b\left(a+bx^2\right)}$$

input
```
Int[x^10/(a + b*x^2)^2,x]
```

output
```
-1/2*x^9/(b*(a + b*x^2)) + (9*(-((a^3*x)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5
)/(5*b^2) + x^7/(7*b) + (a^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)))/(2
*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{-\frac{1}{7} b^{3} x^{7}+\frac{2}{5} a\, b^{2} x^{5}-a^{2} b\, x^{3}+4 a^{3} x}{b^{5}}+\frac{a^{4} \left(-\frac{x}{2 \left(b\, x^{2}+a \right)}+\frac{9 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{5}}$	76
risch	$\frac{x^{7}}{7 b^{2}}-\frac{2 a\, x^{5}}{5 b^{3}}+\frac{a^{2} x^{3}}{b^{4}}-\frac{4 a^{3} x}{b^{5}}-\frac{a^{4} x}{2 b^{5} \left(b\, x^{2}+a \right)}+\frac{9 \sqrt{-a b}\, a^{3} \ln \left(-\sqrt{-a b}\, x+a \right)}{4 b^{6}}-\frac{9 \sqrt{-a b}\, a^{3} \ln \left(\sqrt{-a b}\, x+a \right)}{4 b^{6}}$	107

input
```
int(x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/7*b^3*x^7+2/5*a*b^2*x^5-a^2*b*x^3+4*a^3*x)+a^4/b^5*(-1/2*x/(b*x
^2+a)+9/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.41

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{20\,b^4x^9 - 36\,ab^3x^7 + 84\,a^2b^2x^5 - 420\,a^3bx^3 - 630\,a^4x + 315\,(a^3bx^2 + a^4)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{140\,(b^6x^2 + ab^5)}, \; 1 -\right.$$

input
```
integrate(x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/140*(20*b^4*x^9 - 36*a*b^3*x^7 + 84*a^2*b^2*x^5 - 420*a^3*b*x^3 - 630*a
^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)))/(b^6*x^2 + a*b^5), 1/70*(10*b^4*x^9 - 18*a*b^3*x^7 + 42*a^2
*b^2*x^5 - 210*a^3*b*x^3 - 315*a^4*x + 315*(a^3*b*x^2 + a^4)*sqrt(a/b)*arc
tan(b*x*sqrt(a/b)/a))/(b^6*x^2 + a*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx = -\frac{a^4x}{2ab^5+2b^6x^2} - \frac{4a^3x}{b^5} + \frac{a^2x^3}{b^4} - \frac{2ax^5}{5b^3} - \frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{9\sqrt{-\frac{a^7}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^7}{b^{11}}}}{a^3}\right)}{4} + \frac{x^7}{7b^2}$$

input
```
integrate(x**10/(b*x**2+a)**2,x)
```

output
```
-a**4*x/(2*a*b**5 + 2*b**6*x**2) - 4*a**3*x/b**5 + a**2*x**3/b**4 - 2*a*x*
*5/(5*b**3) - 9*sqrt(-a**7/b**11)*log(x - b**5*sqrt(-a**7/b**11)/a**3)/4 +
 9*sqrt(-a**7/b**11)*log(x + b**5*sqrt(-a**7/b**11)/a**3)/4 + x**7/(7*b**2
)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx = -\frac{a^4x}{2\,(b^6x^2+ab^5)} + \frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{5\,b^3x^7 - 14\,ab^2x^5 + 35\,a^2bx^3 - 140\,a^3x}{35\,b^5}$$

input
```
integrate(x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a^4*x/(b^6*x^2 + a*b^5) + 9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
5) + 1/35*(5*b^3*x^7 - 14*a*b^2*x^5 + 35*a^2*b*x^3 - 140*a^3*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx = \frac{9\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} - \frac{a^4x}{2\,(bx^2+a)b^5} + \frac{5\,b^{12}x^7 - 14\,ab^{11}x^5 + 35\,a^2b^{10}x^3 - 140\,a^3b^9x}{35\,b^{14}}$$

input
```
integrate(x^10/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
9/2*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) - 1/2*a^4*x/((b*x^2 + a)*b^5
) + 1/35*(5*b^12*x^7 - 14*a*b^11*x^5 + 35*a^2*b^10*x^3 - 140*a^3*b^9*x)/b^
14
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx = \frac{x^7}{7\,b^2} - \frac{2\,a\,x^5}{5\,b^3} - \frac{4\,a^3\,x}{b^5} + \frac{9\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{11/2}} + \frac{a^2\,x^3}{b^4} - \frac{a^4\,x}{2\,\left(b^6\,x^2+a\,b^5\right)}$$

input
```
int(x^10/(a + b*x^2)^2,x)
```

output
```
x^7/(7*b^2) - (2*a*x^5)/(5*b^3) - (4*a^3*x)/b^5 + (9*a^(7/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(11/2)) + (a^2*x^3)/b^4 - (a^4*x)/(2*(a*b^5 + b^6*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{x^{10}}{(a+bx^2)^2}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4 + 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2 - 315a^4bx - 210a^3b^2x^3 + 42a^2b^3x^5 - 18a\,b^4}{70b^6\,(b\,x^2+a)}$$

input
```
int(x^10/(b*x^2+a)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 + 315*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 - 315*a**4*b*x - 210*a**3*b*
*2*x**3 + 42*a**2*b**3*x**5 - 18*a*b**4*x**7 + 10*b**5*x**9)/(70*b**6*(a +
 b*x**2))
```

3.159 $\int \frac{x^8}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 78

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = \frac{3a^2x}{b^4} - \frac{2ax^3}{3b^3} + \frac{x^5}{5b^2} + \frac{a^3x}{2b^4(a+bx^2)} - \frac{7a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

output
```
3*a^2*x/b^4-2/3*a*x^3/b^3+1/5*x^5/b^2+1/2*a^3*x/b^4/(b*x^2+a)-7/2*a^(5/2)*
arctan(b^(1/2)*x/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = \frac{x\left(90a^2 - 20abx^2 + 6b^2x^4 + \frac{15a^3}{a+bx^2}\right)}{30b^4} - \frac{7a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{9/2}}$$

input
```
Integrate[x^8/(a + b*x^2)^2,x]
```

output
```
(x*(90*a^2 - 20*a*b*x^2 + 6*b^2*x^4 + (15*a^3)/(a + b*x^2)))/(30*b^4) - (7
*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(9/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7\int \frac{x^6}{bx^2+a}dx}{2b}-\frac{x^7}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{254}$$

$$\frac{7\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3(bx^2+a)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b\,(a+bx^2)}$$

input

```
Int[x^8/(a + b*x^2)^2,x]
```

output

```
-1/2*x^7/(b*(a + b*x^2)) + (7*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) -
 (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/(2*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{1}{5}b^2x^5-\frac{2}{3}ab\,x^3+3a^2x}{b^4}-\frac{a^3\left(-\frac{x}{2\left(b\,x^2+a\right)}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{b^4}$	65
risch	$\frac{x^5}{5b^2}-\frac{2a\,x^3}{3b^3}+\frac{3a^2x}{b^4}+\frac{a^3x}{2b^4\left(b\,x^2+a\right)}+\frac{7\sqrt{-ab}\,a^2\ln\left(-\sqrt{-ab}\,x-a\right)}{4b^5}-\frac{7\sqrt{-ab}\,a^2\ln\left(\sqrt{-ab}\,x-a\right)}{4b^5}$	101

input
```
int(x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/5*b^2*x^5-2/3*a*b*x^3+3*a^2*x)-a^3/b^4*(-1/2*x/(b*x^2+a)+7/2/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.44

$$\int \frac{x^8}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{12\,b^3x^7 - 28\,ab^2x^5 + 140\,a^2bx^3 + 210\,a^3x + 105\,(a^2bx^2 + a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{60\,(b^5x^2 + ab^4)}, \frac{6\,b^3x^7 - 14\,ab\right.$$

input
```
integrate(x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*b^3*x^7 - 28*a*b^2*x^5 + 140*a^2*b*x^3 + 210*a^3*x + 105*(a^2*b*
x^2 + a^3)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^
5*x^2 + a*b^4), 1/30*(6*b^3*x^7 - 14*a*b^2*x^5 + 70*a^2*b*x^3 + 105*a^3*x
- 105*(a^2*b*x^2 + a^3)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^2 + a*b^
4)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.59

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = \frac{a^3x}{2ab^4+2b^5x^2} + \frac{3a^2x}{b^4} - \frac{2ax^3}{3b^3} + \frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x - \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} - \frac{7\sqrt{-\frac{a^5}{b^9}}\log\left(x + \frac{b^4\sqrt{-\frac{a^5}{b^9}}}{a^2}\right)}{4} + \frac{x^5}{5b^2}$$

input
```
integrate(x**8/(b*x**2+a)**2,x)
```

output
```
a**3*x/(2*a*b**4 + 2*b**5*x**2) + 3*a**2*x/b**4 - 2*a*x**3/(3*b**3) + 7*sq
rt(-a**5/b**9)*log(x - b**4*sqrt(-a**5/b**9)/a**2)/4 - 7*sqrt(-a**5/b**9)*
log(x + b**4*sqrt(-a**5/b**9)/a**2)/4 + x**5/(5*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = \frac{a^3x}{2\,(b^5x^2+ab^4)} - \frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,b^2x^5-10\,abx^3+45\,a^2x}{15\,b^4}$$

input
```
integrate(x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a^3*x/(b^5*x^2 + a*b^4) - 7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4
) + 1/15*(3*b^2*x^5 - 10*a*b*x^3 + 45*a^2*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = -\frac{7\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{a^3x}{2\,(bx^2+a)b^4} + \frac{3\,b^8x^5-10\,ab^7x^3+45\,a^2b^6x}{15\,b^{10}}$$

input
```
integrate(x^8/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-7/2*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/2*a^3*x/((b*x^2 + a)*b^
4) + 1/15*(3*b^8*x^5 - 10*a*b^7*x^3 + 45*a^2*b^6*x)/b^10
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{x^8}{(a+bx^2)^2}\,dx = \frac{x^5}{5\,b^2} - \frac{2\,a\,x^3}{3\,b^3} + \frac{3\,a^2\,x}{b^4} - \frac{7\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{9/2}} + \frac{a^3\,x}{2\,\left(b^5\,x^2+a\,b^4\right)}$$

input
```
int(x^8/(a + b*x^2)^2,x)
```

output
```
x^5/(5*b^2) - (2*a*x^3)/(3*b^3) + (3*a^2*x)/b^4 - (7*a^(5/2)*atan((b^(1/2)
*x)/a^(1/2)))/(2*b^(9/2)) + (a^3*x)/(2*(a*b^4 + b^5*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.23

$$\int \frac{x^8}{(a+bx^2)^2} dx$$
$$= \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3 - 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2 + 105a^3bx + 70a^2b^2x^3 - 14a\,b^3x^5 + 6b^4x^7}{30b^5\,(b\,x^2+a)}$$

input
```
int(x^8/(b*x^2+a)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 - 105*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 105*a**3*b*x + 70*a**2*
b**2*x**3 - 14*a*b**3*x**5 + 6*b**4*x**7)/(30*b**5*(a + b*x**2))
```

3.160 $\int \frac{x^6}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^6}{(a+bx^2)^2} dx = -\frac{2ax}{b^3} + \frac{x^3}{3b^2} - \frac{a^2x}{2b^3(a+bx^2)} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

output
```
-2*a*x/b^3+1/3*x^3/b^2-1/2*a^2*x/b^3/(b*x^2+a)+5/2*a^(3/2)*arctan(b^(1/2)*
x/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{x^6}{(a+bx^2)^2} dx = \frac{x\left(-12a+2bx^2-\frac{3a^2}{a+bx^2}\right)}{6b^3} + \frac{5a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2)^2,x]
```

output
```
(x*(-12*a + 2*b*x^2 - (3*a^2)/(a + b*x^2)))/(6*b^3) + (5*a^(3/2)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*b^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{bx^2+a}dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{254}$$

$$\frac{5\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}$$

input
```
Int[x^6/(a + b*x^2)^2,x]
```

output
```
-1/2*x^5/(b*(a + b*x^2)) + (5*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/(2*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{-\frac{1}{3} b \,x^{3}+2 a x}{b^{3}}+\frac{a^{2} \left(-\frac{x}{2 \left(b \,x^{2}+a \right)}+\frac{5 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{3}}$	54
risch	$\frac{x^{3}}{3 b^{2}}-\frac{2 a x}{b^{3}}-\frac{a^{2} x}{2 b^{3} \left(b \,x^{2}+a \right)}+\frac{5 \sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x+a \right)}{4 b^{4}}-\frac{5 \sqrt{-a b}\, a \ln \left(\sqrt{-a b}\, x+a \right)}{4 b^{4}}$	82

input
```
int(x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/3*b*x^3+2*a*x)+a^2/b^3*(-1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.52

$$\int \frac{x^6}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,b^2x^5 - 20\,abx^3 - 30\,a^2x + 15\,(abx^2+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{12\,(b^4x^2+ab^3)}, \frac{2\,b^2x^5 - 10\,abx^3 - 15\,a^2x + 15}{6\,(b^4x^2}\right.$$

input
```
integrate(x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*b^2*x^5 - 20*a*b*x^3 - 30*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(-a/b)*l
og((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^4*x^2 + a*b^3), 1/6*(2*
b^2*x^5 - 10*a*b*x^3 - 15*a^2*x + 15*(a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*
sqrt(a/b)/a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.65

$$\int \frac{x^6}{(a+bx^2)^2}\,dx = -\frac{a^2x}{2ab^3+2b^4x^2} - \frac{2ax}{b^3} - \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x-\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{5\sqrt{-\frac{a^3}{b^7}}\log\left(x+\frac{b^3\sqrt{-\frac{a^3}{b^7}}}{a}\right)}{4} + \frac{x^3}{3b^2}$$

input
```
integrate(x**6/(b*x**2+a)**2,x)
```

output
```
-a**2*x/(2*a*b**3 + 2*b**4*x**2) - 2*a*x/b**3 - 5*sqrt(-a**3/b**7)*log(x -
 b**3*sqrt(-a**3/b**7)/a)/4 + 5*sqrt(-a**3/b**7)*log(x + b**3*sqrt(-a**3/b
**7)/a)/4 + x**3/(3*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.91

$$\int \frac{x^6}{(a+bx^2)^2}\,dx = -\frac{a^2x}{2\,(b^4x^2+ab^3)} + \frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{bx^3-6\,ax}{3\,b^3}$$

input
```
integrate(x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*x/(b^4*x^2 + a*b^3) + 5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^
3) + 1/3*(b*x^3 - 6*a*x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{x^6}{(a+bx^2)^2}\,dx = \frac{5\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{a^2x}{2\,(bx^2+a)b^3} + \frac{b^4x^3-6\,ab^3x}{3\,b^6}$$

input
```
integrate(x^6/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
5/2*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*a^2*x/((b*x^2 + a)*b^3
) + 1/3*(b^4*x^3 - 6*a*b^3*x)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(a+bx^2)^2}\,dx = \frac{x^3}{3\,b^2} + \frac{5\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{7/2}} - \frac{a^2\,x}{2\,\left(b^4\,x^2+a\,b^3\right)} - \frac{2\,a\,x}{b^3}$$

input
```
int(x^6/(a + b*x^2)^2,x)
```

output
```
x^3/(3*b^2) + (5*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(7/2)) - (a^2*x)/
(2*(a*b^3 + b^4*x^2)) - (2*a*x)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{x^6}{(a+bx^2)^2}\,dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2-15a^2bx-10a\,b^2x^3+2b^3x^5}{6b^4\,(b\,x^2+a)}$$

input
```
int(x^6/(b*x^2+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 15*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 - 15*a**2*b*x - 10*a*b**2*x**3 +
2*b**3*x**5)/(6*b**4*(a + b*x**2))
```

3.161 $\int \frac{x^4}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{x^4}{(a+bx^2)^2} dx = \frac{x}{b^2} + \frac{ax}{2b^2(a+bx^2)} - \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}$$

output
```
x/b^2+1/2*a*x/b^2/(b*x^2+a)-3/2*a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^2} dx = \frac{x}{b^2} + \frac{ax}{2b^2(a+bx^2)} - \frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^2,x]
```

output
```
x/b^2 + (a*x)/(2*b^2*(a + b*x^2)) - (3*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*b^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\int \frac{x^2}{bx^2+a}dx}{2b}-\frac{x^3}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{3\left(\frac{x}{b}-\frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{2b}-\frac{x^3}{2b\,(a+bx^2)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\,(a+bx^2)}$$

input
```
Int[x^4/(a + b*x^2)^2,x]
```

output
```
-1/2*x^3/(b*(a + b*x^2)) + (3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/b^(3/2)))/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{x}{b^{2}}-\frac{a \left(-\frac{x}{2 \left(b \,x^{2}+a \right)}+\frac{3 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^{2}}$	42
risch	$\frac{x}{b^{2}}+\frac{a x}{2 b^{2} \left(b \,x^{2}+a \right)}+\frac{3 \sqrt{-a b}\, \ln \left(-\sqrt{-a b}\, x -a \right)}{4 b^{3}}-\frac{3 \sqrt{-a b}\, \ln \left(\sqrt{-a b}\, x -a \right)}{4 b^{3}}$	72

input
```
int(x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2-a/b^2*(-1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.67

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = \left[\frac{4\,bx^3+3\,(bx^2+a)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+6\,ax}{4\,(b^3x^2+ab^2)}, \frac{2\,bx^3-3\,(bx^2+a)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+3\,ax}{2\,(b^3x^2+ab^2)}\right]$$

input
```
integrate(x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*b*x^3 + 3*(b*x^2 + a)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 6*a*x)/(b^3*x^2 + a*b^2), 1/2*(2*b*x^3 - 3*(b*x^2 + a)*sq
rt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3*a*x)/(b^3*x^2 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = \frac{ax}{2ab^2+2b^3x^2} + \frac{3\sqrt{-\frac{a}{b^5}}\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4} - \frac{3\sqrt{-\frac{a}{b^5}}\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{4} + \frac{x}{b^2}$$

input
```
integrate(x**4/(b*x**2+a)**2,x)
```

output
```
a*x/(2*a*b**2 + 2*b**3*x**2) + 3*sqrt(-a/b**5)*log(-b**2*sqrt(-a/b**5) + x
)/4 - 3*sqrt(-a/b**5)*log(b**2*sqrt(-a/b**5) + x)/4 + x/b**2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = \frac{ax}{2\,(b^3x^2+ab^2)} - \frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{x}{b^2}$$

input
```
integrate(x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a*x/(b^3*x^2 + a*b^2) - 3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) +
x/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = -\frac{3\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{ax}{2\,(bx^2+a)b^2} + \frac{x}{b^2}$$

input
```
integrate(x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-3/2*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*a*x/((b*x^2 + a)*b^2) +
 x/b^2
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = \frac{x}{b^2} + \frac{a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)} - \frac{3\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}$$

input
```
int(x^4/(a + b*x^2)^2,x)
```

output
```
x/b^2 + (a*x)/(2*(a*b^2 + b^3*x^2)) - (3*a^(1/2)*atan((b^(1/2)*x)/a^(1/2))
)/(2*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int \frac{x^4}{(a+bx^2)^2}\,dx = \frac{-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a-3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2+3abx+2b^2x^3}{2b^3\,(b\,x^2+a)}$$

input
```
int(x^4/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a - 3*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**2 + 3*a*b*x + 2*b**2*x**3)/(2*b**3*(a +
 b*x**2))
```

3.162 $\int \frac{x^2}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{x^2}{(a+bx^2)^2} \, dx = -\frac{x}{2b(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

output
```
-1/2*x/b/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^2} \, dx = -\frac{x}{2b(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^2,x]
```

output
```
-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^2} dx$$

$$\Big\downarrow \ 252$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2b} - \frac{x}{2b\,(a+bx^2)}$$

$$\Big\downarrow \ 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}} - \frac{x}{2b\,(a+bx^2)}$$

input
```
Int[x^2/(a + b*x^2)^2,x]
```

output
```
-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{x}{2b(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	36
risch	$-\frac{x}{2b(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,b}$	62

input
```
int(x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/b/(b*x^2+a)+1/2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = \left[-\frac{2\,abx+(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{4\,(ab^3x^2+a^2b^2)}, -\frac{abx-(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\,(ab^3x^2+a^2b^2)}\right]$$

input
```
integrate(x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x + (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^3*x^2 + a^2*b^2), -1/2*(a*b*x - (b*x^2 + a)*sqrt(a*b)*ar
ctan(sqrt(a*b)*x/a))/(a*b^3*x^2 + a^2*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = -\frac{x}{2ab+2b^2x^2} - \frac{\sqrt{-\frac{1}{ab^3}}\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{ab^3}}\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{4}$$

input
```
integrate(x**2/(b*x**2+a)**2,x)
```

output
```
-x/(2*a*b + 2*b**2*x**2) - sqrt(-1/(a*b**3))*log(-a*b*sqrt(-1/(a*b**3)) +
x)/4 + sqrt(-1/(a*b**3))*log(a*b*sqrt(-1/(a*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = -\frac{x}{2\,(b^2x^2+ab)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b}$$

input
```
integrate(x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(b^2*x^2 + a*b) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b} - \frac{x}{2\,(bx^2+a)b}$$

input
```
integrate(x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*x/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{3/2}} - \frac{x}{2\,b\,(b\,x^2+a)}$$

input
```
int(x^2/(a + b*x^2)^2,x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2)) - x/(2*b*(a + b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int \frac{x^2}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2 - abx}{2a\,b^2\,(b\,x^2+a)}$$

input
```
int(x^2/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 - a*b*x)/(2*a*b**2*(a + b*x**2))
```

3.163 $\int \frac{1}{(a+bx^2)^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 45

$$\int \frac{1}{(a+bx^2)^2} dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(b*x^2+a)+1/2*arctan(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^2} dx = \frac{x}{2a(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-2),x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a\,(a+bx^2)}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\,(a+bx^2)}$$

input
```
Int[(a + b*x^2)^(-2),x]
```

output
```
x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(b\,x^2+a)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	36
risch	$\frac{x}{2a(b\,x^2+a)}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{4\sqrt{-ab}\,a}$	62

input
```
int(1/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \frac{1}{\left(a+bx^2\right)^2}\,dx = \left[\frac{2\,abx-(bx^2+a)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)}, \frac{abx+(bx^2+a)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2a^2+2abx^2} - \frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(b*x**2+a)**2,x)
```

output
```
x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) +
x)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2\,(abx^2+a^2)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a}$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a} + \frac{x}{2(bx^2+a)a}$$

input
```
integrate(1/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{x}{2a\,(b\,x^2+a)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^2,x)
```

output
```
x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int \frac{1}{(a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a + \sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b\,x^2 + abx}{2a^2b\,(b\,x^2+a)}$$

input
```
int(1/(b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a + sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b*x**2 + a*b*x)/(2*a**2*b*(a + b*x**2))
```

3.164 $\int \frac{1}{x^2(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{1}{a^2x}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/a^2/x-1/2*b*x/a^2/(b*x^2+a)-3/2*b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(5/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{1}{a^2x}-\frac{bx}{2a^2\left(a+bx^2\right)}-\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^2),x]
```

output
```
-(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3 \int \frac{1}{x^2 (bx^2+a)}dx}{2a}+\frac{1}{2ax (a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{3 \left(-\frac{b \int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax (a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{3 \left(-\frac{\sqrt{b} \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax (a+bx^2)}$$

input
```
Int[1/(x^2*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{b\left(\frac{x}{2b\,x^{2}+2a}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{1}{a^{2}x}$	45
risch	$\frac{-\frac{3b\,x^{2}}{2a^{2}}-\frac{1}{a}}{x\left(b\,x^{2}+a\right)}+\frac{3\left(\sum_{\_R=\operatorname{RootOf}\left(a^{5}\_Z^{2}+b\right)}\_R\ln\left(\left(3\_R^{2}a^{5}+2b\right)x+a^{3}\_R\right)\right)}{4}$	68

input
```
int(1/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-b/a^2*(1/2*x/(b*x^2+a)+3/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^2 (a + bx^2)^2} \, dx = \left[-\frac{6 \, bx^2 - 3 \, (bx^3 + ax) \sqrt{-\frac{b}{a}} \log \left(\frac{bx^2 - 2 \, ax \sqrt{-\frac{b}{a}} - a}{bx^2 + a} \right) + 4 \, a}{4 \, (a^2 bx^3 + a^3 x)}, -\frac{3 \, bx^2 + 3 \, (bx^3 + ax) \sqrt{\frac{b}{a}} \arctan \left(x \sqrt{\frac{b}{a}} \right) + 2 \, a}{2 \, (a^2 bx^3 + a^3 x)} \right]$$

input
```
integrate(1/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a)
- a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a
*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^2 (a + bx^2)^2} \, dx = \frac{3\sqrt{-\frac{b}{a^5}} \log \left(-\frac{a^3 \sqrt{-\frac{b}{a^5}}}{b} + x \right)}{4} - \frac{3\sqrt{-\frac{b}{a^5}} \log \left(\frac{a^3 \sqrt{-\frac{b}{a^5}}}{b} + x \right)}{4} + \frac{-2a - 3bx^2}{2a^3 x + 2a^2 bx^3}$$

input
```
integrate(1/x**2/(b*x**2+a)**2,x)
```

output
```
3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**
3*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 (a+bx^2)^2} \, dx = -\frac{3 \, bx^2 + 2 \, a}{2 \, (a^2 bx^3 + a^3 x)} - \frac{3 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab} a^2}$$

input
```
integrate(1/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sq
rt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^2 (a+bx^2)^2} \, dx = -\frac{3 \, b \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab} a^2} - \frac{3 \, bx^2 + 2 \, a}{2 \, (bx^3 + ax) a^2}$$

input
```
integrate(1/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
 + a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2 (a+bx^2)^2} \, dx = -\frac{\frac{1}{a} + \frac{3 \, b \, x^2}{2 \, a^2}}{b \, x^3 + a \, x} - \frac{3 \, \sqrt{b} \, \mathrm{atan}\left(\frac{\sqrt{b} \, x}{\sqrt{a}}\right)}{2 \, a^{5/2}}$$

input
```
int(1/(x^2*(a + b*x^2)^2),x)
```

output
```
- (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^
(1/2)))/(2*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 (a+bx^2)^2} \, dx = \frac{-3\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ax - 3\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,x^3 - 2a^2 - 3ab\,x^2}{2a^3x\,(b\,x^2+a)}$$

input
```
int(1/x^2/(b*x^2+a)^2,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*x - 3*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x**3 - 2*a**2 - 3*a*b*x**2)/(2*a**3*x*(a
 + b*x**2))
```

3.165 $\int \frac{1}{x^4(a+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{x^4 (a+bx^2)^2} dx = -\frac{1}{3a^2x^3} + \frac{2b}{a^3x} + \frac{b^2x}{2a^3(a+bx^2)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-1/3/a^2/x^3+2*b/a^3/x+1/2*b^2*x/a^3/(b*x^2+a)+5/2*b^(3/2)*arctan(b^(1/2)*
x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a+bx^2)^2} dx = -\frac{1}{3a^2x^3} + \frac{2b}{a^3x} + \frac{b^2x}{2a^3(a+bx^2)} + \frac{5b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{x^4(bx^2+a)}dx}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}$$

input
```
Int[1/(x^4*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{b^{2}\left(\frac{x}{2 b \,x^{2}+2 a}+\frac{5 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{3}}-\frac{1}{3 a^{2} x^{3}}+\frac{2 b}{a^{3} x}$	55
risch	$\frac{\frac{5 b^{2} x^{4}}{2 a^{3}}+\frac{5 b \,x^{2}}{3 a^{2}}-\frac{1}{3 a}}{x^{3} \left(b \,x^{2}+a \right)}+\frac{5 \left(\sum_{\_R =\operatorname{RootOf}\left(a^{7} \_Z^{2}+b^{3}\right)}\_R \ln\left(\left(3 \_R^{2} a^{7}+2 b^{3}\right) x -a^{4} b \_R \right)\right)}{4}$	85

input
```
int(1/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
b^2/a^3*(1/2*x/(b*x^2+a)+5/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3/a^2/
x^3+2*b/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^4 (a+bx^2)^2} dx = \left[\frac{30 b^2 x^4 + 20 abx^2 + 15 (b^2 x^5 + abx^3) \sqrt{-\frac{b}{a}} \log \left(\frac{bx^2 + 2 ax\sqrt{-\frac{b}{a}} - a}{bx^2 + a} \right) - 4 a^2}{12 (a^3 bx^5 + a^4 x^3)}, \frac{15 b^2 x^4 + 10 abx^2 + 15 (b^2 x^5 + \right.$$

input
```
integrate(1/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*
x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1
/6*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sq
rt(b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{1}{x^4 (a+bx^2)^2} dx = -\frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(-\frac{a^4 \sqrt{-\frac{b^3}{a^7}}}{b^2} + x \right)}{4} + \frac{5\sqrt{-\frac{b^3}{a^7}} \log \left(\frac{a^4 \sqrt{-\frac{b^3}{a^7}}}{b^2} + x \right)}{4} + \frac{-2a^2 + 10abx^2 + 15b^2 x^4}{6a^4 x^3 + 6a^3 bx^5}$$

input
```
integrate(1/x**4/(b*x**2+a)**2,x)
```

output
```
-5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/
a**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*
b**2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx = \frac{15 \, b^2 x^4 + 10 \, abx^2 - 2 \, a^2}{6 \, (a^3 bx^5 + a^4 x^3)} + \frac{5 \, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab} a^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx = \frac{5 \, b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab} a^3} + \frac{b^2 x}{2 \, (bx^2 + a) a^3} + \frac{6 \, bx^2 - a}{3 \, a^3 x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3
) + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx = \frac{\frac{5 \, b \, x^2}{3 \, a^2} - \frac{1}{3 \, a} + \frac{5 \, b^2 \, x^4}{2 \, a^3}}{b \, x^5 + a \, x^3} + \frac{5 \, b^{3/2} \, \mathrm{atan}\left(\frac{\sqrt{b} \, x}{\sqrt{a}}\right)}{2 \, a^{7/2}}$$

input
```
int(1/(x^4*(a + b*x^2)^2),x)
```

output
```
((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b
^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^4 (a+bx^2)^2} \, dx = \frac{15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) ab\,x^3 + 15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^2x^5 - 2a^3 + 10a^2b\,x^2 + 15a\,b^2x^4}{6a^4x^3\left(b\,x^2+a\right)}$$

input
```
int(1/x^4/(b*x^2+a)^2,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 + 15*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 2*a**3 + 10*a**2*b*x**2 +
15*a*b**2*x**4)/(6*a**4*x**3*(a + b*x**2))
```

3.166 $\int \frac{1}{x^6(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 80

$$\int \frac{1}{x^6(a+bx^2)^2}\,dx = -\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}-\frac{b^3x}{2a^4(a+bx^2)}-\frac{7b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

output
```
-1/5/a^2/x^5+2/3*b/a^3/x^3-3*b^2/a^4/x-1/2*b^3*x/a^4/(b*x^2+a)-7/2*b^(5/2)
*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(a+bx^2)^2}\,dx = -\frac{1}{5a^2x^5}+\frac{2b}{3a^3x^3}-\frac{3b^2}{a^4x}-\frac{b^3x}{2a^4(a+bx^2)}-\frac{7b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^2),x]
```

output
```
-1/5*1/(a^2*x^5) + (2*b)/(3*a^3*x^3) - (3*b^2)/(a^4*x) - (b^3*x)/(2*a^4*(a
 + b*x^2)) - (7*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(9/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{x^6(bx^2+a)}dx}{2a}+\frac{1}{2ax^5(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\int \frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}$$

input
```
Int[1/(x^6*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x^5*(a + b*x^2)) + (7*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(
1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{b^{3}\left(\frac{x}{2b\,x^{2}+2a}+\frac{7\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{4}}-\frac{1}{5a^{2}x^{5}}+\frac{2b}{3a^{3}x^{3}}-\frac{3b^{2}}{a^{4}x}$	67
risch	$\frac{-\frac{7b^{3}x^{6}}{2a^{4}}-\frac{7b^{2}x^{4}}{3a^{3}}+\frac{7b\,x^{2}}{15a^{2}}-\frac{1}{5a}}{x^{5}\left(b\,x^{2}+a\right)}+\frac{7\sqrt{-ab}\,b^{2}\ln\left(-bx+\sqrt{-ab}\right)}{4a^{5}}-\frac{7\sqrt{-ab}\,b^{2}\ln\left(-bx-\sqrt{-ab}\right)}{4a^{5}}$	106

input
```
int(1/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-b^3/a^4*(1/2*x/(b*x^2+a)+7/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5/a^2
/x^5+2/3*b/a^3/x^3-3*b^2/a^4/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^6\,(a+bx^2)^2}\,dx$$
$$= \left[-\frac{210\,b^3x^6+140\,ab^2x^4-28\,a^2bx^2+12\,a^3-105\,(b^3x^7+ab^2x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)}{60\,(a^4bx^7+a^5x^5)},\right.$$
$$\left.-\frac{105\,b^3x^6+70\,ab^2x^4-14\,a^2bx^2+6\,a^3+105\,(b^3x^7+ab^2x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)}{30\,(a^4bx^7+a^5x^5)}\right]$$

input
```
integrate(1/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/60*(210*b^3*x^6 + 140*a*b^2*x^4 - 28*a^2*b*x^2 + 12*a^3 - 105*(b^3*x^7
 + a*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/
(a^4*b*x^7 + a^5*x^5), -1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 +
6*a^3 + 105*(b^3*x^7 + a*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b*x^
7 + a^5*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^6 (a+bx^2)^2}\, dx = \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4} - \frac{7\sqrt{-\frac{b^5}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^5}{a^9}}}{b^3}+x\right)}{4} + \frac{-6a^3+14a^2bx^2-70ab^2x^4-105b^3x^6}{30a^5x^5+30a^4bx^7}$$

input
```
integrate(1/x**6/(b*x**2+a)**2,x)
```

output
```
7*sqrt(-b**5/a**9)*log(-a**5*sqrt(-b**5/a**9)/b**3 + x)/4 - 7*sqrt(-b**5/a
**9)*log(a**5*sqrt(-b**5/a**9)/b**3 + x)/4 + (-6*a**3 + 14*a**2*b*x**2 - 7
0*a*b**2*x**4 - 105*b**3*x**6)/(30*a**5*x**5 + 30*a**4*b*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^6 (a+bx^2)^2}\, dx = -\frac{105\, b^3x^6+70\, ab^2x^4-14\, a^2bx^2+6\, a^3}{30\, (a^4bx^7+a^5x^5)} - \frac{7\, b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\, \sqrt{ab}a^4}$$

input
```
integrate(1/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/30*(105*b^3*x^6 + 70*a*b^2*x^4 - 14*a^2*b*x^2 + 6*a^3)/(a^4*b*x^7 + a^5
*x^5) - 7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6 \left(a+bx^2\right)^2}\,dx = -\frac{7\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4} - \frac{b^3x}{2\,(bx^2+a)a^4} - \frac{45\,b^2x^4-10\,abx^2+3\,a^2}{15\,a^4x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-7/2*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) - 1/2*b^3*x/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*x^4 - 10*a*b*x^2 + 3*a^2)/(a^4*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^6 \left(a+bx^2\right)^2}\,dx = -\frac{\frac{1}{5\,a}-\frac{7\,b\,x^2}{15\,a^2}+\frac{7\,b^2\,x^4}{3\,a^3}+\frac{7\,b^3\,x^6}{2\,a^4}}{b\,x^7+a\,x^5} - \frac{7\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{9/2}}$$

input
```
int(1/(x^6*(a + b*x^2)^2),x)
```

output
```
- (1/(5*a) - (7*b*x^2)/(15*a^2) + (7*b^2*x^4)/(3*a^3) + (7*b^3*x^6)/(2*a^4
))/(a*x^5 + b*x^7) - (7*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^6 \left(a+bx^2\right)^2}\,dx$$

$$= \frac{-105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^5 - 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3x^7 - 6a^4 + 14a^3b\,x^2 - 70a^2b^2x^4 - 105a\,b^3x^6}{30a^5x^5\,(b\,x^2+a)}$$

input
```
int(1/x^6/(b*x^2+a)^2,x)
```

output
```
( - 105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**5 - 105*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**7 - 6*a**4 + 14*a**3*b
*x**2 - 70*a**2*b**2*x**4 - 105*a*b**3*x**6)/(30*a**5*x**5*(a + b*x**2))
```

3.167 $\int \frac{1}{x^8(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{1}{x^8 (a+bx^2)^2}\,dx = -\frac{1}{7a^2x^7}+\frac{2b}{5a^3x^5}-\frac{b^2}{a^4x^3}+\frac{4b^3}{a^5x}+\frac{b^4x}{2a^5(a+bx^2)}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

output
```
-1/7/a^2/x^7+2/5*b/a^3/x^5-b^2/a^4/x^3+4*b^3/a^5/x+1/2*b^4*x/a^5/(b*x^2+a)
+9/2*b^(7/2)*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^8 (a+bx^2)^2}\,dx = -\frac{1}{7a^2x^7}+\frac{2b}{5a^3x^5}-\frac{b^2}{a^4x^3}+\frac{4b^3}{a^5x}+\frac{b^4x}{2a^5(a+bx^2)}+\frac{9b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

input
```
Integrate[1/(x^8*(a + b*x^2)^2),x]
```

output
```
-1/7*1/(a^2*x^7) + (2*b)/(5*a^3*x^5) - b^2/(a^4*x^3) + (4*b^3)/(a^5*x) + (
b^4*x)/(2*a^5*(a + b*x^2)) + (9*b^(7/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^
(11/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {253, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \; 253$$

$$\frac{9\int \frac{1}{x^8(bx^2+a)}dx}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{9\left(-\frac{b\int \frac{1}{x^6\left(bx^2+a\right)}dx}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{9\left(-\frac{b\left(-\frac{b\int \frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

$$\Big\downarrow \; 264$$

$$\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

$\Big\downarrow$ 264

$$\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

$\Big\downarrow$ 218

$$\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}$$

input
```
Int[1/(x^8*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x^7*(a + b*x^2)) + (9*(-1/7*1/(a*x^7) - (b*(-1/5*1/(a*x^5) - (b*(-1
/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3
/2)))/a))/a))/a))/(2*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{b^{4}\left(\frac{x}{2 b\,x^{2}+2 a}+\frac{9 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{a^{5}}-\frac{1}{7 a^{2} x^{7}}+\frac{2 b}{5 a^{3} x^{5}}-\frac{b^{2}}{a^{4} x^{3}}+\frac{4 b^{3}}{a^{5} x}$	77
risch	$\frac{\frac{9 b^{4} x^{8}}{2 a^{5}}+\frac{3 b^{3} x^{6}}{a^{4}}-\frac{3 b^{2} x^{4}}{5 a^{3}}+\frac{9 b\,x^{2}}{35 a^{2}}-\frac{1}{7 a}}{x^{7} \left(b\,x^{2}+a\right)}+\frac{9 \sqrt{-a b}\, b^{3} \ln\left(-b x-\sqrt{-a b}\right)}{4 a^{6}}-\frac{9 \sqrt{-a b}\, b^{3} \ln\left(-b x+\sqrt{-a b}\right)}{4 a^{6}}$	117

input
```
int(1/x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
b^4/a^5*(1/2*x/(b*x^2+a)+9/2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/7/a^2/
x^7+2/5*b/a^3/x^5-b^2/a^4/x^3+4*b^3/a^5/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.42

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = \left[\frac{630 \, b^4 x^8 + 420 \, ab^3 x^6 - 84 \, a^2 b^2 x^4 + 36 \, a^3 b x^2 - 20 \, a^4 + 315 \, (b^4 x^9 + ab^3 x^7) \sqrt{-\frac{b}{a}} \log \left(\frac{bx^2 + 2 \, ax \sqrt{-\frac{b}{a}} - a}{bx^2 + a} \right)}{140 \, (a^5 b x^9 + a^6 x^7)}, \right.$$

input
```
integrate(1/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/140*(630*b^4*x^8 + 420*a*b^3*x^6 - 84*a^2*b^2*x^4 + 36*a^3*b*x^2 - 20*a
^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) -
a)/(b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/70*(315*b^4*x^8 + 210*a*b^3*x^6
- 42*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4 + 315*(b^4*x^9 + a*b^3*x^7)*sqrt(
b/a)*arctan(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = -\frac{9\sqrt{-\frac{b^7}{a^{11}}} \log \left(-\frac{a^6 \sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x \right)}{4} + \frac{9\sqrt{-\frac{b^7}{a^{11}}} \log \left(\frac{a^6 \sqrt{-\frac{b^7}{a^{11}}}}{b^4} + x \right)}{4} + \frac{-10a^4 + 18a^3 bx^2 - 42a^2 b^2 x^4 + 210ab^3 x^6 + 315b^4 x^8}{70a^6 x^7 + 70a^5 bx^9}$$

input
```
integrate(1/x**8/(b*x**2+a)**2,x)
```

output
```
-9*sqrt(-b**7/a**11)*log(-a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + 9*sqrt(-b**
7/a**11)*log(a**6*sqrt(-b**7/a**11)/b**4 + x)/4 + (-10*a**4 + 18*a**3*b*x*
*2 - 42*a**2*b**2*x**4 + 210*a*b**3*x**6 + 315*b**4*x**8)/(70*a**6*x**7 +
70*a**5*b*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = \frac{315 \, b^4x^8 + 210 \, ab^3x^6 - 42 \, a^2b^2x^4 + 18 \, a^3bx^2 - 10 \, a^4}{70 \, (a^5bx^9 + a^6x^7)} + \frac{9 \, b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab}a^5}$$

input
```
integrate(1/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/70*(315*b^4*x^8 + 210*a*b^3*x^6 - 42*a^2*b^2*x^4 + 18*a^3*b*x^2 - 10*a^4
)/(a^5*b*x^9 + a^6*x^7) + 9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = \frac{9 \, b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2 \, \sqrt{ab}a^5} + \frac{b^4x}{2 \, (bx^2 + a)a^5} + \frac{140 \, b^3x^6 - 35 \, ab^2x^4 + 14 \, a^2bx^2 - 5 \, a^3}{35 \, a^5x^7}$$

input
```
integrate(1/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
9/2*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) + 1/2*b^4*x/((b*x^2 + a)*a^5
) + 1/35*(140*b^3*x^6 - 35*a*b^2*x^4 + 14*a^2*b*x^2 - 5*a^3)/(a^5*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = \frac{\frac{9\,b\,x^2}{35\,a^2}-\frac{1}{7\,a}-\frac{3\,b^2\,x^4}{5\,a^3}+\frac{3\,b^3\,x^6}{a^4}+\frac{9\,b^4\,x^8}{2\,a^5}}{b\,x^9+a\,x^7}+\frac{9\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{11/2}}$$

input
```
int(1/(x^8*(a + b*x^2)^2),x)
```

output
```
((9*b*x^2)/(35*a^2) - 1/(7*a) - (3*b^2*x^4)/(5*a^3) + (3*b^3*x^6)/a^4 + (9
*b^4*x^8)/(2*a^5))/(a*x^7 + b*x^9) + (9*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))
/(2*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^8 (a+bx^2)^2} \, dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x^7+315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^4x^9-10a^5+18a^4b\,x^2-42a^3b^2x^4+210a^2b^3x^6}{70a^6x^7\,(b\,x^2+a)}$$

input
```
int(1/x^8/(b*x^2+a)^2,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**7 + 315*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**9 - 10*a**5 + 18*a**4*b*x
**2 - 42*a**3*b**2*x**4 + 210*a**2*b**3*x**6 + 315*a*b**4*x**8)/(70*a**6*x
**7*(a + b*x**2))
```

3.168 $\int \frac{x^{15}}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{x^{15}}{(a+bx^2)^3} \, dx = \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{x^{10}}{10b^3} + \frac{a^7}{4b^8(a+bx^2)^2} - \frac{7a^6}{2b^8(a+bx^2)} - \frac{21a^5\log(a+bx^2)}{2b^8}$$

output
```
15/2*a^4*x^2/b^7-5/2*a^3*x^4/b^6+a^2*x^6/b^5-3/8*a*x^8/b^4+1/10*x^10/b^3+1
/4*a^7/b^8/(b*x^2+a)^2-7/2*a^6/b^8/(b*x^2+a)-21/2*a^5*ln(b*x^2+a)/b^8
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{x^{15}}{(a+bx^2)^3} \, dx = \frac{300a^4bx^2 - 100a^3b^2x^4 + 40a^2b^3x^6 - 15ab^4x^8 + 4b^5x^{10} + \frac{10a^7}{(a+bx^2)^2} - \frac{140a^6}{a+bx^2} - 420a^5\log(a+bx^2)}{40b^8}$$

input
```
Integrate[x^15/(a + b*x^2)^3,x]
```

output
```
(300*a^4*b*x^2 - 100*a^3*b^2*x^4 + 40*a^2*b^3*x^6 - 15*a*b^4*x^8 + 4*b^5*x
^10 + (10*a^7)/(a + b*x^2)^2 - (140*a^6)/(a + b*x^2) - 420*a^5*Log[a + b*x
^2])/(40*b^8)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^{14}}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{x^8}{b^3}-\frac{3ax^6}{b^4}+\frac{6a^2x^4}{b^5}-\frac{10a^3x^2}{b^6}-\frac{21a^5}{b^7\,(bx^2+a)}+\frac{7a^6}{b^7\,(bx^2+a)^2}-\frac{a^7}{b^7\,(bx^2+a)^3}+\frac{15a^4}{b^7}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{a^7}{2b^8\,(a+bx^2)^2}-\frac{7a^6}{b^8\,(a+bx^2)}-\frac{21a^5\log\left(a+bx^2\right)}{b^8}+\frac{15a^4x^2}{b^7}-\frac{5a^3x^4}{b^6}+\frac{2a^2x^6}{b^5}-\frac{3ax^8}{4b^4}+\frac{x^{10}}{5b^3}\right)$$

input
```
Int[x^15/(a + b*x^2)^3,x]
```

output
```
((15*a^4*x^2)/b^7 - (5*a^3*x^4)/b^6 + (2*a^2*x^6)/b^5 - (3*a*x^8)/(4*b^4)
+ x^10/(5*b^3) + a^7/(2*b^8*(a + b*x^2)^2) - (7*a^6)/(b^8*(a + b*x^2)) - (
21*a^5*Log[a + b*x^2])/b^8)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

method	result
risch	$\frac{x^{10}}{10b^3}-\frac{3a\,x^8}{8b^4}+\frac{a^2x^6}{b^5}-\frac{5a^3x^4}{2b^6}+\frac{15a^4x^2}{2b^7}+\frac{-\frac{7a^6x^2}{2}-\frac{13a^7}{4b}}{b^7\left(b\,x^2+a\right)^2}-\frac{21a^5\ln\left(b\,x^2+a\right)}{2b^8}$
norman	$\frac{\frac{x^{14}}{10b}-\frac{7a\,x^{12}}{40b^2}+\frac{7a^2x^{10}}{20b^3}-\frac{63a^7}{4b^8}-\frac{7a^3x^8}{8b^4}+\frac{7a^4x^6}{2b^5}-\frac{21a^6x^2}{b^7}}{\left(b\,x^2+a\right)^2}-\frac{21a^5\ln\left(b\,x^2+a\right)}{2b^8}$
default	$\frac{\frac{1}{10}b^4x^{10}-\frac{3}{8}a\,b^3x^8+a^2b^2x^6-\frac{5}{2}a^3b\,x^4+\frac{15}{2}a^4x^2}{b^7}-\frac{a^5\left(-\frac{a^2}{2b\left(b\,x^2+a\right)^2}+\frac{7a}{b\left(b\,x^2+a\right)}+\frac{21\ln\left(b\,x^2+a\right)}{b}\right)}{2b^7}$
parallelrisch	$-\frac{-4b^7x^{14}+7a\,b^6x^{12}-14a^2b^5x^{10}+35a^3b^4x^8-140a^4b^3x^6+420\ln\left(b\,x^2+a\right)x^4a^5b^2+840\ln\left(b\,x^2+a\right)x^2a^6b+840a^6b\,x^2+420\ln}{40b^8\left(b\,x^2+a\right)^2}$

input
```
int(x^15/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*x^10/b^3-3/8*a*x^8/b^4+a^2*x^6/b^5-5/2*a^3*x^4/b^6+15/2*a^4*x^2/b^7+(
-7/2*a^6*x^2-13/4*a^7/b)/b^7/(b*x^2+a)^2-21/2*a^5*ln(b*x^2+a)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.20

$$\int \frac{x^{15}}{(a+bx^2)^3}\,dx = \frac{4\,b^7x^{14} - 7\,ab^6x^{12} + 14\,a^2b^5x^{10} - 35\,a^3b^4x^8 + 140\,a^4b^3x^6 + 500\,a^5b^2x^4 + 160\,a^6bx^2 - 130\,a^7 - 420\,(a^5b^2x}{40\,(b^{10}x^4 + 2\,ab^9x^2 + a^2b^8)}$$

input
```
integrate(x^15/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/40*(4*b^7*x^14 - 7*a*b^6*x^12 + 14*a^2*b^5*x^10 - 35*a^3*b^4*x^8 + 140*a
^4*b^3*x^6 + 500*a^5*b^2*x^4 + 160*a^6*b*x^2 - 130*a^7 - 420*(a^5*b^2*x^4
+ 2*a^6*b*x^2 + a^7)*log(b*x^2 + a))/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04

$$\int \frac{x^{15}}{(a+bx^2)^3}\,dx = -\frac{21a^5\log(a+bx^2)}{2b^8} + \frac{15a^4x^2}{2b^7} - \frac{5a^3x^4}{2b^6} + \frac{a^2x^6}{b^5} - \frac{3ax^8}{8b^4} + \frac{-13a^7 - 14a^6bx^2}{4a^2b^8 + 8ab^9x^2 + 4b^{10}x^4} + \frac{x^{10}}{10b^3}$$

input
```
integrate(x**15/(b*x**2+a)**3,x)
```

output
```
-21*a**5*log(a + b*x**2)/(2*b**8) + 15*a**4*x**2/(2*b**7) - 5*a**3*x**4/(2
*b**6) + a**2*x**6/b**5 - 3*a*x**8/(8*b**4) + (-13*a**7 - 14*a**6*b*x**2)/
(4*a**2*b**8 + 8*a*b**9*x**2 + 4*b**10*x**4) + x**10/(10*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{x^{15}}{(a+bx^2)^3}\,dx = -\frac{14\,a^6bx^2+13\,a^7}{4\,(b^{10}x^4+2\,ab^9x^2+a^2b^8)} - \frac{21\,a^5\log\left(bx^2+a\right)}{2\,b^8} + \frac{4\,b^4x^{10}-15\,ab^3x^8+40\,a^2b^2x^6-100\,a^3bx^4+300\,a^4x^2}{40\,b^7}$$

input
```
integrate(x^15/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(14*a^6*b*x^2 + 13*a^7)/(b^10*x^4 + 2*a*b^9*x^2 + a^2*b^8) - 21/2*a^5
*log(b*x^2 + a)/b^8 + 1/40*(4*b^4*x^10 - 15*a*b^3*x^8 + 40*a^2*b^2*x^6 - 1
00*a^3*b*x^4 + 300*a^4*x^2)/b^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{x^{15}}{(a+bx^2)^3}\,dx = -\frac{21\,a^5\log\left(|bx^2+a|\right)}{2\,b^8} + \frac{63\,a^5b^2x^4+112\,a^6bx^2+50\,a^7}{4\,(bx^2+a)^2b^8} + \frac{4\,b^{12}x^{10}-15\,ab^{11}x^8+40\,a^2b^{10}x^6-100\,a^3b^9x^4+300\,a^4b^8x^2}{40\,b^{15}}$$

input
```
integrate(x^15/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-21/2*a^5*log(abs(b*x^2 + a))/b^8 + 1/4*(63*a^5*b^2*x^4 + 112*a^6*b*x^2 +
50*a^7)/((b*x^2 + a)^2*b^8) + 1/40*(4*b^12*x^10 - 15*a*b^11*x^8 + 40*a^2*b
^10*x^6 - 100*a^3*b^9*x^4 + 300*a^4*b^8*x^2)/b^15
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx = \frac{x^{10}}{10\,b^3} - \frac{\frac{13\,a^7}{4\,b} + \frac{7\,a^6\,x^2}{2}}{a^2\,b^7 + 2\,a\,b^8\,x^2 + b^9\,x^4} - \frac{3\,a\,x^8}{8\,b^4} - \frac{21\,a^5\,\ln\left(b\,x^2+a\right)}{2\,b^8} + \frac{a^2\,x^6}{b^5} - \frac{5\,a^3\,x^4}{2\,b^6} + \frac{15\,a^4\,x^2}{2\,b^7}$$

input
```
int(x^15/(a + b*x^2)^3,x)
```

output
```
x^10/(10*b^3) - ((13*a^7)/(4*b) + (7*a^6*x^2)/2)/(a^2*b^7 + b^9*x^4 + 2*a*
b^8*x^2) - (3*a*x^8)/(8*b^4) - (21*a^5*log(a + b*x^2))/(2*b^8) + (a^2*x^6)
/b^5 - (5*a^3*x^4)/(2*b^6) + (15*a^4*x^2)/(2*b^7)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.24

$$\int \frac{x^{15}}{\left(a+bx^2\right)^3}\,dx = \frac{-420\log\left(b\,x^2+a\right)a^7 - 840\log\left(b\,x^2+a\right)a^6 b\,x^2 - 420\log\left(b\,x^2+a\right)a^5 b^2 x^4 - 210a^7 + 420a^5 b^2 x^4 + 140a^4}{40b^8\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^15/(b*x^2+a)^3,x)
```

output
```
( - 420*log(a + b*x**2)*a**7 - 840*log(a + b*x**2)*a**6*b*x**2 - 420*log(a
 + b*x**2)*a**5*b**2*x**4 - 210*a**7 + 420*a**5*b**2*x**4 + 140*a**4*b**3*
x**6 - 35*a**3*b**4*x**8 + 14*a**2*b**5*x**10 - 7*a*b**6*x**12 + 4*b**7*x*
*14)/(40*b**8*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.169 $\int \frac{x^{13}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{x^{13}}{(a+bx^2)^3} dx = -\frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{x^8}{8b^3} - \frac{a^6}{4b^7(a+bx^2)^2} + \frac{3a^5}{b^7(a+bx^2)} + \frac{15a^4\log(a+bx^2)}{2b^7}$$

output
```
-5*a^3*x^2/b^6+3/2*a^2*x^4/b^5-1/2*a*x^6/b^4+1/8*x^8/b^3-1/4*a^6/b^7/(b*x^
2+a)^2+3*a^5/b^7/(b*x^2+a)+15/2*a^4*ln(b*x^2+a)/b^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.85

$$\int \frac{x^{13}}{(a+bx^2)^3} dx = \frac{-40a^3bx^2 + 12a^2b^2x^4 - 4ab^3x^6 + b^4x^8 - \frac{2a^6}{(a+bx^2)^2} + \frac{24a^5}{a+bx^2} + 60a^4\log(a+bx^2)}{8b^7}$$

input
```
Integrate[x^13/(a + b*x^2)^3,x]
```

output
```
(-40*a^3*b*x^2 + 12*a^2*b^2*x^4 - 4*a*b^3*x^6 + b^4*x^8 - (2*a^6)/(a + b*x
^2)^2 + (24*a^5)/(a + b*x^2) + 60*a^4*Log[a + b*x^2])/(8*b^7)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(\frac{a^6}{b^6(bx^2+a)^3}-\frac{6a^5}{b^6(bx^2+a)^2}+\frac{15a^4}{b^6(bx^2+a)}-\frac{10a^3}{b^6}+\frac{6x^2a^2}{b^5}-\frac{3x^4a}{b^4}+\frac{x^6}{b^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{a^6}{2b^7(a+bx^2)^2}+\frac{6a^5}{b^7(a+bx^2)}+\frac{15a^4\log(a+bx^2)}{b^7}-\frac{10a^3x^2}{b^6}+\frac{3a^2x^4}{b^5}-\frac{ax^6}{b^4}+\frac{x^8}{4b^3}\right)$$

input
```
Int[x^13/(a + b*x^2)^3,x]
```

output
```
((-10*a^3*x^2)/b^6 + (3*a^2*x^4)/b^5 - (a*x^6)/b^4 + x^8/(4*b^3) - a^6/(2*
b^7*(a + b*x^2)^2) + (6*a^5)/(b^7*(a + b*x^2)) + (15*a^4*Log[a + b*x^2])/b
^7)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87

method	result	s
norman	$\frac{\frac{15a^5x^2}{b^6}+\frac{x^{12}}{8b}-\frac{a x^{10}}{4b^2}+\frac{5a^2x^8}{8b^3}+\frac{45a^6}{4b^7}-\frac{5a^3x^6}{2b^4}}{(b x^2+a)^2}+\frac{15a^4\ln(b x^2+a)}{2b^7}$	8
risch	$\frac{x^8}{8b^3}-\frac{a x^6}{2b^4}+\frac{3a^2x^4}{2b^5}-\frac{5a^3x^2}{b^6}+\frac{3a^5x^2+\frac{11a^6}{4b}}{b^6(b x^2+a)^2}+\frac{15a^4\ln(b x^2+a)}{2b^7}$	8
default	$-\frac{-\frac{1}{8}b^3x^8+\frac{1}{2}a b^2x^6-\frac{3}{2}a^2b x^4+5a^3x^2}{b^6}+\frac{a^4\left(-\frac{a^2}{2b\left(b x^2+a\right)^2}+\frac{6a}{b\left(b x^2+a\right)}+\frac{15\ln\left(b x^2+a\right)}{b}\right)}{2b^6}$	9
parallelrisch	$\frac{b^6x^{12}-2a b^5x^{10}+5a^2b^4x^8-20a^3x^6b^3+60\ln(b x^2+a)x^4a^4b^2+120\ln(b x^2+a)x^2a^5b+120a^5b x^2+60\ln(b x^2+a)a^6+90a^6}{8b^7(b x^2+a)^2}$	1

input
```
int(x^13/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(15*a^5/b^6*x^2+1/8/b*x^12-1/4*a/b^2*x^10+5/8*a^2/b^3*x^8+45/4*a^6/b^7-5/2
*a^3/b^4*x^6)/(b*x^2+a)^2+15/2*a^4*ln(b*x^2+a)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.25

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{b^6x^{12} - 2\,ab^5x^{10} + 5\,a^2b^4x^8 - 20\,a^3b^3x^6 - 68\,a^4b^2x^4 - 16\,a^5bx^2 + 22\,a^6 + 60\,(a^4b^2x^4 + 2\,a^5bx^2 + a^6)\log(}{8\,(b^9x^4 + 2\,ab^8x^2 + a^2b^7)}$$

input
```
integrate(x^13/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*(b^6*x^12 - 2*a*b^5*x^10 + 5*a^2*b^4*x^8 - 20*a^3*b^3*x^6 - 68*a^4*b^2
*x^4 - 16*a^5*b*x^2 + 22*a^6 + 60*(a^4*b^2*x^4 + 2*a^5*b*x^2 + a^6)*log(b*
x^2 + a))/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{15a^4\log(a+bx^2)}{2b^7} - \frac{5a^3x^2}{b^6} + \frac{3a^2x^4}{2b^5} - \frac{ax^6}{2b^4} + \frac{11a^6 + 12a^5bx^2}{4a^2b^7 + 8ab^8x^2 + 4b^9x^4} + \frac{x^8}{8b^3}$$

input
```
integrate(x**13/(b*x**2+a)**3,x)
```

output
```
15*a**4*log(a + b*x**2)/(2*b**7) - 5*a**3*x**2/b**6 + 3*a**2*x**4/(2*b**5)
 - a*x**6/(2*b**4) + (11*a**6 + 12*a**5*b*x**2)/(4*a**2*b**7 + 8*a*b**8*x*
*2 + 4*b**9*x**4) + x**8/(8*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{12\,a^5bx^2+11\,a^6}{4\,(b^9x^4+2\,ab^8x^2+a^2b^7)}+\frac{15\,a^4\log\left(bx^2+a\right)}{2\,b^7}+\frac{b^3x^8-4\,ab^2x^6+12\,a^2bx^4-40\,a^3x^2}{8\,b^6}$$

input
```
integrate(x^13/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(12*a^5*b*x^2 + 11*a^6)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7) + 15/2*a^4*l
og(b*x^2 + a)/b^7 + 1/8*(b^3*x^8 - 4*a*b^2*x^6 + 12*a^2*b*x^4 - 40*a^3*x^2
)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{15\,a^4\log\left(|bx^2+a|\right)}{2\,b^7}-\frac{45\,a^4b^2x^4+78\,a^5bx^2+34\,a^6}{4\,(bx^2+a)^2b^7}+\frac{b^9x^8-4\,ab^8x^6+12\,a^2b^7x^4-40\,a^3b^6x^2}{8\,b^{12}}$$

input
```
integrate(x^13/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
15/2*a^4*log(abs(b*x^2 + a))/b^7 - 1/4*(45*a^4*b^2*x^4 + 78*a^5*b*x^2 + 34
*a^6)/((b*x^2 + a)^2*b^7) + 1/8*(b^9*x^8 - 4*a*b^8*x^6 + 12*a^2*b^7*x^4 -
40*a^3*b^6*x^2)/b^12
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{\frac{11\,a^6}{4\,b}+3\,a^5\,x^2}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}+\frac{x^8}{8\,b^3}-\frac{a\,x^6}{2\,b^4}+\frac{15\,a^4\,\ln\left(b\,x^2+a\right)}{2\,b^7}+\frac{3\,a^2\,x^4}{2\,b^5}-\frac{5\,a^3\,x^2}{b^6}$$

input
```
int(x^13/(a + b*x^2)^3,x)
```

output
```
((11*a^6)/(4*b) + 3*a^5*x^2)/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + x^8/(8*b^
3) - (a*x^6)/(2*b^4) + (15*a^4*log(a + b*x^2))/(2*b^7) + (3*a^2*x^4)/(2*b^
5) - (5*a^3*x^2)/b^6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.29

$$\int \frac{x^{13}}{(a+bx^2)^3}\,dx = \frac{60\log(b\,x^2+a)\,a^6+120\log(b\,x^2+a)\,a^5b\,x^2+60\log(b\,x^2+a)\,a^4b^2x^4+30a^6-60a^4b^2x^4-20a^3b^3x^6+5}{8b^7\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^13/(b*x^2+a)^3,x)
```

output
```
(60*log(a + b*x**2)*a**6 + 120*log(a + b*x**2)*a**5*b*x**2 + 60*log(a + b*
x**2)*a**4*b**2*x**4 + 30*a**6 - 60*a**4*b**2*x**4 - 20*a**3*b**3*x**6 + 5
*a**2*b**4*x**8 - 2*a*b**5*x**10 + b**6*x**12)/(8*b**7*(a**2 + 2*a*b*x**2
+ b**2*x**4))
```

3.170 $\int \frac{x^{11}}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{x^{11}}{(a+bx^2)^3} \, dx = \frac{3a^2x^2}{b^5} - \frac{3ax^4}{4b^4} + \frac{x^6}{6b^3} + \frac{a^5}{4b^6(a+bx^2)^2} - \frac{5a^4}{2b^6(a+bx^2)} - \frac{5a^3\log(a+bx^2)}{b^6}$$

output
```
3*a^2*x^2/b^5-3/4*a*x^4/b^4+1/6*x^6/b^3+1/4*a^5/b^6/(b*x^2+a)^2-5/2*a^4/b^
6/(b*x^2+a)-5*a^3*ln(b*x^2+a)/b^6
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(a+bx^2)^3} \, dx = \frac{36a^2bx^2 - 9ab^2x^4 + 2b^3x^6 + \frac{3a^5}{(a+bx^2)^2} - \frac{30a^4}{a+bx^2} - 60a^3\log(a+bx^2)}{12b^6}$$

input
```
Integrate[x^11/(a + b*x^2)^3,x]
```

output
```
(36*a^2*b*x^2 - 9*a*b^2*x^4 + 2*b^3*x^6 + (3*a^5)/(a + b*x^2)^2 - (30*a^4)
/(a + b*x^2) - 60*a^3*Log[a + b*x^2])/(12*b^6)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^2)^3}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{x^{10}}{(bx^2+a)^3}dx^2$$
$$\Big\downarrow \text{ 49}$$
$$\frac{1}{2}\int \left(-\frac{a^5}{b^5(bx^2+a)^3}+\frac{5a^4}{b^5(bx^2+a)^2}-\frac{10a^3}{b^5(bx^2+a)}+\frac{6a^2}{b^5}-\frac{3x^2a}{b^4}+\frac{x^4}{b^3}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2}\left(\frac{a^5}{2b^6(a+bx^2)^2}-\frac{5a^4}{b^6(a+bx^2)}-\frac{10a^3\log(a+bx^2)}{b^6}+\frac{6a^2x^2}{b^5}-\frac{3ax^4}{2b^4}+\frac{x^6}{3b^3}\right)$$

input
```
Int[x^11/(a + b*x^2)^3,x]
```

output
```
((6*a^2*x^2)/b^5 - (3*a*x^4)/(2*b^4) + x^6/(3*b^3) + a^5/(2*b^6*(a + b*x^2
)^2) - (5*a^4)/(b^6*(a + b*x^2)) - (10*a^3*Log[a + b*x^2])/b^6)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

method	result	size
norman	$\frac{\frac{x^{10}}{6b}-\frac{5a\,x^{8}}{12b^{2}}+\frac{5a^{2}x^{6}}{3b^{3}}-\frac{15a^{5}}{2b^{6}}-\frac{10a^{4}x^{2}}{b^{5}}}{\left(b\,x^{2}+a\right)^{2}}-\frac{5a^{3}\ln\left(b\,x^{2}+a\right)}{b^{6}}$	76
risch	$\frac{x^{6}}{6b^{3}}-\frac{3a\,x^{4}}{4b^{4}}+\frac{3a^{2}x^{2}}{b^{5}}+\frac{-\frac{5a^{4}x^{2}}{2}-\frac{9a^{5}}{4b}}{b^{5}\left(b\,x^{2}+a\right)^{2}}-\frac{5a^{3}\ln\left(b\,x^{2}+a\right)}{b^{6}}$	76
default	$\frac{\frac{1}{6}b^{2}x^{6}-\frac{3}{4}ab\,x^{4}+3a^{2}x^{2}}{b^{5}}-\frac{a^{3}\left(-\frac{a^{2}}{2b\left(b\,x^{2}+a\right)^{2}}+\frac{5a}{b\left(b\,x^{2}+a\right)}+\frac{10\ln\left(b\,x^{2}+a\right)}{b}\right)}{2b^{5}}$	84
parallelrisch	$-\frac{-2b^{5}x^{10}+5a\,b^{4}x^{8}-20a^{2}b^{3}x^{6}+60\ln\left(b\,x^{2}+a\right)x^{4}a^{3}b^{2}+120\ln\left(b\,x^{2}+a\right)x^{2}a^{4}b+120a^{4}b\,x^{2}+60\ln\left(b\,x^{2}+a\right)a^{5}+90a^{5}}{12b^{6}\left(b\,x^{2}+a\right)^{2}}$	107

input
```
int(x^11/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/6*x^10/b-5/12*a/b^2*x^8+5/3*a^2/b^3*x^6-15/2*a^5/b^6-10*a^4/b^5*x^2)/(b
*x^2+a)^2-5*a^3*ln(b*x^2+a)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.32

$$\int \frac{x^{11}}{(a+bx^2)^3}\,dx = \frac{2\,b^5x^{10}-5\,ab^4x^8+20\,a^2b^3x^6+63\,a^3b^2x^4+6\,a^4bx^2-27\,a^5-60\,(a^3b^2x^4+2\,a^4bx^2+a^5)\log\left(bx^2+a\right)}{12\,(b^8x^4+2\,ab^7x^2+a^2b^6)}$$

input
```
integrate(x^11/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/12*(2*b^5*x^10 - 5*a*b^4*x^8 + 20*a^2*b^3*x^6 + 63*a^3*b^2*x^4 + 6*a^4*b
*x^2 - 27*a^5 - 60*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*log(b*x^2 + a))/(b^8*
x^4 + 2*a*b^7*x^2 + a^2*b^6)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx=-\frac{5a^3\log\left(a+bx^2\right)}{b^6}+\frac{3a^2x^2}{b^5}-\frac{3ax^4}{4b^4}+\frac{-9a^5-10a^4bx^2}{4a^2b^6+8ab^7x^2+4b^8x^4}+\frac{x^6}{6b^3}$$

input
```
integrate(x**11/(b*x**2+a)**3,x)
```

output
```
-5*a**3*log(a + b*x**2)/b**6 + 3*a**2*x**2/b**5 - 3*a*x**4/(4*b**4) + (-9*
a**5 - 10*a**4*b*x**2)/(4*a**2*b**6 + 8*a*b**7*x**2 + 4*b**8*x**4) + x**6/
(6*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{x^{11}}{\left(a+bx^2\right)^3}\,dx=-\frac{10\,a^4bx^2+9\,a^5}{4\left(b^8x^4+2\,ab^7x^2+a^2b^6\right)}-\frac{5\,a^3\log\left(bx^2+a\right)}{b^6}+\frac{2\,b^2x^6-9\,abx^4+36\,a^2x^2}{12\,b^5}$$

input
```
integrate(x^11/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(10*a^4*b*x^2 + 9*a^5)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) - 5*a^3*log(
b*x^2 + a)/b^6 + 1/12*(2*b^2*x^6 - 9*a*b*x^4 + 36*a^2*x^2)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int \frac{x^{11}}{(a+bx^2)^3}\,dx = -\frac{5\,a^3\log\left(|bx^2+a|\right)}{b^6} + \frac{30\,a^3b^2x^4+50\,a^4bx^2+21\,a^5}{4\,(bx^2+a)^2b^6} + \frac{2\,b^6x^6-9\,ab^5x^4+36\,a^2b^4x^2}{12\,b^9}$$

input
```
integrate(x^11/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-5*a^3*log(abs(b*x^2 + a))/b^6 + 1/4*(30*a^3*b^2*x^4 + 50*a^4*b*x^2 + 21*a
^5)/((b*x^2 + a)^2*b^6) + 1/12*(2*b^6*x^6 - 9*a*b^5*x^4 + 36*a^2*b^4*x^2)/
b^9
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{x^{11}}{(a+bx^2)^3}\,dx = \frac{x^6}{6\,b^3} - \frac{\frac{9\,a^5}{4\,b}+\frac{5\,a^4\,x^2}{2}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4} - \frac{3\,a\,x^4}{4\,b^4} - \frac{5\,a^3\,\ln\left(b\,x^2+a\right)}{b^6} + \frac{3\,a^2\,x^2}{b^5}$$

input
```
int(x^11/(a + b*x^2)^3,x)
```

output
```
x^6/(6*b^3) - ((9*a^5)/(4*b) + (5*a^4*x^2)/2)/(a^2*b^5 + b^7*x^4 + 2*a*b^6
*x^2) - (3*a*x^4)/(4*b^4) - (5*a^3*log(a + b*x^2))/b^6 + (3*a^2*x^2)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{x^{11}}{(a+bx^2)^3}\,dx = \frac{-60\log(b\,x^2+a)\,a^5-120\log(b\,x^2+a)\,a^4b\,x^2-60\log(b\,x^2+a)\,a^3b^2x^4-30a^5+60a^3b^2x^4+20a^2b^3x^6 -}{12b^6\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^11/(b*x^2+a)^3,x)
```

output
```
( - 60*log(a + b*x**2)*a**5 - 120*log(a + b*x**2)*a**4*b*x**2 - 60*log(a +
 b*x**2)*a**3*b**2*x**4 - 30*a**5 + 60*a**3*b**2*x**4 + 20*a**2*b**3*x**6
- 5*a*b**4*x**8 + 2*b**5*x**10)/(12*b**6*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.171 $\int \frac{x^9}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x^9}{(a+bx^2)^3} dx = -\frac{3ax^2}{2b^4} + \frac{x^4}{4b^3} - \frac{a^4}{4b^5(a+bx^2)^2} + \frac{2a^3}{b^5(a+bx^2)} + \frac{3a^2\log(a+bx^2)}{b^5}$$

output
```
-3/2*a*x^2/b^4+1/4*x^4/b^3-1/4*a^4/b^5/(b*x^2+a)^2+2*a^3/b^5/(b*x^2+a)+3*a
^2*ln(b*x^2+a)/b^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{(a+bx^2)^3} dx = \frac{-6abx^2 + b^2x^4 - \frac{a^4}{(a+bx^2)^2} + \frac{8a^3}{a+bx^2} + 12a^2\log(a+bx^2)}{4b^5}$$

input
```
Integrate[x^9/(a + b*x^2)^3,x]
```

output
```
(-6*a*b*x^2 + b^2*x^4 - a^4/(a + b*x^2)^2 + (8*a^3)/(a + b*x^2) + 12*a^2*L
og[a + b*x^2])/(4*b^5)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4\,(bx^2+a)^3}-\frac{4a^3}{b^4\,(bx^2+a)^2}+\frac{6a^2}{b^4\,(bx^2+a)}-\frac{3a}{b^4}+\frac{x^2}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{2b^5\,(a+bx^2)^2}+\frac{4a^3}{b^5\,(a+bx^2)}+\frac{6a^2\log\left(a+bx^2\right)}{b^5}-\frac{3ax^2}{b^4}+\frac{x^4}{2b^3}\right)$$

input
```
Int[x^9/(a + b*x^2)^3,x]
```

output
```
((-3*a*x^2)/b^4 + x^4/(2*b^3) - a^4/(2*b^5*(a + b*x^2)^2) + (4*a^3)/(b^5*(
a + b*x^2)) + (6*a^2*Log[a + b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

method	result	size
norman	$\frac{\frac{x^8}{4b}-\frac{a\,x^6}{b^2}+\frac{9a^4}{2b^5}+\frac{6a^3x^2}{b^4}}{\left(b\,x^2+a\right)^2}+\frac{3a^2\ln\left(b\,x^2+a\right)}{b^5}$	65
default	$\frac{\left(-b\,x^2+3a\right)^2}{4b^5}+\frac{a^2\left(-\frac{a^2}{2b\left(b\,x^2+a\right)^2}+\frac{4a}{b\left(b\,x^2+a\right)}+\frac{6\ln\left(b\,x^2+a\right)}{b}\right)}{2b^4}$	73
risch	$\frac{x^4}{4b^3}-\frac{3a\,x^2}{2b^4}+\frac{9a^2}{4b^5}+\frac{2a^3x^2+\frac{7a^4}{4b}}{b^4\left(b\,x^2+a\right)^2}+\frac{3a^2\ln\left(b\,x^2+a\right)}{b^5}$	73
parallelrisch	$\frac{b^4x^8-4a\,b^3x^6+12\ln\left(b\,x^2+a\right)x^4a^2b^2+24\ln\left(b\,x^2+a\right)x^2a^3b+24a^3b\,x^2+12a^4\ln\left(b\,x^2+a\right)+18a^4}{4b^5\left(b\,x^2+a\right)^2}$	95

input
```
int(x^9/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/4/b*x^8-a/b^2*x^6+9/2*a^4/b^5+6*a^3*x^2/b^4)/(b*x^2+a)^2+3*a^2*ln(b*x^2
+a)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.39

$$\int \frac{x^9}{\left(a+bx^2\right)^3}\,dx = \frac{b^4x^8-4\,ab^3x^6-11\,a^2b^2x^4+2\,a^3bx^2+7\,a^4+12\,\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)\log\left(bx^2+a\right)}{4\,\left(b^7x^4+2\,ab^6x^2+a^2b^5\right)}$$

input
```
integrate(x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(b^4*x^8 - 4*a*b^3*x^6 - 11*a^2*b^2*x^4 + 2*a^3*b*x^2 + 7*a^4 + 12*(a^
2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*log(b*x^2 + a))/(b^7*x^4 + 2*a*b^6*x^2 + a^
2*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{3a^2\log(a+bx^2)}{b^5} - \frac{3ax^2}{2b^4} + \frac{7a^4+8a^3bx^2}{4a^2b^5+8ab^6x^2+4b^7x^4} + \frac{x^4}{4b^3}$$

input
```
integrate(x**9/(b*x**2+a)**3,x)
```

output
```
3*a**2*log(a + b*x**2)/b**5 - 3*a*x**2/(2*b**4) + (7*a**4 + 8*a**3*b*x**2)
/(4*a**2*b**5 + 8*a*b**6*x**2 + 4*b**7*x**4) + x**4/(4*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{8\,a^3bx^2+7\,a^4}{4\,(b^7x^4+2\,ab^6x^2+a^2b^5)} + \frac{3\,a^2\log(bx^2+a)}{b^5} + \frac{bx^4-6\,ax^2}{4\,b^4}$$

input
```
integrate(x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(8*a^3*b*x^2 + 7*a^4)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) + 3*a^2*log(b*
x^2 + a)/b^5 + 1/4*(b*x^4 - 6*a*x^2)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{3\,a^2\log\left(|bx^2+a|\right)}{b^5} + \frac{b^3x^4 - 6\,ab^2x^2}{4\,b^6} - \frac{18\,a^2b^2x^4 + 28\,a^3bx^2 + 11\,a^4}{4\,(bx^2+a)^2b^5}$$

input
```
integrate(x^9/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3*a^2*log(abs(b*x^2 + a))/b^5 + 1/4*(b^3*x^4 - 6*a*b^2*x^2)/b^6 - 1/4*(18*
a^2*b^2*x^4 + 28*a^3*b*x^2 + 11*a^4)/((b*x^2 + a)^2*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{\frac{7\,a^4}{4\,b} + 2\,a^3\,x^2}{a^2\,b^4 + 2\,a\,b^5\,x^2 + b^6\,x^4} + \frac{x^4}{4\,b^3} - \frac{3\,a\,x^2}{2\,b^4} + \frac{3\,a^2\,\ln\left(b\,x^2+a\right)}{b^5}$$

input
```
int(x^9/(a + b*x^2)^3,x)
```

output
```
((7*a^4)/(4*b) + 2*a^3*x^2)/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + x^4/(4*b^3
) - (3*a*x^2)/(2*b^4) + (3*a^2*log(a + b*x^2))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{x^9}{(a+bx^2)^3}\,dx = \frac{12\log(b\,x^2+a)\,a^4 + 24\log(b\,x^2+a)\,a^3b\,x^2 + 12\log(b\,x^2+a)\,a^2b^2x^4 + 6a^4 - 12a^2b^2x^4 - 4a\,b^3x^6 + b^4x^8}{4b^5\left(b^2x^4 + 2ab\,x^2 + a^2\right)}$$

input
```
int(x^9/(b*x^2+a)^3,x)
```

output
```
(12*log(a + b*x**2)*a**4 + 24*log(a + b*x**2)*a**3*b*x**2 + 12*log(a + b*x
**2)*a**2*b**2*x**4 + 6*a**4 - 12*a**2*b**2*x**4 - 4*a*b**3*x**6 + b**4*x*
*8)/(4*b**5*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.172 $\int \frac{x^7}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = \frac{x^2}{2b^3}+\frac{a^3}{4b^4(a+bx^2)^2}-\frac{3a^2}{2b^4(a+bx^2)}-\frac{3a\log(a+bx^2)}{2b^4}$$

output
```
1/2*x^2/b^3+1/4*a^3/b^4/(b*x^2+a)^2-3/2*a^2/b^4/(b*x^2+a)-3/2*a*ln(b*x^2+a
)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = -\frac{-2bx^2+\frac{a^2(5a+6bx^2)}{(a+bx^2)^2}+6a\log(a+bx^2)}{4b^4}$$

input
```
Integrate[x^7/(a + b*x^2)^3,x]
```

output
```
-1/4*(-2*b*x^2 + (a^2*(5*a + 6*b*x^2))/(a + b*x^2)^2 + 6*a*Log[a + b*x^2])
/b^4
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^3} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^3}+\frac{3a^2}{b^3(bx^2+a)^2}-\frac{3a}{b^3(bx^2+a)}+\frac{1}{b^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a^3}{2b^4(a+bx^2)^2}-\frac{3a^2}{b^4(a+bx^2)}-\frac{3a\log(a+bx^2)}{b^4}+\frac{x^2}{b^3}\right)$$

input
```
Int[x^7/(a + b*x^2)^3,x]
```

output
```
(x^2/b^3 + a^3/(2*b^4*(a + b*x^2)^2) - (3*a^2)/(b^4*(a + b*x^2)) - (3*a*Lo
g[a + b*x^2])/b^4)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
norman	$\frac{\frac{x^6}{2b}-\frac{9a^3}{4b^4}-\frac{3a^2x^2}{b^3}}{\left(b\,x^2+a\right)^2}-\frac{3a\ln\left(b\,x^2+a\right)}{2b^4}$	54
risch	$\frac{x^2}{2b^3}+\frac{-\frac{3a^2x^2}{2}-\frac{5a^3}{4b}}{b^3\left(b\,x^2+a\right)^2}-\frac{3a\ln\left(b\,x^2+a\right)}{2b^4}$	54
default	$\frac{x^2}{2b^3}-\frac{a\left(-\frac{a^2}{2b\left(b\,x^2+a\right)^2}+\frac{3a}{b\left(b\,x^2+a\right)}+\frac{3\ln\left(b\,x^2+a\right)}{b}\right)}{2b^3}$	62
parallelrisch	$-\frac{-2b^3x^6+6\ln\left(b\,x^2+a\right)x^4a\,b^2+12\ln\left(b\,x^2+a\right)x^2a^2b+12a^2b\,x^2+6a^3\ln\left(b\,x^2+a\right)+9a^3}{4b^4\left(b\,x^2+a\right)^2}$	85

input
```
int(x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2/b*x^6-9/4*a^3/b^4-3*a^2*x^2/b^3)/(b*x^2+a)^2-3/2*a*ln(b*x^2+a)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.40

$$\int\frac{x^7}{\left(a+bx^2\right)^3}\,dx=\frac{2\,b^3x^6+4\,ab^2x^4-4\,a^2bx^2-5\,a^3-6\,\left(ab^2x^4+2\,a^2bx^2+a^3\right)\log\left(bx^2+a\right)}{4\,\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

input
```
integrate(x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*x^6 + 4*a*b^2*x^4 - 4*a^2*b*x^2 - 5*a^3 - 6*(a*b^2*x^4 + 2*a^2*
b*x^2 + a^3)*log(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = -\frac{3a\log(a+bx^2)}{2b^4} + \frac{-5a^3-6a^2bx^2}{4a^2b^4+8ab^5x^2+4b^6x^4} + \frac{x^2}{2b^3}$$

input
```
integrate(x**7/(b*x**2+a)**3,x)
```

output
```
-3*a*log(a + b*x**2)/(2*b**4) + (-5*a**3 - 6*a**2*b*x**2)/(4*a**2*b**4 + 8
*a*b**5*x**2 + 4*b**6*x**4) + x**2/(2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = -\frac{6\,a^2bx^2+5\,a^3}{4\,(b^6x^4+2\,ab^5x^2+a^2b^4)} + \frac{x^2}{2\,b^3} - \frac{3\,a\log(bx^2+a)}{2\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*a^2*b*x^2 + 5*a^3)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 1/2*x^2/b^3
 - 3/2*a*log(b*x^2 + a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = \frac{x^2}{2\,b^3} - \frac{3\,a\log\left(|bx^2+a|\right)}{2\,b^4} + \frac{9\,ab^2x^4+12\,a^2bx^2+4\,a^3}{4\,(bx^2+a)^2b^4}$$

input
```
integrate(x^7/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*x^2/b^3 - 3/2*a*log(abs(b*x^2 + a))/b^4 + 1/4*(9*a*b^2*x^4 + 12*a^2*b*
x^2 + 4*a^3)/((b*x^2 + a)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = \frac{x^2}{2\,b^3} - \frac{\frac{5\,a^3}{4\,b}+\frac{3\,a^2\,x^2}{2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} - \frac{3\,a\,\ln\left(b\,x^2+a\right)}{2\,b^4}$$

input
```
int(x^7/(a + b*x^2)^3,x)
```

output
```
x^2/(2*b^3) - ((5*a^3)/(4*b) + (3*a^2*x^2)/2)/(a^2*b^3 + b^5*x^4 + 2*a*b^4
*x^2) - (3*a*log(a + b*x^2))/(2*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.46

$$\int \frac{x^7}{(a+bx^2)^3}\,dx = \frac{-6\log(b\,x^2+a)\,a^3-12\log(b\,x^2+a)\,a^2b\,x^2-6\log(b\,x^2+a)\,a\,b^2x^4-3a^3+6a\,b^2x^4+2b^3x^6}{4b^4\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^7/(b*x^2+a)^3,x)
```

output
```
( - 6*log(a + b*x**2)*a**3 - 12*log(a + b*x**2)*a**2*b*x**2 - 6*log(a + b*
x**2)*a*b**2*x**4 - 3*a**3 + 6*a*b**2*x**4 + 2*b**3*x**6)/(4*b**4*(a**2 +
2*a*b*x**2 + b**2*x**4))
```

3.173 $\int \frac{x^5}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int \frac{x^5}{(a+bx^2)^3} dx = -\frac{a^2}{4b^3(a+bx^2)^2} + \frac{a}{b^3(a+bx^2)} + \frac{\log(a+bx^2)}{2b^3}$$

output
```
-1/4*a^2/b^3/(b*x^2+a)^2+a/b^3/(b*x^2+a)+1/2*ln(b*x^2+a)/b^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(a+bx^2)^3} dx = \frac{\frac{a(3a+4bx^2)}{(a+bx^2)^2} + 2\log(a+bx^2)}{4b^3}$$

input
```
Integrate[x^5/(a + b*x^2)^3,x]
```

output
```
((a*(3*a + 4*b*x^2))/(a + b*x^2)^2 + 2*Log[a + b*x^2])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^3}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^3}-\frac{2a}{b^2(bx^2+a)^2}+\frac{1}{b^2(bx^2+a)}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{2b^3(a+bx^2)^2}+\frac{2a}{b^3(a+bx^2)}+\frac{\log(a+bx^2)}{b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^3,x]
```

output
```
(-1/2*a^2/(b^3*(a + b*x^2)^2) + (2*a)/(b^3*(a + b*x^2)) + Log[a + b*x^2]/b
^3)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

method	result	size
norman	$\frac{\frac{a\,x^{2}}{b^{2}}+\frac{3a^{2}}{4b^{3}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	42
risch	$\frac{\frac{a\,x^{2}}{b^{2}}+\frac{3a^{2}}{4b^{3}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	42
default	$-\frac{a^{2}}{4b^{3}\left(b\,x^{2}+a\right)^{2}}+\frac{a}{b^{3}\left(b\,x^{2}+a\right)}+\frac{\ln\left(b\,x^{2}+a\right)}{2b^{3}}$	46
parallelrisch	$\frac{2b^{2}\ln\left(b\,x^{2}+a\right)x^{4}+4\ln\left(b\,x^{2}+a\right)x^{2}ab+4ab\,x^{2}+2\ln\left(b\,x^{2}+a\right)a^{2}+3a^{2}}{4b^{3}\left(b\,x^{2}+a\right)^{2}}$	72

input
```
int(x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(a*x^2/b^2+3/4*a^2/b^3)/(b*x^2+a)^2+1/2*ln(b*x^2+a)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.41

$$\int\frac{x^5}{(a+bx^2)^3}\,dx=\frac{4\,abx^2+3\,a^2+2\,(b^2x^4+2\,abx^2+a^2)\log\left(bx^2+a\right)}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate(x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*a*b*x^2 + 3*a^2 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a))/(b^
5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(a+bx^2)^3}\,dx = \frac{3a^2+4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{\log(a+bx^2)}{2b^3}$$

input
```
integrate(x**5/(b*x**2+a)**3,x)
```

output
```
(3*a**2 + 4*a*b*x**2)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + log(a
+ b*x**2)/(2*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{(a+bx^2)^3}\,dx = \frac{4\,abx^2+3\,a^2}{4\,(b^5x^4+2\,ab^4x^2+a^2b^3)}+\frac{\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(4*a*b*x^2 + 3*a^2)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + 1/2*log(b*x^2
+ a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{(a+bx^2)^3}\,dx = \frac{\log(|bx^2+a|)}{2\,b^3}-\frac{3\,bx^4+2\,ax^2}{4\,(bx^2+a)^2b^2}$$

input
```
integrate(x^5/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b^3 - 1/4*(3*b*x^4 + 2*a*x^2)/((b*x^2 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.06

$$\int \frac{x^5}{(a+bx^2)^3}\,dx = \frac{\frac{3\,a^2}{4\,b^3}+\frac{a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\ln\left(b\,x^2+a\right)}{2\,b^3}$$

input
```
int(x^5/(a + b*x^2)^3,x)
```

output
```
((3*a^2)/(4*b^3) + (a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) + log(a + b*x^
2)/(2*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{x^5}{(a+bx^2)^3}\,dx = \frac{2\log(b\,x^2+a)\,a^2+4\log(b\,x^2+a)\,ab\,x^2+2\log(b\,x^2+a)\,b^2x^4+a^2-2b^2x^4}{4b^3\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^5/(b*x^2+a)^3,x)
```

output
```
(2*log(a + b*x**2)*a**2 + 4*log(a + b*x**2)*a*b*x**2 + 2*log(a + b*x**2)*b
**2*x**4 + a**2 - 2*b**2*x**4)/(4*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.174 $\int \frac{x^3}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^3}{(a+bx^2)^3} dx = \frac{x^4}{4a(a+bx^2)^2}$$

output
```
1/4*x^4/a/(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.26

$$\int \frac{x^3}{(a+bx^2)^3} dx = -\frac{a+2bx^2}{4b^2(a+bx^2)^2}$$

input
```
Integrate[x^3/(a + b*x^2)^3,x]
```

output
```
-1/4*(a + 2*b*x^2)/(b^2*(a + b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^3}\,dx$$
$$\Big\downarrow \ 242$$
$$\frac{x^4}{4a\,(a+bx^2)^2}$$

input
```
Int[x^3/(a + b*x^2)^3,x]
```

output
```
x^4/(4*a*(a + b*x^2)^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

method	result	size
gosper	$-\frac{2b\,x^{2}+a}{4\left(b\,x^{2}+a\right)^{2} b^{2}}$	23
orering	$-\frac{2b\,x^{2}+a}{4\left(b\,x^{2}+a\right)^{2} b^{2}}$	23
parallelrisch	$\frac{-2b\,x^{2}-a}{4b^{2} \left(b\,x^{2}+a\right)^{2}}$	25
norman	$\frac{-\frac{x^{2}}{2b}-\frac{a}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	26
risch	$\frac{-\frac{x^{2}}{2b}-\frac{a}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}$	26
default	$\frac{a}{4b^{2} \left(b\,x^{2}+a\right)^{2}}-\frac{1}{2b^{2} \left(b\,x^{2}+a\right)}$	31

input
```
int(x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*b*x^2+a)/(b*x^2+a)^2/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = -\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = \frac{-a-2bx^2}{4a^2b^2+8ab^3x^2+4b^4x^4}$$

input
```
integrate(x**3/(b*x**2+a)**3,x)
```

output
```
(-a - 2*b*x**2)/(4*a**2*b**2 + 8*a*b**3*x**2 + 4*b**4*x**4)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.89

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = -\frac{2\,bx^2+a}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = -\frac{2\,bx^2+a}{4\,(bx^2+a)^2b^2}$$

input
```
integrate(x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/4*(2*b*x^2 + a)/((b*x^2 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = -\frac{\frac{a}{4\,b^2}+\frac{x^2}{2\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(x^3/(a + b*x^2)^3,x)
```

output
```
-(a/(4*b^2) + x^2/(2*b))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \frac{x^3}{(a+bx^2)^3}\,dx = \frac{x^4}{4a\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^3/(b*x^2+a)^3,x)
```

output
```
x**4/(4*a*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.175 $\int \frac{x}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{(a+bx^2)^3} \, dx = -\frac{1}{4b(a+bx^2)^2}$$

output
```
-1/4/b/(b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)^3} \, dx = -\frac{1}{4b(a+bx^2)^2}$$

input
```
Integrate[x/(a + b*x^2)^3,x]
```

output
```
-1/4*1/(b*(a + b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{4b(a+bx^2)^2}$$

input
```
Int[x/(a + b*x^2)^3,x]
```

output
```
-1/4*1/(b*(a + b*x^2)^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{4b(b\,x^2+a)^2}$	15
derivativedivides	$-\frac{1}{4b(b\,x^2+a)^2}$	15
default	$-\frac{1}{4b(b\,x^2+a)^2}$	15
norman	$-\frac{1}{4b(b\,x^2+a)^2}$	15
risch	$-\frac{1}{4b(b\,x^2+a)^2}$	15
parallelrisch	$-\frac{1}{4b(b\,x^2+a)^2}$	15
orering	$-\frac{1}{4b(b\,x^2+a)^2}$	15

input
```
int(x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/(b*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

input
```
integrate(x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4a^2b+8ab^2x^2+4b^3x^4}$$

input
```
integrate(x/(b*x**2+a)**3,x)
```

output
```
-1/(4*a**2*b + 8*a*b**2*x**2 + 4*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4\,(bx^2+a)^2b}$$

input
```
integrate(x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4/((b*x^2 + a)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4\,(bx^2+a)^2b}$$

input
```
integrate(x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/4/((b*x^2 + a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4\,a^2\,b+8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

input
```
int(x/(a + b*x^2)^3,x)
```

output
```
-1/(4*a^2*b + 4*b^3*x^4 + 8*a*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \frac{x}{(a+bx^2)^3}\,dx = -\frac{1}{4b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x/(b*x^2+a)^3,x)
```

output
```
( - 1)/(4*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.176 $\int \frac{1}{x(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{1}{x(a+bx^2)^3} \, dx = \frac{1}{4a(a+bx^2)^2} + \frac{1}{2a^2(a+bx^2)} + \frac{\log(x)}{a^3} - \frac{\log(a+bx^2)}{2a^3}$$

output
```
1/4/a/(b*x^2+a)^2+1/2/a^2/(b*x^2+a)+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+bx^2)^3} \, dx = \frac{\frac{a(3a+2bx^2)}{(a+bx^2)^2} + 4\log(x) - 2\log(a+bx^2)}{4a^3}$$

input
```
Integrate[1/(x*(a + b*x^2)^3),x]
```

output
```
((a*(3*a + 2*b*x^2))/(a + b*x^2)^2 + 4*Log[x] - 2*Log[a + b*x^2])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+a\right)^3}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(-\frac{b}{a^3\left(bx^2+a\right)}-\frac{b}{a^2\left(bx^2+a\right)^2}-\frac{b}{a\left(bx^2+a\right)^3}+\frac{1}{a^3x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(a+bx^2\right)}{a^3}+\frac{\log\left(x^2\right)}{a^3}+\frac{1}{a^2\left(a+bx^2\right)}+\frac{1}{2a\left(a+bx^2\right)^2}\right)$$

input
```
Int[1/(x*(a + b*x^2)^3),x]
```

output
```
(1/(2*a*(a + b*x^2)^2) + 1/(a^2*(a + b*x^2)) + Log[x^2]/a^3 - Log[a + b*x^
2]/a^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\frac{b\,x^{2}}{2 a^{2}}+\frac{3}{4 a}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(b\,x^{2}+a\right)}{2 a^{3}}$	46
norman	$\frac{-\frac{b\,x^{2}}{a^{2}}-\frac{3 b^{2} x^{4}}{4 a^{3}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(b\,x^{2}+a\right)}{2 a^{3}}$	52
default	$-\frac{b\left(-\frac{a^{2}}{2 b\left(b\,x^{2}+a\right)^{2}}-\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{\ln\left(b\,x^{2}+a\right)}{b}\right)}{2 a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	59
parallelrisch	$\frac{4 b^{2} \ln\left(x\right) x^{4}-2 b^{2} \ln\left(b\,x^{2}+a\right) x^{4}-3 b^{2} x^{4}+8 a b \ln\left(x\right) x^{2}-4 \ln\left(b\,x^{2}+a\right) x^{2} a b-4 a b\,x^{2}+4 a^{2} \ln\left(x\right)-2 \ln\left(b\,x^{2}+a\right) a^{2}}{4 a^{3} \left(b\,x^{2}+a\right)^{2}}$	101

input
```
int(1/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/2*b/a^2*x^2+3/4/a)/(b*x^2+a)^2+ln(x)/a^3-1/2*ln(b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx = \frac{2\,abx^{2}+3\,a^{2}-2\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\log\left(bx^{2}+a\right)+4\,\left(b^{2}x^{4}+2\,abx^{2}+a^{2}\right)\log\left(x\right)}{4\,\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)}$$

input
```
integrate(1/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*a*b*x^2 + 3*a^2 - 2*(b^2*x^4 + 2*a*b*x^2 + a^2)*log(b*x^2 + a) + 4*
(b^2*x^4 + 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx = \frac{3a+2bx^2}{4a^4+8a^3bx^2+4a^2b^2x^4}+\frac{\log\left(x\right)}{a^3}-\frac{\log\left(\frac{a}{b}+x^2\right)}{2a^3}$$

input
```
integrate(1/x/(b*x**2+a)**3,x)
```

output
```
(3*a + 2*b*x**2)/(4*a**4 + 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a**3
 - log(a/b + x**2)/(2*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx = \frac{2\,bx^2+3\,a}{4\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2+a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate(1/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 + a)
/a^3 + 1/2*log(x^2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx=\frac{\log\left(x^2\right)}{2\,a^3}-\frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^3}+\frac{3\,b^2x^4+8\,abx^2+6\,a^2}{4\,\left(bx^2+a\right)^2a^3}$$

input
```
integrate(1/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 + a))/a^3 + 1/4*(3*b^2*x^4 + 8*a*b*x^
2 + 6*a^2)/((b*x^2 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx=\frac{\ln\left(x\right)}{a^3}+\frac{\frac{3}{4\,a}+\frac{b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\ln\left(b\,x^2+a\right)}{2\,a^3}$$

input
```
int(1/(x*(a + b*x^2)^3),x)
```

output
```
log(x)/a^3 + (3/(4*a) + (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - log
(a + b*x^2)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.02

$$\int \frac{1}{x\left(a+bx^2\right)^3}\,dx$$
$$=\frac{-2\log(b\,x^2+a)\,a^2-4\log(b\,x^2+a)\,ab\,x^2-2\log(b\,x^2+a)\,b^2x^4+4\log(x)\,a^2+8\log(x)\,ab\,x^2+4\log(x)}{4a^3\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(1/x/(b*x^2+a)^3,x)
```

output
```
( - 2*log(a + b*x**2)*a**2 - 4*log(a + b*x**2)*a*b*x**2 - 2*log(a + b*x**2
)*b**2*x**4 + 4*log(x)*a**2 + 8*log(x)*a*b*x**2 + 4*log(x)*b**2*x**4 + 2*a
**2 - b**2*x**4)/(4*a**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.177 $\int \frac{1}{x^3 (a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx = -\frac{1}{2a^3x^2} - \frac{b}{4a^2 (a+bx^2)^2} - \frac{b}{a^3 (a+bx^2)} - \frac{3b \log(x)}{a^4} + \frac{3b \log (a+bx^2)}{2a^4}$$

output
```
-1/2/a^3/x^2-1/4*b/a^2/(b*x^2+a)^2-b/a^3/(b*x^2+a)-3*b*ln(x)/a^4+3/2*b*ln(
b*x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx = -\frac{\frac{a(2a^2+9abx^2+6b^2x^4)}{x^2(a+bx^2)^2} + 12b \log(x) - 6b \log (a+bx^2)}{4a^4}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/4*((a*(2*a^2 + 9*a*b*x^2 + 6*b^2*x^4))/(x^2*(a + b*x^2)^2) + 12*b*Log[x
] - 6*b*Log[a + b*x^2])/a^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^3} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{3b^2}{a^4 (bx^2+a)} + \frac{2b^2}{a^3 (bx^2+a)^2} + \frac{b^2}{a^2 (bx^2+a)^3} - \frac{3b}{a^4 x^2} + \frac{1}{a^3 x^4} \right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{3b \log(x^2)}{a^4} + \frac{3b \log(a+bx^2)}{a^4} - \frac{2b}{a^3 (a+bx^2)} - \frac{1}{a^3 x^2} - \frac{b}{2a^2 (a+bx^2)^2} \right)$$

input
```
Int[1/(x^3*(a + b*x^2)^3),x]
```

output
```
(-(1/(a^3*x^2)) - b/(2*a^2*(a + b*x^2)^2) - (2*b)/(a^3*(a + b*x^2)) - (3*b
*Log[x^2])/a^4 + (3*b*Log[a + b*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97

method	result
norman	$\frac{\frac{3b^2x^4}{a^3}-\frac{1}{2a}+\frac{9b^3x^6}{4a^4}}{x^2(b\,x^2+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(b\,x^2+a)}{2a^4}$
risch	$\frac{-\frac{3b^2x^4}{2a^3}-\frac{9b\,x^2}{4a^2}-\frac{1}{2a}}{x^2(b\,x^2+a)^2}-\frac{3b\ln(x)}{a^4}+\frac{3b\ln(-b\,x^2-a)}{2a^4}$
default	$\frac{b^2\left(-\frac{a^2}{2b\left(b\,x^2+a\right)^2}-\frac{2a}{b\left(b\,x^2+a\right)}+\frac{3\ln\left(b\,x^2+a\right)}{b}\right)}{2a^4}-\frac{1}{2a^3x^2}-\frac{3b\ln(x)}{a^4}$
parallelrisch	$-\frac{12b^3\ln(x)x^6-6b^3\ln(b\,x^2+a)x^6-9b^3x^6+24a\,b^2\ln(x)x^4-12\ln(b\,x^2+a)x^4a\,b^2-12a\,b^2x^4+12a^2b\ln(x)x^2-6\ln(b\,x^2+a)x^2a}{4a^4x^2(b\,x^2+a)^2}$

input
```
int(1/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(3*b^2/a^3*x^4-1/2/a+9/4*b^3/a^4*x^6)/x^2/(b*x^2+a)^2-3*b*ln(x)/a^4+3/2*b*
ln(b*x^2+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^3 (a+bx^2)^3} dx = -\frac{6 ab^2x^4 + 9 a^2bx^2 + 2 a^3 - 6 (b^3x^6 + 2 ab^2x^4 + a^2bx^2) \log (bx^2 + a) + 12 (b^3x^6 + 2 ab^2x^4 + a^2bx^2) \log (}{4 (a^4b^2x^6 + 2 a^5bx^4 + a^6x^2)}$$

input
```
integrate(1/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3 - 6*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b
*x^2)*log(b*x^2 + a) + 12*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4
*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3 (a+bx^2)^3} dx = \frac{-2a^2 - 9abx^2 - 6b^2x^4}{4a^5x^2 + 8a^4bx^4 + 4a^3b^2x^6} - \frac{3b \log (x)}{a^4} + \frac{3b \log \left(\frac{a}{b} + x^2\right)}{2a^4}$$

input
```
integrate(1/x**3/(b*x**2+a)**3,x)
```

output
```
(-2*a**2 - 9*a*b*x**2 - 6*b**2*x**4)/(4*a**5*x**2 + 8*a**4*b*x**4 + 4*a**3
*b**2*x**6) - 3*b*log(x)/a**4 + 3*b*log(a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3 (a+bx^2)^3} dx = -\frac{6 b^2x^4 + 9 abx^2 + 2 a^2}{4 (a^3b^2x^6 + 2 a^4bx^4 + a^5x^2)} + \frac{3 b \log (bx^2 + a)}{2 a^4} - \frac{3 b \log (x^2)}{2 a^4}$$

input
```
integrate(1/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)
 + 3/2*b*log(b*x^2 + a)/a^4 - 3/2*b*log(x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx = -\frac{3 \, b \log \left(x^{2}\right)}{2 \, a^{4}} + \frac{3 \, b \log \left({\left| b x^{2} + a \right|}\right)}{2 \, a^{4}} - \frac{9 \, b^{3} x^{4} + 22 \, a b^{2} x^{2} + 14 \, a^{2} b}{4 \, {\left(b x^{2} + a\right)}^{2} a^{4}} + \frac{3 \, b x^{2} - a}{2 \, a^{4} x^{2}}$$

input
```
integrate(1/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/2*b*log(x^2)/a^4 + 3/2*b*log(abs(b*x^2 + a))/a^4 - 1/4*(9*b^3*x^4 + 22*
a*b^2*x^2 + 14*a^2*b)/((b*x^2 + a)^2*a^4) + 1/2*(3*b*x^2 - a)/(a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx = \frac{3\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4} - \frac{\frac{1}{2\,a}+\frac{9\,b\,x^2}{4\,a^2}+\frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6} - \frac{3\,b\,\ln\left(x\right)}{a^4}$$

input
```
int(1/(x^3*(a + b*x^2)^3),x)
```

output
```
(3*b*log(a + b*x^2))/(2*a^4) - (1/(2*a) + (9*b*x^2)/(4*a^2) + (3*b^2*x^4)/
(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4) - (3*b*log(x))/a^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.99

$$\int \frac{1}{x^3 (a+bx^2)^3} \, dx$$
$$= \frac{6\log(b\,x^2+a)\,a^2b\,x^2+12\log(b\,x^2+a)\,a\,b^2x^4+6\log(b\,x^2+a)\,b^3x^6-12\log(x)\,a^2b\,x^2-24\log(x)\,a\,b^2x^4}{4a^4x^2\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/x^3/(b*x^2+a)^3,x)
```

output
```
(6*log(a + b*x**2)*a**2*b*x**2 + 12*log(a + b*x**2)*a*b**2*x**4 + 6*log(a
+ b*x**2)*b**3*x**6 - 12*log(x)*a**2*b*x**2 - 24*log(x)*a*b**2*x**4 - 12*l
og(x)*b**3*x**6 - 2*a**3 - 6*a**2*b*x**2 + 3*b**3*x**6)/(4*a**4*x**2*(a**2
 + 2*a*b*x**2 + b**2*x**4))
```

3.178 $\int \frac{1}{x^5 (a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 86

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx = -\frac{1}{4a^3x^4} + \frac{3b}{2a^4x^2} + \frac{b^2}{4a^3 (a+bx^2)^2} + \frac{3b^2}{2a^4 (a+bx^2)} + \frac{6b^2 \log(x)}{a^5} - \frac{3b^2 \log(a+bx^2)}{a^5}$$

output
```
-1/4/a^3/x^4+3/2*b/a^4/x^2+1/4*b^2/a^3/(b*x^2+a)^2+3/2*b^2/a^4/(b*x^2+a)+6
*b^2*ln(x)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx = \frac{\frac{a(-a^3+4a^2bx^2+18ab^2x^4+12b^3x^6)}{x^4(a+bx^2)^2} + 24b^2 \log(x) - 12b^2 \log(a+bx^2)}{4a^5}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^3),x]
```

output
```
((a*(-a^3 + 4*a^2*b*x^2 + 18*a*b^2*x^4 + 12*b^3*x^6))/(x^4*(a + b*x^2)^2)
+ 24*b^2*Log[x] - 12*b^2*Log[a + b*x^2])/(4*a^5)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int \frac{1}{x^6 (bx^2+a)^3} dx^2$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{2} \int \left(-\frac{6b^3}{a^5 (bx^2+a)} - \frac{3b^3}{a^4 (bx^2+a)^2} - \frac{b^3}{a^3 (bx^2+a)^3} + \frac{6b^2}{a^5x^2} - \frac{3b}{a^4x^4} + \frac{1}{a^3x^6}\right) dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{6b^2 \log (x^2)}{a^5} - \frac{6b^2 \log (a+bx^2)}{a^5} + \frac{3b^2}{a^4 (a+bx^2)} + \frac{3b}{a^4x^2} + \frac{b^2}{2a^3 (a+bx^2)^2} - \frac{1}{2a^3x^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)^3),x]
```

output
```
(-1/2*1/(a^3*x^4) + (3*b)/(a^4*x^2) + b^2/(2*a^3*(a + b*x^2)^2) + (3*b^2)/
(a^4*(a + b*x^2)) + (6*b^2*Log[x^2])/a^5 - (6*b^2*Log[a + b*x^2])/a^5)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

method	result
norman	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{4a}-\frac{6b^3x^6}{a^4}-\frac{9b^4x^8}{2a^5}}{x^4(b\,x^2+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{3b^2\ln(b\,x^2+a)}{a^5}$
risch	$\frac{\frac{3b^3x^6}{a^4}+\frac{9b^2x^4}{2a^3}+\frac{b\,x^2}{a^2}-\frac{1}{4a}}{x^4(b\,x^2+a)^2}+\frac{6b^2\ln(x)}{a^5}-\frac{3b^2\ln(b\,x^2+a)}{a^5}$
default	$-\frac{b^3\left(-\frac{a^2}{2b(b\,x^2+a)^2}-\frac{3a}{b(b\,x^2+a)}+\frac{6\ln(b\,x^2+a)}{b}\right)}{2a^5}-\frac{1}{4a^3x^4}+\frac{3b}{2a^4x^2}+\frac{6b^2\ln(x)}{a^5}$
parallelrisch	$\frac{24b^4\ln(x)x^8-12b^4\ln(b\,x^2+a)x^8-18b^4x^8+48\ln(x)x^6a\,b^3-24\ln(b\,x^2+a)x^6a\,b^3-24a\,b^3x^6+24\ln(x)x^4a^2b^2-12\ln(b\,x^2+a)x}{4a^5x^4(b\,x^2+a)^2}$

input
```
int(1/x^5/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x^2-1/4/a-6*b^3/a^4*x^6-9/2*b^4/a^5*x^8)/x^4/(b*x^2+a)^2+6*b^2*ln(x
)/a^5-3*b^2*ln(b*x^2+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx$$
$$= \frac{12\, ab^3x^6 + 18\, a^2b^2x^4 + 4\, a^3bx^2 - a^4 - 12\, (b^4x^8 + 2\, ab^3x^6 + a^2b^2x^4) \log (bx^2 + a) + 24\, (b^4x^8 + 2\, ab^3x^6 +}{4\, (a^5b^2x^8 + 2\, a^6bx^6 + a^7x^4)}$$

input
```
integrate(1/x^5/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(12*a*b^3*x^6 + 18*a^2*b^2*x^4 + 4*a^3*b*x^2 - a^4 - 12*(b^4*x^8 + 2*a
*b^3*x^6 + a^2*b^2*x^4)*log(b*x^2 + a) + 24*(b^4*x^8 + 2*a*b^3*x^6 + a^2*b
^2*x^4)*log(x))/(a^5*b^2*x^8 + 2*a^6*b*x^6 + a^7*x^4)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx = \frac{-a^3 + 4a^2bx^2 + 18ab^2x^4 + 12b^3x^6}{4a^6x^4 + 8a^5bx^6 + 4a^4b^2x^8} + \frac{6b^2 \log (x)}{a^5} - \frac{3b^2 \log \left(\frac{a}{b} + x^2\right)}{a^5}$$

input
```
integrate(1/x**5/(b*x**2+a)**3,x)
```

output
```
(-a**3 + 4*a**2*b*x**2 + 18*a*b**2*x**4 + 12*b**3*x**6)/(4*a**6*x**4 + 8*a
**5*b*x**6 + 4*a**4*b**2*x**8) + 6*b**2*log(x)/a**5 - 3*b**2*log(a/b + x**
2)/a**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx = \frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\,\left(a^4b^2x^8+2\,a^5bx^6+a^6x^4\right)} - \frac{3\,b^2\log\left(bx^2+a\right)}{a^5} + \frac{3\,b^2\log\left(x^2\right)}{a^5}$$

input
```
integrate(1/x^5/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(12*b^3*x^6 + 18*a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/(a^4*b^2*x^8 + 2*a^5*b
*x^6 + a^6*x^4) - 3*b^2*log(b*x^2 + a)/a^5 + 3*b^2*log(x^2)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx = \frac{3\,b^2\log\left(x^2\right)}{a^5} - \frac{3\,b^2\log\left(|bx^2+a|\right)}{a^5} + \frac{12\,b^3x^6+18\,ab^2x^4+4\,a^2bx^2-a^3}{4\,\left(bx^4+ax^2\right)^2a^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3*b^2*log(x^2)/a^5 - 3*b^2*log(abs(b*x^2 + a))/a^5 + 1/4*(12*b^3*x^6 + 18*
a*b^2*x^4 + 4*a^2*b*x^2 - a^3)/((b*x^4 + a*x^2)^2*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^5\left(a+bx^2\right)^3}\,dx = \frac{\frac{b\,x^2}{a^2}-\frac{1}{4\,a}+\frac{9\,b^2\,x^4}{2\,a^3}+\frac{3\,b^3\,x^6}{a^4}}{a^2\,x^4+2\,a\,b\,x^6+b^2\,x^8} - \frac{3\,b^2\,\ln\left(b\,x^2+a\right)}{a^5} + \frac{6\,b^2\,\ln\left(x\right)}{a^5}$$

input
```
int(1/(x^5*(a + b*x^2)^3),x)
```

output
```
((b*x^2)/a^2 - 1/(4*a) + (9*b^2*x^4)/(2*a^3) + (3*b^3*x^6)/a^4)/(a^2*x^4 +
 b^2*x^8 + 2*a*b*x^6) - (3*b^2*log(a + b*x^2))/a^5 + (6*b^2*log(x))/a^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.72

$$\int \frac{1}{x^5 (a+bx^2)^3} \, dx$$
$$= \frac{-12\log(b\,x^2+a)\,a^2b^2x^4 - 24\log(b\,x^2+a)\,a\,b^3x^6 - 12\log(b\,x^2+a)\,b^4x^8 + 24\log(x)\,a^2b^2x^4 + 48\log(x)}{4a^5x^4\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/x^5/(b*x^2+a)^3,x)
```

output
```
( - 12*log(a + b*x**2)*a**2*b**2*x**4 - 24*log(a + b*x**2)*a*b**3*x**6 - 1
2*log(a + b*x**2)*b**4*x**8 + 24*log(x)*a**2*b**2*x**4 + 48*log(x)*a*b**3*
x**6 + 24*log(x)*b**4*x**8 - a**4 + 4*a**3*b*x**2 + 12*a**2*b**2*x**4 - 6*
b**4*x**8)/(4*a**5*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.179 $\int \frac{1}{x^7(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 95

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx=-\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{b^3}{4a^4\left(a+bx^2\right)^2}-\frac{2b^3}{a^5\left(a+bx^2\right)}-\frac{10b^3\log(x)}{a^6}+\frac{5b^3\log\left(a+bx^2\right)}{a^6}$$

output
```
-1/6/a^3/x^6+3/4*b/a^4/x^4-3*b^2/a^5/x^2-1/4*b^3/a^4/(b*x^2+a)^2-2*b^3/a^5
/(b*x^2+a)-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx=-\frac{\frac{a\left(2a^4-5a^3bx^2+20a^2b^2x^4+90ab^3x^6+60b^4x^8\right)}{x^6\left(a+bx^2\right)^2}+120b^3\log(x)-60b^3\log\left(a+bx^2\right)}{12a^6}$$

input
```
Integrate[1/(x^7*(a + b*x^2)^3),x]
```

output
```
-1/12*((a*(2*a^4 - 5*a^3*b*x^2 + 20*a^2*b^2*x^4 + 90*a*b^3*x^6 + 60*b^4*x^
8))/(x^6*(a + b*x^2)^2) + 120*b^3*Log[x] - 60*b^3*Log[a + b*x^2])/a^6
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)^3} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^8 (bx^2+a)^3}dx^2$$
$$\Big\downarrow \text{54}$$
$$\frac{1}{2}\int \left(\frac{10b^4}{a^6 (bx^2+a)}+\frac{4b^4}{a^5 (bx^2+a)^2}+\frac{b^4}{a^4 (bx^2+a)^3}-\frac{10b^3}{a^6x^2}+\frac{6b^2}{a^5x^4}-\frac{3b}{a^4x^6}+\frac{1}{a^3x^8}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(-\frac{10b^3\log(x^2)}{a^6}+\frac{10b^3\log(a+bx^2)}{a^6}-\frac{4b^3}{a^5 (a+bx^2)}-\frac{6b^2}{a^5x^2}-\frac{b^3}{2a^4 (a+bx^2)^2}+\frac{3b}{2a^4x^4}-\frac{1}{3a^3x^6}\right)$$

input
```
Int[1/(x^7*(a + b*x^2)^3),x]
```

output
```
(-1/3*1/(a^3*x^6) + (3*b)/(2*a^4*x^4) - (6*b^2)/(a^5*x^2) - b^3/(2*a^4*(a
+ b*x^2)^2) - (4*b^3)/(a^5*(a + b*x^2)) - (10*b^3*Log[x^2])/a^6 + (10*b^3*
Log[a + b*x^2])/a^6)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

method	result
norman	$\frac{-\frac{1}{6a}+\frac{5b\,x^2}{12a^2}-\frac{5b^2x^4}{3a^3}+\frac{10b^4x^8}{a^5}+\frac{15b^5x^{10}}{2a^6}}{x^6(b\,x^2+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{5b^3\ln(b\,x^2+a)}{a^6}$
risch	$\frac{-\frac{5b^4x^8}{a^5}-\frac{15b^3x^6}{2a^4}-\frac{5b^2x^4}{3a^3}+\frac{5b\,x^2}{12a^2}-\frac{1}{6a}}{x^6(b\,x^2+a)^2}-\frac{10b^3\ln(x)}{a^6}+\frac{5b^3\ln(-b\,x^2-a)}{a^6}$
default	$\frac{b^4\left(-\frac{a^2}{2b\left(b\,x^2+a\right)^2}-\frac{4a}{b\left(b\,x^2+a\right)}+\frac{10\ln\left(b\,x^2+a\right)}{b}\right)}{2a^6}-\frac{1}{6a^3x^6}+\frac{3b}{4a^4x^4}-\frac{3b^2}{a^5x^2}-\frac{10b^3\ln(x)}{a^6}$
parallelrisch	$-\frac{120\ln(x)x^{10}b^5-60\ln(b\,x^2+a)x^{10}b^5-90b^5x^{10}+240a\,b^4\ln(x)x^8-120\ln(b\,x^2+a)x^8a\,b^4-120a\,b^4x^8+120\ln(x)x^6a^2b^3-60\,\mathrm{l}}{12a^6x^6(b\,x^2+a)^2}$

input
```
int(1/x^7/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/6/a+5/12*b/a^2*x^2-5/3*b^2/a^3*x^4+10*b^4/a^5*x^8+15/2*b^5/a^6*x^10)/x
^6/(b*x^2+a)^2-10*b^3*ln(x)/a^6+5*b^3*ln(b*x^2+a)/a^6
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.53

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx = -\frac{60\,ab^4x^8+90\,a^2b^3x^6+20\,a^3b^2x^4-5\,a^4bx^2+2\,a^5-60\left(b^5x^{10}+2\,ab^4x^8+a^2b^3x^6\right)\log\left(bx^2+a\right)+120}{12\left(a^6b^2x^{10}+2\,a^7bx^8+a^8x^6\right)}$$

input
```
integrate(1/x^7/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/12*(60*a*b^4*x^8 + 90*a^2*b^3*x^6 + 20*a^3*b^2*x^4 - 5*a^4*b*x^2 + 2*a^
5 - 60*(b^5*x^10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(b*x^2 + a) + 120*(b^5*x^
10 + 2*a*b^4*x^8 + a^2*b^3*x^6)*log(x))/(a^6*b^2*x^10 + 2*a^7*b*x^8 + a^8*
x^6)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^7\left(a+bx^2\right)^3}\,dx = \frac{-2a^4+5a^3bx^2-20a^2b^2x^4-90ab^3x^6-60b^4x^8}{12a^7x^6+24a^6bx^8+12a^5b^2x^{10}} - \frac{10b^3\log\left(x\right)}{a^6} + \frac{5b^3\log\left(\frac{a}{b}+x^2\right)}{a^6}$$

input
```
integrate(1/x**7/(b*x**2+a)**3,x)
```

output
```
(-2*a**4 + 5*a**3*b*x**2 - 20*a**2*b**2*x**4 - 90*a*b**3*x**6 - 60*b**4*x**
*8)/(12*a**7*x**6 + 24*a**6*b*x**8 + 12*a**5*b**2*x**10) - 10*b**3*log(x)/
a**6 + 5*b**3*log(a/b + x**2)/a**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^7 (a+bx^2)^3} \, dx = -\frac{60 \, b^4 x^8 + 90 \, a b^3 x^6 + 20 \, a^2 b^2 x^4 - 5 \, a^3 b x^2 + 2 \, a^4}{12 \, (a^5 b^2 x^{10} + 2 \, a^6 b x^8 + a^7 x^6)} + \frac{5 \, b^3 \log \left(b x^2 + a\right)}{a^6} - \frac{5 \, b^3 \log \left(x^2\right)}{a^6}$$

input
```
integrate(1/x^7/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/12*(60*b^4*x^8 + 90*a*b^3*x^6 + 20*a^2*b^2*x^4 - 5*a^3*b*x^2 + 2*a^4)/(
a^5*b^2*x^10 + 2*a^6*b*x^8 + a^7*x^6) + 5*b^3*log(b*x^2 + a)/a^6 - 5*b^3*l
og(x^2)/a^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^7 (a+bx^2)^3} \, dx = -\frac{5 \, b^3 \log \left(x^2\right)}{a^6} + \frac{5 \, b^3 \log \left(|b x^2 + a|\right)}{a^6} - \frac{30 \, b^5 x^4 + 68 \, a b^4 x^2 + 39 \, a^2 b^3}{4 \, (b x^2 + a)^2 a^6} + \frac{110 \, b^3 x^6 - 36 \, a b^2 x^4 + 9 \, a^2 b x^2 - 2 \, a^3}{12 \, a^6 x^6}$$

input
```
integrate(1/x^7/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-5*b^3*log(x^2)/a^6 + 5*b^3*log(abs(b*x^2 + a))/a^6 - 1/4*(30*b^5*x^4 + 68
*a*b^4*x^2 + 39*a^2*b^3)/((b*x^2 + a)^2*a^6) + 1/12*(110*b^3*x^6 - 36*a*b^
2*x^4 + 9*a^2*b*x^2 - 2*a^3)/(a^6*x^6)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^7 (a+bx^2)^3} \, dx = \frac{5\, b^3\, \ln\left(b\, x^2+a\right)}{a^6} - \frac{\frac{1}{6\, a} - \frac{5\, b\, x^2}{12\, a^2} + \frac{5\, b^2\, x^4}{3\, a^3} + \frac{15\, b^3\, x^6}{2\, a^4} + \frac{5\, b^4\, x^8}{a^5}}{a^2\, x^6 + 2\, a\, b\, x^8 + b^2\, x^{10}} - \frac{10\, b^3\, \ln\left(x\right)}{a^6}$$

input
```
int(1/(x^7*(a + b*x^2)^3),x)
```

output
```
(5*b^3*log(a + b*x^2))/a^6 - (1/(6*a) - (5*b*x^2)/(12*a^2) + (5*b^2*x^4)/(
3*a^3) + (15*b^3*x^6)/(2*a^4) + (5*b^4*x^8)/a^5)/(a^2*x^6 + b^2*x^10 + 2*a
*b*x^8) - (10*b^3*log(x))/a^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^7 (a+bx^2)^3} \, dx = \frac{60 \log(b\, x^2+a)\, a^2b^3x^6 + 120 \log(b\, x^2+a)\, a\, b^4x^8 + 60 \log(b\, x^2+a)\, b^5x^{10} - 120 \log(x)\, a^2b^3x^6 - 240 \log(x}{12a^6x^6 \left(b^2x^4 + 2ab\, x^2 + a^2\right)}$$

input
```
int(1/x^7/(b*x^2+a)^3,x)
```

output
```
(60*log(a + b*x**2)*a**2*b**3*x**6 + 120*log(a + b*x**2)*a*b**4*x**8 + 60*
log(a + b*x**2)*b**5*x**10 - 120*log(x)*a**2*b**3*x**6 - 240*log(x)*a*b**4
*x**8 - 120*log(x)*b**5*x**10 - 2*a**5 + 5*a**4*b*x**2 - 20*a**3*b**2*x**4
 - 60*a**2*b**3*x**6 + 30*b**5*x**10)/(12*a**6*x**6*(a**2 + 2*a*b*x**2 + b
**2*x**4))
```

3.180 $\int \frac{1}{x^9(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{1}{x^9(a+bx^2)^3}\,dx = -\frac{1}{8a^3x^8}+\frac{b}{2a^4x^6}-\frac{3b^2}{2a^5x^4}+\frac{5b^3}{a^6x^2}+\frac{b^4}{4a^5(a+bx^2)^2}+\frac{5b^4}{2a^6(a+bx^2)}+\frac{15b^4\log(x)}{a^7}-\frac{15b^4\log(a+bx^2)}{2a^7}$$

output
```
-1/8/a^3/x^8+1/2*b/a^4/x^6-3/2*b^2/a^5/x^4+5*b^3/a^6/x^2+1/4*b^4/a^5/(b*x^
2+a)^2+5/2*b^4/a^6/(b*x^2+a)+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^9(a+bx^2)^3}\,dx = \frac{\frac{a(-a^5+2a^4bx^2-5a^3b^2x^4+20a^2b^3x^6+90ab^4x^8+60b^5x^{10})}{x^8(a+bx^2)^2}+120b^4\log(x)-60b^4\log(a+bx^2)}{8a^7}$$

input
```
Integrate[1/(x^9*(a + b*x^2)^3),x]
```

output
```
((a*(-a^5 + 2*a^4*b*x^2 - 5*a^3*b^2*x^4 + 20*a^2*b^3*x^6 + 90*a*b^4*x^8 +
60*b^5*x^10))/(x^8*(a + b*x^2)^2) + 120*b^4*Log[x] - 60*b^4*Log[a + b*x^2]
)/(8*a^7)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^9 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^{10} (bx^2+a)^3} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(-\frac{15b^5}{a^7 (bx^2+a)} - \frac{5b^5}{a^6 (bx^2+a)^2} - \frac{b^5}{a^5 (bx^2+a)^3} + \frac{15b^4}{a^7 x^2} - \frac{10b^3}{a^6 x^4} + \frac{6b^2}{a^5 x^6} - \frac{3b}{a^4 x^8} + \frac{1}{a^3 x^{10}}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{15b^4 \log(x^2)}{a^7} - \frac{15b^4 \log(a+bx^2)}{a^7} + \frac{5b^4}{a^6 (a+bx^2)} + \frac{10b^3}{a^6 x^2} + \frac{b^4}{2a^5 (a+bx^2)^2} - \frac{3b^2}{a^5 x^4} + \frac{b}{a^4 x^6} - \frac{1}{4a^3 x^8}\right)$$

input
```
Int[1/(x^9*(a + b*x^2)^3),x]
```

output
```
(-1/4*1/(a^3*x^8) + b/(a^4*x^6) - (3*b^2)/(a^5*x^4) + (10*b^3)/(a^6*x^2) +
 b^4/(2*a^5*(a + b*x^2)^2) + (5*b^4)/(a^6*(a + b*x^2)) + (15*b^4*Log[x^2])
/a^7 - (15*b^4*Log[a + b*x^2])/a^7)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

method	result
norman	$\frac{-\frac{15 b^{5} x^{10}}{a^{6}}-\frac{1}{8 a}+\frac{b\,x^{2}}{4 a^{2}}-\frac{5 b^{2} x^{4}}{8 a^{3}}+\frac{5 b^{3} x^{6}}{2 a^{4}}-\frac{45 b^{6} x^{12}}{4 a^{7}}}{x^{8} \left(b\,x^{2}+a \right)^{2}}+\frac{15 b^{4} \ln \left(x \right)}{a^{7}}-\frac{15 b^{4} \ln \left(b\,x^{2}+a \right)}{2 a^{7}}$
risch	$\frac{\frac{15 b^{5} x^{10}}{2 a^{6}}+\frac{45 b^{4} x^{8}}{4 a^{5}}+\frac{5 b^{3} x^{6}}{2 a^{4}}-\frac{5 b^{2} x^{4}}{8 a^{3}}+\frac{b\,x^{2}}{4 a^{2}}-\frac{1}{8 a}}{x^{8} \left(b\,x^{2}+a \right)^{2}}+\frac{15 b^{4} \ln \left(x \right)}{a^{7}}-\frac{15 b^{4} \ln \left(b\,x^{2}+a \right)}{2 a^{7}}$
default	$-\frac{b^{5} \left(-\frac{a^{2}}{2 b \left(b\,x^{2}+a \right)^{2}}-\frac{5 a}{b \left(b\,x^{2}+a \right)}+\frac{15 \ln \left(b\,x^{2}+a \right)}{b}\right)}{2 a^{7}}-\frac{1}{8 a^{3} x^{8}}+\frac{15 b^{4} \ln \left(x \right)}{a^{7}}+\frac{5 b^{3}}{a^{6} x^{2}}-\frac{3 b^{2}}{2 a^{5} x^{4}}+\frac{b}{2 a^{4} x^{6}}$
parallelrisch	$\frac{120 \ln \left(x \right) x^{12} b^{6}-60 \ln \left(b\,x^{2}+a \right) x^{12} b^{6}-90 b^{6} x^{12}+240 \ln \left(x \right) x^{10} a\,b^{5}-120 \ln \left(b\,x^{2}+a \right) x^{10} a\,b^{5}-120 a\,b^{5} x^{10}+120 \ln \left(x \right) x^{8} a^{2} b^{4}-60 \ln}{8 a^{7} x^{8} \left(b\,x^{2}+a \right)^{2}}$

input
```
int(1/x^9/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-15*b^5/a^6*x^10-1/8/a+1/4*b/a^2*x^2-5/8*b^2/a^3*x^4+5/2*b^3/a^4*x^6-45/4
*b^6/a^7*x^12)/x^8/(b*x^2+a)^2+15*b^4*ln(x)/a^7-15/2*b^4*ln(b*x^2+a)/a^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.39

$$\int \frac{1}{x^9 (a+bx^2)^3} \, dx = \frac{60 \, ab^5x^{10} + 90 \, a^2b^4x^8 + 20 \, a^3b^3x^6 - 5 \, a^4b^2x^4 + 2 \, a^5bx^2 - a^6 - 60 \left(b^6x^{12} + 2 \, ab^5x^{10} + a^2b^4x^8\right) \log \left(bx^2 + a\right)}{8 \left(a^7b^2x^{12} + 2 \, a^8bx^{10} + a^9x^8\right)}$$

input
```
integrate(1/x^9/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/8*(60*a*b^5*x^10 + 90*a^2*b^4*x^8 + 20*a^3*b^3*x^6 - 5*a^4*b^2*x^4 + 2*a
^5*b*x^2 - a^6 - 60*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(b*x^2 + a)
 + 120*(b^6*x^12 + 2*a*b^5*x^10 + a^2*b^4*x^8)*log(x))/(a^7*b^2*x^12 + 2*a
^8*b*x^10 + a^9*x^8)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^9 (a+bx^2)^3} \, dx = \frac{-a^5 + 2a^4bx^2 - 5a^3b^2x^4 + 20a^2b^3x^6 + 90ab^4x^8 + 60b^5x^{10}}{8a^8x^8 + 16a^7bx^{10} + 8a^6b^2x^{12}} + \frac{15b^4 \log(x)}{a^7} - \frac{15b^4 \log\left(\frac{a}{b} + x^2\right)}{2a^7}$$

input
```
integrate(1/x**9/(b*x**2+a)**3,x)
```

output
```
(-a**5 + 2*a**4*b*x**2 - 5*a**3*b**2*x**4 + 20*a**2*b**3*x**6 + 90*a*b**4*
x**8 + 60*b**5*x**10)/(8*a**8*x**8 + 16*a**7*b*x**10 + 8*a**6*b**2*x**12)
+ 15*b**4*log(x)/a**7 - 15*b**4*log(a/b + x**2)/(2*a**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^9 (a+bx^2)^3}\,dx = \frac{60\,b^5x^{10}+90\,ab^4x^8+20\,a^2b^3x^6-5\,a^3b^2x^4+2\,a^4bx^2-a^5}{8\,(a^6b^2x^{12}+2\,a^7bx^{10}+a^8x^8)} - \frac{15\,b^4\log(bx^2+a)}{2\,a^7} + \frac{15\,b^4\log(x^2)}{2\,a^7}$$

input
```
integrate(1/x^9/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(60*b^5*x^10 + 90*a*b^4*x^8 + 20*a^2*b^3*x^6 - 5*a^3*b^2*x^4 + 2*a^4*b
*x^2 - a^5)/(a^6*b^2*x^12 + 2*a^7*b*x^10 + a^8*x^8) - 15/2*b^4*log(b*x^2 +
 a)/a^7 + 15/2*b^4*log(x^2)/a^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^9 (a+bx^2)^3}\,dx = \frac{15\,b^4\log(x^2)}{2\,a^7} - \frac{15\,b^4\log(|bx^2+a|)}{2\,a^7} + \frac{45\,b^6x^4+100\,ab^5x^2+56\,a^2b^4}{4\,(bx^2+a)^2a^7} - \frac{125\,b^4x^8-40\,ab^3x^6+12\,a^2b^2x^4-4\,a^3bx^2+a^4}{8\,a^7x^8}$$

input
```
integrate(1/x^9/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
15/2*b^4*log(x^2)/a^7 - 15/2*b^4*log(abs(b*x^2 + a))/a^7 + 1/4*(45*b^6*x^4
 + 100*a*b^5*x^2 + 56*a^2*b^4)/((b*x^2 + a)^2*a^7) - 1/8*(125*b^4*x^8 - 40
*a*b^3*x^6 + 12*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)/(a^7*x^8)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^9 \left(a+bx^2\right)^3}\,dx = \frac{\frac{b\,x^2}{4\,a^2}-\frac{1}{8\,a}-\frac{5\,b^2\,x^4}{8\,a^3}+\frac{5\,b^3\,x^6}{2\,a^4}+\frac{45\,b^4\,x^8}{4\,a^5}+\frac{15\,b^5\,x^{10}}{2\,a^6}}{a^2\,x^8+2\,a\,b\,x^{10}+b^2\,x^{12}} - \frac{15\,b^4\,\ln\left(b\,x^2+a\right)}{2\,a^7}+\frac{15\,b^4\,\ln\left(x\right)}{a^7}$$

input
```
int(1/(x^9*(a + b*x^2)^3),x)
```

output
```
((b*x^2)/(4*a^2) - 1/(8*a) - (5*b^2*x^4)/(8*a^3) + (5*b^3*x^6)/(2*a^4) + (
45*b^4*x^8)/(4*a^5) + (15*b^5*x^10)/(2*a^6))/(a^2*x^8 + b^2*x^12 + 2*a*b*x
^10) - (15*b^4*log(a + b*x^2))/(2*a^7) + (15*b^4*log(x))/a^7
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^9 \left(a+bx^2\right)^3}\,dx$$

$$= \frac{-60 \log(b\,x^2+a)\,a^2b^4x^8 - 120 \log(b\,x^2+a)\,a\,b^5x^{10} - 60 \log(b\,x^2+a)\,b^6x^{12} + 120 \log(x)\,a^2b^4x^8 + 240\,\mathrm{lo}}{8a^7x^8\left(b^2x^4+2ab\,x^2\right.}$$

input
```
int(1/x^9/(b*x^2+a)^3,x)
```

output
```
( - 60*log(a + b*x**2)*a**2*b**4*x**8 - 120*log(a + b*x**2)*a*b**5*x**10 -
 60*log(a + b*x**2)*b**6*x**12 + 120*log(x)*a**2*b**4*x**8 + 240*log(x)*a*
b**5*x**10 + 120*log(x)*b**6*x**12 - a**6 + 2*a**5*b*x**2 - 5*a**4*b**2*x*
*4 + 20*a**3*b**3*x**6 + 60*a**2*b**4*x**8 - 30*b**6*x**12)/(8*a**7*x**8*(
a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.181 $\int \frac{x^{12}}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = -\frac{10a^3x}{b^6}+\frac{2a^2x^3}{b^5}-\frac{3ax^5}{5b^4}+\frac{x^7}{7b^3}+\frac{a^5x}{4b^6(a+bx^2)^2}-\frac{21a^4x}{8b^6(a+bx^2)}+\frac{99a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

output
```
-10*a^3*x/b^6+2*a^2*x^3/b^5-3/5*a*x^5/b^4+1/7*x^7/b^3+1/4*a^5*x/b^6/(b*x^2
+a)^2-21/8*a^4*x/b^6/(b*x^2+a)+99/8*a^(7/2)*arctan(b^(1/2)*x/a^(1/2))/b^(1
3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = -\frac{3465a^5x+5775a^4bx^3+1848a^3b^2x^5-264a^2b^3x^7+88ab^4x^9-40b^5x^{11}}{280b^6(a+bx^2)^2}+\frac{99a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

input
```
Integrate[x^12/(a + b*x^2)^3,x]
```

output
```
-1/280*(3465*a^5*x + 5775*a^4*b*x^3 + 1848*a^3*b^2*x^5 - 264*a^2*b^3*x^7 +
 88*a*b^4*x^9 - 40*b^5*x^11)/(b^6*(a + b*x^2)^2) + (99*a^(7/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{11\int \frac{x^{10}}{(bx^2+a)^2}dx}{4b}-\frac{x^{11}}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{11\left(\frac{9\int \frac{x^8}{bx^2+a}dx}{2b}-\frac{x^9}{2b(a+bx^2)}\right)}{4b}-\frac{x^{11}}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{254}$$

$$\frac{11\left(\frac{9\int \left(\frac{x^6}{b}-\frac{ax^4}{b^2}+\frac{a^2x^2}{b^3}+\frac{a^4}{b^4(bx^2+a)}-\frac{a^3}{b^4}\right)dx}{2b}-\frac{x^9}{2b(a+bx^2)}\right)}{4b}-\frac{x^{11}}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{11\left(\frac{9\left(\frac{a^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}-\frac{a^3x}{b^4}+\frac{a^2x^3}{3b^3}-\frac{ax^5}{5b^2}+\frac{x^7}{7b}\right)}{2b}-\frac{x^9}{2b(a+bx^2)}\right)}{4b}-\frac{x^{11}}{4b\left(a+bx^2\right)^2}$$

input
```
Int[x^12/(a + b*x^2)^3,x]
```

output
```
-1/4*x^11/(b*(a + b*x^2)^2) + (11*(-1/2*x^9/(b*(a + b*x^2)) + (9*(-((a^3*x
)/b^4) + (a^2*x^3)/(3*b^3) - (a*x^5)/(5*b^2) + x^7/(7*b) + (a^(7/2)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.78

method	result	siz
default	$-\frac{-\frac{1}{7}b^{3}x^{7}+\frac{3}{5}a\,b^{2}x^{5}-2a^{2}b\,x^{3}+10a^{3}x}{b^{6}}+\frac{a^{4}\left(\frac{-\frac{21}{8}b\,x^{3}-\frac{19}{8}a x}{\left(b\,x^{2}+a\right)^{2}}+\frac{99\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^{6}}$	85
risch	$\frac{x^{7}}{7b^{3}}-\frac{3a\,x^{5}}{5b^{4}}+\frac{2a^{2}x^{3}}{b^{5}}-\frac{10a^{3}x}{b^{6}}+\frac{-\frac{21}{8}a^{4}b\,x^{3}-\frac{19}{8}a^{5}x}{b^{6}\left(b\,x^{2}+a\right)^{2}}+\frac{99\sqrt{-ab}\,a^{3}\ln\left(-\sqrt{-ab}\,x+a\right)}{16b^{7}}-\frac{99\sqrt{-ab}\,a^{3}\ln\left(\sqrt{-ab}\,x+a\right)}{16b^{7}}$	11

input
```
int(x^12/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/7*b^3*x^7+3/5*a*b^2*x^5-2*a^2*b*x^3+10*a^3*x)+a^4/b^6*((-21/8*b
*x^3-19/8*a*x)/(b*x^2+a)^2+99/8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.55

$$\int \frac{x^{12}}{\left(a+bx^2\right)^3}\,dx$$

$$= \left[\frac{80\,b^5x^{11}-176\,ab^4x^9+528\,a^2b^3x^7-3696\,a^3b^2x^5-11550\,a^4bx^3-6930\,a^5x+3465\,(a^3b^2x^4+2\,a^4bx^2+}{560\,(b^8x^4+2\,ab^7x^2+a^2b^6)}\right.$$

input
```
integrate(x^12/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/560*(80*b^5*x^11 - 176*a*b^4*x^9 + 528*a^2*b^3*x^7 - 3696*a^3*b^2*x^5 -
 11550*a^4*b*x^3 - 6930*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5)*sqr
t(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^8*x^4 + 2*a*b^
7*x^2 + a^2*b^6), 1/280*(40*b^5*x^11 - 88*a*b^4*x^9 + 264*a^2*b^3*x^7 - 18
48*a^3*b^2*x^5 - 5775*a^4*b*x^3 - 3465*a^5*x + 3465*(a^3*b^2*x^4 + 2*a^4*b
*x^2 + a^5)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^8*x^4 + 2*a*b^7*x^2 + a^
2*b^6)]
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.49

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = -\frac{10a^3x}{b^6} + \frac{2a^2x^3}{b^5} - \frac{3ax^5}{5b^4} - \frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x - \frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16} + \frac{99\sqrt{-\frac{a^7}{b^{13}}}\log\left(x + \frac{b^6\sqrt{-\frac{a^7}{b^{13}}}}{a^3}\right)}{16} + \frac{-19a^5x - 21a^4bx^3}{8a^2b^6 + 16ab^7x^2 + 8b^8x^4} + \frac{x^7}{7b^3}$$

input
```
integrate(x**12/(b*x**2+a)**3,x)
```

output
```
-10*a**3*x/b**6 + 2*a**2*x**3/b**5 - 3*a*x**5/(5*b**4) - 99*sqrt(-a**7/b**
13)*log(x - b**6*sqrt(-a**7/b**13)/a**3)/16 + 99*sqrt(-a**7/b**13)*log(x +
 b**6*sqrt(-a**7/b**13)/a**3)/16 + (-19*a**5*x - 21*a**4*b*x**3)/(8*a**2*b
**6 + 16*a*b**7*x**2 + 8*b**8*x**4) + x**7/(7*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = -\frac{21\,a^4bx^3 + 19\,a^5x}{8\,(b^8x^4 + 2\,ab^7x^2 + a^2b^6)} + \frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6} + \frac{5\,b^3x^7 - 21\,ab^2x^5 + 70\,a^2bx^3 - 350\,a^3x}{35\,b^6}$$

input
```
integrate(x^12/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(21*a^4*b*x^3 + 19*a^5*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6) + 99/8*a^
4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/35*(5*b^3*x^7 - 21*a*b^2*x^5 +
 70*a^2*b*x^3 - 350*a^3*x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = \frac{99\,a^4\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6} - \frac{21\,a^4bx^3+19\,a^5x}{8\,(bx^2+a)^2b^6} + \frac{5\,b^{18}x^7-21\,ab^{17}x^5+70\,a^2b^{16}x^3-350\,a^3b^{15}x}{35\,b^{21}}$$

input
```
integrate(x^12/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
99/8*a^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^6) - 1/8*(21*a^4*b*x^3 + 19*a^
5*x)/((b*x^2 + a)^2*b^6) + 1/35*(5*b^18*x^7 - 21*a*b^17*x^5 + 70*a^2*b^16*
x^3 - 350*a^3*b^15*x)/b^21
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx = \frac{x^7}{7\,b^3} - \frac{\frac{19\,a^5\,x}{8}+\frac{21\,b\,a^4\,x^3}{8}}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4} - \frac{3\,a\,x^5}{5\,b^4} - \frac{10\,a^3\,x}{b^6} + \frac{99\,a^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{13/2}} + \frac{2\,a^2\,x^3}{b^5}$$

input
```
int(x^12/(a + b*x^2)^3,x)
```

output
```
x^7/(7*b^3) - ((19*a^5*x)/8 + (21*a^4*b*x^3)/8)/(a^2*b^6 + b^8*x^4 + 2*a*b
^7*x^2) - (3*a*x^5)/(5*b^4) - (10*a^3*x)/b^6 + (99*a^(7/2)*atan((b^(1/2)*x
)/a^(1/2)))/(8*b^(13/2)) + (2*a^2*x^3)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

$$\int \frac{x^{12}}{(a+bx^2)^3}\,dx$$
$$= \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^5 + 6930\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4b\,x^2 + 3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b^2x^4 - 3465a}{280b^7\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int(x^12/(b*x^2+a)^3,x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5 + 6930*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b*x**2 + 3465*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**2*x**4 - 3465*a**5*b*x - 5775*a**4*b**2
*x**3 - 1848*a**3*b**3*x**5 + 264*a**2*b**4*x**7 - 88*a*b**5*x**9 + 40*b**
6*x**11)/(280*b**7*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.182 $\int \frac{x^{10}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 96

$$\int \frac{x^{10}}{(a+bx^2)^3} dx = \frac{6a^2x}{b^5} - \frac{ax^3}{b^4} + \frac{x^5}{5b^3} - \frac{a^4x}{4b^5(a+bx^2)^2} + \frac{17a^3x}{8b^5(a+bx^2)} - \frac{63a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

output
```
6*a^2*x/b^5-a*x^3/b^4+1/5*x^5/b^3-1/4*a^4*x/b^5/(b*x^2+a)^2+17/8*a^3*x/b^5
/(b*x^2+a)-63/8*a^(5/2)*arctan(b^(1/2)*x/a^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.92

$$\int \frac{x^{10}}{(a+bx^2)^3} dx = \frac{315a^4x + 525a^3bx^3 + 168a^2b^2x^5 - 24ab^3x^7 + 8b^4x^9}{40b^5(a+bx^2)^2} - \frac{63a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{11/2}}$$

input
```
Integrate[x^10/(a + b*x^2)^3,x]
```

output
```
(315*a^4*x + 525*a^3*b*x^3 + 168*a^2*b^2*x^5 - 24*a*b^3*x^7 + 8*b^4*x^9)/(
40*b^5*(a + b*x^2)^2) - (63*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(11/
2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{9\int \frac{x^8}{(bx^2+a)^2}dx}{4b}-\frac{x^9}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \; 252$$

$$\frac{9\left(\frac{7\int \frac{x^6}{bx^2+a}dx}{2b}-\frac{x^7}{2b(a+bx^2)}\right)}{4b}-\frac{x^9}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \; 254$$

$$\frac{9\left(\frac{7\int \left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b(a+bx^2)}\right)}{4b}-\frac{x^9}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{9\left(\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b(a+bx^2)}\right)}{4b}-\frac{x^9}{4b\,(a+bx^2)^2}$$

input
```
Int[x^10/(a + b*x^2)^3,x]
```

output
```
-1/4*x^9/(b*(a + b*x^2)^2) + (9*(-1/2*x^7/(b*(a + b*x^2)) + (7*((a^2*x)/b^
3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^
(7/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\frac{1}{5} b^{2} x^{5}-a b \,x^{3}+6 a^{2} x}{b^{5}}-\frac{a^{3} \left(\frac{-\frac{17}{8} b \,x^{3}-\frac{15}{8} a x}{\left(b \,x^{2}+a \right)^{2}}+\frac{63 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{5}}$	74
risch	$\frac{x^{5}}{5 b^{3}}-\frac{a \,x^{3}}{b^{4}}+\frac{6 a^{2} x}{b^{5}}+\frac{\frac{17}{8} a^{3} b \,x^{3}+\frac{15}{8} a^{4} x}{b^{5} \left(b \,x^{2}+a \right)^{2}}+\frac{63 \sqrt{-a b}\, a^{2} \ln \left(-\sqrt{-a b}\, x -a \right)}{16 b^{6}}-\frac{63 \sqrt{-a b}\, a^{2} \ln \left(\sqrt{-a b}\, x -a \right)}{16 b^{6}}$	112

input
```
int(x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*b^2*x^5-a*b*x^3+6*a^2*x)-1/b^5*a^3*((-17/8*b*x^3-15/8*a*x)/(b*x
^2+a)^2+63/8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.67

$$\int \frac{x^{10}}{(a+bx^2)^3}\,dx = \left[\frac{16\,b^4x^9 - 48\,ab^3x^7 + 336\,a^2b^2x^5 + 1050\,a^3bx^3 + 630\,a^4x + 315\,(a^2b^2x^4 + 2\,a^3bx^2 + a^4)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2}{b}\right.}{80\,(b^7x^4 + 2\,ab^6x^2 + a^2b^5)}\right.$$

input
```
integrate(x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/80*(16*b^4*x^9 - 48*a*b^3*x^7 + 336*a^2*b^2*x^5 + 1050*a^3*b*x^3 + 630*
a^4*x + 315*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(-a/b)*log((b*x^2 - 2*b*
x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5), 1/40*(8
*b^4*x^9 - 24*a*b^3*x^7 + 168*a^2*b^2*x^5 + 525*a^3*b*x^3 + 315*a^4*x - 31
5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^
7*x^4 + 2*a*b^6*x^2 + a^2*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.50

$$\int \frac{x^{10}}{(a+bx^2)^3}\,dx = \frac{6a^2x}{b^5} - \frac{ax^3}{b^4} + \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x - \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} - \frac{63\sqrt{-\frac{a^5}{b^{11}}}\log\left(x + \frac{b^5\sqrt{-\frac{a^5}{b^{11}}}}{a^2}\right)}{16} + \frac{15a^4x + 17a^3bx^3}{8a^2b^5 + 16ab^6x^2 + 8b^7x^4} + \frac{x^5}{5b^3}$$

input
```
integrate(x**10/(b*x**2+a)**3,x)
```

output
```
6*a**2*x/b**5 - a*x**3/b**4 + 63*sqrt(-a**5/b**11)*log(x - b**5*sqrt(-a**5
/b**11)/a**2)/16 - 63*sqrt(-a**5/b**11)*log(x + b**5*sqrt(-a**5/b**11)/a**
2)/16 + (15*a**4*x + 17*a**3*b*x**3)/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*b**
7*x**4) + x**5/(5*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{x^{10}}{(a+bx^2)^3}\,dx = \frac{17\,a^3bx^3+15\,a^4x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)} - \frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{b^2x^5-5\,abx^3+30\,a^2x}{5\,b^5}$$

input
```
integrate(x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(17*a^3*b*x^3 + 15*a^4*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) - 63/8*a^3
*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/5*(b^2*x^5 - 5*a*b*x^3 + 30*a^2
*x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.88

$$\int \frac{x^{10}}{(a+bx^2)^3}\,dx = -\frac{63\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5} + \frac{17\,a^3bx^3+15\,a^4x}{8\,(bx^2+a)^2b^5} + \frac{b^{12}x^5-5\,ab^{11}x^3+30\,a^2b^{10}x}{5\,b^{15}}$$

input
```
integrate(x^10/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-63/8*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^5) + 1/8*(17*a^3*b*x^3 + 15*a
^4*x)/((b*x^2 + a)^2*b^5) + 1/5*(b^12*x^5 - 5*a*b^11*x^3 + 30*a^2*b^10*x)/
b^15
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx = \frac{\frac{15\,a^4\,x}{8}+\frac{17\,b\,a^3\,x^3}{8}}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}+\frac{x^5}{5\,b^3}-\frac{a\,x^3}{b^4}+\frac{6\,a^2\,x}{b^5}-\frac{63\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{11/2}}$$

input
```
int(x^10/(a + b*x^2)^3,x)
```

output
```
((15*a^4*x)/8 + (17*a^3*b*x^3)/8)/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + x^5/
(5*b^3) - (a*x^3)/b^4 + (6*a^2*x)/b^5 - (63*a^(5/2)*atan((b^(1/2)*x)/a^(1/
2)))/(8*b^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.51

$$\int \frac{x^{10}}{\left(a+bx^2\right)^3}\,dx = \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^4-630\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3b\,x^2-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^4+315a^4b}{40b^6\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^10/(b*x^2+a)^3,x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4 - 630*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b*x**2 - 315*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**4 + 315*a**4*b*x + 525*a**3*b**2*x
**3 + 168*a**2*b**3*x**5 - 24*a*b**4*x**7 + 8*b**5*x**9)/(40*b**6*(a**2 +
2*a*b*x**2 + b**2*x**4))
```

3.183 $\int \frac{x^8}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{x^8}{(a+bx^2)^3} dx = -\frac{3ax}{b^4} + \frac{x^3}{3b^3} + \frac{a^3x}{4b^4(a+bx^2)^2} - \frac{13a^2x}{8b^4(a+bx^2)} + \frac{35a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

output
```
-3*a*x/b^4+1/3*x^3/b^3+1/4*a^3*x/b^4/(b*x^2+a)^2-13/8*a^2*x/b^4/(b*x^2+a)+
35/8*a^(3/2)*arctan(b^(1/2)*x/a^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{(a+bx^2)^3} dx = -\frac{105a^3x + 175a^2bx^3 + 56ab^2x^5 - 8b^3x^7}{24b^4(a+bx^2)^2} + \frac{35a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{9/2}}$$

input
```
Integrate[x^8/(a + b*x^2)^3,x]
```

output
```
-1/24*(105*a^3*x + 175*a^2*b*x^3 + 56*a*b^2*x^5 - 8*b^3*x^7)/(b^4*(a + b*x
^2)^2) + (35*a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(9/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{7\int \frac{x^6}{(bx^2+a)^2}dx}{4b}-\frac{x^7}{4b(a+bx^2)^2}$$

$$\Big\downarrow \ 252$$

$$\frac{7\left(\frac{5\int \frac{x^4}{bx^2+a}dx}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}$$

$$\Big\downarrow \ 254$$

$$\frac{7\left(\frac{5\int \left(\frac{a^2}{b^2(bx^2+a)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{7\left(\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b(a+bx^2)}\right)}{4b}-\frac{x^7}{4b(a+bx^2)^2}$$

input
```
Int[x^8/(a + b*x^2)^3,x]
```

output
```
-1/4*x^7/(b*(a + b*x^2)^2) + (7*(-1/2*x^5/(b*(a + b*x^2)) + (5*(-((a*x)/b^
2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2)))/(2*b)))/(
4*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{-\frac{1}{3} b \,x^{3}+3 a x}{b^{4}}+\frac{a^{2} \left(\frac{-\frac{13}{8} b \,x^{3}-\frac{11}{8} a x}{\left(b \,x^{2}+a \right)^{2}}+\frac{35 \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{8 \sqrt{a b}}\right)}{b^{4}}$	63
risch	$\frac{x^{3}}{3 b^{3}}-\frac{3 a x}{b^{4}}+\frac{-\frac{13}{8} a^{2} b \,x^{3}-\frac{11}{8} a^{3} x}{b^{4} \left(b \,x^{2}+a \right)^{2}}+\frac{35 \sqrt{-a b}\, a \ln \left(-\sqrt{-a b}\, x +a \right)}{16 b^{5}}-\frac{35 \sqrt{-a b}\, a \ln \left(\sqrt{-a b}\, x +a \right)}{16 b^{5}}$	93

input
```
int(x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/3*b*x^3+3*a*x)+a^2/b^4*((-13/8*b*x^3-11/8*a*x)/(b*x^2+a)^2+35/8
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.71

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = \left[\frac{16\,b^3x^7 - 112\,ab^2x^5 - 350\,a^2bx^3 - 210\,a^3x + 105\,(ab^2x^4 + 2\,a^2bx^2 + a^3)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{48\,(b^6x^4 + 2\,ab^5x^2 + a^2b^4)}, \ 8\,b\right.$$

input
```
integrate(x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*b^3*x^7 - 112*a*b^2*x^5 - 350*a^2*b*x^3 - 210*a^3*x + 105*(a*b^2
*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b
*x^2 + a)))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/24*(8*b^3*x^7 - 56*a*b^2*
x^5 - 175*a^2*b*x^3 - 105*a^3*x + 105*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = -\frac{3ax}{b^4} - \frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x - \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} + \frac{35\sqrt{-\frac{a^3}{b^9}}\log\left(x + \frac{b^4\sqrt{-\frac{a^3}{b^9}}}{a}\right)}{16} + \frac{-11a^3x - 13a^2bx^3}{8a^2b^4 + 16ab^5x^2 + 8b^6x^4} + \frac{x^3}{3b^3}$$

input
```
integrate(x**8/(b*x**2+a)**3,x)
```

output
```
-3*a*x/b**4 - 35*sqrt(-a**3/b**9)*log(x - b**4*sqrt(-a**3/b**9)/a)/16 + 35
*sqrt(-a**3/b**9)*log(x + b**4*sqrt(-a**3/b**9)/a)/16 + (-11*a**3*x - 13*a
**2*b*x**3)/(8*a**2*b**4 + 16*a*b**5*x**2 + 8*b**6*x**4) + x**3/(3*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = -\frac{13\,a^2bx^3+11\,a^3x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)} + \frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} + \frac{bx^3-9\,ax}{3\,b^4}$$

input
```
integrate(x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(13*a^2*b*x^3 + 11*a^3*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4) + 35/8*a^
2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1/3*(b*x^3 - 9*a*x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = \frac{35\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^4} - \frac{13\,a^2bx^3+11\,a^3x}{8\,(bx^2+a)^2b^4} + \frac{b^6x^3-9\,ab^5x}{3\,b^9}$$

input
```
integrate(x^8/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
35/8*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) - 1/8*(13*a^2*b*x^3 + 11*a^
3*x)/((b*x^2 + a)^2*b^4) + 1/3*(b^6*x^3 - 9*a*b^5*x)/b^9
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{x^8}{(a+bx^2)^3}\,dx = \frac{x^3}{3\,b^3} - \frac{\frac{11\,a^3\,x}{8}+\frac{13\,b\,a^2\,x^3}{8}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + \frac{35\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{9/2}} - \frac{3\,a\,x}{b^4}$$

input
```
int(x^8/(a + b*x^2)^3,x)
```

output
```
x^3/(3*b^3) - ((11*a^3*x)/8 + (13*a^2*b*x^3)/8)/(a^2*b^4 + b^6*x^4 + 2*a*b
^5*x^2) + (35*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(9/2)) - (3*a*x)/b^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{x^8}{(a+bx^2)^3}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^3 + 210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^2 + 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^4 - 105a^3bx -}{24b^5\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int(x^8/(b*x^2+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3 + 210*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**2 + 105*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**4 - 105*a**3*b*x - 175*a**2*b**2*x**3 -
56*a*b**3*x**5 + 8*b**4*x**7)/(24*b**5*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.184 $\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 71

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx=\frac{x}{b^3}-\frac{a^2x}{4b^3\left(a+bx^2\right)^2}+\frac{9ax}{8b^3\left(a+bx^2\right)}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

output
```
x/b^3-1/4*a^2*x/b^3/(b*x^2+a)^2+9/8*a*x/b^3/(b*x^2+a)-15/8*a^(1/2)*arctan(
b^(1/2)*x/a^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx=\frac{15a^2x+25abx^3+8b^2x^5}{8b^3\left(a+bx^2\right)^2}-\frac{15\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2)^3,x]
```

output
```
(15*a^2*x + 25*a*b*x^3 + 8*b^2*x^5)/(8*b^3*(a + b*x^2)^2) - (15*Sqrt[a]*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^3} dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^4}{(bx^2+a)^2}dx}{4b} - \frac{x^5}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{3\int \frac{x^2}{bx^2+a}dx}{2b} - \frac{x^3}{2b(a+bx^2)}\right)}{4b} - \frac{x^5}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{5\left(\frac{3\left(\frac{x}{b} - \frac{a\int \frac{1}{bx^2+a}dx}{b}\right)}{2b} - \frac{x^3}{2b(a+bx^2)}\right)}{4b} - \frac{x^5}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5\left(\frac{3\left(\frac{x}{b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b} - \frac{x^3}{2b(a+bx^2)}\right)}{4b} - \frac{x^5}{4b(a+bx^2)^2}$$

input
```
Int[x^6/(a + b*x^2)^3,x]
```

output
```
-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3/(b*(a + b*x^2)) + (3*(x/b - (Sqr
t[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{x}{b^3}-\frac{a\left(\frac{-\frac{9}{8}b\,x^3-\frac{7}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{15\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{b^3}$	51
risch	$\frac{x}{b^3}+\frac{\frac{9}{8}ab\,x^3+\frac{7}{8}a^2x}{b^3\left(b\,x^2+a\right)^2}+\frac{15\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x-a\right)}{16b^4}-\frac{15\sqrt{-ab}\,\ln\left(\sqrt{-ab}\,x-a\right)}{16b^4}$	83

input
```
int(x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
x/b^3-a/b^3*((-9/8*b*x^3-7/8*a*x)/(b*x^2+a)^2+15/8/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.85

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx = \left[\frac{16\,b^2x^5+50\,abx^3+30\,a^2x+15\,(b^2x^4+2\,abx^2+a^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)}{16\,(b^5x^4+2\,ab^4x^2+a^2b^3)}, \frac{8\,b^2x^5+25\,abx^3+15\right.$$

input
```
integrate(x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*b^2*x^5 + 50*a*b*x^3 + 30*a^2*x + 15*(b^2*x^4 + 2*a*b*x^2 + a^2)
*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^5*x^4 + 2*
a*b^4*x^2 + a^2*b^3), 1/8*(8*b^2*x^5 + 25*a*b*x^3 + 15*a^2*x - 15*(b^2*x^4
 + 2*a*b*x^2 + a^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^5*x^4 + 2*a*b^4*
x^2 + a^2*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.51

$$\int \frac{x^6}{\left(a+bx^2\right)^3}\,dx = \frac{15\sqrt{-\frac{a}{b^7}}\log\left(-b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16} - \frac{15\sqrt{-\frac{a}{b^7}}\log\left(b^3\sqrt{-\frac{a}{b^7}}+x\right)}{16} + \frac{7a^2x+9abx^3}{8a^2b^3+16ab^4x^2+8b^5x^4} + \frac{x}{b^3}$$

input
```
integrate(x**6/(b*x**2+a)**3,x)
```

output
```
15*sqrt(-a/b**7)*log(-b**3*sqrt(-a/b**7) + x)/16 - 15*sqrt(-a/b**7)*log(b*
*3*sqrt(-a/b**7) + x)/16 + (7*a**2*x + 9*a*b*x**3)/(8*a**2*b**3 + 16*a*b**
4*x**2 + 8*b**5*x**4) + x/b**3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.96

$$\int \frac{x^6}{(a+bx^2)^3}\,dx = \frac{9\,abx^3+7\,a^2x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)} - \frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{x}{b^3}$$

input
```
integrate(x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(9*a*b*x^3 + 7*a^2*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) - 15/8*a*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{x^6}{(a+bx^2)^3}\,dx = -\frac{15\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{x}{b^3} + \frac{9\,abx^3+7\,a^2x}{8\,(bx^2+a)^2b^3}$$

input
```
integrate(x^6/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-15/8*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + x/b^3 + 1/8*(9*a*b*x^3 + 7
*a^2*x)/((b*x^2 + a)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.90

$$\int \frac{x^6}{(a+bx^2)^3}\,dx = \frac{\frac{7\,a^2\,x}{8}+\frac{9\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{x}{b^3}-\frac{15\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

input
```
int(x^6/(a + b*x^2)^3,x)
```

output
```
((7*a^2*x)/8 + (9*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) + x/b^3 -
(15*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

$$\int \frac{x^6}{(a+bx^2)^3}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+15a^2bx+25a}{8b^4\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^6/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 - 30*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 - 15*sqrt(b)*sqrt(a)*atan((b*x
)/(sqrt(b)*sqrt(a)))*b**2*x**4 + 15*a**2*b*x + 25*a*b**2*x**3 + 8*b**3*x**
5)/(8*b**4*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.185 $\int \frac{x^4}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^2)^3} dx = -\frac{x^3}{4b(a+bx^2)^2} - \frac{3x}{8b^2(a+bx^2)} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}$$

output
```
-1/4*x^3/b/(b*x^2+a)^2-3/8*x/b^2/(b*x^2+a)+3/8*arctan(b^(1/2)*x/a^(1/2))/a
^(1/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{x^4}{(a+bx^2)^3} dx = -\frac{3ax+5bx^3}{8b^2(a+bx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^3,x]
```

output
```
-1/8*(3*a*x + 5*b*x^3)/(b^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*Sqrt[a]*b^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\int \frac{x^2}{(bx^2+a)^2}dx}{4b}-\frac{x^3}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2b}-\frac{x}{2b(a+bx^2)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b(a+bx^2)}\right)}{4b}-\frac{x^3}{4b(a+bx^2)^2}$$

input
```
Int[x^4/(a + b*x^2)^3,x]
```

output
```
-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(b*(a + b*x^2)) + ArcTan[(Sqrt[b]*
x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{-\frac{5x^3}{8b}-\frac{3ax}{8b^2}}{\left(b\,x^2+a\right)^2}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}$	47
risch	$\frac{-\frac{5x^3}{8b}-\frac{3ax}{8b^2}}{\left(b\,x^2+a\right)^2}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^2}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,b^2}$	73

input
```
int(x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-5/8*x^3/b-3/8*a*x/b^2)/(b*x^2+a)^2+3/8/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = \left[-\frac{10\,ab^2x^3+6\,a^2bx+3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)},\ -\frac{5\,ab^2x^3+3\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}\right]$$

input
```
integrate(x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(10*a*b^2*x^3 + 6*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^5*x^4 + 2*a^2*b^4*x^
2 + a^3*b^3), -1/8*(5*a*b^2*x^3 + 3*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2
)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.72

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{ab^5}}\log\left(-ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{ab^5}}\log\left(ab^2\sqrt{-\frac{1}{ab^5}}+x\right)}{16} + \frac{-3ax-5bx^3}{8a^2b^2+16ab^3x^2+8b^4x^4}$$

input
```
integrate(x**4/(b*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a*b**5))*log(-a*b**2*sqrt(-1/(a*b**5)) + x)/16 + 3*sqrt(-1/(a*
b**5))*log(a*b**2*sqrt(-1/(a*b**5)) + x)/16 + (-3*a*x - 5*b*x**3)/(8*a**2*
b**2 + 16*a*b**3*x**2 + 8*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = -\frac{5\,bx^3+3\,ax}{8\,(b^4x^4+2\,ab^3x^2+a^2b^2)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2}$$

input
```
integrate(x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 3/8*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^2} - \frac{5\,bx^3+3\,ax}{8\,(bx^2+a)^2b^2}$$

input
```
integrate(x^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/8*(5*b*x^3 + 3*a*x)/((b*x^2
+ a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}} - \frac{\frac{5\,x^3}{8\,b}+\frac{3\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(x^4/(a + b*x^2)^3,x)
```

output
```
(3*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2)) - ((5*x^3)/(8*b) + (3*a*
x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4-3a^2bx-5a\,b^2x^3}{8a\,b^3\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^4/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 - 3*a**2*b*x - 5*a*b**2*x**3)/(8*a*b**3*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.186 $\int \frac{x^2}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 65

$$\int \frac{x^2}{(a+bx^2)^3} \, dx = -\frac{x}{4b(a+bx^2)^2} + \frac{x}{8ab(a+bx^2)} + \frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

output
```
-1/4*x/b/(b*x^2+a)^2+1/8*x/a/b/(b*x^2+a)+1/8*arctan(b^(1/2)*x/a^(1/2))/a^(
3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(a+bx^2)^3} \, dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-a+bx^2)}{(a+bx^2)^2} + \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-a + b*x^2))/(a + b*x^2)^2 + ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(8*a^(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{(bx^2+a)^2}dx}{4b}-\frac{x}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}}{4b}-\frac{x}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}}{4b}-\frac{x}{4b(a+bx^2)^2}$$

input
```
Int[x^2/(a + b*x^2)^3,x]
```

output
```
-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/(2*a^(3/2)*Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x^3}{8a}-\frac{x}{8b}}{\left(b\,x^2+a\right)^2}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8ab\sqrt{ab}}$	49
risch	$\frac{\frac{x^3}{8a}-\frac{x}{8b}}{\left(b\,x^2+a\right)^2}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}+\frac{\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,ba}$	78

input
```
int(x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8/a*x^3-1/8*x/b)/(b*x^2+a)^2+1/8/a/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int \frac{x^2}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{2\,ab^2x^3-2\,a^2bx-(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}, \frac{ab^2x^3-a^2bx+(b^2x^4+2\,abx^2+a^2}{8\,(a^2b^4x^4+2\,a^3b^3x^2+}\right.$$

input
```
integrate(x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*a*b^2*x^3 - 2*a^2*b*x - (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2
+ a^4*b^2), 1/8*(a*b^2*x^3 - a^2*b*x + (b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a*
b)*arctan(sqrt(a*b)*x/a))/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(51) = 102.

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{-\frac{1}{a^3b^3}}+x\right)}{16} + \frac{-ax+bx^3}{8a^3b+16a^2b^2x^2+8ab^3x^4}$$

input
```
integrate(x**2/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**3*b**3))*log(-a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + sqrt(-1/(
a**3*b**3))*log(a**2*b*sqrt(-1/(a**3*b**3)) + x)/16 + (-a*x + b*x**3)/(8*a
**3*b + 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = \frac{bx^3-ax}{8\,(ab^3x^4+2\,a^2b^2x^2+a^3b)} + \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab}$$

input
```
integrate(x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 1/8*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab} + \frac{bx^3-ax}{8\,(bx^2+a)^2ab}$$

input
```
integrate(x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/8*(b*x^3 - a*x)/((b*x^2 + a)
^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}} - \frac{\frac{x}{8\,b}-\frac{x^3}{8\,a}}{a^2+2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(x^2/(a + b*x^2)^3,x)
```

output
```
atan((b^(1/2)*x)/a^(1/2))/(8*a^(3/2)*b^(3/2)) - (x/(8*b) - x^3/(8*a))/(a^2
 + b^2*x^4 + 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \frac{x^2}{(a+bx^2)^3}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+2\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4-a^2bx+a\,b^2x^3}{8a^2b^2\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^2/(b*x^2+a)^3,x)
```

output
```
(sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 2*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)
*sqrt(a)))*b**2*x**4 - a**2*b*x + a*b**2*x**3)/(8*a**2*b**2*(a**2 + 2*a*b*
x**2 + b**2*x**4))
```

3.187 $\int \frac{1}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 9, antiderivative size = 62

$$\int \frac{1}{(a+bx^2)^3} dx = \frac{x}{4a(a+bx^2)^2} + \frac{3x}{8a^2(a+bx^2)} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*x/a/(b*x^2+a)^2+3/8*x/a^2/(b*x^2+a)+3/8*arctan(b^(1/2)*x/a^(1/2))/a^(5
/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^2)^3} dx = \frac{5ax+3bx^3}{8a^2(a+bx^2)^2} + \frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(-3),x]
```

output
```
(5*a*x + 3*b*x^3)/(8*a^2*(a + b*x^2)^2) + (3*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
(8*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{1}{(bx^2+a)^2}dx}{4a}+\frac{x}{4a\,(a+bx^2)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(\frac{\int \frac{1}{bx^2+a}dx}{2a}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a\,(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a+bx^2)}\right)}{4a}+\frac{x}{4a\,(a+bx^2)^2}$$

input
```
Int[(a + b*x^2)^(-3),x]
```

output
```
x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[
a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4a(b\,x^{2}+a)^{2}}+\frac{\frac{3x}{8a\left(b\,x^{2}+a\right)}+\frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	57
risch	$\frac{\frac{3b\,x^{3}}{8a^{2}}+\frac{5x}{8a}}{\left(b\,x^{2}+a\right)^{2}}-\frac{3\ln\left(bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^{2}}+\frac{3\ln\left(-bx+\sqrt{-ab}\right)}{16\sqrt{-ab}\,a^{2}}$	73

input
```
int(1/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(b*x^2+a)^2+3/4/a*(1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.03

$$\int \frac{1}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{6\,ab^2x^3+10\,a^2bx-3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}x-a}{bx^2+a}\right)}{16\,(a^3b^3x^4+2\,a^4b^2x^2+a^5b)}, \frac{3\,ab^2x^3+5\,a^2bx+3\,(b^2x^4+2\,ab}{8\,(a^3b^3x^4+2\,a^4}\right.$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(6*a*b^2*x^3 + 10*a^2*b*x - 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a*b)
*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^3*x^4 + 2*a^4*b^2*x
^2 + a^5*b), 1/8*(3*a*b^2*x^3 + 5*a^2*b*x + 3*(b^2*x^4 + 2*a*b*x^2 + a^2)*
sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+bx^2)^3}\,dx = -\frac{3\sqrt{-\frac{1}{a^5b}}\log\left(-a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{-\frac{1}{a^5b}}\log\left(a^3\sqrt{-\frac{1}{a^5b}}+x\right)}{16} + \frac{5ax+3bx^3}{8a^4+16a^3bx^2+8a^2b^2x^4}$$

input
```
integrate(1/(b*x**2+a)**3,x)
```

output
```
-3*sqrt(-1/(a**5*b))*log(-a**3*sqrt(-1/(a**5*b)) + x)/16 + 3*sqrt(-1/(a**5
*b))*log(a**3*sqrt(-1/(a**5*b)) + x)/16 + (5*a*x + 3*b*x**3)/(8*a**4 + 16*
a**3*b*x**2 + 8*a**2*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\,bx^3+5\,ax}{8\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2}$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3/8*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^2} + \frac{3\,bx^3+5\,ax}{8\,(bx^2+a)^2a^2}$$

input
```
integrate(1/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/8*(3*b*x^3 + 5*a*x)/((b*x^2
+ a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{\frac{5\,x}{8\,a}+\frac{3\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{3\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^3,x)
```

output
```
((5*x)/(8*a) + (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*atan((b
^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{1}{(a+bx^2)^3}\,dx = \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2+6\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^4+5a^2bx+3a\,b^2x^3}{8a^3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2 + 6*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**2 + 3*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*b**2*x**4 + 5*a**2*b*x + 3*a*b**2*x**3)/(8*a**3*b*(a**2 + 2
*a*b*x**2 + b**2*x**4))
```

3.188 $\int \frac{1}{x^2(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{1}{x^2(a+bx^2)^3}\,dx = -\frac{1}{a^3x} - \frac{bx}{4a^2(a+bx^2)^2} - \frac{7bx}{8a^3(a+bx^2)} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-1/a^3/x-1/4*b*x/a^2/(b*x^2+a)^2-7/8*b*x/a^3/(b*x^2+a)-15/8*b^(1/2)*arctan
(b^(1/2)*x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2(a+bx^2)^3}\,dx = -\frac{8a^2+25abx^2+15b^2x^4}{8a^3x(a+bx^2)^2} - \frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^3),x]
```

output
```
-1/8*(8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)/(a^3*x*(a + b*x^2)^2) - (15*Sqrt[b]
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5 \int \frac{1}{x^2 (bx^2+a)^2} dx}{4a} + \frac{1}{4ax (a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{5 \left(\frac{3 \int \frac{1}{x^2 (bx^2+a)} dx}{2a} + \frac{1}{2ax(a+bx^2)} \right)}{4a} + \frac{1}{4ax (a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{5 \left(\frac{3 \left(-\frac{b \int \frac{1}{bx^2+a} dx}{a} - \frac{1}{ax} \right)}{2a} + \frac{1}{2ax(a+bx^2)} \right)}{4a} + \frac{1}{4ax (a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{5 \left(\frac{3 \left(-\frac{\sqrt{b} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax} \right)}{2a} + \frac{1}{2ax(a+bx^2)} \right)}{4a} + \frac{1}{4ax (a+bx^2)^2}$$

input
```
Int[1/(x^2*(a + b*x^2)^3),x]
```

output
```
1/(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sq
rt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{b\left(\frac{\frac{7}{8}b\,x^{3}+\frac{9}{8}a x}{\left(b\,x^{2}+a\right)^{2}}+\frac{15\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{3}}-\frac{1}{a^{3}x}$	54
risch	$\frac{-\frac{15b^{2}x^{4}}{8a^{3}}-\frac{25b\,x^{2}}{8a^{2}}-\frac{1}{a}}{x\left(b\,x^{2}+a\right)^{2}}+\frac{15\sqrt{-ab}\,\ln\left(-bx+\sqrt{-ab}\right)}{16a^{4}}-\frac{15\sqrt{-ab}\,\ln\left(-bx-\sqrt{-ab}\right)}{16a^{4}}$	89

input
```
int(1/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*b*((7/8*b*x^3+9/8*a*x)/(b*x^2+a)^2+15/8/(a*b)^(1/2)*arctan(b*x/(a*b
)^(1/2)))-1/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.81

$$\int \frac{1}{x^2 (a+bx^2)^3}\,dx = \left[-\frac{30\,b^2x^4+50\,abx^2-15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+16\,a^2}{16\,(a^3b^2x^5+2\,a^4bx^3+a^5x)},\ -\frac{15\,b^2x^4+25\,abx^2+15\,(b^2x^5+2\,abx^3+a^2x)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+8\,a^2}{8\,(a^3b^2x^5+2\,a^4bx^3+a^5x)}\right]$$

input
```
integrate(1/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(30*b^2*x^4 + 50*a*b*x^2 - 15*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(-b
/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 16*a^2)/(a^3*b^2*x^5
 + 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 + 25*a*b*x^2 + 15*(b^2*x^5 + 2*a
*b*x^3 + a^2*x)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 8*a^2)/(a^3*b^2*x^5 + 2*a^
4*b*x^3 + a^5*x)]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.61

$$\int \frac{1}{x^2 (a+bx^2)^3}\,dx = \frac{15\sqrt{-\frac{b}{a^7}}\log\left(-\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16} - \frac{15\sqrt{-\frac{b}{a^7}}\log\left(\frac{a^4\sqrt{-\frac{b}{a^7}}}{b}+x\right)}{16} + \frac{-8a^2-25abx^2-15b^2x^4}{8a^5x+16a^4bx^3+8a^3b^2x^5}$$

input
```
integrate(1/x**2/(b*x**2+a)**3,x)
```

output
```
15*sqrt(-b/a**7)*log(-a**4*sqrt(-b/a**7)/b + x)/16 - 15*sqrt(-b/a**7)*log(
a**4*sqrt(-b/a**7)/b + x)/16 + (-8*a**2 - 25*a*b*x**2 - 15*b**2*x**4)/(8*a
**5*x + 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{8\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)} - \frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

input
```
integrate(1/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
 - 15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3} - \frac{7\,b^2x^3+9\,abx}{8\left(bx^2+a\right)^2a^3} - \frac{1}{a^3x}$$

input
```
integrate(1/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-15/8*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/8*(7*b^2*x^3 + 9*a*b*x)/
((b*x^2 + a)^2*a^3) - 1/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a+bx^2)^3} \, dx = -\frac{\frac{1}{a}+\frac{25\,b\,x^2}{8\,a^2}+\frac{15\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{15\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

input
```
int(1/(x^2*(a + b*x^2)^3),x)
```

output
```
- (1/a + (25*b*x^2)/(8*a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a
*b*x^3) - (15*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.68

$$\int \frac{1}{x^2 (a+bx^2)^3} \, dx = \frac{-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2x-30\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)ab\,x^3-15\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^2x^5-8a^3-25a^2b}{8a^4x\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/x^2/(b*x^2+a)^3,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*x - 30*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b*x**3 - 15*sqrt(b)*sqrt(a)*atan((b
*x)/(sqrt(b)*sqrt(a)))*b**2*x**5 - 8*a**3 - 25*a**2*b*x**2 - 15*a*b**2*x**
4)/(8*a**4*x*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.189 $\int \frac{1}{x^4(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{1}{x^4(a+bx^2)^3}\,dx = -\frac{1}{3a^3x^3} + \frac{3b}{a^4x} + \frac{b^2x}{4a^3(a+bx^2)^2} + \frac{11b^2x}{8a^4(a+bx^2)} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

output
```
-1/3/a^3/x^3+3*b/a^4/x+1/4*b^2*x/a^3/(b*x^2+a)^2+11/8*b^2*x/a^4/(b*x^2+a)+
35/8*b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4(a+bx^2)^3}\,dx = \frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^4x^3(a+bx^2)^2} + \frac{35b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^3),x]
```

output
```
(-8*a^3 + 56*a^2*b*x^2 + 175*a*b^2*x^4 + 105*b^3*x^6)/(24*a^4*x^3*(a + b*x
^2)^2) + (35*b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \int \frac{1}{x^4 (bx^2+a)^2}dx}{4a}+\frac{1}{4ax^3 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{7 \left(\frac{5 \int \frac{1}{x^4 \left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^3 (a+bx^2)}\right)}{4a}+\frac{1}{4ax^3 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{7 \left(\frac{5 \left(-\frac{b \int \frac{1}{x^2 \left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3 (a+bx^2)}\right)}{4a}+\frac{1}{4ax^3 \left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3(a+bx^2)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}$$

input
```
Int[1/(x^4*(a + b*x^2)^3),x]
```

output
```
1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3*(a + b*x^2)) + (5*(-1/3*1/(a*x^
3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/
(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{b^{2}\left(\frac{\frac{11}{8}b\,x^{3}+\frac{13}{8}a x}{\left(b\,x^{2}+a\right)^{2}}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^{4}}-\frac{1}{3a^{3}x^{3}}+\frac{3b}{a^{4}x}$	64
risch	$\frac{\frac{35b^{3}x^{6}}{8a^{4}}+\frac{175b^{2}x^{4}}{24a^{3}}+\frac{7b\,x^{2}}{3a^{2}}-\frac{1}{3a}}{x^{3}\left(b\,x^{2}+a\right)^{2}}+\frac{35\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)}{16a^{5}}-\frac{35\sqrt{-ab}\,b\ln\left(-bx+\sqrt{-ab}\right)}{16a^{5}}$	102

input
```
int(1/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*b^2*((11/8*b*x^3+13/8*a*x)/(b*x^2+a)^2+35/8/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))-1/3/a^3/x^3+3*b/a^4/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.74

$$\int \frac{1}{x^{4}\left(a+bx^{2}\right)^{3}}\,dx$$
$$=\left[\frac{210\,b^{3}x^{6}+350\,ab^{2}x^{4}+112\,a^{2}bx^{2}-16\,a^{3}+105\left(b^{3}x^{7}+2\,ab^{2}x^{5}+a^{2}bx^{3}\right)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)}{48\left(a^{4}b^{2}x^{7}+2\,a^{5}bx^{5}+a^{6}x^{3}\right)},\ 1\ldots\right.$$

input
```
integrate(1/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(210*b^3*x^6 + 350*a*b^2*x^4 + 112*a^2*b*x^2 - 16*a^3 + 105*(b^3*x^7
 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/
(b*x^2 + a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3), 1/24*(105*b^3*x^6 + 1
75*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3 + 105*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x
^3)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(-\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16} + \frac{35\sqrt{-\frac{b^3}{a^9}}\log\left(\frac{a^5\sqrt{-\frac{b^3}{a^9}}}{b^2}+x\right)}{16} + \frac{-8a^3+56a^2bx^2+175ab^2x^4+105b^3x^6}{24a^6x^3+48a^5bx^5+24a^4b^2x^7}$$

input
```
integrate(1/x**4/(b*x**2+a)**3,x)
```

output
```
-35*sqrt(-b**3/a**9)*log(-a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + 35*sqrt(-b*
*3/a**9)*log(a**5*sqrt(-b**3/a**9)/b**2 + x)/16 + (-8*a**3 + 56*a**2*b*x**
2 + 175*a*b**2*x**4 + 105*b**3*x**6)/(24*a**6*x**3 + 48*a**5*b*x**5 + 24*a
**4*b**2*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.99

$$\int \frac{1}{x^4\left(a+bx^2\right)^3}\,dx = \frac{105\,b^3x^6+175\,ab^2x^4+56\,a^2bx^2-8\,a^3}{24\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)} + \frac{35\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4}$$

input
```
integrate(1/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/24*(105*b^3*x^6 + 175*a*b^2*x^4 + 56*a^2*b*x^2 - 8*a^3)/(a^4*b^2*x^7 + 2
*a^5*b*x^5 + a^6*x^3) + 35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 (a+bx^2)^3}\,dx = \frac{35\,b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}a^4} + \frac{11\,b^3x^3 + 13\,ab^2x}{8\,(bx^2+a)^2a^4} + \frac{9\,bx^2-a}{3\,a^4x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
35/8*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4) + 1/8*(11*b^3*x^3 + 13*a*b^
2*x)/((b*x^2 + a)^2*a^4) + 1/3*(9*b*x^2 - a)/(a^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (a+bx^2)^3}\,dx = \frac{\frac{7\,b\,x^2}{3\,a^2} - \frac{1}{3\,a} + \frac{175\,b^2\,x^4}{24\,a^3} + \frac{35\,b^3\,x^6}{8\,a^4}}{a^2\,x^3 + 2\,a\,b\,x^5 + b^2\,x^7} + \frac{35\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{9/2}}$$

input
```
int(1/(x^4*(a + b*x^2)^3),x)
```

output
```
((7*b*x^2)/(3*a^2) - 1/(3*a) + (175*b^2*x^4)/(24*a^3) + (35*b^3*x^6)/(8*a^
4))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5) + (35*b^(3/2)*atan((b^(1/2)*x)/a^(1/2)
))/(8*a^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^4 (a+bx^2)^3}\,dx = \frac{105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b\,x^3 + 210\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^2x^5 + 105\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^3x^7 - 8a^4 + 5}{24a^5x^3\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int(1/x^4/(b*x^2+a)^3,x)
```

output
```
(105*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b*x**3 + 210*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**2*x**5 + 105*sqrt(b)*sqrt(a)
*atan((b*x)/(sqrt(b)*sqrt(a)))*b**3*x**7 - 8*a**4 + 56*a**3*b*x**2 + 175*a
**2*b**2*x**4 + 105*a*b**3*x**6)/(24*a**5*x**3*(a**2 + 2*a*b*x**2 + b**2*x
**4))
```

3.190 $\int \frac{1}{x^6(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int \frac{1}{x^6 (a+bx^2)^3} dx = -\frac{1}{5a^3x^5} + \frac{b}{a^4x^3} - \frac{6b^2}{a^5x} - \frac{b^3x}{4a^4(a+bx^2)^2} - \frac{15b^3x}{8a^5(a+bx^2)} - \frac{63b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

output
```
-1/5/a^3/x^5+b/a^4/x^3-6*b^2/a^5/x-1/4*b^3*x/a^4/(b*x^2+a)^2-15/8*b^3*x/a^
5/(b*x^2+a)-63/8*b^(5/2)*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^6 (a+bx^2)^3} dx = -\frac{8a^4 - 24a^3bx^2 + 168a^2b^2x^4 + 525ab^3x^6 + 315b^4x^8}{40a^5x^5(a+bx^2)^2} - \frac{63b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/40*(8*a^4 - 24*a^3*b*x^2 + 168*a^2*b^2*x^4 + 525*a*b^3*x^6 + 315*b^4*x^
8)/(a^5*x^5*(a + b*x^2)^2) - (63*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(11/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {253, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9\int \frac{1}{x^6(bx^2+a)^2}dx}{4a}+\frac{1}{4ax^5 (a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{9\left(\frac{7\int \frac{1}{x^6\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5 (a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(\frac{7\left(-\frac{b\int \frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5 (a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\int \frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

$\downarrow$ 264

$$\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int \frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}$$

$\downarrow$ 218

$$\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5(a+bx^2)}\right)}{4a}+\frac{1}{4ax^5(a+bx^2)^2}$$

input
```
Int[1/(x^6*(a + b*x^2)^3),x]
```

output
```
1/(4*a*x^5*(a + b*x^2)^2) + (9*(1/(2*a*x^5*(a + b*x^2)) + (7*(-1/5*1/(a*x^
5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/a^(3/2)))/a))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{b^3\left(\frac{\frac{15}{8}b\,x^3+\frac{17}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{63\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^5}-\frac{1}{5a^3x^5}+\frac{b}{a^4x^3}-\frac{6b^2}{a^5x}$	75
risch	$\frac{-\frac{63b^4x^8}{8a^5}-\frac{105b^3x^6}{8a^4}-\frac{21b^2x^4}{5a^3}+\frac{3b\,x^2}{5a^2}-\frac{1}{5a}}{x^5\left(b\,x^2+a\right)^2}+\frac{63\left(\sum_{\_R=\mathrm{RootOf}\left(a^{11}\_Z^2+b^5\right)}\_R\ln\left(\left(3\_R^2a^{11}+2b^5\right)x+a^6b^2\_R\right)\right)}{16}$	108

input
```
int(1/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^5*b^3*((15/8*b*x^3+17/8*a*x)/(b*x^2+a)^2+63/8/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))-1/5/a^3/x^5+b/a^4/x^3-6*b^2/a^5/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.72

$$\int \frac{1}{x^6\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{630\,b^4x^8+1050\,ab^3x^6+336\,a^2b^2x^4-48\,a^3bx^2+16\,a^4-315\,(b^4x^9+2\,ab^3x^7+a^2b^2x^5)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2\cdots}{\cdots}\right.}{80\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}\right.$$

$$-\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4+315\,(b^4x^9+2\,ab^3x^7+a^2b^2x^5)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\cdots}\right.}{40\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}$$

input
```
integrate(1/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/80*(630*b^4*x^8 + 1050*a*b^3*x^6 + 336*a^2*b^2*x^4 - 48*a^3*b*x^2 + 16
*a^4 - 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2
*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5),
-1/40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^
4 + 315*(b^4*x^9 + 2*a*b^3*x^7 + a^2*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)
))/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^6 (a+bx^2)^3} dx = \frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(-\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16} - \frac{63\sqrt{-\frac{b^5}{a^{11}}}\log\left(\frac{a^6\sqrt{-\frac{b^5}{a^{11}}}}{b^3}+x\right)}{16} + \frac{-8a^4+24a^3bx^2-168a^2b^2x^4-525ab^3x^6-315b^4x^8}{40a^7x^5+80a^6bx^7+40a^5b^2x^9}$$

input
```
integrate(1/x**6/(b*x**2+a)**3,x)
```

output
```
63*sqrt(-b**5/a**11)*log(-a**6*sqrt(-b**5/a**11)/b**3 + x)/16 - 63*sqrt(-b
**5/a**11)*log(a**6*sqrt(-b**5/a**11)/b**3 + x)/16 + (-8*a**4 + 24*a**3*b*
x**2 - 168*a**2*b**2*x**4 - 525*a*b**3*x**6 - 315*b**4*x**8)/(40*a**7*x**5
 + 80*a**6*b*x**7 + 40*a**5*b**2*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 (a+bx^2)^3} dx = -\frac{315\,b^4x^8+525\,ab^3x^6+168\,a^2b^2x^4-24\,a^3bx^2+8\,a^4}{40\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)} - \frac{63\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

input
```
integrate(1/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/40*(315*b^4*x^8 + 525*a*b^3*x^6 + 168*a^2*b^2*x^4 - 24*a^3*b*x^2 + 8*a^
4)/(a^5*b^2*x^9 + 2*a^6*b*x^7 + a^7*x^5) - 63/8*b^3*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^6 (a + bx^2)^3} \, dx = -\frac{63 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^5} - \frac{15 \, b^4 x^3 + 17 \, ab^3 x}{8 \left(bx^2 + a\right)^2 a^5} - \frac{30 \, b^2 x^4 - 5 \, abx^2 + a^2}{5 \, a^5 x^5}$$

input
```
integrate(1/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-63/8*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5) - 1/8*(15*b^4*x^3 + 17*a*b
^3*x)/((b*x^2 + a)^2*a^5) - 1/5*(30*b^2*x^4 - 5*a*b*x^2 + a^2)/(a^5*x^5)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^6 (a + bx^2)^3} \, dx = -\frac{\frac{1}{5\,a} - \frac{3\,b\,x^2}{5\,a^2} + \frac{21\,b^2\,x^4}{5\,a^3} + \frac{105\,b^3\,x^6}{8\,a^4} + \frac{63\,b^4\,x^8}{8\,a^5}}{a^2\,x^5 + 2\,a\,b\,x^7 + b^2\,x^9} - \frac{63\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{11/2}}$$

input
```
int(1/(x^6*(a + b*x^2)^3),x)
```

output
```
- (1/(5*a) - (3*b*x^2)/(5*a^2) + (21*b^2*x^4)/(5*a^3) + (105*b^3*x^6)/(8*a
^4) + (63*b^4*x^8)/(8*a^5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (63*b^(5/2)*
atan((b^(1/2)*x)/a^(1/2)))/(8*a^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.55

$$\int \frac{1}{x^6 (a+bx^2)^3}\, dx$$
$$= \frac{-315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^2b^2x^5 - 630\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a\,b^3x^7 - 315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)b^4x^9 - 8a^5 +}{40a^6x^5\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int(1/x^6/(b*x^2+a)^3,x)
```

output
```
( - 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**2*x**5 - 630
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**3*x**7 - 315*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**4*x**9 - 8*a**5 + 24*a**4*b*x**2 -
 168*a**3*b**2*x**4 - 525*a**2*b**3*x**6 - 315*a*b**4*x**8)/(40*a**6*x**5*
(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.191 $\int \frac{1}{x^8(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{1}{x^8(a+bx^2)^3}\,dx=-\frac{1}{7a^3x^7}+\frac{3b}{5a^4x^5}-\frac{2b^2}{a^5x^3}+\frac{10b^3}{a^6x}+\frac{b^4x}{4a^5(a+bx^2)^2}+\frac{19b^4x}{8a^6(a+bx^2)}+\frac{99b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

output
```
-1/7/a^3/x^7+3/5*b/a^4/x^5-2*b^2/a^5/x^3+10*b^3/a^6/x+1/4*b^4*x/a^5/(b*x^2
+a)^2+19/8*b^4*x/a^6/(b*x^2+a)+99/8*b^(7/2)*arctan(b^(1/2)*x/a^(1/2))/a^(1
3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^8(a+bx^2)^3}\,dx=\frac{-40a^5+88a^4bx^2-264a^3b^2x^4+1848a^2b^3x^6+5775ab^4x^8+3465b^5x^{10}}{280a^6x^7(a+bx^2)^2}+\frac{99b^{7/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

input
```
Integrate[1/(x^8*(a + b*x^2)^3),x]
```

output
```
(-40*a^5 + 88*a^4*b*x^2 - 264*a^3*b^2*x^4 + 1848*a^2*b^3*x^6 + 5775*a*b^4*
x^8 + 3465*b^5*x^10)/(280*a^6*x^7*(a + b*x^2)^2) + (99*b^(7/2)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(8*a^(13/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {253, 253, 264, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^8 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11 \int \frac{1}{x^8 (bx^2+a)^2}dx}{4a}+\frac{1}{4ax^7 (a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{11 \left(\frac{9 \int \frac{1}{x^8 \left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^7 (a+bx^2)}\right)}{4a}+\frac{1}{4ax^7 (a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11 \left(\frac{9 \left(-\frac{b \int \frac{1}{x^6 \left(bx^2+a\right)}dx}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7 (a+bx^2)}\right)}{4a}+\frac{1}{4ax^7 (a+bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{1}{x^4(bx^2+a)}dx}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}\right)}{4a}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

$\downarrow$ 264

$$\frac{11\left(\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{x^2(bx^2+a)}dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}\right)}{4a}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

$\downarrow$ 264

$$\frac{11\left(\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}\right)}{4a}+\frac{1}{4ax^7\left(a+bx^2\right)^2}$$

$\big\downarrow$ 218

$$\frac{11\left(\frac{9\left(-\frac{b\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{a}-\frac{1}{7ax^7}\right)}{2a}+\frac{1}{2ax^7(a+bx^2)}\right)}{4a}+\frac{1}{4ax^7(a+bx^2)^2}$$

input
```
Int[1/(x^8*(a + b*x^2)^3),x]
```

output
```
1/(4*a*x^7*(a + b*x^2)^2) + (11*(1/(2*a*x^7*(a + b*x^2)) + (9*(-1/7*1/(a*x
^7) - (b*(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/a))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{b^4\left(\frac{\frac{19}{8}b\,x^3+\frac{21}{8}ax}{\left(b\,x^2+a\right)^2}+\frac{99\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^6}-\frac{1}{7a^3x^7}+\frac{3b}{5a^4x^5}-\frac{2b^2}{a^5x^3}+\frac{10b^3}{a^6x}$	86
risch	$\frac{\frac{99b^5x^{10}}{8a^6}+\frac{165b^4x^8}{8a^5}+\frac{33b^3x^6}{5a^4}-\frac{33b^2x^4}{35a^3}+\frac{11b\,x^2}{35a^2}-\frac{1}{7a}}{x^7\left(b\,x^2+a\right)^2}+\frac{99\sqrt{-ab}\,b^3\ln\left(-bx-\sqrt{-ab}\right)}{16a^7}-\frac{99\sqrt{-ab}\,b^3\ln\left(-bx+\sqrt{-ab}\right)}{16a^7}$	128

input
```
int(1/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^6*b^4*((19/8*b*x^3+21/8*a*x)/(b*x^2+a)^2+99/8/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))-1/7/a^3/x^7+3/5*b/a^4/x^5-2*b^2/a^5/x^3+10*b^3/a^6/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^8 (a+bx^2)^3} dx = \left[\frac{6930 b^5 x^{10} + 11550 ab^4 x^8 + 3696 a^2 b^3 x^6 - 528 a^3 b^2 x^4 + 176 a^4 b x^2 - 80 a^5 + 3465 (b^5 x^{11} + 2 ab^4 x^9 + a^2}{560 (a^6 b^2 x^{11} + 2 a^7 b x^9 + a^8 x^7)} \right.$$

input
```
integrate(1/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/560*(6930*b^5*x^10 + 11550*a*b^4*x^8 + 3696*a^2*b^3*x^6 - 528*a^3*b^2*x
^4 + 176*a^4*b*x^2 - 80*a^5 + 3465*(b^5*x^11 + 2*a*b^4*x^9 + a^2*b^3*x^7)*
sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11
+ 2*a^7*b*x^9 + a^8*x^7), 1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2
*b^3*x^6 - 264*a^3*b^2*x^4 + 88*a^4*b*x^2 - 40*a^5 + 3465*(b^5*x^11 + 2*a*
b^4*x^9 + a^2*b^3*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11 + 2*a^
7*b*x^9 + a^8*x^7)]
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^8 (a+bx^2)^3} dx = -\frac{99\sqrt{-\frac{b^7}{a^{13}}} \log \left(-\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{99\sqrt{-\frac{b^7}{a^{13}}} \log \left(\frac{a^7\sqrt{-\frac{b^7}{a^{13}}}}{b^4} + x\right)}{16} + \frac{-40a^5 + 88a^4bx^2 - 264a^3b^2x^4 + 1848a^2b^3x^6 + 5775ab^4x^8 + 3465b^5x^{10}}{280a^8x^7 + 560a^7bx^9 + 280a^6b^2x^{11}}$$

input
```
integrate(1/x**8/(b*x**2+a)**3,x)
```

output
```
-99*sqrt(-b**7/a**13)*log(-a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + 99*sqrt(-
b**7/a**13)*log(a**7*sqrt(-b**7/a**13)/b**4 + x)/16 + (-40*a**5 + 88*a**4*
b*x**2 - 264*a**3*b**2*x**4 + 1848*a**2*b**3*x**6 + 5775*a*b**4*x**8 + 346
5*b**5*x**10)/(280*a**8*x**7 + 560*a**7*b*x**9 + 280*a**6*b**2*x**11)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^8 (a+bx^2)^3} dx = \frac{3465 b^5 x^{10} + 5775 ab^4 x^8 + 1848 a^2 b^3 x^6 - 264 a^3 b^2 x^4 + 88 a^4 b x^2 - 40 a^5}{280 (a^6 b^2 x^{11} + 2 a^7 b x^9 + a^8 x^7)} + \frac{99 b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^6}$$

input
```
integrate(1/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/280*(3465*b^5*x^10 + 5775*a*b^4*x^8 + 1848*a^2*b^3*x^6 - 264*a^3*b^2*x^4
 + 88*a^4*b*x^2 - 40*a^5)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) + 99/8*b^
4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^8 (a+bx^2)^3} dx = \frac{99 b^4 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8 \sqrt{ab} a^6} + \frac{19 b^5 x^3 + 21 ab^4 x}{8 (bx^2+a)^2 a^6} + \frac{350 b^3 x^6 - 70 ab^2 x^4 + 21 a^2 b x^2 - 5 a^3}{35 a^6 x^7}$$

input
```
integrate(1/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
99/8*b^4*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*x^3 + 21*a*b^
4*x)/((b*x^2 + a)^2*a^6) + 1/35*(350*b^3*x^6 - 70*a*b^2*x^4 + 21*a^2*b*x^2
 - 5*a^3)/(a^6*x^7)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^8 (a+bx^2)^3} \, dx = \frac{\frac{11\,b\,x^2}{35\,a^2} - \frac{1}{7\,a} - \frac{33\,b^2\,x^4}{35\,a^3} + \frac{33\,b^3\,x^6}{5\,a^4} + \frac{165\,b^4\,x^8}{8\,a^5} + \frac{99\,b^5\,x^{10}}{8\,a^6}}{a^2\,x^7 + 2\,a\,b\,x^9 + b^2\,x^{11}} + \frac{99\,b^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{13/2}}$$

input
```
int(1/(x^8*(a + b*x^2)^3),x)
```

output
```
((11*b*x^2)/(35*a^2) - 1/(7*a) - (33*b^2*x^4)/(35*a^3) + (33*b^3*x^6)/(5*a
^4) + (165*b^4*x^8)/(8*a^5) + (99*b^5*x^10)/(8*a^6))/(a^2*x^7 + b^2*x^11 +
 2*a*b*x^9) + (99*b^(7/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(13/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^8 (a+bx^2)^3} \, dx = \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^2b^3x^7 + 6930\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a\,b^4x^9 + 3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^5x^{11} - 40}{280a^7x^7\,(b^2x^4 + 2ab\,x^2 + a^2)}$$

input
```
int(1/x^8/(b*x^2+a)^3,x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**3*x**7 + 6930*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**4*x**9 + 3465*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**5*x**11 - 40*a**6 + 88*a**5*b*x**2
 - 264*a**4*b**2*x**4 + 1848*a**3*b**3*x**6 + 5775*a**2*b**4*x**8 + 3465*a
*b**5*x**10)/(280*a**7*x**7*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.192 $\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 216

$$\begin{aligned}\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = {} & \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{x^6}{6b^{10}} - \frac{a^{12}}{18b^{13}(a+bx^2)^9} \\ & + \frac{3a^{11}}{4b^{13}(a+bx^2)^8} - \frac{33a^{10}}{7b^{13}(a+bx^2)^7} + \frac{55a^9}{3b^{13}(a+bx^2)^6} \\ & - \frac{99a^8}{2b^{13}(a+bx^2)^5} + \frac{99a^7}{b^{13}(a+bx^2)^4} - \frac{154a^6}{b^{13}(a+bx^2)^3} \\ & + \frac{198a^5}{b^{13}(a+bx^2)^2} - \frac{495a^4}{2b^{13}(a+bx^2)} - \frac{110a^3\log(a+bx^2)}{b^{13}}\end{aligned}$$

output

```
55/2*a^2*x^2/b^12-5/2*a*x^4/b^11+1/6*x^6/b^10-1/18*a^12/b^13/(b*x^2+a)^9+3
/4*a^11/b^13/(b*x^2+a)^8-33/7*a^10/b^13/(b*x^2+a)^7+55/3*a^9/b^13/(b*x^2+a
)^6-99/2*a^8/b^13/(b*x^2+a)^5+99*a^7/b^13/(b*x^2+a)^4-154*a^6/b^13/(b*x^2+
a)^3+198*a^5/b^13/(b*x^2+a)^2-495/2*a^4/b^13/(b*x^2+a)-110*a^3*ln(b*x^2+a)
/b^13
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.78

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = -\frac{35201a^{12}+289089a^{11}bx^2+1031616a^{10}b^2x^4+2074464a^9b^3x^6+2529576a^8b^4x^8+1831032a^7b^5x^{10}+6}{}$$

input
```
Integrate[x^25/(a + b*x^2)^10,x]
```

output
```
-1/252*(35201*a^12 + 289089*a^11*b*x^2 + 1031616*a^10*b^2*x^4 + 2074464*a^
9*b^3*x^6 + 2529576*a^8*b^4*x^8 + 1831032*a^7*b^5*x^10 + 638568*a^6*b^6*x^
12 - 58968*a^5*b^7*x^14 - 139482*a^4*b^8*x^16 - 43218*a^3*b^9*x^18 - 2772*
a^2*b^10*x^20 + 252*a*b^11*x^22 - 42*b^12*x^24 + 27720*a^3*(a + b*x^2)^9*L
og[a + b*x^2])/(b^13*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^{24}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \ 49$$

$$\frac{1}{2}\int \left(\frac{a^{12}}{b^{12}(bx^2+a)^{10}}-\frac{12a^{11}}{b^{12}(bx^2+a)^9}+\frac{66a^{10}}{b^{12}(bx^2+a)^8}-\frac{220a^9}{b^{12}(bx^2+a)^7}+\frac{495a^8}{b^{12}(bx^2+a)^6}-\frac{792a^7}{b^{12}(bx^2+a)^5}+\frac{}{b^1}\right.$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^{12}}{9b^{13}(a+bx^2)^9}+\frac{3a^{11}}{2b^{13}(a+bx^2)^8}-\frac{66a^{10}}{7b^{13}(a+bx^2)^7}+\frac{110a^9}{3b^{13}(a+bx^2)^6}-\frac{99a^8}{b^{13}(a+bx^2)^5}+\frac{198a^7}{b^{13}(a+bx^2)^4}-\right.$$

input
```
Int[x^25/(a + b*x^2)^10,x]
```

output
```
((55*a^2*x^2)/b^12 - (5*a*x^4)/b^11 + x^6/(3*b^10) - a^12/(9*b^13*(a + b*x
^2)^9) + (3*a^11)/(2*b^13*(a + b*x^2)^8) - (66*a^10)/(7*b^13*(a + b*x^2)^7
) + (110*a^9)/(3*b^13*(a + b*x^2)^6) - (99*a^8)/(b^13*(a + b*x^2)^5) + (19
8*a^7)/(b^13*(a + b*x^2)^4) - (308*a^6)/(b^13*(a + b*x^2)^3) + (396*a^5)/(
b^13*(a + b*x^2)^2) - (495*a^4)/(b^13*(a + b*x^2)) - (220*a^3*Log[a + b*x^
2])/b^13)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

method	result
risch	$\frac{x^6}{6b^{10}} - \frac{5a\,x^4}{2b^{11}} + \frac{55a^2x^2}{2b^{12}} + \frac{-\frac{35201a^{12}}{252b} - \frac{32891a^{11}x^2}{28} - \frac{30371a^{10}b\,x^4}{7} - \frac{27599b^2a^9x^6}{3} - \frac{24519b^3a^8x^8}{2} - 10527a^7b^4x^{10} - 5698a^6b}{b^{12}(b\,x^2+a)^9}$
norman	$\frac{\frac{x^{24}}{6b} - \frac{a\,x^{22}}{b^2} + \frac{11a^2x^{20}}{b^3} - \frac{78419a^{12}}{252b^{13}} - \frac{990a^4x^{16}}{b^5} - \frac{5940a^5x^{14}}{b^6} - \frac{16940a^6x^{12}}{b^7} - \frac{28875a^7x^{10}}{b^8} - \frac{31647a^8x^8}{b^9} - \frac{22638a^9x^6}{b^{10}} - \frac{71874a^{10}x^4}{7b^{11}} - 75}{(b\,x^2+a)^9}$
default	$\frac{\frac{1}{6}b^2x^6 - \frac{5}{2}ab\,x^4 + \frac{55}{2}a^2x^2}{b^{12}} - \frac{a^3\left(\frac{a^9}{9b(b\,x^2+a)^9} + \frac{99a^5}{b(b\,x^2+a)^5} - \frac{198a^4}{b(b\,x^2+a)^4} + \frac{66a^7}{7b(b\,x^2+a)^7} - \frac{3a^8}{2b(b\,x^2+a)^8} - \frac{396a^2}{b(b\,x^2+a)^2} + \frac{495a}{b(b\,x^2+a}\right.}{2b^{12}}$
parallelrisch	$-\frac{78419a^{12} + 27720\ln(b\,x^2+a)x^{18}a^3b^9 + 249480\ln(b\,x^2+a)x^{16}a^4b^8 + 997920\ln(b\,x^2+a)x^{14}a^5b^7 + 2328480\ln(b\,x^2+a)x^{12}a^6}{}$

input
```
int(x^25/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6/b^10-5/2*a*x^4/b^11+55/2*a^2*x^2/b^12+(-35201/252*a^12/b-32891/28*
a^11*x^2-30371/7*a^10*b*x^4-27599/3*b^2*a^9*x^6-24519/2*b^3*a^8*x^8-10527*
a^7*b^4*x^10-5698*a^6*b^5*x^12-1782*a^5*b^6*x^14-495/2*a^4*b^7*x^16)/b^12/
(b*x^2+a)^9-110*a^3*ln(b*x^2+a)/b^13
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.60

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = \frac{42\,b^{12}x^{24} - 252\,ab^{11}x^{22} + 2772\,a^2b^{10}x^{20} + 43218\,a^3b^9x^{18} + 139482\,a^4b^8x^{16} + 58968\,a^5b^7x^{14} - 638568\,a^6b}{}$$

input
```
integrate(x^25/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/252*(42*b^12*x^24 - 252*a*b^11*x^22 + 2772*a^2*b^10*x^20 + 43218*a^3*b^9
*x^18 + 139482*a^4*b^8*x^16 + 58968*a^5*b^7*x^14 - 638568*a^6*b^6*x^12 - 1
831032*a^7*b^5*x^10 - 2529576*a^8*b^4*x^8 - 2074464*a^9*b^3*x^6 - 1031616*
a^10*b^2*x^4 - 289089*a^11*b*x^2 - 35201*a^12 - 27720*(a^3*b^9*x^18 + 9*a^
4*b^8*x^16 + 36*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 126*a^7*b^5*x^10 + 126*a^
8*b^4*x^8 + 84*a^9*b^3*x^6 + 36*a^10*b^2*x^4 + 9*a^11*b*x^2 + a^12)*log(b*
x^2 + a))/(b^22*x^18 + 9*a*b^21*x^16 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12
 + 126*a^4*b^18*x^10 + 126*a^5*b^17*x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^
4 + 9*a^8*b^14*x^2 + a^9*b^13)
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.20

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = -\frac{110a^3\log(a+bx^2)}{b^{13}} + \frac{55a^2x^2}{2b^{12}} - \frac{5ax^4}{2b^{11}} + \frac{-35201a^{12} - 296019a^{11}bx^2 - 1093356a^{10}b^2x^4 - 2318316a^9b^3x^6 - 3089394a^8b^4x^8 - 2652804a^7b^5x^{10} - \cdots}{252a^9b^{13} + 2268a^8b^{14}x^2 + 9072a^7b^{15}x^4 + 21168a^6b^{16}x^6 + 31752a^5b^{17}x^8 + 31752a^4b^{18}x^{10} + 21168a^3b\cdots} + \frac{x^6}{6b^{10}}$$

input
```
integrate(x**25/(b*x**2+a)**10,x)
```

output
```
-110*a**3*log(a + b*x**2)/b**13 + 55*a**2*x**2/(2*b**12) - 5*a*x**4/(2*b**
11) + (-35201*a**12 - 296019*a**11*b*x**2 - 1093356*a**10*b**2*x**4 - 2318
316*a**9*b**3*x**6 - 3089394*a**8*b**4*x**8 - 2652804*a**7*b**5*x**10 - 14
35896*a**6*b**6*x**12 - 449064*a**5*b**7*x**14 - 62370*a**4*b**8*x**16)/(2
52*a**9*b**13 + 2268*a**8*b**14*x**2 + 9072*a**7*b**15*x**4 + 21168*a**6*b
**16*x**6 + 31752*a**5*b**17*x**8 + 31752*a**4*b**18*x**10 + 21168*a**3*b*
*19*x**12 + 9072*a**2*b**20*x**14 + 2268*a*b**21*x**16 + 252*b**22*x**18)
+ x**6/(6*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.12

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = -\frac{62370\,a^4b^8x^{16} + 449064\,a^5b^7x^{14} + 1435896\,a^6b^6x^{12} + 2652804\,a^7b^5x^{10} + 3089394\,a^8b^4x^8 + 2318316\,a^9b^3x^6 + \ldots}{252\,(b^{22}x^{18} + 9\,ab^{21}x^{16} + 36\,a^2b^{20}x^{14} + 84\,a^3b^{19}x^{12} + 126\,a^4b^{18}x^{10} + 126\,a^5b^{17}x^8 + 84\,a^6b^{16}x^6 + \ldots)} - \frac{110\,a^3\log(bx^2+a)}{b^{13}} + \frac{b^2x^6 - 15\,abx^4 + 165\,a^2x^2}{6\,b^{12}}$$

input
```
integrate(x^25/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(62370*a^4*b^8*x^16 + 449064*a^5*b^7*x^14 + 1435896*a^6*b^6*x^12 +
2652804*a^7*b^5*x^10 + 3089394*a^8*b^4*x^8 + 2318316*a^9*b^3*x^6 + 1093356
*a^10*b^2*x^4 + 296019*a^11*b*x^2 + 35201*a^12)/(b^22*x^18 + 9*a*b^21*x^16
 + 36*a^2*b^20*x^14 + 84*a^3*b^19*x^12 + 126*a^4*b^18*x^10 + 126*a^5*b^17*
x^8 + 84*a^6*b^16*x^6 + 36*a^7*b^15*x^4 + 9*a^8*b^14*x^2 + a^9*b^13) - 110
*a^3*log(b*x^2 + a)/b^13 + 1/6*(b^2*x^6 - 15*a*b*x^4 + 165*a^2*x^2)/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.78

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = -\frac{110\,a^3\log(|bx^2+a|)}{b^{13}} + \frac{78419\,a^3b^9x^{18} + 643401\,a^4b^8x^{16} + 2374020\,a^5b^7x^{14} + 5151300\,a^6b^6x^{12} + 7227990\,a^7b^5x^{10} + 6791400\,a^8\ldots}{252\,(bx^2+a)^9b^{13}} + \frac{b^{20}x^6 - 15\,ab^{19}x^4 + 165\,a^2b^{18}x^2}{6\,b^{30}}$$

input
```
integrate(x^25/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-110*a^3*log(abs(b*x^2 + a))/b^13 + 1/252*(78419*a^3*b^9*x^18 + 643401*a^4
*b^8*x^16 + 2374020*a^5*b^7*x^14 + 5151300*a^6*b^6*x^12 + 7227990*a^7*b^5*
x^10 + 6791400*a^8*b^4*x^8 + 4268880*a^9*b^3*x^6 + 1729728*a^10*b^2*x^4 +
409752*a^11*b*x^2 + 43218*a^12)/((b*x^2 + a)^9*b^13) + 1/6*(b^20*x^6 - 15*
a*b^19*x^4 + 165*a^2*b^18*x^2)/b^30
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.12

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = \frac{x^6}{6\,b^{10}} - \frac{\frac{35201\,a^{12}}{252\,b}+\frac{32891\,a^{11}\,x^2}{28}+\frac{30371\,a^{10}\,b\,x^4}{7}+\frac{27599\,a^9\,b^2\,x^6}{3}+\frac{24519\,a^8\,b^3\,x^8}{2}+10527\,a^7\,b^4\,x^{10}+5698\,a^6\,b^5\,x^{12}+17}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+36\,a^2\,b^{1}} - \frac{5\,a\,x^4}{2\,b^{11}} - \frac{110\,a^3\,\ln\left(b\,x^2+a\right)}{b^{13}} + \frac{55\,a^2\,x^2}{2\,b^{12}}$$

input
```
int(x^25/(a + b*x^2)^10,x)
```

output
```
x^6/(6*b^10) - ((35201*a^12)/(252*b) + (32891*a^11*x^2)/28 + (30371*a^10*b
*x^4)/7 + (27599*a^9*b^2*x^6)/3 + (24519*a^8*b^3*x^8)/2 + 10527*a^7*b^4*x^
10 + 5698*a^6*b^5*x^12 + 1782*a^5*b^6*x^14 + (495*a^4*b^7*x^16)/2)/(a^9*b^
12 + b^21*x^18 + 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6
*b^15*x^6 + 126*a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a
^2*b^19*x^14) - (5*a*x^4)/(2*b^11) - (110*a^3*log(a + b*x^2))/b^13 + (55*a
^2*x^2)/(2*b^12)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.87

$$\int \frac{x^{25}}{(a+bx^2)^{10}}\,dx = \frac{-27720\log(b\,x^2+a)\,a^{12}-249480\log(b\,x^2+a)\,a^{11}b\,x^2-997920\log(b\,x^2+a)\,a^{10}b^2x^4-2328480\log(b\,x}{}$$

input
```
int(x^25/(b*x^2+a)^10,x)
```

output
```
( - 27720*log(a + b*x**2)*a**12 - 249480*log(a + b*x**2)*a**11*b*x**2 - 99
7920*log(a + b*x**2)*a**10*b**2*x**4 - 2328480*log(a + b*x**2)*a**9*b**3*x
**6 - 3492720*log(a + b*x**2)*a**8*b**4*x**8 - 3492720*log(a + b*x**2)*a**
7*b**5*x**10 - 2328480*log(a + b*x**2)*a**6*b**6*x**12 - 997920*log(a + b*
x**2)*a**5*b**7*x**14 - 249480*log(a + b*x**2)*a**4*b**8*x**16 - 27720*log
(a + b*x**2)*a**3*b**9*x**18 - 50699*a**12 - 428571*a**11*b*x**2 - 1589544
*a**10*b**2*x**4 - 3376296*a**9*b**3*x**6 - 4482324*a**8*b**4*x**8 - 37837
80*a**7*b**5*x**10 - 1940400*a**6*b**6*x**12 - 498960*a**5*b**7*x**14 + 27
720*a**3*b**9*x**18 + 2772*a**2*b**10*x**20 - 252*a*b**11*x**22 + 42*b**12
*x**24)/(252*b**13*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**
3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 3
6*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.193 $\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 205

$$\begin{aligned}\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = &-\frac{5ax^2}{b^{11}}+\frac{x^4}{4b^{10}}+\frac{a^{11}}{18b^{12}(a+bx^2)^9}-\frac{11a^{10}}{16b^{12}(a+bx^2)^8}\\&+\frac{55a^9}{14b^{12}(a+bx^2)^7}-\frac{55a^8}{4b^{12}(a+bx^2)^6}+\frac{33a^7}{b^{12}(a+bx^2)^5}\\&-\frac{231a^6}{4b^{12}(a+bx^2)^4}+\frac{77a^5}{b^{12}(a+bx^2)^3}-\frac{165a^4}{2b^{12}(a+bx^2)^2}\\&+\frac{165a^3}{2b^{12}(a+bx^2)}+\frac{55a^2\log(a+bx^2)}{2b^{12}}\end{aligned}$$

output
```
-5*a*x^2/b^11+1/4*x^4/b^10+1/18*a^11/b^12/(b*x^2+a)^9-11/16*a^10/b^12/(b*x
^2+a)^8+55/14*a^9/b^12/(b*x^2+a)^7-55/4*a^8/b^12/(b*x^2+a)^6+33*a^7/b^12/(
b*x^2+a)^5-231/4*a^6/b^12/(b*x^2+a)^4+77*a^5/b^12/(b*x^2+a)^3-165/2*a^4/b^
12/(b*x^2+a)^2+165/2*a^3/b^12/(b*x^2+a)+55/2*a^2*ln(b*x^2+a)/b^12
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = \frac{42131a^{11}+351459a^{10}bx^2+1281096a^9b^2x^4+2656584a^8b^3x^6+3402756a^7b^4x^8+2704212a^6b^5x^{10}+122\ldots}{}$$

input
```
Integrate[x^23/(a + b*x^2)^10,x]
```

output
```
(42131*a^11 + 351459*a^10*b*x^2 + 1281096*a^9*b^2*x^4 + 2656584*a^8*b^3*x^
6 + 3402756*a^7*b^4*x^8 + 2704212*a^6*b^5*x^10 + 1220688*a^5*b^6*x^12 + 19
0512*a^4*b^7*x^14 - 77112*a^3*b^8*x^16 - 36288*a^2*b^9*x^18 - 2772*a*b^10*
x^20 + 252*b^11*x^22 + 27720*a^2*(a + b*x^2)^9*Log[a + b*x^2])/(1008*b^12*
(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{22}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{a^{11}}{b^{11}(bx^2+a)^{10}}+\frac{11a^{10}}{b^{11}(bx^2+a)^9}-\frac{55a^9}{b^{11}(bx^2+a)^8}+\frac{165a^8}{b^{11}(bx^2+a)^7}-\frac{330a^7}{b^{11}(bx^2+a)^6}+\frac{462a^6}{b^{11}(bx^2+a)^5}-\ldots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^{11}}{9b^{12}(a+bx^2)^9}-\frac{11a^{10}}{8b^{12}(a+bx^2)^8}+\frac{55a^9}{7b^{12}(a+bx^2)^7}-\frac{55a^8}{2b^{12}(a+bx^2)^6}+\frac{66a^7}{b^{12}(a+bx^2)^5}-\frac{231a^6}{2b^{12}(a+bx^2)^4}+\right.$$

input
```
Int[x^23/(a + b*x^2)^10,x]
```

output
```
((-10*a*x^2)/b^11 + x^4/(2*b^10) + a^11/(9*b^12*(a + b*x^2)^9) - (11*a^10)
/(8*b^12*(a + b*x^2)^8) + (55*a^9)/(7*b^12*(a + b*x^2)^7) - (55*a^8)/(2*b^
12*(a + b*x^2)^6) + (66*a^7)/(b^12*(a + b*x^2)^5) - (231*a^6)/(2*b^12*(a +
 b*x^2)^4) + (154*a^5)/(b^12*(a + b*x^2)^3) - (165*a^4)/(b^12*(a + b*x^2)^
2) + (165*a^3)/(b^12*(a + b*x^2)) + (55*a^2*Log[a + b*x^2])/b^12)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.69

method	result
norman	$\frac{\frac{x^{22}}{4b}-\frac{11a\,x^{20}}{4b^{2}}+\frac{78419a^{11}}{1008b^{12}}+\frac{495a^{3}x^{16}}{2b^{4}}+\frac{1485a^{4}x^{14}}{b^{5}}+\frac{4235a^{5}x^{12}}{b^{6}}+\frac{28875a^{6}x^{10}}{4b^{7}}+\frac{31647a^{7}x^{8}}{4b^{8}}+\frac{11319a^{8}x^{6}}{2b^{9}}+\frac{35937a^{9}x^{4}}{14b^{10}}+\frac{75339a^{10}x^{2}}{112b^{11}}}{\left(b\,x^{2}+a\right)^{9}}$
risch	$\frac{x^{4}}{4b^{10}}-\frac{5a\,x^{2}}{b^{11}}+\frac{25a^{2}}{b^{12}}+\frac{\frac{42131a^{11}}{1008b}+\frac{39611a^{10}x^{2}}{112}+\frac{36839a^{9}b\,x^{4}}{28}+\frac{11253a^{8}b^{2}x^{6}}{4}+\frac{15147a^{7}b^{3}x^{8}}{4}+\frac{13167a^{6}b^{4}x^{10}}{4}+\frac{3619a^{5}b^{5}x^{12}}{2}+}{b^{11}\left(b\,x^{2}+a\right)^{9}}$
default	$\frac{\left(-b\,x^{2}+10a\right)^{2}}{4b^{12}}+\frac{a^{2}\left(\frac{a^{9}}{9b\left(b\,x^{2}+a\right)^{9}}+\frac{66a^{5}}{b\left(b\,x^{2}+a\right)^{5}}-\frac{231a^{4}}{2b\left(b\,x^{2}+a\right)^{4}}+\frac{55a^{7}}{7b\left(b\,x^{2}+a\right)^{7}}-\frac{11a^{8}}{8b\left(b\,x^{2}+a\right)^{8}}-\frac{165a^{2}}{b\left(b\,x^{2}+a\right)^{2}}+\frac{165a}{b\left(b\,x^{2}+a\right)}+55\ln\left(\right.\right.}{2b^{11}}$
parallelrisch	$78419a^{11}+678051a^{10}b\,x^{2}+27720\ln\left(b\,x^{2}+a\right)x^{18}a^{2}b^{9}+249480\ln\left(b\,x^{2}+a\right)x^{16}a^{3}b^{8}+997920\ln\left(b\,x^{2}+a\right)x^{14}a^{4}b^{7}+2328480\ln\left(\right.$

input
```
int(x^23/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(1/4/b*x^22-11/4*a/b^2*x^20+78419/1008*a^11/b^12+495/2*a^3/b^4*x^16+1485*a
^4/b^5*x^14+4235*a^5/b^6*x^12+28875/4*a^6/b^7*x^10+31647/4*a^7/b^8*x^8+113
19/2*a^8/b^9*x^6+35937/14*a^9/b^10*x^4+75339/112*a^10/b^11*x^2)/(b*x^2+a)^
9+55/2*a^2*ln(b*x^2+a)/b^12
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.63

$$\int \frac{x^{23}}{\left(a+bx^2\right)^{10}}\,dx$$
$$= \frac{252\,b^{11}x^{22}-2772\,ab^{10}x^{20}-36288\,a^{2}b^{9}x^{18}-77112\,a^{3}b^{8}x^{16}+190512\,a^{4}b^{7}x^{14}+1220688\,a^{5}b^{6}x^{12}+27042}{}$$

input
```
integrate(x^23/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/1008*(252*b^11*x^22 - 2772*a*b^10*x^20 - 36288*a^2*b^9*x^18 - 77112*a^3*
b^8*x^16 + 190512*a^4*b^7*x^14 + 1220688*a^5*b^6*x^12 + 2704212*a^6*b^5*x^
10 + 3402756*a^7*b^4*x^8 + 2656584*a^8*b^3*x^6 + 1281096*a^9*b^2*x^4 + 351
459*a^10*b*x^2 + 42131*a^11 + 27720*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^
4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8
*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*log(b*x^2 + a))/(b^21*x^1
8 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^1
0 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2
+ a^9*b^12)
```

Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = \frac{55a^2\log{(a+bx^2)}}{2b^{12}} - \frac{5ax^2}{b^{11}} + \frac{42131a^{11} + 356499a^{10}bx^2 + 1326204a^9b^2x^4 + 2835756a^8b^3x^6 + 3817044a^7b^4x^8 + 3318084a^6b^5x^{10} + \cdots}{1008a^9b^{12} + 9072a^8b^{13}x^2 + 36288a^7b^{14}x^4 + 84672a^6b^{15}x^6 + 127008a^5b^{16}x^8 + 127008a^4b^{17}x^{10} + 84672\cdots} + \frac{x^4}{4b^{10}}$$

input
```
integrate(x**23/(b*x**2+a)**10,x)
```

output
```
55*a**2*log(a + b*x**2)/(2*b**12) - 5*a*x**2/b**11 + (42131*a**11 + 356499
*a**10*b*x**2 + 1326204*a**9*b**2*x**4 + 2835756*a**8*b**3*x**6 + 3817044*
a**7*b**4*x**8 + 3318084*a**6*b**5*x**10 + 1823976*a**5*b**6*x**12 + 58212
0*a**4*b**7*x**14 + 83160*a**3*b**8*x**16)/(1008*a**9*b**12 + 9072*a**8*b*
*13*x**2 + 36288*a**7*b**14*x**4 + 84672*a**6*b**15*x**6 + 127008*a**5*b**
16*x**8 + 127008*a**4*b**17*x**10 + 84672*a**3*b**18*x**12 + 36288*a**2*b*
*19*x**14 + 9072*a*b**20*x**16 + 1008*b**21*x**18) + x**4/(4*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.13

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = \frac{83160\,a^3b^8x^{16} + 582120\,a^4b^7x^{14} + 1823976\,a^5b^6x^{12} + 3318084\,a^6b^5x^{10} + 3817044\,a^7b^4x^8 + 2835756\,a^8b^3x}{1008\,(b^{21}x^{18} + 9\,ab^{20}x^{16} + 36\,a^2b^{19}x^{14} + 84\,a^3b^{18}x^{12} + 126\,a^4b^{17}x^{10} + 126\,a^5b^{16}x^8 + 84\,a^6b^{15}} + \frac{55\,a^2\log(bx^2+a)}{2\,b^{12}} + \frac{bx^4 - 20\,ax^2}{4\,b^{11}}$$

input
```
integrate(x^23/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/1008*(83160*a^3*b^8*x^16 + 582120*a^4*b^7*x^14 + 1823976*a^5*b^6*x^12 +
3318084*a^6*b^5*x^10 + 3817044*a^7*b^4*x^8 + 2835756*a^8*b^3*x^6 + 1326204
*a^9*b^2*x^4 + 356499*a^10*b*x^2 + 42131*a^11)/(b^21*x^18 + 9*a*b^20*x^16
+ 36*a^2*b^19*x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x
^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12) + 55/2
*a^2*log(b*x^2 + a)/b^12 + 1/4*(b*x^4 - 20*a*x^2)/b^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.77

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx = \frac{55\,a^2\log(|bx^2+a|)}{2\,b^{12}} + \frac{b^{10}x^4 - 20\,ab^9x^2}{4\,b^{20}} - \frac{78419\,a^2b^9x^{18} + 622611\,a^3b^8x^{16} + 2240964\,a^4b^7x^{14} + 4763220\,a^5b^6x^{12} + 6562710\,a^6b^5x^{10} + 6063750\,a^7}{1008\,(bx^2+a)^9b^{12}}$$

input
```
integrate(x^23/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
55/2*a^2*log(abs(b*x^2 + a))/b^12 + 1/4*(b^10*x^4 - 20*a*b^9*x^2)/b^20 - 1
/1008*(78419*a^2*b^9*x^18 + 622611*a^3*b^8*x^16 + 2240964*a^4*b^7*x^14 + 4
763220*a^5*b^6*x^12 + 6562710*a^6*b^5*x^10 + 6063750*a^7*b^4*x^8 + 3751440
*a^8*b^3*x^6 + 1496880*a^9*b^2*x^4 + 349272*a^10*b*x^2 + 36288*a^11)/((b*x
^2 + a)^9*b^12)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.12

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{\frac{42131\,a^{11}}{1008\,b} + \frac{39611\,a^{10}\,x^2}{112} + \frac{36839\,a^9\,b\,x^4}{28} + \frac{11253\,a^8\,b^2\,x^6}{4} + \frac{15147\,a^7\,b^3\,x^8}{4} + \frac{13167\,a^6\,b^4\,x^{10}}{4} + \frac{3619\,a^5\,b^5\,x^{12}}{2} + \frac{1155\,a^4}{2}}{a^9\,b^{11} + 9\,a^8\,b^{12}\,x^2 + 36\,a^7\,b^{13}\,x^4 + 84\,a^6\,b^{14}\,x^6 + 126\,a^5\,b^{15}\,x^8 + 126\,a^4\,b^{16}\,x^{10} + 84\,a^3\,b^{17}\,x^{12} + 36\,a^2\,b^{18}\,x}$$
$$+ \frac{x^4}{4\,b^{10}} - \frac{5\,a\,x^2}{b^{11}} + \frac{55\,a^2\,\ln\left(b\,x^2+a\right)}{2\,b^{12}}$$

input
```
int(x^23/(a + b*x^2)^10,x)
```

output
```
((42131*a^11)/(1008*b) + (39611*a^10*x^2)/112 + (36839*a^9*b*x^4)/28 + (11
253*a^8*b^2*x^6)/4 + (15147*a^7*b^3*x^8)/4 + (13167*a^6*b^4*x^10)/4 + (361
9*a^5*b^5*x^12)/2 + (1155*a^4*b^6*x^14)/2 + (165*a^3*b^7*x^16)/2)/(a^9*b^1
1 + b^20*x^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*
b^14*x^6 + 126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^
2*b^18*x^14) + x^4/(4*b^10) - (5*a*x^2)/b^11 + (55*a^2*log(a + b*x^2))/(2*
b^12)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.92

$$\int \frac{x^{23}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{27720\log(b\,x^2+a)\,a^{11} + 249480\log(b\,x^2+a)\,a^{10}b\,x^2 + 997920\log(b\,x^2+a)\,a^9b^2x^4 + 2328480\log(b\,x^2+}{}$$

input
```
int(x^23/(b*x^2+a)^10,x)
```

output
```
(27720*log(a + b*x**2)*a**11 + 249480*log(a + b*x**2)*a**10*b*x**2 + 99792
0*log(a + b*x**2)*a**9*b**2*x**4 + 2328480*log(a + b*x**2)*a**8*b**3*x**6
+ 3492720*log(a + b*x**2)*a**7*b**4*x**8 + 3492720*log(a + b*x**2)*a**6*b*
*5*x**10 + 2328480*log(a + b*x**2)*a**5*b**6*x**12 + 997920*log(a + b*x**2
)*a**4*b**7*x**14 + 249480*log(a + b*x**2)*a**3*b**8*x**16 + 27720*log(a +
 b*x**2)*a**2*b**9*x**18 + 50699*a**11 + 428571*a**10*b*x**2 + 1589544*a**
9*b**2*x**4 + 3376296*a**8*b**3*x**6 + 4482324*a**7*b**4*x**8 + 3783780*a*
*6*b**5*x**10 + 1940400*a**5*b**6*x**12 + 498960*a**4*b**7*x**14 - 27720*a
**2*b**9*x**18 - 2772*a*b**10*x**20 + 252*b**11*x**22)/(1008*b**12*(a**9 +
 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**
8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b*
*8*x**16 + b**9*x**18))
```

3.194 $\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 188

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{x^2}{2b^{10}} - \frac{a^{10}}{18b^{11}(a+bx^2)^9} + \frac{5a^9}{8b^{11}(a+bx^2)^8} - \frac{45a^8}{14b^{11}(a+bx^2)^7} + \frac{10a^7}{b^{11}(a+bx^2)^6} - \frac{21a^6}{b^{11}(a+bx^2)^5} + \frac{63a^5}{2b^{11}(a+bx^2)^4} - \frac{35a^4}{b^{11}(a+bx^2)^3} + \frac{30a^3}{b^{11}(a+bx^2)^2} - \frac{45a^2}{2b^{11}(a+bx^2)} - \frac{5a\log(a+bx^2)}{b^{11}}$$

output
```
1/2*x^2/b^10-1/18*a^10/b^11/(b*x^2+a)^9+5/8*a^9/b^11/(b*x^2+a)^8-45/14*a^8
/b^11/(b*x^2+a)^7+10*a^7/b^11/(b*x^2+a)^6-21*a^6/b^11/(b*x^2+a)^5+63/2*a^5
/b^11/(b*x^2+a)^4-35*a^4/b^11/(b*x^2+a)^3+30*a^3/b^11/(b*x^2+a)^2-45/2*a^2
/b^11/(b*x^2+a)-5*a*ln(b*x^2+a)/b^11
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = -\frac{4861a^{10}+41229a^9bx^2+153576a^8b^2x^4+328104a^7b^3x^6+439236a^6b^4x^8+375732a^5b^5x^{10}+197568a^4 \cdots}{504b^{11}(a+ \cdots}$$

input
```
Integrate[x^21/(a + b*x^2)^10,x]
```

output
```
-1/504*(4861*a^10 + 41229*a^9*b*x^2 + 153576*a^8*b^2*x^4 + 328104*a^7*b^3*
x^6 + 439236*a^6*b^4*x^8 + 375732*a^5*b^5*x^10 + 197568*a^4*b^6*x^12 + 544
32*a^3*b^7*x^14 + 2268*a^2*b^8*x^16 - 2268*a*b^9*x^18 - 252*b^10*x^20 + 25
20*a*(a + b*x^2)^9*Log[a + b*x^2])/(b^11*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{20}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{a^{10}}{b^{10}(bx^2+a)^{10}}-\frac{10a^9}{b^{10}(bx^2+a)^9}+\frac{45a^8}{b^{10}(bx^2+a)^8}-\frac{120a^7}{b^{10}(bx^2+a)^7}+\frac{210a^6}{b^{10}(bx^2+a)^6}-\frac{252a^5}{b^{10}(bx^2+a)^5}+\cdots\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^{10}}{9b^{11}(a+bx^2)^9}+\frac{5a^9}{4b^{11}(a+bx^2)^8}-\frac{45a^8}{7b^{11}(a+bx^2)^7}+\frac{20a^7}{b^{11}(a+bx^2)^6}-\frac{42a^6}{b^{11}(a+bx^2)^5}+\frac{63a^5}{b^{11}(a+bx^2)^4}-\right.$$

input
```
Int[x^21/(a + b*x^2)^10,x]
```

output
```
(x^2/b^10 - a^10/(9*b^11*(a + b*x^2)^9) + (5*a^9)/(4*b^11*(a + b*x^2)^8) -
 (45*a^8)/(7*b^11*(a + b*x^2)^7) + (20*a^7)/(b^11*(a + b*x^2)^6) - (42*a^6
)/(b^11*(a + b*x^2)^5) + (63*a^5)/(b^11*(a + b*x^2)^4) - (70*a^4)/(b^11*(a
 + b*x^2)^3) + (60*a^3)/(b^11*(a + b*x^2)^2) - (45*a^2)/(b^11*(a + b*x^2))
 - (10*a*Log[a + b*x^2])/b^11)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.69

method	result
risch	$\frac{x^2}{2b^{10}} + \frac{-\frac{4861a^{10}}{504b} - \frac{4609a^9x^2}{56} - \frac{4329a^8b\,x^4}{14} - 669a^7b^2x^6 - \frac{1827a^6b^3x^8}{2} - \frac{1617a^5b^4x^{10}}{2} - 455a^4b^5x^{12} - 150a^3b^6x^{14} - \frac{45a^2b^7x^{16}}{2}}{b^{10}(b\,x^2+a)^9}$ −
norman	$\frac{\frac{x^{20}}{2b} - \frac{7129a^{10}}{504b^{11}} - \frac{45a^2x^{16}}{b^3} - \frac{270a^3x^{14}}{b^4} - \frac{770a^4x^{12}}{b^5} - \frac{2625a^5x^{10}}{2b^6} - \frac{2877a^6x^8}{2b^7} - \frac{1029a^7x^6}{b^8} - \frac{3267a^8x^4}{7b^9} - \frac{6849a^9x^2}{56b^{10}}}{(b\,x^2+a)^9} - \frac{5a\ln(b\,x^2+a)}{b^{11}}$
default	$\frac{x^2}{2b^{10}} - \frac{a\left(\frac{a^9}{9b(b\,x^2+a)^9} + \frac{42a^5}{b(b\,x^2+a)^5} - \frac{63a^4}{b(b\,x^2+a)^4} + \frac{45a^7}{7b(b\,x^2+a)^7} - \frac{5a^8}{4b(b\,x^2+a)^8} - \frac{60a^2}{b(b\,x^2+a)^2} + \frac{45a}{b(b\,x^2+a)} + \frac{10\ln(b\,x^2+a)}{b} + \frac{}{b(}\right.}{2b^{10}}$
parallelrisch	$-\frac{-252b^{10}x^{20}+61641a^9b\,x^2+725004a^6b^4x^8+661500a^5b^5x^{10}+388080a^4b^6x^{12}+136080a^3b^7x^{14}+22680a^2b^8x^{16}+7129a^{10}+5}{}$

input
```
int(x^21/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2/b^10+(-4861/504*a^10/b-4609/56*a^9*x^2-4329/14*a^8*b*x^4-669*a^7*b
^2*x^6-1827/2*a^6*b^3*x^8-1617/2*a^5*b^4*x^10-455*a^4*b^5*x^12-150*a^3*b^6
*x^14-45/2*a^2*b^7*x^16)/b^10/(b*x^2+a)^9-5*a*ln(b*x^2+a)/b^11
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.71

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{252\,b^{10}x^{20} + 2268\,ab^9x^{18} - 2268\,a^2b^8x^{16} - 54432\,a^3b^7x^{14} - 197568\,a^4b^6x^{12} - 375732\,a^5b^5x^{10} - 439236\,a^6}{504\,(b^{20}x^{18} + 9}$$

input
```
integrate(x^21/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/504*(252*b^10*x^20 + 2268*a*b^9*x^18 - 2268*a^2*b^8*x^16 - 54432*a^3*b^7
*x^14 - 197568*a^4*b^6*x^12 - 375732*a^5*b^5*x^10 - 439236*a^6*b^4*x^8 - 3
28104*a^7*b^3*x^6 - 153576*a^8*b^2*x^4 - 41229*a^9*b*x^2 - 4861*a^10 - 252
0*(a*b^9*x^18 + 9*a^2*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a
^5*b^5*x^10 + 126*a^6*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*
x^2 + a^10)*log(b*x^2 + a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14
+ 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^
6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.24

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = -\frac{5a\log(a+bx^2)}{b^{11}} + \frac{-4861a^{10} - 41481a^9bx^2 - 155844a^8b^2x^4 - 337176a^7b^3x^6 - 460404a^6b^4x^8 - 407484a^5b^5x^{10} - 229\cdots}{504a^9b^{11} + 4536a^8b^{12}x^2 + 18144a^7b^{13}x^4 + 42336a^6b^{14}x^6 + 63504a^5b^{15}x^8 + 63504a^4b^{16}x^{10} + 42336a^3b^{1\cdots}} + \frac{x^2}{2b^{10}}$$

input
```
integrate(x**21/(b*x**2+a)**10,x)
```

output
```
-5*a*log(a + b*x**2)/b**11 + (-4861*a**10 - 41481*a**9*b*x**2 - 155844*a**
8*b**2*x**4 - 337176*a**7*b**3*x**6 - 460404*a**6*b**4*x**8 - 407484*a**5*
b**5*x**10 - 229320*a**4*b**6*x**12 - 75600*a**3*b**7*x**14 - 11340*a**2*b
**8*x**16)/(504*a**9*b**11 + 4536*a**8*b**12*x**2 + 18144*a**7*b**13*x**4
+ 42336*a**6*b**14*x**6 + 63504*a**5*b**15*x**8 + 63504*a**4*b**16*x**10 +
 42336*a**3*b**17*x**12 + 18144*a**2*b**18*x**14 + 4536*a*b**19*x**16 + 50
4*b**20*x**18) + x**2/(2*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = -\frac{11340\,a^2b^8x^{16} + 75600\,a^3b^7x^{14} + 229320\,a^4b^6x^{12} + 407484\,a^5b^5x^{10} + 460404\,a^6b^4x^8 + 337176\,a^7b^3x^6 + \cdots}{504\,(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^2b^{18}x^{14} + 84\,a^3b^{17}x^{12} + 126\,a^4b^{16}x^{10} + 126\,a^5b^{15}x^8 + 84\,a^6b^{14}x^6 + \cdots} + \frac{x^2}{2\,b^{10}} - \frac{5\,a\log(bx^2+a)}{b^{11}}$$

input
```
integrate(x^21/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(11340*a^2*b^8*x^16 + 75600*a^3*b^7*x^14 + 229320*a^4*b^6*x^12 + 40
7484*a^5*b^5*x^10 + 460404*a^6*b^4*x^8 + 337176*a^7*b^3*x^6 + 155844*a^8*b
^2*x^4 + 41481*a^9*b*x^2 + 4861*a^10)/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*
b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a
^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11) + 1/2*x^2/b^10
- 5*a*log(b*x^2 + a)/b^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.74

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{x^2}{2\,b^{10}} - \frac{5\,a\log(|bx^2+a|)}{b^{11}} + \frac{7129\,ab^9x^{18} + 52821\,a^2b^8x^{16} + 181044\,a^3b^7x^{14} + 369516\,a^4b^6x^{12} + 490770\,a^5b^5x^{10} + 437850\,a^6b^4x^8 + \cdots}{504\,(bx^2+a)^9b^{11}}$$

input
```
integrate(x^21/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
1/2*x^2/b^10 - 5*a*log(abs(b*x^2 + a))/b^11 + 1/504*(7129*a*b^9*x^18 + 528
21*a^2*b^8*x^16 + 181044*a^3*b^7*x^14 + 369516*a^4*b^6*x^12 + 490770*a^5*b
^5*x^10 + 437850*a^6*b^4*x^8 + 261660*a^7*b^3*x^6 + 100800*a^8*b^2*x^4 + 2
2680*a^9*b*x^2 + 2268*a^10)/((b*x^2 + a)^9*b^11)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{x^2}{2\,b^{10}} - \frac{\frac{4861\,a^{10}}{504\,b}+\frac{4609\,a^9\,x^2}{56}+\frac{4329\,a^8\,b\,x^4}{14}+669\,a^7\,b^2\,x^6+\frac{1827\,a^6\,b^3\,x^8}{2}+\frac{1617\,a^5\,b^4\,x^{10}}{2}+455\,a^4\,b^5\,x^{12}+150\,a^3}{a^9\,b^{10}+9\,a^8\,b^{11}\,x^2+36\,a^7\,b^{12}\,x^4+84\,a^6\,b^{13}\,x^6+126\,a^5\,b^{14}\,x^8+126\,a^4\,b^{15}\,x^{10}+84\,a^3\,b^{16}\,x^{12}+36\,a^2\,b^{1}} - \frac{5\,a\,\ln\left(b\,x^2+a\right)}{b^{11}}$$

input
```
int(x^21/(a + b*x^2)^10,x)
```

output
```
x^2/(2*b^10) - ((4861*a^10)/(504*b) + (4609*a^9*x^2)/56 + (4329*a^8*b*x^4)
/14 + 669*a^7*b^2*x^6 + (1827*a^6*b^3*x^8)/2 + (1617*a^5*b^4*x^10)/2 + 455
*a^4*b^5*x^12 + 150*a^3*b^6*x^14 + (45*a^2*b^7*x^16)/2)/(a^9*b^10 + b^19*x
^18 + 9*a*b^18*x^16 + 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 +
 126*a^5*b^14*x^8 + 126*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^1
4) - (5*a*log(a + b*x^2))/b^11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.02

$$\int \frac{x^{21}}{(a+bx^2)^{10}}\,dx = \frac{-2520\log(b\,x^2+a)\,a^{10}-22680\log(b\,x^2+a)\,a^9b\,x^2-90720\log(b\,x^2+a)\,a^8b^2x^4-211680\log(b\,x^2+a)}{}$$

input
```
int(x^21/(b*x^2+a)^10,x)
```

output
```
( - 2520*log(a + b*x**2)*a**10 - 22680*log(a + b*x**2)*a**9*b*x**2 - 90720
*log(a + b*x**2)*a**8*b**2*x**4 - 211680*log(a + b*x**2)*a**7*b**3*x**6 -
317520*log(a + b*x**2)*a**6*b**4*x**8 - 317520*log(a + b*x**2)*a**5*b**5*x
**10 - 211680*log(a + b*x**2)*a**4*b**6*x**12 - 90720*log(a + b*x**2)*a**3
*b**7*x**14 - 22680*log(a + b*x**2)*a**2*b**8*x**16 - 2520*log(a + b*x**2)
*a*b**9*x**18 - 4609*a**10 - 38961*a**9*b*x**2 - 144504*a**8*b**2*x**4 - 3
06936*a**7*b**3*x**6 - 407484*a**6*b**4*x**8 - 343980*a**5*b**5*x**10 - 17
6400*a**4*b**6*x**12 - 45360*a**3*b**7*x**14 + 2520*a*b**9*x**18 + 252*b**
10*x**20)/(504*b**11*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b
**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 +
 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.195 $\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 179

$$\begin{aligned}\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = {} & \frac{a^9}{18b^{10}(a+bx^2)^9} - \frac{9a^8}{16b^{10}(a+bx^2)^8} + \frac{18a^7}{7b^{10}(a+bx^2)^7} \\ & - \frac{7a^6}{b^{10}(a+bx^2)^6} + \frac{63a^5}{5b^{10}(a+bx^2)^5} - \frac{63a^4}{4b^{10}(a+bx^2)^4} \\ & + \frac{14a^3}{b^{10}(a+bx^2)^3} - \frac{9a^2}{b^{10}(a+bx^2)^2} + \frac{9a}{2b^{10}(a+bx^2)} + \frac{\log(a+bx^2)}{2b^{10}}\end{aligned}$$

output
```
1/18*a^9/b^10/(b*x^2+a)^9-9/16*a^8/b^10/(b*x^2+a)^8+18/7*a^7/b^10/(b*x^2+a
)^7-7*a^6/b^10/(b*x^2+a)^6+63/5*a^5/b^10/(b*x^2+a)^5-63/4*a^4/b^10/(b*x^2+
a)^4+14*a^3/b^10/(b*x^2+a)^3-9*a^2/b^10/(b*x^2+a)^2+9/2*a/b^10/(b*x^2+a)+1
/2*ln(b*x^2+a)/b^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.65

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{\frac{a(7129a^8+61641a^7bx^2+235224a^6b^2x^4+518616a^5b^3x^6+725004a^4b^4x^8+661500a^3b^5x^{10}+388080a^2b^6x^{12}+136080ab^7x^{14}+22680b^8x^{16})}{(a+bx^2)^9}+25}{5040b^{10}}$$

input
```
Integrate[x^19/(a + b*x^2)^10,x]
```

output
```
((a*(7129*a^8 + 61641*a^7*b*x^2 + 235224*a^6*b^2*x^4 + 518616*a^5*b^3*x^6
+ 725004*a^4*b^4*x^8 + 661500*a^3*b^5*x^10 + 388080*a^2*b^6*x^12 + 136080*
a*b^7*x^14 + 22680*b^8*x^16))/(a + b*x^2)^9 + 2520*Log[a + b*x^2])/(5040*b
^10)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^{18}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(-\frac{a^9}{b^9(bx^2+a)^{10}}+\frac{9a^8}{b^9(bx^2+a)^9}-\frac{36a^7}{b^9(bx^2+a)^8}+\frac{84a^6}{b^9(bx^2+a)^7}-\frac{126a^5}{b^9(bx^2+a)^6}+\frac{126a^4}{b^9(bx^2+a)^5}-\frac{8}{b^9(bx}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^9}{9b^{10}\left(a+bx^2\right)^9}-\frac{9a^8}{8b^{10}\left(a+bx^2\right)^8}+\frac{36a^7}{7b^{10}\left(a+bx^2\right)^7}-\frac{14a^6}{b^{10}\left(a+bx^2\right)^6}+\frac{126a^5}{5b^{10}\left(a+bx^2\right)^5}-\frac{63a^4}{2b^{10}\left(a+bx^2\right)^4}+\right.$$

input
```
Int[x^19/(a + b*x^2)^10,x]
```

output
```
(a^9/(9*b^10*(a + b*x^2)^9) - (9*a^8)/(8*b^10*(a + b*x^2)^8) + (36*a^7)/(7
*b^10*(a + b*x^2)^7) - (14*a^6)/(b^10*(a + b*x^2)^6) + (126*a^5)/(5*b^10*(
a + b*x^2)^5) - (63*a^4)/(2*b^10*(a + b*x^2)^4) + (28*a^3)/(b^10*(a + b*x^
2)^3) - (18*a^2)/(b^10*(a + b*x^2)^2) + (9*a)/(b^10*(a + b*x^2)) + Log[a +
 b*x^2]/b^10)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.67

method	result
norman	$\frac{\frac{7129a^9}{5040b^{10}}+\frac{9a\,x^{16}}{2b^2}+\frac{27a^2x^{14}}{b^3}+\frac{77a^3x^{12}}{b^4}+\frac{525a^4x^{10}}{4b^5}+\frac{2877a^5x^8}{20b^6}+\frac{1029a^6x^6}{10b^7}+\frac{3267a^7x^4}{70b^8}+\frac{6849a^8x^2}{560b^9}}{(b\,x^2+a)^9}+\frac{\ln(b\,x^2+a)}{2b^{10}}$
risch	$\frac{\frac{7129a^9}{5040b^{10}}+\frac{9a\,x^{16}}{2b^2}+\frac{27a^2x^{14}}{b^3}+\frac{77a^3x^{12}}{b^4}+\frac{525a^4x^{10}}{4b^5}+\frac{2877a^5x^8}{20b^6}+\frac{1029a^6x^6}{10b^7}+\frac{3267a^7x^4}{70b^8}+\frac{6849a^8x^2}{560b^9}}{(b\,x^2+a)^9}+\frac{\ln(b\,x^2+a)}{2b^{10}}$
default	$\frac{a^9}{18b^{10}(b\,x^2+a)^9}-\frac{9a^8}{16b^{10}(b\,x^2+a)^8}+\frac{18a^7}{7b^{10}(b\,x^2+a)^7}-\frac{7a^6}{b^{10}(b\,x^2+a)^6}+\frac{63a^5}{5b^{10}(b\,x^2+a)^5}-\frac{63a^4}{4b^{10}(b\,x^2+a)^4}+\frac{14a^3}{b^{10}(b\,x^2}$
parallelrisch	$\frac{22680\ln(b\,x^2+a)x^{16}a\,b^8+90720\ln(b\,x^2+a)x^{14}a^2b^7+211680\ln(b\,x^2+a)x^{12}a^3b^6+317520\ln(b\,x^2+a)x^{10}a^4b^5+317520\ln(b\,x}{}$

input
```
int(x^19/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(7129/5040*a^9/b^10+9/2*a/b^2*x^16+27*a^2/b^3*x^14+77*a^3/b^4*x^12+525/4*a
^4/b^5*x^10+2877/20*a^5/b^6*x^8+1029/10*a^6/b^7*x^6+3267/70*a^7/b^8*x^4+68
49/560*a^8/b^9*x^2)/(b*x^2+a)^9+1/2*ln(b*x^2+a)/b^10
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.68

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{22680\,ab^8x^{16}+136080\,a^2b^7x^{14}+388080\,a^3b^6x^{12}+661500\,a^4b^5x^{10}+725004\,a^5b^4x^8+518616\,a^6b^3x^6+2}{5040\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a}$$

input
```
integrate(x^19/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 661
500*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^
2*x^4 + 61641*a^8*b*x^2 + 7129*a^9 + 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^
2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6
*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a))/(b^19*x^18
+ 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10
+ 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 +
a^9*b^10)
```

Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.22

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{7129a^9 + 61641a^8bx^2 + 235224a^7b^2x^4 + 518616a^6b^3x^6 + 725004a^5b^4x^8 + 661500a^4b^5x^{10} + 3}{5040a^9b^{10} + 45360a^8b^{11}x^2 + 181440a^7b^{12}x^4 + 423360a^6b^{13}x^6 + 635040a^5b^{14}x^8 + 635040a^4b^{15}x^{10} + 4233} + \frac{\log(a+bx^2)}{2b^{10}}$$

input
```
integrate(x**19/(b*x**2+a)**10,x)
```

output
```
(7129*a**9 + 61641*a**8*b*x**2 + 235224*a**7*b**2*x**4 + 518616*a**6*b**3*
x**6 + 725004*a**5*b**4*x**8 + 661500*a**4*b**5*x**10 + 388080*a**3*b**6*x
**12 + 136080*a**2*b**7*x**14 + 22680*a*b**8*x**16)/(5040*a**9*b**10 + 453
60*a**8*b**11*x**2 + 181440*a**7*b**12*x**4 + 423360*a**6*b**13*x**6 + 635
040*a**5*b**14*x**8 + 635040*a**4*b**15*x**10 + 423360*a**3*b**16*x**12 +
181440*a**2*b**17*x**14 + 45360*a*b**18*x**16 + 5040*b**19*x**18) + log(a
+ b*x**2)/(2*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.17

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{22680\,ab^8x^{16} + 136080\,a^2b^7x^{14} + 388080\,a^3b^6x^{12} + 661500\,a^4b^5x^{10} + 725004\,a^5b^4x^8 + 518616\,a^6b^3x^6 + 2}{5040\,(b^{19}x^{18} + 9\,ab^{18}x^{16} + 36\,a^2b^{17}x^{14} + 84\,a^3b^{16}x^{12} + 126\,a^4b^{15}x^{10} + 126\,a^5b^{14}x^8 + 84\,a^6b^{13}x^6 + }$$
$$+ \frac{\log(bx^2+a)}{2\,b^{10}}$$

input
```
integrate(x^19/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/5040*(22680*a*b^8*x^16 + 136080*a^2*b^7*x^14 + 388080*a^3*b^6*x^12 + 661
500*a^4*b^5*x^10 + 725004*a^5*b^4*x^8 + 518616*a^6*b^3*x^6 + 235224*a^7*b^
2*x^4 + 61641*a^8*b*x^2 + 7129*a^9)/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^
17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6
*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10) + 1/2*log(b*x^2 +
 a)/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.66

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{\log(|bx^2+a|)}{2\,b^{10}}$$
$$- \frac{7129\,b^8x^{18} + 41481\,ab^7x^{16} + 120564\,a^2b^6x^{14} + 210756\,a^3b^5x^{12} + 236754\,a^4b^4x^{10} + 173250\,a^5b^3x^8 + 80}{5040\,(bx^2+a)^9b^9}$$

input
```
integrate(x^19/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
1/2*log(abs(b*x^2 + a))/b^10 - 1/5040*(7129*b^8*x^18 + 41481*a*b^7*x^16 +
120564*a^2*b^6*x^14 + 210756*a^3*b^5*x^12 + 236754*a^4*b^4*x^10 + 173250*a
^5*b^3*x^8 + 80220*a^6*b^2*x^6 + 21420*a^7*b*x^4 + 2520*a^8*x^2)/((b*x^2 +
 a)^9*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{\frac{7129\,a^9}{5040\,b^{10}}+\frac{9\,a\,x^{16}}{2\,b^2}+\frac{27\,a^2\,x^{14}}{b^3}+\frac{77\,a^3\,x^{12}}{b^4}+\frac{525\,a^4\,x^{10}}{4\,b^5}+\frac{2877\,a^5\,x^8}{20\,b^6}+\frac{1029\,a^6\,x^6}{10\,b^7}+\frac{3267\,a^7\,x^4}{70\,b^8}+\frac{6849\,a^8\,x^2}{560\,b^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\,b^8} + \frac{\ln\left(b\,x^2+a\right)}{2\,b^{10}}$$

input
```
int(x^19/(a + b*x^2)^10,x)
```

output
```
((7129*a^9)/(5040*b^10) + (9*a*x^16)/(2*b^2) + (27*a^2*x^14)/b^3 + (77*a^3
*x^12)/b^4 + (525*a^4*x^10)/(4*b^5) + (2877*a^5*x^8)/(20*b^6) + (1029*a^6*
x^6)/(10*b^7) + (3267*a^7*x^4)/(70*b^8) + (6849*a^8*x^2)/(560*b^9))/(a^9 +
 b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 +
 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) +
 log(a + b*x^2)/(2*b^10)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.06

$$\int \frac{x^{19}}{(a+bx^2)^{10}}\,dx = \frac{2520\log(b\,x^2+a)\,a^9+22680\log(b\,x^2+a)\,a^8b\,x^2+90720\log(b\,x^2+a)\,a^7b^2x^4+211680\log(b\,x^2+a)\,a^6b}{}$$

input
```
int(x^19/(b*x^2+a)^10,x)
```

output
```
(2520*log(a + b*x**2)*a**9 + 22680*log(a + b*x**2)*a**8*b*x**2 + 90720*log
(a + b*x**2)*a**7*b**2*x**4 + 211680*log(a + b*x**2)*a**6*b**3*x**6 + 3175
20*log(a + b*x**2)*a**5*b**4*x**8 + 317520*log(a + b*x**2)*a**4*b**5*x**10
 + 211680*log(a + b*x**2)*a**3*b**6*x**12 + 90720*log(a + b*x**2)*a**2*b**
7*x**14 + 22680*log(a + b*x**2)*a*b**8*x**16 + 2520*log(a + b*x**2)*b**9*x
**18 + 4609*a**9 + 38961*a**8*b*x**2 + 144504*a**7*b**2*x**4 + 306936*a**6
*b**3*x**6 + 407484*a**5*b**4*x**8 + 343980*a**4*b**5*x**10 + 176400*a**3*
b**6*x**12 + 45360*a**2*b**7*x**14 - 2520*b**9*x**18)/(5040*b**10*(a**9 +
9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8
 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**
8*x**16 + b**9*x**18))
```

3.196 $\int \frac{x^{17}}{(a+bx^2)^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{x^{17}}{(a+bx^2)^{10}} \, dx = \frac{x^{18}}{18a(a+bx^2)^9}$$

output
```
1/18*x^18/a/(b*x^2+a)^9
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 101 vs. 2(19) = 38.

Time = 0.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.32

$$\int \frac{x^{17}}{(a+bx^2)^{10}} \, dx = -\frac{a^8+9a^7bx^2+36a^6b^2x^4+84a^5b^3x^6+126a^4b^4x^8+126a^3b^5x^{10}+84a^2b^6x^{12}+36ab^7x^{14}+9b^8x^{16}}{18b^9(a+bx^2)^9}$$

input
```
Integrate[x^17/(a + b*x^2)^10,x]
```

output
```
-1/18*(a^8 + 9*a^7*b*x^2 + 36*a^6*b^2*x^4 + 84*a^5*b^3*x^6 + 126*a^4*b^4*x
^8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12 + 36*a*b^7*x^14 + 9*b^8*x^16)/(b^9
*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \; 242$$

$$\frac{x^{18}}{18a\,(a+bx^2)^9}$$

input
```
Int[x^17/(a + b*x^2)^10,x]
```

output
```
x^18/(18*a*(a + b*x^2)^9)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$.

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.26

method	result
gosper	$-\frac{9b^{8}x^{16}+36a\,b^{7}x^{14}+84a^{2}b^{6}x^{12}+126a^{3}b^{5}x^{10}+126a^{4}b^{4}x^{8}+84a^{5}b^{3}x^{6}+36a^{6}b^{2}x^{4}+9a^{7}b\,x^{2}+a^{8}}{18\left(b\,x^{2}+a\right)^{9}b^{9}}$
orering	$-\frac{9b^{8}x^{16}+36a\,b^{7}x^{14}+84a^{2}b^{6}x^{12}+126a^{3}b^{5}x^{10}+126a^{4}b^{4}x^{8}+84a^{5}b^{3}x^{6}+36a^{6}b^{2}x^{4}+9a^{7}b\,x^{2}+a^{8}}{18\left(b\,x^{2}+a\right)^{9}b^{9}}$
parallelrisch	$\frac{-9b^{8}x^{16}-36a\,b^{7}x^{14}-84a^{2}b^{6}x^{12}-126a^{3}b^{5}x^{10}-126a^{4}b^{4}x^{8}-84a^{5}b^{3}x^{6}-36a^{6}b^{2}x^{4}-9a^{7}b\,x^{2}-a^{8}}{18b^{9}\left(b\,x^{2}+a\right)^{9}}$
norman	$\frac{-\frac{a^{8}}{18b^{9}}-\frac{a^{7}x^{2}}{2b^{8}}-\frac{2a^{6}x^{4}}{b^{7}}-\frac{14a^{5}x^{6}}{3b^{6}}-\frac{7a^{4}x^{8}}{b^{5}}-\frac{7a^{3}x^{10}}{b^{4}}-\frac{14a^{2}x^{12}}{3b^{3}}-\frac{2a\,x^{14}}{b^{2}}-\frac{x^{16}}{2b}}{\left(b\,x^{2}+a\right)^{9}}$
risch	$\frac{-\frac{a^{8}}{18b^{9}}-\frac{a^{7}x^{2}}{2b^{8}}-\frac{2a^{6}x^{4}}{b^{7}}-\frac{14a^{5}x^{6}}{3b^{6}}-\frac{7a^{4}x^{8}}{b^{5}}-\frac{7a^{3}x^{10}}{b^{4}}-\frac{14a^{2}x^{12}}{3b^{3}}-\frac{2a\,x^{14}}{b^{2}}-\frac{x^{16}}{2b}}{\left(b\,x^{2}+a\right)^{9}}$
default	$-\frac{a^{8}}{18b^{9}\left(b\,x^{2}+a\right)^{9}}-\frac{7a^{4}}{b^{9}\left(b\,x^{2}+a\right)^{5}}+\frac{7a^{3}}{b^{9}\left(b\,x^{2}+a\right)^{4}}-\frac{2a^{6}}{b^{9}\left(b\,x^{2}+a\right)^{7}}+\frac{a^{7}}{2b^{9}\left(b\,x^{2}+a\right)^{8}}+\frac{2a}{b^{9}\left(b\,x^{2}+a\right)^{2}}-\frac{1}{2b^{9}\left(b\,x^{2}+a\right)}-$

input
```
int(x^17/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/18*(9*b^8*x^16+36*a*b^7*x^14+84*a^2*b^6*x^12+126*a^3*b^5*x^10+126*a^4*b
^4*x^8+84*a^5*b^3*x^6+36*a^6*b^2*x^4+9*a^7*b*x^2+a^8)/(b*x^2+a)^9/b^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(17) = 34$.

Time = 0.06 (sec) , antiderivative size = 190, normalized size of antiderivative = 10.00

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = -\frac{9\,b^{8}x^{16}+36\,ab^{7}x^{14}+84\,a^{2}b^{6}x^{12}+126\,a^{3}b^{5}x^{10}+126\,a^{4}b^{4}x^{8}+84\,a^{5}b^{3}x^{6}+36\,a^{6}b^{2}x^{4}+9\,a}{18\,(b^{18}x^{18}+9\,ab^{17}x^{16}+36\,a^{2}b^{16}x^{14}+84\,a^{3}b^{15}x^{12}+126\,a^{4}b^{14}x^{10}+126\,a^{5}b^{13}x^{8}+84\,a^{6}b^{12}x^{6}+36\,a^{7}b^{1}}$$

input
```
integrate(x^17/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^1
8*x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^1
4*x^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10
*x^2 + a^9*b^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(14) = 28.

Time = 0.78 (sec) , antiderivative size = 202, normalized size of antiderivative = 10.63

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = \frac{-a^8 - 9a^7bx^2 - 36a^6b^2x^4 - 84a^5b^3x^6 - 126a^4b^4x^8 - 126a^3b^5x^{10} - 84a^2b^6x^{12} - 36ab^7}{18a^9b^9 + 162a^8b^{10}x^2 + 648a^7b^{11}x^4 + 1512a^6b^{12}x^6 + 2268a^5b^{13}x^8 + 2268a^4b^{14}x^{10} + 1512a^3b^{15}x^{12} + 648a^2}$$

input
```
integrate(x**17/(b*x**2+a)**10,x)
```

output
```
(-a**8 - 9*a**7*b*x**2 - 36*a**6*b**2*x**4 - 84*a**5*b**3*x**6 - 126*a**4*
b**4*x**8 - 126*a**3*b**5*x**10 - 84*a**2*b**6*x**12 - 36*a*b**7*x**14 - 9
*b**8*x**16)/(18*a**9*b**9 + 162*a**8*b**10*x**2 + 648*a**7*b**11*x**4 + 1
512*a**6*b**12*x**6 + 2268*a**5*b**13*x**8 + 2268*a**4*b**14*x**10 + 1512*
a**3*b**15*x**12 + 648*a**2*b**16*x**14 + 162*a*b**17*x**16 + 18*b**18*x**
18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(17) = 34.

Time = 0.04 (sec) , antiderivative size = 190, normalized size of antiderivative = 10.00

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = -\frac{9\,b^8x^{16} + 36\,ab^7x^{14} + 84\,a^2b^6x^{12} + 126\,a^3b^5x^{10} + 126\,a^4b^4x^8 + 84\,a^5b^3x^6 + 36\,a^6b^2x^4 + 9\,a}{18\,(b^{18}x^{18} + 9\,ab^{17}x^{16} + 36\,a^2b^{16}x^{14} + 84\,a^3b^{15}x^{12} + 126\,a^4b^{14}x^{10} + 126\,a^5b^{13}x^8 + 84\,a^6b^{12}x^6 + 36\,a^7b^{1}}$$

input
```
integrate(x^17/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/(b^1
8*x^18 + 9*a*b^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^1
4*x^10 + 126*a^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10
*x^2 + a^9*b^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 5.21

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = -\frac{9\,b^8x^{16}+36\,ab^7x^{14}+84\,a^2b^6x^{12}+126\,a^3b^5x^{10}+126\,a^4b^4x^8+84\,a^5b^3x^6+36\,a^6b^2x^4+9\,a^7bx^2+a^8}{18\,(bx^2+a)^9b^9}$$

input
```
integrate(x^17/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/18*(9*b^8*x^16 + 36*a*b^7*x^14 + 84*a^2*b^6*x^12 + 126*a^3*b^5*x^10 + 1
26*a^4*b^4*x^8 + 84*a^5*b^3*x^6 + 36*a^6*b^2*x^4 + 9*a^7*b*x^2 + a^8)/((b*
x^2 + a)^9*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 10.11

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx = -\frac{a^8+9\,a^7\,b\,x^2+36\,a^6\,b^2\,x^4+84\,a^5\,b^3\,x^6+126\,a^4\,b^4\,x^8+126\,a^3\,b^5\,x^{10}+84\,a^2\,b^6\,x^{1}}{18\,a^9\,b^9+162\,a^8\,b^{10}\,x^2+648\,a^7\,b^{11}\,x^4+1512\,a^6\,b^{12}\,x^6+2268\,a^5\,b^{13}\,x^8+2268\,a^4\,b^{14}\,x^{10}+1512\,a^3\,b^{15}\,x}$$

input
```
int(x^17/(a + b*x^2)^10,x)
```

output
```
-(a^8 + 9*b^8*x^16 + 9*a^7*b*x^2 + 36*a*b^7*x^14 + 36*a^6*b^2*x^4 + 84*a^5
*b^3*x^6 + 126*a^4*b^4*x^8 + 126*a^3*b^5*x^10 + 84*a^2*b^6*x^12)/(18*a^9*b
^9 + 18*b^18*x^18 + 162*a*b^17*x^16 + 162*a^8*b^10*x^2 + 648*a^7*b^11*x^4
+ 1512*a^6*b^12*x^6 + 2268*a^5*b^13*x^8 + 2268*a^4*b^14*x^10 + 1512*a^3*b^
15*x^12 + 648*a^2*b^16*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.53

$$\int \frac{x^{17}}{(a+bx^2)^{10}}\,dx$$

$$= \frac{x^{18}}{18a\,(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a^8b}$$

input
```
int(x^17/(b*x^2+a)^10,x)
```

output
```
x**18/(18*a*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6
+ 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*
b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.197 $\int \frac{x^{15}}{(a+bx^2)^{10}} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^{15}}{(a+bx^2)^{10}} dx = \frac{x^{16}}{18a(a+bx^2)^9} + \frac{x^{16}}{144a^2(a+bx^2)^8}$$

output
```
1/18*x^16/a/(b*x^2+a)^9+1/144*x^16/a^2/(b*x^2+a)^8
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 90 vs. 2(39) = 78.

Time = 0.01 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.31

$$\int \frac{x^{15}}{(a+bx^2)^{10}} dx \\ = -\frac{a^7 + 9a^6bx^2 + 36a^5b^2x^4 + 84a^4b^3x^6 + 126a^3b^4x^8 + 126a^2b^5x^{10} + 84ab^6x^{12} + 36b^7x^{14}}{144b^8(a+bx^2)^9}$$

input
```
Integrate[x^15/(a + b*x^2)^10,x]
```

output
```
-1/144*(a^7 + 9*a^6*b*x^2 + 36*a^5*b^2*x^4 + 84*a^4*b^3*x^6 + 126*a^3*b^4*
x^8 + 126*a^2*b^5*x^10 + 84*a*b^6*x^12 + 36*b^7*x^14)/(b^8*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx$$
$$\Big\downarrow \; 243$$
$$\frac{1}{2}\int \frac{x^{14}}{(bx^2+a)^{10}}dx^2$$
$$\Big\downarrow \; 55$$
$$\frac{1}{2}\left(\frac{\int \frac{x^{14}}{(bx^2+a)^9}dx^2}{9a}+\frac{x^{16}}{9a(a+bx^2)^9}\right)$$
$$\Big\downarrow \; 48$$
$$\frac{1}{2}\left(\frac{x^{16}}{72a^2(a+bx^2)^8}+\frac{x^{16}}{9a(a+bx^2)^9}\right)$$

input
```
Int[x^15/(a + b*x^2)^10,x]
```

output
```
(x^16/(9*a*(a + b*x^2)^9) + x^16/(72*a^2*(a + b*x^2)^8))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.28

method	result
gosper	$-\frac{36 b^7 x^{14}+84 a b^6 x^{12}+126 a^2 b^5 x^{10}+126 a^3 b^4 x^8+84 a^4 b^3 x^6+36 a^5 b^2 x^4+9 a^6 b x^2+a^7}{144 (b x^2+a)^9 b^8}$
orering	$-\frac{36 b^7 x^{14}+84 a b^6 x^{12}+126 a^2 b^5 x^{10}+126 a^3 b^4 x^8+84 a^4 b^3 x^6+36 a^5 b^2 x^4+9 a^6 b x^2+a^7}{144 (b x^2+a)^9 b^8}$
norman	$\frac{-\frac{a^7}{144 b^8}-\frac{a^6 x^2}{16 b^7}-\frac{a^5 x^4}{4 b^6}-\frac{7 a^4 x^6}{12 b^5}-\frac{7 a^3 x^8}{8 b^4}-\frac{7 a^2 x^{10}}{8 b^3}-\frac{7 a x^{12}}{12 b^2}-\frac{x^{14}}{4 b}}{(b x^2+a)^9}$
risch	$\frac{-\frac{a^7}{144 b^8}-\frac{a^6 x^2}{16 b^7}-\frac{a^5 x^4}{4 b^6}-\frac{7 a^4 x^6}{12 b^5}-\frac{7 a^3 x^8}{8 b^4}-\frac{7 a^2 x^{10}}{8 b^3}-\frac{7 a x^{12}}{12 b^2}-\frac{x^{14}}{4 b}}{(b x^2+a)^9}$
parallelrisch	$\frac{-36 b^8 x^{14}-84 a b^7 x^{12}-126 a^2 b^6 x^{10}-126 a^3 b^5 x^8-84 a^4 b^4 x^6-36 a^5 b^3 x^4-9 a^6 x^2 b^2-a^7 b}{144 b^9 (b x^2+a)^9}$
default	$\frac{a^7}{18 b^8 (b x^2+a)^9}+\frac{7 a^3}{2 b^8 (b x^2+a)^5}-\frac{21 a^2}{8 b^8 (b x^2+a)^4}+\frac{3 a^5}{2 b^8 (b x^2+a)^7}-\frac{7 a^6}{16 b^8 (b x^2+a)^8}-\frac{1}{4 b^8 (b x^2+a)^2}+\frac{7 a}{6 b^8 (b x^2+a)^3}$

input
```
int(x^15/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(36*b^7*x^14+84*a*b^6*x^12+126*a^2*b^5*x^10+126*a^3*b^4*x^8+84*a^4*
b^3*x^6+36*a^5*b^2*x^4+9*a^6*b*x^2+a^7)/(b*x^2+a)^9/b^8
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(35) = 70.

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.59

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = -\frac{36\,b^7x^{14}+84\,ab^6x^{12}+126\,a^2b^5x^{10}+126\,a^3b^4x^8+84\,a^4b^3x^6+36\,a^5b^2x^4+9\,a^6bx^2}{144\,(b^{17}x^{18}+9\,ab^{16}x^{16}+36\,a^2b^{15}x^{14}+84\,a^3b^{14}x^{12}+126\,a^4b^{13}x^{10}+126\,a^5b^{12}x^8+84\,a^6b^{11}x^6+36\,a^7b}$$

input
```
integrate(x^15/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^1
6*x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5
*b^12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(31) = 62.

Time = 0.73 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.87

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = \frac{-a^7-9a^6bx^2-36a^5b^2x^4-84a^4b^3x^6-126a^3b^4x^8-126a^2b^5x^{10}-84ab^6x}{144a^9b^8+1296a^8b^9x^2+5184a^7b^{10}x^4+12096a^6b^{11}x^6+18144a^5b^{12}x^8+18144a^4b^{13}x^{10}+12096a^3b^{14}x^{12}}$$

input
```
integrate(x**15/(b*x**2+a)**10,x)
```

output
```
(-a**7 - 9*a**6*b*x**2 - 36*a**5*b**2*x**4 - 84*a**4*b**3*x**6 - 126*a**3*
b**4*x**8 - 126*a**2*b**5*x**10 - 84*a*b**6*x**12 - 36*b**7*x**14)/(144*a*
*9*b**8 + 1296*a**8*b**9*x**2 + 5184*a**7*b**10*x**4 + 12096*a**6*b**11*x*
*6 + 18144*a**5*b**12*x**8 + 18144*a**4*b**13*x**10 + 12096*a**3*b**14*x**
12 + 5184*a**2*b**15*x**14 + 1296*a*b**16*x**16 + 144*b**17*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 179, normalized size of antiderivative = 4.59

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{36\,b^7x^{14} + 84\,ab^6x^{12} + 126\,a^2b^5x^{10} + 126\,a^3b^4x^8 + 84\,a^4b^3x^6 + 36\,a^5b^2x^4 + 9\,a^6bx^2 + \cdots}{144\,(b^{17}x^{18} + 9\,ab^{16}x^{16} + 36\,a^2b^{15}x^{14} + 84\,a^3b^{14}x^{12} + 126\,a^4b^{13}x^{10} + 126\,a^5b^{12}x^8 + 84\,a^6b^{11}x^6 + 36\,a^7b \cdots}$$

input
```
integrate(x^15/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/(b^17*x^18 + 9*a*b^1
6*x^16 + 36*a^2*b^15*x^14 + 84*a^3*b^14*x^12 + 126*a^4*b^13*x^10 + 126*a^5
*b^12*x^8 + 84*a^6*b^11*x^6 + 36*a^7*b^10*x^4 + 9*a^8*b^9*x^2 + a^9*b^8)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = -\frac{36\,b^7x^{14} + 84\,ab^6x^{12} + 126\,a^2b^5x^{10} + 126\,a^3b^4x^8 + 84\,a^4b^3x^6 + 36\,a^5b^2x^4 + 9\,a^6bx^2 + a^7}{144\,(bx^2+a)^9b^8}$$

input
```
integrate(x^15/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/144*(36*b^7*x^14 + 84*a*b^6*x^12 + 126*a^2*b^5*x^10 + 126*a^3*b^4*x^8 +
 84*a^4*b^3*x^6 + 36*a^5*b^2*x^4 + 9*a^6*b*x^2 + a^7)/((b*x^2 + a)^9*b^8)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.64

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = -\frac{a^7+9\,a^6\,b\,x^2+36\,a^5\,b^2\,x^4+84\,a^4\,b^3\,x^6+126\,a^3\,b^4\,x^8+126\,a^2\,b^5\,x^{10}}{144\,a^9\,b^8+1296\,a^8\,b^9\,x^2+5184\,a^7\,b^{10}\,x^4+12096\,a^6\,b^{11}\,x^6+18144\,a^5\,b^{12}\,x^8+18144\,a^4\,b^{13}\,x^{10}+12096}$$

input
```
int(x^15/(a + b*x^2)^10,x)
```

output
```
-(a^7 + 36*b^7*x^14 + 9*a^6*b*x^2 + 84*a*b^6*x^12 + 36*a^5*b^2*x^4 + 84*a^
4*b^3*x^6 + 126*a^3*b^4*x^8 + 126*a^2*b^5*x^10)/(144*a^9*b^8 + 144*b^17*x^
18 + 1296*a*b^16*x^16 + 1296*a^8*b^9*x^2 + 5184*a^7*b^10*x^4 + 12096*a^6*b
^11*x^6 + 18144*a^5*b^12*x^8 + 18144*a^4*b^13*x^10 + 12096*a^3*b^14*x^12 +
 5184*a^2*b^15*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.56

$$\int \frac{x^{15}}{(a+bx^2)^{10}}\,dx = \frac{-36b^7x^{14}-84a\,b^6x^{12}-126a^2b^5x^{10}-126a^3b^4x^8-84a^4b^3x^6-36a^5b^2x^4-9a^6b\,x^2-a^7}{144b^8\left(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a\right.}$$

input
```
int(x^15/(b*x^2+a)^10,x)
```

output
```
( - a**7 - 9*a**6*b*x**2 - 36*a**5*b**2*x**4 - 84*a**4*b**3*x**6 - 126*a**
3*b**4*x**8 - 126*a**2*b**5*x**10 - 84*a*b**6*x**12 - 36*b**7*x**14)/(144*
b**8*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a
**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x*
*14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.198 $\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 58

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx=\frac{x^{14}}{18a\left(a+bx^2\right)^9}+\frac{x^{14}}{72a^2\left(a+bx^2\right)^8}+\frac{x^{14}}{504a^3\left(a+bx^2\right)^7}$$

output
```
1/18*x^14/a/(b*x^2+a)^9+1/72*x^14/a^2/(b*x^2+a)^8+1/504*x^14/a^3/(b*x^2+a)
^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.36

$$\int \frac{x^{13}}{\left(a+bx^2\right)^{10}}\,dx =-\frac{a^6+9a^5bx^2+36a^4b^2x^4+84a^3b^3x^6+126a^2b^4x^8+126ab^5x^{10}+84b^6x^{12}}{504b^7\left(a+bx^2\right)^9}$$

input
```
Integrate[x^13/(a + b*x^2)^10,x]
```

output
```
-1/504*(a^6 + 9*a^5*b*x^2 + 36*a^4*b^2*x^4 + 84*a^3*b^3*x^6 + 126*a^2*b^4*
x^8 + 126*a*b^5*x^10 + 84*b^6*x^12)/(b^7*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {243, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^{12}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(\frac{2\int \frac{x^{12}}{(bx^2+a)^9}dx^2}{9a}+\frac{x^{14}}{9a\left(a+bx^2\right)^9}\right)$$

$$\Big\downarrow \ 55$$

$$\frac{1}{2}\left(\frac{2\left(\frac{\int \frac{x^{12}}{\left(bx^2+a\right)^8}dx^2}{8a}+\frac{x^{14}}{8a(a+bx^2)^8}\right)}{9a}+\frac{x^{14}}{9a\left(a+bx^2\right)^9}\right)$$

$$\Big\downarrow \ 48$$

$$\frac{1}{2}\left(\frac{2\left(\frac{x^{14}}{56a^2(a+bx^2)^7}+\frac{x^{14}}{8a(a+bx^2)^8}\right)}{9a}+\frac{x^{14}}{9a\left(a+bx^2\right)^9}\right)$$

input
```
Int[x^13/(a + b*x^2)^10,x]
```

output
```
(x^14/(9*a*(a + b*x^2)^9) + (2*(x^14/(8*a*(a + b*x^2)^8) + x^14/(56*a^2*(a
 + b*x^2)^7)))/(9*a))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

method	result
gosper	$-\frac{84b^6x^{12}+126a\,b^5x^{10}+126a^2b^4x^8+84a^3x^6b^3+36a^4b^2x^4+9a^5b\,x^2+a^6}{504(b\,x^2+a)^9b^7}$
orering	$-\frac{84b^6x^{12}+126a\,b^5x^{10}+126a^2b^4x^8+84a^3x^6b^3+36a^4b^2x^4+9a^5b\,x^2+a^6}{504(b\,x^2+a)^9b^7}$
norman	$\frac{-\frac{a^6}{504b^7}-\frac{a^5x^2}{56b^6}-\frac{a^4x^4}{14b^5}-\frac{a^3x^6}{6b^4}-\frac{a^2x^8}{4b^3}-\frac{a\,x^{10}}{4b^2}-\frac{x^{12}}{6b}}{(b\,x^2+a)^9}$
risch	$\frac{-\frac{a^6}{504b^7}-\frac{a^5x^2}{56b^6}-\frac{a^4x^4}{14b^5}-\frac{a^3x^6}{6b^4}-\frac{a^2x^8}{4b^3}-\frac{a\,x^{10}}{4b^2}-\frac{x^{12}}{6b}}{(b\,x^2+a)^9}$
parallelrisch	$\frac{-84b^8x^{12}-126a\,b^7x^{10}-126a^2b^6x^8-84a^3b^5x^6-36a^4x^4b^4-9a^5b^3x^2-a^6b^2}{504b^9(b\,x^2+a)^9}$
default	$-\frac{a^6}{18b^7(b\,x^2+a)^9}-\frac{3a^2}{2b^7(b\,x^2+a)^5}+\frac{3a}{4b^7(b\,x^2+a)^4}-\frac{15a^4}{14b^7(b\,x^2+a)^7}+\frac{3a^5}{8b^7(b\,x^2+a)^8}-\frac{1}{6b^7(b\,x^2+a)^3}+\frac{5a^3}{3b^7(b\,x^2+a}$

input
```
int(x^13/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/504*(84*b^6*x^12+126*a*b^5*x^10+126*a^2*b^4*x^8+84*a^3*b^3*x^6+36*a^4*b
^2*x^4+9*a^5*b*x^2+a^6)/(b*x^2+a)^9/b^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.

Time = 0.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.90

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx = -\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\,(b^{16}x^{18}+9\,ab^{15}x^{16}+36\,a^2b^{14}x^{14}+84\,a^3b^{13}x^{12}+126\,a^4b^{12}x^{10}+126\,a^5b^{11}x^8+84\,a^6b^{10}x^6+36\,a^7b}$$

input
```
integrate(x^13/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 +
36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^
14*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6
*b^10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(48) = 96$.

Time = 0.73 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.07

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-a^6 - 9a^5bx^2 - 36a^4b^2x^4 - 84a^3b^3x^6 - 126a^2b^4x^8 - 126ab^5x^{10} - 8}{504a^9b^7 + 4536a^8b^8x^2 + 18144a^7b^9x^4 + 42336a^6b^{10}x^6 + 63504a^5b^{11}x^8 + 63504a^4b^{12}x^{10} + 42336a^3b^{13}x^{12}}$$

input
```
integrate(x**13/(b*x**2+a)**10,x)
```

output
```
(-a**6 - 9*a**5*b*x**2 - 36*a**4*b**2*x**4 - 84*a**3*b**3*x**6 - 126*a**2*
b**4*x**8 - 126*a*b**5*x**10 - 84*b**6*x**12)/(504*a**9*b**7 + 4536*a**8*b
**8*x**2 + 18144*a**7*b**9*x**4 + 42336*a**6*b**10*x**6 + 63504*a**5*b**11
*x**8 + 63504*a**4*b**12*x**10 + 42336*a**3*b**13*x**12 + 18144*a**2*b**14
*x**14 + 4536*a*b**15*x**16 + 504*b**16*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(52) = 104$.

Time = 0.04 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.90

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{84\,b^6x^{12} + 126\,ab^5x^{10} + 126\,a^2b^4x^8 + 84\,a^3b^3x^6 + 36\,a^4b^2x^4 + 9\,a^5bx^2 + a^6}{504\,(b^{16}x^{18} + 9\,ab^{15}x^{16} + 36\,a^2b^{14}x^{14} + 84\,a^3b^{13}x^{12} + 126\,a^4b^{12}x^{10} + 126\,a^5b^{11}x^8 + 84\,a^6b^{10}x^6 + 36\,a^7b}$$

input
```
integrate(x^13/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 +
36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/(b^16*x^18 + 9*a*b^15*x^16 + 36*a^2*b^
14*x^14 + 84*a^3*b^13*x^12 + 126*a^4*b^12*x^10 + 126*a^5*b^11*x^8 + 84*a^6
*b^10*x^6 + 36*a^7*b^9*x^4 + 9*a^8*b^8*x^2 + a^9*b^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx = -\frac{84\,b^6x^{12}+126\,ab^5x^{10}+126\,a^2b^4x^8+84\,a^3b^3x^6+36\,a^4b^2x^4+9\,a^5bx^2+a^6}{504\,(bx^2+a)^9b^7}$$

input
```
integrate(x^13/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/504*(84*b^6*x^12 + 126*a*b^5*x^10 + 126*a^2*b^4*x^8 + 84*a^3*b^3*x^6 +
36*a^4*b^2*x^4 + 9*a^5*b*x^2 + a^6)/((b*x^2 + a)^9*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.93

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx = -\frac{a^6+9\,a^5\,b\,x^2+36\,a^4\,b^2\,x^4+84\,a^3\,b^3\,x^6+126\,a^2\,b^4\,x^8+126\,a}{504\,a^9\,b^7+4536\,a^8\,b^8\,x^2+18144\,a^7\,b^9\,x^4+42336\,a^6\,b^{10}\,x^6+63504\,a^5\,b^{11}\,x^8+63504\,a^4\,b^{12}\,x^{10}+42336}$$

input
```
int(x^13/(a + b*x^2)^10,x)
```

output
```
-(a^6 + 84*b^6*x^12 + 9*a^5*b*x^2 + 126*a*b^5*x^10 + 36*a^4*b^2*x^4 + 84*a
^3*b^3*x^6 + 126*a^2*b^4*x^8)/(504*a^9*b^7 + 504*b^16*x^18 + 4536*a*b^15*x
^16 + 4536*a^8*b^8*x^2 + 18144*a^7*b^9*x^4 + 42336*a^6*b^10*x^6 + 63504*a^
5*b^11*x^8 + 63504*a^4*b^12*x^10 + 42336*a^3*b^13*x^12 + 18144*a^2*b^14*x^
14)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.88

$$\int \frac{x^{13}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-84b^6x^{12}-126a\,b^5x^{10}-126a^2b^4x^8-84a^3b^3x^6-36a^4b^2x^4-9a^5b\,x^2-a^6}{504b^7\left(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a}$$

input
```
int(x^13/(b*x^2+a)^10,x)
```

output
```
( - a**6 - 9*a**5*b*x**2 - 36*a**4*b**2*x**4 - 84*a**3*b**3*x**6 - 126*a**
2*b**4*x**8 - 126*a*b**5*x**10 - 84*b**6*x**12)/(504*b**7*(a**9 + 9*a**8*b
*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a
**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16
+ b**9*x**18))
```

3.199 $\int \frac{x^{11}}{(a+bx^2)^{10}} dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{x^{11}}{(a+bx^2)^{10}} dx = \frac{x^{12}}{18a(a+bx^2)^9} + \frac{x^{12}}{48a^2(a+bx^2)^8} + \frac{x^{12}}{168a^3(a+bx^2)^7} + \frac{x^{12}}{1008a^4(a+bx^2)^6}$$

output
```
1/18*x^12/a/(b*x^2+a)^9+1/48*x^12/a^2/(b*x^2+a)^8+1/168*x^12/a^3/(b*x^2+a)
^7+1/1008*x^12/a^4/(b*x^2+a)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{x^{11}}{(a+bx^2)^{10}} dx = -\frac{a^5 + 9a^4bx^2 + 36a^3b^2x^4 + 84a^2b^3x^6 + 126ab^4x^8 + 126b^5x^{10}}{1008b^6(a+bx^2)^9}$$

input
```
Integrate[x^11/(a + b*x^2)^10,x]
```

output
```
-1/1008*(a^5 + 9*a^4*b*x^2 + 36*a^3*b^2*x^4 + 84*a^2*b^3*x^6 + 126*a*b^4*x
^8 + 126*b^5*x^10)/(b^6*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {243, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^{10}}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \; 55$$

$$\frac{1}{2}\left(\frac{\int \frac{x^{10}}{(bx^2+a)^9}dx^2}{3a}+\frac{x^{12}}{9a(a+bx^2)^9}\right)$$

$$\Big\downarrow \; 55$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{x^{10}}{(bx^2+a)^8}dx^2}{4a}+\frac{x^{12}}{8a(a+bx^2)^8}}{3a}+\frac{x^{12}}{9a(a+bx^2)^9}\right)$$

$$\Big\downarrow \; 55$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\int \frac{x^{10}}{(bx^2+a)^7}dx^2}{7a}+\frac{x^{12}}{7a(a+bx^2)^7}}{4a}+\frac{x^{12}}{8a(a+bx^2)^8}}{3a}+\frac{x^{12}}{9a(a+bx^2)^9}\right)$$

$$\Big\downarrow \; 48$$

$$\frac{1}{2}\left(\frac{\frac{\frac{x^{12}}{42a^2\left(a+bx^2\right)^6}+\frac{x^{12}}{7a\left(a+bx^2\right)^7}}{4a}+\frac{x^{12}}{8a(a+bx^2)^8}}{3a}+\frac{x^{12}}{9a\left(a+bx^2\right)^9}\right)$$

input
```
Int[x^11/(a + b*x^2)^10,x]
```

output
```
(x^12/(9*a*(a + b*x^2)^9) + (x^12/(8*a*(a + b*x^2)^8) + (x^12/(7*a*(a + b*
x^2)^7) + x^12/(42*a^2*(a + b*x^2)^6))/(4*a))/(3*a))/2
```

Defintions of rubi rules used

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

method	result	size
gosper	$-\frac{126b^5x^{10}+126a\,b^4x^8+84a^2b^3x^6+36a^3b^2x^4+9a^4b\,x^2+a^5}{1008(b\,x^2+a)^9b^6}$	67
orering	$-\frac{126b^5x^{10}+126a\,b^4x^8+84a^2b^3x^6+36a^3b^2x^4+9a^4b\,x^2+a^5}{1008(b\,x^2+a)^9b^6}$	67
norman	$\frac{-\frac{a^5}{1008b^6}-\frac{a^4x^2}{112b^5}-\frac{a^3x^4}{28b^4}-\frac{a^2x^6}{12b^3}-\frac{a\,x^8}{8b^2}-\frac{x^{10}}{8b}}{(b\,x^2+a)^9}$	70
risch	$\frac{-\frac{a^5}{1008b^6}-\frac{a^4x^2}{112b^5}-\frac{a^3x^4}{28b^4}-\frac{a^2x^6}{12b^3}-\frac{a\,x^8}{8b^2}-\frac{x^{10}}{8b}}{(b\,x^2+a)^9}$	70
parallelrisch	$\frac{-126b^8x^{10}-126a\,b^7x^8-84a^2x^6b^6-36a^3b^5x^4-9a^4b^4x^2-a^5b^3}{1008b^9(b\,x^2+a)^9}$	74
default	$\frac{a^5}{18b^6(b\,x^2+a)^9}+\frac{a}{2b^6(b\,x^2+a)^5}-\frac{1}{8b^6(b\,x^2+a)^4}+\frac{5a^3}{7b^6(b\,x^2+a)^7}-\frac{5a^4}{16b^6(b\,x^2+a)^8}-\frac{5a^2}{6b^6(b\,x^2+a)^6}$	99

input
```
int(x^11/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/1008*(126*b^5*x^10+126*a*b^4*x^8+84*a^2*b^3*x^6+36*a^3*b^2*x^4+9*a^4*b*
x^2+a^5)/(b*x^2+a)^9/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(69) = 138.

Time = 0.06 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.04

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,(b^{15}x^{18}+9\,ab^{14}x^{16}+36\,a^2b^{13}x^{14}+84\,a^3b^{12}x^{12}+126\,a^4b^{11}x^{10}+126\,a^5b^{10}x^8+84\,a^6b^9x^6+36\,a^7b}$$

input
```
integrate(x^11/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 +
9*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*
b^12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7
*b^8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(65) = 130$.

Time = 0.67 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-a^5 - 9a^4bx^2 - 36a^3b^2x^4 - 84a^2b^3x^6 - 126ab^4x^8 - 126b^5x^{10}}{1008a^9b^6 + 9072a^8b^7x^2 + 36288a^7b^8x^4 + 84672a^6b^9x^6 + 127008a^5b^{10}x^8 + 127008a^4b^{11}x^{10} + 84672a^3b^{12}x^{12} + \dots}$$

input
```
integrate(x**11/(b*x**2+a)**10,x)
```

output
```
(-a**5 - 9*a**4*b*x**2 - 36*a**3*b**2*x**4 - 84*a**2*b**3*x**6 - 126*a*b**
4*x**8 - 126*b**5*x**10)/(1008*a**9*b**6 + 9072*a**8*b**7*x**2 + 36288*a**
7*b**8*x**4 + 84672*a**6*b**9*x**6 + 127008*a**5*b**10*x**8 + 127008*a**4*
b**11*x**10 + 84672*a**3*b**12*x**12 + 36288*a**2*b**13*x**14 + 9072*a*b**
14*x**16 + 1008*b**15*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(69) = 138$.

Time = 0.04 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.04

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{126\,b^5x^{10} + 126\,ab^4x^8 + 84\,a^2b^3x^6 + 36\,a^3b^2x^4 + 9\,a^4bx^2 + a^5}{1008\,(b^{15}x^{18} + 9\,ab^{14}x^{16} + 36\,a^2b^{13}x^{14} + 84\,a^3b^{12}x^{12} + 126\,a^4b^{11}x^{10} + 126\,a^5b^{10}x^8 + 84\,a^6b^9x^6 + 36\,a^7b \dots}$$

input
```
integrate(x^11/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 +
9*a^4*b*x^2 + a^5)/(b^15*x^18 + 9*a*b^14*x^16 + 36*a^2*b^13*x^14 + 84*a^3*
b^12*x^12 + 126*a^4*b^11*x^10 + 126*a^5*b^10*x^8 + 84*a^6*b^9*x^6 + 36*a^7
*b^8*x^4 + 9*a^8*b^7*x^2 + a^9*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx = -\frac{126\,b^5x^{10}+126\,ab^4x^8+84\,a^2b^3x^6+36\,a^3b^2x^4+9\,a^4bx^2+a^5}{1008\,(bx^2+a)^9b^6}$$

input
```
integrate(x^11/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/1008*(126*b^5*x^10 + 126*a*b^4*x^8 + 84*a^2*b^3*x^6 + 36*a^3*b^2*x^4 +
9*a^4*b*x^2 + a^5)/((b*x^2 + a)^9*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx = -\frac{a^5+9\,a^4\,b\,x^2+36\,a^3\,b^2\,x^4+84\,a^2\,b^3\,x^6+126\,a\,b^4\,x^8}{1008\,a^9\,b^6+9072\,a^8\,b^7\,x^2+36288\,a^7\,b^8\,x^4+84672\,a^6\,b^9\,x^6+127008\,a^5\,b^{10}\,x^8+127008\,a^4\,b^{11}\,x^{10}+84}$$

input
```
int(x^11/(a + b*x^2)^10,x)
```

output
```
-(a^5 + 126*b^5*x^10 + 9*a^4*b*x^2 + 126*a*b^4*x^8 + 36*a^3*b^2*x^4 + 84*a
^2*b^3*x^6)/(1008*a^9*b^6 + 1008*b^15*x^18 + 9072*a*b^14*x^16 + 9072*a^8*b
^7*x^2 + 36288*a^7*b^8*x^4 + 84672*a^6*b^9*x^6 + 127008*a^5*b^10*x^8 + 127
008*a^4*b^11*x^10 + 84672*a^3*b^12*x^12 + 36288*a^2*b^13*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.03

$$\int \frac{x^{11}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-126b^5x^{10} - 126a\,b^4x^8 - 84a^2b^3x^6 - 36a^3b^2x^4 - 9a^4b\,x^2 - a^5}{1008b^6\left(b^9x^{18} + 9a\,b^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a\right.}$$

input
```
int(x^11/(b*x^2+a)^10,x)
```

output
```
( - a**5 - 9*a**4*b*x**2 - 36*a**3*b**2*x**4 - 84*a**2*b**3*x**6 - 126*a*b
**4*x**8 - 126*b**5*x**10)/(1008*b**6*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2
*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*
a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.200 $\int \frac{x^9}{(a+bx^2)^{10}} dx$

Optimal result

Integrand size = 13, antiderivative size = 91

$$\int \frac{x^9}{(a+bx^2)^{10}} dx = -\frac{a^4}{18b^5(a+bx^2)^9} + \frac{a^3}{4b^5(a+bx^2)^8} - \frac{3a^2}{7b^5(a+bx^2)^7} + \frac{a}{3b^5(a+bx^2)^6} - \frac{1}{10b^5(a+bx^2)^5}$$

output
```
-1/18*a^4/b^5/(b*x^2+a)^9+1/4*a^3/b^5/(b*x^2+a)^8-3/7*a^2/b^5/(b*x^2+a)^7+
1/3*a/b^5/(b*x^2+a)^6-1/10/b^5/(b*x^2+a)^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{x^9}{(a+bx^2)^{10}} dx = -\frac{a^4+9a^3bx^2+36a^2b^2x^4+84ab^3x^6+126b^4x^8}{1260b^5(a+bx^2)^9}$$

input
```
Integrate[x^9/(a + b*x^2)^10,x]
```

output
```
-1/1260*(a^4 + 9*a^3*b*x^2 + 36*a^2*b^2*x^4 + 84*a*b^3*x^6 + 126*b^4*x^8)/
(b^5*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2)^{10}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4(bx^2+a)^{10}}-\frac{4a^3}{b^4(bx^2+a)^9}+\frac{6a^2}{b^4(bx^2+a)^8}-\frac{4a}{b^4(bx^2+a)^7}+\frac{1}{b^4(bx^2+a)^6}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^4}{9b^5(a+bx^2)^9}+\frac{a^3}{2b^5(a+bx^2)^8}-\frac{6a^2}{7b^5(a+bx^2)^7}+\frac{2a}{3b^5(a+bx^2)^6}-\frac{1}{5b^5(a+bx^2)^5}\right)$$

input
```
Int[x^9/(a + b*x^2)^10,x]
```

output
```
(-1/9*a^4/(b^5*(a + b*x^2)^9) + a^3/(2*b^5*(a + b*x^2)^8) - (6*a^2)/(7*b^5
*(a + b*x^2)^7) + (2*a)/(3*b^5*(a + b*x^2)^6) - 1/(5*b^5*(a + b*x^2)^5))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{126 b^{4} x^{8}+84 a \,b^{3} x^{6}+36 a^{2} b^{2} x^{4}+9 a^{3} b \,x^{2}+a^{4}}{1260 \left(b \,x^{2}+a \right)^{9} b^{5}}$	56
orering	$-\frac{126 b^{4} x^{8}+84 a \,b^{3} x^{6}+36 a^{2} b^{2} x^{4}+9 a^{3} b \,x^{2}+a^{4}}{1260 \left(b \,x^{2}+a \right)^{9} b^{5}}$	56
norman	$\frac{-\frac{a^{4}}{1260 b^{5}}-\frac{a^{3} x^{2}}{140 b^{4}}-\frac{a^{2} x^{4}}{35 b^{3}}-\frac{a \,x^{6}}{15 b^{2}}-\frac{x^{8}}{10 b}}{\left(b \,x^{2}+a \right)^{9}}$	59
risch	$\frac{-\frac{a^{4}}{1260 b^{5}}-\frac{a^{3} x^{2}}{140 b^{4}}-\frac{a^{2} x^{4}}{35 b^{3}}-\frac{a \,x^{6}}{15 b^{2}}-\frac{x^{8}}{10 b}}{\left(b \,x^{2}+a \right)^{9}}$	59
parallelrisch	$\frac{-126 b^{8} x^{8}-84 a \,b^{7} x^{6}-36 a^{2} b^{6} x^{4}-9 a^{3} b^{5} x^{2}-a^{4} b^{4}}{1260 b^{9} \left(b \,x^{2}+a \right)^{9}}$	63
default	$-\frac{a^{4}}{18 b^{5} \left(b \,x^{2}+a \right)^{9}}+\frac{a^{3}}{4 b^{5} \left(b \,x^{2}+a \right)^{8}}-\frac{3 a^{2}}{7 b^{5} \left(b \,x^{2}+a \right)^{7}}+\frac{a}{3 b^{5} \left(b \,x^{2}+a \right)^{6}}-\frac{1}{10 b^{5} \left(b \,x^{2}+a \right)^{5}}$	82

input
```
int(x^9/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/1260*(126*b^4*x^8+84*a*b^3*x^6+36*a^2*b^2*x^4+9*a^3*b*x^2+a^4)/(b*x^2+a
)^9/b^5
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx = -\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7}$$

input
```
integrate(x^9/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/
(b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4
*b^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6
*x^2 + a^9*b^5)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.70

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx = \frac{-a^4-9a^3bx^2-36a^2b^2x^4-84ab^3x^6-126b^4x^8}{1260a^9b^5+11340a^8b^6x^2+45360a^7b^7x^4+105840a^6b^8x^6+158760a^5b^9x^8+158760a^4b^{10}x^{10}+105840a^3b}$$

input
```
integrate(x**9/(b*x**2+a)**10,x)
```

output
```
(-a**4 - 9*a**3*b*x**2 - 36*a**2*b**2*x**4 - 84*a*b**3*x**6 - 126*b**4*x**
8)/(1260*a**9*b**5 + 11340*a**8*b**6*x**2 + 45360*a**7*b**7*x**4 + 105840*
a**6*b**8*x**6 + 158760*a**5*b**9*x**8 + 158760*a**4*b**10*x**10 + 105840*
a**3*b**11*x**12 + 45360*a**2*b**12*x**14 + 11340*a*b**13*x**16 + 1260*b**
14*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,(b^{14}x^{18}+9\,ab^{13}x^{16}+36\,a^2b^{12}x^{14}+84\,a^3b^{11}x^{12}+126\,a^4b^{10}x^{10}+126\,a^5b^9x^8+84\,a^6b^8x^6+36\,a^7b^7}$$

input
```
integrate(x^9/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/
(b^14*x^18 + 9*a*b^13*x^16 + 36*a^2*b^12*x^14 + 84*a^3*b^11*x^12 + 126*a^4
*b^10*x^10 + 126*a^5*b^9*x^8 + 84*a^6*b^8*x^6 + 36*a^7*b^7*x^4 + 9*a^8*b^6
*x^2 + a^9*b^5)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx = -\frac{126\,b^4x^8+84\,ab^3x^6+36\,a^2b^2x^4+9\,a^3bx^2+a^4}{1260\,(bx^2+a)^9b^5}$$

input
```
integrate(x^9/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/1260*(126*b^4*x^8 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4 + 9*a^3*b*x^2 + a^4)/
((b*x^2 + a)^9*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{a^4 + 9\,a^3\,b\,x^2 + 36\,a^2\,b^2\,x^4 + 84\,a\,b^3\,x^6 + 12}{1260\,a^9\,b^5 + 11340\,a^8\,b^6\,x^2 + 45360\,a^7\,b^7\,x^4 + 105840\,a^6\,b^8\,x^6 + 158760\,a^5\,b^9\,x^8 + 158760\,a^4\,b^{10}\,x^{10} + 1}$$

input
```
int(x^9/(a + b*x^2)^10,x)
```

output
```
-(a^4 + 126*b^4*x^8 + 9*a^3*b*x^2 + 84*a*b^3*x^6 + 36*a^2*b^2*x^4)/(1260*a
^9*b^5 + 1260*b^14*x^18 + 11340*a*b^13*x^16 + 11340*a^8*b^6*x^2 + 45360*a^
7*b^7*x^4 + 105840*a^6*b^8*x^6 + 158760*a^5*b^9*x^8 + 158760*a^4*b^10*x^10
 + 105840*a^3*b^11*x^12 + 45360*a^2*b^12*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

$$\int \frac{x^9}{(a+bx^2)^{10}}\,dx$$

$$= \frac{-126b^4x^8 - 84a\,b^3x^6 - 36a^2b^2x^4 - 9a^3b\,x^2 - a^4}{1260b^5\,(b^9x^{18} + 9a\,b^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a}$$

input
```
int(x^9/(b*x^2+a)^10,x)
```

output
```
( - a**4 - 9*a**3*b*x**2 - 36*a**2*b**2*x**4 - 84*a*b**3*x**6 - 126*b**4*x
**8)/(1260*b**5*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x
**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a
**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.201 $\int \frac{x^7}{(a+bx^2)^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int \frac{x^7}{(a+bx^2)^{10}} \, dx = \frac{a^3}{18b^4 (a+bx^2)^9} - \frac{3a^2}{16b^4 (a+bx^2)^8} + \frac{3a}{14b^4 (a+bx^2)^7} - \frac{1}{12b^4 (a+bx^2)^6}$$

output
```
1/18*a^3/b^4/(b*x^2+a)^9-3/16*a^2/b^4/(b*x^2+a)^8+3/14*a/b^4/(b*x^2+a)^7-1
/12/b^4/(b*x^2+a)^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.64

$$\int \frac{x^7}{(a+bx^2)^{10}} \, dx = -\frac{a^3 + 9a^2bx^2 + 36ab^2x^4 + 84b^3x^6}{1008b^4 (a+bx^2)^9}$$

input
```
Integrate[x^7/(a + b*x^2)^10,x]
```

output
```
-1/1008*(a^3 + 9*a^2*b*x^2 + 36*a*b^2*x^4 + 84*b^3*x^6)/(b^4*(a + b*x^2)^9
)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^{10}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^{10}}+\frac{3a^2}{b^3(bx^2+a)^9}-\frac{3a}{b^3(bx^2+a)^8}+\frac{1}{b^3(bx^2+a)^7}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a^3}{9b^4(a+bx^2)^9}-\frac{3a^2}{8b^4(a+bx^2)^8}+\frac{3a}{7b^4(a+bx^2)^7}-\frac{1}{6b^4(a+bx^2)^6}\right)$$

input
```
Int[x^7/(a + b*x^2)^10,x]
```

output
```
(a^3/(9*b^4*(a + b*x^2)^9) - (3*a^2)/(8*b^4*(a + b*x^2)^8) + (3*a)/(7*b^4*
(a + b*x^2)^7) - 1/(6*b^4*(a + b*x^2)^6))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{84b^{3}x^{6}+36a\,b^{2}x^{4}+9a^{2}b\,x^{2}+a^{3}}{1008\left(b\,x^{2}+a\right)^{9}b^{4}}$	45
orering	$-\frac{84b^{3}x^{6}+36a\,b^{2}x^{4}+9a^{2}b\,x^{2}+a^{3}}{1008\left(b\,x^{2}+a\right)^{9}b^{4}}$	45
norman	$\frac{-\frac{a^{3}}{1008b^{4}}-\frac{a^{2}x^{2}}{112b^{3}}-\frac{a\,x^{4}}{28b^{2}}-\frac{x^{6}}{12b}}{\left(b\,x^{2}+a\right)^{9}}$	48
risch	$\frac{-\frac{a^{3}}{1008b^{4}}-\frac{a^{2}x^{2}}{112b^{3}}-\frac{a\,x^{4}}{28b^{2}}-\frac{x^{6}}{12b}}{\left(b\,x^{2}+a\right)^{9}}$	48
parallelrisch	$\frac{-84b^{8}x^{6}-36a\,b^{7}x^{4}-9a^{2}b^{6}x^{2}-a^{3}b^{5}}{1008b^{9}\left(b\,x^{2}+a\right)^{9}}$	52
default	$\frac{a^{3}}{18b^{4}\left(b\,x^{2}+a\right)^{9}}-\frac{3a^{2}}{16b^{4}\left(b\,x^{2}+a\right)^{8}}+\frac{3a}{14b^{4}\left(b\,x^{2}+a\right)^{7}}-\frac{1}{12b^{4}\left(b\,x^{2}+a\right)^{6}}$	65

input
```
int(x^7/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/1008*(84*b^3*x^6+36*a*b^2*x^4+9*a^2*b*x^2+a^3)/(b*x^2+a)^9/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(64) = 128$.

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.88

$$\int \frac{x^7}{\left(a+bx^2\right)^{10}}\,dx =$$

$$-\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\,\left(b^{13}x^{18}+9\,ab^{12}x^{16}+36\,a^2b^{11}x^{14}+84\,a^3b^{10}x^{12}+126\,a^4b^9x^{10}+126\,a^5b^8x^8+84\,a^6b^7x^6+36\,a^7b^6x\right.}$$

input
```
integrate(x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b
^12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^
5*b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(66) = 132$.

Time = 0.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \frac{x^7}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-a^3 - 9a^2bx^2 - 36ab^2x^4 - 84b^3x^6}{1008a^9b^4 + 9072a^8b^5x^2 + 36288a^7b^6x^4 + 84672a^6b^7x^6 + 127008a^5b^8x^8 + 127008a^4b^9x^{10} + 84672a^3b^{10}x^{12}}$$

input
```
integrate(x**7/(b*x**2+a)**10,x)
```

output
```
(-a**3 - 9*a**2*b*x**2 - 36*a*b**2*x**4 - 84*b**3*x**6)/(1008*a**9*b**4 +
9072*a**8*b**5*x**2 + 36288*a**7*b**6*x**4 + 84672*a**6*b**7*x**6 + 127008
*a**5*b**8*x**8 + 127008*a**4*b**9*x**10 + 84672*a**3*b**10*x**12 + 36288*
a**2*b**11*x**14 + 9072*a*b**12*x**16 + 1008*b**13*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(64) = 128$.

Time = 0.04 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.88

$$\int \frac{x^7}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{84\,b^3x^6 + 36\,ab^2x^4 + 9\,a^2bx^2 + a^3}{1008\,(b^{13}x^{18} + 9\,ab^{12}x^{16} + 36\,a^2b^{11}x^{14} + 84\,a^3b^{10}x^{12} + 126\,a^4b^9x^{10} + 126\,a^5b^8x^8 + 84\,a^6b^7x^6 + 36\,a^7b^6}$$

input
```
integrate(x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/(b^13*x^18 + 9*a*b
^12*x^16 + 36*a^2*b^11*x^14 + 84*a^3*b^10*x^12 + 126*a^4*b^9*x^10 + 126*a^
5*b^8*x^8 + 84*a^6*b^7*x^6 + 36*a^7*b^6*x^4 + 9*a^8*b^5*x^2 + a^9*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \frac{x^7}{(a+bx^2)^{10}}\,dx = -\frac{84\,b^3x^6+36\,ab^2x^4+9\,a^2bx^2+a^3}{1008\,(bx^2+a)^9b^4}$$

input
```
integrate(x^7/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/1008*(84*b^3*x^6 + 36*a*b^2*x^4 + 9*a^2*b*x^2 + a^3)/((b*x^2 + a)^9*b^4
)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.89

$$\int \frac{x^7}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{\frac{a^3}{1008\,b^4}+\frac{x^6}{12\,b}+\frac{a\,x^4}{28\,b^2}+\frac{a^2\,x^2}{112\,b^3}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a}$$

input
```
int(x^7/(a + b*x^2)^10,x)
```

output
```
-(a^3/(1008*b^4) + x^6/(12*b) + (a*x^4)/(28*b^2) + (a^2*x^2)/(112*b^3))/(a
^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x
^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^1
4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.86

$$\int \frac{x^7}{(a+bx^2)^{10}}\,dx$$

$$= \frac{-84b^3x^6 - 36a\,b^2x^4 - 9a^2b\,x^2 - a^3}{1008b^4\left(b^9x^{18} + 9a\,b^8x^{16} + 36a^2b^7x^{14} + 84a^3b^6x^{12} + 126a^4b^5x^{10} + 126a^5b^4x^8 + 84a^6b^3x^6 + 36a^7b^2x^4 + 9a\right.}$$

input
```
int(x^7/(b*x^2+a)^10,x)
```

output
```
( - a**3 - 9*a**2*b*x**2 - 36*a*b**2*x**4 - 84*b**3*x**6)/(1008*b**4*(a**9
 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x
**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*
b**8*x**16 + b**9*x**18))
```

3.202 $\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx = -\frac{a^2}{18b^3\left(a+bx^2\right)^9}+\frac{a}{8b^3\left(a+bx^2\right)^8}-\frac{1}{14b^3\left(a+bx^2\right)^7}$$

output
```
-1/18*a^2/b^3/(b*x^2+a)^9+1/8*a/b^3/(b*x^2+a)^8-1/14/b^3/(b*x^2+a)^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx = -\frac{a^2+9abx^2+36b^2x^4}{504b^3\left(a+bx^2\right)^9}$$

input
```
Integrate[x^5/(a + b*x^2)^10,x]
```

output
```
-1/504*(a^2 + 9*a*b*x^2 + 36*b^2*x^4)/(b^3*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{10}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2 (bx^2+a)^{10}}-\frac{2a}{b^2 (bx^2+a)^9}+\frac{1}{b^2 (bx^2+a)^8}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{a^2}{9b^3 (a+bx^2)^9}+\frac{a}{4b^3 (a+bx^2)^8}-\frac{1}{7b^3 (a+bx^2)^7}\right)$$

input
```
Int[x^5/(a + b*x^2)^10,x]
```

output
```
(-1/9*a^2/(b^3*(a + b*x^2)^9) + a/(4*b^3*(a + b*x^2)^8) - 1/(7*b^3*(a + b*
x^2)^7))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{36b^2x^4+9ab\,x^2+a^2}{504(b\,x^2+a)^9b^3}$	34
orering	$-\frac{36b^2x^4+9ab\,x^2+a^2}{504(b\,x^2+a)^9b^3}$	34
norman	$\frac{-\frac{a^2}{504b^3}-\frac{a\,x^2}{56b^2}-\frac{x^4}{14b}}{(b\,x^2+a)^9}$	37
risch	$\frac{-\frac{a^2}{504b^3}-\frac{a\,x^2}{56b^2}-\frac{x^4}{14b}}{(b\,x^2+a)^9}$	37
parallelrisch	$\frac{-36b^8x^4-9a\,b^7x^2-a^2b^6}{504b^9(b\,x^2+a)^9}$	41
default	$-\frac{a^2}{18b^3(b\,x^2+a)^9}+\frac{a}{8b^3(b\,x^2+a)^8}-\frac{1}{14b^3(b\,x^2+a)^7}$	48

input
```
int(x^5/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/504*(36*b^2*x^4+9*a*b*x^2+a^2)/(b*x^2+a)^9/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(47) = 94.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \frac{x^5}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4}$$

input
```
integrate(x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*
b^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*
b^6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(46) = 92$.

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.47

$$\int \frac{x^5}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-a^2-9abx^2-36b^2x^4}{504a^9b^3+4536a^8b^4x^2+18144a^7b^5x^4+42336a^6b^6x^6+63504a^5b^7x^8+63504a^4b^8x^{10}+42336a^3b^9x^{12}+1}$$

input
```
integrate(x**5/(b*x**2+a)**10,x)
```

output
```
(-a**2 - 9*a*b*x**2 - 36*b**2*x**4)/(504*a**9*b**3 + 4536*a**8*b**4*x**2 +
 18144*a**7*b**5*x**4 + 42336*a**6*b**6*x**6 + 63504*a**5*b**7*x**8 + 6350
4*a**4*b**8*x**10 + 42336*a**3*b**9*x**12 + 18144*a**2*b**10*x**14 + 4536*
a*b**11*x**16 + 504*b**12*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.34

$$\int \frac{x^5}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{36\,b^2x^4+9\,abx^2+a^2}{504\,(b^{12}x^{18}+9\,ab^{11}x^{16}+36\,a^2b^{10}x^{14}+84\,a^3b^9x^{12}+126\,a^4b^8x^{10}+126\,a^5b^7x^8+84\,a^6b^6x^6+36\,a^7b^5x^4}$$

input
```
integrate(x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/(b^12*x^18 + 9*a*b^11*x^16 + 36*a^2*
b^10*x^14 + 84*a^3*b^9*x^12 + 126*a^4*b^8*x^10 + 126*a^5*b^7*x^8 + 84*a^6*
b^6*x^6 + 36*a^7*b^5*x^4 + 9*a^8*b^4*x^2 + a^9*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx = -\frac{36\,b^2x^4 + 9\,abx^2 + a^2}{504\,\left(bx^2+a\right)^9 b^3}$$

input
```
integrate(x^5/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/504*(36*b^2*x^4 + 9*a*b*x^2 + a^2)/((b*x^2 + a)^9*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.36

$$\int \frac{x^5}{\left(a+bx^2\right)^{10}}\,dx =$$

$$-\frac{\frac{a^2}{504\,b^3}+\frac{x^4}{14\,b}+\frac{a\,x^2}{56\,b^2}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a}$$

input
```
int(x^5/(a + b*x^2)^10,x)
```

output
```
-(a^2/(504*b^3) + x^4/(14*b) + (a*x^2)/(56*b^2))/(a^9 + b^9*x^18 + 9*a^8*b
*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 +
126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.32

$$\int \frac{x^5}{(a+bx^2)^{10}}\,dx$$

$$= \frac{-36b^2x^4-9ab\,x^2-a^2}{504b^3\,(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a^8}$$

input

```
int(x^5/(b*x^2+a)^10,x)
```

output

```
( - a**2 - 9*a*b*x**2 - 36*b**2*x**4)/(504*b**3*(a**9 + 9*a**8*b*x**2 + 36
*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x
**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**
18))
```

3.203 $\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 34

$$\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx = \frac{a}{18b^2\left(a+bx^2\right)^9} - \frac{1}{16b^2\left(a+bx^2\right)^8}$$

output
```
1/18*a/b^2/(b*x^2+a)^9-1/16/b^2/(b*x^2+a)^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\left(a+bx^2\right)^{10}}\,dx = -\frac{a+9bx^2}{144b^2\left(a+bx^2\right)^9}$$

input
```
Integrate[x^3/(a + b*x^2)^10,x]
```

output
```
-1/144*(a + 9*b*x^2)/(b^2*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{1}{b\,(bx^2+a)^9}-\frac{a}{b\,(bx^2+a)^{10}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{a}{9b^2\,(a+bx^2)^9}-\frac{1}{8b^2\,(a+bx^2)^8}\right)$$

input
```
Int[x^3/(a + b*x^2)^10,x]
```

output
```
(a/(9*b^2*(a + b*x^2)^9) - 1/(8*b^2*(a + b*x^2)^8))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{9b\,x^{2}+a}{144\left(b\,x^{2}+a\right)^{9} b^{2}}$	23
orering	$-\frac{9b\,x^{2}+a}{144\left(b\,x^{2}+a\right)^{9} b^{2}}$	23
norman	$\frac{-\frac{a}{144 b^{2}}-\frac{x^{2}}{16 b}}{\left(b\,x^{2}+a\right)^{9}}$	26
risch	$\frac{-\frac{a}{144 b^{2}}-\frac{x^{2}}{16 b}}{\left(b\,x^{2}+a\right)^{9}}$	26
parallelrisch	$\frac{-9 b^{8} x^{2}-a\,b^{7}}{144 b^{9} \left(b\,x^{2}+a\right)^{9}}$	30
default	$\frac{a}{18 b^{2} \left(b\,x^{2}+a\right)^{9}}-\frac{1}{16 b^{2} \left(b\,x^{2}+a\right)^{8}}$	31

input
```
int(x^3/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/144*(9*b*x^2+a)/(b*x^2+a)^9/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.

Time = 0.06 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.32

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{9\,bx^2+a}{144\,(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4}$$

input
```
integrate(x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3
*b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b
^4*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(29) = 58$.

Time = 0.65 (sec) , antiderivative size = 119, normalized size of antiderivative = 3.50

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx$$

$$= \frac{-a-9bx^2}{144a^9b^2+1296a^8b^3x^2+5184a^7b^4x^4+12096a^6b^5x^6+18144a^5b^6x^8+18144a^4b^7x^{10}+12096a^3b^8x^{12}+51}$$

input
```
integrate(x**3/(b*x**2+a)**10,x)
```

output
```
(-a - 9*b*x**2)/(144*a**9*b**2 + 1296*a**8*b**3*x**2 + 5184*a**7*b**4*x**4
 + 12096*a**6*b**5*x**6 + 18144*a**5*b**6*x**8 + 18144*a**4*b**7*x**10 + 1
2096*a**3*b**8*x**12 + 5184*a**2*b**9*x**14 + 1296*a*b**10*x**16 + 144*b**
11*x**18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(30) = 60$.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 3.32

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{9\,bx^2+a}{144\,(b^{11}x^{18}+9\,ab^{10}x^{16}+36\,a^2b^9x^{14}+84\,a^3b^8x^{12}+126\,a^4b^7x^{10}+126\,a^5b^6x^8+84\,a^6b^5x^6+36\,a^7b^4x^4}$$

input
```
integrate(x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/144*(9*b*x^2 + a)/(b^11*x^18 + 9*a*b^10*x^16 + 36*a^2*b^9*x^14 + 84*a^3
*b^8*x^12 + 126*a^4*b^7*x^10 + 126*a^5*b^6*x^8 + 84*a^6*b^5*x^6 + 36*a^7*b
^4*x^4 + 9*a^8*b^3*x^2 + a^9*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx = -\frac{9\,bx^2+a}{144\,(bx^2+a)^9b^2}$$

input
```
integrate(x^3/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/144*(9*b*x^2 + a)/((b*x^2 + a)^9*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.35

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{\frac{a}{144\,b^2}+\frac{x^2}{16\,b}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a}$$

input
```
int(x^3/(a + b*x^2)^10,x)
```

output
```
-(a/(144*b^2) + x^2/(16*b))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 +
 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84
*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.29

$$\int \frac{x^3}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-9b\,x^2-a}{144b^2\left(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a^8\right.}$$

input
```
int(x^3/(b*x^2+a)^10,x)
```

output
```
( - a - 9*b*x**2)/(144*b**2*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84
*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*
x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.204 $\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx=-\frac{1}{18b\left(a+bx^2\right)^9}$$

output
```
-1/18/b/(b*x^2+a)^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^2\right)^{10}}\,dx=-\frac{1}{18b\left(a+bx^2\right)^9}$$

input
```
Integrate[x/(a + b*x^2)^10,x]
```

output
```
-1/18*1/(b*(a + b*x^2)^9)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{10}} \, dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{18b\,(a+bx^2)^9}$$

input
```
Int[x/(a + b*x^2)^10,x]
```

output
```
-1/18*1/(b*(a + b*x^2)^9)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{18b(b\,x^2+a)^9}$	15
derivativedivides	$-\frac{1}{18b(b\,x^2+a)^9}$	15
default	$-\frac{1}{18b(b\,x^2+a)^9}$	15
norman	$-\frac{1}{18b(b\,x^2+a)^9}$	15
risch	$-\frac{1}{18b(b\,x^2+a)^9}$	15
parallelrisch	$-\frac{1}{18b(b\,x^2+a)^9}$	15
orering	$-\frac{1}{18b(b\,x^2+a)^9}$	15

input
```
int(x/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/18/b/(b*x^2+a)^9
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.44

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18\,(b^{10}x^{18}+9\,ab^9x^{16}+36\,a^2b^8x^{14}+84\,a^3b^7x^{12}+126\,a^4b^6x^{10}+126\,a^5b^5x^8+84\,a^6b^4x^6+36\,a^7b^3x^4+}$$

input
```
integrate(x/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/18/(b^10*x^18 + 9*a*b^9*x^16 + 36*a^2*b^8*x^14 + 84*a^3*b^7*x^12 + 126*
a^4*b^6*x^10 + 126*a^5*b^5*x^8 + 84*a^6*b^4*x^6 + 36*a^7*b^3*x^4 + 9*a^8*b
^2*x^2 + a^9*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(14) = 28$.

Time = 0.66 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.88

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18a^9b+162a^8b^2x^2+648a^7b^3x^4+1512a^6b^4x^6+2268a^5b^5x^8+2268a^4b^6x^{10}+1512a^3b^7x^{12}+648a^2b^8x}$$

input
```
integrate(x/(b*x**2+a)**10,x)
```

output
```
-1/(18*a**9*b + 162*a**8*b**2*x**2 + 648*a**7*b**3*x**4 + 1512*a**6*b**4*x
**6 + 2268*a**5*b**5*x**8 + 2268*a**4*b**6*x**10 + 1512*a**3*b**7*x**12 +
648*a**2*b**8*x**14 + 162*a*b**9*x**16 + 18*b**10*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18\,(bx^2+a)^9b}$$

input
```
integrate(x/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/18/((b*x^2 + a)^9*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18\,(bx^2+a)^9 b}$$

input
```
integrate(x/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-1/18/((b*x^2 + a)^9*b)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{10}}\,dx = -\frac{1}{18\,b\,(b\,x^2+a)^9}$$

input
```
int(x/(a + b*x^2)^10,x)
```

output
```
-1/(18*b*(a + b*x^2)^9)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 6.38

$$\int \frac{x}{(a+bx^2)^{10}}\,dx =$$

$$-\frac{1}{18b\,(b^9x^{18}+9a\,b^8x^{16}+36a^2b^7x^{14}+84a^3b^6x^{12}+126a^4b^5x^{10}+126a^5b^4x^8+84a^6b^3x^6+36a^7b^2x^4+9a$$

input
```
int(x/(b*x^2+a)^10,x)
```

output
```
( - 1)/(18*b*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6
 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2
*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.205 $\int \frac{1}{x(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 166

$$\begin{aligned}\int \frac{1}{x(a+bx^2)^{10}}\,dx = {} & \frac{1}{18a(a+bx^2)^9}+\frac{1}{16a^2(a+bx^2)^8}+\frac{1}{14a^3(a+bx^2)^7} \\ & +\frac{1}{12a^4(a+bx^2)^6}+\frac{1}{10a^5(a+bx^2)^5} \\ & +\frac{1}{8a^6(a+bx^2)^4}+\frac{1}{6a^7(a+bx^2)^3}+\frac{1}{4a^8(a+bx^2)^2} \\ & +\frac{1}{2a^9(a+bx^2)}+\frac{\log(x)}{a^{10}}-\frac{\log(a+bx^2)}{2a^{10}}\end{aligned}$$

output
```
1/18/a/(b*x^2+a)^9+1/16/a^2/(b*x^2+a)^8+1/14/a^3/(b*x^2+a)^7+1/12/a^4/(b*x
^2+a)^6+1/10/a^5/(b*x^2+a)^5+1/8/a^6/(b*x^2+a)^4+1/6/a^7/(b*x^2+a)^3+1/4/a
^8/(b*x^2+a)^2+1/2/a^9/(b*x^2+a)+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{1}{x(a+bx^2)^{10}}\,dx = \frac{\frac{a(7129a^8+41481a^7bx^2+120564a^6b^2x^4+210756a^5b^3x^6+236754a^4b^4x^8+173250a^3b^5x^{10}+80220a^2b^6x^{12}+21420ab^7x^{14}+2520b^8x^{16})}{(a+bx^2)^9}+5040}{5040a^{10}}$$

input
```
Integrate[1/(x*(a + b*x^2)^10),x]
```

output
```
((a*(7129*a^8 + 41481*a^7*b*x^2 + 120564*a^6*b^2*x^4 + 210756*a^5*b^3*x^6
+ 236754*a^4*b^4*x^8 + 173250*a^3*b^5*x^10 + 80220*a^2*b^6*x^12 + 21420*a*
b^7*x^14 + 2520*b^8*x^16))/(a + b*x^2)^9 + 5040*Log[x] - 2520*Log[a + b*x^
2])/(5040*a^10)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{b}{a^{10}(bx^2+a)}-\frac{b}{a^9(bx^2+a)^2}-\frac{b}{a^8(bx^2+a)^3}-\frac{b}{a^7(bx^2+a)^4}-\frac{b}{a^6(bx^2+a)^5}-\frac{b}{a^5(bx^2+a)^6}-\frac{b}{a^4(b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(a+bx^2\right)}{a^{10}}+\frac{\log\left(x^2\right)}{a^{10}}+\frac{1}{a^9\left(a+bx^2\right)}+\frac{1}{2a^8\left(a+bx^2\right)^2}+\frac{1}{3a^7\left(a+bx^2\right)^3}+\frac{1}{4a^6\left(a+bx^2\right)^4}+\frac{1}{5a^5\left(a+bx\right.}\right.$$

input
```
Int[1/(x*(a + b*x^2)^10),x]
```

output
```
(1/(9*a*(a + b*x^2)^9) + 1/(8*a^2*(a + b*x^2)^8) + 1/(7*a^3*(a + b*x^2)^7)
 + 1/(6*a^4*(a + b*x^2)^6) + 1/(5*a^5*(a + b*x^2)^5) + 1/(4*a^6*(a + b*x^2
)^4) + 1/(3*a^7*(a + b*x^2)^3) + 1/(2*a^8*(a + b*x^2)^2) + 1/(a^9*(a + b*x
^2)) + Log[x^2]/a^10 - Log[a + b*x^2]/a^10)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.74

method	result
risch	$\frac{\frac{7129}{5040a}+\frac{4609b\,x^{2}}{560a^{2}}+\frac{3349b^{2}x^{4}}{140a^{3}}+\frac{2509b^{3}x^{6}}{60a^{4}}+\frac{1879b^{4}x^{8}}{40a^{5}}+\frac{275b^{5}x^{10}}{8a^{6}}+\frac{191b^{6}x^{12}}{12a^{7}}+\frac{17b^{7}x^{14}}{4a^{8}}+\frac{b^{8}x^{16}}{2a^{9}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{\ln\left(x\right)}{a^{10}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{10}}$
norman	$\frac{-\frac{9b\,x^{2}}{2a^{2}}-\frac{27b^{2}x^{4}}{a^{3}}-\frac{77b^{3}x^{6}}{a^{4}}-\frac{525b^{4}x^{8}}{4a^{5}}-\frac{2877b^{5}x^{10}}{20a^{6}}-\frac{1029b^{6}x^{12}}{10a^{7}}-\frac{3267b^{7}x^{14}}{70a^{8}}-\frac{6849b^{8}x^{16}}{560a^{9}}-\frac{7129b^{9}x^{18}}{5040a^{10}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{\ln\left(x\right)}{a^{10}}-\frac{\ln\left(b\,x^{2}+a\right)}{2a^{10}}$
default	$-\frac{b\left(-\frac{a^{9}}{9b\left(b\,x^{2}+a\right)^{9}}-\frac{a^{5}}{5b\left(b\,x^{2}+a\right)^{5}}-\frac{a^{4}}{4b\left(b\,x^{2}+a\right)^{4}}-\frac{a^{7}}{7b\left(b\,x^{2}+a\right)^{7}}-\frac{a^{8}}{8b\left(b\,x^{2}+a\right)^{8}}-\frac{a^{2}}{2b\left(b\,x^{2}+a\right)^{2}}-\frac{a}{b\left(b\,x^{2}+a\right)}+\frac{\ln\left(b\,x^{2}+a\right)}{b}-\frac{a^{3}}{3b\left(b\,x^{2}\right.}}{2a^{10}}$
parallelrisch	$\frac{423360\ln\left(x\right)x^{12}a^{3}b^{6}+635040\ln\left(x\right)x^{10}a^{4}b^{5}+635040\ln\left(x\right)x^{8}a^{5}b^{4}+423360\ln\left(x\right)x^{6}a^{6}b^{3}+181440\ln\left(x\right)x^{4}a^{7}b^{2}-22680\ln\left(b\,x^{2}+\right.}{}$

input
```
int(1/x/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(7129/5040/a+4609/560*b/a^2*x^2+3349/140*b^2/a^3*x^4+2509/60*b^3/a^4*x^6+1
879/40*b^4/a^5*x^8+275/8*b^5/a^6*x^10+191/12*b^6/a^7*x^12+17/4*b^7/a^8*x^1
4+1/2*b^8/a^9*x^16)/(b*x^2+a)^9+ln(x)/a^10-1/2*ln(b*x^2+a)/a^10
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(146) = 292$.

Time = 0.07 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.40

$$\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx$$
$$= \frac{2520\,ab^8x^{16}+21420\,a^2b^7x^{14}+80220\,a^3b^6x^{12}+173250\,a^4b^5x^{10}+236754\,a^5b^4x^8+210756\,a^6b^3x^6+1205}{}$$

input
```
integrate(1/x/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/5040*(2520*a*b^8*x^16 + 21420*a^2*b^7*x^14 + 80220*a^3*b^6*x^12 + 173250
*a^4*b^5*x^10 + 236754*a^5*b^4*x^8 + 210756*a^6*b^3*x^6 + 120564*a^7*b^2*x
^4 + 41481*a^8*b*x^2 + 7129*a^9 - 2520*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b
^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^
3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log(b*x^2 + a) + 5040*(b^9*x^1
8 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 +
126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*log
(x))/(a^10*b^9*x^18 + 9*a^11*b^8*x^16 + 36*a^12*b^7*x^14 + 84*a^13*b^6*x^1
2 + 126*a^14*b^5*x^10 + 126*a^15*b^4*x^8 + 84*a^16*b^3*x^6 + 36*a^17*b^2*x
^4 + 9*a^18*b*x^2 + a^19)
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.34

$$\int \frac{1}{x\left(a+bx^2\right)^{10}}\,dx = \frac{7129a^8 + 41481a^7bx^2 + 120564a^6b^2x^4 + 210756a^5b^3x^6 + 236754a^4b^4x^8 + 173250a^3b^5x^{10} + \cdots}{5040a^{18} + 45360a^{17}bx^2 + 181440a^{16}b^2x^4 + 423360a^{15}b^3x^6 + 635040a^{14}b^4x^8 + 635040a^{13}b^5x^{10} + 423360a\cdots} + \frac{\log(x)}{a^{10}} - \frac{\log\left(\frac{a}{b}+x^2\right)}{2a^{10}}$$

input
```
integrate(1/x/(b*x**2+a)**10,x)
```

output
```
(7129*a**8 + 41481*a**7*b*x**2 + 120564*a**6*b**2*x**4 + 210756*a**5*b**3*
x**6 + 236754*a**4*b**4*x**8 + 173250*a**3*b**5*x**10 + 80220*a**2*b**6*x*
*12 + 21420*a*b**7*x**14 + 2520*b**8*x**16)/(5040*a**18 + 45360*a**17*b*x*
*2 + 181440*a**16*b**2*x**4 + 423360*a**15*b**3*x**6 + 635040*a**14*b**4*x
**8 + 635040*a**13*b**5*x**10 + 423360*a**12*b**6*x**12 + 181440*a**11*b**
7*x**14 + 45360*a**10*b**8*x**16 + 5040*a**9*b**9*x**18) + log(x)/a**10 -
log(a/b + x**2)/(2*a**10)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.29

$$\int \frac{1}{x\,(a+bx^2)^{10}}\,dx = \frac{2520\,b^8x^{16}+21420\,ab^7x^{14}+80220\,a^2b^6x^{12}+173250\,a^3b^5x^{10}+236754\,a^4b^4x^8+210756\,a^5b^3x^6+120564}{5040\,(a^9b^9x^{18}+9\,a^{10}b^8x^{16}+36\,a^{11}b^7x^{14}+84\,a^{12}b^6x^{12}+126\,a^{13}b^5x^{10}+126\,a^{14}b^4x^8+84\,a^{15}b^3x^6+3} - \frac{\log(bx^2+a)}{2\,a^{10}} + \frac{\log(x^2)}{2\,a^{10}}$$

input
```
integrate(1/x/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/5040*(2520*b^8*x^16 + 21420*a*b^7*x^14 + 80220*a^2*b^6*x^12 + 173250*a^3
*b^5*x^10 + 236754*a^4*b^4*x^8 + 210756*a^5*b^3*x^6 + 120564*a^6*b^2*x^4 +
 41481*a^7*b*x^2 + 7129*a^8)/(a^9*b^9*x^18 + 9*a^10*b^8*x^16 + 36*a^11*b^7
*x^14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 126*a^14*b^4*x^8 + 84*a^15*
b^3*x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18) - 1/2*log(b*x^2 + a)/a^10
 + 1/2*log(x^2)/a^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.82

$$\int \frac{1}{x\,(a+bx^2)^{10}}\,dx = \frac{\log(x^2)}{2\,a^{10}} - \frac{\log(|bx^2+a|)}{2\,a^{10}} + \frac{7129\,b^9x^{18}+66681\,ab^8x^{16}+278064\,a^2b^7x^{14}+679056\,a^3b^6x^{12}+1071504\,a^4b^5x^{10}+1135008\,a^5b^4x^8+8}{5040\,(bx^2+a)^9a^{10}}$$

input
```
integrate(1/x/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^10 - 1/2*log(abs(b*x^2 + a))/a^10 + 1/5040*(7129*b^9*x^18 +
 66681*a*b^8*x^16 + 278064*a^2*b^7*x^14 + 679056*a^3*b^6*x^12 + 1071504*a^
4*b^5*x^10 + 1135008*a^5*b^4*x^8 + 809592*a^6*b^3*x^6 + 377208*a^7*b^2*x^4
 + 105642*a^8*b*x^2 + 14258*a^9)/((b*x^2 + a)^9*a^10)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.27

$$\int \frac{1}{x(a+bx^2)^{10}}\,dx = \frac{\ln(x)}{a^{10}} + \frac{\frac{7129}{5040\,a}+\frac{4609\,b\,x^2}{560\,a^2}+\frac{3349\,b^2\,x^4}{140\,a^3}+\frac{2509\,b^3\,x^6}{60\,a^4}+\frac{1879\,b^4\,x^8}{40\,a^5}+\frac{275\,b^5\,x^{10}}{8\,a^6}+\frac{191\,b^6\,x^{12}}{12\,a^7}+\frac{17\,b^7\,x^{14}}{4\,a^8}+\frac{b^8\,x^{16}}{2\,a^9}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a} - \frac{\ln(b\,x^2+a)}{2\,a^{10}}$$

input
```
int(1/(x*(a + b*x^2)^10),x)
```

output
```
log(x)/a^10 + (7129/(5040*a) + (4609*b*x^2)/(560*a^2) + (3349*b^2*x^4)/(14
0*a^3) + (2509*b^3*x^6)/(60*a^4) + (1879*b^4*x^8)/(40*a^5) + (275*b^5*x^10
)/(8*a^6) + (191*b^6*x^12)/(12*a^7) + (17*b^7*x^14)/(4*a^8) + (b^8*x^16)/(
2*a^9))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84
*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a
^2*b^7*x^14) - log(a + b*x^2)/(2*a^10)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.92

$$\int \frac{1}{x(a+bx^2)^{10}}\,dx = \frac{-2520\log(b\,x^2+a)\,b^9x^{18}-2520\log(b\,x^2+a)\,a^9-280b^9x^{18}+5040\log(x)\,b^9x^{18}+38961a^8b\,x^2+11048}{}$$

input
```
int(1/x/(b*x^2+a)^10,x)
```

output

```
( - 2520*log(a + b*x**2)*a**9 - 22680*log(a + b*x**2)*a**8*b*x**2 - 90720*
log(a + b*x**2)*a**7*b**2*x**4 - 211680*log(a + b*x**2)*a**6*b**3*x**6 - 3
17520*log(a + b*x**2)*a**5*b**4*x**8 - 317520*log(a + b*x**2)*a**4*b**5*x*
*10 - 211680*log(a + b*x**2)*a**3*b**6*x**12 - 90720*log(a + b*x**2)*a**2*
b**7*x**14 - 22680*log(a + b*x**2)*a*b**8*x**16 - 2520*log(a + b*x**2)*b**
9*x**18 + 5040*log(x)*a**9 + 45360*log(x)*a**8*b*x**2 + 181440*log(x)*a**7
*b**2*x**4 + 423360*log(x)*a**6*b**3*x**6 + 635040*log(x)*a**5*b**4*x**8 +
 635040*log(x)*a**4*b**5*x**10 + 423360*log(x)*a**3*b**6*x**12 + 181440*lo
g(x)*a**2*b**7*x**14 + 45360*log(x)*a*b**8*x**16 + 5040*log(x)*b**9*x**18
+ 6849*a**9 + 38961*a**8*b*x**2 + 110484*a**7*b**2*x**4 + 187236*a**6*b**3
*x**6 + 201474*a**5*b**4*x**8 + 137970*a**4*b**5*x**10 + 56700*a**3*b**6*x
**12 + 11340*a**2*b**7*x**14 - 280*b**9*x**18)/(5040*a**10*(a**9 + 9*a**8*
b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*
a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16
 + b**9*x**18))
```

3.206 $\int \frac{1}{x^3(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 184

$$\begin{aligned}\int \frac{1}{x^3(a+bx^2)^{10}}\,dx = &-\frac{1}{2a^{10}x^2}-\frac{b}{18a^2(a+bx^2)^9}-\frac{b}{8a^3(a+bx^2)^8}\\&-\frac{3b}{14a^4(a+bx^2)^7}-\frac{b}{3a^5(a+bx^2)^6}-\frac{b}{2a^6(a+bx^2)^5}\\&-\frac{3b}{4a^7(a+bx^2)^4}-\frac{7b}{6a^8(a+bx^2)^3}-\frac{2b}{a^9(a+bx^2)^2}\\&-\frac{9b}{2a^{10}(a+bx^2)}-\frac{10b\log(x)}{a^{11}}+\frac{5b\log(a+bx^2)}{a^{11}}\end{aligned}$$

output
```
-1/2/a^10/x^2-1/18*b/a^2/(b*x^2+a)^9-1/8*b/a^3/(b*x^2+a)^8-3/14*b/a^4/(b*x
^2+a)^7-1/3*b/a^5/(b*x^2+a)^6-1/2*b/a^6/(b*x^2+a)^5-3/4*b/a^7/(b*x^2+a)^4-
7/6*b/a^8/(b*x^2+a)^3-2*b/a^9/(b*x^2+a)^2-9/2*b/a^10/(b*x^2+a)-10*b*ln(x)/
a^11+5*b*ln(b*x^2+a)/a^11
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3 (a+bx^2)^{10}}\,dx = -\frac{\frac{a(252a^9+7129a^8bx^2+41481a^7b^2x^4+120564a^6b^3x^6+210756a^5b^4x^8+236754a^4b^5x^{10}+173250a^3b^6x^{12}+80220a^2b^7x^{14}+21420ab^8x^{16}+2520b}{x^2(a+bx^2)^9}}{504a^{11}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^10),x]
```

output
```
-1/504*((a*(252*a^9 + 7129*a^8*b*x^2 + 41481*a^7*b^2*x^4 + 120564*a^6*b^3*
x^6 + 210756*a^5*b^4*x^8 + 236754*a^4*b^5*x^10 + 173250*a^3*b^6*x^12 + 802
20*a^2*b^7*x^14 + 21420*a*b^8*x^16 + 2520*b^9*x^18))/(x^2*(a + b*x^2)^9) +
 5040*b*Log[x] - 2520*b*Log[a + b*x^2])/a^11
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+a)^{10}}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{10b^2}{a^{11}(bx^2+a)}+\frac{9b^2}{a^{10}(bx^2+a)^2}+\frac{8b^2}{a^9(bx^2+a)^3}+\frac{7b^2}{a^8(bx^2+a)^4}+\frac{6b^2}{a^7(bx^2+a)^5}+\frac{5b^2}{a^6(bx^2+a)^6}+\frac{4}{a^5(bx}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{10b\log\left(x^2\right)}{a^{11}}+\frac{10b\log\left(a+bx^2\right)}{a^{11}}-\frac{9b}{a^{10}\left(a+bx^2\right)}-\frac{1}{a^{10}x^2}-\frac{4b}{a^9\left(a+bx^2\right)^2}-\frac{7b}{3a^8\left(a+bx^2\right)^3}-\frac{3b}{2a^7\left(a+bx^2\right.}\right.$$

input
```
Int[1/(x^3*(a + b*x^2)^10),x]
```

output
```
(-(1/(a^10*x^2)) - b/(9*a^2*(a + b*x^2)^9) - b/(4*a^3*(a + b*x^2)^8) - (3*
b)/(7*a^4*(a + b*x^2)^7) - (2*b)/(3*a^5*(a + b*x^2)^6) - b/(a^6*(a + b*x^2
)^5) - (3*b)/(2*a^7*(a + b*x^2)^4) - (7*b)/(3*a^8*(a + b*x^2)^3) - (4*b)/(
a^9*(a + b*x^2)^2) - (9*b)/(a^10*(a + b*x^2)) - (10*b*Log[x^2])/a^11 + (10
*b*Log[a + b*x^2])/a^11)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.77

method	result
norman	$\frac{-\frac{1}{2a}+\frac{45b^2x^4}{a^3}+\frac{270b^3x^6}{a^4}+\frac{770b^4x^8}{a^5}+\frac{2625b^5x^{10}}{2a^6}+\frac{2877b^6x^{12}}{2a^7}+\frac{1029b^7x^{14}}{a^8}+\frac{3267b^8x^{16}}{7a^9}+\frac{6849b^9x^{18}}{56a^{10}}+\frac{7129b^{10}x^{20}}{504a^{11}}}{x^2(b\,x^2+a)^9}-\frac{10b\ln(x)}{a^{11}}+$
risch	$\frac{-\frac{1}{2a}-\frac{7129b\,x^2}{504a^2}-\frac{4609b^2x^4}{56a^3}-\frac{3349b^3x^6}{14a^4}-\frac{2509b^4x^8}{6a^5}-\frac{1879b^5x^{10}}{4a^6}-\frac{1375b^6x^{12}}{4a^7}-\frac{955b^7x^{14}}{6a^8}-\frac{85b^8x^{16}}{2a^9}-\frac{5b^9x^{18}}{a^{10}}}{x^2(b\,x^2+a)^9}-\frac{10b\ln(x)}{a^{11}}+\frac{5b\ln}{}$
default	$\frac{b^2\left(-\frac{a^9}{9b\left(b\,x^2+a\right)^9}-\frac{a^5}{b\left(b\,x^2+a\right)^5}-\frac{3a^4}{2b\left(b\,x^2+a\right)^4}-\frac{3a^7}{7b\left(b\,x^2+a\right)^7}-\frac{a^8}{4b\left(b\,x^2+a\right)^8}-\frac{4a^2}{b\left(b\,x^2+a\right)^2}-\frac{9a}{b\left(b\,x^2+a\right)}+\frac{10\ln\left(b\,x^2+a\right)}{b}-\frac{7a^3}{3b\left(b\,x^2+\right.}\right.}{2a^{11}}$
parallelrisch	$-\frac{-7129b^{10}x^{20}-388080a^6b^4x^8-661500a^5b^5x^{10}-725004a^4b^6x^{12}-518616a^3b^7x^{14}-235224a^2b^8x^{16}-61641a\,b^9x^{18}+252a^{10}}{}$

input
```
int(1/x^3/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/2/a+45*b^2/a^3*x^4+270*b^3/a^4*x^6+770*b^4/a^5*x^8+2625/2*b^5/a^6*x^10
+2877/2*b^6/a^7*x^12+1029*b^7/a^8*x^14+3267/7*b^8/a^9*x^16+6849/56*b^9/a^1
0*x^18+7129/504*b^10/a^11*x^20)/x^2/(b*x^2+a)^9-10*b*ln(x)/a^11+5*b*ln(b*x
^2+a)/a^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(166) = 332$.

Time = 0.08 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3\left(a+bx^2\right)^{10}}\,dx =$$
$$-\frac{2520\,ab^9x^{18}+21420\,a^2b^8x^{16}+80220\,a^3b^7x^{14}+173250\,a^4b^6x^{12}+236754\,a^5b^5x^{10}+210756\,a^6b^4x^8+1}{}$$

input
```
integrate(1/x^3/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/504*(2520*a*b^9*x^18 + 21420*a^2*b^8*x^16 + 80220*a^3*b^7*x^14 + 173250
*a^4*b^6*x^12 + 236754*a^5*b^5*x^10 + 210756*a^6*b^4*x^8 + 120564*a^7*b^3*
x^6 + 41481*a^8*b^2*x^4 + 7129*a^9*b*x^2 + 252*a^10 - 2520*(b^10*x^20 + 9*
a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^
5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*
log(b*x^2 + a) + 5040*(b^10*x^20 + 9*a*b^9*x^18 + 36*a^2*b^8*x^16 + 84*a^3
*b^7*x^14 + 126*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 84*a^6*b^4*x^8 + 36*a^7*
b^3*x^6 + 9*a^8*b^2*x^4 + a^9*b*x^2)*log(x))/(a^11*b^9*x^20 + 9*a^12*b^8*x
^18 + 36*a^13*b^7*x^16 + 84*a^14*b^6*x^14 + 126*a^15*b^5*x^12 + 126*a^16*b
^4*x^10 + 84*a^17*b^3*x^8 + 36*a^18*b^2*x^6 + 9*a^19*b*x^4 + a^20*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^3 (a+bx^2)^{10}}\,dx = \frac{-252a^9 - 7129a^8bx^2 - 41481a^7b^2x^4 - 120564a^6b^3x^6 - 210756a^5b^4x^8 - 236754a^4b^5x^{10} - 173250a^3b^6}{504a^{19}x^2 + 4536a^{18}bx^4 + 18144a^{17}b^2x^6 + 42336a^{16}b^3x^8 + 63504a^{15}b^4x^{10} + 63504a^{14}b^5x^{12} + 42336a^{13}b^6} - \frac{10b\log(x)}{a^{11}} + \frac{5b\log\left(\frac{a}{b}+x^2\right)}{a^{11}}$$

input
```
integrate(1/x**3/(b*x**2+a)**10,x)
```

output
```
(-252*a**9 - 7129*a**8*b*x**2 - 41481*a**7*b**2*x**4 - 120564*a**6*b**3*x*
*6 - 210756*a**5*b**4*x**8 - 236754*a**4*b**5*x**10 - 173250*a**3*b**6*x**
12 - 80220*a**2*b**7*x**14 - 21420*a*b**8*x**16 - 2520*b**9*x**18)/(504*a*
*19*x**2 + 4536*a**18*b*x**4 + 18144*a**17*b**2*x**6 + 42336*a**16*b**3*x*
*8 + 63504*a**15*b**4*x**10 + 63504*a**14*b**5*x**12 + 42336*a**13*b**6*x*
*14 + 18144*a**12*b**7*x**16 + 4536*a**11*b**8*x**18 + 504*a**10*b**9*x**2
0) - 10*b*log(x)/a**11 + 5*b*log(a/b + x**2)/a**11
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3 (a+bx^2)^{10}} dx = -\frac{2520 b^9 x^{18} + 21420 ab^8 x^{16} + 80220 a^2 b^7 x^{14} + 173250 a^3 b^6 x^{12} + 236754 a^4 b^5 x^{10} + 210756 a^5 b^4 x^8 + 120}{504 (a^{10} b^9 x^{20} + 9 a^{11} b^8 x^{18} + 36 a^{12} b^7 x^{16} + 84 a^{13} b^6 x^{14} + 126 a^{14} b^5 x^{12} + 126 a^{15} b^4 x^{10} + 84 a^1} + \frac{5 b \log (bx^2 + a)}{a^{11}} - \frac{5 b \log (x^2)}{a^{11}}$$

input
```
integrate(1/x^3/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/504*(2520*b^9*x^18 + 21420*a*b^8*x^16 + 80220*a^2*b^7*x^14 + 173250*a^3
*b^6*x^12 + 236754*a^4*b^5*x^10 + 210756*a^5*b^4*x^8 + 120564*a^6*b^3*x^6
+ 41481*a^7*b^2*x^4 + 7129*a^8*b*x^2 + 252*a^9)/(a^10*b^9*x^20 + 9*a^11*b^
8*x^18 + 36*a^12*b^7*x^16 + 84*a^13*b^6*x^14 + 126*a^14*b^5*x^12 + 126*a^1
5*b^4*x^10 + 84*a^16*b^3*x^8 + 36*a^17*b^2*x^6 + 9*a^18*b*x^4 + a^19*x^2)
+ 5*b*log(b*x^2 + a)/a^11 - 5*b*log(x^2)/a^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^3 (a+bx^2)^{10}} dx = -\frac{5 b \log (x^2)}{a^{11}} + \frac{5 b \log (|bx^2 + a|)}{a^{11}} + \frac{10 bx^2 - a}{2 a^{11} x^2} - \frac{7129 b^{10} x^{18} + 66429 ab^9 x^{16} + 275796 a^2 b^8 x^{14} + 669984 a^3 b^7 x^{12} + 1050336 a^4 b^6 x^{10} + 1103256 a^5 b^5 x^8 +}{504 (bx^2 + a)^9 a^{11}}$$

input
```
integrate(1/x^3/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-5*b*log(x^2)/a^11 + 5*b*log(abs(b*x^2 + a))/a^11 + 1/2*(10*b*x^2 - a)/(a^
11*x^2) - 1/504*(7129*b^10*x^18 + 66429*a*b^9*x^16 + 275796*a^2*b^8*x^14 +
 669984*a^3*b^7*x^12 + 1050336*a^4*b^6*x^10 + 1103256*a^5*b^5*x^8 + 777840
*a^6*b^4*x^6 + 356040*a^7*b^3*x^4 + 96570*a^8*b^2*x^2 + 11990*a^9*b)/((b*x
^2 + a)^9*a^11)
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3 (a+bx^2)^{10}}\,dx = \frac{5\,b\,\ln\left(b\,x^2+a\right)}{a^{11}} - \frac{\frac{1}{2\,a}+\frac{7129\,b\,x^2}{504\,a^2}+\frac{4609\,b^2\,x^4}{56\,a^3}+\frac{3349\,b^3\,x^6}{14\,a^4}+\frac{2509\,b^4\,x^8}{6\,a^5}+\frac{1879\,b^5\,x^{10}}{4\,a^6}+\frac{1375\,b^6\,x^{12}}{4\,a^7}+\frac{955\,b^7\,x^{14}}{6\,a^8}+\frac{85\,b^8\,x^{16}}{2\,a^9}+}{a^9\,x^2+9\,a^8\,b\,x^4+36\,a^7\,b^2\,x^6+84\,a^6\,b^3\,x^8+126\,a^5\,b^4\,x^{10}+126\,a^4\,b^5\,x^{12}+84\,a^3\,b^6\,x^{14}+36\,a^2\,b^7\,x^{16}+} - \frac{10\,b\,\ln\left(x\right)}{a^{11}}$$

input
```
int(1/(x^3*(a + b*x^2)^10),x)
```

output
```
(5*b*log(a + b*x^2))/a^11 - (1/(2*a) + (7129*b*x^2)/(504*a^2) + (4609*b^2*
x^4)/(56*a^3) + (3349*b^3*x^6)/(14*a^4) + (2509*b^4*x^8)/(6*a^5) + (1879*b
^5*x^10)/(4*a^6) + (1375*b^6*x^12)/(4*a^7) + (955*b^7*x^14)/(6*a^8) + (85*
b^8*x^16)/(2*a^9) + (5*b^9*x^18)/a^10)/(a^9*x^2 + b^9*x^20 + 9*a^8*b*x^4 +
 9*a*b^8*x^18 + 36*a^7*b^2*x^6 + 84*a^6*b^3*x^8 + 126*a^5*b^4*x^10 + 126*a
^4*b^5*x^12 + 84*a^3*b^6*x^14 + 36*a^2*b^7*x^16) - (10*b*log(x))/a^11
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 511, normalized size of antiderivative = 2.78

$$\int \frac{1}{x^3 (a+bx^2)^{10}}\,dx = \frac{2520\log(b\,x^2+a)\,a^9b\,x^2+22680\log(b\,x^2+a)\,a^8b^2x^4+90720\log(b\,x^2+a)\,a^7b^3x^6+211680\log(b\,x^2+a}{}$$

input
```
int(1/x^3/(b*x^2+a)^10,x)
```

output
```
(2520*log(a + b*x**2)*a**9*b*x**2 + 22680*log(a + b*x**2)*a**8*b**2*x**4 +
 90720*log(a + b*x**2)*a**7*b**3*x**6 + 211680*log(a + b*x**2)*a**6*b**4*x
**8 + 317520*log(a + b*x**2)*a**5*b**5*x**10 + 317520*log(a + b*x**2)*a**4
*b**6*x**12 + 211680*log(a + b*x**2)*a**3*b**7*x**14 + 90720*log(a + b*x**
2)*a**2*b**8*x**16 + 22680*log(a + b*x**2)*a*b**9*x**18 + 2520*log(a + b*x
**2)*b**10*x**20 - 5040*log(x)*a**9*b*x**2 - 45360*log(x)*a**8*b**2*x**4 -
 181440*log(x)*a**7*b**3*x**6 - 423360*log(x)*a**6*b**4*x**8 - 635040*log(
x)*a**5*b**5*x**10 - 635040*log(x)*a**4*b**6*x**12 - 423360*log(x)*a**3*b*
*7*x**14 - 181440*log(x)*a**2*b**8*x**16 - 45360*log(x)*a*b**9*x**18 - 504
0*log(x)*b**10*x**20 - 252*a**10 - 6849*a**9*b*x**2 - 38961*a**8*b**2*x**4
 - 110484*a**7*b**3*x**6 - 187236*a**6*b**4*x**8 - 201474*a**5*b**5*x**10
- 137970*a**4*b**6*x**12 - 56700*a**3*b**7*x**14 - 11340*a**2*b**8*x**16 +
 280*b**10*x**20)/(504*a**11*x**2*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**
4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3
*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.207 $\int \frac{1}{x^5(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 217

$$\begin{aligned}\int \frac{1}{x^5 (a+bx^2)^{10}}\,dx = &-\frac{1}{4a^{10}x^4}+\frac{5b}{a^{11}x^2}+\frac{b^2}{18a^3 (a+bx^2)^9}+\frac{3b^2}{16a^4 (a+bx^2)^8}\\ &+\frac{3b^2}{7a^5 (a+bx^2)^7}+\frac{5b^2}{6a^6 (a+bx^2)^6}+\frac{3b^2}{2a^7 (a+bx^2)^5}\\ &+\frac{21b^2}{8a^8 (a+bx^2)^4}+\frac{14b^2}{3a^9 (a+bx^2)^3}+\frac{9b^2}{a^{10} (a+bx^2)^2}\\ &+\frac{45b^2}{2a^{11} (a+bx^2)}+\frac{55b^2\log(x)}{a^{12}}-\frac{55b^2\log(a+bx^2)}{2a^{12}}\end{aligned}$$

output
```
-1/4/a^10/x^4+5*b/a^11/x^2+1/18*b^2/a^3/(b*x^2+a)^9+3/16*b^2/a^4/(b*x^2+a)
^8+3/7*b^2/a^5/(b*x^2+a)^7+5/6*b^2/a^6/(b*x^2+a)^6+3/2*b^2/a^7/(b*x^2+a)^5
+21/8*b^2/a^8/(b*x^2+a)^4+14/3*b^2/a^9/(b*x^2+a)^3+9*b^2/a^10/(b*x^2+a)^2+
45/2*b^2/a^11/(b*x^2+a)+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^5 (a+bx^2)^{10}}\,dx = \frac{\frac{a(-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b^6x^{12}+1905750a^3b^7x^{14}+882420a^2b^8x^{16}}{x^4(a+bx^2)^9}}{1008a^{12}}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^10),x]
```

output
```
((a*(-252*a^10 + 2772*a^9*b*x^2 + 78419*a^8*b^2*x^4 + 456291*a^7*b^3*x^6 +
 1326204*a^6*b^4*x^8 + 2318316*a^5*b^5*x^10 + 2604294*a^4*b^6*x^12 + 19057
50*a^3*b^7*x^14 + 882420*a^2*b^8*x^16 + 235620*a*b^9*x^18 + 27720*b^10*x^2
0))/(x^4*(a + b*x^2)^9) + 55440*b^2*Log[x] - 27720*b^2*Log[a + b*x^2])/(10
08*a^12)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{1}{x^6 (bx^2+a)^{10}}\,dx^2$$

$$\Big\downarrow \ 54$$

$$\frac{1}{2}\int \bigg(-\frac{55b^3}{a^{12}(bx^2+a)}-\frac{45b^3}{a^{11}(bx^2+a)^2}-\frac{36b^3}{a^{10}(bx^2+a)^3}-\frac{28b^3}{a^9(bx^2+a)^4}-\frac{21b^3}{a^8(bx^2+a)^5}-\frac{15b^3}{a^7(bx^2+a)^6}-\frac{}{a^6(}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{55b^2\log\left(x^2\right)}{a^{12}}-\frac{55b^2\log\left(a+bx^2\right)}{a^{12}}+\frac{45b^2}{a^{11}\left(a+bx^2\right)}+\frac{10b}{a^{11}x^2}+\frac{18b^2}{a^{10}\left(a+bx^2\right)^2}-\frac{1}{2a^{10}x^4}+\frac{28b^2}{3a^9\left(a+bx^2\right)^3}+\right.$$

input
```
Int[1/(x^5*(a + b*x^2)^10),x]
```

output
```
(-1/2*1/(a^10*x^4) + (10*b)/(a^11*x^2) + b^2/(9*a^3*(a + b*x^2)^9) + (3*b^
2)/(8*a^4*(a + b*x^2)^8) + (6*b^2)/(7*a^5*(a + b*x^2)^7) + (5*b^2)/(3*a^6*
(a + b*x^2)^6) + (3*b^2)/(a^7*(a + b*x^2)^5) + (21*b^2)/(4*a^8*(a + b*x^2)
^4) + (28*b^2)/(3*a^9*(a + b*x^2)^3) + (18*b^2)/(a^10*(a + b*x^2)^2) + (45
*b^2)/(a^11*(a + b*x^2)) + (55*b^2*Log[x^2])/a^12 - (55*b^2*Log[a + b*x^2]
)/a^12)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.71

method	result
norman	$\frac{-\frac{1}{4a}+\frac{11b\,x^2}{4a^2}-\frac{495b^3x^6}{2a^4}-\frac{1485b^4x^8}{a^5}-\frac{4235b^5x^{10}}{a^6}-\frac{28875b^6x^{12}}{4a^7}-\frac{31647b^7x^{14}}{4a^8}-\frac{11319b^8x^{16}}{2a^9}-\frac{35937b^9x^{18}}{14a^{10}}-\frac{75339b^{10}x^{20}}{112a^{11}}-\frac{78419b^{11}x^{2}}{1008a^{12}}}{x^4(b\,x^2+a)^9}$
risch	$\frac{-\frac{1}{4a}+\frac{11b\,x^2}{4a^2}+\frac{78419b^2x^4}{1008a^3}+\frac{50699b^3x^6}{112a^4}+\frac{36839b^4x^8}{28a^5}+\frac{27599b^5x^{10}}{12a^6}+\frac{20669b^6x^{12}}{8a^7}+\frac{15125b^7x^{14}}{8a^8}+\frac{10505b^8x^{16}}{12a^9}+\frac{935b^9x^{18}}{4a^{10}}+\frac{55b^{10}x^{20}}{2a^{11}}}{x^4(b\,x^2+a)^9}$
default	$-\frac{b^3\left(-\frac{a^9}{9b\left(b\,x^2+a\right)^9}-\frac{3a^5}{b\left(b\,x^2+a\right)^5}-\frac{21a^4}{4b\left(b\,x^2+a\right)^4}-\frac{6a^7}{7b\left(b\,x^2+a\right)^7}-\frac{3a^8}{8b\left(b\,x^2+a\right)^8}-\frac{18a^2}{b\left(b\,x^2+a\right)^2}-\frac{45a}{b\left(b\,x^2+a\right)}+\frac{55\ln\left(b\,x^2+a\right)}{b}-\frac{2}{3b(b\,x}\right.}{2a^{12}}$
parallelrisch	$55440\ln(x)x^4a^9b^2+6985440\ln(x)x^{14}a^4b^7+6985440\ln(x)x^{12}a^5b^6+4656960\ln(x)x^{10}a^6b^5-252a^{11}+2772a^{10}b\,x^2+4656960\ln$

input
```
int(1/x^5/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/4/a+11/4*b/a^2*x^2-495/2*b^3/a^4*x^6-1485*b^4/a^5*x^8-4235*b^5/a^6*x^1
0-28875/4*b^6/a^7*x^12-31647/4*b^7/a^8*x^14-11319/2*b^8/a^9*x^16-35937/14*
b^9/a^10*x^18-75339/112*b^10/a^11*x^20-78419/1008*b^11/a^12*x^22)/x^4/(b*x
^2+a)^9+55*b^2*ln(x)/a^12-55/2*b^2*ln(b*x^2+a)/a^12
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(197) = 394.

Time = 0.08 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^5\left(a+bx^2\right)^{10}}\,dx$$
$$=\frac{27720\,ab^{10}x^{20}+235620\,a^2b^9x^{18}+882420\,a^3b^8x^{16}+1905750\,a^4b^7x^{14}+2604294\,a^5b^6x^{12}+2318316\,a^6b^5x}{}$$

input
```
integrate(1/x^5/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
1/1008*(27720*a*b^10*x^20 + 235620*a^2*b^9*x^18 + 882420*a^3*b^8*x^16 + 19
05750*a^4*b^7*x^14 + 2604294*a^5*b^6*x^12 + 2318316*a^6*b^5*x^10 + 1326204
*a^7*b^4*x^8 + 456291*a^8*b^3*x^6 + 78419*a^9*b^2*x^4 + 2772*a^10*b*x^2 -
252*a^11 - 27720*(b^11*x^22 + 9*a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8
*x^16 + 126*a^4*b^7*x^14 + 126*a^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4
*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x^4)*log(b*x^2 + a) + 55440*(b^11*x^22 + 9*
a*b^10*x^20 + 36*a^2*b^9*x^18 + 84*a^3*b^8*x^16 + 126*a^4*b^7*x^14 + 126*a
^5*b^6*x^12 + 84*a^6*b^5*x^10 + 36*a^7*b^4*x^8 + 9*a^8*b^3*x^6 + a^9*b^2*x
^4)*log(x))/(a^12*b^9*x^22 + 9*a^13*b^8*x^20 + 36*a^14*b^7*x^18 + 84*a^15*
b^6*x^16 + 126*a^16*b^5*x^14 + 126*a^17*b^4*x^12 + 84*a^18*b^3*x^10 + 36*a
^19*b^2*x^8 + 9*a^20*b*x^6 + a^21*x^4)
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^5\left(a+bx^2\right)^{10}}\,dx = \frac{-252a^{10}+2772a^9bx^2+78419a^8b^2x^4+456291a^7b^3x^6+1326204a^6b^4x^8+2318316a^5b^5x^{10}+2604294a^4b}{1008a^{20}x^4+9072a^{19}bx^6+36288a^{18}b^2x^8+84672a^{17}b^3x^{10}+127008a^{16}b^4x^{12}+127008a^{15}b^5x^{14}+} + \frac{55b^2\log(x)}{a^{12}} - \frac{55b^2\log\left(\frac{a}{b}+x^2\right)}{2a^{12}}$$

input
```
integrate(1/x**5/(b*x**2+a)**10,x)
```

output
```
(-252*a**10 + 2772*a**9*b*x**2 + 78419*a**8*b**2*x**4 + 456291*a**7*b**3*x
**6 + 1326204*a**6*b**4*x**8 + 2318316*a**5*b**5*x**10 + 2604294*a**4*b**6
*x**12 + 1905750*a**3*b**7*x**14 + 882420*a**2*b**8*x**16 + 235620*a*b**9*
x**18 + 27720*b**10*x**20)/(1008*a**20*x**4 + 9072*a**19*b*x**6 + 36288*a*
*18*b**2*x**8 + 84672*a**17*b**3*x**10 + 127008*a**16*b**4*x**12 + 127008*
a**15*b**5*x**14 + 84672*a**14*b**6*x**16 + 36288*a**13*b**7*x**18 + 9072*
a**12*b**8*x**20 + 1008*a**11*b**9*x**22) + 55*b**2*log(x)/a**12 - 55*b**2
*log(a/b + x**2)/(2*a**12)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^5 (a+bx^2)^{10}} dx = \frac{27720 b^{10}x^{20} + 235620 ab^9x^{18} + 882420 a^2b^8x^{16} + 1905750 a^3b^7x^{14} + 2604294 a^4b^6x^{12} + 2318316 a^5b^5x^{10}}{1008 (a^{11}b^9x^{22} + 9 a^{12}b^8x^{20} + 36 a^{13}b^7x^{18} + 84 a^{14}b^6x^{16} + 126 a^{15}b^5x^{14} + 126 a^{16}b} - \frac{55 b^2 \log(bx^2+a)}{2 a^{12}} + \frac{55 b^2 \log(x^2)}{2 a^{12}}$$

input
```
integrate(1/x^5/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/1008*(27720*b^10*x^20 + 235620*a*b^9*x^18 + 882420*a^2*b^8*x^16 + 190575
0*a^3*b^7*x^14 + 2604294*a^4*b^6*x^12 + 2318316*a^5*b^5*x^10 + 1326204*a^6
*b^4*x^8 + 456291*a^7*b^3*x^6 + 78419*a^8*b^2*x^4 + 2772*a^9*b*x^2 - 252*a
^10)/(a^11*b^9*x^22 + 9*a^12*b^8*x^20 + 36*a^13*b^7*x^18 + 84*a^14*b^6*x^1
6 + 126*a^15*b^5*x^14 + 126*a^16*b^4*x^12 + 84*a^17*b^3*x^10 + 36*a^18*b^2
*x^8 + 9*a^19*b*x^6 + a^20*x^4) - 55/2*b^2*log(b*x^2 + a)/a^12 + 55/2*b^2*
log(x^2)/a^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^5 (a+bx^2)^{10}} dx = \frac{55 b^2 \log(x^2)}{2 a^{12}} - \frac{55 b^2 \log(|bx^2+a|)}{2 a^{12}} - \frac{165 b^2x^4 - 20 abx^2 + a^2}{4 a^{12}x^4} + \frac{78419 b^{11}x^{18} + 728451 ab^{10}x^{16} + 3013596 a^2b^9x^{14} + 7290444 a^3b^8x^{12} + 11372256 a^4b^7x^{10} + 11871216 a}{1008 (bx^2+a)^9 a^{12}}$$

input
```
integrate(1/x^5/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
55/2*b^2*log(x^2)/a^12 - 55/2*b^2*log(abs(b*x^2 + a))/a^12 - 1/4*(165*b^2*
x^4 - 20*a*b*x^2 + a^2)/(a^12*x^4) + 1/1008*(78419*b^11*x^18 + 728451*a*b^
10*x^16 + 3013596*a^2*b^9*x^14 + 7290444*a^3*b^8*x^12 + 11372256*a^4*b^7*x
^10 + 11871216*a^5*b^6*x^8 + 8302224*a^6*b^5*x^6 + 3757680*a^7*b^4*x^4 + 1
001790*a^8*b^3*x^2 + 120550*a^9*b^2)/((b*x^2 + a)^9*a^12)
```

Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5 (a+bx^2)^{10}} dx$$
$$= \frac{\frac{11 b x^2}{4 a^2} - \frac{1}{4 a} + \frac{78419 b^2 x^4}{1008 a^3} + \frac{50699 b^3 x^6}{112 a^4} + \frac{36839 b^4 x^8}{28 a^5} + \frac{27599 b^5 x^{10}}{12 a^6} + \frac{20669 b^6 x^{12}}{8 a^7} + \frac{15125 b^7 x^{14}}{8 a^8} + \frac{10505 b^8 x^{16}}{12 a^9} + \frac{935 b}{4 a}}{a^9 x^4 + 9 a^8 b x^6 + 36 a^7 b^2 x^8 + 84 a^6 b^3 x^{10} + 126 a^5 b^4 x^{12} + 126 a^4 b^5 x^{14} + 84 a^3 b^6 x^{16} + 36 a^2 b^7 x^{18} + 9}$$
$$- \frac{55 b^2 \ln(b x^2 + a)}{2 a^{12}} + \frac{55 b^2 \ln(x)}{a^{12}}$$

input
```
int(1/(x^5*(a + b*x^2)^10),x)
```

output
```
((11*b*x^2)/(4*a^2) - 1/(4*a) + (78419*b^2*x^4)/(1008*a^3) + (50699*b^3*x^
6)/(112*a^4) + (36839*b^4*x^8)/(28*a^5) + (27599*b^5*x^10)/(12*a^6) + (206
69*b^6*x^12)/(8*a^7) + (15125*b^7*x^14)/(8*a^8) + (10505*b^8*x^16)/(12*a^9
) + (935*b^9*x^18)/(4*a^10) + (55*b^10*x^20)/(2*a^11))/(a^9*x^4 + b^9*x^22
 + 9*a^8*b*x^6 + 9*a*b^8*x^20 + 36*a^7*b^2*x^8 + 84*a^6*b^3*x^10 + 126*a^5
*b^4*x^12 + 126*a^4*b^5*x^14 + 84*a^3*b^6*x^16 + 36*a^2*b^7*x^18) - (55*b^
2*log(a + b*x^2))/(2*a^12) + (55*b^2*log(x))/a^12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.42

$$\int \frac{1}{x^5 (a+bx^2)^{10}} dx$$
$$= \frac{-27720 \log(b x^2 + a) a^9 b^2 x^4 - 249480 \log(b x^2 + a) a^8 b^3 x^6 - 997920 \log(b x^2 + a) a^7 b^4 x^8 - 2328480 \log(}{}$$

input
```
int(1/x^5/(b*x^2+a)^10,x)
```

output

```
( - 27720*log(a + b*x**2)*a**9*b**2*x**4 - 249480*log(a + b*x**2)*a**8*b**
3*x**6 - 997920*log(a + b*x**2)*a**7*b**4*x**8 - 2328480*log(a + b*x**2)*a
**6*b**5*x**10 - 3492720*log(a + b*x**2)*a**5*b**6*x**12 - 3492720*log(a +
 b*x**2)*a**4*b**7*x**14 - 2328480*log(a + b*x**2)*a**3*b**8*x**16 - 99792
0*log(a + b*x**2)*a**2*b**9*x**18 - 249480*log(a + b*x**2)*a*b**10*x**20 -
 27720*log(a + b*x**2)*b**11*x**22 + 55440*log(x)*a**9*b**2*x**4 + 498960*
log(x)*a**8*b**3*x**6 + 1995840*log(x)*a**7*b**4*x**8 + 4656960*log(x)*a**
6*b**5*x**10 + 6985440*log(x)*a**5*b**6*x**12 + 6985440*log(x)*a**4*b**7*x
**14 + 4656960*log(x)*a**3*b**8*x**16 + 1995840*log(x)*a**2*b**9*x**18 + 4
98960*log(x)*a*b**10*x**20 + 55440*log(x)*b**11*x**22 - 252*a**11 + 2772*a
**10*b*x**2 + 75339*a**9*b**2*x**4 + 428571*a**8*b**3*x**6 + 1215324*a**7*
b**4*x**8 + 2059596*a**6*b**5*x**10 + 2216214*a**5*b**6*x**12 + 1517670*a*
*4*b**7*x**14 + 623700*a**3*b**8*x**16 + 124740*a**2*b**9*x**18 - 3080*b**
11*x**22)/(1008*a**12*x**4*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*
a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x
**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.208 $\int \frac{1}{x^7(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 226

$$\begin{aligned}\int \frac{1}{x^7(a+bx^2)^{10}}\,dx = & -\frac{1}{6a^{10}x^6}+\frac{5b}{2a^{11}x^4}-\frac{55b^2}{2a^{12}x^2}-\frac{b^3}{18a^4(a+bx^2)^9}-\frac{b^3}{4a^5(a+bx^2)^8}\\ & -\frac{5b^3}{7a^6(a+bx^2)^7}-\frac{5b^3}{3a^7(a+bx^2)^6}-\frac{7b^3}{2a^8(a+bx^2)^5}\\ & -\frac{7b^3}{a^9(a+bx^2)^4}-\frac{14b^3}{a^{10}(a+bx^2)^3}-\frac{30b^3}{a^{11}(a+bx^2)^2}\\ & -\frac{165b^3}{2a^{12}(a+bx^2)}-\frac{220b^3\log(x)}{a^{13}}+\frac{110b^3\log(a+bx^2)}{a^{13}}\end{aligned}$$

output
```
-1/6/a^10/x^6+5/2*b/a^11/x^4-55/2*b^2/a^12/x^2-1/18*b^3/a^4/(b*x^2+a)^9-1/
4*b^3/a^5/(b*x^2+a)^8-5/7*b^3/a^6/(b*x^2+a)^7-5/3*b^3/a^7/(b*x^2+a)^6-7/2*
b^3/a^8/(b*x^2+a)^5-7*b^3/a^9/(b*x^2+a)^4-14*b^3/a^10/(b*x^2+a)^3-30*b^3/a
^11/(b*x^2+a)^2-165/2*b^3/a^12/(b*x^2+a)-220*b^3*ln(x)/a^13+110*b^3*ln(b*x
^2+a)/a^13
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^7 (a+bx^2)^{10}} dx = -\frac{\frac{a(42a^{11}-252a^{10}bx^2+2772a^9b^2x^4+78419a^8b^3x^6+456291a^7b^4x^8+1326204a^6b^5x^{10}+2318316a^5b^6x^{12}+2604294a^4b^7x^{14}+1905750a^3b^8x^{16}+}{x^6(a+bx^2)^9}}{252a^{13}}$$

input
```
Integrate[1/(x^7*(a + b*x^2)^10),x]
```

output
```
-1/252*((a*(42*a^11 - 252*a^10*b*x^2 + 2772*a^9*b^2*x^4 + 78419*a^8*b^3*x^
6 + 456291*a^7*b^4*x^8 + 1326204*a^6*b^5*x^10 + 2318316*a^5*b^6*x^12 + 260
4294*a^4*b^7*x^14 + 1905750*a^3*b^8*x^16 + 882420*a^2*b^9*x^18 + 235620*a*
b^10*x^20 + 27720*b^11*x^22))/(x^6*(a + b*x^2)^9) + 55440*b^3*Log[x] - 277
20*b^3*Log[a + b*x^2])/a^13
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^7 (a+bx^2)^{10}} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^8 (bx^2+a)^{10}} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{220b^4}{a^{13}(bx^2+a)} + \frac{165b^4}{a^{12}(bx^2+a)^2} + \frac{120b^4}{a^{11}(bx^2+a)^3} + \frac{84b^4}{a^{10}(bx^2+a)^4} + \frac{56b^4}{a^9(bx^2+a)^5} + \frac{35b^4}{a^8(bx^2+a)^6} + \frac{}{a^7(b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{220b^3\log\left(x^2\right)}{a^{13}}+\frac{220b^3\log\left(a+bx^2\right)}{a^{13}}-\frac{165b^3}{a^{12}\left(a+bx^2\right)}-\frac{55b^2}{a^{12}x^2}-\frac{60b^3}{a^{11}\left(a+bx^2\right)^2}+\frac{5b}{a^{11}x^4}-\frac{28b^3}{a^{10}\left(a+bx^2\right)^3}\right.$$

input
```
Int[1/(x^7*(a + b*x^2)^10),x]
```

output
```
(-1/3*1/(a^10*x^6) + (5*b)/(a^11*x^4) - (55*b^2)/(a^12*x^2) - b^3/(9*a^4*(
a + b*x^2)^9) - b^3/(2*a^5*(a + b*x^2)^8) - (10*b^3)/(7*a^6*(a + b*x^2)^7)
 - (10*b^3)/(3*a^7*(a + b*x^2)^6) - (7*b^3)/(a^8*(a + b*x^2)^5) - (14*b^3)
/(a^9*(a + b*x^2)^4) - (28*b^3)/(a^10*(a + b*x^2)^3) - (60*b^3)/(a^11*(a +
 b*x^2)^2) - (165*b^3)/(a^12*(a + b*x^2)) - (220*b^3*Log[x^2])/a^13 + (220
*b^3*Log[a + b*x^2])/a^13)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

method	result
norman	$\frac{\frac{b\,x^2}{a^2}-\frac{1}{6a}-\frac{11b^2x^4}{a^3}+\frac{990b^4x^8}{a^5}+\frac{5940b^5x^{10}}{a^6}+\frac{16940b^6x^{12}}{a^7}+\frac{28875b^7x^{14}}{a^8}+\frac{31647b^8x^{16}}{a^9}+\frac{22638b^9x^{18}}{a^{10}}+\frac{71874b^{10}x^{20}}{7a^{11}}+\frac{75339b^{11}x^{22}}{28a^{12}}+\frac{78}{}}{x^6(b\,x^2+a)^9}$
risch	$\frac{-\frac{1}{6a}+\frac{b\,x^2}{a^2}-\frac{11b^2x^4}{a^3}-\frac{78419b^3x^6}{252a^4}-\frac{50699b^4x^8}{28a^5}-\frac{36839b^5x^{10}}{7a^6}-\frac{27599b^6x^{12}}{3a^7}-\frac{20669b^7x^{14}}{2a^8}-\frac{15125b^8x^{16}}{2a^9}-\frac{10505b^9x^{18}}{3a^{10}}-\frac{935b^{10}x^{20}}{a^{11}}-1}{x^6(b\,x^2+a)^9}$
default	$\frac{b^4\left(-\frac{a^9}{9b(b\,x^2+a)^9}-\frac{7a^5}{b(b\,x^2+a)^5}-\frac{14a^4}{b(b\,x^2+a)^4}-\frac{10a^7}{7b(b\,x^2+a)^7}-\frac{a^8}{2b(b\,x^2+a)^8}-\frac{60a^2}{b(b\,x^2+a)^2}-\frac{165a}{b(b\,x^2+a)}+\frac{220\ln(b\,x^2+a)}{b}-\frac{28a^3}{b(b\,x^2+}\right.}{2a^{13}}$
parallelrisch	$-\frac{42a^{12}+498960\ln(x)x^{22}a\,b^{11}-249480\ln(b\,x^2+a)x^{22}a\,b^{11}+1995840\ln(x)x^{20}a^2b^{10}-997920\ln(b\,x^2+a)x^{20}a^2b^{10}+4656960}{}$

input
```
int(1/x^7/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x^2-1/6/a-11*b^2/a^3*x^4+990*b^4/a^5*x^8+5940*b^5/a^6*x^10+16940*b^
6/a^7*x^12+28875*b^7/a^8*x^14+31647*b^8/a^9*x^16+22638*b^9/a^10*x^18+71874
/7*b^10/a^11*x^20+75339/28*b^11/a^12*x^22+78419/252*b^12/a^13*x^24)/x^6/(b
*x^2+a)^9-220*b^3*ln(x)/a^13+110*b^3*ln(b*x^2+a)/a^13
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. 2(208) = 416.

Time = 0.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^7\left(a+bx^2\right)^{10}}\,dx = -\frac{27720\,ab^{11}x^{22}+235620\,a^2b^{10}x^{20}+882420\,a^3b^9x^{18}+1905750\,a^4b^8x^{16}+2604294\,a^5b^7x^{14}+2318316\,a^6}{}$$

input
```
integrate(1/x^7/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
-1/252*(27720*a*b^11*x^22 + 235620*a^2*b^10*x^20 + 882420*a^3*b^9*x^18 + 1
905750*a^4*b^8*x^16 + 2604294*a^5*b^7*x^14 + 2318316*a^6*b^6*x^12 + 132620
4*a^7*b^5*x^10 + 456291*a^8*b^4*x^8 + 78419*a^9*b^3*x^6 + 2772*a^10*b^2*x^
4 - 252*a^11*b*x^2 + 42*a^12 - 27720*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b
^10*x^20 + 84*a^3*b^9*x^18 + 126*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*
b^6*x^12 + 36*a^7*b^5*x^10 + 9*a^8*b^4*x^8 + a^9*b^3*x^6)*log(b*x^2 + a) +
 55440*(b^12*x^24 + 9*a*b^11*x^22 + 36*a^2*b^10*x^20 + 84*a^3*b^9*x^18 + 1
26*a^4*b^8*x^16 + 126*a^5*b^7*x^14 + 84*a^6*b^6*x^12 + 36*a^7*b^5*x^10 + 9
*a^8*b^4*x^8 + a^9*b^3*x^6)*log(x))/(a^13*b^9*x^24 + 9*a^14*b^8*x^22 + 36*
a^15*b^7*x^20 + 84*a^16*b^6*x^18 + 126*a^17*b^5*x^16 + 126*a^18*b^4*x^14 +
 84*a^19*b^3*x^12 + 36*a^20*b^2*x^10 + 9*a^21*b*x^8 + a^22*x^6)
```

Sympy [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^7 (a+bx^2)^{10}}\,dx = \frac{-42a^{11} + 252a^{10}bx^2 - 2772a^9b^2x^4 - 78419a^8b^3x^6 - 456291a^7b^4x^8 - 1326204a^6b^5x^{10} - 2318316a^5b^6x^{12}}{252a^{21}x^6 + 2268a^{20}bx^8 + 9072a^{19}b^2x^{10} + 21168a^{18}b^3x^{12} + 31752a^{17}b^4x^{14} + 31752a^{16}b} - \frac{220b^3\log(x)}{a^{13}} + \frac{110b^3\log\left(\frac{a}{b}+x^2\right)}{a^{13}}$$

input
```
integrate(1/x**7/(b*x**2+a)**10,x)
```

output
```
(-42*a**11 + 252*a**10*b*x**2 - 2772*a**9*b**2*x**4 - 78419*a**8*b**3*x**6
 - 456291*a**7*b**4*x**8 - 1326204*a**6*b**5*x**10 - 2318316*a**5*b**6*x**
12 - 2604294*a**4*b**7*x**14 - 1905750*a**3*b**8*x**16 - 882420*a**2*b**9*
x**18 - 235620*a*b**10*x**20 - 27720*b**11*x**22)/(252*a**21*x**6 + 2268*a
**20*b*x**8 + 9072*a**19*b**2*x**10 + 21168*a**18*b**3*x**12 + 31752*a**17
*b**4*x**14 + 31752*a**16*b**5*x**16 + 21168*a**15*b**6*x**18 + 9072*a**14
*b**7*x**20 + 2268*a**13*b**8*x**22 + 252*a**12*b**9*x**24) - 220*b**3*log
(x)/a**13 + 110*b**3*log(a/b + x**2)/a**13
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^7 (a+bx^2)^{10}}\,dx = -\frac{27720\,b^{11}x^{22} + 235620\,ab^{10}x^{20} + 882420\,a^2b^9x^{18} + 1905750\,a^3b^8x^{16} + 2604294\,a^4b^7x^{14} + 2318316\,a^5b^6}{252\,(a^{12}b^9x^{24} + 9\,a^{13}b^8x^{22} + 36\,a^{14}b^7x^{20} + 84\,a^{15}b^6x^{18} + 126\,a^{16}b^5x^{16} +} + \frac{110\,b^3\log(bx^2+a)}{a^{13}} - \frac{110\,b^3\log(x^2)}{a^{13}}$$

input
```
integrate(1/x^7/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/252*(27720*b^11*x^22 + 235620*a*b^10*x^20 + 882420*a^2*b^9*x^18 + 19057
50*a^3*b^8*x^16 + 2604294*a^4*b^7*x^14 + 2318316*a^5*b^6*x^12 + 1326204*a^
6*b^5*x^10 + 456291*a^7*b^4*x^8 + 78419*a^8*b^3*x^6 + 2772*a^9*b^2*x^4 - 2
52*a^10*b*x^2 + 42*a^11)/(a^12*b^9*x^24 + 9*a^13*b^8*x^22 + 36*a^14*b^7*x^
20 + 84*a^15*b^6*x^18 + 126*a^16*b^5*x^16 + 126*a^17*b^4*x^14 + 84*a^18*b^
3*x^12 + 36*a^19*b^2*x^10 + 9*a^20*b*x^8 + a^21*x^6) + 110*b^3*log(b*x^2 +
 a)/a^13 - 110*b^3*log(x^2)/a^13
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^7 (a+bx^2)^{10}}\,dx = -\frac{110\,b^3\log(x^2)}{a^{13}} + \frac{110\,b^3\log(|bx^2+a|)}{a^{13}} + \frac{1210\,b^3x^6 - 165\,ab^2x^4 + 15\,a^2bx^2 - a^3}{6\,a^{13}x^6} - \frac{78419\,b^{12}x^{18} + 726561\,ab^{11}x^{16} + 2996964\,a^2b^{10}x^{14} + 7225764\,a^3b^9x^{12} + 11226726\,a^4b^8x^{10} + 11663316}{252\,(bx^2+a)^9a^{13}}$$

input
```
integrate(1/x^7/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-110*b^3*log(x^2)/a^13 + 110*b^3*log(abs(b*x^2 + a))/a^13 + 1/6*(1210*b^3*
x^6 - 165*a*b^2*x^4 + 15*a^2*b*x^2 - a^3)/(a^13*x^6) - 1/252*(78419*b^12*x
^18 + 726561*a*b^11*x^16 + 2996964*a^2*b^10*x^14 + 7225764*a^3*b^9*x^12 +
11226726*a^4*b^8*x^10 + 11663316*a^5*b^7*x^8 + 8108184*a^6*b^6*x^6 + 36412
56*a^7*b^5*x^4 + 960210*a^8*b^4*x^2 + 113620*a^9*b^3)/((b*x^2 + a)^9*a^13)
```

Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^7 (a+bx^2)^{10}} \, dx = \frac{110\, b^3 \, \ln\left(b\, x^2+a\right)}{a^{13}} - \frac{\frac{1}{6\, a} - \frac{b\, x^2}{a^2} + \frac{11\, b^2\, x^4}{a^3} + \frac{78419\, b^3\, x^6}{252\, a^4} + \frac{50699\, b^4\, x^8}{28\, a^5} + \frac{36839\, b^5\, x^{10}}{7\, a^6} + \frac{27599\, b^6\, x^{12}}{3\, a^7} + \frac{20669\, b^7\, x^{14}}{2\, a^8} + \frac{15125\, b^8\, x^{16}}{2\, a^9} + \frac{10505\, b^9\, x^{1}}{3\, a^{10}}}{a^9\, x^6 + 9\, a^8\, b\, x^8 + 36\, a^7\, b^2\, x^{10} + 84\, a^6\, b^3\, x^{12} + 126\, a^5\, b^4\, x^{14} + 126\, a^4\, b^5\, x^{16} + 84\, a^3\, b^6\, x^{18} + 36\, a^2\, b^7\, x} - \frac{220\, b^3 \, \ln\left(x\right)}{a^{13}}$$

input
```
int(1/(x^7*(a + b*x^2)^10),x)
```

output
```
(110*b^3*log(a + b*x^2))/a^13 - (1/(6*a) - (b*x^2)/a^2 + (11*b^2*x^4)/a^3
+ (78419*b^3*x^6)/(252*a^4) + (50699*b^4*x^8)/(28*a^5) + (36839*b^5*x^10)/
(7*a^6) + (27599*b^6*x^12)/(3*a^7) + (20669*b^7*x^14)/(2*a^8) + (15125*b^8
*x^16)/(2*a^9) + (10505*b^9*x^18)/(3*a^10) + (935*b^10*x^20)/a^11 + (110*b
^11*x^22)/a^12)/(a^9*x^6 + b^9*x^24 + 9*a^8*b*x^8 + 9*a*b^8*x^22 + 36*a^7*
b^2*x^10 + 84*a^6*b^3*x^12 + 126*a^5*b^4*x^14 + 126*a^4*b^5*x^16 + 84*a^3*
b^6*x^18 + 36*a^2*b^7*x^20) - (220*b^3*log(x))/a^13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 537, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^7 (a+bx^2)^{10}} \, dx = \frac{27720 \log(b\, x^2+a)\, b^{12} x^{24} - 55440 \log(x)\, b^{12} x^{24} - 1517670 a^4 b^8 x^{16} + 252 a^{11} b\, x^2 - 2772 a^{10} b^2 x^4 - 75339 a}{}$$

input
```
int(1/x^7/(b*x^2+a)^10,x)
```

output
```
(27720*log(a + b*x**2)*a**9*b**3*x**6 + 249480*log(a + b*x**2)*a**8*b**4*x
**8 + 997920*log(a + b*x**2)*a**7*b**5*x**10 + 2328480*log(a + b*x**2)*a**
6*b**6*x**12 + 3492720*log(a + b*x**2)*a**5*b**7*x**14 + 3492720*log(a + b
*x**2)*a**4*b**8*x**16 + 2328480*log(a + b*x**2)*a**3*b**9*x**18 + 997920*
log(a + b*x**2)*a**2*b**10*x**20 + 249480*log(a + b*x**2)*a*b**11*x**22 +
27720*log(a + b*x**2)*b**12*x**24 - 55440*log(x)*a**9*b**3*x**6 - 498960*l
og(x)*a**8*b**4*x**8 - 1995840*log(x)*a**7*b**5*x**10 - 4656960*log(x)*a**
6*b**6*x**12 - 6985440*log(x)*a**5*b**7*x**14 - 6985440*log(x)*a**4*b**8*x
**16 - 4656960*log(x)*a**3*b**9*x**18 - 1995840*log(x)*a**2*b**10*x**20 -
498960*log(x)*a*b**11*x**22 - 55440*log(x)*b**12*x**24 - 42*a**12 + 252*a*
*11*b*x**2 - 2772*a**10*b**2*x**4 - 75339*a**9*b**3*x**6 - 428571*a**8*b**
4*x**8 - 1215324*a**7*b**5*x**10 - 2059596*a**6*b**6*x**12 - 2216214*a**5*
b**7*x**14 - 1517670*a**4*b**8*x**16 - 623700*a**3*b**9*x**18 - 124740*a**
2*b**10*x**20 + 3080*b**12*x**24)/(252*a**13*x**6*(a**9 + 9*a**8*b*x**2 +
36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5
*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x
**18))
```

3.209 $\int \frac{x^{24}}{(a+bx^2)^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 238

$$\int \frac{x^{24}}{(a+bx^2)^{10}} \, dx = \frac{55a^2x}{b^{12}} - \frac{10ax^3}{3b^{11}} + \frac{x^5}{5b^{10}} + \frac{a^{11}x}{18b^{12}(a+bx^2)^9} - \frac{199a^{10}x}{288b^{12}(a+bx^2)^8} + \frac{763a^9x}{192b^{12}(a+bx^2)^7} - \frac{32321a^8x}{2304b^{12}(a+bx^2)^6} + \frac{784949a^7x}{23040b^{12}(a+bx^2)^5} - \frac{1242571a^6x}{20480b^{12}(a+bx^2)^4} + \frac{10225523a^5x}{122880b^{12}(a+bx^2)^3} - \frac{9238669a^4x}{98304b^{12}(a+bx^2)^2} + \frac{6981491a^3x}{65536b^{12}(a+bx^2)} - \frac{7436429a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{25/2}}$$

output

```
55*a^2*x/b^12-10/3*a*x^3/b^11+1/5*x^5/b^10+1/18*a^11*x/b^12/(b*x^2+a)^9-19
9/288*a^10*x/b^12/(b*x^2+a)^8+763/192*a^9*x/b^12/(b*x^2+a)^7-32321/2304*a^
8*x/b^12/(b*x^2+a)^6+784949/23040*a^7*x/b^12/(b*x^2+a)^5-1242571/20480*a^6
*x/b^12/(b*x^2+a)^4+10225523/122880*a^5*x/b^12/(b*x^2+a)^3-9238669/98304*a
^4*x/b^12/(b*x^2+a)^2+6981491/65536*a^3*x/b^12/(b*x^2+a)-7436429/65536*a^(
5/2)*arctan(b^(1/2)*x/a^(1/2))/b^(25/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.70

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{b}x(334639305a^{11}+2900207310a^{10}bx^2+11110024926a^9b^2x^4+24648575094a^8b^3x^6+34810986496a^7b^4x^8+32314857354a^6b^5x^{10}+19562592546\cdots}{(a+bx^2)^9}}{2949\cdots}$$

input
```
Integrate[x^24/(a + b*x^2)^10,x]
```

output
```
((Sqrt[b]*x*(334639305*a^11 + 2900207310*a^10*b*x^2 + 11110024926*a^9*b^2*
x^4 + 24648575094*a^8*b^3*x^6 + 34810986496*a^7*b^4*x^8 + 32314857354*a^6*
b^5*x^10 + 19562592546*a^5*b^6*x^12 + 7323998514*a^4*b^7*x^14 + 1469632311
*a^3*b^8*x^16 + 94961664*a^2*b^9*x^18 - 4521984*a*b^10*x^20 + 589824*b^11*
x^22))/(a + b*x^2)^9 - 334639305*a^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(294
9120*b^(25/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 252, 252, 252, 252, 252, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{23\int \frac{x^{22}}{(bx^2+a)^9}dx}{18b} - \frac{x^{23}}{18b\,(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{23\left(\frac{21\int \frac{x^{20}}{\left(bx^2+a\right)^8}dx}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{23\left(\frac{21\left(\frac{19\int \frac{x^{18}}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\int \frac{x^{16}}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \textbf{252}$$

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\int\frac{x^{14}}{\left(bx^2+a\right)^5}dx}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{23}}{18b\left(a+bx^2\right)^9}$$

$\downarrow$ 252

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\int\frac{x^{12}}{\left(bx^2+a\right)^4}\,dx}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{23}}{18b\,(a+bx^2)^9}$$

↓ 252

$$\left(\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\int\frac{x^{10}}{\left(bx^2+a\right)^3}\,dx}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{21}}{16b(a+bx^2)^8}\right)$$

↓ 252

$$\left(\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{x^8}{\left(bx^2+a\right)^2}\,dx}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{19}}{14b\left(a+bx^2\right)^7}\right.\right.$$

↓ 252

$$\left(\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{x^6}{bx^2+a}\,dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{17}}{12b\left(a+bx^2\right)^6}\right.\right.$$

↓ 254

$$\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\left(\frac{x^4}{b}-\frac{ax^2}{b^2}-\frac{a^3}{b^3\left(bx^2+a\right)}+\frac{a^2}{b^3}\right)dx}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+bx^2\right)^5}$$

↓ 2009

$$\left(\left(\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2x}{b^3}-\frac{ax^3}{3b^2}+\frac{x^5}{5b}\right)}{2b}-\frac{x^7}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^9}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^{11}}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{13}}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^{15}}{10b\left(a+b\ldots\right.}\right.\right.\right.$$

input
```
Int[x^24/(a + b*x^2)^10,x]
```

output
```
-1/18*x^23/(b*(a + b*x^2)^9) + (23*(-1/16*x^21/(b*(a + b*x^2)^8) + (21*(-1
/14*x^19/(b*(a + b*x^2)^7) + (19*(-1/12*x^17/(b*(a + b*x^2)^6) + (17*(-1/1
0*x^15/(b*(a + b*x^2)^5) + (3*(-1/8*x^13/(b*(a + b*x^2)^4) + (13*(-1/6*x^1
1/(b*(a + b*x^2)^3) + (11*(-1/4*x^9/(b*(a + b*x^2)^2) + (9*(-1/2*x^7/(b*(a
 + b*x^2)) + (7*((a^2*x)/b^3 - (a*x^3)/(3*b^2) + x^5/(5*b) - (a^(5/2)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)))/(2*b)))/(4*b)))/(6*b)))/(8*b)))/(2*b)))
/(12*b)))/(14*b)))/(16*b)))/(18*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.63

method	result
default	$\frac{\frac{1}{5}b^2x^5-\frac{10}{3}ab\,x^3+55a^2x}{b^{12}}-\frac{a^3\left(\frac{-\frac{3831949}{65536}a^8x-\frac{48340777}{98304}a^7b\,x^3-\frac{297702839}{163840}a^6b^2x^5-\frac{631790371}{163840}a^5b^3x^7-\frac{463199}{90}a^4b^4x^9-\frac{725918941}{163840}a^3b^5 x}{\left(b\,x^2+a\right)^9}\right.}{b^{12}}$
risch	$\frac{x^5}{5b^{10}}-\frac{10a\,x^3}{3b^{11}}+\frac{55a^2x}{b^{12}}+\frac{\frac{3831949}{65536}a^{11}x+\frac{48340777}{98304}a^{10}b\,x^3+\frac{297702839}{163840}b^2a^9x^5+\frac{631790371}{163840}b^3a^8x^7+\frac{463199}{90}a^7b^4x^9+\frac{725918941}{163840}a^6b^5}{b^{12}(b\,x^2+a)^9}$

input
```
int(x^24/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
1/b^12*(1/5*b^2*x^5-10/3*a*b*x^3+55*a^2*x)-1/b^12*a^3*((-3831949/65536*a^8
*x-48340777/98304*a^7*b*x^3-297702839/163840*a^6*b^2*x^5-631790371/163840*
a^5*b^3*x^7-463199/90*a^4*b^4*x^9-725918941/163840*a^3*b^5*x^11-394553929/
163840*a^2*b^6*x^13-74539223/98304*a*b^7*x^15-6981491/65536*b^8*x^17)/(b*x
^2+a)^9+7436429/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 718, normalized size of antiderivative = 3.02

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate(x^24/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/5898240*(1179648*b^11*x^23 - 9043968*a*b^10*x^21 + 189923328*a^2*b^9*x^
19 + 2939264622*a^3*b^8*x^17 + 14647997028*a^4*b^7*x^15 + 39125185092*a^5*
b^6*x^13 + 64629714708*a^6*b^5*x^11 + 69621972992*a^7*b^4*x^9 + 4929715018
8*a^8*b^3*x^7 + 22220049852*a^9*b^2*x^5 + 5800414620*a^10*b*x^3 + 66927861
0*a^11*x + 334639305*(a^2*b^9*x^18 + 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84
*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 126*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a
^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b)
 - a)/(b*x^2 + a)))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3
*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a
^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12), 1/2949120*(589824*b^11*x^23 - 45
21984*a*b^10*x^21 + 94961664*a^2*b^9*x^19 + 1469632311*a^3*b^8*x^17 + 7323
998514*a^4*b^7*x^15 + 19562592546*a^5*b^6*x^13 + 32314857354*a^6*b^5*x^11
+ 34810986496*a^7*b^4*x^9 + 24648575094*a^8*b^3*x^7 + 11110024926*a^9*b^2*
x^5 + 2900207310*a^10*b*x^3 + 334639305*a^11*x - 334639305*(a^2*b^9*x^18 +
 9*a^3*b^8*x^16 + 36*a^4*b^7*x^14 + 84*a^5*b^6*x^12 + 126*a^6*b^5*x^10 + 1
26*a^7*b^4*x^8 + 84*a^8*b^3*x^6 + 36*a^9*b^2*x^4 + 9*a^10*b*x^2 + a^11)*sq
rt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*
x^14 + 84*a^3*b^18*x^12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^
15*x^6 + 36*a^7*b^14*x^4 + 9*a^8*b^13*x^2 + a^9*b^12)]
```

Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.32

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \frac{55a^2x}{b^{12}} - \frac{10ax^3}{3b^{11}} + \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x - \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} - \frac{7436429\sqrt{-\frac{a^5}{b^{25}}}\log\left(x + \frac{b^{12}\sqrt{-\frac{a^5}{b^{25}}}}{a^2}\right)}{131072} + \frac{172437705a^{11}x + 1450223310a^{10}bx^3 + 5358651102a^9b^2x^5 + 11372226678a^8b^3x^7 + 15178104832a^7}{2949120a^9b^{12} + 26542080a^8b^{13}x^2 + 106168320a^7b^{14}x^4 + 247726080a^6b^{15}x^6 + 371589120a^5b^{16}x^8 + 371} + \frac{x^5}{5b^{10}}$$

input
```
integrate(x**24/(b*x**2+a)**10,x)
```

output
```
55*a**2*x/b**12 - 10*a*x**3/(3*b**11) + 7436429*sqrt(-a**5/b**25)*log(x -
b**12*sqrt(-a**5/b**25)/a**2)/131072 - 7436429*sqrt(-a**5/b**25)*log(x + b
**12*sqrt(-a**5/b**25)/a**2)/131072 + (172437705*a**11*x + 1450223310*a**1
0*b*x**3 + 5358651102*a**9*b**2*x**5 + 11372226678*a**8*b**3*x**7 + 151781
04832*a**7*b**4*x**9 + 13066540938*a**6*b**5*x**11 + 7101970722*a**5*b**6*
x**13 + 2236176690*a**4*b**7*x**15 + 314167095*a**3*b**8*x**17)/(2949120*a
**9*b**12 + 26542080*a**8*b**13*x**2 + 106168320*a**7*b**14*x**4 + 2477260
80*a**6*b**15*x**6 + 371589120*a**5*b**16*x**8 + 371589120*a**4*b**17*x**1
0 + 247726080*a**3*b**18*x**12 + 106168320*a**2*b**19*x**14 + 26542080*a*b
**20*x**16 + 2949120*b**21*x**18) + x**5/(5*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.04

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \frac{314167095\,a^3b^8x^{17} + 2236176690\,a^4b^7x^{15} + 7101970722\,a^5b^6x^{13} + 13066540938\,a^6b^5x^{11} + 15178104832\,a}{2949120\,(b^{21}x^{18} + 9\,ab^{20}x^{16} + 36\,a^2b^{19}x^{14} + 84\,a^3b^{18}x^{12} + 126\,a^4b^{17}x^{10} +} - \frac{7436429\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{12}} + \frac{3\,b^2x^5 - 50\,abx^3 + 825\,a^2x}{15\,b^{12}}$$

input
```
integrate(x^24/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/2949120*(314167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a
^5*b^6*x^13 + 13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 1137222
6678*a^8*b^3*x^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437
705*a^11*x)/(b^21*x^18 + 9*a*b^20*x^16 + 36*a^2*b^19*x^14 + 84*a^3*b^18*x^
12 + 126*a^4*b^17*x^10 + 126*a^5*b^16*x^8 + 84*a^6*b^15*x^6 + 36*a^7*b^14*
x^4 + 9*a^8*b^13*x^2 + a^9*b^12) - 7436429/65536*a^3*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^12) + 1/15*(3*b^2*x^5 - 50*a*b*x^3 + 825*a^2*x)/b^12
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.68

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = -\frac{7436429\,a^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{12}} + \frac{314167095\,a^3b^8x^{17} + 2236176690\,a^4b^7x^{15} + 7101970722\,a^5b^6x^{13} + 13066540938\,a^6b^5x^{11} + 1517810483}{2949120\,(bx^2 +} + \frac{3\,b^{40}x^5 - 50\,ab^{39}x^3 + 825\,a^2b^{38}x}{15\,b^{50}}$$

input
```
integrate(x^24/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-7436429/65536*a^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^12) + 1/2949120*(314
167095*a^3*b^8*x^17 + 2236176690*a^4*b^7*x^15 + 7101970722*a^5*b^6*x^13 +
13066540938*a^6*b^5*x^11 + 15178104832*a^7*b^4*x^9 + 11372226678*a^8*b^3*x
^7 + 5358651102*a^9*b^2*x^5 + 1450223310*a^10*b*x^3 + 172437705*a^11*x)/((
b*x^2 + a)^9*b^12) + 1/15*(3*b^40*x^5 - 50*a*b^39*x^3 + 825*a^2*b^38*x)/b^
50
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.01

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \frac{\frac{3831949\,a^{11}\,x}{65536}+\frac{48340777\,a^{10}\,b\,x^3}{98304}+\frac{297702839\,a^9\,b^2\,x^5}{163840}+\frac{631790371\,a^8\,b^3\,x^7}{163840}+\frac{463199\,a^7\,b^4\,x^9}{90}+\frac{725918941\,a^6\,b^5\,x^{11}}{163840}+\frac{394553929}{1638}}{a^9\,b^{12}+9\,a^8\,b^{13}\,x^2+36\,a^7\,b^{14}\,x^4+84\,a^6\,b^{15}\,x^6+126\,a^5\,b^{16}\,x^8+126\,a^4\,b^{17}\,x^{10}+84\,a^3\,b^{18}\,x^{12}+30} + \frac{x^5}{5\,b^{10}} - \frac{10\,a\,x^3}{3\,b^{11}} + \frac{55\,a^2\,x}{b^{12}} - \frac{7436429\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{25/2}}$$

input
```
int(x^24/(a + b*x^2)^10,x)
```

output
```
((3831949*a^11*x)/65536 + (48340777*a^10*b*x^3)/98304 + (297702839*a^9*b^2
*x^5)/163840 + (631790371*a^8*b^3*x^7)/163840 + (463199*a^7*b^4*x^9)/90 +
(725918941*a^6*b^5*x^11)/163840 + (394553929*a^5*b^6*x^13)/163840 + (74539
223*a^4*b^7*x^15)/98304 + (6981491*a^3*b^8*x^17)/65536)/(a^9*b^12 + b^21*x
^18 + 9*a*b^20*x^16 + 9*a^8*b^13*x^2 + 36*a^7*b^14*x^4 + 84*a^6*b^15*x^6 +
 126*a^5*b^16*x^8 + 126*a^4*b^17*x^10 + 84*a^3*b^18*x^12 + 36*a^2*b^19*x^1
4) + x^5/(5*b^10) - (10*a*x^3)/(3*b^11) + (55*a^2*x)/b^12 - (7436429*a^(5/
2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(25/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.05

$$\int \frac{x^{24}}{(a+bx^2)^{10}}\,dx = \frac{-334639305\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^{11} - 3011753745\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^{10}b\,x^2 - 12047014980\sqrt{b}\,\sqrt{a}\,ata}{}$$

input
```
int(x^24/(b*x^2+a)^10,x)
```

output
```
( - 334639305*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**11 - 301175
3745*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**10*b*x**2 - 12047014
980*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9*b**2*x**4 - 2810970
1620*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b**3*x**6 - 421645
52430*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**4*x**8 - 42164
552430*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**5*x**10 - 281
09701620*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**6*x**12 - 1
2047014980*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**7*x**14 -
 3011753745*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**8*x**16
- 334639305*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**9*x**18
+ 334639305*a**11*b*x + 2900207310*a**10*b**2*x**3 + 11110024926*a**9*b**3
*x**5 + 24648575094*a**8*b**4*x**7 + 34810986496*a**7*b**5*x**9 + 32314857
354*a**6*b**6*x**11 + 19562592546*a**5*b**7*x**13 + 7323998514*a**4*b**8*x
**15 + 1469632311*a**3*b**9*x**17 + 94961664*a**2*b**10*x**19 - 4521984*a*
b**11*x**21 + 589824*b**12*x**23)/(2949120*b**13*(a**9 + 9*a**8*b*x**2 + 3
6*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*
x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x*
*18))
```

3.210 $\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 225

$$\begin{aligned}\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx =& -\frac{10ax}{b^{11}}+\frac{x^3}{3b^{10}}-\frac{a^{10}x}{18b^{11}(a+bx^2)^9}+\frac{181a^9x}{288b^{11}(a+bx^2)^8}\\&-\frac{625a^8x}{192b^{11}(a+bx^2)^7}+\frac{23555a^7x}{2304b^{11}(a+bx^2)^6}-\frac{100243a^6x}{4608b^{11}(a+bx^2)^5}\\&+\frac{136301a^5x}{4096b^{11}(a+bx^2)^4}-\frac{938245a^4x}{24576b^{11}(a+bx^2)^3}+\frac{3418855a^3x}{98304b^{11}(a+bx^2)^2}\\&-\frac{1987865a^2x}{65536b^{11}(a+bx^2)}+\frac{1616615a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{23/2}}\end{aligned}$$

output
```
-10*a*x/b^11+1/3*x^3/b^10-1/18*a^10*x/b^11/(b*x^2+a)^9+181/288*a^9*x/b^11/
(b*x^2+a)^8-625/192*a^8*x/b^11/(b*x^2+a)^7+23555/2304*a^7*x/b^11/(b*x^2+a)
^6-100243/4608*a^6*x/b^11/(b*x^2+a)^5+136301/4096*a^5*x/b^11/(b*x^2+a)^4-9
38245/24576*a^4*x/b^11/(b*x^2+a)^3+3418855/98304*a^3*x/b^11/(b*x^2+a)^2-19
87865/65536*a^2*x/b^11/(b*x^2+a)+1616615/65536*a^(3/2)*arctan(b^(1/2)*x/a^
(1/2))/b^(23/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.69

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{b}x(-14549535a^{10}-126095970a^9bx^2-483044562a^8b^2x^4-1071677178a^7b^3x^6-1513521152a^6b^4x^8-1404993798a^5b^5x^{10}-850547502a^4b^6x^{12}-}{(a+bx^2)^9}}{589824b^{23/2}}$$

input
```
Integrate[x^22/(a + b*x^2)^10,x]
```

output
```
((Sqrt[b]*x*(-14549535*a^10 - 126095970*a^9*b*x^2 - 483044562*a^8*b^2*x^4
- 1071677178*a^7*b^3*x^6 - 1513521152*a^6*b^4*x^8 - 1404993798*a^5*b^5*x^1
0 - 850547502*a^4*b^6*x^12 - 318434718*a^3*b^7*x^14 - 63897057*a^2*b^8*x^1
6 - 4128768*a*b^9*x^18 + 196608*b^10*x^20))/(a + b*x^2)^9 + 14549535*a^(3/
2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*b^(23/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 252, 252, 252, 252, 252, 252, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{7\int \frac{x^{20}}{(bx^2+a)^9}dx}{6b} - \frac{x^{21}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{19\int \frac{x^{18}}{\left(bx^2+a\right)^8}dx}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{19\left(\frac{17\int \frac{x^{16}}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\int \frac{x^{14}}{\left(bx^2+a\right)^6}dx}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\int\frac{x^{12}}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\int\frac{x^{10}}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{21}}{18b\left(a+bx^2\right)^9}$$

↓ 252

$$7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\int\frac{x^8}{\left(bx^2+a\right)^3}\,dx}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{19}}{16b(a+bx^2)^8}\right.$$

↓ 252

$$\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int\frac{x^6}{\left(bx^2+a\right)^2}\,dx}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{17}}{14b\left(a+bx^2\right)^7}\right.\right.$$

↓ 252

$$\left(\left(17\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{x^4}{bx^2+a}\,dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^{15}}{12b\left(a+bx^2\right)^6}\right.\right.\right.$$

↓ 254

$$\left(\left(\left(\left(5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\left(\frac{a^2}{b^2\left(bx^2+a\right)}-\frac{a}{b^2}+\frac{x^2}{b}\right)dx}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}\right)\right.\right.\right.\right.$$

↓ 2009

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{ax}{b^2}+\frac{x^3}{3b}\right)}{2b}-\frac{x^5}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^7}{4b\left(a+bx^2\right)^2}\right)}{2b}-\frac{x^9}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^{11}}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{13}}{10b\left(a+bx^2\right)^5}$$

input
```
Int[x^22/(a + b*x^2)^10,x]
```

output
```
-1/18*x^21/(b*(a + b*x^2)^9) + (7*(-1/16*x^19/(b*(a + b*x^2)^8) + (19*(-1/
14*x^17/(b*(a + b*x^2)^7) + (17*(-1/12*x^15/(b*(a + b*x^2)^6) + (5*(-1/10*
x^13/(b*(a + b*x^2)^5) + (13*(-1/8*x^11/(b*(a + b*x^2)^4) + (11*(-1/6*x^9/
(b*(a + b*x^2)^3) + (3*(-1/4*x^7/(b*(a + b*x^2)^2) + (7*(-1/2*x^5/(b*(a +
b*x^2)) + (5*(-((a*x)/b^2) + x^3/(3*b) + (a^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/b^(5/2)))/(2*b)))/(4*b)))/(2*b)))/(8*b)))/(10*b)))/(4*b)))/(14*b)))/(
16*b)))/(6*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.62

method	result
default	$-\frac{-\frac{1}{3} b \,x^{3}+10 a x}{b^{11}}+\frac{a^{2} \left(\frac{-\frac{961255}{65536} a^{8} x -\frac{12201403}{98304} a^{7} b \,x^{3}-\frac{15137633}{32768} a^{6} b^{2} x^{5}-\frac{32405717}{32768} a^{5} b^{3} x^{7}-\frac{24013}{18} a^{4} b^{4} x^{9}-\frac{38143787}{32768} a^{3} b^{5} x^{11}-\frac{21103775}{32768}}{\left(b \,x^{2}+a \right)^{9}}\right.}{b^{11}}$
risch	$\frac{x^{3}}{3 b^{10}}-\frac{10 a x}{b^{11}}+\frac{-\frac{961255}{65536} a^{10} x -\frac{12201403}{98304} a^{9} b \,x^{3}-\frac{15137633}{32768} a^{8} b^{2} x^{5}-\frac{32405717}{32768} a^{7} b^{3} x^{7}-\frac{24013}{18} a^{6} b^{4} x^{9}-\frac{38143787}{32768} a^{5} b^{5} x^{11}-\frac{21103775}{32768} a^{4}}{b^{11} \left(b \,x^{2}+a \right)^{9}}$

input
```
int(x^22/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-1/b^11*(-1/3*b*x^3+10*a*x)+1/b^11*a^2*((-961255/65536*a^8*x-12201403/9830
4*a^7*b*x^3-15137633/32768*a^6*b^2*x^5-32405717/32768*a^5*b^3*x^7-24013/18
*a^4*b^4*x^9-38143787/32768*a^3*b^5*x^11-21103775/32768*a^2*b^6*x^13-20435
525/98304*a*b^7*x^15-1987865/65536*b^8*x^17)/(b*x^2+a)^9+1616615/65536/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.08

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$$

$$= \left[\frac{393216\, b^{10}x^{21} - 8257536\, ab^9x^{19} - 127794114\, a^2b^8x^{17} - 636869436\, a^3b^7x^{15} - 1701095004\, a^4b^6x^{13} - 28}{} \right.$$

input
```
integrate(x^22/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/1179648*(393216*b^10*x^21 - 8257536*a*b^9*x^19 - 127794114*a^2*b^8*x^17
 - 636869436*a^3*b^7*x^15 - 1701095004*a^4*b^6*x^13 - 2809987596*a^5*b^5*x
^11 - 3027042304*a^6*b^4*x^9 - 2143354356*a^7*b^3*x^7 - 966089124*a^8*b^2*
x^5 - 252191940*a^9*b*x^3 - 29099070*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2
*b^8*x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6
*b^4*x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(-a/b
)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)))/(b^20*x^18 + 9*a*b^19*x
^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^
15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11), 1
/589824*(196608*b^10*x^21 - 4128768*a*b^9*x^19 - 63897057*a^2*b^8*x^17 - 3
18434718*a^3*b^7*x^15 - 850547502*a^4*b^6*x^13 - 1404993798*a^5*b^5*x^11 -
 1513521152*a^6*b^4*x^9 - 1071677178*a^7*b^3*x^7 - 483044562*a^8*b^2*x^5 -
 126095970*a^9*b*x^3 - 14549535*a^10*x + 14549535*(a*b^9*x^18 + 9*a^2*b^8*
x^16 + 36*a^3*b^7*x^14 + 84*a^4*b^6*x^12 + 126*a^5*b^5*x^10 + 126*a^6*b^4*
x^8 + 84*a^7*b^3*x^6 + 36*a^8*b^2*x^4 + 9*a^9*b*x^2 + a^10)*sqrt(a/b)*arct
an(b*x*sqrt(a/b)/a))/(b^20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^
3*b^17*x^12 + 126*a^4*b^16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*
a^7*b^13*x^4 + 9*a^8*b^12*x^2 + a^9*b^11)]
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.33

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{10ax}{b^{11}} - \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x-\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} + \frac{1616615\sqrt{-\frac{a^3}{b^{23}}}\log\left(x+\frac{b^{11}\sqrt{-\frac{a^3}{b^{23}}}}{a}\right)}{131072} + \frac{-8651295a^{10}x - 73208418a^9bx^3 - 272477394a^8b^2x^5 - 583302906a^7b^3x^7 - 786857984a^6b^4x^9 - 6}{589824a^9b^{11} + 5308416a^8b^{12}x^2 + 21233664a^7b^{13}x^4 + 49545216a^6b^{14}x^6 + 74317824a^5b^{15}x^8 + 7431782}$$
$$+ \frac{x^3}{3b^{10}}$$

input
```
integrate(x**22/(b*x**2+a)**10,x)
```

output
```
-10*a*x/b**11 - 1616615*sqrt(-a**3/b**23)*log(x - b**11*sqrt(-a**3/b**23)/
a)/131072 + 1616615*sqrt(-a**3/b**23)*log(x + b**11*sqrt(-a**3/b**23)/a)/1
31072 + (-8651295*a**10*x - 73208418*a**9*b*x**3 - 272477394*a**8*b**2*x**
5 - 583302906*a**7*b**3*x**7 - 786857984*a**6*b**4*x**9 - 686588166*a**5*b
**5*x**11 - 379867950*a**4*b**6*x**13 - 122613150*a**3*b**7*x**15 - 178907
85*a**2*b**8*x**17)/(589824*a**9*b**11 + 5308416*a**8*b**12*x**2 + 2123366
4*a**7*b**13*x**4 + 49545216*a**6*b**14*x**6 + 74317824*a**5*b**15*x**8 +
74317824*a**4*b**16*x**10 + 49545216*a**3*b**17*x**12 + 21233664*a**2*b**1
8*x**14 + 5308416*a*b**19*x**16 + 589824*b**20*x**18) + x**3/(3*b**10)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx = -\frac{17890785\,a^2b^8x^{17} + 122613150\,a^3b^7x^{15} + 379867950\,a^4b^6x^{13} + 686588166\,a^5b^5x^{11} + 786857984\,a^6b^4x^9}{589824\,(b^{20}x^{18} + 9\,ab^{19}x^{16} + 36\,a^2b^{18}x^{14} + 84\,a^3b^{17}x^{12} + 126\,a^4b^{16}x^{10} + 126\,a^5} + \frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{11}} + \frac{bx^3 - 30\,ax}{3\,b^{11}}$$

input
```
integrate(x^22/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/589824*(17890785*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*
b^6*x^13 + 686588166*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*
b^3*x^7 + 272477394*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/(b^
20*x^18 + 9*a*b^19*x^16 + 36*a^2*b^18*x^14 + 84*a^3*b^17*x^12 + 126*a^4*b^
16*x^10 + 126*a^5*b^15*x^8 + 84*a^6*b^14*x^6 + 36*a^7*b^13*x^4 + 9*a^8*b^1
2*x^2 + a^9*b^11) + 1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^1
1) + 1/3*(b*x^3 - 30*a*x)/b^11
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.67

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx = \frac{1616615\,a^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{11}} - \frac{17890785\,a^2b^8x^{17}+122613150\,a^3b^7x^{15}+379867950\,a^4b^6x^{13}+686588166\,a^5b^5x^{11}+786857984\,a^6b^4x^9}{589824\,(bx^2+a)^9b^{11}} + \frac{b^{20}x^3-30\,ab^{19}x}{3\,b^{30}}$$

input
```
integrate(x^22/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
1616615/65536*a^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^11) - 1/589824*(17890
785*a^2*b^8*x^17 + 122613150*a^3*b^7*x^15 + 379867950*a^4*b^6*x^13 + 68658
8166*a^5*b^5*x^11 + 786857984*a^6*b^4*x^9 + 583302906*a^7*b^3*x^7 + 272477
394*a^8*b^2*x^5 + 73208418*a^9*b*x^3 + 8651295*a^10*x)/((b*x^2 + a)^9*b^11
) + 1/3*(b^20*x^3 - 30*a*b^19*x)/b^30
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.03

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx = \frac{x^3}{3\,b^{10}} - \frac{\frac{961255\,a^{10}\,x}{65536}+\frac{12201403\,a^9\,b\,x^3}{98304}+\frac{15137633\,a^8\,b^2\,x^5}{32768}+\frac{32405717\,a^7\,b^3\,x^7}{32768}+\frac{24013\,a^6\,b^4\,x^9}{18}+\frac{38143787\,a^5\,b^5\,x^{11}}{32768}+\frac{21103775\,a^4\,b^6}{32768}}{a^9\,b^{11}+9\,a^8\,b^{12}\,x^2+36\,a^7\,b^{13}\,x^4+84\,a^6\,b^{14}\,x^6+126\,a^5\,b^{15}\,x^8+126\,a^4\,b^{16}\,x^{10}+84\,a^3\,b^{17}\,x^{12}+36} + \frac{1616615\,a^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{23/2}} - \frac{10\,a\,x}{b^{11}}$$

input
```
int(x^22/(a + b*x^2)^10,x)
```

output
```
x^3/(3*b^10) - ((961255*a^10*x)/65536 + (12201403*a^9*b*x^3)/98304 + (1513
7633*a^8*b^2*x^5)/32768 + (32405717*a^7*b^3*x^7)/32768 + (24013*a^6*b^4*x^
9)/18 + (38143787*a^5*b^5*x^11)/32768 + (21103775*a^4*b^6*x^13)/32768 + (2
0435525*a^3*b^7*x^15)/98304 + (1987865*a^2*b^8*x^17)/65536)/(a^9*b^11 + b^
20*x^18 + 9*a*b^19*x^16 + 9*a^8*b^12*x^2 + 36*a^7*b^13*x^4 + 84*a^6*b^14*x
^6 + 126*a^5*b^15*x^8 + 126*a^4*b^16*x^10 + 84*a^3*b^17*x^12 + 36*a^2*b^18
*x^14) + (1616615*a^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(23/2)) - (1
0*a*x)/b^11
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 475, normalized size of antiderivative = 2.11

$$\int \frac{x^{22}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{14549535\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^{10} + 130945815\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^{9}b\,x^{2} + 523783260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{}$$

input
```
int(x^22/(b*x^2+a)^10,x)
```

output
```
(14549535*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**10 + 130945815*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9*b*x**2 + 523783260*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b**2*x**4 + 1222160940*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**3*x**6 + 1833241410*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**4*x**8 + 1833241410*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**5*x**10 + 1222160940*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**6*x**12 + 523783260*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**7*x**14 + 130945815*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**8*x**16 + 14549535*sqrt
(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**9*x**18 - 14549535*a**10*b*
x - 126095970*a**9*b**2*x**3 - 483044562*a**8*b**3*x**5 - 1071677178*a**7*
b**4*x**7 - 1513521152*a**6*b**5*x**9 - 1404993798*a**5*b**6*x**11 - 85054
7502*a**4*b**7*x**13 - 318434718*a**3*b**8*x**15 - 63897057*a**2*b**9*x**1
7 - 4128768*a*b**10*x**19 + 196608*b**11*x**21)/(589824*b**12*(a**9 + 9*a*
*8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 1
26*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x*
*16 + b**9*x**18))
```

3.211 $\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 211

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = \frac{x}{b^{10}} + \frac{a^9x}{18b^{10}(a+bx^2)^9} - \frac{163a^8x}{288b^{10}(a+bx^2)^8} + \frac{3505a^7x}{1344b^{10}(a+bx^2)^7} - \frac{115715a^6x}{16128b^{10}(a+bx^2)^6} + \frac{422803a^5x}{32256b^{10}(a+bx^2)^5} - \frac{480365a^4x}{28672b^{10}(a+bx^2)^4} + \frac{379795a^3x}{24576b^{10}(a+bx^2)^3} - \frac{1050145a^2x}{98304b^{10}(a+bx^2)^2} + \frac{424415ax}{65536b^{10}(a+bx^2)} - \frac{230945\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536b^{21/2}}$$

output
```
x/b^10+1/18*a^9*x/b^10/(b*x^2+a)^9-163/288*a^8*x/b^10/(b*x^2+a)^8+3505/134
4*a^7*x/b^10/(b*x^2+a)^7-115715/16128*a^6*x/b^10/(b*x^2+a)^6+422803/32256*
a^5*x/b^10/(b*x^2+a)^5-480365/28672*a^4*x/b^10/(b*x^2+a)^4+379795/24576*a^
3*x/b^10/(b*x^2+a)^3-1050145/98304*a^2*x/b^10/(b*x^2+a)^2+424415/65536*a*x
/b^10/(b*x^2+a)-230945/65536*a^(1/2)*arctan(b^(1/2)*x/a^(1/2))/b^(21/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{b}x(14549535a^9+126095970a^8bx^2+483044562a^7b^2x^4+1071677178a^6b^3x^6+1513521152a^5b^4x^8+1404993798a^4b^5x^{10}+850547502a^3b^6x^{12}+31}{(a+bx^2)^9}}{4128768b^{21/2}}$$

input
```
Integrate[x^20/(a + b*x^2)^10,x]
```

output
```
((Sqrt[b]*x*(14549535*a^9 + 126095970*a^8*b*x^2 + 483044562*a^7*b^2*x^4 +
1071677178*a^6*b^3*x^6 + 1513521152*a^5*b^4*x^8 + 1404993798*a^4*b^5*x^10
+ 850547502*a^3*b^6*x^12 + 318434718*a^2*b^7*x^14 + 63897057*a*b^8*x^16 +
4128768*b^9*x^18))/(a + b*x^2)^9 - 14549535*Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(4128768*b^(21/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 252, 252, 252, 252, 252, 252, 252, 252, 262, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{19\int \frac{x^{18}}{(bx^2+a)^9}dx}{18b} - \frac{x^{19}}{18b\,(a+bx^2)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{19\left(\frac{17\int \frac{x^{16}}{\left(bx^2+a\right)^8}dx}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{19\left(\frac{17\left(\frac{15\int \frac{x^{14}}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\int \frac{x^{12}}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\int\frac{x^{10}}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow 252$$

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{x^8}{\left(bx^2+a\right)^4}\,dx}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{19}}{18b\left(a+bx^2\right)^9}$$

↓ 252

$$\left(\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{x^6}{\left(bx^2+a\right)^3}\,dx}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{17}}{16b(a+bx^2)^8}\right.$$

↓ 252

$$\left(17\,\frac{\left(15\,\frac{\left(13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\,\frac{\left(\frac{5\int\frac{x^4}{\left(bx^2+a\right)^2}\,dx}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{15}}{14b\left(a+bx^2\right)^7}\right)\right.$$

↓ 252

$$\left(\left(15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{x^2}{bx^2+a}\,dx}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{13}}{12b\left(a+bx^2\right)^6}\right.\right.\right.$$

↓ 262

$$\Bigg(\Bigg(\Bigg(13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{a\int\frac{1}{bx^2+a}\,dx}{b}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right)$$

↓ 218

$$\left(\left(\left(\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x}{b}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}\right)}{2b}-\frac{x^3}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^5}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^7}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^9}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^{11}}{10b\left(a+bx^2\right)^5}\right.\right.\right.\right.$$

input
```
Int[x^20/(a + b*x^2)^10,x]
```

output
```
-1/18*x^19/(b*(a + b*x^2)^9) + (19*(-1/16*x^17/(b*(a + b*x^2)^8) + (17*(-1
/14*x^15/(b*(a + b*x^2)^7) + (15*(-1/12*x^13/(b*(a + b*x^2)^6) + (13*(-1/1
0*x^11/(b*(a + b*x^2)^5) + (11*(-1/8*x^9/(b*(a + b*x^2)^4) + (9*(-1/6*x^7/
(b*(a + b*x^2)^3) + (7*(-1/4*x^5/(b*(a + b*x^2)^2) + (5*(-1/2*x^3/(b*(a +
b*x^2)) + (3*(x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)))/(2*b))
)/(4*b)))/(6*b)))/(8*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.61

method	result
default	$\frac{x}{b^{10}}-\frac{a\left(\frac{-\frac{165409}{65536}a^{8}x-\frac{2117549}{98304}a^{7}b\,x^{3}-\frac{2654039}{32768}a^{6}b^{2}x^{5}-\frac{40270037}{229376}a^{5}b^{3}x^{7}-\frac{30313}{126}a^{4}b^{4}x^{9}-\frac{49153835}{229376}a^{3}b^{5}x^{11}-\frac{3997865}{32768}a^{2}b^{6}x^{13}-\frac{4042835}{98304}}{\left(b\,x^{2}+a\right)^{9}}\right.}{b^{10}}$
risch	$\frac{x}{b^{10}}+\frac{\frac{165409}{65536}a^{9}x+\frac{2117549}{98304}a^{8}b\,x^{3}+\frac{2654039}{32768}a^{7}b^{2}x^{5}+\frac{40270037}{229376}a^{6}b^{3}x^{7}+\frac{30313}{126}a^{5}b^{4}x^{9}+\frac{49153835}{229376}a^{4}b^{5}x^{11}+\frac{3997865}{32768}a^{3}b^{6}x^{13}+\frac{4042835}{98304}a}{b^{10}\left(b\,x^{2}+a\right)^{9}}$

input
```
int(x^20/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
x/b^10-1/b^10*a*((-165409/65536*a^8*x-2117549/98304*a^7*b*x^3-2654039/3276
8*a^6*b^2*x^5-40270037/229376*a^5*b^3*x^7-30313/126*a^4*b^4*x^9-49153835/2
29376*a^3*b^5*x^11-3997865/32768*a^2*b^6*x^13-4042835/98304*a*b^7*x^15-424
415/65536*b^8*x^17)/(b*x^2+a)^9+230945/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.15

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$$

$$= \left[\frac{8257536\,b^{9}x^{19}+127794114\,ab^{8}x^{17}+636869436\,a^{2}b^{7}x^{15}+1701095004\,a^{3}b^{6}x^{13}+2809987596\,a^{4}b^{5}x^{11}+}{}\right.$$

input
```
integrate(x^20/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(8257536*b^9*x^19 + 127794114*a*b^8*x^17 + 636869436*a^2*b^7*x^
15 + 1701095004*a^3*b^6*x^13 + 2809987596*a^4*b^5*x^11 + 3027042304*a^5*b^
4*x^9 + 2143354356*a^6*b^3*x^7 + 966089124*a^7*b^2*x^5 + 252191940*a^8*b*x
^3 + 29099070*a^9*x + 14549535*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14
+ 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 +
36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a
^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36
*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10), 1/4128768*(4128768*b^9*x^19 +
63897057*a*b^8*x^17 + 318434718*a^2*b^7*x^15 + 850547502*a^3*b^6*x^13 + 14
04993798*a^4*b^5*x^11 + 1513521152*a^5*b^4*x^9 + 1071677178*a^6*b^3*x^7 +
483044562*a^7*b^2*x^5 + 126095970*a^8*b*x^3 + 14549535*a^9*x - 14549535*(b
^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x
^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^
9)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a))/(b^19*x^18 + 9*a*b^18*x^16 + 36*a^2*
b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15*x^10 + 126*a^5*b^14*x^8 + 84*a
^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*x^2 + a^9*b^10)]
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.30

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(-b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072} - \frac{230945\sqrt{-\frac{a}{b^{21}}}\log\left(b^{10}\sqrt{-\frac{a}{b^{21}}}+x\right)}{131072}$$
$$+ \frac{10420767a^9x + 88937058a^8bx^3 + 334408914a^7b^2x^5 + 724860666a^6b^3x^7 + 993296384a^5b^4\ldots}{4128768a^9b^{10} + 37158912a^8b^{11}x^2 + 148635648a^7b^{12}x^4 + 346816512a^6b^{13}x^6 + 520224768a^5b^{14}x^8 + 520\ldots}$$
$$+ \frac{x}{b^{10}}$$

input
```
integrate(x**20/(b*x**2+a)**10,x)
```

output
```
230945*sqrt(-a/b**21)*log(-b**10*sqrt(-a/b**21) + x)/131072 - 230945*sqrt(
-a/b**21)*log(b**10*sqrt(-a/b**21) + x)/131072 + (10420767*a**9*x + 889370
58*a**8*b*x**3 + 334408914*a**7*b**2*x**5 + 724860666*a**6*b**3*x**7 + 993
296384*a**5*b**4*x**9 + 884769030*a**4*b**5*x**11 + 503730990*a**3*b**6*x*
*13 + 169799070*a**2*b**7*x**15 + 26738145*a*b**8*x**17)/(4128768*a**9*b**
10 + 37158912*a**8*b**11*x**2 + 148635648*a**7*b**12*x**4 + 346816512*a**6
*b**13*x**6 + 520224768*a**5*b**14*x**8 + 520224768*a**4*b**15*x**10 + 346
816512*a**3*b**16*x**12 + 148635648*a**2*b**17*x**14 + 37158912*a*b**18*x*
*16 + 4128768*b**19*x**18) + x/b**10
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.05

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = \frac{26738145\,ab^8x^{17}+169799070\,a^2b^7x^{15}+503730990\,a^3b^6x^{13}+884769030\,a^4b^5x^{11}+993296384\,a^5b^4x^9+\cdots}{4128768\,(b^{19}x^{18}+9\,ab^{18}x^{16}+36\,a^2b^{17}x^{14}+84\,a^3b^{16}x^{12}+126\,a^4b^{15}x^{10}+126\,a^5b^{1\cdots}} - \frac{230945\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{10}}+\frac{x}{b^{10}}$$

input
```
integrate(x^20/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/4128768*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^
6*x^13 + 884769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^
3*x^7 + 334408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/(b^19
*x^18 + 9*a*b^18*x^16 + 36*a^2*b^17*x^14 + 84*a^3*b^16*x^12 + 126*a^4*b^15
*x^10 + 126*a^5*b^14*x^8 + 84*a^6*b^13*x^6 + 36*a^7*b^12*x^4 + 9*a^8*b^11*
x^2 + a^9*b^10) - 230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) +
x/b^10
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.62

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = -\frac{230945\,a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}b^{10}} + \frac{x}{b^{10}} + \frac{26738145\,ab^8x^{17} + 169799070\,a^2b^7x^{15} + 503730990\,a^3b^6x^{13} + 884769030\,a^4b^5x^{11} + 993296384\,a^5b^4x^9}{4128768\,(bx^2+a)^9b^{10}}$$

input
```
integrate(x^20/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-230945/65536*a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^10) + x/b^10 + 1/412876
8*(26738145*a*b^8*x^17 + 169799070*a^2*b^7*x^15 + 503730990*a^3*b^6*x^13 +
 884769030*a^4*b^5*x^11 + 993296384*a^5*b^4*x^9 + 724860666*a^6*b^3*x^7 +
334408914*a^7*b^2*x^5 + 88937058*a^8*b*x^3 + 10420767*a^9*x)/((b*x^2 + a)^
9*b^10)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx = \frac{\frac{165409\,a^9\,x}{65536} + \frac{2117549\,a^8\,b\,x^3}{98304} + \frac{2654039\,a^7\,b^2\,x^5}{32768} + \frac{40270037\,a^6\,b^3\,x^7}{229376} + \frac{30313\,a^5\,b^4\,x^9}{126} + \frac{49153835\,a^4\,b^5\,x^{11}}{229376} + \frac{3997865\,a^3\,b^6\,x^{13}}{32768} +}{a^9\,b^{10} + 9\,a^8\,b^{11}\,x^2 + 36\,a^7\,b^{12}\,x^4 + 84\,a^6\,b^{13}\,x^6 + 126\,a^5\,b^{14}\,x^8 + 126\,a^4\,b^{15}\,x^{10} + 84\,a^3\,b^{16}\,x^{12} + 36\,a^2\,b^{1}} + \frac{x}{b^{10}} - \frac{230945\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,b^{21/2}}$$

input
```
int(x^20/(a + b*x^2)^10,x)
```

output
```
((165409*a^9*x)/65536 + (2117549*a^8*b*x^3)/98304 + (424415*a*b^8*x^17)/65
536 + (2654039*a^7*b^2*x^5)/32768 + (40270037*a^6*b^3*x^7)/229376 + (30313
*a^5*b^4*x^9)/126 + (49153835*a^4*b^5*x^11)/229376 + (3997865*a^3*b^6*x^13
)/32768 + (4042835*a^2*b^7*x^15)/98304)/(a^9*b^10 + b^19*x^18 + 9*a*b^18*x
^16 + 9*a^8*b^11*x^2 + 36*a^7*b^12*x^4 + 84*a^6*b^13*x^6 + 126*a^5*b^14*x^
8 + 126*a^4*b^15*x^10 + 84*a^3*b^16*x^12 + 36*a^2*b^17*x^14) + x/b^10 - (2
30945*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*b^(21/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 461, normalized size of antiderivative = 2.18

$$\int \frac{x^{20}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-14549535\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 - 130945815\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 - 523783260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{}$$

input
```
int(x^20/(b*x^2+a)^10,x)
```

output
```
( - 14549535*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 - 13094581
5*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 - 523783260*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 - 1222160940*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 - 1833241410*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 - 1833241410*sq
rt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 - 1222160940*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 - 523783260*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 - 130945815*s
qrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**8*x**16 - 14549535*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**9*x**18 + 14549535*a**9*b*x +
126095970*a**8*b**2*x**3 + 483044562*a**7*b**3*x**5 + 1071677178*a**6*b**4
*x**7 + 1513521152*a**5*b**5*x**9 + 1404993798*a**4*b**6*x**11 + 850547502
*a**3*b**7*x**13 + 318434718*a**2*b**8*x**15 + 63897057*a*b**9*x**17 + 412
8768*b**10*x**19)/(4128768*b**11*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4
 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*
b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.212 $\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 197

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx = -\frac{x^{17}}{18b(a+bx^2)^9} - \frac{17x^{15}}{288b^2(a+bx^2)^8} - \frac{85x^{13}}{1344b^3(a+bx^2)^7} - \frac{1105x^{11}}{16128b^4(a+bx^2)^6} - \frac{2431x^9}{32256b^5(a+bx^2)^5} - \frac{2431x^7}{28672b^6(a+bx^2)^4} - \frac{2431x^5}{24576b^7(a+bx^2)^3} - \frac{12155x^3}{98304b^8(a+bx^2)^2} - \frac{12155x}{65536b^9(a+bx^2)} + \frac{12155\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536\sqrt{a}b^{19/2}}$$

output
```
-1/18*x^17/b/(b*x^2+a)^9-17/288*x^15/b^2/(b*x^2+a)^8-85/1344*x^13/b^3/(b*x
^2+a)^7-1105/16128*x^11/b^4/(b*x^2+a)^6-2431/32256*x^9/b^5/(b*x^2+a)^5-243
1/28672*x^7/b^6/(b*x^2+a)^4-2431/24576*x^5/b^7/(b*x^2+a)^3-12155/98304*x^3
/b^8/(b*x^2+a)^2-12155/65536*x/b^9/(b*x^2+a)+12155/65536*arctan(b^(1/2)*x/
a^(1/2))/a^(1/2)/b^(19/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.68

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{-\frac{\sqrt{b}x(765765a^8+6636630a^7bx^2+25423398a^6b^2x^4+56404062a^5b^3x^6+79659008a^4b^4x^8+73947042a^3b^5x^{10}+44765658a^2b^6x^{12}+16759722ab^7x}{(a+bx^2)^9}}{4128768b^{19/2}}$$

input
```
Integrate[x^18/(a + b*x^2)^10,x]
```

output
```
(-((Sqrt[b]*x*(765765*a^8 + 6636630*a^7*b*x^2 + 25423398*a^6*b^2*x^4 + 564
04062*a^5*b^3*x^6 + 79659008*a^4*b^4*x^8 + 73947042*a^3*b^5*x^10 + 4476565
8*a^2*b^6*x^12 + 16759722*a*b^7*x^14 + 3363003*b^8*x^16))/(a + b*x^2)^9) +
 (765765*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(4128768*b^(19/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.32, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 252, 252, 252, 252, 252, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$$
$$\Big\downarrow \text{252}$$
$$\frac{17\int \frac{x^{16}}{(bx^2+a)^9}dx}{18b}-\frac{x^{17}}{18b(a+bx^2)^9}$$
$$\Big\downarrow \text{252}$$
$$\frac{17\left(\frac{15\int \frac{x^{14}}{(bx^2+a)^8}dx}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{17}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{17\left(\frac{15\left(\frac{13\int \frac{x^{12}}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{17}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\int \frac{x^{10}}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{17}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{x^8}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{17}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{x^6}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^7}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{17}}{18b\left(a+bx^2\right)^9}$$

$\downarrow$ 252

$$17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{x^4}{\left(bx^2+a\right)^3}dx}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{15}}{16b(a+bx^2)^8}\right.$$

$\downarrow$ 252

$$\left(\left(15\,\frac{\left(13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\,\frac{\left(5\,\frac{\left(3\int\frac{x^2}{\left(bx^2+a\right)^2}\,dx\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{13}}{14b\left(a+bx^2\right)^7}\right)\right.$$

↓ 252

$$\left(\left(13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}\,dx}{2b}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right.\right.\right.$$

↓ 218

$$\left(\left(13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}-\frac{x}{2b\left(a+bx^2\right)}\right)}{4b}-\frac{x^3}{4b\left(a+bx^2\right)^2}\right)}{6b}-\frac{x^5}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^7}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^9}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^{11}}{12b\left(a+bx^2\right)^6}\right.\right.\right.$$

input
```
Int[x^18/(a + b*x^2)^10,x]
```

output
```
-1/18*x^17/(b*(a + b*x^2)^9) + (17*(-1/16*x^15/(b*(a + b*x^2)^8) + (15*(-1
/14*x^13/(b*(a + b*x^2)^7) + (13*(-1/12*x^11/(b*(a + b*x^2)^6) + (11*(-1/1
0*x^9/(b*(a + b*x^2)^5) + (9*(-1/8*x^7/(b*(a + b*x^2)^4) + (7*(-1/6*x^5/(b
*(a + b*x^2)^3) + (5*(-1/4*x^3/(b*(a + b*x^2)^2) + (3*(-1/2*x/(b*(a + b*x^
2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))))/(4*b)))/(6*b)))/(8
*b)))/(10*b)))/(12*b)))/(14*b)))/(16*b)))/(18*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

method	result
default	$\frac{-\frac{12155a^8x}{65536b^9}-\frac{158015a^7x^3}{98304b^8}-\frac{201773a^6x^5}{32768b^7}-\frac{3133559a^5x^7}{229376b^6}-\frac{2431a^4x^9}{126b^5}-\frac{4108169a^3x^{11}}{229376b^4}-\frac{355283a^2x^{13}}{32768b^3}-\frac{399041a\,x^{15}}{98304b^2}-\frac{53381x^{17}}{65536b}}{(b\,x^2+a)^9}+\frac{12155\,\mathrm{arc}}{6553}$
risch	$\frac{-\frac{12155a^8x}{65536b^9}-\frac{158015a^7x^3}{98304b^8}-\frac{201773a^6x^5}{32768b^7}-\frac{3133559a^5x^7}{229376b^6}-\frac{2431a^4x^9}{126b^5}-\frac{4108169a^3x^{11}}{229376b^4}-\frac{355283a^2x^{13}}{32768b^3}-\frac{399041a\,x^{15}}{98304b^2}-\frac{53381x^{17}}{65536b}}{(b\,x^2+a)^9}-\frac{12155\ln(}{13107}$

input
```
int(x^18/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-12155/65536*a^8/b^9*x-158015/98304*a^7/b^8*x^3-201773/32768*a^6/b^7*x^5-
3133559/229376*a^5/b^6*x^7-2431/126*a^4/b^5*x^9-4108169/229376*a^3/b^4*x^1
1-355283/32768*a^2/b^3*x^13-399041/98304*a/b^2*x^15-53381/65536/b*x^17)/(b
*x^2+a)^9+12155/65536/b^9/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.30

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx = \left[-\frac{6726006\,ab^9x^{17} + 33519444\,a^2b^8x^{15} + 89531316\,a^3b^7x^{13} + 147894084\,a^4b^6x^{11} + 159318016\,a^5b^5x^9 + \cdots}{8257536\cdots}\right.$$
$$-\frac{3363003\,ab^9x^{17} + 16759722\,a^2b^8x^{15} + 44765658\,a^3b^7x^{13} + 73947042\,a^4b^6x^{11} + 79659008\,a^5b^5x^9 + 564\cdots}{4128768\,(ab^{19}x\cdots}$$

input
```
integrate(x^18/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[-1/8257536*(6726006*a*b^9*x^17 + 33519444*a^2*b^8*x^15 + 89531316*a^3*b^7
*x^13 + 147894084*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 112808124*a^6*b^4
*x^7 + 50846796*a^7*b^3*x^5 + 13273260*a^8*b^2*x^3 + 1531530*a^9*b*x + 765
765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4
*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^
2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a*b^19
*x^18 + 9*a^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^
15*x^10 + 126*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^1
1*x^2 + a^10*b^10), -1/4128768*(3363003*a*b^9*x^17 + 16759722*a^2*b^8*x^15
 + 44765658*a^3*b^7*x^13 + 73947042*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 +
56404062*a^6*b^4*x^7 + 25423398*a^7*b^3*x^5 + 6636630*a^8*b^2*x^3 + 765765
*a^9*b*x - 765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*
x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^
4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a*b^19*x^18 + 9*a
^2*b^18*x^16 + 36*a^3*b^17*x^14 + 84*a^4*b^16*x^12 + 126*a^5*b^15*x^10 + 1
26*a^6*b^14*x^8 + 84*a^7*b^13*x^6 + 36*a^8*b^12*x^4 + 9*a^9*b^11*x^2 + a^1
0*b^10)]
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(-ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072} + \frac{12155\sqrt{-\frac{1}{ab^{19}}}\log\left(ab^9\sqrt{-\frac{1}{ab^{19}}}+x\right)}{131072}$$
$$+ \frac{-765765a^8x - 6636630a^7bx^3 - 25423398a^6b^2x^5 - 56404062a^5b^3x^7 - 79659008a^4b^4\cdots}{4128768a^9b^9 + 37158912a^8b^{10}x^2 + 148635648a^7b^{11}x^4 + 346816512a^6b^{12}x^6 + 520224768a^5b^{13}x^8 + 520\cdots}$$

input
```
integrate(x**18/(b*x**2+a)**10,x)
```

output
```
-12155*sqrt(-1/(a*b**19))*log(-a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + 121
55*sqrt(-1/(a*b**19))*log(a*b**9*sqrt(-1/(a*b**19)) + x)/131072 + (-765765
*a**8*x - 6636630*a**7*b*x**3 - 25423398*a**6*b**2*x**5 - 56404062*a**5*b*
*3*x**7 - 79659008*a**4*b**4*x**9 - 73947042*a**3*b**5*x**11 - 44765658*a*
*2*b**6*x**13 - 16759722*a*b**7*x**15 - 3363003*b**8*x**17)/(4128768*a**9*
b**9 + 37158912*a**8*b**10*x**2 + 148635648*a**7*b**11*x**4 + 346816512*a*
*6*b**12*x**6 + 520224768*a**5*b**13*x**8 + 520224768*a**4*b**14*x**10 + 3
46816512*a**3*b**15*x**12 + 148635648*a**2*b**16*x**14 + 37158912*a*b**17*
x**16 + 4128768*b**18*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx =$$
$$-\frac{3363003\,b^8x^{17} + 16759722\,ab^7x^{15} + 44765658\,a^2b^6x^{13} + 73947042\,a^3b^5x^{11} + 79659008\,a^4b^4x^9 + 56404\cdots}{4128768\,(b^{18}x^{18} + 9\,ab^{17}x^{16} + 36\,a^2b^{16}x^{14} + 84\,a^3b^{15}x^{12} + 126\,a^4b^{14}x^{10} + 126\,a^5b^{13}x^8\cdots}$$
$$+ \frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}b^9}$$

input
```
integrate(x^18/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/4128768*(3363003*b^8*x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13
 + 73947042*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 2
5423398*a^6*b^2*x^5 + 6636630*a^7*b*x^3 + 765765*a^8*x)/(b^18*x^18 + 9*a*b
^17*x^16 + 36*a^2*b^16*x^14 + 84*a^3*b^15*x^12 + 126*a^4*b^14*x^10 + 126*a
^5*b^13*x^8 + 84*a^6*b^12*x^6 + 36*a^7*b^11*x^4 + 9*a^8*b^10*x^2 + a^9*b^9
) + 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.62

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx = \frac{12155\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}b^9} - \frac{3363003\,b^8x^{17}+16759722\,ab^7x^{15}+44765658\,a^2b^6x^{13}+73947042\,a^3b^5x^{11}+79659008\,a^4b^4x^9+56404\cdots}{4128768\,(bx^2+a)^9b^9}$$

input
```
integrate(x^18/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^9) - 1/4128768*(3363003*b^8
*x^17 + 16759722*a*b^7*x^15 + 44765658*a^2*b^6*x^13 + 73947042*a^3*b^5*x^1
1 + 79659008*a^4*b^4*x^9 + 56404062*a^5*b^3*x^7 + 25423398*a^6*b^2*x^5 + 6
636630*a^7*b*x^3 + 765765*a^8*x)/((b*x^2 + a)^9*b^9)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx = \frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\sqrt{a}\,b^{19/2}} - \frac{\frac{53381\,x^{17}}{65536\,b}+\frac{399041\,a\,x^{15}}{98304\,b^2}+\frac{12155\,a^8\,x}{65536\,b^9}+\frac{355283\,a^2\,x^{13}}{32768\,b^3}+\frac{4108169\,a^3\,x^{11}}{229376\,b^4}+\frac{2431\,a^4\,x^9}{126\,b^5}+\frac{3133559\,a^5\,x^7}{229376\,b^6}+\frac{201773\,a^6\,x^5}{32768\,b^7}+\cdots}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a\cdots}$$

input
```
int(x^18/(a + b*x^2)^10,x)
```

output
```
(12155*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(1/2)*b^(19/2)) - ((53381*x^17)
/(65536*b) + (399041*a*x^15)/(98304*b^2) + (12155*a^8*x)/(65536*b^9) + (35
5283*a^2*x^13)/(32768*b^3) + (4108169*a^3*x^11)/(229376*b^4) + (2431*a^4*x
^9)/(126*b^5) + (3133559*a^5*x^7)/(229376*b^6) + (201773*a^6*x^5)/(32768*b
^7) + (158015*a^7*x^3)/(98304*b^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^
8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x
^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.31

$$\int \frac{x^{18}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{765765\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 6891885\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 27567540\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2}{}$$

input
```
int(x^18/(b*x^2+a)^10,x)
```

output
```
(765765*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 6891885*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 27567540*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 64324260*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 96486390*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 96486390*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 64324260*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 27567540*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 6891885*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a*b**8*x**16 + 765765*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*b**9*x**18 - 765765*a**9*b*x - 6636630*a**8*b**2*x**3
- 25423398*a**7*b**3*x**5 - 56404062*a**6*b**4*x**7 - 79659008*a**5*b**5*x
**9 - 73947042*a**4*b**6*x**11 - 44765658*a**3*b**7*x**13 - 16759722*a**2*
b**8*x**15 - 3363003*a*b**9*x**17)/(4128768*a*b**10*(a**9 + 9*a**8*b*x**2
+ 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b*
*5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9
*x**18))
```

3.213 $\int \frac{x^{16}}{(a+bx^2)^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 198

$$\int \frac{x^{16}}{(a+bx^2)^{10}} \, dx = -\frac{x^{15}}{18b(a+bx^2)^9} - \frac{5x^{13}}{96b^2(a+bx^2)^8} - \frac{65x^{11}}{1344b^3(a+bx^2)^7} - \frac{715x^9}{16128b^4(a+bx^2)^6} - \frac{143x^7}{3584b^5(a+bx^2)^5} - \frac{143x^5}{4096b^6(a+bx^2)^4} - \frac{715x^3}{24576b^7(a+bx^2)^3} - \frac{715x}{32768b^8(a+bx^2)^2} + \frac{715x}{65536ab^8(a+bx^2)} + \frac{715\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{3/2}b^{17/2}}$$

output
```
-1/18*x^15/b/(b*x^2+a)^9-5/96*x^13/b^2/(b*x^2+a)^8-65/1344*x^11/b^3/(b*x^2
+a)^7-715/16128*x^9/b^4/(b*x^2+a)^6-143/3584*x^7/b^5/(b*x^2+a)^5-143/4096*
x^5/b^6/(b*x^2+a)^4-715/24576*x^3/b^7/(b*x^2+a)^3-715/32768*x/b^8/(b*x^2+a
)^2+715/65536*x/a/b^8/(b*x^2+a)+715/65536*arctan(b^(1/2)*x/a^(1/2))/a^(3/2
)/b^(17/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.70

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}-985866ab^7x^{14}+450}{(a+bx^2)^9}}{4128768a^{3/2}b^{17/2}}$$

input
```
Integrate[x^16/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
 3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 263327
4*a^2*b^6*x^12 - 985866*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 4504
5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(3/2)*b^(17/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.31, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 252, 252, 252, 252, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^{14}}{(bx^2+a)^9}dx}{6b} - \frac{x^{15}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{5\left(\frac{13\int \frac{x^{12}}{(bx^2+a)^8}dx}{16b} - \frac{x^{13}}{16b(a+bx^2)^8}\right)}{6b} - \frac{x^{15}}{18b(a+bx^2)^9}$$

$$\Big\downarrow 252$$

$$\frac{5\left(\frac{13\left(\frac{11\int\frac{x^{10}}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{13}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow 252$$

$$\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\int\frac{x^8}{\left(bx^2+a\right)^6}dx}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{13}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow 252$$

$$\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\int\frac{x^6}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{13}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{x^4}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^5}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{13}}{16b(a+bx^2)^8}\right)}{6b}-\frac{x^{15}}{18b\left(a+bx^2\right)^9}$$

↓ 252

$$5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\int\frac{x^2}{\left(bx^2+a\right)^3}\,dx}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{13}}{16b(a+bx^2)^8}\right.$$

↓ 252

$$\left(13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right.\right.$$

↓ 215

$$\left(13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(\frac{\frac{\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}}{4b}-\frac{x}{4b\left(a+bx^2\right)^2}}{2b}-\frac{x^3}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^5}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^7}{10b\left(a+bx^2\right)^5}\right)}{4b}-\frac{x^9}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^{11}}{14b\left(a+bx^2\right)^7}\right.\right.$$

↓ 218

$$\left(\frac{13 \left(\frac{11 \left(\frac{3 \left(\frac{7 \left(\frac{5 \left(\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)}}{4b} - \frac{x}{4b\left(a+bx^2\right)^2} \right)}{2b} - \frac{x^3}{6b\left(a+bx^2\right)^3} \right)}{8b} - \frac{x^5}{8b\left(a+bx^2\right)^4} \right)}{10b} - \frac{x^7}{10b\left(a+bx^2\right)^5} \right)}{4b} - \frac{x^9}{12b\left(a+bx^2\right)^6} \right)}{14b} - \frac{x^{11}}{14b\left(a+bx^2\right)^7} \right.$$

input
```
Int[x^16/(a + b*x^2)^10,x]
```

output
```
-1/18*x^15/(b*(a + b*x^2)^9) + (5*(-1/16*x^13/(b*(a + b*x^2)^8) + (13*(-1/
14*x^11/(b*(a + b*x^2)^7) + (11*(-1/12*x^9/(b*(a + b*x^2)^6) + (3*(-1/10*x
^7/(b*(a + b*x^2)^5) + (7*(-1/8*x^5/(b*(a + b*x^2)^4) + (5*(-1/6*x^3/(b*(a
 + b*x^2)^3) + (-1/4*x/(b*(a + b*x^2)^2) + (x/(2*a*(a + b*x^2)) + ArcTan[(
Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]))/(4*b))/(2*b)))/(8*b)))/(10*b)))/(
4*b)))/(14*b)))/(16*b)))/(6*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.63

method	result
default	$\frac{-\frac{715a^{7}x}{65536b^{8}}-\frac{9295a^{6}x^{3}}{98304b^{7}}-\frac{11869a^{5}x^{5}}{32768b^{6}}-\frac{184327a^{4}x^{7}}{229376b^{5}}-\frac{143a^{3}x^{9}}{126b^{4}}-\frac{241657a^{2}x^{11}}{229376b^{3}}-\frac{20899a\,x^{13}}{32768b^{2}}-\frac{23473x^{15}}{98304b}+\frac{715x^{17}}{65536a}}{\left(b\,x^{2}+a\right)^{9}}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a\,b^{8}\sqrt{ab}}$
risch	$\frac{-\frac{715a^{7}x}{65536b^{8}}-\frac{9295a^{6}x^{3}}{98304b^{7}}-\frac{11869a^{5}x^{5}}{32768b^{6}}-\frac{184327a^{4}x^{7}}{229376b^{5}}-\frac{143a^{3}x^{9}}{126b^{4}}-\frac{241657a^{2}x^{11}}{229376b^{3}}-\frac{20899a\,x^{13}}{32768b^{2}}-\frac{23473x^{15}}{98304b}+\frac{715x^{17}}{65536a}}{\left(b\,x^{2}+a\right)^{9}}-\frac{715\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^{8}a}+$

input
```
int(x^16/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-715/65536*a^7/b^8*x-9295/98304*a^6/b^7*x^3-11869/32768*a^5/b^6*x^5-18432
7/229376*a^4/b^5*x^7-143/126*a^3/b^4*x^9-241657/229376*a^2/b^3*x^11-20899/
32768*a/b^2*x^13-23473/98304/b*x^15+715/65536/a*x^17)/(b*x^2+a)^9+715/6553
6/a/b^8/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.30

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$$

$$= \left[\frac{90090\,ab^9x^{17} - 1971732\,a^2b^8x^{15} - 5266548\,a^3b^7x^{13} - 8699652\,a^4b^6x^{11} - 9371648\,a^5b^5x^9 - 6635772\,a^6b}{8257536\,(a^2b^{18}x^{18} +}\right.$$

input
```
integrate(x^16/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(90090*a*b^9*x^17 - 1971732*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
 - 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 2990
988*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a
^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^2*b^18*x^18 + 9*a^3*b
^17*x^16 + 36*a^4*b^16*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a
^7*b^13*x^8 + 84*a^8*b^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b
^9), 1/4128768*(45045*a*b^9*x^17 - 985866*a^2*b^8*x^15 - 2633274*a^3*b^7*x
^13 - 4349826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 - 1
495494*a^7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18
+ 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 12
6*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^18*x^18 + 9*a^3*b^17*x^16 + 36*a^4*b^16
*x^14 + 84*a^5*b^15*x^12 + 126*a^6*b^14*x^10 + 126*a^7*b^13*x^8 + 84*a^8*b
^12*x^6 + 36*a^9*b^11*x^4 + 9*a^10*b^10*x^2 + a^11*b^9)]
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(-a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072} + \frac{715\sqrt{-\frac{1}{a^3b^{17}}}\log\left(a^2b^8\sqrt{-\frac{1}{a^3b^{17}}}+x\right)}{131072}$$
$$+ \frac{-45045a^8x - 390390a^7bx^3 - 1495494a^6b^2x^5 - 3317886a^5b^3x^7 - 4685824a^4}{4128768a^{10}b^8 + 37158912a^9b^9x^2 + 148635648a^8b^{10}x^4 + 346816512a^7b^{11}x^6 + 520224768a^6b^{12}x^8 + 520}$$

input
```
integrate(x**16/(b*x**2+a)**10,x)
```

output
```
-715*sqrt(-1/(a**3*b**17))*log(-a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/13107
2 + 715*sqrt(-1/(a**3*b**17))*log(a**2*b**8*sqrt(-1/(a**3*b**17)) + x)/131
072 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 33178
86*a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 263
3274*a**2*b**6*x**13 - 985866*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a*
*10*b**8 + 37158912*a**9*b**9*x**2 + 148635648*a**8*b**10*x**4 + 346816512
*a**7*b**11*x**6 + 520224768*a**6*b**12*x**8 + 520224768*a**5*b**13*x**10
+ 346816512*a**4*b**14*x**12 + 148635648*a**3*b**15*x**14 + 37158912*a**2*
b**16*x**16 + 4128768*a*b**17*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.11

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\,b^8x^{17} - 985866\,ab^7x^{15} - 2633274\,a^2b^6x^{13} - 4349826\,a^3b^5x^{11} - 4685824\,a^4b^4x^9 - 3317886\,a^5b^3x^7 -}{4128768\,(ab^{17}x^{18} + 9\,a^2b^{16}x^{16} + 36\,a^3b^{15}x^{14} + 84\,a^4b^{14}x^{12} + 126\,a^5b^{13}x^{10} + 126\,a^6b^{12}x^8 + 84\,a^7b}$$
$$+ \frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}ab^8}$$

input
```
integrate(x^16/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/4128768*(45045*b^8*x^17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 434
9826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^
6*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a*b^17*x^18 + 9*a^2*b^16*x^16
 + 36*a^3*b^15*x^14 + 84*a^4*b^14*x^12 + 126*a^5*b^13*x^10 + 126*a^6*b^12*
x^8 + 84*a^7*b^11*x^6 + 36*a^8*b^10*x^4 + 9*a^9*b^9*x^2 + a^10*b^8) + 715/
65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx = \frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}ab^8} + \frac{45045\,b^8x^{17} - 985866\,ab^7x^{15} - 2633274\,a^2b^6x^{13} - 4349826\,a^3b^5x^{11} - 4685824\,a^4b^4x^9 - 3317886\,a^5b^3x}{4128768\,(bx^2+a)^9ab^8}$$

input
```
integrate(x^16/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^8) + 1/4128768*(45045*b^8*x
^17 - 985866*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 - 46
85824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a*b^8)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.05

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx = \frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{3/2}\,b^{17/2}} - \frac{\frac{23473\,x^{15}}{98304\,b} - \frac{715\,x^{17}}{65536\,a} + \frac{20899\,a\,x^{13}}{32768\,b^2} + \frac{715\,a^7\,x}{65536\,b^8} + \frac{241657\,a^2\,x^{11}}{229376\,b^3} + \frac{143\,a^3\,x^9}{126\,b^4} + \frac{184327\,a^4\,x^7}{229376\,b^5} + \frac{11869\,a^5\,x^5}{32768\,b^6} + \frac{9295}{983}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a}$$

input
```
int(x^16/(a + b*x^2)^10,x)
```

output
```
(715*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(3/2)*b^(17/2)) - ((23473*x^15)/(
98304*b) - (715*x^17)/(65536*a) + (20899*a*x^13)/(32768*b^2) + (715*a^7*x)
/(65536*b^8) + (241657*a^2*x^11)/(229376*b^3) + (143*a^3*x^9)/(126*b^4) +
(184327*a^4*x^7)/(229376*b^5) + (11869*a^5*x^5)/(32768*b^6) + (9295*a^6*x^
3)/(98304*b^7))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*
x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^1
2 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.30

$$\int \frac{x^{16}}{(a+bx^2)^{10}}\,dx = \frac{45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 405405\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8 b\,x^2 + 1621620\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7 b^2 x^4 -$$

input
```
int(x^16/(b*x^2+a)^10,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 405405*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 1621620*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 3783780*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 5675670*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 5675670*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 3783780*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 1621620*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 405405*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**8*x**16 + 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**9*x**18 - 45045*a**9*b*x - 390390*a**8*b**2*x**3 - 1495494*a*
*7*b**3*x**5 - 3317886*a**6*b**4*x**7 - 4685824*a**5*b**5*x**9 - 4349826*a
**4*b**6*x**11 - 2633274*a**3*b**7*x**13 - 985866*a**2*b**8*x**15 + 45045*
a*b**9*x**17)/(4128768*a**2*b**9*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4
 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*
b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.214 $\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 199

$$\begin{aligned}\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = &-\frac{x^{13}}{18b(a+bx^2)^9}-\frac{13x^{11}}{288b^2(a+bx^2)^8}-\frac{143x^9}{4032b^3(a+bx^2)^7}\\&-\frac{143x^7}{5376b^4(a+bx^2)^6}-\frac{143x^5}{7680b^5(a+bx^2)^5}-\frac{143x^3}{12288b^6(a+bx^2)^4}\\&-\frac{143x}{24576b^7(a+bx^2)^3}+\frac{143x}{98304ab^7(a+bx^2)^2}\\&+\frac{143x}{65536a^2b^7(a+bx^2)}+\frac{143\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{5/2}b^{15/2}}\end{aligned}$$

output
```
-1/18*x^13/b/(b*x^2+a)^9-13/288*x^11/b^2/(b*x^2+a)^8-143/4032*x^9/b^3/(b*x
^2+a)^7-143/5376*x^7/b^4/(b*x^2+a)^6-143/7680*x^5/b^5/(b*x^2+a)^5-143/1228
8*x^3/b^6/(b*x^2+a)^4-143/24576*x/b^7/(b*x^2+a)^3+143/98304*x/a/b^7/(b*x^2
+a)^2+143/65536*x/a^2/b^7/(b*x^2+a)+143/65536*arctan(b^(1/2)*x/a^(1/2))/a^
(5/2)/b^(15/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-45045a^8-390390a^7bx^2-1495494a^6b^2x^4-3317886a^5b^3x^6-4685824a^4b^4x^8-4349826a^3b^5x^{10}-2633274a^2b^6x^{12}+390390ab^7x^{14}+450}{(a+bx^2)^9}}{20643840a^{5/2}b^{15/2}}$$

input
```
Integrate[x^14/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 - 1495494*a^6*b^2*x^4 -
 3317886*a^5*b^3*x^6 - 4685824*a^4*b^4*x^8 - 4349826*a^3*b^5*x^10 - 263327
4*a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 4504
5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(5/2)*b^(15/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 252, 252, 252, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{13\int \frac{x^{12}}{(bx^2+a)^9}dx}{18b}-\frac{x^{13}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{13\left(\frac{11\int \frac{x^{10}}{(bx^2+a)^8}dx}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{13}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{9\int \frac{x^8}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{13}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{x^6}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{13}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\int \frac{x^4}{\left(bx^2+a\right)^5}dx}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{13}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 252

$$\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{\frac{3\int\frac{x^2}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x^3}{8b\left(a+bx^2\right)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{13}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 252$$

$$13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{3\left(\frac{\int \frac{1}{\left(bx^2+a\right)^3}dx}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b\left(a+bx^2\right)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2)^8}\right.$$

↓ 215

$$13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\,\frac{3\,\frac{\left(\frac{\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b\left(a+bx^2\right)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^{11}}{16b(a+bx^2}$$

↓ 215

$$\left(\left(11\left(\frac{9\left(\frac{7\left(\frac{\frac{3\left(\frac{3\left(\frac{\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b\left(a+bx^2\right)^4}}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right)\right.\right.$$

↓ 218

$$\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{3\left(\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}}{6b}-\frac{x}{6b\left(a+bx^2\right)^3}\right)}{8b}-\frac{x^3}{8b\left(a+bx^2\right)^4}\right)}{2b}-\frac{x^5}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^7}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^9}{14b\left(a+bx^2\right)^7}\right.$$

input
```
Int[x^14/(a + b*x^2)^10,x]
```

output
```
-1/18*x^13/(b*(a + b*x^2)^9) + (13*(-1/16*x^11/(b*(a + b*x^2)^8) + (11*(-1
/14*x^9/(b*(a + b*x^2)^7) + (9*(-1/12*x^7/(b*(a + b*x^2)^6) + (7*(-1/10*x^
5/(b*(a + b*x^2)^5) + (-1/8*x^3/(b*(a + b*x^2)^4) + (3*(-1/6*x/(b*(a + b*x
^2)^3) + (x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b
]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a))/(6*b)))/(8*b))/(2*b)))/(12*b)))
/(14*b)))/(16*b)))/(18*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

method	result
default	$\frac{-\frac{143a^6x}{65536b^7}-\frac{1859a^5x^3}{98304b^6}-\frac{11869a^4x^5}{163840b^5}-\frac{184327a^3x^7}{1146880b^4}-\frac{143a^2x^9}{630b^3}-\frac{241657a\,x^{11}}{1146880b^2}-\frac{20899x^{13}}{163840b}+\frac{1859x^{15}}{98304a}+\frac{143b\,x^{17}}{65536a^2}}{(b\,x^2+a)^9}+\frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^2b^7\sqrt{ab}}$
risch	$\frac{-\frac{143a^6x}{65536b^7}-\frac{1859a^5x^3}{98304b^6}-\frac{11869a^4x^5}{163840b^5}-\frac{184327a^3x^7}{1146880b^4}-\frac{143a^2x^9}{630b^3}-\frac{241657a\,x^{11}}{1146880b^2}-\frac{20899x^{13}}{163840b}+\frac{1859x^{15}}{98304a}+\frac{143b\,x^{17}}{65536a^2}}{(b\,x^2+a)^9}-\frac{143\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^7a^2}+$

input
```
int(x^14/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-143/65536*a^6/b^7*x-1859/98304*a^5/b^6*x^3-11869/163840*a^4/b^5*x^5-1843
27/1146880*a^3/b^4*x^7-143/630*a^2/b^3*x^9-241657/1146880*a/b^2*x^11-20899
/163840/b*x^13+1859/98304/a*x^15+143/65536*b/a^2*x^17)/(b*x^2+a)^9+143/655
36/a^2/b^7/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.29

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = \left[\frac{90090\,ab^9x^{17}+780780\,a^2b^8x^{15}-5266548\,a^3b^7x^{13}-8699652\,a^4b^6x^{11}-9371648\,a^5b^5x^9-6635772\,a^6b^4}{41287680\,(a^3b^{17}x^{18}+}\right.$$

input
```
integrate(x^14/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 - 5266548*a^3*b^7*x^13
 - 8699652*a^4*b^6*x^11 - 9371648*a^5*b^5*x^9 - 6635772*a^6*b^4*x^7 - 2990
988*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a
^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^17*x^18 + 9*a^4*b
^16*x^16 + 36*a^5*b^15*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a
^8*b^12*x^8 + 84*a^9*b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b
^8), 1/20643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 - 2633274*a^3*b^7*
x^13 - 4349826*a^4*b^6*x^11 - 4685824*a^5*b^5*x^9 - 3317886*a^6*b^4*x^7 -
1495494*a^7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18
 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 1
26*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^17*x^18 + 9*a^4*b^16*x^16 + 36*a^5*b^1
5*x^14 + 84*a^6*b^14*x^12 + 126*a^7*b^13*x^10 + 126*a^8*b^12*x^8 + 84*a^9*
b^11*x^6 + 36*a^10*b^10*x^4 + 9*a^11*b^9*x^2 + a^12*b^8)]
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.46

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(-a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072} + \frac{143\sqrt{-\frac{1}{a^5b^{15}}}\log\left(a^3b^7\sqrt{-\frac{1}{a^5b^{15}}}+x\right)}{131072} + \frac{-45045a^8x - 390390a^7bx^3 - 1495494a^6b^2x^5 - 3317886a^5b^3x^7 - 46858}{20643840a^{11}b^7 + 185794560a^{10}b^8x^2 + 743178240a^9b^9x^4 + 1734082560a^8b^{10}x^6 + 2601123840a^7b^{11}x^8 +}$$

input
```
integrate(x**14/(b*x**2+a)**10,x)
```

output
```
-143*sqrt(-1/(a**5*b**15))*log(-a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/13107
2 + 143*sqrt(-1/(a**5*b**15))*log(a**3*b**7*sqrt(-1/(a**5*b**15)) + x)/131
072 + (-45045*a**8*x - 390390*a**7*b*x**3 - 1495494*a**6*b**2*x**5 - 33178
86*a**5*b**3*x**7 - 4685824*a**4*b**4*x**9 - 4349826*a**3*b**5*x**11 - 263
3274*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a
**11*b**7 + 185794560*a**10*b**8*x**2 + 743178240*a**9*b**9*x**4 + 1734082
560*a**8*b**10*x**6 + 2601123840*a**7*b**11*x**8 + 2601123840*a**6*b**12*x
**10 + 1734082560*a**5*b**13*x**12 + 743178240*a**4*b**14*x**14 + 18579456
0*a**3*b**15*x**16 + 20643840*a**2*b**16*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.11

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} - 2633274\,a^2b^6x^{13} - 4349826\,a^3b^5x^{11} - 4685824\,a^4b^4x^9 - 3317886\,a^5b^3x^7 -}{20643840\,(a^2b^{16}x^{18} + 9\,a^3b^{15}x^{16} + 36\,a^4b^{14}x^{14} + 84\,a^5b^{13}x^{12} + 126\,a^6b^{12}x^{10} + 126\,a^7b^{11}x^8 + 84\,a^8} + \frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^2b^7}$$

input
```
integrate(x^14/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 43
49826*a^3*b^5*x^11 - 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a
^6*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^2*b^16*x^18 + 9*a^3*b^15*x
^16 + 36*a^4*b^14*x^14 + 84*a^5*b^13*x^12 + 126*a^6*b^12*x^10 + 126*a^7*b^
11*x^8 + 84*a^8*b^10*x^6 + 36*a^9*b^9*x^4 + 9*a^10*b^8*x^2 + a^11*b^7) + 1
43/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = \frac{143\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^2b^7} + \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} - 2633274\,a^2b^6x^{13} - 4349826\,a^3b^5x^{11} - 4685824\,a^4b^4x^9 - 3317886\,a^5b^3x^7}{20643840\,(bx^2+a)^9a^2b^7}$$

input
```
integrate(x^14/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^7) + 1/20643840*(45045*b^
8*x^17 + 390390*a*b^7*x^15 - 2633274*a^2*b^6*x^13 - 4349826*a^3*b^5*x^11 -
 4685824*a^4*b^4*x^9 - 3317886*a^5*b^3*x^7 - 1495494*a^6*b^2*x^5 - 390390*
a^7*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^2*b^7)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.03

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx = \frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{5/2}\,b^{15/2}} - \frac{\frac{20899\,x^{13}}{163840\,b} - \frac{1859\,x^{15}}{98304\,a} + \frac{241657\,a\,x^{11}}{1146880\,b^2} + \frac{143\,a^6\,x}{65536\,b^7} - \frac{143\,b\,x^{17}}{65536\,a^2} + \frac{143\,a^2\,x^9}{630\,b^3} + \frac{184327\,a^3\,x^7}{1146880\,b^4} + \frac{11869\,a^4\,x^5}{163840\,b^5} + \frac{1859\,a}{9830}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a}$$

input
```
int(x^14/(a + b*x^2)^10,x)
```

output
```
(143*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(5/2)*b^(15/2)) - ((20899*x^13)/(
163840*b) - (1859*x^15)/(98304*a) + (241657*a*x^11)/(1146880*b^2) + (143*a
^6*x)/(65536*b^7) - (143*b*x^17)/(65536*a^2) + (143*a^2*x^9)/(630*b^3) + (
184327*a^3*x^7)/(1146880*b^4) + (11869*a^4*x^5)/(163840*b^5) + (1859*a^5*x
^3)/(98304*b^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2
*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^
12 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.29

$$\int \frac{x^{14}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 405405\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 1621620\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2x^4 -}{}$$

input
```
int(x^14/(b*x^2+a)^10,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 405405*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 1621620*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 3783780*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 5675670*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 5675670*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 3783780*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 1621620*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 405405*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**8*x**16 + 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**9*x**18 - 45045*a**9*b*x - 390390*a**8*b**2*x**3 - 1495494*a*
*7*b**3*x**5 - 3317886*a**6*b**4*x**7 - 4685824*a**5*b**5*x**9 - 4349826*a
**4*b**6*x**11 - 2633274*a**3*b**7*x**13 + 390390*a**2*b**8*x**15 + 45045*
a*b**9*x**17)/(20643840*a**3*b**8*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**
4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3
*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.215 $\int \frac{x^{12}}{(a+bx^2)^{10}} dx$

Optimal result

Integrand size = 13, antiderivative size = 200

$$\begin{aligned}\int \frac{x^{12}}{(a+bx^2)^{10}} dx =& -\frac{x^{11}}{18b(a+bx^2)^9} - \frac{11x^9}{288b^2(a+bx^2)^8} - \frac{11x^7}{448b^3(a+bx^2)^7} \\ & - \frac{11x^5}{768b^4(a+bx^2)^6} - \frac{11x^3}{1536b^5(a+bx^2)^5} - \frac{11x}{4096b^6(a+bx^2)^4} \\ & + \frac{11x}{24576ab^6(a+bx^2)^3} + \frac{55x}{98304a^2b^6(a+bx^2)^2} \\ & + \frac{55x}{65536a^3b^6(a+bx^2)} + \frac{55\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{7/2}b^{13/2}}\end{aligned}$$

output
```
-1/18*x^11/b/(b*x^2+a)^9-11/288*x^9/b^2/(b*x^2+a)^8-11/448*x^7/b^3/(b*x^2+
a)^7-11/768*x^5/b^4/(b*x^2+a)^6-11/1536*x^3/b^5/(b*x^2+a)^5-11/4096*x/b^6/
(b*x^2+a)^4+11/24576*x/a/b^6/(b*x^2+a)^3+55/98304*x/a^2/b^6/(b*x^2+a)^2+55
/65536*x/a^3/b^6/(b*x^2+a)+55/65536*arctan(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(1
3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{b}x(-3465a^8-30030a^7bx^2-115038a^6b^2x^4-255222a^5b^3x^6-360448a^4b^4x^8-334602a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9} \cdots}{4128768a^{7/2}b^{13/2}}$$

input
```
Integrate[x^12/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 - 25
5222*a^5*b^3*x^6 - 360448*a^4*b^4*x^8 - 334602*a^3*b^5*x^10 + 115038*a^2*b
^6*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(7/2)*b^(13/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 252, 252, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$$
$$\Big\downarrow \ 252$$
$$\frac{11\int \frac{x^{10}}{(bx^2+a)^9}dx}{18b}-\frac{x^{11}}{18b(a+bx^2)^9}$$
$$\Big\downarrow \ 252$$
$$\frac{11\left(\frac{9\int \frac{x^8}{(bx^2+a)^8}dx}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b(a+bx^2)^9}$$

$$\Big\downarrow \ \textbf{252}$$

$$\frac{11\left(\frac{9\left(\frac{\int \frac{x^6}{\left(bx^2+a\right)^7}dx}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ \textbf{252}$$

$$\frac{11\left(\frac{9\left(\frac{\frac{5\int \frac{x^4}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ \textbf{252}$$

$$\frac{11\left(\frac{9\left(\frac{\frac{5\left(\frac{3\int \frac{x^2}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ \textbf{252}$$

$$\frac{11\left(\frac{9\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\left(bx^2+a\right)^4}dx}{8b}-\frac{x}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{11\left(\frac{9\left(\frac{\frac{5\left(\frac{3\left(\frac{\frac{5\int\frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^{11}}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 215

$$11\left(\frac{9\left(\frac{5\left(\frac{3\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^9}{16b(a+bx^2)}\right.$$

↓ 215

$$11\left(\frac{9\left(\frac{5\left(\frac{3\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b} -$$

↓ 218

$$
\frac{11\left(\frac{9\left(\frac{5\left(\frac{3\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}}{8b}-\frac{x}{8b\left(a+bx^2\right)^4}\right)}{10b}-\frac{x^3}{10b\left(a+bx^2\right)^5}\right)}{12b}-\frac{x^5}{12b\left(a+bx^2\right)^6}}{2b}-\frac{x^7}{14b\left(a+bx^2\right)^7}\right)}{16b}
$$

input
```
Int[x^12/(a + b*x^2)^10,x]
```

output
```
-1/18*x^11/(b*(a + b*x^2)^9) + (11*(-1/16*x^9/(b*(a + b*x^2)^8) + (9*(-1/1
4*x^7/(b*(a + b*x^2)^7) + (-1/12*x^5/(b*(a + b*x^2)^6) + (5*(-1/10*x^3/(b*
(a + b*x^2)^5) + (3*(-1/8*x/(b*(a + b*x^2)^4) + (x/(6*a*(a + b*x^2)^3) + (
5*(x/(4*a*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sq
rt[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a))/(8*b)))/(10*b)))/(12*b))/(2*b)
))/(16*b)))/(18*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

method	result
default	$\frac{-\frac{55a^5x}{65536b^6}-\frac{715a^4x^3}{98304b^5}-\frac{913a^3x^5}{32768b^4}-\frac{14179a^2x^7}{229376b^3}-\frac{11a\,x^9}{126b^2}-\frac{18589x^{11}}{229376b}+\frac{913x^{13}}{32768a}+\frac{715b\,x^{15}}{98304a^2}+\frac{55b^2x^{17}}{65536a^3}}{(b\,x^2+a)^9}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^3b^6\sqrt{ab}}$
risch	$\frac{-\frac{55a^5x}{65536b^6}-\frac{715a^4x^3}{98304b^5}-\frac{913a^3x^5}{32768b^4}-\frac{14179a^2x^7}{229376b^3}-\frac{11a\,x^9}{126b^2}-\frac{18589x^{11}}{229376b}+\frac{913x^{13}}{32768a}+\frac{715b\,x^{15}}{98304a^2}+\frac{55b^2x^{17}}{65536a^3}}{(b\,x^2+a)^9}-\frac{55\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^6a^3}+\frac{55\ln(-bx+}{131072\sqrt{-a}}$

input
```
int(x^12/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-55/65536*a^5/b^6*x-715/98304*a^4/b^5*x^3-913/32768*a^3/b^4*x^5-14179/229
376*a^2/b^3*x^7-11/126*a/b^2*x^9-18589/229376/b*x^11+913/32768/a*x^13+715/
98304*b/a^2*x^15+55/65536*b^2/a^3*x^17)/(b*x^2+a)^9+55/65536/a^3/b^6/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.27

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$$
$$= \left[\frac{6930\,ab^9x^{17} + 60060\,a^2b^8x^{15} + 230076\,a^3b^7x^{13} - 669204\,a^4b^6x^{11} - 720896\,a^5b^5x^9 - 510444\,a^6b^4x^7 - 23}{8257536\,(a^4b^{16}x^{18} + 9\,a^5b^1}\right.$$

input
```
integrate(x^12/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 - 6
69204*a^4*b^6*x^11 - 720896*a^5*b^5*x^9 - 510444*a^6*b^4*x^7 - 230076*a^7*
b^3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16
 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8
+ 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x
^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 +
36*a^6*b^14*x^14 + 84*a^7*b^13*x^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8
 + 84*a^10*b^10*x^6 + 36*a^11*b^9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7), 1/4128
768*(3465*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 - 334602*a
^4*b^6*x^11 - 360448*a^5*b^5*x^9 - 255222*a^6*b^4*x^7 - 115038*a^7*b^3*x^5
 - 30030*a^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a
^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^
6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)
*x/a))/(a^4*b^16*x^18 + 9*a^5*b^15*x^16 + 36*a^6*b^14*x^14 + 84*a^7*b^13*x
^12 + 126*a^8*b^12*x^10 + 126*a^9*b^11*x^8 + 84*a^10*b^10*x^6 + 36*a^11*b^
9*x^4 + 9*a^12*b^8*x^2 + a^13*b^7)]
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.46

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(-a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072} + \frac{55\sqrt{-\frac{1}{a^7b^{13}}}\log\left(a^4b^6\sqrt{-\frac{1}{a^7b^{13}}}+x\right)}{131072}$$
$$+ \frac{-3465a^8x - 30030a^7bx^3 - 115038a^6b^2x^5 - 255222a^5b^3x^7 - 360448a^4}{4128768a^{12}b^6 + 37158912a^{11}b^7x^2 + 148635648a^{10}b^8x^4 + 346816512a^9b^9x^6 + 520224768a^8b^{10}x^8 + 520}$$

input
```
integrate(x**12/(b*x**2+a)**10,x)
```

output
```
-55*sqrt(-1/(a**7*b**13))*log(-a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/131072
 + 55*sqrt(-1/(a**7*b**13))*log(a**4*b**6*sqrt(-1/(a**7*b**13)) + x)/13107
2 + (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 - 255222*a**
5*b**3*x**7 - 360448*a**4*b**4*x**9 - 334602*a**3*b**5*x**11 + 115038*a**2
*b**6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**12*b**6 +
37158912*a**11*b**7*x**2 + 148635648*a**10*b**8*x**4 + 346816512*a**9*b**9
*x**6 + 520224768*a**8*b**10*x**8 + 520224768*a**7*b**11*x**10 + 346816512
*a**6*b**12*x**12 + 148635648*a**5*b**13*x**14 + 37158912*a**4*b**14*x**16
 + 4128768*a**3*b**15*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.10

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx$$
$$= \frac{3465\,b^8x^{17} + 30030\,ab^7x^{15} + 115038\,a^2b^6x^{13} - 334602\,a^3b^5x^{11} - 360448\,a^4b^4x^9 - 255222\,a^5b^3x^7 - 11503}{4128768\,(a^3b^{15}x^{18} + 9\,a^4b^{14}x^{16} + 36\,a^5b^{13}x^{14} + 84\,a^6b^{12}x^{12} + 126\,a^7b^{11}x^{10} + 126\,a^8b^{10}x^8 + 84\,a^9b^9x^6 +}$$
$$+ \frac{55\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^3b^6}$$

input
```
integrate(x^12/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602
*a^3*b^5*x^11 - 360448*a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x
^5 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^3*b^15*x^18 + 9*a^4*b^14*x^16 + 36*a
^5*b^13*x^14 + 84*a^6*b^12*x^12 + 126*a^7*b^11*x^10 + 126*a^8*b^10*x^8 + 8
4*a^9*b^9*x^6 + 36*a^10*b^8*x^4 + 9*a^11*b^7*x^2 + a^12*b^6) + 55/65536*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx = \frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^3b^6} + \frac{3465\,b^8x^{17}+30030\,ab^7x^{15}+115038\,a^2b^6x^{13}-334602\,a^3b^5x^{11}-360448\,a^4b^4x^9-255222\,a^5b^3x^7-115}{4128768\,(bx^2+a)^9a^3b^6}$$

input
```
integrate(x^12/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^6) + 1/4128768*(3465*b^8*x
^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 - 334602*a^3*b^5*x^11 - 36044
8*a^4*b^4*x^9 - 255222*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3
- 3465*a^8*x)/((b*x^2 + a)^9*a^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx = \frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{7/2}\,b^{13/2}} - \frac{\frac{18589\,x^{11}}{229376\,b}-\frac{913\,x^{13}}{32768\,a}+\frac{11\,a\,x^9}{126\,b^2}+\frac{55\,a^5\,x}{65536\,b^6}-\frac{715\,b\,x^{15}}{98304\,a^2}+\frac{14179\,a^2\,x^7}{229376\,b^3}+\frac{913\,a^3\,x^5}{32768\,b^4}+\frac{715\,a^4\,x^3}{98304\,b^5}-\frac{55\,b^2\,x^{17}}{65536\,a^3}}{a^9+9\,a^8\,b\,x^2+36\,a^7\,b^2\,x^4+84\,a^6\,b^3\,x^6+126\,a^5\,b^4\,x^8+126\,a^4\,b^5\,x^{10}+84\,a^3\,b^6\,x^{12}+36\,a^2\,b^7\,x^{14}+9\,a}$$

input
```
int(x^12/(a + b*x^2)^10,x)
```

output
```
(55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(7/2)*b^(13/2)) - ((18589*x^11)/(2
29376*b) - (913*x^13)/(32768*a) + (11*a*x^9)/(126*b^2) + (55*a^5*x)/(65536
*b^6) - (715*b*x^15)/(98304*a^2) + (14179*a^2*x^7)/(229376*b^3) + (913*a^3
*x^5)/(32768*b^4) + (715*a^4*x^3)/(98304*b^5) - (55*b^2*x^17)/(65536*a^3))
/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^
3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*
x^14)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.28

$$\int \frac{x^{12}}{(a+bx^2)^{10}}\,dx = \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9+31185\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2+124740\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2x^4+29}{}$$

input
```
int(x^12/(b*x^2+a)^10,x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 31185*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 124740*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 291060*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 436590*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 436590*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 291060*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**3*b**6*x**12 + 124740*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**2*b**7*x**14 + 31185*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**8*x**16 + 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**9*x**18 - 3465*a**9*b*x - 30030*a**8*b**2*x**3 - 115038*a**7*b**3*x**5
- 255222*a**6*b**4*x**7 - 360448*a**5*b**5*x**9 - 334602*a**4*b**6*x**11 +
 115038*a**3*b**7*x**13 + 30030*a**2*b**8*x**15 + 3465*a*b**9*x**17)/(4128
768*a**4*b**7*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**
6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**
2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.216 $\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 201

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = -\frac{x^9}{18b(a+bx^2)^9} - \frac{x^7}{32b^2(a+bx^2)^8} - \frac{x^5}{64b^3(a+bx^2)^7} - \frac{5x^3}{768b^4(a+bx^2)^6} - \frac{x}{512b^5(a+bx^2)^5} + \frac{x}{4096ab^5(a+bx^2)^4} + \frac{7x}{24576a^2b^5(a+bx^2)^3} + \frac{35x}{98304a^3b^5(a+bx^2)^2} + \frac{35x}{65536a^4b^5(a+bx^2)} + \frac{35\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{9/2}b^{11/2}}$$

output

```
-1/18*x^9/b/(b*x^2+a)^9-1/32*x^7/b^2/(b*x^2+a)^8-1/64*x^5/b^3/(b*x^2+a)^7-
5/768*x^3/b^4/(b*x^2+a)^6-1/512*x/b^5/(b*x^2+a)^5+1/4096*x/a/b^5/(b*x^2+a)
^4+7/24576*x/a^2/b^5/(b*x^2+a)^3+35/98304*x/a^3/b^5/(b*x^2+a)^2+35/65536*x
/a^4/b^5/(b*x^2+a)+35/65536*arctan(b^(1/2)*x/a^(1/2))/a^(9/2)/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.69

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6-32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16})}{(a+bx^2)^9}+315\,\mathrm{ar}}{589824a^{9/2}b^{11/2}}$$

input
```
Integrate[x^10/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202
*a^5*b^3*x^6 - 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12
 + 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(589824*a^(9/2)*b^(11/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 252, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^{10}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{x^8}{(bx^2+a)^9}dx}{2b}-\frac{x^9}{18b\,(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{7\int \frac{x^6}{(bx^2+a)^8}dx}{16b}-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\,(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{\frac{5\int \frac{x^4}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}}{16b}\right)-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{\frac{5\left(\frac{\int \frac{x^2}{\left(bx^2+a\right)^6}dx}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}}{16b}\right)-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{\frac{5\left(\frac{\frac{\int \frac{1}{\left(bx^2+a\right)^5}dx}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}}{16b}\right)-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{7\left(\frac{5\left(\frac{\frac{\frac{7\int \frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 215

$$\frac{\frac{7\left(\frac{5\left(\frac{\frac{\frac{7\left(\frac{5\int \frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 215

$$\frac{7\left(\frac{5\left(\frac{\frac{\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^7}{16b(a+bx^2)^8}}{2b}-\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{7\left(\frac{5\left(\frac{\frac{\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{}{16b(}}{2b}$$

$$\frac{x^9}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{7\left(\frac{5\left(\frac{\frac{\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}}{10b}-\frac{x}{10b\left(a+bx^2\right)^5}}{4b}-\frac{x^3}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^5}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{[illegible]}{16[illegible]}}{2b}$$

$$\frac{x^9}{18b\left(a+bx^2\right)^9}$$

input
```
Int[x^10/(a + b*x^2)^10,x]
```

output
```
-1/18*x^9/(b*(a + b*x^2)^9) + (-1/16*x^7/(b*(a + b*x^2)^8) + (7*(-1/14*x^5
/(b*(a + b*x^2)^7) + (5*(-1/12*x^3/(b*(a + b*x^2)^6) + (-1/10*x/(b*(a + b*
x^2)^5) + (x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*
(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*
a^(3/2)*Sqrt[b])))/(4*a)))/(6*a)))/(8*a))/(10*b))/(4*b)))/(14*b)))/(16*b))
/(2*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

method	result
default	$\frac{-\frac{35a^4x}{65536b^5}-\frac{455a^3x^3}{98304b^4}-\frac{581a^2x^5}{32768b^3}-\frac{1289a\,x^7}{32768b^2}-\frac{x^9}{18b}+\frac{1289x^{11}}{32768a}+\frac{581b\,x^{13}}{32768a^2}+\frac{455b^2x^{15}}{98304a^3}+\frac{35b^3x^{17}}{65536a^4}}{(b\,x^2+a)^9}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^4b^5\sqrt{ab}}$
risch	$\frac{-\frac{35a^4x}{65536b^5}-\frac{455a^3x^3}{98304b^4}-\frac{581a^2x^5}{32768b^3}-\frac{1289a\,x^7}{32768b^2}-\frac{x^9}{18b}+\frac{1289x^{11}}{32768a}+\frac{581b\,x^{13}}{32768a^2}+\frac{455b^2x^{15}}{98304a^3}+\frac{35b^3x^{17}}{65536a^4}}{(b\,x^2+a)^9}-\frac{35\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^5a^4}+\frac{35\ln\left(-bx+\sqrt{-}\right.}{131072\sqrt{-ab}\,b^5}$

input
```
int(x^10/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-35/65536*a^4*x/b^5-455/98304*a^3/b^4*x^3-581/32768*a^2/b^3*x^5-1289/3276
8*a/b^2*x^7-1/18*x^9/b+1289/32768/a*x^11+581/32768*b/a^2*x^13+455/98304*b^
2/a^3*x^15+35/65536*b^3/a^4*x^17)/(b*x^2+a)^9+35/65536/a^4/b^5/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.25

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx$$
$$= \left[\frac{630\,ab^9x^{17} + 5460\,a^2b^8x^{15} + 20916\,a^3b^7x^{13} + 46404\,a^4b^6x^{11} - 65536\,a^5b^5x^9 - 46404\,a^6b^4x^7 - 20916\,a^7b}{1179648\,(a^5b^{15}x^{18} + 9\,a^6b^{14}x^{16} -}\right.$$

input
```
integrate(x^10/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 4640
4*a^4*b^6*x^11 - 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5
 - 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*
b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b
^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^15*x^18 + 9*a^6*b^14*x^16 + 36*a^7*b^1
3*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^10 + 126*a^10*b^10*x^8 + 84*a^1
1*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^2 + a^14*b^6), 1/589824*(315*a*
b^9*x^17 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 - 3
2768*a^5*b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^
3 - 315*a^9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*
b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^
2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^15*x^18
 + 9*a^6*b^14*x^16 + 36*a^7*b^13*x^14 + 84*a^8*b^12*x^12 + 126*a^9*b^11*x^
10 + 126*a^10*b^10*x^8 + 84*a^11*b^9*x^6 + 36*a^12*b^8*x^4 + 9*a^13*b^7*x^
2 + a^14*b^6)]
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.45

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = -\frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(-a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072} + \frac{35\sqrt{-\frac{1}{a^9b^{11}}}\log\left(a^5b^5\sqrt{-\frac{1}{a^9b^{11}}}+x\right)}{131072} + \frac{-315a^8x - 2730a^7bx^3 - 10458a^6b^2x^5 - 23202a^5b^3x^7 - 32768a^4b^4x^9 +}{589824a^{13}b^5 + 5308416a^{12}b^6x^2 + 21233664a^{11}b^7x^4 + 49545216a^{10}b^8x^6 + 74317824a^9b^9x^8 + 74317824}$$

input
```
integrate(x**10/(b*x**2+a)**10,x)
```

output
```
-35*sqrt(-1/(a**9*b**11))*log(-a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/131072
 + 35*sqrt(-1/(a**9*b**11))*log(a**5*b**5*sqrt(-1/(a**9*b**11)) + x)/13107
2 + (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b*
*3*x**7 - 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x
**13 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**13*b**5 + 5308416*a*
*12*b**6*x**2 + 21233664*a**11*b**7*x**4 + 49545216*a**10*b**8*x**6 + 7431
7824*a**9*b**9*x**8 + 74317824*a**8*b**10*x**10 + 49545216*a**7*b**11*x**1
2 + 21233664*a**6*b**12*x**14 + 5308416*a**5*b**13*x**16 + 589824*a**4*b**
14*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.10

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = \frac{315\,b^8x^{17} + 2730\,ab^7x^{15} + 10458\,a^2b^6x^{13} + 23202\,a^3b^5x^{11} - 32768\,a^4b^4x^9 - 23202\,a^5b^3x^7 - 10458\,a^6b^2}{589824\,(a^4b^{14}x^{18} + 9\,a^5b^{13}x^{16} + 36\,a^6b^{12}x^{14} + 84\,a^7b^{11}x^{12} + 126\,a^8b^{10}x^{10} + 126\,a^9b^9x^8 + 84\,a^{10}b^8x^6 + 36} + \frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^4b^5}$$

input
```
integrate(x^10/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*
b^5*x^11 - 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 273
0*a^7*b*x^3 - 315*a^8*x)/(a^4*b^14*x^18 + 9*a^5*b^13*x^16 + 36*a^6*b^12*x^
14 + 84*a^7*b^11*x^12 + 126*a^8*b^10*x^10 + 126*a^9*b^9*x^8 + 84*a^10*b^8*
x^6 + 36*a^11*b^7*x^4 + 9*a^12*b^6*x^2 + a^13*b^5) + 35/65536*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^4*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.64

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = \frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^4b^5} + \frac{315\,b^8x^{17} + 2730\,ab^7x^{15} + 10458\,a^2b^6x^{13} + 23202\,a^3b^5x^{11} - 32768\,a^4b^4x^9 - 23202\,a^5b^3x^7 - 10458\,a^6b^2}{589824\,(bx^2+a)^9a^4b^5}$$

input
```
integrate(x^10/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b^5) + 1/589824*(315*b^8*x^1
7 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 - 32768*a^4*
b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8
*x)/((b*x^2 + a)^9*a^4*b^5)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.02

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx = \frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{9/2}\,b^{11/2}} - \frac{\frac{x^9}{18\,b} - \frac{1289\,x^{11}}{32768\,a} + \frac{1289\,a\,x^7}{32768\,b^2} + \frac{35\,a^4\,x}{65536\,b^5} - \frac{581\,b\,x^{13}}{32768\,a^2} + \frac{581\,a^2\,x^5}{32768\,b^3} + \frac{455\,a^3\,x^3}{98304\,b^4} - \frac{455\,b^2\,x^{15}}{98304\,a^3} - \frac{35\,b^3\,x^{17}}{65536\,a^4}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a}$$

input
```
int(x^10/(a + b*x^2)^10,x)
```

output
```
(35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(9/2)*b^(11/2)) - (x^9/(18*b) - (1
289*x^11)/(32768*a) + (1289*a*x^7)/(32768*b^2) + (35*a^4*x)/(65536*b^5) -
(581*b*x^13)/(32768*a^2) + (581*a^2*x^5)/(32768*b^3) + (455*a^3*x^3)/(9830
4*b^4) - (455*b^2*x^15)/(98304*a^3) - (35*b^3*x^17)/(65536*a^4))/(a^9 + b^
9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 12
6*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.27

$$\int \frac{x^{10}}{(a+bx^2)^{10}}\,dx$$

$$= \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 2835\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 11340\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2x^4 + 26460}{}$$

input
```
int(x^10/(b*x^2+a)^10,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 2835*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 11340*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 26460*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 39690*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**5*b**4*x**8 + 39690*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**4*b**5*x**10 + 26460*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**3*b**6*x**12 + 11340*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**2*b**7*x**14 + 2835*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a
*b**8*x**16 + 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**9*x**18
 - 315*a**9*b*x - 2730*a**8*b**2*x**3 - 10458*a**7*b**3*x**5 - 23202*a**6*
b**4*x**7 - 32768*a**5*b**5*x**9 + 23202*a**4*b**6*x**11 + 10458*a**3*b**7
*x**13 + 2730*a**2*b**8*x**15 + 315*a*b**9*x**17)/(589824*a**5*b**6*(a**9
+ 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x*
*8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b
**8*x**16 + b**9*x**18))
```

3.217 $\int \frac{x^8}{\left(a+bx^2\right)^{10}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 202

$$\int \frac{x^8}{\left(a+bx^2\right)^{10}} \, dx = -\frac{x^7}{18b\left(a+bx^2\right)^9}-\frac{7x^5}{288b^2\left(a+bx^2\right)^8}-\frac{5x^3}{576b^3\left(a+bx^2\right)^7}-\frac{5x}{2304b^4\left(a+bx^2\right)^6}+\frac{x}{4608ab^4\left(a+bx^2\right)^5}+\frac{x}{4096a^2b^4\left(a+bx^2\right)^4}+\frac{7x}{24576a^3b^4\left(a+bx^2\right)^3}+\frac{35x}{98304a^4b^4\left(a+bx^2\right)^2}+\frac{35x}{65536a^5b^4\left(a+bx^2\right)}+\frac{35\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{11/2}b^{9/2}}$$

output
```
-1/18*x^7/b/(b*x^2+a)^9-7/288*x^5/b^2/(b*x^2+a)^8-5/576*x^3/b^3/(b*x^2+a)^
7-5/2304*x/b^4/(b*x^2+a)^6+1/4608*x/a/b^4/(b*x^2+a)^5+1/4096*x/a^2/b^4/(b*
x^2+a)^4+7/24576*x/a^3/b^4/(b*x^2+a)^3+35/98304*x/a^4/b^4/(b*x^2+a)^2+35/6
5536*x/a^5/b^4/(b*x^2+a)+35/65536*arctan(b^(1/2)*x/a^(1/2))/a^(11/2)/b^(9/
2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-315a^8-2730a^7bx^2-10458a^6b^2x^4-23202a^5b^3x^6+32768a^4b^4x^8+23202a^3b^5x^{10}+10458a^2b^6x^{12}+2730ab^7x^{14}+315b^8x^{16})}{(a+bx^2)^9}+315\,\mathrm{ar}}{589824a^{11/2}b^{9/2}}$$

input
```
Integrate[x^8/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-315*a^8 - 2730*a^7*b*x^2 - 10458*a^6*b^2*x^4 - 23202
*a^5*b^3*x^6 + 32768*a^4*b^4*x^8 + 23202*a^3*b^5*x^10 + 10458*a^2*b^6*x^12
 + 2730*a*b^7*x^14 + 315*b^8*x^16))/(a + b*x^2)^9 + 315*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(589824*a^(11/2)*b^(9/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 252, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7\int \frac{x^6}{(bx^2+a)^9}dx}{18b}-\frac{x^7}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{7\left(\frac{5\int \frac{x^4}{(bx^2+a)^8}dx}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b(a+bx^2)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{x^2}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\left(bx^2+a\right)^6}dx}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\frac{9\int \frac{1}{\left(bx^2+a\right)^5}dx}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{9\left(\frac{7\int \frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^7}{18b\left(a+bx^2\right)^9}$$

↓ 215

$$\left(7\left(\frac{5\left(\frac{3\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^5}{16b(a+b}\right.$$

$\downarrow$ 215

$$\Bigg(\Bigg(5\,\frac{3\,\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}$$

↓ 218

$$\Bigg(\Bigg(5\Bigg(\frac{3\left(\frac{\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}}{12b}-\frac{x}{12b\left(a+bx^2\right)^6}\right)}{14b}-\frac{x^3}{14b\left(a+bx^2\right)^7}$$

input
```
Int[x^8/(a + b*x^2)^10,x]
```

output
```
-1/18*x^7/(b*(a + b*x^2)^9) + (7*(-1/16*x^5/(b*(a + b*x^2)^8) + (5*(-1/14*
x^3/(b*(a + b*x^2)^7) + (3*(-1/12*x/(b*(a + b*x^2)^6) + (x/(10*a*(a + b*x^
2)^5) + (9*(x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a
*(a + b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2
*a^(3/2)*Sqrt[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a))/(12*b)))/(14*b)))/(16*
b)))/(18*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

method	result
default	$\frac{-\frac{35a^3x}{65536b^4}-\frac{455a^2x^3}{98304b^3}-\frac{581a\,x^5}{32768b^2}-\frac{1289x^7}{32768b}+\frac{x^9}{18a}+\frac{1289b\,x^{11}}{32768a^2}+\frac{581b^2x^{13}}{32768a^3}+\frac{455b^3x^{15}}{98304a^4}+\frac{35b^4x^{17}}{65536a^5}}{(b\,x^2+a)^9}+\frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^5b^4\sqrt{ab}}$
risch	$\frac{-\frac{35a^3x}{65536b^4}-\frac{455a^2x^3}{98304b^3}-\frac{581a\,x^5}{32768b^2}-\frac{1289x^7}{32768b}+\frac{x^9}{18a}+\frac{1289b\,x^{11}}{32768a^2}+\frac{581b^2x^{13}}{32768a^3}+\frac{455b^3x^{15}}{98304a^4}+\frac{35b^4x^{17}}{65536a^5}}{(b\,x^2+a)^9}-\frac{35\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^4a^5}+\frac{35\ln\left(-bx+\sqrt{-\right.}{131072\sqrt{-ab}\,b^4$

input
```
int(x^8/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-35/65536*a^3*x/b^4-455/98304*a^2*x^3/b^3-581/32768*a/b^2*x^5-1289/32768/
b*x^7+1/18/a*x^9+1289/32768*b/a^2*x^11+581/32768*b^2/a^3*x^13+455/98304*b^
3/a^4*x^15+35/65536*b^4/a^5*x^17)/(b*x^2+a)^9+35/65536/a^5/b^4/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.24

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx$$
$$= \left[\frac{630\,ab^9x^{17}+5460\,a^2b^8x^{15}+20916\,a^3b^7x^{13}+46404\,a^4b^6x^{11}+65536\,a^5b^5x^9-46404\,a^6b^4x^7-20916\,a^7b}{1179648\,(a^6b^{14}x^{18}+9\,a^7b^{13}x^{16}-}\right.$$

input
```
integrate(x^8/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/1179648*(630*a*b^9*x^17 + 5460*a^2*b^8*x^15 + 20916*a^3*b^7*x^13 + 4640
4*a^4*b^6*x^11 + 65536*a^5*b^5*x^9 - 46404*a^6*b^4*x^7 - 20916*a^7*b^3*x^5
 - 5460*a^8*b^2*x^3 - 630*a^9*b*x - 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*
b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b
^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqr
t(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^14*x^18 + 9*a^7*b^13*x^16 + 36*a^8*b^1
2*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x^10 + 126*a^11*b^9*x^8 + 84*a^1
2*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^2 + a^15*b^5), 1/589824*(315*a*
b^9*x^17 + 2730*a^2*b^8*x^15 + 10458*a^3*b^7*x^13 + 23202*a^4*b^6*x^11 + 3
2768*a^5*b^5*x^9 - 23202*a^6*b^4*x^7 - 10458*a^7*b^3*x^5 - 2730*a^8*b^2*x^
3 - 315*a^9*b*x + 315*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*
b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^
2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^6*b^14*x^18
 + 9*a^7*b^13*x^16 + 36*a^8*b^12*x^14 + 84*a^9*b^11*x^12 + 126*a^10*b^10*x
^10 + 126*a^11*b^9*x^8 + 84*a^12*b^8*x^6 + 36*a^13*b^7*x^4 + 9*a^14*b^6*x^
2 + a^15*b^5)]
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.44

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(-a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072} + \frac{35\sqrt{-\frac{1}{a^{11}b^9}}\log\left(a^6b^4\sqrt{-\frac{1}{a^{11}b^9}}+x\right)}{131072} + \frac{-315a^8x - 2730a^7bx^3 - 10458a^6b^2x^5 - 23202a^5b^3x^7 + 32768a^4b^4x^9 +}{589824a^{14}b^4 + 5308416a^{13}b^5x^2 + 21233664a^{12}b^6x^4 + 49545216a^{11}b^7x^6 + 74317824a^{10}b^8x^8 + 7431782}$$

input
```
integrate(x**8/(b*x**2+a)**10,x)
```

output
```
-35*sqrt(-1/(a**11*b**9))*log(-a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/131072
 + 35*sqrt(-1/(a**11*b**9))*log(a**6*b**4*sqrt(-1/(a**11*b**9)) + x)/13107
2 + (-315*a**8*x - 2730*a**7*b*x**3 - 10458*a**6*b**2*x**5 - 23202*a**5*b*
*3*x**7 + 32768*a**4*b**4*x**9 + 23202*a**3*b**5*x**11 + 10458*a**2*b**6*x
**13 + 2730*a*b**7*x**15 + 315*b**8*x**17)/(589824*a**14*b**4 + 5308416*a*
*13*b**5*x**2 + 21233664*a**12*b**6*x**4 + 49545216*a**11*b**7*x**6 + 7431
7824*a**10*b**8*x**8 + 74317824*a**9*b**9*x**10 + 49545216*a**8*b**10*x**1
2 + 21233664*a**7*b**11*x**14 + 5308416*a**6*b**12*x**16 + 589824*a**5*b**
13*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx$$
$$= \frac{315\,b^8x^{17} + 2730\,ab^7x^{15} + 10458\,a^2b^6x^{13} + 23202\,a^3b^5x^{11} + 32768\,a^4b^4x^9 - 23202\,a^5b^3x^7 - 10458\,a^6b^2}{589824\,(a^5b^{13}x^{18} + 9\,a^6b^{12}x^{16} + 36\,a^7b^{11}x^{14} + 84\,a^8b^{10}x^{12} + 126\,a^9b^9x^{10} + 126\,a^{10}b^8x^8 + 84\,a^{11}b^7x^6 + 36} + \frac{35\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^5b^4}$$

input
```
integrate(x^8/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/589824*(315*b^8*x^17 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*
b^5*x^11 + 32768*a^4*b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 273
0*a^7*b*x^3 - 315*a^8*x)/(a^5*b^13*x^18 + 9*a^6*b^12*x^16 + 36*a^7*b^11*x^
14 + 84*a^8*b^10*x^12 + 126*a^9*b^9*x^10 + 126*a^10*b^8*x^8 + 84*a^11*b^7*
x^6 + 36*a^12*b^6*x^4 + 9*a^13*b^5*x^2 + a^14*b^4) + 35/65536*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^5*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx = \frac{35\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^5b^4} + \frac{315\,b^8x^{17} + 2730\,ab^7x^{15} + 10458\,a^2b^6x^{13} + 23202\,a^3b^5x^{11} + 32768\,a^4b^4x^9 - 23202\,a^5b^3x^7 - 10458\,a^6b^2}{589824\,(bx^2+a)^9a^5b^4}$$

input
```
integrate(x^8/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
35/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5*b^4) + 1/589824*(315*b^8*x^1
7 + 2730*a*b^7*x^15 + 10458*a^2*b^6*x^13 + 23202*a^3*b^5*x^11 + 32768*a^4*
b^4*x^9 - 23202*a^5*b^3*x^7 - 10458*a^6*b^2*x^5 - 2730*a^7*b*x^3 - 315*a^8
*x)/((b*x^2 + a)^9*a^5*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.01

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx = \frac{\frac{x^9}{18\,a} - \frac{1289\,x^7}{32768\,b} - \frac{581\,a\,x^5}{32768\,b^2} - \frac{35\,a^3\,x}{65536\,b^4} + \frac{1289\,b\,x^{11}}{32768\,a^2} - \frac{455\,a^2\,x^3}{98304\,b^3} + \frac{581\,b^2\,x^{13}}{32768\,a^3} + \frac{455\,b^3\,x^{15}}{98304\,a^4} + \frac{35\,b^4\,x^{17}}{65536\,a^5}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8} + \frac{35\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{11/2}\,b^{9/2}}$$

input
```
int(x^8/(a + b*x^2)^10,x)
```

output
```
(x^9/(18*a) - (1289*x^7)/(32768*b) - (581*a*x^5)/(32768*b^2) - (35*a^3*x)/
(65536*b^4) + (1289*b*x^11)/(32768*a^2) - (455*a^2*x^3)/(98304*b^3) + (581
*b^2*x^13)/(32768*a^3) + (455*b^3*x^15)/(98304*a^4) + (35*b^4*x^17)/(65536
*a^5))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*
a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^
2*b^7*x^14) + (35*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(11/2)*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.26

$$\int \frac{x^8}{(a+bx^2)^{10}}\,dx = \frac{315\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 2835\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8 b\,x^2 + 11340\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7 b^2 x^4 + 26460}{}$$

input
```
int(x^8/(b*x^2+a)^10,x)
```

output
```
(315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 2835*sqrt(b)*sqr
t(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 11340*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 26460*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 39690*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a**5*b**4*x**8 + 39690*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**4*b**5*x**10 + 26460*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a**3*b**6*x**12 + 11340*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a
)))*a**2*b**7*x**14 + 2835*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a
*b**8*x**16 + 315*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**9*x**18
 - 315*a**9*b*x - 2730*a**8*b**2*x**3 - 10458*a**7*b**3*x**5 - 23202*a**6*
b**4*x**7 + 32768*a**5*b**5*x**9 + 23202*a**4*b**6*x**11 + 10458*a**3*b**7
*x**13 + 2730*a**2*b**8*x**15 + 315*a*b**9*x**17)/(589824*a**6*b**5*(a**9
+ 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x*
*8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b
**8*x**16 + b**9*x**18))
```

3.218 $\int \frac{x^6}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 203

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = -\frac{x^5}{18b(a+bx^2)^9} - \frac{5x^3}{288b^2(a+bx^2)^8} - \frac{5x}{1344b^3(a+bx^2)^7} + \frac{5x}{16128ab^3(a+bx^2)^6} + \frac{11x}{32256a^2b^3(a+bx^2)^5} + \frac{11x}{28672a^3b^3(a+bx^2)^4} + \frac{11x}{24576a^4b^3(a+bx^2)^3} + \frac{55x}{98304a^5b^3(a+bx^2)^2} + \frac{55x}{65536a^6b^3(a+bx^2)} + \frac{55\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{13/2}b^{7/2}}$$

output
```
-1/18*x^5/b/(b*x^2+a)^9-5/288*x^3/b^2/(b*x^2+a)^8-5/1344*x/b^3/(b*x^2+a)^7
+5/16128*x/a/b^3/(b*x^2+a)^6+11/32256*x/a^2/b^3/(b*x^2+a)^5+11/28672*x/a^3
/b^3/(b*x^2+a)^4+11/24576*x/a^4/b^3/(b*x^2+a)^3+55/98304*x/a^5/b^3/(b*x^2+
a)^2+55/65536*x/a^6/b^3/(b*x^2+a)+55/65536*arctan(b^(1/2)*x/a^(1/2))/a^(13
/2)/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-3465a^8-30030a^7bx^2-115038a^6b^2x^4+334602a^5b^3x^6+360448a^4b^4x^8+255222a^3b^5x^{10}+115038a^2b^6x^{12}+30030ab^7x^{14}+3465b^8x^{16})}{(a+bx^2)^9}}{4128768a^{13/2}b^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-3465*a^8 - 30030*a^7*b*x^2 - 115038*a^6*b^2*x^4 + 33
4602*a^5*b^3*x^6 + 360448*a^4*b^4*x^8 + 255222*a^3*b^5*x^10 + 115038*a^2*b
^6*x^12 + 30030*a*b^7*x^14 + 3465*b^8*x^16))/(a + b*x^2)^9 + 3465*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(13/2)*b^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 252, 215, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{5\int \frac{x^4}{(bx^2+a)^9}dx}{18b}-\frac{x^5}{18b(a+bx^2)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{5\left(\frac{3\int \frac{x^2}{(bx^2+a)^8}dx}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b(a+bx^2)^9}$$

$$\Big\downarrow \ 252$$

$$\frac{5\left(\frac{3\left(\frac{\int \frac{1}{\left(bx^2+a\right)^7}dx}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{5\left(\frac{3\left(\frac{\frac{11\int \frac{1}{\left(bx^2+a\right)^6}dx}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{5\left(\frac{3\left(\frac{\frac{11\left(\frac{9\int \frac{1}{\left(bx^2+a\right)^5}dx}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{5\left(\frac{3\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{5\left(\frac{3\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+bx^2)^8}\right)}{18b}-\frac{x^5}{18b\left(a+bx^2\right)^9}$$

↓ 215

$$5\left(\frac{3\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right)}{16b}-\frac{x^3}{16b(a+}\right.$$

↓ 215

$$\left(\left(3\,\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}}{14b}-\frac{x}{14b\left(a+bx^2\right)^7}\right.\right.$$

↓ 218

$$\left(3 \left(\frac{\frac{11 \left(\frac{9 \left(\frac{7 \left(\frac{5 \left(\frac{3 \left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{x}{2a\left(a+bx^2\right)} \right)}{4a} + \frac{x}{4a\left(a+bx^2\right)^2} \right)}{6a} + \frac{x}{6a\left(a+bx^2\right)^3} \right)}{8a} + \frac{x}{8a\left(a+bx^2\right)^4} \right)}{10a} + \frac{x}{10a\left(a+bx^2\right)^5} \right)}{12a} + \frac{x}{12a\left(a+bx^2\right)^6}}{14b} - \frac{x}{14b\left(a+bx^2\right)} \right.\right.$$

input
```
Int[x^6/(a + b*x^2)^10,x]
```

output
```
-1/18*x^5/(b*(a + b*x^2)^9) + (5*(-1/16*x^3/(b*(a + b*x^2)^8) + (3*(-1/14*
x/(b*(a + b*x^2)^7) + (x/(12*a*(a + b*x^2)^6) + (11*(x/(10*a*(a + b*x^2)^5
) + (9*(x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*(a
+ b*x^2)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(
3/2)*Sqrt[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a))/(14*b)))/(16*b)))
/(18*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

method	result
default	$\frac{-\frac{55a^{2}x}{65536b^{3}}-\frac{715a\,x^{3}}{98304b^{2}}-\frac{913x^{5}}{32768b}+\frac{18589x^{7}}{229376a}+\frac{11b\,x^{9}}{126a^{2}}+\frac{14179b^{2}x^{11}}{229376a^{3}}+\frac{913b^{3}x^{13}}{32768a^{4}}+\frac{715b^{4}x^{15}}{98304a^{5}}+\frac{55b^{5}x^{17}}{65536a^{6}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{55\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^{6}b^{3}\sqrt{ab}}$
risch	$\frac{-\frac{55a^{2}x}{65536b^{3}}-\frac{715a\,x^{3}}{98304b^{2}}-\frac{913x^{5}}{32768b}+\frac{18589x^{7}}{229376a}+\frac{11b\,x^{9}}{126a^{2}}+\frac{14179b^{2}x^{11}}{229376a^{3}}+\frac{913b^{3}x^{13}}{32768a^{4}}+\frac{715b^{4}x^{15}}{98304a^{5}}+\frac{55b^{5}x^{17}}{65536a^{6}}}{\left(b\,x^{2}+a\right)^{9}}-\frac{55\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b^{3}a^{6}}+\frac{55\ln(-bx+}{131072\sqrt{-a}}$

input
```
int(x^6/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-55/65536*a^2*x/b^3-715/98304*a*x^3/b^2-913/32768*x^5/b+18589/229376/a*x^
7+11/126*b/a^2*x^9+14179/229376*b^2/a^3*x^11+913/32768*b^3/a^4*x^13+715/98
304*b^4/a^5*x^15+55/65536*b^5/a^6*x^17)/(b*x^2+a)^9+55/65536/a^6/b^3/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.22

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx$$

$$= \left[\frac{6930\,ab^9x^{17} + 60060\,a^2b^8x^{15} + 230076\,a^3b^7x^{13} + 510444\,a^4b^6x^{11} + 720896\,a^5b^5x^9 + 669204\,a^6b^4x^7 - 23}{8257536\,(a^7b^{13}x^{18} + 9\,a^8b^1}\right.$$

input
```
integrate(x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(6930*a*b^9*x^17 + 60060*a^2*b^8*x^15 + 230076*a^3*b^7*x^13 + 5
10444*a^4*b^6*x^11 + 720896*a^5*b^5*x^9 + 669204*a^6*b^4*x^7 - 230076*a^7*
b^3*x^5 - 60060*a^8*b^2*x^3 - 6930*a^9*b*x - 3465*(b^9*x^18 + 9*a*b^8*x^16
 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8
+ 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*b)*log((b*x
^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 +
36*a^9*b^11*x^14 + 84*a^10*b^10*x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^
8 + 84*a^13*b^7*x^6 + 36*a^14*b^6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4), 1/4128
768*(3465*a*b^9*x^17 + 30030*a^2*b^8*x^15 + 115038*a^3*b^7*x^13 + 255222*a
^4*b^6*x^11 + 360448*a^5*b^5*x^9 + 334602*a^6*b^4*x^7 - 115038*a^7*b^3*x^5
 - 30030*a^8*b^2*x^3 - 3465*a^9*b*x + 3465*(b^9*x^18 + 9*a*b^8*x^16 + 36*a
^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^
6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)
*x/a))/(a^7*b^13*x^18 + 9*a^8*b^12*x^16 + 36*a^9*b^11*x^14 + 84*a^10*b^10*
x^12 + 126*a^11*b^9*x^10 + 126*a^12*b^8*x^8 + 84*a^13*b^7*x^6 + 36*a^14*b^
6*x^4 + 9*a^15*b^5*x^2 + a^16*b^4)]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.43

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(-a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072} + \frac{55\sqrt{-\frac{1}{a^{13}b^7}}\log\left(a^7b^3\sqrt{-\frac{1}{a^{13}b^7}}+x\right)}{131072}$$
$$+ \frac{-3465a^8x - 30030a^7bx^3 - 115038a^6b^2x^5 + 334602a^5b^3x^7 + 360448a^4}{4128768a^{15}b^3 + 37158912a^{14}b^4x^2 + 148635648a^{13}b^5x^4 + 346816512a^{12}b^6x^6 + 520224768a^{11}b^7x^8 + 520}$$

input
```
integrate(x**6/(b*x**2+a)**10,x)
```

output
```
-55*sqrt(-1/(a**13*b**7))*log(-a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/131072
 + 55*sqrt(-1/(a**13*b**7))*log(a**7*b**3*sqrt(-1/(a**13*b**7)) + x)/13107
2 + (-3465*a**8*x - 30030*a**7*b*x**3 - 115038*a**6*b**2*x**5 + 334602*a**
5*b**3*x**7 + 360448*a**4*b**4*x**9 + 255222*a**3*b**5*x**11 + 115038*a**2
*b**6*x**13 + 30030*a*b**7*x**15 + 3465*b**8*x**17)/(4128768*a**15*b**3 +
37158912*a**14*b**4*x**2 + 148635648*a**13*b**5*x**4 + 346816512*a**12*b**
6*x**6 + 520224768*a**11*b**7*x**8 + 520224768*a**10*b**8*x**10 + 34681651
2*a**9*b**9*x**12 + 148635648*a**8*b**10*x**14 + 37158912*a**7*b**11*x**16
 + 4128768*a**6*b**12*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx$$
$$= \frac{3465\,b^8x^{17} + 30030\,ab^7x^{15} + 115038\,a^2b^6x^{13} + 255222\,a^3b^5x^{11} + 360448\,a^4b^4x^9 + 334602\,a^5b^3x^7 - 11503}{4128768\,(a^6b^{12}x^{18} + 9\,a^7b^{11}x^{16} + 36\,a^8b^{10}x^{14} + 84\,a^9b^9x^{12} + 126\,a^{10}b^8x^{10} + 126\,a^{11}b^7x^8 + 84\,a^{12}b^6x^6 +}$$
$$+ \frac{55\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^6b^3}$$

input
```
integrate(x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/4128768*(3465*b^8*x^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222
*a^3*b^5*x^11 + 360448*a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x
^5 - 30030*a^7*b*x^3 - 3465*a^8*x)/(a^6*b^12*x^18 + 9*a^7*b^11*x^16 + 36*a
^8*b^10*x^14 + 84*a^9*b^9*x^12 + 126*a^10*b^8*x^10 + 126*a^11*b^7*x^8 + 84
*a^12*b^6*x^6 + 36*a^13*b^5*x^4 + 9*a^14*b^4*x^2 + a^15*b^3) + 55/65536*ar
ctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = \frac{55\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^6b^3} + \frac{3465\,b^8x^{17} + 30030\,ab^7x^{15} + 115038\,a^2b^6x^{13} + 255222\,a^3b^5x^{11} + 360448\,a^4b^4x^9 + 334602\,a^5b^3x^7 - 115}{4128768\,(bx^2+a)^9a^6b^3}$$

input
```
integrate(x^6/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
55/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6*b^3) + 1/4128768*(3465*b^8*x
^17 + 30030*a*b^7*x^15 + 115038*a^2*b^6*x^13 + 255222*a^3*b^5*x^11 + 36044
8*a^4*b^4*x^9 + 334602*a^5*b^3*x^7 - 115038*a^6*b^2*x^5 - 30030*a^7*b*x^3
- 3465*a^8*x)/((b*x^2 + a)^9*a^6*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx = \frac{\frac{18589\,x^7}{229376\,a} - \frac{913\,x^5}{32768\,b} - \frac{715\,a\,x^3}{98304\,b^2} - \frac{55\,a^2\,x}{65536\,b^3} + \frac{11\,b\,x^9}{126\,a^2} + \frac{14179\,b^2\,x^{11}}{229376\,a^3} + \frac{913\,b^3\,x^{13}}{32768\,a^4} + \frac{715\,b^4\,x^{15}}{98304\,a^5} + \frac{55\,b^5\,x^{17}}{65536\,a^6}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8} + \frac{55\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{13/2}\,b^{7/2}}$$

input
```
int(x^6/(a + b*x^2)^10,x)
```

output
```
((18589*x^7)/(229376*a) - (913*x^5)/(32768*b) - (715*a*x^3)/(98304*b^2) -
(55*a^2*x)/(65536*b^3) + (11*b*x^9)/(126*a^2) + (14179*b^2*x^11)/(229376*a
^3) + (913*b^3*x^13)/(32768*a^4) + (715*b^4*x^15)/(98304*a^5) + (55*b^5*x^
17)/(65536*a^6))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2
*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 + 84*a^3*b^6*x^
12 + 36*a^2*b^7*x^14) + (55*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(13/2)*b^(
7/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.25

$$\int \frac{x^6}{(a+bx^2)^{10}}\,dx$$
$$= \frac{3465\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 31185\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 124740\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2x^4 + 29}{}$$

input
```
int(x^6/(b*x^2+a)^10,x)
```

output
```
(3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 31185*sqrt(b)*s
qrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 124740*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 291060*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 436590*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 436590*sqrt(b)*sqrt(a)*atan((b*x)/(
sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 291060*sqrt(b)*sqrt(a)*atan((b*x)/(sqr
t(b)*sqrt(a)))*a**3*b**6*x**12 + 124740*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b
)*sqrt(a)))*a**2*b**7*x**14 + 31185*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sq
rt(a)))*a*b**8*x**16 + 3465*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**9*x**18 - 3465*a**9*b*x - 30030*a**8*b**2*x**3 - 115038*a**7*b**3*x**5
+ 334602*a**6*b**4*x**7 + 360448*a**5*b**5*x**9 + 255222*a**4*b**6*x**11 +
 115038*a**3*b**7*x**13 + 30030*a**2*b**8*x**15 + 3465*a*b**9*x**17)/(4128
768*a**7*b**4*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**
6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**
2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.219 $\int \frac{x^4}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 204

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx = -\frac{x^3}{18b(a+bx^2)^9} - \frac{x}{96b^2(a+bx^2)^8} + \frac{x}{1344ab^2(a+bx^2)^7} + \frac{13x}{16128a^2b^2(a+bx^2)^6} + \frac{143x}{161280a^3b^2(a+bx^2)^5} + \frac{143x}{143360a^4b^2(a+bx^2)^4} + \frac{143x}{122880a^5b^2(a+bx^2)^3} + \frac{143x}{98304a^6b^2(a+bx^2)^2} + \frac{143x}{65536a^7b^2(a+bx^2)} + \frac{143\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{15/2}b^{5/2}}$$

output
```
-1/18*x^3/b/(b*x^2+a)^9-1/96*x/b^2/(b*x^2+a)^8+1/1344*x/a/b^2/(b*x^2+a)^7+
13/16128*x/a^2/b^2/(b*x^2+a)^6+143/161280*x/a^3/b^2/(b*x^2+a)^5+143/143360
*x/a^4/b^2/(b*x^2+a)^4+143/122880*x/a^5/b^2/(b*x^2+a)^3+143/98304*x/a^6/b^
2/(b*x^2+a)^2+143/65536*x/a^7/b^2/(b*x^2+a)+143/65536*arctan(b^(1/2)*x/a^(
1/2))/a^(15/2)/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-45045a^8-390390a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+450}{(a+bx^2)^9}}{20643840a^{15/2}b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 - 390390*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
 4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 149549
4*a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 4504
5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(20643840*a^(15/2)*b^(5/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 252, 215, 215, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{x^2}{(bx^2+a)^9}dx}{6b} - \frac{x^3}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\int \frac{1}{(bx^2+a)^8}dx}{16b} - \frac{x}{16b(a+bx^2)^8}}{6b} - \frac{x^3}{18b(a+bx^2)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\frac{13\int \frac{1}{\left(bx^2+a\right)^7}dx}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2)^8}}{6b}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\frac{13\left(\frac{11\int \frac{1}{\left(bx^2+a\right)^6}dx}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2)^8}}{6b}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\frac{13\left(\frac{11\left(\frac{9\int \frac{1}{\left(bx^2+a\right)^5}dx}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2)^8}}{6b}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2)^8}}{6b}-\frac{x^3}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2)^8}}{6b}-$$

$$\frac{x^3}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}}{16b}-\frac{x}{16b(a+bx^2}}{6b}$$

$$\frac{x^3}{18b\left(a+bx^2\right)^9}$$

↓ 215

$$13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right.$$

↓ 218

$$13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right.$$

input
```
Int[x^4/(a + b*x^2)^10,x]
```

output
```
-1/18*x^3/(b*(a + b*x^2)^9) + (-1/16*x/(b*(a + b*x^2)^8) + (x/(14*a*(a + b
*x^2)^7) + (13*(x/(12*a*(a + b*x^2)^6) + (11*(x/(10*a*(a + b*x^2)^5) + (9*
(x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*(a + b*x^2
)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sq
rt[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a)))/(14*a))/(16*b))/(6*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.60

method	result
default	$\frac{-\frac{143 a x}{65536 b^{2}}-\frac{1859 x^{3}}{98304 b}+\frac{20899 x^{5}}{163840 a}+\frac{241657 b\,x^{7}}{1146880 a^{2}}+\frac{143 b^{2} x^{9}}{630 a^{3}}+\frac{184327 b^{3} x^{11}}{1146880 a^{4}}+\frac{11869 b^{4} x^{13}}{163840 a^{5}}+\frac{1859 b^{5} x^{15}}{98304 a^{6}}+\frac{143 b^{6} x^{17}}{65536 a^{7}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{143 \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{65536 a^{7} b^{2} \sqrt{a b}}$
risch	$\frac{-\frac{143 a x}{65536 b^{2}}-\frac{1859 x^{3}}{98304 b}+\frac{20899 x^{5}}{163840 a}+\frac{241657 b\,x^{7}}{1146880 a^{2}}+\frac{143 b^{2} x^{9}}{630 a^{3}}+\frac{184327 b^{3} x^{11}}{1146880 a^{4}}+\frac{11869 b^{4} x^{13}}{163840 a^{5}}+\frac{1859 b^{5} x^{15}}{98304 a^{6}}+\frac{143 b^{6} x^{17}}{65536 a^{7}}}{\left(b\,x^{2}+a\right)^{9}}-\frac{143 \ln\left(b x+\sqrt{-a b}\right)}{131072 \sqrt{-a b}\, b^{2} a^{7}}+$

input
```
int(x^4/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-143/65536*a*x/b^2-1859/98304*x^3/b+20899/163840/a*x^5+241657/1146880*b/a
^2*x^7+143/630*b^2/a^3*x^9+184327/1146880*b^3/a^4*x^11+11869/163840*b^4/a^
5*x^13+1859/98304*b^5/a^6*x^15+143/65536*b^6/a^7*x^17)/(b*x^2+a)^9+143/655
36/a^7/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.21

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$

$$= \left[\frac{90090\,ab^9x^{17} + 780780\,a^2b^8x^{15} + 2990988\,a^3b^7x^{13} + 6635772\,a^4b^6x^{11} + 9371648\,a^5b^5x^9 + 8699652\,a^6b^4\ldots}{41287680\,(a^8b^{12}x^{18} + \ldots}\right.$$

input
```
integrate(x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/41287680*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13
 + 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 5266
548*a^7*b^3*x^5 - 780780*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a
^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^8*b^12*x^18 + 9*a^9*b
^11*x^16 + 36*a^10*b^10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*
a^13*b^7*x^8 + 84*a^14*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b
^3), 1/20643840*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*
x^13 + 3317886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 +
2633274*a^7*b^3*x^5 - 390390*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18
 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 1
26*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^8*b^12*x^18 + 9*a^9*b^11*x^16 + 36*a^10*b^
10*x^14 + 84*a^11*b^9*x^12 + 126*a^12*b^8*x^10 + 126*a^13*b^7*x^8 + 84*a^1
4*b^6*x^6 + 36*a^15*b^5*x^4 + 9*a^16*b^4*x^2 + a^17*b^3)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.43

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(-a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072} + \frac{143\sqrt{-\frac{1}{a^{15}b^5}}\log\left(a^8b^2\sqrt{-\frac{1}{a^{15}b^5}}+x\right)}{131072} + \frac{-45045a^8x - 390390a^7bx^3 + 2633274a^6b^2x^5 + 4349826a^5b^3x^7 + 46858}{20643840a^{16}b^2 + 185794560a^{15}b^3x^2 + 743178240a^{14}b^4x^4 + 1734082560a^{13}b^5x^6 + 2601123840a^{12}b^6x^8}$$

input
```
integrate(x**4/(b*x**2+a)**10,x)
```

output
```
-143*sqrt(-1/(a**15*b**5))*log(-a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/13107
2 + 143*sqrt(-1/(a**15*b**5))*log(a**8*b**2*sqrt(-1/(a**15*b**5)) + x)/131
072 + (-45045*a**8*x - 390390*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 43498
26*a**5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 149
5494*a**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(20643840*a
**16*b**2 + 185794560*a**15*b**3*x**2 + 743178240*a**14*b**4*x**4 + 173408
2560*a**13*b**5*x**6 + 2601123840*a**12*b**6*x**8 + 2601123840*a**11*b**7*
x**10 + 1734082560*a**10*b**8*x**12 + 743178240*a**9*b**9*x**14 + 18579456
0*a**8*b**10*x**16 + 20643840*a**7*b**11*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} + 1495494\,a^2b^6x^{13} + 3317886\,a^3b^5x^{11} + 4685824\,a^4b^4x^9 + 4349826\,a^5b^3x^7 +}{20643840\,(a^7b^{11}x^{18} + 9\,a^8b^{10}x^{16} + 36\,a^9b^9x^{14} + 84\,a^{10}b^8x^{12} + 126\,a^{11}b^7x^{10} + 126\,a^{12}b^6x^8 + 84\,a^{13}} + \frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^7b^2}$$

input
```
integrate(x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/20643840*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 33
17886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a
^6*b^2*x^5 - 390390*a^7*b*x^3 - 45045*a^8*x)/(a^7*b^11*x^18 + 9*a^8*b^10*x
^16 + 36*a^9*b^9*x^14 + 84*a^10*b^8*x^12 + 126*a^11*b^7*x^10 + 126*a^12*b^
6*x^8 + 84*a^13*b^5*x^6 + 36*a^14*b^4*x^4 + 9*a^15*b^3*x^2 + a^16*b^2) + 1
43/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.63

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx = \frac{143\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^7b^2} + \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} + 1495494\,a^2b^6x^{13} + 3317886\,a^3b^5x^{11} + 4685824\,a^4b^4x^9 + 4349826\,a^5b^3x^7}{20643840\,(bx^2+a)^9a^7b^2}$$

input
```
integrate(x^4/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
143/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7*b^2) + 1/20643840*(45045*b^
8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 +
 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 - 390390*
a^7*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^7*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx = \frac{\frac{20899\,x^5}{163840\,a} - \frac{1859\,x^3}{98304\,b} + \frac{241657\,b\,x^7}{1146880\,a^2} + \frac{143\,b^2\,x^9}{630\,a^3} + \frac{184327\,b^3\,x^{11}}{1146880\,a^4} + \frac{11869\,b^4\,x^{13}}{163840\,a^5} + \frac{1859\,b^5\,x^{15}}{98304\,a^6} + \frac{143\,b^6\,x^{17}}{65536\,a^7} - \frac{143\,a}{65536\,b}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8} + \frac{143\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{15/2}\,b^{5/2}}$$

input
```
int(x^4/(a + b*x^2)^10,x)
```

output
```
((20899*x^5)/(163840*a) - (1859*x^3)/(98304*b) + (241657*b*x^7)/(1146880*a
^2) + (143*b^2*x^9)/(630*a^3) + (184327*b^3*x^11)/(1146880*a^4) + (11869*b
^4*x^13)/(163840*a^5) + (1859*b^5*x^15)/(98304*a^6) + (143*b^6*x^17)/(6553
6*a^7) - (143*a*x)/(65536*b^2))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^
16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10
+ 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (143*atan((b^(1/2)*x)/a^(1/2)))/(65
536*a^(15/2)*b^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.24

$$\int \frac{x^4}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 405405\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 1621620\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2x^4 -}{}$$

input
```
int(x^4/(b*x^2+a)^10,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 405405*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 1621620*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 3783780*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 5675670*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 5675670*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 3783780*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 1621620*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 405405*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**8*x**16 + 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**9*x**18 - 45045*a**9*b*x - 390390*a**8*b**2*x**3 + 2633274*a*
*7*b**3*x**5 + 4349826*a**6*b**4*x**7 + 4685824*a**5*b**5*x**9 + 3317886*a
**4*b**6*x**11 + 1495494*a**3*b**7*x**13 + 390390*a**2*b**8*x**15 + 45045*
a*b**9*x**17)/(20643840*a**8*b**3*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**
4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3
*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.220 $\int \frac{x^2}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 205

$$\begin{aligned}\int \frac{x^2}{(a+bx^2)^{10}}\,dx = &-\frac{x}{18b(a+bx^2)^9}+\frac{x}{288ab(a+bx^2)^8}+\frac{5x}{1344a^2b(a+bx^2)^7}\\&+\frac{65x}{16128a^3b(a+bx^2)^6}+\frac{143x}{32256a^4b(a+bx^2)^5}\\&+\frac{143x}{28672a^5b(a+bx^2)^4}+\frac{143x}{24576a^6b(a+bx^2)^3}\\&+\frac{715x}{98304a^7b(a+bx^2)^2}+\frac{715x}{65536a^8b(a+bx^2)}+\frac{715\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{17/2}b^{3/2}}\end{aligned}$$

output
```
-1/18*x/b/(b*x^2+a)^9+1/288*x/a/b/(b*x^2+a)^8+5/1344*x/a^2/b/(b*x^2+a)^7+6
5/16128*x/a^3/b/(b*x^2+a)^6+143/32256*x/a^4/b/(b*x^2+a)^5+143/28672*x/a^5/
b/(b*x^2+a)^4+143/24576*x/a^6/b/(b*x^2+a)^3+715/98304*x/a^7/b/(b*x^2+a)^2+
715/65536*x/a^8/b/(b*x^2+a)+715/65536*arctan(b^(1/2)*x/a^(1/2))/a^(17/2)/b
^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx = \frac{\frac{\sqrt{a}\sqrt{b}x(-45045a^8+985866a^7bx^2+2633274a^6b^2x^4+4349826a^5b^3x^6+4685824a^4b^4x^8+3317886a^3b^5x^{10}+1495494a^2b^6x^{12}+390390ab^7x^{14}+450}{(a+bx^2)^9}}{4128768a^{17/2}b^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^10,x]
```

output
```
((Sqrt[a]*Sqrt[b]*x*(-45045*a^8 + 985866*a^7*b*x^2 + 2633274*a^6*b^2*x^4 +
 4349826*a^5*b^3*x^6 + 4685824*a^4*b^4*x^8 + 3317886*a^3*b^5*x^10 + 149549
4*a^2*b^6*x^12 + 390390*a*b^7*x^14 + 45045*b^8*x^16))/(a + b*x^2)^9 + 4504
5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(4128768*a^(17/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 215, 215, 215, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{10}}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\int \frac{1}{(bx^2+a)^9}dx}{18b} - \frac{x}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{15\int \frac{1}{(bx^2+a)^8}dx}{16a} + \frac{x}{16a(a+bx^2)^8}}{18b} - \frac{x}{18b(a+bx^2)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{15\left(\frac{13\int \frac{1}{\left(bx^2+a\right)^7}dx}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}}{18b}-\frac{x}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{15\left(\frac{13\left(\frac{11\int \frac{1}{\left(bx^2+a\right)^6}dx}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}}{18b}-\frac{x}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\int \frac{1}{\left(bx^2+a\right)^5}dx}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}}{18b}-\frac{x}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}}{18b}-\frac{x}{18b\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int \frac{1}{\left(bx^2+a\right)^3}dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}}{18b}-\frac{x}{18b\left(a+bx^2\right)^9}$$

$\Big\downarrow$ 215

$$\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right.$$

↓ 215

$$\left(\left(13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\,\frac{\left(5\,\frac{\left(3\,\frac{\left(\frac{\int \frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right.\right.$$

↓ 218

$$\left(\left(13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right.\right.\right.$$

input
```
Int[x^2/(a + b*x^2)^10,x]
```

output
```
-1/18*x/(b*(a + b*x^2)^9) + (x/(16*a*(a + b*x^2)^8) + (15*(x/(14*a*(a + b*
x^2)^7) + (13*(x/(12*a*(a + b*x^2)^6) + (11*(x/(10*a*(a + b*x^2)^5) + (9*(
x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*(a + b*x^2)
^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqr
t[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a)))/(14*a)))/(16*a))/(18*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.60

method	result
default	$\frac{-\frac{715x}{65536b}+\frac{23473x^{3}}{98304a}+\frac{20899b\,x^{5}}{32768a^{2}}+\frac{241657b^{2}x^{7}}{229376a^{3}}+\frac{143b^{3}x^{9}}{126a^{4}}+\frac{184327b^{4}x^{11}}{229376a^{5}}+\frac{11869b^{5}x^{13}}{32768a^{6}}+\frac{9295b^{6}x^{15}}{98304a^{7}}+\frac{715b^{7}x^{17}}{65536a^{8}}}{\left(b\,x^{2}+a\right)^{9}}+\frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536a^{8}b\sqrt{ab}}$
risch	$\frac{-\frac{715x}{65536b}+\frac{23473x^{3}}{98304a}+\frac{20899b\,x^{5}}{32768a^{2}}+\frac{241657b^{2}x^{7}}{229376a^{3}}+\frac{143b^{3}x^{9}}{126a^{4}}+\frac{184327b^{4}x^{11}}{229376a^{5}}+\frac{11869b^{5}x^{13}}{32768a^{6}}+\frac{9295b^{6}x^{15}}{98304a^{7}}+\frac{715b^{7}x^{17}}{65536a^{8}}}{\left(b\,x^{2}+a\right)^{9}}-\frac{715\ln\left(bx+\sqrt{-ab}\right)}{131072\sqrt{-ab}\,b\,a^{8}}+$

input
```
int(x^2/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(-715/65536*x/b+23473/98304/a*x^3+20899/32768*b/a^2*x^5+241657/229376*b^2/
a^3*x^7+143/126*b^3/a^4*x^9+184327/229376*b^4/a^5*x^11+11869/32768*b^5/a^6
*x^13+9295/98304*b^6/a^7*x^15+715/65536*b^7/a^8*x^17)/(b*x^2+a)^9+715/6553
6/a^8/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 654, normalized size of antiderivative = 3.19

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx$$
$$= \left[\frac{90090\,ab^9x^{17} + 780780\,a^2b^8x^{15} + 2990988\,a^3b^7x^{13} + 6635772\,a^4b^6x^{11} + 9371648\,a^5b^5x^9 + 8699652\,a^6b^4}{8257536\,(a^9b^{11}x^{18} +}\right.$$

input
```
integrate(x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(90090*a*b^9*x^17 + 780780*a^2*b^8*x^15 + 2990988*a^3*b^7*x^13
+ 6635772*a^4*b^6*x^11 + 9371648*a^5*b^5*x^9 + 8699652*a^6*b^4*x^7 + 52665
48*a^7*b^3*x^5 + 1971732*a^8*b^2*x^3 - 90090*a^9*b*x - 45045*(b^9*x^18 + 9
*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 126*a
^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^9*b^11*x^18 + 9*a^10*
b^10*x^16 + 36*a^11*b^9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*
a^14*b^6*x^8 + 84*a^15*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b
^2), 1/4128768*(45045*a*b^9*x^17 + 390390*a^2*b^8*x^15 + 1495494*a^3*b^7*x
^13 + 3317886*a^4*b^6*x^11 + 4685824*a^5*b^5*x^9 + 4349826*a^6*b^4*x^7 + 2
633274*a^7*b^3*x^5 + 985866*a^8*b^2*x^3 - 45045*a^9*b*x + 45045*(b^9*x^18
+ 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*b^5*x^10 + 12
6*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2 + a^9)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a))/(a^9*b^11*x^18 + 9*a^10*b^10*x^16 + 36*a^11*b^
9*x^14 + 84*a^12*b^8*x^12 + 126*a^13*b^7*x^10 + 126*a^14*b^6*x^8 + 84*a^15
*b^5*x^6 + 36*a^16*b^4*x^4 + 9*a^17*b^3*x^2 + a^18*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.40

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx$$
$$= -\frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(-a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072} + \frac{715\sqrt{-\frac{1}{a^{17}b^3}}\log\left(a^9b\sqrt{-\frac{1}{a^{17}b^3}}+x\right)}{131072}$$
$$+ \frac{-45045a^8x + 985866a^7bx^3 + 2633274a^6b^2x^5 + 4349826a^5b^3x^7 + 4685824a^4}{4128768a^{17}b + 37158912a^{16}b^2x^2 + 148635648a^{15}b^3x^4 + 346816512a^{14}b^4x^6 + 520224768a^{13}b^5x^8 + 520}$$

input
```
integrate(x**2/(b*x**2+a)**10,x)
```

output
```
-715*sqrt(-1/(a**17*b**3))*log(-a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
 715*sqrt(-1/(a**17*b**3))*log(a**9*b*sqrt(-1/(a**17*b**3)) + x)/131072 +
(-45045*a**8*x + 985866*a**7*b*x**3 + 2633274*a**6*b**2*x**5 + 4349826*a**
5*b**3*x**7 + 4685824*a**4*b**4*x**9 + 3317886*a**3*b**5*x**11 + 1495494*a
**2*b**6*x**13 + 390390*a*b**7*x**15 + 45045*b**8*x**17)/(4128768*a**17*b
+ 37158912*a**16*b**2*x**2 + 148635648*a**15*b**3*x**4 + 346816512*a**14*b
**4*x**6 + 520224768*a**13*b**5*x**8 + 520224768*a**12*b**6*x**10 + 346816
512*a**11*b**7*x**12 + 148635648*a**10*b**8*x**14 + 37158912*a**9*b**9*x**
16 + 4128768*a**8*b**10*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} + 1495494\,a^2b^6x^{13} + 3317886\,a^3b^5x^{11} + 4685824\,a^4b^4x^9 + 4349826\,a^5b^3x^7 +}{4128768\,(a^8b^{10}x^{18} + 9\,a^9b^9x^{16} + 36\,a^{10}b^8x^{14} + 84\,a^{11}b^7x^{12} + 126\,a^{12}b^6x^{10} + 126\,a^{13}b^5x^8 + 84\,a^{14}}$$
$$+ \frac{715\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^8b}$$

input
```
integrate(x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
1/4128768*(45045*b^8*x^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 331
7886*a^3*b^5*x^11 + 4685824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^
6*b^2*x^5 + 985866*a^7*b*x^3 - 45045*a^8*x)/(a^8*b^10*x^18 + 9*a^9*b^9*x^1
6 + 36*a^10*b^8*x^14 + 84*a^11*b^7*x^12 + 126*a^12*b^6*x^10 + 126*a^13*b^5
*x^8 + 84*a^14*b^4*x^6 + 36*a^15*b^3*x^4 + 9*a^16*b^2*x^2 + a^17*b) + 715/
65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx = \frac{715\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\sqrt{ab}a^8b} + \frac{45045\,b^8x^{17} + 390390\,ab^7x^{15} + 1495494\,a^2b^6x^{13} + 3317886\,a^3b^5x^{11} + 4685824\,a^4b^4x^9 + 4349826\,a^5b^3x^7}{4128768\,(bx^2+a)^9a^8b}$$

input
```
integrate(x^2/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
715/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^8*b) + 1/4128768*(45045*b^8*x
^17 + 390390*a*b^7*x^15 + 1495494*a^2*b^6*x^13 + 3317886*a^3*b^5*x^11 + 46
85824*a^4*b^4*x^9 + 4349826*a^5*b^3*x^7 + 2633274*a^6*b^2*x^5 + 985866*a^7
*b*x^3 - 45045*a^8*x)/((b*x^2 + a)^9*a^8*b)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx = \frac{\frac{23473\,x^3}{98304\,a} - \frac{715\,x}{65536\,b} + \frac{20899\,b\,x^5}{32768\,a^2} + \frac{241657\,b^2\,x^7}{229376\,a^3} + \frac{143\,b^3\,x^9}{126\,a^4} + \frac{184327\,b^4\,x^{11}}{229376\,a^5} + \frac{11869\,b^5\,x^{13}}{32768\,a^6} + \frac{9295\,b^6\,x^{15}}{98304\,a^7} + \frac{715\,b^7}{65536}}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8} + \frac{715\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{17/2}\,b^{3/2}}$$

input
```
int(x^2/(a + b*x^2)^10,x)
```

output
```
((23473*x^3)/(98304*a) - (715*x)/(65536*b) + (20899*b*x^5)/(32768*a^2) + (
241657*b^2*x^7)/(229376*a^3) + (143*b^3*x^9)/(126*a^4) + (184327*b^4*x^11)
/(229376*a^5) + (11869*b^5*x^13)/(32768*a^6) + (9295*b^6*x^15)/(98304*a^7)
 + (715*b^7*x^17)/(65536*a^8))/(a^9 + b^9*x^18 + 9*a^8*b*x^2 + 9*a*b^8*x^1
6 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 + 126*a^4*b^5*x^10 +
 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (715*atan((b^(1/2)*x)/a^(1/2)))/(655
36*a^(17/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.22

$$\int \frac{x^2}{(a+bx^2)^{10}}\,dx$$
$$= \frac{45045\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a^9 + 405405\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a^8 b\,x^2 + 1621620\sqrt{b}\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\sqrt{a}}\right)a^7 b^2 x^4 -}{}$$

input
```
int(x^2/(b*x^2+a)^10,x)
```

output
```
(45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 405405*sqrt(b)
*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 1621620*sqrt(b)*sqrt(
a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 3783780*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 5675670*sqrt(b)*sqrt(a)*ata
n((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 5675670*sqrt(b)*sqrt(a)*atan((
b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 3783780*sqrt(b)*sqrt(a)*atan((b*
x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 1621620*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 405405*sqrt(b)*sqrt(a)*atan((b*x)/(s
qrt(b)*sqrt(a)))*a*b**8*x**16 + 45045*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*
sqrt(a)))*b**9*x**18 - 45045*a**9*b*x + 985866*a**8*b**2*x**3 + 2633274*a*
*7*b**3*x**5 + 4349826*a**6*b**4*x**7 + 4685824*a**5*b**5*x**9 + 3317886*a
**4*b**6*x**11 + 1495494*a**3*b**7*x**13 + 390390*a**2*b**8*x**15 + 45045*
a*b**9*x**17)/(4128768*a**9*b**2*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4
 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**3*
b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.221 $\int \frac{1}{(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 181

$$\begin{aligned}\int \frac{1}{(a+bx^2)^{10}}\,dx &= \frac{x}{18a\,(a+bx^2)^9}+\frac{17x}{288a^2\,(a+bx^2)^8}+\frac{85x}{1344a^3\,(a+bx^2)^7}\\&\quad+\frac{1105x}{16128a^4\,(a+bx^2)^6}+\frac{2431x}{32256a^5\,(a+bx^2)^5}\\&\quad+\frac{2431x}{28672a^6\,(a+bx^2)^4}+\frac{2431x}{24576a^7\,(a+bx^2)^3}+\frac{12155x}{98304a^8\,(a+bx^2)^2}\\&\quad+\frac{12155x}{65536a^9\,(a+bx^2)}+\frac{12155\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{19/2}\sqrt{b}}\end{aligned}$$

output
```
1/18*x/a/(b*x^2+a)^9+17/288*x/a^2/(b*x^2+a)^8+85/1344*x/a^3/(b*x^2+a)^7+11
05/16128*x/a^4/(b*x^2+a)^6+2431/32256*x/a^5/(b*x^2+a)^5+2431/28672*x/a^6/(
b*x^2+a)^4+2431/24576*x/a^7/(b*x^2+a)^3+12155/98304*x/a^8/(b*x^2+a)^2+1215
5/65536*x/a^9/(b*x^2+a)+12155/65536*arctan(b^(1/2)*x/a^(1/2))/a^(19/2)/b^(
1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^2)^{10}}\,dx$$

$$= \frac{\frac{3363003a^8x+16759722a^7bx^3+44765658a^6b^2x^5+73947042a^5b^3x^7+79659008a^4b^4x^9+56404062a^3b^5x^{11}+25423398a^2b^6x^{13}+6636630ab^7x^{15}+7}{a^9(a+bx^2)^9}}{4128768}$$

input
```
Integrate[(a + b*x^2)^(-10),x]
```

output
```
((3363003*a^8*x + 16759722*a^7*b*x^3 + 44765658*a^6*b^2*x^5 + 73947042*a^5
*b^3*x^7 + 79659008*a^4*b^4*x^9 + 56404062*a^3*b^5*x^11 + 25423398*a^2*b^6
*x^13 + 6636630*a*b^7*x^15 + 765765*b^8*x^17)/(a^9*(a + b*x^2)^9) + (76576
5*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(19/2)*Sqrt[b]))/4128768
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.35, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.111$, Rules used = {215, 215, 215, 215, 215, 215, 215, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{10}}\,dx$$

$$\Big\downarrow \; 215$$

$$\frac{17\int \frac{1}{(bx^2+a)^9}dx}{18a}+\frac{x}{18a(a+bx^2)^9}$$

$$\Big\downarrow \; 215$$

$$\frac{17\left(\frac{15\int \frac{1}{(bx^2+a)^8}dx}{16a}+\frac{x}{16a(a+bx^2)^8}\right)}{18a}+\frac{x}{18a(a+bx^2)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{17\left(\frac{15\left(\frac{13\int\frac{1}{\left(bx^2+a\right)^7}dx}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}\right)}{18a}+\frac{x}{18a\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\int\frac{1}{\left(bx^2+a\right)^6}dx}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}\right)}{18a}+\frac{x}{18a\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{215}$$

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{1}{\left(bx^2+a\right)^5}dx}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}\right)}{18a}+\frac{x}{18a\left(a+bx^2\right)^9}$$

$$\Big\downarrow \ 215$$

$$\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int \frac{1}{\left(bx^2+a\right)^4}dx}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}\right)}{18a}+\frac{x}{18a\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 215$$

$$17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{\left(bx^2+a\right)^3}\,dx}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right)}{16a}+\frac{x}{16a(a+bx^2)^8}\right)$$

↓ 215

$$\left(\left(15\,\frac{\left(13\,\frac{\left(11\,\frac{\left(9\,\frac{\left(7\,\frac{\left(5\,\left(\frac{3\int\frac{1}{\left(bx^2+a\right)^2}\,dx}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)}{14a}+\frac{x}{14a\left(a+bx^2\right)^7}\right.\right.$$

↓ 215

$$\left(\left(\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int\frac{1}{bx^2+a}\,dx}{2a}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}\right)\right.\right.$$

↓ 218

$$\Bigg(\Bigg(\Bigg(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a+bx^2\right)^2}\right)}{6a}+\frac{x}{6a\left(a+bx^2\right)^3}\right)}{8a}+\frac{x}{8a\left(a+bx^2\right)^4}\right)}{10a}+\frac{x}{10a\left(a+bx^2\right)^5}\right)}{12a}+\frac{x}{12a\left(a+bx^2\right)^6}$$

input
```
Int[(a + b*x^2)^(-10),x]
```

output
```
x/(18*a*(a + b*x^2)^9) + (17*(x/(16*a*(a + b*x^2)^8) + (15*(x/(14*a*(a + b
*x^2)^7) + (13*(x/(12*a*(a + b*x^2)^6) + (11*(x/(10*a*(a + b*x^2)^5) + (9*
(x/(8*a*(a + b*x^2)^4) + (7*(x/(6*a*(a + b*x^2)^3) + (5*(x/(4*a*(a + b*x^2
)^2) + (3*(x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sq
rt[b])))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(12*a)))/(14*a)))/(16*a)))/(18*a
)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

method	result
risch	$\frac{\frac{53381x}{65536a}+\frac{399041b\,x^{3}}{98304a^{2}}+\frac{355283b^{2}x^{5}}{32768a^{3}}+\frac{4108169b^{3}x^{7}}{229376a^{4}}+\frac{2431b^{4}x^{9}}{126a^{5}}+\frac{3133559b^{5}x^{11}}{229376a^{6}}+\frac{201773b^{6}x^{13}}{32768a^{7}}+\frac{158015b^{7}x^{15}}{98304a^{8}}+\frac{12155b^{8}x^{17}}{65536a^{9}}}{\left(b\,x^{2}+a\right)^{9}} - \frac{12155\ln\left(bx\cdots\right.}{131072\sqrt{\cdots}}$ $9\left(\frac{7x}{48a\left(b\,x^{2}+a\right)^{3}}\cdots\right.$ $11\left(\frac{9x}{80a\left(b\,x^{2}+a\right)^{4}}+\cdots\right.$ $13\left(\frac{11x}{120a\left(b\,x^{2}+a\right)^{5}}+\cdots\right.$ $15\left(\frac{13x}{168a\left(b\,x^{2}+a\right)^{6}}+\cdots\right.$

input
```
int(1/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
(53381/65536*x/a+399041/98304*b/a^2*x^3+355283/32768*b^2/a^3*x^5+4108169/2
29376*b^3/a^4*x^7+2431/126*b^4/a^5*x^9+3133559/229376*b^5/a^6*x^11+201773/
32768*b^6/a^7*x^13+158015/98304*b^7/a^8*x^15+12155/65536*b^8/a^9*x^17)/(b*
x^2+a)^9-12155/131072/(-a*b)^(1/2)/a^9*ln(b*x+(-a*b)^(1/2))+12155/131072/(
-a*b)^(1/2)/a^9*ln(-b*x+(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(153) = 306$.

Time = 0.08 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.59

$$\int \frac{1}{(a+bx^2)^{10}}\,dx$$
$$= \left[\frac{1531530\,ab^9x^{17} + 13273260\,a^2b^8x^{15} + 50846796\,a^3b^7x^{13} + 112808124\,a^4b^6x^{11} + 159318016\,a^5b^5x^9 + 14}{8257536\,(}\right.$$

input
```
integrate(1/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/8257536*(1531530*a*b^9*x^17 + 13273260*a^2*b^8*x^15 + 50846796*a^3*b^7*
x^13 + 112808124*a^4*b^6*x^11 + 159318016*a^5*b^5*x^9 + 147894084*a^6*b^4*
x^7 + 89531316*a^7*b^3*x^5 + 33519444*a^8*b^2*x^3 + 6726006*a^9*b*x - 7657
65*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^12 + 126*a^4*
b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4 + 9*a^8*b*x^2
 + a^9)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^10*b^
10*x^18 + 9*a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14
*b^6*x^10 + 126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*
b^2*x^2 + a^19*b), 1/4128768*(765765*a*b^9*x^17 + 6636630*a^2*b^8*x^15 + 2
5423398*a^3*b^7*x^13 + 56404062*a^4*b^6*x^11 + 79659008*a^5*b^5*x^9 + 7394
7042*a^6*b^4*x^7 + 44765658*a^7*b^3*x^5 + 16759722*a^8*b^2*x^3 + 3363003*a
^9*b*x + 765765*(b^9*x^18 + 9*a*b^8*x^16 + 36*a^2*b^7*x^14 + 84*a^3*b^6*x^
12 + 126*a^4*b^5*x^10 + 126*a^5*b^4*x^8 + 84*a^6*b^3*x^6 + 36*a^7*b^2*x^4
+ 9*a^8*b*x^2 + a^9)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^10*b^10*x^18 + 9*
a^11*b^9*x^16 + 36*a^12*b^8*x^14 + 84*a^13*b^7*x^12 + 126*a^14*b^6*x^10 +
126*a^15*b^5*x^8 + 84*a^16*b^4*x^6 + 36*a^17*b^3*x^4 + 9*a^18*b^2*x^2 + a^
19*b)]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx^2)^{10}}\,dx = -\frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(-a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072} + \frac{12155\sqrt{-\frac{1}{a^{19}b}}\log\left(a^{10}\sqrt{-\frac{1}{a^{19}b}}+x\right)}{131072} + \frac{3363003a^8x + 16759722a^7bx^3 + 44765658a^6b^2x^5 + 73947042a^5b^3x^7 + 79659008a^4}{4128768a^{18} + 37158912a^{17}bx^2 + 148635648a^{16}b^2x^4 + 346816512a^{15}b^3x^6 + 520224768a^{14}b^4x^8 + 52022}$$

input
```
integrate(1/(b*x**2+a)**10,x)
```

output

```
-12155*sqrt(-1/(a**19*b))*log(-a**10*sqrt(-1/(a**19*b)) + x)/131072 + 1215
5*sqrt(-1/(a**19*b))*log(a**10*sqrt(-1/(a**19*b)) + x)/131072 + (3363003*a
**8*x + 16759722*a**7*b*x**3 + 44765658*a**6*b**2*x**5 + 73947042*a**5*b**
3*x**7 + 79659008*a**4*b**4*x**9 + 56404062*a**3*b**5*x**11 + 25423398*a**
2*b**6*x**13 + 6636630*a*b**7*x**15 + 765765*b**8*x**17)/(4128768*a**18 +
37158912*a**17*b*x**2 + 148635648*a**16*b**2*x**4 + 346816512*a**15*b**3*x
**6 + 520224768*a**14*b**4*x**8 + 520224768*a**13*b**5*x**10 + 346816512*a
**12*b**6*x**12 + 148635648*a**11*b**7*x**14 + 37158912*a**10*b**8*x**16 +
 4128768*a**9*b**9*x**18)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a+bx^2)^{10}}\,dx$$
$$= \frac{765765\,b^8x^{17} + 6636630\,ab^7x^{15} + 25423398\,a^2b^6x^{13} + 56404062\,a^3b^5x^{11} + 79659008\,a^4b^4x^9 + 73947042\,a}{4128768\,(a^9b^9x^{18} + 9\,a^{10}b^8x^{16} + 36\,a^{11}b^7x^{14} + 84\,a^{12}b^6x^{12} + 126\,a^{13}b^5x^{10} + 126\,a^{14}b^4x^8 +}$$
$$+ \frac{12155\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^9}$$

input

```
integrate(1/(b*x^2+a)^10,x, algorithm="maxima")
```

output

```
1/4128768*(765765*b^8*x^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 +
56404062*a^3*b^5*x^11 + 79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 4476
5658*a^6*b^2*x^5 + 16759722*a^7*b*x^3 + 3363003*a^8*x)/(a^9*b^9*x^18 + 9*a
^10*b^8*x^16 + 36*a^11*b^7*x^14 + 84*a^12*b^6*x^12 + 126*a^13*b^5*x^10 + 1
26*a^14*b^4*x^8 + 84*a^15*b^3*x^6 + 36*a^16*b^2*x^4 + 9*a^17*b*x^2 + a^18)
 + 12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.67

$$\int \frac{1}{(a+bx^2)^{10}}\,dx = \frac{12155 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^9} + \frac{765765\,b^8x^{17} + 6636630\,ab^7x^{15} + 25423398\,a^2b^6x^{13} + 56404062\,a^3b^5x^{11} + 79659008\,a^4b^4x^9 + 7394704}{4128768\,(bx^2+a)^9a^9}$$

input
```
integrate(1/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
12155/65536*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^9) + 1/4128768*(765765*b^8*
x^17 + 6636630*a*b^7*x^15 + 25423398*a^2*b^6*x^13 + 56404062*a^3*b^5*x^11
+ 79659008*a^4*b^4*x^9 + 73947042*a^5*b^3*x^7 + 44765658*a^6*b^2*x^5 + 167
59722*a^7*b*x^3 + 3363003*a^8*x)/((b*x^2 + a)^9*a^9)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+bx^2)^{10}}\,dx = \frac{\frac{53381\,x}{65536\,a} + \frac{399041\,b\,x^3}{98304\,a^2} + \frac{355283\,b^2\,x^5}{32768\,a^3} + \frac{4108169\,b^3\,x^7}{229376\,a^4} + \frac{2431\,b^4\,x^9}{126\,a^5} + \frac{3133559\,b^5\,x^{11}}{229376\,a^6} + \frac{201773\,b^6\,x^{13}}{32768\,a^7} + \frac{158015\,b^7\,x^{15}}{98304\,a^8} + 12}{a^9 + 9\,a^8\,b\,x^2 + 36\,a^7\,b^2\,x^4 + 84\,a^6\,b^3\,x^6 + 126\,a^5\,b^4\,x^8 + 126\,a^4\,b^5\,x^{10} + 84\,a^3\,b^6\,x^{12} + 36\,a^2\,b^7\,x^{14} + 9\,a\,b^8} + \frac{12155\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{19/2}\,\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^10,x)
```

output
```
((53381*x)/(65536*a) + (399041*b*x^3)/(98304*a^2) + (355283*b^2*x^5)/(3276
8*a^3) + (4108169*b^3*x^7)/(229376*a^4) + (2431*b^4*x^9)/(126*a^5) + (3133
559*b^5*x^11)/(229376*a^6) + (201773*b^6*x^13)/(32768*a^7) + (158015*b^7*x
^15)/(98304*a^8) + (12155*b^8*x^17)/(65536*a^9))/(a^9 + b^9*x^18 + 9*a^8*b
*x^2 + 9*a*b^8*x^16 + 36*a^7*b^2*x^4 + 84*a^6*b^3*x^6 + 126*a^5*b^4*x^8 +
126*a^4*b^5*x^10 + 84*a^3*b^6*x^12 + 36*a^2*b^7*x^14) + (12155*atan((b^(1/
2)*x)/a^(1/2)))/(65536*a^(19/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.52

$$\int \frac{1}{(a+bx^2)^{10}}\,dx$$
$$= \frac{765765\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^9 + 6891885\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^8b\,x^2 + 27567540\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)a^7b^2}{}$$

input
```
int(1/(b*x^2+a)^10,x)
```

output
```
(765765*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9 + 6891885*sqrt(
b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**2 + 27567540*sqrt(b)*sq
rt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**4 + 64324260*sqrt(b)*sqrt
(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**6 + 96486390*sqrt(b)*sqrt(a
)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**8 + 96486390*sqrt(b)*sqrt(a)*
atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**10 + 64324260*sqrt(b)*sqrt(a)*a
tan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**12 + 27567540*sqrt(b)*sqrt(a)*at
an((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**7*x**14 + 6891885*sqrt(b)*sqrt(a)*atan
((b*x)/(sqrt(b)*sqrt(a)))*a*b**8*x**16 + 765765*sqrt(b)*sqrt(a)*atan((b*x)
/(sqrt(b)*sqrt(a)))*b**9*x**18 + 3363003*a**9*b*x + 16759722*a**8*b**2*x**
3 + 44765658*a**7*b**3*x**5 + 73947042*a**6*b**4*x**7 + 79659008*a**5*b**5
*x**9 + 56404062*a**4*b**6*x**11 + 25423398*a**3*b**7*x**13 + 6636630*a**2
*b**8*x**15 + 765765*a*b**9*x**17)/(4128768*a**10*b*(a**9 + 9*a**8*b*x**2
+ 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b*
*5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9
*x**18))
```

3.222 $\int \frac{1}{x^2(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 198

$$\int \frac{1}{x^2(a+bx^2)^{10}}\,dx = -\frac{1}{a^{10}x} - \frac{bx}{18a^2(a+bx^2)^9} - \frac{35bx}{288a^3(a+bx^2)^8} - \frac{271bx}{1344a^4(a+bx^2)^7} - \frac{4867bx}{16128a^5(a+bx^2)^6} - \frac{13933bx}{32256a^6(a+bx^2)^5} - \frac{17517bx}{28672a^7(a+bx^2)^4} - \frac{21613bx}{24576a^8(a+bx^2)^3} - \frac{132641bx}{98304a^9(a+bx^2)^2} - \frac{165409bx}{65536a^{10}(a+bx^2)} - \frac{230945\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{21/2}}$$

output
```
-1/a^10/x-1/18*b*x/a^2/(b*x^2+a)^9-35/288*b*x/a^3/(b*x^2+a)^8-271/1344*b*x
/a^4/(b*x^2+a)^7-4867/16128*b*x/a^5/(b*x^2+a)^6-13933/32256*b*x/a^6/(b*x^2
+a)^5-17517/28672*b*x/a^7/(b*x^2+a)^4-21613/24576*b*x/a^8/(b*x^2+a)^3-1326
41/98304*b*x/a^9/(b*x^2+a)^2-165409/65536*b*x/a^10/(b*x^2+a)-230945/65536*
b^(1/2)*arctan(b^(1/2)*x/a^(1/2))/a^(21/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{10}}\,dx = \frac{-\frac{\sqrt{a}\left(4128768a^9+63897057a^8bx^2+318434718a^7b^2x^4+850547502a^6b^3x^6+1404993798a^5b^4x^8+1513521152a^4b^5x^{10}+1071677178a^3b^6x^{12}+483\right.}{x(a+bx^2)^9}}{4128768a^{21/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^10),x]
```

output
```
(-((Sqrt[a]*(4128768*a^9 + 63897057*a^8*b*x^2 + 318434718*a^7*b^2*x^4 + 85
0547502*a^6*b^3*x^6 + 1404993798*a^5*b^4*x^8 + 1513521152*a^4*b^5*x^10 + 1
071677178*a^3*b^6*x^12 + 483044562*a^2*b^7*x^14 + 126095970*a*b^8*x^16 + 1
4549535*b^9*x^18))/(x*(a + b*x^2)^9)) - 14549535*Sqrt[b]*ArcTan[(Sqrt[b]*x
)/Sqrt[a]])/(4128768*a^(21/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {253, 253, 253, 253, 253, 253, 253, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{10}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{19\int \frac{1}{x^2(bx^2+a)^9}dx}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 253$$

$$\frac{19\left(\frac{17\int \frac{1}{x^2\left(bx^2+a\right)^8}dx}{16a}+\frac{1}{16ax(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{19\left(\frac{17\left(\frac{15\int \frac{1}{x^2\left(bx^2+a\right)^7}dx}{14a}+\frac{1}{14ax\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\int \frac{1}{x^2\left(bx^2+a\right)^6}dx}{12a}+\frac{1}{12ax\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\int\frac{1}{x^2\left(bx^2+a\right)^5}dx}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 253$$

$$\frac{19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{1}{x^2\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax\left(a+bx^2\right)^9}$$

$\big\downarrow$ 253

$$19\left(\frac{17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{1}{x^2\left(bx^2+a\right)^3}\,dx}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax(a+\cdots}\right.$$

↓ 253

$$\left(17\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\int\frac{1}{x^2\left(bx^2+a\right)^2}\,dx}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax\left(a+b\cdots\right.}\right.\right.$$

↓ 253

$$\left(\left(\left(\frac{15\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int\frac{1}{x^2\left(bx^2+a\right)}\,dx}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax\left(a+bx\right.}\right.\right.\right.$$

$\downarrow$ 264

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{b\int\frac{1}{bx^2+a}\,dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\Bigg)$$

↓ 218

$$\left(\left(\left(\left(13\left(\frac{11\left(\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax\left(a+bx^2\right)^5}\right.\right.\right.\right.$$

input
```
Int[1/(x^2*(a + b*x^2)^10),x]
```

output
```
1/(18*a*x*(a + b*x^2)^9) + (19*(1/(16*a*x*(a + b*x^2)^8) + (17*(1/(14*a*x*
(a + b*x^2)^7) + (15*(1/(12*a*x*(a + b*x^2)^6) + (13*(1/(10*a*x*(a + b*x^2
)^5) + (11*(1/(8*a*x*(a + b*x^2)^4) + (9*(1/(6*a*x*(a + b*x^2)^3) + (7*(1/
(4*a*x*(a + b*x^2)^2) + (5*(1/(2*a*x*(a + b*x^2)) + (3*(-(1/(a*x)) - (Sqrt
[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)))/(8*a)))
/(10*a)))/(12*a)))/(14*a)))/(16*a)))/(18*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.66

method	result
default	$-\frac{b\left(\frac{\frac{424415}{65536}a^{8}x+\frac{4042835}{98304}a^{7}b\,x^{3}+\frac{3997865}{32768}a^{6}b^{2}x^{5}+\frac{49153835}{229376}a^{5}b^{3}x^{7}+\frac{30313}{126}a^{4}b^{4}x^{9}+\frac{40270037}{229376}a^{3}b^{5}x^{11}+\frac{2654039}{32768}a^{2}b^{6}x^{13}+\frac{2117549}{98304}a\,b^{7}x^{15}}{\left(b\,x^{2}+a\right)^{9}}\right.}{a^{10}}$
risch	$\frac{-\frac{1}{a}-\frac{1014239b\,x^{2}}{65536a^{2}}-\frac{7581779b^{2}x^{4}}{98304a^{3}}-\frac{6750377b^{3}x^{6}}{32768a^{4}}-\frac{780552111b^{4}x^{8}}{229376a^{5}}-\frac{46189b^{5}x^{10}}{126a^{6}}-\frac{595376211b^{6}x^{12}}{229376a^{7}}-\frac{3833687b^{7}x^{14}}{32768a^{8}}-\frac{3002285b^{8}x^{16}}{98304a^{9}}-\frac{230945b^{9}x}{65536a^{1}}}{x(b\,x^{2}+a)^{9}}$

input
```
int(1/x^2/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-b/a^10*((424415/65536*a^8*x+4042835/98304*a^7*b*x^3+3997865/32768*a^6*b^2
*x^5+49153835/229376*a^5*b^3*x^7+30313/126*a^4*b^4*x^9+40270037/229376*a^3
*b^5*x^11+2654039/32768*a^2*b^6*x^13+2117549/98304*a*b^7*x^15+165409/65536
*b^8*x^17)/(b*x^2+a)^9+230945/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1
/a^10/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 664, normalized size of antiderivative = 3.35

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx$$

$$= \left[-\frac{29099070\,b^9x^{18} + 252191940\,ab^8x^{16} + 966089124\,a^2b^7x^{14} + 2143354356\,a^3b^6x^{12} + 3027042304\,a^4b^5x^{1}}{} \right.$$

$$-\frac{14549535\,b^9x^{18} + 126095970\,ab^8x^{16} + 483044562\,a^2b^7x^{14} + 1071677178\,a^3b^6x^{12} + 1513521152\,a^4b^5x^{10}}{}$$

input
```
integrate(1/x^2/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[-1/8257536*(29099070*b^9*x^18 + 252191940*a*b^8*x^16 + 966089124*a^2*b^7*
x^14 + 2143354356*a^3*b^6*x^12 + 3027042304*a^4*b^5*x^10 + 2809987596*a^5*
b^4*x^8 + 1701095004*a^6*b^3*x^6 + 636869436*a^7*b^2*x^4 + 127794114*a^8*b
*x^2 + 8257536*a^9 - 14549535*(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 +
 84*a^3*b^6*x^13 + 126*a^4*b^5*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 3
6*a^7*b^2*x^5 + 9*a^8*b*x^3 + a^9*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b
/a) - a)/(b*x^2 + a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15
 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x
^7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x), -1/4128768*(14549535*b^9*x^
18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x^14 + 1071677178*a^3*b^6*x^
12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b^4*x^8 + 850547502*a^6*b^3*
x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^2 + 4128768*a^9 + 14549535*
(b^9*x^19 + 9*a*b^8*x^17 + 36*a^2*b^7*x^15 + 84*a^3*b^6*x^13 + 126*a^4*b^5
*x^11 + 126*a^5*b^4*x^9 + 84*a^6*b^3*x^7 + 36*a^7*b^2*x^5 + 9*a^8*b*x^3 +
a^9*x)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 3
6*a^12*b^7*x^15 + 84*a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9
+ 84*a^16*b^3*x^7 + 36*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x)]
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.42

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx = \frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(-\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072} - \frac{230945\sqrt{-\frac{b}{a^{21}}}\log\left(\frac{a^{11}\sqrt{-\frac{b}{a^{21}}}}{b}+x\right)}{131072} + \frac{-4128768a^9 - 63897057a^8bx^2 - 318434718a^7b^2x^4 - 850547502a^6b^3x^6 - 1404993798a^5b^4x^8 - 1513}{4128768a^{19}x + 37158912a^{18}bx^3 + 148635648a^{17}b^2x^5 + 346816512a^{16}b^3x^7 + 520224768a^{15}b^4x^9 + 5202}$$

input
```
integrate(1/x**2/(b*x**2+a)**10,x)
```

output
```
230945*sqrt(-b/a**21)*log(-a**11*sqrt(-b/a**21)/b + x)/131072 - 230945*sqr
t(-b/a**21)*log(a**11*sqrt(-b/a**21)/b + x)/131072 + (-4128768*a**9 - 6389
7057*a**8*b*x**2 - 318434718*a**7*b**2*x**4 - 850547502*a**6*b**3*x**6 - 1
404993798*a**5*b**4*x**8 - 1513521152*a**4*b**5*x**10 - 1071677178*a**3*b*
*6*x**12 - 483044562*a**2*b**7*x**14 - 126095970*a*b**8*x**16 - 14549535*b
**9*x**18)/(4128768*a**19*x + 37158912*a**18*b*x**3 + 148635648*a**17*b**2
*x**5 + 346816512*a**16*b**3*x**7 + 520224768*a**15*b**4*x**9 + 520224768*
a**14*b**5*x**11 + 346816512*a**13*b**6*x**13 + 148635648*a**12*b**7*x**15
 + 37158912*a**11*b**8*x**17 + 4128768*a**10*b**9*x**19)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx = -\frac{14549535\,b^9x^{18} + 126095970\,ab^8x^{16} + 483044562\,a^2b^7x^{14} + 1071677178\,a^3b^6x^{12} + 1513521152\,a^4b^5x^{10}}{4128768\,(a^{10}b^9x^{19} + 9\,a^{11}b^8x^{17} + 36\,a^{12}b^7x^{15} + 84\,a^{13}b^6x^{13} + 126\,a^{14}b^5} - \frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{10}}$$

input
```
integrate(1/x^2/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/4128768*(14549535*b^9*x^18 + 126095970*a*b^8*x^16 + 483044562*a^2*b^7*x
^14 + 1071677178*a^3*b^6*x^12 + 1513521152*a^4*b^5*x^10 + 1404993798*a^5*b
^4*x^8 + 850547502*a^6*b^3*x^6 + 318434718*a^7*b^2*x^4 + 63897057*a^8*b*x^
2 + 4128768*a^9)/(a^10*b^9*x^19 + 9*a^11*b^8*x^17 + 36*a^12*b^7*x^15 + 84*
a^13*b^6*x^13 + 126*a^14*b^5*x^11 + 126*a^15*b^4*x^9 + 84*a^16*b^3*x^7 + 3
6*a^17*b^2*x^5 + 9*a^18*b*x^3 + a^19*x) - 230945/65536*b*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^10)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx = -\frac{230945\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{10}} - \frac{1}{a^{10}x} - \frac{10420767\,b^9x^{17} + 88937058\,ab^8x^{15} + 334408914\,a^2b^7x^{13} + 724860666\,a^3b^6x^{11} + 993296384\,a^4b^5x^9 + 88}{4128768\,(bx^2+a)^9a^{10}}$$

input
```
integrate(1/x^2/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-230945/65536*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^10) - 1/(a^10*x) - 1/41
28768*(10420767*b^9*x^17 + 88937058*a*b^8*x^15 + 334408914*a^2*b^7*x^13 +
724860666*a^3*b^6*x^11 + 993296384*a^4*b^5*x^9 + 884769030*a^5*b^4*x^7 + 5
03730990*a^6*b^3*x^5 + 169799070*a^7*b^2*x^3 + 26738145*a^8*b*x)/((b*x^2 +
 a)^9*a^10)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+bx^2)^{10}}\,dx = -\frac{\frac{1}{a} + \frac{1014239\,b\,x^2}{65536\,a^2} + \frac{7581779\,b^2\,x^4}{98304\,a^3} + \frac{6750377\,b^3\,x^6}{32768\,a^4} + \frac{78055211\,b^4\,x^8}{229376\,a^5} + \frac{46189\,b^5\,x^{10}}{126\,a^6} + \frac{59537621\,b^6\,x^{12}}{229376\,a^7} + \frac{3833687\,b^7\,x^{14}}{32768\,a^8} + 300}{a^9\,x + 9\,a^8\,b\,x^3 + 36\,a^7\,b^2\,x^5 + 84\,a^6\,b^3\,x^7 + 126\,a^5\,b^4\,x^9 + 126\,a^4\,b^5\,x^{11} + 84\,a^3\,b^6\,x^{13} + 36\,a^2\,b^7\,x^{15} +} - \frac{230945\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{21/2}}$$

input
```
int(1/(x^2*(a + b*x^2)^10),x)
```

output
```
- (1/a + (1014239*b*x^2)/(65536*a^2) + (7581779*b^2*x^4)/(98304*a^3) + (67
50377*b^3*x^6)/(32768*a^4) + (78055211*b^4*x^8)/(229376*a^5) + (46189*b^5*
x^10)/(126*a^6) + (59537621*b^6*x^12)/(229376*a^7) + (3833687*b^7*x^14)/(3
2768*a^8) + (3002285*b^8*x^16)/(98304*a^9) + (230945*b^9*x^18)/(65536*a^10
))/(a^9*x + b^9*x^19 + 9*a^8*b*x^3 + 9*a*b^8*x^17 + 36*a^7*b^2*x^5 + 84*a^
6*b^3*x^7 + 126*a^5*b^4*x^9 + 126*a^4*b^5*x^11 + 84*a^3*b^6*x^13 + 36*a^2*
b^7*x^15) - (230945*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^(21/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.34

$$\int \frac{1}{x^2 (a+bx^2)^{10}} dx = \frac{-14549535\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^9 x - 130945815\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^8 b\, x^3 - 523783260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{b}{\sqrt{b}}\right.}{}$$

input
```
int(1/x^2/(b*x^2+a)^10,x)
```

output
```
( - 14549535*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9*x - 130945
815*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b*x**3 - 523783260*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**2*x**5 - 1222160940*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**3*x**7 - 1833241410*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**4*x**9 - 1833241410*
sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**5*x**11 - 1222160940
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**6*x**13 - 523783260
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**7*x**15 - 130945815
*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**8*x**17 - 14549535*sqr
t(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**9*x**19 - 4128768*a**10 - 63
897057*a**9*b*x**2 - 318434718*a**8*b**2*x**4 - 850547502*a**7*b**3*x**6 -
 1404993798*a**6*b**4*x**8 - 1513521152*a**5*b**5*x**10 - 1071677178*a**4*
b**6*x**12 - 483044562*a**3*b**7*x**14 - 126095970*a**2*b**8*x**16 - 14549
535*a*b**9*x**18)/(4128768*a**11*x*(a**9 + 9*a**8*b*x**2 + 36*a**7*b**2*x*
*4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*a**4*b**5*x**10 + 84*a**
3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16 + b**9*x**18))
```

3.223 $\int \frac{1}{x^4(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 227

$$\begin{aligned}\int \frac{1}{x^4(a+bx^2)^{10}}\,dx = &-\frac{1}{3a^{10}x^3}+\frac{10b}{a^{11}x}+\frac{b^2x}{18a^3(a+bx^2)^9}+\frac{53b^2x}{288a^4(a+bx^2)^8}\\ &+\frac{79b^2x}{192a^5(a+bx^2)^7}+\frac{1795b^2x}{2304a^6(a+bx^2)^6}+\frac{6253b^2x}{4608a^7(a+bx^2)^5}\\ &+\frac{9325b^2x}{4096a^8(a+bx^2)^4}+\frac{93947b^2x}{24576a^9(a+bx^2)^3}+\frac{666343b^2x}{98304a^{10}(a+bx^2)^2}\\ &+\frac{961255b^2x}{65536a^{11}(a+bx^2)}+\frac{1616615b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{23/2}}\end{aligned}$$

output
```
-1/3/a^10/x^3+10*b/a^11/x+1/18*b^2*x/a^3/(b*x^2+a)^9+53/288*b^2*x/a^4/(b*x
^2+a)^8+79/192*b^2*x/a^5/(b*x^2+a)^7+1795/2304*b^2*x/a^6/(b*x^2+a)^6+6253/
4608*b^2*x/a^7/(b*x^2+a)^5+9325/4096*b^2*x/a^8/(b*x^2+a)^4+93947/24576*b^2
*x/a^9/(b*x^2+a)^3+666343/98304*b^2*x/a^10/(b*x^2+a)^2+961255/65536*b^2*x/
a^11/(b*x^2+a)+1616615/65536*b^(3/2)*arctan(b^(1/2)*x/a^(1/2))/a^(23/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.69

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx$$
$$= \frac{\frac{\sqrt{a}(-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+1404993798a^5b^5x^{10}+1513521152a^4b^6x^{12}+107167}{x^3(a+bx^2)^9}}{589824a^{23/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^10),x]
```

output
```
((Sqrt[a]*(-196608*a^10 + 4128768*a^9*b*x^2 + 63897057*a^8*b^2*x^4 + 31843
4718*a^7*b^3*x^6 + 850547502*a^6*b^4*x^8 + 1404993798*a^5*b^5*x^10 + 15135
21152*a^4*b^6*x^12 + 1071677178*a^3*b^7*x^14 + 483044562*a^2*b^8*x^16 + 12
6095970*a*b^9*x^18 + 14549535*b^10*x^20))/(x^3*(a + b*x^2)^9) + 14549535*b
^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(589824*a^(23/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.30, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {253, 253, 253, 253, 253, 253, 253, 253, 253, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx$$
$$\Big\downarrow \text{ 253}$$
$$\frac{7\int \frac{1}{x^4(bx^2+a)^9}dx}{6a} + \frac{1}{18ax^3(a+bx^2)^9}$$
$$\Big\downarrow \text{ 253}$$

$$\frac{7\left(\frac{19\int \frac{1}{x^4\left(bx^2+a\right)^8}dx}{16a}+\frac{1}{16ax^3(a+bx^2)^8}\right)}{6a}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

$\downarrow$ 253

$$\frac{7\left(\frac{19\left(\frac{17\int \frac{1}{x^4\left(bx^2+a\right)^7}dx}{14a}+\frac{1}{14ax^3\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^3(a+bx^2)^8}\right)}{6a}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

$\downarrow$ 253

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\int \frac{1}{x^4\left(bx^2+a\right)^6}dx}{4a}+\frac{1}{12ax^3\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^3\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^3(a+bx^2)^8}\right)}{6a}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

$\downarrow$ 253

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\int \frac{1}{x^4\left(bx^2+a\right)^5}dx}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}\right)}{4a}+\frac{1}{12ax^3\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^3\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^3(a+bx^2)^8}\right)}{6a}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

$\big\downarrow$ 253

$$\frac{7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\int\frac{1}{x^4\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}\right)}{4a}+\frac{1}{12ax^3\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^3\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^3(a+bx^2)^8}\right)}{6a}+\frac{1}{18ax^3\left(a+bx^2\right)^9}$$

$\downarrow$ 253

$$7\left(\frac{19\left(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\int\frac{1}{x^4\left(bx^2+a\right)^3}\,dx}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}\right)}{4a}+\frac{1}{12ax^3\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^3\left(a+bx^2\right)^7}\right)}{16a}+\frac{}{16a\cdots}\right.$$

↓ 253

$$\left(\left(19\,\frac{\left(17\,\frac{\left(5\,\frac{\left(13\,\frac{\left(11\,\frac{\left(3\,\frac{\left(\frac{7\int\frac{1}{x^4\left(bx^2+a\right)^2}\,dx}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}\right)}{4a}+\frac{1}{12ax^3\left(a+bx^2\right)^6}\right)}{14a}+\frac{\cdots}{14ax\cdots}\right)\cdots\right)$$

↓ 253

$$\Bigg(\Bigg(\Bigg(\frac{17\left(\frac{5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\int\frac{1}{x^4\left(bx^2+a\right)}\,dx}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx^2\right)^5}\right)}{4a}+\frac{}{12ax^3\cdots}$$

↓ 264

$$5\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\int\frac{1}{x^2\left(bx^2+a\right)}dx}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)}{10a}+\frac{1}{10ax^3\left(a+bx\right.}$$

↓ 264

$$\left(\left(\left(\left(\left(\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(\frac{5\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}\,dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}\right)\right.\right.\right.\right.$$

↓ 218

$$\frac{13\left(\frac{11\left(\frac{3\left(\frac{7\left(5\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)\right)}{2a}+\frac{1}{2ax^3\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^3\left(a+bx^2\right)^2}\right)}{2a}+\frac{1}{6ax^3\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^3\left(a+bx^2\right)^4}$$

input
```
Int[1/(x^4*(a + b*x^2)^10),x]
```

output
```
1/(18*a*x^3*(a + b*x^2)^9) + (7*(1/(16*a*x^3*(a + b*x^2)^8) + (19*(1/(14*a
*x^3*(a + b*x^2)^7) + (17*(1/(12*a*x^3*(a + b*x^2)^6) + (5*(1/(10*a*x^3*(a
 + b*x^2)^5) + (13*(1/(8*a*x^3*(a + b*x^2)^4) + (11*(1/(6*a*x^3*(a + b*x^2
)^3) + (3*(1/(4*a*x^3*(a + b*x^2)^2) + (7*(1/(2*a*x^3*(a + b*x^2)) + (5*(-
1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(
3/2)))/a))/(2*a)))/(4*a)))/(2*a)))/(8*a)))/(10*a)))/(4*a)))/(14*a)))/(16*a
)))/(6*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.62

method	result
default	$\frac{b^2\left(\frac{\frac{1987865}{65536}a^8x+\frac{20435525}{98304}a^7b\,x^3+\frac{21103775}{32768}a^6b^2x^5+\frac{38143787}{32768}a^5b^3x^7+\frac{24013}{18}a^4b^4x^9+\frac{32405717}{32768}a^3b^5x^{11}+\frac{15137633}{32768}a^2b^6x^{13}+\frac{12201403}{98304}a\,b^7}{\left(b\,x^2+a\right)^9}\right.}{a^{11}}$
risch	$\frac{-\frac{1}{3a}+\frac{7b\,x^2}{a^2}+\frac{7099673b^2x^4}{65536a^3}+\frac{53072453b^3x^6}{98304a^4}+\frac{472526639b^4x^8}{32768a^5}+\frac{780552211b^5x^{10}}{32768a^6}+\frac{461896b^6x^{12}}{18a^7}+\frac{595376621b^7x^{14}}{32768a^8}+\frac{268358809b^8x^{16}}{32768a^9}+\frac{210159956b^9}{98304a^{10}}}{x^3\left(b\,x^2+a\right)^9}$

input
```
int(1/x^4/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
b^2/a^11*((1987865/65536*a^8*x+20435525/98304*a^7*b*x^3+21103775/32768*a^6
*b^2*x^5+38143787/32768*a^5*b^3*x^7+24013/18*a^4*b^4*x^9+32405717/32768*a^
3*b^5*x^11+15137633/32768*a^2*b^6*x^13+12201403/98304*a*b^7*x^15+961255/65
536*b^8*x^17)/(b*x^2+a)^9+1616615/65536/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))-1/3/a^10/x^3+10*b/a^11/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.08

$$\int \frac{1}{x^4\left(a+bx^2\right)^{10}}\,dx$$

$$= \left[\frac{29099070\,b^{10}x^{20}+252191940\,ab^9x^{18}+966089124\,a^2b^8x^{16}+2143354356\,a^3b^7x^{14}+3027042304\,a^4b^6x^{12}}{}\right.$$

input
```
integrate(1/x^4/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[1/1179648*(29099070*b^10*x^20 + 252191940*a*b^9*x^18 + 966089124*a^2*b^8*
x^16 + 2143354356*a^3*b^7*x^14 + 3027042304*a^4*b^6*x^12 + 2809987596*a^5*
b^5*x^10 + 1701095004*a^6*b^4*x^8 + 636869436*a^7*b^3*x^6 + 127794114*a^8*
b^2*x^4 + 8257536*a^9*b*x^2 - 393216*a^10 + 14549535*(b^10*x^21 + 9*a*b^9*
x^19 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b^5*
x^11 + 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqrt(-
b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^11*b^9*x^21 + 9*a
^12*b^8*x^19 + 36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 1
26*a^16*b^4*x^11 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20
*x^3), 1/589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2
*b^8*x^16 + 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798
*a^5*b^5*x^10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a
^8*b^2*x^4 + 4128768*a^9*b*x^2 - 196608*a^10 + 14549535*(b^10*x^21 + 9*a*b
^9*x^19 + 36*a^2*b^8*x^17 + 84*a^3*b^7*x^15 + 126*a^4*b^6*x^13 + 126*a^5*b
^5*x^11 + 84*a^6*b^4*x^9 + 36*a^7*b^3*x^7 + 9*a^8*b^2*x^5 + a^9*b*x^3)*sqr
t(b/a)*arctan(x*sqrt(b/a)))/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 + 36*a^13*b^7
*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^11 + 84*a^17
*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3)]
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.34

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx$$

$$= -\frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\log\left(-\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072} + \frac{1616615\sqrt{-\frac{b^3}{a^{23}}}\log\left(\frac{a^{12}\sqrt{-\frac{b^3}{a^{23}}}}{b^2}+x\right)}{131072} + \frac{-196608a^{10}+4128768a^9bx^2+63897057a^8b^2x^4+318434718a^7b^3x^6+850547502a^6b^4x^8+140499379\ldots}{589824a^{20}x^3+5308416a^{19}bx^5+21233664a^{18}b^2x^7+49545216a^{17}b^3x^9+74317824a^{16}b^4x^{11}\ldots}$$

input
```
integrate(1/x**4/(b*x**2+a)**10,x)
```

output

```
-1616615*sqrt(-b**3/a**23)*log(-a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 +
 1616615*sqrt(-b**3/a**23)*log(a**12*sqrt(-b**3/a**23)/b**2 + x)/131072 +
(-196608*a**10 + 4128768*a**9*b*x**2 + 63897057*a**8*b**2*x**4 + 318434718
*a**7*b**3*x**6 + 850547502*a**6*b**4*x**8 + 1404993798*a**5*b**5*x**10 +
1513521152*a**4*b**6*x**12 + 1071677178*a**3*b**7*x**14 + 483044562*a**2*b
**8*x**16 + 126095970*a*b**9*x**18 + 14549535*b**10*x**20)/(589824*a**20*x
**3 + 5308416*a**19*b*x**5 + 21233664*a**18*b**2*x**7 + 49545216*a**17*b**
3*x**9 + 74317824*a**16*b**4*x**11 + 74317824*a**15*b**5*x**13 + 49545216*
a**14*b**6*x**15 + 21233664*a**13*b**7*x**17 + 5308416*a**12*b**8*x**19 +
589824*a**11*b**9*x**21)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx = \frac{14549535\,b^{10}x^{20} + 126095970\,ab^9x^{18} + 483044562\,a^2b^8x^{16} + 1071677178\,a^3b^7x^{14} + 1513521152\,a^4b^6x^{12} + \cdots}{589824\,(a^{11}b^9x^{21} + 9\,a^{12}b^8x^{19} + 36\,a^{13}b^7x^{17} + 84\,a^{14}b^6x^{15} + \cdots} + \frac{1616615\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{11}}$$

input

```
integrate(1/x^4/(b*x^2+a)^10,x, algorithm="maxima")
```

output

```
1/589824*(14549535*b^10*x^20 + 126095970*a*b^9*x^18 + 483044562*a^2*b^8*x^
16 + 1071677178*a^3*b^7*x^14 + 1513521152*a^4*b^6*x^12 + 1404993798*a^5*b^
5*x^10 + 850547502*a^6*b^4*x^8 + 318434718*a^7*b^3*x^6 + 63897057*a^8*b^2*
x^4 + 4128768*a^9*b*x^2 - 196608*a^10)/(a^11*b^9*x^21 + 9*a^12*b^8*x^19 +
36*a^13*b^7*x^17 + 84*a^14*b^6*x^15 + 126*a^15*b^5*x^13 + 126*a^16*b^4*x^1
1 + 84*a^17*b^3*x^9 + 36*a^18*b^2*x^7 + 9*a^19*b*x^5 + a^20*x^3) + 1616615
/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.65

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx = \frac{1616615\,b^2 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{11}} + \frac{30\,bx^2 - a}{3\,a^{11}x^3} + \frac{8651295\,b^{10}x^{17} + 73208418\,ab^9x^{15} + 272477394\,a^2b^8x^{13} + 583302906\,a^3b^7x^{11} + 786857984\,a^4b^6x^9 + 68}{589824\,(bx^2+a)^9a^{11}}$$

input
```
integrate(1/x^4/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
1616615/65536*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^11) + 1/3*(30*b*x^2 -
 a)/(a^11*x^3) + 1/589824*(8651295*b^10*x^17 + 73208418*a*b^9*x^15 + 27247
7394*a^2*b^8*x^13 + 583302906*a^3*b^7*x^11 + 786857984*a^4*b^6*x^9 + 68658
8166*a^5*b^5*x^7 + 379867950*a^6*b^4*x^5 + 122613150*a^7*b^3*x^3 + 1789078
5*a^8*b^2*x)/((b*x^2 + a)^9*a^11)
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^4 (a+bx^2)^{10}}\,dx = \frac{\frac{7\,b\,x^2}{a^2} - \frac{1}{3\,a} + \frac{7099673\,b^2\,x^4}{65536\,a^3} + \frac{53072453\,b^3\,x^6}{98304\,a^4} + \frac{47252639\,b^4\,x^8}{32768\,a^5} + \frac{78055211\,b^5\,x^{10}}{32768\,a^6} + \frac{46189\,b^6\,x^{12}}{18\,a^7} + \frac{59537621\,b^7\,x^{14}}{32768\,a^8} + \frac{26835809}{32768}}{a^9\,x^3 + 9\,a^8\,b\,x^5 + 36\,a^7\,b^2\,x^7 + 84\,a^6\,b^3\,x^9 + 126\,a^5\,b^4\,x^{11} + 126\,a^4\,b^5\,x^{13} + 84\,a^3\,b^6\,x^{15} + 36\,a^2\,b} + \frac{1616615\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{23/2}}$$

input
```
int(1/(x^4*(a + b*x^2)^10),x)
```

output
```
((7*b*x^2)/a^2 - 1/(3*a) + (7099673*b^2*x^4)/(65536*a^3) + (53072453*b^3*x
^6)/(98304*a^4) + (47252639*b^4*x^8)/(32768*a^5) + (78055211*b^5*x^10)/(32
768*a^6) + (46189*b^6*x^12)/(18*a^7) + (59537621*b^7*x^14)/(32768*a^8) + (
26835809*b^8*x^16)/(32768*a^9) + (21015995*b^9*x^18)/(98304*a^10) + (16166
15*b^10*x^20)/(65536*a^11))/(a^9*x^3 + b^9*x^21 + 9*a^8*b*x^5 + 9*a*b^8*x^
19 + 36*a^7*b^2*x^7 + 84*a^6*b^3*x^9 + 126*a^5*b^4*x^11 + 126*a^4*b^5*x^13
 + 84*a^3*b^6*x^15 + 36*a^2*b^7*x^17) + (1616615*b^(3/2)*atan((b^(1/2)*x)/
a^(1/2)))/(65536*a^(23/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.11

$$\int \frac{1}{x^4 (a + bx^2)^{10}} \, dx$$
$$= \frac{14549535\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^9 b\, x^3 + 130945815\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^8 b^2 x^5 + 523783260\sqrt{b}\,\sqrt{a}\,atan\left(\frac{}{}\right.}{}$$

input
```
int(1/x^4/(b*x^2+a)^10,x)
```

output
```
(14549535*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9*b*x**3 + 1309
45815*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b**2*x**5 + 52378
3260*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**3*x**7 + 122216
0940*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**4*x**9 + 183324
1410*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**5*x**11 + 18332
41410*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**6*x**13 + 1222
160940*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b**7*x**15 + 523
783260*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2*b**8*x**17 + 130
945815*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b**9*x**19 + 145495
35*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**10*x**21 - 196608*a**1
1 + 4128768*a**10*b*x**2 + 63897057*a**9*b**2*x**4 + 318434718*a**8*b**3*x
**6 + 850547502*a**7*b**4*x**8 + 1404993798*a**6*b**5*x**10 + 1513521152*a
**5*b**6*x**12 + 1071677178*a**4*b**7*x**14 + 483044562*a**3*b**8*x**16 +
126095970*a**2*b**9*x**18 + 14549535*a*b**10*x**20)/(589824*a**12*x**3*(a*
*9 + 9*a**8*b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4
*x**8 + 126*a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*
a*b**8*x**16 + b**9*x**18))
```

3.224 $\int \frac{1}{x^6(a+bx^2)^{10}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 240

$$\int \frac{1}{x^6(a+bx^2)^{10}}\,dx = -\frac{1}{5a^{10}x^5}+\frac{10b}{3a^{11}x^3}-\frac{55b^2}{a^{12}x}-\frac{b^3x}{18a^4(a+bx^2)^9} -\frac{71b^3x}{288a^5(a+bx^2)^8}-\frac{133b^3x}{192a^6(a+bx^2)^7}-\frac{3649b^3x}{2304a^7(a+bx^2)^6} -\frac{74699b^3x}{23040a^8(a+bx^2)^5}-\frac{128459b^3x}{20480a^9(a+bx^2)^4} -\frac{1472653b^3x}{122880a^{10}(a+bx^2)^3}-\frac{2357389b^3x}{98304a^{11}(a+bx^2)^2} -\frac{3831949b^3x}{65536a^{12}(a+bx^2)}-\frac{7436429b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{65536a^{25/2}}$$

output
```
-1/5/a^10/x^5+10/3*b/a^11/x^3-55*b^2/a^12/x-1/18*b^3*x/a^4/(b*x^2+a)^9-71/
288*b^3*x/a^5/(b*x^2+a)^8-133/192*b^3*x/a^6/(b*x^2+a)^7-3649/2304*b^3*x/a^
7/(b*x^2+a)^6-74699/23040*b^3*x/a^8/(b*x^2+a)^5-128459/20480*b^3*x/a^9/(b*
x^2+a)^4-1472653/122880*b^3*x/a^10/(b*x^2+a)^3-2357389/98304*b^3*x/a^11/(b
*x^2+a)^2-3831949/65536*b^3*x/a^12/(b*x^2+a)-7436429/65536*b^(5/2)*arctan(
b^(1/2)*x/a^(1/2))/a^(25/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^6 (a+bx^2)^{10}} dx = \frac{-\frac{\sqrt{a}(589824a^{11}-4521984a^{10}bx^2+94961664a^9b^2x^4+1469632311a^8b^3x^6+7323998514a^7b^4x^8+19562592546a^6b^5x^{10}+32314857354a^5b^6x^{12}+3}{x^5(a+bx^2)^9}}{294}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^10),x]
```

output
```
(-((Sqrt[a]*(589824*a^11 - 4521984*a^10*b*x^2 + 94961664*a^9*b^2*x^4 + 146
9632311*a^8*b^3*x^6 + 7323998514*a^7*b^4*x^8 + 19562592546*a^6*b^5*x^10 +
32314857354*a^5*b^6*x^12 + 34810986496*a^4*b^7*x^14 + 24648575094*a^3*b^8*
x^16 + 11110024926*a^2*b^9*x^18 + 2900207310*a*b^10*x^20 + 334639305*b^11*
x^22))/(x^5*(a + b*x^2)^9)) - 334639305*b^(5/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(2949120*a^(25/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {253, 253, 253, 253, 253, 253, 253, 253, 253, 264, 264, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{10}} dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23 \int \frac{1}{x^6 (bx^2+a)^9}dx}{18a}+\frac{1}{18ax^5 (a+bx^2)^9}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{23\left(\frac{21\int\frac{1}{x^6\left(bx^2+a\right)^8}dx}{16a}+\frac{1}{16ax^5(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{23\left(\frac{21\left(\frac{19\int\frac{1}{x^6\left(bx^2+a\right)^7}dx}{14a}+\frac{1}{14ax^5\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^5(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\int\frac{1}{x^6\left(bx^2+a\right)^6}dx}{12a}+\frac{1}{12ax^5\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^5\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^5(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

$$\Big\downarrow \text{253}$$

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\int\frac{1}{x^6\left(bx^2+a\right)^5}\,dx}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax^5\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^5\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^5(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

$$\Big\downarrow \; 253$$

$$\frac{23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\int\frac{1}{x^6\left(bx^2+a\right)^4}dx}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax^5\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^5\left(a+bx^2\right)^7}\right)}{16a}+\frac{1}{16ax^5(a+bx^2)^8}\right)}{18a}+\frac{1}{18ax^5\left(a+bx^2\right)^9}$$

$\downarrow$ 253

$$23\left(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\int\frac{1}{x^6\left(bx^2+a\right)^3}dx}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax^5\left(a+bx^2\right)^6}\right)}{14a}+\frac{1}{14ax^5\left(a+bx^2\right)^7}\right)}{16a}+\frac{}{16\ldots}\right.$$

↓ 253

$$\Bigg(\Bigg(\frac{21\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\int\frac{1}{x^6\left(bx^2+a\right)^2}\,dx}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}\right)}{12a}+\frac{1}{12ax^5\left(a+bx^2\right)^6}\right)}{14a}+\frac{}{14\cdots}$$

↓ 253

$$\left(\left(\left(\frac{19\left(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\int\frac{1}{x^6\left(bx^2+a\right)}dx}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+bx^2\right)^5}\right)}{12a}+\frac{}{12a\cdots}\right.\right.\right.$$

↓ 264

$$\Bigg(\Bigg(\Bigg(\Bigg(\frac{17\left(\frac{3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\int\frac{1}{x^4\left(bx^2+a\right)}dx}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}\right)}{2a}+\frac{1}{10ax^5\left(a+\right.}$$

↓ 264

$$3\left(\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\int\frac{1}{x^2\left(bx^2+a\right)}\,dx}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{8a}+\frac{1}{8ax^5\left(a+bx^2\right)^4}$$

↓ 264

$$\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{b\int\frac{1}{bx^2+a}\,dx}{a}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}\right)}{}$$

↓ 218

$$\frac{13\left(\frac{11\left(\frac{9\left(\frac{7\left(-\frac{b\left(-\frac{b\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{a}-\frac{1}{3ax^3}\right)}{a}-\frac{1}{5ax^5}\right)}{2a}+\frac{1}{2ax^5\left(a+bx^2\right)}\right)}{4a}+\frac{1}{4ax^5\left(a+bx^2\right)^2}\right)}{6a}+\frac{1}{6ax^5\left(a+bx^2\right)^3}$$

input
```
Int[1/(x^6*(a + b*x^2)^10),x]
```

output
```
1/(18*a*x^5*(a + b*x^2)^9) + (23*(1/(16*a*x^5*(a + b*x^2)^8) + (21*(1/(14*
a*x^5*(a + b*x^2)^7) + (19*(1/(12*a*x^5*(a + b*x^2)^6) + (17*(1/(10*a*x^5*
(a + b*x^2)^5) + (3*(1/(8*a*x^5*(a + b*x^2)^4) + (13*(1/(6*a*x^5*(a + b*x^
2)^3) + (11*(1/(4*a*x^5*(a + b*x^2)^2) + (9*(1/(2*a*x^5*(a + b*x^2)) + (7*
(-1/5*1/(a*x^5) - (b*(-1/3*1/(a*x^3) - (b*(-(1/(a*x)) - (Sqrt[b]*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/a^(3/2)))/a))/a))/(2*a)))/(4*a)))/(6*a)))/(8*a)))/(2*a
)))/(12*a)))/(14*a)))/(16*a)))/(18*a)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.64

method	result
default	$-\frac{b^3\left(\frac{\frac{6981491}{65536}a^8x+\frac{74539223}{98304}a^7bx^3+\frac{394553929}{163840}a^6b^2x^5+\frac{725918941}{163840}a^5b^3x^7+\frac{463199}{90}a^4b^4x^9+\frac{631790371}{163840}a^3b^5x^{11}+\frac{297702839}{163840}a^2b^6x^{13}+\frac{48340}{983}\cdots}{(bx^2+a)^9}\right.}{a^{12}}$
risch	$\frac{-\frac{1}{5a}+\frac{23bx^2}{15a^2}-\frac{161b^2x^4}{5a^3}-\frac{163292479b^3x^6}{327680a^4}-\frac{1220666419b^4x^8}{491520a^5}-\frac{10868106697b^5x^{10}}{163840a^6}-\frac{17952698853b^6x^{12}}{163840a^7}-\frac{1062347b^7x^{14}}{90a^8}-\frac{1369365283b^8x^{16}}{163840a^9}-617\cdots}{x^5(bx^2+a)^9}$

input
```
int(1/x^6/(b*x^2+a)^10,x,method=_RETURNVERBOSE)
```

output
```
-b^3/a^12*((6981491/65536*a^8*x+74539223/98304*a^7*b*x^3+394553929/163840*
a^6*b^2*x^5+725918941/163840*a^5*b^3*x^7+463199/90*a^4*b^4*x^9+631790371/1
63840*a^3*b^5*x^11+297702839/163840*a^2*b^6*x^13+48340777/98304*a*b^7*x^15
+3831949/65536*b^8*x^17)/(b*x^2+a)^9+7436429/65536/(a*b)^(1/2)*arctan(b*x/
(a*b)^(1/2)))-1/5/a^10/x^5+10/3*b/a^11/x^3-55*b^2/a^12/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 726, normalized size of antiderivative = 3.02

$$\int \frac{1}{x^6\left(a+bx^2\right)^{10}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^6/(b*x^2+a)^10,x, algorithm="fricas")
```

output
```
[-1/5898240*(669278610*b^11*x^22 + 5800414620*a*b^10*x^20 + 22220049852*a^
2*b^9*x^18 + 49297150188*a^3*b^8*x^16 + 69621972992*a^4*b^7*x^14 + 6462971
4708*a^5*b^6*x^12 + 39125185092*a^6*b^5*x^10 + 14647997028*a^7*b^4*x^8 + 2
939264622*a^8*b^3*x^6 + 189923328*a^9*b^2*x^4 - 9043968*a^10*b*x^2 + 11796
48*a^11 - 334639305*(b^11*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*
b^8*x^17 + 126*a^4*b^7*x^15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*
b^4*x^9 + 9*a^8*b^3*x^7 + a^9*b^2*x^5)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(
-b/a) - a)/(b*x^2 + a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^
19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^
3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5), -1/2949120*(334639305
*b^11*x^22 + 2900207310*a*b^10*x^20 + 11110024926*a^2*b^9*x^18 + 246485750
94*a^3*b^8*x^16 + 34810986496*a^4*b^7*x^14 + 32314857354*a^5*b^6*x^12 + 19
562592546*a^6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 1469632311*a^8*b^3*x^6 +
 94961664*a^9*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a^11 + 334639305*(b^11
*x^23 + 9*a*b^10*x^21 + 36*a^2*b^9*x^19 + 84*a^3*b^8*x^17 + 126*a^4*b^7*x^
15 + 126*a^5*b^6*x^13 + 84*a^6*b^5*x^11 + 36*a^7*b^4*x^9 + 9*a^8*b^3*x^7 +
 a^9*b^2*x^5)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^12*b^9*x^23 + 9*a^13*b^8*x
^21 + 36*a^14*b^7*x^19 + 84*a^15*b^6*x^17 + 126*a^16*b^5*x^15 + 126*a^17*b
^4*x^13 + 84*a^18*b^3*x^11 + 36*a^19*b^2*x^9 + 9*a^20*b*x^7 + a^21*x^5)]
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^6 (a+bx^2)^{10}}\,dx = \frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(-\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072} - \frac{7436429\sqrt{-\frac{b^5}{a^{25}}}\log\left(\frac{a^{13}\sqrt{-\frac{b^5}{a^{25}}}}{b^3}+x\right)}{131072} + \frac{-589824a^{11}+4521984a^{10}bx^2-94961664a^9b^2x^4-1469632311a^8b^3x^6-7323998514a^7b^4x^8-1956259}{2949120a^{21}x^5+26542080a^{20}bx^7+106168320a^{19}b^2x^9+247726080a^{18}b^3x^{11}+}$$

input
```
integrate(1/x**6/(b*x**2+a)**10,x)
```

output
```
7436429*sqrt(-b**5/a**25)*log(-a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 -
7436429*sqrt(-b**5/a**25)*log(a**13*sqrt(-b**5/a**25)/b**3 + x)/131072 + (
-589824*a**11 + 4521984*a**10*b*x**2 - 94961664*a**9*b**2*x**4 - 146963231
1*a**8*b**3*x**6 - 7323998514*a**7*b**4*x**8 - 19562592546*a**6*b**5*x**10
 - 32314857354*a**5*b**6*x**12 - 34810986496*a**4*b**7*x**14 - 24648575094
*a**3*b**8*x**16 - 11110024926*a**2*b**9*x**18 - 2900207310*a*b**10*x**20
- 334639305*b**11*x**22)/(2949120*a**21*x**5 + 26542080*a**20*b*x**7 + 106
168320*a**19*b**2*x**9 + 247726080*a**18*b**3*x**11 + 371589120*a**17*b**4
*x**13 + 371589120*a**16*b**5*x**15 + 247726080*a**15*b**6*x**17 + 1061683
20*a**14*b**7*x**19 + 26542080*a**13*b**8*x**21 + 2949120*a**12*b**9*x**23
)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^6 (a+bx^2)^{10}} \, dx = -\frac{334639305 \, b^{11} x^{22} + 2900207310 \, ab^{10} x^{20} + 11110024926 \, a^2 b^9 x^{18} + 24648575094 \, a^3 b^8 x^{16} + 3481098649}{2949120 \, (a^{12} b^9 x^{23} + 9 \, a^{13} b^8 x^{21} + 36 \, a^{14} b^7 x^{1}} - \frac{7436429 \, b^3 \arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536 \, \sqrt{ab} a^{12}}$$

input
```
integrate(1/x^6/(b*x^2+a)^10,x, algorithm="maxima")
```

output
```
-1/2949120*(334639305*b^11*x^22 + 2900207310*a*b^10*x^20 + 11110024926*a^2
*b^9*x^18 + 24648575094*a^3*b^8*x^16 + 34810986496*a^4*b^7*x^14 + 32314857
354*a^5*b^6*x^12 + 19562592546*a^6*b^5*x^10 + 7323998514*a^7*b^4*x^8 + 146
9632311*a^8*b^3*x^6 + 94961664*a^9*b^2*x^4 - 4521984*a^10*b*x^2 + 589824*a
^11)/(a^12*b^9*x^23 + 9*a^13*b^8*x^21 + 36*a^14*b^7*x^19 + 84*a^15*b^6*x^1
7 + 126*a^16*b^5*x^15 + 126*a^17*b^4*x^13 + 84*a^18*b^3*x^11 + 36*a^19*b^2
*x^9 + 9*a^20*b*x^7 + a^21*x^5) - 7436429/65536*b^3*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*a^12)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^6 (a+bx^2)^{10}}\,dx = -\frac{7436429\,b^3\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{65536\,\sqrt{ab}a^{12}} - \frac{825\,b^2x^4 - 50\,abx^2 + 3\,a^2}{15\,a^{12}x^5} - \frac{172437705\,b^{11}x^{17} + 1450223310\,ab^{10}x^{15} + 5358651102\,a^2b^9x^{13} + 11372226678\,a^3b^8x^{11} + 15178104832}{2949120\,(bx^2+}$$

input
```
integrate(1/x^6/(b*x^2+a)^10,x, algorithm="giac")
```

output
```
-7436429/65536*b^3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^12) - 1/15*(825*b^2*
x^4 - 50*a*b*x^2 + 3*a^2)/(a^12*x^5) - 1/2949120*(172437705*b^11*x^17 + 14
50223310*a*b^10*x^15 + 5358651102*a^2*b^9*x^13 + 11372226678*a^3*b^8*x^11
+ 15178104832*a^4*b^7*x^9 + 13066540938*a^5*b^6*x^7 + 7101970722*a^6*b^5*x
^5 + 2236176690*a^7*b^4*x^3 + 314167095*a^8*b^3*x)/((b*x^2 + a)^9*a^12)
```

Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^6 (a+bx^2)^{10}}\,dx = -\frac{\frac{1}{5\,a} - \frac{23\,b\,x^2}{15\,a^2} + \frac{161\,b^2\,x^4}{5\,a^3} + \frac{163292479\,b^3\,x^6}{327680\,a^4} + \frac{1220666419\,b^4\,x^8}{491520\,a^5} + \frac{1086810697\,b^5\,x^{10}}{163840\,a^6} + \frac{1795269853\,b^6\,x^{12}}{163840\,a^7} + \frac{1062347\,b^7\,x^{14}}{90\,a^8} +}{a^9\,x^5 + 9\,a^8\,b\,x^7 + 36\,a^7\,b^2\,x^9 + 84\,a^6\,b^3\,x^{11} + 126\,a^5\,b^4\,x^{13} + 126\,a^4\,b^5\,x^{15} + 84\,a^3} - \frac{7436429\,b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{65536\,a^{25/2}}$$

input
```
int(1/(x^6*(a + b*x^2)^10),x)
```

output
```
- (1/(5*a) - (23*b*x^2)/(15*a^2) + (161*b^2*x^4)/(5*a^3) + (163292479*b^3*
x^6)/(327680*a^4) + (1220666419*b^4*x^8)/(491520*a^5) + (1086810697*b^5*x^
10)/(163840*a^6) + (1795269853*b^6*x^12)/(163840*a^7) + (1062347*b^7*x^14)
/(90*a^8) + (1369365283*b^8*x^16)/(163840*a^9) + (617223607*b^9*x^18)/(163
840*a^10) + (96673577*b^10*x^20)/(98304*a^11) + (7436429*b^11*x^22)/(65536
*a^12))/(a^9*x^5 + b^9*x^23 + 9*a^8*b*x^7 + 9*a*b^8*x^21 + 36*a^7*b^2*x^9
+ 84*a^6*b^3*x^11 + 126*a^5*b^4*x^13 + 126*a^4*b^5*x^15 + 84*a^3*b^6*x^17
+ 36*a^2*b^7*x^19) - (7436429*b^(5/2)*atan((b^(1/2)*x)/a^(1/2)))/(65536*a^
(25/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^6 (a+bx^2)^{10}} \, dx$$
$$= \frac{-334639305\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^9b^2x^5 - 3011753745\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) a^8b^3x^7 - 12047014980\sqrt{b}\,\sqrt{a}}{}$$

input
```
int(1/x^6/(b*x^2+a)^10,x)
```

output
```
( - 334639305*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**9*b**2*x**5
 - 3011753745*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**8*b**3*x**7
 - 12047014980*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**7*b**4*x**
9 - 28109701620*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**6*b**5*x*
*11 - 42164552430*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**5*b**6*
x**13 - 42164552430*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**4*b**
7*x**15 - 28109701620*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**3*b
**8*x**17 - 12047014980*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a**2
*b**9*x**19 - 3011753745*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*a*b
**10*x**21 - 334639305*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b**11
*x**23 - 589824*a**12 + 4521984*a**11*b*x**2 - 94961664*a**10*b**2*x**4 -
1469632311*a**9*b**3*x**6 - 7323998514*a**8*b**4*x**8 - 19562592546*a**7*b
**5*x**10 - 32314857354*a**6*b**6*x**12 - 34810986496*a**5*b**7*x**14 - 24
648575094*a**4*b**8*x**16 - 11110024926*a**3*b**9*x**18 - 2900207310*a**2*
b**10*x**20 - 334639305*a*b**11*x**22)/(2949120*a**13*x**5*(a**9 + 9*a**8*
b*x**2 + 36*a**7*b**2*x**4 + 84*a**6*b**3*x**6 + 126*a**5*b**4*x**8 + 126*
a**4*b**5*x**10 + 84*a**3*b**6*x**12 + 36*a**2*b**7*x**14 + 9*a*b**8*x**16
 + b**9*x**18))
```

3.225 $\int \frac{1}{x(1+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{1}{x(1+bx^2)}\,dx=\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

output
```
ln(x)-1/2*ln(b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx^2)}\,dx=\log(x)-\frac{1}{2}\log\left(1+bx^2\right)$$

input
```
Integrate[1/(x*(1 + b*x^2)),x]
```

output
```
Log[x] - Log[1 + b*x^2]/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+1\right)}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+1\right)}dx^2$$

$$\Big\downarrow \; 47$$

$$\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \; 14$$

$$\frac{1}{2}\left(\log\left(x^2\right)-b\int \frac{1}{bx^2+1}dx^2\right)$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(\log\left(x^2\right)-\log\left(bx^2+1\right)\right)$$

input
```
Int[1/(x*(1 + b*x^2)),x]
```

output
```
(Log[x^2] - Log[1 + b*x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
norman	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
risch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
parallelrisch	$\ln\left(x\right)-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	14
meijerg	$\ln\left(x\right)+\frac{\ln\left(b\right)}{2}-\frac{\ln\left(b\,x^{2}+1\right)}{2}$	18

input
```
int(1/x/(b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
ln(x)-1/2*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+bx^2)}\,dx = -\frac{1}{2}\log\left(bx^2+1\right)+\log\left(x\right)$$

input
```
integrate(1/x/(b*x^2+1),x, algorithm="fricas")
```

output
```
-1/2*log(b*x^2 + 1) + log(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(1+bx^2)}\,dx = \log\left(x\right)-\frac{\log\left(x^2+\frac{1}{b}\right)}{2}$$

input
```
integrate(1/x/(b*x**2+1),x)
```

output
```
log(x) - log(x**2 + 1/b)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{1}{x(1+bx^2)}\,dx = -\frac{1}{2}\log\left(bx^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(b*x^2+1),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{1}{x(1+bx^2)}\,dx = \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

input
```
integrate(1/x/(b*x^2+1),x, algorithm="giac")
```

output
```
1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(1+bx^2)}\,dx = \ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

input
```
int(1/(x*(b*x^2 + 1)),x)
```

output
```
log(x) - log((3*b*x^2)/2 + 3/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{x(1+bx^2)}\,dx = -\frac{\log(b\,x^2+1)}{2} + \log(x)$$

input
```
int(1/x/(b*x^2+1),x)
```

output
```
( - log(b*x**2 + 1) + 2*log(x))/2
```

3.226 $\int \frac{1}{x(-1+bx^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{1}{x(-1+bx^2)} dx = -\log(x) + \frac{1}{2}\log(1-bx^2)$$

output
```
-ln(x)+1/2*ln(-b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(-1+bx^2)} dx = -\log(x) + \frac{1}{2}\log(1-bx^2)$$

input
```
Integrate[1/(x*(-1 + b*x^2)),x]
```

output
```
-Log[x] + Log[1 - b*x^2]/2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {243, 25, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (bx^2-1)} \, dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int -\frac{1}{x^2 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{1}{2} \int \frac{1}{x^2 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 47}$$
$$\frac{1}{2} \left(-b \int \frac{1}{1-bx^2}dx^2-\int \frac{1}{x^2}dx^2\right)$$
$$\Big\downarrow \text{ 14}$$
$$\frac{1}{2} \left(-b \int \frac{1}{1-bx^2}dx^2-\log \left(x^2\right)\right)$$
$$\Big\downarrow \text{ 16}$$
$$\frac{1}{2} \left(\log \left(1-bx^2\right)-\log \left(x^2\right)\right)$$

input
```
Int[1/(x*(-1 + b*x^2)),x]
```

output
```
(-Log[x^2] + Log[1 - b*x^2])/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
default	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
norman	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
parallelrisch	$-\ln\left(x\right)+\frac{\ln\left(b\,x^{2}-1\right)}{2}$	16
risch	$-\ln\left(x\right)+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	17
meijerg	$-\ln\left(x\right)-\frac{\ln\left(-b\right)}{2}+\frac{\ln\left(-b\,x^{2}+1\right)}{2}$	23

input
```
int(1/x/(b*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
-ln(x)+1/2*ln(b*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(-1+bx^2)}\,dx = \frac{1}{2}\log\left(bx^2-1\right) - \log\left(x\right)$$

input
```
integrate(1/x/(b*x^2-1),x, algorithm="fricas")
```

output
```
1/2*log(b*x^2 - 1) - log(x)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x(-1+bx^2)}\,dx = -\log\left(x\right) + \frac{\log\left(x^2-\frac{1}{b}\right)}{2}$$

input
```
integrate(1/x/(b*x**2-1),x)
```

output
```
-log(x) + log(x**2 - 1/b)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(-1+bx^2)}\,dx = \frac{1}{2}\log\left(bx^2-1\right) - \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(b*x^2-1),x, algorithm="maxima")
```

output
```
1/2*log(b*x^2 - 1) - 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(-1+bx^2)}\,dx = -\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

input
```
integrate(1/x/(b*x^2-1),x, algorithm="giac")
```

output
```
-1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(-1+bx^2)}\,dx = \frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln\left(x\right)$$

input
```
int(1/(x*(b*x^2 - 1)),x)
```

output
```
log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{x(-1+bx^2)}\,dx = \frac{\log\left(-\sqrt{b}+bx\right)}{2}+\frac{\log\left(\sqrt{b}+bx\right)}{2}-\log(x)$$

input
```
int(1/x/(b*x^2-1),x)
```

output
```
(log( - sqrt(b) + b*x) + log(sqrt(b) + b*x) - 2*log(x))/2
```

3.227 $\int \frac{1}{x^3(1+bx^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \frac{1}{x^3 (1 + bx^2)} dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1 + bx^2\right)$$

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (1 + bx^2)} dx = -\frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1 + bx^2\right)$$

input
```
Integrate[1/(x^3*(1 + b*x^2)),x]
```

output
```
-1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (bx^2+1)}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^4 (bx^2+1)}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{b^2}{bx^2+1}-\frac{b}{x^2}+\frac{1}{x^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-b\log (x^2)+b\log (bx^2+1)-\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(1 + b*x^2)),x]
```

output
```
(-x^(-2) - b*Log[x^2] + b*Log[1 + b*x^2])/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(b\,x^2+1)}{2}$	23
norman	$-\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(b\,x^2+1)}{2}$	23
risch	$-\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(-b\,x^2-1)}{2}$	24
meijerg	$\frac{b\left(-\frac{1}{x^2 b}-2\ln(x)-\ln(b)+\ln(b\,x^2+1)\right)}{2}$	29
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln(b\,x^2+1)x^2+1}{2x^2}$	30

input
```
int(1/x^3/(b*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(1+bx^2)}\,dx = \frac{bx^2\log(bx^2+1) - 2\,bx^2\log(x) - 1}{2\,x^2}$$

input
```
integrate(1/x^3/(b*x^2+1),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (1 + bx^2)} \, dx = -b \log (x) + \frac{b \log \left(x^2 + \frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(b*x**2+1),x)
```

output
```
-b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (1 + bx^2)} \, dx = \frac{1}{2} \, b \log \left(bx^2 + 1\right) - \frac{1}{2} \, b \log \left(x^2\right) - \frac{1}{2 \, x^2}$$

input
```
integrate(1/x^3/(b*x^2+1),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 + 1) - 1/2*b*log(x^2) - 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (1 + bx^2)} \, dx = -\frac{1}{2} \, b \log \left(x^2\right) + \frac{1}{2} \, b \log \left(\left|bx^2 + 1\right|\right) + \frac{bx^2 - 1}{2 \, x^2}$$

input
```
integrate(1/x^3/(b*x^2+1),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3 (1 + bx^2)}\, dx = \frac{b \ln (b\, x^2 + 1)}{2} - b\, \ln (x) - \frac{1}{2\, x^2}$$

input
```
int(1/(x^3*(b*x^2 + 1)),x)
```

output
```
(b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (1 + bx^2)}\, dx = \frac{\log(b\, x^2 + 1)\, b\, x^2 - 2 \log(x)\, b\, x^2 - 1}{2x^2}$$

input
```
int(1/x^3/(b*x^2+1),x)
```

output
```
(log(b*x**2 + 1)*b*x**2 - 2*log(x)*b*x**2 - 1)/(2*x**2)
```

3.228 $\int \frac{1}{x^3(-1+bx^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{1}{x^3(-1+bx^2)}\,dx = \frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

output
```
1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(-1+bx^2)}\,dx = \frac{1}{2x^2} - b\log(x) + \frac{1}{2}b\log\left(1-bx^2\right)$$

input
```
Integrate[1/(x^3*(-1 + b*x^2)),x]
```

output
```
1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {243, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (bx^2-1)} \, dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2} \int -\frac{1}{x^4 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 25}$$
$$-\frac{1}{2} \int \frac{1}{x^4 (1-bx^2)}dx^2$$
$$\Big\downarrow \text{ 54}$$
$$-\frac{1}{2} \int \left(-\frac{b^2}{bx^2-1}+\frac{b}{x^2}+\frac{1}{x^4}\right)dx^2$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{2} \left(-b \log \left(x^2\right)+b \log \left(1-bx^2\right)+\frac{1}{x^2}\right)$$

input
```
Int[1/(x^3*(-1 + b*x^2)),x]
```

output
```
(x^(-2) - b*Log[x^2] + b*Log[1 - b*x^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

method	result	size
default	$\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(b\,x^2-1)}{2}$	23
norman	$\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(b\,x^2-1)}{2}$	23
risch	$\frac{1}{2x^2} - b\ln(x) + \frac{b\ln(-b\,x^2+1)}{2}$	24
parallelrisch	$-\frac{2b\ln(x)x^2-b\ln(b\,x^2-1)x^2-1}{2x^2}$	30
meijerg	$\frac{b\left(\frac{1}{x^2 b}-2\ln(x)-\ln(-b)+\ln(-b\,x^2+1)\right)}{2}$	31

input
```
int(1/x^3/(b*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = \frac{bx^2 \log(bx^2 - 1) - 2\,bx^2 \log(x) + 1}{2\,x^2}$$

input
```
integrate(1/x^3/(b*x^2-1),x, algorithm="fricas")
```

output
```
1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = -b \log(x) + \frac{b \log\left(x^2 - \frac{1}{b}\right)}{2} + \frac{1}{2x^2}$$

input
```
integrate(1/x**3/(b*x**2-1),x)
```

output
```
-b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = \frac{1}{2} b \log(bx^2 - 1) - \frac{1}{2} b \log(x^2) + \frac{1}{2\,x^2}$$

input
```
integrate(1/x^3/(b*x^2-1),x, algorithm="maxima")
```

output
```
1/2*b*log(b*x^2 - 1) - 1/2*b*log(x^2) + 1/2/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = -\frac{1}{2} b \log \left(x^2\right) + \frac{1}{2} b \log \left(\left|bx^2 - 1\right|\right) + \frac{bx^2 + 1}{2 x^2}$$

input
```
integrate(1/x^3/(b*x^2-1),x, algorithm="giac")
```

output
```
-1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = \frac{b \ln \left(b x^2 - 1\right)}{2} - b \ln (x) + \frac{1}{2 x^2}$$

input
```
int(1/(x^3*(b*x^2 - 1)),x)
```

output
```
(b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \frac{1}{x^3 (-1 + bx^2)} \, dx = \frac{\log \left(-\sqrt{b} + bx\right) b x^2 + \log \left(\sqrt{b} + bx\right) b x^2 - 2 \log (x) b x^2 + 1}{2x^2}$$

input
```
int(1/x^3/(b*x^2-1),x)
```

output
```
(log( - sqrt(b) + b*x)*b*x**2 + log(sqrt(b) + b*x)*b*x**2 - 2*log(x)*b*x**
2 + 1)/(2*x**2)
```

3.229 $\int \frac{1}{x(1+bx^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{x(1+bx^2)^2} dx = \frac{1}{2(1+bx^2)} + \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

output
```
1/(2*b*x^2+2)+ln(x)-1/2*ln(b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{x(1+bx^2)^2} dx = \frac{1}{2+2bx^2} + \log(x) - \frac{1}{2}\log\left(1+bx^2\right)$$

input
```
Integrate[1/(x*(1 + b*x^2)^2),x]
```

output
```
(2 + 2*b*x^2)^(-1) + Log[x] - Log[1 + b*x^2]/2
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2+1\right)^2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(bx^2+1\right)^2}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(-\frac{b}{bx^2+1}-\frac{b}{\left(bx^2+1\right)^2}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{bx^2+1}-\log\left(bx^2+1\right)+\log\left(x^2\right)\right)$$

input
```
Int[1/(x*(1 + b*x^2)^2),x]
```

output
```
((1 + b*x^2)^(-1) + Log[x^2] - Log[1 + b*x^2])/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
risch	$\frac{1}{2b\,x^2+2}+\ln\left(x\right)-\frac{\ln\left(b\,x^2+1\right)}{2}$	25
norman	$-\frac{b\,x^2}{2(b\,x^2+1)}+\ln\left(x\right)-\frac{\ln\left(b\,x^2+1\right)}{2}$	29
default	$-\frac{b\left(-\frac{1}{b\left(b\,x^2+1\right)}+\frac{\ln\left(b\,x^2+1\right)}{b}\right)}{2}+\ln\left(x\right)$	34
meijerg	$\frac{1}{2}+\ln\left(x\right)+\frac{\ln(b)}{2}-\frac{b\,x^2}{2b\,x^2+2}-\frac{\ln\left(b\,x^2+1\right)}{2}$	35
parallelrisch	$\frac{2b\ln(x)x^2-b\ln\left(b\,x^2+1\right)x^2-b\,x^2+2\ln(x)-\ln\left(b\,x^2+1\right)}{2b\,x^2+2}$	55

input
```
int(1/x/(b*x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/(b*x^2+1)+ln(x)-1/2*ln(b*x^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(1+bx^2\right)^2}\,dx = -\frac{\left(bx^2+1\right)\log\left(bx^2+1\right)-2\left(bx^2+1\right)\log\left(x\right)-1}{2\left(bx^2+1\right)}$$

input
```
integrate(1/x/(b*x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 + 1)*log(b*x^2 + 1) - 2*(b*x^2 + 1)*log(x) - 1)/(b*x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{x(1+bx^2)^2}\,dx = \log(x) - \frac{\log\left(x^2+\frac{1}{b}\right)}{2} + \frac{1}{2bx^2+2}$$

input
```
integrate(1/x/(b*x**2+1)**2,x)
```

output
```
log(x) - log(x**2 + 1/b)/2 + 1/(2*b*x**2 + 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(1+bx^2)^2}\,dx = \frac{1}{2(bx^2+1)} - \frac{1}{2}\log\left(bx^2+1\right) + \frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(b*x^2+1)^2,x, algorithm="maxima")
```

output
```
1/2/(b*x^2 + 1) - 1/2*log(b*x^2 + 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{1}{x(1+bx^2)^2}\,dx = \frac{bx^2+2}{2(bx^2+1)} + \frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(|bx^2+1|\right)$$

input
```
integrate(1/x/(b*x^2+1)^2,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + 2)/(b*x^2 + 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(1+bx^2\right)^2}\,dx = \ln\left(x\right) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2} + \frac{1}{2\,\left(b\,x^2+1\right)}$$

input
```
int(1/(x*(b*x^2 + 1)^2),x)
```

output
```
log(x) - log((3*b*x^2)/2 + 3/2)/2 + 1/(2*(b*x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int \frac{1}{x\left(1+bx^2\right)^2}\,dx = \frac{-\log(b\,x^2+1)\,b\,x^2-\log(b\,x^2+1)+2\log(x)\,b\,x^2+2\log(x)-b\,x^2}{2b\,x^2+2}$$

input
```
int(1/x/(b*x^2+1)^2,x)
```

output
```
( - log(b*x**2 + 1)*b*x**2 - log(b*x**2 + 1) + 2*log(x)*b*x**2 + 2*log(x)
- b*x**2)/(2*(b*x**2 + 1))
```

3.230 $\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx=\frac{1}{2\left(1-bx^2\right)}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)$$

output
```
1/(-2*b*x^2+2)+ln(x)-1/2*ln(-b*x^2+1)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx=\frac{1}{2-2bx^2}+\log(x)-\frac{1}{2}\log\left(1-bx^2\right)$$

input
```
Integrate[1/(x*(-1 + b*x^2)^2),x]
```

output
```
(2 - 2*b*x^2)^(-1) + Log[x] - Log[1 - b*x^2]/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(bx^2-1\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(1-bx^2\right)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(-\frac{b}{bx^2-1}+\frac{b}{\left(bx^2-1\right)^2}+\frac{1}{x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{1-bx^2}-\log\left(1-bx^2\right)+\log\left(x^2\right)\right)$$

input
```
Int[1/(x*(-1 + b*x^2)^2),x]
```

output
```
((1 - b*x^2)^(-1) + Log[x^2] - Log[1 - b*x^2])/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{1}{2(b\,x^2-1)}-\frac{\ln(b\,x^2-1)}{2}+\ln(x)$	25
norman	$-\frac{b\,x^2}{2(b\,x^2-1)}-\frac{\ln(b\,x^2-1)}{2}+\ln(x)$	29
default	$\ln(x)-\frac{b\left(\frac{1}{b\left(b\,x^2-1\right)}+\frac{\ln\left(b\,x^2-1\right)}{b}\right)}{2}$	33
meijerg	$\frac{1}{2}+\ln(x)+\frac{\ln(-b)}{2}+\frac{b\,x^2}{-2b\,x^2+2}-\frac{\ln(-b\,x^2+1)}{2}$	37
parallelrisch	$\frac{2b\ln(x)x^2-b\ln(b\,x^2-1)x^2-b\,x^2-2\ln(x)+\ln(b\,x^2-1)}{2b\,x^2-2}$	53

input
```
int(1/x/(b*x^2-1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(b*x^2-1)-1/2*ln(b*x^2-1)+ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\,(-1+bx^2)^2}\,dx = -\frac{(bx^2-1)\log(bx^2-1)-2\,(bx^2-1)\log(x)+1}{2\,(bx^2-1)}$$

input
```
integrate(1/x/(b*x^2-1)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 - 1)*log(b*x^2 - 1) - 2*(b*x^2 - 1)*log(x) + 1)/(b*x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx=\log\left(x\right)-\frac{\log\left(x^2-\frac{1}{b}\right)}{2}-\frac{1}{2bx^2-2}$$

input
```
integrate(1/x/(b*x**2-1)**2,x)
```

output
```
log(x) - log(x**2 - 1/b)/2 - 1/(2*b*x**2 - 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx=-\frac{1}{2\left(bx^2-1\right)}-\frac{1}{2}\log\left(bx^2-1\right)+\frac{1}{2}\log\left(x^2\right)$$

input
```
integrate(1/x/(b*x^2-1)^2,x, algorithm="maxima")
```

output
```
-1/2/(b*x^2 - 1) - 1/2*log(b*x^2 - 1) + 1/2*log(x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx=\frac{bx^2-2}{2\left(bx^2-1\right)}+\frac{1}{2}\log\left(x^2\right)-\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

input
```
integrate(1/x/(b*x^2-1)^2,x, algorithm="giac")
```

output
```
1/2*(b*x^2 - 2)/(b*x^2 - 1) + 1/2*log(x^2) - 1/2*log(abs(b*x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx = \ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}-\frac{3}{2}\right)}{2} - \frac{1}{2\,\left(b\,x^2-1\right)}$$

input
```
int(1/(x*(b*x^2 - 1)^2),x)
```

output
```
log(x) - log((3*b*x^2)/2 - 3/2)/2 - 1/(2*(b*x^2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \frac{1}{x\left(-1+bx^2\right)^2}\,dx = \frac{-\log\left(-\sqrt{b}+bx\right)b\,x^2+\log\left(-\sqrt{b}+bx\right)-\log\left(\sqrt{b}+bx\right)b\,x^2+\log\left(\sqrt{b}+bx\right)+2\log(x)\,b\,x^2-2\log(x}{2b\,x^2-2}$$

input
```
int(1/x/(b*x^2-1)^2,x)
```

output
```
( - log( - sqrt(b) + b*x)*b*x**2 + log( - sqrt(b) + b*x) - log(sqrt(b) + b
*x)*b*x**2 + log(sqrt(b) + b*x) + 2*log(x)*b*x**2 - 2*log(x) - b*x**2)/(2*
(b*x**2 - 1))
```

3.231 $\int \frac{x}{-1+x^2} dx$

Optimal result

Integrand size = 9, antiderivative size = 12

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(1-x^2\right)$$

output
```
1/2*ln(-x^2+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{x}{-1+x^2} dx = \frac{1}{2}\log\left(-1+x^2\right)$$

input
```
Integrate[x/(-1 + x^2),x]
```

output
```
Log[-1 + x^2]/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{x^2-1} \, dx$$

$$\Big\downarrow \; 240$$

$$\frac{1}{2} \log \left(1-x^2\right)$$

input
```
Int[x/(-1 + x^2),x]
```

output
```
Log[1 - x^2]/2
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\ln(x^2-1)}{2}$	9
risch	$\frac{\ln(x^2-1)}{2}$	9
meijerg	$\frac{\ln(-x^2+1)}{2}$	11
default	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
norman	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14
parallelrisch	$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}$	14

input
```
int(1/(x^2-1)*x,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx=\frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(x/(x^2-1),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{x}{-1+x^2}\,dx = \frac{\log(x^2-1)}{2}$$

input
```
integrate(x/(x**2-1),x)
```

output
```
log(x**2 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log(x^2-1)$$

input
```
integrate(x/(x^2-1),x, algorithm="maxima")
```

output
```
1/2*log(x^2 - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log(|x^2-1|)$$

input
```
integrate(x/(x^2-1),x, algorithm="giac")
```

output
```
1/2*log(abs(x^2 - 1))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{x}{-1+x^2}\,dx = \frac{\ln(x^2-1)}{2}$$

input
```
int(x/(x^2 - 1),x)
```

output
```
log(x^2 - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{x}{-1+x^2}\,dx = \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

input
```
int(x/(x^2-1),x)
```

output
```
(log(x - 1) + log(x + 1))/2
```

3.232 $\int \frac{x^2}{(1+x^2)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{x^2}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(1+x^2)^2} dx = -\frac{x}{2(1+x^2)} + \frac{\arctan(x)}{2}$$

input
```
Integrate[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(x^2+1)^2} dx$$
$$\Big\downarrow \text{252}$$
$$\frac{1}{2}\int \frac{1}{x^2+1}dx - \frac{x}{2(x^2+1)}$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan(x)}{2} - \frac{x}{2(x^2+1)}$$

input
```
Int[x^2/(1 + x^2)^2,x]
```

output
```
-1/2*x/(1 + x^2) + ArcTan[x]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
meijerg	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
risch	$-\frac{x}{2(x^2+1)}+\frac{\arctan(x)}{2}$	16
parallelrisch	$-\frac{i\ln(x-i)x^2-i\ln(x+i)x^2+i\ln(x-i)-i\ln(x+i)+2x}{4(x^2+1)}$	52

input
```
int(x^2/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(x^2+1)+1/2*arctan(x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{(x^2+1)\arctan(x)-x}{2(x^2+1)}$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{(1+x^2)^2}\,dx = -\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

input
```
integrate(x**2/(x**2+1)**2,x)
```

output
```
-x/(2*x**2 + 2) + atan(x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(1+x^2)^2} \, dx = -\frac{x}{2(x^2+1)} + \frac{1}{2} \arctan(x)$$

input
```
integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

output
```
-1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{(1+x^2)^2} \, dx = \frac{\mathrm{atan}(x)}{2} - \frac{x}{2\,(x^2+1)}$$

input
```
int(x^2/(x^2 + 1)^2,x)
```

output
```
atan(x)/2 - x/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \frac{atan(x)\,x^2 + atan(x) - x}{2x^2 + 2}$$

input

```
int(x^2/(x^2+1)^2,x)
```

output

```
(atan(x)*x**2 + atan(x) - x)/(2*(x**2 + 1))
```

3.233 $\int x^2(4-x^2)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int x^2 (4-x^2)^2 \, dx = \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}$$

output
```
16/3*x^3-8/5*x^5+1/7*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2 (4-x^2)^2 \, dx = \frac{16x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}$$

input
```
Integrate[x^2*(4 - x^2)^2,x]
```

output
```
(16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(4-x^2\right)^2 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(x^6-8 x^4+16 x^2\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^7}{7}-\frac{8 x^5}{5}+\frac{16 x^3}{3}$$

input
```
Int[x^2*(4 - x^2)^2,x]
```

output
```
(16*x^3)/3 - (8*x^5)/5 + x^7/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7$	17
norman	$\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7$	17
risch	$\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7$	17
parallelrisch	$\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7$	17
gosper	$\frac{x^3(15x^4-168x^2+560)}{105}$	18
orering	$\frac{x^3(15x^4-168x^2+560)(-x^2+4)^2}{105(x-2)^2(2+x)^2}$	37

input
```
int(x^2*(-x^2+4)^2,x,method=_RETURNVERBOSE)
```

output
```
16/3*x^3-8/5*x^5+1/7*x^7
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2(4-x^2)^2\,dx = \frac{1}{7}x^7 - \frac{8}{5}x^5 + \frac{16}{3}x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="fricas")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4 - x^2)^2 \, dx = \frac{x^7}{7} - \frac{8x^5}{5} + \frac{16x^3}{3}$$

input
```
integrate(x**2*(-x**2+4)**2,x)
```

output
```
x**7/7 - 8*x**5/5 + 16*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="maxima")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int x^2 (4 - x^2)^2 \, dx = \frac{1}{7} x^7 - \frac{8}{5} x^5 + \frac{16}{3} x^3$$

input
```
integrate(x^2*(-x^2+4)^2,x, algorithm="giac")
```

output
```
1/7*x^7 - 8/5*x^5 + 16/3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4-x^2)^2 \, dx = \frac{x^3 (15 x^4 - 168 x^2 + 560)}{105}$$

input
```
int(x^2*(x^2 - 4)^2,x)
```

output
```
(x^3*(15*x^4 - 168*x^2 + 560))/105
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int x^2 (4-x^2)^2 \, dx = \frac{x^3(15x^4 - 168x^2 + 560)}{105}$$

input
```
int(x^2*(-x^2+4)^2,x)
```

output
```
(x**3*(15*x**4 - 168*x**2 + 560))/105
```

3.234 $\int \frac{x}{(1-x^2)^5} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8(1-x^2)^4}$$

output
```
1/8/(-x^2+1)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8(-1+x^2)^4}$$

input
```
Integrate[x/(1 - x^2)^5,x]
```

output
```
1/(8*(-1 + x^2)^4)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(1-x^2)^5} \, dx$$

$$\Big\downarrow \; 241$$

$$\frac{1}{8\,(1-x^2)^4}$$

input
```
Int[x/(1 - x^2)^5,x]
```

output
```
1/(8*(1 - x^2)^4)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{1}{8(x^2-1)^4}$	10
default	$\frac{1}{8(x^2-1)^4}$	10
norman	$\frac{1}{8(x^2-1)^4}$	10
risch	$\frac{1}{8(x^2-1)^4}$	10
parallelrisch	$\frac{1}{8(x^2-1)^4}$	10
derivativedivides	$\frac{1}{8(-x^2+1)^4}$	12
orering	$-\frac{(1+x)(-1+x)}{8(-x^2+1)^5}$	18
meijerg	$\frac{x^2(-x^6+4x^4-6x^2+4)}{8(-x^2+1)^4}$	32

input
```
int(x/(-x^2+1)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/(x^2-1)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(x^8-4\,x^6+6\,x^4-4\,x^2+1)}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="fricas")
```

output
```
1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

input
```
integrate(x/(-x**2+1)**5,x)
```

output
```
1/(8*x**8 - 32*x**6 + 48*x**4 - 32*x**2 + 8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(x^2-1)^4}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="maxima")
```

output
```
1/8/(x^2 - 1)^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5}\,dx = \frac{1}{8\,(x^2-1)^4}$$

input
```
integrate(x/(-x^2+1)^5,x, algorithm="giac")
```

output
```
1/8/(x^2 - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8 (x^2-1)^4}$$

input
```
int(-x/(x^2 - 1)^5,x)
```

output
```
1/(8*(x^2 - 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \frac{x}{(1-x^2)^5} \, dx = \frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

input
```
int(x/(-x^2+1)^5,x)
```

output
```
1/(8*(x**8 - 4*x**6 + 6*x**4 - 4*x**2 + 1))
```

3.235 $\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}\right.$

Optimal result

Integrand size = 73, antiderivative size = 13

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2} -\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(1-x^2\right)^4}$$

output
```
1/8/(-x^2+1)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2} -\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(-1+x^2\right)^4}$$

input
```
Integrate[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/
(256*(-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3)
 - 5/(256*(1 + x)^2),x]
```

output
```
1/(8*(-1 + x^2)^4)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 81 vs. 2(13) = 26.

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.23, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\frac{5}{256(x+1)^2}-\frac{5}{128(x+1)^3}-\frac{3}{64(x+1)^4}-\frac{1}{32(x+1)^5}+\frac{5}{256(x-1)^2}-\frac{5}{128(x-1)^3}+\frac{3}{64(x-1)^4}-\frac{1}{32(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{256(x+1)}+\frac{5}{256(x+1)^2}+\frac{1}{64(x+1)^3}+\frac{1}{128(x+1)^4}+\frac{5}{256(1-x)}+\frac{5}{256(1-x)^2}+\frac{1}{64(1-x)^3}+\frac{1}{128(1-x)^4}$$

input
```
Int[-1/32*1/(-1 + x)^5 + 3/(64*(-1 + x)^4) - 5/(128*(-1 + x)^3) + 5/(256*(
-1 + x)^2) - 1/(32*(1 + x)^5) - 3/(64*(1 + x)^4) - 5/(128*(1 + x)^3) - 5/(
256*(1 + x)^2),x]
```

output
```
1/(128*(1 - x)^4) + 1/(64*(1 - x)^3) + 5/(256*(1 - x)^2) + 5/(256*(1 - x))
 + 1/(128*(1 + x)^4) + 1/(64*(1 + x)^3) + 5/(256*(1 + x)^2) + 5/(256*(1 +
x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result
gosper	$\frac{1}{8(-1+x)^{4}(1+x)^{4}}$
norman	$\frac{1}{8(-1+x)^{4}(1+x)^{4}}$
risch	$\frac{1}{8(-1+x)^{4}(1+x)^{4}}$
parallelrisch	$\frac{1}{8(-1+x)^{4}(1+x)^{4}}$
default	$\frac{1}{128(-1+x)^{4}}-\frac{1}{64(-1+x)^{3}}+\frac{5}{256(-1+x)^{2}}-\frac{5}{256(-1+x)}+\frac{1}{128(1+x)^{4}}+\frac{1}{64(1+x)^{3}}+\frac{5}{256(1+x)^{2}}+\frac{5}{256(1+x)}$
orering	$-\frac{(1+x)(-1+x)\left(-\frac{1}{32(-1+x)^{5}}+\frac{3}{64(-1+x)^{4}}-\frac{5}{128(-1+x)^{3}}+\frac{5}{256(-1+x)^{2}}-\frac{1}{32(1+x)^{5}}-\frac{3}{64(1+x)^{4}}-\frac{5}{128(1+x)^{3}}-\frac{5}{256(1+x)^{2}}\right)}{8x}$
meijerg	$\frac{x\left(-x^{3}+4x^{2}-6x+4\right)}{128(1-x)^{4}}+\frac{x\left(x^{2}-3x+3\right)}{64(1-x)^{3}}+\frac{5x(-x+2)}{256(1-x)^{2}}+\frac{5x}{256(1-x)}-\frac{x\left(x^{3}+4x^{2}+6x+4\right)}{128(1+x)^{4}}-\frac{x\left(x^{2}+3x+3\right)}{64(1+x)^{3}}-\frac{5x(2+x}{256(1+x}$

input
```
int(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^
5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/(-1+x)^4/(1+x)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="fricas")
```

output
```
1/8/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(8) = 16$.

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2}\right) dx = \frac{1}{8x^8 - 32x^6 + 48x^4 - 32x^2 + 8}$$

input
```
integrate(-1/32/(-1+x)**5+3/64/(-1+x)**4-5/128/(-1+x)**3+5/256/(-1+x)**2-1
/32/(1+x)**5-3/64/(1+x)**4-5/128/(1+x)**3-5/256/(1+x)**2,x)
```

output
```
1/(8*x***8 - 32*x***6 + 48*x***4 - 32*x***2 + 8)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \left(-\frac{1}{32(-1+x)^5} + \frac{3}{64(-1+x)^4} - \frac{5}{128(-1+x)^3} + \frac{5}{256(-1+x)^2} - \frac{1}{32(1+x)^5} - \frac{3}{64(1+x)^4} - \frac{5}{128(1+x)^3} - \frac{5}{256(1+x)^2}\right) dx = \frac{5}{256\,(x+1)} - \frac{5}{256\,(x-1)} + \frac{5}{256\,(x+1)^2} + \frac{5}{256\,(x-1)^2} + \frac{1}{64\,(x+1)^3} - \frac{1}{64\,(x-1)^3} + \frac{1}{128\,(x+1)^4} + \frac{1}{128\,(x-1)^4}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="maxima")
```

output
```
5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(
x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(9) = 18$.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)\,dx=\frac{5}{256\,(x+1)}-\frac{5}{256\,(x-1)}+\frac{5}{256\,(x+1)^2}+\frac{5}{256\,(x-1)^2}+\frac{1}{64\,(x+1)^3}-\frac{1}{64\,(x-1)^3}+\frac{1}{128\,(x+1)^4}+\frac{1}{128\,(x-1)^4}$$

input
```
integrate(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/
(1+x)^5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x, algorithm="giac")
```

output
```
5/256/(x + 1) - 5/256/(x - 1) + 5/256/(x + 1)^2 + 5/256/(x - 1)^2 + 1/64/(
x + 1)^3 - 1/64/(x - 1)^3 + 1/128/(x + 1)^4 + 1/128/(x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right)\,dx=\frac{1}{8\,(x^2-1)^4}$$

input
```
int(5/(256*(x - 1)^2) - 5/(256*(x + 1)^2) - 5/(128*(x - 1)^3) - 5/(128*(x
+ 1)^3) + 3/(64*(x - 1)^4) - 3/(64*(x + 1)^4) - 1/(32*(x - 1)^5) - 1/(32*(
x + 1)^5),x)
```

output
```
1/(8*(x^2 - 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

$$\int \left(-\frac{1}{32(-1+x)^5}+\frac{3}{64(-1+x)^4}-\frac{5}{128(-1+x)^3}+\frac{5}{256(-1+x)^2}-\frac{1}{32(1+x)^5}-\frac{3}{64(1+x)^4}-\frac{5}{128(1+x)^3}-\frac{5}{256(1+x)^2}\right) dx = \frac{1}{8x^8-32x^6+48x^4-32x^2+8}$$

input
```
int(-1/32/(-1+x)^5+3/64/(-1+x)^4-5/128/(-1+x)^3+5/256/(-1+x)^2-1/32/(1+x)^
5-3/64/(1+x)^4-5/128/(1+x)^3-5/256/(1+x)^2,x)
```

output
```
1/(8*(x**8 - 4*x**6 + 6*x**4 - 4*x**2 + 1))
```

3.236 $\int \frac{x^3}{a-bx^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 28

$$\int \frac{x^3}{a-bx^2} \, dx = -\frac{x^2}{2b} - \frac{a \log (a-bx^2)}{2b^2}$$

output
```
-1/2*x^2/b-1/2*a*ln(-b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{a-bx^2} \, dx = -\frac{x^2}{2b} - \frac{a \log (a-bx^2)}{2b^2}$$

input
```
Integrate[x^3/(a - b*x^2),x]
```

output
```
-1/2*x^2/b - (a*Log[a - b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a-bx^2}dx$$
$$\Big\downarrow \textbf{243}$$
$$\frac{1}{2}\int \frac{x^2}{a-bx^2}dx^2$$
$$\Big\downarrow \textbf{49}$$
$$\frac{1}{2}\int \left(-\frac{a}{b\left(bx^2-a\right)}-\frac{1}{b}\right)dx^2$$
$$\Big\downarrow \textbf{2009}$$
$$\frac{1}{2}\left(-\frac{a\log\left(a-bx^2\right)}{b^2}-\frac{x^2}{b}\right)$$

input
```
Int[x^3/(a - b*x^2),x]
```

output
```
(-(x^2/b) - (a*Log[a - b*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

method	result	size
parallelrisch	$-\frac{b\,x^{2}+a\ln\left(b\,x^{2}-a\right)}{2b^{2}}$	24
default	$-\frac{x^{2}}{2b}-\frac{a\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	25
norman	$-\frac{x^{2}}{2b}-\frac{a\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	25
risch	$-\frac{x^{2}}{2b}-\frac{a\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	25

input
```
int(x^3/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a*ln(b*x^2-a))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a-bx^2}\,dx=-\frac{bx^2+a\log\left(bx^2-a\right)}{2\,b^2}$$

input
```
integrate(x^3/(-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2 + a*log(b*x^2 - a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{a-bx^2}\,dx = -\frac{a\log\left(-a+bx^2\right)}{2b^2} - \frac{x^2}{2b}$$

input
```
integrate(x**3/(-b*x**2+a),x)
```

output
```
-a*log(-a + b*x**2)/(2*b**2) - x**2/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{a-bx^2}\,dx = -\frac{x^2}{2\,b} - \frac{a\log\left(bx^2-a\right)}{2\,b^2}$$

input
```
integrate(x^3/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*x^2/b - 1/2*a*log(b*x^2 - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^3}{a-bx^2}\,dx = -\frac{x^2}{2\,b} - \frac{a\log\left(|bx^2-a|\right)}{2\,b^2}$$

input
```
integrate(x^3/(-b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*x^2/b - 1/2*a*log(abs(b*x^2 - a))/b^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{a-bx^2} \, dx = -\frac{b\,x^2 + a\,\ln\left(b\,x^2 - a\right)}{2\,b^2}$$

input
```
int(x^3/(a - b*x^2),x)
```

output
```
-(b*x^2 + a*log(b*x^2 - a))/(2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{x^3}{a-bx^2} \, dx = \frac{-\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a-b\,x^2}{2b^2}$$

input
```
int(x^3/(-b*x^2+a),x)
```

output
```
( - (log( - sqrt(b)*sqrt(a) - b*x)*a + log(sqrt(b)*sqrt(a) - b*x)*a + b*x*
*2))/(2*b**2)
```

3.237 $\int \frac{x^2}{a-bx^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 31

$$\int \frac{x^2}{a-bx^2} dx = -\frac{x}{b} + \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

output
```
-x/b+a^(1/2)*arctanh(b^(1/2)*x/a^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a-bx^2} dx = -\frac{x}{b} + \frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
Integrate[x^2/(a - b*x^2),x]
```

output
```
-(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a-bx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{a\int \frac{1}{a-bx^2}dx}{b}-\frac{x}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

input
```
Int[x^2/(a - b*x^2),x]
```

output
```
-(x/b) + (Sqrt[a]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{x}{b}+\frac{a\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{b\sqrt{ab}}$	27
risch	$-\frac{x}{b}-\frac{\sqrt{ab}\,\ln\left(\sqrt{ab}\,x-a\right)}{2b^{2}}+\frac{\sqrt{ab}\,\ln\left(-\sqrt{ab}\,x-a\right)}{2b^{2}}$	53

input
```
int(x^2/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-x/b+a/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.58

$$\int \frac{x^2}{a-bx^2}\,dx = \left[\frac{\sqrt{\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{\frac{a}{b}}+a}{bx^2-a}\right)-2\,x}{2\,b}, -\frac{\sqrt{-\frac{a}{b}}\arctan\left(\frac{bx\sqrt{-\frac{a}{b}}}{a}\right)+x}{b}\right]$$

input
```
integrate(x^2/(-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a/b)*log((b*x^2 + 2*b*x*sqrt(a/b) + a)/(b*x^2 - a)) - 2*x)/b, -
(sqrt(-a/b)*arctan(b*x*sqrt(-a/b)/a) + x)/b]
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{a-bx^2}\,dx = -\frac{\sqrt{\frac{a}{b^3}}\log\left(-b\sqrt{\frac{a}{b^3}}+x\right)}{2}+\frac{\sqrt{\frac{a}{b^3}}\log\left(b\sqrt{\frac{a}{b^3}}+x\right)}{2}-\frac{x}{b}$$

input
```
integrate(x**2/(-b*x**2+a),x)
```

output
```
-sqrt(a/b**3)*log(-b*sqrt(a/b**3) + x)/2 + sqrt(a/b**3)*log(b*sqrt(a/b**3)
 + x)/2 - x/b
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{x^2}{a-bx^2}\,dx = -\frac{a\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}b}-\frac{x}{b}$$

input
```
integrate(x^2/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*a*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*b) - x/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{a-bx^2}\,dx = -\frac{a\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}b}-\frac{x}{b}$$

input
```
integrate(x^2/(-b*x^2+a),x, algorithm="giac")
```

output
```
-a*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - x/b
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{a-bx^2}\,dx=\frac{\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}-\frac{x}{b}$$

input
```
int(x^2/(a - b*x^2),x)
```

output
```
(a^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/b^(3/2) - x/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{a-bx^2}\,dx=\frac{\sqrt{b}\,\sqrt{a}\,\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)-\sqrt{b}\,\sqrt{a}\,\log\left(\sqrt{b}\,\sqrt{a}-bx\right)-2bx}{2b^2}$$

input
```
int(x^2/(-b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x) - sqrt(b)*sqrt(a)*log(sqrt(
b)*sqrt(a) - b*x) - 2*b*x)/(2*b**2)
```

3.238 $\int \frac{x}{a-bx^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 16

$$\int \frac{x}{a-bx^2} \, dx = -\frac{\log(a-bx^2)}{2b}$$

output
```
-1/2*ln(-b*x^2+a)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{a-bx^2} \, dx = -\frac{\log(a-bx^2)}{2b}$$

input
```
Integrate[x/(a - b*x^2),x]
```

output
```
-1/2*Log[a - b*x^2]/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a-bx^2} \, dx$$

$$\Big\downarrow \; 240$$

$$-\frac{\log \left(a-bx^2\right)}{2b}$$

input
```
Int[x/(a - b*x^2),x]
```

output
```
-1/2*Log[a - b*x^2]/b
```

Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{\ln\left(-b\,x^{2}+a\right)}{2b}$	15
default	$-\frac{\ln\left(-b\,x^{2}+a\right)}{2b}$	15
norman	$-\frac{\ln\left(-b\,x^{2}+a\right)}{2b}$	15
risch	$-\frac{\ln\left(-b\,x^{2}+a\right)}{2b}$	15
parallelrisch	$-\frac{\ln\left(b\,x^{2}-a\right)}{2b}$	16

input
```
int(x/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-b*x^2+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a-bx^2}\,dx = -\frac{\log\left(bx^2-a\right)}{2\,b}$$

input
```
integrate(x/(-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*log(b*x^2 - a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{x}{a-bx^2}\,dx = -\frac{\log\left(-a+bx^2\right)}{2b}$$

input
```
integrate(x/(-b*x**2+a),x)
```

output
```
-log(-a + b*x**2)/(2*b)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a-bx^2}\,dx = -\frac{\log\left(bx^2-a\right)}{2\,b}$$

input
```
integrate(x/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 - a)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{a-bx^2}\,dx = -\frac{\log\left(|bx^2-a|\right)}{2\,b}$$

input
```
integrate(x/(-b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(b*x^2 - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{x}{a-bx^2} \, dx = -\frac{\ln\left(b\,x^2-a\right)}{2\,b}$$

input
```
int(x/(a - b*x^2),x)
```

output
```
-log(b*x^2 - a)/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{x}{a-bx^2} \, dx = \frac{-\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)}{2b}$$

input
```
int(x/(-b*x^2+a),x)
```

output
```
( - (log( - sqrt(b)*sqrt(a) - b*x) + log(sqrt(b)*sqrt(a) - b*x)))/(2*b)
```

3.239 $\int \frac{1}{a-bx^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{1}{a-bx^2} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{a-bx^2} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a-bx^2} \, dx$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

input
```
Int[(a - b*x^2)^(-1),x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$	16
risch	$\frac{\ln\left(bx+\sqrt{ab}\right)}{2\sqrt{ab}}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{2\sqrt{ab}}$	37

input
```
int(1/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{1}{a-bx^2}\,dx = \left[\frac{\sqrt{ab}\log\left(\frac{bx^2+2\sqrt{ab}x+a}{bx^2-a}\right)}{2\,ab}, -\frac{\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{ab}\right]$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a))/(a*b), -sqrt(-
a*b)*arctan(sqrt(-a*b)*x/a)/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(22) = 44$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\sqrt{\frac{1}{ab}}\log\left(-a\sqrt{\frac{1}{ab}}+x\right)}{2} + \frac{\sqrt{\frac{1}{ab}}\log\left(a\sqrt{\frac{1}{ab}}+x\right)}{2}$$

input
```
integrate(1/(-b*x**2+a),x)
```

output
```
-sqrt(1/(a*b))*log(-a*sqrt(1/(a*b)) + x)/2 + sqrt(1/(a*b))*log(a*sqrt(1/(a
*b)) + x)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{2\sqrt{ab}}$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/sqrt(a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{a-bx^2}\,dx = -\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}}$$

input
```
integrate(1/(-b*x^2+a),x, algorithm="giac")
```

output
```
-arctan(b*x/sqrt(-a*b))/sqrt(-a*b)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{a-bx^2}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2),x)
```

output
```
atanh((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{a-bx^2}\,dx=\frac{\sqrt{b}\,\sqrt{a}\left(\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)\right)}{2ab}$$

input
```
int(1/(-b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*(log( - sqrt(b)*sqrt(a) - b*x) - log(sqrt(b)*sqrt(a) - b*
x)))/(2*a*b)
```

3.240 $\int \frac{1}{x(a-bx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

output
```
ln(x)/a-1/2*ln(-b*x^2+a)/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx=\frac{\log(x)}{a}-\frac{\log\left(a-bx^2\right)}{2a}$$

input
```
Integrate[1/(x*(a - b*x^2)),x]
```

output
```
Log[x]/a - Log[a - b*x^2]/(2*a)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a-bx^2)}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2(a-bx^2)}dx^2$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{a-bx^2}dx^2}{a}+\frac{\int \frac{1}{x^2}dx^2}{a}\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(\frac{b\int \frac{1}{a-bx^2}dx^2}{a}+\frac{\log(x^2)}{a}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{\log(x^2)}{a}-\frac{\log(a-bx^2)}{a}\right)$$

input
```
Int[1/(x*(a - b*x^2)),x]
```

output
```
(Log[x^2]/a - Log[a - b*x^2]/a)/2
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^{2}+a)}{2a}$	22
norman	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^{2}+a)}{2a}$	22
risch	$\frac{\ln(x)}{a}-\frac{\ln(-b\,x^{2}+a)}{2a}$	22
parallelrisch	$\frac{2\ln(x)-\ln(b\,x^{2}-a)}{2a}$	23

input
```
int(1/x/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
ln(x)/a-1/2*ln(-b*x^2+a)/a
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx = -\frac{\log\left(bx^2-a\right)-2\,\log\left(x\right)}{2\,a}$$

input
```
integrate(1/x/(-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(log(b*x^2 - a) - 2*log(x))/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx = \frac{\log\left(x\right)}{a} - \frac{\log\left(-\frac{a}{b}+x^2\right)}{2a}$$

input
```
integrate(1/x/(-b*x**2+a),x)
```

output
```
log(x)/a - log(-a/b + x**2)/(2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a-bx^2\right)}\,dx = -\frac{\log\left(bx^2-a\right)}{2\,a} + \frac{\log\left(x^2\right)}{2\,a}$$

input
```
integrate(1/x/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*log(b*x^2 - a)/a + 1/2*log(x^2)/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{1}{x(a-bx^2)}\,dx = \frac{\log(x^2)}{2a} - \frac{\log(|bx^2-a|)}{2a}$$

input
```
integrate(1/x/(-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*log(x^2)/a - 1/2*log(abs(b*x^2 - a))/a
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{x(a-bx^2)}\,dx = \frac{\ln(x)}{a} - \frac{\ln(a-b\,x^2)}{2\,a}$$

input
```
int(1/(x*(a - b*x^2)),x)
```

output
```
log(x)/a - log(a - b*x^2)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int \frac{1}{x(a-bx^2)}\,dx = \frac{-\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)+2\log(x)}{2a}$$

input
```
int(1/x/(-b*x^2+a),x)
```

output
```
( - log( - sqrt(b)*sqrt(a) - b*x) - log(sqrt(b)*sqrt(a) - b*x) + 2*log(x))
/(2*a)
```

3.241 $\int \frac{1}{x^2(a-bx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \frac{1}{x^2(a-bx^2)}\,dx = -\frac{1}{ax} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-1/a/x+b^(1/2)*arctanh(b^(1/2)*x/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a-bx^2)}\,dx = -\frac{1}{ax} + \frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)),x]
```

output
```
-(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a - bx^2)} \, dx$$
$$\Big\downarrow \text{264}$$
$$\frac{b \int \frac{1}{a-bx^2} dx}{a} - \frac{1}{ax}$$
$$\Big\downarrow \text{221}$$
$$\frac{\sqrt{b} \text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{ax}$$

input
```
Int[1/(x^2*(a - b*x^2)),x]
```

output
```
-(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{b \operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{a\sqrt{ab}}-\frac{1}{ax}$	29
risch	$-\frac{1}{ax}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(a^{3}\_Z^{2}-b\right)}\_R\ln\left(\left(3\_R^{2}a^{3}-2b\right)x+a^{2}\_R\right)\right)}{2}$	50

input
```
int(1/x^2/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
b/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))-1/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.48

$$\int \frac{1}{x^2\left(a-bx^2\right)}\,dx=\left[\frac{x\sqrt{\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{\frac{b}{a}}+a}{bx^2-a}\right)-2}{2\,ax},-\frac{x\sqrt{-\frac{b}{a}}\arctan\left(x\sqrt{-\frac{b}{a}}\right)+1}{ax}\right]$$

input
```
integrate(1/x^2/(-b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) - 2)/(a*x
), -(x*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 1)/(a*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^2 (a - bx^2)}\, dx = -\frac{\sqrt{\frac{b}{a^3}} \log \left(-\frac{a^2\sqrt{\frac{b}{a^3}}}{b} + x\right)}{2} + \frac{\sqrt{\frac{b}{a^3}} \log \left(\frac{a^2\sqrt{\frac{b}{a^3}}}{b} + x\right)}{2} - \frac{1}{ax}$$

input
```
integrate(1/x**2/(-b*x**2+a),x)
```

output
```
-sqrt(b/a**3)*log(-a**2*sqrt(b/a**3)/b + x)/2 + sqrt(b/a**3)*log(a**2*sqrt
(b/a**3)/b + x)/2 - 1/(a*x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^2 (a - bx^2)}\, dx = -\frac{b \log \left(\frac{bx - \sqrt{ab}}{bx + \sqrt{ab}}\right)}{2\sqrt{ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a) - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2 (a - bx^2)}\, dx = -\frac{b \arctan \left(\frac{bx}{\sqrt{-ab}}\right)}{\sqrt{-ab}a} - \frac{1}{ax}$$

input
```
integrate(1/x^2/(-b*x^2+a),x, algorithm="giac")
```

output
```
-b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^2(a-bx^2)}\,dx = \frac{\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}} - \frac{1}{a\,x}$$

input
```
int(1/(x^2*(a - b*x^2)),x)
```

output
```
(b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/a^(3/2) - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2(a-bx^2)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)x - \sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}-bx\right)x - 2a}{2a^2x}$$

input
```
int(1/x^2/(-b*x^2+a),x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*x - sqrt(b)*sqrt(a)*log(sqr
t(b)*sqrt(a) - b*x)*x - 2*a)/(2*a**2*x)
```

3.242 $\int \frac{1}{x^3(a-bx^2)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{1}{x^3(a-bx^2)}\,dx = -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log(a-bx^2)}{2a^2}$$

output
```
-1/2/a/x^2+b*ln(x)/a^2-1/2*b*ln(-b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3(a-bx^2)}\,dx = -\frac{1}{2ax^2}+\frac{b\log(x)}{a^2}-\frac{b\log(a-bx^2)}{2a^2}$$

input
```
Integrate[1/(x^3*(a - b*x^2)),x]
```

output
```
-1/2*1/(a*x^2) + (b*Log[x])/a^2 - (b*Log[a - b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^2)} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{1}{x^4 (a-bx^2)} dx^2$$

$$\Big\downarrow \ 54$$

$$\frac{1}{2} \int \left(\frac{b^2}{a^2 (a-bx^2)} + \frac{b}{a^2 x^2} + \frac{1}{a x^4}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(\frac{b \log(x^2)}{a^2} - \frac{b \log(a-bx^2)}{a^2} - \frac{1}{a x^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^2)),x]
```

output
```
(-(1/(a*x^2)) + (b*Log[x^2])/a^2 - (b*Log[a - b*x^2])/a^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{1}{2a\,x^{2}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32
norman	$-\frac{1}{2a\,x^{2}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32
risch	$-\frac{1}{2a\,x^{2}}+\frac{b\ln\left(x\right)}{a^{2}}-\frac{b\ln\left(-b\,x^{2}+a\right)}{2a^{2}}$	32
parallelrisch	$\frac{2b\ln\left(x\right)x^{2}-b\ln\left(b\,x^{2}-a\right)x^{2}-a}{2a^{2}x^{2}}$	37

input
```
int(1/x^3/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/x^2+b*ln(x)/a^2-1/2*b*ln(-b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3\left(a-bx^2\right)}\,dx=-\frac{bx^2\log\left(bx^2-a\right)-2\,bx^2\log\left(x\right)+a}{2\,a^2x^2}$$

input
```
integrate(1/x^3/(-b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*log(b*x^2 - a) - 2*b*x^2*log(x) + a)/(a^2*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3 (a - bx^2)}\, dx = -\frac{1}{2ax^2} + \frac{b \log(x)}{a^2} - \frac{b \log\left(-\frac{a}{b} + x^2\right)}{2a^2}$$

input
```
integrate(1/x**3/(-b*x**2+a),x)
```

output
```
-1/(2*a*x**2) + b*log(x)/a**2 - b*log(-a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a - bx^2)}\, dx = -\frac{b \log(bx^2 - a)}{2\, a^2} + \frac{b \log(x^2)}{2\, a^2} - \frac{1}{2\, ax^2}$$

input
```
integrate(1/x^3/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*b*log(b*x^2 - a)/a^2 + 1/2*b*log(x^2)/a^2 - 1/2/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (a - bx^2)}\, dx = \frac{b \log(x^2)}{2\, a^2} - \frac{b \log(|bx^2 - a|)}{2\, a^2} - \frac{bx^2 + a}{2\, a^2 x^2}$$

input
```
integrate(1/x^3/(-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*b*log(x^2)/a^2 - 1/2*b*log(abs(b*x^2 - a))/a^2 - 1/2*(b*x^2 + a)/(a^2*
x^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^3 (a-bx^2)} \, dx = \frac{b \ln(x)}{a^2} - \frac{b \ln(a - b x^2)}{2 a^2} - \frac{1}{2 a x^2}$$

input
```
int(1/(x^3*(a - b*x^2)),x)
```

output
```
(b*log(x))/a^2 - (b*log(a - b*x^2))/(2*a^2) - 1/(2*a*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^3 (a-bx^2)} \, dx = \frac{-\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) b\, x^2 - \log\left(\sqrt{b}\,\sqrt{a} - bx\right) b\, x^2 + 2 \log(x)\, b\, x^2 - a}{2a^2x^2}$$

input
```
int(1/x^3/(-b*x^2+a),x)
```

output
```
( - log( - sqrt(b)*sqrt(a) - b*x)*b*x**2 - log(sqrt(b)*sqrt(a) - b*x)*b*x*
*2 + 2*log(x)*b*x**2 - a)/(2*a**2*x**2)
```

3.243 $\int \frac{x^3}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{x^3}{(a-bx^2)^2} dx = \frac{a}{2b^2(a-bx^2)} + \frac{\log(a-bx^2)}{2b^2}$$

output
```
1/2*a/b^2/(-b*x^2+a)+1/2*ln(-b*x^2+a)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(a-bx^2)^2} dx = \frac{\frac{a}{a-bx^2} + \log(a-bx^2)}{2b^2}$$

input
```
Integrate[x^3/(a - b*x^2)^2,x]
```

output
```
(a/(a - b*x^2) + Log[a - b*x^2])/(2*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{x^2}{(a-bx^2)^2} dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{2} \int \left(\frac{a}{b(bx^2-a)^2}+\frac{1}{b(bx^2-a)}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{a}{b^2(a-bx^2)}+\frac{\log(a-bx^2)}{b^2}\right)$$

input
```
Int[x^3/(a - b*x^2)^2,x]
```

output
```
(a/(b^2*(a - b*x^2)) + Log[a - b*x^2]/b^2)/2
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{a}{2b^{2}\left(-b\,x^{2}+a\right)}+\frac{\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	32
norman	$\frac{a}{2b^{2}\left(-b\,x^{2}+a\right)}+\frac{\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	32
risch	$\frac{a}{2b^{2}\left(-b\,x^{2}+a\right)}+\frac{\ln\left(-b\,x^{2}+a\right)}{2b^{2}}$	32
parallelrisch	$\frac{b\ln\left(b\,x^{2}-a\right)x^{2}-a\ln\left(b\,x^{2}-a\right)-a}{2b^{2}\left(b\,x^{2}-a\right)}$	49

input
```
int(x^3/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/b^2/(-b*x^2+a)+1/2*ln(-b*x^2+a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = \frac{(bx^2-a)\log\left(bx^2-a\right)-a}{2\left(b^3x^2-ab^2\right)}$$

input
```
integrate(x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/2*((b*x^2 - a)*log(b*x^2 - a) - a)/(b^3*x^2 - a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = -\frac{a}{-2ab^2+2b^3x^2}+\frac{\log(-a+bx^2)}{2b^2}$$

input
```
integrate(x**3/(-b*x**2+a)**2,x)
```

output
```
-a/(-2*a*b**2 + 2*b**3*x**2) + log(-a + b*x**2)/(2*b**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = -\frac{a}{2\,(b^3x^2-ab^2)}+\frac{\log(bx^2-a)}{2\,b^2}$$

input
```
integrate(x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*a/(b^3*x^2 - a*b^2) + 1/2*log(b*x^2 - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = -\frac{\frac{\log\left(\frac{|bx^2-a|}{(bx^2-a)^2|b|}\right)}{b}+\frac{a}{(bx^2-a)b}}{2\,b}$$

input
```
integrate(x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(log(abs(b*x^2 - a)/((b*x^2 - a)^2*abs(b)))/b + a/((b*x^2 - a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = \frac{\ln(b\,x^2-a)}{2\,b^2}+\frac{a}{2\,b^2\,(a-b\,x^2)}$$

input
```
int(x^3/(a - b*x^2)^2,x)
```

output
```
log(b*x^2 - a)/(2*b^2) + a/(2*b^2*(a - b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.37

$$\int \frac{x^3}{(a-bx^2)^2}\,dx = \frac{\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a-\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2+\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2+b\,x^2}{2b^2\,(-b\,x^2+a)}$$

input
```
int(x^3/(-b*x^2+a)^2,x)
```

output
```
(log( - sqrt(b)*sqrt(a) - b*x)*a - log( - sqrt(b)*sqrt(a) - b*x)*b*x**2 +
log(sqrt(b)*sqrt(a) - b*x)*a - log(sqrt(b)*sqrt(a) - b*x)*b*x**2 + b*x**2)
/(2*b**2*(a - b*x**2))
```

3.244 $\int \frac{x^2}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{x^2}{(a-bx^2)^2} dx = \frac{x}{2b(a-bx^2)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

output
```
1/2*x/b/(-b*x^2+a)-1/2*arctanh(b^(1/2)*x/a^(1/2))/a^(1/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{x^2}{(a-bx^2)^2} dx = -\frac{x}{2b(-a+bx^2)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

input
```
Integrate[x^2/(a - b*x^2)^2,x]
```

output
```
-1/2*x/(b*(-a + b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \ 252$$

$$\frac{x}{2b(a-bx^2)} - \frac{\int \frac{1}{a-bx^2}dx}{2b}$$

$$\Big\downarrow \ 221$$

$$\frac{x}{2b(a-bx^2)} - \frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{3/2}}$$

input
```
Int[x^2/(a - b*x^2)^2,x]
```

output
```
x/(2*b*(a - b*x^2)) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*Sqrt[a]*b^(3/2))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2b(-b\,x^{2}+a)}-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}$	37
risch	$\frac{x}{2b(-b\,x^{2}+a)}+\frac{\ln\left(bx-\sqrt{ab}\right)}{4\sqrt{ab}\,b}-\frac{\ln\left(-bx-\sqrt{ab}\right)}{4\sqrt{ab}\,b}$	63

input
```
int(x^2/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/b/(-b*x^2+a)-1/2/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.76

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = \left[-\frac{2\,abx-(bx^2-a)\sqrt{ab}\log\left(\frac{bx^2-2\sqrt{ab}x+a}{bx^2-a}\right)}{4\,(ab^3x^2-a^2b^2)},\right.$$
$$\left.-\frac{abx-(bx^2-a)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{2\,(ab^3x^2-a^2b^2)}\right]$$

input
```
integrate(x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 - 2*sqrt(a*b)*x + a)/(b*
x^2 - a)))/(a*b^3*x^2 - a^2*b^2), -1/2*(a*b*x - (b*x^2 - a)*sqrt(-a*b)*arc
tan(sqrt(-a*b)*x/a))/(a*b^3*x^2 - a^2*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = -\frac{x}{-2ab+2b^2x^2} + \frac{\sqrt{\frac{1}{ab^3}}\log\left(-ab\sqrt{\frac{1}{ab^3}}+x\right)}{4} - \frac{\sqrt{\frac{1}{ab^3}}\log\left(ab\sqrt{\frac{1}{ab^3}}+x\right)}{4}$$

input
```
integrate(x**2/(-b*x**2+a)**2,x)
```

output
```
-x/(-2*a*b + 2*b**2*x**2) + sqrt(1/(a*b**3))*log(-a*b*sqrt(1/(a*b**3)) + x
)/4 - sqrt(1/(a*b**3))*log(a*b*sqrt(1/(a*b**3)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = -\frac{x}{2\,(b^2x^2-ab)} + \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}b}$$

input
```
integrate(x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(b^2*x^2 - a*b) + 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqr
t(a*b)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}b} - \frac{x}{2\,(bx^2-a)b}$$

input
```
integrate(x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*b) - 1/2*x/((b*x^2 - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = \frac{x}{2\,b\,(a-b\,x^2)} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int(x^2/(a - b*x^2)^2,x)
```

output
```
x/(2*b*(a - b*x^2)) - atanh((b^(1/2)*x)/a^(1/2))/(2*a^(1/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \frac{x^2}{(a-bx^2)^2}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a+\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2+\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a-\sqrt{b}\,\sqrt{a}}{4a\,b^2\left(-b\,x^2+a\right)}$$

input
```
int(x^2/(-b*x^2+a)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a + sqrt(b)*sqrt(a)*log(
 - sqrt(b)*sqrt(a) - b*x)*b*x**2 + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b
*x)*a - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*b*x**2 + 2*a*b*x)/(4*a*
b**2*(a - b*x**2))
```

3.245 $\int \frac{x}{(a-bx^2)^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(a-bx^2)^2} \, dx = \frac{1}{2b(a-bx^2)}$$

output
```
1/2/b/(-b*x^2+a)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a-bx^2)^2} \, dx = \frac{1}{2b(a-bx^2)}$$

input
```
Integrate[x/(a - b*x^2)^2,x]
```

output
```
1/(2*b*(a - b*x^2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{2b\left(a-bx^2\right)}$$

input
```
Int[x/(a - b*x^2)^2,x]
```

output
```
1/(2*b*(a - b*x^2))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{1}{2b(-b\,x^2+a)}$	16
derivativedivides	$\frac{1}{2b(-b\,x^2+a)}$	16
default	$\frac{1}{2b(-b\,x^2+a)}$	16
norman	$\frac{1}{2b(-b\,x^2+a)}$	16
risch	$\frac{1}{2b(-b\,x^2+a)}$	16
orering	$\frac{1}{2b(-b\,x^2+a)}$	16
parallelrisch	$-\frac{1}{2b(b\,x^2-a)}$	17

input
```
int(x/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/b/(-b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^2}\,dx = -\frac{1}{2\,(b^2x^2-ab)}$$

input
```
integrate(x/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2/(b^2*x^2 - a*b)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a-bx^2)^2}\,dx = -\frac{1}{-2ab+2b^2x^2}$$

input
```
integrate(x/(-b*x**2+a)**2,x)
```

output
```
-1/(-2*a*b + 2*b**2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^2}\,dx = -\frac{1}{2\,(bx^2-a)b}$$

input
```
integrate(x/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2/((b*x^2 - a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^2}\,dx = -\frac{1}{2\,(bx^2-a)b}$$

input
```
integrate(x/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2/((b*x^2 - a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a-bx^2)^2}\,dx = \frac{1}{2\,b\,(a-b\,x^2)}$$

input
```
int(x/(a - b*x^2)^2,x)
```

output
```
1/(2*b*(a - b*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \frac{x}{(a-bx^2)^2}\,dx = \frac{x^2}{2a\,(-b\,x^2+a)}$$

input
```
int(x/(-b*x^2+a)^2,x)
```

output
```
x**2/(2*a*(a - b*x**2))
```

3.246 $\int \frac{1}{(a-bx^2)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{(a-bx^2)^2} dx = \frac{x}{2a(a-bx^2)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

output
```
1/2*x/a/(-b*x^2+a)+1/2*arctanh(b^(1/2)*x/a^(1/2))/a^(3/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a-bx^2)^2} dx = -\frac{x}{2a(-a+bx^2)} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-2),x]
```

output
```
-1/2*x/(a*(-a + b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^2}dx$$
$$\Big\downarrow \ 215$$
$$\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}$$
$$\Big\downarrow \ 221$$
$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}$$

input
```
Int[(a - b*x^2)^(-2),x]
```

output
```
x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x}{2a(-b\,x^2+a)}+\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$	37
risch	$\frac{x}{2a(-b\,x^2+a)}+\frac{\ln\left(bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}-\frac{\ln\left(-bx+\sqrt{ab}\right)}{4\sqrt{ab}\,a}$	59

input
```
int(1/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.74

$$\int \frac{1}{(a-bx^2)^2}\,dx = \left[-\frac{2\,abx-(bx^2-a)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{4\,(a^2b^2x^2-a^3b)}, -\frac{abx+(bx^2-a)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{2\,(a^2b^2x^2-a^3b)}\right]$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*b*x - (b*x^2 - a)*sqrt(a*b)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*
x^2 - a)))/(a^2*b^2*x^2 - a^3*b), -1/2*(a*b*x + (b*x^2 - a)*sqrt(-a*b)*arc
tan(sqrt(-a*b)*x/a))/(a^2*b^2*x^2 - a^3*b)]
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{x}{-2a^2+2abx^2} - \frac{\sqrt{\frac{1}{a^3b}}\log\left(-a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4} + \frac{\sqrt{\frac{1}{a^3b}}\log\left(a^2\sqrt{\frac{1}{a^3b}}+x\right)}{4}$$

input
```
integrate(1/(-b*x**2+a)**2,x)
```

output
```
-x/(-2*a**2 + 2*a*b*x**2) - sqrt(1/(a**3*b))*log(-a**2*sqrt(1/(a**3*b)) +
x)/4 + sqrt(1/(a**3*b))*log(a**2*sqrt(1/(a**3*b)) + x)/4
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{x}{2(abx^2-a^2)} - \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{4\sqrt{ab}a}$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x/(a*b*x^2 - a^2) - 1/4*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqr
t(a*b)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a-bx^2)^2}\,dx = -\frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\sqrt{-ab}a} - \frac{x}{2(bx^2-a)a}$$

input
```
integrate(1/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a) - 1/2*x/((b*x^2 - a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{\left(a-bx^2\right)^2}\,dx=\frac{x}{2\,a\,\left(a-b\,x^2\right)}+\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^2,x)
```

output
```
x/(2*a*(a - b*x^2)) + atanh((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \frac{1}{\left(a-bx^2\right)^2}\,dx$$
$$=\frac{\sqrt{b}\,\sqrt{a}\,\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a-\sqrt{b}\,\sqrt{a}\,\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2-\sqrt{b}\,\sqrt{a}\,\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a+\sqrt{b}\,\sqrt{a}\,\mathrm{lo}}{4a^2b\left(-b\,x^2+a\right)}$$

input
```
int(1/(-b*x^2+a)^2,x)
```

output
```
(sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a - sqrt(b)*sqrt(a)*log( -
sqrt(b)*sqrt(a) - b*x)*b*x**2 - sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)
*a + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*b*x**2 + 2*a*b*x)/(4*a**2*
b*(a - b*x**2))
```

3.247 $\int \frac{1}{x(a-bx^2)^2} dx$

Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{1}{x(a-bx^2)^2} dx = \frac{1}{2a(a-bx^2)} + \frac{\log(x)}{a^2} - \frac{\log(a-bx^2)}{2a^2}$$

output
```
1/2/a/(-b*x^2+a)+ln(x)/a^2-1/2*ln(-b*x^2+a)/a^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{x(a-bx^2)^2} dx = \frac{\frac{a}{a-bx^2} + 2\log(x) - \log(a-bx^2)}{2a^2}$$

input
```
Integrate[1/(x*(a - b*x^2)^2),x]
```

output
```
(a/(a - b*x^2) + 2*Log[x] - Log[a - b*x^2])/(2*a^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a-bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(a-bx^2\right)^2}dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2}\int \left(\frac{b}{a^2\left(a-bx^2\right)}+\frac{b}{a\left(a-bx^2\right)^2}+\frac{1}{a^2x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{\log\left(a-bx^2\right)}{a^2}+\frac{\log\left(x^2\right)}{a^2}+\frac{1}{a\left(a-bx^2\right)}\right)$$

input
```
Int[1/(x*(a - b*x^2)^2),x]
```

output
```
(1/(a*(a - b*x^2)) + Log[x^2]/a^2 - Log[a - b*x^2]/a^2)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{1}{2a(-b\,x^{2}+a)}+\frac{\ln(x)}{a^{2}}-\frac{\ln(-b\,x^{2}+a)}{2a^{2}}$	37
norman	$\frac{b\,x^{2}}{2a^{2}(-b\,x^{2}+a)}+\frac{\ln(x)}{a^{2}}-\frac{\ln(-b\,x^{2}+a)}{2a^{2}}$	41
default	$\frac{b\left(-\frac{\ln\left(-b\,x^{2}+a\right)}{b}+\frac{a}{b\left(-b\,x^{2}+a\right)}\right)}{2a^{2}}+\frac{\ln(x)}{a^{2}}$	44
parallelrisch	$\frac{2b\ln(x)x^{2}-b\ln(b\,x^{2}-a)\,x^{2}-b\,x^{2}-2a\ln(x)+a\ln(b\,x^{2}-a)}{2a^{2}(b\,x^{2}-a)}$	65

input
```
int(1/x/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a/(-b*x^2+a)+ln(x)/a^2-1/2*ln(-b*x^2+a)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx=-\frac{(bx^2-a)\log\left(bx^2-a\right)-2\left(bx^2-a\right)\log\left(x\right)+a}{2\left(a^2bx^2-a^3\right)}$$

input
```
integrate(1/x/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*x^2 - a)*log(b*x^2 - a) - 2*(b*x^2 - a)*log(x) + a)/(a^2*b*x^2 -
a^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx=-\frac{1}{-2a^2+2abx^2}+\frac{\log\left(x\right)}{a^2}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^2}$$

input
```
integrate(1/x/(-b*x**2+a)**2,x)
```

output
```
-1/(-2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(-a/b + x**2)/(2*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx=-\frac{1}{2\left(abx^2-a^2\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^2}+\frac{\log\left(x^2\right)}{2\,a^2}$$

input
```
integrate(1/x/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2/(a*b*x^2 - a^2) - 1/2*log(b*x^2 - a)/a^2 + 1/2*log(x^2)/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx=\frac{\log\left(x^2\right)}{2\,a^2}-\frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^2}+\frac{bx^2-2\,a}{2\left(bx^2-a\right)a^2}$$

input
```
integrate(1/x/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 - a))/a^2 + 1/2*(b*x^2 - 2*a)/((b*x^2
 - a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx=\frac{\ln\left(x\right)}{a^2}+\frac{1}{2\,a\,\left(a-b\,x^2\right)}-\frac{\ln\left(a-b\,x^2\right)}{2\,a^2}$$

input
```
int(1/(x*(a - b*x^2)^2),x)
```

output
```
log(x)/a^2 + 1/(2*a*(a - b*x^2)) - log(a - b*x^2)/(2*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.40

$$\int \frac{1}{x\left(a-bx^2\right)^2}\,dx$$
$$=\frac{-\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a+\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2-\log\left(\sqrt{b}\,\sqrt{a}-bx\right)a+\log\left(\sqrt{b}\,\sqrt{a}-bx\right)b\,x^2+2\,\mathrm{lo}}{2a^2\left(-b\,x^2+a\right)}$$

input
```
int(1/x/(-b*x^2+a)^2,x)
```

output
```
( - log( - sqrt(b)*sqrt(a) - b*x)*a + log( - sqrt(b)*sqrt(a) - b*x)*b*x**2
 - log(sqrt(b)*sqrt(a) - b*x)*a + log(sqrt(b)*sqrt(a) - b*x)*b*x**2 + 2*lo
g(x)*a - 2*log(x)*b*x**2 + b*x**2)/(2*a**2*(a - b*x**2))
```

3.248 $\int \frac{1}{x^2(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{1}{x^2(a-bx^2)^2}\,dx = -\frac{1}{a^2x}+\frac{bx}{2a^2(a-bx^2)}+\frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-1/a^2/x+1/2*b*x/a^2/(-b*x^2+a)+3/2*b^(1/2)*arctanh(b^(1/2)*x/a^(1/2))/a^(
5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^2(a-bx^2)^2}\,dx = -\frac{1}{a^2x}-\frac{bx}{2a^2(-a+bx^2)}+\frac{3\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^2),x]
```

output
```
-(1/(a^2*x)) - (b*x)/(2*a^2*(-a + b*x^2)) + (3*Sqrt[b]*ArcTanh[(Sqrt[b]*x)
/Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^2} \, dx$$
$$\Big\downarrow \text{253}$$
$$\frac{3\int \frac{1}{x^2(a-bx^2)}dx}{2a}+\frac{1}{2ax(a-bx^2)}$$
$$\Big\downarrow \text{264}$$
$$\frac{3\left(\frac{b\int \frac{1}{a-bx^2}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a-bx^2)}$$
$$\Big\downarrow \text{221}$$
$$\frac{3\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a-bx^2)}$$

input
```
Int[1/(x^2*(a - b*x^2)^2),x]
```

output
```
1/(2*a*x*(a - b*x^2)) + (3*(-(1/(a*x)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt
[a]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{b\left(\frac{x}{-2b\,x^{2}+2a}+\frac{3\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^{2}}-\frac{1}{a^{2}x}$	45
risch	$\frac{\frac{3b\,x^{2}}{2a^{2}}-\frac{1}{a}}{x\left(-b\,x^{2}+a\right)}+\frac{3\sqrt{ab}\,\ln\left(bx+\sqrt{ab}\right)}{4a^{3}}-\frac{3\sqrt{ab}\,\ln\left(bx-\sqrt{ab}\right)}{4a^{3}}$	73

input
```
int(1/x^2/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
b/a^2*(1/2*x/(-b*x^2+a)+3/2/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))-1/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.55

$$\int \frac{1}{x^2 (a - bx^2)^2} \, dx = \left[-\frac{6 \, bx^2 - 3 \, (bx^3 - ax)\sqrt{\frac{b}{a}} \log \left(\frac{bx^2 + 2 \, ax\sqrt{\frac{b}{a}} + a}{bx^2 - a} \right) - 4 \, a}{4 \, (a^2 bx^3 - a^3 x)}, -\frac{3 \, bx^2 + 3 \, (bx^3 - ax)\sqrt{-\frac{b}{a}} \arctan \left(x\sqrt{-\frac{b}{a}} \right) - 2 \, a}{2 \, (a^2 bx^3 - a^3 x)} \right]$$

input
```
integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(6*b*x^2 - 3*(b*x^3 - a*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) +
a)/(b*x^2 - a)) - 4*a)/(a^2*b*x^3 - a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 - a*x
)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) - 2*a)/(a^2*b*x^3 - a^3*x)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{1}{x^2 (a - bx^2)^2} \, dx = -\frac{3\sqrt{\frac{b}{a^5}} \log \left(-\frac{a^3\sqrt{\frac{b}{a^5}}}{b} + x \right)}{4} + \frac{3\sqrt{\frac{b}{a^5}} \log \left(\frac{a^3\sqrt{\frac{b}{a^5}}}{b} + x \right)}{4} + \frac{2a - 3bx^2}{-2a^3x + 2a^2bx^3}$$

input
```
integrate(1/x**2/(-b*x**2+a)**2,x)
```

output
```
-3*sqrt(b/a**5)*log(-a**3*sqrt(b/a**5)/b + x)/4 + 3*sqrt(b/a**5)*log(a**3*
sqrt(b/a**5)/b + x)/4 + (2*a - 3*b*x**2)/(-2*a**3*x + 2*a**2*b*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^2 \left(a - bx^2\right)^2} \, dx = -\frac{3\,bx^2 - 2\,a}{2\,\left(a^2bx^3 - a^3x\right)} - \frac{3\,b\log\left(\frac{bx - \sqrt{ab}}{bx + \sqrt{ab}}\right)}{4\,\sqrt{ab}a^2}$$

input
```
integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*b*x^2 - 2*a)/(a^2*b*x^3 - a^3*x) - 3/4*b*log((b*x - sqrt(a*b))/(b*
x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^2 \left(a - bx^2\right)^2} \, dx = -\frac{3\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{2\,\sqrt{-ab}a^2} - \frac{3\,bx^2 - 2\,a}{2\,(bx^3 - ax)a^2}$$

input
```
integrate(1/x^2/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
-3/2*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/2*(3*b*x^2 - 2*a)/((b*x
^3 - a*x)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 \left(a - bx^2\right)^2} \, dx = \frac{3\,\sqrt{b}\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{\frac{1}{a} - \frac{3\,b\,x^2}{2\,a^2}}{a\,x - b\,x^3}$$

input
```
int(1/(x^2*(a - b*x^2)^2),x)
```

output
```
(3*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) - (1/a - (3*b*x^2)/(2*a
^2))/(a*x - b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

$$\int \frac{1}{x^2 (a - bx^2)^2} \, dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) ax - 3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) b\,x^3 - 3\sqrt{b}\,\sqrt{a}\log\left(\sqrt{b}\,\sqrt{a} - bx\right) ax + 3\sqrt{}}{4a^3 x\,(-b\,x^2 + a)}$$

input
```
int(1/x^2/(-b*x^2+a)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a*x - 3*sqrt(b)*sqrt(a)*l
og( - sqrt(b)*sqrt(a) - b*x)*b*x**3 - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a
) - b*x)*a*x + 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*b*x**3 - 4*a**
2 + 6*a*b*x**2)/(4*a**3*x*(a - b*x**2))
```

3.249 $\int \frac{1}{x^3(a-bx^2)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 52

$$\int \frac{1}{x^3(a-bx^2)^2}\,dx = -\frac{1}{2a^2x^2}+\frac{b}{2a^2(a-bx^2)}+\frac{2b\log(x)}{a^3}-\frac{b\log(a-bx^2)}{a^3}$$

output
```
-1/2/a^2/x^2+1/2*b/a^2/(-b*x^2+a)+2*b*ln(x)/a^3-b*ln(-b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3(a-bx^2)^2}\,dx = \frac{-\frac{a}{x^2}+\frac{ab}{a-bx^2}+4b\log(x)-2b\log(a-bx^2)}{2a^3}$$

input
```
Integrate[1/(x^3*(a - b*x^2)^2),x]
```

output
```
(-(a/x^2) + (a*b)/(a - b*x^2) + 4*b*Log[x] - 2*b*Log[a - b*x^2])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^2)^2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (a-bx^2)^2} dx^2$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{2} \int \left(\frac{2b^2}{a^3 (a-bx^2)} + \frac{b^2}{a^2 (a-bx^2)^2} + \frac{2b}{a^3 x^2} + \frac{1}{a^2 x^4}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{2b \log (x^2)}{a^3} - \frac{2b \log (a-bx^2)}{a^3} + \frac{b}{a^2 (a-bx^2)} - \frac{1}{a^2 x^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^2)^2),x]
```

output
```
(-(1/(a^2*x^2)) + b/(a^2*(a - b*x^2)) + (2*b*Log[x^2])/a^3 - (2*b*Log[a -
b*x^2])/a^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

method	result	size
risch	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{1}{2 a}}{x^{2} \left(-b\,x^{2}+a \right)}+\frac{2 b \ln \left(x \right)}{a^{3}}-\frac{b \ln \left(-b\,x^{2}+a \right)}{a^{3}}$	53
norman	$\frac{-\frac{1}{2 a}+\frac{b^{2} x^{4}}{a^{3}}}{x^{2} \left(-b\,x^{2}+a \right)}+\frac{2 b \ln \left(x \right)}{a^{3}}-\frac{b \ln \left(-b\,x^{2}+a \right)}{a^{3}}$	55
default	$\frac{b^{2} \left(-\frac{2 \ln \left(-b\,x^{2}+a \right)}{b}+\frac{a}{b \left(-b\,x^{2}+a \right)}\right)}{2 a^{3}}-\frac{1}{2 a^{2} x^{2}}+\frac{2 b \ln \left(x \right)}{a^{3}}$	56
parallelrisch	$\frac{4 b^{2} \ln \left(x \right) x^{4}-2 \ln \left(b\,x^{2}-a \right) x^{4} b^{2}-2 b^{2} x^{4}-4 a b \ln \left(x \right) x^{2}+2 \ln \left(b\,x^{2}-a \right) x^{2} a b +a^{2}}{2 a^{3} x^{2} \left(b\,x^{2}-a \right)}$	86

input
```
int(1/x^3/(-b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(b/a^2*x^2-1/2/a)/x^2/(-b*x^2+a)+2*b*ln(x)/a^3-b*ln(-b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3 \left(a-bx^2\right)^2}\,dx = -\frac{2\,abx^2-a^2+2\,(b^2x^4-abx^2)\log\left(bx^2-a\right)-4\,(b^2x^4-abx^2)\log\left(x\right)}{2\,(a^3bx^4-a^4x^2)}$$

input
```
integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*x^2 - a^2 + 2*(b^2*x^4 - a*b*x^2)*log(b*x^2 - a) - 4*(b^2*x^4
- a*b*x^2)*log(x))/(a^3*b*x^4 - a^4*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^3 (a-bx^2)^2} \, dx = \frac{a-2bx^2}{-2a^3x^2+2a^2bx^4} + \frac{2b\log(x)}{a^3} - \frac{b\log\left(-\frac{a}{b}+x^2\right)}{a^3}$$

input
```
integrate(1/x**3/(-b*x**2+a)**2,x)
```

output
```
(a - 2*b*x**2)/(-2*a**3*x**2 + 2*a**2*b*x**4) + 2*b*log(x)/a**3 - b*log(-a
/b + x**2)/a**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3 (a-bx^2)^2} \, dx = -\frac{2\,bx^2-a}{2\,(a^2bx^4-a^3x^2)} - \frac{b\log(bx^2-a)}{a^3} + \frac{b\log(x^2)}{a^3}$$

input
```
integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*b*x^2 - a)/(a^2*b*x^4 - a^3*x^2) - b*log(b*x^2 - a)/a^3 + b*log(x^
2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (a-bx^2)^2}\,dx = \frac{b\log(x^2)}{a^3} - \frac{b\log(|bx^2-a|)}{a^3} - \frac{2\,bx^2-a}{2\,(bx^4-ax^2)a^2}$$

input
```
integrate(1/x^3/(-b*x^2+a)^2,x, algorithm="giac")
```

output
```
b*log(x^2)/a^3 - b*log(abs(b*x^2 - a))/a^3 - 1/2*(2*b*x^2 - a)/((b*x^4 - a
*x^2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a-bx^2)^2}\,dx = \frac{2\,b\,\ln(x)}{a^3} - \frac{b\,\ln(a-b\,x^2)}{a^3} - \frac{\frac{1}{2\,a}-\frac{b\,x^2}{a^2}}{a\,x^2-b\,x^4}$$

input
```
int(1/(x^3*(a - b*x^2)^2),x)
```

output
```
(2*b*log(x))/a^3 - (b*log(a - b*x^2))/a^3 - (1/(2*a) - (b*x^2)/a^2)/(a*x^2
 - b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 (a-bx^2)^2}\,dx = \frac{-2\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2+2\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^2x^4-2\log\left(\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2+2\log\left(\sqrt{b}\,\sqrt{a}-\right.}{2a^3x^2\left(-b\,x^2+a\right)}$$

input
```
int(1/x^3/(-b*x^2+a)^2,x)
```

output

```
( - 2*log( - sqrt(b)*sqrt(a) - b*x)*a*b*x**2 + 2*log( - sqrt(b)*sqrt(a) -
b*x)*b**2*x**4 - 2*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**2 + 2*log(sqrt(b)*sqr
t(a) - b*x)*b**2*x**4 + 4*log(x)*a*b*x**2 - 4*log(x)*b**2*x**4 - a**2 + 2*
b**2*x**4)/(2*a**3*x**2*(a - b*x**2))
```

3.250 $\int \frac{x^3}{(a-bx^2)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 20

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = \frac{x^4}{4a\,(a-bx^2)^2}$$

output
```
1/4*x^4/a/(-b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = -\frac{a-2bx^2}{4b^2\,(a-bx^2)^2}$$

input
```
Integrate[x^3/(a - b*x^2)^3,x]
```

output
```
-1/4*(a - 2*b*x^2)/(b^2*(a - b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a-bx^2)^3}\,dx$$
$$\Big\downarrow \; 242$$
$$\frac{x^4}{4a\,(a-bx^2)^2}$$

input
```
Int[x^3/(a - b*x^2)^3,x]
```

output
```
x^4/(4*a*(a - b*x^2)^2)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

method	result	size
gosper	$-\frac{-2b\,x^{2}+a}{4\left(-b\,x^{2}+a\right)^{2} b^{2}}$	24
orering	$-\frac{-2b\,x^{2}+a}{4\left(-b\,x^{2}+a\right)^{2} b^{2}}$	24
norman	$\frac{\frac{x^{2}}{2b}-\frac{a}{4b^{2}}}{\left(-b\,x^{2}+a\right)^{2}}$	27
risch	$\frac{\frac{x^{2}}{2b}-\frac{a}{4b^{2}}}{\left(-b\,x^{2}+a\right)^{2}}$	27
parallelrisch	$\frac{2b\,x^{2}-a}{4b^{2}\left(b\,x^{2}-a\right)^{2}}$	27
default	$\frac{a}{4b^{2}\left(-b\,x^{2}+a\right)^{2}}-\frac{1}{2b^{2}\left(-b\,x^{2}+a\right)}$	33

input
```
int(x^3/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*x^2+a)/(-b*x^2+a)^2/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = \frac{2\,bx^2-a}{4\,(b^4x^4-2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = -\frac{a-2bx^2}{4a^2b^2-8ab^3x^2+4b^4x^4}$$

input
```
integrate(x**3/(-b*x**2+a)**3,x)
```

output
```
-(a - 2*b*x**2)/(4*a**2*b**2 - 8*a*b**3*x**2 + 4*b**4*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = \frac{2\,bx^2-a}{4\,(b^4x^4-2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4*(2*b*x^2 - a)/(b^4*x^4 - 2*a*b^3*x^2 + a^2*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = \frac{2\,bx^2-a}{4\,(bx^2-a)^2b^2}$$

input
```
integrate(x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/4*(2*b*x^2 - a)/((b*x^2 - a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = -\frac{\frac{a}{4\,b^2}-\frac{x^2}{2\,b}}{a^2-2\,a\,b\,x^2+b^2\,x^4}$$

input
```
int(x^3/(a - b*x^2)^3,x)
```

output
```
-(a/(4*b^2) - x^2/(2*b))/(a^2 + b^2*x^4 - 2*a*b*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.40

$$\int \frac{x^3}{(a-bx^2)^3}\,dx = \frac{x^4}{4a\,(b^2x^4-2ab\,x^2+a^2)}$$

input
```
int(x^3/(-b*x^2+a)^3,x)
```

output
```
x**4/(4*a*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.251 $\int \frac{x^2}{(a-bx^2)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{x^2}{(a-bx^2)^3} \, dx = \frac{x}{4b(a-bx^2)^2} - \frac{x}{8ab(a-bx^2)} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

output
```
1/4*x/b/(-b*x^2+a)^2-1/8*x/a/b/(-b*x^2+a)-1/8*arctanh(b^(1/2)*x/a^(1/2))/a
^(3/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(a-bx^2)^3} \, dx = \frac{x(a+bx^2)}{8ab(a-bx^2)^2} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{3/2}}$$

input
```
Integrate[x^2/(a - b*x^2)^3,x]
```

output
```
(x*(a + b*x^2))/(8*a*b*(a - b*x^2)^2) - ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(8*a^
(3/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {252, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^3}dx$$

$$\Big\downarrow \ 252$$

$$\frac{x}{4b(a-bx^2)^2}-\frac{\int \frac{1}{(a-bx^2)^2}dx}{4b}$$

$$\Big\downarrow \ 215$$

$$\frac{x}{4b(a-bx^2)^2}-\frac{\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}}{4b}$$

$$\Big\downarrow \ 221$$

$$\frac{x}{4b(a-bx^2)^2}-\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}}{4b}$$

input
```
Int[x^2/(a - b*x^2)^3,x]
```

output
```
x/(4*b*(a - b*x^2)^2) - (x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]
]/(2*a^(3/2)*Sqrt[b]))/(4*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\frac{x^3}{8a}+\frac{x}{8b}}{\left(-b\,x^2+a\right)^2}-\frac{\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8ab\sqrt{ab}}$	50
risch	$\frac{\frac{x^3}{8a}+\frac{x}{8b}}{\left(-b\,x^2+a\right)^2}+\frac{\ln\left(bx-\sqrt{ab}\right)}{16\sqrt{ab}\,ba}-\frac{\ln\left(-bx-\sqrt{ab}\right)}{16\sqrt{ab}\,ba}$	79

input
```
int(x^2/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8/a*x^3+1/8*x/b)/(-b*x^2+a)^2-1/8/a/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.81

$$\int \frac{x^2}{(a-bx^2)^3}\,dx = \left[\frac{2\,ab^2x^3+2\,a^2bx+(b^2x^4-2\,abx^2+a^2)\sqrt{ab}\log\left(\frac{bx^2-2\,\sqrt{ab}x+a}{bx^2-a}\right)}{16\,(a^2b^4x^4-2\,a^3b^3x^2+a^4b^2)},\ \frac{ab^2x^3+a^2bx+(b^2x^4-2\,abx^2+a^2)\sqrt{\cdot}}{8\,(a^2b^4x^4-2\,a^3b^3x^2+\cdot}\right.$$

input
```
integrate(x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(2*a*b^2*x^3 + 2*a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)*log
((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 +
a^4*b^2), 1/8*(a*b^2*x^3 + a^2*b*x + (b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(-a*b
)*arctan(sqrt(-a*b)*x/a))/(a^2*b^4*x^4 - 2*a^3*b^3*x^2 + a^4*b^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(51) = 102.

Time = 0.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.57

$$\int \frac{x^2}{(a-bx^2)^3}\,dx = \frac{\sqrt{\frac{1}{a^3b^3}}\log\left(-a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16} - \frac{\sqrt{\frac{1}{a^3b^3}}\log\left(a^2b\sqrt{\frac{1}{a^3b^3}}+x\right)}{16} - \frac{-ax-bx^3}{8a^3b-16a^2b^2x^2+8ab^3x^4}$$

input
```
integrate(x**2/(-b*x**2+a)**3,x)
```

output
```
sqrt(1/(a**3*b**3))*log(-a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - sqrt(1/(a**3
*b**3))*log(a**2*b*sqrt(1/(a**3*b**3)) + x)/16 - (-a*x - b*x**3)/(8*a**3*b
 - 16*a**2*b**2*x**2 + 8*a*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{x^2}{(a-bx^2)^3}\,dx = \frac{bx^3+ax}{8\,(ab^3x^4-2\,a^2b^2x^2+a^3b)} + \frac{\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}ab}$$

input
```
integrate(x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(b*x^3 + a*x)/(a*b^3*x^4 - 2*a^2*b^2*x^2 + a^3*b) + 1/16*log((b*x - sq
rt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(a-bx^2)^3}\,dx = \frac{\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\,\sqrt{-ab}ab} + \frac{bx^3+ax}{8\,(bx^2-a)^2ab}$$

input
```
integrate(x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a*b) + 1/8*(b*x^3 + a*x)/((b*x^2 -
a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a-bx^2)^3}\,dx = \frac{\frac{x}{8\,b}+\frac{x^3}{8\,a}}{a^2-2\,a\,b\,x^2+b^2\,x^4} - \frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

input
```
int(x^2/(a - b*x^2)^3,x)
```

output
```
(x/(8*b) + x^3/(8*a))/(a^2 + b^2*x^4 - 2*a*b*x^2) - atanh((b^(1/2)*x)/a^(1
/2))/(8*a^(3/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.63

$$\int \frac{x^2}{(a-bx^2)^3}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2+2\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2-\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^2x^4+}{16a^2b^2\,(}$$

input
```
int(x^2/(-b*x^2+a)^3,x)
```

output
```
( - sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**2 + 2*sqrt(b)*sqrt(a)
*log( - sqrt(b)*sqrt(a) - b*x)*a*b*x**2 - sqrt(b)*sqrt(a)*log( - sqrt(b)*s
qrt(a) - b*x)*b**2*x**4 + sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**2
- 2*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**2 + sqrt(b)*sqrt(a)*
log(sqrt(b)*sqrt(a) - b*x)*b**2*x**4 + 2*a**2*b*x + 2*a*b**2*x**3)/(16*a**
2*b**2*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.252 $\int \frac{x}{(a-bx^2)^3} dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(a-bx^2)^3} dx = \frac{1}{4b(a-bx^2)^2}$$

output
```
1/4/b/(-b*x^2+a)^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a-bx^2)^3} dx = \frac{1}{4b(a-bx^2)^2}$$

input
```
Integrate[x/(a - b*x^2)^3,x]
```

output
```
1/(4*b*(a - b*x^2)^2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{4b(a-bx^2)^2}$$

input
```
Int[x/(a - b*x^2)^3,x]
```

output
```
1/(4*b*(a - b*x^2)^2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{1}{4b(-b\,x^2+a)^2}$	16
derivativedivides	$\frac{1}{4b(-b\,x^2+a)^2}$	16
default	$\frac{1}{4b(-b\,x^2+a)^2}$	16
norman	$\frac{1}{4b(-b\,x^2+a)^2}$	16
risch	$\frac{1}{4b(-b\,x^2+a)^2}$	16
orering	$\frac{1}{4b(-b\,x^2+a)^2}$	16
parallelrisch	$\frac{1}{4b(b\,x^2-a)^2}$	17

input
```
int(x/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/b/(-b*x^2+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4\,(b^3x^4-2\,ab^2x^2+a^2b)}$$

input
```
integrate(x/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/4/(b^3*x^4 - 2*a*b^2*x^2 + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4a^2b - 8ab^2x^2 + 4b^3x^4}$$

input
```
integrate(x/(-b*x**2+a)**3,x)
```

output
```
1/(4*a**2*b - 8*a*b**2*x**2 + 4*b**3*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4\,(bx^2-a)^2b}$$

input
```
integrate(x/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/4/((b*x^2 - a)^2*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4\,(bx^2-a)^2b}$$

input
```
integrate(x/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/4/((b*x^2 - a)^2*b)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4\,a^2\,b-8\,a\,b^2\,x^2+4\,b^3\,x^4}$$

input
```
int(x/(a - b*x^2)^3,x)
```

output
```
1/(4*a^2*b + 4*b^3*x^4 - 8*a*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \frac{x}{(a-bx^2)^3}\,dx = \frac{1}{4b\,(b^2x^4-2ab\,x^2+a^2)}$$

input
```
int(x/(-b*x^2+a)^3,x)
```

output
```
1/(4*b*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.253 $\int \frac{1}{(a-bx^2)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{x}{4a(a-bx^2)^2} + \frac{3x}{8a^2(a-bx^2)} + \frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

output
```
1/4*x/a/(-b*x^2+a)^2+3/8*x/a^2/(-b*x^2+a)+3/8*arctanh(b^(1/2)*x/a^(1/2))/a
^(5/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{5ax-3bx^3}{8a^2(a-bx^2)^2} + \frac{3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}$$

input
```
Integrate[(a - b*x^2)^(-3),x]
```

output
```
(5*a*x - 3*b*x^3)/(8*a^2*(a - b*x^2)^2) + (3*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(5/2)*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\int \frac{1}{(a-bx^2)^2}dx}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\left(\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}$$

input
```
Int[(a - b*x^2)^(-3),x]
```

output
```
x/(4*a*(a - b*x^2)^2) + (3*(x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt
[a]]/(2*a^(3/2)*Sqrt[b])))/(4*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{x}{4a\left(-b\,x^{2}+a\right)^{2}}+\frac{\frac{3x}{8a\left(-b\,x^{2}+a\right)}+\frac{3\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{a}$	59
risch	$\frac{-\frac{3b\,x^{3}}{8a^{2}}+\frac{5x}{8a}}{\left(-b\,x^{2}+a\right)^{2}}+\frac{3\ln\left(bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^{2}}-\frac{3\ln\left(-bx+\sqrt{ab}\right)}{16\sqrt{ab}\,a^{2}}$	70

input
```
int(1/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x/a/(-b*x^2+a)^2+3/4/a*(1/2*x/a/(-b*x^2+a)+1/2/a/(a*b)^(1/2)*arctanh(b
*x/(a*b)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.94

$$\int \frac{1}{\left(a-bx^2\right)^3}\,dx=\left[-\frac{6\,ab^2x^3-10\,a^2bx-3\,(b^2x^4-2\,abx^2+a^2)\sqrt{ab}\log\left(\frac{bx^2+2\,\sqrt{ab}x+a}{bx^2-a}\right)}{16\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)},\right.$$
$$\left.-\frac{3\,ab^2x^3-5\,a^2bx+3\,(b^2x^4-2\,abx^2+a^2)\sqrt{-ab}\arctan\left(\frac{\sqrt{-ab}x}{a}\right)}{8\,(a^3b^3x^4-2\,a^4b^2x^2+a^5b)}\right]$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(6*a*b^2*x^3 - 10*a^2*b*x - 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*sqrt(a*b)
*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^3*b^3*x^4 - 2*a^4*b^2*x^
2 + a^5*b), -1/8*(3*a*b^2*x^3 - 5*a^2*b*x + 3*(b^2*x^4 - 2*a*b*x^2 + a^2)*
sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^3*b^3*x^4 - 2*a^4*b^2*x^2 + a^5*b)]
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\sqrt{\frac{1}{a^5b}}\log\left(-a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16} + \frac{3\sqrt{\frac{1}{a^5b}}\log\left(a^3\sqrt{\frac{1}{a^5b}}+x\right)}{16} - \frac{-5ax+3bx^3}{8a^4-16a^3bx^2+8a^2b^2x^4}$$

input
```
integrate(1/(-b*x**2+a)**3,x)
```

output
```
-3*sqrt(1/(a**5*b))*log(-a**3*sqrt(1/(a**5*b)) + x)/16 + 3*sqrt(1/(a**5*b)
)*log(a**3*sqrt(1/(a**5*b)) + x)/16 - (-5*a*x + 3*b*x**3)/(8*a**4 - 16*a**
3*b*x**2 + 8*a**2*b**2*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\,bx^3-5\,ax}{8\,(a^2b^2x^4-2\,a^3bx^2+a^4)} - \frac{3\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{16\,\sqrt{ab}a^2}$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*b*x^3 - 5*a*x)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 3/16*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{1}{(a-bx^2)^3}\,dx = -\frac{3\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{8\sqrt{-ab}a^2} - \frac{3\,bx^3-5\,ax}{8\,(bx^2-a)^2a^2}$$

input
```
integrate(1/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/8*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^2) - 1/8*(3*b*x^3 - 5*a*x)/((b*x
^2 - a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a-bx^2)^3}\,dx = \frac{\frac{5\,x}{8\,a}-\frac{3\,b\,x^3}{8\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4} + \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^3,x)
```

output
```
((5*x)/(8*a) - (3*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) + (3*atanh((
b^(1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.78

$$\int \frac{1}{(a-bx^2)^3}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2-6\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2+3\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^2x^4 -}{16a^3b}$$

input
```
int(1/(-b*x^2+a)^3,x)
```

output
```
(3*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**2 - 6*sqrt(b)*sqrt(a)*
log( - sqrt(b)*sqrt(a) - b*x)*a*b*x**2 + 3*sqrt(b)*sqrt(a)*log( - sqrt(b)*
sqrt(a) - b*x)*b**2*x**4 - 3*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*
*2 + 6*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**2 - 3*sqrt(b)*sqr
t(a)*log(sqrt(b)*sqrt(a) - b*x)*b**2*x**4 + 10*a**2*b*x - 6*a*b**2*x**3)/(
16*a**3*b*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.254 $\int \frac{1}{x(a-bx^2)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx=\frac{1}{4a\left(a-bx^2\right)^2}+\frac{1}{2a^2\left(a-bx^2\right)}+\frac{\log(x)}{a^3}-\frac{\log\left(a-bx^2\right)}{2a^3}$$

output
```
1/4/a/(-b*x^2+a)^2+1/2/a^2/(-b*x^2+a)+ln(x)/a^3-1/2*ln(-b*x^2+a)/a^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx=\frac{\frac{a(3a-2bx^2)}{(a-bx^2)^2}+4\log(x)-2\log\left(a-bx^2\right)}{4a^3}$$

input
```
Integrate[1/(x*(a - b*x^2)^3),x]
```

output
```
((a*(3*a - 2*b*x^2))/(a - b*x^2)^2 + 4*Log[x] - 2*Log[a - b*x^2])/(4*a^3)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(a-bx^2\right)^3}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^2\left(a-bx^2\right)^3}dx^2$$

$$\Big\downarrow \; 54$$

$$\frac{1}{2}\int \left(\frac{b}{a^3\left(a-bx^2\right)}+\frac{b}{a^2\left(a-bx^2\right)^2}+\frac{b}{a\left(a-bx^2\right)^3}+\frac{1}{a^3x^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{\log\left(a-bx^2\right)}{a^3}+\frac{\log\left(x^2\right)}{a^3}+\frac{1}{a^2\left(a-bx^2\right)}+\frac{1}{2a\left(a-bx^2\right)^2}\right)$$

input
```
Int[1/(x*(a - b*x^2)^3),x]
```

output
```
(1/(2*a*(a - b*x^2)^2) + 1/(a^2*(a - b*x^2)) + Log[x^2]/a^3 - Log[a - b*x^
2]/a^3)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{-\frac{b\,x^{2}}{2a^{2}}+\frac{3}{4a}}{\left(-b\,x^{2}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(-b\,x^{2}+a\right)}{2a^{3}}$	48
norman	$\frac{\frac{b\,x^{2}}{a^{2}}-\frac{3b^{2}x^{4}}{4a^{3}}}{\left(-b\,x^{2}+a\right)^{2}}+\frac{\ln\left(x\right)}{a^{3}}-\frac{\ln\left(-b\,x^{2}+a\right)}{2a^{3}}$	53
default	$\frac{b\left(\frac{a^{2}}{2b\left(-b\,x^{2}+a\right)^{2}}-\frac{\ln\left(-b\,x^{2}+a\right)}{b}+\frac{a}{b\left(-b\,x^{2}+a\right)}\right)}{2a^{3}}+\frac{\ln\left(x\right)}{a^{3}}$	62
parallelrisch	$\frac{4b^{2}\ln\left(x\right)x^{4}-2\ln\left(b\,x^{2}-a\right)x^{4}b^{2}-3b^{2}x^{4}-8ab\ln\left(x\right)x^{2}+4\ln\left(b\,x^{2}-a\right)x^{2}ab+4ab\,x^{2}+4a^{2}\ln\left(x\right)-2\ln\left(b\,x^{2}-a\right)a^{2}}{4a^{3}\left(b\,x^{2}-a\right)^{2}}$	109

input
```
int(1/x/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b/a^2*x^2+3/4/a)/(-b*x^2+a)^2+ln(x)/a^3-1/2*ln(-b*x^2+a)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = -\frac{2\,abx^{2}-3\,a^{2}+2\left(b^{2}x^{4}-2\,abx^{2}+a^{2}\right)\log\left(bx^{2}-a\right)-4\left(b^{2}x^{4}-2\,abx^{2}+a^{2}\right)\log\left(x\right)}{4\left(a^{3}b^{2}x^{4}-2\,a^{4}bx^{2}+a^{5}\right)}$$

input
```
integrate(1/x/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*b*x^2 - 3*a^2 + 2*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(b*x^2 - a) - 4
*(b^2*x^4 - 2*a*b*x^2 + a^2)*log(x))/(a^3*b^2*x^4 - 2*a^4*b*x^2 + a^5)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.98

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = -\frac{-3a+2bx^2}{4a^4-8a^3bx^2+4a^2b^2x^4}+\frac{\log\left(x\right)}{a^3}-\frac{\log\left(-\frac{a}{b}+x^2\right)}{2a^3}$$

input
```
integrate(1/x/(-b*x**2+a)**3,x)
```

output
```
-(-3*a + 2*b*x**2)/(4*a**4 - 8*a**3*b*x**2 + 4*a**2*b**2*x**4) + log(x)/a*
*3 - log(-a/b + x**2)/(2*a**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = -\frac{2\,bx^2-3\,a}{4\left(a^2b^2x^4-2\,a^3bx^2+a^4\right)}-\frac{\log\left(bx^2-a\right)}{2\,a^3}+\frac{\log\left(x^2\right)}{2\,a^3}$$

input
```
integrate(1/x/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x^2 - 3*a)/(a^2*b^2*x^4 - 2*a^3*b*x^2 + a^4) - 1/2*log(b*x^2 - a
)/a^3 + 1/2*log(x^2)/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = \frac{\log\left(x^2\right)}{2\,a^3} - \frac{\log\left(\left|bx^2-a\right|\right)}{2\,a^3} + \frac{3\,b^2x^4-8\,abx^2+6\,a^2}{4\left(bx^2-a\right)^2a^3}$$

input
```
integrate(1/x/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^3 - 1/2*log(abs(b*x^2 - a))/a^3 + 1/4*(3*b^2*x^4 - 8*a*b*x^
2 + 6*a^2)/((b*x^2 - a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx = \frac{\ln(x)}{a^3} + \frac{\frac{3}{4\,a}-\frac{b\,x^2}{2\,a^2}}{a^2-2\,a\,b\,x^2+b^2\,x^4} - \frac{\ln\left(a-b\,x^2\right)}{2\,a^3}$$

input
```
int(1/(x*(a - b*x^2)^3),x)
```

output
```
log(x)/a^3 + (3/(4*a) - (b*x^2)/(2*a^2))/(a^2 + b^2*x^4 - 2*a*b*x^2) - log
(a - b*x^2)/(2*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.05

$$\int \frac{1}{x\left(a-bx^2\right)^3}\,dx$$
$$= \frac{-2\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2+4\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)ab\,x^2-2\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^2x^4-2\log\left(\sqrt{b}\,\sqrt{a}-b\right.}{4a^3\left(\right.}$$

input
```
int(1/x/(-b*x^2+a)^3,x)
```

output
```
( - 2*log( - sqrt(b)*sqrt(a) - b*x)*a**2 + 4*log( - sqrt(b)*sqrt(a) - b*x)
*a*b*x**2 - 2*log( - sqrt(b)*sqrt(a) - b*x)*b**2*x**4 - 2*log(sqrt(b)*sqrt
(a) - b*x)*a**2 + 4*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**2 - 2*log(sqrt(b)*sq
rt(a) - b*x)*b**2*x**4 + 4*log(x)*a**2 - 8*log(x)*a*b*x**2 + 4*log(x)*b**2
*x**4 + 2*a**2 - b**2*x**4)/(4*a**3*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.255 $\int \frac{1}{x^2(a-bx^2)^3} dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \frac{1}{x^2\left(a-bx^2\right)^3} dx = -\frac{1}{a^3x} + \frac{bx}{4a^2\left(a-bx^2\right)^2} + \frac{7bx}{8a^3\left(a-bx^2\right)} + \frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

output
```
-1/a^3/x+1/4*b*x/a^2/(-b*x^2+a)^2+7/8*b*x/a^3/(-b*x^2+a)+15/8*b^(1/2)*arct
anh(b^(1/2)*x/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^2\left(a-bx^2\right)^3} dx = \frac{-8a^2+25abx^2-15b^2x^4}{8a^3x\left(a-bx^2\right)^2} + \frac{15\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^3),x]
```

output
```
(-8*a^2 + 25*a*b*x^2 - 15*b^2*x^4)/(8*a^3*x*(a - b*x^2)^2) + (15*Sqrt[b]*A
rcTanh[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {253, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{x^2(a-bx^2)^2}dx}{4a}+\frac{1}{4ax(a-bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{3\int \frac{1}{x^2\left(a-bx^2\right)}dx}{2a}+\frac{1}{2ax(a-bx^2)}\right)}{4a}+\frac{1}{4ax(a-bx^2)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{a-bx^2}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a-bx^2)}\right)}{4a}+\frac{1}{4ax(a-bx^2)^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{5\left(\frac{3\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax(a-bx^2)}\right)}{4a}+\frac{1}{4ax(a-bx^2)^2}$$

input
```
Int[1/(x^2*(a - b*x^2)^3),x]
```

output
```
1/(4*a*x*(a - b*x^2)^2) + (5*(1/(2*a*x*(a - b*x^2)) + (3*(-(1/(a*x)) + (Sq
rt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{b\left(\frac{-\frac{7}{8}b\,x^{3}+\frac{9}{8}a x}{\left(-b\,x^{2}+a\right)^{2}}+\frac{15\,\operatorname{arctanh}\left(\frac{b x}{\sqrt{a b}}\right)}{8\sqrt{a b}}\right)}{a^{3}}-\frac{1}{a^{3}x}$	54
risch	$\frac{-\frac{15 b^{2} x^{4}}{8 a^{3}}+\frac{25 b\,x^{2}}{8 a^{2}}-\frac{1}{a}}{x\left(-b\,x^{2}+a\right)^{2}}+\frac{15\left(\sum_{\_R=\operatorname{RootOf}\left(a^{7}\_Z^{2}-b\right)}\_R\ln\left(\left(3\_R^{2}a^{7}-2b\right)x+a^{4}\_R\right)\right)}{16}$	82

input
```
int(1/x^2/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*b*((-7/8*b*x^3+9/8*a*x)/(-b*x^2+a)^2+15/8/(a*b)^(1/2)*arctanh(b*x/(a
*b)^(1/2)))-1/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (a-bx^2)^3} dx = \left[-\frac{30 b^2x^4 - 50 abx^2 - 15 (b^2x^5 - 2 abx^3 + a^2x) \sqrt{\frac{b}{a}} \log \left(\frac{bx^2+2 ax\sqrt{\frac{b}{a}}+a}{bx^2-a} \right) + 16 a^2}{16 (a^3b^2x^5 - 2 a^4bx^3 + a^5x)}, -\frac{15 b^2x^4 - 25 abx^2 + 15 (b^2x^5 - 2 abx^3 + a^2x) \sqrt{-\frac{b}{a}} \arctan \left(x\sqrt{-\frac{b}{a}} \right) + 8 a^2}{8 (a^3b^2x^5 - 2 a^4bx^3 + a^5x)} \right]$$

input
```
integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(30*b^2*x^4 - 50*a*b*x^2 - 15*(b^2*x^5 - 2*a*b*x^3 + a^2*x)*sqrt(b/
a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)) + 16*a^2)/(a^3*b^2*x^5 -
 2*a^4*b*x^3 + a^5*x), -1/8*(15*b^2*x^4 - 25*a*b*x^2 + 15*(b^2*x^5 - 2*a*b
*x^3 + a^2*x)*sqrt(-b/a)*arctan(x*sqrt(-b/a)) + 8*a^2)/(a^3*b^2*x^5 - 2*a^
4*b*x^3 + a^5*x)]
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2 (a-bx^2)^3} dx = -\frac{15\sqrt{\frac{b}{a^7}} \log \left(-\frac{a^4\sqrt{\frac{b}{a^7}}}{b} + x \right)}{16} + \frac{15\sqrt{\frac{b}{a^7}} \log \left(\frac{a^4\sqrt{\frac{b}{a^7}}}{b} + x \right)}{16} - \frac{8a^2 - 25abx^2 + 15b^2x^4}{8a^5x - 16a^4bx^3 + 8a^3b^2x^5}$$

input
```
integrate(1/x**2/(-b*x**2+a)**3,x)
```

output
```
-15*sqrt(b/a**7)*log(-a**4*sqrt(b/a**7)/b + x)/16 + 15*sqrt(b/a**7)*log(a*
*4*sqrt(b/a**7)/b + x)/16 - (8*a**2 - 25*a*b*x**2 + 15*b**2*x**4)/(8*a**5*
x - 16*a**4*b*x**3 + 8*a**3*b**2*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2 (a-bx^2)^3} \, dx = -\frac{15 \, b^2 x^4 - 25 \, abx^2 + 8 \, a^2}{8 \, (a^3 b^2 x^5 - 2 \, a^4 b x^3 + a^5 x)} - \frac{15 \, b \log \left(\frac{bx - \sqrt{ab}}{bx + \sqrt{ab}} \right)}{16 \, \sqrt{ab} a^3}$$

input
```
integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*b^2*x^4 - 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 - 2*a^4*b*x^3 + a^5*x)
 - 15/16*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a-bx^2)^3} \, dx = -\frac{15 \, b \arctan \left(\frac{bx}{\sqrt{-ab}} \right)}{8 \, \sqrt{-ab} a^3} - \frac{7 \, b^2 x^3 - 9 \, abx}{8 \, (bx^2 - a)^2 a^3} - \frac{1}{a^3 x}$$

input
```
integrate(1/x^2/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
-15/8*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3) - 1/8*(7*b^2*x^3 - 9*a*b*x
)/((b*x^2 - a)^2*a^3) - 1/(a^3*x)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a - bx^2)^3} \, dx = \frac{15 \sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\, a^{7/2}} - \frac{\frac{1}{a} - \frac{25\, b\, x^2}{8\, a^2} + \frac{15\, b^2\, x^4}{8\, a^3}}{a^2\, x - 2\, a\, b\, x^3 + b^2\, x^5}$$

input
```
int(1/(x^2*(a - b*x^2)^3),x)
```

output
```
(15*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) - (1/a - (25*b*x^2)/(8
*a^2) + (15*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 - 2*a*b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.53

$$\int \frac{1}{x^2 (a - bx^2)^3} \, dx$$
$$= \frac{15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) a^2 x - 30\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) ab\, x^3 + 15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a} - bx\right) b}{}$$

input
```
int(1/x^2/(-b*x^2+a)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**2*x - 30*sqrt(b)*sqrt
(a)*log( - sqrt(b)*sqrt(a) - b*x)*a*b*x**3 + 15*sqrt(b)*sqrt(a)*log( - sqr
t(b)*sqrt(a) - b*x)*b**2*x**5 - 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b
*x)*a**2*x + 30*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b*x**3 - 15*s
qrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*b**2*x**5 - 16*a**3 + 50*a**2*b*
x**2 - 30*a*b**2*x**4)/(16*a**4*x*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.256 $\int \frac{1}{x^3(a-bx^2)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 69

$$\int \frac{1}{x^3(a-bx^2)^3}\,dx = -\frac{1}{2a^3x^2}+\frac{b}{4a^2(a-bx^2)^2}+\frac{b}{a^3(a-bx^2)}+\frac{3b\log(x)}{a^4}-\frac{3b\log(a-bx^2)}{2a^4}$$

output
```
-1/2/a^3/x^2+1/4*b/a^2/(-b*x^2+a)^2+b/a^3/(-b*x^2+a)+3*b*ln(x)/a^4-3/2*b*l
n(-b*x^2+a)/a^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3(a-bx^2)^3}\,dx = \frac{\frac{a(-2a^2+9abx^2-6b^2x^4)}{(ax-bx^3)^2}+12b\log(x)-6b\log(a-bx^2)}{4a^4}$$

input
```
Integrate[1/(x^3*(a - b*x^2)^3),x]
```

output
```
((a*(-2*a^2 + 9*a*b*x^2 - 6*b^2*x^4))/(a*x - b*x^3)^2 + 12*b*Log[x] - 6*b*
Log[a - b*x^2])/(4*a^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4 (a-bx^2)^3}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{3b^2}{a^4 (a-bx^2)}+\frac{2b^2}{a^3 (a-bx^2)^2}+\frac{b^2}{a^2 (a-bx^2)^3}+\frac{3b}{a^4x^2}+\frac{1}{a^3x^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3b\log (x^2)}{a^4}-\frac{3b\log (a-bx^2)}{a^4}+\frac{2b}{a^3 (a-bx^2)}-\frac{1}{a^3x^2}+\frac{b}{2a^2 (a-bx^2)^2}\right)$$

input
```
Int[1/(x^3*(a - b*x^2)^3),x]
```

output
```
(-(1/(a^3*x^2)) + b/(2*a^2*(a - b*x^2)^2) + (2*b)/(a^3*(a - b*x^2)) + (3*b
*Log[x^2])/a^4 - (3*b*Log[a - b*x^2])/a^4)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

method	result
risch	$\frac{-\frac{3 b^2 x^4}{2 a^3}+\frac{9 b x^2}{4 a^2}-\frac{1}{2 a}}{x^2 (-b x^2+a)^2}+\frac{3 b \ln(x)}{a^4}-\frac{3 b \ln(-b x^2+a)}{2 a^4}$
norman	$\frac{-\frac{1}{2 a}+\frac{3 b^2 x^4}{a^3}-\frac{9 b^3 x^6}{4 a^4}}{x^2 (-b x^2+a)^2}+\frac{3 b \ln(x)}{a^4}-\frac{3 b \ln(-b x^2+a)}{2 a^4}$
default	$\frac{b^2 \left(\frac{a^2}{2 b (-b x^2+a)^2}-\frac{3 \ln(-b x^2+a)}{b}+\frac{2 a}{b (-b x^2+a)}\right)}{2 a^4}-\frac{1}{2 a^3 x^2}+\frac{3 b \ln(x)}{a^4}$
parallelrisch	$\frac{12 b^3 \ln(x) x^6-6 \ln(b x^2-a) x^6 b^3-9 b^3 x^6-24 a b^2 \ln(x) x^4+12 \ln(b x^2-a) x^4 a b^2+12 a b^2 x^4+12 a^2 b \ln(x) x^2-6 \ln(b x^2-a) x^2 a^2 b}{4 a^4 x^2 (b x^2-a)^2}$

input
```
int(1/x^3/(-b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(-3/2*b^2/a^3*x^4+9/4*b/a^2*x^2-1/2/a)/x^2/(-b*x^2+a)^2+3*b*ln(x)/a^4-3/2*
b*ln(-b*x^2+a)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx = -\frac{6\,ab^2x^4 - 9\,a^2bx^2 + 2\,a^3 + 6\,(b^3x^6 - 2\,ab^2x^4 + a^2bx^2)\log(bx^2-a) - 12\,(b^3x^6 - 2\,ab^2x^4 + a^2bx^2)\log(}{4\,(a^4b^2x^6 - 2\,a^5bx^4 + a^6x^2)}$$

input
```
integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3 + 6*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b
*x^2)*log(b*x^2 - a) - 12*(b^3*x^6 - 2*a*b^2*x^4 + a^2*b*x^2)*log(x))/(a^4
*b^2*x^6 - 2*a^5*b*x^4 + a^6*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx = -\frac{2a^2 - 9abx^2 + 6b^2x^4}{4a^5x^2 - 8a^4bx^4 + 4a^3b^2x^6} + \frac{3b\log(x)}{a^4} - \frac{3b\log\left(-\frac{a}{b} + x^2\right)}{2a^4}$$

input
```
integrate(1/x**3/(-b*x**2+a)**3,x)
```

output
```
-(2*a**2 - 9*a*b*x**2 + 6*b**2*x**4)/(4*a**5*x**2 - 8*a**4*b*x**4 + 4*a**3
*b**2*x**6) + 3*b*log(x)/a**4 - 3*b*log(-a/b + x**2)/(2*a**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx = -\frac{6\,b^2x^4 - 9\,abx^2 + 2\,a^2}{4\,(a^3b^2x^6 - 2\,a^4bx^4 + a^5x^2)} - \frac{3\,b\log(bx^2-a)}{2\,a^4} + \frac{3\,b\log(x^2)}{2\,a^4}$$

input
```
integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(6*b^2*x^4 - 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 - 2*a^4*b*x^4 + a^5*x^2)
 - 3/2*b*log(b*x^2 - a)/a^4 + 3/2*b*log(x^2)/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx = \frac{3 \, b \log \left(x^{2}\right)}{2 \, a^{4}} - \frac{3 \, b \log \left({\left| b x^{2} - a \right|}\right)}{2 \, a^{4}} + \frac{9 \, b^{3} x^{4} - 22 \, a b^{2} x^{2} + 14 \, a^{2} b}{4 \, {\left(b x^{2} - a\right)}^{2} a^{4}} - \frac{3 \, b x^{2} + a}{2 \, a^{4} x^{2}}$$

input
```
integrate(1/x^3/(-b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/2*b*log(x^2)/a^4 - 3/2*b*log(abs(b*x^2 - a))/a^4 + 1/4*(9*b^3*x^4 - 22*a
*b^2*x^2 + 14*a^2*b)/((b*x^2 - a)^2*a^4) - 1/2*(3*b*x^2 + a)/(a^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx = \frac{3\,b\,\ln\left(x\right)}{a^4} - \frac{3\,b\,\ln\left(a-b\,x^2\right)}{2\,a^4} - \frac{\frac{1}{2\,a}-\frac{9\,b\,x^2}{4\,a^2}+\frac{3\,b^2\,x^4}{2\,a^3}}{a^2\,x^2-2\,a\,b\,x^4+b^2\,x^6}$$

input
```
int(1/(x^3*(a - b*x^2)^3),x)
```

output
```
(3*b*log(x))/a^4 - (3*b*log(a - b*x^2))/(2*a^4) - (1/(2*a) - (9*b*x^2)/(4*
a^2) + (3*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 - 2*a*b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.96

$$\int \frac{1}{x^3 (a-bx^2)^3} \, dx$$
$$= \frac{-6\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2b\,x^2+12\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a\,b^2x^4-6\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)b^3x^6-6\log\left(\sqrt{b}\,\sqrt{\right.}\right.}{}$$

input
```
int(1/x^3/(-b*x^2+a)^3,x)
```

output
```
( - 6*log( - sqrt(b)*sqrt(a) - b*x)*a**2*b*x**2 + 12*log( - sqrt(b)*sqrt(a
) - b*x)*a*b**2*x**4 - 6*log( - sqrt(b)*sqrt(a) - b*x)*b**3*x**6 - 6*log(s
qrt(b)*sqrt(a) - b*x)*a**2*b*x**2 + 12*log(sqrt(b)*sqrt(a) - b*x)*a*b**2*x
**4 - 6*log(sqrt(b)*sqrt(a) - b*x)*b**3*x**6 + 12*log(x)*a**2*b*x**2 - 24*
log(x)*a*b**2*x**4 + 12*log(x)*b**3*x**6 - 2*a**3 + 6*a**2*b*x**2 - 3*b**3
*x**6)/(4*a**4*x**2*(a**2 - 2*a*b*x**2 + b**2*x**4))
```

3.257 $\int \frac{x^3}{(a-bx^2)^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = \frac{a}{8b^2(a-bx^2)^4} - \frac{1}{6b^2(a-bx^2)^3}$$

output
```
1/8*a/b^2/(-b*x^2+a)^4-1/6/b^2/(-b*x^2+a)^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = -\frac{a-4bx^2}{24b^2(a-bx^2)^4}$$

input
```
Integrate[x^3/(a - b*x^2)^5,x]
```

output
```
-1/24*(a - 4*b*x^2)/(b^2*(a - b*x^2)^4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a-bx^2)^5}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(a-bx^2)^5}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a}{b(a-bx^2)^5}-\frac{1}{b(a-bx^2)^4}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a}{4b^2(a-bx^2)^4}-\frac{1}{3b^2(a-bx^2)^3}\right)$$

input
```
Int[x^3/(a - b*x^2)^5,x]
```

output
```
(a/(4*b^2*(a - b*x^2)^4) - 1/(3*b^2*(a - b*x^2)^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{-4b\,x^{2}+a}{24\left(-b\,x^{2}+a\right)^{4} b^{2}}$	24
orering	$-\frac{-4b\,x^{2}+a}{24\left(-b\,x^{2}+a\right)^{4} b^{2}}$	24
norman	$\frac{\frac{x^{2}}{6b}-\frac{a}{24b^{2}}}{\left(-b\,x^{2}+a\right)^{4}}$	27
risch	$\frac{\frac{x^{2}}{6b}-\frac{a}{24b^{2}}}{\left(-b\,x^{2}+a\right)^{4}}$	27
parallelrisch	$\frac{4b^{3}x^{2}-a\,b^{2}}{24b^{4}\left(b\,x^{2}-a\right)^{4}}$	32
default	$\frac{a}{8b^{2}\left(-b\,x^{2}+a\right)^{4}}-\frac{1}{6b^{2}\left(-b\,x^{2}+a\right)^{3}}$	33

input
```
int(x^3/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-4*b*x^2+a)/(-b*x^2+a)^4/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = \frac{4\,bx^2-a}{24\,(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate(x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2
+ a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = -\frac{a-4bx^2}{24a^4b^2-96a^3b^3x^2+144a^2b^4x^4-96ab^5x^6+24b^6x^8}$$

input
```
integrate(x**3/(-b*x**2+a)**5,x)
```

output
```
-(a - 4*b*x**2)/(24*a**4*b**2 - 96*a**3*b**3*x**2 + 144*a**2*b**4*x**4 - 9
6*a*b**5*x**6 + 24*b**6*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.67

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = \frac{4\,bx^2-a}{24\,(b^6x^8-4\,ab^5x^6+6\,a^2b^4x^4-4\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate(x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/24*(4*b*x^2 - a)/(b^6*x^8 - 4*a*b^5*x^6 + 6*a^2*b^4*x^4 - 4*a^3*b^3*x^2
+ a^4*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = \frac{\frac{4}{(bx^2-a)^3b}+\frac{3a}{(bx^2-a)^4b}}{24\,b}$$

input
```
integrate(x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/24*(4/((b*x^2 - a)^3*b) + 3*a/((b*x^2 - a)^4*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = -\frac{\frac{a}{24\,b^2}-\frac{x^2}{6\,b}}{a^4-4\,a^3\,b\,x^2+6\,a^2\,b^2\,x^4-4\,a\,b^3\,x^6+b^4\,x^8}$$

input
```
int(x^3/(a - b*x^2)^5,x)
```

output
```
-(a/(24*b^2) - x^2/(6*b))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a
^2*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.58

$$\int \frac{x^3}{(a-bx^2)^5}\,dx = \frac{4b\,x^2-a}{24b^2\,(b^4x^8-4a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2+a^4)}$$

input
```
int(x^3/(-b*x^2+a)^5,x)
```

output
```
( - a + 4*b*x**2)/(24*b**2*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2*x**4 - 4*a*
b**3*x**6 + b**4*x**8))
```

3.258 $\int \frac{x^2}{(a-bx^2)^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{x^2}{(a-bx^2)^5} dx = \frac{x}{8b(a-bx^2)^4} - \frac{x}{48ab(a-bx^2)^3} - \frac{5x}{192a^2b(a-bx^2)^2} - \frac{5x}{128a^3b(a-bx^2)} - \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

output
```
1/8*x/b/(-b*x^2+a)^4-1/48*x/a/b/(-b*x^2+a)^3-5/192*x/a^2/b/(-b*x^2+a)^2-5/
128*x/a^3/b/(-b*x^2+a)-5/128*arctanh(b^(1/2)*x/a^(1/2))/a^(7/2)/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{(a-bx^2)^5} dx = \frac{15a^3x + 73a^2bx^3 - 55ab^2x^5 + 15b^3x^7}{384a^3b(a-bx^2)^4} - \frac{5\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{7/2}b^{3/2}}$$

input
```
Integrate[x^2/(a - b*x^2)^5,x]
```

output
```
(15*a^3*x + 73*a^2*b*x^3 - 55*a*b^2*x^5 + 15*b^3*x^7)/(384*a^3*b*(a - b*x^
2)^4) - (5*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/(128*a^(7/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {252, 215, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^5}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{x}{8b(a-bx^2)^4}-\frac{\int \frac{1}{(a-bx^2)^4}dx}{8b}$$

$$\Big\downarrow \text{215}$$

$$\frac{x}{8b(a-bx^2)^4}-\frac{\frac{5\int \frac{1}{(a-bx^2)^3}dx}{6a}+\frac{x}{6a(a-bx^2)^3}}{8b}$$

$$\Big\downarrow \text{215}$$

$$\frac{x}{8b(a-bx^2)^4}-\frac{\frac{5\left(\frac{3\int \frac{1}{(a-bx^2)^2}dx}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}}{8b}$$

$$\Big\downarrow \text{215}$$

$$\frac{x}{8b(a-bx^2)^4}-\frac{\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a(a-bx^2)}\right)}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}}{8b}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{8b\left(a-bx^2\right)^4}-\frac{\frac{5\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}}{8b}$$

input
```
Int[x^2/(a - b*x^2)^5,x]
```

output
```
x/(8*b*(a - b*x^2)^4) - (x/(6*a*(a - b*x^2)^3) + (5*(x/(4*a*(a - b*x^2)^2)
 + (3*(x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[
b])))/(4*a)))/(6*a))/(8*b)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\frac{5b^2x^7}{128a^3}-\frac{55b\,x^5}{384a^2}+\frac{73x^3}{384a}+\frac{5x}{128b}}{\left(-b\,x^2+a\right)^4}-\frac{5\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128a^3b\sqrt{ab}}$	70
risch	$\frac{\frac{5b^2x^7}{128a^3}-\frac{55b\,x^5}{384a^2}+\frac{73x^3}{384a}+\frac{5x}{128b}}{\left(-b\,x^2+a\right)^4}+\frac{5\ln\left(bx-\sqrt{ab}\right)}{256\sqrt{ab}\,b\,a^3}-\frac{5\ln\left(-bx-\sqrt{ab}\right)}{256\sqrt{ab}\,b\,a^3}$	99

input
```
int(x^2/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(5/128*b^2/a^3*x^7-55/384*b/a^2*x^5+73/384/a*x^3+5/128*x/b)/(-b*x^2+a)^4-5
/128/a^3/b/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.97

$$\int \frac{x^2}{\left(a-bx^2\right)^5}\,dx = \left[\frac{30\,ab^4x^7-110\,a^2b^3x^5+146\,a^3b^2x^3+30\,a^4bx+15\,(b^4x^8-4\,ab^3x^6+6\,a^2b^2x^4-4\,a^3bx^2+a^4)\sqrt{ab}\log\left(\right.}{768\,(a^4b^6x^8-4\,a^5b^5x^6+6\,a^6b^4x^4-4\,a^7b^3x^2+a^8b^2)}\right.$$

input
```
integrate(x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
[1/768*(30*a*b^4*x^7 - 110*a^2*b^3*x^5 + 146*a^3*b^2*x^3 + 30*a^4*b*x + 15
*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b)*log
((b*x^2 - 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^4*b^6*x^8 - 4*a^5*b^5*x^6 +
6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2), 1/384*(15*a*b^4*x^7 - 55*a^2*b^3
*x^5 + 73*a^3*b^2*x^3 + 15*a^4*b*x + 15*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2
*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^4*b^6*x^8
- 4*a^5*b^5*x^6 + 6*a^6*b^4*x^4 - 4*a^7*b^3*x^2 + a^8*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{(a-bx^2)^5}\,dx = \frac{5\sqrt{\frac{1}{a^7b^3}}\log\left(-a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256} - \frac{5\sqrt{\frac{1}{a^7b^3}}\log\left(a^4b\sqrt{\frac{1}{a^7b^3}}+x\right)}{256} - \frac{-15a^3x-73a^2bx^3+55ab^2x^5-15b^3x^7}{384a^7b-1536a^6b^2x^2+2304a^5b^3x^4-1536a^4b^4x^6+384a^3b^5x^8}$$

input
```
integrate(x**2/(-b*x**2+a)**5,x)
```

output
```
5*sqrt(1/(a**7*b**3))*log(-a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - 5*sqrt(1/
(a**7*b**3))*log(a**4*b*sqrt(1/(a**7*b**3)) + x)/256 - (-15*a**3*x - 73*a*
*2*b*x**3 + 55*a*b**2*x**5 - 15*b**3*x**7)/(384*a**7*b - 1536*a**6*b**2*x*
*2 + 2304*a**5*b**3*x**4 - 1536*a**4*b**4*x**6 + 384*a**3*b**5*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{(a-bx^2)^5}\,dx = \frac{15\,b^3x^7-55\,ab^2x^5+73\,a^2bx^3+15\,a^3x}{384\,(a^3b^5x^8-4\,a^4b^4x^6+6\,a^5b^3x^4-4\,a^6b^2x^2+a^7b)} + \frac{5\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}a^3b}$$

input
```
integrate(x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/384*(15*b^3*x^7 - 55*a*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/(a^3*b^5*x^8 -
 4*a^4*b^4*x^6 + 6*a^5*b^3*x^4 - 4*a^6*b^2*x^2 + a^7*b) + 5/256*log((b*x -
 sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^3*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a-bx^2)^5}\,dx = \frac{5\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\sqrt{-ab}a^3b} + \frac{15\,b^3x^7 - 55\,ab^2x^5 + 73\,a^2bx^3 + 15\,a^3x}{384\,(bx^2-a)^4a^3b}$$

input
```
integrate(x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
5/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^3*b) + 1/384*(15*b^3*x^7 - 55*a
*b^2*x^5 + 73*a^2*b*x^3 + 15*a^3*x)/((b*x^2 - a)^4*a^3*b)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a-bx^2)^5}\,dx = \frac{\frac{5\,x}{128\,b} + \frac{73\,x^3}{384\,a} - \frac{55\,b\,x^5}{384\,a^2} + \frac{5\,b^2\,x^7}{128\,a^3}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8} - \frac{5\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{7/2}\,b^{3/2}}$$

input
```
int(x^2/(a - b*x^2)^5,x)
```

output
```
((5*x)/(128*b) + (73*x^3)/(384*a) - (55*b*x^5)/(384*a^2) + (5*b^2*x^7)/(12
8*a^3))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4) - (5*a
tanh((b^(1/2)*x)/a^(1/2)))/(128*a^(7/2)*b^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.01

$$\int \frac{x^2}{(a-bx^2)^5}\,dx = \frac{-15\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^4 + 60\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^3b\,x^2 - 90\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)}{}$$

input
```
int(x^2/(-b*x^2+a)^5,x)
```

output
```
( - 15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**4 + 60*sqrt(b)*sqr
t(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**3*b*x**2 - 90*sqrt(b)*sqrt(a)*log( -
 sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 + 60*sqrt(b)*sqrt(a)*log( - sqrt(b)
*sqrt(a) - b*x)*a*b**3*x**6 - 15*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) -
b*x)*b**4*x**8 + 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**4 - 60*s
qrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**3*b*x**2 + 90*sqrt(b)*sqrt(a)
*log(sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 - 60*sqrt(b)*sqrt(a)*log(sqrt(b
)*sqrt(a) - b*x)*a*b**3*x**6 + 15*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*
x)*b**4*x**8 + 30*a**4*b*x + 146*a**3*b**2*x**3 - 110*a**2*b**3*x**5 + 30*
a*b**4*x**7)/(768*a**4*b**2*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2*x**4 - 4*a
*b**3*x**6 + b**4*x**8))
```

3.259 $\int \frac{x}{(a-bx^2)^5} dx$

Optimal result

Integrand size = 12, antiderivative size = 17

$$\int \frac{x}{(a-bx^2)^5} dx = \frac{1}{8b(a-bx^2)^4}$$

output
```
1/8/b/(-b*x^2+a)^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a-bx^2)^5} dx = \frac{1}{8b(a-bx^2)^4}$$

input
```
Integrate[x/(a - b*x^2)^5,x]
```

output
```
1/(8*b*(a - b*x^2)^4)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a-bx^2)^5} \, dx$$

$$\Big\downarrow \; 241$$

$$\frac{1}{8b(a-bx^2)^4}$$

input
```
Int[x/(a - b*x^2)^5,x]
```

output
```
1/(8*b*(a - b*x^2)^4)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
derivativedivides	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
default	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
norman	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
risch	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
orering	$\frac{1}{8b(-b\,x^{2}+a)^{4}}$	16
parallelrisch	$\frac{1}{8b(b\,x^{2}-a)^{4}}$	17

input
```
int(x/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/b/(-b*x^2+a)^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(16) = 32.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8\,(b^5x^8-4\,ab^4x^6+6\,a^2b^3x^4-4\,a^3b^2x^2+a^4b)}$$

input
```
integrate(x/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
1/8/(b^5*x^8 - 4*a*b^4*x^6 + 6*a^2*b^3*x^4 - 4*a^3*b^2*x^2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.88

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8a^4b - 32a^3b^2x^2 + 48a^2b^3x^4 - 32ab^4x^6 + 8b^5x^8}$$

input
```
integrate(x/(-b*x**2+a)**5,x)
```

output
```
1/(8*a**4*b - 32*a**3*b**2*x**2 + 48*a**2*b**3*x**4 - 32*a*b**4*x**6 + 8*b
**5*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8\,(bx^2-a)^4b}$$

input
```
integrate(x/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
1/8/((b*x^2 - a)^4*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8\,(bx^2-a)^4b}$$

input
```
integrate(x/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/8/((b*x^2 - a)^4*b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.82

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8\,a^4\,b-32\,a^3\,b^2\,x^2+48\,a^2\,b^3\,x^4-32\,a\,b^4\,x^6+8\,b^5\,x^8}$$

input
```
int(x/(a - b*x^2)^5,x)
```

output
```
1/(8*a^4*b + 8*b^5*x^8 - 32*a*b^4*x^6 - 32*a^3*b^2*x^2 + 48*a^2*b^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.76

$$\int \frac{x}{(a-bx^2)^5}\,dx = \frac{1}{8b\,(b^4x^8-4a\,b^3x^6+6a^2b^2x^4-4a^3b\,x^2+a^4)}$$

input
```
int(x/(-b*x^2+a)^5,x)
```

output
```
1/(8*b*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2*x**4 - 4*a*b**3*x**6 + b**4*x**
8))
```

3.260 $\int \frac{1}{(a-bx^2)^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 100

$$\int \frac{1}{(a-bx^2)^5}\,dx = \frac{x}{8a(a-bx^2)^4}+\frac{7x}{48a^2(a-bx^2)^3}+\frac{35x}{192a^3(a-bx^2)^2}+\frac{35x}{128a^4(a-bx^2)}+\frac{35\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{9/2}\sqrt{b}}$$

output
```
1/8*x/a/(-b*x^2+a)^4+7/48*x/a^2/(-b*x^2+a)^3+35/192*x/a^3/(-b*x^2+a)^2+35/
128*x/a^4/(-b*x^2+a)+35/128*arctanh(b^(1/2)*x/a^(1/2))/a^(9/2)/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a-bx^2)^5}\,dx = \frac{\frac{\sqrt{a}x(279a^3-511a^2bx^2+385ab^2x^4-105b^3x^6)}{(a-bx^2)^4}+\frac{105\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}}{384a^{9/2}}$$

input
```
Integrate[(a - b*x^2)^(-5),x]
```

output
```
((Sqrt[a]*x*(279*a^3 - 511*a^2*b*x^2 + 385*a*b^2*x^4 - 105*b^3*x^6))/(a -
b*x^2)^4 + (105*ArcTanh[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(384*a^(9/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {215, 215, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^5}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{7\int \frac{1}{(a-bx^2)^4}dx}{8a}+\frac{x}{8a(a-bx^2)^4}$$

$$\Big\downarrow \text{215}$$

$$\frac{7\left(\frac{5\int \frac{1}{(a-bx^2)^3}dx}{6a}+\frac{x}{6a(a-bx^2)^3}\right)}{8a}+\frac{x}{8a(a-bx^2)^4}$$

$$\Big\downarrow \text{215}$$

$$\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{(a-bx^2)^2}dx}{4a}+\frac{x}{4a(a-bx^2)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}\right)}{8a}+\frac{x}{8a(a-bx^2)^4}$$

$$\Big\downarrow \text{215}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\int \frac{1}{a-bx^2}dx}{2a}+\frac{x}{2a\left(a-bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}\right)}{8a}+\frac{x}{8a\left(a-bx^2\right)^4}$$

$$\Big\downarrow \text{221}$$

$$\frac{7\left(\frac{5\left(\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a-bx^2\right)}\right)}{4a}+\frac{x}{4a\left(a-bx^2\right)^2}\right)}{6a}+\frac{x}{6a(a-bx^2)^3}\right)}{8a}+\frac{x}{8a\left(a-bx^2\right)^4}$$

input
```
Int[(a - b*x^2)^(-5),x]
```

output
```
x/(8*a*(a - b*x^2)^4) + (7*(x/(6*a*(a - b*x^2)^3) + (5*(x/(4*a*(a - b*x^2)
^2) + (3*(x/(2*a*(a - b*x^2)) + ArcTanh[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sq
rt[b])))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{-\frac{35b^3x^7}{128a^4}+\frac{385b^2x^5}{384a^3}-\frac{511b\,x^3}{384a^2}+\frac{93x}{128a}}{\left(-b\,x^2+a\right)^4}+\frac{35\ln\left(bx+\sqrt{ab}\right)}{256\sqrt{ab}\,a^4}-\frac{35\ln\left(-bx+\sqrt{ab}\right)}{256\sqrt{ab}\,a^4}$	92
default	$\frac{x}{8a\left(-b\,x^2+a\right)^4}+\frac{\frac{7x}{48a\left(-b\,x^2+a\right)^3}+\frac{7\left(\frac{5x}{24a\left(-b\,x^2+a\right)^2}+\frac{5\left(\frac{3x}{8a\left(-b\,x^2+a\right)}+\frac{3\,\mathrm{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{8a\sqrt{ab}}\right)}{6a}\right)}{8a}}{a}$	103

input
```
int(1/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(-35/128*b^3/a^4*x^7+385/384*b^2/a^3*x^5-511/384*b/a^2*x^3+93/128*x/a)/(-b
*x^2+a)^4+35/256/(a*b)^(1/2)/a^4*ln(b*x+(a*b)^(1/2))-35/256/(a*b)^(1/2)/a^
4*ln(-b*x+(a*b)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.20

$$\int \frac{1}{(a-bx^2)^5}\,dx$$
$$= \left[-\frac{210\,ab^4x^7 - 770\,a^2b^3x^5 + 1022\,a^3b^2x^3 - 558\,a^4bx - 105\,(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4)\sqrt{}}{768\,(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b)}\right.$$
$$-\frac{105\,ab^4x^7 - 385\,a^2b^3x^5 + 511\,a^3b^2x^3 - 279\,a^4bx + 105\,(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4)\sqrt{-}}{384\,(a^5b^5x^8 - 4\,a^6b^4x^6 + 6\,a^7b^3x^4 - 4\,a^8b^2x^2 + a^9b)}$$

input
```
integrate(1/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
[-1/768*(210*a*b^4*x^7 - 770*a^2*b^3*x^5 + 1022*a^3*b^2*x^3 - 558*a^4*b*x
- 105*(b^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(a*b
)*log((b*x^2 + 2*sqrt(a*b)*x + a)/(b*x^2 - a)))/(a^5*b^5*x^8 - 4*a^6*b^4*x
^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b), -1/384*(105*a*b^4*x^7 - 385*a
^2*b^3*x^5 + 511*a^3*b^2*x^3 - 279*a^4*b*x + 105*(b^4*x^8 - 4*a*b^3*x^6 +
6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*sqrt(-a*b)*arctan(sqrt(-a*b)*x/a))/(a^5
*b^5*x^8 - 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 - 4*a^8*b^2*x^2 + a^9*b)]
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a-bx^2)^5}\,dx = -\frac{35\sqrt{\frac{1}{a^9b}}\log\left(-a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256}+\frac{35\sqrt{\frac{1}{a^9b}}\log\left(a^5\sqrt{\frac{1}{a^9b}}+x\right)}{256} - \frac{-279a^3x+511a^2bx^3-385ab^2x^5+105b^3x^7}{384a^8-1536a^7bx^2+2304a^6b^2x^4-1536a^5b^3x^6+384a^4b^4x^8}$$

input
```
integrate(1/(-b*x**2+a)**5,x)
```

output
```
-35*sqrt(1/(a**9*b))*log(-a**5*sqrt(1/(a**9*b)) + x)/256 + 35*sqrt(1/(a**9
*b))*log(a**5*sqrt(1/(a**9*b)) + x)/256 - (-279*a**3*x + 511*a**2*b*x**3 -
 385*a*b**2*x**5 + 105*b**3*x**7)/(384*a**8 - 1536*a**7*b*x**2 + 2304*a**6
*b**2*x**4 - 1536*a**5*b**3*x**6 + 384*a**4*b**4*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a-bx^2)^5}\,dx = -\frac{105\,b^3x^7-385\,ab^2x^5+511\,a^2bx^3-279\,a^3x}{384\,(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8)} - \frac{35\,\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}a^4}$$

input
```
integrate(1/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
-1/384*(105*b^3*x^7 - 385*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/(a^4*b^4*
x^8 - 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 35/256*log((b*x
 - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a-bx^2)^5}\,dx = -\frac{35\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\sqrt{-ab}a^4} - \frac{105\,b^3x^7 - 385\,ab^2x^5 + 511\,a^2bx^3 - 279\,a^3x}{384\,(bx^2-a)^4a^4}$$

input
```
integrate(1/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
-35/128*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^4) - 1/384*(105*b^3*x^7 - 385
*a*b^2*x^5 + 511*a^2*b*x^3 - 279*a^3*x)/((b*x^2 - a)^4*a^4)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a-bx^2)^5}\,dx = \frac{\frac{93\,x}{128\,a} - \frac{511\,b\,x^3}{384\,a^2} + \frac{385\,b^2\,x^5}{384\,a^3} - \frac{35\,b^3\,x^7}{128\,a^4}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8} + \frac{35\,\mathrm{atanh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128\,a^{9/2}\,\sqrt{b}}$$

input
```
int(1/(a - b*x^2)^5,x)
```

output
```
((93*x)/(128*a) - (511*b*x^3)/(384*a^2) + (385*b^2*x^5)/(384*a^3) - (35*b^
3*x^7)/(128*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x
^4) + (35*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(9/2)*b^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.28

$$\int \frac{1}{(a-bx^2)^5}\,dx$$
$$= \frac{105\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^4-420\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^3b\,x^2+630\sqrt{b}\,\sqrt{a}\log\left(-\sqrt{b}\,\sqrt{a}-bx\right.}{}$$

input
```
int(1/(-b*x^2+a)^5,x)
```

output
```
(105*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**4 - 420*sqrt(b)*sqrt
(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**3*b*x**2 + 630*sqrt(b)*sqrt(a)*log( -
 sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 - 420*sqrt(b)*sqrt(a)*log( - sqrt(b
)*sqrt(a) - b*x)*a*b**3*x**6 + 105*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a)
- b*x)*b**4*x**8 - 105*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**4 + 4
20*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**3*b*x**2 - 630*sqrt(b)*sq
rt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 + 420*sqrt(b)*sqrt(a)*log(
sqrt(b)*sqrt(a) - b*x)*a*b**3*x**6 - 105*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(
a) - b*x)*b**4*x**8 + 558*a**4*b*x - 1022*a**3*b**2*x**3 + 770*a**2*b**3*x
**5 - 210*a*b**4*x**7)/(768*a**5*b*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2*x**
4 - 4*a*b**3*x**6 + b**4*x**8))
```

3.261 $\int \frac{1}{x(a-bx^2)^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{1}{x\,(a-bx^2)^5}\,dx = \frac{1}{8a\,(a-bx^2)^4}+\frac{1}{6a^2\,(a-bx^2)^3}+\frac{1}{4a^3\,(a-bx^2)^2}+\frac{1}{2a^4\,(a-bx^2)}+\frac{\log(x)}{a^5}-\frac{\log(a-bx^2)}{2a^5}$$

output
```
1/8/a/(-b*x^2+a)^4+1/6/a^2/(-b*x^2+a)^3+1/4/a^3/(-b*x^2+a)^2+1/2/a^4/(-b*x
^2+a)+ln(x)/a^5-1/2*ln(-b*x^2+a)/a^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{1}{x\,(a-bx^2)^5}\,dx = \frac{\frac{a(25a^3-52a^2bx^2+42ab^2x^4-12b^3x^6)}{(a-bx^2)^4}+24\log(x)-12\log(a-bx^2)}{24a^5}$$

input
```
Integrate[1/(x*(a - b*x^2)^5),x]
```

output
```
((a*(25*a^3 - 52*a^2*b*x^2 + 42*a*b^2*x^4 - 12*b^3*x^6))/(a - b*x^2)^4 + 2
4*Log[x] - 12*Log[a - b*x^2])/(24*a^5)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a-bx^2)^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(a-bx^2)^5}dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}\int \left(\frac{b}{a^5(a-bx^2)}+\frac{b}{a^4(a-bx^2)^2}+\frac{b}{a^3(a-bx^2)^3}+\frac{b}{a^2(a-bx^2)^4}+\frac{b}{a(a-bx^2)^5}+\frac{1}{a^5x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{\log(a-bx^2)}{a^5}+\frac{\log(x^2)}{a^5}+\frac{1}{a^4(a-bx^2)}+\frac{1}{2a^3(a-bx^2)^2}+\frac{1}{3a^2(a-bx^2)^3}+\frac{1}{4a(a-bx^2)^4}\right)$$

input
```
Int[1/(x*(a - b*x^2)^5),x]
```

output
```
(1/(4*a*(a - b*x^2)^4) + 1/(3*a^2*(a - b*x^2)^3) + 1/(2*a^3*(a - b*x^2)^2)
 + 1/(a^4*(a - b*x^2)) + Log[x^2]/a^5 - Log[a - b*x^2]/a^5)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
risch	$\frac{-\frac{b^3 x^6}{2a^4}+\frac{7b^2 x^4}{4a^3}-\frac{13b x^2}{6a^2}+\frac{25}{24a}}{\left(-b x^2+a\right)^4}+\frac{\ln(x)}{a^5}-\frac{\ln\left(-b x^2+a\right)}{2a^5}$
norman	$\frac{\frac{2b x^2}{a^2}-\frac{9b^2 x^4}{2a^3}+\frac{11b^3 x^6}{3a^4}-\frac{25b^4 x^8}{24a^5}}{\left(-b x^2+a\right)^4}+\frac{\ln(x)}{a^5}-\frac{\ln\left(-b x^2+a\right)}{2a^5}$
default	$\frac{b\left(\frac{a^2}{2b\left(-b x^2+a\right)^2}-\frac{\ln\left(-b x^2+a\right)}{b}+\frac{a^3}{3b\left(-b x^2+a\right)^3}+\frac{a}{b\left(-b x^2+a\right)}+\frac{a^4}{4b\left(-b x^2+a\right)^4}\right)}{2a^5}+\frac{\ln(x)}{a^5}$
parallelrisch	$\frac{24b^4 \ln(x) x^8-12\ln\left(b x^2-a\right) x^8 b^4-25b^4 x^8-96\ln(x) x^6 a b^3+48\ln\left(b x^2-a\right) x^6 a b^3+88a b^3 x^6+144\ln(x) x^4 a^2 b^2-72\ln\left(b x^2-a\right)}{24a^5\left(b x^2-a\right)^4}$

input
```
int(1/x/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*b^3/a^4*x^6+7/4*b^2/a^3*x^4-13/6*b/a^2*x^2+25/24/a)/(-b*x^2+a)^4+ln(
x)/a^5-1/2*ln(-b*x^2+a)/a^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(85) = 170$.

Time = 0.07 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.98

$$\int \frac{1}{x\left(a-bx^2\right)^5}\,dx = -\frac{12\,ab^3x^6 - 42\,a^2b^2x^4 + 52\,a^3bx^2 - 25\,a^4 + 12\left(b^4x^8 - 4\,ab^3x^6 + 6\,a^2b^2x^4 - 4\,a^3bx^2 + a^4\right)\log\left(bx^2 - a\right)}{24\left(a^5b^4x^8 - 4\,a^6b^3x^6 + 6\,a^7b^2x^4 - 4\,a^8bx^2 + a^9\right)}$$

input
```
integrate(1/x/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
-1/24*(12*a*b^3*x^6 - 42*a^2*b^2*x^4 + 52*a^3*b*x^2 - 25*a^4 + 12*(b^4*x^8
 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(b*x^2 - a) - 24*(b
^4*x^8 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4 - 4*a^3*b*x^2 + a^4)*log(x))/(a^5*b^4
*x^8 - 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 - 4*a^8*b*x^2 + a^9)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{1}{x\left(a-bx^2\right)^5}\,dx = -\frac{-25a^3 + 52a^2bx^2 - 42ab^2x^4 + 12b^3x^6}{24a^8 - 96a^7bx^2 + 144a^6b^2x^4 - 96a^5b^3x^6 + 24a^4b^4x^8} + \frac{\log\left(x\right)}{a^5} - \frac{\log\left(-\frac{a}{b} + x^2\right)}{2a^5}$$

input
```
integrate(1/x/(-b*x**2+a)**5,x)
```

output
```
-(-25*a**3 + 52*a**2*b*x**2 - 42*a*b**2*x**4 + 12*b**3*x**6)/(24*a**8 - 96
*a**7*b*x**2 + 144*a**6*b**2*x**4 - 96*a**5*b**3*x**6 + 24*a**4*b**4*x**8)
 + log(x)/a**5 - log(-a/b + x**2)/(2*a**5)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int \frac{1}{x(a-bx^2)^5}\,dx = -\frac{12\,b^3x^6-42\,ab^2x^4+52\,a^2bx^2-25\,a^3}{24\,(a^4b^4x^8-4\,a^5b^3x^6+6\,a^6b^2x^4-4\,a^7bx^2+a^8)} - \frac{\log(bx^2-a)}{2\,a^5}+\frac{\log(x^2)}{2\,a^5}$$

input
```
integrate(1/x/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
-1/24*(12*b^3*x^6 - 42*a*b^2*x^4 + 52*a^2*b*x^2 - 25*a^3)/(a^4*b^4*x^8 - 4
*a^5*b^3*x^6 + 6*a^6*b^2*x^4 - 4*a^7*b*x^2 + a^8) - 1/2*log(b*x^2 - a)/a^5
 + 1/2*log(x^2)/a^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93

$$\int \frac{1}{x(a-bx^2)^5}\,dx = \frac{\log(x^2)}{2\,a^5}-\frac{\log(|bx^2-a|)}{2\,a^5} + \frac{25\,b^4x^8-112\,ab^3x^6+192\,a^2b^2x^4-152\,a^3bx^2+50\,a^4}{24\,(bx^2-a)^4a^5}$$

input
```
integrate(1/x/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
1/2*log(x^2)/a^5 - 1/2*log(abs(b*x^2 - a))/a^5 + 1/24*(25*b^4*x^8 - 112*a*
b^3*x^6 + 192*a^2*b^2*x^4 - 152*a^3*b*x^2 + 50*a^4)/((b*x^2 - a)^4*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\,(a-bx^2)^5}\,dx = \frac{\ln(x)}{a^5} + \frac{\frac{25}{24\,a} - \frac{13\,b\,x^2}{6\,a^2} + \frac{7\,b^2\,x^4}{4\,a^3} - \frac{b^3\,x^6}{2\,a^4}}{a^4 - 4\,a^3\,b\,x^2 + 6\,a^2\,b^2\,x^4 - 4\,a\,b^3\,x^6 + b^4\,x^8} - \frac{\ln(a-b\,x^2)}{2\,a^5}$$

input
```
int(1/(x*(a - b*x^2)^5),x)
```

output
```
log(x)/a^5 + (25/(24*a) - (13*b*x^2)/(6*a^2) + (7*b^2*x^4)/(4*a^3) - (b^3*
x^6)/(2*a^4))/(a^4 + b^4*x^8 - 4*a^3*b*x^2 - 4*a*b^3*x^6 + 6*a^2*b^2*x^4)
- log(a - b*x^2)/(2*a^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.65

$$\int \frac{1}{x\,(a-bx^2)^5}\,dx = \frac{-12\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^4 + 48\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^3b\,x^2 - 72\log\left(-\sqrt{b}\,\sqrt{a}-bx\right)a^2b^2x^4 + 48\log\left(-\right.}{}$$

input
```
int(1/x/(-b*x^2+a)^5,x)
```

output
```
( - 12*log( - sqrt(b)*sqrt(a) - b*x)*a**4 + 48*log( - sqrt(b)*sqrt(a) - b*
x)*a**3*b*x**2 - 72*log( - sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 + 48*log(
 - sqrt(b)*sqrt(a) - b*x)*a*b**3*x**6 - 12*log( - sqrt(b)*sqrt(a) - b*x)*b
**4*x**8 - 12*log(sqrt(b)*sqrt(a) - b*x)*a**4 + 48*log(sqrt(b)*sqrt(a) - b
*x)*a**3*b*x**2 - 72*log(sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**4 + 48*log(sq
rt(b)*sqrt(a) - b*x)*a*b**3*x**6 - 12*log(sqrt(b)*sqrt(a) - b*x)*b**4*x**8
 + 24*log(x)*a**4 - 96*log(x)*a**3*b*x**2 + 144*log(x)*a**2*b**2*x**4 - 96
*log(x)*a*b**3*x**6 + 24*log(x)*b**4*x**8 + 22*a**4 - 40*a**3*b*x**2 + 24*
a**2*b**2*x**4 - 3*b**4*x**8)/(24*a**5*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2
*x**4 - 4*a*b**3*x**6 + b**4*x**8))
```

3.262 $\int \frac{1}{x^2(a-bx^2)^5}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 112

$$\int \frac{1}{x^2 (a-bx^2)^5}\,dx = -\frac{1}{a^5 x} + \frac{bx}{8a^2(a-bx^2)^4} + \frac{5bx}{16a^3(a-bx^2)^3} + \frac{41bx}{64a^4(a-bx^2)^2} + \frac{187bx}{128a^5(a-bx^2)} + \frac{315\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}$$

output
```
-1/a^5/x+1/8*b*x/a^2/(-b*x^2+a)^4+5/16*b*x/a^3/(-b*x^2+a)^3+41/64*b*x/a^4/
(-b*x^2+a)^2+187/128*b*x/a^5/(-b*x^2+a)+315/128*b^(1/2)*arctanh(b^(1/2)*x/
a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a-bx^2)^5}\,dx = \frac{\frac{\sqrt{a}(-128a^4+837a^3bx^2-1533a^2b^2x^4+1155ab^3x^6-315b^4x^8)}{x(a-bx^2)^4} + 315\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128a^{11/2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^5),x]
```

output
```
((Sqrt[a]*(-128*a^4 + 837*a^3*b*x^2 - 1533*a^2*b^2*x^4 + 1155*a*b^3*x^6 -
315*b^4*x^8))/(x*(a - b*x^2)^4) + 315*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a]]
)/(128*a^(11/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {253, 253, 253, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^5} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9 \int \frac{1}{x^2 (a-bx^2)^4}dx}{8a}+\frac{1}{8ax (a-bx^2)^4}$$

$$\Big\downarrow \text{253}$$

$$\frac{9 \left(\frac{7 \int \frac{1}{x^2 \left(a-bx^2\right)^3}dx}{6a}+\frac{1}{6ax(a-bx^2)^3}\right)}{8a}+\frac{1}{8ax (a-bx^2)^4}$$

$$\Big\downarrow \text{253}$$

$$\frac{9 \left(\frac{7 \left(\frac{5 \int \frac{1}{x^2 \left(a-bx^2\right)^2}dx}{4a}+\frac{1}{4ax\left(a-bx^2\right)^2}\right)}{6a}+\frac{1}{6ax(a-bx^2)^3}\right)}{8a}+\frac{1}{8ax (a-bx^2)^4}$$

$$\Big\downarrow \text{253}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{1}{x^2\left(a-bx^2\right)}dx}{2a}+\frac{1}{2ax\left(a-bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a-bx^2\right)^2}\right)}{6a}+\frac{1}{6ax(a-bx^2)^3}\right)}{8a}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

$$\Big\downarrow \; 264$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{a-bx^2}dx}{a}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a-bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a-bx^2\right)^2}\right)}{6a}+\frac{1}{6ax(a-bx^2)^3}\right)}{8a}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

$$\Big\downarrow \; 221$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}\right)}{2a}+\frac{1}{2ax\left(a-bx^2\right)}\right)}{4a}+\frac{1}{4ax\left(a-bx^2\right)^2}\right)}{6a}+\frac{1}{6ax(a-bx^2)^3}\right)}{8a}+\frac{1}{8ax\left(a-bx^2\right)^4}$$

input
```
Int[1/(x^2*(a - b*x^2)^5),x]
```

output
```
1/(8*a*x*(a - b*x^2)^4) + (9*(1/(6*a*x*(a - b*x^2)^3) + (7*(1/(4*a*x*(a -
b*x^2)^2) + (5*(1/(2*a*x*(a - b*x^2)) + (3*(-(1/(a*x)) + (Sqrt[b]*ArcTanh[
(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)))/(2*a)))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{b\left(\frac{-\frac{187}{128}b^3x^7+\frac{643}{128}a\,b^2x^5-\frac{765}{128}a^2b\,x^3+\frac{325}{128}a^3x}{\left(-b\,x^2+a\right)^4}+\frac{315\,\operatorname{arctanh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\sqrt{ab}}\right)}{a^5}-\frac{1}{a^5x}$	76
risch	$\frac{-\frac{315b^4x^8}{128a^5}+\frac{1155b^3x^6}{128a^4}-\frac{1533b^2x^4}{128a^3}+\frac{837b\,x^2}{128a^2}-\frac{1}{a}}{x\left(-b\,x^2+a\right)^4}+\frac{315\left(\sum_{\_R=\operatorname{RootOf}\left(a^{11}\_Z^2-b\right)}\_R\ln\left(\left(3\_R^2a^{11}-2b\right)x+a^6\_R\right)\right)}{256}$	104

input
```
int(1/x^2/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*b*((-187/128*b^3*x^7+643/128*a*b^2*x^5-765/128*a^2*b*x^3+325/128*a^3
*x)/(-b*x^2+a)^4+315/128/(a*b)^(1/2)*arctanh(b*x/(a*b)^(1/2)))-1/a^5/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.98

$$\int \frac{1}{x^2\left(a-bx^2\right)^5}\,dx$$

$$=\left[-\frac{630\,b^4x^8-2310\,ab^3x^6+3066\,a^2b^2x^4-1674\,a^3bx^2+256\,a^4-315\,(b^4x^9-4\,ab^3x^7+6\,a^2b^2x^5-4\,a^3b}{256\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)}\right.$$

$$-\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4+315\,(b^4x^9-4\,ab^3x^7+6\,a^2b^2x^5-4\,a^3bx^3}{128\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)}$$

input
```
integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
[-1/256*(630*b^4*x^8 - 2310*a*b^3*x^6 + 3066*a^2*b^2*x^4 - 1674*a^3*b*x^2
+ 256*a^4 - 315*(b^4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4
*x)*sqrt(b/a)*log((b*x^2 + 2*a*x*sqrt(b/a) + a)/(b*x^2 - a)))/(a^5*b^4*x^9
 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*x), -1/128*(315*b^4*x
^8 - 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 + 128*a^4 + 315*(b^
4*x^9 - 4*a*b^3*x^7 + 6*a^2*b^2*x^5 - 4*a^3*b*x^3 + a^4*x)*sqrt(-b/a)*arct
an(x*sqrt(-b/a)))/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x
^3 + a^9*x)]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^2 (a-bx^2)^5} \, dx = -\frac{315\sqrt{\frac{b}{a^{11}}}\log\left(-\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256} + \frac{315\sqrt{\frac{b}{a^{11}}}\log\left(\frac{a^6\sqrt{\frac{b}{a^{11}}}}{b}+x\right)}{256} - \frac{128a^4 - 837a^3bx^2 + 1533a^2b^2x^4 - 1155ab^3x^6 + 315b^4x^8}{128a^9x - 512a^8bx^3 + 768a^7b^2x^5 - 512a^6b^3x^7 + 128a^5b^4x^9}$$

input
```
integrate(1/x**2/(-b*x**2+a)**5,x)
```

output
```
-315*sqrt(b/a**11)*log(-a**6*sqrt(b/a**11)/b + x)/256 + 315*sqrt(b/a**11)*
log(a**6*sqrt(b/a**11)/b + x)/256 - (128*a**4 - 837*a**3*b*x**2 + 1533*a**
2*b**2*x***4 - 1155*a*b**3*x**6 + 315*b**4*x**8)/(128*a**9*x - 512*a**8*b*x
**3 + 768*a**7*b**2*x**5 - 512*a**6*b**3*x**7 + 128*a**5*b**4*x**9)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^2(a-bx^2)^5}\,dx = -\frac{315\,b^4x^8-1155\,ab^3x^6+1533\,a^2b^2x^4-837\,a^3bx^2+128\,a^4}{128\,(a^5b^4x^9-4\,a^6b^3x^7+6\,a^7b^2x^5-4\,a^8bx^3+a^9x)} - \frac{315\,b\log\left(\frac{bx-\sqrt{ab}}{bx+\sqrt{ab}}\right)}{256\,\sqrt{ab}a^5}$$

input
```
integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
-1/128*(315*b^4*x^8 - 1155*a*b^3*x^6 + 1533*a^2*b^2*x^4 - 837*a^3*b*x^2 +
128*a^4)/(a^5*b^4*x^9 - 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 - 4*a^8*b*x^3 + a^9*
x) - 315/256*b*log((b*x - sqrt(a*b))/(b*x + sqrt(a*b)))/(sqrt(a*b)*a^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2(a-bx^2)^5}\,dx = -\frac{315\,b\arctan\left(\frac{bx}{\sqrt{-ab}}\right)}{128\,\sqrt{-ab}a^5} - \frac{1}{a^5x} - \frac{187\,b^4x^7-643\,ab^3x^5+765\,a^2b^2x^3-325\,a^3bx}{128\,(bx^2-a)^4a^5}$$

input
```
integrate(1/x^2/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
-315/128*b*arctan(b*x/sqrt(-a*b))/(sqrt(-a*b)*a^5) - 1/(a^5*x) - 1/128*(18
7*b^4*x^7 - 643*a*b^3*x^5 + 765*a^2*b^2*x^3 - 325*a^3*b*x)/((b*x^2 - a)^4*
a^5)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int \frac{1}{x^2 (a-bx^2)^5} \, dx = \frac{315 \sqrt{b} \operatorname{atanh}\left(\frac{\sqrt{b}\, x}{\sqrt{a}}\right)}{128 a^{11/2}} - \frac{\frac{1}{a} - \frac{837 b x^2}{128 a^2} + \frac{1533 b^2 x^4}{128 a^3} - \frac{1155 b^3 x^6}{128 a^4} + \frac{315 b^4 x^8}{128 a^5}}{a^4 x - 4 a^3 b x^3 + 6 a^2 b^2 x^5 - 4 a b^3 x^7 + b^4 x^9}$$

input
```
int(1/(x^2*(a - b*x^2)^5),x)
```

output
```
(315*b^(1/2)*atanh((b^(1/2)*x)/a^(1/2)))/(128*a^(11/2)) - (1/a - (837*b*x^
2)/(128*a^2) + (1533*b^2*x^4)/(128*a^3) - (1155*b^3*x^6)/(128*a^4) + (315*
b^4*x^8)/(128*a^5))/(a^4*x + b^4*x^9 - 4*a^3*b*x^3 - 4*a*b^3*x^7 + 6*a^2*b
^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.01

$$\int \frac{1}{x^2 (a-bx^2)^5} \, dx = \frac{315\sqrt{b}\sqrt{a}\log\left(-\sqrt{b}\sqrt{a}-bx\right)a^4x - 1260\sqrt{b}\sqrt{a}\log\left(-\sqrt{b}\sqrt{a}-bx\right)a^3bx^3 + 1890\sqrt{b}\sqrt{a}\log\left(-\sqrt{b}\sqrt{a}\right.}{}$$

input
```
int(1/x^2/(-b*x^2+a)^5,x)
```

output
```
(315*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**4*x - 1260*sqrt(b)*s
qrt(a)*log( - sqrt(b)*sqrt(a) - b*x)*a**3*b*x**3 + 1890*sqrt(b)*sqrt(a)*lo
g( - sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**5 - 1260*sqrt(b)*sqrt(a)*log( - s
qrt(b)*sqrt(a) - b*x)*a*b**3*x**7 + 315*sqrt(b)*sqrt(a)*log( - sqrt(b)*sqr
t(a) - b*x)*b**4*x**9 - 315*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**
4*x + 1260*sqrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**3*b*x**3 - 1890*s
qrt(b)*sqrt(a)*log(sqrt(b)*sqrt(a) - b*x)*a**2*b**2*x**5 + 1260*sqrt(b)*sq
rt(a)*log(sqrt(b)*sqrt(a) - b*x)*a*b**3*x**7 - 315*sqrt(b)*sqrt(a)*log(sqr
t(b)*sqrt(a) - b*x)*b**4*x**9 - 256*a**5 + 1674*a**4*b*x**2 - 3066*a**3*b*
*2*x**4 + 2310*a**2*b**3*x**6 - 630*a*b**4*x**8)/(256*a**6*x*(a**4 - 4*a**
3*b*x**2 + 6*a**2*b**2*x**4 - 4*a*b**3*x**6 + b**4*x**8))
```

3.263 $\int \frac{1}{x^3(a-bx^2)^5} dx$

Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{x^3 (a-bx^2)^5} dx = -\frac{1}{2a^5x^2} + \frac{b}{8a^2 (a-bx^2)^4} + \frac{b}{3a^3 (a-bx^2)^3} + \frac{3b}{4a^4 (a-bx^2)^2} + \frac{2b}{a^5 (a-bx^2)} + \frac{5b\log(x)}{a^6} - \frac{5b\log(a-bx^2)}{2a^6}$$

output
```
-1/2/a^5/x^2+1/8*b/a^2/(-b*x^2+a)^4+1/3*b/a^3/(-b*x^2+a)^3+3/4*b/a^4/(-b*x
^2+a)^2+2*b/a^5/(-b*x^2+a)+5*b*ln(x)/a^6-5/2*b*ln(-b*x^2+a)/a^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^3 (a-bx^2)^5} dx = \frac{\frac{a(-12a^4+125a^3bx^2-260a^2b^2x^4+210ab^3x^6-60b^4x^8)}{x^2(a-bx^2)^4} + 120b\log(x) - 60b\log(a-bx^2)}{24a^6}$$

input
```
Integrate[1/(x^3*(a - b*x^2)^5),x]
```

output
```
((a*(-12*a^4 + 125*a^3*b*x^2 - 260*a^2*b^2*x^4 + 210*a*b^3*x^6 - 60*b^4*x^
8))/(x^2*(a - b*x^2)^4) + 120*b*Log[x] - 60*b*Log[a - b*x^2])/(24*a^6)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (a-bx^2)^5} dx^2$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2} \int \left(\frac{5b^2}{a^6 (a-bx^2)} + \frac{4b^2}{a^5 (a-bx^2)^2} + \frac{3b^2}{a^4 (a-bx^2)^3} + \frac{2b^2}{a^3 (a-bx^2)^4} + \frac{b^2}{a^2 (a-bx^2)^5} + \frac{5b}{a^6 x^2} + \frac{1}{a^5 x^4}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{5b \log (x^2)}{a^6} - \frac{5b \log (a-bx^2)}{a^6} + \frac{4b}{a^5 (a-bx^2)} - \frac{1}{a^5 x^2} + \frac{3b}{2a^4 (a-bx^2)^2} + \frac{2b}{3a^3 (a-bx^2)^3} + \frac{b}{4a^2 (a-bx^2)^4}\right)$$

input
```
Int[1/(x^3*(a - b*x^2)^5),x]
```

output
```
(-(1/(a^5*x^2)) + b/(4*a^2*(a - b*x^2)^4) + (2*b)/(3*a^3*(a - b*x^2)^3) +
(3*b)/(2*a^4*(a - b*x^2)^2) + (4*b)/(a^5*(a - b*x^2)) + (5*b*Log[x^2])/a^6
 - (5*b*Log[a - b*x^2])/a^6)/2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

method	result
risch	$\frac{-\frac{5b^4x^8}{2a^5}+\frac{35b^3x^6}{4a^4}-\frac{65b^2x^4}{6a^3}+\frac{125bx^2}{24a^2}-\frac{1}{2a}}{x^2(-bx^2+a)^4}+\frac{5b\ln(x)}{a^6}-\frac{5b\ln(-bx^2+a)}{2a^6}$
norman	$\frac{-\frac{1}{2a}+\frac{10b^2x^4}{a^3}-\frac{45b^3x^6}{2a^4}+\frac{55b^4x^8}{3a^5}-\frac{125b^5x^{10}}{24a^6}}{x^2(-bx^2+a)^4}+\frac{5b\ln(x)}{a^6}-\frac{5b\ln(-bx^2+a)}{2a^6}$
default	$\frac{b^2\left(\frac{3a^2}{2b\left(-bx^2+a\right)^2}-\frac{5\ln\left(-bx^2+a\right)}{b}+\frac{2a^3}{3b\left(-bx^2+a\right)^3}+\frac{4a}{b\left(-bx^2+a\right)}+\frac{a^4}{4b\left(-bx^2+a\right)^4}\right)}{2a^6}-\frac{1}{2a^5x^2}+\frac{5b\ln(x)}{a^6}$
parallelrisch	$\frac{120\ln(x)x^{10}b^5-60\ln(bx^2-a)x^{10}b^5-125b^5x^{10}-480ab^4\ln(x)x^8+240\ln(bx^2-a)x^8ab^4+440ab^4x^8+720\ln(x)x^6a^2b^3-360\,\mathrm{l}}{24a^6x}$

input
```
int(1/x^3/(-b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
(-5/2*b^4/a^5*x^8+35/4*b^3/a^4*x^6-65/6*b^2/a^3*x^4+125/24*b/a^2*x^2-1/2/a
)/x^2/(-b*x^2+a)^4+5*b*ln(x)/a^6-5/2*b*ln(-b*x^2+a)/a^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(100) = 200$.

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = -\frac{60\,ab^4x^8 - 210\,a^2b^3x^6 + 260\,a^3b^2x^4 - 125\,a^4bx^2 + 12\,a^5 + 60\,(b^5x^{10} - 4\,ab^4x^8 + 6\,a^2b^3x^6 - 4\,a^3b^2x^4 +}{24\,(a^6b^4x^{10} - 4\,a^7b^3x^8 + 6\,a^8b^2x^6 -}$$

input
```
integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="fricas")
```

output
```
-1/24*(60*a*b^4*x^8 - 210*a^2*b^3*x^6 + 260*a^3*b^2*x^4 - 125*a^4*b*x^2 +
12*a^5 + 60*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^3*b^2*x^4 + a^4*
b*x^2)*log(b*x^2 - a) - 120*(b^5*x^10 - 4*a*b^4*x^8 + 6*a^2*b^3*x^6 - 4*a^
3*b^2*x^4 + a^4*b*x^2)*log(x))/(a^6*b^4*x^10 - 4*a^7*b^3*x^8 + 6*a^8*b^2*x
^6 - 4*a^9*b*x^4 + a^10*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = -\frac{12a^4 - 125a^3bx^2 + 260a^2b^2x^4 - 210ab^3x^6 + 60b^4x^8}{24a^9x^2 - 96a^8bx^4 + 144a^7b^2x^6 - 96a^6b^3x^8 + 24a^5b^4x^{10}} + \frac{5b\log(x)}{a^6} - \frac{5b\log\left(-\frac{a}{b} + x^2\right)}{2a^6}$$

input
```
integrate(1/x**3/(-b*x**2+a)**5,x)
```

output
```
-(12*a**4 - 125*a**3*b*x**2 + 260*a**2*b**2*x**4 - 210*a*b**3*x**6 + 60*b*
*4*x**8)/(24*a**9*x**2 - 96*a**8*b*x**4 + 144*a**7*b**2*x**6 - 96*a**6*b**
3*x**8 + 24*a**5*b**4*x**10) + 5*b*log(x)/a**6 - 5*b*log(-a/b + x**2)/(2*a
**6)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = -\frac{60 \, b^4 x^8 - 210 \, a b^3 x^6 + 260 \, a^2 b^2 x^4 - 125 \, a^3 b x^2 + 12 \, a^4}{24 \, (a^5 b^4 x^{10} - 4 \, a^6 b^3 x^8 + 6 \, a^7 b^2 x^6 - 4 \, a^8 b x^4 + a^9 x^2)} - \frac{5 \, b \log (b x^2 - a)}{2 \, a^6} + \frac{5 \, b \log (x^2)}{2 \, a^6}$$

input
```
integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="maxima")
```

output
```
-1/24*(60*b^4*x^8 - 210*a*b^3*x^6 + 260*a^2*b^2*x^4 - 125*a^3*b*x^2 + 12*a
^4)/(a^5*b^4*x^10 - 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6 - 4*a^8*b*x^4 + a^9*x^2)
 - 5/2*b*log(b*x^2 - a)/a^6 + 5/2*b*log(x^2)/a^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = \frac{5 \, b \log (x^2)}{2 \, a^6} - \frac{5 \, b \log (|b x^2 - a|)}{2 \, a^6} - \frac{5 \, b x^2 + a}{2 \, a^6 x^2} + \frac{125 \, b^5 x^8 - 548 \, a b^4 x^6 + 912 \, a^2 b^3 x^4 - 688 \, a^3 b^2 x^2 + 202 \, a^4 b}{24 \, (b x^2 - a)^4 a^6}$$

input
```
integrate(1/x^3/(-b*x^2+a)^5,x, algorithm="giac")
```

output
```
5/2*b*log(x^2)/a^6 - 5/2*b*log(abs(b*x^2 - a))/a^6 - 1/2*(5*b*x^2 + a)/(a^
6*x^2) + 1/24*(125*b^5*x^8 - 548*a*b^4*x^6 + 912*a^2*b^3*x^4 - 688*a^3*b^2
*x^2 + 202*a^4*b)/((b*x^2 - a)^4*a^6)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = \frac{5\,b\,\ln(x)}{a^6} - \frac{5\,b\,\ln(a-b\,x^2)}{2\,a^6} - \frac{\frac{1}{2\,a} - \frac{125\,b\,x^2}{24\,a^2} + \frac{65\,b^2\,x^4}{6\,a^3} - \frac{35\,b^3\,x^6}{4\,a^4} + \frac{5\,b^4\,x^8}{2\,a^5}}{a^4\,x^2 - 4\,a^3\,b\,x^4 + 6\,a^2\,b^2\,x^6 - 4\,a\,b^3\,x^8 + b^4\,x^{10}}$$

input
```
int(1/(x^3*(a - b*x^2)^5),x)
```

output
```
(5*b*log(x))/a^6 - (5*b*log(a - b*x^2))/(2*a^6) - (1/(2*a) - (125*b*x^2)/(
24*a^2) + (65*b^2*x^4)/(6*a^3) - (35*b^3*x^6)/(4*a^4) + (5*b^4*x^8)/(2*a^5
))/(a^4*x^2 + b^4*x^10 - 4*a^3*b*x^4 - 4*a*b^3*x^8 + 6*a^2*b^2*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.43

$$\int \frac{1}{x^3 (a-bx^2)^5} \, dx = \frac{-60 \log\left(-\sqrt{b}\,\sqrt{a} - bx\right) a^4 b\, x^2 + 240 \log\left(-\sqrt{b}\,\sqrt{a} - bx\right) a^3 b^2 x^4 - 360 \log\left(-\sqrt{b}\,\sqrt{a} - bx\right) a^2 b^3 x^6 + 240}{}$$

input
```
int(1/x^3/(-b*x^2+a)^5,x)
```

output
```
( - 60*log( - sqrt(b)*sqrt(a) - b*x)*a**4*b*x**2 + 240*log( - sqrt(b)*sqrt
(a) - b*x)*a**3*b**2*x**4 - 360*log( - sqrt(b)*sqrt(a) - b*x)*a**2*b**3*x*
*6 + 240*log( - sqrt(b)*sqrt(a) - b*x)*a*b**4*x**8 - 60*log( - sqrt(b)*sqr
t(a) - b*x)*b**5*x**10 - 60*log(sqrt(b)*sqrt(a) - b*x)*a**4*b*x**2 + 240*l
og(sqrt(b)*sqrt(a) - b*x)*a**3*b**2*x**4 - 360*log(sqrt(b)*sqrt(a) - b*x)*
a**2*b**3*x**6 + 240*log(sqrt(b)*sqrt(a) - b*x)*a*b**4*x**8 - 60*log(sqrt(
b)*sqrt(a) - b*x)*b**5*x**10 + 120*log(x)*a**4*b*x**2 - 480*log(x)*a**3*b*
*2*x**4 + 720*log(x)*a**2*b**3*x**6 - 480*log(x)*a*b**4*x**8 + 120*log(x)*
b**5*x**10 - 12*a**5 + 110*a**4*b*x**2 - 200*a**3*b**2*x**4 + 120*a**2*b**
3*x**6 - 15*b**5*x**10)/(24*a**6*x**2*(a**4 - 4*a**3*b*x**2 + 6*a**2*b**2*
x**4 - 4*a*b**3*x**6 + b**4*x**8))
```

3.264 $\int x^{7/2}(a+bx^2)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int x^{7/2}\left(a+bx^2\right)\,dx=\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

output
```
2/9*a*x^(9/2)+2/13*b*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{7/2}\left(a+bx^2\right)\,dx=\frac{2}{117}\left(13ax^{9/2}+9bx^{13/2}\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^2),x]
```

output
```
(2*(13*a*x^(9/2) + 9*b*x^(13/2)))/117
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(a+bx^2)\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(ax^{7/2}+bx^{11/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}ax^{9/2}+\frac{2}{13}bx^{13/2}$$

input
```
Int[x^(7/2)*(a + b*x^2),x]
```

output
```
(2*a*x^(9/2))/9 + (2*b*x^(13/2))/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{9}{2}}}{9}+\frac{2b\,x^{\frac{13}{2}}}{13}$	14
default	$\frac{2a\,x^{\frac{9}{2}}}{9}+\frac{2b\,x^{\frac{13}{2}}}{13}$	14
gosper	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16
trager	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16
risch	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16
orering	$\frac{2x^{\frac{9}{2}}\left(9b\,x^{2}+13a\right)}{117}$	16

input
```
int(x^(7/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/9*a*x^(9/2)+2/13*b*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{7/2}\left(a+bx^2\right)\,dx=\frac{2}{117}\left(9\,bx^6+13\,ax^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a),x, algorithm="fricas")
```

output
```
2/117*(9*b*x^6 + 13*a*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{7/2}(a+bx^2)\;dx = \frac{2ax^{\frac{9}{2}}}{9} + \frac{2bx^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(7/2)*(b*x**2+a),x)
```

output
```
2*a*x**(9/2)/9 + 2*b*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/2}(a+bx^2)\;dx = \frac{2}{13}\,bx^{\frac{13}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a),x, algorithm="maxima")
```

output
```
2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{7/2}(a+bx^2)\;dx = \frac{2}{13}\,bx^{\frac{13}{2}} + \frac{2}{9}\,ax^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a),x, algorithm="giac")
```

output
```
2/13*b*x^(13/2) + 2/9*a*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{7/2}\left(a+bx^2\right)\,dx=\frac{2\,x^{9/2}\,\left(9\,b\,x^2+13\,a\right)}{117}$$

input
```
int(x^(7/2)*(a + b*x^2),x)
```

output
```
(2*x^(9/2)*(13*a + 9*b*x^2))/117
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(a+bx^2\right)\,dx=\frac{2\sqrt{x}\,x^4(9b\,x^2+13a)}{117}$$

input
```
int(x^(7/2)*(b*x^2+a),x)
```

output
```
(2*sqrt(x)*x**4*(13*a + 9*b*x**2))/117
```

3.265 $\int x^{5/2}(a+bx^2)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int x^{5/2}\left(a+bx^2\right)\,dx=\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

output
```
2/7*a*x^(7/2)+2/11*b*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/2}\left(a+bx^2\right)\,dx=\frac{2}{77}x^{7/2}\left(11a+7bx^2\right)$$

input
```
Integrate[x^(5/2)*(a + b*x^2),x]
```

output
```
(2*x^(7/2)*(11*a + 7*b*x^2))/77
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(ax^{5/2}+bx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}ax^{7/2}+\frac{2}{11}bx^{11/2}$$

input
```
Int[x^(5/2)*(a + b*x^2),x]
```

output
```
(2*a*x^(7/2))/7 + (2*b*x^(11/2))/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
default	$\frac{2a\,x^{\frac{7}{2}}}{7}+\frac{2b\,x^{\frac{11}{2}}}{11}$	14
gosper	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16
trager	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16
risch	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16
orering	$\frac{2x^{\frac{7}{2}}\left(7b\,x^{2}+11a\right)}{77}$	16

input
```
int(x^(5/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/7*a*x^(7/2)+2/11*b*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{5/2}\left(a+bx^2\right)\,dx=\frac{2}{77}\left(7\,bx^5+11\,ax^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a),x, algorithm="fricas")
```

output
```
2/77*(7*b*x^5 + 11*a*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{5/2}(a+bx^2)\,dx = \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(5/2)*(b*x**2+a),x)
```

output
```
2*a*x**(7/2)/7 + 2*b*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+bx^2)\,dx = \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a),x, algorithm="maxima")
```

output
```
2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{5/2}(a+bx^2)\,dx = \frac{2}{11}\,bx^{\frac{11}{2}} + \frac{2}{7}\,ax^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a),x, algorithm="giac")
```

output
```
2/11*b*x^(11/2) + 2/7*a*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{5/2}\left(a+bx^2\right)\,dx=\frac{2\,x^{7/2}\,\left(7\,b\,x^2+11\,a\right)}{77}$$

input
```
int(x^(5/2)*(a + b*x^2),x)
```

output
```
(2*x^(7/2)*(11*a + 7*b*x^2))/77
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^2\right)\,dx=\frac{2\sqrt{x}\,x^3(7b\,x^2+11a)}{77}$$

input
```
int(x^(5/2)*(b*x^2+a),x)
```

output
```
(2*sqrt(x)*x**3*(11*a + 7*b*x**2))/77
```

3.266 $\int x^{3/2}(a+bx^2)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int x^{3/2}(a+bx^2)\,dx=\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

output
```
2/5*a*x^(5/2)+2/9*b*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{3/2}(a+bx^2)\,dx=\frac{2}{45}\left(9ax^{5/2}+5bx^{9/2}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^2),x]
```

output
```
(2*(9*a*x^(5/2) + 5*b*x^(9/2)))/45
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)\,dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(ax^{3/2}+bx^{7/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{5}ax^{5/2}+\frac{2}{9}bx^{9/2}$$

input
```
Int[x^(3/2)*(a + b*x^2),x]
```

output
```
(2*a*x^(5/2))/5 + (2*b*x^(9/2))/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
default	$\frac{2a\,x^{\frac{5}{2}}}{5}+\frac{2b\,x^{\frac{9}{2}}}{9}$	14
gosper	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16
trager	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16
risch	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16
orering	$\frac{2x^{\frac{5}{2}}\left(5b\,x^{2}+9a\right)}{45}$	16

input
```
int(x^(3/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/5*a*x^(5/2)+2/9*b*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int x^{3/2}\left(a+bx^2\right)\,dx=\frac{2}{45}\left(5\,bx^4+9\,ax^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a),x, algorithm="fricas")
```

output
```
2/45*(5*b*x^4 + 9*a*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int x^{3/2}\left(a+bx^2\right)\,dx=\frac{2ax^{\frac{5}{2}}}{5}+\frac{2bx^{\frac{9}{2}}}{9}$$

input
```
integrate(x**(3/2)*(b*x**2+a),x)
```

output
```
2*a*x**(5/2)/5 + 2*b*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}\left(a+bx^2\right)\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a),x, algorithm="maxima")
```

output
```
2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int x^{3/2}\left(a+bx^2\right)\,dx=\frac{2}{9}\,bx^{\frac{9}{2}}+\frac{2}{5}\,ax^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a),x, algorithm="giac")
```

output
```
2/9*b*x^(9/2) + 2/5*a*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int x^{3/2}(a+bx^2)\ dx=\frac{2\,x^{5/2}\,(5\,b\,x^2+9\,a)}{45}$$

input
```
int(x^(3/2)*(a + b*x^2),x)
```

output
```
(2*x^(5/2)*(9*a + 5*b*x^2))/45
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int x^{3/2}(a+bx^2)\ dx=\frac{2\sqrt{x}\,x^2(5b\,x^2+9a)}{45}$$

input
```
int(x^(3/2)*(b*x^2+a),x)
```

output
```
(2*sqrt(x)*x**2*(9*a + 5*b*x**2))/45
```

3.267 $\int \sqrt{x}(a+bx^2)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \sqrt{x}(a+bx^2)\,dx=\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

output
```
2/3*a*x^(3/2)+2/7*b*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \sqrt{x}(a+bx^2)\,dx=\frac{2}{21}\left(7ax^{3/2}+3bx^{7/2}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^2),x]
```

output
```
(2*(7*a*x^(3/2) + 3*b*x^(7/2)))/21
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right) \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a\sqrt{x}+bx^{5/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}ax^{3/2}+\frac{2}{7}bx^{7/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2),x]
```

output
```
(2*a*x^(3/2))/3 + (2*b*x^(7/2))/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{2a x^{\frac{3}{2}}}{3}+\frac{2b x^{\frac{7}{2}}}{7}$	14
default	$\frac{2a x^{\frac{3}{2}}}{3}+\frac{2b x^{\frac{7}{2}}}{7}$	14
gosper	$\frac{2x^{\frac{3}{2}}\left(3b x^{2}+7a\right)}{21}$	16
trager	$\frac{2x^{\frac{3}{2}}\left(3b x^{2}+7a\right)}{21}$	16
risch	$\frac{2x^{\frac{3}{2}}\left(3b x^{2}+7a\right)}{21}$	16
orering	$\frac{2x^{\frac{3}{2}}\left(3b x^{2}+7a\right)}{21}$	16

input
```
int(x^(1/2)*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*x^(3/2)+2/7*b*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \sqrt{x}\left(a+bx^2\right)\,dx=\frac{2}{21}\left(3\,bx^3+7\,ax\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x^2+a),x, algorithm="fricas")
```

output
```
2/21*(3*b*x^3 + 7*a*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sqrt{x}(a+bx^2)\,dx=\frac{2ax^{\frac{3}{2}}}{3}+\frac{2bx^{\frac{7}{2}}}{7}$$

input
```
integrate(x**(1/2)*(b*x**2+a),x)
```

output
```
2*a*x**(3/2)/3 + 2*b*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx^2)\,dx=\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a),x, algorithm="maxima")
```

output
```
2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \sqrt{x}(a+bx^2)\,dx=\frac{2}{7}\,bx^{\frac{7}{2}}+\frac{2}{3}\,ax^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a),x, algorithm="giac")
```

output
```
2/7*b*x^(7/2) + 2/3*a*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}(a+bx^2)\ dx = \frac{2\,x^{3/2}\,(3\,b\,x^2+7\,a)}{21}$$

input
```
int(x^(1/2)*(a + b*x^2),x)
```

output
```
(2*x^(3/2)*(7*a + 3*b*x^2))/21
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sqrt{x}(a+bx^2)\ dx = \frac{2\sqrt{x}\,x(3b\,x^2+7a)}{21}$$

input
```
int(x^(1/2)*(b*x^2+a),x)
```

output
```
(2*sqrt(x)*x*(7*a + 3*b*x**2))/21
```

3.268 $\int \frac{a+bx^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+bx^2}{\sqrt{x}} \, dx = 2a\sqrt{x} + \frac{2}{5}bx^{5/2}$$

output
```
2*a*x^(1/2)+2/5*b*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{a+bx^2}{\sqrt{x}} \, dx = \frac{2}{5}\left(5a\sqrt{x} + bx^{5/2}\right)$$

input
```
Integrate[(a + b*x^2)/Sqrt[x],x]
```

output
```
(2*(5*a*Sqrt[x] + b*x^(5/2)))/5
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a}{\sqrt{x}}+bx^{3/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a\sqrt{x}+\frac{2}{5}bx^{5/2}$$

input
```
Int[(a + b*x^2)/Sqrt[x],x]
```

output
```
2*a*Sqrt[x] + (2*b*x^(5/2))/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2a\sqrt{x}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
default	$2a\sqrt{x}+\frac{2b\,x^{\frac{5}{2}}}{5}$	14
gosper	$\frac{2\sqrt{x}\,(b\,x^{2}+5a)}{5}$	15
trager	$\left(\frac{2b\,x^{2}}{5}+2a\right)\sqrt{x}$	15
risch	$\frac{2\sqrt{x}\,(b\,x^{2}+5a)}{5}$	15
orering	$\frac{2\sqrt{x}\,(b\,x^{2}+5a)}{5}$	15

input
```
int((b*x^2+a)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*x^(1/2)+2/5*b*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx=\frac{2}{5}\left(bx^2+5\,a\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)/x^(1/2),x, algorithm="fricas")
```

output
```
2/5*(b*x^2 + 5*a)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx = 2a\sqrt{x} + \frac{2bx^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x**2+a)/x**(1/2),x)
```

output
```
2*a*sqrt(x) + 2*b*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx = \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((b*x^2+a)/x^(1/2),x, algorithm="maxima")
```

output
```
2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx = \frac{2}{5}\,bx^{\frac{5}{2}} + 2\,a\sqrt{x}$$

input
```
integrate((b*x^2+a)/x^(1/2),x, algorithm="giac")
```

output
```
2/5*b*x^(5/2) + 2*a*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(b\,x^2+5\,a)}{5}$$

input
```
int((a + b*x^2)/x^(1/2),x)
```

output
```
(2*x^(1/2)*(5*a + b*x^2))/5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(b\,x^2+5a)}{5}$$

input
```
int((b*x^2+a)/x^(1/2),x)
```

output
```
(2*sqrt(x)*(5*a + b*x**2))/5
```

3.269 $\int \frac{a+bx^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+bx^2}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}}+\frac{2}{3}bx^{3/2}$$

output
```
-2*a/x^(1/2)+2/3*b*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^{3/2}}\,dx = -\frac{2(3a-bx^2)}{3\sqrt{x}}$$

input
```
Integrate[(a + b*x^2)/x^(3/2),x]
```

output
```
(-2*(3*a - b*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^{3/2}}\,dx$$
$$\Big\downarrow \; 244$$
$$\int \left(\frac{a}{x^{3/2}}+b\sqrt{x}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{2}{3}bx^{3/2}-\frac{2a}{\sqrt{x}}$$

input
```
Int[(a + b*x^2)/x^(3/2),x]
```

output
```
(-2*a)/Sqrt[x] + (2*b*x^(3/2))/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a}{\sqrt{x}}+\frac{2b\,x^{\frac{3}{2}}}{3}$	14
default	$-\frac{2a}{\sqrt{x}}+\frac{2b\,x^{\frac{3}{2}}}{3}$	14
gosper	$-\frac{2(-b\,x^2+3a)}{3\sqrt{x}}$	16
trager	$-\frac{2(-b\,x^2+3a)}{3\sqrt{x}}$	16
risch	$-\frac{2(-b\,x^2+3a)}{3\sqrt{x}}$	16
orering	$-\frac{2(-b\,x^2+3a)}{3\sqrt{x}}$	16

input
```
int((b*x^2+a)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a/x^(1/2)+2/3*b*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{a+bx^2}{x^{3/2}}\,dx=\frac{2\,(bx^2-3\,a)}{3\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)/x^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b*x^2 - 3*a)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^{3/2}}\,dx = -\frac{2a}{\sqrt{x}} + \frac{2bx^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x**2+a)/x**(3/2),x)
```

output
```
-2*a/sqrt(x) + 2*b*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{3/2}}\,dx = \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)/x^(3/2),x, algorithm="maxima")
```

output
```
2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{3/2}}\,dx = \frac{2}{3}\,bx^{\frac{3}{2}} - \frac{2\,a}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)/x^(3/2),x, algorithm="giac")
```

output
```
2/3*b*x^(3/2) - 2*a/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^{3/2}}\,dx=-\frac{6\,a-2\,b\,x^2}{3\,\sqrt{x}}$$

input
```
int((a + b*x^2)/x^(3/2),x)
```

output
```
-(6*a - 2*b*x^2)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^{3/2}}\,dx=\frac{\frac{2b\,x^2}{3}-2a}{\sqrt{x}}$$

input
```
int((b*x^2+a)/x^(3/2),x)
```

output
```
(2*( - 3*a + b*x**2))/(3*sqrt(x))
```

3.270 $\int \frac{a+bx^2}{x^{5/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+bx^2}{x^{5/2}} \, dx = -\frac{2a}{3x^{3/2}} + 2b\sqrt{x}$$

output
```
-2/3*a/x^(3/2)+2*b*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^{5/2}} \, dx = -\frac{2(a-3bx^2)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x^2)/x^(5/2),x]
```

output
```
(-2*(a - 3*b*x^2))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^{5/2}}\,dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a}{x^{5/2}}+\frac{b}{\sqrt{x}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$2b\sqrt{x}-\frac{2a}{3x^{3/2}}$$

input
```
Int[(a + b*x^2)/x^(5/2),x]
```

output
```
(-2*a)/(3*x^(3/2)) + 2*b*Sqrt[x]
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14
derivativedivides	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$	14
default	$-\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$	14
trager	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14
risch	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14
orering	$-\frac{2(-3b\,x^2+a)}{3x^{\frac{3}{2}}}$	14

input
```
int((b*x^2+a)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*x^2+a)/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = \frac{2\,(3\,bx^2-a)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b*x^2 - a)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = -\frac{2a}{3x^{\frac{3}{2}}}+2b\sqrt{x}$$

input
```
integrate((b*x**2+a)/x**(5/2),x)
```

output
```
-2*a/(3*x**(3/2)) + 2*b*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = 2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)/x^(5/2),x, algorithm="maxima")
```

output
```
2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = 2\,b\sqrt{x}-\frac{2\,a}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)/x^(5/2),x, algorithm="giac")
```

output
```
2*b*sqrt(x) - 2/3*a/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = -\frac{2\,a-6\,b\,x^2}{3\,x^{3/2}}$$

input
```
int((a + b*x^2)/x^(5/2),x)
```

output
```
-(2*a - 6*b*x^2)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^{5/2}}\,dx = \frac{2b\,x^2-\frac{2a}{3}}{\sqrt{x}\,x}$$

input
```
int((b*x^2+a)/x^(5/2),x)
```

output
```
(2*( - a + 3*b*x**2))/(3*sqrt(x)*x)
```

3.271 $\int \frac{a+bx^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{2a}{5x^{5/2}} - \frac{2b}{\sqrt{x}}$$

output
```
-2/5*a/x^(5/2)-2*b/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{2(a+5bx^2)}{5x^{5/2}}$$

input
```
Integrate[(a + b*x^2)/x^(7/2),x]
```

output
```
(-2*(a + 5*b*x^2))/(5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx^2}{x^{7/2}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a}{x^{7/2}}+\frac{b}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a}{5x^{5/2}}-\frac{2b}{\sqrt{x}}$$

input
```
Int[(a + b*x^2)/x^(7/2),x]
```

output
```
(-2*a)/(5*x^(5/2)) - (2*b)/Sqrt[x]
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
gosper	$-\frac{2(5b\,x^{2}+a)}{5x^{\frac{5}{2}}}$	14
derivativedivides	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$	14
default	$-\frac{2a}{5x^{\frac{5}{2}}}-\frac{2b}{\sqrt{x}}$	14
trager	$-\frac{2(5b\,x^{2}+a)}{5x^{\frac{5}{2}}}$	14
risch	$-\frac{2(5b\,x^{2}+a)}{5x^{\frac{5}{2}}}$	14
orering	$-\frac{2(5b\,x^{2}+a)}{5x^{\frac{5}{2}}}$	14

input
```
int((b*x^2+a)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(5*b*x^2+a)/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{7/2}}\,dx=-\frac{2\left(5\,bx^2+a\right)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)/x^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(5*b*x^2 + a)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{2a}{5x^{\frac{5}{2}}} - \frac{2b}{\sqrt{x}}$$

input
```
integrate((b*x**2+a)/x**(7/2),x)
```

output
```
-2*a/(5*x**(5/2)) - 2*b/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)/x^(7/2),x, algorithm="maxima")
```

output
```
-2/5*(5*b*x^2 + a)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{2\,(5\,bx^2+a)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)/x^(7/2),x, algorithm="giac")
```

output
```
-2/5*(5*b*x^2 + a)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = -\frac{10\,b\,x^2+2\,a}{5\,x^{5/2}}$$

input
```
int((a + b*x^2)/x^(7/2),x)
```

output
```
-(2*a + 10*b*x^2)/(5*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{a+bx^2}{x^{7/2}}\,dx = \frac{-2b\,x^2-\frac{2a}{5}}{\sqrt{x}\,x^2}$$

input
```
int((b*x^2+a)/x^(7/2),x)
```

output
```
(2*( - a - 5*b*x**2))/(5*sqrt(x)*x**2)
```

3.272 $\int x^{7/2}(a+bx^2)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int x^{7/2}\left(a+bx^2\right)^2\,dx=\frac{2}{9}a^2x^{9/2}+\frac{4}{13}abx^{13/2}+\frac{2}{17}b^2x^{17/2}$$

output
```
2/9*a^2*x^(9/2)+4/13*a*b*x^(13/2)+2/17*b^2*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{7/2}\left(a+bx^2\right)^2\,dx=\frac{2x^{9/2}(221a^2+306abx^2+117b^2x^4)}{1989}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^2,x]
```

output
```
(2*x^(9/2)*(221*a^2 + 306*a*b*x^2 + 117*b^2*x^4))/1989
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2} \left(a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2 x^{7/2}+2 a b x^{11/2}+b^2 x^{15/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9} a^2 x^{9/2}+\frac{4}{13} a b x^{13/2}+\frac{2}{17} b^2 x^{17/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^2,x]
```

output
```
(2*a^2*x^(9/2))/9 + (4*a*b*x^(13/2))/13 + (2*b^2*x^(17/2))/17
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4ab\,x^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}$	25
default	$\frac{2a^2x^{\frac{9}{2}}}{9}+\frac{4ab\,x^{\frac{13}{2}}}{13}+\frac{2b^2x^{\frac{17}{2}}}{17}$	25
gosper	$\frac{2x^{\frac{9}{2}}\left(117b^2x^4+306ab\,x^2+221a^2\right)}{1989}$	27
trager	$\frac{2x^{\frac{9}{2}}\left(117b^2x^4+306ab\,x^2+221a^2\right)}{1989}$	27
risch	$\frac{2x^{\frac{9}{2}}\left(117b^2x^4+306ab\,x^2+221a^2\right)}{1989}$	27
orering	$\frac{2x^{\frac{9}{2}}\left(117b^2x^4+306ab\,x^2+221a^2\right)}{1989}$	27

input
```
int(x^(7/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9*a^2*x^(9/2)+4/13*a*b*x^(13/2)+2/17*b^2*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{7/2}\left(a+bx^2\right)^2\,dx=\frac{2}{1989}\left(117\,b^2x^8+306\,abx^6+221\,a^2x^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/1989*(117*b^2*x^8 + 306*a*b*x^6 + 221*a^2*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{7/2}(a+bx^2)^2\,dx = \frac{2a^2x^{\frac{9}{2}}}{9} + \frac{4abx^{\frac{13}{2}}}{13} + \frac{2b^2x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**2,x)
```

output
```
2*a**2*x**(9/2)/9 + 4*a*b*x**(13/2)/13 + 2*b**2*x**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{7/2}(a+bx^2)^2\,dx = \frac{2}{17}\,b^2x^{\frac{17}{2}} + \frac{4}{13}\,abx^{\frac{13}{2}} + \frac{2}{9}\,a^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{7/2}(a+bx^2)^2\,dx = \frac{2}{17}\,b^2x^{\frac{17}{2}} + \frac{4}{13}\,abx^{\frac{13}{2}} + \frac{2}{9}\,a^2x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/17*b^2*x^(17/2) + 4/13*a*b*x^(13/2) + 2/9*a^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int x^{7/2} \left(a+bx^2\right)^2 \, dx = x^{9/2} \left(\frac{2\,a^2}{9}+\frac{4\,a\,b\,x^2}{13}+\frac{2\,b^2\,x^4}{17}\right)$$

input
```
int(x^(7/2)*(a + b*x^2)^2,x)
```

output
```
x^(9/2)*((2*a^2)/9 + (2*b^2*x^4)/17 + (4*a*b*x^2)/13)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{7/2} \left(a+bx^2\right)^2 \, dx = \frac{2\sqrt{x}\,x^4(117b^2x^4+306ab\,x^2+221a^2)}{1989}$$

input
```
int(x^(7/2)*(b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x**4*(221*a**2 + 306*a*b*x**2 + 117*b**2*x**4))/1989
```

3.273 $\int x^{5/2}(a+bx^2)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

output
```
2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*b^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2x^{7/2}(165a^2+210abx^2+77b^2x^4)}{1155}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^2,x]
```

output
```
(2*x^(7/2)*(165*a^2 + 210*a*b*x^2 + 77*b^2*x^4))/1155
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^2x^{5/2}+2abx^{9/2}+b^2x^{13/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}a^2x^{7/2}+\frac{4}{11}abx^{11/2}+\frac{2}{15}b^2x^{15/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^2,x]
```

output
```
(2*a^2*x^(7/2))/7 + (4*a*b*x^(11/2))/11 + (2*b^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2b^{2}x^{\frac{15}{2}}}{15}$	25
default	$\frac{2a^{2}x^{\frac{7}{2}}}{7}+\frac{4ab\,x^{\frac{11}{2}}}{11}+\frac{2b^{2}x^{\frac{15}{2}}}{15}$	25
gosper	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27
trager	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27
risch	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27
orering	$\frac{2x^{\frac{7}{2}}\left(77b^{2}x^{4}+210ab\,x^{2}+165a^{2}\right)}{1155}$	27

input
```
int(x^(5/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^2*x^(7/2)+4/11*a*b*x^(11/2)+2/15*b^2*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{5/2}\left(a+bx^{2}\right)^{2}\,dx=\frac{2}{1155}\left(77\,b^{2}x^{7}+210\,abx^{5}+165\,a^{2}x^{3}\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/1155*(77*b^2*x^7 + 210*a*b*x^5 + 165*a^2*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2a^2x^{\frac{7}{2}}}{7}+\frac{4abx^{\frac{11}{2}}}{11}+\frac{2b^2x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**2,x)
```

output
```
2*a**2*x**(7/2)/7 + 4*a*b*x**(11/2)/11 + 2*b**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2}{15}\,b^2x^{\frac{15}{2}}+\frac{4}{11}\,abx^{\frac{11}{2}}+\frac{2}{7}\,a^2x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/15*b^2*x^(15/2) + 4/11*a*b*x^(11/2) + 2/7*a^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2\,x^{7/2}\,\left(165\,a^2+210\,a\,b\,x^2+77\,b^2\,x^4\right)}{1155}$$

input
```
int(x^(5/2)*(a + b*x^2)^2,x)
```

output
```
(2*x^(7/2)*(165*a^2 + 77*b^2*x^4 + 210*a*b*x^2))/1155
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(a+bx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^3(77b^2x^4+210ab\,x^2+165a^2)}{1155}$$

input
```
int(x^(5/2)*(b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x**3*(165*a**2 + 210*a*b*x**2 + 77*b**2*x**4))/1155
```

3.274 $\int x^{3/2}(a+bx^2)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int x^{3/2}\left(a+bx^2\right)^2\,dx=\frac{2}{5}a^2x^{5/2}+\frac{4}{9}abx^{9/2}+\frac{2}{13}b^2x^{13/2}$$

output
```
2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*b^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int x^{3/2}\left(a+bx^2\right)^2\,dx=\frac{2}{585}x^{5/2}\left(117a^2+130abx^2+45b^2x^4\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^2,x]
```

output
```
(2*x^(5/2)*(117*a^2 + 130*a*b*x^2 + 45*b^2*x^4))/585
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2 x^{3/2}+2 a b x^{7/2}+b^2 x^{11/2}\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5} a^2 x^{5/2}+\frac{4}{9} a b x^{9/2}+\frac{2}{13} b^2 x^{13/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^2,x]
```

output
```
(2*a^2*x^(5/2))/5 + (4*a*b*x^(9/2))/9 + (2*b^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}$	25
default	$\frac{2a^2x^{\frac{5}{2}}}{5}+\frac{4ab\,x^{\frac{9}{2}}}{9}+\frac{2b^2x^{\frac{13}{2}}}{13}$	25
gosper	$\frac{2x^{\frac{5}{2}}\left(45b^2x^4+130ab\,x^2+117a^2\right)}{585}$	27
trager	$\frac{2x^{\frac{5}{2}}\left(45b^2x^4+130ab\,x^2+117a^2\right)}{585}$	27
risch	$\frac{2x^{\frac{5}{2}}\left(45b^2x^4+130ab\,x^2+117a^2\right)}{585}$	27
orering	$\frac{2x^{\frac{5}{2}}\left(45b^2x^4+130ab\,x^2+117a^2\right)}{585}$	27

input
```
int(x^(3/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^2*x^(5/2)+4/9*a*b*x^(9/2)+2/13*b^2*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int x^{3/2}\left(a+bx^2\right)^2\,dx=\frac{2}{585}\left(45\,b^2x^6+130\,abx^4+117\,a^2x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/585*(45*b^2*x^6 + 130*a*b*x^4 + 117*a^2*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int x^{3/2}(a+bx^2)^2\,dx = \frac{2a^2x^{\frac{5}{2}}}{5} + \frac{4abx^{\frac{9}{2}}}{9} + \frac{2b^2x^{\frac{13}{2}}}{13}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**2,x)
```

output
```
2*a**2*x**(5/2)/5 + 4*a*b*x**(9/2)/9 + 2*b**2*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx^2)^2\,dx = \frac{2}{13}\,b^2x^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int x^{3/2}(a+bx^2)^2\,dx = \frac{2}{13}\,b^2x^{\frac{13}{2}} + \frac{4}{9}\,abx^{\frac{9}{2}} + \frac{2}{5}\,a^2x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/13*b^2*x^(13/2) + 4/9*a*b*x^(9/2) + 2/5*a^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int x^{3/2}\left(a+bx^2\right)^2\,dx=\frac{2\,x^{5/2}\,\left(117\,a^2+130\,a\,b\,x^2+45\,b^2\,x^4\right)}{585}$$

input
```
int(x^(3/2)*(a + b*x^2)^2,x)
```

output
```
(2*x^(5/2)*(117*a^2 + 45*b^2*x^4 + 130*a*b*x^2))/585
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(a+bx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^2(45b^2x^4+130ab\,x^2+117a^2)}{585}$$

input
```
int(x^(3/2)*(b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x**2*(117*a**2 + 130*a*b*x**2 + 45*b**2*x**4))/585
```

3.275 $\int \sqrt{x}(a+bx^2)^2 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \sqrt{x}(a+bx^2)^2 \, dx = \frac{2}{3}a^2x^{3/2} + \frac{4}{7}abx^{7/2} + \frac{2}{11}b^2x^{11/2}$$

output
```
2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*b^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sqrt{x}(a+bx^2)^2 \, dx = \frac{2}{231}x^{3/2}(77a^2 + 66abx^2 + 21b^2x^4)$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^2,x]
```

output
```
(2*x^(3/2)*(77*a^2 + 66*a*b*x^2 + 21*b^2*x^4))/231
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2\sqrt{x}+2abx^{5/2}+b^2x^{9/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^2x^{3/2}+\frac{4}{7}abx^{7/2}+\frac{2}{11}b^2x^{11/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^2,x]
```

output
```
(2*a^2*x^(3/2))/3 + (4*a*b*x^(7/2))/7 + (2*b^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{11}{2}}}{11}$	25
default	$\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4ab\,x^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{11}{2}}}{11}$	25
gosper	$\frac{2x^{\frac{3}{2}}(21b^2x^4+66ab\,x^2+77a^2)}{231}$	27
trager	$\frac{2x^{\frac{3}{2}}(21b^2x^4+66ab\,x^2+77a^2)}{231}$	27
risch	$\frac{2x^{\frac{3}{2}}(21b^2x^4+66ab\,x^2+77a^2)}{231}$	27
orering	$\frac{2x^{\frac{3}{2}}(21b^2x^4+66ab\,x^2+77a^2)}{231}$	27

input
```
int(x^(1/2)*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*a^2*x^(3/2)+4/7*a*b*x^(7/2)+2/11*b^2*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2}{231}\left(21\,b^2x^5+66\,abx^3+77\,a^2x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
2/231*(21*b^2*x^5 + 66*a*b*x^3 + 77*a^2*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2a^2x^{\frac{3}{2}}}{3}+\frac{4abx^{\frac{7}{2}}}{7}+\frac{2b^2x^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(1/2)*(b*x**2+a)**2,x)
```

output
```
2*a**2*x**(3/2)/3 + 4*a*b*x**(7/2)/7 + 2*b**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2}{11}\,b^2x^{\frac{11}{2}}+\frac{4}{7}\,abx^{\frac{7}{2}}+\frac{2}{3}\,a^2x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
2/11*b^2*x^(11/2) + 4/7*a*b*x^(7/2) + 2/3*a^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2\,x^{3/2}\,\left(77\,a^2+66\,a\,b\,x^2+21\,b^2\,x^4\right)}{231}$$

input
```
int(x^(1/2)*(a + b*x^2)^2,x)
```

output
```
(2*x^(3/2)*(77*a^2 + 21*b^2*x^4 + 66*a*b*x^2))/231
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{x}\left(a+bx^2\right)^2\,dx=\frac{2\sqrt{x}\,x(21b^2x^4+66ab\,x^2+77a^2)}{231}$$

input
```
int(x^(1/2)*(b*x^2+a)^2,x)
```

output
```
(2*sqrt(x)*x*(77*a**2 + 66*a*b*x**2 + 21*b**2*x**4))/231
```

3.276 $\int \frac{(a+bx^2)^2}{\sqrt{x}} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+bx^2)^2}{\sqrt{x}} dx = 2a^2\sqrt{x} + \frac{4}{5}abx^{5/2} + \frac{2}{9}b^2x^{9/2}$$

output
```
2*a^2*x^(1/2)+4/5*a*b*x^(5/2)+2/9*b^2*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{\sqrt{x}} dx = \frac{2}{45}\sqrt{x}\left(45a^2 + 18abx^2 + 5b^2x^4\right)$$

input
```
Integrate[(a + b*x^2)^2/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(45*a^2 + 18*a*b*x^2 + 5*b^2*x^4))/45
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^2}{\sqrt{x}}+2abx^{3/2}+b^2x^{7/2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^2\sqrt{x}+\frac{4}{5}abx^{5/2}+\frac{2}{9}b^2x^{9/2}$$

input
```
Int[(a + b*x^2)^2/Sqrt[x],x]
```

output
```
2*a^2*Sqrt[x] + (4*a*b*x^(5/2))/5 + (2*b^2*x^(9/2))/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
default	$2a^2\sqrt{x}+\frac{4ab\,x^{\frac{5}{2}}}{5}+\frac{2b^2x^{\frac{9}{2}}}{9}$	25
trager	$\left(\frac{2}{9}b^2x^4+\frac{4}{5}ab\,x^2+2a^2\right)\sqrt{x}$	26
gosper	$\frac{2\sqrt{x}\,(5b^2x^4+18ab\,x^2+45a^2)}{45}$	27
risch	$\frac{2\sqrt{x}\,(5b^2x^4+18ab\,x^2+45a^2)}{45}$	27
orering	$\frac{2\sqrt{x}\,(5b^2x^4+18ab\,x^2+45a^2)}{45}$	27

input
```
int((b*x^2+a)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^2*x^(1/2)+4/5*a*b*x^(5/2)+2/9*b^2*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx=\frac{2}{45}\left(5\,b^2x^4+18\,abx^2+45\,a^2\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/45*(5*b^2*x^4 + 18*a*b*x^2 + 45*a^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} + \frac{4abx^{\frac{5}{2}}}{5} + \frac{2b^2x^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**2+a)**2/x**(1/2),x)
```

output
```
2*a**2*sqrt(x) + 4*a*b*x**(5/2)/5 + 2*b**2*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx = \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((b*x^2+a)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx = \frac{2}{9}\,b^2x^{\frac{9}{2}} + \frac{4}{5}\,abx^{\frac{5}{2}} + 2\,a^2\sqrt{x}$$

input
```
integrate((b*x^2+a)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/9*b^2*x^(9/2) + 4/5*a*b*x^(5/2) + 2*a^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx = \frac{2\,\sqrt{x}\,(45\,a^2+18\,a\,b\,x^2+5\,b^2\,x^4)}{45}$$

input
```
int((a + b*x^2)^2/x^(1/2),x)
```

output
```
(2*x^(1/2)*(45*a^2 + 5*b^2*x^4 + 18*a*b*x^2))/45
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(5b^2x^4+18ab\,x^2+45a^2)}{45}$$

input
```
int((b*x^2+a)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*(45*a**2 + 18*a*b*x**2 + 5*b**2*x**4))/45
```

3.277 $\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + \frac{4}{3}abx^{3/2} + \frac{2}{7}b^2x^{7/2}$$

output
```
-2*a^2/x^(1/2)+4/3*a*b*x^(3/2)+2/7*b^2*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = -\frac{2(21a^2-14abx^2-3b^2x^4)}{21\sqrt{x}}$$

input
```
Integrate[(a + b*x^2)^2/x^(3/2),x]
```

output
```
(-2*(21*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(21*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(\frac{a^2}{x^{3/2}}+2ab\sqrt{x}+b^2x^{5/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a^2}{\sqrt{x}}+\frac{4}{3}abx^{3/2}+\frac{2}{7}b^2x^{7/2}$$

input
```
Int[(a + b*x^2)^2/x^(3/2),x]
```

output
```
(-2*a^2)/Sqrt[x] + (4*a*b*x^(3/2))/3 + (2*b^2*x^(7/2))/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a^{2}}{\sqrt{x}}+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}x^{\frac{7}{2}}}{7}$	25
default	$-\frac{2a^{2}}{\sqrt{x}}+\frac{4ab\,x^{\frac{3}{2}}}{3}+\frac{2b^{2}x^{\frac{7}{2}}}{7}$	25
gosper	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27
trager	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27
risch	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27
orering	$-\frac{2\left(-3b^{2}x^{4}-14ab\,x^{2}+21a^{2}\right)}{21\sqrt{x}}$	27

input
```
int((b*x^2+a)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^2/x^(1/2)+4/3*a*b*x^(3/2)+2/7*b^2*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{\left(a+bx^2\right)^2}{x^{3/2}}\,dx=\frac{2\,\left(3\,b^2x^4+14\,abx^2-21\,a^2\right)}{21\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/21*(3*b^2*x^4 + 14*a*b*x^2 - 21*a^2)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = -\frac{2a^2}{\sqrt{x}} + \frac{4abx^{\frac{3}{2}}}{3} + \frac{2b^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**2+a)**2/x**(3/2),x)
```

output
```
-2*a**2/sqrt(x) + 4*a*b*x**(3/2)/3 + 2*b**2*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = \frac{2}{7}\,b^2x^{\frac{7}{2}} + \frac{4}{3}\,abx^{\frac{3}{2}} - \frac{2\,a^2}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/7*b^2*x^(7/2) + 4/3*a*b*x^(3/2) - 2*a^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = \frac{-42\,a^2+28\,a\,b\,x^2+6\,b^2\,x^4}{21\,\sqrt{x}}$$

input
```
int((a + b*x^2)^2/x^(3/2),x)
```

output
```
(6*b^2*x^4 - 42*a^2 + 28*a*b*x^2)/(21*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2}{x^{3/2}}\,dx = \frac{\frac{2}{7}b^2x^4+\frac{4}{3}ab\,x^2-2a^2}{\sqrt{x}}$$

input
```
int((b*x^2+a)^2/x^(3/2),x)
```

output
```
(2*( - 21*a**2 + 14*a*b*x**2 + 3*b**2*x**4))/(21*sqrt(x))
```

3.278 $\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{3/2}} + 4ab\sqrt{x} + \frac{2}{5}b^2x^{5/2}$$

output
```
-2/3*a^2/x^(3/2)+4*a*b*x^(1/2)+2/5*b^2*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = -\frac{2(5a^2 - 30abx^2 - 3b^2x^4)}{15x^{3/2}}$$

input
```
Integrate[(a + b*x^2)^2/x^(5/2),x]
```

output
```
(-2*(5*a^2 - 30*a*b*x^2 - 3*b^2*x^4))/(15*x^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^2}{x^{5/2}}+\frac{2ab}{\sqrt{x}}+b^2x^{3/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^2}{3x^{3/2}}+4ab\sqrt{x}+\frac{2}{5}b^2x^{5/2}$$

input
```
Int[(a + b*x^2)^2/x^(5/2),x]
```

output
```
(-2*a^2)/(3*x^(3/2)) + 4*a*b*Sqrt[x] + (2*b^2*x^(5/2))/5
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{5}{2}}}{5}$	25
default	$-\frac{2a^2}{3x^{\frac{3}{2}}}+4ab\sqrt{x}+\frac{2b^2x^{\frac{5}{2}}}{5}$	25
gosper	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27
trager	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27
risch	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27
orering	$-\frac{2(-3b^2x^4-30ab\,x^2+5a^2)}{15x^{\frac{3}{2}}}$	27

input
```
int((b*x^2+a)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2/x^(3/2)+4*a*b*x^(1/2)+2/5*b^2*x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = \frac{2\,(3\,b^2x^4+30\,abx^2-5\,a^2)}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^4 + 30*a*b*x^2 - 5*a^2)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = -\frac{2a^2}{3x^{\frac{3}{2}}} + 4ab\sqrt{x} + \frac{2b^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((b*x**2+a)**2/x**(5/2),x)
```

output
```
-2*a**2/(3*x**(3/2)) + 4*a*b*sqrt(x) + 2*b**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = \frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx = \frac{2}{5}\,b^2x^{\frac{5}{2}} + 4\,ab\sqrt{x} - \frac{2\,a^2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/5*b^2*x^(5/2) + 4*a*b*sqrt(x) - 2/3*a^2/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx=\frac{-10\,a^2+60\,a\,b\,x^2+6\,b^2\,x^4}{15\,x^{3/2}}$$

input
```
int((a + b*x^2)^2/x^(5/2),x)
```

output
```
(6*b^2*x^4 - 10*a^2 + 60*a*b*x^2)/(15*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^{5/2}}\,dx=\frac{\frac{2}{5}b^2x^4+4ab\,x^2-\frac{2}{3}a^2}{\sqrt{x}\,x}$$

input
```
int((b*x^2+a)^2/x^(5/2),x)
```

output
```
(2*( - 5*a**2 + 30*a*b*x**2 + 3*b**2*x**4))/(15*sqrt(x)*x)
```

3.279 $\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{5/2}} - \frac{4ab}{\sqrt{x}} + \frac{2}{3}b^2x^{3/2}$$

output
```
-2/5*a^2/x^(5/2)-4*a*b/x^(1/2)+2/3*b^2*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = \frac{2(-3a^2 - 30abx^2 + 5b^2x^4)}{15x^{5/2}}$$

input
```
Integrate[(a + b*x^2)^2/x^(7/2),x]
```

output
```
(2*(-3*a^2 - 30*a*b*x^2 + 5*b^2*x^4))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx$$
$$\Big\downarrow \ 244$$
$$\int \left(\frac{a^2}{x^{7/2}}+\frac{2ab}{x^{3/2}}+b^2\sqrt{x}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{2a^2}{5x^{5/2}}-\frac{4ab}{\sqrt{x}}+\frac{2}{3}b^2x^{3/2}$$

input
```
Int[(a + b*x^2)^2/x^(7/2),x]
```

output
```
(-2*a^2)/(5*x^(5/2)) - (4*a*b)/Sqrt[x] + (2*b^2*x^(3/2))/3
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

method	result	size
derivativedivides	$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3}$	25
default	$-\frac{2a^2}{5x^{\frac{5}{2}}}-\frac{4ab}{\sqrt{x}}+\frac{2b^2x^{\frac{3}{2}}}{3}$	25
gosper	$-\frac{2(-5b^2x^4+30ab\,x^2+3a^2)}{15x^{\frac{5}{2}}}$	27
trager	$-\frac{2(-5b^2x^4+30ab\,x^2+3a^2)}{15x^{\frac{5}{2}}}$	27
risch	$-\frac{2(-5b^2x^4+30ab\,x^2+3a^2)}{15x^{\frac{5}{2}}}$	27
orering	$-\frac{2(-5b^2x^4+30ab\,x^2+3a^2)}{15x^{\frac{5}{2}}}$	27

input
```
int((b*x^2+a)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^2/x^(5/2)-4*a*b/x^(1/2)+2/3*b^2*x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx=\frac{2\,(5\,b^2x^4-30\,abx^2-3\,a^2)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*b^2*x^4 - 30*a*b*x^2 - 3*a^2)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = -\frac{2a^2}{5x^{\frac{5}{2}}} - \frac{4ab}{\sqrt{x}} + \frac{2b^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((b*x**2+a)**2/x**(7/2),x)
```

output
```
-2*a**2/(5*x**(5/2)) - 4*a*b/sqrt(x) + 2*b**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = \frac{2}{3}\,b^2x^{\frac{3}{2}} - \frac{2\,(10\,abx^2+a^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = \frac{2}{3}\,b^2x^{\frac{3}{2}} - \frac{2\,(10\,abx^2+a^2)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/3*b^2*x^(3/2) - 2/5*(10*a*b*x^2 + a^2)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = -\frac{6\,a^2+60\,a\,b\,x^2-10\,b^2\,x^4}{15\,x^{5/2}}$$

input
```
int((a + b*x^2)^2/x^(7/2),x)
```

output
```
-(6*a^2 - 10*b^2*x^4 + 60*a*b*x^2)/(15*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^2}{x^{7/2}}\,dx = \frac{\frac{2}{3}b^2x^4-4ab\,x^2-\frac{2}{5}a^2}{\sqrt{x}\,x^2}$$

input
```
int((b*x^2+a)^2/x^(7/2),x)
```

output
```
(2*( - 3*a**2 - 30*a*b*x**2 + 5*b**2*x**4))/(15*sqrt(x)*x**2)
```

3.280 $\int x^{7/2}(a+bx^2)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{7/2}(a+bx^2)^3\,dx=\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

output
```
2/9*a^3*x^(9/2)+6/13*a^2*b*x^(13/2)+6/17*a*b^2*x^(17/2)+2/21*b^3*x^(21/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^{7/2}(a+bx^2)^3\,dx=\frac{2x^{9/2}(1547a^3+3213a^2bx^2+2457ab^2x^4+663b^3x^6)}{13923}$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^3,x]
```

output
```
(2*x^(9/2)*(1547*a^3 + 3213*a^2*b*x^2 + 2457*a*b^2*x^4 + 663*b^3*x^6))/139
23
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^3\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^3x^{7/2}+3a^2bx^{11/2}+3ab^2x^{15/2}+b^3x^{19/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}a^3x^{9/2}+\frac{6}{13}a^2bx^{13/2}+\frac{6}{17}ab^2x^{17/2}+\frac{2}{21}b^3x^{21/2}$$

input
```
Int[x^(7/2)*(a + b*x^2)^3,x]
```

output
```
(2*a^3*x^(9/2))/9 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2*x^(17/2))/17 + (2*b^3
*x^(21/2))/21
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{9}{2}}}{9}+\frac{6a^2b\,x^{\frac{13}{2}}}{13}+\frac{6a\,b^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{21}{2}}}{21}$	36
default	$\frac{2a^3x^{\frac{9}{2}}}{9}+\frac{6a^2b\,x^{\frac{13}{2}}}{13}+\frac{6a\,b^2x^{\frac{17}{2}}}{17}+\frac{2b^3x^{\frac{21}{2}}}{21}$	36
gosper	$\frac{2x^{\frac{9}{2}}(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3)}{13923}$	38
trager	$\frac{2x^{\frac{9}{2}}(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3)}{13923}$	38
risch	$\frac{2x^{\frac{9}{2}}(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3)}{13923}$	38
orering	$\frac{2x^{\frac{9}{2}}(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3)}{13923}$	38

input
```
int(x^(7/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/9*a^3*x^(9/2)+6/13*a^2*b*x^(13/2)+6/17*a*b^2*x^(17/2)+2/21*b^3*x^(21/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{7/2}(a+bx^2)^3\,dx = \frac{2}{13923}\left(663\,b^3x^{10}+2457\,ab^2x^8+3213\,a^2bx^6+1547\,a^3x^4\right)\sqrt{x}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/13923*(663*b^3*x^10 + 2457*a*b^2*x^8 + 3213*a^2*b*x^6 + 1547*a^3*x^4)*sq
rt(x)
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{7/2}(a+bx^2)^3\,dx = \frac{2a^3x^{\frac{9}{2}}}{9} + \frac{6a^2bx^{\frac{13}{2}}}{13} + \frac{6ab^2x^{\frac{17}{2}}}{17} + \frac{2b^3x^{\frac{21}{2}}}{21}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**3,x)
```

output
```
2*a**3*x**(9/2)/9 + 6*a**2*b*x**(13/2)/13 + 6*a*b**2*x**(17/2)/17 + 2*b**3
*x**(21/2)/21
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx^2)^3\,dx = \frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^
(9/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}(a+bx^2)^3\,dx = \frac{2}{21}\,b^3x^{\frac{21}{2}} + \frac{6}{17}\,ab^2x^{\frac{17}{2}} + \frac{6}{13}\,a^2bx^{\frac{13}{2}} + \frac{2}{9}\,a^3x^{\frac{9}{2}}$$

input
```
integrate(x^(7/2)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/21*b^3*x^(21/2) + 6/17*a*b^2*x^(17/2) + 6/13*a^2*b*x^(13/2) + 2/9*a^3*x^
(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{7/2}\left(a+bx^2\right)^3\,dx=\frac{2\,a^3\,x^{9/2}}{9}+\frac{2\,b^3\,x^{21/2}}{21}+\frac{6\,a^2\,b\,x^{13/2}}{13}+\frac{6\,a\,b^2\,x^{17/2}}{17}$$

input
```
int(x^(7/2)*(a + b*x^2)^3,x)
```

output
```
(2*a^3*x^(9/2))/9 + (2*b^3*x^(21/2))/21 + (6*a^2*b*x^(13/2))/13 + (6*a*b^2
*x^(17/2))/17
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{7/2}\left(a+bx^2\right)^3\,dx=\frac{2\sqrt{x}\,x^4(663b^3x^6+2457a\,b^2x^4+3213a^2b\,x^2+1547a^3)}{13923}$$

input
```
int(x^(7/2)*(b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x**4*(1547*a**3 + 3213*a**2*b*x**2 + 2457*a*b**2*x**4 + 663*b**
3*x**6))/13923
```

3.281 $\int x^{5/2}(a+bx^2)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx=\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

output
```
2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*b^2*x^(15/2)+2/19*b^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx=\frac{2x^{7/2}(1045a^3+1995a^2bx^2+1463ab^2x^4+385b^3x^6)}{7315}$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^3,x]
```

output
```
(2*x^(7/2)*(1045*a^3 + 1995*a^2*b*x^2 + 1463*a*b^2*x^4 + 385*b^3*x^6))/731
5
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(a^3x^{5/2}+3a^2bx^{9/2}+3ab^2x^{13/2}+b^3x^{17/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{7}a^3x^{7/2}+\frac{6}{11}a^2bx^{11/2}+\frac{2}{5}ab^2x^{15/2}+\frac{2}{19}b^3x^{19/2}$$

input
```
Int[x^(5/2)*(a + b*x^2)^3,x]
```

output
```
(2*a^3*x^(7/2))/7 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2*x^(15/2))/5 + (2*b^3*
x^(19/2))/19
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2b\,x^{\frac{11}{2}}}{11}+\frac{2a\,b^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{19}{2}}}{19}$	36
default	$\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2b\,x^{\frac{11}{2}}}{11}+\frac{2a\,b^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{19}{2}}}{19}$	36
gosper	$\frac{2x^{\frac{7}{2}}(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$	38
trager	$\frac{2x^{\frac{7}{2}}(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$	38
risch	$\frac{2x^{\frac{7}{2}}(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$	38
orering	$\frac{2x^{\frac{7}{2}}(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$	38

input
```
int(x^(5/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/7*a^3*x^(7/2)+6/11*a^2*b*x^(11/2)+2/5*a*b^2*x^(15/2)+2/19*b^3*x^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx=\frac{2}{7315}\left(385\,b^3x^9+1463\,ab^2x^7+1995\,a^2bx^5+1045\,a^3x^3\right)\sqrt{x}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/7315*(385*b^3*x^9 + 1463*a*b^2*x^7 + 1995*a^2*b*x^5 + 1045*a^3*x^3)*sqrt
(x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{5/2}(a+bx^2)^3\,dx=\frac{2a^3x^{\frac{7}{2}}}{7}+\frac{6a^2bx^{\frac{11}{2}}}{11}+\frac{2ab^2x^{\frac{15}{2}}}{5}+\frac{2b^3x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**3,x)
```

output
```
2*a**3*x**(7/2)/7 + 6*a**2*b*x**(11/2)/11 + 2*a*b**2*x**(15/2)/5 + 2*b**3*
x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx^2)^3\,dx=\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(
7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}(a+bx^2)^3\,dx=\frac{2}{19}\,b^3x^{\frac{19}{2}}+\frac{2}{5}\,ab^2x^{\frac{15}{2}}+\frac{6}{11}\,a^2bx^{\frac{11}{2}}+\frac{2}{7}\,a^3x^{\frac{7}{2}}$$

input
```
integrate(x^(5/2)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/19*b^3*x^(19/2) + 2/5*a*b^2*x^(15/2) + 6/11*a^2*b*x^(11/2) + 2/7*a^3*x^(
7/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx=\frac{2\,a^3\,x^{7/2}}{7}+\frac{2\,b^3\,x^{19/2}}{19}+\frac{6\,a^2\,b\,x^{11/2}}{11}+\frac{2\,a\,b^2\,x^{15/2}}{5}$$

input
```
int(x^(5/2)*(a + b*x^2)^3,x)
```

output
```
(2*a^3*x^(7/2))/7 + (2*b^3*x^(19/2))/19 + (6*a^2*b*x^(11/2))/11 + (2*a*b^2
*x^(15/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{5/2}\left(a+bx^2\right)^3\,dx=\frac{2\sqrt{x}\,x^3(385b^3x^6+1463a\,b^2x^4+1995a^2b\,x^2+1045a^3)}{7315}$$

input
```
int(x^(5/2)*(b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x**3*(1045*a**3 + 1995*a**2*b*x**2 + 1463*a*b**2*x**4 + 385*b**
3*x**6))/7315
```

3.282 $\int x^{3/2}(a+bx^2)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{3/2}\left(a+bx^2\right)^3\,dx=\frac{2}{5}a^3x^{5/2}+\frac{2}{3}a^2bx^{9/2}+\frac{6}{13}ab^2x^{13/2}+\frac{2}{17}b^3x^{17/2}$$

output
```
2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*b^2*x^(13/2)+2/17*b^3*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^{3/2}\left(a+bx^2\right)^3\,dx=\frac{2x^{5/2}(663a^3+1105a^2bx^2+765ab^2x^4+195b^3x^6)}{3315}$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^3,x]
```

output
```
(2*x^(5/2)*(663*a^3 + 1105*a^2*b*x^2 + 765*a*b^2*x^4 + 195*b^3*x^6))/3315
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2} \left(a+b x^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^3 x^{3/2}+3 a^2 b x^{7/2}+3 a b^2 x^{11/2}+b^3 x^{15/2}\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5} a^3 x^{5/2}+\frac{2}{3} a^2 b x^{9/2}+\frac{6}{13} a b^2 x^{13/2}+\frac{2}{17} b^3 x^{17/2}$$

input
```
Int[x^(3/2)*(a + b*x^2)^3,x]
```

output
```
(2*a^3*x^(5/2))/5 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x^(13/2))/13 + (2*b^3*x
^(17/2))/17
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$	36
default	$\frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2b\,x^{\frac{9}{2}}}{3}+\frac{6a\,b^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$	36
gosper	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38
trager	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38
risch	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38
orering	$\frac{2x^{\frac{5}{2}}\left(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3\right)}{3315}$	38

input
```
int(x^(3/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/5*a^3*x^(5/2)+2/3*a^2*b*x^(9/2)+6/13*a*b^2*x^(13/2)+2/17*b^3*x^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^{3/2}\left(a+bx^2\right)^3\,dx=\frac{2}{3315}\left(195\,b^3x^8+765\,ab^2x^6+1105\,a^2bx^4+663\,a^3x^2\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/3315*(195*b^3*x^8 + 765*a*b^2*x^6 + 1105*a^2*b*x^4 + 663*a^3*x^2)*sqrt(x
)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int x^{3/2}(a+bx^2)^3\,dx = \frac{2a^3x^{\frac{5}{2}}}{5}+\frac{2a^2bx^{\frac{9}{2}}}{3}+\frac{6ab^2x^{\frac{13}{2}}}{13}+\frac{2b^3x^{\frac{17}{2}}}{17}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**3,x)
```

output
```
2*a**3*x**(5/2)/5 + 2*a**2*b*x**(9/2)/3 + 6*a*b**2*x**(13/2)/13 + 2*b**3*x
**(17/2)/17
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^2)^3\,dx = \frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5
/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^2)^3\,dx = \frac{2}{17}\,b^3x^{\frac{17}{2}}+\frac{6}{13}\,ab^2x^{\frac{13}{2}}+\frac{2}{3}\,a^2bx^{\frac{9}{2}}+\frac{2}{5}\,a^3x^{\frac{5}{2}}$$

input
```
integrate(x^(3/2)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/17*b^3*x^(17/2) + 6/13*a*b^2*x^(13/2) + 2/3*a^2*b*x^(9/2) + 2/5*a^3*x^(5
/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int x^{3/2}(a+bx^2)^3\,dx=\frac{2\,a^3\,x^{5/2}}{5}+\frac{2\,b^3\,x^{17/2}}{17}+\frac{2\,a^2\,b\,x^{9/2}}{3}+\frac{6\,a\,b^2\,x^{13/2}}{13}$$

input
```
int(x^(3/2)*(a + b*x^2)^3,x)
```

output
```
(2*a^3*x^(5/2))/5 + (2*b^3*x^(17/2))/17 + (2*a^2*b*x^(9/2))/3 + (6*a*b^2*x
^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int x^{3/2}(a+bx^2)^3\,dx=\frac{2\sqrt{x}\,x^2(195b^3x^6+765a\,b^2x^4+1105a^2b\,x^2+663a^3)}{3315}$$

input
```
int(x^(3/2)*(b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x**2*(663*a**3 + 1105*a**2*b*x**2 + 765*a*b**2*x**4 + 195*b**3*
x**6))/3315
```

3.283 $\int \sqrt{x}(a+bx^2)^3\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{x}(a+bx^2)^3\,dx=\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

output
```
2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*b^2*x^(11/2)+2/15*b^3*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \sqrt{x}(a+bx^2)^3\,dx=\frac{2x^{3/2}(385a^3+495a^2bx^2+315ab^2x^4+77b^3x^6)}{1155}$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^3,x]
```

output
```
(2*x^(3/2)*(385*a^3 + 495*a^2*b*x^2 + 315*a*b^2*x^4 + 77*b^3*x^6))/1155
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3\sqrt{x}+3a^2bx^{5/2}+3ab^2x^{9/2}+b^3x^{13/2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}a^3x^{3/2}+\frac{6}{7}a^2bx^{7/2}+\frac{6}{11}ab^2x^{11/2}+\frac{2}{15}b^3x^{15/2}$$

input
```
Int[Sqrt[x]*(a + b*x^2)^3,x]
```

output
```
(2*a^3*x^(3/2))/3 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x^(11/2))/11 + (2*b^3*x
^(15/2))/15
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{15}{2}}}{15}$	36
default	$\frac{2a^{3}x^{\frac{3}{2}}}{3}+\frac{6a^{2}b\,x^{\frac{7}{2}}}{7}+\frac{6a\,b^{2}x^{\frac{11}{2}}}{11}+\frac{2b^{3}x^{\frac{15}{2}}}{15}$	36
gosper	$\frac{2x^{\frac{3}{2}}\left(77b^{3}x^{6}+315a\,b^{2}x^{4}+495a^{2}b\,x^{2}+385a^{3}\right)}{1155}$	38
trager	$\frac{2x^{\frac{3}{2}}\left(77b^{3}x^{6}+315a\,b^{2}x^{4}+495a^{2}b\,x^{2}+385a^{3}\right)}{1155}$	38
risch	$\frac{2x^{\frac{3}{2}}\left(77b^{3}x^{6}+315a\,b^{2}x^{4}+495a^{2}b\,x^{2}+385a^{3}\right)}{1155}$	38
orering	$\frac{2x^{\frac{3}{2}}\left(77b^{3}x^{6}+315a\,b^{2}x^{4}+495a^{2}b\,x^{2}+385a^{3}\right)}{1155}$	38

input
```
int(x^(1/2)*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*a^3*x^(3/2)+6/7*a^2*b*x^(7/2)+6/11*a*b^2*x^(11/2)+2/15*b^3*x^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.75

$$\int \sqrt{x}\left(a+bx^2\right)^3\,dx=\frac{2}{1155}\left(77\,b^3x^7+315\,ab^2x^5+495\,a^2bx^3+385\,a^3x\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
2/1155*(77*b^3*x^7 + 315*a*b^2*x^5 + 495*a^2*b*x^3 + 385*a^3*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \sqrt{x}(a+bx^2)^3\,dx=\frac{2a^3x^{\frac{3}{2}}}{3}+\frac{6a^2bx^{\frac{7}{2}}}{7}+\frac{6ab^2x^{\frac{11}{2}}}{11}+\frac{2b^3x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(1/2)*(b*x**2+a)**3,x)
```

output
```
2*a**3*x**(3/2)/3 + 6*a**2*b*x**(7/2)/7 + 6*a*b**2*x**(11/2)/11 + 2*b**3*x
**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^2)^3\,dx=\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3
/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(a+bx^2)^3\,dx=\frac{2}{15}\,b^3x^{\frac{15}{2}}+\frac{6}{11}\,ab^2x^{\frac{11}{2}}+\frac{6}{7}\,a^2bx^{\frac{7}{2}}+\frac{2}{3}\,a^3x^{\frac{3}{2}}$$

input
```
integrate(x^(1/2)*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
2/15*b^3*x^(15/2) + 6/11*a*b^2*x^(11/2) + 6/7*a^2*b*x^(7/2) + 2/3*a^3*x^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.69

$$\int \sqrt{x}\left(a+bx^2\right)^3\,dx=\frac{2\,a^3\,x^{3/2}}{3}+\frac{2\,b^3\,x^{15/2}}{15}+\frac{6\,a^2\,b\,x^{7/2}}{7}+\frac{6\,a\,b^2\,x^{11/2}}{11}$$

input
```
int(x^(1/2)*(a + b*x^2)^3,x)
```

output
```
(2*a^3*x^(3/2))/3 + (2*b^3*x^(15/2))/15 + (6*a^2*b*x^(7/2))/7 + (6*a*b^2*x
^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \sqrt{x}\left(a+bx^2\right)^3\,dx=\frac{2\sqrt{x}\,x\left(77b^3x^6+315a\,b^2x^4+495a^2b\,x^2+385a^3\right)}{1155}$$

input
```
int(x^(1/2)*(b*x^2+a)^3,x)
```

output
```
(2*sqrt(x)*x*(385*a**3 + 495*a**2*b*x**2 + 315*a*b**2*x**4 + 77*b**3*x**6)
)/1155
```

3.284 $\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + \frac{6}{5}a^2bx^{5/2} + \frac{2}{3}ab^2x^{9/2} + \frac{2}{13}b^3x^{13/2}$$

output
```
2*a^3*x^(1/2)+6/5*a^2*b*x^(5/2)+2/3*a*b^2*x^(9/2)+2/13*b^3*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = \frac{2}{195}\sqrt{x}\left(195a^3 + 117a^2bx^2 + 65ab^2x^4 + 15b^3x^6\right)$$

input
```
Integrate[(a + b*x^2)^3/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(195*a^3 + 117*a^2*b*x^2 + 65*a*b^2*x^4 + 15*b^3*x^6))/195
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^3}{\sqrt{x}}+3a^2bx^{3/2}+3ab^2x^{7/2}+b^3x^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2a^3\sqrt{x}+\frac{6}{5}a^2bx^{5/2}+\frac{2}{3}ab^2x^{9/2}+\frac{2}{13}b^3x^{13/2}$$

input
```
Int[(a + b*x^2)^3/Sqrt[x],x]
```

output
```
2*a^3*Sqrt[x] + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/2))/3 + (2*b^3*x^(13/2
))/13
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$2a^3\sqrt{x}+\frac{6a^2b\,x^{\frac{5}{2}}}{5}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$	36
default	$2a^3\sqrt{x}+\frac{6a^2b\,x^{\frac{5}{2}}}{5}+\frac{2a\,b^2x^{\frac{9}{2}}}{3}+\frac{2b^3x^{\frac{13}{2}}}{13}$	36
trager	$\left(\frac{2}{13}b^3x^6+\frac{2}{3}a\,b^2x^4+\frac{6}{5}a^2b\,x^2+2a^3\right)\sqrt{x}$	37
gosper	$\frac{2\sqrt{x}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)}{195}$	38
risch	$\frac{2\sqrt{x}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)}{195}$	38
orering	$\frac{2\sqrt{x}\,(15b^3x^6+65a\,b^2x^4+117a^2b\,x^2+195a^3)}{195}$	38

input
```
int((b*x^2+a)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a^3*x^(1/2)+6/5*a^2*b*x^(5/2)+2/3*a*b^2*x^(9/2)+2/13*b^3*x^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx=\frac{2}{195}\left(15\,b^3x^6+65\,ab^2x^4+117\,a^2bx^2+195\,a^3\right)\sqrt{x}$$

input
```
integrate((b*x^2+a)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/195*(15*b^3*x^6 + 65*a*b^2*x^4 + 117*a^2*b*x^2 + 195*a^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = 2a^3\sqrt{x} + \frac{6a^2bx^{\frac{5}{2}}}{5} + \frac{2ab^2x^{\frac{9}{2}}}{3} + \frac{2b^3x^{\frac{13}{2}}}{13}$$

input
```
integrate((b*x**2+a)**3/x**(1/2),x)
```

output
```
2*a**3*sqrt(x) + 6*a**2*b*x**(5/2)/5 + 2*a*b**2*x**(9/2)/3 + 2*b**3*x**(13
/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((b*x^2+a)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = \frac{2}{13}\,b^3x^{\frac{13}{2}} + \frac{2}{3}\,ab^2x^{\frac{9}{2}} + \frac{6}{5}\,a^2bx^{\frac{5}{2}} + 2\,a^3\sqrt{x}$$

input
```
integrate((b*x^2+a)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/13*b^3*x^(13/2) + 2/3*a*b^2*x^(9/2) + 6/5*a^2*b*x^(5/2) + 2*a^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = 2\,a^3\,\sqrt{x} + \frac{2\,b^3\,x^{13/2}}{13} + \frac{6\,a^2\,b\,x^{5/2}}{5} + \frac{2\,a\,b^2\,x^{9/2}}{3}$$

input
```
int((a + b*x^2)^3/x^(1/2),x)
```

output
```
2*a^3*x^(1/2) + (2*b^3*x^(13/2))/13 + (6*a^2*b*x^(5/2))/5 + (2*a*b^2*x^(9/
2))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,(15b^3x^6 + 65a\,b^2x^4 + 117a^2b\,x^2 + 195a^3)}{195}$$

input
```
int((b*x^2+a)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*(195*a**3 + 117*a**2*b*x**2 + 65*a*b**2*x**4 + 15*b**3*x**6))/1
95
```

3.285 $\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + 2a^2bx^{3/2} + \frac{6}{7}ab^2x^{7/2} + \frac{2}{11}b^3x^{11/2}$$

output
```
-2*a^3/x^(1/2)+2*a^2*b*x^(3/2)+6/7*a*b^2*x^(7/2)+2/11*b^3*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = -\frac{2(77a^3 - 77a^2bx^2 - 33ab^2x^4 - 7b^3x^6)}{77\sqrt{x}}$$

input
```
Integrate[(a + b*x^2)^3/x^(3/2),x]
```

output
```
(-2*(77*a^3 - 77*a^2*b*x^2 - 33*a*b^2*x^4 - 7*b^3*x^6))/(77*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(\frac{a^3}{x^{3/2}}+3a^2b\sqrt{x}+3ab^2x^{5/2}+b^3x^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2a^3}{\sqrt{x}}+2a^2bx^{3/2}+\frac{6}{7}ab^2x^{7/2}+\frac{2}{11}b^3x^{11/2}$$

input
```
Int[(a + b*x^2)^3/x^(3/2),x]
```

output
```
(-2*a^3)/Sqrt[x] + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2))/7 + (2*b^3*x^(11/2)
)/11
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2a^3}{\sqrt{x}}+2a^2b\,x^{\frac{3}{2}}+\frac{6a\,b^2x^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
default	$-\frac{2a^3}{\sqrt{x}}+2a^2b\,x^{\frac{3}{2}}+\frac{6a\,b^2x^{\frac{7}{2}}}{7}+\frac{2b^3x^{\frac{11}{2}}}{11}$	36
gosper	$-\frac{2(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77\sqrt{x}}$	38
trager	$-\frac{2(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77\sqrt{x}}$	38
risch	$-\frac{2(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77\sqrt{x}}$	38
orering	$-\frac{2(-7b^3x^6-33a\,b^2x^4-77a^2b\,x^2+77a^3)}{77\sqrt{x}}$	38

input
```
int((b*x^2+a)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*a^3/x^(1/2)+2*a^2*b*x^(3/2)+6/7*a*b^2*x^(7/2)+2/11*b^3*x^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = \frac{2\,(7\,b^3x^6+33\,ab^2x^4+77\,a^2bx^2-77\,a^3)}{77\,\sqrt{x}}$$

input
```
integrate((b*x^2+a)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/77*(7*b^3*x^6 + 33*a*b^2*x^4 + 77*a^2*b*x^2 - 77*a^3)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = -\frac{2a^3}{\sqrt{x}} + 2a^2bx^{\frac{3}{2}} + \frac{6ab^2x^{\frac{7}{2}}}{7} + \frac{2b^3x^{\frac{11}{2}}}{11}$$

input
```
integrate((b*x**2+a)**3/x**(3/2),x)
```

output
```
-2*a**3/sqrt(x) + 2*a**2*b*x**(3/2) + 6*a*b**2*x**(7/2)/7 + 2*b**3*x**(11/
2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = \frac{2}{11}\,b^3x^{\frac{11}{2}} + \frac{6}{7}\,ab^2x^{\frac{7}{2}} + 2\,a^2bx^{\frac{3}{2}} - \frac{2\,a^3}{\sqrt{x}}$$

input
```
integrate((b*x^2+a)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/11*b^3*x^(11/2) + 6/7*a*b^2*x^(7/2) + 2*a^2*b*x^(3/2) - 2*a^3/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = \frac{2\,b^3\,x^{11/2}}{11} - \frac{2\,a^3}{\sqrt{x}} + 2\,a^2\,b\,x^{3/2} + \frac{6\,a\,b^2\,x^{7/2}}{7}$$

input
```
int((a + b*x^2)^3/x^(3/2),x)
```

output
```
(2*b^3*x^(11/2))/11 - (2*a^3)/x^(1/2) + 2*a^2*b*x^(3/2) + (6*a*b^2*x^(7/2)
)/7
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^3}{x^{3/2}}\,dx = \frac{\frac{2}{11}b^3x^6 + \frac{6}{7}a\,b^2x^4 + 2a^2b\,x^2 - 2a^3}{\sqrt{x}}$$

input
```
int((b*x^2+a)^3/x^(3/2),x)
```

output
```
(2*( - 77*a**3 + 77*a**2*b*x**2 + 33*a*b**2*x**4 + 7*b**3*x**6))/(77*sqrt(
x))
```

3.286 $\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{3/2}} + 6a^2b\sqrt{x} + \frac{6}{5}ab^2x^{5/2} + \frac{2}{9}b^3x^{9/2}$$

output
```
-2/3*a^3/x^(3/2)+6*a^2*b*x^(1/2)+6/5*a*b^2*x^(5/2)+2/9*b^3*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = -\frac{2(15a^3 - 135a^2bx^2 - 27ab^2x^4 - 5b^3x^6)}{45x^{3/2}}$$

input
```
Integrate[(a + b*x^2)^3/x^(5/2),x]
```

output
```
(-2*(15*a^3 - 135*a^2*b*x^2 - 27*a*b^2*x^4 - 5*b^3*x^6))/(45*x^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 244$$

$$\int \left(\frac{a^3}{x^{5/2}}+\frac{3a^2b}{\sqrt{x}}+3ab^2x^{3/2}+b^3x^{7/2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2a^3}{3x^{3/2}}+6a^2b\sqrt{x}+\frac{6}{5}ab^2x^{5/2}+\frac{2}{9}b^3x^{9/2}$$

input
```
Int[(a + b*x^2)^3/x^(5/2),x]
```

output
```
(-2*a^3)/(3*x^(3/2)) + 6*a^2*b*Sqrt[x] + (6*a*b^2*x^(5/2))/5 + (2*b^3*x^(9
/2))/9
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}+6a^{2}b\sqrt{x}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
default	$-\frac{2a^{3}}{3x^{\frac{3}{2}}}+6a^{2}b\sqrt{x}+\frac{6a\,b^{2}x^{\frac{5}{2}}}{5}+\frac{2b^{3}x^{\frac{9}{2}}}{9}$	36
gosper	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38
trager	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38
risch	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38
orering	$-\frac{2\left(-5b^{3}x^{6}-27a\,b^{2}x^{4}-135a^{2}b\,x^{2}+15a^{3}\right)}{45x^{\frac{3}{2}}}$	38

input
```
int((b*x^2+a)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^3/x^(3/2)+6*a^2*b*x^(1/2)+6/5*a*b^2*x^(5/2)+2/9*b^3*x^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = \frac{2\,(5\,b^3x^6+27\,ab^2x^4+135\,a^2bx^2-15\,a^3)}{45\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/45*(5*b^3*x^6 + 27*a*b^2*x^4 + 135*a^2*b*x^2 - 15*a^3)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = -\frac{2a^3}{3x^{\frac{3}{2}}} + 6a^2b\sqrt{x} + \frac{6ab^2x^{\frac{5}{2}}}{5} + \frac{2b^3x^{\frac{9}{2}}}{9}$$

input
```
integrate((b*x**2+a)**3/x**(5/2),x)
```

output
```
-2*a**3/(3*x**(3/2)) + 6*a**2*b*sqrt(x) + 6*a*b**2*x**(5/2)/5 + 2*b**3*x**
(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = \frac{2}{9}\,b^3x^{\frac{9}{2}} + \frac{6}{5}\,ab^2x^{\frac{5}{2}} + 6\,a^2b\sqrt{x} - \frac{2\,a^3}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/9*b^3*x^(9/2) + 6/5*a*b^2*x^(5/2) + 6*a^2*b*sqrt(x) - 2/3*a^3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = \frac{2\,b^3\,x^{9/2}}{9} - \frac{2\,a^3}{3\,x^{3/2}} + 6\,a^2\,b\,\sqrt{x} + \frac{6\,a\,b^2\,x^{5/2}}{5}$$

input
```
int((a + b*x^2)^3/x^(5/2),x)
```

output
```
(2*b^3*x^(9/2))/9 - (2*a^3)/(3*x^(3/2)) + 6*a^2*b*x^(1/2) + (6*a*b^2*x^(5/
2))/5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^3}{x^{5/2}}\,dx = \frac{\frac{2}{9}b^3x^6 + \frac{6}{5}a\,b^2x^4 + 6a^2b\,x^2 - \frac{2}{3}a^3}{\sqrt{x}\,x}$$

input
```
int((b*x^2+a)^3/x^(5/2),x)
```

output
```
(2*( - 15*a**3 + 135*a**2*b*x**2 + 27*a*b**2*x**4 + 5*b**3*x**6))/(45*sqrt
(x)*x)
```

3.287 $\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = -\frac{2a^3}{5x^{5/2}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{3/2} + \frac{2}{7}b^3x^{7/2}$$

output
```
-2/5*a^3/x^(5/2)-6*a^2*b/x^(1/2)+2*a*b^2*x^(3/2)+2/7*b^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = -\frac{2(7a^3+105a^2bx^2-35ab^2x^4-5b^3x^6)}{35x^{5/2}}$$

input
```
Integrate[(a + b*x^2)^3/x^(7/2),x]
```

output
```
(-2*(7*a^3 + 105*a^2*b*x^2 - 35*a*b^2*x^4 - 5*b^3*x^6))/(35*x^(5/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx$$
$$\Big\downarrow \text{244}$$
$$\int \left(\frac{a^3}{x^{7/2}}+\frac{3a^2b}{x^{3/2}}+3ab^2\sqrt{x}+b^3x^{5/2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2a^3}{5x^{5/2}}-\frac{6a^2b}{\sqrt{x}}+2ab^2x^{3/2}+\frac{2}{7}b^3x^{7/2}$$

input
```
Int[(a + b*x^2)^3/x^(7/2),x]
```

output
```
(-2*a^3)/(5*x^(5/2)) - (6*a^2*b)/Sqrt[x] + 2*a*b^2*x^(3/2) + (2*b^3*x^(7/2
))/7
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{7}{2}}}{7}$	36
default	$-\frac{2a^3}{5x^{\frac{5}{2}}}-\frac{6a^2b}{\sqrt{x}}+2a\,b^2x^{\frac{3}{2}}+\frac{2b^3x^{\frac{7}{2}}}{7}$	36
gosper	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38
trager	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38
risch	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38
orering	$-\frac{2(-5b^3x^6-35a\,b^2x^4+105a^2b\,x^2+7a^3)}{35x^{\frac{5}{2}}}$	38

input
```
int((b*x^2+a)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*a^3/x^(5/2)-6*a^2*b/x^(1/2)+2*a*b^2*x^(3/2)+2/7*b^3*x^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = \frac{2\,(5\,b^3x^6+35\,ab^2x^4-105\,a^2bx^2-7\,a^3)}{35\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^3*x^6 + 35*a*b^2*x^4 - 105*a^2*b*x^2 - 7*a^3)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = -\frac{2a^3}{5x^{\frac{5}{2}}} - \frac{6a^2b}{\sqrt{x}} + 2ab^2x^{\frac{3}{2}} + \frac{2b^3x^{\frac{7}{2}}}{7}$$

input
```
integrate((b*x**2+a)**3/x**(7/2),x)
```

output
```
-2*a**3/(5*x**(5/2)) - 6*a**2*b/sqrt(x) + 2*a*b**2*x**(3/2) + 2*b**3*x**(7
/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + 2\,ab^2x^{\frac{3}{2}} - \frac{2\,(15\,a^2bx^2 + a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = \frac{2}{7}\,b^3x^{\frac{7}{2}} + 2\,ab^2x^{\frac{3}{2}} - \frac{2\,(15\,a^2bx^2 + a^3)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate((b*x^2+a)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/7*b^3*x^(7/2) + 2*a*b^2*x^(3/2) - 2/5*(15*a^2*b*x^2 + a^3)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = -\frac{14\,a^3+210\,a^2\,b\,x^2-70\,a\,b^2\,x^4-10\,b^3\,x^6}{35\,x^{5/2}}$$

input
```
int((a + b*x^2)^3/x^(7/2),x)
```

output
```
-(14*a^3 - 10*b^3*x^6 + 210*a^2*b*x^2 - 70*a*b^2*x^4)/(35*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^3}{x^{7/2}}\,dx = \frac{\frac{2}{7}b^3x^6+2a\,b^2x^4-6a^2b\,x^2-\frac{2}{5}a^3}{\sqrt{x}\,x^2}$$

input
```
int((b*x^2+a)^3/x^(7/2),x)
```

output
```
(2*( - 7*a**3 - 105*a**2*b*x**2 + 35*a*b**2*x**4 + 5*b**3*x**6))/(35*sqrt(
x)*x**2)
```

3.288 $\int \frac{x^{7/2}}{a+bx^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 159

$$\int \frac{x^{7/2}}{a+bx^2} \, dx = -\frac{2a\sqrt{x}}{b^2} + \frac{2x^{5/2}}{5b} - \frac{a^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} + \frac{a^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{9/4}} + \frac{a^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{9/4}}$$

output
```
-2*a*x^(1/2)/b^2+2/5*x^(5/2)/b-1/2*a^(5/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2
)/a^(1/4))*2^(1/2)/b^(9/4)+1/2*a^(5/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^
(1/4))*2^(1/2)/b^(9/4)+1/2*a^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)
/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.81

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{4\sqrt[4]{b}\sqrt{x}(-5a+bx^2)-5\sqrt{2}a^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+5\sqrt{2}a^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{10b^{9/4}}$$

input
```
Integrate[x^(7/2)/(a + b*x^2),x]
```

output
```
(4*b^(1/4)*Sqrt[x]*(-5*a + b*x^2) - 5*Sqrt[2]*a^(5/4)*ArcTan[(Sqrt[a] - Sq
rt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*a^(5/4)*ArcTanh[(S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(10*b^(9/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.56, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\int \frac{x^{3/2}}{bx^2+a}dx}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\int\frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{b}$$

$\Big\downarrow$ 755

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int\frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 1476

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 1082

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 217

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$\downarrow$ 1479

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$\downarrow$ 25

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2x^{5/2}}{5b}-\frac{a\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 1103$$

$$\frac{2x^{5/2}}{5b} - \frac{a\left(\frac{2\sqrt{x}}{b} - \frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}} + \frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{b}$$

input
```
Int[x^(7/2)/(a + b*x^2),x]
```

output
```
(2*x^(5/2))/(5*b) - (a*((2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(
1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{2\left(-\frac{b\,x^{\frac{5}{2}}}{5}+a\sqrt{x}\right)}{b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}$
default	$-\frac{2\left(-\frac{b\,x^{\frac{5}{2}}}{5}+a\sqrt{x}\right)}{b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}$
risch	$-\frac{2\left(-b\,x^{2}+5a\right)\sqrt{x}}{5b^{2}}+\frac{a\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}}$

input
```
int(x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*(-1/5*b*x^(5/2)+a*x^(1/2))+1/4*a/b^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/
b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)
^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1
/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05

$$\int\frac{x^{7/2}}{a+bx^2}\,dx=\frac{5\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\log\left(b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}+a\sqrt{x}\right)+5i\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}\log\left(i\,b^2\left(-\frac{a^5}{b^9}\right)^{\frac{1}{4}}+a\sqrt{x}\right)-5i\,b^2\left(-\right.}{}$$

input
```
integrate(x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/10*(5*b^2*(-a^5/b^9)^(1/4)*log(b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 5*I*b
^2*(-a^5/b^9)^(1/4)*log(I*b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*I*b^2*(-a^
5/b^9)^(1/4)*log(-I*b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) - 5*b^2*(-a^5/b^9)^(
1/4)*log(-b^2*(-a^5/b^9)^(1/4) + a*sqrt(x)) + 4*(b*x^2 - 5*a)*sqrt(x))/b^2
```

Sympy [A] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{5}{2}} \\ \frac{2x^{\frac{9}{2}}}{9a} \\ \frac{2x^{\frac{5}{2}}}{5b} \\ -\frac{2a\sqrt{x}}{b^2} - \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2} + \frac{a\sqrt[4]{-\frac{a}{b}}\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2} + \frac{2x^{\frac{5}{2}}}{5b} \end{cases}$$

input
```
integrate(x**(7/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a), Eq(b, 0)
), (2*x**(5/2)/(5*b), Eq(a, 0)), (-2*a*sqrt(x)/b**2 - a*(-a/b)**(1/4)*log(
sqrt(x) - (-a/b)**(1/4))/(2*b**2) + a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**
(1/4))/(2*b**2) + a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b**2 + 2*x**
(5/2)/(5*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.22

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{2\left(bx^{\frac{5}{2}}-5\,a\sqrt{x}\right)}{5\,b^2} + \frac{\frac{2\,\sqrt{2}a^{\frac{3}{2}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}a^{\frac{3}{2}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{5}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \frac{\sqrt{2}a^{\frac{5}{4}}\,\mathrm{lo}}{}}{4\,b^2}$$

input
```
integrate(x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
2/5*(b*x^(5/2) - 5*a*sqrt(x))/b^2 + 1/4*(2*sqrt(2)*a^(3/2)*arctan(1/2*sqrt
(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/s
qrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*a^(3/2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1
/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(
b)) + sqrt(2)*a^(5/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sq
rt(a))/b^(1/4) - sqrt(2)*a^(5/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sq
rt(b)*x + sqrt(a))/b^(1/4))/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.23

$$\begin{aligned}\int \frac{x^{7/2}}{a+bx^2}\,dx &= \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,a\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3} \\ &+ \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,a\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^3} \\ &+ \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,a\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3} \\ &- \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\,a\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^3} + \frac{2\left(b^4x^{\frac{5}{2}}-5\,ab^3\sqrt{x}\right)}{5\,b^5}\end{aligned}$$

input
```
integrate(x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/b^3 + 1/2*sqrt(2)*(a*b^3)^(1/4)*a*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 + 1/4*sqrt(2)*(a*b^3)^(
1/4)*a*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 - 1/4*sqrt(2)*
(a*b^3)^(1/4)*a*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 2/
5*(b^4*x^(5/2) - 5*a*b^3*sqrt(x))/b^5
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.42

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{2\,x^{5/2}}{5\,b} - \frac{2\,a\,\sqrt{x}}{b^2} - \frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{{\left(-a\right)}^{1/4}}\right)}{b^{9/4}} + \frac{{\left(-a\right)}^{5/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/4}}\right)\,1{}\mathrm{i}}{b^{9/4}}$$

input
```
int(x^(7/2)/(a + b*x^2),x)
```

output
```
(2*x^(5/2))/(5*b) - (2*a*x^(1/2))/b^2 - ((-a)^(5/4)*atan((b^(1/4)*x^(1/2))
/(-a)^(1/4)))/b^(9/4) + ((-a)^(5/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*
1i)/b^(9/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int \frac{x^{7/2}}{a+bx^2}\,dx = \frac{-10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-5b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\log\left(-\right.}{}$$

input
```
int(x^(7/2)/(b*x^2+a),x)
```

output
```
( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*b**(3/4)*a**(1/4)*sqrt(2)*
atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sq
rt(2)))*a - 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*s
qrt(2) + sqrt(a) + sqrt(b)*x)*a + 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*
b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 40*sqrt(x)*a*b + 8*sq
rt(x)*b**2*x**2)/(20*b**3)
```

3.289 $\int \frac{x^{5/2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2x^{3/2}}{3b}+\frac{a^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}-\frac{a^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{7/4}}+\frac{a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{7/4}}$$

output
```
2/3*x^(3/2)/b+1/2*a^(3/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2
)/b^(7/4)-1/2*a^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/b^
(7/4)+1/2*a^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)
*x))*2^(1/2)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{4b^{3/4}x^{3/2}+3\sqrt{2}a^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}a^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{6b^{7/4}}$$

input
```
Integrate[x^(5/2)/(a + b*x^2),x]
```

output
```
(4*b^(3/4)*x^(3/2) + 3*Sqrt[2]*a^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*a^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*
b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(6*b^(7/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{3/2}}{3b}-\frac{a\int \frac{\sqrt{x}}{bx^2+a}dx}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\int \frac{x}{bx^2+a}d\sqrt{x}}{b}$$

$$\Big\downarrow \text{826}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}$$

$\big\downarrow$ 27

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{b}$$

$\big\downarrow$ 1103

$$\frac{2x^{3/2}}{3b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{b}$$

input

```
Int[x^(5/2)/(a + b*x^2),x]
```

output
```
(2*x^(3/2))/(3*b) - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)
]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a]
+ Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4))
)/(2*Sqrt[b])))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116
default	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116
risch	$\frac{2x^{\frac{3}{2}}}{3b}-\frac{a\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	116

input
```
int(x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)/b-1/4*a/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(
1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = -\frac{3\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)-3i\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(i\,b^5\left(-\frac{a^3}{b^7}\right)^{\frac{3}{4}}+a^2\sqrt{x}\right)+3i\,b\left(-\frac{a^3}{b^7}\right)^{\frac{1}{4}}\log\left(-i\,b^5\right.}{6\,b}$$

input
```
integrate(x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*b*(-a^3/b^7)^(1/4)*log(b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 3*I*b
*(-a^3/b^7)^(1/4)*log(I*b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) + 3*I*b*(-a^3/
b^7)^(1/4)*log(-I*b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 3*b*(-a^3/b^7)^(1/
4)*log(-b^5*(-a^3/b^7)^(3/4) + a^2*sqrt(x)) - 4*x^(3/2))/b
```

Sympy [A] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \begin{cases} \tilde{\infty}x^{\frac{3}{2}} & \text{for } a=0\wedge b=0 \\ \frac{2x^{\frac{7}{2}}}{7a} & \text{for } b=0 \\ \frac{2x^{\frac{3}{2}}}{3b} & \text{for } a=0 \\ -\frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}+\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b^2\sqrt[4]{-\frac{a}{b}}}+\frac{2x^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a), Eq(b, 0)
), (2*x**(3/2)/(3*b), Eq(a, 0)), (-a*log(sqrt(x) - (-a/b)**(1/4))/(2*b**2*
(-a/b)**(1/4)) + a*log(sqrt(x) + (-a/b)**(1/4))/(2*b**2*(-a/b)**(1/4)) - a
*atan(sqrt(x)/(-a/b)**(1/4))/(b**2*(-a/b)**(1/4)) + 2*x**(3/2)/(3*b), True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.26

$$\int \frac{x^{5/2}}{a+bx^2}\,dx =$$

$$-\frac{a\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right.}{4b}$$

$$+\frac{2x^{\frac{3}{2}}}{3b}$$

input
```
integrate(x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*a*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/b + 2/3*x^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.20

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2\,x^{\frac{3}{2}}}{3\,b} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^4} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^4}$$

input
```
integrate(x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
2/3*x^(3/2)/b - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b
)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/b^4 - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-
1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^4 + 1/4*sqrt(
2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^4 - 1/
4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/
b^4
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.36

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{2\,x^{3/2}}{3\,b} + \frac{(-a)^{3/4}\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{b^{7/4}} - \frac{(-a)^{3/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{b^{7/4}}$$

input
```
int(x^(5/2)/(a + b*x^2),x)
```

output
```
(2*x^(3/2))/(3*b) + ((-a)^(3/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4
) - ((-a)^(3/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(7/4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99

$$\int \frac{x^{5/2}}{a+bx^2}\,dx = \frac{6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-3b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\log\left(-\sqrt{x}\right.}{12b^2}$$

input
```
int(x^(5/2)/(b*x^2+a),x)
```

output
```
(6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
 - 3*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
sqrt(a) + sqrt(b)*x) + 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + 8*sqrt(x)*b*x)/(12*b**2)
```

3.290 $\int \frac{x^{3/2}}{a+bx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{x^{3/2}}{a+bx^2} dx = \frac{2\sqrt{x}}{b} + \frac{\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}b^{5/4}} - \frac{\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}b^{5/4}}$$

output
```
2*x^(1/2)/b+1/2*a^(1/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/
b^(5/4)-1/2*a^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/b^(5
/4)-1/2*a^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x
))*2^(1/2)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = \frac{4\sqrt[4]{b}\sqrt{x}+\sqrt{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-\sqrt{2}\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2b^{5/4}}$$

input
```
Integrate[x^(3/2)/(a + b*x^2),x]
```

output
```
(4*b^(1/4)*Sqrt[x] + Sqrt[2]*a^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])] - Sqrt[2]*a^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1
/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2*b^(5/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.56, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{a+bx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{bx^2+a}d\sqrt{x}}{b}$$

$$\Big\downarrow \text{755}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}$$

input
```
Int[x^(3/2)/(a + b*x^2),x]
```

output
```
(2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[a])))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115
default	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115
risch	$\frac{2\sqrt{x}}{b}-\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b}$	115

input
```
int(x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b-1/4/b*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(
a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/
(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}}{a+bx^2}\,dx =$$

$$-\frac{b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)+i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}\log\left(-i\,b\left(-\frac{a}{b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-}{2\,b}$$

input
```
integrate(x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(b*(-a/b^5)^(1/4)*log(b*(-a/b^5)^(1/4) + sqrt(x)) + I*b*(-a/b^5)^(1/4
)*log(I*b*(-a/b^5)^(1/4) + sqrt(x)) - I*b*(-a/b^5)^(1/4)*log(-I*b*(-a/b^5)
^(1/4) + sqrt(x)) - b*(-a/b^5)^(1/4)*log(-b*(-a/b^5)^(1/4) + sqrt(x)) - 4*
sqrt(x))/b
```

Sympy [A] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.75

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=\begin{cases}\tilde{\infty}\sqrt{x} & \text{for } a=0 \wedge \\ \frac{2x^{\frac{5}{2}}}{5a} & \text{for } b=0 \\ \frac{2\sqrt{x}}{b} & \text{for } a=0 \\ \frac{2\sqrt{x}}{b}+\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b}-\frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b} & \text{otherwise}\end{cases}$$

input
```
integrate(x**(3/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a), Eq(b, 0))
, (2*sqrt(x)/b, Eq(a, 0)), (2*sqrt(x)/b + (-a/b)**(1/4)*log(sqrt(x) - (-a/
b)**(1/4))/(2*b) - (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*b) - (-a/
b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/b, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.26

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=-\frac{\frac{2\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}}-\frac{\sqrt{2}a^{\frac{1}{4}}\ldots}{\ldots}}{4b}+\frac{2\sqrt{x}}{b}$$

input
```
integrate(x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sq
rt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sq
rt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a
^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4) - sqrt(2)*a^(1/4)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/b^(1/4))/b + 2*s
qrt(x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.21

$$\int \frac{x^{3/2}}{a+bx^2}\,dx = -\frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,b^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,b^2} + \frac{2\sqrt{x}}{b}$$

input
```
integrate(x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^2 - 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 1/4*sqrt(2)*(a*b^
3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^2 + 2*sqrt(x)
/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.37

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=\frac{2\sqrt{x}}{b}-\frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}}-\frac{(-a)^{1/4}\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{b^{5/4}}$$

input
```
int(x^(3/2)/(a + b*x^2),x)
```

output
```
(2*x^(1/2))/b - ((-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4) -
((-a)^(1/4)*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/b^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.99

$$\int \frac{x^{3/2}}{a+bx^2}\,dx=\frac{2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\log\left(-\sqrt{x}\,b\right.}{4b^2}$$

input
```
int(x^(3/2)/(b*x^2+a),x)
```

output
```
(2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*s
qrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
 + b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x) - b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + 8*sqrt(x)*b)/(4*b**2)
```

3.291 $\int \frac{\sqrt{x}}{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 137

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

output
```
-1/2*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)+1/2
*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(1/4)/b^(3/4)-1/2*arc
tanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(1/4)/
b^(3/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}\sqrt[4]{a}b^{3/4}}$$

input
```
Integrate[Sqrt[x]/(a + b*x^2),x]
```

output
```
-((ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTa
nh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(1
/4)*b^(3/4)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.56, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{x}{bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)$$

input
```
Int[Sqrt[x]/(a + b*x^2),x]
```

output
```
2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/
4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1
/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	106
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	106

input
```
int(x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(-\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(-1/(a*b^3))^(1/4)*log(a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*I*(-1
/(a*b^3))^(1/4)*log(I*a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) + 1/2*I*(-1/(a*b
^3))^(1/4)*log(-I*a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(-1/(a*b^3))^(
1/4)*log(-a*b^2*(-1/(a*b^3))^(3/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ -\frac{2}{b\sqrt{x}} & \text{for } a=0 \\ \frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} - \frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2b\sqrt[4]{-\frac{a}{b}}} + \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{b\sqrt[4]{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0))
, (-2/(b*sqrt(x)), Eq(a, 0)), (log(sqrt(x) - (-a/b)**(1/4))/(2*b*(-a/b)**(
1/4)) - log(sqrt(x) + (-a/b)**(1/4))/(2*b*(-a/b)**(1/4)) + atan(sqrt(x)/(-
a/b)**(1/4))/(b*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{1}{4}}b^{\frac{3}{4}}}$$

input
```
integrate(x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - 1/4*sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + 1/4*sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(
3/4))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab^3}$$

input
```
integrate(x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)*(a*b^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) + 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b
^3)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)-\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{1/4}\,b^{3/4}}$$

input
```
int(x^(1/2)/(a + b*x^2),x)
```

output
```
(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))
/((-a)^(1/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{a+bx^2}\,dx = \frac{\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+\log\left(-\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+\sqrt{a}+\sqrt{b}\,x\right)-\log\left(\sqrt{x}\right.\right.}{4b^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int(x^(1/2)/(b*x^2+a),x)
```

output
```
(b**(1/4)*a**(3/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) - log(sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)))/(4*a*b)
```

3.292 $\int \frac{1}{\sqrt{x}(a+bx^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

output
```
-1/2*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)+1/2
*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(3/4)/b^(1/4)+1/2*arc
tanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/
b^(1/4)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{-\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{3/4}\sqrt[4]{b}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)),x]
```

output
```
(-ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + ArcTan
h[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(Sqrt[2]*a^(3/
4)*b^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.57, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 266}$$

$$2\int \frac{1}{bx^2+a}d\sqrt{x}$$

$$\Big\downarrow \text{ 755}$$

$$2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \; 27$$

$$2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

$$\Big\downarrow \; 1103$$

$$2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)),x]
```

output
```
2*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/
4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1
/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] +
Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/
4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a}$	106
default	$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a}$	106

input
```
int(1/x^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/
(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-i\,a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}\log\left(-a\left(-\frac{1}{a^3b}\right)^{\frac{1}{4}}+\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(-1/(a^3*b))^(1/4)*log(a*(-1/(a^3*b))^(1/4) + sqrt(x)) + 1/2*I*(-1/(a^
3*b))^(1/4)*log(I*a*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*I*(-1/(a^3*b))^(1/
4)*log(-I*a*(-1/(a^3*b))^(1/4) + sqrt(x)) - 1/2*(-1/(a^3*b))^(1/4)*log(-a*
(-1/(a^3*b))^(1/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} & \text{for } a=0 \wedge b=0 \\ -\frac{2}{3bx^{\frac{3}{2}}} & \text{for } a=0 \\ \frac{2\sqrt{x}}{a} & \text{for } b=0 \\ -\frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a} + \frac{\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*b*x**(3/2)), Eq(a, 0
)), (2*sqrt(x)/a, Eq(b, 0)), (-(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/
(2*a) + (-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(2*a) + (-a/b)**(1/4)*a
tan(sqrt(x)/(-a/b)**(1/4))/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{4\,a^{\frac{3}{4}}b^{\frac{1}{4}}}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 1/4*sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - 1/4*sqrt(2
)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(
1/4))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,ab}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b) + 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(s
qrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b) + 1/4*sqrt(2)*(a*b^3)^(
1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b) - 1/4*sqrt(2)*
(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.27

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx = -\frac{\operatorname{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)+\operatorname{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{(-a)^{3/4}\,b^{1/4}}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)),x)
```

output
```
-(atan((b^(1/4)*x^(1/2))/(-a)^(1/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))
)/((-a)^(3/4)*b^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)}\,dx$$
$$= \frac{\sqrt{2}\left(-2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+2atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\log\left(-\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+\sqrt{a}+\sqrt{b}\,x\right)+\log\left(\sqrt{x}\right.\right.}{4b^{\frac{1}{4}}a^{\frac{3}{4}}}$$

input
```
int(1/x^(1/2)/(b*x^2+a),x)
```

output
```
(b**(3/4)*a**(1/4)*sqrt(2)*( - 2*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) + 2*atan((b**(1/4)*a**(1/4)*sqrt(
2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + log(sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)))/(4*a*b)
```

3.293 $\int \frac{1}{x^{3/2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{1}{x^{3/2}\,(a+bx^2)}\,dx=-\frac{2}{a\sqrt{x}}+\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}}-\frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{5/4}}+\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{5/4}}$$

output
```
-2/a/x^(1/2)+1/2*b^(1/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)
/a^(5/4)-1/2*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(
5/4)+1/2*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*
x))*2^(1/2)/a^(5/4)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx=\frac{-\frac{4\sqrt[4]{a}}{\sqrt{x}}+\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+\sqrt{2}\sqrt[4]{b}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{2a^{5/4}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(1/4))/Sqrt[x] + Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt
[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b
^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(2*a^(5/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.57, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(a+bx^2)}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ 1476$$

$$-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ 1082$$

$$-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ 217$$

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$$\Big\downarrow \ 1479$$

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$\downarrow$ 25

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$\downarrow$ 27

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

$\downarrow$ 1103

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}$$

input

```
Int[1/(x^(3/2)*(a + b*x^2)),x]
```

output
```
-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(
Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4
)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(
2*Sqrt[b])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{a\sqrt{x}}$	115
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{a\sqrt{x}}$	115
risch	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{a\sqrt{x}}$	115

input
```
int(1/x^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))-2/a/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = -\frac{ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)-i\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(i\,a^4\left(-\frac{b}{a^5}\right)^{\frac{3}{4}}+b\sqrt{x}\right)+i\,ax\left(-\frac{b}{a^5}\right)^{\frac{1}{4}}\log\left(-i\,a^4\left(-\frac{b}{a^5}\right.\right.}{2\,ax}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/2*(a*x*(-b/a^5)^(1/4)*log(a^4*(-b/a^5)^(3/4) + b*sqrt(x)) - I*a*x*(-b/a
^5)^(1/4)*log(I*a^4*(-b/a^5)^(3/4) + b*sqrt(x)) + I*a*x*(-b/a^5)^(1/4)*log
(-I*a^4*(-b/a^5)^(3/4) + b*sqrt(x)) - a*x*(-b/a^5)^(1/4)*log(-a^4*(-b/a^5)
^(3/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x)
```

Sympy [A] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} & \text{for}\: a=0 \wedge b=0 \\ -\frac{2}{5bx^{\frac{5}{2}}} & \text{for}\: a=0 \\ -\frac{2}{a\sqrt{x}} & \text{for}\: b=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}+\frac{\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a\sqrt[4]{-\frac{a}{b}}}-\frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a\sqrt[4]{-\frac{a}{b}}}-\frac{2}{a\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (-2/(5*b*x**(5/2)), Eq(a, 0
)), (-2/(a*sqrt(x)), Eq(b, 0)), (-log(sqrt(x) - (-a/b)**(1/4))/(2*a*(-a/b)
**(1/4)) + log(sqrt(x) + (-a/b)**(1/4))/(2*a*(-a/b)**(1/4)) - atan(sqrt(x)
/(-a/b)**(1/4))/(a*(-a/b)**(1/4)) - 2/(a*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = -\frac{b\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{4a} - \frac{2}{a\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2
)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(s
qrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(
1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*lo
g(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)
))/a - 2/(a*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = -\frac{2}{a\sqrt{x}} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2a^2b^2} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2b^2} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4a^2b^2}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-2/(a*sqrt(x)) - 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/
b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) - 1/2*sqrt(2)*(a*b^3)^(3/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2
) + 1/4*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a
/b))/(a^2*b^2) - 1/4*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(a^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^{3/2}(a+bx^2)}\,dx = \frac{(-b)^{1/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}} - \frac{(-b)^{1/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{5/4}} - \frac{2}{a\sqrt{x}}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)),x)
```

output
```
((-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - ((-b)^(1/4)*ata
n(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(5/4) - 2/(a*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)}\,dx = \frac{2\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-\sqrt{x}\,b^{\frac{1}{4}}$$

input

```
int(1/x^(3/2)/(b*x^2+a),x)
```

output

```
(2*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2))) - 2*sqrt(x)*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2))) - sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/
4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) + sqrt(x)*b**(1/4)*a**(3/4)*sqr
t(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x) - 8*a)/(
4*sqrt(x)*a**2)
```

3.294 $\int \frac{1}{x^{5/2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 149

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx=-\frac{2}{3ax^{3/2}}+\frac{b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{7/4}}-\frac{b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{7/4}}$$

output
```
-2/3/a/x^(3/2)+1/2*b^(3/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/
2)/a^(7/4)-1/2*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a
^(7/4)-1/2*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2
)*x))*2^(1/2)/a^(7/4)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^{5/2}(a+bx^2)}\,dx=\frac{-\frac{4a^{3/4}}{x^{3/2}}+3\sqrt{2}b^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-3\sqrt{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{6a^{7/4}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(3/4))/x^(3/2) + 3*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/
4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(6*a^(7/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.55, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 1476$$

$$-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 217$$

$$-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 1479$$

$$-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)),x]
```

output
```
-2/(3*a*x^(3/2)) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/
(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]
/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))
/(2*Sqrt[a])))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$-\frac{2}{3 a \,x^{\frac{3}{2}}}-\frac{b \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x -\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a^{2}}$	116
default	$-\frac{2}{3 a \,x^{\frac{3}{2}}}-\frac{b \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x -\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a^{2}}$	116
risch	$-\frac{2}{3 a \,x^{\frac{3}{2}}}-\frac{b \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln \left(\frac{x +\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}{x -\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{x}\, \sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2 \arctan \left(\frac{\sqrt{2}\, \sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4 a^{2}}$	116

input
```
int(1/x^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/x^(3/2)-1/4*b/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^
(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2
^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx =$$
$$-\frac{3\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)+3i\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}+b\sqrt{x}\right)-3i\,ax^2\left(-\frac{b^3}{a^7}\right)^{\frac{1}{4}}\log\left(\right.}{6\,ax^2}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/6*(3*a*x^2*(-b^3/a^7)^(1/4)*log(a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 3*I
*a*x^2*(-b^3/a^7)^(1/4)*log(I*a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*I*a*x^
2*(-b^3/a^7)^(1/4)*log(-I*a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) - 3*a*x^2*(-b^
3/a^7)^(1/4)*log(-a^2*(-b^3/a^7)^(1/4) + b*sqrt(x)) + 4*sqrt(x))/(a*x^2)
```

Sympy [A] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ -\frac{2}{7bx^{\frac{7}{2}}} \\ -\frac{2}{3ax^{\frac{3}{2}}} \\ -\frac{2}{3ax^{\frac{3}{2}}}+\frac{b\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2}-\frac{b\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2}-\frac{b\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2} \end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (-2/(7*b*x**(7/2)), Eq(a, 0
)), (-2/(3*a*x**(3/2)), Eq(b, 0)), (-2/(3*a*x**(3/2)) + b*(-a/b)**(1/4)*lo
g(sqrt(x) - (-a/b)**(1/4))/(2*a**2) - b*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)
**(1/4))/(2*a**2) - b*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/a**2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{5/2}(a+bx^2)}\,dx =$$

$$-\frac{\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(-\right.}{}}{4a}$$

$$-\frac{2}{3ax^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*
sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2
)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt
(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^
(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))
/a - 2/3/(a*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx = -\frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^2} - \frac{2}{3\,ax^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/a^2 - 1/2*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^2 - 1/4*sqrt(2)*(a*b^3)^(1/4
)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 + 1/4*sqrt(2)*(a*b^
3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^2 - 2/3/(a*x^
(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)}\,dx = \frac{(-b)^{3/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}} - \frac{2}{3\,a\,x^{3/2}} + \frac{(-b)^{3/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{7/4}}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)),x)
```

output
```
((-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4) - 2/(3*a*x^(3/2))
+ ((-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(7/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2}\,(a+bx^2)}\,dx = \frac{6\sqrt{x}\,b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x-6\sqrt{x}\,b^{\frac{3}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x+3\sqrt{}}{}$$

input
```
int(1/x^(5/2)/(b*x^2+a),x)
```

output
```
(6*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*s
qrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*x - 6*sqrt(x)*b**(3/4)*a**(1/
4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*
a**(1/4)*sqrt(2)))*x + 3*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*
b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*x - 3*sqrt(x)*b**(3/4)*a*
*(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x
)*x - 8*a)/(12*sqrt(x)*a**2*x)
```

3.295 $\int \frac{1}{x^{7/2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 160

$$\int \frac{1}{x^{7/2}(a+bx^2)}\,dx = -\frac{2}{5ax^{5/2}}+\frac{2b}{a^2\sqrt{x}}-\frac{b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}+\frac{b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}a^{9/4}}-\frac{b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt{2}a^{9/4}}$$

output
```
-2/5/a/x^(5/2)+2*b/a^2/x^(1/2)-1/2*b^(5/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2
)/a^(1/4))*2^(1/2)/a^(9/4)+1/2*b^(5/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^
(1/4))*2^(1/2)/a^(9/4)-1/2*b^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)
/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^{7/2}(a+bx^2)}\,dx = \frac{-\frac{4\sqrt[4]{a}(a-5bx^2)}{x^{5/2}} - 5\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 5\sqrt{2}b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{10a^{9/4}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)),x]
```

output
```
((-4*a^(1/4)*(a - 5*b*x^2))/x^(5/2) - 5*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] -
Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*Sqrt[2]*b^(5/4)*ArcTanh[
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(10*a^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.55, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\int \frac{1}{x^{3/2}(bx^2+a)}dx}{a} - \frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a} - \frac{2}{a\sqrt{x}}\right)}{a} - \frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{b\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 826

$$-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 1476

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 1082

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 217

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 1479

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}$$

$$\frac{2}{5ax^{5/2}}$$

$\Big\downarrow$ 25

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)),x]
```

output
```
-2/(5*a*x^(5/2)) - (b*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1
/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[S
qrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(
1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2(-5b\,x^{2}+a)}{5a^{2}x^{\frac{5}{2}}}+\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124
derivativedivides	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^{2}\sqrt{x}}$	125
default	$\frac{b\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{4a^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{2}{5a\,x^{\frac{5}{2}}}+\frac{2b}{a^{2}\sqrt{x}}$	125

input
```
int(1/x^(7/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-5*b*x^2+a)/a^2/x^(5/2)+1/4*b/a^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(
1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/
2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*
x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{7/2}\left(a+bx^{2}\right)}\,dx=\frac{5\,a^{2}x^{3}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(a^{7}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{3}{4}}+b^{4}\sqrt{x}\right)-5i\,a^{2}x^{3}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{1}{4}}\log\left(i\,a^{7}\left(-\frac{b^{5}}{a^{9}}\right)^{\frac{3}{4}}+b^{4}\sqrt{x}\right.}{}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/10*(5*a^2*x^3*(-b^5/a^9)^(1/4)*log(a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) -
 5*I*a^2*x^3*(-b^5/a^9)^(1/4)*log(I*a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) +
5*I*a^2*x^3*(-b^5/a^9)^(1/4)*log(-I*a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) -
5*a^2*x^3*(-b^5/a^9)^(1/4)*log(-a^7*(-b^5/a^9)^(3/4) + b^4*sqrt(x)) + 4*(5
*b*x^2 - a)*sqrt(x))/(a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 30.96 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} & \text{for } a=0 \\ -\frac{2}{9bx^{\frac{9}{2}}} & \text{for } a=0 \\ -\frac{2}{5ax^{\frac{5}{2}}} & \text{for } b=0 \\ -\frac{2}{5ax^{\frac{5}{2}}} + \frac{b\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} - \frac{b\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{2a^2\sqrt[4]{-\frac{a}{b}}} + \frac{b\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{a^2\sqrt[4]{-\frac{a}{b}}} + \frac{2b}{a^2\sqrt{x}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a),x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(9*b*x**(9/2)), Eq(a, 0
)), (-2/(5*a*x**(5/2)), Eq(b, 0)), (-2/(5*a*x**(5/2)) + b*log(sqrt(x) - (-
a/b)**(1/4))/(2*a**2*(-a/b)**(1/4)) - b*log(sqrt(x) + (-a/b)**(1/4))/(2*a*
*2*(-a/b)**(1/4)) + b*atan(sqrt(x)/(-a/b)**(1/4))/(a**2*(-a/b)**(1/4)) + 2
*b/(a**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx = \frac{b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}\right.}{a^{\frac{1}{4}}b^{\frac{3}{4}}}\right.}{4\,a^2} + \frac{2\left(5\,bx^2-a\right)}{5\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)
*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(
2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(
sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*l
og(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4
)))/a^2 + 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx = \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,a^3b} - \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b} + \frac{\sqrt{2}\left(ab^3\right)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{4\,a^3b} + \frac{2\left(5\,bx^2-a\right)}{5\,a^2x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^3*b) + 1/2*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) - 1/4*sqrt(2)*(a*b^
3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) + 1/4*sq
rt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3
*b) + 2/5*(5*b*x^2 - a)/(a^2*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx=\frac{(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{a^{9/4}}-\frac{\frac{2}{5\,a}-\frac{2\,b\,x^2}{a^2}}{x^{5/2}}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)),x)
```

output
```
((-b)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - ((-b)^(5/4)*ata
n(((-b)^(1/4)*x^(1/2))/a^(1/4)))/a^(9/4) - (2/(5*a) - (2*b*x^2)/a^2)/x^(5/
2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)}\,dx=\frac{-10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2}{}$$

input
```
int(1/x^(7/2)/(b*x^2+a),x)
```

output

```
( - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 10*sqrt(x)*b**(1
/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/
(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 + 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*
log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 5
*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) +
 sqrt(a) + sqrt(b)*x)*b*x**2 - 8*a**2 + 40*a*b*x**2)/(20*sqrt(x)*a**3*x**2
)
```

3.296 $\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \frac{5\sqrt{x}}{2b^2}-\frac{x^{5/2}}{2b(a+bx^2)}+\frac{5\sqrt[4]{a}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}b^{9/4}}-\frac{5\sqrt[4]{a}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}b^{9/4}}$$

output
```
5/2*x^(1/2)/b^2-1/2*x^(5/2)/b/(b*x^2+a)+5/8*a^(1/4)*arctan(1-2^(1/2)*b^(1/
4)*x^(1/2)/a^(1/4))*2^(1/2)/b^(9/4)-5/8*a^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x
^(1/2)/a^(1/4))*2^(1/2)/b^(9/4)-5/8*a^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4
)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{b}\sqrt{x}(5a+4bx^2)}{a+bx^2}+5\sqrt{2}\sqrt[4]{a}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-5\sqrt{2}\sqrt[4]{a}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{8b^{9/4}}$$

input

```
Integrate[x^(7/2)/(a + b*x^2)^2,x]
```

output

```
((4*b^(1/4)*Sqrt[x]*(5*a + 4*b*x^2))/(a + b*x^2) + 5*Sqrt[2]*a^(1/4)*ArcTa
n[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 5*Sqrt[2]*a^(
1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(8*
b^(9/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5\int \frac{x^{3/2}}{bx^2+a}dx}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{262}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{a\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\int \frac{1}{bx^2+a}d\sqrt{x}}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b(a+bx^2)}$$

$\downarrow$ 1479

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}$$

$$\frac{x^{5/2}}{2b(a+bx^2)}$$

$\downarrow$ 25

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{2\sqrt{x}}{b}-\frac{2a\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{b}\right)}{4b}-\frac{x^{5/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[x^(7/2)/(a + b*x^2)^2,x]
```

output
```
-1/2*x^(5/2)/(b*(a + b*x^2)) + (5*((2*Sqrt[x])/b - (2*a*((-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) +
(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*
a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/b))/(4*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	si
derivativedivides	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{4\left(b\,x^2+a\right)}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{b^2}$	13
default	$\frac{2\sqrt{x}}{b^2}-\frac{2a\left(-\frac{\sqrt{x}}{4\left(b\,x^2+a\right)}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{b^2}$	13
risch	$\frac{2\sqrt{x}}{b^2}-\frac{a\left(-\frac{\sqrt{x}}{2\left(b\,x^2+a\right)}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a}\right)}{b^2}$	13

input
```
int(x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)/b^2-2*a/b^2*(-1/4*x^(1/2)/(b*x^2+a)+5/32*(a/b)^(1/4)/a*2^(1/2)*(
ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = -\frac{5\,(b^3x^2+ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(5\,b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)+5\,(i\,b^3x^2+i\,ab^2)\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}\log\left(5i\,b^2\left(-\frac{a}{b^9}\right)^{\frac{1}{4}}+5\,\sqrt{x}\right)+5}{}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(5*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*log(5*b^2*(-a/b^9)^(1/4) + 5*sqrt
(x)) + 5*(I*b^3*x^2 + I*a*b^2)*(-a/b^9)^(1/4)*log(5*I*b^2*(-a/b^9)^(1/4) +
 5*sqrt(x)) + 5*(-I*b^3*x^2 - I*a*b^2)*(-a/b^9)^(1/4)*log(-5*I*b^2*(-a/b^9
)^(1/4) + 5*sqrt(x)) - 5*(b^3*x^2 + a*b^2)*(-a/b^9)^(1/4)*log(-5*b^2*(-a/b
^9)^(1/4) + 5*sqrt(x)) - 4*(4*b*x^2 + 5*a)*sqrt(x))/(b^3*x^2 + a*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(163) = 326$.

Time = 81.88 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.91

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \begin{cases} \tilde{\infty}\sqrt{x} \\ \frac{2x^{\frac{9}{2}}}{9a^2} \\ \frac{2\sqrt{x}}{b^2} \\ \frac{20a\sqrt{x}}{8ab^2+8b^3x^2} + \frac{5a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2} - \frac{5a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2+8b^3x^2} - \frac{10a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right.}{8ab^2+8b^3x^2} \end{cases}$$

input
```
integrate(x**(7/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(9/2)/(9*a**2), Eq(b,
0)), (2*sqrt(x)/b**2, Eq(a, 0)), (20*a*sqrt(x)/(8*a*b**2 + 8*b**3*x**2) +
5*a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) -
5*a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) -
10*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) +
16*b*x**(5/2)/(8*a*b**2 + 8*b**3*x**2) + 5*b*x**2*(-a/b)**(1/4)*log(sqrt(x
) - (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 5*b*x**2*(-a/b)**(1/4)*log(s
qrt(x) + (-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2) - 10*b*x**2*(-a/b)**(1/4)
*atan(sqrt(x)/(-a/b)**(1/4))/(8*a*b**2 + 8*b**3*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \frac{a\sqrt{x}}{2\,(b^3x^2+ab^2)} - \frac{5\left(\frac{2\,\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\,\sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}a^{\frac{1}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{b^{\frac{1}{4}}} - \sqrt{}\right.}{16\,b^2} + \frac{2\,\sqrt{x}}{b^2}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*a*sqrt(x)/(b^3*x^2 + a*b^2) - 5/16*(2*sqrt(2)*sqrt(a)*arctan(1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sq
rt(sqrt(a)*sqrt(b)) + 2*sqrt(2)*sqrt(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/sqrt(sqrt(a)*sqrt(b
)) + sqrt(2)*a^(1/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqr
t(a))/b^(1/4) - sqrt(2)*a^(1/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqr
t(b)*x + sqrt(a))/b^(1/4))/b^2 + 2*sqrt(x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = -\frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^3} - \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,b^3} - \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^3} + \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,b^3} + \frac{a\sqrt{x}}{2\,(bx^2+a)b^2} + \frac{2\sqrt{x}}{b^2}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/b^3 - 5/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/b^3 - 5/16*sqrt(2)*(a*b^3)^(1/
4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 5/16*sqrt(2)*(a*
b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/b^3 + 1/2*a*s
qrt(x)/((b*x^2 + a)*b^2) + 2*sqrt(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.45

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \frac{2\sqrt{x}}{b^2} - \frac{5\,(-a)^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4\,b^{9/4}} + \frac{a\sqrt{x}}{2\,(b^3\,x^2+a\,b^2)} + \frac{(-a)^{1/4}\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}\,1\mathrm{i}}{(-a)^{1/4}}\right)\,5\mathrm{i}}{4\,b^{9/4}}$$

input
```
int(x^(7/2)/(a + b*x^2)^2,x)
```

output
```
(2*x^(1/2))/b^2 - (5*(-a)^(1/4)*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*b^(
9/4)) + ((-a)^(1/4)*atan((b^(1/4)*x^(1/2)*1i)/(-a)^(1/4))*5i)/(4*b^(9/4))
+ (a*x^(1/2))/(2*(a*b^2 + b^3*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

$$\int \frac{x^{7/2}}{(a+bx^2)^2}\,dx = \frac{10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2-10b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,a}{}$$

input
```
int(x^(7/2)/(b*x^2+a)^2,x)
```

output
```
(10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*b**(3/4)*a**(1/4)*sqrt(2)*ata
n((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(
2)))*b*x**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 10*b**(3/4)*a**(1/4
)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a
**(1/4)*sqrt(2)))*b*x**2 + 5*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(
1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 5*b**(3/4)*a**(1/4)*sqrt(
2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2
- 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt
(a) + sqrt(b)*x)*a - 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 40*sqrt(x)*a*b + 32*sqrt(x)*b
**2*x**2)/(16*b**3*(a + b*x**2))
```

3.297 $\int \frac{x^{5/2}}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 165

$$\int \frac{x^{5/2}}{(a+bx^2)^2} \, dx = -\frac{x^{3/2}}{2b(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}\sqrt[4]{a}b^{7/4}}$$

output
```
-1/2*x^(3/2)/b/(b*x^2+a)-3/8*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(
1/2)/a^(1/4)/b^(7/4)+3/8*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)
/a^(1/4)/b^(7/4)-3/8*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1
/2)*x))*2^(1/2)/a^(1/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{-\frac{4b^{3/4}x^{3/2}}{a+bx^2} - \frac{3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{a}} - \frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{a}}}{8b^{7/4}}$$

input
```
Integrate[x^(5/2)/(a + b*x^2)^2,x]
```

output
```
((-4*b^(3/4)*x^(3/2))/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x
)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(1/4) - (3*Sqrt[2]*ArcTanh[(Sqrt[2
]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(1/4))/(8*b^(7/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \ 252$$

$$\frac{3\int \frac{\sqrt{x}}{bx^2+a}dx}{4b} - \frac{x^{3/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \ 266$$

$$\frac{3\int \frac{x}{bx^2+a}d\sqrt{x}}{2b} - \frac{x^{3/2}}{2b(a+bx^2)}$$

$$\Big\downarrow \ 826$$

$$\frac{3\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 1476

$$\frac{3\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 1082

$$\frac{3\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 217

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\downarrow$ 1479

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{2b}-\frac{x^{3/2}}{2b\left(a+bx^2\right)}$$

input
```
Int[x^(5/2)/(a + b*x^2)^2,x]
```

output
```
-1/2*x^(3/2)/(b*(a + b*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])
/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x]
)/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log
[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)
*b^(1/4)))/(2*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2b(b\,x^{2}+a)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124
default	$-\frac{x^{\frac{3}{2}}}{2b(b\,x^{2}+a)}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b^{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	124

input
```
int(x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(3/2)/b/(b*x^2+a)+3/16/b^2/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.21

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx=\frac{3\,(b^2x^2+ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(ab^5\left(-\frac{1}{ab^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\,(i\,b^2x^2+i\,ab)\left(-\frac{1}{ab^7}\right)^{\frac{1}{4}}\log\left(i\,ab^5\left(-\frac{1}{ab^7}\right.\right.}{}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(3*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log(a*b^5*(-1/(a*b^7))^(3/4) + s
qrt(x)) - 3*(I*b^2*x^2 + I*a*b)*(-1/(a*b^7))^(1/4)*log(I*a*b^5*(-1/(a*b^7)
)^(3/4) + sqrt(x)) - 3*(-I*b^2*x^2 - I*a*b)*(-1/(a*b^7))^(1/4)*log(-I*a*b^
5*(-1/(a*b^7))^(3/4) + sqrt(x)) - 3*(b^2*x^2 + a*b)*(-1/(a*b^7))^(1/4)*log
(-a*b^5*(-1/(a*b^7))^(3/4) + sqrt(x)) - 4*x^(3/2))/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(151) = 302$.

Time = 52.33 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.38

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} \\ \frac{2x^{\frac{7}{2}}}{7a^2} \\ -\frac{2}{b^2\sqrt{x}} \\ \frac{3a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} - \frac{3a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} + \frac{6a\,\mathrm{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b^3x^2\sqrt[4]{-\frac{a}{b}}} - \frac{4bx^{\frac{3}{2}}\sqrt[4]{}}{8ab^2\sqrt[4]{-\frac{a}{b}}+8b} \end{cases}$$

input
```
integrate(x**(5/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(7/2)/(7*a**2), Eq(b,
0)), (-2/(b**2*sqrt(x)), Eq(a, 0)), (3*a*log(sqrt(x) - (-a/b)**(1/4))/(8*a
*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) - 3*a*log(sqrt(x) + (-a/b
)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*a*atan(
sqrt(x)/(-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)
) - 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b
)**(1/4)) + 3*b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4)
+ 8*b**3*x**2*(-a/b)**(1/4)) - 3*b*x**2*log(sqrt(x) + (-a/b)**(1/4))/(8*a*
b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)) + 6*b*x**2*atan(sqrt(x)/(-
a/b)**(1/4))/(8*a*b**2*(-a/b)**(1/4) + 8*b**3*x**2*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(b^2x^2+ab)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\sqrt{}\right.}{}\right.}{16\,b}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*x^(3/2)/(b^2*x^2 + a*b) + 3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)
*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)
*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4
) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(
b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(
a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*
x + sqrt(a))/(a^(1/4)*b^(3/4)))/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = -\frac{x^{\frac{3}{2}}}{2\,(bx^2+a)b} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^4} - \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4} + \frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^4}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*x^(3/2)/((b*x^2 + a)*b) + 3/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a*b^4) + 3/8*sqrt(2)*(a*
b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/
4))/(a*b^4) - 3/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) +
 x + sqrt(a/b))/(a*b^4) + 3/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*
(a/b)^(1/4) + x + sqrt(a/b))/(a*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}} - \frac{x^{3/2}}{2\,b\,(b\,x^2+a)} - \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{1/4}\,b^{7/4}}$$

input
```
int(x^(5/2)/(a + b*x^2)^2,x)
```

output
```
(3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^(7/4)) - x^(3/2)/(2
*b*(a + b*x^2)) - (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(1/4)*b^
(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.86

$$\int \frac{x^{5/2}}{(a+bx^2)^2}\,dx = \frac{-6b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 6b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 6b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,ata}{}$$

input
```
int(x^(5/2)/(b*x^2+a)^2,x)
```

output
```
( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 6*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*b*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 6*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b*x**2 + 3*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + 3*b**(1/4)*a**(3/4)*sqrt(2
)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 -
 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a - 3*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 8*sqrt(x)*a*b*x)/(16*a*b**2*(a
 + b*x**2))
```

3.298 $\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 165

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = -\frac{\sqrt{x}}{2b(a+bx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

output
```
-1/2*x^(1/2)/b/(b*x^2+a)-1/8*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(
1/2)/a^(3/4)/b^(5/4)+1/8*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)
/a^(3/4)/b^(5/4)+1/8*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1
/2)*x))*2^(1/2)/a^(3/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{-\frac{4\sqrt[4]{b}\sqrt{x}}{a+bx^2} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}}}{8b^{5/4}}$$

input
```
Integrate[x^(3/2)/(a + b*x^2)^2,x]
```

output
```
((-4*b^(1/4)*Sqrt[x])/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/
(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (Sqrt[2]*ArcTanh[(Sqrt[2]*a^
(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/4))/(8*b^(5/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{4b} - \frac{\sqrt{x}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{266}$$

$$\frac{\int \frac{1}{bx^2+a}d\sqrt{x}}{2b} - \frac{\sqrt{x}}{2b(a+bx^2)}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1476$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1082$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 217$$

$$\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 1479$$

$$\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b\left(a+bx^2\right)}$$

input
```
Int[x^(3/2)/(a + b*x^2)^2,x]
```

output
```
-1/2*Sqrt[x]/(b*(a + b*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a]))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{\sqrt{x}}{2b(b\,x^2+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ba}$	127
default	$-\frac{\sqrt{x}}{2b(b\,x^2+a)}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ba}$	127

input
```
int(x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/b/(b*x^2+a)+1/16/b*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x
^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2
*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2
)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.16

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{(b^2x^2+ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(ab\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}+\sqrt{x}\right)-(-i\,b^2x^2-i\,ab)\left(-\frac{1}{a^3b^5}\right)^{\frac{1}{4}}\log\left(i\,ab\left(-\frac{1}{a^3b^5}\right)\cdots\right.}{}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log(a*b*(-1/(a^3*b^5))^(1/4) + s
qrt(x)) - (-I*b^2*x^2 - I*a*b)*(-1/(a^3*b^5))^(1/4)*log(I*a*b*(-1/(a^3*b^5
))^(1/4) + sqrt(x)) - (I*b^2*x^2 + I*a*b)*(-1/(a^3*b^5))^(1/4)*log(-I*a*b*
(-1/(a^3*b^5))^(1/4) + sqrt(x)) - (b^2*x^2 + a*b)*(-1/(a^3*b^5))^(1/4)*log
(-a*b*(-1/(a^3*b^5))^(1/4) + sqrt(x)) - 4*sqrt(x))/(b^2*x^2 + a*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(146) = 292$.

Time = 29.30 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.96

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{3}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a^2} \\ -\frac{2}{3b^2x^{\frac{3}{2}}} \\ -\frac{4a\sqrt{x}}{8a^2b+8ab^2x^2} - \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b+8ab^2x^2} + \frac{a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b+8ab^2x^2} + \frac{2a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right.}{8a^2b+8ab^2x^2} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/x**(3/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**2), Eq(b,
 0)), (-2/(3*b**2*x**(3/2)), Eq(a, 0)), (-4*a*sqrt(x)/(8*a**2*b + 8*a*b**2
*x**2) - a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2
*x**2) + a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b + 8*a*b**2
*x**2) + 2*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b + 8*a*b**
2*x**2) - b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b + 8*
a*b**2*x**2) + b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**2*b
 + 8*a*b**2*x**2) + 2*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*
a**2*b + 8*a*b**2*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{\right.}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{16\,b} - \frac{\sqrt{x}}{2\,(b^2x^2+ab)}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))
/b - 1/2*sqrt(x)/(b^2*x^2 + a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,ab^2} + \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^2} - \frac{\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,ab^2} - \frac{\sqrt{x}}{2\,(bx^2+a)b}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^2) + 1/16*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^2) - 1/16*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
*b^2) - 1/2*sqrt(x)/((b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = -\frac{\sqrt{x}}{2\,b\,(b\,x^2+a)} - \frac{\operatorname{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{3/4}\,b^{5/4}} - \frac{\operatorname{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{3/4}\,b^{5/4}}$$

input
```
int(x^(3/2)/(a + b*x^2)^2,x)
```

output
```
- x^(1/2)/(2*b*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(
3/4)*b^(5/4)) - atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(3/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.84

$$\int \frac{x^{3/2}}{(a+bx^2)^2}\,dx = \frac{-2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) - 2b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2 + 2b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,ata}{}$$

input
```
int(x^(3/2)/(b*x^2+a)^2,x)
```

output
```
( - 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 2*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*b*x**2 + 2*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b*x**2 - b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - b**(3/4)*a**(1/4)*sqrt(2)*lo
g( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + b**
(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*a + b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt
(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 8*sqrt(x)*a*b)/(16*a*b**2*(a + b*x**2)
)
```

3.299 $\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 165

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{3/2}}{2a(a+bx^2)} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} + \frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{5/4}b^{3/4}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{5/4}b^{3/4}}$$

output
```
1/2*x^(3/2)/a/(b*x^2+a)-1/8*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1
/2)/a^(5/4)/b^(3/4)+1/8*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/
a^(5/4)/b^(3/4)-1/8*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/
2)*x))*2^(1/2)/a^(5/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{\frac{4\sqrt[4]{a}x^{3/2}}{a+bx^2} - \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}}{8a^{5/4}}$$

input
```
Integrate[Sqrt[x]/(a + b*x^2)^2,x]
```

output
```
((4*a^(1/4)*x^(3/2))/(a + b*x^2) - (Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(
Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (Sqrt[2]*ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4))/(8*a^(5/4))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{\int \frac{\sqrt{x}}{bx^2+a}dx}{4a} + \frac{x^{3/2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{\int \frac{x}{bx^2+a}d\sqrt{x}}{2a} + \frac{x^{3/2}}{2a(a+bx^2)}$$

$$\Big\downarrow \text{ 826}$$

$$\frac{\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a\left(a+bx^2\right)}$$

input

```
Int[Sqrt[x]/(a + b*x^2)^2,x]
```

output

```
x^(3/2)/(2*a*(a + b*x^2)) + ((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/
4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1
/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*
a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/
4)))/(2*Sqrt[b]))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{2a(b\,x^{2}+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	127
default	$\frac{x^{\frac{3}{2}}}{2a(b\,x^{2}+a)}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	127

input
```
int(x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)/a/(b*x^2+a)+1/16/a/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{(abx^2+a^2)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-\left(i\,abx^2+i\,a^2\right)\left(-\frac{1}{a^5b^3}\right)^{\frac{1}{4}}\log\left(i\,a^4b^2\left(-\frac{1}{a^5b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-}{8}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*((a*b*x^2 + a^2)*(-1/(a^5*b^3))^(1/4)*log(a^4*b^2*(-1/(a^5*b^3))^(3/4)
 + sqrt(x)) - (I*a*b*x^2 + I*a^2)*(-1/(a^5*b^3))^(1/4)*log(I*a^4*b^2*(-1/(
a^5*b^3))^(3/4) + sqrt(x)) - (-I*a*b*x^2 - I*a^2)*(-1/(a^5*b^3))^(1/4)*log
(-I*a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(x)) - (a*b*x^2 + a^2)*(-1/(a^5*b^3
))^(1/4)*log(-a^4*b^2*(-1/(a^5*b^3))^(3/4) + sqrt(x)) + 4*x^(3/2))/(a*b*x^
2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(146) = 292.

Time = 17.50 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.42

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{5}{2}}} \\ \frac{2x^{\frac{3}{2}}}{3a^2} \\ -\frac{2}{5b^2x^{\frac{5}{2}}} \\ \frac{a\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}-\frac{a\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{2a\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+\frac{4bx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}}{8a^2b\sqrt[4]{-\frac{a}{b}}+8ab^2x^2\sqrt[4]{-\frac{a}{b}}}+ \end{cases}$$

input
```
integrate(x**(1/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/x**(5/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**2), Eq(b,
 0)), (-2/(5*b**2*x**(5/2)), Eq(a, 0)), (a*log(sqrt(x) - (-a/b)**(1/4))/(8
*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - a*log(sqrt(x) + (-a
/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*a*a
tan(sqrt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)*
*(1/4)) + 4*b*x**(3/2)*(-a/b)**(1/4)/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x*
*2*(-a/b)**(1/4)) + b*x**2*log(sqrt(x) - (-a/b)**(1/4))/(8*a**2*b*(-a/b)**
(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) - b*x**2*log(sqrt(x) + (-a/b)**(1/4))
/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)) + 2*b*x**2*atan(sq
rt(x)/(-a/b)**(1/4))/(8*a**2*b*(-a/b)**(1/4) + 8*a*b**2*x**2*(-a/b)**(1/4)
), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{\frac{3}{2}}}{2\,(abx^2+a^2)} + \frac{\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \sqrt{2}\log\left(-\sqrt{2}\right.}{16\,a}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*x^(3/2)/(a*b*x^2 + a^2) + 1/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*
a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*
sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4)
 - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b
)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a
^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x
 + sqrt(a))/(a^(1/4)*b^(3/4)))/a
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{\frac{3}{2}}}{2\,(bx^2+a)a} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b^3} - \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3} + \frac{\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b^3}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*x^(3/2)/((b*x^2 + a)*a) + 1/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^3) + 1/8*sqrt(2)*(a
*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1
/4))/(a^2*b^3) - 1/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4
) + x + sqrt(a/b))/(a^2*b^3) + 1/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqr
t(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx = \frac{x^{3/2}}{2\,a\,\left(b\,x^2+a\right)} - \frac{\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{5/4}\,b^{3/4}} + \frac{\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{5/4}\,b^{3/4}}$$

input
```
int(x^(1/2)/(a + b*x^2)^2,x)
```

output
```
x^(3/2)/(2*a*(a + b*x^2)) - atan((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/
4)*b^(3/4)) + atanh((b^(1/4)*x^(1/2))/(-a)^(1/4))/(4*(-a)^(5/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.85

$$\int \frac{\sqrt{x}}{(a+bx^2)^2}\,dx$$
$$= \frac{-2b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-2b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+2b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right.}{}$$

input
```
int(x^(1/2)/(b*x^2+a)^2,x)
```

output
```
( - 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 2*b**(1/4)*a**(3/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*b*x**2 + 2*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 2*b**(1/4)*a**(3/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b*x**2 + b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a + b**(1/4)*a**(3/4)*sqrt(2)*lo
g( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 - b**
(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + s
qrt(b)*x)*a - b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt
(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 8*sqrt(x)*a*b*x)/(16*a**2*b*(a + b*x**
2))
```

3.300 $\int \frac{1}{\sqrt{x}(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 165

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\sqrt{x}}{2a\,(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{7/4}\sqrt[4]{b}}$$

output
```
1/2*x^(1/2)/a/(b*x^2+a)-3/8*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1
/2)/a^(7/4)/b^(1/4)+3/8*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/
a^(7/4)/b^(1/4)+3/8*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/
2)*x))*2^(1/2)/a^(7/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx=\frac{\frac{4a^{3/4}\sqrt{x}}{a+bx^2}-\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}}+\frac{3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}}{8a^{7/4}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

output
```
((4*a^(3/4)*Sqrt[x])/(a + b*x^2) - (3*Sqrt[2]*ArcTan[(Sqrt[a] - Sqrt[b]*x)
/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(1/4) + (3*Sqrt[2]*ArcTanh[(Sqrt[2]
*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(1/4))/(8*a^(7/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.46, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{\sqrt{x}(bx^2+a)}dx}{4a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\int \frac{1}{bx^2+a}d\sqrt{x}}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a\left(a+bx^2\right)}$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)^2),x]
```

output
```
Sqrt[x]/(2*a*(a + b*x^2)) + (3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^
(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a
^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^
(1/4)))/(2*Sqrt[a])))/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\sqrt{x}}{2a(b\,x^2+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2}$	124
default	$\frac{\sqrt{x}}{2a(b\,x^2+a)}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a^2}$	124

input
```
int(1/x^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/a/(b*x^2+a)+3/16/a^2*(a/b)^(1/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*
arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)
-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{3\,(abx^2+a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(-i\,abx^2-i\,a^2)\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}\log\left(i\,a^2\left(-\frac{1}{a^7b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-3\,(}{8\,(abx}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(3*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(a^2*(-1/(a^7*b))^(1/4) + sqr
t(x)) - 3*(-I*a*b*x^2 - I*a^2)*(-1/(a^7*b))^(1/4)*log(I*a^2*(-1/(a^7*b))^(
1/4) + sqrt(x)) - 3*(I*a*b*x^2 + I*a^2)*(-1/(a^7*b))^(1/4)*log(-I*a^2*(-1/
(a^7*b))^(1/4) + sqrt(x)) - 3*(a*b*x^2 + a^2)*(-1/(a^7*b))^(1/4)*log(-a^2*
(-1/(a^7*b))^(1/4) + sqrt(x)) + 4*sqrt(x))/(a*b*x^2 + a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(151) = 302$.

Time = 24.67 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ \frac{2\sqrt{x}}{a^2} \\ -\frac{2}{7b^2x^{\frac{7}{2}}} \\ \frac{4a\sqrt{x}}{8a^3+8a^2bx^2} - \frac{3a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3+8a^2bx^2} + \frac{3a\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3+8a^2bx^2} + \frac{6a\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3+8a^2bx^2} - \frac{3bx^2\sqrt[4]{-\frac{a}{b}}\log}{8a^3+8} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**2, Eq(b, 0)),
 (-2/(7*b**2*x**(7/2)), Eq(a, 0)), (4*a*sqrt(x)/(8*a**3 + 8*a**2*b*x**2) -
 3*a*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) +
 3*a*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) +
 6*a*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**2) -
3*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3 + 8*a**2*b*x**
2) + 3*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3 + 8*a**2*
b*x**2) + 6*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3 + 8*a
**2*b*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^2}\,dx$$

$$= \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right.}{16\,a}$$

$$+\frac{\sqrt{x}}{2\left(abx^2+a^2\right)}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
3/16*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sq
rt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqr
t(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/
4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(
-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))
/a + 1/2*sqrt(x)/(a*b*x^2 + a^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^2b} + \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b} - \frac{3\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^2b} + \frac{\sqrt{x}}{2\,(bx^2+a)a}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt
(x))/(a/b)^(1/4))/(a^2*b) + 3/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*
(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b) + 3/16*sqrt(2)*(a*b
^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b) - 3/16*
sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a
^2*b) + 1/2*sqrt(x)/((b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{\sqrt{x}}{2\,a\,\left(b\,x^2+a\right)} + \frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}} + \frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{4\,(-a)^{7/4}\,b^{1/4}}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)^2),x)
```

output
```
x^(1/2)/(2*a*(a + b*x^2)) + (3*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)
^(7/4)*b^(1/4)) + (3*atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(4*(-a)^(7/4)*b^
(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.85

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^2}\,dx = \frac{-6b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)-6b^{\frac{7}{4}}a^{\frac{1}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2+6b^{\frac{3}{4}}a^{\frac{5}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}+2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)}{}$$

input
```
int(1/x^(1/2)/(b*x^2+a)^2,x)
```

output
```
( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a - 6*b**(3/4)*a**(1/4)*sqrt(2)*at
an((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt
(2)))*b*x**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 6*b**(3/4)*a**(1/4)
*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a*
*(1/4)*sqrt(2)))*b*x**2 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1
/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a - 3*b**(3/4)*a**(1/4)*sqrt(2
)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 +
 3*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a + 3*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 8*sqrt(x)*a*b)/(16*a**2*b*(a +
 b*x**2))
```

3.301 $\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int \frac{1}{x^{3/2}(a+bx^2)^2}\,dx = -\frac{5}{2a^2\sqrt{x}}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}+\frac{5\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}} - \frac{5\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{9/4}}+\frac{5\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{9/4}}$$

output
```
-5/2/a^2/x^(1/2)+1/2/a/x^(1/2)/(b*x^2+a)+5/8*b^(1/4)*arctan(1-2^(1/2)*b^(1
/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(9/4)-5/8*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*
x^(1/2)/a^(1/4))*2^(1/2)/a^(9/4)+5/8*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/
4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{-\frac{4\sqrt[4]{a}(4a+5bx^2)}{\sqrt{x}(a+bx^2)}+5\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+5\sqrt{2}\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{8a^{9/4}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
((-4*a^(1/4)*(4*a + 5*b*x^2))/(Sqrt[x]*(a + b*x^2)) + 5*Sqrt[2]*b^(1/4)*Ar
cTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 5*Sqrt[2]*
b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/
(8*a^(9/4))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{x^{3/2}(bx^2+a)}dx}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$$\Big\downarrow \ 826$$

$$\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$$\Big\downarrow \ 1476$$

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$$\Big\downarrow \ 1082$$

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$$\Big\downarrow \ 217$$

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$\downarrow$ 1479

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}$$

$$\frac{1}{2a\sqrt{x}\,(a+bx^2)}$$

$\downarrow$ 25

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)^2),x]
```

output
```
1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sq
rt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (S
qrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) -
(-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*
a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]
*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{2}{a^{2}\sqrt{x}}-\frac{2b\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}$	136
default	$-\frac{2}{a^{2}\sqrt{x}}-\frac{2b\left(\frac{x^{\frac{3}{2}}}{4b\,x^{2}+4a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}$	136
risch	$-\frac{2}{a^{2}\sqrt{x}}-\frac{b\left(\frac{x^{\frac{3}{2}}}{2b\,x^{2}+2a}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{2}}$	136

input
```
int(1/x^(3/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/a^2/x^(1/2)-2*b/a^2*(1/4*x^(3/2)/(b*x^2+a)+5/32/b/(a/b)^(1/4)*2^(1/2)*(
ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1
/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)
/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = -\frac{5\left(a^2bx^3+a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(125\,a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}+125\,b\sqrt{x}\right)+5\left(-i\,a^2bx^3-i\,a^3x\right)\left(-\frac{b}{a^9}\right)^{\frac{1}{4}}\log\left(125i\,a^7\left(-\frac{b}{a^9}\right)^{\frac{3}{4}}\right.}{}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(5*(a^2*b*x^3 + a^3*x)*(-b/a^9)^(1/4)*log(125*a^7*(-b/a^9)^(3/4) + 12
5*b*sqrt(x)) + 5*(-I*a^2*b*x^3 - I*a^3*x)*(-b/a^9)^(1/4)*log(125*I*a^7*(-b
/a^9)^(3/4) + 125*b*sqrt(x)) + 5*(I*a^2*b*x^3 + I*a^3*x)*(-b/a^9)^(1/4)*lo
g(-125*I*a^7*(-b/a^9)^(3/4) + 125*b*sqrt(x)) - 5*(a^2*b*x^3 + a^3*x)*(-b/a
^9)^(1/4)*log(-125*a^7*(-b/a^9)^(3/4) + 125*b*sqrt(x)) + 4*(5*b*x^2 + 4*a)
*sqrt(x))/(a^2*b*x^3 + a^3*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 512 vs. 2(165) = 330.

Time = 50.60 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.89

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{9}{2}}} \\ -\frac{2}{a^2\sqrt{x}} \\ -\frac{2}{9b^2x^{\frac{9}{2}}} \\ -\frac{5a\sqrt{x}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\frac{a}{b}}}+\frac{5a\sqrt{x}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\frac{a}{b}}}-\frac{10a\sqrt{x}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{8a^3\sqrt{x}\sqrt[4]{-\frac{a}{b}}+8a^2bx^{\frac{5}{2}}\sqrt[4]{-\frac{a}{b}}} \end{cases}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (-2/(a**2*sqrt(x)), Eq(b, 0
)), (-2/(9*b**2*x**(9/2)), Eq(a, 0)), (-5*a*sqrt(x)*log(sqrt(x) - (-a/b)**
(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) +
5*a*sqrt(x)*log(sqrt(x) + (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8
*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 10*a*sqrt(x)*atan(sqrt(x)/(-a/b)**(1/4))
/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 16*a*(
-a/b)**(1/4)/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/
4)) - 5*b*x**(5/2)*log(sqrt(x) - (-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1
/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) + 5*b*x**(5/2)*log(sqrt(x) + (-a/b)
**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4))
- 10*b*x**(5/2)*atan(sqrt(x)/(-a/b)**(1/4))/(8*a**3*sqrt(x)*(-a/b)**(1/4)
+ 8*a**2*b*x**(5/2)*(-a/b)**(1/4)) - 20*b*x**2*(-a/b)**(1/4)/(8*a**3*sqrt(
x)*(-a/b)**(1/4) + 8*a**2*b*x**(5/2)*(-a/b)**(1/4)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = -\frac{5\,bx^2+4\,a}{2\left(a^2bx^{\frac{5}{2}}+a^3\sqrt{x}\right)} - \frac{5\,b\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\right.}{}\right.}{16\,a^2}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(5*b*x^2 + 4*a)/(a^2*b*x^(5/2) + a^3*sqrt(x)) - 5/16*b*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)
*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqr
t(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x)
+ sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^
(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = -\frac{5\,bx^2+4\,a}{2\left(bx^{\frac{5}{2}}+a\sqrt{x}\right)a^2} - \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2} - \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^3b^2} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2} - \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^3b^2}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(5*b*x^2 + 4*a)/((b*x^(5/2) + a*sqrt(x))*a^2) - 5/8*sqrt(2)*(a*b^3)^(
3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^
3*b^2) - 5/8*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4
) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^2) + 5/16*sqrt(2)*(a*b^3)^(3/4)*log(sqr
t(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2) - 5/16*sqrt(2)*(a*b^3)
^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{5\,{(-b)}^{1/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}} - \frac{5\,{(-b)}^{1/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{9/4}} - \frac{\frac{2}{a}+\frac{5\,b\,x^2}{2\,a^2}}{a\,\sqrt{x}+b\,x^{5/2}}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)^2),x)
```

output
```
(5*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (5*(-b)^(
1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(9/4)) - (2/a + (5*b*x^2)/(2
*a^2))/(a*x^(1/2) + b*x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^2}\,dx = \frac{10\sqrt{x}\,b^{\frac{1}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)+10\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2-}{}$$

input
```
int(1/x^(3/2)/(b*x^2+a)^2,x)
```

output
```
(10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a + 10*sqrt(x)*b**(1/4)*a**(
3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4
)*a**(1/4)*sqrt(2)))*b*x**2 - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a - 10*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**2 - 5*sqrt(x)*b**(1
/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
sqrt(b)*x)*a - 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)
*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**2 + 5*sqrt(x)*b**(1/4)*a**(3
/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a
 + 5*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(
2) + sqrt(a) + sqrt(b)*x)*b*x**2 - 32*a**2 - 40*a*b*x**2)/(16*sqrt(x)*a**3
*(a + b*x**2))
```

3.302 $\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx=-\frac{7}{6a^2x^{3/2}}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}+\frac{7b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{11/4}}-\frac{7b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{11/4}}$$

output
```
-7/6/a^2/x^(3/2)+1/2/a/x^(3/2)/(b*x^2+a)+7/8*b^(3/4)*arctan(1-2^(1/2)*b^(1
/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(11/4)-7/8*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)
*x^(1/2)/a^(1/4))*2^(1/2)/a^(11/4)-7/8*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(
1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx=\frac{-\frac{4a^{3/4}(4a+7bx^2)}{x^{3/2}(a+bx^2)}+21\sqrt{2}b^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-21\sqrt{2}b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{24a^{11/4}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
((-4*a^(3/4)*(4*a + 7*b*x^2))/(x^(3/2)*(a + b*x^2)) + 21*Sqrt[2]*b^(3/4)*A
rcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 21*Sqrt[2
]*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]
)/(24*a^(11/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.47, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{x^{5/2}(bx^2+a)}dx}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 755

$$\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1476

$$\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1082

$$\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 217

$$\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\downarrow$ 1479

$$\frac{7\left(-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3}}\right)}{4a}$$

$$\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\downarrow$ 25

$$\frac{7\left(-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}$$

$$\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 27

$$\frac{7\left(-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+$$

$$\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

$\Big\downarrow$ 1103

$$\frac{7\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}$$

input
```
Int[1/(x^(5/2)*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x^(3/2)*(a + b*x^2)) + (7*(-2/(3*a*x^(3/2)) - (2*b*((-(ArcTan[1 - (
Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 +
(Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])
+ (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2
]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[
b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{2b\left(\frac{\sqrt{x}}{4b\,x^2+4a}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{a^2}-\frac{2}{3a^2x^{\frac{3}{2}}}$
default	$-\frac{2b\left(\frac{\sqrt{x}}{4b\,x^2+4a}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32a}\right)}{a^2}-\frac{2}{3a^2x^{\frac{3}{2}}}$
risch	$-\frac{2}{3a^2x^{\frac{3}{2}}}-\frac{b\left(\frac{\sqrt{x}}{2b\,x^2+2a}+\frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16a}\right)}{a^2}$

input
```
int(1/x^(5/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*b/a^2*(1/4*x^(1/2)/(b*x^2+a)+7/32*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1
/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2
)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x
^(1/2)-1)))-2/3/a^2/x^(3/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = -\frac{21\left(a^2bx^4+a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7\,a^3\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}+7\,b\sqrt{x}\right)+21\left(i\,a^2bx^4+i\,a^3x^2\right)\left(-\frac{b^3}{a^{11}}\right)^{\frac{1}{4}}\log\left(7i\,a^3\left(-\frac{b^3}{a^{11}}\right.\right.}{}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/24*(21*(a^2*b*x^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*a^3*(-b^3/a^11)^(1
/4) + 7*b*sqrt(x)) + 21*(I*a^2*b*x^4 + I*a^3*x^2)*(-b^3/a^11)^(1/4)*log(7*
I*a^3*(-b^3/a^11)^(1/4) + 7*b*sqrt(x)) + 21*(-I*a^2*b*x^4 - I*a^3*x^2)*(-b
^3/a^11)^(1/4)*log(-7*I*a^3*(-b^3/a^11)^(1/4) + 7*b*sqrt(x)) - 21*(a^2*b*x
^4 + a^3*x^2)*(-b^3/a^11)^(1/4)*log(-7*a^3*(-b^3/a^11)^(1/4) + 7*b*sqrt(x)
) + 4*(7*b*x^2 + 4*a)*sqrt(x))/(a^2*b*x^4 + a^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(165) = 330$.

Time = 113.71 (sec) , antiderivative size = 425, normalized size of antiderivative = 2.40

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} \\ -\frac{2}{3a^2x^{\frac{3}{2}}} \\ -\frac{2}{11b^2x^{\frac{11}{2}}} \\ -\frac{16a^2}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}+\frac{21abx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{21abx^{\frac{3}{2}}\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{24a^4x^{\frac{3}{2}}+24a^3bx^{\frac{7}{2}}}-\frac{42\ldots}{} \end{cases}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)**2,x)
```

output
```
Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(b, 0)), (-2/(3*a**2*x**(3/2)), Eq(
b, 0)), (-2/(11*b**2*x**(11/2)), Eq(a, 0)), (-16*a**2/(24*a**4*x**(3/2) +
24*a**3*b*x**(7/2)) + 21*a*b*x**(3/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**
(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*a*b*x**(3/2)*(-a/b)**(
1/4)*log(sqrt(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2))
- 42*a*b*x**(3/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3
/2) + 24*a**3*b*x**(7/2)) - 28*a*b*x**2/(24*a**4*x**(3/2) + 24*a**3*b*x**(
7/2)) + 21*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(24*a*
*4*x**(3/2) + 24*a**3*b*x**(7/2)) - 21*b**2*x**(7/2)*(-a/b)**(1/4)*log(sqr
t(x) + (-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a**3*b*x**(7/2)) - 42*b**2*x*
*(7/2)*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(24*a**4*x**(3/2) + 24*a*
*3*b*x**(7/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = -\frac{7\,bx^2+4\,a}{6\left(a^2bx^{\frac{7}{2}}+a^3x^{\frac{3}{2}}\right)} - \frac{7\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}} - \sqrt{2}b^{\frac{3}{4}}\right)}{16\,a^2}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/6*(7*b*x^2 + 4*a)/(a^2*b*x^(7/2) + a^3*x^(3/2)) - 7/16*(2*sqrt(2)*b*arc
tan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(
2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(s
qrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4
)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^
(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/a^(3/4))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx = -\frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3} - \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8a^3} - \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3} + \frac{7\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16a^3} - \frac{b\sqrt{x}}{2(bx^2+a)a^2} - \frac{2}{3a^2x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/a^3 - 7/8*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)*(sq
rt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^3 - 7/16*sqrt(2)*(a*b^3)^(1/
4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 + 7/16*sqrt(2)*(a*
b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^3 - 1/2*b*s
qrt(x)/((b*x^2 + a)*a^2) - 2/3/(a^2*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^{5/2}(a+bx^2)^2}\,dx = \frac{7(-b)^{3/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{4a^{11/4}} - \frac{\frac{2}{3a}+\frac{7bx^2}{6a^2}}{a\,x^{3/2}+b\,x^{7/2}} + \frac{7(-b)^{3/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{4a^{11/4}}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)^2),x)
```

output
```
(7*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(11/4)) - (2/(3*a)
+ (7*b*x^2)/(6*a^2))/(a*x^(3/2) + b*x^(7/2)) + (7*(-b)^(3/4)*atanh(((-b)^(
1/4)*x^(1/2))/a^(1/4)))/(4*a^(11/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.90

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^2}\,dx = \frac{42\sqrt{x}\, b^{\frac{3}{4}} a^{\frac{5}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) x + 42\sqrt{x}\, b^{\frac{7}{4}} a^{\frac{1}{4}}\sqrt{2}\, atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right) x^3 -}{}$$

input
```
int(1/x^(5/2)/(b*x^2+a)^2,x)
```

output
```
(42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*x + 42*sqrt(x)*b**(3/4)*a*
*(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1
/4)*a**(1/4)*sqrt(2)))*b*x**3 - 42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan(
(b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)
))*a*x - 42*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt
(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b*x**3 + 21*sqrt(x)*
b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a*x + 21*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*
b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**3 - 21*sqrt(x)*b**(3
/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*a*x - 21*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b*x**3 - 32*a**2 - 56*a*b*x**2)/(48*
sqrt(x)*a**3*x*(a + b*x**2))
```

3.303 $\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 190

$$\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx = -\frac{9}{10a^2x^{5/2}}+\frac{9b}{2a^3\sqrt{x}} + \frac{1}{2ax^{5/2}(a+bx^2)} - \frac{9b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} + \frac{9b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{4\sqrt{2}a^{13/4}} - \frac{9b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{4\sqrt{2}a^{13/4}}$$

output
```
-9/10/a^2/x^(5/2)+9/2*b/a^3/x^(1/2)+1/2/a/x^(5/2)/(b*x^2+a)-9/8*b^(5/4)*ar
ctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(13/4)+9/8*b^(5/4)*arcta
n(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(13/4)-9/8*b^(5/4)*arctanh(
2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx=\frac{\frac{4\sqrt[4]{a}(-4a^2+36abx^2+45b^2x^4)}{x^{5/2}(a+bx^2)}-45\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-45\sqrt{2}b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}}{}\right.}{40a^{13/4}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)^2),x]
```

output
```
((4*a^(1/4)*(-4*a^2 + 36*a*b*x^2 + 45*b^2*x^4))/(x^(5/2)*(a + b*x^2)) - 45
*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqr
t[x])] - 45*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqr
t[a] + Sqrt[b]*x)])/(40*a^(13/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{9\int \frac{1}{x^{7/2}(bx^2+a)}dx}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{9\left(-\frac{b\int \frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{9\left(-\frac{b\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 266$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 826$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1476$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1479}$$

$$
9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{}}\right)}{a}\right)
$$

$$
\overline{\qquad 4a \qquad}
$$

$$
\frac{1}{2ax^{5/2}\left(a+bx^2\right)}
$$

$\downarrow$ 25

$$9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}\right)$$

$$\overline{4a}$$

$$\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$$\downarrow 27$$

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+$$

$$\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

$\downarrow$ 1103

$$\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}- \ldots\right)}{4a}$$

$$\frac{1}{2ax^{5/2}\left(a+bx^2\right)}$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)^2),x]
```

output
```
1/(2*a*x^(5/2)*(a + b*x^2)) + (9*(-2/(5*a*x^(5/2)) - (b*(-2/(a*Sqrt[x]) -
(2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^
(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b
^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
 + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/
a))/(4*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{2(-10b\,x^2+a)}{5a^3x^{\frac{5}{2}}}+\frac{b^2\left(\frac{x^{\frac{3}{2}}}{2b\,x^2+2a}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{16b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}$
derivativedivides	$\frac{2b^2\left(\frac{x^{\frac{3}{2}}}{4b\,x^2+4a}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}-\frac{2}{5a^2x^{\frac{5}{2}}}+\frac{4b}{a^3\sqrt{x}}$
default	$\frac{2b^2\left(\frac{x^{\frac{3}{2}}}{4b\,x^2+4a}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{32b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^3}-\frac{2}{5a^2x^{\frac{5}{2}}}+\frac{4b}{a^3\sqrt{x}}$

input
```
int(1/x^(7/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-10*b*x^2+a)/a^3/x^(5/2)+1/a^3*b^2*(1/2*x^(3/2)/(b*x^2+a)+9/16/b/(a/
b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^
(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1
)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.38

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^2}\,dx = \frac{45\left(a^3bx^5+a^4x^3\right)\left(-\frac{b^5}{a^{13}}\right)^{\frac{1}{4}}\log\left(729\,a^{10}\left(-\frac{b^5}{a^{13}}\right)^{\frac{3}{4}}+729\,b^4\sqrt{x}\right)-45\left(i\,a^3bx^5+i\,a^4x\right.}{}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/40*(45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*log(729*a^10*(-b^5/a^13)^
(3/4) + 729*b^4*sqrt(x)) - 45*(I*a^3*b*x^5 + I*a^4*x^3)*(-b^5/a^13)^(1/4)*
log(729*I*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sqrt(x)) - 45*(-I*a^3*b*x^5 - I
*a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*I*a^10*(-b^5/a^13)^(3/4) + 729*b^4*sq
rt(x)) - 45*(a^3*b*x^5 + a^4*x^3)*(-b^5/a^13)^(1/4)*log(-729*a^10*(-b^5/a^
13)^(3/4) + 729*b^4*sqrt(x)) + 4*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)*sqrt(x)
)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\,(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{7/2}\,(a+bx^2)^2}\,dx = \frac{45\,b^2x^4+36\,abx^2-4\,a^2}{10\left(a^3bx^{\frac{9}{2}}+a^4x^{\frac{5}{2}}\right)} + \frac{9\,b^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(-\cdots\right.}{}\cdots\right.}{16\,a^3}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/10*(45*b^2*x^4 + 36*a*b*x^2 - 4*a^2)/(a^3*b*x^(9/2) + a^4*x^(5/2)) + 9/1
6*b^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*s
qrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)
*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sq
rt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1
/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log
(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4))
)/a^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{7/2}(a+bx^2)^2}\,dx = \frac{b^2x^{\frac{3}{2}}}{2\,(bx^2+a)a^3} + \frac{9\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b} + \frac{9\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{8\,a^4b} - \frac{9\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b} + \frac{9\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{16\,a^4b} + \frac{2\,(10\,bx^2-a)}{5\,a^3x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*b^2*x^(3/2)/((b*x^2 + a)*a^3) + 9/8*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*s
qrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b) + 9/8*sqrt(2
)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b
)^(1/4))/(a^4*b) - 9/16*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1
/4) + x + sqrt(a/b))/(a^4*b) + 9/16*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqr
t(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b) + 2/5*(10*b*x^2 - a)/(a^3*x^(5/2
))
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^{7/2} (a+bx^2)^2} \, dx = \frac{\frac{18\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{9\,b^2\,x^4}{2\,a^3}}{a\,x^{5/2}+b\,x^{9/2}} - \frac{9\,(-b)^{5/4}\,\mathrm{atan}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}} + \frac{9\,(-b)^{5/4}\,\mathrm{atanh}\left(\frac{(-b)^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{4\,a^{13/4}}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)^2),x)
```

output
```
((18*b*x^2)/(5*a^2) - 2/(5*a) + (9*b^2*x^4)/(2*a^3))/(a*x^(5/2) + b*x^(9/2
)) - (9*(-b)^(5/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4)) + (9*(
-b)^(5/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(4*a^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.87

$$\int \frac{1}{x^{7/2} (a+bx^2)^2} \, dx = \frac{-90\sqrt{x}\,b^{\frac{5}{4}}a^{\frac{7}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x^2-90\sqrt{x}\,b^{\frac{9}{4}}a^{\frac{3}{4}}\sqrt{2}\,atan\left(\frac{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}-2\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}\sqrt{2}}\right)x}{}$$

input
```
int(1/x^(7/2)/(b*x^2+a)^2,x)
```

output
```
( - 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) -
 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 90*sqrt(x)*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sq
rt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1
/4)*sqrt(2)))*a*b*x**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/
4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2
*x**4 + 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 45*sqrt(x)*b**(1/4)*a**(3/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b
**2*x**4 - 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 45*sqrt(x)*b**(1/4)*a**(3/4)
*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2
*x**4 - 32*a**3 + 288*a**2*b*x**2 + 360*a*b**2*x**4)/(80*sqrt(x)*a**4*x**2
*(a + b*x**2))
```

3.304 $\int \frac{x^{7/2}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 186

$$\int \frac{x^{7/2}}{(a+bx^2)^3} dx = -\frac{x^{5/2}}{4b(a+bx^2)^2} - \frac{5\sqrt{x}}{16b^2(a+bx^2)} - \frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} + \frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{3/4}b^{9/4}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{3/4}b^{9/4}}$$

output
```
-1/4*x^(5/2)/b/(b*x^2+a)^2-5/16*x^(1/2)/b^2/(b*x^2+a)-5/64*arctan(1-2^(1/2
)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+5/64*arctan(1+2^(1/2)*b
^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(3/4)/b^(9/4)+5/64*arctanh(2^(1/2)*a^(1/
4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(3/4)/b^(9/4)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{b}\sqrt{x}(5a+9bx^2)}{(a+bx^2)^2} - \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{a^{3/4}} + \frac{5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{a^{3/4}}}{64b^{9/4}}$$

input
```
Integrate[x^(7/2)/(a + b*x^2)^3,x]
```

output
```
((-4*b^(1/4)*Sqrt[x]*(5*a + 9*b*x^2))/(a + b*x^2)^2 - (5*Sqrt[2]*ArcTan[(S
qrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/a^(3/4) + (5*Sqrt[
2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/a^(3/
4))/(64*b^(9/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {252, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{5\int \frac{x^{3/2}}{(bx^2+a)^2}dx}{8b} - \frac{x^{5/2}}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \; 252$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{4b} - \frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b} - \frac{x^{5/2}}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{\int \frac{1}{bx^2+a}d\sqrt{x}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{5\left(\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}$$

$$\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-$$

$$\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-$$

$$\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}}{2b}-\frac{\sqrt{x}}{2b(a+bx^2)}\right)}{8b}-\frac{x^{5/2}}{4b\left(a+bx^2\right)^2}$$

input
```
Int[x^(7/2)/(a + b*x^2)^3,x]
```

output
```
-1/4*x^(5/2)/(b*(a + b*x^2)^2) + (5*(-1/2*Sqrt[x]/(b*(a + b*x^2)) + ((-(Ar
cTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + A
rcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*
x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[
x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]))/(2*b)))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^2}}{(bx^2+a)^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a}$	139
default	$\frac{-\frac{9x^{\frac{5}{2}}}{16b}-\frac{5a\sqrt{x}}{16b^2}}{(bx^2+a)^2}+\frac{5\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b^2a}$	139

input
```
int(x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-9/32*x^(5/2)/b-5/32*a*x^(1/2)/b^2)/(b*x^2+a)^2+5/128/b^2*(a/b)^(1/4)/a
*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^
(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arct
an(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.48

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = \frac{5\left(b^4x^4+2\,ab^3x^2+a^2b^2\right)\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}\log\left(ab^2\left(-\frac{1}{a^3b^9}\right)^{\frac{1}{4}}+\sqrt{x}\right)-5\left(-i\,b^4x^4-2i\,ab^3x^2-i\right.}{}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(a*b^2*(
-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(-I*b^4*x^4 - 2*I*a*b^3*x^2 - I*a^2*b^2
)*(-1/(a^3*b^9))^(1/4)*log(I*a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(I*
b^4*x^4 + 2*I*a*b^3*x^2 + I*a^2*b^2)*(-1/(a^3*b^9))^(1/4)*log(-I*a*b^2*(-1
/(a^3*b^9))^(1/4) + sqrt(x)) - 5*(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)*(-1/(a^
3*b^9))^(1/4)*log(-a*b^2*(-1/(a^3*b^9))^(1/4) + sqrt(x)) - 4*(9*b*x^2 + 5*
a)*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.17

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = -\frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\,(b^4x^4+2\,ab^3x^2+a^2b^2)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{\right.}}{}\right)}{128\,b^2}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) + 5/12
8*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(
x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arc
tan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a
)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*
b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sq
rt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/b^
2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3} + \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,ab^3} + \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3} - \frac{5\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,ab^3} - \frac{9\,bx^{\frac{5}{2}}+5\,a\sqrt{x}}{16\,(bx^2+a)^2b^2}$$

input
```
integrate(x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a*b^3) + 5/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2
)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a*b^3) + 5/128*sqrt(2)*(
a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a*b^3) - 5/
128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b)
)/(a*b^3) - 1/16*(9*b*x^(5/2) + 5*a*sqrt(x))/((b*x^2 + a)^2*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.47

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = -\frac{\frac{9\,x^{5/2}}{16\,b}+\frac{5\,a\,\sqrt{x}}{16\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{3/4}\,b^{9/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{3/4}\,b^{9/4}}$$

input
```
int(x^(7/2)/(a + b*x^2)^3,x)
```

output
```
- ((9*x^(5/2))/(16*b) + (5*a*x^(1/2))/(16*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2
) - (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4)) - (5*at
anh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(3/4)*b^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.56

$$\int \frac{x^{7/2}}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(7/2)/(b*x^2+a)^3,x)
```

output
```
( - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b*x**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 10*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 20*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b*x**2 + 10*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 5*b**(3/4)
*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*a**2 - 10*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 5*b**(3/4)*a**(1/4)*sqrt(2)*l
og( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 +
 5*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a**2 + 10*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a
**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 5*b**(3/4)*a**(1/4)*sqrt
(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4
 - 40*sqrt(x)*a**2*b - 72*sqrt(x)*a*b**2*x**2)/(128*a*b**3*(a**2 + 2*a*b*x
**2 + b**2*x**4))
```

3.305 $\int \frac{x^{5/2}}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{x^{5/2}}{(a+bx^2)^3} \, dx = -\frac{x^{3/2}}{4b(a+bx^2)^2} + \frac{3x^{3/2}}{16ab(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{5/4}b^{7/4}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{5/4}b^{7/4}}$$

output
```
-1/4*x^(3/2)/b/(b*x^2+a)^2+3/16*x^(3/2)/a/b/(b*x^2+a)-3/64*arctan(1-2^(1/2
)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(5/4)/b^(7/4)+3/64*arctan(1+2^(1/2)*b
^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(5/4)/b^(7/4)-3/64*arctanh(2^(1/2)*a^(1/
4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(5/4)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{-\frac{4\sqrt[4]{a}b^{3/4}x^{3/2}(a-3bx^2)}{(a+bx^2)^2} - 3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right) - 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{64a^{5/4}b^{7/4}}$$

input
```
Integrate[x^(5/2)/(a + b*x^2)^3,x]
```

output
```
((-4*a^(1/4)*b^(3/4)*x^(3/2)*(a - 3*b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcT
an[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 3*Sqrt[2]*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(5/4
)*b^(7/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {252, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3\int \frac{\sqrt{x}}{(bx^2+a)^2}dx}{8b} - \frac{x^{3/2}}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{3\left(\frac{\int \frac{\sqrt{x}}{bx^2+a}dx}{4a} + \frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b} - \frac{x^{3/2}}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\left(\frac{\int \frac{x}{bx^2+a}d\sqrt{x}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}$$

$$\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 25

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-$$

$$\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 27

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-$$

$$\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8b}-\frac{x^{3/2}}{4b\left(a+bx^2\right)^2}$$

input

```
Int[x^(5/2)/(a + b*x^2)^3,x]
```

output

```
-1/4*x^(3/2)/(b*(a + b*x^2)^2) + (3*(x^(3/2)/(2*a*(a + b*x^2)) + ((-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/
(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]))/(2*a)))/(8*b)
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(b\,x^2+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a\,b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	138
default	$\frac{\frac{3x^{\frac{7}{2}}}{16a}-\frac{x^{\frac{3}{2}}}{16b}}{(b\,x^2+a)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a\,b^2\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	138

input
```
int(x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(3/32/a*x^(7/2)-1/32*x^(3/2)/b)/(b*x^2+a)^2+3/128/a/b^2/(a/b)^(1/4)*2^(1
/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.53

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\left(-\frac{1}{a^5b^7}\right)^{\frac{1}{4}}\log\left(a^4b^5\left(-\frac{1}{a^5b^7}\right)^{\frac{3}{4}}+\sqrt{x}\right)-3\,(i\,ab^3x^4+2i\,a^2b^2x^2}{}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^5*b^7))^(1/4)*log(a^4*b
^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) - 3*(I*a*b^3*x^4 + 2*I*a^2*b^2*x^2 + I*
a^3*b)*(-1/(a^5*b^7))^(1/4)*log(I*a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x))
- 3*(-I*a*b^3*x^4 - 2*I*a^2*b^2*x^2 - I*a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-I
*a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 +
a^3*b)*(-1/(a^5*b^7))^(1/4)*log(-a^4*b^5*(-1/(a^5*b^7))^(3/4) + sqrt(x)) +
 4*(3*b*x^3 - a*x)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(5/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\,(ab^3x^4+2\,a^2b^2x^2+a^3b)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(-\sqrt{\right.}{\right.}}{128\,ab}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(3*b*x^(7/2) - a*x^(3/2))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/(a*
b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{3\,bx^{\frac{7}{2}}-ax^{\frac{3}{2}}}{16\,(bx^2+a)^2ab}+\frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}+\frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^4}-\frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}+\frac{3\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^4}$$

input
```
integrate(x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/16*(3*b*x^(7/2) - a*x^(3/2))/((b*x^2 + a)^2*a*b) + 3/64*sqrt(2)*(a*b^3)^
(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a
^2*b^4) + 3/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1
/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^4) - 3/128*sqrt(2)*(a*b^3)^(3/4)*log(
sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4) + 3/128*sqrt(2)*(a*
b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

$$\int \frac{x^{5/2}}{(a+bx^2)^3}\,dx = \frac{\frac{3\,x^{7/2}}{16\,a}-\frac{x^{3/2}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{5/4}\,b^{7/4}}$$

input
```
int(x^(5/2)/(a + b*x^2)^3,x)
```

output
```
((3*x^(7/2))/(16*a) - x^(3/2)/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (3*ata
n((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4)) + (3*atanh((b^(1/
4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(5/4)*b^(7/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.53

$$\int \frac{x^{5/2}}{(a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(x^(5/2)/(b*x^2+a)^3,x)
```

output
```
( - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a*b*x**2 - 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 6*b**
(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 12*b**(1/4)*a**(3/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*x**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 3*b**(1/4)*a**
(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)
*x)*a**2 + 6*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sq
rt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 3*b**(1/4)*a**(3/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 - 3*b*
*(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
sqrt(b)*x)*a**2 - 6*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 3*b**(1/4)*a**(3/4)*sqrt(2)*lo
g(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 - 8*s
qrt(x)*a**2*b*x + 24*sqrt(x)*a*b**2*x**3)/(128*a**2*b**2*(a**2 + 2*a*b*x**
2 + b**2*x**4))
```

3.306 $\int \frac{x^{3/2}}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{x^{3/2}}{(a+bx^2)^3} dx = -\frac{\sqrt{x}}{4b(a+bx^2)^2} + \frac{\sqrt{x}}{16ab(a+bx^2)} - \frac{3\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{7/4}b^{5/4}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{7/4}b^{5/4}}$$

output
```
-1/4*x^(1/2)/b/(b*x^2+a)^2+1/16*x^(1/2)/a/b/(b*x^2+a)-3/64*arctan(1-2^(1/2
)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+3/64*arctan(1+2^(1/2)*b
^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(7/4)/b^(5/4)+3/64*arctanh(2^(1/2)*a^(1/
4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(7/4)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.72

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx=\frac{\frac{4a^{3/4}\sqrt[4]{b}\sqrt{x}(-3a+bx^2)}{(a+bx^2)^2}-3\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{64a^{7/4}b^{5/4}}$$

input
```
Integrate[x^(3/2)/(a + b*x^2)^3,x]
```

output
```
((4*a^(3/4)*b^(1/4)*Sqrt[x]*(-3*a + b*x^2))/(a + b*x^2)^2 - 3*Sqrt[2]*ArcT
an[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] + 3*Sqrt[2]*Ar
cTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(64*a^(7/4
)*b^(5/4))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.43, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {252, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{\sqrt{x}(bx^2+a)^2}dx}{8b}-\frac{\sqrt{x}}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{3\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\,(a+bx^2)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{\frac{3\int \frac{1}{bx^2+a}d\sqrt{x}}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 25

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 1103

$$\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}}{8b}-\frac{\sqrt{x}}{4b\left(a+bx^2\right)^2}$$

input
```
Int[x^(3/2)/(a + b*x^2)^3,x]
```

output
```
-1/4*Sqrt[x]/(b*(a + b*x^2)^2) + (Sqrt[x]/(2*a*(a + b*x^2)) + (3*((-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/
(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(2*a))/(8*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^{2}b}$	138
default	$\frac{\frac{x^{\frac{5}{2}}}{16a}-\frac{3\sqrt{x}}{16b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\,\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^{2}b}$	138

input
```
int(x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/32/a*x^(5/2)-3/32*x^(1/2)/b)/(b*x^2+a)^2+3/128/a^2/b*(a/b)^(1/4)*2^(1
/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)*x^(1/2)
*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^
(1/2)/(a/b)^(1/4)*x^(1/2)-1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.48

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)\big(-\frac{1}{a^7b^5}\big)^{\frac{1}{4}}\log\Big(a^2b\big(-\frac{1}{a^7b^5}\big)^{\frac{1}{4}}+\sqrt{x}\Big)-3\,(-i\,ab^3x^4-2i\,a^2b^2x^2}{}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)*(-1/(a^7*b^5))^(1/4)*log(a^2*b
*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(-I*a*b^3*x^4 - 2*I*a^2*b^2*x^2 - I*a
^3*b)*(-1/(a^7*b^5))^(1/4)*log(I*a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3
*(I*a*b^3*x^4 + 2*I*a^2*b^2*x^2 + I*a^3*b)*(-1/(a^7*b^5))^(1/4)*log(-I*a^2
*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) - 3*(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
*(-1/(a^7*b^5))^(1/4)*log(-a^2*b*(-1/(a^7*b^5))^(1/4) + sqrt(x)) + 4*(b*x^
2 - 3*a)*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 666 vs. $2(170) = 340$.

Time = 159.35 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.52

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \begin{cases} \frac{\tilde{\infty}}{x^{\frac{7}{2}}} \\ \frac{2x^{\frac{5}{2}}}{5a^3} \\ -\frac{2}{7b^3x^{\frac{7}{2}}} \\ -\frac{12a^2\sqrt{x}}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} - \frac{3a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} + \frac{3a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{64a^4b+128a^3b^2x^2+64a^2b^3x^4} + \frac{6a^2\sqrt[4]{}}{64a^4b} \end{cases}$$

input
```
integrate(x**(3/2)/(b*x**2+a)**3,x)
```

output
```
Piecewise((zoo/x**(7/2), Eq(a, 0) & Eq(b, 0)), (2*x**(5/2)/(5*a**3), Eq(b,
 0)), (-2/(7*b**3*x**(7/2)), Eq(a, 0)), (-12*a**2*sqrt(x)/(64*a**4*b + 128
*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 3*a**2*(-a/b)**(1/4)*log(sqrt(x) -
(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 3*a*
*2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x
**2 + 64*a**2*b**3*x**4) + 6*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4)
)/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 4*a*b*x**(5/2)/(6
4*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) - 6*a*b*x**2*(-a/b)**(1
/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2
*b**3*x**4) + 6*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a*
*4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 12*a*b*x**2*(-a/b)**(1/4)
*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**
3*x**4) - 3*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*
b + 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4) + 3*b**2*x**4*(-a/b)**(1/4)*lo
g(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b + 128*a**3*b**2*x**2 + 64*a**2*b**3*
x**4) + 6*b**2*x**4*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b +
 128*a**3*b**2*x**2 + 64*a**2*b**3*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{bx^{\frac{5}{2}} - 3\,a\sqrt{x}}{16\,(ab^3x^4 + 2\,a^2b^2x^2 + a^3b)} + \frac{3\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}} - \frac{\sqrt{2}\log\left(-\sqrt{2}\right.}{}\right.}{128\,ab}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(b*x^(5/2) - 3*a*sqrt(x))/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + 3/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/(a*
b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.12

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^2b^2} + \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^2} - \frac{3\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^2b^2} + \frac{bx^{\frac{5}{2}}-3\,a\sqrt{x}}{16\,(bx^2+a)^2ab}$$

input
```
integrate(x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqr
t(x))/(a/b)^(1/4))/(a^2*b^2) + 3/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^2*b^2) + 3/128*sqrt(
2)*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^2*b^2
) - 3/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqr
t(a/b))/(a^2*b^2) + 1/16*(b*x^(5/2) - 3*a*sqrt(x))/((b*x^2 + a)^2*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.45

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \frac{\frac{x^{5/2}}{16\,a}-\frac{3\,\sqrt{x}}{16\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{3\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{7/4}\,b^{5/4}}+\frac{3\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{7/4}\,b^{5/4}}$$

input
```
int(x^(3/2)/(a + b*x^2)^3,x)
```

output
```
(x^(5/2)/(16*a) - (3*x^(1/2))/(16*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (3*ata
n((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4)) + (3*atanh((b^(1/
4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(7/4)*b^(5/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.52

$$\int \frac{x^{3/2}}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(3/2)/(b*x^2+a)^3,x)
```

output

```
( - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x
)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 12*b**(3/4)*a**(1/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a*b*x**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*s
qrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 6*b**
(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b)
)/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 12*b**(3/4)*a**(1/4)*sqrt(2)*atan((b
**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))
*a*b*x**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) +
2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 3*b**(3/4)*a**
(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)
*x)*a**2 - 6*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sq
rt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 3*b**(3/4)*a**(1/4)*sqrt(2)*log( -
 sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 + 3*b*
*(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) +
sqrt(b)*x)*a**2 + 6*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4
)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 3*b**(3/4)*a**(1/4)*sqrt(2)*lo
g(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 - 24*
sqrt(x)*a**2*b + 8*sqrt(x)*a*b**2*x**2)/(128*a**2*b**2*(a**2 + 2*a*b*x**2
+ b**2*x**4))
```

3.307 $\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 186

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{x^{3/2}}{4a(a+bx^2)^2}+\frac{5x^{3/2}}{16a^2(a+bx^2)}-\frac{5\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}+\frac{5\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}-\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{9/4}b^{3/4}}$$

output
```
1/4*x^(3/2)/a/(b*x^2+a)^2+5/16*x^(3/2)/a^2/(b*x^2+a)-5/64*arctan(1-2^(1/2)
*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(9/4)/b^(3/4)+5/64*arctan(1+2^(1/2)*b^
(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(9/4)/b^(3/4)-5/64*arctanh(2^(1/2)*a^(1/4
)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(9/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{\frac{4\sqrt[4]{a}x^{3/2}(9a+5bx^2)}{(a+bx^2)^2} - \frac{5\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{b^{3/4}} - \frac{5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{b^{3/4}}}{64a^{9/4}}$$

input
```
Integrate[Sqrt[x]/(a + b*x^2)^3,x]
```

output
```
((4*a^(1/4)*x^(3/2)*(9*a + 5*b*x^2))/(a + b*x^2)^2 - (5*Sqrt[2]*ArcTan[(Sq
rt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])])/b^(3/4) - (5*Sqrt[2
]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/b^(3/4
))/(64*a^(9/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {253, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{\sqrt{x}}{(bx^2+a)^2}dx}{8a} + \frac{x^{3/2}}{4a\,(a+bx^2)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{\int \frac{\sqrt{x}}{bx^2+a}dx}{4a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{\int \frac{x}{bx^2+a}d\sqrt{x}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{5\left(\frac{\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5\left(\frac{\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}$$

$$\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+$$

$$\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{5\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}}{2a}+\frac{x^{3/2}}{2a(a+bx^2)}\right)}{8a}+\frac{x^{3/2}}{4a\left(a+bx^2\right)^2}$$

input
```
Int[Sqrt[x]/(a + b*x^2)^3,x]
```

output
```
x^(3/2)/(4*a*(a + b*x^2)^2) + (5*(x^(3/2)/(2*a*(a + b*x^2)) + ((-(ArcTan[1
 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[
1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[
b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sq
rt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + S
qrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]))/(2*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{x^{\frac{3}{2}}}{4a(b\,x^{2}+a)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16a\left(b\,x^{2}+a\right)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$	150
default	$\frac{x^{\frac{3}{2}}}{4a(b\,x^{2}+a)^{2}}+\frac{\frac{5x^{\frac{3}{2}}}{16a\left(b\,x^{2}+a\right)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128ab\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a}$	150

input
```
int(x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(3/2)/a/(b*x^2+a)^2+5/4/a*(1/4*x^(3/2)/a/(b*x^2+a)+1/32/a/b/(a/b)^(1
/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log\left(a^7b^2\left(-\frac{1}{a^9b^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)-5\,(i\,a^2b^2x^4+2i\,a^3bx^2+i\,a^4)\left(-\frac{1}{a^9b^3}\right)^{\frac{1}{4}}\log}{}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^9*b^3))^(1/4)*log(a^7*b^2
*(-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*(I*a^2*b^2*x^4 + 2*I*a^3*b*x^2 + I*a^
4)*(-1/(a^9*b^3))^(1/4)*log(I*a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*
(-I*a^2*b^2*x^4 - 2*I*a^3*b*x^2 - I*a^4)*(-1/(a^9*b^3))^(1/4)*log(-I*a^7*b
^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) - 5*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(
-1/(a^9*b^3))^(1/4)*log(-a^7*b^2*(-1/(a^9*b^3))^(3/4) + sqrt(x)) + 4*(5*b*
x^3 + 9*a*x)*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 887 vs. 2(172) = 344.

Time = 100.50 (sec) , antiderivative size = 887, normalized size of antiderivative = 4.77

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)/(b*x**2+a)**3,x)
```

output
```
Piecewise((zoo/x**(9/2), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a**3), Eq(b,
 0)), (-2/(9*b**3*x**(9/2)), Eq(a, 0)), (5*a**2*log(sqrt(x) - (-a/b)**(1/4
))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b
**3*x**4*(-a/b)**(1/4)) - 5*a**2*log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b*(
-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)
**(1/4)) + 10*a**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) +
128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 36*a
*b*x**(3/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-
a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*a*b*x**2*log(sqrt(x) -
 (-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4
) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) - 10*a*b*x**2*log(sqrt(x) + (-a/b)**(
1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**
2*b**3*x**4*(-a/b)**(1/4)) + 20*a*b*x**2*atan(sqrt(x)/(-a/b)**(1/4))/(64*a
**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4
*(-a/b)**(1/4)) + 20*b**2*x**(7/2)*(-a/b)**(1/4)/(64*a**4*b*(-a/b)**(1/4)
+ 128*a**3*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 5*
b**2*x**4*log(sqrt(x) - (-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3
*b**2*x**2*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) - 5*b**2*x**4*
log(sqrt(x) + (-a/b)**(1/4))/(64*a**4*b*(-a/b)**(1/4) + 128*a**3*b**2*x**2
*(-a/b)**(1/4) + 64*a**2*b**3*x**4*(-a/b)**(1/4)) + 10*b**2*x**4*atan(s...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{5\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log\left(\right.}{}\right.}{128\,a^2}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 5/128
*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x
))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)
*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b
^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqr
t(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{5\,bx^{\frac{7}{2}}+9\,ax^{\frac{3}{2}}}{16\,(bx^2+a)^2a^2} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b^3} - \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3} + \frac{5\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b^3}$$

input
```
integrate(x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/16*(5*b*x^(7/2) + 9*a*x^(3/2))/((b*x^2 + a)^2*a^2) + 5/64*sqrt(2)*(a*b^3
)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/
(a^3*b^3) + 5/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^
(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b^3) - 5/128*sqrt(2)*(a*b^3)^(3/4)*lo
g(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3) + 5/128*sqrt(2)*(
a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \frac{\frac{9\,x^{3/2}}{16\,a}+\frac{5\,b\,x^{7/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{5\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}}-\frac{5\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{9/4}\,b^{3/4}}$$

input
```
int(x^(1/2)/(a + b*x^2)^3,x)
```

output
```
((9*x^(3/2))/(16*a) + (5*b*x^(7/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
+ (5*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4)) - (5*atan
h((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(9/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.57

$$\int \frac{\sqrt{x}}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(x^(1/2)/(b*x^2+a)^3,x)
```

output

```
( - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 20*b**(1/4)*a**(3/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b*x**2 - 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 10*
b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 20*b**(1/4)*a**(3/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b*x**2 + 10*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 5*b**(1/4)
*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqr
t(b)*x)*a**2 + 10*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 5*b**(1/4)*a**(3/4)*sqrt(2)*l
og( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 -
 5*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*a**2 - 10*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a
**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 5*b**(1/4)*a**(3/4)*sqrt
(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4
 + 72*sqrt(x)*a**2*b*x + 40*sqrt(x)*a*b**2*x**3)/(128*a**3*b*(a**2 + 2*a*b
*x**2 + b**2*x**4))
```

3.308 $\int \frac{1}{\sqrt{x}\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 186

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\sqrt{x}}{4a\,(a+bx^2)^2} + \frac{7\sqrt{x}}{16a^2\,(a+bx^2)} - \frac{21\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}} + \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{11/4}\sqrt[4]{b}}$$

output
```
1/4*x^(1/2)/a/(b*x^2+a)^2+7/16*x^(1/2)/a^2/(b*x^2+a)-21/64*arctan(1-2^(1/2
)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(11/4)/b^(1/4)+21/64*arctan(1+2^(1/2)
*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(11/4)/b^(1/4)+21/64*arctanh(2^(1/2)*a
^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(11/4)/b^(1/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\frac{4a^{3/4}\sqrt{x}(11a+7bx^2)}{(a+bx^2)^2} - \frac{21\sqrt{2}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)}{\sqrt[4]{b}} + \frac{21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{\sqrt[4]{b}}}{64a^{11/4}}$$

input
```
Integrate[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

output
```
((4*a^(3/4)*Sqrt[x]*(11*a + 7*b*x^2))/(a + b*x^2)^2 - (21*Sqrt[2]*ArcTan[(
Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])]])/b^(1/4) + (21*Sqr
t[2]*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)]])/b^(
1/4))/(64*a^(11/4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {253, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7\int \frac{1}{\sqrt{x}(bx^2+a)^2}dx}{8a} + \frac{\sqrt{x}}{4a\,(a+bx^2)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7\left(\frac{3\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{4a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 266

$$\frac{7\left(\frac{3\int \frac{1}{bx^2+a}d\sqrt{x}}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 755

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{7\left(\frac{3\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{7\left(\frac{3\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{7\left(\frac{3\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{7\left(\frac{3\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+b\ldots}\right.}{8a}$$

$$\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\big\downarrow$ 25

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}$$

$$\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\big\downarrow$ 27

$$\frac{7\left(\frac{3\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int \frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

$\big\downarrow$ 1103

$$\frac{7\left(\frac{3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{2a}+\frac{\sqrt{x}}{2a(a+bx^2)}\right)}{8a}+\frac{\sqrt{x}}{4a\left(a+bx^2\right)^2}$$

input
```
Int[1/(Sqrt[x]*(a + b*x^2)^3),x]
```

output
```
Sqrt[x]/(4*a*(a + b*x^2)^2) + (7*(Sqrt[x]/(2*a*(a + b*x^2)) + (3*((-(ArcTa
n[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcT
an[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/
(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
+ Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a])))/(2*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\sqrt{x}}{4a(b\,x^2+a)^2}+\frac{\frac{7\sqrt{x}}{16a\left(b\,x^2+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^2}}{a}$
default	$\frac{\sqrt{x}}{4a(b\,x^2+a)^2}+\frac{\frac{7\sqrt{x}}{16a\left(b\,x^2+a\right)}+\frac{21\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a^2}}{a}$

input
```
int(1/x^(1/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^(1/2)/a/(b*x^2+a)^2+7/4/a*(1/4*x^(1/2)/a/(b*x^2+a)+3/32/a^2*(a/b)^(1
/4)*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x-(a/b)^(1/4)
*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*a
rctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{21\,(a^2b^2x^4+2\,a^3bx^2+a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\log\left(a^3\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}+\sqrt{x}\right)-21\,(-i\,a^2b^2x^4-2i\,a^3bx^2-i\,a^4)\left(-\frac{1}{a^{11}b}\right)^{\frac{1}{4}}\,\mathrm{l}}{}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/64*(21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(1/4)*log(a^3*(-1
/(a^11*b))^(1/4) + sqrt(x)) - 21*(-I*a^2*b^2*x^4 - 2*I*a^3*b*x^2 - I*a^4)*
(-1/(a^11*b))^(1/4)*log(I*a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) - 21*(I*a^2*b
^2*x^4 + 2*I*a^3*b*x^2 + I*a^4)*(-1/(a^11*b))^(1/4)*log(-I*a^3*(-1/(a^11*b
))^(1/4) + sqrt(x)) - 21*(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)*(-1/(a^11*b))^(
1/4)*log(-a^3*(-1/(a^11*b))^(1/4) + sqrt(x)) + 4*(7*b*x^2 + 11*a)*sqrt(x))
/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(172) = 344$.

Time = 147.06 (sec) , antiderivative size = 627, normalized size of antiderivative = 3.37

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}}{x^{\frac{11}{2}}} \\ \frac{2\sqrt{x}}{a^3} \\ -\frac{2}{11b^3x^{\frac{11}{2}}} \\ \frac{44a^2\sqrt{x}}{64a^5+128a^4bx^2+64a^3b^2x^4} - \frac{21a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}-\sqrt[4]{-\frac{a}{b}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4} + \frac{21a^2\sqrt[4]{-\frac{a}{b}}\log\left(\sqrt{x}+\sqrt[4]{-\frac{a}{b}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4} + \frac{42a^2\sqrt[4]{-\frac{a}{b}}\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{-\frac{a}{b}}}\right)}{64a^5+128a^4bx^2+64a^3b^2x^4} \end{cases}$$

input
```
integrate(1/x**(1/2)/(b*x**2+a)**3,x)
```

output
```
Piecewise((zoo/x**(11/2), Eq(a, 0) & Eq(b, 0)), (2*sqrt(x)/a**3, Eq(b, 0))
, (-2/(11*b**3*x**(11/2)), Eq(a, 0)), (44*a**2*sqrt(x)/(64*a**5 + 128*a**4
*b*x**2 + 64*a**3*b**2*x**4) - 21*a**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b)*
*(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 21*a**2*(-a/b)**
(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b*
*2*x**4) + 42*a**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64*a**5 + 12
8*a**4*b*x**2 + 64*a**3*b**2*x**4) + 28*a*b*x**(5/2)/(64*a**5 + 128*a**4*b
*x**2 + 64*a**3*b**2*x**4) - 42*a*b*x**2*(-a/b)**(1/4)*log(sqrt(x) - (-a/b
)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) + 42*a*b*x**2*(-
a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a
**3*b**2*x**4) + 84*a*b*x**2*(-a/b)**(1/4)*atan(sqrt(x)/(-a/b)**(1/4))/(64
*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4) - 21*b**2*x**4*(-a/b)**(1/4)*
log(sqrt(x) - (-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**
4) + 21*b**2*x**4*(-a/b)**(1/4)*log(sqrt(x) + (-a/b)**(1/4))/(64*a**5 + 12
8*a**4*b*x**2 + 64*a**3*b**2*x**4) + 42*b**2*x**4*(-a/b)**(1/4)*atan(sqrt(
x)/(-a/b)**(1/4))/(64*a**5 + 128*a**4*b*x**2 + 64*a**3*b**2*x**4), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\,(a^2b^2x^4+2\,a^3bx^2+a^4)} + \frac{21\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}-\frac{\sqrt{2}\log\left(-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}\right)}{128\,a^2}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 21/1
28*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt
(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(
a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))) + sqrt(2)*log(sqrt(2)*a^(1/4)
*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)) - sqrt(2)*log(-s
qrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(3/4)*b^(1/4)))/a
^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b} + \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^3b} + \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b} - \frac{21\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^3b} + \frac{7\,bx^{\frac{5}{2}}+11\,a\sqrt{x}}{16\,(bx^2+a)^2a^2}$$

input
```
integrate(1/x^(1/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*sq
rt(x))/(a/b)^(1/4))/(a^3*b) + 21/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^3*b) + 21/128*sqrt(2
)*(a*b^3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^3*b) -
 21/128*sqrt(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(
a/b))/(a^3*b) + 1/16*(7*b*x^(5/2) + 11*a*sqrt(x))/((b*x^2 + a)^2*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \frac{\frac{11\,\sqrt{x}}{16\,a}+\frac{7\,b\,x^{5/2}}{16\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{21\,\mathrm{atan}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{11/4}\,b^{1/4}} - \frac{21\,\mathrm{atanh}\left(\frac{b^{1/4}\,\sqrt{x}}{(-a)^{1/4}}\right)}{32\,(-a)^{11/4}\,b^{1/4}}$$

input
```
int(1/(x^(1/2)*(a + b*x^2)^3),x)
```

output
```
((11*x^(1/2))/(16*a) + (7*b*x^(5/2))/(16*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)
 - (21*atan((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4)) - (21*
atanh((b^(1/4)*x^(1/2))/(-a)^(1/4)))/(32*(-a)^(11/4)*b^(1/4))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.56

$$\int \frac{1}{\sqrt{x}\,(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x^(1/2)/(b*x^2+a)^3,x)
```

output
```
( - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(
x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 84*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*a*b*x**2 - 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)
*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 + 42*
b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt
(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 84*b**(3/4)*a**(1/4)*sqrt(2)*atan
((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2
)))*a*b*x**2 + 42*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2
) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 21*b**(3/4
)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sq
rt(b)*x)*a**2 - 42*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 21*b**(3/4)*a**(1/4)*sqrt(2)
*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4
 + 21*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sq
rt(a) + sqrt(b)*x)*a**2 + 42*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4
)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 21*b**(3/4)*a**(1/4)*
sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*
x**4 + 88*sqrt(x)*a**2*b + 56*sqrt(x)*a*b**2*x**2)/(128*a**3*b*(a**2 + 2*a
*b*x**2 + b**2*x**4))
```

3.309 $\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 198

$$\int \frac{1}{x^{3/2}(a+bx^2)^3}\,dx = -\frac{45}{16a^3\sqrt{x}} + \frac{1}{4a\sqrt{x}(a+bx^2)^2} + \frac{9}{16a^2\sqrt{x}(a+bx^2)} + \frac{45\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}} - \frac{45\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{13/4}} + \frac{45\sqrt[4]{b}\,\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{13/4}}$$

output
```
-45/16/a^3/x^(1/2)+1/4/a/x^(1/2)/(b*x^2+a)^2+9/16/a^2/x^(1/2)/(b*x^2+a)+45
/64*b^(1/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(13/4)-45/
64*b^(1/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(13/4)+45/6
4*b^(1/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^(
1/2)/a^(13/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx=\frac{-\frac{4\sqrt[4]{a}\left(32a^2+81abx^2+45b^2x^4\right)}{\sqrt{x}\left(a+bx^2\right)^2}+45\sqrt{2}\sqrt[4]{b}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)+45\sqrt{2}\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{64a^{13/4}}$$

input
```
Integrate[1/(x^(3/2)*(a + b*x^2)^3),x]
```

output
```
((-4*a^(1/4)*(32*a^2 + 81*a*b*x^2 + 45*b^2*x^4))/(Sqrt[x]*(a + b*x^2)^2) +
 45*Sqrt[2]*b^(1/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4)*
Sqrt[x])] + 45*Sqrt[2]*b^(1/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x])/(
Sqrt[a] + Sqrt[b]*x)])/(64*a^(13/4))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {253, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{9\int \frac{1}{x^{3/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{9\left(\frac{5\int \frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2a\sqrt{x}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{9\left(\frac{5\left(-\frac{b\int \frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \; 266$$

$$\frac{9\left(\frac{5\left(-\frac{2b\int \frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \; 826$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}$$

$$\Big\downarrow \; 1476$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}+\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1479$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}\right)}{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\downarrow 25$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\cdots\right)}{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

$$\Big\downarrow 27$$

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{1}{2a\sqrt{x}(a+bx^2)}\right)}{8a}$$

$$\frac{1}{4a\sqrt{x}\,(a+bx^2)^2}$$

$\downarrow$ 1103

$$\frac{9\left(\frac{5\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{4a}+\frac{}{2a}\cdots\right)}{8a}$$

$$\frac{1}{4a\sqrt{x}\left(a+bx^2\right)^2}$$

input
```
Int[1/(x^(3/2)*(a + b*x^2)^3),x]
```

output
```
1/(4*a*Sqrt[x]*(a + b*x^2)^2) + (9*(1/(2*a*Sqrt[x]*(a + b*x^2)) + (5*(-2/(
a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt[
2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sqrt
[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^
(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sqrt[
2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sq
rt[b])))/a))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2b\left(\frac{\frac{13b\,x^{\frac{7}{2}}}{32}+\frac{17a\,x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}-\frac{2}{a^{3}\sqrt{x}}$
default	$-\frac{2b\left(\frac{\frac{13b\,x^{\frac{7}{2}}}{32}+\frac{17a\,x^{\frac{3}{2}}}{32}}{\left(b\,x^{2}+a\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}-\frac{2}{a^{3}\sqrt{x}}$
risch	$-\frac{2}{a^{3}\sqrt{x}}-\frac{b\left(\frac{\frac{13b\,x^{\frac{7}{2}}}{16}+\frac{17a\,x^{\frac{3}{2}}}{16}}{\left(b\,x^{2}+a\right)^{2}}+\frac{45\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128b\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{a^{3}}$

input
```
int(1/x^(3/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/a^3*b*((13/32*b*x^(7/2)+17/32*a*x^(3/2))/(b*x^2+a)^2+45/256/b/(a/b)^(1/
4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*
x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*ar
ctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))-2/a^3/x^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.43

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{45\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)\left(-\frac{b}{a^{13}}\right)^{\frac{1}{4}}\log\left(91125\,a^{10}\left(-\frac{b}{a^{13}}\right)^{\frac{3}{4}}+91125\,b\sqrt{x}\right)+45\left(-i\,a^3b^2x^5-2i\,a^4bx^3-\right.}{}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/64*(45*(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(91125*a^
10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) + 45*(-I*a^3*b^2*x^5 - 2*I*a^4*b*x^3
 - I*a^5*x)*(-b/a^13)^(1/4)*log(91125*I*a^10*(-b/a^13)^(3/4) + 91125*b*sqr
t(x)) + 45*(I*a^3*b^2*x^5 + 2*I*a^4*b*x^3 + I*a^5*x)*(-b/a^13)^(1/4)*log(-
91125*I*a^10*(-b/a^13)^(3/4) + 91125*b*sqrt(x)) - 45*(a^3*b^2*x^5 + 2*a^4*
b*x^3 + a^5*x)*(-b/a^13)^(1/4)*log(-91125*a^10*(-b/a^13)^(3/4) + 91125*b*s
qrt(x)) + 4*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^5 + 2*a
^4*b*x^3 + a^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate(1/x**(3/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx=-\frac{45\,b^2x^4+81\,abx^2+32\,a^2}{16\,\left(a^3b^2x^{\frac{9}{2}}+2\,a^4bx^{\frac{5}{2}}+a^5\sqrt{x}\right)}-\frac{45\,b\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}-\frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}}+\frac{\sqrt{2}\log\left(\cdots\right.}{}\right.}{128\,a^3}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/16*(45*b^2*x^4 + 81*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(9/2) + 2*a^4*b*x^(5/2
) + a^5*sqrt(x)) - 45/128*b*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/4)
*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b)
)*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) - 2*sq
rt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) - sq
rt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*
b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt
(a))/(a^(1/4)*b^(3/4)))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{3/2}\left(a+bx^2\right)^3}\,dx = -\frac{2}{a^3\sqrt{x}} - \frac{45\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2} - \frac{45\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4b^2} + \frac{45\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2} - \frac{45\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4b^2} - \frac{13\,b^2x^{\frac{7}{2}}+17\,abx^{\frac{3}{2}}}{16\left(bx^2+a\right)^2a^3}$$

input
```
integrate(1/x^(3/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-2/(a^3*sqrt(x)) - 45/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)
*(a/b)^(1/4) + 2*sqrt(x))/(a/b)^(1/4))/(a^4*b^2) - 45/64*sqrt(2)*(a*b^3)^(
3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a
^4*b^2) + 45/128*sqrt(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x
 + sqrt(a/b))/(a^4*b^2) - 45/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x
)*(a/b)^(1/4) + x + sqrt(a/b))/(a^4*b^2) - 1/16*(13*b^2*x^(7/2) + 17*a*b*x
^(3/2))/((b*x^2 + a)^2*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{3/2} (a + bx^2)^3} \, dx = \frac{45 (-b)^{1/4} \operatorname{atanh}\left(\frac{(-b)^{1/4} \sqrt{x}}{a^{1/4}}\right)}{32 a^{13/4}} - \frac{45 (-b)^{1/4} \operatorname{atan}\left(\frac{(-b)^{1/4} \sqrt{x}}{a^{1/4}}\right)}{32 a^{13/4}} - \frac{\frac{2}{a} + \frac{81 b x^2}{16 a^2} + \frac{45 b^2 x^4}{16 a^3}}{a^2 \sqrt{x} + b^2 x^{9/2} + 2 a b x^{5/2}}$$

input
```
int(1/(x^(3/2)*(a + b*x^2)^3),x)
```

output
```
(45*(-b)^(1/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (45*(-
b)^(1/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(13/4)) - (2/a + (81*b*
x^2)/(16*a^2) + (45*b^2*x^4)/(16*a^3))/(a^2*x^(1/2) + b^2*x^(9/2) + 2*a*b*
x^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 506, normalized size of antiderivative = 2.56

$$\int \frac{1}{x^{3/2} (a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^(3/2)/(b*x^2+a)^3,x)
```

output
```
(90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*
sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 + 180*sqrt(x)*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*a
tan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqr
t(2)))*b**2*x**4 - 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**
(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2 - 180
*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqr
t(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**2 - 90*sqrt(x)*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*b**2*x**4 - 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*
log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2 - 90*
sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
 + sqrt(a) + sqrt(b)*x)*a*b*x**2 - 45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*lo
g( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**4 +
45*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
 + sqrt(a) + sqrt(b)*x)*a**2 + 90*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sq
rt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**2 + 45*sqrt(
x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(
a) + sqrt(b)*x)*b**2*x**4 - 256*a**3 - 648*a**2*b*x**2 - 360*a*b**2*x**...
```

3.310 $\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 198

$$\begin{aligned}\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx &= -\frac{77}{48a^3x^{3/2}}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}\\ &\quad+\frac{11}{16a^2x^{3/2}\left(a+bx^2\right)}+\frac{77b^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}}\\ &\quad-\frac{77b^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{15/4}}-\frac{77b^{3/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{15/4}}\end{aligned}$$

output
```
-77/48/a^3/x^(3/2)+1/4/a/x^(3/2)/(b*x^2+a)^2+11/16/a^2/x^(3/2)/(b*x^2+a)+7
7/64*b^(3/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(15/4)-77
/64*b^(3/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/4))*2^(1/2)/a^(15/4)-77/
64*b^(3/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2)/(a^(1/2)+b^(1/2)*x))*2^
(1/2)/a^(15/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{-\frac{4a^{3/4}\left(32a^2+121abx^2+77b^2x^4\right)}{x^{3/2}\left(a+bx^2\right)^2}+231\sqrt{2}b^{3/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-231\sqrt{2}b^{3/4}\text{arctanh}}{192a^{15/4}}$$

input
```
Integrate[1/(x^(5/2)*(a + b*x^2)^3),x]
```

output
```
((-4*a^(3/4)*(32*a^2 + 121*a*b*x^2 + 77*b^2*x^4))/(x^(3/2)*(a + b*x^2)^2)
+ 231*Sqrt[2]*b^(3/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sqrt[2]*a^(1/4)*b^(1/4
)*Sqrt[x])] - 231*Sqrt[2]*b^(3/4)*ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x]
)/(Sqrt[a] + Sqrt[b]*x)])/(192*a^(15/4))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.46, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {253, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11\int \frac{1}{x^{5/2}\left(bx^2+a\right)^2}dx}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{11\left(\frac{7\int \frac{1}{x^{5/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ax^{3/2}\left(a+bx^2\right)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{7\left(-\frac{b\int \frac{1}{\sqrt{x}\left(bx^2+a\right)}dx}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{11\left(\frac{7\left(-\frac{2b\int \frac{1}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int\frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int\frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\int \frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int \frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1479$$

$$11\left(\frac{7\left(-\frac{2b\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{\cdots}{3ax\cdots}\cdots\right)}{4a}\cdots\right)}{8a}\cdots$$

$$\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$\downarrow$ 25

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}\right)}{8a}$$

$$\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{a}}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\frac{1}{2ax^{3/2}(a-}\right.}{8a}$$

$$\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

$\downarrow$ 1103

$$\frac{11\left(\frac{7\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}+\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{a}}\right)}{a}-\frac{2}{3ax^{3/2}}\right)}{4a}+\right.}{8a}$$

$$\frac{1}{4ax^{3/2}\left(a+bx^2\right)^2}$$

input

```
Int[1/(x^(5/2)*(a + b*x^2)^3),x]
```

output

```
1/(4*a*x^(3/2)*(a + b*x^2)^2) + (11*(1/(2*a*x^(3/2)*(a + b*x^2)) + (7*(-2/
(3*a*x^(3/2)) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(Sq
rt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[x])/a^(1/4)]/(S
qrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[a]) + (-1/2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4)) + Log[Sqrt[a] + Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]*a^(1/4)*b^(1/4)))/(2
*Sqrt[a])))/a))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.73

method	result
derivativedivides	$-\frac{2}{3a^3x^{\frac{3}{2}}}-\frac{2b\left(\frac{\frac{15b\,x^{\frac{5}{2}}}{32}+\frac{19a\sqrt{x}}{32}}{\left(b\,x^2+a\right)^2}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a}\right)}{a^3}$
default	$-\frac{2}{3a^3x^{\frac{3}{2}}}-\frac{2b\left(\frac{\frac{15b\,x^{\frac{5}{2}}}{32}+\frac{19a\sqrt{x}}{32}}{\left(b\,x^2+a\right)^2}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{256a}\right)}{a^3}$
risch	$-\frac{2}{3a^3x^{\frac{3}{2}}}-\frac{b\left(\frac{\frac{15b\,x^{\frac{5}{2}}}{16}+\frac{19a\sqrt{x}}{16}}{\left(b\,x^2+a\right)^2}+\frac{77\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)\right)}{128a}\right)}{a^3}$

input
```
int(1/x^(5/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/a^3/x^(3/2)-2/a^3*b*((15/32*b*x^(5/2)+19/32*a*x^(1/2))/(b*x^2+a)^2+77
/256*(a/b)^(1/4)/a*2^(1/2)*(ln((x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2))
/(x-(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(2^(1/2)/(a/b)^(1/4)
*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.57

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{231\left(a^3b^2x^6+2\,a^4bx^4+a^5x^2\right)\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}\log\left(77\,a^4\left(-\frac{b^3}{a^{15}}\right)^{\frac{1}{4}}+77\,b\sqrt{x}\right)+231\left(i\,a^3b^2x^6+2i\,a^4bx^4+i\,a^5x\right.}{}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/192*(231*(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(77
*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) + 231*(I*a^3*b^2*x^6 + 2*I*a^4*b*x^
4 + I*a^5*x^2)*(-b^3/a^15)^(1/4)*log(77*I*a^4*(-b^3/a^15)^(1/4) + 77*b*sqr
t(x)) + 231*(-I*a^3*b^2*x^6 - 2*I*a^4*b*x^4 - I*a^5*x^2)*(-b^3/a^15)^(1/4)
*log(-77*I*a^4*(-b^3/a^15)^(1/4) + 77*b*sqrt(x)) - 231*(a^3*b^2*x^6 + 2*a^
4*b*x^4 + a^5*x^2)*(-b^3/a^15)^(1/4)*log(-77*a^4*(-b^3/a^15)^(1/4) + 77*b*
sqrt(x)) + 4*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)*sqrt(x))/(a^3*b^2*x^6 + 2
*a^4*b*x^4 + a^5*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(5/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = -\frac{77\,b^2x^4+121\,abx^2+32\,a^2}{48\left(a^3b^2x^{\frac{11}{2}}+2\,a^4bx^{\frac{7}{2}}+a^5x^{\frac{3}{2}}\right)} - \frac{77\left(\frac{2\sqrt{2}b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{2\sqrt{2}b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\sqrt{b}\sqrt{x}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}+\frac{\sqrt{2}b^{\frac{3}{4}}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{3}{4}}}-\cdots\right)}{128\,a^3}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/48*(77*b^2*x^4 + 121*a*b*x^2 + 32*a^2)/(a^3*b^2*x^(11/2) + 2*a^4*b*x^(7
/2) + a^5*x^(3/2)) - 77/128*(2*sqrt(2)*b*arctan(1/2*sqrt(2)*(sqrt(2)*a^(1/
4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(b))) + 2*sqrt(2)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) -
2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b)))
+ sqrt(2)*b^(3/4)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b)*x + sqrt(a
))/a^(3/4) - sqrt(2)*b^(3/4)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/a^(3/4))/a^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = -\frac{77\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4} - \frac{77\,\sqrt{2}(ab^3)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^4} - \frac{77\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4} + \frac{77\,\sqrt{2}(ab^3)^{\frac{1}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^4} - \frac{15\,b^2x^{\frac{5}{2}}+19\,ab\sqrt{x}}{16\,(bx^2+a)^2a^3} - \frac{2}{3\,a^3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*s
qrt(x))/(a/b)^(1/4))/a^4 - 77/64*sqrt(2)*(a*b^3)^(1/4)*arctan(-1/2*sqrt(2)
*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/a^4 - 77/128*sqrt(2)*(a*b^
3)^(1/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 + 77/128*sqr
t(2)*(a*b^3)^(1/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/a^4 -
 1/16*(15*b^2*x^(5/2) + 19*a*b*sqrt(x))/((b*x^2 + a)^2*a^3) - 2/3/(a^3*x^(
3/2))
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \frac{77\left(-b\right)^{3/4}\operatorname{atan}\left(\frac{\left(-b\right)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}} - \frac{\frac{2}{3\,a}+\frac{121\,b\,x^2}{48\,a^2}+\frac{77\,b^2\,x^4}{48\,a^3}}{a^2\,x^{3/2}+b^2\,x^{11/2}+2\,a\,b\,x^{7/2}} + \frac{77\left(-b\right)^{3/4}\operatorname{atanh}\left(\frac{\left(-b\right)^{1/4}\sqrt{x}}{a^{1/4}}\right)}{32\,a^{15/4}}$$

input
```
int(1/(x^(5/2)*(a + b*x^2)^3),x)
```

output
```
(77*(-b)^(3/4)*atan(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(15/4)) - (2/(3*a
) + (121*b*x^2)/(48*a^2) + (77*b^2*x^4)/(48*a^3))/(a^2*x^(3/2) + b^2*x^(11
/2) + 2*a*b*x^(7/2)) + (77*(-b)^(3/4)*atanh(((-b)^(1/4)*x^(1/2))/a^(1/4)))
/(32*a^(15/4))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 513, normalized size of antiderivative = 2.59

$$\int \frac{1}{x^{5/2}\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/x^(5/2)/(b*x^2+a)^3,x)
```

output
```
(462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2
*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*x + 924*sqrt(x)*b**(3/
4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(
b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**3 + 462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(
2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)
*sqrt(2)))*b**2*x**5 - 462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4
)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*
x - 924*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
+ 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b*x**3 - 462*sqrt(x)*b
**(3/4)*a**(1/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(
b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**2*x**5 + 231*sqrt(x)*b**(3/4)*a**(1/4)
*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a
**2*x + 462*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(
1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b*x**3 + 231*sqrt(x)*b**(3/4)*a**(1/
4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)
*b**2*x**5 - 231*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a**2*x - 462*sqrt(x)*b**(3/4)*a**(1/
4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*
b*x**3 - 231*sqrt(x)*b**(3/4)*a**(1/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/
4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**2*x**5 - 256*a**3 - 968*a**2*b*x**...
```

3.311 $\int \frac{1}{x^{7/2}(a+bx^2)^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 211

$$\int \frac{1}{x^{7/2}(a+bx^2)^3} dx = -\frac{117}{80a^3x^{5/2}} + \frac{117b}{16a^4\sqrt{x}} + \frac{1}{4ax^{5/2}(a+bx^2)^2} + \frac{13}{16a^2x^{5/2}(a+bx^2)} - \frac{117b^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}} + \frac{117b^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{32\sqrt{2}a^{17/4}} - \frac{117b^{5/4}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}{\sqrt{a}+\sqrt{b}x}\right)}{32\sqrt{2}a^{17/4}}$$

output

```
-117/80/a^3/x^(5/2)+117/16*b/a^4/x^(1/2)+1/4/a/x^(5/2)/(b*x^2+a)^2+13/16/a
^2/x^(5/2)/(b*x^2+a)-117/64*b^(5/4)*arctan(1-2^(1/2)*b^(1/4)*x^(1/2)/a^(1/
4))*2^(1/2)/a^(17/4)+117/64*b^(5/4)*arctan(1+2^(1/2)*b^(1/4)*x^(1/2)/a^(1/
4))*2^(1/2)/a^(17/4)-117/64*b^(5/4)*arctanh(2^(1/2)*a^(1/4)*b^(1/4)*x^(1/2
)/(a^(1/2)+b^(1/2)*x))*2^(1/2)/a^(17/4)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx=\frac{\frac{4\sqrt[4]{a}(-32a^3+416a^2bx^2+1053ab^2x^4+585b^3x^6)}{x^{5/2}(a+bx^2)^2}-585\sqrt{2}b^{5/4}\arctan\left(\frac{\sqrt{a}-\sqrt{b}x}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}}\right)-585\sqrt{2}b}{320a^{17/4}}$$

input
```
Integrate[1/(x^(7/2)*(a + b*x^2)^3),x]
```

output
```
((4*a^(1/4)*(-32*a^3 + 416*a^2*b*x^2 + 1053*a*b^2*x^4 + 585*b^3*x^6))/(x^(
5/2)*(a + b*x^2)^2) - 585*Sqrt[2]*b^(5/4)*ArcTan[(Sqrt[a] - Sqrt[b]*x)/(Sq
rt[2]*a^(1/4)*b^(1/4)*Sqrt[x])] - 585*Sqrt[2]*b^(5/4)*ArcTanh[(Sqrt[2]*a^(
1/4)*b^(1/4)*Sqrt[x])/(Sqrt[a] + Sqrt[b]*x)])/(320*a^(17/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {253, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{13\int \frac{1}{x^{7/2}(bx^2+a)^2}dx}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{9\int \frac{1}{x^{7/2}\left(bx^2+a\right)}dx}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{13\left(\frac{9\left(-\frac{b\int\frac{1}{x^{3/2}\left(bx^2+a\right)}dx}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 264$$

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{b\int\frac{\sqrt{x}}{bx^2+a}dx}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 266$$

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\int\frac{x}{bx^2+a}d\sqrt{x}}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 826$$

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\int \frac{\sqrt{b}x+\sqrt{a}}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 1476

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int \frac{1}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}+\frac{\int \frac{1}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

↓ 1082

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\int\frac{1}{-x-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int\frac{1}{-x-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

↓ 217

$$\frac{13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\int\frac{\sqrt{a}-\sqrt{b}x}{bx^2+a}d\sqrt{x}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}+\frac{1}{2ax^{5/2}(a+bx^2)}\right)}{8a}+$$

$$\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

↓ 1479

$$13\left(\frac{\left(9\left(-\frac{\left(b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}\cdots}{a}\cdots}{4a}\cdots$$

↓ 25

$$13 \quad \left(\frac{\left(9 - \frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} - \frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}} - \frac{\frac{\int \frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{b}\left(x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}} + \frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}\right)}{\sqrt[4]{b}\left(x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}\right)}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a} - \frac{2}{a\sqrt{x}} \cdots \right)}{a} \cdots \right)}{4a} \cdots$$

↓ 27

$$13\;\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt[4]{a}-2\sqrt[4]{b}\sqrt{x}}{x-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt{2}\sqrt[4]{a}\sqrt{b}}+\frac{\int\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}+\sqrt[4]{a}}{x+\frac{\sqrt{2}\sqrt[4]{a}\sqrt{x}}{\sqrt[4]{b}}+\frac{\sqrt{a}}{\sqrt{b}}}d\sqrt{x}}{2\sqrt[4]{a}\sqrt{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\frac{2}{5ax^{5/2}}\right)}{4a}$$

↓ 1103

$$13\left(\frac{9\left(-\frac{b\left(-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}+1\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}-\frac{\frac{\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}-\frac{\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\sqrt{x}+\sqrt{a}+\sqrt{b}x\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}}}{2\sqrt{b}}\right)}{a}-\frac{2}{a\sqrt{x}}\right)}{a}-\cdots\right)}{4a}\cdots\right)}{8a}$$

$$\frac{1}{4ax^{5/2}\left(a+bx^2\right)^2}$$

input
```
Int[1/(x^(7/2)*(a + b*x^2)^3),x]
```

output
```
1/(4*a*x^(5/2)*(a + b*x^2)^2) + (13*(1/(2*a*x^(5/2)*(a + b*x^2)) + (9*(-2/
(5*a*x^(5/2)) - (b*(-2/(a*Sqrt[x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*
Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4))) + ArcTan[1 + (Sqrt[2]*b^(1/4)
*Sqrt[x])/a^(1/4)]/(Sqrt[2]*a^(1/4)*b^(1/4)))/(2*Sqrt[b]) - (-1/2*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(Sqrt[2]*a^(1/4)*b^(1/4
)) + Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*Sqrt[x] + Sqrt[b]*x]/(2*Sqrt[2]
*a^(1/4)*b^(1/4)))/(2*Sqrt[b])))/a))/a))/(4*a)))/(8*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{2(-15b\,x^2+a)}{5a^4x^{\frac{5}{2}}}+\frac{b^2\left(\frac{\frac{21b\,x^{\frac{7}{2}}}{16}+\frac{25a\,x^{\frac{3}{2}}}{16}}{(b\,x^2+a)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\right.}{128b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^4}$
derivativedivides	$-\frac{2}{5a^3x^{\frac{5}{2}}}+\frac{6b}{a^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{21b\,x^{\frac{7}{2}}}{32}+\frac{25a\,x^{\frac{3}{2}}}{32}}{(b\,x^2+a)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^4}$
default	$-\frac{2}{5a^3x^{\frac{5}{2}}}+\frac{6b}{a^4\sqrt{x}}+\frac{2b^2\left(\frac{\frac{21b\,x^{\frac{7}{2}}}{32}+\frac{25a\,x^{\frac{3}{2}}}{32}}{(b\,x^2+a)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}{x+\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{x}\,\sqrt{2}+\sqrt{\frac{a}{b}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{}}{\left(\frac{a}{b}\right)}\right.}{256b\left(\frac{a}{b}\right)^{\frac{1}{4}}}}{a^4}$

input
```
int(1/x^(7/2)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(-15*b*x^2+a)/a^4/x^(5/2)+1/a^4*b^2*(2*(21/32*b*x^(7/2)+25/32*a*x^(3/
2))/(b*x^2+a)^2+117/128/b/(a/b)^(1/4)*2^(1/2)*(ln((x-(a/b)^(1/4)*x^(1/2)*2
^(1/2)+(a/b)^(1/2))/(x+(a/b)^(1/4)*x^(1/2)*2^(1/2)+(a/b)^(1/2)))+2*arctan(
2^(1/2)/(a/b)^(1/4)*x^(1/2)+1)+2*arctan(2^(1/2)/(a/b)^(1/4)*x^(1/2)-1)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^{7/2}\left(a+bx^2\right)^3}\,dx = \frac{585\left(a^4b^2x^7+2\,a^5bx^5+a^6x^3\right)\left(-\frac{b^5}{a^{17}}\right)^{\frac{1}{4}}\log\left(1601613\,a^{13}\left(-\frac{b^5}{a^{17}}\right)^{\frac{3}{4}}+1601613\,b^4\sqrt{x}\right.}{}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/320*(585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)*log(160
1613*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) - 585*(I*a^4*b^2*x^7 +
2*I*a^5*b*x^5 + I*a^6*x^3)*(-b^5/a^17)^(1/4)*log(1601613*I*a^13*(-b^5/a^17
)^(3/4) + 1601613*b^4*sqrt(x)) - 585*(-I*a^4*b^2*x^7 - 2*I*a^5*b*x^5 - I*a
^6*x^3)*(-b^5/a^17)^(1/4)*log(-1601613*I*a^13*(-b^5/a^17)^(3/4) + 1601613*
b^4*sqrt(x)) - 585*(a^4*b^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)*(-b^5/a^17)^(1/4)
*log(-1601613*a^13*(-b^5/a^17)^(3/4) + 1601613*b^4*sqrt(x)) + 4*(585*b^3*x
^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)*sqrt(x))/(a^4*b^2*x^7 + 2*a^
5*b*x^5 + a^6*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^{7/2}\,(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/x**(7/2)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^{7/2}\,(a+bx^2)^3}\,dx = \frac{585\,b^3x^6 + 1053\,ab^2x^4 + 416\,a^2bx^2 - 32\,a^3}{80\left(a^4b^2x^{\frac{13}{2}} + 2\,a^5bx^{\frac{9}{2}} + a^6x^{\frac{5}{2}}\right)} + \frac{117\,b^2\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}+2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} + \frac{2\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}-2\,\sqrt{b}\sqrt{x}\right)}{2\,\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\sqrt{2}\log\left(\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\sqrt{x}+\sqrt{b}x+\sqrt{a}\right)}{a^{\frac{1}{4}}b^{\frac{3}{4}}} + \frac{\sqrt{2}\log}{}\right.}{128\,a^4}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/80*(585*b^3*x^6 + 1053*a*b^2*x^4 + 416*a^2*b*x^2 - 32*a^3)/(a^4*b^2*x^(1
3/2) + 2*a^5*b*x^(9/2) + a^6*x^(5/2)) + 117/128*b^2*(2*sqrt(2)*arctan(1/2*
sqrt(2)*(sqrt(2)*a^(1/4)*b^(1/4) + 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2
)*a^(1/4)*b^(1/4) - 2*sqrt(b)*sqrt(x))/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a
)*sqrt(b))*sqrt(b)) - sqrt(2)*log(sqrt(2)*a^(1/4)*b^(1/4)*sqrt(x) + sqrt(b
)*x + sqrt(a))/(a^(1/4)*b^(3/4)) + sqrt(2)*log(-sqrt(2)*a^(1/4)*b^(1/4)*sq
rt(x) + sqrt(b)*x + sqrt(a))/(a^(1/4)*b^(3/4)))/a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^{7/2}(a+bx^2)^3}\,dx = \frac{117\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b} + \frac{117\,\sqrt{2}(ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}-2\,\sqrt{x}\right)}{2\,\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{64\,a^5b} - \frac{117\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b} + \frac{117\,\sqrt{2}(ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{\frac{a}{b}}\right)}{128\,a^5b} + \frac{21\,b^3x^{\frac{7}{2}}+25\,ab^2x^{\frac{3}{2}}}{16\,(bx^2+a)^2a^4} + \frac{2\,(15\,bx^2-a)}{5\,a^4x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(a/b)^(1/4) + 2*s
qrt(x))/(a/b)^(1/4))/(a^5*b) + 117/64*sqrt(2)*(a*b^3)^(3/4)*arctan(-1/2*sq
rt(2)*(sqrt(2)*(a/b)^(1/4) - 2*sqrt(x))/(a/b)^(1/4))/(a^5*b) - 117/128*sqr
t(2)*(a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + sqrt(a/b))/(a^5*b
) + 117/128*sqrt(2)*(a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(a/b)^(1/4) + x + s
qrt(a/b))/(a^5*b) + 1/16*(21*b^3*x^(7/2) + 25*a*b^2*x^(3/2))/((b*x^2 + a)^
2*a^4) + 2/5*(15*b*x^2 - a)/(a^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^{7/2} (a+bx^2)^3} \, dx = \frac{\frac{26\,b\,x^2}{5\,a^2}-\frac{2}{5\,a}+\frac{1053\,b^2\,x^4}{80\,a^3}+\frac{117\,b^3\,x^6}{16\,a^4}}{a^2\,x^{5/2}+b^2\,x^{13/2}+2\,a\,b\,x^{9/2}} - \frac{117\,{(-b)}^{5/4}\,\mathrm{atan}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}} + \frac{117\,{(-b)}^{5/4}\,\mathrm{atanh}\left(\frac{{(-b)}^{1/4}\,\sqrt{x}}{a^{1/4}}\right)}{32\,a^{17/4}}$$

input
```
int(1/(x^(7/2)*(a + b*x^2)^3),x)
```

output
```
((26*b*x^2)/(5*a^2) - 2/(5*a) + (1053*b^2*x^4)/(80*a^3) + (117*b^3*x^6)/(1
6*a^4))/(a^2*x^(5/2) + b^2*x^(13/2) + 2*a*b*x^(9/2)) - (117*(-b)^(5/4)*ata
n(((-b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4)) + (117*(-b)^(5/4)*atanh(((-
b)^(1/4)*x^(1/2))/a^(1/4)))/(32*a^(17/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^{7/2} (a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/x^(7/2)/(b*x^2+a)^3,x)
```

output
```
( - 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2)
 - 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a**2*b*x**2 - 2340*sqrt
(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*
sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b**2*x**4 - 1170*sqrt(x)*b**(1/4)*
a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*sqrt(2) - 2*sqrt(x)*sqrt(b))/(b**
(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2
)*atan((b**(1/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*
sqrt(2)))*a**2*b*x**2 + 2340*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1
/4)*a**(1/4)*sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*a*b
**2*x**4 + 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*atan((b**(1/4)*a**(1/4)*
sqrt(2) + 2*sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4)*sqrt(2)))*b**3*x**6 + 585*
sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2)
 + sqrt(a) + sqrt(b)*x)*a**2*b*x**2 + 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(
2)*log( - sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*
x**4 + 585*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log( - sqrt(x)*b**(1/4)*a**(1
/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*b**3*x**6 - 585*sqrt(x)*b**(1/4)*a**(3/
4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*
*2*b*x**2 - 1170*sqrt(x)*b**(1/4)*a**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a*
*(1/4)*sqrt(2) + sqrt(a) + sqrt(b)*x)*a*b**2*x**4 - 585*sqrt(x)*b**(1/4)*a
**(3/4)*sqrt(2)*log(sqrt(x)*b**(1/4)*a**(1/4)*sqrt(2) + sqrt(a) + sqrt(...
```

3.312 $\int \frac{x^{7/2}}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 82

$$\int \frac{x^{7/2}}{1+x^2} \, dx = -2\sqrt{x} + \frac{2x^{5/2}}{5} - \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
-2*x^(1/2)+2/5*x^(5/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1
+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}}{1+x^2} \, dx = \frac{2}{5}\sqrt{x}\left(-5+x^2\right) + \frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} + \frac{\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[x^(7/2)/(1 + x^2),x]
```

output
```
(2*Sqrt[x]*(-5 + x^2))/5 + ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.48, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {262, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{x^2+1} \, dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{5/2}}{5}-\int \frac{x^{3/2}}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\int \frac{1}{\sqrt{x}\left(x^2+1\right)}dx+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{1}{x^2+1}d\sqrt{x}+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{755}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\frac{2x^{5/2}}{5}-2\sqrt{x}$$

input
```
Int[x^(7/2)/(1 + x^2),x]
```

output
```
-2*Sqrt[x] + (2*x^(5/2))/5 + 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) +
ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x
]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
default	$\frac{2x^{\frac{5}{2}}}{5}-2\sqrt{x}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
risch	$\frac{2(x^2-5)\sqrt{x}}{5}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
meijerg	$-\frac{2\sqrt{x}\,(-9x^2+45)}{45}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\cdots\right.}{2}$
trager	$\left(\frac{2x^2}{5}-2\right)\sqrt{x}+\frac{\operatorname{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5 x-\operatorname{RootOf}\left(\_Z^4+1\right)^5-2\operatorname{RootOf}\left(\_Z^4+1\right)^3 x+\cdots}{\operatorname{RootOf}\left(\_Z^4+1\right)^2 x-\operatorname{RootOf}\left(\_Z^4+\cdots\right.}\right.}{2}$

input
```
int(x^(7/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)-2*x^(1/2)+1/4*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(
1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{2}{5}\left(x^2-5\right)\sqrt{x}+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(7/2)/(x^2+1),x, algorithm="fricas")
```

output
```
2/5*(x^2 - 5)*sqrt(x) + 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) + 1/2*sqrt
(2)*arctan(sqrt(2)*sqrt(x) - 1) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1)
 - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

input
```
integrate(x**(7/2)/(x**2+1),x)
```

output
```
2*x**(5/2)/5 - 2*sqrt(x) - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + s
qrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) -
 1)/2 + sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - 2\sqrt{x}$$

input
```
integrate(x^(7/2)/(x^2+1),x, algorithm="maxima")
```

output
```
2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(
2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - 2\sqrt{x}$$

input
```
integrate(x^(7/2)/(x^2+1),x, algorithm="giac")
```

output
```
2/5*x^(5/2) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(
2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2*sqrt(x
)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{2\,x^{5/2}}{5} - 2\sqrt{x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^(7/2)/(x^2 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2*x^(1/2) + (2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.91

$$\int \frac{x^{7/2}}{1+x^2}\,dx = \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)}{2} + \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)}{2} + \frac{2\sqrt{x}\,x^2}{5} - 2\sqrt{x} - \frac{\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{2}\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)}{4}$$

input
```
int(x^(7/2)/(x^2+1),x)
```

output
```
(10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) + 10*sqrt(2)*atan((2*sqrt(
x) + sqrt(2))/sqrt(2)) + 8*sqrt(x)*x**2 - 40*sqrt(x) - 5*sqrt(2)*log( - sq
rt(x)*sqrt(2) + x + 1) + 5*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1))/20
```

3.313 $\int \frac{x^{5/2}}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{x^{5/2}}{1+x^2} \, dx = \frac{2x^{3/2}}{3} + \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
2/3*x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^
(1/2))*2^(1/2)+1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{x^{5/2}}{1+x^2} \, dx = \frac{2x^{3/2}}{3} - \frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[x^(5/2)/(1 + x^2),x]
```

output
```
(2*x^(3/2))/3 - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + ArcTanh[(Sqrt
[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {262, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^{3/2}}{3}-\int \frac{\sqrt{x}}{x^2+1}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2x^{3/2}}{3}-2\int \frac{x}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{826}$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1479}$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \; 25$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{2x^{3/2}}{3}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)$$

input

```
Int[x^(5/2)/(1 + x^2),x]
```

output

```
(2*x^(3/2))/3 - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sq
rt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Lo
g[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
risch	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
meijerg	$\frac{2x^{\frac{3}{2}}}{3}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\cdots\right)}{\left(x^{2}\right)^{\frac{3}{4}}}\cdots\right)}{2}$
trager	$\frac{2x^{\frac{3}{2}}}{3}-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x+\mathrm{RootOf}\left(\_Z^{4}+\cdots\right)\cdots}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1}\cdots\right)}{2}$

input
```
int(x^(5/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)-1/4*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2
*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{x^{5/2}}{1+x^2}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right) + \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(5/2)/(x^2+1),x, algorithm="fricas")
```

output
```
2/3*x^(3/2) - 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) - 1/2*sqrt(2)*arctan
(sqrt(2)*sqrt(x) - 1) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) - 1/4*sqr
t(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.32

$$\int \frac{x^{5/2}}{1+x^2}\,dx = \frac{2x^{\frac{3}{2}}}{3} - \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

input
```
integrate(x**(5/2)/(x**2+1),x)
```

output
```
2*x**(3/2)/3 - sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4
*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt
(2)*atan(sqrt(2)*sqrt(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x^{5/2}}{1+x^2}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(5/2)/(x^2+1),x, algorithm="maxima")
```

output
```
2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(
2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{x^{5/2}}{1+x^2}\,dx = \frac{2}{3}\,x^{\frac{3}{2}} - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(5/2)/(x^2+1),x, algorithm="giac")
```

output
```
2/3*x^(3/2) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*
sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(
2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.56

$$\int \frac{x^{5/2}}{1+x^2}\,dx = \frac{2\,x^{3/2}}{3} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^(5/2)/(x^2 + 1),x)
```

output
```
(2*x^(3/2))/3 - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) -
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{x^{5/2}}{1+x^2}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)}{2} - \frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)}{2} + \frac{2\sqrt{x}\,x}{3} - \frac{\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)}{4} + \frac{\sqrt{2}\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)}{4}$$

input
```
int(x^(5/2)/(x^2+1),x)
```

output
```
( - 6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) - 6*sqrt(2)*atan((2*sqrt
(x) + sqrt(2))/sqrt(2)) + 8*sqrt(x)*x - 3*sqrt(2)*log( - sqrt(x)*sqrt(2) +
 x + 1) + 3*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1))/12
```

3.314 $\int \frac{x^{3/2}}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 74

$$\int \frac{x^{3/2}}{1+x^2} \, dx = 2\sqrt{x} + \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
2*x^(1/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(1
/2))*2^(1/2)-1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.73

$$\int \frac{x^{3/2}}{1+x^2} \, dx = 2\sqrt{x} - \frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[x^(3/2)/(1 + x^2),x]
```

output
```
2*Sqrt[x] - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] - ArcTanh[(Sqrt[2]*
Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{x^2+1} \, dx$$

$$\Big\downarrow \text{262}$$

$$2\sqrt{x}-\int \frac{1}{\sqrt{x}\,(x^2+1)}dx$$

$$\Big\downarrow \text{266}$$

$$2\sqrt{x}-2\int \frac{1}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{755}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{1082}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{217}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{1479}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[x^(3/2)/(1 + x^2),x]
```

output
```
2*Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2
]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1
 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result
derivativedivides	$2\sqrt{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
default	$2\sqrt{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
risch	$2\sqrt{x}-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
meijerg	$2\sqrt{x}-\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{\cdots}}{2+\cdots}\right)}{\left(x^{2}\right)\cdots}\cdots\right.}{2}$
trager	$2\sqrt{x}-\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}+2\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}-\operatorname{RootOf}\left(\_Z^{4}+\cdots\right.}{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}-x-1}\cdots\right.}{2}$

input
```
int(x^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)-1/4*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+2*a
rctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}}{1+x^2} dx = -\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right) - \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + 2\sqrt{x}$$

input
```
integrate(x^(3/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) - 1/2*sqrt(2)*arctan(sqrt(2)*sqrt
(x) - 1) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) + 1/4*sqrt(2)*log(-sqr
t(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{x^{3/2}}{1+x^2} dx = 2\sqrt{x} + \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

input
```
integrate(x**(3/2)/(x**2+1),x)
```

output
```
2*sqrt(x) + sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sq
rt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)
*atan(sqrt(2)*sqrt(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}}{1+x^2} dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + 2\sqrt{x}$$

input
```
integrate(x^(3/2)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{x^{3/2}}{1+x^2} dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + 2\sqrt{x}$$

input
```
integrate(x^(3/2)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}}{1+x^2}\,dx = 2\sqrt{x}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^(3/2)/(x^2 + 1),x)
```

output
```
2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2^(1
/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{1+x^2}\,dx = -\frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)}{2}-\frac{\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)}{2} + 2\sqrt{x}+\frac{\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)}{4}-\frac{\sqrt{2}\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)}{4}$$

input
```
int(x^(3/2)/(x^2+1),x)
```

output
```
( - 2*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) - 2*sqrt(2)*atan((2*sqrt
(x) + sqrt(2))/sqrt(2)) + 8*sqrt(x) + sqrt(2)*log( - sqrt(x)*sqrt(2) + x +
 1) - sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1))/4
```

3.315 $\int \frac{\sqrt{x}}{1+x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\sqrt{x}}{1+x^2}\,dx=-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)-1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{x}}{1+x^2}\,dx=\frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[Sqrt[x]/(1 + x^2),x]
```

output
```
(ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/
Sqrt[2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{x^2+1}\,dx$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{x}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[Sqrt[x]/(1 + x^2),x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/S
qrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
meijerg	$\frac{x^{\frac{3}{2}}\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{4\left(x^{2}\right)^{\frac{3}{4}}}+\frac{x^{\frac{3}{2}}\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}-\frac{x^{\frac{3}{2}}\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{4\left(x^{2}\right)^{\frac{3}{4}}}+\frac{x^{\frac{3}{2}}\sqrt{2}\,\mathrm{ar}}{}$
trager	$-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x+\mathrm{RootOf}\left(\_Z^{4}+1\right)x+\mathrm{Roo}}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1}\right.}{2}$

input
```
int(x^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^
(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) + 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(
x) - 1) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) + 1/4*sqrt(2)*log(-sqrt
(2)*sqrt(x) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

input
```
integrate(x**(1/2)/(x**2+1),x)
```

output
```
sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2
)*sqrt(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(x^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{x}}{1+x^2}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(x^(1/2)/(x^2 + 1),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{x}}{1+x^2}\,dx$$
$$= \frac{\sqrt{2}\left(2atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+2atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)+\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)-\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)\right)}{4}$$

input
```
int(x^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*(2*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) + 2*atan((2*sqrt(x) + sqrt
(2))/sqrt(2)) + log( - sqrt(x)*sqrt(2) + x + 1) - log(sqrt(x)*sqrt(2) + x
+ 1)))/4
```

3.316 $\int \frac{1}{\sqrt{x}(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 66

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx=-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1+2^(1/2)*x^(1/2))*2^(1/
2)+1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx=\frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(Sqrt[x]*(1 + x^2)),x]
```

output
```
(ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/
Sqrt[2]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(x^2+1)}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\int \frac{1}{x^2+1}d\sqrt{x}$$

$$\Big\downarrow \text{ 755}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 1476}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{ 1082}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 1479}$$

$$2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow 27$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow 1103$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[1/(Sqrt[x]*(1 + x^2)),x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/S
qrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*S
qrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}$
meijerg	$-\frac{\sqrt{x}\,\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{4(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x}\,\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{4(x^2)^{\frac{1}{4}}}+\frac{\sqrt{x}\,\sqrt{}}{}$
trager	$\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\left(\_Z^4+1\right)x-\mathrm{RootO}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-1}\right.}{2}$

input
```
int(1/x^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+2*arctan(1+2^
(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}+1\right)+\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\sqrt{x}-1\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(1/x^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*sqrt(x) + 1) + 1/2*sqrt(2)*arctan(sqrt(2)*sqrt(
x) - 1) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x + 1) - 1/4*sqrt(2)*log(-sqrt
(2)*sqrt(x) + x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.36

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx = -\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}$$

input
```
integrate(1/x**(1/2)/(x**2+1),x)
```

output
```
-sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(
x) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(
2)*sqrt(x) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(1/x^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)$$

input
```
integrate(1/x^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^(1/2)*(x^2 + 1)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{x}\,(1+x^2)}\,dx$$
$$= \frac{\sqrt{2}\left(2atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+2atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)-\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)+\log\left(\sqrt{x}\,\sqrt{2}+x+1\right)\right)}{4}$$

input
```
int(1/x^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*(2*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) + 2*atan((2*sqrt(x) + sqrt
(2))/sqrt(2)) - log( - sqrt(x)*sqrt(2) + x + 1) + log(sqrt(x)*sqrt(2) + x
+ 1)))/4
```

3.317 $\int \frac{1}{x^{3/2}(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{x^{3/2}(1+x^2)}\,dx = -\frac{2}{\sqrt{x}}+\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
-2/x^(1/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x^(
1/2))*2^(1/2)+1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2}(1+x^2)}\,dx = -\frac{2}{\sqrt{x}}-\frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x^(3/2)*(1 + x^2)),x]
```

output
```
-2/Sqrt[x] - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] + ArcTanh[(Sqrt[2]
*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2}(x^2+1)}dx$$

$$\Big\downarrow \text{264}$$

$$-\int \frac{\sqrt{x}}{x^2+1}dx-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{266}$$

$$-2\int \frac{x}{x^2+1}d\sqrt{x}-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$-2\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1476}$$

$$-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1082}$$

$$-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{217}$$

$$-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \; 1479$$

$$-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \; 25$$

$$-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \; 27$$

$$-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \; 1103$$

$$-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\frac{2}{\sqrt{x}}$$

input
```
Int[1/(x^(3/2)*(1 + x^2)),x]
```

output
```
-2/Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[
2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1
 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{\sqrt{x}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{\sqrt{x}}$
risch	$-\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{\sqrt{x}}$
meijerg	$-\frac{2}{\sqrt{x}}-\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(\ldots}{2+\sqrt{2}\ldots}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}\ldots\right)}{2}$
trager	$-\frac{2}{\sqrt{x}}+\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}-2\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}x+\operatorname{RootOf}\left(\_Z^{4}+1\ldots\right.}{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1\ldots}\right.}{2}$

input
```
int(1/x^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2
^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^{3/2}(1+x^2)}\,dx = -\frac{2\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{x}+1\right)+2\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}x\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\sqrt{2}x\log\left(-\sqrt{2}\sqrt{x}\right.}{4x}$$

input
```
integrate(1/x^(3/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(2)*x*arctan(sqrt(2)*sqrt(x) + 1) + 2*sqrt(2)*x*arctan(sqrt(2)
*sqrt(x) - 1) - sqrt(2)*x*log(sqrt(2)*sqrt(x) + x + 1) + sqrt(2)*x*log(-sq
rt(2)*sqrt(x) + x + 1) + 8*sqrt(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33

$$\int \frac{1}{x^{3/2}(1+x^2)}\,dx = -\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4}+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2}-\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2}-\frac{2}{\sqrt{x}}$$

input
```
integrate(1/x**(3/2)/(x**2+1),x)
```

output
```
-sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 + sqrt(2)*log(4*sqrt(2)*sqrt(
x) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(
2)*sqrt(x) + 1)/2 - 2/sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^{3/2}\,(1+x^2)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)+\frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^{3/2}\,(1+x^2)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right)-\frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right)+\frac{1}{4}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{1}{4}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}$$

input
```
integrate(1/x^(3/2)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx = -\frac{2}{\sqrt{x}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^(3/2)*(x^2 + 1)),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) - 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) - 2/x^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)}\,dx = \frac{-2\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)-2\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)-\sqrt{x}\,\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x+1\right)}{4\sqrt{x}}$$

input
```
int(1/x^(3/2)/(x^2+1),x)
```

output
```
( - 2*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) - 2*sqrt(x)*sqrt
(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) - sqrt(x)*sqrt(2)*log( - sqrt(x)*s
qrt(2) + x + 1) + sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1) - 8)/(4*sqr
t(x))
```

3.318 $\int \frac{1}{x^{5/2}(1+x^2)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 76

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{2}{3x^{3/2}}+\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
-2/3/x^(3/2)-1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctan(1+2^(1/2)*x
^(1/2))*2^(1/2)-1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{2}{3x^{3/2}}-\frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x^(5/2)*(1 + x^2)),x]
```

output
```
-2/(3*x^(3/2)) - ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2] - ArcTanh[(Sqr
t[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.50, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}(x^2+1)}dx$$

$$\Big\downarrow \text{264}$$

$$-\int \frac{1}{\sqrt{x}(x^2+1)}dx-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-2\int \frac{1}{x^2+1}d\sqrt{x}-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{755}$$

$$-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{1476}$$

$$-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{1082}$$

$$-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \text{1479}$$

$$-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \ 25$$

$$-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

$$\Big\downarrow \ 1103$$

$$-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}$$

input
```
Int[1/(x^(5/2)*(1 + x^2)),x]
```

output
```
-2/(3*x^(3/2)) - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + S
qrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] +
Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$
default	$-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$
risch	$-\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{4}-\frac{2}{3x^{\frac{3}{2}}}$
meijerg	$-\frac{2}{3x^{\frac{3}{2}}}-\frac{\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\quad}{2+}\right)}{\left(x^{2}\right.}\right.}{2}$
trager	$-\frac{2}{3x^{\frac{3}{2}}}-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x+\mathrm{RootOf}\left(\_Z^{4}\right.}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1}\right.}{2}$

input
```
int(1/x^(5/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+2*arctan(1+2
^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/3/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{6\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{x}+1\right)+6\sqrt{2}x^2\arctan\left(\sqrt{2}\sqrt{x}-1\right)+3\sqrt{2}x^2\log\left(\sqrt{2}\sqrt{x}+x+1\right)-3\sqrt{2}x^2\log\left(\right.}{12\,x^2}$$

input
```
integrate(1/x^(5/2)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(2)*x^2*arctan(sqrt(2)*sqrt(x) + 1) + 6*sqrt(2)*x^2*arctan(sq
rt(2)*sqrt(x) - 1) + 3*sqrt(2)*x^2*log(sqrt(2)*sqrt(x) + x + 1) - 3*sqrt(2
)*x^2*log(-sqrt(2)*sqrt(x) + x + 1) + 8*sqrt(x))/x^2
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2} - \frac{2}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(5/2)/(x**2+1),x)
```

output
```
sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 - sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 - sqrt(2)*atan(sqrt(2
)*sqrt(x) + 1)/2 - 2/(3*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) -\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{3x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(x^2+1),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 1/2*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = -\frac{2}{3\,x^{3/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^(5/2)*(x^2 + 1)),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 + 1i/2) - 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 - 1i/2) - 2/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^{5/2}(1+x^2)}\,dx = \frac{-6\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x - 6\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x + 3\sqrt{x}\,\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+x\right.}{12\sqrt{x}\,x}$$

input
```
int(1/x^(5/2)/(x^2+1),x)
```

output
```
( - 6*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x - 6*sqrt(x)*sq
rt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x + 3*sqrt(x)*sqrt(2)*log( - sqr
t(x)*sqrt(2) + x + 1)*x - 3*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x
 - 8)/(12*sqrt(x)*x)
```

3.319 $\int \frac{1}{x^{7/2}(1+x^2)} dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{1}{x^{7/2}(1+x^2)} dx = -\frac{2}{5x^{5/2}} + \frac{2}{\sqrt{x}} - \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

output
```
-2/5/x^(5/2)+2/x^(1/2)+1/2*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/2*arctan(1
+2^(1/2)*x^(1/2))*2^(1/2)-1/2*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^{7/2}(1+x^2)} dx = \frac{2(-1+5x^2)}{5x^{5/2}} + \frac{\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{\sqrt{2}}$$

input
```
Integrate[1/(x^(7/2)*(1 + x^2)),x]
```

output
```
(2*(-1 + 5*x^2))/(5*x^(5/2)) + ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])]/Sqrt[2]
- ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)]/Sqrt[2]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}(x^2+1)}dx$$

$$\Big\downarrow \text{264}$$

$$-\int \frac{1}{x^{3/2}(x^2+1)}dx-\frac{2}{5x^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$\int \frac{\sqrt{x}}{x^2+1}dx-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{x}{x^2+1}d\sqrt{x}-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{25}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int\frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}$$

input
```
Int[1/(x^(7/2)*(1 + x^2)),x]
```

output
```
-2/(5*x^(5/2)) + 2/Sqrt[x] + 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) +
ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*
Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{5x^{\frac{5}{2}}}+\frac{2}{\sqrt{x}}$
default	$\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}-\frac{2}{5x^{\frac{5}{2}}}+\frac{2}{\sqrt{x}}$
risch	$\frac{2x^{2}-\frac{2}{5}}{x^{\frac{5}{2}}}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{4}$
meijerg	$\frac{2}{\sqrt{x}}-\frac{2}{5x^{\frac{5}{2}}}+\frac{x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\right.}{\left(\right.}\right.}{2}$
trager	$\frac{2x^{2}-\frac{2}{5}}{x^{\frac{5}{2}}}-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x+\mathrm{RootOf}\left(\_Z^{4}+\right.}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}+x+1}\right.}{2}$

input
```
int(1/x^(7/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^
(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/5/x^(5/2)+2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^{7/2}\left(1+x^{2}\right)}\,dx=\frac{10\sqrt{2}x^{3}\arctan\left(\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}x^{3}\arctan\left(\sqrt{2}\sqrt{x}-1\right)-5\sqrt{2}x^{3}\log\left(\sqrt{2}\sqrt{x}+\right.}{20\,x^{3}}$$

input
```
integrate(1/x^(7/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/20*(10*sqrt(2)*x^3*arctan(sqrt(2)*sqrt(x) + 1) + 10*sqrt(2)*x^3*arctan(s
qrt(2)*sqrt(x) - 1) - 5*sqrt(2)*x^3*log(sqrt(2)*sqrt(x) + x + 1) + 5*sqrt(
2)*x^3*log(-sqrt(2)*sqrt(x) + x + 1) + 8*(5*x^2 - 1)*sqrt(x))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^{7/2}(1+x^2)}\,dx = \frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{4} - \frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{4} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{2} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{2} + \frac{2}{\sqrt{x}} - \frac{2}{5x^{\frac{5}{2}}}$$

input
```
integrate(1/x**(7/2)/(x**2+1),x)
```

output
```
sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/4 - sqrt(2)*log(4*sqrt(2)*sqrt(x
) + 4*x + 4)/4 + sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/2 + sqrt(2)*atan(sqrt(2
)*sqrt(x) + 1)/2 + 2/sqrt(x) - 2/(5*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{7/2}(1+x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{2\,(5\,x^2-1)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^{7/2}(1+x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{2(5x^2-1)}{5x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/2*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/4*sqrt(2)*log(sqrt(2)*sqrt(x) + x
 + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2/5*(5*x^2 - 1)/x^(5/2
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^{7/2}(1+x^2)}\,dx = \frac{2x^2-\frac{2}{5}}{x^{5/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(1/(x^(7/2)*(x^2 + 1)),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/2 - 1i/2) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/2 + 1i/2) + (2*x^2 - 2/5)/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.19

$$\int \frac{1}{x^{7/2}\,(1+x^2)}\,dx = \frac{10\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 + 10\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2 + 5\sqrt{x}\,\sqrt{2}\log\left(-\sqrt{x}\,\sqrt{2}+\right.}{20\sqrt{x}\,x^2}$$

input
```
int(1/x^(7/2)/(x^2+1),x)
```

output
```
(10*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 10*sqrt(x)*
sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 5*sqrt(x)*sqrt(2)*log(
- sqrt(x)*sqrt(2) + x + 1)*x**2 - 5*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) +
x + 1)*x**2 + 40*x**2 - 8)/(20*sqrt(x)*x**2)
```

3.320 $\int \frac{x^{7/2}}{(1+x^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = \frac{5\sqrt{x}}{2} - \frac{x^{5/2}}{2(1+x^2)} + \frac{5\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{5\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
5/2*x^(1/2)-x^(5/2)/(2*x^2+2)-5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*a
rctan(1+2^(1/2)*x^(1/2))*2^(1/2)-5/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2
)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = \frac{1}{8}\left(\frac{4\sqrt{x}(5+4x^2)}{1+x^2} - 5\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(7/2)/(1 + x^2)^2,x]
```

output
```
((4*Sqrt[x]*(5 + 4*x^2))/(1 + x^2) - 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sq
rt[x])] - 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {252, 262, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5}{4}\int \frac{x^{3/2}}{x^2+1}dx-\frac{x^{5/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{4}\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}(x^2+1)}dx\right)-\frac{x^{5/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{266}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\int \frac{1}{x^2+1}d\sqrt{x}\right)-\frac{x^{5/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)\right)-\frac{x^{5/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)\right)-\frac{x^{5/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int\frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)\right)-\frac{x^{5/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)-\frac{x^{5/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.$$
$$\frac{x^{5/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.$$
$$\frac{x^{5/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int\frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.$$
$$\frac{x^{5/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{ 1103}$$

$$\frac{5}{4}\left(2\sqrt{x}-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right.\right.\right.$$
$$\frac{x^{5/2}}{2\left(x^2+1\right)}$$

input
```
Int[x^(7/2)/(1 + x^2)^2,x]
```

output
```
-1/2*x^(5/2)/(1 + x^2) + (5*(2*Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]
/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]
*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

method	result
derivativedivides	$2\sqrt{x}+\frac{\sqrt{x}}{2x^{2}+2}-\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$2\sqrt{x}+\frac{\sqrt{x}}{2x^{2}+2}-\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$\frac{\left(4x^{2}+5\right)\sqrt{x}}{2x^{2}+2}-\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$\frac{\sqrt{x}\left(36x^{2}+45\right)}{18x^{2}+18}-\frac{5\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\,a}{}\right.}{8}$
trager	$\frac{\left(4x^{2}+5\right)\sqrt{x}}{2x^{2}+2}-\frac{5\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{5}+2\operatorname{RootOf}\left(\_Z^{4}+1\right)^{3}-\operatorname{RootOf}\left(\right.}{\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}x-\operatorname{RootOf}\left(\_Z^{4}+1\right)^{2}}\right.}{8}$

input
```
int(x^(7/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)+1/2*x^(1/2)/(x^2+1)-5/16*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^
(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int\frac{x^{7/2}}{\left(1+x^{2}\right)^{2}}\,dx=$$
$$-\frac{10\sqrt{2}(x^{2}+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}(x^{2}+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+5\sqrt{2}(x^{2}+1)\log\left(\sqrt{2}\sqrt{x}+x\right.}{16\left(x^{2}+1\right)}$$

input
```
integrate(x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/16*(10*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 10*sqrt(2)*(x^2
+ 1)*arctan(sqrt(2)*sqrt(x) - 1) + 5*sqrt(2)*(x^2 + 1)*log(sqrt(2)*sqrt(x)
 + x + 1) - 5*sqrt(2)*(x^2 + 1)*log(-sqrt(2)*sqrt(x) + x + 1) - 8*(4*x^2 +
 5)*sqrt(x))/(x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(87) = 174$.

Time = 1.10 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.80

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = \frac{32x^{\frac{5}{2}}}{16x^2+16} + \frac{40\sqrt{x}}{16x^2+16} + \frac{5\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} - \frac{5\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} - \frac{10\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16} - \frac{10\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16} + \frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} - \frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} - \frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16} - \frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}$$

input
```
integrate(x**(7/2)/(x**2+1)**2,x)
```

output
```
32*x**(5/2)/(16*x**2 + 16) + 40*sqrt(x)/(16*x**2 + 16) + 5*sqrt(2)*x**2*lo
g(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*sqrt(2)*x**2*log(4*sqrt
(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(
x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**
2 + 16) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 5*s
qrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 10*sqrt(2)*atan(s
qrt(2)*sqrt(x) - 1)/(16*x**2 + 16) - 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/
(16*x**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = -\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) -\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}+\frac{\sqrt{x}}{2\,(x^2+1)}$$

input
```
integrate(x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqr
t(x)/(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = -\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) -\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)+\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)+2\sqrt{x}+\frac{\sqrt{x}}{2\,(x^2+1)}$$

input
```
integrate(x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 2*sqrt(x) + 1/2*sqr
t(x)/(x^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = \frac{\sqrt{x}}{2\,(x^2+1)} + 2\sqrt{x} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)$$

input
```
int(x^(7/2)/(x^2 + 1)^2,x)
```

output
```
x^(1/2)/(2*(x^2 + 1)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 -
5i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/8 + 5i/8) + 2*x^(1/2
)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.62

$$\int \frac{x^{7/2}}{(1+x^2)^2}\,dx = \frac{-10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 - 10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right) - 10\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2 - 10\sqrt{2}\,ata$$

input
```
int(x^(7/2)/(x^2+1)^2,x)
```

output
```
( - 10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 - 10*sqrt(2)*atan(
(2*sqrt(x) - sqrt(2))/sqrt(2)) - 10*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqr
t(2))*x**2 - 10*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) + 32*sqrt(x)*x
**2 + 40*sqrt(x) + 5*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 + 5*sqrt
(2)*log( - sqrt(x)*sqrt(2) + x + 1) - 5*sqrt(2)*log(sqrt(x)*sqrt(2) + x +
1)*x**2 - 5*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1))/(16*(x**2 + 1))
```

3.321 $\int \frac{x^{5/2}}{(1+x^2)^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{x^{5/2}}{(1+x^2)^2} dx = -\frac{x^{3/2}}{2(1+x^2)} - \frac{3\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
-1/2*x^(3/2)/(x^2+1)+3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2
^(1/2)*x^(1/2))*2^(1/2)-3/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int \frac{x^{5/2}}{(1+x^2)^2} dx = \frac{1}{8}\left(-\frac{4x^{3/2}}{1+x^2} + 3\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(5/2)/(1 + x^2)^2,x]
```

output
```
((-4*x^(3/2))/(1 + x^2) + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - 3
*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3}{4}\int \frac{\sqrt{x}}{x^2+1}dx-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{266}$$

$$\frac{3}{2}\int \frac{x}{x^2+1}d\sqrt{x}-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \ 1479$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\frac{x^{3/2}}{2\left(x^2+1\right)}$$

input
```
Int[x^(5/2)/(1 + x^2)^2,x]
```

output
```
-1/2*x^(3/2)/(1 + x^2) + (3*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + Arc
Tan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqr
t[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{x^{\frac{3}{2}}}{2(x^{2}+1)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$-\frac{x^{\frac{3}{2}}}{2(x^{2}+1)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$-\frac{x^{\frac{3}{2}}}{2(x^{2}+1)}+\frac{3\sqrt{2}\,\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$-\frac{x^{\frac{3}{2}}}{2(x^{2}+1)}+\frac{3x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}{2-\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}}\right)}{\left(x^{2}\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,\left(x^{2}\right)^{\frac{1}{4}}+\sqrt{x^{2}}\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\ldots\right.}{\left(\ldots\right.}\right.}{8}$
trager	$-\frac{x^{\frac{3}{2}}}{2(x^{2}+1)}+\frac{3\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{5}-2\,\mathrm{RootOf}\left(\_Z^{4}+1\right)^{3}x+\mathrm{RootOf}\left(\ldots\right.}{\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}x-\mathrm{RootOf}\left(\_Z^{4}+1\right)^{2}\ldots}\right.}{8}$

input
```
int(x^(5/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(3/2)/(x^2+1)+3/16*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(
1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^{5/2}}{(1+x^2)^2}\,dx = \frac{6\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+6\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-3\sqrt{2}(x^2+1)\log}{16\,(x^2+1)}$$

input
```
integrate(x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(6*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 6*sqrt(2)*(x^2 + 1
)*arctan(sqrt(2)*sqrt(x) - 1) - 3*sqrt(2)*(x^2 + 1)*log(sqrt(2)*sqrt(x) +
x + 1) + 3*sqrt(2)*(x^2 + 1)*log(-sqrt(2)*sqrt(x) + x + 1) - 8*x^(3/2))/(x
^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(78) = 156.

Time = 0.72 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{x^{5/2}}{(1+x^2)^2}\,dx &= -\frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &\quad -\frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &\quad +\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16} \\ &\quad +\frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &\quad +\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}\end{aligned}$$

input
```
integrate(x**(5/2)/(x**2+1)**2,x)
```

output
```
-8*x**(3/2)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x +
 4)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x
**2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sq
rt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + 3*sqrt(2)*log(-4*sqr
t(2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) +
 4*x + 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 +
16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{x^{5/2}}{(1+x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{x^{\frac{3}{2}}}{2\,(x^2+1)}$$

input
```
integrate(x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{x^{5/2}}{(1+x^2)^2}\,dx = \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{x^{\frac{3}{2}}}{2\,(x^2+1)}$$

input
```
integrate(x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*x^(3/2)/(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

$$\int \frac{x^{5/2}}{(1+x^2)^2}\,dx = -\frac{x^{3/2}}{2\,(x^2+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right)$$

input
```
int(x^(5/2)/(x^2 + 1)^2,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 - 3i/8) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 + 3i/8) - x^(3/2)/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.71

$$\int \frac{x^{5/2}}{(1+x^2)^2}\,dx = \frac{6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+6\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2+6\sqrt{2}\,atan\left(\frac{2\sqrt{x}\cdots}{\sqrt{\cdots}}\right.}{}$$

input
```
int(x^(5/2)/(x^2+1)^2,x)
```

output
```
(6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sq
rt(x) - sqrt(2))/sqrt(2)) + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*
x**2 + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) - 8*sqrt(x)*x + 3*sqr
t(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 + 3*sqrt(2)*log( - sqrt(x)*sqrt(
2) + x + 1) - 3*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 - 3*sqrt(2)*log(
sqrt(x)*sqrt(2) + x + 1))/(16*(x**2 + 1))
```

3.322 $\int \frac{x^{3/2}}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{x^{3/2}}{(1+x^2)^2} \, dx = -\frac{\sqrt{x}}{2(1+x^2)} - \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
-1/2*x^(1/2)/(x^2+1)+1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2
^(1/2)*x^(1/2))*2^(1/2)+1/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{x^{3/2}}{(1+x^2)^2} \, dx = \frac{1}{8}\left(-\frac{4\sqrt{x}}{1+x^2} + \sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(3/2)/(1 + x^2)^2,x]
```

output
```
((-4*Sqrt[x])/(1 + x^2) + Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + Sqr
t[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(x^2+1)^2}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{4}\int \frac{1}{\sqrt{x}(x^2+1)}dx-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{2}\int \frac{1}{x^2+1}d\sqrt{x}-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2(x^2+1)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

input
```
Int[x^(3/2)/(1 + x^2)^2,x]
```

output
```
-1/2*Sqrt[x]/(1 + x^2) + ((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan
[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt
[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result
derivativedivides	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$-\frac{\sqrt{x}}{2(x^2+1)}+\frac{\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan \cdots}{\cdots}\right.}{8}$
trager	$-\frac{\sqrt{x}}{2(x^2+1)}-\frac{\operatorname{RootOf}(\_Z^4+1)^3\ln\left(\frac{\operatorname{RootOf}(\_Z^4+1)^5x-\operatorname{RootOf}(\_Z^4+1)^5+2\operatorname{RootOf}(\_Z^4+1)^3-\operatorname{RootOf}(\_Z \cdots}{\operatorname{RootOf}(\_Z^4+1)^2x-\operatorname{RootOf}(\_Z^4+1)^2-x-\cdots}\right.}{8}$

input
```
int(x^(3/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/(x^2+1)+1/16*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(
1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = \frac{2\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+2\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+\sqrt{2}(x^2+1)\log}{16\,(x^2+1)}$$

input
```
integrate(x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 2*sqrt(2)*(x^2 + 1
)*arctan(sqrt(2)*sqrt(x) - 1) + sqrt(2)*(x^2 + 1)*log(sqrt(2)*sqrt(x) + x
+ 1) - sqrt(2)*(x^2 + 1)*log(-sqrt(2)*sqrt(x) + x + 1) - 8*sqrt(x))/(x^2 +
 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(73) = 146.

Time = 0.59 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.86

$$\begin{aligned}\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = &-\frac{8\sqrt{x}}{16x^2+16}-\frac{\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}\\&+\frac{\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}\\&+\frac{2\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}\\&+\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}\end{aligned}$$

input
```
integrate(x**(3/2)/(x**2+1)**2,x)
```

output
```
-8*sqrt(x)/(16*x**2 + 16) - sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 +
 16) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - sqrt(2)*log(-4*sqrt(2)*sq
rt(x) + 4*x + 4)/(16*x**2 + 16) + sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*s
qrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{\sqrt{x}}{2\,(x^2+1)}$$

input
```
integrate(x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{\sqrt{x}}{2\,(x^2+1)}$$

input
```
integrate(x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = -\frac{\sqrt{x}}{2\,(x^2+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right)$$

input
```
int(x^(3/2)/(x^2 + 1)^2,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 + 1i/8) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 - 1i/8) - x^(1/2)/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.68

$$\int \frac{x^{3/2}}{(1+x^2)^2}\,dx = \frac{2\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 + 2\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2 + 2\sqrt{2}\,atan\left(\frac{2\sqrt{x}\cdots}{\sqrt{\cdots}}\right)\cdots}{}$$

input
```
int(x^(3/2)/(x^2+1)^2,x)
```

output
```
(2*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 2*sqrt(2)*atan((2*sq
rt(x) - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*
x**2 + 2*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) - 8*sqrt(x) - sqrt(2)
*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - sqrt(2)*log( - sqrt(x)*sqrt(2) + x
 + 1) + sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + sqrt(2)*log(sqrt(x)*sq
rt(2) + x + 1))/(16*(x**2 + 1))
```

3.323 $\int \frac{\sqrt{x}}{(1+x^2)^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{\sqrt{x}}{(1+x^2)^2} \, dx = \frac{x^{3/2}}{2(1+x^2)} - \frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
x^(3/2)/(2*x^2+2)+1/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+1/8*arctan(1+2^(1
/2)*x^(1/2))*2^(1/2)-1/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{x}}{(1+x^2)^2} \, dx = \frac{1}{8}\left(\frac{4x^{3/2}}{1+x^2} + \sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - \sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[Sqrt[x]/(1 + x^2)^2,x]
```

output
```
((4*x^(3/2))/(1 + x^2) + Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - Sqrt
[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(x^2+1)^2}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{4}\int \frac{\sqrt{x}}{x^2+1}dx+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{2}\int \frac{x}{x^2+1}d\sqrt{x}+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}-\frac{\arctan(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2(x^2+1)}$$

$$\Big\downarrow \ 1479$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}$$

input
```
Int[Sqrt[x]/(1 + x^2)^2,x]
```

output
```
x^(3/2)/(2*(1 + x^2)) + ((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]
) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
default	$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
risch	$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
meijerg	$\frac{2x^{\frac{3}{2}}}{4x^2+4}+\frac{x^{\frac{3}{2}}\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{16\left(x^2\right)^{\frac{3}{4}}}+\frac{x^{\frac{3}{2}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{8\left(x^2\right)^{\frac{3}{4}}}-\frac{x^{\frac{3}{2}}\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{16\left(x^2\right)^{\frac{3}{4}}}-$
trager	$\frac{x^{\frac{3}{2}}}{2x^2+2}-\frac{\operatorname{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^4+1\right)^5x-\operatorname{RootOf}\left(\_Z^4+1\right)^5+2\operatorname{RootOf}\left(\_Z^4+1\right)^3-\operatorname{RootOf}\left(\_Z^4+1\right)}{\operatorname{RootOf}\left(\_Z^4+1\right)^2x-\operatorname{RootOf}\left(\_Z^4+1\right)^2-x-1}\right.}{8}$

input
```
int(x^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)/(x^2+1)+1/16*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1
/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{2\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+2\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-\sqrt{2}(x^2+1)\log\left(\sqrt{2}\sqrt{x}+x+1\right.}{16\,(x^2+1)}$$

input
```
integrate(x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(2*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 2*sqrt(2)*(x^2 + 1
)*arctan(sqrt(2)*sqrt(x) - 1) - sqrt(2)*(x^2 + 1)*log(sqrt(2)*sqrt(x) + x
+ 1) + sqrt(2)*(x^2 + 1)*log(-sqrt(2)*sqrt(x) + x + 1) + 8*x^(3/2))/(x^2 +
 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(73) = 146.

Time = 0.47 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.86

$$\begin{aligned}\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = {} & \frac{8x^{\frac{3}{2}}}{16x^2+16}+\frac{\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ & -\frac{\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ & +\frac{2\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16} \\ & +\frac{\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16}-\frac{\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ & +\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16}+\frac{2\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}\end{aligned}$$

input
```
integrate(x**(1/2)/(x**2+1)**2,x)
```

output
```
8*x**(3/2)/(16*x**2 + 16) + sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) - sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**2 +
 16) + 2*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*sqrt(2)
*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) + sqrt(2)*log(-4*sqrt(2)*sq
rt(x) + 4*x + 4)/(16*x**2 + 16) - sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)
/(16*x**2 + 16) + 2*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 2*s
qrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{x^{\frac{3}{2}}}{2\,(x^2+1)}$$

input
```
integrate(x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{1}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{1}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{x^{\frac{3}{2}}}{2\,(x^2+1)}$$

input
```
integrate(x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 1/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 1/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 1/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{x^{3/2}}{2\,(x^2+1)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}-\frac{1}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{1}{8}+\frac{1}{8}\mathrm{i}\right)$$

input
```
int(x^(1/2)/(x^2 + 1)^2,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(1/8 - 1i/8) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(1/8 + 1i/8) + x^(3/2)/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{x}}{(1+x^2)^2}\,dx = \frac{2\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+2\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+2\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2+2\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)+8\sqrt{x}\,x+}{}$$

input
```
int(x^(1/2)/(x^2+1)^2,x)
```

output
```
(2*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 2*sqrt(2)*atan((2*sq
rt(x) - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*
x**2 + 2*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) + 8*sqrt(x)*x + sqrt(
2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 + sqrt(2)*log( - sqrt(x)*sqrt(2) +
 x + 1) - sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 - sqrt(2)*log(sqrt(x)*
sqrt(2) + x + 1))/(16*(x**2 + 1))
```

3.324 $\int \frac{1}{\sqrt{x}(1+x^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 90

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{\sqrt{x}}{2(1+x^2)} - \frac{3\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
x^(1/2)/(2*x^2+2)+3/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+3/8*arctan(1+2^(1
/2)*x^(1/2))*2^(1/2)+3/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{1}{8}\left(\frac{4\sqrt{x}}{1+x^2} + 3\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(Sqrt[x]*(1 + x^2)^2),x]
```

output
```
((4*Sqrt[x])/(1 + x^2) + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] + 3*
Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\left(x^2+1\right)^2}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3}{4}\int \frac{1}{\sqrt{x}\left(x^2+1\right)}dx+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{3}{2}\int \frac{1}{x^2+1}d\sqrt{x}+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}$$

input
```
Int[1/(Sqrt[x]*(1 + x^2)^2),x]
```

output
```
Sqrt[x]/(2*(1 + x^2)) + (3*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcT
an[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sq
rt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$\frac{\sqrt{x}}{2x^2+2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$\frac{2\sqrt{x}}{4x^2+4}-\frac{3\sqrt{x}\,\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{16(x^2)^{\frac{1}{4}}}+\frac{3\sqrt{x}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{8(x^2)^{\frac{1}{4}}}+\frac{3\sqrt{x}\,\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x}\cdots\right)}{16(x^2)^{\frac{1}{4}}}$
trager	$\frac{\sqrt{x}}{2x^2+2}-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\left(\_Z^4+\cdots\right)}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x-1}\cdots\right)}{8}$

input
```
int(1/x^(1/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(1/2)/(x^2+1)+3/16*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(1
/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{6\,\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+6\,\sqrt{2}(x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+3\,\sqrt{2}(x^2+1)\log\left(\sqrt{2}\sqrt{x}+x+\right.}{16\,(x^2+1)}$$

input
```
integrate(1/x^(1/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/16*(6*sqrt(2)*(x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 6*sqrt(2)*(x^2 + 1
)*arctan(sqrt(2)*sqrt(x) - 1) + 3*sqrt(2)*(x^2 + 1)*log(sqrt(2)*sqrt(x) +
x + 1) - 3*sqrt(2)*(x^2 + 1)*log(-sqrt(2)*sqrt(x) + x + 1) + 8*sqrt(x))/(x
^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(78) = 156.

Time = 0.52 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx &= \frac{8\sqrt{x}}{16x^2+16} - \frac{3\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &+ \frac{3\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &+ \frac{6\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16} + \frac{6\sqrt{2}x^2\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16} \\ &- \frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} + \frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^2+16} \\ &+ \frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^2+16} + \frac{6\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^2+16}\end{aligned}$$

input
```
integrate(1/x**(1/2)/(x**2+1)**2,x)
```

output
```
8*sqrt(x)/(16*x**2 + 16) - 3*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4
)/(16*x**2 + 16) + 3*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**
2 + 16) + 6*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16) + 6*sqrt
(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16) - 3*sqrt(2)*log(-4*sqrt(
2)*sqrt(x) + 4*x + 4)/(16*x**2 + 16) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4
*x + 4)/(16*x**2 + 16) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**2 + 16
) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**2 + 16)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = {} & \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) \\
& + \frac{3}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) \\
& + \frac{3}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) \\
& - \frac{3}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{\sqrt{x}}{2\,(x^2+1)}
\end{aligned}
$$

input
```
integrate(1/x^(1/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{3}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) + \frac{3}{8}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{3}{16}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{3}{16}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{\sqrt{x}}{2\,(x^2+1)}$$

input
```
integrate(1/x^(1/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
3/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) - 3/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*sqrt(x)/(x^2 + 1
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx = \frac{\sqrt{x}}{2\,\left(x^2+1\right)} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}+\frac{3}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{3}{8}-\frac{3}{8}\mathrm{i}\right)$$

input
```
int(1/(x^(1/2)*(x^2 + 1)^2),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/8 + 3i/8) + 2^(1/2)*atan(2^(
1/2)*x^(1/2)*(1/2 + 1i/2))*(3/8 - 3i/8) + x^(1/2)/(2*(x^2 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^2}\,dx$$
$$= \frac{6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 + 6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right) + 6\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^2 + 6\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right) + 8\sqrt{x} - 3}{}$$

input
```
int(1/x^(1/2)/(x^2+1)^2,x)
```

output
```
(6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sq
rt(x) - sqrt(2))/sqrt(2)) + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*
x**2 + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) + 8*sqrt(x) - 3*sqrt(
2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - 3*sqrt(2)*log( - sqrt(x)*sqrt(2)
 + x + 1) + 3*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 3*sqrt(2)*log(sq
rt(x)*sqrt(2) + x + 1))/(16*(x**2 + 1))
```

3.325 $\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx = -\frac{5}{2\sqrt{x}}+\frac{1}{2\sqrt{x}(1+x^2)}+\frac{5\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{5\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
-5/2/x^(1/2)+1/2/x^(1/2)/(x^2+1)-5/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-5/
8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+5/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(
1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{3/2}(1+x^2)^2}\,dx = \frac{1}{8}\left(-\frac{4(4+5x^2)}{\sqrt{x}(1+x^2)}-5\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)+5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(3/2)*(1 + x^2)^2),x]
```

output
```
((-4*(4 + 5*x^2))/(Sqrt[x]*(1 + x^2)) - 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]
*Sqrt[x])] + 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/8
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.34, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} \left(x^2+1\right)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5}{4} \int \frac{1}{x^{3/2} \left(x^2+1\right)}dx+\frac{1}{2 \sqrt{x} \left(x^2+1\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{4} \left(-\int \frac{\sqrt{x}}{x^2+1}dx-\frac{2}{\sqrt{x}}\right)+\frac{1}{2 \sqrt{x} \left(x^2+1\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{5}{4} \left(-2 \int \frac{x}{x^2+1}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)+\frac{1}{2 \sqrt{x} \left(x^2+1\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{5}{4} \left(-2 \left(\frac{1}{2} \int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2} \int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2 \sqrt{x} \left(x^2+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{4} \left(-2 \left(\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x-\sqrt{2} \sqrt{x}+1}d\sqrt{x}+\frac{1}{2} \int \frac{1}{x+\sqrt{2} \sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2} \int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2 \sqrt{x} \left(x^2+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right. \frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\right. \frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right. \frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)\right.$$
$$\frac{1}{2\sqrt{x}\left(x^2+1\right)}$$

input
```
Int[1/(x^(3/2)*(1 + x^2)^2),x]
```

output
```
1/(2*Sqrt[x]*(1 + x^2)) + (5*(-2/Sqrt[x] - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x
]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sq
rt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{2}{\sqrt{x}}-\frac{x^{\frac{3}{2}}}{2(x^2+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
default	$-\frac{2}{\sqrt{x}}-\frac{x^{\frac{3}{2}}}{2(x^2+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
risch	$-\frac{5x^2+4}{2\sqrt{x}\,(x^2+1)}-\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$-\frac{2(5x^2+4)}{\sqrt{x}\,(4x^2+4)}-\frac{5x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\mathrm{arc}}{}\right.}{8}$
trager	$-\frac{5x^2+4}{2\sqrt{x}\,(x^2+1)}+\frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\mathrm{RootO}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)}\right.}{8}$

input
```
int(1/x^(3/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)-1/2*x^(3/2)/(x^2+1)-5/16*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2
^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)
))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx =$$
$$-\frac{10\sqrt{2}(x^3+x)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}(x^3+x)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-5\sqrt{2}(x^3+x)\log\left(\sqrt{2}\sqrt{x}+x\right.}{16\left(x^3+x\right)}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/16*(10*sqrt(2)*(x^3 + x)*arctan(sqrt(2)*sqrt(x) + 1) + 10*sqrt(2)*(x^3
+ x)*arctan(sqrt(2)*sqrt(x) - 1) - 5*sqrt(2)*(x^3 + x)*log(sqrt(2)*sqrt(x)
 + x + 1) + 5*sqrt(2)*(x^3 + x)*log(-sqrt(2)*sqrt(x) + x + 1) + 8*(5*x^2 +
 4)*sqrt(x))/(x^3 + x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(88) = 176$.

Time = 0.70 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.70

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx = -\frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} + \frac{5\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{5\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} + \frac{5\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{10\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{40x^2}{16x^{\frac{5}{2}}+16\sqrt{x}} - \frac{32}{16x^{\frac{5}{2}}+16\sqrt{x}}$$

input
```
integrate(1/x**(3/2)/(x**2+1)**2,x)
```

output
```
-5*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2) + 16*sq
rt(x)) + 5*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*x**(5/2)
+ 16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(16*x**(5/2)
 + 16*sqrt(x)) - 10*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(16*x**(5/2
) + 16*sqrt(x)) - 5*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(16*
x**(5/2) + 16*sqrt(x)) + 5*sqrt(2)*sqrt(x)*log(4*sqrt(2)*sqrt(x) + 4*x + 4
)/(16*x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1
)/(16*x**(5/2) + 16*sqrt(x)) - 10*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1
)/(16*x**(5/2) + 16*sqrt(x)) - 40*x**2/(16*x**(5/2) + 16*sqrt(x)) - 32/(16
*x**(5/2) + 16*sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx = -\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{5x^2+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^
(5/2) + sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx = -\frac{5}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{5}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{5}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{5}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{5x^2+4}{2\left(x^{\frac{5}{2}}+\sqrt{x}\right)}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-5/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 5/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) - 5/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*(5*x^2 + 4)/(x^
(5/2) + sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx=-\frac{\frac{5\,x^2}{2}+2}{\sqrt{x}+x^{5/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}+\frac{5}{8}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{5}{8}-\frac{5}{8}\mathrm{i}\right)$$

input
```
int(1/(x^(3/2)*(x^2 + 1)^2),x)
```

output
```
- ((5*x^2)/2 + 2)/(x^(1/2) + x^(5/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2
- 1i/2))*(5/8 - 5i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/8 +
5i/8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.76

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^2}\,dx=\frac{-10\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2-10\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)-10\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)}{}$$

input
```
int(1/x^(3/2)/(x^2+1)^2,x)
```

output
```
( - 10*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 - 10*sqrt(
x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2)) - 10*sqrt(x)*sqrt(2)*atan((
2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 - 10*sqrt(x)*sqrt(2)*atan((2*sqrt(x) +
sqrt(2))/sqrt(2)) - 5*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2
 - 5*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) + 5*sqrt(x)*sqrt(2)*l
og(sqrt(x)*sqrt(2) + x + 1)*x**2 + 5*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) +
 x + 1) - 40*x**2 - 32)/(16*sqrt(x)*(x**2 + 1))
```

3.326 $\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx = -\frac{7}{6x^{3/2}}+\frac{1}{2x^{3/2}(1+x^2)} + \frac{7\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{7\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}} - \frac{7\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
-7/6/x^(3/2)+1/2/x^(3/2)/(x^2+1)-7/8*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)-7/
8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-7/8*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(
1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx = \frac{1}{24}\left(-\frac{4(4+7x^2)}{x^{3/2}(1+x^2)} - 21\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(5/2)*(1 + x^2)^2),x]
```

output
```
((-4*(4 + 7*x^2))/(x^(3/2)*(1 + x^2)) - 21*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2
]*Sqrt[x])] - 21*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/24
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2} \left(x^2+1\right)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7}{4} \int \frac{1}{x^{5/2} \left(x^2+1\right)}dx+\frac{1}{2 x^{3/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{7}{4} \left(-\int \frac{1}{\sqrt{x} \left(x^2+1\right)}dx-\frac{2}{3 x^{3/2}}\right)+\frac{1}{2 x^{3/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{7}{4} \left(-2 \int \frac{1}{x^2+1}d\sqrt{x}-\frac{2}{3 x^{3/2}}\right)+\frac{1}{2 x^{3/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{755}$$

$$\frac{7}{4} \left(-2 \left(\frac{1}{2} \int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2} \int \frac{x+1}{x^2+1}d\sqrt{x}\right)-\frac{2}{3 x^{3/2}}\right)+\frac{1}{2 x^{3/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{7}{4} \left(-2 \left(\frac{1}{2} \int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2} \left(\frac{1}{2} \int \frac{1}{x-\sqrt{2} \sqrt{x}+1}d\sqrt{x}+\frac{1}{2} \int \frac{1}{x+\sqrt{2} \sqrt{x}+1}d\sqrt{x}\right)\right)-\frac{2}{3 x^{3/2}}\right)+\frac{1}{2 x^{3/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{1479}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.$$
$$\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{}{3}\right.$$
$$\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)\right.$$
$$\frac{1}{2x^{3/2}\left(x^2+1\right)}$$

input
```
Int[1/(x^(5/2)*(1 + x^2)^2),x]
```

output
```
1/(2*x^(3/2)*(1 + x^2)) + (7*(-2/(3*x^(3/2)) - 2*((-(ArcTan[1 - Sqrt[2]*Sq
rt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log[1 - S
qrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)
))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{\sqrt{x}}{2(x^2+1)}-\frac{7\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}-\frac{2}{3x^{\frac{3}{2}}}$
default	$-\frac{\sqrt{x}}{2(x^2+1)}-\frac{7\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}-\frac{2}{3x^{\frac{3}{2}}}$
risch	$-\frac{7x^2+4}{6(x^2+1)x^{\frac{3}{2}}}-\frac{7\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{16}$
meijerg	$-\frac{2(7x^2+4)}{3x^{\frac{3}{2}}(4x^2+4)}-\frac{7\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}}{}\right.}{8}$
trager	$-\frac{7x^2+4}{6(x^2+1)x^{\frac{3}{2}}}+\frac{7\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}(}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2}\right.}{8}$

input
```
int(1/x^(5/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(1/2)/(x^2+1)-7/16*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-2^(1/2)*x^(
1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/3/x^(
3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.16

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^2}\,dx =$$
$$-\frac{42\sqrt{2}(x^4+x^2)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+42\sqrt{2}(x^4+x^2)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+21\sqrt{2}(x^4+x^2)\log\left(\sqrt{2}\sqrt{x}\right.}{48\left(x^4+x^2\right)}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
-1/48*(42*sqrt(2)*(x^4 + x^2)*arctan(sqrt(2)*sqrt(x) + 1) + 42*sqrt(2)*(x^
4 + x^2)*arctan(sqrt(2)*sqrt(x) - 1) + 21*sqrt(2)*(x^4 + x^2)*log(sqrt(2)*
sqrt(x) + x + 1) - 21*sqrt(2)*(x^4 + x^2)*log(-sqrt(2)*sqrt(x) + x + 1) +
8*(7*x^2 + 4)*sqrt(x))/(x^4 + x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(88) = 176$.

Time = 1.07 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.70

$$\int \frac{1}{x^{5/2}(1+x^2)^2}\,dx = \frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{21\sqrt{2}x^{\frac{7}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{7}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} + \frac{21\sqrt{2}x^{\frac{3}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{21\sqrt{2}x^{\frac{3}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{42\sqrt{2}x^{\frac{3}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{56x^2}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}} - \frac{32}{48x^{\frac{7}{2}}+48x^{\frac{3}{2}}}$$

input
```
integrate(1/x**(5/2)/(x**2+1)**2,x)
```

output
```
21*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2) + 48*x*
*(3/2)) - 21*sqrt(2)*x**(7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(48*x**(7/2
) + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(48*x**(7
/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) + 1)/(48*x**
(7/2) + 48*x**(3/2)) + 21*sqrt(2)*x**(3/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(48*x**(7/2) + 48*x**(3/2)) - 21*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(2)
*sqrt(x) - 1)/(48*x**(7/2) + 48*x**(3/2)) - 42*sqrt(2)*x**(3/2)*atan(sqrt(
2)*sqrt(x) + 1)/(48*x**(7/2) + 48*x**(3/2)) - 56*x**2/(48*x**(7/2) + 48*x*
*(3/2)) - 32/(48*x**(7/2) + 48*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^{5/2} (1+x^2)^2} \, dx = -\frac{7}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}\left(\sqrt{2}+2 \sqrt{x}\right)\right) - \frac{7}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}\left(\sqrt{2}-2 \sqrt{x}\right)\right) - \frac{7}{16} \sqrt{2} \log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{7}{16} \sqrt{2} \log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{7 x^2+4}{6 \left(x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
-7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/6*(7*x^2 + 4)/(x^
(7/2) + x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{5/2} (1+x^2)^2} \, dx = -\frac{7}{8} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2}\left(\sqrt{2}+2 \sqrt{x}\right)\right) - \frac{7}{8} \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2}\left(\sqrt{2}-2 \sqrt{x}\right)\right) - \frac{7}{16} \sqrt{2} \log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{7}{16} \sqrt{2} \log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{\sqrt{x}}{2 (x^2+1)} - \frac{2}{3 x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
-7/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 7/8*sqrt(2)*arcta
n(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 7/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
 x + 1) + 7/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/2*sqrt(x)/(x^2 +
1) - 2/3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{5/2} (1+x^2)^2} \, dx = -\frac{\frac{7x^2}{6}+\frac{2}{3}}{x^{3/2}+x^{7/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{8}-\frac{7}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{7}{8}+\frac{7}{8}\mathrm{i}\right)$$

input
```
int(1/(x^(5/2)*(x^2 + 1)^2),x)
```

output
```
- ((7*x^2)/6 + 2/3)/(x^(3/2) + x^(7/2)) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/
2 - 1i/2))*(7/8 + 7i/8) - 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(7/8
- 7i/8)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^{5/2} (1+x^2)^2} \, dx = \frac{-42\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^3 - 42\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x - 42\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right.}{}$$

input
```
int(1/x^(5/2)/(x^2+1)^2,x)
```

output
```
( - 42*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**3 - 42*sqrt(
x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x - 42*sqrt(x)*sqrt(2)*atan
((2*sqrt(x) + sqrt(2))/sqrt(2))*x**3 - 42*sqrt(x)*sqrt(2)*atan((2*sqrt(x)
+ sqrt(2))/sqrt(2))*x + 21*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)
*x**3 + 21*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x - 21*sqrt(x)*
sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**3 - 21*sqrt(x)*sqrt(2)*log(sqrt(x)
*sqrt(2) + x + 1)*x - 56*x**2 - 32)/(48*sqrt(x)*x*(x**2 + 1))
```

3.327 $\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 108

$$\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx = -\frac{9}{10x^{5/2}}+\frac{9}{2\sqrt{x}}+\frac{1}{2x^{5/2}(1+x^2)} - \frac{9\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}+\frac{9\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{4\sqrt{2}}-\frac{9\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{4\sqrt{2}}$$

output
```
-9/10/x^(5/2)+9/2/x^(1/2)+1/2/x^(5/2)/(x^2+1)+9/8*arctan(-1+2^(1/2)*x^(1/2
))*2^(1/2)+9/8*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-9/8*arctanh(2^(1/2)*x^(1/
2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx = \frac{1}{40}\left(\frac{4(-4+36x^2+45x^4)}{x^{5/2}(1+x^2)} + 45\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-45\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(7/2)*(1 + x^2)^2),x]
```

output
```
((4*(-4 + 36*x^2 + 45*x^4))/(x^(5/2)*(1 + x^2)) + 45*Sqrt[2]*ArcTan[(-1 +
x)/(Sqrt[2]*Sqrt[x])] - 45*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/40
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2} \left(x^2+1\right)^2} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9}{4} \int \frac{1}{x^{7/2} \left(x^2+1\right)}dx+\frac{1}{2 x^{5/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{9}{4} \left(-\int \frac{1}{x^{3/2} \left(x^2+1\right)}dx-\frac{2}{5 x^{5/2}}\right)+\frac{1}{2 x^{5/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{9}{4} \left(\int \frac{\sqrt{x}}{x^2+1}dx-\frac{2}{5 x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2 x^{5/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{266}$$

$$\frac{9}{4} \left(2 \int \frac{x}{x^2+1}d\sqrt{x}-\frac{2}{5 x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2 x^{5/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{826}$$

$$\frac{9}{4} \left(2 \left(\frac{1}{2} \int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2} \int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5 x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2 x^{5/2} \left(x^2+1\right)}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \; 1082$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \; 1479$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\right.$$
$$\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{}{5}\right.$$
$$\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

$$\Big\downarrow \text{1103}$$

$$\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\right.$$
$$\frac{1}{2x^{5/2}\left(x^2+1\right)}$$

input
```
Int[1/(x^(7/2)*(1 + x^2)^2),x]
```

output
```
1/(2*x^(5/2)*(1 + x^2)) + (9*(-2/(5*x^(5/2)) + 2/Sqrt[x] + 2*((-(ArcTan[1
- Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Lo
g[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*S
qrt[2]))/2)))/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

method	result
derivativedivides	$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}-\frac{2}{5x^{\frac{5}{2}}}+\frac{4}{\sqrt{x}}$
default	$\frac{x^{\frac{3}{2}}}{2x^2+2}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}-\frac{2}{5x^{\frac{5}{2}}}+\frac{4}{\sqrt{x}}$
risch	$\frac{45x^4+36x^2-4}{10(x^2+1)x^{\frac{5}{2}}}+\frac{9\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{16}$
meijerg	$-\frac{2(-45x^4-36x^2+4)}{5x^{\frac{5}{2}}(4x^2+4)}+\frac{9x^{\frac{3}{2}}\Big(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\cdots}{8}$
trager	$\frac{45x^4+36x^2-4}{10(x^2+1)x^{\frac{5}{2}}}-\frac{9\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\Big(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\cdots}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2\cdots}\cdots}{8}$

input
```
int(1/x^(7/2)/(x^2+1)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^(3/2)/(x^2+1)+9/16*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1
/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2)))-2/5/x^(5
/2)+4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{90\sqrt{2}(x^5+x^3)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+90\sqrt{2}(x^5+x^3)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-45\sqrt{2}(}{}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="fricas")
```

output
```
1/80*(90*sqrt(2)*(x^5 + x^3)*arctan(sqrt(2)*sqrt(x) + 1) + 90*sqrt(2)*(x^5
 + x^3)*arctan(sqrt(2)*sqrt(x) - 1) - 45*sqrt(2)*(x^5 + x^3)*log(sqrt(2)*s
qrt(x) + x + 1) + 45*sqrt(2)*(x^5 + x^3)*log(-sqrt(2)*sqrt(x) + x + 1) + 8
*(45*x^4 + 36*x^2 - 4)*sqrt(x))/(x^5 + x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(97) = 194.

Time = 1.91 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.56

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} - \frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{45\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} - \frac{45\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{90\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{360x^4}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} + \frac{288x^2}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}} - \frac{32}{80x^{\frac{9}{2}}+80x^{\frac{5}{2}}}$$

input
```
integrate(1/x**(7/2)/(x**2+1)**2,x)
```

output
```
45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2) + 80*x*
*(5/2)) - 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(80*x**(9/2
) + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(80*x**(9
/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(80*x**
(9/2) + 80*x**(5/2)) + 45*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x +
4)/(80*x**(9/2) + 80*x**(5/2)) - 45*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(2)
*sqrt(x) - 1)/(80*x**(9/2) + 80*x**(5/2)) + 90*sqrt(2)*x**(5/2)*atan(sqrt(
2)*sqrt(x) + 1)/(80*x**(9/2) + 80*x**(5/2)) + 360*x**4/(80*x**(9/2) + 80*x
**(5/2)) + 288*x**2/(80*x**(9/2) + 80*x**(5/2)) - 32/(80*x**(9/2) + 80*x**
(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^{7/2}(1+x^2)^2}\,dx = \frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{45\,x^4+36\,x^2-4}{10\left(x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="maxima")
```

output
```
9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/10*(45*x^4 + 36*x^
2 - 4)/(x^(9/2) + x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{9}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{9}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{9}{16}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{9}{16}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{x^{\frac{3}{2}}}{2\left(x^2+1\right)} + \frac{2\left(10\,x^2-1\right)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^2,x, algorithm="giac")
```

output
```
9/8*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 9/8*sqrt(2)*arctan
(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 9/16*sqrt(2)*log(sqrt(2)*sqrt(x) +
x + 1) + 9/16*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/2*x^(3/2)/(x^2 + 1
) + 2/5*(10*x^2 - 1)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{\frac{9\,x^4}{2}+\frac{18\,x^2}{5}-\frac{2}{5}}{x^{5/2}+x^{9/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{8}-\frac{9}{8}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{9}{8}+\frac{9}{8}\mathrm{i}\right)$$

input
```
int(1/(x^(7/2)*(x^2 + 1)^2),x)
```

output
```
((18*x^2)/5 + (9*x^4)/2 - 2/5)/(x^(5/2) + x^(9/2)) + 2^(1/2)*atan(2^(1/2)*
x^(1/2)*(1/2 - 1i/2))*(9/8 - 9i/8) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1
i/2))*(9/8 + 9i/8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.80

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^2}\,dx = \frac{90\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4+90\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+90\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right.}{}$$

input
```
int(1/x^(7/2)/(x^2+1)^2,x)
```

output
```
(90*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 90*sqrt(x)*
sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 90*sqrt(x)*sqrt(2)*atan
((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 90*sqrt(x)*sqrt(2)*atan((2*sqrt(x)
+ sqrt(2))/sqrt(2))*x**2 + 45*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x +
 1)*x**4 + 45*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - 45*sq
rt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**4 - 45*sqrt(x)*sqrt(2)*log(s
qrt(x)*sqrt(2) + x + 1)*x**2 + 360*x**4 + 288*x**2 - 32)/(80*sqrt(x)*x**2*
(x**2 + 1))
```

3.328 $\int \frac{x^{7/2}}{(1+x^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{x^{7/2}}{(1+x^2)^3} \, dx = -\frac{x^{5/2}}{4(1+x^2)^2} - \frac{5\sqrt{x}}{16(1+x^2)} - \frac{5\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-1/4*x^(5/2)/(x^2+1)^2-5*x^(1/2)/(16*x^2+16)+5/64*arctan(-1+2^(1/2)*x^(1/2
))*2^(1/2)+5/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+5/64*arctanh(2^(1/2)*x^(
1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

$$\int \frac{x^{7/2}}{(1+x^2)^3} \, dx = \frac{1}{64}\left(-\frac{4\sqrt{x}(5+9x^2)}{(1+x^2)^2} + 5\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 5\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(7/2)/(1 + x^2)^3,x]
```

output
```
((-4*Sqrt[x]*(5 + 9*x^2))/(1 + x^2)^2 + 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]
*Sqrt[x])] + 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 252, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5}{8}\int \frac{x^{3/2}}{(x^2+1)^2}dx-\frac{x^{5/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{5}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{x}(x^2+1)}dx-\frac{\sqrt{x}}{2(x^2+1)}\right)-\frac{x^{5/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{5}{8}\left(\frac{1}{2}\int \frac{1}{x^2+1}d\sqrt{x}-\frac{\sqrt{x}}{2(x^2+1)}\right)-\frac{x^{5/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)-\frac{\sqrt{x}}{2(x^2+1)}\right)-\frac{x^{5/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)-\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 1082}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)-\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)-\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 1479}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)-\frac{}{2(}\right.$$
$$\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$

$$\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)-\right.$$

$$\frac{x^{5/2}}{4\left(x^2+1\right)^2}$$

input

```
Int[x^(7/2)/(1 + x^2)^3,x]
```

output

```
-1/4*x^(5/2)/(1 + x^2)^2 + (5*(-1/2*Sqrt[x]/(1 + x^2) + ((-(ArcTan[1 - Sqr
t[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Lo
g[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[
2]))/2)/2))/8
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{(9x^2+5)\sqrt{x}}{16(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
derivativedivides	$\frac{-\frac{9x^{\frac{5}{2}}}{16}-\frac{5\sqrt{x}}{16}}{(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
default	$\frac{-\frac{9x^{\frac{5}{2}}}{16}-\frac{5\sqrt{x}}{16}}{(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\,\sqrt{x}+1}{x-\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
meijerg	$-\frac{\sqrt{x}\,(81x^2+45)}{144(x^2+1)^2}+\frac{5\sqrt{x}\left(-\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{1}{4}}}+\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{1}{4}}}+\cdots\right.}{64}$
trager	$-\frac{(9x^2+5)\sqrt{x}}{16(x^2+1)^2}+\frac{5\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\mathrm{RootOf}\cdots}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)\cdots}\right.}{64}$

input
```
int(x^(7/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(9*x^2+5)/(x^2+1)^2*x^(1/2)+5/128*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/
(x-2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(
1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = \frac{10\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+}{1}$$

input
```
integrate(x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(10*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 10*sqrt(
2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) - 1) + 5*sqrt(2)*(x^4 + 2*x^2
+ 1)*log(sqrt(2)*sqrt(x) + x + 1) - 5*sqrt(2)*(x^4 + 2*x^2 + 1)*log(-sqrt(
2)*sqrt(x) + x + 1) - 8*(9*x^2 + 5)*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(94) = 188.

Time = 2.23 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.54

$$\begin{aligned}\int \frac{x^{7/2}}{(1+x^2)^3}\,dx &= -\frac{72x^{\frac{5}{2}}}{128x^4+256x^2+128}-\frac{40\sqrt{x}}{128x^4+256x^2+128}\\ &-\frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}\\ &+\frac{10\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}\\ &-\frac{10\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}\\ &+\frac{20\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{20\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}\\ &-\frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}+\frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128}\\ &+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128}+\frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}\end{aligned}$$

input
```
integrate(x**(7/2)/(x**2+1)**3,x)
```

output
```
-72*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 40*sqrt(x)/(128*x**4 + 256*x**2
 + 128) - 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256
*x**2 + 128) - 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
 + 256*x**2 + 128) + 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128
*x**4 + 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x
**4 + 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**
4 + 256*x**2 + 128) - 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**
4 + 256*x**2 + 128) + 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
 + 256*x**2 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = \frac{5}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) + \frac{5}{64}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) + \frac{5}{128}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{5}{128}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{9\,x^{\frac{5}{2}}+5\,\sqrt{x}}{16\,(x^4+2\,x^2+1)}$$

input
```
integrate(x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2)
+ 5*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = \frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{9\,x^{\frac{5}{2}}+5\sqrt{x}}{16\,(x^2+1)^2}$$

input
```
integrate(x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 5/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) - 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(9*x^(5/2)
+ 5*sqrt(x))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = -\frac{\frac{5\sqrt{x}}{16}+\frac{9\,x^{5/2}}{16}}{x^4+2\,x^2+1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right)$$

input
```
int(x^(7/2)/(x^2 + 1)^3,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 + 5i/64) + 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 - 5i/64) - ((5*x^(1/2))/16 + (9*x^(5/2)
)/16)/(2*x^2 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.26

$$\int \frac{x^{7/2}}{(1+x^2)^3}\,dx = \frac{10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4+20\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+10\sqrt{2}\,atan(}{}$$

input
```
int(x^(7/2)/(x^2+1)^3,x)
```

output
```
(10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 20*sqrt(2)*atan((2*
sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/s
qrt(2)) + 10*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 20*sqrt(2)
*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 10*sqrt(2)*atan((2*sqrt(x) + s
qrt(2))/sqrt(2)) - 72*sqrt(x)*x**2 - 40*sqrt(x) - 5*sqrt(2)*log( - sqrt(x)
*sqrt(2) + x + 1)*x**4 - 10*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 -
 5*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) + 5*sqrt(2)*log(sqrt(x)*sqrt(2)
 + x + 1)*x**4 + 10*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 5*sqrt(2)*
log(sqrt(x)*sqrt(2) + x + 1))/(128*(x**4 + 2*x**2 + 1))
```

3.329 $\int \frac{x^{5/2}}{(1+x^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{x^{5/2}}{(1+x^2)^3} dx = -\frac{x^{3/2}}{4(1+x^2)^2} + \frac{3x^{3/2}}{16(1+x^2)} - \frac{3\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-1/4*x^(3/2)/(x^2+1)^2+3*x^(3/2)/(16*x^2+16)+3/64*arctan(-1+2^(1/2)*x^(1/2
))*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-3/64*arctanh(2^(1/2)*x^(
1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

$$\int \frac{x^{5/2}}{(1+x^2)^3} dx = \frac{1}{64}\left(\frac{4x^{3/2}(-1+3x^2)}{(1+x^2)^2} + 3\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(5/2)/(1 + x^2)^3,x]
```

output
```
((4*x^(3/2)*(-1 + 3*x^2))/(1 + x^2)^2 + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]
*Sqrt[x])] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}}{(x^2+1)^3}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{3}{8}\int \frac{\sqrt{x}}{(x^2+1)^2}dx-\frac{x^{3/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{3}{8}\left(\frac{1}{4}\int \frac{\sqrt{x}}{x^2+1}dx+\frac{x^{3/2}}{2(x^2+1)}\right)-\frac{x^{3/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{3}{8}\left(\frac{1}{2}\int \frac{x}{x^2+1}d\sqrt{x}+\frac{x^{3/2}}{2(x^2+1)}\right)-\frac{x^{3/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2(x^2+1)}\right)-\frac{x^{3/2}}{4(x^2+1)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)-\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)-\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)-\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{}{2}\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

input
```
Int[x^(5/2)/(1 + x^2)^3,x]
```

output
```
-1/4*x^(3/2)/(1 + x^2)^2 + (3*(x^(3/2)/(2*(1 + x^2)) + ((-(ArcTan[1 - Sqrt
[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 -
Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]
))/2)/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x^{\frac{3}{2}}(3x^2-1)}{16(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
derivativedivides	$\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
default	$\frac{\frac{3x^{\frac{7}{2}}}{16}-\frac{x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
meijerg	$-\frac{x^{\frac{3}{2}}(-21x^2+7)}{112(x^2+1)^2}+\frac{3x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\ldots\right.}{64}$
trager	$\frac{x^{\frac{3}{2}}(3x^2-1)}{16(x^2+1)^2}-\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}\left(\ldots\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2-x\ldots}\right.}{64}$

input
```
int(x^(5/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*x^(3/2)*(3*x^2-1)/(x^2+1)^2+3/128*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(
x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = \frac{6\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+6\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-3}{12}$$

input
```
integrate(x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(6*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 6*sqrt(2)
*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) - 1) - 3*sqrt(2)*(x^4 + 2*x^2 +
1)*log(sqrt(2)*sqrt(x) + x + 1) + 3*sqrt(2)*(x^4 + 2*x^2 + 1)*log(-sqrt(2)
*sqrt(x) + x + 1) + 8*(3*x^3 - x)*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(94) = 188.

Time = 1.77 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.54

$$\begin{aligned}\int \frac{x^{5/2}}{(1+x^2)^3}\,dx &= \frac{24x^{\frac{7}{2}}}{128x^4+256x^2+128} - \frac{8x^{\frac{3}{2}}}{128x^4+256x^2+128} \\ &+ \frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} \\ &+ \frac{6\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} \\ &+ \frac{6\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{6\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} \\ &+ \frac{12\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{12\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} \\ &+ \frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} \\ &+ \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}\end{aligned}$$

input
```
integrate(x**(5/2)/(x**2+1)**3,x)
```

output
```
24*x**(7/2)/(128*x**4 + 256*x**2 + 128) - 8*x**(3/2)/(128*x**4 + 256*x**2
+ 128) + 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*
x**2 + 128) - 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
256*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x*
*2 + 128) + 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) - 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
 + 256*x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 +
 256*x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 2
56*x**2 + 128) + 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) - 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 +
 128) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = \frac{3}{64}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}+2\,\sqrt{x}\right)\right) + \frac{3}{64}\,\sqrt{2}\arctan\left(-\frac{1}{2}\,\sqrt{2}\left(\sqrt{2}-2\,\sqrt{x}\right)\right) - \frac{3}{128}\,\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{128}\,\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{3\,x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16\,(x^4+2\,x^2+1)}$$

input
```
integrate(x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2)
- x^(3/2))/(x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = \frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{3x^{\frac{7}{2}}-x^{\frac{3}{2}}}{16\,(x^2+1)^2}$$

input
```
integrate(x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 3/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) + 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(3*x^(7/2)
- x^(3/2))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = -\frac{\frac{x^{3/2}}{16}-\frac{3x^{7/2}}{16}}{x^4+2x^2+1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right)$$

input
```
int(x^(5/2)/(x^2 + 1)^3,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 - 3i/64) + 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 + 3i/64) - (x^(3/2)/16 - (3*x^(7/2))/16
)/(2*x^2 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.27

$$\int \frac{x^{5/2}}{(1+x^2)^3}\,dx = \frac{6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4+12\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+6\sqrt{2}\,atan\left(\frac{2\sqrt{}}{}\right.}{}$$

input
```
int(x^(5/2)/(x^2+1)^3,x)
```

output
```
(6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 12*sqrt(2)*atan((2*s
qrt(x) - sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqr
t(2)) + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 12*sqrt(2)*at
an((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(
2))/sqrt(2)) + 24*sqrt(x)*x**3 - 8*sqrt(x)*x + 3*sqrt(2)*log( - sqrt(x)*sq
rt(2) + x + 1)*x**4 + 6*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 + 3*s
qrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) - 3*sqrt(2)*log(sqrt(x)*sqrt(2) + x
 + 1)*x**4 - 6*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 - 3*sqrt(2)*log(s
qrt(x)*sqrt(2) + x + 1))/(128*(x**4 + 2*x**2 + 1))
```

3.330 $\int \frac{x^{3/2}}{(1+x^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{x^{3/2}}{(1+x^2)^3} dx = -\frac{\sqrt{x}}{4(1+x^2)^2} + \frac{\sqrt{x}}{16(1+x^2)} - \frac{3\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{3\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-1/4*x^(1/2)/(x^2+1)^2+x^(1/2)/(16*x^2+16)+3/64*arctan(-1+2^(1/2)*x^(1/2))
*2^(1/2)+3/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+3/64*arctanh(2^(1/2)*x^(1/
2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.66

$$\int \frac{x^{3/2}}{(1+x^2)^3} dx = \frac{1}{64}\left(\frac{4\sqrt{x}(-3+x^2)}{(1+x^2)^2} + 3\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[x^(3/2)/(1 + x^2)^3,x]
```

output
```
((4*Sqrt[x]*(-3 + x^2))/(1 + x^2)^2 + 3*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*S
qrt[x])] + 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {252, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{8}\int \frac{1}{\sqrt{x}\,(x^2+1)^2}dx-\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{1}{8}\left(\frac{3}{4}\int \frac{1}{\sqrt{x}\,(x^2+1)}dx+\frac{\sqrt{x}}{2\,(x^2+1)}\right)-\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{8}\left(\frac{3}{2}\int \frac{1}{x^2+1}d\sqrt{x}+\frac{\sqrt{x}}{2\,(x^2+1)}\right)-\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)+\frac{\sqrt{x}}{2\,(x^2+1)}\right)-\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)+\frac{\sqrt{x}}{2\,(x^2+1)}\right)-\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)-\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{}{2(}\right.$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}\right.$$

input
```
Int[x^(3/2)/(1 + x^2)^3,x]
```

output
```
-1/4*Sqrt[x]/(1 + x^2)^2 + (Sqrt[x]/(2*(1 + x^2)) + (3*((-(ArcTan[1 - Sqrt
[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*Log
[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2
]))/2))/2)/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

method	result
risch	$\frac{(x^2-3)\sqrt{x}}{16(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
derivativedivides	$\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
default	$\frac{\frac{x^{\frac{5}{2}}}{16}-\frac{3\sqrt{x}}{16}}{(x^2+1)^2}+\frac{3\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
meijerg	$-\frac{\sqrt{x}\,(-5x^2+15)}{80(x^2+1)^2}+\frac{3\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}+\cdots}{64}$
trager	$\frac{(x^2-3)\sqrt{x}}{16(x^2+1)^2}+\frac{3\,\mathrm{RootOf}\left(\_Z^4+1\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\mathrm{RootOf}\left(\cdots\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2+\cdots}\right.}{64}$

input
```
int(x^(3/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(x^2-3)/(x^2+1)^2*x^(1/2)+3/128*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)/(x-
2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1/2
)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx=\frac{6\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+6\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+3\cdots}{128\cdots}$$

input
```
integrate(x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(6*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 6*sqrt(2)
*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) - 1) + 3*sqrt(2)*(x^4 + 2*x^2 +
1)*log(sqrt(2)*sqrt(x) + x + 1) - 3*sqrt(2)*(x^4 + 2*x^2 + 1)*log(-sqrt(2)
*sqrt(x) + x + 1) + 8*(x^2 - 3)*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(92) = 184$.

Time = 1.30 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.54

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = \frac{8x^{\frac{5}{2}}}{128x^4+256x^2+128} - \frac{24\sqrt{x}}{128x^4+256x^2+128} - \frac{3\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{3\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} - \frac{6\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{12\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{12\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} - \frac{3\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{3\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{6\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

input
```
integrate(x**(3/2)/(x**2+1)**3,x)
```

output
```
8*x**(5/2)/(128*x**4 + 256*x**2 + 128) - 24*sqrt(x)/(128*x**4 + 256*x**2 +
 128) - 3*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256*x
**2 + 128) + 3*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 2
56*x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 6*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**
2 + 128) - 6*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 6*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 +
256*x**2 + 128) + 12*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 25
6*x**2 + 128) - 3*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 25
6*x**2 + 128) + 3*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*x**2 +
128) + 6*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 + 128)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = \frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{x^{\frac{5}{2}}-3\sqrt{x}}{16\,(x^4+2\,x^2+1)}$$

input
```
integrate(x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) -
3*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = \frac{3}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{3}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{3}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{3}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{x^{\frac{5}{2}}-3\sqrt{x}}{16\,(x^2+1)^2}$$

input
```
integrate(x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
3/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 3/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 3/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) - 3/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(x^(5/2) -
3*sqrt(x))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = -\frac{\frac{3\sqrt{x}}{16}-\frac{x^{5/2}}{16}}{x^4+2x^2+1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}+\frac{3}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{3}{64}-\frac{3}{64}\mathrm{i}\right)$$

input
```
int(x^(3/2)/(x^2 + 1)^3,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(3/64 + 3i/64) + 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(3/64 - 3i/64) - ((3*x^(1/2))/16 - x^(5/2)/16
)/(2*x^2 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.26

$$\int \frac{x^{3/2}}{(1+x^2)^3}\,dx = \frac{6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4 + 12\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 + 6\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right) + 6\sqrt{2}\,atan\left(\frac{2\sqrt{}}{}\right.}{}$$

input
```
int(x^(3/2)/(x^2+1)^3,x)
```

output
```
(6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 12*sqrt(2)*atan((2*s
qrt(x) - sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqr
t(2)) + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 12*sqrt(2)*at
an((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 6*sqrt(2)*atan((2*sqrt(x) + sqrt(
2))/sqrt(2)) + 8*sqrt(x)*x**2 - 24*sqrt(x) - 3*sqrt(2)*log( - sqrt(x)*sqrt
(2) + x + 1)*x**4 - 6*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - 3*sqr
t(2)*log( - sqrt(x)*sqrt(2) + x + 1) + 3*sqrt(2)*log(sqrt(x)*sqrt(2) + x +
 1)*x**4 + 6*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 3*sqrt(2)*log(sqr
t(x)*sqrt(2) + x + 1))/(128*(x**4 + 2*x**2 + 1))
```

3.331 $\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{x^{3/2}}{4(1+x^2)^2} + \frac{5x^{3/2}}{16(1+x^2)} - \frac{5\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{5\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} - \frac{5\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
1/4*x^(3/2)/(x^2+1)^2+5*x^(3/2)/(16*x^2+16)+5/64*arctan(-1+2^(1/2)*x^(1/2)
)*2^(1/2)+5/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-5/64*arctanh(2^(1/2)*x^(1
/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{1}{64}\left(\frac{4x^{3/2}(9+5x^2)}{(1+x^2)^2} + 5\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) - 5\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[Sqrt[x]/(1 + x^2)^3,x]
```

output
```
((4*x^(3/2)*(9 + 5*x^2))/(1 + x^2)^2 + 5*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2]*
Sqrt[x])] - 5*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {253, 253, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{(x^2+1)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5}{8} \int \frac{\sqrt{x}}{(x^2+1)^2} dx + \frac{x^{3/2}}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{5}{8} \left(\frac{1}{4} \int \frac{\sqrt{x}}{x^2+1} dx + \frac{x^{3/2}}{2 (x^2+1)} \right) + \frac{x^{3/2}}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{5}{8} \left(\frac{1}{2} \int \frac{x}{x^2+1} d\sqrt{x} + \frac{x^{3/2}}{2 (x^2+1)} \right) + \frac{x^{3/2}}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{5}{8} \left(\frac{1}{2} \left(\frac{1}{2} \int \frac{x+1}{x^2+1} d\sqrt{x} - \frac{1}{2} \int \frac{1-x}{x^2+1} d\sqrt{x} \right) + \frac{x^{3/2}}{2 (x^2+1)} \right) + \frac{x^{3/2}}{4 (x^2+1)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)+\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1082

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)+\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 217

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)+\frac{x^{3/2}}{2\left(x^2+1\right)}\right)+\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1479

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\right.$$

$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 25

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\right.$$

$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 27

$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$
$$\Big\downarrow \; 1103$$
$$\frac{5}{8}\left(\frac{1}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+\right.$$
$$\frac{x^{3/2}}{4\left(x^2+1\right)^2}$$

input
```
Int[Sqrt[x]/(1 + x^2)^3,x]
```

output
```
x^(3/2)/(4*(1 + x^2)^2) + (5*(x^(3/2)/(2*(1 + x^2)) + ((-(ArcTan[1 - Sqrt[
2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - S
qrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2])
)/2)/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

method	result
risch	$\frac{x^{\frac{3}{2}}(5x^2+9)}{16(x^2+1)^2}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
derivativedivides	$\frac{x^{\frac{3}{2}}}{4(x^2+1)^2}+\frac{5x^{\frac{3}{2}}}{16(x^2+1)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
default	$\frac{x^{\frac{3}{2}}}{4(x^2+1)^2}+\frac{5x^{\frac{3}{2}}}{16(x^2+1)}+\frac{5\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
meijerg	$\frac{x^{\frac{3}{2}}(15x^2+27)}{48(x^2+1)^2}+\frac{5x^{\frac{3}{2}}\sqrt{2}\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{128(x^2)^{\frac{3}{4}}}+\frac{5x^{\frac{3}{2}}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{64(x^2)^{\frac{3}{4}}}-\frac{5x^{\frac{3}{2}}\sqrt{2}\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}\right.}{128(x^2)^{\frac{3}{4}}}$
trager	$\frac{x^{\frac{3}{2}}(5x^2+9)}{16(x^2+1)^2}+\frac{5\operatorname{RootOf}(\_Z^4+1)\ln\left(-\frac{\operatorname{RootOf}(\_Z^4+1)^5x-\operatorname{RootOf}(\_Z^4+1)^5+2\operatorname{RootOf}(\_Z^4+1)^3-\operatorname{RootOf}(}{\operatorname{RootOf}(\_Z^4+1)^2x-\operatorname{RootOf}(\_Z^4+1)^2-}\right.}{64}$

input
```
int(x^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*x^(3/2)*(5*x^2+9)/(x^2+1)^2+5/128*2^(1/2)*(ln((x-2^(1/2)*x^(1/2)+1)/(
x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.25

$$\int\frac{\sqrt{x}}{(1+x^2)^3}\,dx$$
$$=\frac{10\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+10\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-5\sqrt{2}(x^4+2x^2}{128\,(x^4+2x^2+}$$

input
```
integrate(x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(10*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 10*sqrt(
2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) - 1) - 5*sqrt(2)*(x^4 + 2*x^2
+ 1)*log(sqrt(2)*sqrt(x) + x + 1) + 5*sqrt(2)*(x^4 + 2*x^2 + 1)*log(-sqrt(
2)*sqrt(x) + x + 1) + 8*(5*x^3 + 9*x)*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(94) = 188$.

Time = 1.04 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.54

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{40x^{\frac{7}{2}}}{128x^4+256x^2+128} + \frac{72x^{\frac{3}{2}}}{128x^4+256x^2+128} + \frac{5\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{5\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{10\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{10\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} + \frac{10\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{10\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{20\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{20\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} + \frac{5\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} - \frac{5\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{10\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

input
```
integrate(x**(1/2)/(x**2+1)**3,x)
```

output
```
40*x**(7/2)/(128*x**4 + 256*x**2 + 128) + 72*x**(3/2)/(128*x**4 + 256*x**2
 + 128) + 5*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) - 5*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 +
 256*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 10*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256
*x**2 + 128) + 10*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
 + 256*x**2 + 128) - 10*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128
*x**4 + 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*x
**4 + 256*x**2 + 128) + 20*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x**
4 + 256*x**2 + 128) + 5*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**
4 + 256*x**2 + 128) - 5*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
 + 256*x**2 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 256*
x**2 + 128) + 10*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2 +
128)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16(x^4+2x^2+1)}$$

input
```
integrate(x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2)
+ 9*x^(3/2))/(x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{5}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{5}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{5}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{5}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{5x^{\frac{7}{2}}+9x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}$$

input
```
integrate(x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
5/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 5/64*sqrt(2)*arct
an(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 5/128*sqrt(2)*log(sqrt(2)*sqrt(x)
 + x + 1) + 5/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(5*x^(7/2)
+ 9*x^(3/2))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{\frac{9x^{3/2}}{16}+\frac{5x^{7/2}}{16}}{x^4+2x^2+1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}-\frac{5}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{5}{64}+\frac{5}{64}\mathrm{i}\right)$$

input
```
int(x^(1/2)/(x^2 + 1)^3,x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(5/64 - 5i/64) + 2^(1/2)*atan(2
^(1/2)*x^(1/2)*(1/2 + 1i/2))*(5/64 + 5i/64) + ((9*x^(3/2))/16 + (5*x^(7/2)
)/16)/(2*x^2 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.27

$$\int \frac{\sqrt{x}}{(1+x^2)^3}\,dx = \frac{10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4 + 20\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2 + 10\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right) + 10\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^4 + 2}{}$$

input
```
int(x^(1/2)/(x^2+1)^3,x)
```

output
```
(10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 20*sqrt(2)*atan((2*
sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 10*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/s
qrt(2)) + 10*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 20*sqrt(2)
*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 10*sqrt(2)*atan((2*sqrt(x) + s
qrt(2))/sqrt(2)) + 40*sqrt(x)*x**3 + 72*sqrt(x)*x + 5*sqrt(2)*log( - sqrt(
x)*sqrt(2) + x + 1)*x**4 + 10*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2
 + 5*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) - 5*sqrt(2)*log(sqrt(x)*sqrt(
2) + x + 1)*x**4 - 10*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 - 5*sqrt(2
)*log(sqrt(x)*sqrt(2) + x + 1))/(128*(x**4 + 2*x**2 + 1))
```

3.332 $\int \frac{1}{\sqrt{x}(1+x^2)^3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 106

$$\int \frac{1}{\sqrt{x}(1+x^2)^3} \, dx = \frac{\sqrt{x}}{4(1+x^2)^2} + \frac{7\sqrt{x}}{16(1+x^2)} - \frac{21\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{21\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}} + \frac{21\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
1/4*x^(1/2)/(x^2+1)^2+7*x^(1/2)/(16*x^2+16)+21/64*arctan(-1+2^(1/2)*x^(1/2
))*2^(1/2)+21/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+21/64*arctanh(2^(1/2)*x
^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{x}(1+x^2)^3} \, dx = \frac{1}{64}\left(\frac{4\sqrt{x}(11+7x^2)}{(1+x^2)^2} + 21\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right) + 21\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(Sqrt[x]*(1 + x^2)^3),x]
```

output
```
((4*Sqrt[x]*(11 + 7*x^2))/(1 + x^2)^2 + 21*Sqrt[2]*ArcTan[(-1 + x)/(Sqrt[2
]*Sqrt[x])] + 21*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/64
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {253, 253, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\,(x^2+1)^3}\,dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7}{8}\int \frac{1}{\sqrt{x}\,(x^2+1)^2}dx+\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{7}{8}\left(\frac{3}{4}\int \frac{1}{\sqrt{x}\,(x^2+1)}dx+\frac{\sqrt{x}}{2\,(x^2+1)}\right)+\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{7}{8}\left(\frac{3}{2}\int \frac{1}{x^2+1}d\sqrt{x}+\frac{\sqrt{x}}{2\,(x^2+1)}\right)+\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 755}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)+\frac{\sqrt{x}}{2\,(x^2+1)}\right)+\frac{\sqrt{x}}{4\,(x^2+1)^2}$$

$$\Big\downarrow \text{ 1476}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int\frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int\frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{\sqrt{x}}{2\left(x^2+1\right)}\right)+\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(-\frac{\int-\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right.$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)+\frac{}{2(}\right.$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{7}{8}\left(\frac{3}{2}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right)+$$
$$\frac{\sqrt{x}}{4\left(x^2+1\right)^2}$$

input
```
Int[1/(Sqrt[x]*(1 + x^2)^3),x]
```

output
```
Sqrt[x]/(4*(1 + x^2)^2) + (7*(Sqrt[x]/(2*(1 + x^2)) + (3*((-(ArcTan[1 - Sq
rt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (-1/2*L
og[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt
[2]))/2))/2))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

method	result
risch	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}+\frac{21\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
derivativedivides	$\frac{\sqrt{x}}{4(x^2+1)^2}+\frac{7\sqrt{x}}{16(x^2+1)}+\frac{21\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
default	$\frac{\sqrt{x}}{4(x^2+1)^2}+\frac{7\sqrt{x}}{16(x^2+1)}+\frac{21\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
meijerg	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}-\frac{21\sqrt{x}\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{128(x^2)^{\frac{1}{4}}}+\frac{21\sqrt{x}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{64(x^2)^{\frac{1}{4}}}+\frac{21\sqrt{x}\sqrt{2}\ln\left(1+\sqrt{2}\left(\right.\right.}{128(x^2)^{\frac{1}{4}}}$
trager	$\frac{(7x^2+11)\sqrt{x}}{16(x^2+1)^2}-\frac{21\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-\mathrm{RootOf}}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z^4+1\right)^2}\right.}{64}$

input
```
int(1/x^(1/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(7*x^2+11)/(x^2+1)^2*x^(1/2)+21/128*2^(1/2)*(ln((x+2^(1/2)*x^(1/2)+1)
/(x-2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2^(1/2)*x^
(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = \frac{42\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+42\sqrt{2}(x^4+2x^2+1)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+21\sqrt{2}(x^4+2x^2}{128\left(x^4+2x^2-\right.}$$

input
```
integrate(1/x^(1/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/128*(42*sqrt(2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) + 1) + 42*sqrt(
2)*(x^4 + 2*x^2 + 1)*arctan(sqrt(2)*sqrt(x) - 1) + 21*sqrt(2)*(x^4 + 2*x^2
 + 1)*log(sqrt(2)*sqrt(x) + x + 1) - 21*sqrt(2)*(x^4 + 2*x^2 + 1)*log(-sqr
t(2)*sqrt(x) + x + 1) + 8*(7*x^2 + 11)*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(94) = 188$.

Time = 1.29 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.54

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = \frac{56x^{\frac{5}{2}}}{128x^4+256x^2+128} + \frac{88\sqrt{x}}{128x^4+256x^2+128} - \frac{21\sqrt{2}x^4\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{21\sqrt{2}x^4\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{42\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{42\sqrt{2}x^4\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} - \frac{42\sqrt{2}x^2\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{42\sqrt{2}x^2\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{84\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{84\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128} - \frac{21\sqrt{2}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{21\sqrt{2}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^4+256x^2+128} + \frac{42\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^4+256x^2+128} + \frac{42\sqrt{2}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^4+256x^2+128}$$

input
```
integrate(1/x**(1/2)/(x**2+1)**3,x)
```

output
```
56*x**(5/2)/(128*x**4 + 256*x**2 + 128) + 88*sqrt(x)/(128*x**4 + 256*x**2
+ 128) - 21*sqrt(2)*x**4*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4 + 256
*x**2 + 128) + 21*sqrt(2)*x**4*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**4
+ 256*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 +
256*x**2 + 128) + 42*sqrt(2)*x**4*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 25
6*x**2 + 128) - 42*sqrt(2)*x**2*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**
4 + 256*x**2 + 128) + 42*sqrt(2)*x**2*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(12
8*x**4 + 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) - 1)/(128*
x**4 + 256*x**2 + 128) + 84*sqrt(2)*x**2*atan(sqrt(2)*sqrt(x) + 1)/(128*x*
*4 + 256*x**2 + 128) - 21*sqrt(2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x
**4 + 256*x**2 + 128) + 21*sqrt(2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x
**4 + 256*x**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**4 + 2
56*x**2 + 128) + 42*sqrt(2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**4 + 256*x**2
 + 128)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$
\begin{aligned}
\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = {} & \frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) \\
& + \frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) \\
& + \frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) \\
& - \frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{7\,x^{\frac{5}{2}}+11\sqrt{x}}{16\,(x^4+2\,x^2+1)}
\end{aligned}
$$

input
```
integrate(1/x^(1/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt
(x) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5
/2) + 11*sqrt(x))/(x^4 + 2*x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = \frac{21}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{21}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{21}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{21}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{7x^{\frac{5}{2}}+11\sqrt{x}}{16\,(x^2+1)^2}$$

input
```
integrate(1/x^(1/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
21/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 21/64*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 21/128*sqrt(2)*log(sqrt(2)*sqrt
(x) + x + 1) - 21/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(7*x^(5
/2) + 11*sqrt(x))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx = \frac{\frac{11\sqrt{x}}{16}+\frac{7x^{5/2}}{16}}{x^4+2x^2+1} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{21}{64}+\frac{21}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(\frac{21}{64}-\frac{21}{64}\mathrm{i}\right)$$

input
```
int(1/(x^(1/2)*(x^2 + 1)^3),x)
```

output
```
2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(21/64 + 21i/64) + 2^(1/2)*atan
(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(21/64 - 21i/64) + ((11*x^(1/2))/16 + (7*x^
(5/2))/16)/(2*x^2 + x^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sqrt{x}\,(1+x^2)^3}\,dx$$
$$= \frac{42\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4+84\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2+42\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)+42\sqrt{2}\,atan\left(\frac{2\sqrt{x}+\sqrt{2}}{\sqrt{2}}\right)x^4+8}{}$$

input
```
int(1/x^(1/2)/(x^2+1)^3,x)
```

output
```
(42*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 84*sqrt(2)*atan((2*
sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 42*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/s
qrt(2)) + 42*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**4 + 84*sqrt(2)
*atan((2*sqrt(x) + sqrt(2))/sqrt(2))*x**2 + 42*sqrt(2)*atan((2*sqrt(x) + s
qrt(2))/sqrt(2)) + 56*sqrt(x)*x**2 + 88*sqrt(x) - 21*sqrt(2)*log( - sqrt(x
)*sqrt(2) + x + 1)*x**4 - 42*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2
- 21*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1) + 21*sqrt(2)*log(sqrt(x)*sqrt
(2) + x + 1)*x**4 + 42*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 21*sqrt
(2)*log(sqrt(x)*sqrt(2) + x + 1))/(128*(x**4 + 2*x**2 + 1))
```

3.333 $\int \frac{1}{x^{3/2}(1+x^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx=-\frac{45}{16\sqrt{x}}+\frac{1}{4\sqrt{x}\left(1+x^2\right)^2}+\frac{9}{16\sqrt{x}\left(1+x^2\right)}+\frac{45\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{45\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{45\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-45/16/x^(1/2)+1/4/x^(1/2)/(x^2+1)^2+9/16/x^(1/2)/(x^2+1)-45/64*arctan(-1+
2^(1/2)*x^(1/2))*2^(1/2)-45/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)+45/64*arc
tanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx=\frac{1}{64}\left(-\frac{4(32+81x^2+45x^4)}{\sqrt{x}\left(1+x^2\right)^2}-45\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)+45\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(3/2)*(1 + x^2)^3),x]
```

output
```
((-4*(32 + 81*x^2 + 45*x^4))/(Sqrt[x]*(1 + x^2)^2) - 45*Sqrt[2]*ArcTan[(-1
 + x)/(Sqrt[2]*Sqrt[x])] + 45*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)])/
64
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {253, 253, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{3/2} (x^2+1)^3} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{9}{8} \int \frac{1}{x^{3/2} (x^2+1)^2} dx + \frac{1}{4\sqrt{x} (x^2+1)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{9}{8} \left(\frac{5}{4} \int \frac{1}{x^{3/2} (x^2+1)} dx + \frac{1}{2\sqrt{x} (x^2+1)} \right) + \frac{1}{4\sqrt{x} (x^2+1)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{9}{8} \left(\frac{5}{4} \left(-\int \frac{\sqrt{x}}{x^2+1} dx - \frac{2}{\sqrt{x}} \right) + \frac{1}{2\sqrt{x} (x^2+1)} \right) + \frac{1}{4\sqrt{x} (x^2+1)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{9}{8} \left(\frac{5}{4} \left(-2 \int \frac{x}{x^2+1} d\sqrt{x} - \frac{2}{\sqrt{x}} \right) + \frac{1}{2\sqrt{x} (x^2+1)} \right) + \frac{1}{4\sqrt{x} (x^2+1)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{9}{8} \left(\frac{5}{4} \left(-2 \left(\frac{1}{2} \int \frac{x+1}{x^2+1} d\sqrt{x} - \frac{1}{2} \int \frac{1-x}{x^2+1} d\sqrt{x} \right) - \frac{2}{\sqrt{x}} \right) + \frac{1}{2\sqrt{x} (x^2+1)} \right) + \frac{1}{4\sqrt{x} (x^2+1)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{1}{2}\int\frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int\frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2\sqrt{x}\left(x^2+\right.}$$

$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1082$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int\frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int\frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2\sqrt{x}\left(x^2+1\right)}\right)$$

$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int\frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)+\frac{1}{2\sqrt{x}\left(x^2+1\right)}\right)+$$

$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int-\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right.\right.\right.$$

$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{9}{8}\left(\frac{5}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right.\right.\right.$$
$$\frac{1}{4\sqrt{x}\left(x^2+1\right)^2}$$

input
```
Int[1/(x^(3/2)*(1 + x^2)^3),x]
```

output
```
1/(4*Sqrt[x]*(1 + x^2)^2) + (9*(1/(2*Sqrt[x]*(1 + x^2)) + (5*(-2/Sqrt[x] -
 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[x]]/
Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*S
qrt[x] + x]/(2*Sqrt[2]))/2)))/4))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
derivativedivides	$-\frac{2}{\sqrt{x}}-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{(x^2+1)^2}-\frac{45\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
default	$-\frac{2}{\sqrt{x}}-\frac{2\left(\frac{13x^{\frac{7}{2}}}{32}+\frac{17x^{\frac{3}{2}}}{32}\right)}{(x^2+1)^2}-\frac{45\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\,\sqrt{x}+1}{x+\sqrt{2}\,\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\,\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\,\sqrt{x}\right)\right)}{128}$
meijerg	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}-\frac{45x^{\frac{3}{2}}\left(\frac{\sqrt{2}\,\ln\left(1-\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\frac{\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,(x^2)^{\frac{1}{4}}}{2-\sqrt{2}\,(x^2)^{\frac{1}{4}}}\right)}{(x^2)^{\frac{3}{4}}}-\frac{\sqrt{2}\,\ln\left(1+\sqrt{2}\,(x^2)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2(x^2)^{\frac{3}{4}}}+\cdots\right)}{64}$
trager	$-\frac{45x^4+81x^2+32}{16\sqrt{x}\,(x^2+1)^2}+\frac{45\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3x+\cdots}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z\cdots\right.}\right.}{64}$

input
```
int(1/x^(3/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*(45*x^4+81*x^2+32)/x^(1/2)/(x^2+1)^2-45/128*2^(1/2)*(ln((x-2^(1/2)*x
^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+2
^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = -\frac{90\sqrt{2}(x^5+2\,x^3+x)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+90\sqrt{2}(x^5+2\,x^3+x)\arctan\left(\sqrt{2}\sqrt{x}-1\right)-45\sqrt{2}(x^5+2\,x^3}{128\,(x^5+2}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/128*(90*sqrt(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(x) + 1) + 90*sqrt
(2)*(x^5 + 2*x^3 + x)*arctan(sqrt(2)*sqrt(x) - 1) - 45*sqrt(2)*(x^5 + 2*x^
3 + x)*log(sqrt(2)*sqrt(x) + x + 1) + 45*sqrt(2)*(x^5 + 2*x^3 + x)*log(-sq
rt(2)*sqrt(x) + x + 1) + 8*(45*x^4 + 81*x^2 + 32)*sqrt(x))/(x^5 + 2*x^3 +
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(104) = 208$.

Time = 1.72 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.68

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = -\frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} + \frac{45\sqrt{2}x^{\frac{9}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{90\sqrt{2}x^{\frac{9}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} + \frac{90\sqrt{2}x^{\frac{5}{2}}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{180\sqrt{2}x^{\frac{5}{2}}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{45\sqrt{2}\sqrt{x}\log\left(-4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} + \frac{45\sqrt{2}\sqrt{x}\log\left(4\sqrt{2}\sqrt{x}+4x+4\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}-1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{90\sqrt{2}\sqrt{x}\operatorname{atan}\left(\sqrt{2}\sqrt{x}+1\right)}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{360x^4}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{648x^2}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}} - \frac{256}{128x^{\frac{9}{2}}+256x^{\frac{5}{2}}+128\sqrt{x}}$$

input

```
integrate(1/x**(3/2)/(x**2+1)**3,x)
```

output
```
-45*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256
*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x
 + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*sqrt(2)*x**(9/2)*at
an(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 90*s
qrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 256*x**(5/2) + 1
28*sqrt(x)) - 90*sqrt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x
**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 90*sqrt(2)*x**(5/2)*log(4*sqrt(2)*
sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 180*sqrt(
2)*x**(5/2)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5/2) + 128*s
qrt(x)) - 180*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**(9/2) + 2
56*x**(5/2) + 128*sqrt(x)) - 45*sqrt(2)*sqrt(x)*log(-4*sqrt(2)*sqrt(x) + 4
*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) + 45*sqrt(2)*sqrt(x)*l
og(4*sqrt(2)*sqrt(x) + 4*x + 4)/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)
) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) - 1)/(128*x**(9/2) + 256*x**(5
/2) + 128*sqrt(x)) - 90*sqrt(2)*sqrt(x)*atan(sqrt(2)*sqrt(x) + 1)/(128*x**
(9/2) + 256*x**(5/2) + 128*sqrt(x)) - 360*x**4/(128*x**(9/2) + 256*x**(5/2
) + 128*sqrt(x)) - 648*x**2/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x)) -
256/(128*x**(9/2) + 256*x**(5/2) + 128*sqrt(x))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{3/2}(1+x^2)^3}\,dx = -\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) + \frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) - \frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{45x^4+81x^2+32}{16\left(x^{\frac{9}{2}}+2x^{\frac{5}{2}}+\sqrt{x}\right)}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(45*x^
4 + 81*x^2 + 32)/(x^(9/2) + 2*x^(5/2) + sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = -\frac{45}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right)-\frac{45}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right)+\frac{45}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right)-\frac{45}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right)-\frac{2}{\sqrt{x}}-\frac{13\,x^{\frac{7}{2}}+17\,x^{\frac{3}{2}}}{16\left(x^2+1\right)^2}$$

input
```
integrate(1/x^(3/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
-45/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 45/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) + 45/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) - 45/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 2/sqrt(x) -
 1/16*(13*x^(7/2) + 17*x^(3/2))/(x^2 + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = -\frac{\frac{45\,x^4}{16}+\frac{81\,x^2}{16}+2}{\sqrt{x}+2\,x^{5/2}+x^{9/2}}+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{45}{64}+\frac{45}{64}\mathrm{i}\right)+\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(-\frac{45}{64}-\frac{45}{64}\mathrm{i}\right)$$

input
```
int(1/(x^(3/2)*(x^2 + 1)^3),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(45/64 - 45i/64) - 2^(1/2)*at
an(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(45/64 + 45i/64) - ((81*x^2)/16 + (45*x^4
)/16 + 2)/(x^(1/2) + 2*x^(5/2) + x^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^{3/2}\left(1+x^2\right)^3}\,dx = \frac{-90\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4-180\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^2-90\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-}{\sqrt{}}\right.}{}$$

input
```
int(1/x^(3/2)/(x^2+1)^3,x)
```

output
```
( - 90*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 - 180*sqrt
(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 - 90*sqrt(x)*sqrt(2)*
atan((2*sqrt(x) - sqrt(2))/sqrt(2)) - 90*sqrt(x)*sqrt(2)*atan((2*sqrt(x) +
 sqrt(2))/sqrt(2))*x**4 - 180*sqrt(x)*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/s
qrt(2))*x**2 - 90*sqrt(x)*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)) - 45
*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**4 - 90*sqrt(x)*sqrt(2)
*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - 45*sqrt(x)*sqrt(2)*log( - sqrt(x)*
sqrt(2) + x + 1) + 45*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**4 +
90*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 45*sqrt(x)*sqrt(2)*
log(sqrt(x)*sqrt(2) + x + 1) - 360*x**4 - 648*x**2 - 256)/(128*sqrt(x)*(x*
*4 + 2*x**2 + 1))
```

3.334 $\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 115

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = -\frac{77}{48x^{3/2}}+\frac{1}{4x^{3/2}\left(1+x^2\right)^2}+\frac{11}{16x^{3/2}\left(1+x^2\right)} + \frac{77\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{77\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-77/48/x^(3/2)+1/4/x^(3/2)/(x^2+1)^2+11/16/x^(3/2)/(x^2+1)-77/64*arctan(-1
+2^(1/2)*x^(1/2))*2^(1/2)-77/64*arctan(1+2^(1/2)*x^(1/2))*2^(1/2)-77/64*ar
ctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = \frac{1}{192}\left(-\frac{4(32+121x^2+77x^4)}{x^{3/2}\left(1+x^2\right)^2} - 231\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-231\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(5/2)*(1 + x^2)^3),x]
```

output
```
((-4*(32 + 121*x^2 + 77*x^4))/(x^(3/2)*(1 + x^2)^2) - 231*Sqrt[2]*ArcTan[(
-1 + x)/(Sqrt[2]*Sqrt[x])] - 231*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[x])/(1 + x)
])/192
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {253, 253, 264, 266, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{5/2}\left(x^2+1\right)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11}{8}\int \frac{1}{x^{5/2}\left(x^2+1\right)^2}dx+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{11}{8}\left(\frac{7}{4}\int \frac{1}{x^{5/2}\left(x^2+1\right)}dx+\frac{1}{2x^{3/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-\int \frac{1}{\sqrt{x}\left(x^2+1\right)}dx-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\int \frac{1}{x^2+1}d\sqrt{x}-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}(x\right.$$

$$\frac{1}{4x^{3/2}(x^2+1)^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}(x^2+1\right.$$

$$\frac{1}{4x^{3/2}(x^2+1)^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}+\frac{1}{2}\left(\frac{\arctan(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}-\frac{\arctan(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}\right)\right)-\frac{2}{3x^{3/2}}\right)+\frac{1}{2x^{3/2}(x^2+1)}\right)$$

$$\frac{1}{4x^{3/2}(x^2+1)^2}$$

$$\Big\downarrow \text{1479}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}(\sqrt{2}\sqrt{x}+1)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}-\frac{\arctan(1-\sqrt{2}\sqrt{x}}{\sqrt{2}}\right.\right.\right.\right.$$

$$\frac{1}{4x^{3/2}(x^2+1)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}(\sqrt{2}\sqrt{x}+1)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan(\sqrt{2}\sqrt{x}+1)}{\sqrt{2}}-\frac{\arctan(1-\sqrt{2}\sqrt{x})}{\sqrt{2}}\right)\right)\right.\right.$$

$$\frac{1}{4x^{3/2}(x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

$\Big\downarrow$ 1103

$$\frac{11}{8}\left(\frac{7}{4}\left(-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{1}{4x^{3/2}\left(x^2+1\right)^2}$$

input
```
Int[1/(x^(5/2)*(1 + x^2)^3),x]
```

output
```
1/(4*x^(3/2)*(1 + x^2)^2) + (11*(1/(2*x^(3/2)*(1 + x^2)) + (7*(-2/(3*x^(3/
2)) - 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt
[x]]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*Sqrt[x] + x]/Sqrt[2] + Log[1 + Sqr
t[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/4))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}-\frac{77\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
derivativedivides	$-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{(x^2+1)^2}-\frac{77\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}-\frac{2}{3x^{\frac{3}{2}}}$
default	$-\frac{2\left(\frac{15x^{\frac{5}{2}}}{32}+\frac{19\sqrt{x}}{32}\right)}{(x^2+1)^2}-\frac{77\sqrt{2}\left(\ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}-\frac{2}{3x^{\frac{3}{2}}}$
meijerg	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}-\frac{77\sqrt{x}\left(-\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{1}{4}}}\right.}{64}$
trager	$-\frac{77x^4+121x^2+32}{48(x^2+1)^2x^{\frac{3}{2}}}+\frac{77\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5+2\,\mathrm{RootOf}\left(\_Z^4+1\right)^3-}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{RootOf}\left(\_Z\right.}\right.}{64}$

input
```
int(1/x^(5/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(77*x^4+121*x^2+32)/(x^2+1)^2/x^(3/2)-77/128*2^(1/2)*(ln((x+2^(1/2)*
x^(1/2)+1)/(x-2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*arctan(-1+
2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx =$$
$$-\frac{462\sqrt{2}(x^6+2x^4+x^2)\arctan\left(\sqrt{2}\sqrt{x}+1\right)+462\sqrt{2}(x^6+2x^4+x^2)\arctan\left(\sqrt{2}\sqrt{x}-1\right)+231\sqrt{2}(x^6 \cdots}{384\,(x \cdots}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
-1/384*(462*sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(x) + 1) + 462*
sqrt(2)*(x^6 + 2*x^4 + x^2)*arctan(sqrt(2)*sqrt(x) - 1) + 231*sqrt(2)*(x^6
 + 2*x^4 + x^2)*log(sqrt(2)*sqrt(x) + x + 1) - 231*sqrt(2)*(x^6 + 2*x^4 +
x^2)*log(-sqrt(2)*sqrt(x) + x + 1) + 8*(77*x^4 + 121*x^2 + 32)*sqrt(x))/(x
^6 + 2*x^4 + x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(104) = 208.

Time = 2.70 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.68

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**(5/2)/(x**2+1)**3,x)
```

output
```
231*sqrt(2)*x**(11/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 7
68*x**(7/2) + 384*x**(3/2)) - 231*sqrt(2)*x**(11/2)*log(4*sqrt(2)*sqrt(x)
+ 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**
(11/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3
/2)) - 462*sqrt(2)*x**(11/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 76
8*x**(7/2) + 384*x**(3/2)) + 462*sqrt(2)*x**(7/2)*log(-4*sqrt(2)*sqrt(x) +
 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(
7/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*
x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x) - 1)/(384*x**(11/2)
+ 768*x**(7/2) + 384*x**(3/2)) - 924*sqrt(2)*x**(7/2)*atan(sqrt(2)*sqrt(x)
 + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) + 231*sqrt(2)*x**(3/2)
*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11/2) + 768*x**(7/2) + 384*x**
(3/2)) - 231*sqrt(2)*x**(3/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(384*x**(11
/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(3/2)*atan(sqrt(2)*sqr
t(x) - 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 462*sqrt(2)*x**(
3/2)*atan(sqrt(2)*sqrt(x) + 1)/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2
)) - 616*x**4/(384*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 968*x**2/(38
4*x**(11/2) + 768*x**(7/2) + 384*x**(3/2)) - 256/(384*x**(11/2) + 768*x**(
7/2) + 384*x**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = -\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{77\,x^4+121\,x^2+32}{48\left(x^{\frac{11}{2}}+2\,x^{\frac{7}{2}}+x^{\frac{3}{2}}\right)}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
-77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/48*(77*x^
4 + 121*x^2 + 32)/(x^(11/2) + 2*x^(7/2) + x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx = -\frac{77}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) - \frac{77}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{77}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{77}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) - \frac{15\,x^{\frac{5}{2}}+19\sqrt{x}}{16\,(x^2+1)^2} - \frac{2}{3\,x^{\frac{3}{2}}}$$

input
```
integrate(1/x^(5/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
-77/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) - 77/64*sqrt(2)*a
rctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 77/128*sqrt(2)*log(sqrt(2)*sqr
t(x) + x + 1) + 77/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) - 1/16*(15*x^
(5/2) + 19*sqrt(x))/(x^2 + 1)^2 - 2/3/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^{5/2}(1+x^2)^3}\,dx = -\frac{\frac{77\,x^4}{48}+\frac{121\,x^2}{48}+\frac{2}{3}}{x^{3/2}+2\,x^{7/2}+x^{11/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}-\frac{77}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\sqrt{x}\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\left(-\frac{77}{64}+\frac{77}{64}\mathrm{i}\right)$$

input
```
int(1/(x^(5/2)*(x^2 + 1)^3),x)
```

output
```
- 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(77/64 + 77i/64) - 2^(1/2)*at
an(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(77/64 - 77i/64) - ((121*x^2)/48 + (77*x^
4)/48 + 2/3)/(x^(3/2) + 2*x^(7/2) + x^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.38

$$\int \frac{1}{x^{5/2}\left(1+x^2\right)^3}\,dx = \frac{-462\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^5-924\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^3-462\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}}{}\right.}{}$$

input

```
int(1/x^(5/2)/(x^2+1)^3,x)
```

output

```
( - 462*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**5 - 924*sqr
t(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**3 - 462*sqrt(x)*sqrt(2
)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x - 462*sqrt(x)*sqrt(2)*atan((2*sqrt
(x) + sqrt(2))/sqrt(2))*x**5 - 924*sqrt(x)*sqrt(2)*atan((2*sqrt(x) + sqrt(
2))/sqrt(2))*x**3 - 462*sqrt(x)*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/sqrt(2)
)*x + 231*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**5 + 462*sqrt(
x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**3 + 231*sqrt(x)*sqrt(2)*log(
 - sqrt(x)*sqrt(2) + x + 1)*x - 231*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) +
x + 1)*x**5 - 462*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**3 - 231*
sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x - 616*x**4 - 968*x**2 - 256
)/(384*sqrt(x)*x*(x**4 + 2*x**2 + 1))
```

3.335 $\int \frac{1}{x^{7/2}(1+x^2)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 124

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = -\frac{117}{80x^{5/2}}+\frac{117}{16\sqrt{x}}+\frac{1}{4x^{5/2}\left(1+x^2\right)^2}+\frac{13}{16x^{5/2}\left(1+x^2\right)} -\frac{117\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}+\frac{117\arctan\left(1+\sqrt{2}\sqrt{x}\right)}{32\sqrt{2}}-\frac{117\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)}{32\sqrt{2}}$$

output
```
-117/80/x^(5/2)+117/16/x^(1/2)+1/4/x^(5/2)/(x^2+1)^2+13/16/x^(5/2)/(x^2+1)
+117/64*arctan(-1+2^(1/2)*x^(1/2))*2^(1/2)+117/64*arctan(1+2^(1/2)*x^(1/2)
)*2^(1/2)-117/64*arctanh(2^(1/2)*x^(1/2)/(1+x))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{1}{320}\left(\frac{4\left(-32+416x^2+1053x^4+585x^6\right)}{x^{5/2}\left(1+x^2\right)^2} +585\sqrt{2}\arctan\left(\frac{-1+x}{\sqrt{2}\sqrt{x}}\right)-585\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x}}{1+x}\right)\right)$$

input
```
Integrate[1/(x^(7/2)*(1 + x^2)^3),x]
```

output
```
((4*(-32 + 416*x^2 + 1053*x^4 + 585*x^6))/(x^(5/2)*(1 + x^2)^2) + 585*Sqrt
[2]*ArcTan[(-1 + x)/(Sqrt[2]*Sqrt[x])] - 585*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt
[x])/(1 + x)])/320
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.31, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {253, 253, 264, 264, 266, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^{7/2}\left(x^2+1\right)^3}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{13}{8}\int \frac{1}{x^{7/2}\left(x^2+1\right)^2}dx+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{253}$$

$$\frac{13}{8}\left(\frac{9}{4}\int \frac{1}{x^{7/2}\left(x^2+1\right)}dx+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{13}{8}\left(\frac{9}{4}\left(-\int \frac{1}{x^{3/2}\left(x^2+1\right)}dx-\frac{2}{5x^{5/2}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{13}{8}\left(\frac{9}{4}\left(\int \frac{\sqrt{x}}{x^2+1}dx-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{266}$$

$$\frac{13}{8}\left(\frac{9}{4}\left(2\int \frac{x}{x^2+1}d\sqrt{x}-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \text{826}$$

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\int \frac{x+1}{x^2+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$\downarrow$ 1476

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$\downarrow$ 1082

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-x-1}d\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-x-1}d\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$\downarrow$ 217

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-x}{x^2+1}d\sqrt{x}\right)-\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{1}{2x^{5/2}\left(x^2+1\right)}\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$\downarrow$ 1479

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$\downarrow$ 25

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\sqrt{x}+1\right)}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)\right)\right)\right)+\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2\sqrt{x}}{x-\sqrt{2}\sqrt{x}+1}d\sqrt{x}}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right.\right.\right.\right.$$
$$\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

$$\Big\downarrow \; 1103$$

$$\frac{13}{8}\left(\frac{9}{4}\left(2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}\sqrt{x}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}\sqrt{x}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(x-\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}-\frac{\log\left(x+\sqrt{2}\sqrt{x}+1\right)}{2\sqrt{2}}\right)\right.\right.\right.$$
$$\frac{1}{4x^{5/2}\left(x^2+1\right)^2}$$

input
```
Int[1/(x^(7/2)*(1 + x^2)^3),x]
```

output
```
1/(4*x^(5/2)*(1 + x^2)^2) + (13*(1/(2*x^(5/2)*(1 + x^2)) + (9*(-2/(5*x^(5/
2)) + 2/Sqrt[x] + 2*((-(ArcTan[1 - Sqrt[2]*Sqrt[x]]/Sqrt[2]) + ArcTan[1 +
Sqrt[2]*Sqrt[x]]/Sqrt[2])/2 + (Log[1 - Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]) -
Log[1 + Sqrt[2]*Sqrt[x] + x]/(2*Sqrt[2]))/2)))/4))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

method	result
risch	$\frac{585x^6+1053x^4+416x^2-32}{80(x^2+1)^2x^{\frac{5}{2}}}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
derivativedivides	$-\frac{2}{5x^{\frac{5}{2}}}+\frac{6}{\sqrt{x}}+\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
default	$-\frac{2}{5x^{\frac{5}{2}}}+\frac{6}{\sqrt{x}}+\frac{\frac{21x^{\frac{7}{2}}}{16}+\frac{25x^{\frac{3}{2}}}{16}}{(x^2+1)^2}+\frac{117\sqrt{2}\left(\ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)+2\arctan\left(1+\sqrt{2}\sqrt{x}\right)+2\arctan\left(-1+\sqrt{2}\sqrt{x}\right)\right)}{128}$
meijerg	$-\frac{-585x^6-1053x^4-416x^2+32}{80x^{\frac{5}{2}}(x^2+1)^2}+\frac{117x^{\frac{3}{2}}\left(\frac{\sqrt{2}\ln\left(1-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}+\sqrt{x^2}\right)}{2\left(x^2\right)^{\frac{3}{4}}}+\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}{2-\sqrt{2}\left(x^2\right)^{\frac{1}{4}}}\right)}{\left(x^2\right)^{\frac{3}{4}}}-\frac{\sqrt{2}\ln\left(1+\sqrt{2}\left(x^2\right.\right.}{2\left(x^2\right)^{\frac{3}{4}}}\right.}{64}$
trager	$\frac{585x^6+1053x^4+416x^2-32}{80(x^2+1)^2x^{\frac{5}{2}}}-\frac{117\,\mathrm{RootOf}\left(\_Z^4+1\right)^3\ln\left(\frac{\mathrm{RootOf}\left(\_Z^4+1\right)^5x-\mathrm{RootOf}\left(\_Z^4+1\right)^5-2\,\mathrm{RootOf}\left(\_Z\right.}{\mathrm{RootOf}\left(\_Z^4+1\right)^2x-\mathrm{Roo}}\right.}{64}$

input
```
int(1/x^(7/2)/(x^2+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/80*(585*x^6+1053*x^4+416*x^2-32)/(x^2+1)^2/x^(5/2)+117/128*2^(1/2)*(ln((
x-2^(1/2)*x^(1/2)+1)/(x+2^(1/2)*x^(1/2)+1))+2*arctan(1+2^(1/2)*x^(1/2))+2*
arctan(-1+2^(1/2)*x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^{7/2} (1+x^2)^3} \, dx = \frac{1170 \sqrt{2}(x^7 + 2x^5 + x^3) \arctan \left(\sqrt{2}\sqrt{x} + 1\right) + 1170 \sqrt{2}(x^7 + 2x^5 + x^3) \arctan \left(\sqrt{2}\right.}{}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="fricas")
```

output
```
1/640*(1170*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(x) + 1) + 1170
*sqrt(2)*(x^7 + 2*x^5 + x^3)*arctan(sqrt(2)*sqrt(x) - 1) - 585*sqrt(2)*(x^
7 + 2*x^5 + x^3)*log(sqrt(2)*sqrt(x) + x + 1) + 585*sqrt(2)*(x^7 + 2*x^5 +
 x^3)*log(-sqrt(2)*sqrt(x) + x + 1) + 8*(585*x^6 + 1053*x^4 + 416*x^2 - 32
)*sqrt(x))/(x^7 + 2*x^5 + x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(112) = 224.

Time = 4.79 (sec) , antiderivative size = 678, normalized size of antiderivative = 5.47

$$\int \frac{1}{x^{7/2} (1+x^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**(7/2)/(x**2+1)**3,x)
```

output
```
585*sqrt(2)*x**(13/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1
280*x**(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(13/2)*log(4*sqrt(2)*sqrt(x)
 + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*
x**(13/2)*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x
**(5/2)) + 1170*sqrt(2)*x**(13/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2)
 + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(9/2)*log(-4*sqrt(2)*sq
rt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 1170*sqr
t(2)*x**(9/2)*log(4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**(9
/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sqrt(2)*sqrt(x) - 1)/(640
*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 2340*sqrt(2)*x**(9/2)*atan(sq
rt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 585*sq
rt(2)*x**(5/2)*log(-4*sqrt(2)*sqrt(x) + 4*x + 4)/(640*x**(13/2) + 1280*x**
(9/2) + 640*x**(5/2)) - 585*sqrt(2)*x**(5/2)*log(4*sqrt(2)*sqrt(x) + 4*x +
 4)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 1170*sqrt(2)*x**(5/2)
*atan(sqrt(2)*sqrt(x) - 1)/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2))
+ 1170*sqrt(2)*x**(5/2)*atan(sqrt(2)*sqrt(x) + 1)/(640*x**(13/2) + 1280*x*
*(9/2) + 640*x**(5/2)) + 4680*x**6/(640*x**(13/2) + 1280*x**(9/2) + 640*x*
*(5/2)) + 8424*x**4/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) + 3328*
x**2/(640*x**(13/2) + 1280*x**(9/2) + 640*x**(5/2)) - 256/(640*x**(13/2) +
 1280*x**(9/2) + 640*x**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{117}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{585\,x^6+1053\,x^4+416\,x^2-32}{80\left(x^{\frac{13}{2}}+2\,x^{\frac{9}{2}}+x^{\frac{5}{2}}\right)}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="maxima")
```

output
```
117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*s
qrt(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/80*(58
5*x^6 + 1053*x^4 + 416*x^2 - 32)/(x^(13/2) + 2*x^(9/2) + x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{117}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{x}\right)\right) + \frac{117}{64}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{x}\right)\right) - \frac{117}{128}\sqrt{2}\log\left(\sqrt{2}\sqrt{x}+x+1\right) + \frac{117}{128}\sqrt{2}\log\left(-\sqrt{2}\sqrt{x}+x+1\right) + \frac{21\,x^{\frac{7}{2}}+25\,x^{\frac{3}{2}}}{16\,\left(x^2+1\right)^2} + \frac{2\,(15\,x^2-1)}{5\,x^{\frac{5}{2}}}$$

input
```
integrate(1/x^(7/2)/(x^2+1)^3,x, algorithm="giac")
```

output
```
117/64*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(x))) + 117/64*sqrt(2)*
arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(x))) - 117/128*sqrt(2)*log(sqrt(2)*s
qrt(x) + x + 1) + 117/128*sqrt(2)*log(-sqrt(2)*sqrt(x) + x + 1) + 1/16*(21
*x^(7/2) + 25*x^(3/2))/(x^2 + 1)^2 + 2/5*(15*x^2 - 1)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{\frac{117\,x^6}{16}+\frac{1053\,x^4}{80}+\frac{26\,x^2}{5}-\frac{2}{5}}{x^{5/2}+2\,x^{9/2}+x^{13/2}} + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}-\frac{117}{64}\mathrm{i}\right) + \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\sqrt{x}\,\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)\right)\,\left(\frac{117}{64}+\frac{117}{64}\mathrm{i}\right)$$

input
```
int(1/(x^(7/2)*(x^2 + 1)^3),x)
```

output
```
((26*x^2)/5 + (1053*x^4)/80 + (117*x^6)/16 - 2/5)/(x^(5/2) + 2*x^(9/2) + x
^(13/2)) + 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 - 1i/2))*(117/64 - 117i/64) +
 2^(1/2)*atan(2^(1/2)*x^(1/2)*(1/2 + 1i/2))*(117/64 + 117i/64)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.31

$$\int \frac{1}{x^{7/2}\left(1+x^2\right)^3}\,dx = \frac{1170\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^6+2340\sqrt{x}\,\sqrt{2}\,atan\left(\frac{2\sqrt{x}-\sqrt{2}}{\sqrt{2}}\right)x^4+1170\sqrt{x}\,\sqrt{2}\,atan\left(\right.}{}$$

input
```
int(1/x^(7/2)/(x^2+1)^3,x)
```

output
```
(1170*sqrt(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**6 + 2340*sqrt
(x)*sqrt(2)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**4 + 1170*sqrt(x)*sqrt(2
)*atan((2*sqrt(x) - sqrt(2))/sqrt(2))*x**2 + 1170*sqrt(x)*sqrt(2)*atan((2*
sqrt(x) + sqrt(2))/sqrt(2))*x**6 + 2340*sqrt(x)*sqrt(2)*atan((2*sqrt(x) +
sqrt(2))/sqrt(2))*x**4 + 1170*sqrt(x)*sqrt(2)*atan((2*sqrt(x) + sqrt(2))/s
qrt(2))*x**2 + 585*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**6 +
1170*sqrt(x)*sqrt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**4 + 585*sqrt(x)*sq
rt(2)*log( - sqrt(x)*sqrt(2) + x + 1)*x**2 - 585*sqrt(x)*sqrt(2)*log(sqrt(
x)*sqrt(2) + x + 1)*x**6 - 1170*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x +
1)*x**4 - 585*sqrt(x)*sqrt(2)*log(sqrt(x)*sqrt(2) + x + 1)*x**2 + 4680*x**
6 + 8424*x**4 + 3328*x**2 - 256)/(640*sqrt(x)*x**2*(x**4 + 2*x**2 + 1))
```

3.336 $\int \frac{\sqrt{x}}{a-bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 58

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx=-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

output
```
-arctan(b^(1/4)*x^(1/2)/a^(1/4))/a^(1/4)/b^(3/4)+arctanh(b^(1/4)*x^(1/2)/a
^(1/4))/a^(1/4)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx=\frac{-\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)+\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}b^{3/4}}$$

input
```
Integrate[Sqrt[x]/(a - b*x^2),x]
```

output
```
(-ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sqrt[x])/a^(1/4)])/
(a^(1/4)*b^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {266, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{a-bx^2}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\int \frac{x}{a-bx^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 827}$$

$$2\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{b}x}d\sqrt{x}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}x+\sqrt{a}}d\sqrt{x}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{ 218}$$

$$2\left(\frac{\int \frac{1}{\sqrt{a}-\sqrt{b}x}d\sqrt{x}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}\right)$$

$$\Big\downarrow \text{ 221}$$

$$2\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}b^{3/4}}\right)$$

input
```
Int[Sqrt[x]/(a - b*x^2),x]
```

output
```
2*(-1/2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)]/(a^(1/4)*b^(3/4)) + ArcTanh[(b^(
1/4)*Sqrt[x])/a^(1/4)]/(2*a^(1/4)*b^(3/4)))
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	58
default	$-\frac{2\arctan\left(\frac{\sqrt{x}}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{x}+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sqrt{x}-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2b\left(\frac{a}{b}\right)^{\frac{1}{4}}}$	58

input
```
int(x^(1/2)/(-b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(a/b)^(1/4)*(2*arctan(x^(1/2)/(a/b)^(1/4))-ln((x^(1/2)+(a/b)^(1/4))
/(x^(1/2)-(a/b)^(1/4))))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}i\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(i\,ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) + \frac{1}{2}i\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-i\,ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right) - \frac{1}{2}\left(\frac{1}{ab^3}\right)^{\frac{1}{4}}\log\left(-ab^2\left(\frac{1}{ab^3}\right)^{\frac{3}{4}}+\sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(-b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*(1/(a*b^3))^(1/4)*log(a*b^2*(1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*I*(1/(a
*b^3))^(1/4)*log(I*a*b^2*(1/(a*b^3))^(3/4) + sqrt(x)) + 1/2*I*(1/(a*b^3))^
(1/4)*log(-I*a*b^2*(1/(a*b^3))^(3/4) + sqrt(x)) - 1/2*(1/(a*b^3))^(1/4)*lo
g(-a*b^2*(1/(a*b^3))^(3/4) + sqrt(x))
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = \begin{cases} \frac{\tilde{\infty}}{\sqrt{x}} & \text{for } a=0 \wedge b=0 \\ \frac{2x^{\frac{3}{2}}}{3a} & \text{for } b=0 \\ \frac{2}{b\sqrt{x}} & \text{for } a=0 \\ -\frac{\log\left(\sqrt{x}-\sqrt[4]{\frac{a}{b}}\right)}{2b\sqrt[4]{\frac{a}{b}}} + \frac{\log\left(\sqrt{x}+\sqrt[4]{\frac{a}{b}}\right)}{2b\sqrt[4]{\frac{a}{b}}} - \frac{\operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt[4]{\frac{a}{b}}}\right)}{b\sqrt[4]{\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(-b*x**2+a),x)
```

output
```
Piecewise((zoo/sqrt(x), Eq(a, 0) & Eq(b, 0)), (2*x**(3/2)/(3*a), Eq(b, 0))
, (2/(b*sqrt(x)), Eq(a, 0)), (-log(sqrt(x) - (a/b)**(1/4))/(2*b*(a/b)**(1/
4)) + log(sqrt(x) + (a/b)**(1/4))/(2*b*(a/b)**(1/4)) - atan(sqrt(x)/(a/b)*
*(1/4))/(b*(a/b)**(1/4)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}} - \frac{\log\left(\frac{\sqrt{b}\sqrt{x}-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sqrt{x}+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}$$

input
```
integrate(x^(1/2)/(-b*x^2+a),x, algorithm="maxima")
```

output
```
-arctan(sqrt(b)*sqrt(x)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(sqrt(a)*sqrt(b))*sqrt
(b)) - 1/2*log((sqrt(b)*sqrt(x) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sqrt(x)
+ sqrt(sqrt(a)*sqrt(b))))/(sqrt(sqrt(a)*sqrt(b))*sqrt(b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(38) = 76$.

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.34

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} + \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sqrt{x}\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2\,ab^3} - \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}}\log\left(\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^3} + \frac{\sqrt{2}(-ab^3)^{\frac{3}{4}}\log\left(-\sqrt{2}\sqrt{x}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+x+\sqrt{-\frac{a}{b}}\right)}{4\,ab^3}$$

input
```
integrate(x^(1/2)/(-b*x^2+a),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sq
rt(x))/(-a/b)^(1/4))/(a*b^3) + 1/2*sqrt(2)*(-a*b^3)^(3/4)*arctan(-1/2*sqrt
(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sqrt(x))/(-a/b)^(1/4))/(a*b^3) - 1/4*sqrt(2)
*(-a*b^3)^(3/4)*log(sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt(-a/b))/(a*b^3)
 + 1/4*sqrt(2)*(-a*b^3)^(3/4)*log(-sqrt(2)*sqrt(x)*(-a/b)^(1/4) + x + sqrt
(-a/b))/(a*b^3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = -\frac{\mathrm{atan}\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)-\mathrm{atanh}\left(\frac{b^{1/4}\sqrt{x}}{a^{1/4}}\right)}{a^{1/4}\,b^{3/4}}$$

input
```
int(x^(1/2)/(a - b*x^2),x)
```

output
```
-(atan((b^(1/4)*x^(1/2))/a^(1/4)) - atanh((b^(1/4)*x^(1/2))/a^(1/4)))/(a^(
1/4)*b^(3/4))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}}{a-bx^2}\,dx = \frac{-2atan\left(\frac{\sqrt{x}\,\sqrt{b}}{b^{\frac{1}{4}}a^{\frac{1}{4}}}\right)+\log\left(a^{\frac{1}{4}}+\sqrt{x}\,b^{\frac{1}{4}}\right)-\log\left(-a^{\frac{1}{4}}+\sqrt{x}\,b^{\frac{1}{4}}\right)}{2b^{\frac{3}{4}}a^{\frac{1}{4}}}$$

input
```
int(x^(1/2)/(-b*x^2+a),x)
```

output
```
(b**(1/4)*a**(3/4)*( - 2*atan((sqrt(x)*sqrt(b))/(b**(1/4)*a**(1/4))) + log
(a**(1/4) + sqrt(x)*b**(1/4)) - log( - a**(1/4) + sqrt(x)*b**(1/4))))/(2*a
*b)
```

3.337 $\int \frac{\sqrt{x}}{1-x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\sqrt{x}}{1-x^2} \, dx = -\arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

output
```
-arctan(x^(1/2))+arctanh(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{1-x^2} \, dx = -\arctan\left(\sqrt{x}\right) + \operatorname{arctanh}\left(\sqrt{x}\right)$$

input
```
Integrate[Sqrt[x]/(1 - x^2),x]
```

output
```
-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{1-x^2}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\int \frac{x}{1-x^2}d\sqrt{x}$$

$$\Big\downarrow \text{ 827}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{1}{2}\int \frac{1}{x+1}d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 216}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-x}d\sqrt{x}-\frac{\arctan\left(\sqrt{x}\right)}{2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\text{arctanh}\left(\sqrt{x}\right)}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}\right)$$

input
```
Int[Sqrt[x]/(1 - x^2),x]
```

output
```
2*(-1/2*ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]/2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

method	result	size
derivativedivides	$-\frac{\ln\left(-1+\sqrt{x}\right)}{2}+\frac{\ln\left(1+\sqrt{x}\right)}{2}-\arctan\left(\sqrt{x}\right)$	24
default	$-\frac{\ln\left(-1+\sqrt{x}\right)}{2}+\frac{\ln\left(1+\sqrt{x}\right)}{2}-\arctan\left(\sqrt{x}\right)$	24
meijerg	$-\frac{x^{\frac{3}{2}}\left(\ln\left(1-\left(x^{2}\right)^{\frac{1}{4}}\right)-\ln\left(1+\left(x^{2}\right)^{\frac{1}{4}}\right)+2\arctan\left(\left(x^{2}\right)^{\frac{1}{4}}\right)\right)}{2\left(x^{2}\right)^{\frac{3}{4}}}$	40
trager	$-\frac{\ln\left(\frac{-1-x+2\sqrt{x}}{-1+x}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x}-\operatorname{RootOf}\left(\_Z^{2}+1\right)}{1+x}\right)}{2}$	58

input
```
int(x^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-1+x^(1/2))+1/2*ln(1+x^(1/2))-arctan(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = -\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
-arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = -\frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2} - \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
integrate(x**(1/2)/(-x**2+1),x)
```

output
```
-log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 - atan(sqrt(x))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(11) = 22$.

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = -\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\sqrt{x}-1\right)$$

input
```
integrate(x^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
-arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = -\arctan\left(\sqrt{x}\right) + \frac{1}{2}\log\left(\sqrt{x}+1\right) - \frac{1}{2}\log\left(\left|\sqrt{x}-1\right|\right)$$

input
```
integrate(x^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
-arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = \operatorname{atanh}\left(\sqrt{x}\right) - \operatorname{atan}\left(\sqrt{x}\right)$$

input
```
int(-x^(1/2)/(x^2 - 1),x)
```

output
```
atanh(x^(1/2)) - atan(x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{x}}{1-x^2}\,dx = -atan\left(\sqrt{x}\right) - \frac{\log\left(\sqrt{x}-1\right)}{2} + \frac{\log\left(\sqrt{x}+1\right)}{2}$$

input
```
int(x^(1/2)/(-x^2+1),x)
```

output
```
( - 2*atan(sqrt(x)) - log(sqrt(x) - 1) + log(sqrt(x) + 1))/2
```

3.338 $\int \frac{(cx)^{4/3}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 235

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = \frac{3c\sqrt[3]{cx}}{b} - \frac{\sqrt[6]{a}c^{4/3}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{b^{7/6}} + \frac{\sqrt[6]{a}c^{4/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2b^{7/6}} - \frac{\sqrt[6]{a}c^{4/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2b^{7/6}} - \frac{\sqrt{3}\sqrt[6]{a}c^{4/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2b^{7/6}}$$

output
```
3*c*(c*x)^(1/3)/b-a^(1/6)*c^(4/3)*arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/
3))/b^(7/6)-1/2*a^(1/6)*c^(4/3)*arctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1
/6)/c^(1/3))/b^(7/6)-1/2*a^(1/6)*c^(4/3)*arctan(3^(1/2)+2*b^(1/6)*(c*x)^(1
/3)/a^(1/6)/c^(1/3))/b^(7/6)-1/2*3^(1/2)*a^(1/6)*c^(4/3)*arctanh(3^(1/2)*a
^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3)/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3)))/
b^(7/6)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = \frac{(cx)^{4/3}\left(6\sqrt[6]{b}\sqrt[3]{x}+\sqrt[6]{a}\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-2\sqrt[6]{a}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-\sqrt{3}\sqrt[6]{a}\text{arc}}{2b^{7/6}x^{4/3}}$$

input
```
Integrate[(c*x)^(4/3)/(a + b*x^2),x]
```

output
```
((c*x)^(4/3)*(6*b^(1/6)*x^(1/3) + a^(1/6)*ArcTan[a^(1/6)/(b^(1/6)*x^(1/3))
 - (b^(1/6)*x^(1/3))/a^(1/6)] - 2*a^(1/6)*ArcTan[(b^(1/6)*x^(1/3))/a^(1/6)
] - Sqrt[3]*a^(1/6)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3))/(a^(1/3) + b
^(1/3)*x^(2/3))]))/(2*b^(7/6)*x^(4/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {262, 266, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{ac^2\int \frac{1}{(cx)^{2/3}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\int \frac{1}{bx^2+a}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \text{753}$$

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{c^{2/3}\int\frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3a^{5/6}}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\right.}\right.}{b}$$

$\downarrow$ 27

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{c^{2/3}\int\frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}}}{6a^{5/6}}\right.}{b}$$

$\downarrow$ 218

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int\frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\arctan}{3}\right.}{b}$$

$\downarrow$ 1142

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int\frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}\right.}{b}$$

$\downarrow$ 25

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\cdots\right)}{\cdots}$$

$\downarrow$ 27

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}\right)}{6a^{5/6}}+\cdots\right)}{\cdots}$$

$\downarrow$ 1082

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\int \frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{\cdots}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{\cdots}}\cdots\right)}{\cdots}\right)}{b}$$

$\downarrow$ 217

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\cdots}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\cdots}\right.}{\cdots}\right.}{b}$$

$\Big\downarrow$ 1103

$$\frac{3c\sqrt[3]{cx}}{b}-\frac{3ac\left(\frac{\sqrt[3]{c}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}+1\right)\right)}{\sqrt[6]{b}}\cdots\right.}{\cdots}\right.}{b}$$

input
```
Int[(c*x)^(4/3)/(a + b*x^2),x]
```

output
```
(3*c*(c*x)^(1/3))/b - (3*a*c*((c^(1/3)*ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1/
6)*c^(1/3))])/(3*a^(5/6)*b^(1/6)) + (c^(1/3)*(-(ArcTan[Sqrt[3]*(1 - (2*b^(
1/6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1
/3)*c^(2/3) - Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^
(2/3)])/(2*b^(1/6))))/(6*a^(5/6)) + (c^(1/3)*(ArcTan[Sqrt[3]*(1 + (2*b^(1/
6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3)
*c^(2/3) + Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/
3)])/(2*b^(1/6))))/(6*a^(5/6))))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{c\left(12(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\left(\left(\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)-\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)\right)\sqrt{}\right.}{4b}$
derivativedivides	$3c\left(\frac{(cx)^{\frac{1}{3}}}{b}-\frac{\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}{}\right.}{}\right.$
default	$3c\left(\frac{(cx)^{\frac{1}{3}}}{b}-\frac{\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}{}\right.}{}\right.$

input
```
int((c*x)^(4/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*(12*(c*x)^(1/3)+(a*c^2/b)^(1/6)*((ln((c*x)^(2/3)-3^(1/2)*(a*c^2/b)^(
1/6)*(c*x)^(1/3)+(a*c^2/b)^(1/3))-ln((c*x)^(2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(
c*x)^(1/3)+(a*c^2/b)^(1/3)))*3^(1/2)-2*arctan(2*(c*x)^(1/3)/(a*c^2/b)^(1/6
)+3^(1/2))-2*arctan(2*(c*x)^(1/3)/(a*c^2/b)^(1/6)-3^(1/2))-4*arctan((c*x)^
(1/3)/(a*c^2/b)^(1/6))))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.23

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = -\frac{\left(-\frac{ac^8}{b^7}\right)^{\frac{1}{6}}\left(\sqrt{-3}b+b\right)\log\left((cx)^{\frac{1}{3}}c+\frac{1}{2}\left(-\frac{ac^8}{b^7}\right)^{\frac{1}{6}}\left(\sqrt{-3}b+b\right)\right)-\left(-\frac{ac^8}{b^7}\right)^{\frac{1}{6}}\left(\sqrt{-3}b+b\right)\log\left((cx)^{\frac{1}{3}}c-\frac{1}{2}\right.}{}$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*((-a*c^8/b^7)^(1/6)*(sqrt(-3)*b + b)*log((c*x)^(1/3)*c + 1/2*(-a*c^8/
b^7)^(1/6)*(sqrt(-3)*b + b)) - (-a*c^8/b^7)^(1/6)*(sqrt(-3)*b + b)*log((c*
x)^(1/3)*c - 1/2*(-a*c^8/b^7)^(1/6)*(sqrt(-3)*b + b)) + (-a*c^8/b^7)^(1/6)
*(sqrt(-3)*b - b)*log((c*x)^(1/3)*c + 1/2*(-a*c^8/b^7)^(1/6)*(sqrt(-3)*b -
 b)) - (-a*c^8/b^7)^(1/6)*(sqrt(-3)*b - b)*log((c*x)^(1/3)*c - 1/2*(-a*c^8
/b^7)^(1/6)*(sqrt(-3)*b - b)) + 2*(-a*c^8/b^7)^(1/6)*b*log((c*x)^(1/3)*c +
 (-a*c^8/b^7)^(1/6)*b) - 2*(-a*c^8/b^7)^(1/6)*b*log((c*x)^(1/3)*c - (-a*c^
8/b^7)^(1/6)*b) - 12*(c*x)^(1/3)*c)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.66

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = -\frac{7\sqrt[6]{a}c^{\frac{4}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} - \frac{7i\sqrt[6]{a}c^{\frac{4}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} - \frac{7\sqrt[6]{a}c^{\frac{4}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} + \frac{7\sqrt[6]{a}c^{\frac{4}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} + \frac{7i\sqrt[6]{a}c^{\frac{4}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} + \frac{7\sqrt[6]{a}c^{\frac{4}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{7}{6}\right)}{12b^{\frac{7}{6}}\Gamma\left(\frac{13}{6}\right)} + \frac{7c^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{7}{6}\right)}{2b\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(4/3)/(b*x**2+a),x)
```

output
```
-7*a**(1/6)*c**(4/3)*exp(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*p
i/6)/a**(1/6))*gamma(7/6)/(12*b**(7/6)*gamma(13/6)) - 7*I*a**(1/6)*c**(4/3
)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/2)/a**(1/6))*gamma(7/6)/(12*b**
(7/6)*gamma(13/6)) - 7*a**(1/6)*c**(4/3)*exp(I*pi/6)*log(1 - b**(1/6)*x**(
1/3)*exp_polar(5*I*pi/6)/a**(1/6))*gamma(7/6)/(12*b**(7/6)*gamma(13/6)) +
7*a**(1/6)*c**(4/3)*exp(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(7*I*
pi/6)/a**(1/6))*gamma(7/6)/(12*b**(7/6)*gamma(13/6)) + 7*I*a**(1/6)*c**(4/
3)*log(1 - b**(1/6)*x**(1/3)*exp_polar(3*I*pi/2)/a**(1/6))*gamma(7/6)/(12*
b**(7/6)*gamma(13/6)) + 7*a**(1/6)*c**(4/3)*exp(I*pi/6)*log(1 - b**(1/6)*x
**(1/3)*exp_polar(11*I*pi/6)/a**(1/6))*gamma(7/6)/(12*b**(7/6)*gamma(13/6)
) + 7*c**(4/3)*x**(1/3)*gamma(7/6)/(2*b*gamma(13/6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(157) = 314$.

Time = 0.15 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.37

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = \frac{\frac{12\,(cx)^{\frac{1}{3}}c^2}{b} - \frac{\left(\frac{\sqrt{3}c^4\log\left(\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}c^4\log\left(-\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{5}{6}}b^{\frac{1}{6}}} + \frac{4c^4}{\cdots}}{\cdots}}{4\cdots}$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(12*(c*x)^(1/3)*c^2/b - (sqrt(3)*c^4*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(
1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(5/6)*b^(1/6)
) - sqrt(3)*c^4*log(-sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/
3)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(5/6)*b^(1/6)) + 4*c^4*arctan((c*x)^(
1/3)*b^(1/3)/sqrt((a*c^2)^(1/3)*b^(1/3)))/((a*c^2)^(2/3)*sqrt((a*c^2)^(1/3
)*b^(1/3))) + 2*(a*c^2)^(1/3)*c^2*arctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) +
2*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(a*sqrt((a*c^2)^(1/3)*
b^(1/3))) + 2*(a*c^2)^(1/3)*c^2*arctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2
*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(a*sqrt((a*c^2)^(1/3)*b
^(1/3))))*a/b)/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx =$$

$$-\frac{\frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}c^2\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{b^2}-\frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}c^2\log\left(-\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{b^2}-\frac{12\,(cx)^{\frac{1}{3}}c^2}{b}+\cdots}{4\,c}$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(3)*(a*b^5*c^2)^(1/6)*c^2*log(sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3
) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/b^2 - sqrt(3)*(a*b^5*c^2)^(1/6)*c^2*log
(-sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/b^2
 - 12*(c*x)^(1/3)*c^2/b + 2*(a*b^5*c^2)^(1/6)*c^2*arctan((sqrt(3)*(a*c^2/b
)^(1/6) + 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/b^2 + 2*(a*b^5*c^2)^(1/6)*c^2*ar
ctan(-(sqrt(3)*(a*c^2/b)^(1/6) - 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/b^2 + 4*(
a*b^5*c^2)^(1/6)*c^2*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))/b^2)/c
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.17

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = \frac{3\,c\,(c\,x)^{1/3}}{b} + \frac{(-a)^{1/6}\,c^{4/3}\,\mathrm{atan}\left(\frac{b^{1/6}\,(c\,x)^{1/3}\,1\mathrm{i}}{(-a)^{1/6}\,c^{1/3}}\right)\,1\mathrm{i}}{b^{7/6}}$$
$$-\frac{(-a)^{1/6}\,c^{4/3}\,\ln\left(486\,(-a)^{25/6}\,b^{5/6}\,c^{37/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+486\,a^4\,b\,c^{12}\,(c\,x)^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{7/6}}$$
$$-\frac{(-a)^{1/6}\,c^{4/3}\,\ln\left(486\,(-a)^{25/6}\,b^{5/6}\,c^{37/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+486\,a^4\,b\,c^{12}\,(c\,x)^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{7/6}}$$
$$+\frac{(-a)^{1/6}\,c^{4/3}\,\ln\left(972\,(-a)^{25/6}\,b^{5/6}\,c^{37/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-486\,a^4\,b\,c^{12}\,(c\,x)^{1/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{7/6}}$$
$$+\frac{(-a)^{1/6}\,c^{4/3}\,\ln\left(972\,(-a)^{25/6}\,b^{5/6}\,c^{37/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-486\,a^4\,b\,c^{12}\,(c\,x)^{1/3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{7/6}}$$

input
```
int((c*x)^(4/3)/(a + b*x^2),x)
```

output
```
(3*c*(c*x)^(1/3))/b + ((-a)^(1/6)*c^(4/3)*atan((b^(1/6)*(c*x)^(1/3)*1i)/((
-a)^(1/6)*c^(1/3)))*1i)/b^(7/6) - ((-a)^(1/6)*c^(4/3)*log(486*(-a)^(25/6)*
b^(5/6)*c^(37/3)*((3^(1/2)*1i)/2 - 1/2) + 486*a^4*b*c^12*(c*x)^(1/3))*((3^
(1/2)*1i)/2 - 1/2))/(2*b^(7/6)) - ((-a)^(1/6)*c^(4/3)*log(486*(-a)^(25/6)*
b^(5/6)*c^(37/3)*((3^(1/2)*1i)/2 + 1/2) + 486*a^4*b*c^12*(c*x)^(1/3))*((3^
(1/2)*1i)/2 + 1/2))/(2*b^(7/6)) + ((-a)^(1/6)*c^(4/3)*log(972*(-a)^(25/6)*
b^(5/6)*c^(37/3)*((3^(1/2)*1i)/4 - 1/4) - 486*a^4*b*c^12*(c*x)^(1/3))*((3^
(1/2)*1i)/4 - 1/4))/b^(7/6) + ((-a)^(1/6)*c^(4/3)*log(972*(-a)^(25/6)*b^(5
/6)*c^(37/3)*((3^(1/2)*1i)/4 + 1/4) - 486*a^4*b*c^12*(c*x)^(1/3))*((3^(1/2
)*1i)/4 + 1/4))/b^(7/6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.73

$$\int \frac{(cx)^{4/3}}{a+bx^2}\,dx = \frac{c^{\frac{4}{3}}\left(2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-4b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{x^{\frac{1}{3}}b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)+b^{\frac{1}{6}}a^{\frac{1}{6}}\cdots}{4b\cdots}$$

input
```
int((c*x)^(4/3)/(b*x^2+a),x)
```

output
```
(c**(1/3)*c*(2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1
/3)*b**(1/3))/(b**(1/6)*a**(1/6))) - 2*b**(1/6)*a**(1/6)*atan((b**(1/6)*a*
*(1/6)*sqrt(3) + 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) - 4*b**(1/6)*a*
*(1/6)*atan((x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) + b**(1/6)*a**(1/6)*s
qrt(3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**
(1/3)) - b**(1/6)*a**(1/6)*sqrt(3)*log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3)
+ a**(1/3) + x**(2/3)*b**(1/3)) + 12*x**(1/3)*b**(1/3)))/(4*b**(1/3)*b)
```

3.339 $\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = -\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a}b^{2/3}} - \frac{\sqrt[3]{c}\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[3]{a}b^{2/3}} + \frac{\sqrt[3]{c}\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{4/3}\right)}{4\sqrt[3]{a}b^{2/3}}$$

output
```
-1/2*3^(1/2)*c^(1/3)*arctan(1/3*(1-2*b^(1/3)*(c*x)^(2/3)/a^(1/3)/c^(2/3))*
3^(1/2))/a^(1/3)/b^(2/3)-1/2*c^(1/3)*ln(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3
))/a^(1/3)/b^(2/3)+1/4*c^(1/3)*ln(a^(2/3)*c^(4/3)-a^(1/3)*b^(1/3)*c^(2/3)*
(c*x)^(2/3)+b^(2/3)*(c*x)^(4/3))/a^(1/3)/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx$$
$$= \frac{\sqrt[3]{cx}\left(-2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{2/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{2/3}\right)+\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{x}+\sqrt[3]{b}x^{2/3}\right)+\log\right.}{4\sqrt[3]{a}b^{2/3}\sqrt[3]{x}}$$

input
```
Integrate[(c*x)^(1/3)/(a + b*x^2),x]
```

output
```
((c*x)^(1/3)*(-2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(2/3))/a^(1/3))/Sqrt[3]]
 - 2*Log[a^(1/3) + b^(1/3)*x^(2/3)] + Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6
)*x^(1/3) + b^(1/3)*x^(2/3)] + Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/
3) + b^(1/3)*x^(2/3)]))/(4*a^(1/3)*b^(2/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {266, 27, 807, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx$$
$$\Big\downarrow \; 266$$
$$\frac{3\int \frac{c^3x}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{c}$$
$$\Big\downarrow \; 27$$
$$3c\int \frac{cx}{bx^2c^2+ac^2}d\sqrt[3]{cx}$$

$$\Big\downarrow \text{807}$$

$$\frac{3}{2}c\int \frac{(cx)^{2/3}}{ac^2+bxc}d(cx)^{2/3}$$

$$\Big\downarrow \text{821}$$

$$\frac{3}{2}c\left(\frac{\int \frac{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{3}{2}c\left(\frac{\int \frac{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log}{}\right.$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log(}{}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\right.$$

$\Big\downarrow$ 1082

$$\frac{3}{2}c\left(\frac{\frac{3\int \frac{1}{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}-4}d\left(1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)$$

$\Big\downarrow$ 217

$$\frac{3}{2}c\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)$$

$\Big\downarrow$ 1103

$$\frac{3}{2}c\left(\frac{\frac{\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{2/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)$$

input
```
Int[(c*x)^(1/3)/(a + b*x^2),x]
```

output
```
(3*c*(-1/3*Log[a^(1/3)*c^(2/3) + b^(1/3)*(c*x)^(2/3)]/(a^(1/3)*b^(2/3)*c^(
2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x)^(2/3))/(a^(1/3)*c^(2/3)))
/Sqrt[3]])/b^(1/3)) + Log[a^(2/3)*c^(4/3) + b^(2/3)*(c*x)^(2/3) - a^(1/3)*
b^(1/3)*c^(2/3)*(c*x)^(2/3)]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3)*c^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{c\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right)-2\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)+\ln\left(cx(cx)^{\frac{1}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)\right)}{4b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}$
derivativedivides	$3c\left(-\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{12b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right)$
default	$3c\left(-\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{12b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right)$

input
```
int((c*x)^(1/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*(2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(c*x)^(2/3)-(a*c^2/b)^(1/3))/(a*c^2
/b)^(1/3))-2*ln((c*x)^(2/3)+(a*c^2/b)^(1/3))+ln(c*x*(c*x)^(1/3)-(a*c^2/b)^
(1/3)*(c*x)^(2/3)+(a*c^2/b)^(2/3)))/b/(a*c^2/b)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = \frac{1}{2}\sqrt{3}\left(-\frac{c}{ab^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(cx)^{\frac{2}{3}}b\left(-\frac{c}{ab^2}\right)^{\frac{1}{3}}+\sqrt{3}c}{3c}\right) - \frac{1}{4}\left(-\frac{c}{ab^2}\right)^{\frac{1}{3}}\log\left(-(cx)^{\frac{2}{3}}ab\left(-\frac{c}{ab^2}\right)^{\frac{2}{3}}+(cx)^{\frac{1}{3}}cx-ac\left(-\frac{c}{ab^2}\right)^{\frac{1}{3}}\right) + \frac{1}{2}\left(-\frac{c}{ab^2}\right)^{\frac{1}{3}}\log\left(ab\left(-\frac{c}{ab^2}\right)^{\frac{2}{3}}+(cx)^{\frac{2}{3}}\right)$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*(-c/(a*b^2))^(1/3)*arctan(1/3*(2*sqrt(3)*(c*x)^(2/3)*b*(-c/(a*
b^2))^(1/3) + sqrt(3)*c)/c) - 1/4*(-c/(a*b^2))^(1/3)*log(-(c*x)^(2/3)*a*b*
(-c/(a*b^2))^(2/3) + (c*x)^(1/3)*c*x - a*c*(-c/(a*b^2))^(1/3)) + 1/2*(-c/(
a*b^2))^(1/3)*log(a*b*(-c/(a*b^2))^(2/3) + (c*x)^(2/3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = -\frac{\sqrt[3]{c}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt[3]{c}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)} - \frac{\sqrt[3]{c}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{2}{3}\right)}{3\sqrt[3]{a}b^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((c*x)**(1/3)/(b*x**2+a),x)
```

output
```
-c**(1/3)*exp(-2*I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(I*pi/3)/a**(1
/3))*gamma(2/3)/(3*a**(1/3)*b**(2/3)*gamma(5/3)) - c**(1/3)*log(1 - b**(1/
3)*x**(2/3)*exp_polar(I*pi)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*b**(2/3)*gamm
a(5/3)) - c**(1/3)*exp(2*I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(5*I*p
i/3)/a**(1/3))*gamma(2/3)/(3*a**(1/3)*b**(2/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = \frac{\frac{2\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(cx)^{\frac{2}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}b}+\frac{c^2\log\left((cx)^{\frac{4}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}b}-\frac{2\,c^2\log\left((cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}b}}{4\,c}$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) - (a*c^2/b)^(1/3))/(a
*c^2/b)^(1/3))/((a*c^2/b)^(1/3)*b) + c^2*log((c*x)^(4/3) - (a*c^2/b)^(1/3)
*(c*x)^(2/3) + (a*c^2/b)^(2/3))/((a*c^2/b)^(1/3)*b) - 2*c^2*log((c*x)^(2/3
) + (a*c^2/b)^(1/3))/((a*c^2/b)^(1/3)*b))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = -\frac{\frac{2\left(-\frac{ac^2}{b}\right)^{\frac{2}{3}}\log\left(\left|(cx)^{\frac{2}{3}}-\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right|\right)}{a}+\frac{2\sqrt{3}(-ab^2c^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(cx)^{\frac{2}{3}}+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{ab^2}-\frac{(-ab^2c^2)^{\frac{2}{3}}\log\left((cx)^{\frac{1}{3}}cx+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}(\right.}{ab^2}}{4\,c}$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(2*(-a*c^2/b)^(2/3)*log(abs((c*x)^(2/3) - (-a*c^2/b)^(1/3)))/a + 2*sq
rt(3)*(-a*b^2*c^2)^(2/3)*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) + (-a*c^2/b)^(1
/3))/(-a*c^2/b)^(1/3))/(a*b^2) - (-a*b^2*c^2)^(2/3)*log((c*x)^(1/3)*c*x +
(-a*c^2/b)^(1/3)*(c*x)^(2/3) + (-a*c^2/b)^(2/3))/(a*b^2))/c
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = \frac{(-c)^{1/3}\ln\left(81\,a\,b^2\,c^6+81\,a^{2/3}\,b^{7/3}\,(-c)^{16/3}\,(c\,x)^{2/3}\right)}{2\,a^{1/3}\,b^{2/3}} - \frac{(-c)^{1/3}\ln\left(81\,a\,b^2\,c^6-81\,a^{2/3}\,b^{7/3}\,(-c)^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(c\,x)^{2/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{1/3}\,b^{2/3}} + \frac{(-c)^{1/3}\ln\left(81\,a\,b^2\,c^6+162\,a^{2/3}\,b^{7/3}\,(-c)^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(c\,x)^{2/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{a^{1/3}\,b^{2/3}}$$

input
```
int((c*x)^(1/3)/(a + b*x^2),x)
```

output
```
((-c)^(1/3)*log(81*a*b^2*c^6 + 81*a^(2/3)*b^(7/3)*(-c)^(16/3)*(c*x)^(2/3))
)/(2*a^(1/3)*b^(2/3)) - ((-c)^(1/3)*log(81*a*b^2*c^6 - 81*a^(2/3)*b^(7/3)*
(-c)^(16/3)*((3^(1/2)*1i)/2 + 1/2)*(c*x)^(2/3))*((3^(1/2)*1i)/2 + 1/2))/(2
*a^(1/3)*b^(2/3)) + ((-c)^(1/3)*log(81*a*b^2*c^6 + 162*a^(2/3)*b^(7/3)*(-c
)^(16/3)*((3^(1/2)*1i)/4 - 1/4)*(c*x)^(2/3))*((3^(1/2)*1i)/4 - 1/4))/(a^(1
/3)*b^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[3]{cx}}{a+bx^2}\,dx = \frac{c^{\frac{1}{3}}\left(-2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2\log\left(a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{1}{3}}\right)+\log\left(-x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+\right.\right.}{4b^{\frac{2}{3}}a^{\frac{1}{3}}}$$

input
```
int((c*x)^(1/3)/(b*x^2+a),x)
```

output
```
(c**(1/3)*( - 2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1/3)*b**(1
/3))/(b**(1/6)*a**(1/6))) - 2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*
x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) - 2*log(a**(1/3) + x**(2/3)*b**(1/
3)) + log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(
1/3)) + log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1
/3))))/(4*b**(2/3)*a**(1/3))
```

3.340 $\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 221

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{a^{5/6}\sqrt[6]{b}c^{2/3}} - \frac{\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{5/6}\sqrt[6]{b}c^{2/3}} + \frac{\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{5/6}\sqrt[6]{b}c^{2/3}} + \frac{\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2a^{5/6}\sqrt[6]{b}c^{2/3}}$$

output
```
arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(5/6)/b^(1/6)/c^(2/3)+1/2*ar
ctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(5/6)/b^(1/6)/c^(2/
3)+1/2*arctan(3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(5/6)/b^(1/
6)/c^(2/3)+1/2*3^(1/2)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3)
/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3)))/a^(5/6)/b^(1/6)/c^(2/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{x^{2/3}\left(-\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)+2\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)+\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{a}+}\right.\right.}{2a^{5/6}\sqrt[6]{b}(cx)^{2/3}}$$

input
```
Integrate[1/((c*x)^(2/3)*(a + b*x^2)),x]
```

output
```
(x^(2/3)*(-ArcTan[a^(1/6)/(b^(1/6)*x^(1/3)) - (b^(1/6)*x^(1/3))/a^(1/6)] +
 2*ArcTan[(b^(1/6)*x^(1/3))/a^(1/6)] + Sqrt[3]*ArcTanh[(Sqrt[3]*a^(1/6)*b^
(1/6)*x^(1/3))/(a^(1/3) + b^(1/3)*x^(2/3))]))/(2*a^(5/6)*b^(1/6)*(c*x)^(2/
3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.38, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {266, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\int \frac{1}{bx^2+a}d\sqrt[3]{cx}}{c}$$

$$\Big\downarrow \text{ 753}$$

$$\frac{3\left(\frac{c^{2/3}\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{}}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\right.}}{3a^{5}}\right.}{c}$$

$\big\downarrow$ 27

$$3\left(\frac{\frac{c^{2/3}\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}}}{6a^{5/6}}}{c}\right.$$

$\big\downarrow$ 218

$$3\left(\frac{\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\arctan\Big(}{3a^{5/}}}{c}\right.$$

$\big\downarrow$ 1142

$$3\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\right.$$

$\big\downarrow$ 25

$$3\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\right.$$

$\big\downarrow$ 27

$$3\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int\frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}\right)}{6a^{5/6}}+\right.$$

$\downarrow$ 1082

$$3\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\int\frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}}\right.}{}\right.$$

$$c$$

$\downarrow$ 217

$$3\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}\right.}{}\right.$$

$$c$$

$\downarrow$ 1103

$$3\left(\frac{\sqrt[3]{c}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}+1\right)\right)}{\sqrt[6]{b}}\right.}{}\right.$$

$$c$$

input
```
Int[1/((c*x)^(2/3)*(a + b*x^2)),x]
```

output
```
(3*((c^(1/3)*ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1/6)*c^(1/3))])/(3*a^(5/6)*b
^(1/6)) + (c^(1/3)*(-(ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*(c*x)^(1/3))/(Sqrt[3]
*a^(1/6)*c^(1/3)))]/b^(1/6)) - (Sqrt[3]*Log[a^(1/3)*c^(2/3) - Sqrt[3]*a^(1
/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(2*b^(1/6))))/(6*a
^(5/6)) + (c^(1/3)*(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*(c*x)^(1/3))/(Sqrt[3]*a
^(1/6)*c^(1/3)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3)*c^(2/3) + Sqrt[3]*a^(1/6)
*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(2*b^(1/6))))/(6*a^(5
/6))))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{\left(-\frac{\sqrt{3}\,\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{2}+\frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}-(cx)^{\frac{2}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{2}+\arctan\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)\cdots}{\left(\frac{a\,c\cdots}{b}\right.}\right.\cdots}{2ac}$
derivativedivides	$3c\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\operatorname{arc}\cdots}{\cdots}\right.$
default	$3c\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\operatorname{arc}\cdots}{\cdots}\right.$

input
```
int(1/(c*x)^(2/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1/2*3^(1/2)*ln((c*x)^(2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)+(a*c
^2/b)^(1/3))+1/2*3^(1/2)*ln(3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)-(c*x)^(2/3
)-(a*c^2/b)^(1/3))+arctan((3^(1/2)*(a*c^2/b)^(1/6)-2*(c*x)^(1/3))/(a*c^2/b
)^(1/6))-2*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))-arctan((3^(1/2)*(a*c^2/b)^(
1/6)+2*(c*x)^(1/3))/(a*c^2/b)^(1/6)))*(a*c^2/b)^(1/6)/a/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.30

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}ac+ac\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}ac+ac\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(\frac{1}{2}\left(\sqrt{-3}ac-ac\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right) - \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(-\frac{1}{2}\left(\sqrt{-3}ac-ac\right)\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right) + \frac{1}{2}\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(ac\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right) - \frac{1}{2}\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}\log\left(-ac\left(-\frac{1}{a^5bc^4}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}\right)$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(-3) + 1)*(-1/(a^5*b*c^4))^(1/6)*log(1/2*(sqrt(-3)*a*c + a*c)*(-1
/(a^5*b*c^4))^(1/6) + (c*x)^(1/3)) - 1/4*(sqrt(-3) + 1)*(-1/(a^5*b*c^4))^(
1/6)*log(-1/2*(sqrt(-3)*a*c + a*c)*(-1/(a^5*b*c^4))^(1/6) + (c*x)^(1/3)) +
 1/4*(sqrt(-3) - 1)*(-1/(a^5*b*c^4))^(1/6)*log(1/2*(sqrt(-3)*a*c - a*c)*(-
1/(a^5*b*c^4))^(1/6) + (c*x)^(1/3)) - 1/4*(sqrt(-3) - 1)*(-1/(a^5*b*c^4))^
(1/6)*log(-1/2*(sqrt(-3)*a*c - a*c)*(-1/(a^5*b*c^4))^(1/6) + (c*x)^(1/3))
+ 1/2*(-1/(a^5*b*c^4))^(1/6)*log(a*c*(-1/(a^5*b*c^4))^(1/6) + (c*x)^(1/3))
 - 1/2*(-1/(a^5*b*c^4))^(1/6)*log(-a*c*(-1/(a^5*b*c^4))^(1/6) + (c*x)^(1/3
))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.61

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)} + \frac{i\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)} + \frac{e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)} - \frac{e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)} - \frac{i\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)} - \frac{e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{1}{6}\right)}{12a^{\frac{5}{6}}\sqrt[6]{b}c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate(1/(c*x)**(2/3)/(b*x**2+a),x)
```

output
```
exp(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/6)/a**(1/6))*gamma(
1/6)/(12*a**(5/6)*b**(1/6)*c**(2/3)*gamma(7/6)) + I*log(1 - b**(1/6)*x**(1
/3)*exp_polar(I*pi/2)/a**(1/6))*gamma(1/6)/(12*a**(5/6)*b**(1/6)*c**(2/3)*
gamma(7/6)) + exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(5*I*pi/6)/a*
*(1/6))*gamma(1/6)/(12*a**(5/6)*b**(1/6)*c**(2/3)*gamma(7/6)) - exp(5*I*pi
/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(7*I*pi/6)/a**(1/6))*gamma(1/6)/(12
*a**(5/6)*b**(1/6)*c**(2/3)*gamma(7/6)) - I*log(1 - b**(1/6)*x**(1/3)*exp_
polar(3*I*pi/2)/a**(1/6))*gamma(1/6)/(12*a**(5/6)*b**(1/6)*c**(2/3)*gamma(
7/6)) - exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(11*I*pi/6)/a**(1/6
))*gamma(1/6)/(12*a**(5/6)*b**(1/6)*c**(2/3)*gamma(7/6))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(145) = 290$.

Time = 0.12 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.33

$$\int \frac{1}{(cx)^{2/3}\,(a+bx^2)}\,dx = \frac{\frac{\sqrt{3}c^2\log\left(\sqrt{3}(ac^2)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+(ac^2)^{\frac{1}{3}}\right)}{(ac^2)^{\frac{5}{6}}b^{\frac{1}{6}}} - \frac{\sqrt{3}c^2\log\left(-\sqrt{3}(ac^2)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+(ac^2)^{\frac{1}{3}}\right)}{(ac^2)^{\frac{5}{6}}b^{\frac{1}{6}}}}{}$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(sqrt(3)*c^2*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/
3)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(5/6)*b^(1/6)) - sqrt(3)*c^2*log(-sqr
t(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^(1/
3))/((a*c^2)^(5/6)*b^(1/6)) + 4*c^2*arctan((c*x)^(1/3)*b^(1/3)/sqrt((a*c^2
)^(1/3)*b^(1/3)))/((a*c^2)^(2/3)*sqrt((a*c^2)^(1/3)*b^(1/3))) + 2*(a*c^2)^
(1/3)*arctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) + 2*(c*x)^(1/3)*b^(1/3))/sqrt(
(a*c^2)^(1/3)*b^(1/3)))/(a*sqrt((a*c^2)^(1/3)*b^(1/3))) + 2*(a*c^2)^(1/3)*
arctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^
2)^(1/3)*b^(1/3)))/(a*sqrt((a*c^2)^(1/3)*b^(1/3))))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.22

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{4\,abc} - \frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}\log\left(-\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{4\,abc} + \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}+2\,(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{2\,abc} + \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}-2\,(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{2\,abc} + \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{abc}$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*sqrt(3)*(a*b^5*c^2)^(1/6)*log(sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c
*x)^(2/3) + (a*c^2/b)^(1/3))/(a*b*c) - 1/4*sqrt(3)*(a*b^5*c^2)^(1/6)*log(-
sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/(a*b*
c) + 1/2*(a*b^5*c^2)^(1/6)*arctan((sqrt(3)*(a*c^2/b)^(1/6) + 2*(c*x)^(1/3)
)/(a*c^2/b)^(1/6))/(a*b*c) + 1/2*(a*b^5*c^2)^(1/6)*arctan(-(sqrt(3)*(a*c^2
/b)^(1/6) - 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(a*b*c) + (a*b^5*c^2)^(1/6)*ar
ctan((c*x)^(1/3)/(a*c^2/b)^(1/6))/(a*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.32

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = -\frac{\operatorname{atanh}\left(\frac{b^{1/6}\,(c\,x)^{1/3}}{(-a)^{1/6}\,c^{1/3}}\right)}{(-a)^{5/6}\,b^{1/6}\,c^{2/3}}$$

$$-\frac{\operatorname{atan}\left(\frac{b^{29/6}\,c^{10/3}\,(c\,x)^{1/3}\,243\mathrm{i}}{(-a)^{5/6}\left(\frac{243\,b^{14/3}\,c^{11/3}}{(-a)^{2/3}}-\frac{\sqrt{3}\,b^{14/3}\,c^{11/3}\,243\mathrm{i}}{(-a)^{2/3}}\right)}-\frac{243\,\sqrt{3}\,b^{29/6}\,c^{10/3}\,(c\,x)^{1/3}}{(-a)^{5/6}\left(\frac{243\,b^{14/3}\,c^{11/3}}{(-a)^{2/3}}-\frac{\sqrt{3}\,b^{14/3}\,c^{11/3}\,243\mathrm{i}}{(-a)^{2/3}}\right)}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-a)^{5/6}\,b^{1/6}\,c^{2/3}}$$

$$+\frac{\operatorname{atan}\left(\frac{b^{29/6}\,c^{10/3}\,(c\,x)^{1/3}\,243\mathrm{i}}{(-a)^{5/6}\left(\frac{243\,b^{14/3}\,c^{11/3}}{(-a)^{2/3}}+\frac{\sqrt{3}\,b^{14/3}\,c^{11/3}\,243\mathrm{i}}{(-a)^{2/3}}\right)}+\frac{243\,\sqrt{3}\,b^{29/6}\,c^{10/3}\,(c\,x)^{1/3}}{(-a)^{5/6}\left(\frac{243\,b^{14/3}\,c^{11/3}}{(-a)^{2/3}}+\frac{\sqrt{3}\,b^{14/3}\,c^{11/3}\,243\mathrm{i}}{(-a)^{2/3}}\right)}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,(-a)^{5/6}\,b^{1/6}\,c^{2/3}}$$

input
```
int(1/((c*x)^(2/3)*(a + b*x^2)),x)
```

output
```
(atan((b^(29/6)*c^(10/3)*(c*x)^(1/3)*243i)/((-a)^(5/6)*((243*b^(14/3)*c^(1
1/3))/(-a)^(2/3) + (3^(1/2)*b^(14/3)*c^(11/3)*243i)/(-a)^(2/3))) + (243*3^
(1/2)*b^(29/6)*c^(10/3)*(c*x)^(1/3))/((-a)^(5/6)*((243*b^(14/3)*c^(11/3))/
(-a)^(2/3) + (3^(1/2)*b^(14/3)*c^(11/3)*243i)/(-a)^(2/3))))*(3^(1/2)*1i -
1)*1i)/(2*(-a)^(5/6)*b^(1/6)*c^(2/3)) - (atan((b^(29/6)*c^(10/3)*(c*x)^(1/
3)*243i)/((-a)^(5/6)*((243*b^(14/3)*c^(11/3))/(-a)^(2/3) - (3^(1/2)*b^(14/
3)*c^(11/3)*243i)/(-a)^(2/3))) - (243*3^(1/2)*b^(29/6)*c^(10/3)*(c*x)^(1/3
))/((-a)^(5/6)*((243*b^(14/3)*c^(11/3))/(-a)^(2/3) - (3^(1/2)*b^(14/3)*c^(
11/3)*243i)/(-a)^(2/3))))*(3^(1/2)*1i + 1)*1i)/(2*(-a)^(5/6)*b^(1/6)*c^(2/
3)) - atanh((b^(1/6)*(c*x)^(1/3))/((-a)^(1/6)*c^(1/3)))/((-a)^(5/6)*b^(1/6
)*c^(2/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)}\,dx = \frac{-2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+4atan\left(\frac{x^{\frac{1}{3}}b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)-\sqrt{3}\log\left(-x\right.}{4b^{\frac{1}{6}}a^{\frac{5}{6}}c^{\frac{2}{3}}}$$

input
```
int(1/(c*x)^(2/3)/(b*x^2+a),x)
```

output
```
(b**(1/6)*a**(1/6)*( - 2*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1/3)*b**(
1/3))/(b**(1/6)*a**(1/6))) + 2*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*x**(1/3
)*b**(1/3))/(b**(1/6)*a**(1/6))) + 4*atan((x**(1/3)*b**(1/3))/(b**(1/6)*a*
*(1/6))) - sqrt(3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) +
x**(2/3)*b**(1/3)) + sqrt(3)*log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(
1/3) + x**(2/3)*b**(1/3))))/(4*c**(2/3)*b**(1/3)*a)
```

3.341 $\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 189

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = -\frac{3}{2ac(cx)^{2/3}} + \frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{2a^{4/3}c^{5/3}} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2a^{4/3}c^{5/3}} - \frac{\sqrt[3]{b}\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{4/3}\right)}{4a^{4/3}c^{5/3}}$$

output
```
-3/2/a/c/(c*x)^(2/3)+1/2*3^(1/2)*b^(1/3)*arctan(1/3*(1-2*b^(1/3)*(c*x)^(2/
3)/a^(1/3)/c^(2/3))*3^(1/2))/a^(4/3)/c^(5/3)+1/2*b^(1/3)*ln(a^(1/3)*c^(2/3
)+b^(1/3)*(c*x)^(2/3))/a^(4/3)/c^(5/3)-1/4*b^(1/3)*ln(a^(2/3)*c^(4/3)-a^(1
/3)*b^(1/3)*c^(2/3)*(c*x)^(2/3)+b^(2/3)*(c*x)^(4/3))/a^(4/3)/c^(5/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.06

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = \frac{x\left(-6\sqrt[3]{a}+2\sqrt{3}\sqrt[3]{b}x^{2/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{2/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{b}x^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{2/3}\right)-\cdots\right)}{4\cdots}$$

input
```
Integrate[1/((c*x)^(5/3)*(a + b*x^2)),x]
```

output
```
(x*(-6*a^(1/3) + 2*Sqrt[3]*b^(1/3)*x^(2/3)*ArcTan[(1 - (2*b^(1/3)*x^(2/3))
/a^(1/3))/Sqrt[3]] + 2*b^(1/3)*x^(2/3)*Log[a^(1/3) + b^(1/3)*x^(2/3)] - b^
(1/3)*x^(2/3)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3) + b^(1/3)*x^(2
/3)] - b^(1/3)*x^(2/3)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3) + b^(
1/3)*x^(2/3)]))/(4*a^(4/3)*(c*x)^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {264, 266, 27, 807, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\int \frac{\sqrt[3]{cx}}{bx^2+a}dx}{ac^2}-\frac{3}{2ac(cx)^{2/3}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{3b\int \frac{c^3x}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{ac^3}-\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 27

$$-\frac{3b\int \frac{cx}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{ac}-\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 807

$$-\frac{3b\int \frac{(cx)^{2/3}}{ac^2+bxc}d(cx)^{2/3}}{2ac}-\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 821

$$-\frac{3b\left(\frac{\int \frac{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}\right)}{2ac}-\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 16

$$-\frac{3b\left(\frac{\int \frac{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)}{2ac}-\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 1142

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c \ldots\right.}{3\sqrt[3]{\ldots}}\ldots\right)}{2ac}$$

$$\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 25

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2\ldots}\right.}{3\sqrt[3]{\ldots}}\right.}{2ac}$$

$$\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 27

$$-\frac{3b\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}\ldots\right.}{3\ldots}\right.}{2ac}$$

$$\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 1082

$$-\frac{3b\left(\frac{\frac{3\int \frac{1}{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}-4}d\left(1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)}{2ac}-\ldots$$

$$\frac{3}{2ac(cx)^{2/3}}$$

$\downarrow$ 217

$$-\frac{3b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)}{2ac}-\frac{3}{2ac(cx)^{2/3}}$$

$$\Big\downarrow \; 1103$$

$$-\frac{3b\left(\frac{\frac{\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{2/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}c^{2/3}}-\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3\sqrt[3]{a}b^{2/3}c^{2/3}}\right)}{2ac}-\frac{3}{2ac(cx)^{2/3}}$$

input

```
Int[1/((c*x)^(5/3)*(a + b*x^2)),x]
```

output

```
-3/(2*a*c*(c*x)^(2/3)) - (3*b*(-1/3*Log[a^(1/3)*c^(2/3) + b^(1/3)*(c*x)^(2
/3)]/(a^(1/3)*b^(2/3)*c^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x)^
(2/3))/(a^(1/3)*c^(2/3)))/Sqrt[3]])/b^(1/3)) + Log[a^(2/3)*c^(4/3) + b^(2/
3)*(c*x)^(2/3) - a^(1/3)*b^(1/3)*c^(2/3)*(c*x)^(2/3)]/(2*b^(1/3)))/(3*a^(1
/3)*b^(1/3)*c^(2/3))))/(2*a*c)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{2\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(cx)^{\frac{2}{3}}}{3\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}-\frac{\sqrt{3}}{3}\right)(cx)^{\frac{2}{3}}-2\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)(cx)^{\frac{2}{3}}+\ln\left(cx(cx)^{\frac{1}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)(c}{4(cx)^{\frac{2}{3}}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}ac}$
derivativedivides	$3c\left(-\frac{1}{2a\,c^{2}(cx)^{\frac{2}{3}}}-\frac{b\left(-\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right.}{2a\,c^{2}}\right.$
default	$3c\left(-\frac{1}{2a\,c^{2}(cx)^{\frac{2}{3}}}-\frac{b\left(-\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right.}{3}\right.}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right.}{2a\,c^{2}}\right.$

input
```
int(1/(c*x)^(5/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*3^(1/2)*arctan(2/3*3^(1/2)*(c*x)^(2/3)/(a*c^2/b)^(1/3)-1/3*3^(1/2)
)*(c*x)^(2/3)-2*ln((c*x)^(2/3)+(a*c^2/b)^(1/3))*(c*x)^(2/3)+ln(c*x*(c*x)^(
1/3)-(a*c^2/b)^(1/3)*(c*x)^(2/3)+(a*c^2/b)^(2/3))*(c*x)^(2/3)+6*(a*c^2/b)^
(1/3))/(c*x)^(2/3)/(a*c^2/b)^(1/3)/a/c
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.83

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = -\frac{2\sqrt{3}cx\left(\frac{b}{ac^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2}{3}\sqrt{3}(cx)^{\frac{2}{3}}\left(\frac{b}{ac^2}\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2\,cx\left(\frac{b}{ac^2}\right)^{\frac{1}{3}}\log\left(ac^2\left(\frac{b}{ac^2}\right)^{\frac{2}{3}}+(cx)^{\frac{2}{3}}b\right)+cx\left(\frac{b}{ac^2}\right)^{\frac{1}{3}}\log}{4\,ac^2x}$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*c*x*(b/(a*c^2))^(1/3)*arctan(2/3*sqrt(3)*(c*x)^(2/3)*(b/(a
*c^2))^(1/3) - 1/3*sqrt(3)) - 2*c*x*(b/(a*c^2))^(1/3)*log(a*c^2*(b/(a*c^2)
)^(2/3) + (c*x)^(2/3)*b) + c*x*(b/(a*c^2))^(1/3)*log(-(c*x)^(2/3)*a*c*(b/(
a*c^2))^(2/3) + (c*x)^(1/3)*b*x + a*c*(b/(a*c^2))^(1/3)) + 6*(c*x)^(1/3))/
(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.08

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}e^{-\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{6a^{\frac{4}{3}}c^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{6a^{\frac{4}{3}}c^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)} - \frac{\sqrt[3]{b}e^{\frac{2i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(-\frac{1}{3}\right)}{6a^{\frac{4}{3}}c^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(c*x)**(5/3)/(b*x**2+a),x)
```

output
```
gamma(-1/3)/(2*a*c**(5/3)*x**(2/3)*gamma(2/3)) - b**(1/3)*exp(-2*I*pi/3)*l
og(1 - b**(1/3)*x**(2/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(-1/3)/(6*a**(4/
3)*c**(5/3)*gamma(2/3)) - b**(1/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(I*p
i)/a**(1/3))*gamma(-1/3)/(6*a**(4/3)*c**(5/3)*gamma(2/3)) - b**(1/3)*exp(2
*I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(-1/
3)/(6*a**(4/3)*c**(5/3)*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = -\frac{\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(cx)^{\frac{2}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}a}+\frac{\log\left((cx)^{\frac{4}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}a}-\frac{2\,\log\left((cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}a}+\frac{6}{(cx)^{\frac{2}{3}}a}}{4\,c}$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) - (a*c^2/b)^(1/3))/(a*c^
2/b)^(1/3))/((a*c^2/b)^(1/3)*a) + log((c*x)^(4/3) - (a*c^2/b)^(1/3)*(c*x)^
(2/3) + (a*c^2/b)^(2/3))/((a*c^2/b)^(1/3)*a) - 2*log((c*x)^(2/3) + (a*c^2/
b)^(1/3))/((a*c^2/b)^(1/3)*a) + 6/((c*x)^(2/3)*a))/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

$$\int \frac{1}{(cx)^{5/3}\,(a+bx^2)}\,dx = \frac{\frac{2\left(-\frac{ac^2}{b}\right)^{\frac{2}{3}} b\log\left(\left|(cx)^{\frac{2}{3}}-\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right|\right)}{a^2c^2} - \frac{6}{(cx)^{\frac{2}{3}}a} + \frac{2\sqrt{3}(-ab^2c^2)^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{a^2bc^2}}{4c}$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(2*(-a*c^2/b)^(2/3)*b*log(abs((c*x)^(2/3) - (-a*c^2/b)^(1/3)))/(a^2*c^
2) - 6/((c*x)^(2/3)*a) + 2*sqrt(3)*(-a*b^2*c^2)^(2/3)*arctan(1/3*sqrt(3)*(
2*(c*x)^(2/3) + (-a*c^2/b)^(1/3))/(-a*c^2/b)^(1/3))/(a^2*b*c^2) - (-a*b^2*
c^2)^(2/3)*log((c*x)^(1/3)*c*x + (-a*c^2/b)^(1/3)*(c*x)^(2/3) + (-a*c^2/b)
^(2/3))/(a^2*b*c^2))/c
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\begin{aligned}\int \frac{1}{(cx)^{5/3}\,(a+bx^2)}\,dx &= \frac{b^{1/3}\,\ln\left(81\,a^4\,b^6\,c^5+81\,a^{11/3}\,b^{19/3}\,c^{13/3}\,{\left(c\,x\right)}^{2/3}\right)}{2\,a^{4/3}\,c^{5/3}} \\ &- \frac{3}{2\,a\,c\,{\left(c\,x\right)}^{2/3}} \\ &- \frac{b^{1/3}\,\ln\left(81\,a^4\,b^6\,c^5-81\,a^{11/3}\,b^{19/3}\,c^{13/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(c\,x\right)}^{2/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{4/3}\,c^{5/3}} \\ &+ \frac{b^{1/3}\,\ln\left(81\,a^4\,b^6\,c^5+162\,a^{11/3}\,b^{19/3}\,c^{13/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{\left(c\,x\right)}^{2/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{a^{4/3}\,c^{5/3}}\end{aligned}$$

input
```
int(1/((c*x)^(5/3)*(a + b*x^2)),x)
```

output
```
(b^(1/3)*log(81*a^4*b^6*c^5 + 81*a^(11/3)*b^(19/3)*c^(13/3)*(c*x)^(2/3)))/
(2*a^(4/3)*c^(5/3)) - 3/(2*a*c*(c*x)^(2/3)) - (b^(1/3)*log(81*a^4*b^6*c^5
- 81*a^(11/3)*b^(19/3)*c^(13/3)*((3^(1/2)*1i)/2 + 1/2)*(c*x)^(2/3))*((3^(1
/2)*1i)/2 + 1/2))/(2*a^(4/3)*c^(5/3)) + (b^(1/3)*log(81*a^4*b^6*c^5 + 162*
a^(11/3)*b^(19/3)*c^(13/3)*((3^(1/2)*1i)/4 - 1/4)*(c*x)^(2/3))*((3^(1/2)*1
i)/4 - 1/4))/(a^(4/3)*c^(5/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)}\,dx = \frac{2x^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2x^{\frac{2}{3}}b^{\frac{1}{3}}\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-6a^{\frac{1}{3}}+2x^{\frac{2}{3}}b^{\frac{1}{3}}}{}$$

input
```
int(1/(c*x)^(5/3)/(b*x^2+a),x)
```

output
```
(c**(1/3)*(2*x**(2/3)*b**(1/3)*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2
*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) + 2*x**(2/3)*b**(1/3)*sqrt(3)*ata
n((b**(1/6)*a**(1/6)*sqrt(3) + 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) -
 6*a**(1/3) + 2*x**(2/3)*b**(1/3)*log(a**(1/3) + x**(2/3)*b**(1/3)) - x**(
2/3)*b**(1/3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2
/3)*b**(1/3)) - x**(2/3)*b**(1/3)*log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) +
 a**(1/3) + x**(2/3)*b**(1/3))))/(4*x**(2/3)*a**(1/3)*a*c**2)
```

3.342 $\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx = -\frac{3}{5ac(cx)^{5/3}} - \frac{b^{5/6}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{a^{11/6}c^{8/3}} + \frac{b^{5/6}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{11/6}c^{8/3}} - \frac{b^{5/6}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{11/6}c^{8/3}} - \frac{\sqrt{3}b^{5/6}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2a^{11/6}c^{8/3}}$$

output
```
-3/5/a/c/(c*x)^(5/3)-b^(5/6)*arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a
^(11/6)/c^(8/3)-1/2*b^(5/6)*arctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/
c^(1/3))/a^(11/6)/c^(8/3)-1/2*b^(5/6)*arctan(3^(1/2)+2*b^(1/6)*(c*x)^(1/3)
/a^(1/6)/c^(1/3))/a^(11/6)/c^(8/3)-1/2*3^(1/2)*b^(5/6)*arctanh(3^(1/2)*a^(
1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3)/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3)))/a^
(11/6)/c^(8/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.67

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx = \frac{x\left(-6a^{5/6}+5b^{5/6}x^{5/3}\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-10b^{5/6}x^{5/3}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right.\right.}{10a^{11/6}(cx)^{8/3}}$$

input
```
Integrate[1/((c*x)^(8/3)*(a + b*x^2)),x]
```

output
```
(x*(-6*a^(5/6) + 5*b^(5/6)*x^(5/3)*ArcTan[a^(1/6)/(b^(1/6)*x^(1/3)) - (b^(
1/6)*x^(1/3))/a^(1/6)] - 10*b^(5/6)*x^(5/3)*ArcTan[(b^(1/6)*x^(1/3))/a^(1/
6)] - 5*Sqrt[3]*b^(5/6)*x^(5/3)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3))/
(a^(1/3) + b^(1/3)*x^(2/3))]))/(10*a^(11/6)*(c*x)^(8/3))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.37, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {264, 266, 753, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx$$

$$\Big\downarrow \; 264$$

$$-\frac{b\int \frac{1}{(cx)^{2/3}(bx^2+a)}dx}{ac^2}-\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 266$$

$$-\frac{3b\int \frac{1}{bx^2+a}d\sqrt[3]{cx}}{ac^3}-\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 753$$

$$-\frac{3b\left(\frac{c^{2/3}\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}\cdots}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\cdots\right.}\cdots}{\cdots}\cdots\right.}{ac^3}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$\downarrow$ 27

$$-\frac{3b\left(\frac{c^{2/3}\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3a^{2/3}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\cdots}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\cdots}\cdots}{6a^{5/6}}\cdots\right.}{ac^3}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$\downarrow$ 218

$$-\frac{3b\left(\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\int \frac{2\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6a^{5/6}}+\frac{\sqrt[3]{c}\arctan\cdots}{3\cdots}\cdots\right.}{ac^3}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$\downarrow$ 1142

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}\cdots\right.}{\cdots}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 25$$

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\cdots\right)}{\cdots}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}\right)}{6a^{5/6}}\cdots\right)}{\cdots}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 1082$$

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\int \frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\cdots}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\cdots}\cdots\right)}{\cdots}\cdots\right)}{ac^3}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \; 217$$

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\cdots}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\cdots}\cdots\right.}{\cdots}\cdots}{ac^3\cdots}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

$\Big\downarrow$ 1103

$$-\frac{3b\left(\frac{\sqrt[3]{c}\left(-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}\right)}{6a^{5/6}}+\frac{\sqrt[3]{c}\left(\frac{\arctan\left(\sqrt{3}\left(\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}+1\cdots\right.\right.}{\sqrt[6]{b}}\cdots\right.}{\cdots}\cdots}{ac^3\cdots}$$

$$\frac{3}{5ac(cx)^{5/3}}$$

input
```
Int[1/((c*x)^(8/3)*(a + b*x^2)),x]
```

output
```
-3/(5*a*c*(c*x)^(5/3)) - (3*b*((c^(1/3)*ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1
/6)*c^(1/3))])/(3*a^(5/6)*b^(1/6)) + (c^(1/3)*(-(ArcTan[Sqrt[3]*(1 - (2*b^
(1/6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6)) - (Sqrt[3]*Log[a^(
1/3)*c^(2/3) - Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)
^(2/3)])/(2*b^(1/6))))/(6*a^(5/6)) + (c^(1/3)*(ArcTan[Sqrt[3]*(1 + (2*b^(1
/6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6) + (Sqrt[3]*Log[a^(1/3
)*c^(2/3) + Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2
/3)])/(2*b^(1/6))))/(6*a^(5/6))))/(a*c^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.09

method	result
derivativedivides	$3c\left(-\frac{1}{5a\,c^{2}(cx)^{\frac{5}{3}}}-\frac{\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\cdots\right.}{}\right.$
default	$3c\left(-\frac{1}{5a\,c^{2}(cx)^{\frac{5}{3}}}-\frac{\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{6a\,c^{2}}+\cdots\right.}{}\right.$
pseudoelliptic	$\frac{-\frac{b\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)(cx)^{\frac{5}{3}}}{2}+\frac{b\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}\ln\left(\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}-(cx)^{\frac{2}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)(\cdots}{2}}{}$

input
```
int(1/(c*x)^(8/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
3*c*(-1/5/a/c^2/(c*x)^(5/3)-(1/12/a/c^2*3^(1/2)*(a*c^2/b)^(1/6)*ln((c*x)^(
2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)+(a*c^2/b)^(1/3))+1/6/a/c^2*(a*c^2
/b)^(1/6)*arctan(2*(c*x)^(1/3)/(a*c^2/b)^(1/6)+3^(1/2))+1/3/a/c^2*(a*c^2/b
)^(1/6)*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))-1/12/a/c^2*3^(1/2)*(a*c^2/b)^(
1/6)*ln((c*x)^(2/3)-3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)+(a*c^2/b)^(1/3))+1
/6/a/c^2*(a*c^2/b)^(1/6)*arctan(2*(c*x)^(1/3)/(a*c^2/b)^(1/6)-3^(1/2)))*b/
a/c^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(159) = 318.

Time = 0.08 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.82

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx =$$

$$-\frac{10\,ac^3x^2\left(-\frac{b^5}{a^{11}c^{16}}\right)^{\frac{1}{6}}\log\left(a^2c^3\left(-\frac{b^5}{a^{11}c^{16}}\right)^{\frac{1}{6}}+(cx)^{\frac{1}{3}}b\right)-10\,ac^3x^2\left(-\frac{b^5}{a^{11}c^{16}}\right)^{\frac{1}{6}}\log\left(-a^2c^3\left(-\frac{b^5}{a^{11}c^{16}}\right)^{\frac{1}{6}}+(cx\right.}{}$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/20*(10*a*c^3*x^2*(-b^5/(a^11*c^16))^(1/6)*log(a^2*c^3*(-b^5/(a^11*c^16)
)^(1/6) + (c*x)^(1/3)*b) - 10*a*c^3*x^2*(-b^5/(a^11*c^16))^(1/6)*log(-a^2*
c^3*(-b^5/(a^11*c^16))^(1/6) + (c*x)^(1/3)*b) + 5*(sqrt(-3)*a*c^3*x^2 + a*
c^3*x^2)*(-b^5/(a^11*c^16))^(1/6)*log((c*x)^(1/3)*b + 1/2*(sqrt(-3)*a^2*c^
3 + a^2*c^3)*(-b^5/(a^11*c^16))^(1/6)) - 5*(sqrt(-3)*a*c^3*x^2 + a*c^3*x^2
)*(-b^5/(a^11*c^16))^(1/6)*log((c*x)^(1/3)*b - 1/2*(sqrt(-3)*a^2*c^3 + a^2
*c^3)*(-b^5/(a^11*c^16))^(1/6)) + 5*(sqrt(-3)*a*c^3*x^2 - a*c^3*x^2)*(-b^5
/(a^11*c^16))^(1/6)*log((c*x)^(1/3)*b + 1/2*(sqrt(-3)*a^2*c^3 - a^2*c^3)*(
-b^5/(a^11*c^16))^(1/6)) - 5*(sqrt(-3)*a*c^3*x^2 - a*c^3*x^2)*(-b^5/(a^11*
c^16))^(1/6)*log((c*x)^(1/3)*b - 1/2*(sqrt(-3)*a^2*c^3 - a^2*c^3)*(-b^5/(a
^11*c^16))^(1/6)) + 12*(c*x)^(1/3))/(a*c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.68

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx = \frac{\Gamma\left(-\frac{5}{6}\right)}{2ac^{\frac{8}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{1}{6}\right)} + \frac{5b^{\frac{5}{6}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)} + \frac{5ib^{\frac{5}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)} + \frac{5b^{\frac{5}{6}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)} - \frac{5b^{\frac{5}{6}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)} - \frac{5ib^{\frac{5}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)} - \frac{5b^{\frac{5}{6}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{5}{6}\right)}{12a^{\frac{11}{6}}c^{\frac{8}{3}}\Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(8/3)/(b*x**2+a),x)
```

output
```
gamma(-5/6)/(2*a*c**(8/3)*x**(5/3)*gamma(1/6)) + 5*b**(5/6)*exp(5*I*pi/6)*
log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/6)/a**(1/6))*gamma(-5/6)/(12*a**(
11/6)*c**(8/3)*gamma(1/6)) + 5*I*b**(5/6)*log(1 - b**(1/6)*x**(1/3)*exp_po
lar(I*pi/2)/a**(1/6))*gamma(-5/6)/(12*a**(11/6)*c**(8/3)*gamma(1/6)) + 5*b
**(5/6)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(5*I*pi/6)/a**(1/6)
)*gamma(-5/6)/(12*a**(11/6)*c**(8/3)*gamma(1/6)) - 5*b**(5/6)*exp(5*I*pi/6
)*log(1 - b**(1/6)*x**(1/3)*exp_polar(7*I*pi/6)/a**(1/6))*gamma(-5/6)/(12*
a**(11/6)*c**(8/3)*gamma(1/6)) - 5*I*b**(5/6)*log(1 - b**(1/6)*x**(1/3)*ex
p_polar(3*I*pi/2)/a**(1/6))*gamma(-5/6)/(12*a**(11/6)*c**(8/3)*gamma(1/6))
 - 5*b**(5/6)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(11*I*pi/6)/a
**(1/6))*gamma(-5/6)/(12*a**(11/6)*c**(8/3)*gamma(1/6))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.30

$$\int \frac{1}{(cx)^{8/3}\,(a+bx^2)}\,dx = -\frac{\frac{5\left(\frac{\sqrt{3}b^{\frac{5}{6}}\log\left(\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{5}{6}}}-\frac{\sqrt{3}b^{\frac{5}{6}}\log\left(-\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{5}{6}}}+\frac{4b\arctan\left(\frac{(cx)^{\frac{1}{3}}b^{\frac{1}{3}}}{\sqrt{\left(ac^2\right)^{\frac{1}{3}}b^{\frac{1}{3}}}}\right)}{\left(ac^2\right)^{\frac{2}{3}}\sqrt{\left(ac^2\right)^{\frac{1}{3}}b^{\frac{1}{3}}}}+\frac{2\left(ac^2\right)^{\frac{1}{3}}\cdots}{\cdots}\right.}{a}}{20\,c}$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/20*(5*(sqrt(3)*b^(5/6)*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) +
(c*x)^(2/3)*b^(1/3) + (a*c^2)^(1/3))/(a*c^2)^(5/6) - sqrt(3)*b^(5/6)*log(-
sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^
(1/3))/(a*c^2)^(5/6) + 4*b*arctan((c*x)^(1/3)*b^(1/3)/sqrt((a*c^2)^(1/3)*b
^(1/3)))/((a*c^2)^(2/3)*sqrt((a*c^2)^(1/3)*b^(1/3))) + 2*(a*c^2)^(1/3)*b*a
rctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) + 2*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^2)
^(1/3)*b^(1/3)))/(a*c^2*sqrt((a*c^2)^(1/3)*b^(1/3))) + 2*(a*c^2)^(1/3)*b*a
rctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^2
)^(1/3)*b^(1/3)))/(a*c^2*sqrt((a*c^2)^(1/3)*b^(1/3))))/a + 12/((c*x)^(5/3)
*a))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.14

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx = -\frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{4\,a^2c^3} + \frac{\sqrt{3}(ab^5c^2)^{\frac{1}{6}}\log\left(-\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{4\,a^2c^3} - \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}+2\,(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{2\,a^2c^3} - \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(-\frac{\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}-2\,(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{2\,a^2c^3} - \frac{(ab^5c^2)^{\frac{1}{6}}\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{a^2c^3} - \frac{3}{5\,(cx)^{\frac{2}{3}}\,ac^2x}$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*(a*b^5*c^2)^(1/6)*log(sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (
c*x)^(2/3) + (a*c^2/b)^(1/3))/(a^2*c^3) + 1/4*sqrt(3)*(a*b^5*c^2)^(1/6)*lo
g(-sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/(a
^2*c^3) - 1/2*(a*b^5*c^2)^(1/6)*arctan((sqrt(3)*(a*c^2/b)^(1/6) + 2*(c*x)^
(1/3))/(a*c^2/b)^(1/6))/(a^2*c^3) - 1/2*(a*b^5*c^2)^(1/6)*arctan(-(sqrt(3)
*(a*c^2/b)^(1/6) - 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(a^2*c^3) - (a*b^5*c^2)
^(1/6)*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))/(a^2*c^3) - 3/5/((c*x)^(2/3)*a*
c^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.20

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)}\,dx = -\frac{3}{5\,a\,c\,(c\,x)^{5/3}} - \frac{(-b)^{5/6}\,\mathrm{atan}\left(\frac{(-b)^{1/6}\,(c\,x)^{1/3}\,1\mathrm{i}}{a^{1/6}\,c^{1/3}}\right)\,1\mathrm{i}}{a^{11/6}\,c^{8/3}}$$
$$- \frac{(-b)^{5/6}\,\ln\left(486\,a^{31/6}\,(-b)^{53/6}\,c^{16/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+486\,a^5\,b^9\,c^5\,(c\,x)^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{11/6}\,c^{8/3}}$$
$$- \frac{(-b)^{5/6}\,\ln\left(486\,a^{31/6}\,(-b)^{53/6}\,c^{16/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)+486\,a^5\,b^9\,c^5\,(c\,x)^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,a^{11/6}\,c^{8/3}}$$
$$+ \frac{(-b)^{5/6}\,\ln\left(972\,a^{31/6}\,(-b)^{53/6}\,c^{16/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-486\,a^5\,b^9\,c^5\,(c\,x)^{1/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{a^{11/6}\,c^{8/3}}$$
$$+ \frac{(-b)^{5/6}\,\ln\left(972\,a^{31/6}\,(-b)^{53/6}\,c^{16/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-486\,a^5\,b^9\,c^5\,(c\,x)^{1/3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{a^{11/6}\,c^{8/3}}$$

input
```
int(1/((c*x)^(8/3)*(a + b*x^2)),x)
```

output
```
((-b)^(5/6)*log(972*a^(31/6)*(-b)^(53/6)*c^(16/3)*((3^(1/2)*1i)/4 - 1/4) -
 486*a^5*b^9*c^5*(c*x)^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(a^(11/6)*c^(8/3)) -
 ((-b)^(5/6)*atan(((-b)^(1/6)*(c*x)^(1/3)*1i)/(a^(1/6)*c^(1/3)))*1i)/(a^(1
1/6)*c^(8/3)) - ((-b)^(5/6)*log(486*a^(31/6)*(-b)^(53/6)*c^(16/3)*((3^(1/2
)*1i)/2 - 1/2) + 486*a^5*b^9*c^5*(c*x)^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*a
^(11/6)*c^(8/3)) - ((-b)^(5/6)*log(486*a^(31/6)*(-b)^(53/6)*c^(16/3)*((3^(
1/2)*1i)/2 + 1/2) + 486*a^5*b^9*c^5*(c*x)^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(
2*a^(11/6)*c^(8/3)) - 3/(5*a*c*(c*x)^(5/3)) + ((-b)^(5/6)*log(972*a^(31/6)
*(-b)^(53/6)*c^(16/3)*((3^(1/2)*1i)/4 + 1/4) - 486*a^5*b^9*c^5*(c*x)^(1/3)
)*((3^(1/2)*1i)/4 + 1/4))/(a^(11/6)*c^(8/3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.80

$$\int \frac{1}{(cx)^{8/3}\left(a+bx^2\right)}\,dx = \frac{10x^{\frac{5}{3}}b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-10x^{\frac{5}{3}}b^{\frac{7}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-20x^{\frac{5}{3}}b^{\frac{7}{6}}a^{\frac{1}{6}}\,ata}{}$$

input
```
int(1/(c*x)^(8/3)/(b*x^2+a),x)
```

output
```
(c**(1/3)*(10*x**(2/3)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) -
 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6)))*b*x - 10*x**(2/3)*b**(1/6)*a**(
1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(
1/6)))*b*x - 20*x**(2/3)*b**(1/6)*a**(1/6)*atan((x**(1/3)*b**(1/3))/(b**(1
/6)*a**(1/6)))*b*x + 5*x**(2/3)*b**(1/6)*a**(1/6)*sqrt(3)*log( - x**(1/3)*
b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3))*b*x - 5*x**(2/3)
*b**(1/6)*a**(1/6)*sqrt(3)*log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/
3) + x**(2/3)*b**(1/3))*b*x - 12*b**(1/3)*a))/(20*x**(2/3)*b**(1/3)*a**2*c
**3*x)
```

3.343 $\int \frac{(cx)^{8/3}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 237

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{3c(cx)^{5/3}}{5b} - \frac{a^{5/6}c^{8/3}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{b^{11/6}} + \frac{a^{5/6}c^{8/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2b^{11/6}} - \frac{a^{5/6}c^{8/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2b^{11/6}} + \frac{\sqrt{3}a^{5/6}c^{8/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2b^{11/6}}$$

output
```
3/5*c*(c*x)^(5/3)/b-a^(5/6)*c^(8/3)*arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(
1/3))/b^(11/6)-1/2*a^(5/6)*c^(8/3)*arctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a
^(1/6)/c^(1/3))/b^(11/6)-1/2*a^(5/6)*c^(8/3)*arctan(3^(1/2)+2*b^(1/6)*(c*x
)^(1/3)/a^(1/6)/c^(1/3))/b^(11/6)+1/2*3^(1/2)*a^(5/6)*c^(8/3)*arctanh(3^(1
/2)*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3)/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/
3)))/b^(11/6)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{(cx)^{8/3}\left(6b^{5/6}x^{5/3}+5a^{5/6}\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-10a^{5/6}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)+5\sqrt{3}\right.}{10b^{11/6}x^{8/3}}$$

input
```
Integrate[(c*x)^(8/3)/(a + b*x^2),x]
```

output
```
((c*x)^(8/3)*(6*b^(5/6)*x^(5/3) + 5*a^(5/6)*ArcTan[a^(1/6)/(b^(1/6)*x^(1/3
)) - (b^(1/6)*x^(1/3))/a^(1/6)] - 10*a^(5/6)*ArcTan[(b^(1/6)*x^(1/3))/a^(1
/6)] + 5*Sqrt[3]*a^(5/6)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3))/(a^(1/3
) + b^(1/3)*x^(2/3))]))/(10*b^(11/6)*x^(8/3))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {262, 266, 27, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{ac^2\int \frac{(cx)^{2/3}}{bx^2+a}dx}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac\int \frac{c^2(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\int \frac{(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \text{824}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\left(\frac{\int\frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}+\frac{\int-\frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\int-\frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\ldots\right.}\ldots}{3\sqrt[6]{a}b^{2/3}\ldots}\right.}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\left(\frac{\int\frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}-\frac{\int\frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int\frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(\ldots}\ldots}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}\right.}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\left(-\frac{\int\frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int\frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\ldots}{\sqrt[6]{a}\ldots}\right.}{3\sqrt[6]{a}b^{5/6}\ldots}\right.}{b}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int\frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int-\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\ldots\right.}{\ldots}$$

$$\Big\downarrow \text{25}$$

$$\frac{3c(cx)^{5/3}}{5b}-$$

$$3ac^3\left(-\frac{\frac{\sqrt{3}\int\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int\frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\ldots\right.$$

$\downarrow$ 27

$$\frac{3c(cx)^{5/3}}{5b}-$$

$$3ac^3\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int\frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\ldots\right.$$

$\downarrow$ 1082

$$\frac{3c(cx)^{5/3}}{5b}-$$

$$\frac{3ac^3\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\int\frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}+2\sqrt[6]{\ldots}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c\ldots}}\ldots}{\ldots}\right.}{b}$$

$\downarrow$ 217

$$\frac{3c(cx)^{5/3}}{5b}-$$

$$\frac{3ac^3\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}+2\sqrt[6]{b}\ldots}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\ldots}\ldots}{\ldots}\right.}{b}$$

$\downarrow$ 1103

$$\frac{3c(cx)^{5/3}}{5b}-\frac{3ac^3\left(-\frac{-\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\dots}{2\sqrt[6]{b}}}{6\sqrt[6]{a}\dots}\right.}{b}$$

input
```
Int[(c*x)^(8/3)/(a + b*x^2),x]
```

output
```
(3*c*(c*x)^(5/3))/(5*b) - (3*a*c^3*(ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1/6)*
c^(1/3))]/(3*a^(1/6)*b^(5/6)*c^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*(c
*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3)*c^(2
/3) - Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/
(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)*c^(1/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/
6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3
)*c^(2/3) + Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2
/3)])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)*c^(1/3))))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$\frac{c^2\left(\frac{\sqrt{3}\,\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{3}}\right)ac}{2}-\frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}-(cx)^{\frac{2}{3}}-\left(\frac{a\,c^2}{b}\right)^{\frac{1}{3}}\right)ac}{2}+\frac{6bx(cx)^{\frac{2}{3}}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}{5}\right.}{2\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}b^2}$
derivativedivides	$3c\left(\frac{(cx)^{\frac{5}{3}}}{5b}-\frac{\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^2}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}}{b}\right.$
default	$3c\left(\frac{(cx)^{\frac{5}{3}}}{5b}-\frac{\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^2}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}}{b}\right.$
risch	$\frac{3x^2c^3}{5b(cx)^{\frac{1}{3}}}+\left(-\frac{a\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}\right)}{b^2\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^2}{b}\right)^{\frac{1}{3}}\right)}{4b\,c^2}-\frac{a\arctan\left(\right.}{2b^2\left(\right.}\right.$

input
```
int((c*x)^(8/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2*(1/2*3^(1/2)*ln((c*x)^(2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)+(a
*c^2/b)^(1/3))*a*c-1/2*3^(1/2)*ln(3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)-(c*x
)^(2/3)-(a*c^2/b)^(1/3))*a*c+6/5*b*x*(c*x)^(2/3)*(a*c^2/b)^(1/6)+arctan((3
^(1/2)*(a*c^2/b)^(1/6)-2*(c*x)^(1/3))/(a*c^2/b)^(1/6))*a*c-2*arctan((c*x)^
(1/3)/(a*c^2/b)^(1/6))*a*c-arctan((3^(1/2)*(a*c^2/b)^(1/6)+2*(c*x)^(1/3))/
(a*c^2/b)^(1/6))*a*c)/(a*c^2/b)^(1/6)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 368 vs. $2(157) = 314$.

Time = 0.08 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.55

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{12\,(cx)^{\frac{2}{3}}\,c^2x - 10\left(-\frac{a^5c^{16}}{b^{11}}\right)^{\frac{1}{6}} b\log\left((cx)^{\frac{1}{3}}\,a^4c^{13} + \left(-\frac{a^5c^{16}}{b^{11}}\right)^{\frac{5}{6}} b^9\right) + 10\left(-\frac{a^5c^{16}}{b^{11}}\right)^{\frac{1}{6}} b\log\left((c$$

input
```
integrate((c*x)^(8/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/20*(12*(c*x)^(2/3)*c^2*x - 10*(-a^5*c^16/b^11)^(1/6)*b*log((c*x)^(1/3)*a
^4*c^13 + (-a^5*c^16/b^11)^(5/6)*b^9) + 10*(-a^5*c^16/b^11)^(1/6)*b*log((c
*x)^(1/3)*a^4*c^13 - (-a^5*c^16/b^11)^(5/6)*b^9) + 5*(-a^5*c^16/b^11)^(1/6
)*(sqrt(-3)*b - b)*log((c*x)^(1/3)*a^4*c^13 + 1/2*(-a^5*c^16/b^11)^(5/6)*(
sqrt(-3)*b^9 + b^9)) - 5*(-a^5*c^16/b^11)^(1/6)*(sqrt(-3)*b - b)*log((c*x)
^(1/3)*a^4*c^13 - 1/2*(-a^5*c^16/b^11)^(5/6)*(sqrt(-3)*b^9 + b^9)) + 5*(-a
^5*c^16/b^11)^(1/6)*(sqrt(-3)*b + b)*log((c*x)^(1/3)*a^4*c^13 + 1/2*(-a^5*
c^16/b^11)^(5/6)*(sqrt(-3)*b^9 - b^9)) - 5*(-a^5*c^16/b^11)^(1/6)*(sqrt(-3
)*b + b)*log((c*x)^(1/3)*a^4*c^13 - 1/2*(-a^5*c^16/b^11)^(5/6)*(sqrt(-3)*b
^9 - b^9)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.65

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = -\frac{11a^{\frac{5}{6}}c^{\frac{8}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} - \frac{11ia^{\frac{5}{6}}c^{\frac{8}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} - \frac{11a^{\frac{5}{6}}c^{\frac{8}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} + \frac{11a^{\frac{5}{6}}c^{\frac{8}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} + \frac{11ia^{\frac{5}{6}}c^{\frac{8}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} + \frac{11a^{\frac{5}{6}}c^{\frac{8}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{11}{6}\right)}{12b^{\frac{11}{6}}\Gamma\left(\frac{17}{6}\right)} + \frac{11c^{\frac{8}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{11}{6}\right)}{10b\Gamma\left(\frac{17}{6}\right)}$$

input
```
integrate((c*x)**(8/3)/(b*x**2+a),x)
```

output
```
-11*a**(5/6)*c**(8/3)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi
/6)/a**(1/6))*gamma(11/6)/(12*b**(11/6)*gamma(17/6)) - 11*I*a**(5/6)*c**(8
/3)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/2)/a**(1/6))*gamma(11/6)/(12*
b**(11/6)*gamma(17/6)) - 11*a**(5/6)*c**(8/3)*exp(5*I*pi/6)*log(1 - b**(1/
6)*x**(1/3)*exp_polar(5*I*pi/6)/a**(1/6))*gamma(11/6)/(12*b**(11/6)*gamma(
17/6)) + 11*a**(5/6)*c**(8/3)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_po
lar(7*I*pi/6)/a**(1/6))*gamma(11/6)/(12*b**(11/6)*gamma(17/6)) + 11*I*a**(
5/6)*c**(8/3)*log(1 - b**(1/6)*x**(1/3)*exp_polar(3*I*pi/2)/a**(1/6))*gamm
a(11/6)/(12*b**(11/6)*gamma(17/6)) + 11*a**(5/6)*c**(8/3)*exp(5*I*pi/6)*lo
g(1 - b**(1/6)*x**(1/3)*exp_polar(11*I*pi/6)/a**(1/6))*gamma(11/6)/(12*b**
(11/6)*gamma(17/6)) + 11*c**(8/3)*x**(5/3)*gamma(11/6)/(10*b*gamma(17/6))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.23

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{\frac{5\,ac^4\left(\frac{\sqrt{3}\log\left(\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\frac{\sqrt{3}\left(ac^2\right)}{\sqrt{(}}\right.}{b^{\frac{2}{3}}\sqrt{\left(ac\right.}}}{b}}{20\,c}$$

input
```
integrate((c*x)^(8/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/20*(5*a*c^4*(sqrt(3)*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*
x)^(2/3)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(1/6)*b^(5/6)) - sqrt(3)*log(-s
qrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^(
1/3))/((a*c^2)^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) +
2*(c*x)^(1/3)*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^
(1/3)*b^(1/3))) - 2*arctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2*(c*x)^(1/3)
*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^(1/3)*b^(1/3)
)) - 4*arctan((c*x)^(1/3)*b^(1/3)/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sq
rt((a*c^2)^(1/3)*b^(1/3))))/b + 12*(c*x)^(5/3)*c^2/b)/c
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.15

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{1}{20}\,c^2\left(\frac{12\,(cx)^{\frac{2}{3}}\,x}{b}-\frac{20\left(\frac{ac^2}{b}\right)^{\frac{5}{6}}\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{bc}+\frac{5\sqrt{3}(ab^5c^2)^{\frac{5}{6}}\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\right.}{b^6c}\right.$$

input
```
integrate((c*x)^(8/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/20*c^2*(12*(c*x)^(2/3)*x/b - 20*(a*c^2/b)^(5/6)*arctan((c*x)^(1/3)/(a*c^
2/b)^(1/6))/(b*c) + 5*sqrt(3)*(a*b^5*c^2)^(5/6)*log(sqrt(3)*(a*c^2/b)^(1/6
)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/(b^6*c) - 5*sqrt(3)*(a*b^5*
c^2)^(5/6)*log(-sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2
/b)^(1/3))/(b^6*c) - 10*(a*b^5*c^2)^(5/6)*arctan((sqrt(3)*(a*c^2/b)^(1/6)
+ 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(b^6*c) - 10*(a*b^5*c^2)^(5/6)*arctan(-(
sqrt(3)*(a*c^2/b)^(1/6) - 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(b^6*c))
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.15

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{3\,c\,(c\,x)^{5/3}}{5\,b} + \frac{(-a)^{5/6}\,c^{8/3}\,\mathrm{atan}\left(\frac{b^{1/6}\,(c\,x)^{1/3}\,1\mathrm{i}}{(-a)^{1/6}\,c^{1/3}}\right)\,1\mathrm{i}}{b^{11/6}} - \frac{(-a)^{5/6}\,c^{8/3}\,\ln\left(972\,a^6\,c^{15}+972\,(-a)^{35/6}\,b^{1/6}\,c^{44/3}\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(c\,x)^{1/3}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{11/6}} - \frac{(-a)^{5/6}\,c^{8/3}\,\ln\left(972\,a^6\,c^{15}+972\,(-a)^{35/6}\,b^{1/6}\,c^{44/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)(c\,x)^{1/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{11/6}} + \frac{(-a)^{5/6}\,c^{8/3}\,\ln\left(972\,a^6\,c^{15}-1944\,(-a)^{35/6}\,b^{1/6}\,c^{44/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(c\,x)^{1/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{11/6}} + \frac{(-a)^{5/6}\,c^{8/3}\,\ln\left(972\,a^6\,c^{15}-1944\,(-a)^{35/6}\,b^{1/6}\,c^{44/3}\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)(c\,x)^{1/3}\right)\left(\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{11/6}}$$

input
```
int((c*x)^(8/3)/(a + b*x^2),x)
```

output
```
(3*c*(c*x)^(5/3))/(5*b) + ((-a)^(5/6)*c^(8/3)*atan((b^(1/6)*(c*x)^(1/3)*1i
)/((-a)^(1/6)*c^(1/3)))*1i)/b^(11/6) - ((-a)^(5/6)*c^(8/3)*log(972*a^6*c^1
5 + 972*(-a)^(35/6)*b^(1/6)*c^(44/3)*((3^(1/2)*1i)/2 - 1/2)*(c*x)^(1/3))*(
(3^(1/2)*1i)/2 - 1/2))/(2*b^(11/6)) - ((-a)^(5/6)*c^(8/3)*log(972*a^6*c^15
 + 972*(-a)^(35/6)*b^(1/6)*c^(44/3)*((3^(1/2)*1i)/2 + 1/2)*(c*x)^(1/3))*((
3^(1/2)*1i)/2 + 1/2))/(2*b^(11/6)) + ((-a)^(5/6)*c^(8/3)*log(972*a^6*c^15
- 1944*(-a)^(35/6)*b^(1/6)*c^(44/3)*((3^(1/2)*1i)/4 - 1/4)*(c*x)^(1/3))*((
3^(1/2)*1i)/4 - 1/4))/b^(11/6) + ((-a)^(5/6)*c^(8/3)*log(972*a^6*c^15 - 19
44*(-a)^(35/6)*b^(1/6)*c^(44/3)*((3^(1/2)*1i)/4 + 1/4)*(c*x)^(1/3))*((3^(1
/2)*1i)/4 + 1/4))/b^(11/6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{8/3}}{a+bx^2}\,dx = \frac{c^{\frac{8}{3}}\left(10b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-10b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-20b^{\frac{1}{6}}a^{\frac{7}{6}}atan\left(\frac{x^{\frac{1}{3}}b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)-5\cdots}{}$$

input
```
int((c*x)^(8/3)/(b*x^2+a),x)
```

output
```
(c**(2/3)*c**2*(10*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x
**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6)))*a - 10*b**(1/6)*a**(1/6)*atan((b**(
1/6)*a**(1/6)*sqrt(3) + 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6)))*a - 20*b
**(1/6)*a**(1/6)*atan((x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6)))*a - 5*b**(1
/6)*a**(1/6)*sqrt(3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3)
+ x**(2/3)*b**(1/3))*a + 5*b**(1/6)*a**(1/6)*sqrt(3)*log(x**(1/3)*b**(1/6)
*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3))*a + 12*x**(2/3)*a**(1/3)
*b*x))/(20*a**(1/3)*b**2)
```

3.344 $\int \frac{(cx)^{5/3}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 187

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{3c(cx)^{2/3}}{2b} + \frac{\sqrt{3}\sqrt[3]{a}c^{5/3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{2b^{4/3}} - \frac{\sqrt[3]{a}c^{5/3}\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2b^{4/3}} + \frac{\sqrt[3]{a}c^{5/3}\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{4/3}\right)}{4b^{4/3}}$$

output
```
3/2*c*(c*x)^(2/3)/b+1/2*3^(1/2)*a^(1/3)*c^(5/3)*arctan(1/3*(1-2*b^(1/3)*(c
*x)^(2/3)/a^(1/3)/c^(2/3))*3^(1/2))/b^(4/3)-1/2*a^(1/3)*c^(5/3)*ln(a^(1/3)
*c^(2/3)+b^(1/3)*(c*x)^(2/3))/b^(4/3)+1/4*a^(1/3)*c^(5/3)*ln(a^(2/3)*c^(4/
3)-a^(1/3)*b^(1/3)*c^(2/3)*(c*x)^(2/3)+b^(2/3)*(c*x)^(4/3))/b^(4/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.01

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{(cx)^{5/3}\left(6\sqrt[3]{b}x^{2/3}+2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{2/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{2/3}\right)+\sqrt[3]{a}\log\left(\right.\right.}{4b^{4/3}x^{5/3}}$$

input
```
Integrate[(c*x)^(5/3)/(a + b*x^2),x]
```

output
```
((c*x)^(5/3)*(6*b^(1/3)*x^(2/3) + 2*Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*b^(1/3)
*x^(2/3))/a^(1/3))/Sqrt[3]] - 2*a^(1/3)*Log[a^(1/3) + b^(1/3)*x^(2/3)] + a
^(1/3)*Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3) + b^(1/3)*x^(2/3)] +
a^(1/3)*Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3) + b^(1/3)*x^(2/3)]))
/(4*b^(4/3)*x^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.03, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {262, 266, 27, 807, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/3}}{a+bx^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{ac^2\int \frac{1}{\sqrt[3]{cx}(bx^2+a)}dx}{b}$$

$$\Big\downarrow \text{266}$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac\int \frac{c^2\sqrt[3]{cx}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\int \frac{\sqrt[3]{cx}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \; 807$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\int \frac{1}{ac^2+bxc}d(cx)^{2/3}}{2b}$$

$$\Big\downarrow \; 750$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\left(\frac{\int \frac{2\sqrt[3]{a}c^{2/3}-\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}\right)}{2b}$$

$$\Big\downarrow \; 16$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\left(\frac{\int \frac{2\sqrt[3]{a}c^{2/3}-\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)}{2b}$$

$$\Big\downarrow \; 1142$$

$$\frac{3c(cx)^{2/3}}{2b}-$$

$$\frac{3ac^3\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}\right.}{3a}\right.}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3c(cx)^{2/3}}{2b}-3ac^3\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int\frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{\int\frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2}\right.}{3a^{2}}\right.}{2b}$$

$\downarrow$ 27

$$\frac{\frac{3c(cx)^{2/3}}{2b}-3ac^3\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int\frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{1}{2}\int\frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}\right.}{3}\right.}{2b}$$

$\downarrow$ 1082

$$\frac{\frac{3c(cx)^{2/3}}{2b}-3ac^3\left(\frac{\frac{1}{2}\int\frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{3\int\frac{1}{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}-4}d\left(1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)}{2b}$$

$\downarrow$ 217

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)}{2b}$$

$$\Big\downarrow \; 1103$$

$$\frac{3c(cx)^{2/3}}{2b}-\frac{3ac^3\left(\frac{-\frac{\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{2/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)}{2b}$$

input
```
Int[(c*x)^(5/3)/(a + b*x^2),x]
```

output
```
(3*c*(c*x)^(2/3))/(2*b) - (3*a*c^3*(Log[a^(1/3)*c^(2/3) + b^(1/3)*(c*x)^(2
/3)]/(3*a^(2/3)*b^(1/3)*c^(4/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x
)^(2/3))/(a^(1/3)*c^(2/3)))/Sqrt[3]])/b^(1/3)) - Log[a^(2/3)*c^(4/3) + b^(
2/3)*(c*x)^(2/3) - a^(1/3)*b^(1/3)*c^(2/3)*(c*x)^(2/3)]/(2*b^(1/3)))/(3*a^
(2/3)*c^(4/3))))/(2*b)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{c\left(a c^2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}-\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}}\right)+a c^2\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)-\frac{a c^2\ln\left(cx(cx)^{\frac{1}{3}}-\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right.\right.}{2}\right.}{2\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}b^2}$
derivativedivides	$3c\left(\frac{(cx)^{\frac{2}{3}}}{2b}-\frac{\left(\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}\right)a}{2b}\right.$
default	$3c\left(\frac{(cx)^{\frac{2}{3}}}{2b}-\frac{\left(\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}\right)}{6b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{2}{3}}}\right)a}{2b}\right.$

input
```
int((c*x)^(5/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c/(a*c^2/b)^(2/3)*(a*c^2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(c*x)^(2/3)-(a
*c^2/b)^(1/3))/(a*c^2/b)^(1/3))+a*c^2*ln((c*x)^(2/3)+(a*c^2/b)^(1/3))-1/2*
a*c^2*ln(c*x*(c*x)^(1/3)-(a*c^2/b)^(1/3)*(c*x)^(2/3)+(a*c^2/b)^(2/3))-3*(c
*x)^(2/3)*(a*c^2/b)^(2/3)*b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.84

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{2\sqrt{3}\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}} c \arctan\left(-\frac{\sqrt{3}ac^2-2\sqrt{3}\left(-\frac{ac^2}{b}\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}b}{3\,ac^2}\right) - \left(-\frac{ac^2}{b}\right)^{\frac{1}{3}} c\log\left((cx)^{\frac{1}{3}}\,cx + \left(-\frac{ac^2}{b}\right)^{\frac{1}{3}} \cdots\right.}{4\,b}$$

input
```
integrate((c*x)^(5/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(2*sqrt(3)*(-a*c^2/b)^(1/3)*c*arctan(-1/3*(sqrt(3)*a*c^2 - 2*sqrt(3)*(
-a*c^2/b)^(2/3)*(c*x)^(2/3)*b)/(a*c^2)) - (-a*c^2/b)^(1/3)*c*log((c*x)^(1/
3)*c*x + (-a*c^2/b)^(1/3)*(c*x)^(2/3) + (-a*c^2/b)^(2/3)) + 2*(-a*c^2/b)^(
1/3)*c*log((c*x)^(2/3) - (-a*c^2/b)^(1/3)) + 6*(c*x)^(2/3)*c)/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{2\sqrt[3]{a}c^{\frac{5}{3}}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3b^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)} - \frac{2\sqrt[3]{a}c^{\frac{5}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3b^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{2\sqrt[3]{a}c^{\frac{5}{3}}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3b^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)} + \frac{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}{b\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((c*x)**(5/3)/(b*x**2+a),x)
```

output
```
2*a**(1/3)*c**(5/3)*exp(-I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(I*pi/
3)/a**(1/3))*gamma(4/3)/(3*b**(4/3)*gamma(7/3)) - 2*a**(1/3)*c**(5/3)*log(
1 - b**(1/3)*x**(2/3)*exp_polar(I*pi)/a**(1/3))*gamma(4/3)/(3*b**(4/3)*gam
ma(7/3)) + 2*a**(1/3)*c**(5/3)*exp(I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_p
olar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*b**(4/3)*gamma(7/3)) + 2*c**(5/3)*x
**(2/3)*gamma(4/3)/(b*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.89

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = -\frac{\frac{2\sqrt{3}ac^4\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b^2} - \frac{ac^4\log\left((cx)^{\frac{4}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b^2} + \frac{2ac^4\log\left((cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b^2} - \frac{6(cx)^{\frac{2}{3}}c^2}{b}}{4c}$$

input
```
integrate((c*x)^(5/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*(2*sqrt(3)*a*c^4*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) - (a*c^2/b)^(1/3))
/(a*c^2/b)^(1/3))/((a*c^2/b)^(2/3)*b^2) - a*c^4*log((c*x)^(4/3) - (a*c^2/b
)^(1/3)*(c*x)^(2/3) + (a*c^2/b)^(2/3))/((a*c^2/b)^(2/3)*b^2) + 2*a*c^4*log
((c*x)^(2/3) + (a*c^2/b)^(1/3))/((a*c^2/b)^(2/3)*b^2) - 6*(c*x)^(2/3)*c^2/
b)/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.93

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{\frac{2\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}c^2\log\left(\left|(cx)^{\frac{2}{3}}-\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right|\right)}{b} - \frac{2\sqrt{3}\left(-ab^2c^2\right)^{\frac{1}{3}}c^2\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{b^2} + \frac{6(cx)^{\frac{2}{3}}c^2}{b} - \frac{(-a\cdots}{\cdots}}{4c}$$

input
```
integrate((c*x)^(5/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(2*(-a*c^2/b)^(1/3)*c^2*log(abs((c*x)^(2/3) - (-a*c^2/b)^(1/3)))/b - 2
*sqrt(3)*(-a*b^2*c^2)^(1/3)*c^2*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) + (-a*c^
2/b)^(1/3))/(-a*c^2/b)^(1/3))/b^2 + 6*(c*x)^(2/3)*c^2/b - (-a*b^2*c^2)^(1/
3)*c^2*log((c*x)^(1/3)*c*x + (-a*c^2/b)^(1/3)*(c*x)^(2/3) + (-a*c^2/b)^(2/
3))/b^2)/c
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{3\,c\,(c\,x)^{2/3}}{2\,b} + \frac{(-a)^{1/3}\,c^{5/3}\,\ln\left(162\,(-a)^{10/3}\,b^{2/3}\,c^{29/3}+162\,a^3\,b\,c^9\,(c\,x)^{2/3}\right)}{2\,b^{4/3}} - \frac{(-a)^{1/3}\,c^{5/3}\,\ln\left(162\,(-a)^{10/3}\,b^{2/3}\,c^{29/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-162\,a^3\,b\,c^9\,(c\,x)^{2/3}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2\,b^{4/3}} + \frac{(-a)^{1/3}\,c^{5/3}\,\ln\left(324\,(-a)^{10/3}\,b^{2/3}\,c^{29/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)+162\,a^3\,b\,c^9\,(c\,x)^{2/3}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)}{b^{4/3}}$$

input
```
int((c*x)^(5/3)/(a + b*x^2),x)
```

output
```
(3*c*(c*x)^(2/3))/(2*b) + ((-a)^(1/3)*c^(5/3)*log(162*(-a)^(10/3)*b^(2/3)*
c^(29/3) + 162*a^3*b*c^9*(c*x)^(2/3)))/(2*b^(4/3)) - ((-a)^(1/3)*c^(5/3)*l
og(162*(-a)^(10/3)*b^(2/3)*c^(29/3)*((3^(1/2)*1i)/2 + 1/2) - 162*a^3*b*c^9
*(c*x)^(2/3))*((3^(1/2)*1i)/2 + 1/2))/(2*b^(4/3)) + ((-a)^(1/3)*c^(5/3)*lo
g(324*(-a)^(10/3)*b^(2/3)*c^(29/3)*((3^(1/2)*1i)/4 - 1/4) + 162*a^3*b*c^9*
(c*x)^(2/3))*((3^(1/2)*1i)/4 - 1/4))/b^(4/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^{5/3}}{a+bx^2}\,dx = \frac{c^{\frac{5}{3}}\left(2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)a+2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)a+6x^{\frac{2}{3}}b^{\frac{1}{3}}a^{\frac{2}{3}}-2\log\left(a^{\frac{1}{3}}+x\right.\right.}{4b^{\frac{4}{3}}a^{\frac{2}{3}}}$$

input
```
int((c*x)^(5/3)/(b*x^2+a),x)
```

output
```
(c**(2/3)*c*(2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1/3)*b**(1/
3))/(b**(1/6)*a**(1/6)))*a + 2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2
*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6)))*a + 6*x**(2/3)*b**(1/3)*a**(2/3)
- 2*log(a**(1/3) + x**(2/3)*b**(1/3))*a + log( - x**(1/3)*b**(1/6)*a**(1/6
)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3))*a + log(x**(1/3)*b**(1/6)*a**(1/
6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3))*a))/(4*b**(1/3)*a**(2/3)*b)
```

3.345 $\int \frac{(cx)^{2/3}}{a+bx^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 221

$$\int \frac{(cx)^{2/3}}{a+bx^2} dx = \frac{c^{2/3}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt[6]{a}b^{5/6}} - \frac{c^{2/3}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt[6]{a}b^{5/6}} + \frac{c^{2/3}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2\sqrt[6]{a}b^{5/6}} - \frac{\sqrt{3}c^{2/3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2\sqrt[6]{a}b^{5/6}}$$

output
```
c^(2/3)*arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(1/6)/b^(5/6)+1/2*c^
(2/3)*arctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(1/6)/b^(5/
6)+1/2*c^(2/3)*arctan(3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(1/
6)/b^(5/6)-1/2*3^(1/2)*c^(2/3)*arctanh(3^(1/2)*a^(1/6)*b^(1/6)*c^(1/3)*(c*
x)^(1/3)/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3)))/a^(1/6)/b^(5/6)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.58

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = -\frac{(cx)^{2/3}\left(\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-2\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)+\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[3]{a}+\sqrt[3]{b}x^{2/3}}\right)\right)}{2\sqrt[6]{a}b^{5/6}x^{2/3}}$$

input
```
Integrate[(c*x)^(2/3)/(a + b*x^2),x]
```

output
```
-1/2*((c*x)^(2/3)*(ArcTan[a^(1/6)/(b^(1/6)*x^(1/3)) - (b^(1/6)*x^(1/3))/a^
(1/6)] - 2*ArcTan[(b^(1/6)*x^(1/3))/a^(1/6)] + Sqrt[3]*ArcTanh[(Sqrt[3]*a^
(1/6)*b^(1/6)*x^(1/3))/(a^(1/3) + b^(1/3)*x^(2/3))]))/(a^(1/6)*b^(5/6)*x^(
2/3))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {266, 27, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{2/3}}{a+bx^2}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{3\int \frac{c^2(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{c}$$

$$\Big\downarrow \text{27}$$

$$3c\int \frac{(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}$$

$$\Big\downarrow \text{824}$$

$$3c\left(\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\int -\frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{}}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\right.}}{3\sqrt[6]{a}b^{2/}}\right.$$

$$\downarrow 27$$

$$3c\left(\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}\right.$$

$$\downarrow 218$$

$$3c\left(-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\arctan\left(\frac{\sqrt[6]{}}{\sqrt[6]{}}\right.}{3\sqrt[6]{a}b^{5/6}}\right.$$

$$\downarrow 1142$$

$$3c\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\right.$$

$$\downarrow 25$$

$$3c\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\right.$$

$$\downarrow 27$$

$$3c\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\right.$$

$\downarrow$ 1082

$$3c\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\int \frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{}}}{}\right.$$

$\downarrow$ 217

$$3c\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}}}{}\right.$$

$\downarrow$ 1103

$$3c\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{}\right.}{2\sqrt[6]{b}}}{}\right.$$

input
```
Int[(c*x)^(2/3)/(a + b*x^2),x]
```

output
```
3*c*(ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1/6)*c^(1/3))]/(3*a^(1/6)*b^(5/6)*c^
(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6)*c^(
1/3)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3)*c^(2/3) - Sqrt[3]*a^(1/6)*b^(1/6)*c
^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(2*b^(1/6)))/(6*a^(1/6)*b^(2/3)
*c^(1/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*(c*x)^(1/3))/(Sqrt[3]*a^(1/6
)*c^(1/3)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3)*c^(2/3) + Sqrt[3]*a^(1/6)*b^(
1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(2*b^(1/6)))/(6*a^(1/6)*b
^(2/3)*c^(1/3)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$-\frac{c\left(\frac{\sqrt{3}\,\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{2}-\frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}-(cx)^{\frac{2}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{2}+\arctan\left(\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)}{\left(\frac{a\,c^{2}}{b}\right.}\right.\right.}{2\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}b}$
derivativedivides	$3c\left(\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}\right.$
default	$3c\left(\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^{2}}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}-\sqrt{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{6}}}\right.$

input
```
int((c*x)^(2/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(1/2*3^(1/2)*ln((c*x)^(2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)+(a*
c^2/b)^(1/3))-1/2*3^(1/2)*ln(3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3)-(c*x)^(2/
3)-(a*c^2/b)^(1/3))+arctan((3^(1/2)*(a*c^2/b)^(1/6)-2*(c*x)^(1/3))/(a*c^2/
b)^(1/6))-2*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))-arctan((3^(1/2)*(a*c^2/b)^
(1/6)+2*(c*x)^(1/3))/(a*c^2/b)^(1/6)))/(a*c^2/b)^(1/6)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(145) = 290$.

Time = 0.07 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.50

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = -\frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left((cx)^{\frac{1}{3}}c^3+\frac{1}{2}\left(\sqrt{-3}ab^4+ab^4\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}\right) + \frac{1}{4}\left(\sqrt{-3}-1\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left((cx)^{\frac{1}{3}}c^3-\frac{1}{2}\left(\sqrt{-3}ab^4+ab^4\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}\right) - \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left((cx)^{\frac{1}{3}}c^3+\frac{1}{2}\left(\sqrt{-3}ab^4-ab^4\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}\right) + \frac{1}{4}\left(\sqrt{-3}+1\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left((cx)^{\frac{1}{3}}c^3-\frac{1}{2}\left(\sqrt{-3}ab^4-ab^4\right)\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}\right) + \frac{1}{2}\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left(ab^4\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}+(cx)^{\frac{1}{3}}c^3\right) - \frac{1}{2}\left(-\frac{c^4}{ab^5}\right)^{\frac{1}{6}}\log\left(-ab^4\left(-\frac{c^4}{ab^5}\right)^{\frac{5}{6}}+(cx)^{\frac{1}{3}}c^3\right)$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-3) - 1)*(-c^4/(a*b^5))^(1/6)*log((c*x)^(1/3)*c^3 + 1/2*(sqrt(-
3)*a*b^4 + a*b^4)*(-c^4/(a*b^5))^(5/6)) + 1/4*(sqrt(-3) - 1)*(-c^4/(a*b^5)
)^(1/6)*log((c*x)^(1/3)*c^3 - 1/2*(sqrt(-3)*a*b^4 + a*b^4)*(-c^4/(a*b^5))^
(5/6)) - 1/4*(sqrt(-3) + 1)*(-c^4/(a*b^5))^(1/6)*log((c*x)^(1/3)*c^3 + 1/2
*(sqrt(-3)*a*b^4 - a*b^4)*(-c^4/(a*b^5))^(5/6)) + 1/4*(sqrt(-3) + 1)*(-c^4
/(a*b^5))^(1/6)*log((c*x)^(1/3)*c^3 - 1/2*(sqrt(-3)*a*b^4 - a*b^4)*(-c^4/(
a*b^5))^(5/6)) + 1/2*(-c^4/(a*b^5))^(1/6)*log(a*b^4*(-c^4/(a*b^5))^(5/6) +
 (c*x)^(1/3)*c^3) - 1/2*(-c^4/(a*b^5))^(1/6)*log(-a*b^4*(-c^4/(a*b^5))^(5/
6) + (c*x)^(1/3)*c^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.66

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = \frac{5c^{\frac{2}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)} + \frac{5ic^{\frac{2}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)} + \frac{5c^{\frac{2}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)} - \frac{5c^{\frac{2}{3}}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)} - \frac{5ic^{\frac{2}{3}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)} - \frac{5c^{\frac{2}{3}}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(\frac{5}{6}\right)}{12\sqrt[6]{a}b^{\frac{5}{6}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(2/3)/(b*x**2+a),x)
```

output
```
5*c**(2/3)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/6)/a**(1/6
))*gamma(5/6)/(12*a**(1/6)*b**(5/6)*gamma(11/6)) + 5*I*c**(2/3)*log(1 - b*
*(1/6)*x**(1/3)*exp_polar(I*pi/2)/a**(1/6))*gamma(5/6)/(12*a**(1/6)*b**(5/
6)*gamma(11/6)) + 5*c**(2/3)*exp(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_p
olar(5*I*pi/6)/a**(1/6))*gamma(5/6)/(12*a**(1/6)*b**(5/6)*gamma(11/6)) - 5
*c**(2/3)*exp(I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(7*I*pi/6)/a**(1/
6))*gamma(5/6)/(12*a**(1/6)*b**(5/6)*gamma(11/6)) - 5*I*c**(2/3)*log(1 - b
**(1/6)*x**(1/3)*exp_polar(3*I*pi/2)/a**(1/6))*gamma(5/6)/(12*a**(1/6)*b**
(5/6)*gamma(11/6)) - 5*c**(2/3)*exp(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*ex
p_polar(11*I*pi/6)/a**(1/6))*gamma(5/6)/(12*a**(1/6)*b**(5/6)*gamma(11/6))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.20

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx =$$
$$-\frac{1}{4}c\left(\frac{\sqrt{3}\log\left(\sqrt{3}(ac^2)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+(ac^2)^{\frac{1}{3}}\right)}{(ac^2)^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}(ac^2)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+(ac^2\right.}{(ac^2)^{\frac{1}{6}}b^{\frac{5}{6}}}\right.$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/4*c*(sqrt(3)*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3
)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*
(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^(1/3))/(
(a*c^2)^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) + 2*(c*x)
^(1/3)*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^(1/3)*b
^(1/3))) - 2*arctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2*(c*x)^(1/3)*b^(1/3
))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^(1/3)*b^(1/3))) - 4*
arctan((c*x)^(1/3)*b^(1/3)/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c
^2)^(1/3)*b^(1/3))))
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.16

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = \frac{\frac{4\left(\frac{ac^2}{b}\right)^{\frac{5}{6}}\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}}\right)}{a}-\frac{\sqrt{3}(ab^5c^2)^{\frac{5}{6}}\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{ab^5}+\frac{\sqrt{3}(ab^5c^2)^{\frac{5}{6}}\log\left(-\sqrt{3}\left(\frac{ac^2}{b}\right.\right.}{a}}{}$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/4*(4*(a*c^2/b)^(5/6)*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))/a - sqrt(3)*(a*
b^5*c^2)^(5/6)*log(sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*
c^2/b)^(1/3))/(a*b^5) + sqrt(3)*(a*b^5*c^2)^(5/6)*log(-sqrt(3)*(a*c^2/b)^(
1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/(a*b^5) + 2*(a*b^5*c^2)^
(5/6)*arctan((sqrt(3)*(a*c^2/b)^(1/6) + 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(a
*b^5) + 2*(a*b^5*c^2)^(5/6)*arctan(-(sqrt(3)*(a*c^2/b)^(1/6) - 2*(c*x)^(1/
3))/(a*c^2/b)^(1/6))/(a*b^5))/c
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = \frac{c^{2/3}\,\mathrm{atan}\left(\frac{b^{1/6}\,{\left(c\,x\right)}^{1/3}\,1{}\mathrm{i}}{{\left(-a\right)}^{1/6}\,c^{1/3}}\right)\,1{}\mathrm{i}}{{\left(-a\right)}^{1/6}\,b^{5/6}} - \frac{c^{2/3}\,\ln\left(972\,a^3\,b^3\,c^9-972\,{\left(-a\right)}^{17/6}\,b^{19/6}\,c^{26/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(c\,x\right)}^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,{\left(-a\right)}^{1/6}\,b^{5/6}} - \frac{c^{2/3}\,\ln\left(972\,a^3\,b^3\,c^9-972\,{\left(-a\right)}^{17/6}\,b^{19/6}\,c^{26/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{\left(c\,x\right)}^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,{\left(-a\right)}^{1/6}\,b^{5/6}} + \frac{c^{2/3}\,\ln\left(972\,a^3\,b^3\,c^9+1944\,{\left(-a\right)}^{17/6}\,b^{19/6}\,c^{26/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{\left(c\,x\right)}^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{{\left(-a\right)}^{1/6}\,b^{5/6}} + \frac{c^{2/3}\,\ln\left(972\,a^3\,b^3\,c^9+1944\,{\left(-a\right)}^{17/6}\,b^{19/6}\,c^{26/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{\left(c\,x\right)}^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{{\left(-a\right)}^{1/6}\,b^{5/6}}$$

input
```
int((c*x)^(2/3)/(a + b*x^2),x)
```

output
```
(c^(2/3)*atan((b^(1/6)*(c*x)^(1/3)*1i)/((-a)^(1/6)*c^(1/3)))*1i)/((-a)^(1/
6)*b^(5/6)) - (c^(2/3)*log(972*a^3*b^3*c^9 - 972*(-a)^(17/6)*b^(19/6)*c^(2
6/3)*((3^(1/2)*1i)/2 - 1/2)*(c*x)^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*(-a)^(
1/6)*b^(5/6)) - (c^(2/3)*log(972*a^3*b^3*c^9 - 972*(-a)^(17/6)*b^(19/6)*c^
(26/3)*((3^(1/2)*1i)/2 + 1/2)*(c*x)^(1/3))*((3^(1/2)*1i)/2 + 1/2))/(2*(-a)
^(1/6)*b^(5/6)) + (c^(2/3)*log(972*a^3*b^3*c^9 + 1944*(-a)^(17/6)*b^(19/6)
*c^(26/3)*((3^(1/2)*1i)/4 - 1/4)*(c*x)^(1/3))*((3^(1/2)*1i)/4 - 1/4))/((-a
)^(1/6)*b^(5/6)) + (c^(2/3)*log(972*a^3*b^3*c^9 + 1944*(-a)^(17/6)*b^(19/6
)*c^(26/3)*((3^(1/2)*1i)/4 + 1/4)*(c*x)^(1/3))*((3^(1/2)*1i)/4 + 1/4))/((-
a)^(1/6)*b^(5/6))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^{2/3}}{a+bx^2}\,dx = \frac{c^{\frac{2}{3}}\left(-2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+4atan\left(\frac{x^{\frac{1}{3}}b^{\frac{1}{6}}}{a^{\frac{1}{6}}}\right)+\sqrt{3}\log\left(-x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\right.\right.}{4b^{\frac{5}{6}}a^{\frac{1}{6}}}$$

input
```
int((c*x)^(2/3)/(b*x^2+a),x)
```

output
```
(c**(2/3)*b**(1/6)*a**(1/6)*( - 2*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(
1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) + 2*atan((b**(1/6)*a**(1/6)*sqrt(3) +
2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) + 4*atan((x**(1/3)*b**(1/3))/(b*
*(1/6)*a**(1/6))) + sqrt(3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a*
*(1/3) + x**(2/3)*b**(1/3)) - sqrt(3)*log(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(
3) + a**(1/3) + x**(2/3)*b**(1/3))))/(4*a**(1/3)*b)
```

3.346 $\int \frac{1}{\sqrt[3]{cx}(a+bx^2)} dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{2a^{2/3}\sqrt[3]{b}\sqrt[3]{c}} + \frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2a^{2/3}\sqrt[3]{b}\sqrt[3]{c}} - \frac{\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{4/3}\right)}{4a^{2/3}\sqrt[3]{b}\sqrt[3]{c}}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(1-2*b^(1/3)*(c*x)^(2/3)/a^(1/3)/c^(2/3))*3^(1/2))
/a^(2/3)/b^(1/3)/c^(1/3)+1/2*ln(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3))/a^(2/
3)/b^(1/3)/c^(1/3)-1/4*ln(a^(2/3)*c^(4/3)-a^(1/3)*b^(1/3)*c^(2/3)*(c*x)^(2
/3)+b^(2/3)*(c*x)^(4/3))/a^(2/3)/b^(1/3)/c^(1/3)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = -\frac{\sqrt[3]{x}\left(2\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}x^{2/3}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x^{2/3}\right)+\log\left(\sqrt[3]{a}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{x}+\sqrt[3]{b}x^{2/3}\right)+\log\right.}{4a^{2/3}\sqrt[3]{b}\sqrt[3]{cx}}$$

input
```
Integrate[1/((c*x)^(1/3)*(a + b*x^2)),x]
```

output
```
-1/4*(x^(1/3)*(2*Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*x^(2/3))/a^(1/3))/Sqrt[3]]
 - 2*Log[a^(1/3) + b^(1/3)*x^(2/3)] + Log[a^(1/3) - Sqrt[3]*a^(1/6)*b^(1/6
)*x^(1/3) + b^(1/3)*x^(2/3)] + Log[a^(1/3) + Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/
3) + b^(1/3)*x^(2/3)]))/(a^(2/3)*b^(1/3)*(c*x)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.647, Rules used = {266, 27, 807, 750, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx$$

$$\Big\downarrow \ 266$$

$$\frac{3\int \frac{c^2\sqrt[3]{cx}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{c}$$

$$\Big\downarrow \ 27$$

$$3c\int \frac{\sqrt[3]{cx}}{bx^2c^2+ac^2}d\sqrt[3]{cx}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3}{2}c\int \frac{1}{ac^2+bxc}d(cx)^{2/3}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{3}{2}c\left(\frac{\int \frac{2\sqrt[3]{a}c^{2/3}-\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{2}c\left(\frac{\int \frac{2\sqrt[3]{a}c^{2/3}-\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log}{}\right.$$

$$\Big\downarrow \text{ 25}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}\right)}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{2\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\right.}{}\right.$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}c\left(\frac{\frac{3}{2}\sqrt[3]{a}c^{2/3}\int \frac{1}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}}{3a^{2/3}c^{4/3}}+\right.$$

$\Big\downarrow$ 1082

$$\frac{3}{2}c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}+\frac{3\int \frac{1}{\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}-4}d\left(1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx\right.}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right.$$

$\Big\downarrow$ 217

$$\frac{3}{2}c\left(\frac{\frac{1}{2}\int \frac{\sqrt[3]{a}c^{2/3}-2\sqrt[3]{b}(cx)^{2/3}}{a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}(cx)^{2/3}c^{2/3}+b^{2/3}(cx)^{2/3}}d(cx)^{2/3}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)$$

$\Big\downarrow$ 1103

$$\frac{3}{2}c\left(\frac{-\frac{\log\left(a^{2/3}c^{4/3}-\sqrt[3]{a}\sqrt[3]{b}c^{2/3}(cx)^{2/3}+b^{2/3}(cx)^{2/3}\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a}c^{2/3}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3a^{2/3}c^{4/3}}+\frac{\log\left(\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{3a^{2/3}\sqrt[3]{b}c^{4/3}}\right)$$

input
```
Int[1/((c*x)^(1/3)*(a + b*x^2)),x]
```

output
```
(3*c*(Log[a^(1/3)*c^(2/3) + b^(1/3)*(c*x)^(2/3)]/(3*a^(2/3)*b^(1/3)*c^(4/3
)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*(c*x)^(2/3))/(a^(1/3)*c^(2/3)))/Sq
rt[3]])/b^(1/3)) - Log[a^(2/3)*c^(4/3) + b^(2/3)*(c*x)^(2/3) - a^(1/3)*b^(
1/3)*c^(2/3)*(c*x)^(2/3)]/(2*b^(1/3)))/(3*a^(2/3)*c^(4/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{c\left(2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(2(cx)^{\frac{2}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}\right)+2\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)-\ln\left(cx(cx)^{\frac{1}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)\right)}{4b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}$
derivativedivides	$3c\left(\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{12b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}\right)$
default	$3c\left(\frac{\ln\left((cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left((cx)^{\frac{4}{3}}-\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}\right)}{12b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2(cx)^{\frac{2}{3}}}{\left(\frac{a\,c^{2}}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{6b\left(\frac{a\,c^{2}}{b}\right)^{\frac{2}{3}}}\right)$

input
```
int(1/(c*x)^(1/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*c*(2*3^(1/2)*arctan(1/3*3^(1/2)*(2*(c*x)^(2/3)-(a*c^2/b)^(1/3))/(a*c^2
/b)^(1/3))+2*ln((c*x)^(2/3)+(a*c^2/b)^(1/3))-ln(c*x*(c*x)^(1/3)-(a*c^2/b)^
(1/3)*(c*x)^(2/3)+(a*c^2/b)^(2/3)))/b/(a*c^2/b)^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx$$
$$= \left[\frac{\sqrt{3}abc\sqrt{-\frac{(a^2bc)^{\frac{1}{3}}}{bc}}\log\left(\frac{2\,abcx^2-a^2c-3\,(a^2bc)^{\frac{1}{3}}(cx)^{\frac{2}{3}}a+\sqrt{3}\left(2\,(cx)^{\frac{1}{3}}abcx-(a^2bc)^{\frac{1}{3}}ac+(a^2bc)^{\frac{2}{3}}(cx)^{\frac{2}{3}}\right)\sqrt{-\frac{(a^2bc)^{\frac{1}{3}}}{bc}}}{bx^2+a}\right)-(a^2bc}{4\,a^2bc}\right.$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*a*b*c*sqrt(-(a^2*b*c)^(1/3)/(b*c))*log((2*a*b*c*x^2 - a^2*c
- 3*(a^2*b*c)^(1/3)*(c*x)^(2/3)*a + sqrt(3)*(2*(c*x)^(1/3)*a*b*c*x - (a^2*
b*c)^(1/3)*a*c + (a^2*b*c)^(2/3)*(c*x)^(2/3))*sqrt(-(a^2*b*c)^(1/3)/(b*c))
)/(b*x^2 + a)) - (a^2*b*c)^(2/3)*log((c*x)^(1/3)*a*b*c*x + (a^2*b*c)^(1/3)
*a*c - (a^2*b*c)^(2/3)*(c*x)^(2/3)) + 2*(a^2*b*c)^(2/3)*log((c*x)^(2/3)*a*
b + (a^2*b*c)^(2/3)))/(a^2*b*c), 1/4*(2*sqrt(3)*a*b*c*sqrt((a^2*b*c)^(1/3)
/(b*c))*arctan(-1/3*sqrt(3)*((a^2*b*c)^(1/3)*a*c - 2*(a^2*b*c)^(2/3)*(c*x)
^(2/3))*sqrt((a^2*b*c)^(1/3)/(b*c))/(a^2*c)) - (a^2*b*c)^(2/3)*log((c*x)^(
1/3)*a*b*c*x + (a^2*b*c)^(1/3)*a*c - (a^2*b*c)^(2/3)*(c*x)^(2/3)) + 2*(a^2
*b*c)^(2/3)*log((c*x)^(2/3)*a*b + (a^2*b*c)^(2/3)))/(a^2*b*c)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = -\frac{e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{6a^{\frac{2}{3}}\sqrt[3]{b}\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)} + \frac{\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{6a^{\frac{2}{3}}\sqrt[3]{b}\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)} - \frac{e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{b}x^{\frac{2}{3}}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{1}{3}\right)}{6a^{\frac{2}{3}}\sqrt[3]{b}\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/(c*x)**(1/3)/(b*x**2+a),x)
```

output
```
-exp(-I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(I*pi/3)/a**(1/3))*gamma(
1/3)/(6*a**(2/3)*b**(1/3)*c**(1/3)*gamma(4/3)) + log(1 - b**(1/3)*x**(2/3)
*exp_polar(I*pi)/a**(1/3))*gamma(1/3)/(6*a**(2/3)*b**(1/3)*c**(1/3)*gamma(
4/3)) - exp(I*pi/3)*log(1 - b**(1/3)*x**(2/3)*exp_polar(5*I*pi/3)/a**(1/3)
)*gamma(1/3)/(6*a**(2/3)*b**(1/3)*c**(1/3)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx$$
$$= \frac{\frac{2\sqrt{3}c^2\arctan\left(\frac{\sqrt{3}\left(2\,(cx)^{\frac{2}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b} - \frac{c^2\log\left((cx)^{\frac{4}{3}}-\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b} + \frac{2\,c^2\log\left((cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{\left(\frac{ac^2}{b}\right)^{\frac{2}{3}}b}}{4\,c}$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(2*sqrt(3)*c^2*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) - (a*c^2/b)^(1/3))/(a
*c^2/b)^(1/3))/((a*c^2/b)^(2/3)*b) - c^2*log((c*x)^(4/3) - (a*c^2/b)^(1/3)
*(c*x)^(2/3) + (a*c^2/b)^(2/3))/((a*c^2/b)^(2/3)*b) + 2*c^2*log((c*x)^(2/3
) + (a*c^2/b)^(1/3))/((a*c^2/b)^(2/3)*b))/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = -\frac{\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\log\left(\left|(cx)^{\frac{2}{3}}-\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right|\right)}{2\,ac} + \frac{\sqrt{3}(-ab^2c^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(cx)^{\frac{2}{3}}+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}}\right)}{2\,abc} + \frac{(-ab^2c^2)^{\frac{1}{3}}\log\left((cx)^{\frac{1}{3}}\,cx+\left(-\frac{ac^2}{b}\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}+\left(-\frac{ac^2}{b}\right)^{\frac{2}{3}}\right)}{4\,abc}$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(-a*c^2/b)^(1/3)*log(abs((c*x)^(2/3) - (-a*c^2/b)^(1/3)))/(a*c) + 1/2
*sqrt(3)*(-a*b^2*c^2)^(1/3)*arctan(1/3*sqrt(3)*(2*(c*x)^(2/3) + (-a*c^2/b)
^(1/3))/(-a*c^2/b)^(1/3))/(a*b*c) + 1/4*(-a*b^2*c^2)^(1/3)*log((c*x)^(1/3)
*c*x + (-a*c^2/b)^(1/3)*(c*x)^(2/3) + (-a*c^2/b)^(2/3))/(a*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = \frac{\ln\left(162\,b^4\,c^3\,(c\,x)^{2/3}+162\,a^{1/3}\,b^{11/3}\,c^{11/3}\right)}{2\,a^{2/3}\,b^{1/3}\,c^{1/3}} + \frac{\ln\left(162\,b^4\,c^3\,(c\,x)^{2/3}+81\,a^{1/3}\,b^{11/3}\,c^{11/3}\,(-1+\sqrt{3}\,1\mathrm{i})\right)\,(-1+\sqrt{3}\,1\mathrm{i})}{4\,a^{2/3}\,b^{1/3}\,c^{1/3}} - \frac{\ln\left(162\,b^4\,c^3\,(c\,x)^{2/3}-81\,a^{1/3}\,b^{11/3}\,c^{11/3}\,(1+\sqrt{3}\,1\mathrm{i})\right)\,(1+\sqrt{3}\,1\mathrm{i})}{4\,a^{2/3}\,b^{1/3}\,c^{1/3}}$$

input
```
int(1/((c*x)^(1/3)*(a + b*x^2)),x)
```

output
```
log(162*b^4*c^3*(c*x)^(2/3) + 162*a^(1/3)*b^(11/3)*c^(11/3))/(2*a^(2/3)*b^
(1/3)*c^(1/3)) + (log(162*b^4*c^3*(c*x)^(2/3) + 81*a^(1/3)*b^(11/3)*c^(11/
3)*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/(4*a^(2/3)*b^(1/3)*c^(1/3)) - (log(
162*b^4*c^3*(c*x)^(2/3) - 81*a^(1/3)*b^(11/3)*c^(11/3)*(3^(1/2)*1i + 1))*(
3^(1/2)*1i + 1))/(4*a^(2/3)*b^(1/3)*c^(1/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)}\,dx = \frac{-2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2\sqrt{3}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)+2\log\left(a^{\frac{1}{3}}+x^{\frac{2}{3}}b^{\frac{1}{3}}\right)-\log\left(-x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+a^{\frac{1}{3}}\right.}{4c^{\frac{1}{3}}b^{\frac{1}{3}}a^{\frac{2}{3}}}$$

input
```
int(1/(c*x)^(1/3)/(b*x^2+a),x)
```

output
```
( - 2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1/3)*b**(1/3))/(b**(
1/6)*a**(1/6))) - 2*sqrt(3)*atan((b**(1/6)*a**(1/6)*sqrt(3) + 2*x**(1/3)*b
**(1/3))/(b**(1/6)*a**(1/6))) + 2*log(a**(1/3) + x**(2/3)*b**(1/3)) - log(
 - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3)) - lo
g(x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3)))/(4*c
**(1/3)*b**(1/3)*a**(2/3))
```

3.347 $\int \frac{1}{(cx)^{4/3}(a+bx^2)} dx$

Optimal result

Integrand size = 17, antiderivative size = 237

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)} dx = -\frac{3}{ac\sqrt[3]{cx}} - \frac{\sqrt[6]{b}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{a^{7/6}c^{4/3}} + \frac{\sqrt[6]{b}\arctan\left(\sqrt{3}-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{7/6}c^{4/3}} - \frac{\sqrt[6]{b}\arctan\left(\sqrt{3}+\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[6]{a}\sqrt[3]{c}}\right)}{2a^{7/6}c^{4/3}} + \frac{\sqrt{3}\sqrt[6]{b}\operatorname{arctanh}\left(\frac{\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}\right)}{2a^{7/6}c^{4/3}}$$

output
```
-3/a/c/(c*x)^(1/3)-b^(1/6)*arctan(b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(1/3))/a^(
7/6)/c^(4/3)-1/2*b^(1/6)*arctan(-3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(1/6)/c^(
1/3))/a^(7/6)/c^(4/3)-1/2*b^(1/6)*arctan(3^(1/2)+2*b^(1/6)*(c*x)^(1/3)/a^(
1/6)/c^(1/3))/a^(7/6)/c^(4/3)+1/2*3^(1/2)*b^(1/6)*arctanh(3^(1/2)*a^(1/6)*
b^(1/6)*c^(1/3)*(c*x)^(1/3)/(a^(1/3)*c^(2/3)+b^(1/3)*(c*x)^(2/3)))/a^(7/6)
/c^(4/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.66

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)}\,dx=\frac{x\left(-6\sqrt[6]{a}+\sqrt[6]{b}\sqrt[3]{x}\arctan\left(\frac{\sqrt[6]{a}}{\sqrt[6]{b}\sqrt[3]{x}}-\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)-2\sqrt[6]{b}\sqrt[3]{x}\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{x}}{\sqrt[6]{a}}\right)+\sqrt{\ldots}\right.}{2a^{7/6}(cx)^{4/3}}$$

input
```
Integrate[1/((c*x)^(4/3)*(a + b*x^2)),x]
```

output
```
(x*(-6*a^(1/6) + b^(1/6)*x^(1/3)*ArcTan[a^(1/6)/(b^(1/6)*x^(1/3)) - (b^(1/
6)*x^(1/3))/a^(1/6)] - 2*b^(1/6)*x^(1/3)*ArcTan[(b^(1/6)*x^(1/3))/a^(1/6)]
 + Sqrt[3]*b^(1/6)*x^(1/3)*ArcTanh[(Sqrt[3]*a^(1/6)*b^(1/6)*x^(1/3))/(a^(1
/3) + b^(1/3)*x^(2/3))]))/(2*a^(7/6)*(c*x)^(4/3))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.41, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {264, 266, 27, 824, 27, 218, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)}dx$$

$$\Big\downarrow \; 264$$

$$-\frac{b\int \frac{(cx)^{2/3}}{bx^2+a}dx}{ac^2}-\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \; 266$$

$$-\frac{3b\int \frac{c^2(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{ac^3}-\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \; 27$$

$$-\frac{3b\int \frac{(cx)^{4/3}}{bx^2c^2+ac^2}d\sqrt[3]{cx}}{ac}-\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \text{824}$$

$$-\frac{3b\left(\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}+\frac{\int -\frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{2\left(\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}\right)}d\sqrt[3]{cx}}{3\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\int -\frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{}}{2\left(\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{}\right.}}{3\sqrt[6]{a}b^{2/3}\sqrt[3]{}}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3b\left(\frac{\int \frac{1}{\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{3b^{2/3}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(c}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \text{218}$$

$$-\frac{3b\left(-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}-\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\int \frac{\sqrt[6]{a}\sqrt[3]{c}+\sqrt{3}\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}+\frac{\arctan\left(\frac{\sqrt[6]{b}\sqrt[3]{}}{\sqrt[6]{a}\sqrt[3]{}}\right.}{3\sqrt[6]{a}b^{5/6}\sqrt[3]{}}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$$\Big\downarrow \text{1142}$$

$$-\frac{3b\left(-\frac{-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\sqrt{3}\int -\frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\cdots\right.}{\cdots}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$\downarrow$ 25

$$-\frac{3b\left(-\frac{\frac{\sqrt{3}\int \frac{\sqrt[6]{b}\left(\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}\right)}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{2\sqrt[6]{b}}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\cdots\right.}{\cdots}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$\downarrow$ 27

$$-\frac{3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{1}{2}\sqrt[6]{a}\sqrt[3]{c}\int \frac{1}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\cdots\right.}{\cdots}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$\downarrow$ 1082

$$-\frac{3b\left(-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}-\frac{\int \frac{1}{-(cx)^{2/3}-\frac{1}{3}}d\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)}{\sqrt{3}\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int \frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}+2\sqrt[6]{\cdots}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c\cdots}}\cdots}{\cdots}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$\Big\downarrow$ 217

$$-\frac{3b\left(-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}-2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt[3]{a}c^{2/3}-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{c}+\sqrt[3]{b}(cx)^{2/3}}d\sqrt[3]{cx}+\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}+2\sqrt[6]{b}\sqrt[3]{}}{\sqrt[3]{a}c^{2/3}+\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{cx}\sqrt[3]{}}}{}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

$\Big\downarrow$ 1103

$$-\frac{3b\left(-\frac{\frac{\arctan\left(\sqrt{3}\left(1-\frac{2\sqrt[6]{b}\sqrt[3]{cx}}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{c}}\right)\right)}{\sqrt[6]{b}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\sqrt[3]{b}(cx)^{2/3}\right)}{2\sqrt[6]{b}}}{6\sqrt[6]{a}b^{2/3}\sqrt[3]{c}}-\frac{\frac{\sqrt{3}\log\left(\sqrt{3}\sqrt[6]{a}\sqrt[6]{b}\sqrt[3]{c}\sqrt[3]{cx}+\sqrt[3]{a}c^{2/3}+\right.}{2\sqrt[6]{b}}}{6\sqrt[6]{a}}\right.}{ac}$$

$$\frac{3}{ac\sqrt[3]{cx}}$$

input
```
Int[1/((c*x)^(4/3)*(a + b*x^2)),x]
```

output
```
-3/(a*c*(c*x)^(1/3)) - (3*b*(ArcTan[(b^(1/6)*(c*x)^(1/3))/(a^(1/6)*c^(1/3)
)]/(3*a^(1/6)*b^(5/6)*c^(1/3)) - (ArcTan[Sqrt[3]*(1 - (2*b^(1/6)*(c*x)^(1/
3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6) - (Sqrt[3]*Log[a^(1/3)*c^(2/3) - S
qrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(2*b^(1
/6)))/(6*a^(1/6)*b^(2/3)*c^(1/3)) - (-(ArcTan[Sqrt[3]*(1 + (2*b^(1/6)*(c*x
)^(1/3))/(Sqrt[3]*a^(1/6)*c^(1/3)))]/b^(1/6)) + (Sqrt[3]*Log[a^(1/3)*c^(2/
3) + Sqrt[3]*a^(1/6)*b^(1/6)*c^(1/3)*(c*x)^(1/3) + b^(1/3)*(c*x)^(2/3)])/(
2*b^(1/6)))/(6*a^(1/6)*b^(2/3)*c^(1/3))))/(a*c)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{\frac{\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)\sqrt{3}\,(cx)^{\frac{1}{3}}}{2}-\frac{\ln\left(\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}-(cx)^{\frac{2}{3}}-\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)\sqrt{3}\,(cx)^{\frac{1}{3}}}{2}+\arctan\left(\frac{\sqrt{3}\left(a\right.}{}\right.}{2\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}}$
risch	$-\frac{3}{ac(cx)^{\frac{1}{3}}}+\frac{-\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right)}{a\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}+\frac{b\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}+\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{4a^2c^2}-\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}+\sqrt{3}\right)}{2a\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}}{c}$
derivativedivides	$3c\left(-\frac{1}{a\,c^2(cx)^{\frac{1}{3}}}-\frac{\left(\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^2}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right.}{6b\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right.}{a\,c^2}\right.$
default	$3c\left(-\frac{1}{a\,c^2(cx)^{\frac{1}{3}}}-\frac{\left(\frac{\arctan\left(\frac{(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right)}{3b\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}+\frac{\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{5}{6}}\ln\left((cx)^{\frac{2}{3}}-\sqrt{3}\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+\left(\frac{a c^2}{b}\right)^{\frac{1}{3}}\right)}{12a\,c^2}+\frac{\arctan\left(\frac{2(cx)^{\frac{1}{3}}}{\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right.}{6b\left(\frac{a c^2}{b}\right)^{\frac{1}{6}}}\right.}{a\,c^2}\right.$

input
```
int(1/(c*x)^(4/3)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a*c^2/b)^(1/6)*(1/2*ln((c*x)^(2/3)+3^(1/2)*(a*c^2/b)^(1/6)*(c*x)^(1/3
)+(a*c^2/b)^(1/3))*3^(1/2)*(c*x)^(1/3)-1/2*ln(3^(1/2)*(a*c^2/b)^(1/6)*(c*x
)^(1/3)-(c*x)^(2/3)-(a*c^2/b)^(1/3))*3^(1/2)*(c*x)^(1/3)+arctan((3^(1/2)*(
a*c^2/b)^(1/6)-2*(c*x)^(1/3))/(a*c^2/b)^(1/6))*(c*x)^(1/3)-2*arctan((c*x)^
(1/3)/(a*c^2/b)^(1/6))*(c*x)^(1/3)-arctan((3^(1/2)*(a*c^2/b)^(1/6)+2*(c*x)
^(1/3))/(a*c^2/b)^(1/6))*(c*x)^(1/3)-6*(a*c^2/b)^(1/6))/(c*x)^(1/3)/a/c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(159) = 318$.

Time = 0.08 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.64

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)}\,dx = -\frac{2\,ac^2x\left(-\frac{b}{a^7c^8}\right)^{\frac{1}{6}}\log\left(a^6c^7\left(-\frac{b}{a^7c^8}\right)^{\frac{5}{6}}+(cx)^{\frac{1}{3}}\,b\right)-2\,ac^2x\left(-\frac{b}{a^7c^8}\right)^{\frac{1}{6}}\log\left(-a^6c^7\left(-\frac{b}{a^7c^8}\right)^{\frac{5}{6}}+(cx)^{\frac{1}{3}}\,b\right)-\left(\sqrt{-3}\right.}{}$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(2*a*c^2*x*(-b/(a^7*c^8))^(1/6)*log(a^6*c^7*(-b/(a^7*c^8))^(5/6) + (c
*x)^(1/3)*b) - 2*a*c^2*x*(-b/(a^7*c^8))^(1/6)*log(-a^6*c^7*(-b/(a^7*c^8))^
(5/6) + (c*x)^(1/3)*b) - (sqrt(-3)*a*c^2*x - a*c^2*x)*(-b/(a^7*c^8))^(1/6)
*log(1/2*(sqrt(-3)*a^6*c^7 + a^6*c^7)*(-b/(a^7*c^8))^(5/6) + (c*x)^(1/3)*b
) + (sqrt(-3)*a*c^2*x - a*c^2*x)*(-b/(a^7*c^8))^(1/6)*log(-1/2*(sqrt(-3)*a
^6*c^7 + a^6*c^7)*(-b/(a^7*c^8))^(5/6) + (c*x)^(1/3)*b) - (sqrt(-3)*a*c^2*
x + a*c^2*x)*(-b/(a^7*c^8))^(1/6)*log(1/2*(sqrt(-3)*a^6*c^7 - a^6*c^7)*(-b
/(a^7*c^8))^(5/6) + (c*x)^(1/3)*b) + (sqrt(-3)*a*c^2*x + a*c^2*x)*(-b/(a^7
*c^8))^(1/6)*log(-1/2*(sqrt(-3)*a^6*c^7 - a^6*c^7)*(-b/(a^7*c^8))^(5/6) +
(c*x)^(1/3)*b) + 12*(c*x)^(2/3))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.65

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)}\,dx = \frac{\Gamma\left(-\frac{1}{6}\right)}{2ac^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)} + \frac{\sqrt[6]{b}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)} + \frac{i\sqrt[6]{b}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)} + \frac{\sqrt[6]{b}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{5i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)} - \frac{\sqrt[6]{b}e^{\frac{i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{7i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)} - \frac{i\sqrt[6]{b}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{3i\pi}{2}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)} - \frac{\sqrt[6]{b}e^{\frac{5i\pi}{6}}\log\left(1-\frac{\sqrt[6]{b}\sqrt[3]{x}e^{\frac{11i\pi}{6}}}{\sqrt[6]{a}}\right)\Gamma\left(-\frac{1}{6}\right)}{12a^{\frac{7}{6}}c^{\frac{4}{3}}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate(1/(c*x)**(4/3)/(b*x**2+a),x)
```

output
```
gamma(-1/6)/(2*a*c**(4/3)*x**(1/3)*gamma(5/6)) + b**(1/6)*exp(I*pi/6)*log(
1 - b**(1/6)*x**(1/3)*exp_polar(I*pi/6)/a**(1/6))*gamma(-1/6)/(12*a**(7/6)
*c**(4/3)*gamma(5/6)) + I*b**(1/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(I*p
i/2)/a**(1/6))*gamma(-1/6)/(12*a**(7/6)*c**(4/3)*gamma(5/6)) + b**(1/6)*ex
p(5*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(5*I*pi/6)/a**(1/6))*gamma(
-1/6)/(12*a**(7/6)*c**(4/3)*gamma(5/6)) - b**(1/6)*exp(I*pi/6)*log(1 - b**
(1/6)*x**(1/3)*exp_polar(7*I*pi/6)/a**(1/6))*gamma(-1/6)/(12*a**(7/6)*c**(
4/3)*gamma(5/6)) - I*b**(1/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(3*I*pi/2
)/a**(1/6))*gamma(-1/6)/(12*a**(7/6)*c**(4/3)*gamma(5/6)) - b**(1/6)*exp(5
*I*pi/6)*log(1 - b**(1/6)*x**(1/3)*exp_polar(11*I*pi/6)/a**(1/6))*gamma(-1
/6)/(12*a**(7/6)*c**(4/3)*gamma(5/6))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)}\,dx = \frac{\frac{b\left(\frac{\sqrt{3}\log\left(\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{\sqrt{3}\log\left(-\sqrt{3}\left(ac^2\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}b^{\frac{1}{6}}+(cx)^{\frac{2}{3}}b^{\frac{1}{3}}+\left(ac^2\right)^{\frac{1}{3}}\right)}{\left(ac^2\right)^{\frac{1}{6}}b^{\frac{5}{6}}}-\frac{2\arctan\left(\right.}{}\right.}{a}}{4c}$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/4*(b*(sqrt(3)*log(sqrt(3)*(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3
)*b^(1/3) + (a*c^2)^(1/3))/((a*c^2)^(1/6)*b^(5/6)) - sqrt(3)*log(-sqrt(3)*
(a*c^2)^(1/6)*(c*x)^(1/3)*b^(1/6) + (c*x)^(2/3)*b^(1/3) + (a*c^2)^(1/3))/(
(a*c^2)^(1/6)*b^(5/6)) - 2*arctan((sqrt(3)*(a*c^2)^(1/6)*b^(1/6) + 2*(c*x)
^(1/3)*b^(1/3))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^(1/3)*b
^(1/3))) - 2*arctan(-(sqrt(3)*(a*c^2)^(1/6)*b^(1/6) - 2*(c*x)^(1/3)*b^(1/3
))/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c^2)^(1/3)*b^(1/3))) - 4*
arctan((c*x)^(1/3)*b^(1/3)/sqrt((a*c^2)^(1/3)*b^(1/3)))/(b^(2/3)*sqrt((a*c
^2)^(1/3)*b^(1/3))))/a - 12/((c*x)^(1/3)*a))/c
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.20

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)}\,dx =$$

$$-\frac{\frac{12}{(cx)^{\frac{1}{3}}a}-\frac{\sqrt{3}\left(ab^5c^2\right)^{\frac{5}{6}}\log\left(\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{a^2b^4c^2}+\frac{\sqrt{3}\left(ab^5c^2\right)^{\frac{5}{6}}\log\left(-\sqrt{3}\left(\frac{ac^2}{b}\right)^{\frac{1}{6}}(cx)^{\frac{1}{3}}+(cx)^{\frac{2}{3}}+\left(\frac{ac^2}{b}\right)^{\frac{1}{3}}\right)}{a^2b^4c^2}+\frac{2\left(ab^5c^2\right.}{}}{4c}$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/4*(12/((c*x)^(1/3)*a) - sqrt(3)*(a*b^5*c^2)^(5/6)*log(sqrt(3)*(a*c^2/b)
^(1/6)*(c*x)^(1/3) + (c*x)^(2/3) + (a*c^2/b)^(1/3))/(a^2*b^4*c^2) + sqrt(3
)*(a*b^5*c^2)^(5/6)*log(-sqrt(3)*(a*c^2/b)^(1/6)*(c*x)^(1/3) + (c*x)^(2/3)
 + (a*c^2/b)^(1/3))/(a^2*b^4*c^2) + 2*(a*b^5*c^2)^(5/6)*arctan((sqrt(3)*(a
*c^2/b)^(1/6) + 2*(c*x)^(1/3))/(a*c^2/b)^(1/6))/(a^2*b^4*c^2) + 2*(a*b^5*c
^2)^(5/6)*arctan(-(sqrt(3)*(a*c^2/b)^(1/6) - 2*(c*x)^(1/3))/(a*c^2/b)^(1/6
))/(a^2*b^4*c^2) + 4*(a*b^5*c^2)^(5/6)*arctan((c*x)^(1/3)/(a*c^2/b)^(1/6))
/(a^2*b^4*c^2))/c
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.21

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)}\,dx = -\frac{3}{a\,c\,{(c\,x)}^{1/3}} - \frac{{(-b)}^{1/6}\,\mathrm{atan}\left(\frac{{(-b)}^{1/6}\,{(c\,x)}^{1/3}\,1{}\mathrm{i}}{a^{1/6}\,c^{1/3}}\right)\,1{}\mathrm{i}}{a^{7/6}\,c^{4/3}}$$
$$-\frac{{(-b)}^{1/6}\,\ln\left(972\,a^9\,b^6\,c^{12}-972\,a^{53/6}\,{(-b)}^{37/6}\,c^{35/3}\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{(c\,x)}^{1/3}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{7/6}\,c^{4/3}}$$
$$-\frac{{(-b)}^{1/6}\,\ln\left(972\,a^9\,b^6\,c^{12}-972\,a^{53/6}\,{(-b)}^{37/6}\,c^{35/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,{(c\,x)}^{1/3}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{7/6}\,c^{4/3}}$$
$$+\frac{{(-b)}^{1/6}\,\ln\left(972\,a^9\,b^6\,c^{12}+1944\,a^{53/6}\,{(-b)}^{37/6}\,c^{35/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{(c\,x)}^{1/3}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{a^{7/6}\,c^{4/3}}$$
$$+\frac{{(-b)}^{1/6}\,\ln\left(972\,a^9\,b^6\,c^{12}+1944\,a^{53/6}\,{(-b)}^{37/6}\,c^{35/3}\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\,{(c\,x)}^{1/3}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)}{a^{7/6}\,c^{4/3}}$$

input
```
int(1/((c*x)^(4/3)*(a + b*x^2)),x)
```

output
```
((-b)^(1/6)*log(972*a^9*b^6*c^12 + 1944*a^(53/6)*(-b)^(37/6)*c^(35/3)*((3^
(1/2)*1i)/4 - 1/4)*(c*x)^(1/3))*((3^(1/2)*1i)/4 - 1/4))/(a^(7/6)*c^(4/3))
- ((-b)^(1/6)*atan(((-b)^(1/6)*(c*x)^(1/3)*1i)/(a^(1/6)*c^(1/3)))*1i)/(a^(
7/6)*c^(4/3)) - ((-b)^(1/6)*log(972*a^9*b^6*c^12 - 972*a^(53/6)*(-b)^(37/6
)*c^(35/3)*((3^(1/2)*1i)/2 - 1/2)*(c*x)^(1/3))*((3^(1/2)*1i)/2 - 1/2))/(2*
a^(7/6)*c^(4/3)) - ((-b)^(1/6)*log(972*a^9*b^6*c^12 - 972*a^(53/6)*(-b)^(3
7/6)*c^(35/3)*((3^(1/2)*1i)/2 + 1/2)*(c*x)^(1/3))*((3^(1/2)*1i)/2 + 1/2))/
(2*a^(7/6)*c^(4/3)) - 3/(a*c*(c*x)^(1/3)) + ((-b)^(1/6)*log(972*a^9*b^6*c^
12 + 1944*a^(53/6)*(-b)^(37/6)*c^(35/3)*((3^(1/2)*1i)/4 + 1/4)*(c*x)^(1/3)
)*((3^(1/2)*1i)/4 + 1/4))/(a^(7/6)*c^(4/3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.79

$$\int \frac{1}{(cx)^{4/3}(a+bx^2)}\,dx = \frac{2x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}-2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-2x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\frac{b^{\frac{1}{6}}a^{\frac{1}{6}}\sqrt{3}+2x^{\frac{1}{3}}b^{\frac{1}{3}}}{b^{\frac{1}{6}}a^{\frac{1}{6}}}\right)-4x^{\frac{1}{3}}b^{\frac{1}{6}}a^{\frac{1}{6}}\,atan\left(\right.}{}$$

input
```
int(1/(c*x)^(4/3)/(b*x^2+a),x)
```

output
```
(2*x**(1/3)*b**(1/6)*a**(1/6)*atan((b**(1/6)*a**(1/6)*sqrt(3) - 2*x**(1/3)
*b**(1/3))/(b**(1/6)*a**(1/6))) - 2*x**(1/3)*b**(1/6)*a**(1/6)*atan((b**(1
/6)*a**(1/6)*sqrt(3) + 2*x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) - 4*x**(1
/3)*b**(1/6)*a**(1/6)*atan((x**(1/3)*b**(1/3))/(b**(1/6)*a**(1/6))) - x**(
1/3)*b**(1/6)*a**(1/6)*sqrt(3)*log( - x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3) +
 a**(1/3) + x**(2/3)*b**(1/3)) + x**(1/3)*b**(1/6)*a**(1/6)*sqrt(3)*log(x*
*(1/3)*b**(1/6)*a**(1/6)*sqrt(3) + a**(1/3) + x**(2/3)*b**(1/3)) - 12*a**(
1/3))/(4*x**(1/3)*c**(1/3)*a**(1/3)*a*c)
```

3.348 $\int \frac{x^{2/3}}{1+x^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{x^{2/3}}{1+x^2} \, dx = -\frac{1}{2}\arctan\left(\sqrt{3}-2\sqrt[3]{x}\right)+\frac{1}{2}\arctan\left(\sqrt{3}+2\sqrt[3]{x}\right) + \arctan\left(\sqrt[3]{x}\right)-\frac{1}{2}\sqrt{3}\text{arctanh}\left(\frac{\sqrt{3}\sqrt[3]{x}}{1+x^{2/3}}\right)$$

output
```
1/2*arctan(-3^(1/2)+2*x^(1/3))+1/2*arctan(3^(1/2)+2*x^(1/3))+arctan(x^(1/3
))-1/2*3^(1/2)*arctanh(3^(1/2)*x^(1/3)/(1+x^(2/3)))
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{x^{2/3}}{1+x^2} \, dx = \arctan\left(\sqrt[3]{x}\right)+\frac{1}{2}\left(1-i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt[3]{x}\right) + \frac{1}{2}\left(1+i\sqrt{3}\right)\arctan\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\sqrt[3]{x}\right)$$

input
```
Integrate[x^(2/3)/(1 + x^2),x]
```

output
```
ArcTan[x^(1/3)] + ((1 - I*Sqrt[3])*ArcTan[((1 - I*Sqrt[3])*x^(1/3))/2])/2
+ ((1 + I*Sqrt[3])*ArcTan[((1 + I*Sqrt[3])*x^(1/3))/2])/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {266, 824, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{2/3}}{x^2+1}\,dx$$

$$\Big\downarrow \text{ 266}$$

$$3\int \frac{x^{4/3}}{x^2+1}d\sqrt[3]{x}$$

$$\Big\downarrow \text{ 824}$$

$$3\left(\frac{1}{3}\int \frac{1}{x^{2/3}+1}d\sqrt[3]{x}+\frac{1}{3}\int -\frac{1-\sqrt{3}\sqrt[3]{x}}{2\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)}d\sqrt[3]{x}+\frac{1}{3}\int -\frac{\sqrt{3}\sqrt[3]{x}+1}{2\left(x^{2/3}+\sqrt{3}\sqrt[3]{x}+1\right)}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$3\left(\frac{1}{3}\int \frac{1}{x^{2/3}+1}d\sqrt[3]{x}-\frac{1}{6}\int \frac{1-\sqrt{3}\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{x}+1}{x^{2/3}+\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}\right)$$

$$\Big\downarrow \text{ 216}$$

$$3\left(-\frac{1}{6}\int \frac{1-\sqrt{3}\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\frac{1}{6}\int \frac{\sqrt{3}\sqrt[3]{x}+1}{x^{2/3}+\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}+\frac{1}{3}\arctan\left(\sqrt[3]{x}\right)\right)$$

$$\Big\downarrow \text{ 1142}$$

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}+\frac{1}{2}\sqrt{3}\int -\frac{\sqrt{3}-2\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{1}{x^{2/3}+\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\frac{1}{2}\sqrt{\right.\right.$$

$$\Big\downarrow \text{ 25}$$

$$3\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{x^{2/3}+\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\frac{1}{2}\sqrt{3}\right.\right.$$

$\downarrow$ 1083

$$3\left(\frac{1}{6}\left(-\int\frac{1}{-x^{2/3}-1}d\left(2\sqrt[3]{x}-\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}\right)+\frac{1}{6}\left(-\int\frac{1}{-x^{2/3}-1}d\left(2\sqrt[3]{x}+\sqrt{3}\right)-\right.\right.$$

$\downarrow$ 217

$$3\left(\frac{1}{6}\left(-\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2\sqrt[3]{x}}{x^{2/3}-\sqrt{3}\sqrt[3]{x}+1}d\sqrt[3]{x}-\arctan\left(\sqrt{3}-2\sqrt[3]{x}\right)\right)+\frac{1}{6}\left(\arctan\left(2\sqrt[3]{x}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\int\frac{2\sqrt[3]{x}+}{x^{2/3}+\sqrt{3}}\right.\right.$$

$\downarrow$ 1103

$$3\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\log\left(x^{2/3}-\sqrt{3}\sqrt[3]{x}+1\right)-\arctan\left(\sqrt{3}-2\sqrt[3]{x}\right)\right)+\frac{1}{6}\left(\arctan\left(2\sqrt[3]{x}+\sqrt{3}\right)-\frac{1}{2}\sqrt{3}\log\left(x^{2/3}+\sqrt{3}\sqrt[3]{\right.\right.\right.$$

input
```
Int[x^(2/3)/(1 + x^2),x]
```

output
```
3*(ArcTan[x^(1/3)]/3 + (-ArcTan[Sqrt[3] - 2*x^(1/3)] + (Sqrt[3]*Log[1 - Sq
rt[3]*x^(1/3) + x^(2/3)])/2)/6 + (ArcTan[Sqrt[3] + 2*x^(1/3)] - (Sqrt[3]*L
og[1 + Sqrt[3]*x^(1/3) + x^(2/3)])/2)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 824
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[(2*k
- 1)*m*(Pi/n)] - s*Cos[(2*k - 1)*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k -
 1)*(Pi/n)]*x + s^2*x^2), x] + Int[(r*Cos[(2*k - 1)*m*(Pi/n)] + s*Cos[(2*k
- 1)*(m + 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x]
; 2*(-1)^(m/2)*(r^(m + 2)/(a*n*s^m))   Int[1/(r^2 + s^2*x^2), x] + 2*(r^(m
+ 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGt
Q[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}-\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}+\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)}{2}+\arctan\left(x\right.$
default	$\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}-\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(-\sqrt{3}+2x^{\frac{1}{3}}\right)}{2}-\frac{\sqrt{3}\,\ln\left(x^{\frac{2}{3}}+\sqrt{3}\,x^{\frac{1}{3}}+1\right)}{4}+\frac{\arctan\left(\sqrt{3}+2x^{\frac{1}{3}}\right)}{2}+\arctan\left(x\right.$
meijerg	$\frac{x^{\frac{5}{3}}\sqrt{3}\,\ln\left(1-\sqrt{3}\,(x^2)^{\frac{1}{6}}+(x^2)^{\frac{1}{3}}\right)}{4(x^2)^{\frac{5}{6}}}+\frac{x^{\frac{5}{3}}\arctan\left(\frac{(x^2)^{\frac{1}{6}}}{2-\sqrt{3}\,(x^2)^{\frac{1}{6}}}\right)}{2(x^2)^{\frac{5}{6}}}+\frac{x^{\frac{5}{3}}\arctan\left((x^2)^{\frac{1}{6}}\right)}{(x^2)^{\frac{5}{6}}}-\frac{x^{\frac{5}{3}}\sqrt{3}\,\ln\left(1+\sqrt{3}\,(x^2)\right.}{4(x^2)^{\frac{5}{6}}}$
trager	Expression too large to display

input
```
int(x^(2/3)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/4*3^(1/2)*ln(x^(2/3)-3^(1/2)*x^(1/3)+1)+1/2*arctan(-3^(1/2)+2*x^(1/3))-1
/4*3^(1/2)*ln(x^(2/3)+3^(1/2)*x^(1/3)+1)+1/2*arctan(3^(1/2)+2*x^(1/3))+arc
tan(x^(1/3))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{x^{2/3}}{1+x^2}\,dx = -\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right) + \frac{1}{2}\arctan\left(\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(x^(2/3)/(x^2+1),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)
*x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sq
rt(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \frac{x^{2/3}}{1+x^2}\,dx = \frac{\sqrt{3}\log\left(4x^{\frac{2}{3}} - 4\sqrt{3}\sqrt[3]{x} + 4\right)}{4} - \frac{\sqrt{3}\log\left(4x^{\frac{2}{3}} + 4\sqrt{3}\sqrt[3]{x} + 4\right)}{4} + \operatorname{atan}\left(\sqrt[3]{x}\right) + \frac{\operatorname{atan}\left(2\sqrt[3]{x} - \sqrt{3}\right)}{2} + \frac{\operatorname{atan}\left(2\sqrt[3]{x} + \sqrt{3}\right)}{2}$$

input
```
integrate(x**(2/3)/(x**2+1),x)
```

output
```
sqrt(3)*log(4*x**(2/3) - 4*sqrt(3)*x**(1/3) + 4)/4 - sqrt(3)*log(4*x**(2/3
) + 4*sqrt(3)*x**(1/3) + 4)/4 + atan(x**(1/3)) + atan(2*x**(1/3) - sqrt(3)
)/2 + atan(2*x**(1/3) + sqrt(3))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{x^{2/3}}{1+x^2}\,dx = -\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}} + x^{\frac{2}{3}} + 1\right) + \frac{1}{2}\arctan\left(\sqrt{3} + 2\,x^{\frac{1}{3}}\right) + \frac{1}{2}\arctan\left(-\sqrt{3} + 2\,x^{\frac{1}{3}}\right) + \arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(x^(2/3)/(x^2+1),x, algorithm="maxima")
```

output
```
-1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)
*x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sq
rt(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{x^{2/3}}{1+x^2}\,dx = -\frac{1}{4}\sqrt{3}\log\left(\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{1}{4}\sqrt{3}\log\left(-\sqrt{3}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right) + \frac{1}{2}\arctan\left(\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\frac{1}{2}\arctan\left(-\sqrt{3}+2\,x^{\frac{1}{3}}\right)+\arctan\left(x^{\frac{1}{3}}\right)$$

input
```
integrate(x^(2/3)/(x^2+1),x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*log(sqrt(3)*x^(1/3) + x^(2/3) + 1) + 1/4*sqrt(3)*log(-sqrt(3)
*x^(1/3) + x^(2/3) + 1) + 1/2*arctan(sqrt(3) + 2*x^(1/3)) + 1/2*arctan(-sq
rt(3) + 2*x^(1/3)) + arctan(x^(1/3))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.78

$$\int \frac{x^{2/3}}{1+x^2}\,dx = \operatorname{atan}\left(x^{1/3}\right) - \operatorname{atan}\left(\frac{486\,x^{1/3}}{-243+\sqrt{3}\,243\mathrm{i}}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)-\operatorname{atan}\left(\frac{486\,x^{1/3}}{243+\sqrt{3}\,243\mathrm{i}}\right)\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)$$

input
```
int(x^(2/3)/(x^2 + 1),x)
```

output
```
atan(x^(1/3)) - atan((486*x^(1/3))/(3^(1/2)*243i - 243))*((3^(1/2)*1i)/2 +
 1/2) - atan((486*x^(1/3))/(3^(1/2)*243i + 243))*((3^(1/2)*1i)/2 - 1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int \frac{x^{2/3}}{1+x^2}\,dx = atan\left(x^{\frac{1}{3}}\right) + \frac{atan\left(2x^{\frac{1}{3}}-\sqrt{3}\right)}{2} + \frac{atan\left(2x^{\frac{1}{3}}+\sqrt{3}\right)}{2} + \frac{\sqrt{3}\log\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}\sqrt{3}+1\right)}{4} - \frac{\sqrt{3}\log\left(x^{\frac{2}{3}}+x^{\frac{1}{3}}\sqrt{3}+1\right)}{4}$$

input

```
int(x^(2/3)/(x^2+1),x)
```

output

```
(4*atan(x**(1/3)) + 2*atan(2*x**(1/3) - sqrt(3)) + 2*atan(2*x**(1/3) + sqr
t(3)) + sqrt(3)*log(x**(2/3) - x**(1/3)*sqrt(3) + 1) - sqrt(3)*log(x**(2/3
) + x**(1/3)*sqrt(3) + 1))/4
```

3.349 $\int x^m (a+bx^2)^5 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 97

$$\int x^m (a+bx^2)^5 \, dx = \frac{a^5 x^{1+m}}{1+m} + \frac{5a^4 b x^{3+m}}{3+m} + \frac{10a^3 b^2 x^{5+m}}{5+m} + \frac{10a^2 b^3 x^{7+m}}{7+m} + \frac{5ab^4 x^{9+m}}{9+m} + \frac{b^5 x^{11+m}}{11+m}$$

output
```
a^5*x^(1+m)/(1+m)+5*a^4*b*x^(3+m)/(3+m)+10*a^3*b^2*x^(5+m)/(5+m)+10*a^2*b^
3*x^(7+m)/(7+m)+5*a*b^4*x^(9+m)/(9+m)+b^5*x^(11+m)/(11+m)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.91

$$\int x^m (a+bx^2)^5 \, dx = x^{1+m} \left(\frac{a^5}{1+m} + \frac{5a^4 b x^2}{3+m} + \frac{10a^3 b^2 x^4}{5+m} + \frac{10a^2 b^3 x^6}{7+m} + \frac{5ab^4 x^8}{9+m} + \frac{b^5 x^{10}}{11+m} \right)$$

input
```
Integrate[x^m*(a + b*x^2)^5,x]
```

output
```
x^(1 + m)*(a^5/(1 + m) + (5*a^4*b*x^2)/(3 + m) + (10*a^3*b^2*x^4)/(5 + m)
+ (10*a^2*b^3*x^6)/(7 + m) + (5*a*b^4*x^8)/(9 + m) + (b^5*x^10)/(11 + m))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^5 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^5x^m+5a^4bx^{m+2}+10a^3b^2x^{m+4}+10a^2b^3x^{m+6}+5ab^4x^{m+8}+b^5x^{m+10}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^5x^{m+1}}{m+1}+\frac{5a^4bx^{m+3}}{m+3}+\frac{10a^3b^2x^{m+5}}{m+5}+\frac{10a^2b^3x^{m+7}}{m+7}+\frac{5ab^4x^{m+9}}{m+9}+\frac{b^5x^{m+11}}{m+11}$$

input
```
Int[x^m*(a + b*x^2)^5,x]
```

output
```
(a^5*x^(1 + m))/(1 + m) + (5*a^4*b*x^(3 + m))/(3 + m) + (10*a^3*b^2*x^(5 +
 m))/(5 + m) + (10*a^2*b^3*x^(7 + m))/(7 + m) + (5*a*b^4*x^(9 + m))/(9 + m
) + (b^5*x^(11 + m))/(11 + m)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. 2(97) = 194.

Time = 0.36 (sec) , antiderivative size = 431, normalized size of antiderivative = 4.44

method	result
risch	$\frac{x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}}{}$
orering	$\frac{x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m\,x^{10}}{}$
gosper	$\frac{x^{1+m}(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+1310a\,b^4m^3x^8+1689m}{}$
parallelrisch	$\frac{9129x\,x^m a^5m+5x^9x^m a\,b^4m^5+135x^9x^m a\,b^4m^4+1310x^9x^m a\,b^4m^3+10x^7x^m a^2b^3m^5+5610x^9x^m a\,b^4m^2+290x^7x^m a^2b^3m^4}{}$

input
```
int(x^m*(b*x^2+a)^5,x,method=_RETURNVERBOSE)
```

output
```
x*(b^5*m^5*x^10+25*b^5*m^4*x^10+5*a*b^4*m^5*x^8+230*b^5*m^3*x^10+135*a*b^4
*m^4*x^8+950*b^5*m^2*x^10+10*a^2*b^3*m^5*x^6+1310*a*b^4*m^3*x^8+1689*b^5*m
*x^10+290*a^2*b^3*m^4*x^6+5610*a*b^4*m^2*x^8+945*b^5*x^10+10*a^3*b^2*m^5*x
^4+3020*a^2*b^3*m^3*x^6+10205*a*b^4*m*x^8+310*a^3*b^2*m^4*x^4+13660*a^2*b^
3*m^2*x^6+5775*a*b^4*x^8+5*a^4*b*m^5*x^2+3500*a^3*b^2*m^3*x^4+25770*a^2*b^
3*m*x^6+165*a^4*b*m^4*x^2+17300*a^3*b^2*m^2*x^4+14850*a^2*b^3*x^6+a^5*m^5+
2030*a^4*b*m^3*x^2+34890*a^3*b^2*m*x^4+35*a^5*m^4+11310*a^4*b*m^2*x^2+2079
0*a^3*b^2*x^4+470*a^5*m^3+26765*a^4*b*m*x^2+3010*a^5*m^2+17325*a^4*b*x^2+9
129*a^5*m+10395*a^5)*x^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(97) = 194.

Time = 0.07 (sec) , antiderivative size = 367, normalized size of antiderivative = 3.78

$$\int x^m(a+bx^2)^5\,dx$$
$$=\frac{((b^5m^5+25\,b^5m^4+230\,b^5m^3+950\,b^5m^2+1689\,b^5m+945\,b^5)x^{11}+5\,(ab^4m^5+27\,ab^4m^4+262\,ab^4m^3}{}$$

input
```
integrate(x^m*(b*x^2+a)^5,x, algorithm="fricas")
```

output
```
((b^5*m^5 + 25*b^5*m^4 + 230*b^5*m^3 + 950*b^5*m^2 + 1689*b^5*m + 945*b^5)
*x^11 + 5*(a*b^4*m^5 + 27*a*b^4*m^4 + 262*a*b^4*m^3 + 1122*a*b^4*m^2 + 204
1*a*b^4*m + 1155*a*b^4)*x^9 + 10*(a^2*b^3*m^5 + 29*a^2*b^3*m^4 + 302*a^2*b
^3*m^3 + 1366*a^2*b^3*m^2 + 2577*a^2*b^3*m + 1485*a^2*b^3)*x^7 + 10*(a^3*b
^2*m^5 + 31*a^3*b^2*m^4 + 350*a^3*b^2*m^3 + 1730*a^3*b^2*m^2 + 3489*a^3*b^
2*m + 2079*a^3*b^2)*x^5 + 5*(a^4*b*m^5 + 33*a^4*b*m^4 + 406*a^4*b*m^3 + 22
62*a^4*b*m^2 + 5353*a^4*b*m + 3465*a^4*b)*x^3 + (a^5*m^5 + 35*a^5*m^4 + 47
0*a^5*m^3 + 3010*a^5*m^2 + 9129*a^5*m + 10395*a^5)*x)*x^m/(m^6 + 36*m^5 +
505*m^4 + 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(87) = 174$.

Time = 0.65 (sec) , antiderivative size = 1999, normalized size of antiderivative = 20.61

$$\int x^m (a + bx^2)^5 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**2+a)**5,x)
```

output
```
Piecewise((-a**5/(10*x**10) - 5*a**4*b/(8*x**8) - 5*a**3*b**2/(3*x**6) - 5
*a**2*b**3/(2*x**4) - 5*a*b**4/(2*x**2) + b**5*log(x), Eq(m, -11)), (-a**5
/(8*x**8) - 5*a**4*b/(6*x**6) - 5*a**3*b**2/(2*x**4) - 5*a**2*b**3/x**2 +
5*a*b**4*log(x) + b**5*x**2/2, Eq(m, -9)), (-a**5/(6*x**6) - 5*a**4*b/(4*x
**4) - 5*a**3*b**2/x**2 + 10*a**2*b**3*log(x) + 5*a*b**4*x**2/2 + b**5*x**
4/4, Eq(m, -7)), (-a**5/(4*x**4) - 5*a**4*b/(2*x**2) + 10*a**3*b**2*log(x)
 + 5*a**2*b**3*x**2 + 5*a*b**4*x**4/4 + b**5*x**6/6, Eq(m, -5)), (-a**5/(2
*x**2) + 5*a**4*b*log(x) + 5*a**3*b**2*x**2 + 5*a**2*b**3*x**4/2 + 5*a*b**
4*x**6/6 + b**5*x**8/8, Eq(m, -3)), (a**5*log(x) + 5*a**4*b*x**2/2 + 5*a**
3*b**2*x**4/2 + 5*a**2*b**3*x**6/3 + 5*a*b**4*x**8/8 + b**5*x**10/10, Eq(m
, -1)), (a**5*m**5*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 35*a**5*m**4*x*x**m/(m**6 + 36*m**5 + 505*m**4 +
3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**5*m**3*x*x**m/(m**6 + 3
6*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3010*a**5*
m**2*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
+ 10395) + 9129*a**5*m*x*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 121
39*m**2 + 19524*m + 10395) + 10395*a**5*x*x**m/(m**6 + 36*m**5 + 505*m**4
+ 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5*a**4*b*m**5*x**3*x**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 165*a
**4*b*m**4*x**3*x**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int x^m\left(a+bx^2\right)^5\,dx=\frac{b^5x^{m+11}}{m+11}+\frac{5\,ab^4x^{m+9}}{m+9}+\frac{10\,a^2b^3x^{m+7}}{m+7}+\frac{10\,a^3b^2x^{m+5}}{m+5}+\frac{5\,a^4bx^{m+3}}{m+3}+\frac{a^5x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^5,x, algorithm="maxima")
```

output
```
b^5*x^(m + 11)/(m + 11) + 5*a*b^4*x^(m + 9)/(m + 9) + 10*a^2*b^3*x^(m + 7)
/(m + 7) + 10*a^3*b^2*x^(m + 5)/(m + 5) + 5*a^4*b*x^(m + 3)/(m + 3) + a^5*
x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(97) = 194$.

Time = 0.14 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.57

$$\int x^m (a + bx^2)^5 \, dx$$
$$= \frac{b^5m^5x^{11}x^m + 25\,b^5m^4x^{11}x^m + 5\,ab^4m^5x^9x^m + 230\,b^5m^3x^{11}x^m + 135\,ab^4m^4x^9x^m + 950\,b^5m^2x^{11}x^m + 10}{}$$

input
```
integrate(x^m*(b*x^2+a)^5,x, algorithm="giac")
```

output
```
(b^5*m^5*x^11*x^m + 25*b^5*m^4*x^11*x^m + 5*a*b^4*m^5*x^9*x^m + 230*b^5*m^
3*x^11*x^m + 135*a*b^4*m^4*x^9*x^m + 950*b^5*m^2*x^11*x^m + 10*a^2*b^3*m^5
*x^7*x^m + 1310*a*b^4*m^3*x^9*x^m + 1689*b^5*m*x^11*x^m + 290*a^2*b^3*m^4*
x^7*x^m + 5610*a*b^4*m^2*x^9*x^m + 945*b^5*x^11*x^m + 10*a^3*b^2*m^5*x^5*x
^m + 3020*a^2*b^3*m^3*x^7*x^m + 10205*a*b^4*m*x^9*x^m + 310*a^3*b^2*m^4*x^
5*x^m + 13660*a^2*b^3*m^2*x^7*x^m + 5775*a*b^4*x^9*x^m + 5*a^4*b*m^5*x^3*x
^m + 3500*a^3*b^2*m^3*x^5*x^m + 25770*a^2*b^3*m*x^7*x^m + 165*a^4*b*m^4*x^
3*x^m + 17300*a^3*b^2*m^2*x^5*x^m + 14850*a^2*b^3*x^7*x^m + a^5*m^5*x*x^m
+ 2030*a^4*b*m^3*x^3*x^m + 34890*a^3*b^2*m*x^5*x^m + 35*a^5*m^4*x*x^m + 11
310*a^4*b*m^2*x^3*x^m + 20790*a^3*b^2*x^5*x^m + 470*a^5*m^3*x*x^m + 26765*
a^4*b*m*x^3*x^m + 3010*a^5*m^2*x*x^m + 17325*a^4*b*x^3*x^m + 9129*a^5*m*x*
x^m + 10395*a^5*x*x^m)/(m^6 + 36*m^5 + 505*m^4 + 3480*m^3 + 12139*m^2 + 19
524*m + 10395)
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 389, normalized size of antiderivative = 4.01

$$\int x^m \left(a+bx^2\right)^5 \, dx = \frac{a^5 \, x \, x^m \left(m^5+35 \, m^4+470 \, m^3+3010 \, m^2+9129 \, m+10395\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395} + \frac{b^5 \, x^m \, x^{11} \left(m^5+25 \, m^4+230 \, m^3+950 \, m^2+1689 \, m+945\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395} + \frac{5 \, a \, b^4 \, x^m \, x^9 \left(m^5+27 \, m^4+262 \, m^3+1122 \, m^2+2041 \, m+1155\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395} + \frac{5 \, a^4 \, b \, x^m \, x^3 \left(m^5+33 \, m^4+406 \, m^3+2262 \, m^2+5353 \, m+3465\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395} + \frac{10 \, a^2 \, b^3 \, x^m \, x^7 \left(m^5+29 \, m^4+302 \, m^3+1366 \, m^2+2577 \, m+1485\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395} + \frac{10 \, a^3 \, b^2 \, x^m \, x^5 \left(m^5+31 \, m^4+350 \, m^3+1730 \, m^2+3489 \, m+2079\right)}{m^6+36 \, m^5+505 \, m^4+3480 \, m^3+12139 \, m^2+19524 \, m+10395}$$

input
```
int(x^m*(a + b*x^2)^5,x)
```

output
```
(a^5*x*x^m*(9129*m + 3010*m^2 + 470*m^3 + 35*m^4 + m^5 + 10395))/(19524*m
+ 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (b^5*x^m*x^11*(
1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945))/(19524*m + 12139*m^2 + 3
480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395) + (5*a*b^4*x^m*x^9*(2041*m + 112
2*m^2 + 262*m^3 + 27*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 +
505*m^4 + 36*m^5 + m^6 + 10395) + (5*a^4*b*x^m*x^3*(5353*m + 2262*m^2 + 40
6*m^3 + 33*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
36*m^5 + m^6 + 10395) + (10*a^2*b^3*x^m*x^7*(2577*m + 1366*m^2 + 302*m^3 +
 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5
+ m^6 + 10395) + (10*a^3*b^2*x^m*x^5*(3489*m + 1730*m^2 + 350*m^3 + 31*m^4
 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 +
 10395)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 430, normalized size of antiderivative = 4.43

$$\int x^m \left(a+bx^2\right)^5\,dx$$
$$= \frac{x^m x(b^5m^5x^{10}+25b^5m^4x^{10}+5a\,b^4m^5x^8+230b^5m^3x^{10}+135a\,b^4m^4x^8+950b^5m^2x^{10}+10a^2b^3m^5x^6+13}{}$$

input
```
int(x^m*(b*x^2+a)^5,x)
```

output
```
(x**m*x*(a**5*m**5 + 35*a**5*m**4 + 470*a**5*m**3 + 3010*a**5*m**2 + 9129*
a**5*m + 10395*a**5 + 5*a**4*b*m**5*x**2 + 165*a**4*b*m**4*x**2 + 2030*a**
4*b*m**3*x**2 + 11310*a**4*b*m**2*x**2 + 26765*a**4*b*m*x**2 + 17325*a**4*
b*x**2 + 10*a**3*b**2*m**5*x**4 + 310*a**3*b**2*m**4*x**4 + 3500*a**3*b**2
*m**3*x**4 + 17300*a**3*b**2*m**2*x**4 + 34890*a**3*b**2*m*x**4 + 20790*a*
*3*b**2*x**4 + 10*a**2*b**3*m**5*x**6 + 290*a**2*b**3*m**4*x**6 + 3020*a**
2*b**3*m**3*x**6 + 13660*a**2*b**3*m**2*x**6 + 25770*a**2*b**3*m*x**6 + 14
850*a**2*b**3*x**6 + 5*a*b**4*m**5*x**8 + 135*a*b**4*m**4*x**8 + 1310*a*b*
*4*m**3*x**8 + 5610*a*b**4*m**2*x**8 + 10205*a*b**4*m*x**8 + 5775*a*b**4*x
**8 + b**5*m**5*x**10 + 25*b**5*m**4*x**10 + 230*b**5*m**3*x**10 + 950*b**
5*m**2*x**10 + 1689*b**5*m*x**10 + 945*b**5*x**10))/(m**6 + 36*m**5 + 505*
m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
```

3.350 $\int x^m (a+bx^2)^4 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 79

$$\int x^m (a+bx^2)^4 \, dx = \frac{a^4 x^{1+m}}{1+m} + \frac{4a^3 b x^{3+m}}{3+m} + \frac{6a^2 b^2 x^{5+m}}{5+m} + \frac{4ab^3 x^{7+m}}{7+m} + \frac{b^4 x^{9+m}}{9+m}$$

output
```
a^4*x^(1+m)/(1+m)+4*a^3*b*x^(3+m)/(3+m)+6*a^2*b^2*x^(5+m)/(5+m)+4*a*b^3*x^
(7+m)/(7+m)+b^4*x^(9+m)/(9+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int x^m (a+bx^2)^4 \, dx = x^{1+m} \left(\frac{a^4}{1+m} + \frac{4a^3 b x^2}{3+m} + \frac{6a^2 b^2 x^4}{5+m} + \frac{4ab^3 x^6}{7+m} + \frac{b^4 x^8}{9+m} \right)$$

input
```
Integrate[x^m*(a + b*x^2)^4,x]
```

output
```
x^(1 + m)*(a^4/(1 + m) + (4*a^3*b*x^2)/(3 + m) + (6*a^2*b^2*x^4)/(5 + m) +
 (4*a*b^3*x^6)/(7 + m) + (b^4*x^8)/(9 + m))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^4 dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(a^4x^m+4a^3bx^{m+2}+6a^2b^2x^{m+4}+4ab^3x^{m+6}+b^4x^{m+8}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4x^{m+1}}{m+1}+\frac{4a^3bx^{m+3}}{m+3}+\frac{6a^2b^2x^{m+5}}{m+5}+\frac{4ab^3x^{m+7}}{m+7}+\frac{b^4x^{m+9}}{m+9}$$

input
```
Int[x^m*(a + b*x^2)^4,x]
```

output
```
(a^4*x^(1 + m))/(1 + m) + (4*a^3*b*x^(3 + m))/(3 + m) + (6*a^2*b^2*x^(5 +
m))/(5 + m) + (4*a*b^3*x^(7 + m))/(7 + m) + (b^4*x^(9 + m))/(9 + m)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 3.67

method	result
risch	$\frac{x\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m\right.}{}$
orering	$\frac{x\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2b^2m\right.}{}$
gosper	$\frac{x^{1+m}\left(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176m\,x^8b^4+6a^2b^2m^4x^4+416a\,b^3m^2x^6+105b^4x^8+120a^2\right.}{}$
parallelrisch	$\frac{4x^7x^ma\,b^3m^4+72x^7x^ma\,b^3m^3+416x^7x^ma\,b^3m^2+6x^5x^ma^2b^2m^4+888x^7x^ma\,b^3m+120x^5x^ma^2b^2m^3+780x^5x^ma^2b^2m^2+4}{}$

input
```
int(x^m*(b*x^2+a)^4,x,method=_RETURNVERBOSE)
```

output
```
x*(b^4*m^4*x^8+16*b^4*m^3*x^8+4*a*b^3*m^4*x^6+86*b^4*m^2*x^8+72*a*b^3*m^3*
x^6+176*b^4*m*x^8+6*a^2*b^2*m^4*x^4+416*a*b^3*m^2*x^6+105*b^4*x^8+120*a^2*
b^2*m^3*x^4+888*a*b^3*m*x^6+4*a^3*b*m^4*x^2+780*a^2*b^2*m^2*x^4+540*a*b^3*
x^6+88*a^3*b*m^3*x^2+1800*a^2*b^2*m*x^4+a^4*m^4+656*a^3*b*m^2*x^2+1134*a^2
*b^2*x^4+24*a^4*m^3+1832*a^3*b*m*x^2+206*a^4*m^2+1260*a^3*b*x^2+744*a^4*m+
945*a^4)*x^m/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(79) = 158.

Time = 0.07 (sec) , antiderivative size = 251, normalized size of antiderivative = 3.18

$$\int x^m\left(a+bx^2\right)^4\,dx$$
$$=\frac{\left(\left(b^4m^4+16\,b^4m^3+86\,b^4m^2+176\,b^4m+105\,b^4\right)x^9+4\left(ab^3m^4+18\,ab^3m^3+104\,ab^3m^2+222\,ab^3m+1\right.\right.}{}$$

input
```
integrate(x^m*(b*x^2+a)^4,x, algorithm="fricas")
```

output
```
((b^4*m^4 + 16*b^4*m^3 + 86*b^4*m^2 + 176*b^4*m + 105*b^4)*x^9 + 4*(a*b^3*
m^4 + 18*a*b^3*m^3 + 104*a*b^3*m^2 + 222*a*b^3*m + 135*a*b^3)*x^7 + 6*(a^2
*b^2*m^4 + 20*a^2*b^2*m^3 + 130*a^2*b^2*m^2 + 300*a^2*b^2*m + 189*a^2*b^2)
*x^5 + 4*(a^3*b*m^4 + 22*a^3*b*m^3 + 164*a^3*b*m^2 + 458*a^3*b*m + 315*a^3
*b)*x^3 + (a^4*m^4 + 24*a^4*m^3 + 206*a^4*m^2 + 744*a^4*m + 945*a^4)*x)*x^
m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(70) = 140$.

Time = 0.52 (sec) , antiderivative size = 1221, normalized size of antiderivative = 15.46

$$\int x^m (a + bx^2)^4 \, dx = \text{Too large to display}$$

input
```
integrate(x**m*(b*x**2+a)**4,x)
```

output
```
Piecewise((-a**4/(8*x**8) - 2*a**3*b/(3*x**6) - 3*a**2*b**2/(2*x**4) - 2*a
*b**3/x**2 + b**4*log(x), Eq(m, -9)), (-a**4/(6*x**6) - a**3*b/x**4 - 3*a*
*2*b**2/x**2 + 4*a*b**3*log(x) + b**4*x**2/2, Eq(m, -7)), (-a**4/(4*x**4)
 - 2*a**3*b/x**2 + 6*a**2*b**2*log(x) + 2*a*b**3*x**2 + b**4*x**4/4, Eq(m,
-5)), (-a**4/(2*x**2) + 4*a**3*b*log(x) + 3*a**2*b**2*x**2 + a*b**3*x**4 +
 b**4*x**6/6, Eq(m, -3)), (a**4*log(x) + 2*a**3*b*x**2 + 3*a**2*b**2*x**4/
2 + 2*a*b**3*x**6/3 + b**4*x**8/8, Eq(m, -1)), (a**4*m**4*x*x**m/(m**5 + 2
5*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 24*a**4*m**3*x*x**m/(m**5 +
 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 206*a**4*m**2*x*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 744*a**4*m*x*x**m/(m**
5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 945*a**4*x*x**m/(m**5
+ 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 4*a**3*b*m**4*x**3*x**m/
(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 88*a**3*b*m**3*x**
3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 656*a**3*b*
m**2*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945) + 183
2*a**3*b*m*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m + 945)
 + 1260*a**3*b*x**3*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 + 1689*m +
945) + 6*a**2*b**2*m**4*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 + 950*m**2 +
1689*m + 945) + 120*a**2*b**2*m**3*x**5*x**m/(m**5 + 25*m**4 + 230*m**3 +
950*m**2 + 1689*m + 945) + 780*a**2*b**2*m**2*x**5*x**m/(m**5 + 25*m**4...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

$$\int x^m \left(a+bx^2\right)^4 \, dx = \frac{b^4x^{m+9}}{m+9} + \frac{4\,ab^3x^{m+7}}{m+7} + \frac{6\,a^2b^2x^{m+5}}{m+5} + \frac{4\,a^3bx^{m+3}}{m+3} + \frac{a^4x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^4,x, algorithm="maxima")
```

output
```
b^4*x^(m + 9)/(m + 9) + 4*a*b^3*x^(m + 7)/(m + 7) + 6*a^2*b^2*x^(m + 5)/(m
 + 5) + 4*a^3*b*x^(m + 3)/(m + 3) + a^4*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(79) = 158.

Time = 0.13 (sec) , antiderivative size = 365, normalized size of antiderivative = 4.62

$$\int x^m \left(a+bx^2\right)^4 \, dx = \frac{b^4m^4x^9x^m + 16\,b^4m^3x^9x^m + 4\,ab^3m^4x^7x^m + 86\,b^4m^2x^9x^m + 72\,ab^3m^3x^7x^m + 176\,b^4mx^9x^m + 6\,a^2b^2m^4x}{}$$

input
```
integrate(x^m*(b*x^2+a)^4,x, algorithm="giac")
```

output
```
(b^4*m^4*x^9*x^m + 16*b^4*m^3*x^9*x^m + 4*a*b^3*m^4*x^7*x^m + 86*b^4*m^2*x
^9*x^m + 72*a*b^3*m^3*x^7*x^m + 176*b^4*m*x^9*x^m + 6*a^2*b^2*m^4*x^5*x^m
+ 416*a*b^3*m^2*x^7*x^m + 105*b^4*x^9*x^m + 120*a^2*b^2*m^3*x^5*x^m + 888*
a*b^3*m*x^7*x^m + 4*a^3*b*m^4*x^3*x^m + 780*a^2*b^2*m^2*x^5*x^m + 540*a*b^
3*x^7*x^m + 88*a^3*b*m^3*x^3*x^m + 1800*a^2*b^2*m*x^5*x^m + a^4*m^4*x*x^m
+ 656*a^3*b*m^2*x^3*x^m + 1134*a^2*b^2*x^5*x^m + 24*a^4*m^3*x*x^m + 1832*a
^3*b*m*x^3*x^m + 206*a^4*m^2*x*x^m + 1260*a^3*b*x^3*x^m + 744*a^4*m*x*x^m
+ 945*a^4*x*x^m)/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.44

$$\int x^m (a+bx^2)^4\,dx = \frac{a^4\,x\,x^m\,(m^4+24\,m^3+206\,m^2+744\,m+945)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{b^4\,x^m\,x^9\,(m^4+16\,m^3+86\,m^2+176\,m+105)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{6\,a^2\,b^2\,x^m\,x^5\,(m^4+20\,m^3+130\,m^2+300\,m+189)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{4\,a\,b^3\,x^m\,x^7\,(m^4+18\,m^3+104\,m^2+222\,m+135)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945} + \frac{4\,a^3\,b\,x^m\,x^3\,(m^4+22\,m^3+164\,m^2+458\,m+315)}{m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945}$$

input
```
int(x^m*(a + b*x^2)^4,x)
```

output
```
(a^4*x*x^m*(744*m + 206*m^2 + 24*m^3 + m^4 + 945))/(1689*m + 950*m^2 + 230
*m^3 + 25*m^4 + m^5 + 945) + (b^4*x^m*x^9*(176*m + 86*m^2 + 16*m^3 + m^4 +
 105))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (6*a^2*b^2*x^m*
x^5*(300*m + 130*m^2 + 20*m^3 + m^4 + 189))/(1689*m + 950*m^2 + 230*m^3 +
25*m^4 + m^5 + 945) + (4*a*b^3*x^m*x^7*(222*m + 104*m^2 + 18*m^3 + m^4 + 1
35))/(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5 + 945) + (4*a^3*b*x^m*x^3*
(458*m + 164*m^2 + 22*m^3 + m^4 + 315))/(1689*m + 950*m^2 + 230*m^3 + 25*m
^4 + m^5 + 945)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 289, normalized size of antiderivative = 3.66

$$\int x^m (a+bx^2)^4\,dx = \frac{x^m x(b^4m^4x^8+16b^4m^3x^8+4a\,b^3m^4x^6+86b^4m^2x^8+72a\,b^3m^3x^6+176b^4m\,x^8+6a^2b^2m^4x^4+416a\,b^3m}{}$$

input
```
int(x^m*(b*x^2+a)^4,x)
```

output
```
(x**m*x*(a**4*m**4 + 24*a**4*m**3 + 206*a**4*m**2 + 744*a**4*m + 945*a**4
+ 4*a**3*b*m**4*x**2 + 88*a**3*b*m**3*x**2 + 656*a**3*b*m**2*x**2 + 1832*a
**3*b*m*x**2 + 1260*a**3*b*x**2 + 6*a**2*b**2*m**4*x**4 + 120*a**2*b**2*m*
*3*x**4 + 780*a**2*b**2*m**2*x**4 + 1800*a**2*b**2*m*x**4 + 1134*a**2*b**2
*x**4 + 4*a*b**3*m**4*x**6 + 72*a*b**3*m**3*x**6 + 416*a*b**3*m**2*x**6 +
888*a*b**3*m*x**6 + 540*a*b**3*x**6 + b**4*m**4*x**8 + 16*b**4*m**3*x**8 +
 86*b**4*m**2*x**8 + 176*b**4*m*x**8 + 105*b**4*x**8))/(m**5 + 25*m**4 + 2
30*m**3 + 950*m**2 + 1689*m + 945)
```

3.351 $\int x^m (a+bx^2)^3 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^m \left(a+bx^2\right)^3 \, dx = \frac{a^3 x^{1+m}}{1+m} + \frac{3a^2 b x^{3+m}}{3+m} + \frac{3ab^2 x^{5+m}}{5+m} + \frac{b^3 x^{7+m}}{7+m}$$

output
```
a^3*x^(1+m)/(1+m)+3*a^2*b*x^(3+m)/(3+m)+3*a*b^2*x^(5+m)/(5+m)+b^3*x^(7+m)/
(7+m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int x^m \left(a+bx^2\right)^3 \, dx = x^{1+m} \left(\frac{a^3}{1+m} + \frac{3a^2 b x^2}{3+m} + \frac{3ab^2 x^4}{5+m} + \frac{b^3 x^6}{7+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^3,x]
```

output
```
x^(1 + m)*(a^3/(1 + m) + (3*a^2*b*x^2)/(3 + m) + (3*a*b^2*x^4)/(5 + m) + (
b^3*x^6)/(7 + m))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^3 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^3x^m+3a^2bx^{m+2}+3ab^2x^{m+4}+b^3x^{m+6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3x^{m+1}}{m+1}+\frac{3a^2bx^{m+3}}{m+3}+\frac{3ab^2x^{m+5}}{m+5}+\frac{b^3x^{m+7}}{m+7}$$

input
```
Int[x^m*(a + b*x^2)^3,x]
```

output
```
(a^3*x^(1 + m))/(1 + m) + (3*a^2*b*x^(3 + m))/(3 + m) + (3*a*b^2*x^(5 + m)
)/(5 + m) + (b^3*x^(7 + m))/(7 + m)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

method	result
norman	$\frac{a^3 x\, e^{m\ln(x)}}{1+m}+\frac{b^3 x^7 e^{m\ln(x)}}{7+m}+\frac{3a\, b^2 x^5 e^{m\ln(x)}}{5+m}+\frac{3a^2 b\, x^3 e^{m\ln(x)}}{3+m}$
risch	$\frac{x(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m}{(7+m)(5+m)(3+m)(1+m)}$
orering	$\frac{x(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x^4+a^3m}{(7+m)(5+m)(3+m)(1+m)}$
gosper	$\frac{x^{1+m}(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23m\,x^6b^3+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93m\,x^4a\,b^2+39a^2b\,m^2x^2+63a\,b^2x^4+}{(1+m)(3+m)(5+m)(7+m)}$
parallelrisch	$\frac{x^7x^mb^3m^3+9x^7x^mb^3m^2+23x^7x^mb^3m+3x^5x^ma\,b^2m^3+15x^7x^mb^3+33x^5x^ma\,b^2m^2+93x^5x^ma\,b^2m+3x^3x^ma^2b\,m^3+63x^5}{(7+m)(5+m)(3+m)(}$

input
```
int(x^m*(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3/(1+m)*x*exp(m*ln(x))+b^3/(7+m)*x^7*exp(m*ln(x))+3*a*b^2/(5+m)*x^5*exp(
m*ln(x))+3*a^2*b/(3+m)*x^3*exp(m*ln(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(61) = 122.

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.57

$$\int x^m\left(a+bx^2\right)^3\,dx$$
$$=\frac{((b^3m^3+9\,b^3m^2+23\,b^3m+15\,b^3)x^7+3\,(ab^2m^3+11\,ab^2m^2+31\,ab^2m+21\,ab^2)x^5+3\,(a^2bm^3+13\,a^2b}{m^4+16\,m^3+86\,m^2+176\,m+10}$$

input
```
integrate(x^m*(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
((b^3*m^3 + 9*b^3*m^2 + 23*b^3*m + 15*b^3)*x^7 + 3*(a*b^2*m^3 + 11*a*b^2*m
^2 + 31*a*b^2*m + 21*a*b^2)*x^5 + 3*(a^2*b*m^3 + 13*a^2*b*m^2 + 47*a^2*b*m
 + 35*a^2*b)*x^3 + (a^3*m^3 + 15*a^3*m^2 + 71*a^3*m + 105*a^3)*x)*x^m/(m^4
 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(53) = 106$.

Time = 0.49 (sec) , antiderivative size = 683, normalized size of antiderivative = 11.20

$$\int x^m (a+bx^2)^3 \, dx$$

$$= \begin{cases} -\frac{a^3}{6x^6} - \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} + b^3 \log(x) \\ -\frac{a^3}{4x^4} - \frac{3a^2b}{2x^2} + 3ab^2 \log(x) + \frac{b^3x^2}{2} \\ -\frac{a^3}{2x^2} + 3a^2b \log(x) + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4} \\ a^3 \log(x) + \frac{3a^2bx^2}{2} + \frac{3ab^2x^4}{4} + \frac{b^3x^6}{6} \\ \frac{a^3m^3xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{15a^3m^2xx^m}{m^4+16m^3+86m^2+176m+105} + \frac{71a^3mxx^m}{m^4+16m^3+86m^2+176m+105} + \frac{105a^3xx^m}{m^4+16m^3+86m^2+176m+105} - \end{cases}$$

input
```
integrate(x**m*(b*x**2+a)**3,x)
```

output
```
Piecewise((-a**3/(6*x**6) - 3*a**2*b/(4*x**4) - 3*a*b**2/(2*x**2) + b**3*l
og(x), Eq(m, -7)), (-a**3/(4*x**4) - 3*a**2*b/(2*x**2) + 3*a*b**2*log(x) +
 b**3*x**2/2, Eq(m, -5)), (-a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**
2/2 + b**3*x**4/4, Eq(m, -3)), (a**3*log(x) + 3*a**2*b*x**2/2 + 3*a*b**2*x
**4/4 + b**3*x**6/6, Eq(m, -1)), (a**3*m**3*x*x**m/(m**4 + 16*m**3 + 86*m*
*2 + 176*m + 105) + 15*a**3*m**2*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 71*a**3*m*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 105*a
**3*x*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 3*a**2*b*m**3*x**3*x
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 39*a**2*b*m**2*x**3*x**m/(m
**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 141*a**2*b*m*x**3*x**m/(m**4 + 16
*m**3 + 86*m**2 + 176*m + 105) + 105*a**2*b*x**3*x**m/(m**4 + 16*m**3 + 86
*m**2 + 176*m + 105) + 3*a*b**2*m**3*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 +
 176*m + 105) + 33*a*b**2*m**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
 + 105) + 93*a*b**2*m*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 63*a*b**2*x**5*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + b**3*m**3*
x**7*x**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*b**3*m**2*x**7*x**m
/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*b**3*m*x**7*x**m/(m**4 + 16
*m**3 + 86*m**2 + 176*m + 105) + 15*b**3*x**7*x**m/(m**4 + 16*m**3 + 86*m*
*2 + 176*m + 105), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^m (a + bx^2)^3 \, dx = \frac{b^3 x^{m+7}}{m+7} + \frac{3\,ab^2 x^{m+5}}{m+5} + \frac{3\,a^2 b x^{m+3}}{m+3} + \frac{a^3 x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
b^3*x^(m + 7)/(m + 7) + 3*a*b^2*x^(m + 5)/(m + 5) + 3*a^2*b*x^(m + 3)/(m +
 3) + a^3*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(61) = 122.

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 3.67

$$\int x^m (a + bx^2)^3 \, dx = \frac{b^3 m^3 x^7 x^m + 9\,b^3 m^2 x^7 x^m + 3\,ab^2 m^3 x^5 x^m + 23\,b^3 m x^7 x^m + 33\,ab^2 m^2 x^5 x^m + 15\,b^3 x^7 x^m + 3\,a^2 b m^3 x^3 x^m +}{m^4}$$

input
```
integrate(x^m*(b*x^2+a)^3,x, algorithm="giac")
```

output
```
(b^3*m^3*x^7*x^m + 9*b^3*m^2*x^7*x^m + 3*a*b^2*m^3*x^5*x^m + 23*b^3*m*x^7*
x^m + 33*a*b^2*m^2*x^5*x^m + 15*b^3*x^7*x^m + 3*a^2*b*m^3*x^3*x^m + 93*a*b
^2*m*x^5*x^m + 39*a^2*b*m^2*x^3*x^m + 63*a*b^2*x^5*x^m + a^3*m^3*x*x^m + 1
41*a^2*b*m*x^3*x^m + 15*a^3*m^2*x*x^m + 105*a^2*b*x^3*x^m + 71*a^3*m*x*x^m
 + 105*a^3*x*x^m)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.74

$$\int x^m (a+bx^2)^3 \, dx = x^m \left(\frac{a^3\,x\,(m^3+15\,m^2+71\,m+105)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{b^3\,x^7\,(m^3+9\,m^2+23\,m+15)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{3\,a\,b^2\,x^5\,(m^3+11\,m^2+31\,m+21)}{m^4+16\,m^3+86\,m^2+176\,m+105} + \frac{3\,a^2\,b\,x^3\,(m^3+13\,m^2+47\,m+35)}{m^4+16\,m^3+86\,m^2+176\,m+105} \right)$$

input
```
int(x^m*(a + b*x^2)^3,x)
```

output
```
x^m*((a^3*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 86*m^2 + 16*m^3 + m^4 +
105) + (b^3*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m + 86*m^2 + 16*m^3 + m^4
+ 105) + (3*a*b^2*x^5*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86*m^2 + 16*m^3
 + m^4 + 105) + (3*a^2*b*x^3*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 +
 16*m^3 + m^4 + 105))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.89

$$\int x^m (a+bx^2)^3 \, dx = \frac{x^m x(b^3m^3x^6+9b^3m^2x^6+3a\,b^2m^3x^4+23b^3m\,x^6+33a\,b^2m^2x^4+15b^3x^6+3a^2b\,m^3x^2+93a\,b^2m\,x^4+39}{m^4+16m^3+86m^2+176}$$

input
```
int(x^m*(b*x^2+a)^3,x)
```

output
```
(x**m*x*(a**3*m**3 + 15*a**3*m**2 + 71*a**3*m + 105*a**3 + 3*a**2*b*m**3*x
**2 + 39*a**2*b*m**2*x**2 + 141*a**2*b*m*x**2 + 105*a**2*b*x**2 + 3*a*b**2
*m**3*x**4 + 33*a*b**2*m**2*x**4 + 93*a*b**2*m*x**4 + 63*a*b**2*x**4 + b**
3*m**3*x**6 + 9*b**3*m**2*x**6 + 23*b**3*m*x**6 + 15*b**3*x**6))/(m**4 + 1
6*m**3 + 86*m**2 + 176*m + 105)
```

3.352 $\int x^m (a+bx^2)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int x^m \left(a+bx^2\right)^2 \, dx = \frac{a^2 x^{1+m}}{1+m} + \frac{2abx^{3+m}}{3+m} + \frac{b^2 x^{5+m}}{5+m}$$

output
```
a^2*x^(1+m)/(1+m)+2*a*b*x^(3+m)/(3+m)+b^2*x^(5+m)/(5+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int x^m \left(a+bx^2\right)^2 \, dx = x^{1+m}\left(\frac{a^2}{1+m} + \frac{2abx^2}{3+m} + \frac{b^2x^4}{5+m}\right)$$

input
```
Integrate[x^m*(a + b*x^2)^2,x]
```

output
```
x^(1 + m)*(a^2/(1 + m) + (2*a*b*x^2)/(3 + m) + (b^2*x^4)/(5 + m))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^2 \, dx$$

$$\Big\downarrow \text{244}$$

$$\int \left(a^2x^m+2abx^{m+2}+b^2x^{m+4}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^{m+1}}{m+1}+\frac{2abx^{m+3}}{m+3}+\frac{b^2x^{m+5}}{m+5}$$

input
```
Int[x^m*(a + b*x^2)^2,x]
```

output
```
(a^2*x^(1 + m))/(1 + m) + (2*a*b*x^(3 + m))/(3 + m) + (b^2*x^(5 + m))/(5 +
 m)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

method	result	size
norman	$\frac{a^2 x\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b^2 x^5 \mathrm{e}^{m\ln(x)}}{5+m}+\frac{2ab\,x^3 \mathrm{e}^{m\ln(x)}}{3+m}$	51
risch	$\frac{x(b^2m^2x^4+4m\,x^4b^2+2ab\,m^2x^2+3b^2x^4+12m\,x^2ab+a^2m^2+10ab\,x^2+8m\,a^2+15a^2)x^m}{(5+m)(3+m)(1+m)}$	92
orering	$\frac{x(b^2m^2x^4+4m\,x^4b^2+2ab\,m^2x^2+3b^2x^4+12m\,x^2ab+a^2m^2+10ab\,x^2+8m\,a^2+15a^2)x^m}{(5+m)(3+m)(1+m)}$	92
gosper	$\frac{x^{1+m}(b^2m^2x^4+4m\,x^4b^2+2ab\,m^2x^2+3b^2x^4+12m\,x^2ab+a^2m^2+10ab\,x^2+8m\,a^2+15a^2)}{(1+m)(3+m)(5+m)}$	93
parallelrisch	$\frac{x^5x^mb^2m^2+4x^5x^mb^2m+3x^5x^mb^2+2x^3x^mab\,m^2+12x^3x^mabm+10x^3x^mab+x\,x^ma^2m^2+8x\,x^ma^2m+15x\,x^ma^2}{(5+m)(3+m)(1+m)}$	118

input
```
int(x^m*(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+m)*x*exp(m*ln(x))+b^2/(5+m)*x^5*exp(m*ln(x))+2*a*b/(3+m)*x^3*exp(m*
ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int x^m\left(a+bx^2\right)^2\,dx = \frac{((b^2m^2+4\,b^2m+3\,b^2)x^5+2\,(abm^2+6\,abm+5\,ab)x^3+(a^2m^2+8\,a^2m+15\,a^2)x)x^m}{m^3+9\,m^2+23\,m+15}$$

input
```
integrate(x^m*(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
((b^2*m^2 + 4*b^2*m + 3*b^2)*x^5 + 2*(a*b*m^2 + 6*a*b*m + 5*a*b)*x^3 + (a^
2*m^2 + 8*a^2*m + 15*a^2)*x)*x^m/(m^3 + 9*m^2 + 23*m + 15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 306, normalized size of antiderivative = 7.12

$$\int x^m (a+bx^2)^2 \, dx = \begin{cases} -\frac{a^2}{4x^4} - \frac{ab}{x^2} + b^2 \log(x) \\ -\frac{a^2}{2x^2} + 2ab\log(x) + \frac{b^2x^2}{2} \\ a^2\log(x) + abx^2 + \frac{b^2x^4}{4} \\ \frac{a^2m^2xx^m}{m^3+9m^2+23m+15} + \frac{8a^2mxx^m}{m^3+9m^2+23m+15} + \frac{15a^2xx^m}{m^3+9m^2+23m+15} + \frac{2abm^2x^3x^m}{m^3+9m^2+23m+15} + \frac{12abmx^3x^m}{m^3+9m^2+23m+15} + \frac{10abx^3x^m}{m^3+9m^2+23m+\cdots} \end{cases}$$

input
```
integrate(x**m*(b*x**2+a)**2,x)
```

output
```
Piecewise((-a**2/(4*x**4) - a*b/x**2 + b**2*log(x), Eq(m, -5)), (-a**2/(2*
x**2) + 2*a*b*log(x) + b**2*x**2/2, Eq(m, -3)), (a**2*log(x) + a*b*x**2 +
b**2*x**4/4, Eq(m, -1)), (a**2*m**2*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 8
*a**2*m*x*x**m/(m**3 + 9*m**2 + 23*m + 15) + 15*a**2*x*x**m/(m**3 + 9*m**2
 + 23*m + 15) + 2*a*b*m**2*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 12*a*b*
m*x**3*x**m/(m**3 + 9*m**2 + 23*m + 15) + 10*a*b*x**3*x**m/(m**3 + 9*m**2
+ 23*m + 15) + b**2*m**2*x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 4*b**2*m*
x**5*x**m/(m**3 + 9*m**2 + 23*m + 15) + 3*b**2*x**5*x**m/(m**3 + 9*m**2 +
23*m + 15), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int x^m (a+bx^2)^2 \, dx = \frac{b^2x^{m+5}}{m+5} + \frac{2\,abx^{m+3}}{m+3} + \frac{a^2x^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
b^2*x^(m + 5)/(m + 5) + 2*a*b*x^(m + 3)/(m + 3) + a^2*x^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(43) = 86$.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.72

$$\int x^m (a+bx^2)^2 \, dx = \frac{b^2m^2x^5x^m + 4\,b^2mx^5x^m + 2\,abm^2x^3x^m + 3\,b^2x^5x^m + 12\,abmx^3x^m + a^2m^2xx^m + 10\,abx^3x^m + 8\,a^2mxx^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

input
```
integrate(x^m*(b*x^2+a)^2,x, algorithm="giac")
```

output
```
(b^2*m^2*x^5*x^m + 4*b^2*m*x^5*x^m + 2*a*b*m^2*x^3*x^m + 3*b^2*x^5*x^m + 1
2*a*b*m*x^3*x^m + a^2*m^2*x*x^m + 10*a*b*x^3*x^m + 8*a^2*m*x*x^m + 15*a^2*
x*x^m)/(m^3 + 9*m^2 + 23*m + 15)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.16

$$\int x^m (a+bx^2)^2 \, dx = x^m \left(\frac{a^2\,x\,(m^2+8\,m+15)}{m^3+9\,m^2+23\,m+15} + \frac{b^2\,x^5\,(m^2+4\,m+3)}{m^3+9\,m^2+23\,m+15} + \frac{2\,a\,b\,x^3\,(m^2+6\,m+5)}{m^3+9\,m^2+23\,m+15} \right)$$

input
```
int(x^m*(a + b*x^2)^2,x)
```

output
```
x^m*((a^2*x*(8*m + m^2 + 15))/(23*m + 9*m^2 + m^3 + 15) + (b^2*x^5*(4*m +
m^2 + 3))/(23*m + 9*m^2 + m^3 + 15) + (2*a*b*x^3*(6*m + m^2 + 5))/(23*m +
9*m^2 + m^3 + 15))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.12

$$\int x^m (a+bx^2)^2 \, dx$$
$$= \frac{x^m x(b^2m^2x^4+4b^2m\,x^4+2ab\,m^2x^2+3b^2x^4+12abm\,x^2+a^2m^2+10ab\,x^2+8a^2m+15a^2)}{m^3+9m^2+23m+15}$$

input
```
int(x^m*(b*x^2+a)^2,x)
```

output
```
(x**m*x*(a**2*m**2 + 8*a**2*m + 15*a**2 + 2*a*b*m**2*x**2 + 12*a*b*m*x**2
+ 10*a*b*x**2 + b**2*m**2*x**4 + 4*b**2*m*x**4 + 3*b**2*x**4))/(m**3 + 9*m
**2 + 23*m + 15)
```

3.353 $\int x^m(a+bx^2)\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int x^m(a+bx^2)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}$$

output
```
a*x^(1+m)/(1+m)+b*x^(3+m)/(3+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^m(a+bx^2)\,dx=\frac{ax^{1+m}}{1+m}+\frac{bx^{3+m}}{3+m}$$

input
```
Integrate[x^m*(a + b*x^2),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right) \, dx$$

$$\Big\downarrow \text{ 244}$$

$$\int \left(ax^m+bx^{m+2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax^{m+1}}{m+1}+\frac{bx^{m+3}}{m+3}$$

input
```
Int[x^m*(a + b*x^2),x]
```

output
```
(a*x^(1 + m))/(1 + m) + (b*x^(3 + m))/(3 + m)
```

Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

method	result	size
norman	$\frac{ax\,\mathrm{e}^{m\ln(x)}}{1+m}+\frac{b\,x^3\mathrm{e}^{m\ln(x)}}{3+m}$	30
risch	$\frac{x(bm\,x^2+b\,x^2+am+3a)x^m}{(3+m)(1+m)}$	34
orering	$\frac{x(bm\,x^2+b\,x^2+am+3a)x^m}{(3+m)(1+m)}$	34
gosper	$\frac{x^{1+m}(bm\,x^2+b\,x^2+am+3a)}{(1+m)(3+m)}$	35
parallelrisch	$\frac{x^3x^m bm+x^3x^m b+x\,x^m am+3x\,x^m a}{(3+m)(1+m)}$	44

input
```
int(x^m*(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
a/(1+m)*x*exp(m*ln(x))+b/(3+m)*x^3*exp(m*ln(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^m\left(a+bx^2\right)\,dx=\frac{((bm+b)x^3+(am+3\,a)x)x^m}{m^2+4\,m+3}$$

input
```
integrate(x^m*(b*x^2+a),x, algorithm="fricas")
```

output
```
((b*m + b)*x^3 + (a*m + 3*a)*x)*x^m/(m^2 + 4*m + 3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.76

$$\int x^m (a + bx^2)\ dx = \begin{cases} -\frac{a}{2x^2} + b\log(x) & \text{for } m = -3 \\ a\log(x) + \frac{bx^2}{2} & \text{for } m = -1 \\ \frac{amxx^m}{m^2+4m+3} + \frac{3axx^m}{m^2+4m+3} + \frac{bmx^3x^m}{m^2+4m+3} + \frac{bx^3x^m}{m^2+4m+3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(b*x**2+a),x)
```

output
```
Piecewise((-a/(2*x**2) + b*log(x), Eq(m, -3)), (a*log(x) + b*x**2/2, Eq(m,
 -1)), (a*m*x*x**m/(m**2 + 4*m + 3) + 3*a*x*x**m/(m**2 + 4*m + 3) + b*m*x*
*3*x**m/(m**2 + 4*m + 3) + b*x**3*x**m/(m**2 + 4*m + 3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^m (a + bx^2)\ dx = \frac{bx^{m+3}}{m+3} + \frac{ax^{m+1}}{m+1}$$

input
```
integrate(x^m*(b*x^2+a),x, algorithm="maxima")
```

output
```
b*x^(m + 3)/(m + 3) + a*x^(m + 1)/(m + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int x^m(a+bx^2)\,dx=\frac{bmx^3x^m+bx^3x^m+amxx^m+3\,axx^m}{m^2+4\,m+3}$$

input
```
integrate(x^m*(b*x^2+a),x, algorithm="giac")
```

output
```
(b*m*x^3*x^m + b*x^3*x^m + a*m*x*x^m + 3*a*x*x^m)/(m^2 + 4*m + 3)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int x^m(a+bx^2)\,dx=\frac{x^{m+1}\,(3\,a+a\,m+b\,x^2+b\,m\,x^2)}{m^2+4\,m+3}$$

input
```
int(x^m*(a + b*x^2),x)
```

output
```
(x^(m + 1)*(3*a + a*m + b*x^2 + b*m*x^2))/(4*m + m^2 + 3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int x^m(a+bx^2)\,dx=\frac{x^mx(bm\,x^2+b\,x^2+am+3a)}{m^2+4m+3}$$

input
```
int(x^m*(b*x^2+a),x)
```

output
```
(x**m*x*(a*m + 3*a + b*m*x**2 + b*x**2))/(m**2 + 4*m + 3)
```

3.354 $\int \frac{x^m}{a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{a+bx^2}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}$$

output
```
x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{a+bx^2}\,dx=\frac{x^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}$$

input
```
Integrate[x^m/(a + b*x^2),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a*(1 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{a+bx^2} \, dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a(m+1)}$$

input
```
Int[x^m/(a + b*x^2),x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*(1
 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{b\,x^2+a}dx$$

input
```
int(x^m/(b*x^2+a),x)
```

output
```
int(x^m/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{x^m}{a+bx^2}\,dx = \int \frac{x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral(x^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.26

$$\int \frac{x^m}{a+bx^2}\,dx = \frac{mx^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate(x**m/(b*x**2+a),x)
```

output
```
m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(4*a*gamma(m/2 + 3/2)) + x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{x^m}{a+bx^2}\,dx = \int \frac{x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^m}{a+bx^2}\,dx = \int \frac{x^m}{bx^2+a}\,dx$$

input
```
integrate(x^m/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{a+bx^2}\,dx = \int \frac{x^m}{b\,x^2+a}\,dx$$

input
```
int(x^m/(a + b*x^2),x)
```

output
```
int(x^m/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^m}{a+bx^2}\,dx = \int \frac{x^m}{b\,x^2+a}dx$$

input
```
int(x^m/(b*x^2+a),x)
```

output
```
int(x**m/(a + b*x**2),x)
```

3.355 $\int \frac{x^m}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

output
```
x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \frac{x^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a^2(1+m)}$$

input
```
Integrate[x^m/(a + b*x^2)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a^2*(1 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^2} dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^{m+1} \text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^2(m+1)}$$

input
```
Int[x^m/(a + b*x^2)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^2*
(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)^2}dx$$

input
```
int(x^m/(b*x^2+a)^2,x)
```

output
```
int(x^m/(b*x^2+a)^2,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \int \frac{x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.94 (sec) , antiderivative size = 377, normalized size of antiderivative = 9.67

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = -\frac{am^2x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{2amx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{ax^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{2ax^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} - \frac{bm^2x^2x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{bx^2x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8a^3\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8a^2bx^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate(x**m/(b*x**2+a)**2,x)
```

output
```
-a*m**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(
m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2 + 3/2)) + 2*
a*m*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*g
amma(m/2 + 3/2)) + a*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**2*b*x**2*gamma(m/2
 + 3/2)) + 2*a*x**(m + 1)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a*
*2*b*x**2*gamma(m/2 + 3/2)) - b*m**2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*
a**2*b*x**2*gamma(m/2 + 3/2)) + b*x**2*x**(m + 1)*lerchphi(b*x**2*exp_pola
r(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a**3*gamma(m/2 + 3/2) + 8*a**
2*b*x**2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \int \frac{x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \int \frac{x^m}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^2}\,dx = \int \frac{x^m}{(b\,x^2+a)^2}\,dx$$

input
```
int(x^m/(a + b*x^2)^2,x)
```

output
```
int(x^m/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^2)^2} \, dx = \int \frac{x^m}{b^2x^4 + 2ab\, x^2 + a^2} dx$$

input
```
int(x^m/(b*x^2+a)^2,x)
```

output
```
int(x**m/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.356 $\int \frac{x^m}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \frac{x^m}{(a+bx^2)^3} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(3, \frac{1+m}{2}, \frac{3+m}{2}, -\frac{bx^2}{a}\right)}{a^3(1+m)}$$

output
```
x^(1+m)*hypergeom([3, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05

$$\int \frac{x^m}{(a+bx^2)^3} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(3, \frac{1+m}{2}, 1+\frac{1+m}{2}, -\frac{bx^2}{a}\right)}{a^3(1+m)}$$

input
```
Integrate[x^m/(a + b*x^2)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a^3*(1 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\,\text{Hypergeometric2F1}\left(3,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^3(m+1)}$$

input
```
Int[x^m/(a + b*x^2)^3,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a^3*
(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{(b\,x^2+a)^3}dx$$

input
```
int(x^m/(b*x^2+a)^3,x)
```

output
```
int(x^m/(b*x^2+a)^3,x)
```

Fricas [F]

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.27 (sec) , antiderivative size = 1574, normalized size of antiderivative = 40.36

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**m/(b*x**2+a)**3,x)
```

output
```
a**2*m**3*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamm
a(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) +
 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - 3*a**2*m**2*x**(m + 1)*lerchphi(b*x
**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 +
 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/
2)) - 2*a**2*m**2*x**(m + 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) +
64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) - a*
*2*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a
**3*b**2*x**4*gamma(m/2 + 3/2)) + 8*a**2*m*x**(m + 1)*gamma(m/2 + 1/2)/(32
*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x*
*4*gamma(m/2 + 3/2)) + 3*a**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**
2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 10*a**2*x**(m +
 1)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2
+ 3/2) + 32*a**3*b**2*x**4*gamma(m/2 + 3/2)) + 2*a*b*m**3*x**2*x**(m + 1)*
lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5
*gamma(m/2 + 3/2) + 64*a**4*b*x**2*gamma(m/2 + 3/2) + 32*a**3*b**2*x**4*ga
mma(m/2 + 3/2)) - 6*a*b*m**2*x**2*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(32*a**5*gamma(m/2 + 3/2) + 64*a**...
```

Maxima [F]

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \int \frac{x^m}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \int \frac{x^m}{(b\,x^2+a)^3}\,dx$$

input
```
int(x^m/(a + b*x^2)^3,x)
```

output
```
int(x^m/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^m}{(a+bx^2)^3}\,dx = \int \frac{x^m}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx$$

input
```
int(x^m/(b*x^2+a)^3,x)
```

output
```
int(x**m/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)
```

3.357 $\int \frac{(cx)^{1+m}}{a+bx^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \frac{(cx)^{1+m}}{a+bx^2} dx = \frac{(cx)^{2+m} \text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, -\frac{bx^2}{a}\right)}{ac(2+m)}$$

output
```
(c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c/(2+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.02

$$\int \frac{(cx)^{1+m}}{a+bx^2} dx = \frac{cx^2(cx)^m \text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, 1+\frac{2+m}{2}, -\frac{bx^2}{a}\right)}{a(2+m)}$$

input
```
Integrate[(c*x)^(1 + m)/(a + b*x^2),x]
```

output
```
(c*x^2*(c*x)^m*Hypergeometric2F1[1, (2 + m)/2, 1 + (2 + m)/2, -((b*x^2)/a)
])/(a*(2 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{m+1}}{a+bx^2} \, dx$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+2} \text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{ac(m+2)}$$

input
```
Int[(c*x)^(1 + m)/(a + b*x^2),x]
```

output
```
((c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(
a*c*(2 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{1+m}}{b\,x^2+a}dx$$

input
```
int((c*x)^(1+m)/(b*x^2+a),x)
```

output
```
int((c*x)^(1+m)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m+1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^(m + 1)/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.00

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \frac{c^{m+1}mx^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{c^{m+1}x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((c*x)**(1+m)/(b*x**2+a),x)
```

output
```
c**(m + 1)*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gam
ma(m/2 + 1)/(4*a*gamma(m/2 + 2)) + c**(m + 1)*x**(m + 2)*lerchphi(b*x**2*e
xp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m+1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m+1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(1+m)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^(m + 1)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \int \frac{(c\,x)^{m+1}}{b\,x^2+a}\,dx$$

input
```
int((c*x)^(m + 1)/(a + b*x^2),x)
```

output
```
int((c*x)^(m + 1)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^{1+m}}{a+bx^2}\,dx = \frac{c^m c\left(x^m - \left(\int \frac{x^m}{b\,x^3+ax}dx\right) am\right)}{bm}$$

input
```
int((c*x)^(1+m)/(b*x^2+a),x)
```

output
```
(c**m*c*(x**m - int(x**m/(a*x + b*x**3),x)*a*m))/(b*m)
```

3.358 $\int \frac{(cx)^m}{a+bx^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \frac{(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ac(1+m)}$$

output
```
(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \frac{x(cx)^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}$$

input
```
Integrate[(c*x)^m/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/(
a*(1 + m))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{a+bx^2} \, dx$$
$$\Big\downarrow \ 278$$
$$\frac{(cx)^{m+1} \text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)}$$

input
```
Int[(c*x)^m/(a + b*x^2),x]
```

output
```
((c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(
a*c*(1 + m))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{b\,x^2+a}dx$$

input
```
int((c*x)^m/(b*x^2+a),x)
```

output
```
int((c*x)^m/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^m/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \frac{c^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{c^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m/(b*x**2+a),x)
```

output
```
c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m
/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x**(m + 1)*lerchphi(b*x**2*exp_pol
ar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{a+bx^2}\,dx = \int \frac{(c\,x)^m}{b\,x^2+a}\,dx$$

input
```
int((c*x)^m/(a + b*x^2),x)
```

output
```
int((c*x)^m/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{a+bx^2}\,dx = c^m\left(\int \frac{x^m}{b\,x^2+a}dx\right)$$

input
```
int((c*x)^m/(b*x^2+a),x)
```

output
```
c**m*int(x**m/(a + b*x**2),x)
```

3.359 $\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 38

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\frac{(cx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{2+m}{2},-\frac{bx^2}{a}\right)}{acm}$$

output
```
(c*x)^m*hypergeom([1, 1/2*m],[1+1/2*m],-b*x^2/a)/a/c/m
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\frac{x(cx)^{-1+m} \text{Hypergeometric2F1}\left(1,\frac{m}{2},1+\frac{m}{2},-\frac{bx^2}{a}\right)}{am}$$

input
```
Integrate[(c*x)^(-1 + m)/(a + b*x^2),x]
```

output
```
(x*(c*x)^(-1 + m)*Hypergeometric2F1[1, m/2, 1 + m/2, -((b*x^2)/a)])/(a*m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{m-1}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(cx)^m \text{Hypergeometric2F1}\left(1,\frac{m}{2},\frac{m+2}{2},-\frac{bx^2}{a}\right)}{acm}$$

input
```
Int[(c*x)^(-1 + m)/(a + b*x^2),x]
```

output
```
((c*x)^m*Hypergeometric2F1[1, m/2, (2 + m)/2, -((b*x^2)/a)])/(a*c*m)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{m-1}}{b\,x^2+a}dx$$

input
```
int((c*x)^(m-1)/(b*x^2+a),x)
```

output
```
int((c*x)^(m-1)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\int \frac{(cx)^{m-1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^(m - 1)/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.34

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\frac{a^{\frac{m}{2}}a^{-\frac{m}{2}-1}c^{m-1}mx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}\right)\Gamma\left(\frac{m}{2}\right)}{4\Gamma\left(\frac{m}{2}+1\right)}$$

input
```
integrate((c*x)**(-1+m)/(b*x**2+a),x)
```

output
```
a**(m/2)*a**(-m/2 - 1)*c**(m - 1)*m*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2)*gamma(m/2)/(4*gamma(m/2 + 1))
```

Maxima [F]

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\int \frac{(cx)^{m-1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\int \frac{(cx)^{m-1}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-1+m)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^(m - 1)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\int \frac{(c\,x)^{m-1}}{b\,x^2+a}\,dx$$

input
```
int((c*x)^(m - 1)/(a + b*x^2),x)
```

output
```
int((c*x)^(m - 1)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-1+m}}{a+bx^2}\,dx=\frac{c^m\left(\int \frac{x^m}{b\,x^3+ax}dx\right)}{c}$$

input
```
int((c*x)^(-1+m)/(b*x^2+a),x)
```

output
```
(c**m*int(x**m/(a*x + b*x**3),x))/c
```

3.360 $\int \frac{(cx)^{-2+m}}{a+bx^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{(cx)^{-2+m}}{a+bx^2} dx = -\frac{(cx)^{-1+m} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-1+m), \frac{1+m}{2}, -\frac{bx^2}{a}\right)}{ac(1-m)}$$

output
```
-(c*x)^(-1+m)*hypergeom([1, -1/2+1/2*m],[1/2+1/2*m],-b*x^2/a)/a/c/(1-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{(cx)^{-2+m}}{a+bx^2} dx = \frac{x(cx)^{-2+m} \text{Hypergeometric2F1}\left(1, \frac{1}{2}(-1+m), 1+\frac{1}{2}(-1+m), -\frac{bx^2}{a}\right)}{a(-1+m)}$$

input
```
Integrate[(c*x)^(-2 + m)/(a + b*x^2),x]
```

output
```
(x*(c*x)^(-2 + m)*Hypergeometric2F1[1, (-1 + m)/2, 1 + (-1 + m)/2, -((b*x^
2)/a)])/(a*(-1 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{m-2}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{(cx)^{m-1}\,\text{Hypergeometric2F1}\left(1,\frac{m-1}{2},\frac{m+1}{2},-\frac{bx^2}{a}\right)}{ac(1-m)}$$

input
```
Int[(c*x)^(-2 + m)/(a + b*x^2),x]
```

output
```
-(((c*x)^(-1 + m)*Hypergeometric2F1[1, (-1 + m)/2, (1 + m)/2, -((b*x^2)/a)
])/(a*c*(1 - m)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{-2+m}}{b\,x^2+a}dx$$

input
```
int((c*x)^(-2+m)/(b*x^2+a),x)
```

output
```
int((c*x)^(-2+m)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-2}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^(m - 2)/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \frac{c^{m-2}mx^{m-1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)} - \frac{c^{m-2}x^{m-1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-\frac{1}{2}\right)\Gamma\left(\frac{m}{2}-\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

input
```
integrate((c*x)**(-2+m)/(b*x**2+a),x)
```

output
```
c**(m - 2)*m*x**(m - 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*g
amma(m/2 - 1/2)/(4*a*gamma(m/2 + 1/2)) - c**(m - 2)*x**(m - 1)*lerchphi(b*
x**2*exp_polar(I*pi)/a, 1, m/2 - 1/2)*gamma(m/2 - 1/2)/(4*a*gamma(m/2 + 1/
2))
```

Maxima [F]

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-2}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-2}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-2+m)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^(m - 2)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \int \frac{(c\,x)^{m-2}}{b\,x^2+a}\,dx$$

input
```
int((c*x)^(m - 2)/(a + b*x^2),x)
```

output
```
int((c*x)^(m - 2)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-2+m}}{a+bx^2}\,dx = \frac{c^m\left(\int \frac{x^m}{b\,x^4+a\,x^2}dx\right)}{c^2}$$

input
```
int((c*x)^(-2+m)/(b*x^2+a),x)
```

output
```
(c**m*int(x**m/(a*x**2 + b*x**4),x))/c**2
```

3.361 $\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = -\frac{(cx)^{-2+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-2+m),\frac{m}{2},-\frac{bx^2}{a}\right)}{ac(2-m)}$$

output
```
-(c*x)^(-2+m)*hypergeom([1, -1+1/2*m],[1/2*m],-b*x^2/a)/a/c/(2-m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \frac{x(cx)^{-3+m}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}(-2+m),1+\frac{1}{2}(-2+m),-\frac{bx^2}{a}\right)}{a(-2+m)}$$

input
```
Integrate[(c*x)^(-3 + m)/(a + b*x^2),x]
```

output
```
(x*(c*x)^(-3 + m)*Hypergeometric2F1[1, (-2 + m)/2, 1 + (-2 + m)/2, -((b*x^
2)/a)])/(a*(-2 + m))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{m-3}}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{(cx)^{m-2}\,\text{Hypergeometric2F1}\left(1,\frac{m-2}{2},\frac{m}{2},-\frac{bx^2}{a}\right)}{ac(2-m)}$$

input
```
Int[(c*x)^(-3 + m)/(a + b*x^2),x]
```

output
```
-(((c*x)^(-2 + m)*Hypergeometric2F1[1, (-2 + m)/2, m/2, -((b*x^2)/a)])/(a*
c*(2 - m)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{-3+m}}{b\,x^2+a}dx$$

input
```
int((c*x)^(-3+m)/(b*x^2+a),x)
```

output
```
int((c*x)^(-3+m)/(b*x^2+a),x)
```

Fricas [F]

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-3}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^(m - 3)/(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.89

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \frac{c^{m-3}mx^{m-2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{4a\Gamma\left(\frac{m}{2}\right)} - \frac{c^{m-3}x^{m-2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}-1\right)\Gamma\left(\frac{m}{2}-1\right)}{2a\Gamma\left(\frac{m}{2}\right)}$$

input
```
integrate((c*x)**(-3+m)/(b*x**2+a),x)
```

output
```
c**(m - 3)*m*x**(m - 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 - 1)*gam
ma(m/2 - 1)/(4*a*gamma(m/2)) - c**(m - 3)*x**(m - 2)*lerchphi(b*x**2*exp_p
olar(I*pi)/a, 1, m/2 - 1)*gamma(m/2 - 1)/(2*a*gamma(m/2))
```

Maxima [F]

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-3}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \int \frac{(cx)^{m-3}}{bx^2+a}\,dx$$

input
```
integrate((c*x)^(-3+m)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^(m - 3)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \int \frac{(c\,x)^{m-3}}{b\,x^2+a}\,dx$$

input
```
int((c*x)^(m - 3)/(a + b*x^2),x)
```

output
```
int((c*x)^(m - 3)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{(cx)^{-3+m}}{a+bx^2}\,dx = \frac{c^m\left(\int \frac{x^m}{b\,x^5+a\,x^3}dx\right)}{c^3}$$

input
```
int((c*x)^(-3+m)/(b*x^2+a),x)
```

output
```
(c**m*int(x**m/(a*x**3 + b*x**5),x))/c**3
```

3.362 $\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 36

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{ax^2}{b}\right)}{1+m}$$

output
```
x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-a*x^2/b)/(1+m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2} dx = \frac{x^{1+m} \text{Hypergeometric2F1}\left(2,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{ax^2}{b}\right)}{1+m}$$

input
```
Integrate[x^m/(1 + (a*x^2)/b)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, 1 + (1 + m)/2, -((a*x^2)/b)])/(
1 + m)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ax^2}{b}\right)}{m+1}$$

input
```
Int[x^m/(1 + (a*x^2)/b)^2,x]
```

output
```
(x^(1 + m)*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((a*x^2)/b)])/(1 +
m)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.64 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

method	result	size
meijerg	$\frac{\left(\frac{a}{b}\right)^{-\frac{1}{2}-\frac{m}{2}}\left(\frac{2x^{1+m}\left(\frac{a}{b}\right)^{\frac{1}{2}+\frac{m}{2}}}{2+\frac{2a\,x^{2}}{b}}+\frac{2x^{1+m}\left(\frac{a}{b}\right)^{\frac{1}{2}+\frac{m}{2}}\left(-\frac{m^{2}}{4}+\frac{1}{4}\right)\mathrm{LerchPhi}\left(-\frac{a\,x^{2}}{b},1,\frac{1}{2}+\frac{m}{2}\right)}{1+m}\right)}{2}$	92

input
```
int(x^m/(1+a/b*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(a/b)^(-1/2-1/2*m)*(2*x^(1+m)*(a/b)^(1/2+1/2*m)/(2+2*a/b*x^2)+2/(1+m)*
x^(1+m)*(a/b)^(1/2+1/2*m)*(-1/4*m^2+1/4)*LerchPhi(-a/b*x^2,1,1/2+1/2*m))
```

Fricas [F]

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx = \int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

input
```
integrate(x^m/(1+a*x^2/b)^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^m/(a^2*x^4 + 2*a*b*x^2 + b^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.93 (sec) , antiderivative size = 347, normalized size of antiderivative = 9.64

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx = -\frac{am^2x^2x^{m+1}\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{ax^2x^{m+1}\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} - \frac{bm^2x^{m+1}\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{2bmx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{bx^{m+1}\Phi\left(\frac{ax^2e^{i\pi}}{b},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{2bx^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{8ax^2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)+8b\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate(x**m/(1+a*x**2/b)**2,x)
```

output
```
-a*m**2*x**2*x**(m + 1)*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*g
amma(m/2 + 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + a*x**
2*x**(m + 1)*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 +
1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) - b*m**2*x**(m + 1
)*lerchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x
**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + 2*b*m*x**(m + 1)*gamma(m/2
+ 1/2)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + b*x**(m + 1)*l
erchphi(a*x**2*exp_polar(I*pi)/b, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(8*a*x**2
*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2)) + 2*b*x**(m + 1)*gamma(m/2 + 1/2
)/(8*a*x**2*gamma(m/2 + 3/2) + 8*b*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx=\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

input
```
integrate(x^m/(1+a*x^2/b)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/(a*x^2/b + 1)^2, x)
```

Giac [F]

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx=\int \frac{x^m}{\left(\frac{ax^2}{b}+1\right)^2}\,dx$$

input
```
integrate(x^m/(1+a*x^2/b)^2,x, algorithm="giac")
```

output
```
integrate(x^m/(a*x^2/b + 1)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx=\int \frac{x^m}{\left(\frac{a\,x^2}{b}+1\right)^2}\,dx$$

input
```
int(x^m/((a*x^2)/b + 1)^2,x)
```

output
```
int(x^m/((a*x^2)/b + 1)^2, x)
```

Reduce [F]

$$\int \frac{x^m}{\left(1+\frac{ax^2}{b}\right)^2}\,dx = \left(\int \frac{x^m}{a^2x^4+2ab\,x^2+b^2}dx\right)b^2$$

input
```
int(x^m/(1+a*x^2/b)^2,x)
```

output
```
int(x**m/(a**2*x**4 + 2*a*b*x**2 + b**2),x)*b**2
```

3.363 $\int x^7\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7\sqrt{a+bx^2}\,dx=-\frac{a^3(a+bx^2)^{3/2}}{3b^4}+\frac{3a^2(a+bx^2)^{5/2}}{5b^4}-\frac{3a(a+bx^2)^{7/2}}{7b^4}+\frac{(a+bx^2)^{9/2}}{9b^4}$$

output
```
-1/3*a^3*(b*x^2+a)^(3/2)/b^4+3/5*a^2*(b*x^2+a)^(5/2)/b^4-3/7*a*(b*x^2+a)^(
7/2)/b^4+1/9*(b*x^2+a)^(9/2)/b^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x^7\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(-16a^4+8a^3bx^2-6a^2b^2x^4+5ab^3x^6+35b^4x^8)}{315b^4}$$

input
```
Integrate[x^7*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^4 + 8*a^3*b*x^2 - 6*a^2*b^2*x^4 + 5*a*b^3*x^6 + 35
*b^4*x^8))/(315*b^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^6\sqrt{bx^2+a}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{7/2}}{b^3}-\frac{3a\left(bx^2+a\right)^{5/2}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{3/2}}{b^3}-\frac{a^3\sqrt{bx^2+a}}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^3\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{6a^2\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{2\left(a+bx^2\right)^{9/2}}{9b^4}-\frac{6a\left(a+bx^2\right)^{7/2}}{7b^4}\right)$$

input
```
Int[x^7*Sqrt[a + b*x^2],x]
```

output
```
((-2*a^3*(a + b*x^2)^(3/2))/(3*b^4) + (6*a^2*(a + b*x^2)^(5/2))/(5*b^4) -
(6*a*(a + b*x^2)^(7/2))/(7*b^4) + (2*(a + b*x^2)^(9/2))/(9*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-35b^{3}x^{6}+30a\,b^{2}x^{4}-24a^{2}b\,x^{2}+16a^{3}\right)}{315b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-35b^{3}x^{6}+30a\,b^{2}x^{4}-24a^{2}b\,x^{2}+16a^{3}\right)}{315b^{4}}$	47
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-35b^{3}x^{6}+30a\,b^{2}x^{4}-24a^{2}b\,x^{2}+16a^{3}\right)}{315b^{4}}$	47
trager	$-\frac{\left(-35b^{4}x^{8}-5a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}-8a^{3}b\,x^{2}+16a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{4}}$	58
risch	$-\frac{\left(-35b^{4}x^{8}-5a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}-8a^{3}b\,x^{2}+16a^{4}\right)\sqrt{b\,x^{2}+a}}{315b^{4}}$	58
default	$\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9b}-\frac{2a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}\right)}{7b}\right)}{3b}$	82

input
```
int(x^7*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/315*(b*x^2+a)^(3/2)*(-35*b^3*x^6+30*a*b^2*x^4-24*a^2*b*x^2+16*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7\sqrt{a+bx^2}\,dx = \frac{\left(35\,b^4x^8+5\,ab^3x^6-6\,a^2b^2x^4+8\,a^3bx^2-16\,a^4\right)\sqrt{bx^2+a}}{315\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^4*x^8 + 5*a*b^3*x^6 - 6*a^2*b^2*x^4 + 8*a^3*b*x^2 - 16*a^4)*sq
rt(b*x^2 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.38

$$\int x^7\sqrt{a+bx^2}\,dx = \begin{cases} -\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-16*a**4*sqrt(a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x
**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a
+ b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^7\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}x^6}{9\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}ax^4}{21\,b^2} + \frac{8\,(bx^2+a)^{\frac{3}{2}}a^2x^2}{105\,b^3} - \frac{16\,(bx^2+a)^{\frac{3}{2}}a^3}{315\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(3/2)*x^6/b - 2/21*(b*x^2 + a)^(3/2)*a*x^4/b^2 + 8/105*(b*
x^2 + a)^(3/2)*a^2*x^2/b^3 - 16/315*(b*x^2 + a)^(3/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7\sqrt{a+bx^2}\,dx = \frac{35\,(bx^2+a)^{\frac{9}{2}} - 135\,(bx^2+a)^{\frac{7}{2}}a + 189\,(bx^2+a)^{\frac{5}{2}}a^2 - 105\,(bx^2+a)^{\frac{3}{2}}a^3}{315\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/315*(35*(b*x^2 + a)^(9/2) - 135*(b*x^2 + a)^(7/2)*a + 189*(b*x^2 + a)^(5
/2)*a^2 - 105*(b*x^2 + a)^(3/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^7\sqrt{a+bx^2}\,dx = \sqrt{b\,x^2+a}\left(\frac{x^8}{9} - \frac{16\,a^4}{315\,b^4} + \frac{a\,x^6}{63\,b} - \frac{2\,a^2\,x^4}{105\,b^2} + \frac{8\,a^3\,x^2}{315\,b^3}\right)$$

input
```
int(x^7*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*(x^8/9 - (16*a^4)/(315*b^4) + (a*x^6)/(63*b) - (2*a^2*x^
4)/(105*b^2) + (8*a^3*x^2)/(315*b^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int x^7\sqrt{a+bx^2}\,dx = \frac{\sqrt{b\,x^2+a}\,(35b^4x^8+5a\,b^3x^6-6a^2b^2x^4+8a^3b\,x^2-16a^4)}{315b^4}$$

input
```
int(x^7*(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**4 + 8*a**3*b*x**2 - 6*a**2*b**2*x**4 + 5*a*b**
3*x**6 + 35*b**4*x**8))/(315*b**4)
```

3.364 $\int x^5\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5\sqrt{a+bx^2}\,dx=\frac{a^2(a+bx^2)^{3/2}}{3b^3}-\frac{2a(a+bx^2)^{5/2}}{5b^3}+\frac{(a+bx^2)^{7/2}}{7b^3}$$

output
```
1/3*a^2*(b*x^2+a)^(3/2)/b^3-2/5*a*(b*x^2+a)^(5/2)/b^3+1/7*(b*x^2+a)^(7/2)/
b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int x^5\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(8a^3-4a^2bx^2+3ab^2x^4+15b^3x^6)}{105b^3}$$

input
```
Integrate[x^5*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^3 - 4*a^2*b*x^2 + 3*a*b^2*x^4 + 15*b^3*x^6))/(105*b^
3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4\sqrt{bx^2+a}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{5/2}}{b^2}-\frac{2a\left(bx^2+a\right)^{3/2}}{b^2}+\frac{a^2\sqrt{bx^2+a}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^2\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{2\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{4a\left(a+bx^2\right)^{5/2}}{5b^3}\right)$$

input
```
Int[x^5*Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(a + b*x^2)^(3/2))/(3*b^3) - (4*a*(a + b*x^2)^(5/2))/(5*b^3) + (2*
(a + b*x^2)^(7/2))/(7*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{4}-12 a b\,x^{2}+8 a^{2}\right)}{105 b^{3}}$	36
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{4}-12 a b\,x^{2}+8 a^{2}\right)}{105 b^{3}}$	36
orering	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \left(15 b^{2} x^{4}-12 a b\,x^{2}+8 a^{2}\right)}{105 b^{3}}$	36
trager	$\frac{\left(15 b^{3} x^{6}+3 a\,b^{2} x^{4}-4 a^{2} b\,x^{2}+8 a^{3}\right) \sqrt{b\,x^{2}+a}}{105 b^{3}}$	47
risch	$\frac{\left(15 b^{3} x^{6}+3 a\,b^{2} x^{4}-4 a^{2} b\,x^{2}+8 a^{3}\right) \sqrt{b\,x^{2}+a}}{105 b^{3}}$	47
default	$\frac{x^{4} \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7 b}-\frac{4 a \left(\frac{x^{2} \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5 b}-\frac{2 a \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15 b^{2}}\right)}{7 b}$	58

input
```
int(x^5*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(b*x^2+a)^(3/2)*(15*b^2*x^4-12*a*b*x^2+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^5\sqrt{a+bx^2}\,dx=\frac{(15\,b^3x^6+3\,ab^2x^4-4\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{105\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b^3*x^6 + 3*a*b^2*x^4 - 4*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int x^5\sqrt{a+bx^2}\,dx = \begin{cases} \frac{8a^3\sqrt{a+bx^2}}{105b^3} - \frac{4a^2x^2\sqrt{a+bx^2}}{105b^2} + \frac{ax^4\sqrt{a+bx^2}}{35b} + \frac{x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7,
Ne(b, 0)), (sqrt(a)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}x^4}{7\,b} - \frac{4\,(bx^2+a)^{\frac{3}{2}}ax^2}{35\,b^2} + \frac{8\,(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(3/2)*x^4/b - 4/35*(b*x^2 + a)^(3/2)*a*x^2/b^2 + 8/105*(b*
x^2 + a)^(3/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5\sqrt{a+bx^2}\,dx = \frac{15\,(bx^2+a)^{\frac{7}{2}} - 42\,(bx^2+a)^{\frac{5}{2}}a + 35\,(bx^2+a)^{\frac{3}{2}}a^2}{105\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/105*(15*(b*x^2 + a)^(7/2) - 42*(b*x^2 + a)^(5/2)*a + 35*(b*x^2 + a)^(3/2
)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^5\sqrt{a+bx^2}\,dx = \sqrt{b\,x^2+a}\,\left(\frac{x^6}{7}+\frac{8\,a^3}{105\,b^3}+\frac{a\,x^4}{35\,b}-\frac{4\,a^2\,x^2}{105\,b^2}\right)$$

input
```
int(x^5*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*(x^6/7 + (8*a^3)/(105*b^3) + (a*x^4)/(35*b) - (4*a^2*x^2
)/(105*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int x^5\sqrt{a+bx^2}\,dx = \frac{\sqrt{b\,x^2+a}\,(15b^3x^6+3a\,b^2x^4-4a^2b\,x^2+8a^3)}{105b^3}$$

input
```
int(x^5*(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**3 - 4*a**2*b*x**2 + 3*a*b**2*x**4 + 15*b**3*x**6))
/(105*b**3)
```

3.365 $\int x^3\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3\sqrt{a+bx^2}\,dx=-\frac{a(a+bx^2)^{3/2}}{3b^2}+\frac{(a+bx^2)^{5/2}}{5b^2}$$

output
```
-1/3*a*(b*x^2+a)^(3/2)/b^2+1/5*(b*x^2+a)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(-2a^2+abx^2+3b^2x^4)}{15b^2}$$

input
```
Integrate[x^3*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a^2 + a*b*x^2 + 3*b^2*x^4))/(15*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^2\sqrt{bx^2+a}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{3/2}}{b}-\frac{a\sqrt{bx^2+a}}{b}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{5/2}}{5b^2}-\frac{2a\left(a+bx^2\right)^{3/2}}{3b^2}\right)$$

input
```
Int[x^3*Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(a + b*x^2)^(3/2))/(3*b^2) + (2*(a + b*x^2)^(5/2))/(5*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-3b\,x^{2}+2a\right)}{15b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-3b\,x^{2}+2a\right)}{15b^{2}}$	25
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-3b\,x^{2}+2a\right)}{15b^{2}}$	25
default	$\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15b^{2}}$	34
trager	$-\frac{\left(-3b^{2}x^{4}-ab\,x^{2}+2a^{2}\right)\sqrt{b\,x^{2}+a}}{15b^{2}}$	36
risch	$-\frac{\left(-3b^{2}x^{4}-ab\,x^{2}+2a^{2}\right)\sqrt{b\,x^{2}+a}}{15b^{2}}$	36

input
```
int(x^3*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(3/2)*(-3*b*x^2+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^3\sqrt{a+bx^2}\,dx=\frac{(3\,b^2x^4+abx^2-2\,a^2)\sqrt{bx^2+a}}{15\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*x^4 + a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(31) = 62$.

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.66

$$\int x^3\sqrt{a+bx^2}\,dx = \begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(1
5*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}x^2}{5\,b} - \frac{2\,(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*x^2/b - 2/15*(b*x^2 + a)^(3/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{2}} - 5\,(bx^2+a)^{\frac{3}{2}}a}{15\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/15*(3*(b*x^2 + a)^(5/2) - 5*(b*x^2 + a)^(3/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3\sqrt{a+bx^2}\,dx = \sqrt{b\,x^2+a}\left(\frac{x^4}{5} - \frac{2\,a^2}{15\,b^2} + \frac{a\,x^2}{15\,b}\right)$$

input
```
int(x^3*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*(x^4/5 - (2*a^2)/(15*b^2) + (a*x^2)/(15*b))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3\sqrt{a+bx^2}\,dx = \frac{\sqrt{b\,x^2+a}\,(3b^2x^4+ab\,x^2-2a^2)}{15b^2}$$

input
```
int(x^3*(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a**2 + a*b*x**2 + 3*b**2*x**4))/(15*b**2)
```

3.366 $\int x\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x\sqrt{a+bx^2}\,dx=\frac{(a+bx^2)^{3/2}}{3b}$$

output
```
1/3*(b*x^2+a)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\sqrt{a+bx^2}\,dx=\frac{(a+bx^2)^{3/2}}{3b}$$

input
```
Integrate[x*Sqrt[a + b*x^2],x]
```

output
```
(a + b*x^2)^(3/2)/(3*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx^2)^{3/2}}{3b}$$

input
```
Int[x*Sqrt[a + b*x^2],x]
```

output
```
(a + b*x^2)^(3/2)/(3*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
default	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
trager	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
risch	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15
orering	$\frac{(b\,x^2+a)^{\frac{3}{2}}}{3b}$	15

input
```
int(x*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(b*x^2 + a)^(3/2)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int x\sqrt{a+bx^2}\,dx = \begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b \neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((a*sqrt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)),
(sqrt(a)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(b*x^2 + a)^(3/2)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}}{3\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*x^2 + a)^(3/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt{a+bx^2}\,dx=\frac{{\left(b\,x^2+a\right)}^{3/2}}{3\,b}$$

input
```
int(x*(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(3/2)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x\sqrt{a+bx^2}\,dx=\frac{\sqrt{b\,x^2+a}\,(b\,x^2+a)}{3b}$$

input
```
int(x*(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(a + b*x**2))/(3*b)
```

3.367 $\int \frac{\sqrt{a+bx^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{\sqrt{a+bx^2}}{x} \, dx = \sqrt{a+bx^2} - \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
(b*x^2+a)^(1/2)-a^(1/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{x} \, dx = \sqrt{a+bx^2} - \sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[Sqrt[a + b*x^2]/x,x]
```

output
```
Sqrt[a + b*x^2] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)$$

input
```
Int[Sqrt[a + b*x^2]/x,x]
```

output
```
(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\sqrt{b\,x^{2}+a}-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)$	30
default	$\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)$	39

input
```
int((b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)-a^(1/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = \left[\frac{1}{2}\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+\sqrt{bx^2+a}, \sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\sqrt{bx^2+a}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + sqrt(b*
x^2 + a), sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + sqrt(b*x^2 + a)]
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = -\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{a}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(1/2)/x,x)
```

output
```
-sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
 sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = -\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\sqrt{bx^2+a}$$

input
```
integrate((b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
-sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = \frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{bx^2+a}$$

input
```
integrate((b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

output
```
a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x^2 + a)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = \sqrt{b\,x^2+a} - \sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)$$

input
```
int((a + b*x^2)^(1/2)/x,x)
```

output
```
(a + b*x^2)^(1/2) - a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{a+bx^2}}{x}\,dx = \sqrt{b\,x^2+a} + \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right) - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)$$

input
```
int((b*x^2+a)^(1/2)/x,x)
```

output
```
sqrt(a + b*x**2) + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a)) - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))
```

3.368 $\int \frac{\sqrt{a+bx^2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{\sqrt{a+bx^2}}{x^3} \, dx = -\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*(b*x^2+a)^(1/2)/x^2-1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{x^3} \, dx = -\frac{\sqrt{a+bx^2}}{2x^2} - \frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^3,x]
```

output
```
-1/2*Sqrt[a + b*x^2]/x^2 - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a]
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^3} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^4}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)$$

input
```
Int[Sqrt[a + b*x^2]/x^3,x]
```

output
```
(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{b\,x^{2}+a}}{x^{2} b}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{2}$	41
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2x^{2}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2\sqrt{a}}$	45
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}$	63

input
```
int((b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*b*(-(b*x^2+a)^(1/2)/x^2/b-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx$$
$$= \left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}a}{4ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)-\sqrt{bx^2+a}a}{2ax^2}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
2*sqrt(b*x^2 + a)*a)/(a*x^2), 1/2*(sqrt(-a)*b*x^2*arctan(sqrt(b*x^2 + a)*s
qrt(-a)/a) - sqrt(b*x^2 + a)*a)/(a*x^2)]
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} - \frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(1/2)/x**3,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - b*asinh(sqrt(a)/(sqrt(b)*x))/(2*sqrt
(a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = -\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,\sqrt{a}} + \frac{\sqrt{bx^2+a}b}{2\,a} - \frac{(bx^2+a)^{\frac{3}{2}}}{2\,ax^2}$$

input
```
integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*sqrt(b*x^2 + a)*b/a - 1
/2*(b*x^2 + a)^(3/2)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = \frac{1}{2}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx^2+a}}{bx^2}\right)$$

input
```
integrate((b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x^2 + a)/(b*x^2)
)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = -\frac{\sqrt{b\,x^2+a}}{2\,x^2} - \frac{b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}}$$

input
```
int((a + b*x^2)^(1/2)/x^3,x)
```

output
```
- (a + b*x^2)^(1/2)/(2*x^2) - (b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.68

$$\int \frac{\sqrt{a+bx^2}}{x^3}\,dx = \frac{-\sqrt{b\,x^2+a}\,a+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2-\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2}{2a\,x^2}$$

input
```
int((b*x^2+a)^(1/2)/x^3,x)
```

output
```
( - sqrt(a + b*x**2)*a + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*b*x**2)/(2*a*x**2)
```

3.369 $\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = -\frac{\sqrt{a+bx^2}}{4x^4} - \frac{b\sqrt{a+bx^2}}{8ax^2} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

output
```
-1/4*(b*x^2+a)^(1/2)/x^4-1/8*b*(b*x^2+a)^(1/2)/a/x^2+1/8*b^2*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = \frac{(-2a-bx^2)\sqrt{a+bx^2}}{8ax^4} + \frac{b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{3/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^5,x]
```

output
```
((-2*a - b*x^2)*Sqrt[a + b*x^2])/(8*a*x^4) + (b^2*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(8*a^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^5} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{\sqrt{bx^2+a}}{x^6}dx^2$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2} \left(\frac{1}{4} b \int \frac{1}{x^4 \sqrt{bx^2+a}}dx^2 - \frac{\sqrt{a+bx^2}}{2x^4}\right)$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2} \left(\frac{1}{4} b \left(-\frac{b \int \frac{1}{x^2 \sqrt{bx^2+a}}dx^2}{2a} - \frac{\sqrt{a+bx^2}}{ax^2}\right) - \frac{\sqrt{a+bx^2}}{2x^4}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2} \left(\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a} - \frac{\sqrt{a+bx^2}}{ax^2}\right) - \frac{\sqrt{a+bx^2}}{2x^4}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2} \left(\frac{1}{4} b \left(\frac{b \text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx^2}}{ax^2}\right) - \frac{\sqrt{a+bx^2}}{2x^4}\right)$$

input
```
Int[Sqrt[a + b*x^2]/x^5,x]
```

output
```
(-1/2*Sqrt[a + b*x^2]/x^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)b^{2}x^{4}-\sqrt{b\,x^{2}+a}\left(b\,x^{2}\sqrt{a}+2a^{\frac{3}{2}}\right)}{8a^{\frac{3}{2}}x^{4}}$	56
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(b\,x^{2}+2a)}{8x^{4}a}+\frac{b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8a^{\frac{3}{2}}}$	59
default	$-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{4a\,x^{4}}-\frac{b\left(-\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{2a\,x^{2}}+\frac{b\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)}{2a}\right)}{4a}$	87

input
```
int((b*x^2+a)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^(3/2)*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*b^2*x^4-(b*x^2+a)^(1/2)*(b*x
^2*a^(1/2)+2*a^(3/2)))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = \left[\frac{\sqrt{a}b^2x^4\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(abx^2+2a^2\right)\sqrt{bx^2+a}}{16a^2x^4}, -\frac{\sqrt{-a}b^2x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\left(abx^2+2a^2\right)\sqrt{bx^2+a}}{8a^2x^4}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(sqrt(a)*b^2*x^4*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 - 2*(a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a^2*x^4), -1/8*(sqrt(-a)*b^2*x^4*
arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a
^2*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = -\frac{a}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{b^{\frac{3}{2}}}{8ax\sqrt{\frac{a}{bx^2}+1}} + \frac{b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(1/2)/x**5,x)
```

output
```
-a/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(8*x**3*sqrt(a/(b*x**
2) + 1)) - b**(3/2)/(8*a*x*sqrt(a/(b*x**2) + 1)) + b**2*asinh(sqrt(a)/(sqr
t(b)*x))/(8*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = \frac{b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}b^2}{8\,a^2} + \frac{(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{3}{2}}}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
1/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/8*sqrt(b*x^2 + a)*b^2/a^
2 + 1/8*(b*x^2 + a)^(3/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(3/2)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = -\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{(bx^2+a)^{\frac{3}{2}}b^3+\sqrt{bx^2+a}ab^3}{ab^2x^4}}{8\,b}$$

input
```
integrate((b*x^2+a)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/8*(b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + ((b*x^2 + a)^(3/
2)*b^3 + sqrt(b*x^2 + a)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx = \frac{b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{3/2}} - \frac{\sqrt{b\,x^2+a}}{8\,x^4} - \frac{(b\,x^2+a)^{3/2}}{8\,a\,x^4}$$

input
```
int((a + b*x^2)^(1/2)/x^5,x)
```

output
```
(b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(3/2)) - (a + b*x^2)^(1/2)/(8*
x^4) - (a + b*x^2)^(3/2)/(8*a*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{a+bx^2}}{x^5}\,dx$$
$$= \frac{-2\sqrt{b\,x^2+a}\,a^2 - \sqrt{b\,x^2+a}\,ab\,x^2 - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^4 + \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^4}{8a^2x^4}$$

input
```
int((b*x^2+a)^(1/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2 - sqrt(a + b*x**2)*a*b*x**2 - sqrt(a)*log((sqr
t(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 + sqrt(a)*log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4)/(8*a**2*x**4)
```

3.370 $\int \frac{\sqrt{a+bx^2}}{x^7} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{\sqrt{a+bx^2}}{x^7} \, dx = -\frac{\sqrt{a+bx^2}}{6x^6} - \frac{b\sqrt{a+bx^2}}{24ax^4} + \frac{b^2\sqrt{a+bx^2}}{16a^2x^2} - \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

output
```
-1/6*(b*x^2+a)^(1/2)/x^6-1/24*b*(b*x^2+a)^(1/2)/a/x^4+1/16*b^2*(b*x^2+a)^(
1/2)/a^2/x^2-1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a+bx^2}}{x^7} \, dx = \frac{\sqrt{a+bx^2}(-8a^2-2abx^2+3b^2x^4)}{48a^2x^6} - \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{5/2}}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2 - 2*a*b*x^2 + 3*b^2*x^4))/(48*a^2*x^6) - (b^3*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{bx^2+a}}{x^8}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{6}b\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)$$

input
```
Int[Sqrt[a + b*x^2]/x^7,x]
```

output
```
(-1/3*Sqrt[a + b*x^2]/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqr
t[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*
a)))/6)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\sqrt{b x^2+a}\,(-3b^2x^4+2ab\,x^2+8a^2)}{48x^6a^2}-\frac{b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{16a^{\frac{5}{2}}}$	71
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)b^3x^6+3b^2x^4\sqrt{b\,x^2+a}\,\sqrt{a}-2a^{\frac{3}{2}}b\,x^2\sqrt{b\,x^2+a}-8a^{\frac{5}{2}}\sqrt{b\,x^2+a}}{48a^{\frac{5}{2}}x^6}$	84
default	$-\frac{(b\,x^2+a)^{\frac{3}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{4a\,x^4}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)}{4a}\right)}{2a}$	111

input
```
int((b*x^2+a)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(b*x^2+a)^(1/2)*(-3*b^2*x^4+2*a*b*x^2+8*a^2)/x^6/a^2-1/16/a^(5/2)*b^
3*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.68

$$\int\frac{\sqrt{a+bx^2}}{x^7}\,dx$$
$$=\left[\frac{3\,\sqrt{a}b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,ab^2x^4-2\,a^2bx^2-8\,a^3)\sqrt{bx^2+a}}{96\,a^3x^6},\ \frac{3\,\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{bx^2+a}}{a}\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b^2*x^4 - 2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6), 1/4
8*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*a*b^2*x^4 -
2*a^2*b*x^2 - 8*a^3)*sqrt(b*x^2 + a))/(a^3*x^6)]
```

Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{a+bx^2}}{x^7}\,dx = -\frac{a}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{5\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{3}{2}}}{48ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{5}{2}}}{16a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{5}{2}}}$$

input
```
integrate((b*x**2+a)**(1/2)/x**7,x)
```

output
```
-a/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 5*sqrt(b)/(24*x**5*sqrt(a/(b*x*
*2) + 1)) + b**(3/2)/(48*a*x**3*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(16*a**2*
x*sqrt(a/(b*x**2) + 1)) - b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{\sqrt{a+bx^2}}{x^7}\,dx = -\frac{b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{bx^2+a}b^3}{16\,a^3} - \frac{(bx^2+a)^{\frac{3}{2}}b^2}{16\,a^3x^2} + \frac{(bx^2+a)^{\frac{3}{2}}b}{8\,a^2x^4} - \frac{(bx^2+a)^{\frac{3}{2}}}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
-1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/16*sqrt(b*x^2 + a)*b^3
/a^3 - 1/16*(b*x^2 + a)^(3/2)*b^2/(a^3*x^2) + 1/8*(b*x^2 + a)^(3/2)*b/(a^2
*x^4) - 1/6*(b*x^2 + a)^(3/2)/(a*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^2}}{x^7}\,dx = \frac{1}{48}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx^2+a)^{\frac{5}{2}}-8\,(bx^2+a)^{\frac{3}{2}}a-3\,\sqrt{bx^2+a}a^2}{a^2b^3x^6}\right)$$

input
```
integrate((b*x^2+a)^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/48*b^3*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
a)^(5/2) - 8*(b*x^2 + a)^(3/2)*a - 3*sqrt(b*x^2 + a)*a^2)/(a^2*b^3*x^6))
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+bx^2}}{x^7}\,dx = \frac{(b\,x^2+a)^{5/2}}{16\,a^2\,x^6}-\frac{(b\,x^2+a)^{3/2}}{6\,a\,x^6}-\frac{\sqrt{b\,x^2+a}}{16\,x^6}+\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{5/2}}$$

input
```
int((a + b*x^2)^(1/2)/x^7,x)
```

output
```
(b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i)/(16*a^(5/2)) - (a + b*x^2)^(
1/2)/(16*x^6) - (a + b*x^2)^(3/2)/(6*a*x^6) + (a + b*x^2)^(5/2)/(16*a^2*x^
6)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{a+bx^2}}{x^7}\,dx$$
$$= \frac{-8\sqrt{b\,x^2+a}\,a^3-2\sqrt{b\,x^2+a}\,a^2b\,x^2+3\sqrt{b\,x^2+a}\,a\,b^2x^4+3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^3x^6-3\sqrt{a}\log\left(\sqrt{\right.}}{48a^3x^6}$$

input
```
int((b*x^2+a)^(1/2)/x^7,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3 - 2*sqrt(a + b*x**2)*a**2*b*x**2 + 3*sqrt(a +
b*x**2)*a*b**2*x**4 + 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*
x)/sqrt(a))*b**3*x**6 - 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b
)*x)/sqrt(a))*b**3*x**6)/(48*a**3*x**6)
```

3.371 $\int x^4\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x^4\sqrt{a+bx^2}\,dx=-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{1}{6}x^5\sqrt{a+bx^2}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
-1/16*a^2*x*(b*x^2+a)^(1/2)/b^2+1/24*a*x^3*(b*x^2+a)^(1/2)/b+1/6*x^5*(b*x^
2+a)^(1/2)+1/16*a^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.87

$$\int x^4\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(-3a^2x+2abx^3+8b^2x^5)}{48b^2}+\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

input
```
Integrate[x^4*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2*x + 2*a*b*x^3 + 8*b^2*x^5))/(48*b^2) + (a^3*ArcTa
nh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6}a\int \frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6}x^5\sqrt{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}$$

input
```
Int[x^4*Sqrt[a + b*x^2],x]
```

output
```
(x^5*Sqrt[a + b*x^2])/6 + (a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[
a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))
/(4*b)))/6
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.66

method	result	size
risch	$-\frac{x\left(-8b^{2}x^{4}-2ab\,x^{2}+3a^{2}\right)\sqrt{b\,x^{2}+a}}{48b^{2}}+\frac{a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{5}{2}}}$	62
default	$\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}$	82
pseudoelliptic	$\frac{8b^{\frac{5}{2}}\sqrt{b\,x^{2}+a}\,x^{5}+2a\,b^{\frac{3}{2}}x^{3}\sqrt{b\,x^{2}+a}-3a^{2}x\sqrt{b}\,\sqrt{b\,x^{2}+a}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{3}}{48b^{\frac{5}{2}}}$	82

input
```
int(x^4*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*x*(-8*b^2*x^4-2*a*b*x^2+3*a^2)*(b*x^2+a)^(1/2)/b^2+1/16*a^3/b^(5/2)*
ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.55

$$\int x^{4}\sqrt{a+bx^{2}}\,dx$$
$$=\left[\frac{3\,a^{3}\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)+2\,(8\,b^{3}x^{5}+2\,ab^{2}x^{3}-3\,a^{2}bx)\sqrt{bx^{2}+a}}{96\,b^{3}},\right.$$
$$\left.-\frac{3\,a^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-(8\,b^{3}x^{5}+2\,ab^{2}x^{3}-3\,a^{2}bx)\sqrt{bx^{2}+a}}{48\,b^{3}}\right]$$

input
```
integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 2*a*b^2*x^3 - 3*a^2*b*x)*sqrt(b*x^2 + a))/b^3, -1/48*(3*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 2*a*b^2*x^3 - 3*a
^2*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int x^4\sqrt{a+bx^2}\,dx = -\frac{a^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}} + \frac{5\sqrt{a}x^5}{24\sqrt{1+\frac{bx^2}{a}}} + \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}} + \frac{bx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4*(b*x**2+a)**(1/2),x)
```

output
```
-a**(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - a**(3/2)*x**3/(48*b*sqrt(1 + b*
x**2/a)) + 5*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(5/2)) + b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int x^4\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}x^3}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}ax}{8\,b^2} + \frac{\sqrt{bx^2+a}a^2x}{16\,b^2} + \frac{a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*x^3/b - 1/8*(b*x^2 + a)^(3/2)*a*x/b^2 + 1/16*sqrt(b*
x^2 + a)*a^2*x/b^2 + 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{a+bx^2}\,dx = \frac{1}{48}\left(2\left(4x^2+\frac{a}{b}\right)x^2-\frac{3a^2}{b^2}\right)\sqrt{bx^2+a}\,x - \frac{a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*x^2 + a/b)*x^2 - 3*a^2/b^2)*sqrt(b*x^2 + a)*x - 1/16*a^3*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{a+bx^2}\,dx = \int x^4\,\sqrt{b\,x^2+a}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/2),x)
```

output
```
int(x^4*(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int x^4\sqrt{a+bx^2}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^2bx+2\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^3}{48b^3}$$

input
```
int(x^4*(b*x^2+a)^(1/2),x)
```

output

```
( - 3*sqrt(a + b*x**2)*a**2*b*x + 2*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(
a + b*x**2)*b**3*x**5 + 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3)/(48*b**3)
```

3.372 $\int x^2\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int x^2\sqrt{a+bx^2}\,dx=\frac{ax\sqrt{a+bx^2}}{8b}+\frac{1}{4}x^3\sqrt{a+bx^2}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/8*a*x*(b*x^2+a)^(1/2)/b+1/4*x^3*(b*x^2+a)^(1/2)-1/8*a^2*arctanh(b^(1/2)*
x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int x^2\sqrt{a+bx^2}\,dx=\frac{x\sqrt{a+bx^2}(a+2bx^2)}{8b}-\frac{a^2\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{3/2}}$$

input
```
Integrate[x^2*Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(a + 2*b*x^2))/(8*b) - (a^2*ArcTanh[(Sqrt[b]*x)/(-Sqrt[
a] + Sqrt[a + b*x^2])])/(4*b^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \ 248$$

$$\frac{1}{4}a\int \frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}$$

$$\Big\downarrow \ 224$$

$$\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}$$

input
```
Int[x^2*Sqrt[a + b*x^2],x]
```

output
```
(x^3*Sqrt[a + b*x^2])/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x(2b\,x^2+a)\sqrt{b\,x^2+a}}{8b}-\frac{a^2\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$	49
default	$\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}$	58
pseudoelliptic	$\frac{2\sqrt{b\,x^2+a}\,b^{\frac{3}{2}}x^3+ax\sqrt{b\,x^2+a}\,\sqrt{b}-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a^2}{8b^{\frac{3}{2}}}$	61

input
```
int(x^2*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*x^2+a)*(b*x^2+a)^(1/2)/b-1/8/b^(3/2)*a^2*ln(b^(1/2)*x+(b*x^2+a)
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int x^2\sqrt{a+bx^2}\,dx$$
$$= \left[\frac{a^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3+abx)\sqrt{bx^2+a}}{16\,b^2}, \frac{a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(2\,b^2}{8\,b^2}\right.$$

input
```
integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2, 1/8*(a^2*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (2*b^2*x^3 + a*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.31

$$\int x^2\sqrt{a+bx^2}\,dx = \frac{a^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}} + \frac{3\sqrt{a}x^3}{8\sqrt{1+\frac{bx^2}{a}}} - \frac{a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}} + \frac{bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2*(b*x**2+a)**(1/2),x)
```

output
```
a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*sqrt(a)*x**3/(8*sqrt(1 + b*x**2/a)
) - a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + b*x**5/(4*sqrt(a)*sqrt(1
+ b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

$$\int x^2\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}x}{4\,b} - \frac{\sqrt{bx^2+a}\,ax}{8\,b} - \frac{a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*x/b - 1/8*sqrt(b*x^2 + a)*a*x/b - 1/8*a^2*arcsinh(b*
x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.71

$$\int x^2\sqrt{a+bx^2}\,dx = \frac{1}{8}\,\sqrt{bx^2+a}\left(2\,x^2+\frac{a}{b}\right)x + \frac{a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*x^2 + a/b)*x + 1/8*a^2*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+bx^2}\,dx = \int x^2\,\sqrt{b\,x^2+a}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.86

$$\int x^2\sqrt{a+bx^2}\,dx=\frac{\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3-\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b^2}$$

input
```
int(x^2*(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 - sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b**2)
```

3.373 $\int \sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt{a+bx^2}\,dx=\frac{1}{2}x\sqrt{a+bx^2}+\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/2*x*(b*x^2+a)^(1/2)+1/2*a*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \sqrt{a+bx^2}\,dx=\frac{1}{2}x\sqrt{a+bx^2}-\frac{a\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 - (a*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b]
)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \; 211$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \; 224$$

$$\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}$$

input
```
Int[Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b
])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}$	36
pseudoelliptic	$-\frac{a\left(-\frac{\sqrt{b\,x^{2}+a}\,x}{a}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}\right)}{2}$	42

input
```
int((b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{2\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2-2\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b},\ \frac{\sqrt{bx^2+a}bx-a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b}\right]$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a))/b, 1/2*(sqrt(b*x^2 + a)*b*x - a*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)))/b]
```

Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \sqrt{a+bx^2}\,dx = \frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2} + \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}x + \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*x + 1/2*a*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}\,x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.76

$$\int \sqrt{a+bx^2}\,dx = \frac{x\sqrt{b\,x^2+a}}{2} + \frac{a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2),x)
```

output
```
(x*(a + b*x^2)^(1/2))/2 + (a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \sqrt{a+bx^2}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b}$$

input
```
int((b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b)
```

3.374 $\int \frac{\sqrt{a+bx^2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 42

$$\int \frac{\sqrt{a+bx^2}}{x^2} dx = -\frac{\sqrt{a+bx^2}}{x} + \sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-(b*x^2+a)^(1/2)/x+b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}}{x^2} dx = -\frac{\sqrt{a+bx^2}}{x} - \sqrt{b}\log\left(-\sqrt{b}x + \sqrt{a+bx^2}\right)$$

input
```
Integrate[Sqrt[a + b*x^2]/x^2,x]
```

output
```
-(Sqrt[a + b*x^2]/x) - Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^2}dx$$

$$\Big\downarrow \text{ 247}$$

$$b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}$$

$$\Big\downarrow \text{ 224}$$

$$b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}$$

input
```
Int[Sqrt[a + b*x^2]/x^2,x]
```

output
```
-(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}}{x}+\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)$	36
pseudoelliptic	$\frac{\sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x-\sqrt{b\,x^{2}+a}}{x}$	39
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}$	60

input
```
int((b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)^(1/2)/x+b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = \left[\frac{\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,\sqrt{bx^2+a}}{2\,x},\right.$$
$$\left.-\frac{\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\sqrt{bx^2+a}}{x}\right]$$

input
```
integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*sqrt(b
*x^2 + a))/x, -(sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + sqrt(b*x^2
 + a))/x]
```

Sympy [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = -\frac{\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + \sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{bx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(1/2)/x**2,x)
```

output
```
-sqrt(a)/(x*sqrt(1 + b*x**2/a)) + sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - b*x/(
sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = \sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\sqrt{bx^2+a}}{x}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
sqrt(b)*arcsinh(b*x/sqrt(a*b)) - sqrt(b*x^2 + a)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = -\frac{1}{2}\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{2\,a\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate((b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = -\frac{\sqrt{b\,x^2+a}}{x}-\frac{\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int((a + b*x^2)^(1/2)/x^2,x)
```

output
```
- (a + b*x^2)^(1/2)/x - (b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^
(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{a+bx^2}}{x^2}\,dx = \frac{-\sqrt{b\,x^2+a}+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)x-\sqrt{b}\,x}{x}$$

input
```
int((b*x^2+a)^(1/2)/x^2,x)
```

output
```
( - sqrt(a + b*x**2) + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))
*x - sqrt(b)*x)/x
```

3.375 $\int \frac{\sqrt{a+bx^2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\sqrt{a+bx^2}}{x^4} dx = -\frac{(a+bx^2)^{3/2}}{3ax^3}$$

output
```
-1/3*(b*x^2+a)^(3/2)/a/x^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+bx^2}}{x^4} dx = -\frac{(a+bx^2)^{3/2}}{3ax^3}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^4} \, dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{(a+bx^2)^{3/2}}{3ax^3}$$

input
```
Int[Sqrt[a + b*x^2]/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(3/2)/(a*x^3)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18
trager	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18
risch	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3a\,x^{3}}$	18

input
```
int((b*x^2+a)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(3/2)/a/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = -\frac{(bx^2+a)^{\frac{3}{2}}}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a}$$

input
```
integrate((b*x**2+a)**(1/2)/x**4,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(3*
a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = -\frac{(bx^2+a)^{\frac{3}{2}}}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(b*x^2 + a)^(3/2)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(17) = 34$.

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = \frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}} + a^2 b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a\right)^3}$$

input
```
integrate((b*x^2+a)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) + a^2*b^(3/2))/((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = -\frac{\left(b\,x^2+a\right)^{3/2}}{3\,a\,x^3}$$

input
```
int((a + b*x^2)^(1/2)/x^4,x)
```

output
```
-(a + b*x^2)^(3/2)/(3*a*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\sqrt{a+bx^2}}{x^4}\,dx = \frac{-\sqrt{b\,x^2+a}\,a-\sqrt{b\,x^2+a}\,b\,x^2-\sqrt{b}\,b\,x^3}{3a\,x^3}$$

input
```
int((b*x^2+a)^(1/2)/x^4,x)
```

output
```
( - (sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 + sqrt(b)*b*x**3))/(3*a*
x**3)
```

3.376 $\int \frac{\sqrt{a+bx^2}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sqrt{a+bx^2}}{x^6} dx = -\frac{(a+bx^2)^{3/2}}{5ax^5} + \frac{2b(a+bx^2)^{3/2}}{15a^2x^3}$$

output
```
-1/5*(b*x^2+a)^(3/2)/a/x^5+2/15*b*(b*x^2+a)^(3/2)/a^2/x^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a+bx^2}}{x^6} dx = \frac{\sqrt{a+bx^2}(-3a^2-abx^2+2b^2x^4)}{15a^2x^5}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2 - a*b*x^2 + 2*b^2*x^4))/(15*a^2*x^5)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^6} \, dx$$

$$\Big\downarrow \; 245$$

$$-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

$$\Big\downarrow \; 242$$

$$\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}$$

input
```
Int[Sqrt[a + b*x^2]/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-2b\,x^{2}+3a\right)}{15a^{2}x^{5}}$	28
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-2b\,x^{2}+3a\right)}{15a^{2}x^{5}}$	28
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-2b\,x^{2}+3a\right)}{15a^{2}x^{5}}$	28
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}$	37
trager	$-\frac{\left(-2b^{2}x^{4}+ab\,x^{2}+3a^{2}\right)\sqrt{b\,x^{2}+a}}{15a^{2}x^{5}}$	38
risch	$-\frac{\left(-2b^{2}x^{4}+ab\,x^{2}+3a^{2}\right)\sqrt{b\,x^{2}+a}}{15a^{2}x^{5}}$	38

input
```
int((b*x^2+a)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(3/2)*(-2*b*x^2+3*a)/a^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = \frac{(2\,b^2x^4 - abx^2 - 3\,a^2)\sqrt{bx^2+a}}{15\,a^2x^5}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
1/15*(2*b^2*x^4 - a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a)/(a^2*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15ax^2} + \frac{2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15a^2}$$

input
```
integrate((b*x**2+a)**(1/2)/x**6,x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - b**(3/2)*sqrt(a/(b*x**2) + 1)/(15
*a*x**2) + 2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(15*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}b}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{3}{2}}}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
2/15*(b*x^2 + a)^(3/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(3/2)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.55

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = \frac{4\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{5}{2}} + 5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 ab^{\frac{5}{2}} + 5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 a^2b^{\frac{5}{2}} - a^3b^{\frac{5}{2}}\right)}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

input
```
integrate((b*x^2+a)^(1/2)/x^6,x, algorithm="giac")
```

output
```
4/15*(15*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*a*b^(5/2) + 5*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(5/2) - a^3
*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(3\,a^2+a\,b\,x^2-2\,b^2\,x^4\right)}{15\,a^2\,x^5}$$

input
```
int((a + b*x^2)^(1/2)/x^6,x)
```

output
```
-((a + b*x^2)^(1/2)*(3*a^2 - 2*b^2*x^4 + a*b*x^2))/(15*a^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{a+bx^2}}{x^6}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^2-\sqrt{b\,x^2+a}\,ab\,x^2+2\sqrt{b\,x^2+a}\,b^2x^4-2\sqrt{b}\,b^2x^5}{15a^2x^5}$$

input
```
int((b*x^2+a)^(1/2)/x^6,x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2 - sqrt(a + b*x**2)*a*b*x**2 + 2*sqrt(a + b*x**
2)*b**2*x**4 - 2*sqrt(b)*b**2*x**5)/(15*a**2*x**5)
```

3.377 $\int \frac{\sqrt{a+bx^2}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{\sqrt{a+bx^2}}{x^8} dx = -\frac{(a+bx^2)^{3/2}}{7ax^7} + \frac{4b(a+bx^2)^{3/2}}{35a^2x^5} - \frac{8b^2(a+bx^2)^{3/2}}{105a^3x^3}$$

output
```
-1/7*(b*x^2+a)^(3/2)/a/x^7+4/35*b*(b*x^2+a)^(3/2)/a^2/x^5-8/105*b^2*(b*x^2
+a)^(3/2)/a^3/x^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{a+bx^2}}{x^8} dx = \frac{\sqrt{a+bx^2}(-15a^3 - 3a^2bx^2 + 4ab^2x^4 - 8b^3x^6)}{105a^3x^7}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^8,x]
```

output
```
(Sqrt[a + b*x^2]*(-15*a^3 - 3*a^2*b*x^2 + 4*a*b^2*x^4 - 8*b^3*x^6))/(105*a
^3*x^7)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^8} \, dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b \int \frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b \left(-\frac{2b \int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$-\frac{4b \left(\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}$$

input
```
Int[Sqrt[a + b*x^2]/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(3/2)/(a*x^7) - (4*b*(-1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2
*b*(a + b*x^2)^(3/2))/(15*a^2*x^3)))/(7*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(8b^{2}x^{4}-12ab\,x^{2}+15a^{2}\right)}{105a^{3}x^{7}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(8b^{2}x^{4}-12ab\,x^{2}+15a^{2}\right)}{105a^{3}x^{7}}$	39
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(8b^{2}x^{4}-12ab\,x^{2}+15a^{2}\right)}{105a^{3}x^{7}}$	39
trager	$-\frac{\left(8b^{3}x^{6}-4a\,b^{2}x^{4}+3a^{2}b\,x^{2}+15a^{3}\right)\sqrt{b\,x^{2}+a}}{105a^{3}x^{7}}$	50
risch	$-\frac{\left(8b^{3}x^{6}-4a\,b^{2}x^{4}+3a^{2}b\,x^{2}+15a^{3}\right)\sqrt{b\,x^{2}+a}}{105a^{3}x^{7}}$	50
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7a\,x^{7}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5a\,x^{5}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15a^{2}x^{3}}\right)}{7a}$	61

input
```
int((b*x^2+a)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(b*x^2+a)^(3/2)*(8*b^2*x^4-12*a*b*x^2+15*a^2)/a^3/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = -\frac{(8\,b^3x^6 - 4\,ab^2x^4 + 3\,a^2bx^2 + 15\,a^3)\sqrt{bx^2+a}}{105\,a^3x^7}$$

input
```
integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-1/105*(8*b^3*x^6 - 4*a*b^2*x^4 + 3*a^2*b*x^2 + 15*a^3)*sqrt(b*x^2 + a)/(a
^3*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(61) = 122.

Time = 0.67 (sec) , antiderivative size = 359, normalized size of antiderivative = 5.28

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = &-\frac{15a^5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ &- \frac{33a^4b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ &- \frac{17a^3b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ &- \frac{3a^2b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ &- \frac{12ab^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}} \\ &- \frac{8b^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{105a^5b^4x^6+210a^4b^5x^8+105a^3b^6x^{10}}\end{aligned}$$

input
```
integrate((b*x**2+a)**(1/2)/x**8,x)
```

output
```
-15*a**5*b**(9/2)*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5
*x**8 + 105*a**3*b**6*x**10) - 33*a**4*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)
/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 17*a**3
*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x
**8 + 105*a**3*b**6*x**10) - 3*a**2*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(1
05*a**5*b**4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10) - 12*a*b**(1
7/2)*x**8*sqrt(a/(b*x**2) + 1)/(105*a**5*b**4*x**6 + 210*a**4*b**5*x**8 +
105*a**3*b**6*x**10) - 8*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)/(105*a**5*b*
*4*x**6 + 210*a**4*b**5*x**8 + 105*a**3*b**6*x**10)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = -\frac{8\,(bx^2+a)^{\frac{3}{2}}b^2}{105\,a^3x^3} + \frac{4\,(bx^2+a)^{\frac{3}{2}}b}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{3}{2}}}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*x^3) + 4/35*(b*x^2 + a)^(3/2)*b/(a^2*x^5
) - 1/7*(b*x^2 + a)^(3/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.03

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = \frac{16\left(70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 b^{\frac{7}{2}} + 35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 ab^{\frac{7}{2}} + 21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^2b^{\frac{7}{2}} - 7\left(\sqrt{b}x-\right.\right.}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

input
```
integrate((b*x^2+a)^(1/2)/x^8,x, algorithm="giac")
```

output
```
16/105*(70*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(7/2) + 35*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(7/2) -
 7*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(7/2) + a^4*b^(7/2))/((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = \frac{4\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^3} - \frac{b\,\sqrt{b\,x^2+a}}{35\,a\,x^5} - \frac{\sqrt{b\,x^2+a}}{7\,x^7} - \frac{8\,b^3\,\sqrt{b\,x^2+a}}{105\,a^3\,x}$$

input
```
int((a + b*x^2)^(1/2)/x^8,x)
```

output
```
(4*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^3) - (b*(a + b*x^2)^(1/2))/(35*a*x^5)
 - (a + b*x^2)^(1/2)/(7*x^7) - (8*b^3*(a + b*x^2)^(1/2))/(105*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{a+bx^2}}{x^8}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^3 - 3\sqrt{b\,x^2+a}\,a^2b\,x^2 + 4\sqrt{b\,x^2+a}\,a\,b^2x^4 - 8\sqrt{b\,x^2+a}\,b^3x^6 + 8\sqrt{b}\,b^3x^7}{105a^3x^7}$$

input
```
int((b*x^2+a)^(1/2)/x^8,x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**3 - 3*sqrt(a + b*x**2)*a**2*b*x**2 + 4*sqrt(a +
 b*x**2)*a*b**2*x**4 - 8*sqrt(a + b*x**2)*b**3*x**6 + 8*sqrt(b)*b**3*x**7)
/(105*a**3*x**7)
```

3.378 $\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx=-\frac{(a+bx^2)^{3/2}}{9ax^9}+\frac{2b(a+bx^2)^{3/2}}{21a^2x^7}-\frac{8b^2(a+bx^2)^{3/2}}{105a^3x^5}+\frac{16b^3(a+bx^2)^{3/2}}{315a^4x^3}$$

output
```
-1/9*(b*x^2+a)^(3/2)/a/x^9+2/21*b*(b*x^2+a)^(3/2)/a^2/x^7-8/105*b^2*(b*x^2
+a)^(3/2)/a^3/x^5+16/315*b^3*(b*x^2+a)^(3/2)/a^4/x^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx=\frac{\sqrt{a+bx^2}(-35a^4-5a^3bx^2+6a^2b^2x^4-8ab^3x^6+16b^4x^8)}{315a^4x^9}$$

input
```
Integrate[Sqrt[a + b*x^2]/x^10,x]
```

output
```
(Sqrt[a + b*x^2]*(-35*a^4 - 5*a^3*b*x^2 + 6*a^2*b^2*x^4 - 8*a*b^3*x^6 + 16
*b^4*x^8))/(315*a^4*x^9)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^8}dx}{3a}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{245}$$

$$-\frac{2b\left(-\frac{4b\int \frac{\sqrt{bx^2+a}}{x^6}dx}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)}{3a}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{245}$$

$$-\frac{2b\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^2+a}}{x^4}dx}{5a}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)}{3a}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{3/2}}{15a^2x^3}-\frac{\left(a+bx^2\right)^{3/2}}{5ax^5}\right)}{7a}-\frac{\left(a+bx^2\right)^{3/2}}{7ax^7}\right)}{3a}-\frac{\left(a+bx^2\right)^{3/2}}{9ax^9}$$

input
```
Int[Sqrt[a + b*x^2]/x^10,x]
```

output
```
-1/9*(a + b*x^2)^(3/2)/(a*x^9) - (2*b*(-1/7*(a + b*x^2)^(3/2)/(a*x^7) - (4
*b*(-1/5*(a + b*x^2)^(3/2)/(a*x^5) + (2*b*(a + b*x^2)^(3/2))/(15*a^2*x^3))
)/(7*a)))/(3*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16 b^{3} x^{6}+24 a\,b^{2} x^{4}-30 a^{2} b\,x^{2}+35 a^{3}\right)}{315 x^{9} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16 b^{3} x^{6}+24 a\,b^{2} x^{4}-30 a^{2} b\,x^{2}+35 a^{3}\right)}{315 x^{9} a^{4}}$	50
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\left(-16 b^{3} x^{6}+24 a\,b^{2} x^{4}-30 a^{2} b\,x^{2}+35 a^{3}\right)}{315 x^{9} a^{4}}$	50
trager	$-\frac{\left(-16 b^{4} x^{8}+8 a\,b^{3} x^{6}-6 a^{2} b^{2} x^{4}+5 a^{3} b\,x^{2}+35 a^{4}\right)\sqrt{b\,x^{2}+a}}{315 x^{9} a^{4}}$	61
risch	$-\frac{\left(-16 b^{4} x^{8}+8 a\,b^{3} x^{6}-6 a^{2} b^{2} x^{4}+5 a^{3} b\,x^{2}+35 a^{4}\right)\sqrt{b\,x^{2}+a}}{315 x^{9} a^{4}}$	61
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{9 a\,x^{9}}-\frac{2 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{7 a\,x^{7}}-\frac{4 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{5 a\,x^{5}}+\frac{2 b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{15 a^{2} x^{3}}\right)}{7 a}\right)}{3 a}$	85

input
```
int((b*x^2+a)^(1/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(b*x^2+a)^(3/2)*(-16*b^3*x^6+24*a*b^2*x^4-30*a^2*b*x^2+35*a^3)/x^9/
a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = \frac{(16\,b^4x^8-8\,ab^3x^6+6\,a^2b^2x^4-5\,a^3bx^2-35\,a^4)\sqrt{bx^2+a}}{315\,a^4x^9}$$

input
```
integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="fricas")
```

output
```
1/315*(16*b^4*x^8 - 8*a*b^3*x^6 + 6*a^2*b^2*x^4 - 5*a^3*b*x^2 - 35*a^4)*sq
rt(b*x^2 + a)/(a^4*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(85) = 170.

Time = 0.93 (sec) , antiderivative size = 575, normalized size of antiderivative = 6.25

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = &-\frac{35a^7b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &-\frac{110a^6b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &-\frac{114a^5b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &-\frac{40a^4b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &+\frac{5a^3b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &+\frac{30a^2b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &+\frac{40ab^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}} \\ &+\frac{16b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{315a^7b^9x^8+945a^6b^{10}x^{10}+945a^5b^{11}x^{12}+315a^4b^{12}x^{14}}\end{aligned}$$

input
```
integrate((b*x**2+a)**(1/2)/x**10,x)
```

output
```
-35*a**7*b**(19/2)*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**
10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 110*a**6*b**(21/
2)*x**2*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 +
945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) - 114*a**5*b**(23/2)*x**4*sqr
t(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b*
*11*x**12 + 315*a**4*b**12*x**14) - 40*a**4*b**(25/2)*x**6*sqrt(a/(b*x**2)
 + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 +
315*a**4*b**12*x**14) + 5*a**3*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a*
*7*b**9*x**8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**1
2*x**14) + 30*a**2*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**
8 + 945*a**6*b**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) +
40*a*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b
**10*x**10 + 945*a**5*b**11*x**12 + 315*a**4*b**12*x**14) + 16*b**(33/2)*x
**14*sqrt(a/(b*x**2) + 1)/(315*a**7*b**9*x**8 + 945*a**6*b**10*x**10 + 945
*a**5*b**11*x**12 + 315*a**4*b**12*x**14)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = \frac{16\,(bx^2+a)^{\frac{3}{2}}b^3}{315\,a^4x^3} - \frac{8\,(bx^2+a)^{\frac{3}{2}}b^2}{105\,a^3x^5} + \frac{2\,(bx^2+a)^{\frac{3}{2}}b}{21\,a^2x^7} - \frac{(bx^2+a)^{\frac{3}{2}}}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="maxima")
```

output
```
16/315*(b*x^2 + a)^(3/2)*b^3/(a^4*x^3) - 8/105*(b*x^2 + a)^(3/2)*b^2/(a^3*
x^5) + 2/21*(b*x^2 + a)^(3/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(3/2)/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.80

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = \frac{32\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{9}{2}}+189\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{9}{2}}+84\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{9}{2}}-36\left(\sqrt{b}\right.\right.}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

input
```
integrate((b*x^2+a)^(1/2)/x^10,x, algorithm="giac")
```

output
```
32/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(9/2) + 189*(sqrt(b)*x - sq
rt(b*x^2 + a))^8*a*b^(9/2) + 84*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(9/2
) - 36*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(9/2) + 9*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*a^4*b^(9/2) - a^5*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)^9
```

Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx = \frac{2\,b^2\,\sqrt{b\,x^2+a}}{105\,a^2\,x^5} - \frac{b\,\sqrt{b\,x^2+a}}{63\,a\,x^7} - \frac{\sqrt{b\,x^2+a}}{9\,x^9} - \frac{8\,b^3\,\sqrt{b\,x^2+a}}{315\,a^3\,x^3} + \frac{16\,b^4\,\sqrt{b\,x^2+a}}{315\,a^4\,x}$$

input
```
int((a + b*x^2)^(1/2)/x^10,x)
```

output
```
(2*b^2*(a + b*x^2)^(1/2))/(105*a^2*x^5) - (b*(a + b*x^2)^(1/2))/(63*a*x^7)
 - (a + b*x^2)^(1/2)/(9*x^9) - (8*b^3*(a + b*x^2)^(1/2))/(315*a^3*x^3) + (
16*b^4*(a + b*x^2)^(1/2))/(315*a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{a+bx^2}}{x^{10}}\,dx$$
$$= \frac{-35\sqrt{b\,x^2+a}\,a^4-5\sqrt{b\,x^2+a}\,a^3b\,x^2+6\sqrt{b\,x^2+a}\,a^2b^2x^4-8\sqrt{b\,x^2+a}\,a\,b^3x^6+16\sqrt{b\,x^2+a}\,b^4x^8-16}{315a^4x^9}$$

input
```
int((b*x^2+a)^(1/2)/x^10,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 5*sqrt(a + b*x**2)*a**3*b*x**2 + 6*sqrt(a +
 b*x**2)*a**2*b**2*x**4 - 8*sqrt(a + b*x**2)*a*b**3*x**6 + 16*sqrt(a + b*x
**2)*b**4*x**8 - 16*sqrt(b)*b**4*x**9)/(315*a**4*x**9)
```

3.379 $\int x^7(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^3(a+bx^2)^{5/2}}{5b^4}+\frac{3a^2(a+bx^2)^{7/2}}{7b^4}-\frac{a(a+bx^2)^{9/2}}{3b^4}+\frac{(a+bx^2)^{11/2}}{11b^4}$$

output
```
-1/5*a^3*(b*x^2+a)^(5/2)/b^4+3/7*a^2*(b*x^2+a)^(7/2)/b^4-1/3*a*(b*x^2+a)^(
9/2)/b^4+1/11*(b*x^2+a)^(11/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7\left(a+bx^2\right)^{3/2}\,dx=\frac{(a+bx^2)^{5/2}\left(-16a^3+40a^2bx^2-70ab^2x^4+105b^3x^6\right)}{1155b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(3/2),x]
```

output
```
((a + b*x^2)^(5/2)*(-16*a^3 + 40*a^2*b*x^2 - 70*a*b^2*x^4 + 105*b^3*x^6))/
(1155*b^4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^{3/2} dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{9/2}}{b^3}-\frac{3a\left(bx^2+a\right)^{7/2}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{5/2}}{b^3}-\frac{a^3\left(bx^2+a\right)^{3/2}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{2a^3\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{6a^2\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{2\left(a+bx^2\right)^{11/2}}{11b^4}-\frac{2a\left(a+bx^2\right)^{9/2}}{3b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(3/2),x]
```

output
```
((-2*a^3*(a + b*x^2)^(5/2))/(5*b^4) + (6*a^2*(a + b*x^2)^(7/2))/(7*b^4) -
(2*a*(a + b*x^2)^(9/2))/(3*b^4) + (2*(a + b*x^2)^(11/2))/(11*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(-105 b^{3} x^{6}+70 a\,b^{2} x^{4}-40 a^{2} b\,x^{2}+16 a^{3}\right)}{1155 b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(-105 b^{3} x^{6}+70 a\,b^{2} x^{4}-40 a^{2} b\,x^{2}+16 a^{3}\right)}{1155 b^{4}}$	47
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(-105 b^{3} x^{6}+70 a\,b^{2} x^{4}-40 a^{2} b\,x^{2}+16 a^{3}\right)}{1155 b^{4}}$	47
trager	$-\frac{\left(-105 b^{5} x^{10}-140 a\,b^{4} x^{8}-5 a^{2} b^{3} x^{6}+6 a^{3} b^{2} x^{4}-8 a^{4} b\,x^{2}+16 a^{5}\right) \sqrt{b\,x^{2}+a}}{1155 b^{4}}$	69
risch	$-\frac{\left(-105 b^{5} x^{10}-140 a\,b^{4} x^{8}-5 a^{2} b^{3} x^{6}+6 a^{3} b^{2} x^{4}-8 a^{4} b\,x^{2}+16 a^{5}\right) \sqrt{b\,x^{2}+a}}{1155 b^{4}}$	69
default	$\frac{x^{6} \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{11 b}-\frac{6 a \left(\frac{x^{4} \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9 b}-\frac{4 a \left(\frac{x^{2} \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 b}-\frac{2 a \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 b^{2}}\right)}{9 b}\right)}{11 b}$	82

input
```
int(x^7*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(b*x^2+a)^(5/2)*(-105*b^3*x^6+70*a*b^2*x^4-40*a^2*b*x^2+16*a^3)/b^
4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x^7(a+bx^2)^{3/2}\,dx = \frac{(105\,b^5x^{10}+140\,ab^4x^8+5\,a^2b^3x^6-6\,a^3b^2x^4+8\,a^4bx^2-16\,a^5)\sqrt{bx^2+a}}{1155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/1155*(105*b^5*x^10 + 140*a*b^4*x^8 + 5*a^2*b^3*x^6 - 6*a^3*b^2*x^4 + 8*a
^4*b*x^2 - 16*a^5)*sqrt(b*x^2 + a)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.66

$$\int x^7(a+bx^2)^{3/2}\,dx = \begin{cases} -\frac{16a^5\sqrt{a+bx^2}}{1155b^4}+\frac{8a^4x^2\sqrt{a+bx^2}}{1155b^3}-\frac{2a^3x^4\sqrt{a+bx^2}}{385b^2}+\frac{a^2x^6\sqrt{a+bx^2}}{231b}+\frac{4ax^8\sqrt{a+bx^2}}{33}+\frac{bx^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^8}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**7*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-16*a**5*sqrt(a + b*x**2)/(1155*b**4) + 8*a**4*x**2*sqrt(a + b*
x**2)/(1155*b**3) - 2*a**3*x**4*sqrt(a + b*x**2)/(385*b**2) + a**2*x**6*sq
rt(a + b*x**2)/(231*b) + 4*a*x**8*sqrt(a + b*x**2)/33 + b*x**10*sqrt(a + b
*x**2)/11, Ne(b, 0)), (a**(3/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^7(a+bx^2)^{3/2}\,dx=\frac{(bx^2+a)^{\frac{5}{2}}x^6}{11\,b}-\frac{2\,(bx^2+a)^{\frac{5}{2}}ax^4}{33\,b^2}+\frac{8\,(bx^2+a)^{\frac{5}{2}}a^2x^2}{231\,b^3}-\frac{16\,(bx^2+a)^{\frac{5}{2}}a^3}{1155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/11*(b*x^2 + a)^(5/2)*x^6/b - 2/33*(b*x^2 + a)^(5/2)*a*x^4/b^2 + 8/231*(b
*x^2 + a)^(5/2)*a^2*x^2/b^3 - 16/1155*(b*x^2 + a)^(5/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7(a+bx^2)^{3/2}\,dx=\frac{105\,(bx^2+a)^{\frac{11}{2}}-385\,(bx^2+a)^{\frac{9}{2}}a+495\,(bx^2+a)^{\frac{7}{2}}a^2-231\,(bx^2+a)^{\frac{5}{2}}a^3}{1155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/1155*(105*(b*x^2 + a)^(11/2) - 385*(b*x^2 + a)^(9/2)*a + 495*(b*x^2 + a)
^(7/2)*a^2 - 231*(b*x^2 + a)^(5/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^7 (a+bx^2)^{3/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{4\,a\,x^8}{33}+\frac{b\,x^{10}}{11}-\frac{16\,a^5}{1155\,b^4}+\frac{a^2\,x^6}{231\,b}-\frac{2\,a^3\,x^4}{385\,b^2}+\frac{8\,a^4\,x^2}{1155\,b^3}\right)$$

input
```
int(x^7*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((4*a*x^8)/33 + (b*x^10)/11 - (16*a^5)/(1155*b^4) + (a^2
*x^6)/(231*b) - (2*a^3*x^4)/(385*b^2) + (8*a^4*x^2)/(1155*b^3))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int x^7 (a+bx^2)^{3/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(105b^5x^{10}+140a\,b^4x^8+5a^2b^3x^6-6a^3b^2x^4+8a^4b\,x^2-16a^5)}{1155b^4}$$

input
```
int(x^7*(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**5 + 8*a**4*b*x**2 - 6*a**3*b**2*x**4 + 5*a**2*
b**3*x**6 + 140*a*b**4*x**8 + 105*b**5*x**10))/(1155*b**4)
```

3.380 $\int x^5(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5\left(a+bx^2\right)^{3/2}\,dx=\frac{a^2\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{2a\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{\left(a+bx^2\right)^{9/2}}{9b^3}$$

output
```
1/5*a^2*(b*x^2+a)^(5/2)/b^3-2/7*a*(b*x^2+a)^(7/2)/b^3+1/9*(b*x^2+a)^(9/2)/
b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(a+bx^2\right)^{5/2}\left(8a^2-20abx^2+35b^2x^4\right)}{315b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(3/2),x]
```

output
```
((a + b*x^2)^(5/2)*(8*a^2 - 20*a*b*x^2 + 35*b^2*x^4))/(315*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a+bx^2)^{3/2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4 (bx^2+a)^{3/2} \, dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^{7/2}}{b^2} - \frac{2a(bx^2+a)^{5/2}}{b^2} + \frac{a^2(bx^2+a)^{3/2}}{b^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^2(a+bx^2)^{5/2}}{5b^3} + \frac{2(a+bx^2)^{9/2}}{9b^3} - \frac{4a(a+bx^2)^{7/2}}{7b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(3/2),x]
```

output
```
((2*a^2*(a + b*x^2)^(5/2))/(5*b^3) - (4*a*(a + b*x^2)^(7/2))/(7*b^3) + (2*
(a + b*x^2)^(9/2))/(9*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(35 b^{2} x^{4}-20 a b\,x^{2}+8 a^{2}\right)}{315 b^{3}}$	36
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(35 b^{2} x^{4}-20 a b\,x^{2}+8 a^{2}\right)}{315 b^{3}}$	36
orering	$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(35 b^{2} x^{4}-20 a b\,x^{2}+8 a^{2}\right)}{315 b^{3}}$	36
default	$\frac{x^{4} \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9 b}-\frac{4 a \left(\frac{x^{2} \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 b}-\frac{2 a \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 b^{2}}\right)}{9 b}$	58
trager	$\frac{\left(35 b^{4} x^{8}+50 a\,b^{3} x^{6}+3 a^{2} b^{2} x^{4}-4 a^{3} b\,x^{2}+8 a^{4}\right) \sqrt{b\,x^{2}+a}}{315 b^{3}}$	58
risch	$\frac{\left(35 b^{4} x^{8}+50 a\,b^{3} x^{6}+3 a^{2} b^{2} x^{4}-4 a^{3} b\,x^{2}+8 a^{4}\right) \sqrt{b\,x^{2}+a}}{315 b^{3}}$	58

input
```
int(x^5*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*(b*x^2+a)^(5/2)*(35*b^2*x^4-20*a*b*x^2+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5 \left(a+bx^2\right)^{3/2} \, dx = \frac{(35\,b^4x^8+50\,ab^3x^6+3\,a^2b^2x^4-4\,a^3bx^2+8\,a^4)\sqrt{bx^2+a}}{315\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^4*x^8 + 50*a*b^3*x^6 + 3*a^2*b^2*x^4 - 4*a^3*b*x^2 + 8*a^4)*sq
rt(b*x^2 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(51) = 102$.

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.85

$$\int x^5 (a+bx^2)^{3/2}\, dx = \begin{cases} \frac{8a^4\sqrt{a+bx^2}}{315b^3} - \frac{4a^3x^2\sqrt{a+bx^2}}{315b^2} + \frac{a^2x^4\sqrt{a+bx^2}}{105b} + \frac{10ax^6\sqrt{a+bx^2}}{63} + \frac{bx^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((8*a**4*sqrt(a + b*x**2)/(315*b**3) - 4*a**3*x**2*sqrt(a + b*x**
2)/(315*b**2) + a**2*x**4*sqrt(a + b*x**2)/(105*b) + 10*a*x**6*sqrt(a + b*
x**2)/63 + b*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(3/2)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5 (a+bx^2)^{3/2}\, dx = \frac{(bx^2+a)^{\frac{5}{2}}x^4}{9\,b} - \frac{4\,(bx^2+a)^{\frac{5}{2}}ax^2}{63\,b^2} + \frac{8\,(bx^2+a)^{\frac{5}{2}}a^2}{315\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(5/2)*x^4/b - 4/63*(b*x^2 + a)^(5/2)*a*x^2/b^2 + 8/315*(b*
x^2 + a)^(5/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 (a+bx^2)^{3/2}\,dx = \frac{35\,(bx^2+a)^{\frac{9}{2}} - 90\,(bx^2+a)^{\frac{7}{2}}a + 63\,(bx^2+a)^{\frac{5}{2}}a^2}{315\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/315*(35*(b*x^2 + a)^(9/2) - 90*(b*x^2 + a)^(7/2)*a + 63*(b*x^2 + a)^(5/2
)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5 (a+bx^2)^{3/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{10\,a\,x^6}{63}+\frac{b\,x^8}{9}+\frac{8\,a^4}{315\,b^3}+\frac{a^2\,x^4}{105\,b}-\frac{4\,a^3\,x^2}{315\,b^2}\right)$$

input
```
int(x^5*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((10*a*x^6)/63 + (b*x^8)/9 + (8*a^4)/(315*b^3) + (a^2*x^
4)/(105*b) - (4*a^3*x^2)/(315*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int x^5 (a+bx^2)^{3/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(35b^4x^8+50a\,b^3x^6+3a^2b^2x^4-4a^3b\,x^2+8a^4)}{315b^3}$$

input
```
int(x^5*(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**4 - 4*a**3*b*x**2 + 3*a**2*b**2*x**4 + 50*a*b**3*x
**6 + 35*b**4*x**8))/(315*b**3)
```

3.381 $\int x^3(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3(a+bx^2)^{3/2}\,dx=-\frac{a(a+bx^2)^{5/2}}{5b^2}+\frac{(a+bx^2)^{7/2}}{7b^2}$$

output
```
-1/5*a*(b*x^2+a)^(5/2)/b^2+1/7*(b*x^2+a)^(7/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3(a+bx^2)^{3/2}\,dx=\frac{(a+bx^2)^{5/2}(-2a+5bx^2)}{35b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(3/2),x]
```

output
```
((a + b*x^2)^(5/2)*(-2*a + 5*b*x^2))/(35*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^2 \left(bx^2+a\right)^{3/2} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{5/2}}{b}-\frac{a\left(bx^2+a\right)^{3/2}}{b}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2\left(a+bx^2\right)^{7/2}}{7b^2}-\frac{2a\left(a+bx^2\right)^{5/2}}{5b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(3/2),x]
```

output
```
((-2*a*(a + b*x^2)^(5/2))/(5*b^2) + (2*(a + b*x^2)^(7/2))/(7*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(-5b\,x^2+2a)}{35b^2}$	25
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(-5b\,x^2+2a)}{35b^2}$	25
orering	$-\frac{(b\,x^2+a)^{\frac{5}{2}}(-5b\,x^2+2a)}{35b^2}$	25
default	$\frac{x^2(b\,x^2+a)^{\frac{5}{2}}}{7b}-\frac{2a(b\,x^2+a)^{\frac{5}{2}}}{35b^2}$	34
trager	$-\frac{(-5b^3x^6-8a\,b^2x^4-a^2b\,x^2+2a^3)\sqrt{b\,x^2+a}}{35b^2}$	47
risch	$-\frac{(-5b^3x^6-8a\,b^2x^4-a^2b\,x^2+2a^3)\sqrt{b\,x^2+a}}{35b^2}$	47

input
```
int(x^3*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)^(5/2)*(-5*b*x^2+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^3(a+bx^2)^{3/2}\,dx=\frac{(5\,b^3x^6+8\,ab^2x^4+a^2bx^2-2\,a^3)\sqrt{bx^2+a}}{35\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/35*(5*b^3*x^6 + 8*a*b^2*x^4 + a^2*b*x^2 - 2*a^3)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(31) = 62$.

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.24

$$\int x^3 (a + bx^2)^{3/2} \, dx = \begin{cases} -\frac{2a^3\sqrt{a+bx^2}}{35b^2} + \frac{a^2x^2\sqrt{a+bx^2}}{35b} + \frac{8ax^4\sqrt{a+bx^2}}{35} + \frac{bx^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-2*a**3*sqrt(a + b*x**2)/(35*b**2) + a**2*x**2*sqrt(a + b*x**2)
/(35*b) + 8*a*x**4*sqrt(a + b*x**2)/35 + b*x**6*sqrt(a + b*x**2)/7, Ne(b,
0)), (a**(3/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3 (a + bx^2)^{3/2} \, dx = \frac{(bx^2 + a)^{\frac{5}{2}} x^2}{7\,b} - \frac{2\,(bx^2 + a)^{\frac{5}{2}} a}{35\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*x^2/b - 2/35*(b*x^2 + a)^(5/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a+bx^2)^{3/2}\,dx = \frac{5\,(bx^2+a)^{\frac{7}{2}} - 7\,(bx^2+a)^{\frac{5}{2}}a}{35\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/35*(5*(b*x^2 + a)^(7/2) - 7*(b*x^2 + a)^(5/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^3 (a+bx^2)^{3/2}\,dx = \sqrt{b\,x^2+a}\,\left(\frac{8\,a\,x^4}{35}+\frac{b\,x^6}{7}-\frac{2\,a^3}{35\,b^2}+\frac{a^2\,x^2}{35\,b}\right)$$

input
```
int(x^3*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*a*x^4)/35 + (b*x^6)/7 - (2*a^3)/(35*b^2) + (a^2*x^2)
/(35*b))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int x^3 (a+bx^2)^{3/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(5b^3x^6+8a\,b^2x^4+a^2b\,x^2-2a^3)}{35b^2}$$

input
```
int(x^3*(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a**3 + a**2*b*x**2 + 8*a*b**2*x**4 + 5*b**3*x**6))
/(35*b**2)
```

3.382 $\int x(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x(a+bx^2)^{3/2}\,dx=\frac{(a+bx^2)^{5/2}}{5b}$$

output
```
1/5*(b*x^2+a)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)^{3/2}\,dx=\frac{(a+bx^2)^{5/2}}{5b}$$

input
```
Integrate[x*(a + b*x^2)^(3/2),x]
```

output
```
(a + b*x^2)^(5/2)/(5*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{\left(a+bx^2\right)^{5/2}}{5b}$$

input
```
Int[x*(a + b*x^2)^(3/2),x]
```

output
```
(a + b*x^2)^(5/2)/(5*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
default	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
orering	$\frac{(b\,x^2+a)^{\frac{5}{2}}}{5b}$	15
trager	$\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5b}$	33
risch	$\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5b}$	33

input
```
int(x*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(b*x^2+a)^(5/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x(a+bx^2)^{3/2}\,dx = \frac{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}}{5\,b}$$

input
```
integrate(x*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(12) = 24$.

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.39

$$\int x(a+bx^2)^{3/2}\,dx = \begin{cases} \frac{a^2\sqrt{a+bx^2}}{5b} + \frac{2ax^2\sqrt{a+bx^2}}{5} + \frac{bx^4\sqrt{a+bx^2}}{5} & \text{for } b \neq 0 \\ \frac{a^{\frac{3}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((a**2*sqrt(a + b*x**2)/(5*b) + 2*a*x**2*sqrt(a + b*x**2)/5 + b*x
**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (a**(3/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate(x*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(5/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}}{5\,b}$$

input
```
integrate(x*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/5*(b*x^2 + a)^(5/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{3/2}\,dx=\frac{(b\,x^2+a)^{5/2}}{5\,b}$$

input
```
int(x*(a + b*x^2)^(3/2),x)
```

output
```
(a + b*x^2)^(5/2)/(5*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int x(a+bx^2)^{3/2}\,dx=\frac{\sqrt{b\,x^2+a}\,(b^2x^4+2ab\,x^2+a^2)}{5b}$$

input
```
int(x*(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(a**2 + 2*a*b*x**2 + b**2*x**4))/(5*b)
```

3.383 $\int \frac{(a+bx^2)^{3/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{(a+bx^2)^{3/2}}{x} dx = a\sqrt{a+bx^2} + \frac{1}{3}(a+bx^2)^{3/2} - a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
a*(b*x^2+a)^(1/2)+1/3*(b*x^2+a)^(3/2)-a^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1
/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}}{x} dx = \frac{1}{3}\sqrt{a+bx^2}(4a+bx^2) - a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/x,x]
```

output
```
(Sqrt[a + b*x^2]*(4*a + b*x^2))/3 - a^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}(a+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}(a+bx^2)^{3/2}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/x,x]
```

output
```
((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]]))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

method	result	size
pseudoelliptic	$-a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{\sqrt{b\,x^{2}+a}\,\left(b\,x^{2}+4a\right)}{3}$	41
default	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)$	53

input
```
int((b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-a^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+1/3*(b*x^2+a)^(1/2)*(b*x^2+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.91

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = \left[\frac{1}{2}\,a^{\frac{3}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+\frac{1}{3}\,(bx^2+4\,a)\sqrt{bx^2+a},\sqrt{-a}a\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\frac{1}{3}\,(bx^2+4\,a)\sqrt{bx^2+a}\right]$$

input
```
integrate((b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/2*a^(3/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/3*(b*
x^2 + 4*a)*sqrt(b*x^2 + a), sqrt(-a)*a*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
+ 1/3*(b*x^2 + 4*a)*sqrt(b*x^2 + a)]
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = \frac{4a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}{3}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^2}{a}\right)}{2}-a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)+\frac{\sqrt{a}bx^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

input
```
integrate((b*x**2+a)**(3/2)/x,x)
```

output
```
4*a**(3/2)*sqrt(1 + b*x**2/a)/3 + a**(3/2)*log(b*x**2/a)/2 - a**(3/2)*log(
sqrt(1 + b*x**2/a) + 1) + sqrt(a)*b*x**2*sqrt(1 + b*x**2/a)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = -a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}} + \sqrt{bx^2+a}a$$

input
```
integrate((b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
-a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2) + sqrt(b*x^
2 + a)*a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = \frac{a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}} + \sqrt{bx^2+a}a$$

input
```
integrate((b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

output
```
a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/3*(b*x^2 + a)^(3/2) + sq
rt(b*x^2 + a)*a
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = a\sqrt{b\,x^2+a} - a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + \frac{\left(b\,x^2+a\right)^{3/2}}{3}$$

input
```
int((a + b*x^2)^(3/2)/x,x)
```

output
```
a*(a + b*x^2)^(1/2) - a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (a + b*x^
2)^(3/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^{3/2}}{x}\,dx = \frac{4\sqrt{b\,x^2+a}\,a}{3}+\frac{\sqrt{b\,x^2+a}\,b\,x^2}{3}+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a-\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a$$

input
```
int((b*x^2+a)^(3/2)/x,x)
```

output
```
(4*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 + 3*sqrt(a)*log((sqrt(a +
b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a - 3*sqrt(a)*log((sqrt(a + b*x**2
) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a)/3
```

3.384 $\int \frac{(a+bx^2)^{3/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int \frac{(a+bx^2)^{3/2}}{x^3} dx = b\sqrt{a+bx^2} - \frac{a\sqrt{a+bx^2}}{2x^2} - \frac{3}{2}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
b*(b*x^2+a)^(1/2)-1/2*a*(b*x^2+a)^(1/2)/x^2-3/2*a^(1/2)*b*arctanh((b*x^2+a
)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}}{x^3} dx = \frac{\sqrt{a+bx^2}(-a+2bx^2)}{2x^2} - \frac{3}{2}\sqrt{a}b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(2*x^2) - (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{2}b\int \frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{(a+bx^2)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/x^3,x]
```

output
```
(-((a + b*x^2)^(3/2)/x^2) + (3*b*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]]))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.85

method	result	size
pseudoelliptic	$-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) a b\,x^{2}+\left(-2 b\,x^{2}+a \right) \sqrt{a}\,\sqrt{b\,x^{2}+a}}{2 \sqrt{a}\,x^{2}}$	52
risch	$-\frac{a \sqrt{b\,x^{2}+a}}{2 x^{2}}-\frac{3 \sqrt{a}\,\ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right) b}{2}+b \sqrt{b\,x^{2}+a}$	57
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{2 a\,x^{2}}+\frac{3 b \left(\frac{\left(b\,x^{2}+a \right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2 a}$	77

input
```
int((b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a*b*x^2+(-2*b*x^2+a)*a^(1/2)*(b*x
^2+a)^(1/2))/a^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = \left[\frac{3\,\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,bx^2-a)\sqrt{bx^2+a}}{4\,x^2}, \frac{3\,\sqrt{-a}bx^2\arctan\left(\ldots\right.}{}\right.$$

input
```
integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
+ 2*(2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2, 1/2*(3*sqrt(-a)*b*x^2*arctan(sqrt(
b*x^2 + a)*sqrt(-a)/a) + (2*b*x^2 - a)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 1.37 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = -\frac{3\sqrt{a}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{a^2}{2\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{a\sqrt{b}}{2x\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**3,x)
```

output
```
-3*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - a**2/(2*sqrt(b)*x**3*sqrt(a/(b
*x**2) + 1)) + a*sqrt(b)/(2*x*sqrt(a/(b*x**2) + 1)) + b**(3/2)*x/sqrt(a/(b
*x**2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = -\frac{3}{2}\sqrt{a}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}b + \frac{(bx^2+a)^{\frac{3}{2}}b}{2\,a} - \frac{(bx^2+a)^{\frac{5}{2}}}{2\,ax^2}$$

input
```
integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
-3/2*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*b + 1/2
*(b*x^2 + a)^(3/2)*b/a - 1/2*(b*x^2 + a)^(5/2)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(\frac{3\,a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\sqrt{bx^2+a} - \frac{\sqrt{bx^2+a}a}{bx^2}\right)b$$

input
```
integrate((b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/2*(3*a*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x^2 + a) - s
qrt(b*x^2 + a)*a/(b*x^2))*b
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = b\,\sqrt{b\,x^2+a} - \frac{a\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{3\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}$$

input
```
int((a + b*x^2)^(3/2)/x^3,x)
```

output
```
b*(a + b*x^2)^(1/2) - (a*(a + b*x^2)^(1/2))/(2*x^2) - (3*a^(1/2)*b*atanh((
a + b*x^2)^(1/2)/a^(1/2)))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^{3/2}}{x^3}\,dx = \frac{-\sqrt{b\,x^2+a}\,a + 2\sqrt{b\,x^2+a}\,b\,x^2 + 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2 - 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+ \cdots}{\sqrt{\cdots}}\right. \cdots}{2x^2}$$

input
```
int((b*x^2+a)^(3/2)/x^3,x)
```

output
```
( - sqrt(a + b*x**2)*a + 2*sqrt(a + b*x**2)*b*x**2 + 3*sqrt(a)*log((sqrt(a
 + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b*x**2 - 3*sqrt(a)*log((sqrt(a
+ b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*x**2)/(2*x**2)
```

3.385 $\int \frac{(a+bx^2)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{(a+bx^2)^{3/2}}{x^5} dx = -\frac{a\sqrt{a+bx^2}}{4x^4} - \frac{5b\sqrt{a+bx^2}}{8x^2} - \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

output
```
-1/4*a*(b*x^2+a)^(1/2)/x^4-5/8*b*(b*x^2+a)^(1/2)/x^2-3/8*b^2*arctanh((b*x^
2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}}{x^5} dx = \frac{(-2a-5bx^2)\sqrt{a+bx^2}}{8x^4} - \frac{3b^2 \operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8\sqrt{a}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^5,x]
```

output
```
((-2*a - 5*b*x^2)*Sqrt[a + b*x^2])/(8*x^4) - (3*b^2*ArcTanh[Sqrt[a + b*x^2
]/Sqrt[a]])/(8*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^5} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt[a + b*x^2]/x^2) - (b*ArcTanh[Sq
rt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(5b\,x^{2}+2a\right)}{8x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8\sqrt{a}}$	57
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)b^{2}x^{4}-5b\,x^{2}\sqrt{a}\,\sqrt{b\,x^{2}+a}-2a^{\frac{3}{2}}\sqrt{b\,x^{2}+a}}{8x^{4}\sqrt{a}}$	64
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{4a\,x^{4}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)}{4a}$	101

input
```
int((b*x^2+a)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(5*b*x^2+2*a)/x^4-3/8*b^2/a^(1/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = \left[\frac{3\sqrt{a}b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(5abx^2+2a^2\right)\sqrt{bx^2+a}}{16\,ax^4}, \frac{3\sqrt{-a}b^2x^4\arctan$$

input
```
integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a))/(a*x^4), 1/8*(3*sqrt(-a)*b^2*x
^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (5*a*b*x^2 + 2*a^2)*sqrt(b*x^2 + a
))/(a*x^4)]
```

Sympy [A] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{4x^3}-\frac{5b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{8x}-\frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**5,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(4*x**3) - 5*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(8*x) - 3*b**2*asinh(sqrt(a)/(sqrt(b)*x))/(8*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = -\frac{3\,b^2\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,\sqrt{a}} + \frac{(bx^2+a)^{\frac{3}{2}}b^2}{8\,a^2} + \frac{3\,\sqrt{bx^2+a}b^2}{8\,a} - \frac{(bx^2+a)^{\frac{5}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{5}{2}}}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
-3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/8*(b*x^2 + a)^(3/2)*b^2
/a^2 + 3/8*sqrt(b*x^2 + a)*b^2/a - 1/8*(b*x^2 + a)^(5/2)*b/(a^2*x^2) - 1/4
*(b*x^2 + a)^(5/2)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = \frac{\frac{3\,b^3\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{5\,(bx^2+a)^{\frac{3}{2}}b^3 - 3\,\sqrt{bx^2+a}ab^3}{b^2x^4}}{8\,b}$$

input
```
integrate((b*x^2+a)^(3/2)/x^5,x, algorithm="giac")
```

output
```
1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (5*(b*x^2 + a)^(3/2
)*b^3 - 3*sqrt(b*x^2 + a)*a*b^3)/(b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = \frac{3\,a\,\sqrt{b\,x^2+a}}{8\,x^4} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,\sqrt{a}} - \frac{5\,\left(b\,x^2+a\right)^{3/2}}{8\,x^4}$$

input
```
int((a + b*x^2)^(3/2)/x^5,x)
```

output
```
(3*a*(a + b*x^2)^(1/2))/(8*x^4) - (3*b^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))
/(8*a^(1/2)) - (5*(a + b*x^2)^(3/2))/(8*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^{3/2}}{x^5}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2 - 5\sqrt{b\,x^2+a}\,ab\,x^2 + 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^4 - 3\sqrt{a}\log\Big(\sqrt{b\,x^2}\cdots}{8a\,x^4}$$

input
```
int((b*x^2+a)^(3/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2 - 5*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 - 3*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4)/(8*a*x**4)
```

3.386 $\int \frac{(a+bx^2)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{(a+bx^2)^{3/2}}{x^7} dx = -\frac{a\sqrt{a+bx^2}}{6x^6} - \frac{7b\sqrt{a+bx^2}}{24x^4} - \frac{b^2\sqrt{a+bx^2}}{16ax^2} + \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

output
```
-1/6*a*(b*x^2+a)^(1/2)/x^6-7/24*b*(b*x^2+a)^(1/2)/x^4-1/16*b^2*(b*x^2+a)^(
1/2)/a/x^2+1/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{3/2}}{x^7} dx = \frac{\sqrt{a+bx^2}(-8a^2-14abx^2-3b^2x^4)}{48ax^6} + \frac{b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2 - 14*a*b*x^2 - 3*b^2*x^4))/(48*a*x^6) + (b^3*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^7} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{(bx^2+a)^{3/2}}{x^8} dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2} \left(\frac{1}{2} b \int \frac{\sqrt{bx^2+a}}{x^6} dx^2 - \frac{(a+bx^2)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2} \left(\frac{1}{2} b \left(\frac{1}{4} b \int \frac{1}{x^4\sqrt{bx^2+a}} dx^2 - \frac{\sqrt{a+bx^2}}{2x^4} \right) - \frac{(a+bx^2)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2} \left(\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{b \int \frac{1}{x^2\sqrt{bx^2+a}} dx^2}{2a} - \frac{\sqrt{a+bx^2}}{ax^2} \right) - \frac{\sqrt{a+bx^2}}{2x^4} \right) - \frac{(a+bx^2)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2} \left(\frac{1}{2} b \left(\frac{1}{4} b \left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}} d\sqrt{bx^2+a}}{a} - \frac{\sqrt{a+bx^2}}{ax^2} \right) - \frac{\sqrt{a+bx^2}}{2x^4} \right) - \frac{(a+bx^2)^{3/2}}{3x^6} \right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2} \left(\frac{1}{2} b \left(\frac{1}{4} b \left(\frac{b \text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{a+bx^2}}{ax^2} \right) - \frac{\sqrt{a+bx^2}}{2x^4} \right) - \frac{(a+bx^2)^{3/2}}{3x^6} \right)$$

input
```
Int[(a + b*x^2)^(3/2)/x^7,x]
```

output
```
(-1/3*(a + b*x^2)^(3/2)/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/x^4 + (b*(-(Sqrt[a
+ b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/4))/2)/
2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

method	result	siz
risch	$-\frac{\sqrt{b\,x^2+a}\,(3b^2x^4+14ab\,x^2+8a^2)}{48x^6 a}+\frac{b^3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{16a^{\frac{3}{2}}}$	71
pseudoelliptic	$\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)b^3x^6-3b^2x^4\sqrt{b\,x^2+a}\,\sqrt{a}-14a^{\frac{3}{2}}b\,x^2\sqrt{b\,x^2+a}-8a^{\frac{5}{2}}\sqrt{b\,x^2+a}}{48a^{\frac{3}{2}}x^6}$	84
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)}{2a}\right)}{4a}\right)}{6a}$	12

input
```
int((b*x^2+a)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(b*x^2+a)^(1/2)*(3*b^2*x^4+14*a*b*x^2+8*a^2)/x^6/a+1/16*b^3/a^(3/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.72

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = \left[\frac{3\sqrt{a}b^3x^6\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\,(3\,ab^2x^4+14\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{96\,a^2x^6},\right.$$
$$\left.-\frac{3\sqrt{-a}b^3x^6\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(3\,ab^2x^4+14\,a^2bx^2+8\,a^3)\sqrt{bx^2+a}}{48\,a^2x^6}\right]$$

input
```
integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(3*sqrt(a)*b^3*x^6*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) - 2*(3*a*b^2*x^4 + 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6), -1
/48*(3*sqrt(-a)*b^3*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*a*b^2*x^4
+ 14*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a^2*x^6)]
```

Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = -\frac{a^2}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{11a\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{17b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{b^{\frac{5}{2}}}{16ax\sqrt{\frac{a}{bx^2}+1}} + \frac{b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**7,x)
```

output
```
-a**2/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - 11*a*sqrt(b)/(24*x**5*sqrt(a
/(b*x**2) + 1)) - 17*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) - b**(5/2)/(1
6*a*x*sqrt(a/(b*x**2) + 1)) + b**3*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(3/2)
)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = \frac{b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{3}{2}}} - \frac{(bx^2+a)^{\frac{3}{2}}b^3}{48\,a^3} - \frac{\sqrt{bx^2+a}\,b^3}{16\,a^2} + \frac{(bx^2+a)^{\frac{5}{2}}b^2}{48\,a^3x^2} + \frac{(bx^2+a)^{\frac{5}{2}}b}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{5}{2}}}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
1/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/48*(b*x^2 + a)^(3/2)*b^
3/a^3 - 1/16*sqrt(b*x^2 + a)*b^3/a^2 + 1/48*(b*x^2 + a)^(5/2)*b^2/(a^3*x^2
) + 1/24*(b*x^2 + a)^(5/2)*b/(a^2*x^4) - 1/6*(b*x^2 + a)^(5/2)/(a*x^6)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = -\frac{1}{48}\,b^3\left(\frac{3\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{3\,(bx^2+a)^{\frac{5}{2}}+8\,(bx^2+a)^{\frac{3}{2}}a-3\,\sqrt{bx^2+a}a^2}{ab^3x^6}\right)$$

input
```
integrate((b*x^2+a)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/48*b^3*(3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (3*(b*x^2 + a
)^(5/2) + 8*(b*x^2 + a)^(3/2)*a - 3*sqrt(b*x^2 + a)*a^2)/(a*b^3*x^6))
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = \frac{a\,\sqrt{b\,x^2+a}}{16\,x^6}-\frac{(b\,x^2+a)^{3/2}}{6\,x^6}-\frac{(b\,x^2+a)^{5/2}}{16\,a\,x^6}-\frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}}{16\,a^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)/x^7,x)
```

output
```
(a*(a + b*x^2)^(1/2))/(16*x^6) - (b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))
*1i)/(16*a^(3/2)) - (a + b*x^2)^(3/2)/(6*x^6) - (a + b*x^2)^(5/2)/(16*a*x^
6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^{3/2}}{x^7}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^3-14\sqrt{b\,x^2+a}\,a^2b\,x^2-3\sqrt{b\,x^2+a}\,a\,b^2x^4-3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right.}{48a^2x^6}$$

input

```
int((b*x^2+a)^(3/2)/x^7,x)
```

output

```
( - 8*sqrt(a + b*x**2)*a**3 - 14*sqrt(a + b*x**2)*a**2*b*x**2 - 3*sqrt(a +
 b*x**2)*a*b**2*x**4 - 3*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b**3*x**6 + 3*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(
b)*x)/sqrt(a))*b**3*x**6)/(48*a**2*x**6)
```

3.387 $\int \frac{(a+bx^2)^{3/2}}{x^9} dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{(a+bx^2)^{3/2}}{x^9} dx = -\frac{a\sqrt{a+bx^2}}{8x^8} - \frac{3b\sqrt{a+bx^2}}{16x^6} - \frac{b^2\sqrt{a+bx^2}}{64ax^4} + \frac{3b^3\sqrt{a+bx^2}}{128a^2x^2} - \frac{3b^4\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

output
```
-1/8*a*(b*x^2+a)^(1/2)/x^8-3/16*b*(b*x^2+a)^(1/2)/x^6-1/64*b^2*(b*x^2+a)^(
1/2)/a/x^4+3/128*b^3*(b*x^2+a)^(1/2)/a^2/x^2-3/128*b^4*arctanh((b*x^2+a)^(
1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{3/2}}{x^9} dx = \frac{\sqrt{a+bx^2}(-16a^3-24a^2bx^2-2ab^2x^4+3b^3x^6)}{128a^2x^8} - \frac{3b^4\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^9,x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^3 - 24*a^2*b*x^2 - 2*a*b^2*x^4 + 3*b^3*x^6))/(128*
a^2*x^8) - (3*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(5/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {243, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{3/2}}{x^{10}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{8}b\int \frac{\sqrt{bx^2+a}}{x^8}dx^2-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)$$

input
```
Int[(a + b*x^2)^(3/2)/x^9,x]
```

output
```
(-1/4*(a + b*x^2)^(3/2)/x^8 + (3*b*(-1/3*Sqrt[a + b*x^2]/x^6 + (b*(-1/2*Sq
rt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a)))/6))/8)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{-\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)b^4x^8}{128}+\frac{3\sqrt{bx^2+a}\left(\sqrt{a}\,b^3x^6-\frac{2a^{\frac{3}{2}}b^2x^4}{3}-8a^{\frac{5}{2}}bx^2-\frac{16a^{\frac{7}{2}}}{3}\right)}{128}}{a^{\frac{5}{2}}x^8}$
risch	$-\frac{\sqrt{bx^2+a}\,(-3b^3x^6+2a\,b^2x^4+24a^2bx^2+16a^3)}{128x^8a^2}-\frac{3b^4\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{128a^{\frac{5}{2}}}$
default	$-\frac{(bx^2+a)^{\frac{5}{2}}}{8a\,x^8}-\frac{3b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{6a\,x^6}-\frac{b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{4a\,x^4}+\frac{b\left(-\frac{(bx^2+a)^{\frac{5}{2}}}{2a\,x^2}+\frac{3b\left(\frac{(bx^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{bx^2+a}-\sqrt{a}\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2 \cdots}}{x}\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}\right.}{8a}$

input
```
int((b*x^2+a)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
3/128/a^(5/2)*(-arctanh((b*x^2+a)^(1/2)/a^(1/2))*b^4*x^8+(b*x^2+a)^(1/2)*(
a^(1/2)*b^3*x^6-2/3*a^(3/2)*b^2*x^4-8*a^(5/2)*b*x^2-16/3*a^(7/2)))/x^8
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = \left[\frac{3\sqrt{a}b^4x^8\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\,(3\,ab^3x^6-2\,a^2b^2x^4-24\,a^3bx^2-16\,a^4)\sqrt{bx^2 \cdots}}{256\,a^3x^8}\right.$$

input
```
integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[1/256*(3*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 +
 a))/(a^3*x^8), 1/128*(3*sqrt(-a)*b^4*x^8*arctan(sqrt(b*x^2 + a)*sqrt(-a)/
a) + (3*a*b^3*x^6 - 2*a^2*b^2*x^4 - 24*a^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a)
)/(a^3*x^8)]
```

Sympy [A] (verification not implemented)

Time = 11.82 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = -\frac{a^2}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{5a\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{13b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{5}{2}}}{128ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{7}{2}}}{128a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{5}{2}}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**9,x)
```

output
```
-a**2/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 5*a*sqrt(b)/(16*x**7*sqrt(a/
(b*x**2) + 1)) - 13*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) + b**(5/2)/(12
8*a*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(7/2)/(128*a**2*x*sqrt(a/(b*x**2) +
1)) - 3*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = -\frac{3\,b^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}b^4}{128\,a^4} + \frac{3\sqrt{bx^2+a}b^4}{128\,a^3} - \frac{(bx^2+a)^{\frac{5}{2}}b^3}{128\,a^4x^2} - \frac{(bx^2+a)^{\frac{5}{2}}b^2}{64\,a^3x^4} + \frac{(bx^2+a)^{\frac{5}{2}}b}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{5}{2}}}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
-3/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/128*(b*x^2 + a)^(3/2)
*b^4/a^4 + 3/128*sqrt(b*x^2 + a)*b^4/a^3 - 1/128*(b*x^2 + a)^(5/2)*b^3/(a^
4*x^2) - 1/64*(b*x^2 + a)^(5/2)*b^2/(a^3*x^4) + 1/16*(b*x^2 + a)^(5/2)*b/(
a^2*x^6) - 1/8*(b*x^2 + a)^(5/2)/(a*x^8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = \frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx^2+a)^{\frac{7}{2}}b^5 - 11\,(bx^2+a)^{\frac{5}{2}}ab^5 - 11\,(bx^2+a)^{\frac{3}{2}}a^2b^5 + 3\,\sqrt{bx^2+a}a^3b^5}{a^2b^4x^8}}{128\,b}$$

input
```
integrate((b*x^2+a)^(3/2)/x^9,x, algorithm="giac")
```

output
```
1/128*(3*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 +
 a)^(7/2)*b^5 - 11*(b*x^2 + a)^(5/2)*a*b^5 - 11*(b*x^2 + a)^(3/2)*a^2*b^5
+ 3*sqrt(b*x^2 + a)*a^3*b^5)/(a^2*b^4*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = \frac{3\,a\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{11\,(b\,x^2+a)^{3/2}}{128\,x^8} - \frac{11\,(b\,x^2+a)^{5/2}}{128\,a\,x^8} + \frac{3\,(b\,x^2+a)^{7/2}}{128\,a^2\,x^8} + \frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{128\,a^{5/2}}$$

input
```
int((a + b*x^2)^(3/2)/x^9,x)
```

output
```
(b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(128*a^(5/2)) - (11*(a + b*x
^2)^(3/2))/(128*x^8) + (3*a*(a + b*x^2)^(1/2))/(128*x^8) - (11*(a + b*x^2)
^(5/2))/(128*a*x^8) + (3*(a + b*x^2)^(7/2))/(128*a^2*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^{3/2}}{x^9}\,dx = \frac{-16\sqrt{b\,x^2+a}\,a^4-24\sqrt{b\,x^2+a}\,a^3b\,x^2-2\sqrt{b\,x^2+a}\,a^2b^2x^4+3\sqrt{b\,x^2+a}\,a\,b^3x^6+3\sqrt{}}{128a^3x^8}$$

input
```
int((b*x^2+a)^(3/2)/x^9,x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**4 - 24*sqrt(a + b*x**2)*a**3*b*x**2 - 2*sqrt(a
+ b*x**2)*a**2*b**2*x**4 + 3*sqrt(a + b*x**2)*a*b**3*x**6 + 3*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 - 3*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8)/(128*a**3*x
**8)
```

3.388 $\int x^4(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 115

$$\int x^4\left(a+bx^2\right)^{3/2}\,dx=-\frac{3a^3x\sqrt{a+bx^2}}{128b^2}+\frac{a^2x^3\sqrt{a+bx^2}}{64b}+\frac{1}{16}ax^5\sqrt{a+bx^2}+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}+\frac{3a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
-3/128*a^3*x*(b*x^2+a)^(1/2)/b^2+1/64*a^2*x^3*(b*x^2+a)^(1/2)/b+1/16*a*x^5
*(b*x^2+a)^(1/2)+1/8*x^5*(b*x^2+a)^(3/2)+3/128*a^4*arctanh(b^(1/2)*x/(b*x^
2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.81

$$\int x^4\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}(-3a^3x+2a^2bx^3+24ab^2x^5+16b^3x^7)}{128b^2}+\frac{3a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{64b^{5/2}}$$

input
```
Integrate[x^4*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^3*x + 2*a^2*b*x^3 + 24*a*b^2*x^5 + 16*b^3*x^7))/(12
8*b^2) + (3*a^4*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(64*b^(
5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{3}{8} a \int x^4 \sqrt{bx^2+a}dx+\frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{3}{8} a \left(\frac{1}{6} a \int \frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6} x^5 \sqrt{a+bx^2}\right)+\frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{3}{8} a \left(\frac{1}{6} a \left(\frac{x^3 \sqrt{a+bx^2}}{4 b}-\frac{3 a \int \frac{x^2}{\sqrt{bx^2+a}}dx}{4 b}\right)+\frac{1}{6} x^5 \sqrt{a+bx^2}\right)+\frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{3}{8} a \left(\frac{1}{6} a \left(\frac{x^3 \sqrt{a+bx^2}}{4 b}-\frac{3 a \left(\frac{x \sqrt{a+bx^2}}{2 b}-\frac{a \int \frac{1}{\sqrt{bx^2+a}}dx}{2 b}\right)}{4 b}\right)+\frac{1}{6} x^5 \sqrt{a+bx^2}\right)+\frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}$$

input
```
Int[x^4*(a + b*x^2)^(3/2),x]
```

output
```
(x^5*(a + b*x^2)^(3/2))/8 + (3*a*((x^5*Sqrt[a + b*x^2])/6 + (a*((x^3*Sqrt[
a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/6))/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.63

method	result	size
risch	$-\frac{x\left(-16b^{3}x^{6}-24a\,b^{2}x^{4}-2a^{2}b\,x^{2}+3a^{3}\right)\sqrt{b\,x^{2}+a}}{128b^{2}}+\frac{3a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{5}{2}}}$	73
pseudoelliptic	$\frac{\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{4}}{128}-\frac{3x\left(-\frac{16b^{\frac{7}{2}}x^{6}}{3}-8a\,b^{\frac{5}{2}}x^{4}-\frac{2a^{2}b^{\frac{3}{2}}x^{2}}{3}+a^{3}\sqrt{b}\right)\sqrt{b\,x^{2}+a}}{128}}{b^{\frac{5}{2}}}$	76
default	$\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}$	98

input
```
int(x^4*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/128*x*(-16*b^3*x^6-24*a*b^2*x^4-2*a^2*b*x^2+3*a^3)*(b*x^2+a)^(1/2)/b^2+
3/128/b^(5/2)*a^4*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.46

$$\int x^4(a+bx^2)^{3/2}\,dx = \left[\frac{3\,a^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx)\sqrt{bx^2+a}}{256\,b^3}, -\frac{3\,a^4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(16\,b^4x^7+24\,ab^3x^5+2\,a^2b^2x^3-3\,a^3bx)\sqrt{bx^2+a}}{128\,b^3}\right]$$

input
```
integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/256*(3*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(16*b^4*x^7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b
^3, -1/128*(3*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (16*b^4*x^
7 + 24*a*b^3*x^5 + 2*a^2*b^2*x^3 - 3*a^3*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 10.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.29

$$\int x^4(a+bx^2)^{3/2}\,dx = -\frac{3a^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{a^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13a^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5\sqrt{a}bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{b^2x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4*(b*x**2+a)**(3/2),x)
```

output
```
-3*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**2/a)) - a**(5/2)*x**3/(128*b*sqrt(1
+ b*x**2/a)) + 13*a**(3/2)*x**5/(64*sqrt(1 + b*x**2/a)) + 5*sqrt(a)*b*x**7
/(16*sqrt(1 + b*x**2/a)) + 3*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2))
+ b**2*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.76

$$\int x^4 (a + bx^2)^{3/2}\, dx = \frac{(bx^2 + a)^{\frac{5}{2}} x^3}{8\, b} - \frac{(bx^2 + a)^{\frac{5}{2}} a x}{16\, b^2} + \frac{(bx^2 + a)^{\frac{3}{2}} a^2 x}{64\, b^2} + \frac{3\, \sqrt{bx^2 + a} a^3 x}{128\, b^2} + \frac{3\, a^4 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\, b^{\frac{5}{2}}}$$

input

```
integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output

```
1/8*(b*x^2 + a)^(5/2)*x^3/b - 1/16*(b*x^2 + a)^(5/2)*a*x/b^2 + 1/64*(b*x^2
 + a)^(3/2)*a^2*x/b^2 + 3/128*sqrt(b*x^2 + a)*a^3*x/b^2 + 3/128*a^4*arcsin
h(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.66

$$\int x^4 (a + bx^2)^{3/2}\, dx = \frac{1}{128} \left(2 \left(4 \left(2\, bx^2 + 3\, a\right) x^2 + \frac{a^2}{b}\right) x^2 - \frac{3\, a^3}{b^2}\right) \sqrt{bx^2 + a} x - \frac{3\, a^4 \log\left(\left|-\sqrt{b} x + \sqrt{bx^2 + a}\right|\right)}{128\, b^{\frac{5}{2}}}$$

input

```
integrate(x^4*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output

```
1/128*(2*(4*(2*b*x^2 + 3*a)*x^2 + a^2/b)*x^2 - 3*a^3/b^2)*sqrt(b*x^2 + a)*
x - 3/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a+bx^2)^{3/2} \, dx = \int x^4 \left(b x^2 + a\right)^{3/2} dx$$

input
```
int(x^4*(a + b*x^2)^(3/2),x)
```

output
```
int(x^4*(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.86

$$\int x^4(a+bx^2)^{3/2} \, dx = \frac{-3\sqrt{b x^2+a}\, a^3 b x + 2\sqrt{b x^2+a}\, a^2 b^2 x^3 + 24\sqrt{b x^2+a}\, a b^3 x^5 + 16\sqrt{b x^2+a}\, b^4 x^7 + 3\sqrt{b}\log(}{128 b^3}$$

input
```
int(x^4*(b*x^2+a)^(3/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**3*b*x + 2*sqrt(a + b*x**2)*a**2*b**2*x**3 + 24*s
qrt(a + b*x**2)*a*b**3*x**5 + 16*sqrt(a + b*x**2)*b**4*x**7 + 3*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4)/(128*b**3)
```

3.389 $\int x^2(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int x^2\left(a+bx^2\right)^{3/2}\,dx=\frac{a^2x\sqrt{a+bx^2}}{16b}+\frac{1}{8}ax^3\sqrt{a+bx^2}+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
1/16*a^2*x*(b*x^2+a)^(1/2)/b+1/8*a*x^3*(b*x^2+a)^(1/2)+1/6*x^3*(b*x^2+a)^(
3/2)-1/16*a^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int x^2\left(a+bx^2\right)^{3/2}\,dx=\frac{x\sqrt{a+bx^2}(3a^2+14abx^2+8b^2x^4)}{48b}-\frac{a^3\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(3*a^2 + 14*a*b*x^2 + 8*b^2*x^4))/(48*b) - (a^3*ArcTanh
[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} a \int x^2 \sqrt{bx^2+a} dx + \frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} a \left(\frac{1}{4} a \int \frac{x^2}{\sqrt{bx^2+a}} dx + \frac{1}{4} x^3 \sqrt{a+bx^2}\right) + \frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \int \frac{1}{\sqrt{bx^2+a}} dx}{2b}\right) + \frac{1}{4} x^3 \sqrt{a+bx^2}\right) + \frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \int \frac{1}{1-\frac{bx^2}{bx^2+a}} d\frac{x}{\sqrt{bx^2+a}}}{2b}\right) + \frac{1}{4} x^3 \sqrt{a+bx^2}\right) + \frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x\sqrt{a+bx^2}}{2b} - \frac{a \text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right) + \frac{1}{4} x^3 \sqrt{a+bx^2}\right) + \frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}$$

input
```
Int[x^2*(a + b*x^2)^(3/2),x]
```

output
```
(x^3*(a + b*x^2)^(3/2))/6 + (a*((x^3*Sqrt[a + b*x^2])/4 + (a*((x*Sqrt[a +
b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/4))
/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.68

method	result	size
risch	$\frac{x\left(8b^{2}x^{4}+14ab\,x^{2}+3a^{2}\right)\sqrt{b\,x^{2}+a}}{48b}-\frac{a^{3}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{3}{2}}}$	62
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}$	74
pseudoelliptic	$\frac{8b^{\frac{5}{2}}\sqrt{b\,x^{2}+a}\,x^{5}+14a\,b^{\frac{3}{2}}x^{3}\sqrt{b\,x^{2}+a}+3a^{2}x\sqrt{b}\,\sqrt{b\,x^{2}+a}-3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{3}}{48b^{\frac{3}{2}}}$	82

input
```
int(x^2*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b^2*x^4+14*a*b*x^2+3*a^2)*(b*x^2+a)^(1/2)/b-1/16/b^(3/2)*a^3*ln(
b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.59

$$\int x^2(a+bx^2)^{3/2}\,dx=\left[\frac{3\,a^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3x^5+14\,ab^2x^3+3\,a^2bx)\sqrt{bx^2+a}}{96\,b^2},\frac{3\,a^3}{}\right.$$

input
```
integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/96*(3*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
8*b^3*x^5 + 14*a*b^2*x^3 + 3*a^2*b*x)*sqrt(b*x^2 + a))/b^2, 1/48*(3*a^3*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (8*b^3*x^5 + 14*a*b^2*x^3 + 3*
a^2*b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.31

$$\int x^2(a+bx^2)^{3/2}\,dx = \frac{a^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}} + \frac{17a^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}bx^5}{24\sqrt{1+\frac{bx^2}{a}}} - \frac{a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}} + \frac{b^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2*(b*x**2+a)**(3/2),x)
```

output
```
a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*a**(3/2)*x**3/(48*sqrt(1 + b*x**
2/a)) + 11*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(3/2)) + b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int x^2(a+bx^2)^{3/2}\,dx = \frac{(bx^2+a)^{\frac{5}{2}}x}{6\,b} - \frac{(bx^2+a)^{\frac{3}{2}}ax}{24\,b} - \frac{\sqrt{bx^2+a}a^2x}{16\,b} - \frac{a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*x/b - 1/24*(b*x^2 + a)^(3/2)*a*x/b - 1/16*sqrt(b*x^2
 + a)*a^2*x/b - 1/16*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.69

$$\int x^2 \left(a+bx^2\right)^{3/2} dx = \frac{1}{48} \left(2 \left(4 bx^2 + 7 a\right)x^2 + \frac{3 a^2}{b}\right)\sqrt{bx^2+a}x + \frac{a^3 \log \left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{16 b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*b*x^2 + 7*a)*x^2 + 3*a^2/b)*sqrt(b*x^2 + a)*x + 1/16*a^3*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^2\right)^{3/2} dx = \int x^2 \left(b x^2 + a\right)^{3/2} dx$$

input
```
int(x^2*(a + b*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int x^2 \left(a+bx^2\right)^{3/2} dx = \frac{3\sqrt{b x^2+a}\, a^2 b x + 14\sqrt{b x^2+a}\, a b^2 x^3 + 8\sqrt{b x^2+a}\, b^3 x^5 - 3\sqrt{b} \log\left(\frac{\sqrt{b x^2+a}+\sqrt{b}\, x}{\sqrt{a}}\right) a^3}{48 b^2}$$

input
```
int(x^2*(b*x^2+a)^(3/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a**2*b*x + 14*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
+ b*x**2)*b**3*x**5 - 3*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**3)/(48*b**2)
```

3.390 $\int (a+bx^2)^{3/2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 65

$$\int (a+bx^2)^{3/2} \, dx = \frac{3}{8}ax\sqrt{a+bx^2} + \frac{1}{4}x(a+bx^2)^{3/2} + \frac{3a^2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
3/8*a*x*(b*x^2+a)^(1/2)+1/4*x*(b*x^2+a)^(3/2)+3/8*a^2*arctanh(b^(1/2)*x/(b
*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int (a+bx^2)^{3/2} \, dx = \frac{1}{8}x\sqrt{a+bx^2}(5a+2bx^2) - \frac{3a^2\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(3/2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(5*a + 2*b*x^2))/8 - (3*a^2*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(8*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}$$

input
```
Int[(a + b*x^2)^(3/2),x]
```

output
```
(x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{x\left(2b\,x^{2}+5a\right)\sqrt{b\,x^{2}+a}}{8}+\frac{3a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	48
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}$	52
pseudoelliptic	$\frac{2\sqrt{b\,x^{2}+a}\,b^{\frac{3}{2}}x^{3}+5ax\sqrt{b\,x^{2}+a}\,\sqrt{b}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{2}}{8\sqrt{b}}$	62

input
```
int((b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(2*b*x^2+5*a)*(b*x^2+a)^(1/2)+3/8*a^2*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/
b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.91

$$\int (a+bx^2)^{3/2}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{16\,b}, -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3+5\,abx)\sqrt{bx^2+a}}{8\,b}\right]$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b, -1/8*(3*a^2*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 1.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^{3/2}\,dx = \frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{8}+\frac{\sqrt{a}bx^3\sqrt{1+\frac{bx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(3/2),x)
```

output
```
5*a**(3/2)*x*sqrt(1 + b*x**2/a)/8 + sqrt(a)*b*x**3*sqrt(1 + b*x**2/a)/4 +
3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

$$\int \left(a+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}x+\frac{3}{8}\sqrt{bx^2+a}ax+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*x + 3/8*sqrt(b*x^2 + a)*a*x + 3/8*a^2*arcsinh(b*x/sq
rt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{3/2}\,dx=\frac{1}{8}\left(2\,bx^2+5\,a\right)\sqrt{bx^2+a}x-\frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/8*(2*b*x^2 + 5*a)*sqrt(b*x^2 + a)*x - 3/8*a^2*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/sqrt(b)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/2}\,dx=\frac{x\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2),x)
```

output
```
(x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \left(a+bx^2\right)^{3/2} dx = \frac{5\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b}$$

input
```
int((b*x^2+a)^(3/2),x)
```

output
```
(5*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b)
```

3.391 $\int \frac{(a+bx^2)^{3/2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{(a+bx^2)^{3/2}}{x^2} dx = -\frac{a\sqrt{a+bx^2}}{x} + \frac{1}{2}bx\sqrt{a+bx^2} + \frac{3}{2}a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-a*(b*x^2+a)^(1/2)/x+1/2*b*x*(b*x^2+a)^(1/2)+3/2*a*b^(1/2)*arctanh(b^(1/2)
*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^{3/2}}{x^2} dx = \frac{(-2a+bx^2)\sqrt{a+bx^2}}{2x} + 3a\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^2,x]
```

output
```
((-2*a + b*x^2)*Sqrt[a + b*x^2])/(2*x) + 3*a*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/(
-Sqrt[a] + Sqrt[a + b*x^2])]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$3b\int \sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}$$

$$\Big\downarrow \text{224}$$

$$3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}$$

$$\Big\downarrow \text{219}$$

$$3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}$$

input
```
Int[(a + b*x^2)^(3/2)/x^2,x]
```

output
```
-((a + b*x^2)^(3/2)/x) + 3*b*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2 a \right)}{2 x}+\frac{3 a \sqrt{b}\,\ln \left(\sqrt{b}\,x +\sqrt{b\,x^{2}+a}\right)}{2}$	48
pseudoelliptic	$\frac{x^{2} b \sqrt{b\,x^{2}+a}+3 \sqrt{b}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right) a x -2 a \sqrt{b\,x^{2}+a}}{2 x}$	57
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{a x}+\frac{4 b \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\,x +\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{a}$	76

input
```
int((b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(-b*x^2+2*a)/x+3/2*a*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(
1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = \left[\frac{3\,a\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}(bx^2-2\,a)}{4\,x},\right.$$
$$\left.-\frac{3\,a\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}(bx^2-2\,a)}{2\,x}\right]$$

input
```
integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*(3*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sq
rt(b*x^2 + a)*(b*x^2 - 2*a))/x, -1/2*(3*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - sqrt(b*x^2 + a)*(b*x^2 - 2*a))/x]
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = -\frac{a^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}bx}{2\sqrt{1+\frac{bx^2}{a}}}+\frac{3a\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2}+\frac{b^2x^3}{2\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**2,x)
```

output
```
-a**(3/2)/(x*sqrt(1 + b*x**2/a)) - sqrt(a)*b*x/(2*sqrt(1 + b*x**2/a)) + 3*
a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2 + b**2*x**3/(2*sqrt(a)*sqrt(1 + b*x**
2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = \frac{3}{2}\sqrt{bx^2+a}bx + \frac{3}{2}a\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{3}{2}}}{x}$$

input
```
integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*b*x + 3/2*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - (b*x^2 +
a)^(3/2)/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}bx - \frac{3}{4}a\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2a^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a}$$

input
```
integrate((b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b*x - 3/4*a*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^
2) + 2*a^2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = -\frac{(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)/x^2,x)
```

output
```
-((a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{3/2}}{x^2}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a+4\sqrt{b\,x^2+a}\,b\,x^2+12\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ax-9\sqrt{b}\,ax}{8x}$$

input
```
int((b*x^2+a)^(3/2)/x^2,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a + 4*sqrt(a + b*x**2)*b*x**2 + 12*sqrt(b)*log((sqr
t(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*x - 9*sqrt(b)*a*x)/(8*x)
```

3.392 $\int \frac{(a+bx^2)^{3/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{(a+bx^2)^{3/2}}{x^4} dx = -\frac{a\sqrt{a+bx^2}}{3x^3} - \frac{4b\sqrt{a+bx^2}}{3x} + b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*a*(b*x^2+a)^(1/2)/x^3-4/3*b*(b*x^2+a)^(1/2)/x+b^(3/2)*arctanh(b^(1/2)
*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{3/2}}{x^4} dx = \frac{(-a-4bx^2)\sqrt{a+bx^2}}{3x^3} - b^{3/2}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^4,x]
```

output
```
((-a - 4*b*x^2)*Sqrt[a + b*x^2])/(3*x^3) - b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt
[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{224}$$

$$b\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(3/2)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(3/2)/x^3 + b*(-(Sqrt[a + b*x^2]/x) + Sqrt[b]*ArcTanh[(Sq
rt[b]*x)/Sqrt[a + b*x^2]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.69

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(4 b\,x^{2}+a \right)}{3 x^{3}}+b^{\frac{3}{2}} \ln \left(\sqrt{b}\, x +\sqrt{b\,x^{2}+a}\right)$	44
pseudoelliptic	$\frac{3 b^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right) x^{3}-\sqrt{b\,x^{2}+a}\,\left(4 b\,x^{2}+a \right)}{3 x^{3}}$	51
default	$-\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{3 a\,x^{3}}+\frac{2 b \left(-\frac{\left(b\,x^{2}+a \right)^{\frac{5}{2}}}{a x}+\frac{4 b \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{a}\right)}{3 a}$	100

input
```
int((b*x^2+a)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(4*b*x^2+a)/x^3+b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = \left[\frac{3\,b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(4\,bx^2+a)\sqrt{bx^2+a}}{6\,x^3}, -\frac{3\,\sqrt{-b}bx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(4\,bx^2+a)\sqrt{bx^2+a}}{3\,x^3}\right]$$

input
```
integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(4
*b*x^2 + a)*sqrt(b*x^2 + a))/x^3, -1/3*(3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (4*b*x^2 + a)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2}-\frac{4b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} - \frac{b^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

input
```
integrate((b*x**2+a)**(3/2)/x**4,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 4*b**(3/2)*sqrt(a/(b*x**2) + 1)
/3 - b**(3/2)*log(a/(b*x**2))/2 + b**(3/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = \frac{\sqrt{bx^2+a}b^2x}{a} + b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{2\,(bx^2+a)^{\frac{3}{2}}b}{3\,ax} - \frac{(bx^2+a)^{\frac{5}{2}}}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*b^2*x/a + b^(3/2)*arcsinh(b*x/sqrt(a*b)) - 2/3*(b*x^2 + a)
^(3/2)*b/(a*x) - 1/3*(b*x^2 + a)^(5/2)/(a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(50) = 100.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = -\frac{1}{2}\,b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4ab^{\frac{3}{2}} - 3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}} + 2\,a^3b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(3/2)/x^4,x, algorithm="giac")
```

output
```
-1/2*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 4/3*(3*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a*b^(3/2) - 3*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)
 + 2*a^3*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(3/2)/x^4,x)
```

output
```
int((a + b*x^2)^(3/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{x^4}\,dx = \frac{-\sqrt{b\,x^2+a}\,a-4\sqrt{b\,x^2+a}\,b\,x^2+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^3}{3x^3}$$

input
```
int((b*x^2+a)^(3/2)/x^4,x)
```

output
```
( - sqrt(a + b*x**2)*a - 4*sqrt(a + b*x**2)*b*x**2 + 3*sqrt(b)*log((sqrt(a
 + b*x**2) + sqrt(b)*x)/sqrt(a))*b*x**3)/(3*x**3)
```

3.393 $\int \frac{(a+bx^2)^{3/2}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^2)^{3/2}}{x^6} dx = -\frac{(a+bx^2)^{5/2}}{5ax^5}$$

output
```
-1/5*(b*x^2+a)^(5/2)/a/x^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/2}}{x^6} dx = -\frac{(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/2)/(a*x^5)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^6} \, dx$$

$$\Big\downarrow \; 242$$

$$-\frac{(a+bx^2)^{5/2}}{5ax^5}$$

input
```
Int[(a + b*x^2)^(3/2)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/2)/(a*x^5)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}$	18
default	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}$	18
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}$	18
orering	$-\frac{(b\,x^2+a)^{\frac{5}{2}}}{5a\,x^5}$	18
trager	$-\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5a\,x^5}$	36
risch	$-\frac{(b^2x^4+2ab\,x^2+a^2)\sqrt{b\,x^2+a}}{5a\,x^5}$	36

input
```
int((b*x^2+a)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*(b*x^2+a)^(5/2)/a/x^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.67

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = -\frac{(b^2x^4+2\,abx^2+a^2)\sqrt{bx^2+a}}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-1/5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*x^2 + a)/(a*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{5x^2} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{5a}$$

input
```
integrate((b*x**2+a)**(3/2)/x**6,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 2*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(5*x**2) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(5*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = -\frac{(bx^2+a)^{\frac{5}{2}}}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-1/5*(b*x^2 + a)^(5/2)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.10

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = \frac{2\left(5\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8 b^{\frac{5}{2}} + 10\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 a^2 b^{\frac{5}{2}} + a^4 b^{\frac{5}{2}}\right)}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a\right)^5}$$

input
```
integrate((b*x^2+a)^(3/2)/x^6,x, algorithm="giac")
```

output
```
2/5*(5*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2) + 10*(sqrt(b)*x - sqrt(b*x^
2 + a))^4*a^2*b^(5/2) + a^4*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
^5
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = -\frac{\left(b\,x^2+a\right)^{5/2}}{5\,a\,x^5}$$

input
```
int((a + b*x^2)^(3/2)/x^6,x)
```

output
```
-(a + b*x^2)^(5/2)/(5*a*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.00

$$\int \frac{(a+bx^2)^{3/2}}{x^6}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2-2\sqrt{b\,x^2+a}\,ab\,x^2-\sqrt{b\,x^2+a}\,b^2x^4-\sqrt{b}\,b^2x^5}{5a\,x^5}$$

input
```
int((b*x^2+a)^(3/2)/x^6,x)
```

output
```
( - sqrt(a + b*x**2)*a**2 - 2*sqrt(a + b*x**2)*a*b*x**2 - sqrt(a + b*x**2)
*b**2*x**4 - sqrt(b)*b**2*x**5)/(5*a*x**5)
```

3.394 $\int \frac{(a+bx^2)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^2)^{3/2}}{x^8} dx = -\frac{(a+bx^2)^{5/2}}{7ax^7} + \frac{2b(a+bx^2)^{5/2}}{35a^2x^5}$$

output
```
-1/7*(b*x^2+a)^(5/2)/a/x^7+2/35*b*(b*x^2+a)^(5/2)/a^2/x^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{3/2}}{x^8} dx = \frac{(a+bx^2)^{5/2}(-5a+2bx^2)}{35a^2x^7}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^8,x]
```

output
```
((a + b*x^2)^(5/2)*(-5*a + 2*b*x^2))/(35*a^2*x^7)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx$$

$$\Big\downarrow \ 245$$

$$-\frac{2b\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}}{7ax^7}$$

$$\Big\downarrow \ 242$$

$$\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}$$

input
```
Int[(a + b*x^2)^(3/2)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(5/2)/(a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2b\,x^{2}+5a\right)}{35a^{2}x^{7}}$	28
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2b\,x^{2}+5a\right)}{35a^{2}x^{7}}$	28
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-2b\,x^{2}+5a\right)}{35a^{2}x^{7}}$	28
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7a\,x^{7}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35a^{2}x^{5}}$	37
trager	$-\frac{\left(-2b^{3}x^{6}+a\,b^{2}x^{4}+8a^{2}b\,x^{2}+5a^{3}\right)\sqrt{b\,x^{2}+a}}{35a^{2}x^{7}}$	49
risch	$-\frac{\left(-2b^{3}x^{6}+a\,b^{2}x^{4}+8a^{2}b\,x^{2}+5a^{3}\right)\sqrt{b\,x^{2}+a}}{35a^{2}x^{7}}$	49

input
```
int((b*x^2+a)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/35*(b*x^2+a)^(5/2)*(-2*b*x^2+5*a)/a^2/x^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx=\frac{(2\,b^3x^6-ab^2x^4-8\,a^2bx^2-5\,a^3)\sqrt{bx^2+a}}{35\,a^2x^7}$$

input
```
integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
1/35*(2*b^3*x^6 - a*b^2*x^4 - 8*a^2*b*x^2 - 5*a^3)*sqrt(b*x^2 + a)/(a^2*x^
7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(37) = 74$.

Time = 0.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx = -\frac{a\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6} - \frac{8b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4} - \frac{b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35ax^2} + \frac{2b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^2}$$

input
```
integrate((b*x**2+a)**(3/2)/x**8,x)
```

output
```
-a*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 8*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(35*x**4) - b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*a*x**2) + 2*b**(7/2)*sqrt(a
/(b*x**2) + 1)/(35*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx = \frac{2\,(bx^2+a)^{\frac{5}{2}}b}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{5}{2}}}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
2/35*(b*x^2 + a)^(5/2)*b/(a^2*x^5) - 1/7*(b*x^2 + a)^(5/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 3.77

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx = \frac{4\left(35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}} + 35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}} + 70\left(\sqrt{b}x-\sqrt{bx^2+a}\right)\right.}{35\left(\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}$$

input
```
integrate((b*x^2+a)^(3/2)/x^8,x, algorithm="giac")
```

output
```
4/35*(35*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2) + 35*(sqrt(b)*x - sqrt(b
*x^2 + a))^8*a*b^(7/2) + 70*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(7/2) +
14*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(7/2) + 7*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^4*b^(7/2) - a^5*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^
7
```

Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx = \frac{2\,b^3\,\sqrt{b\,x^2+a}}{35\,a^2\,x} - \frac{8\,b\,\sqrt{b\,x^2+a}}{35\,x^5} - \frac{b^2\,\sqrt{b\,x^2+a}}{35\,a\,x^3} - \frac{a\,\sqrt{b\,x^2+a}}{7\,x^7}$$

input
```
int((a + b*x^2)^(3/2)/x^8,x)
```

output
```
(2*b^3*(a + b*x^2)^(1/2))/(35*a^2*x) - (8*b*(a + b*x^2)^(1/2))/(35*x^5) -
(b^2*(a + b*x^2)^(1/2))/(35*a*x^3) - (a*(a + b*x^2)^(1/2))/(7*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^2)^{3/2}}{x^8}\,dx = \frac{-5\sqrt{b\,x^2+a}\,a^3 - 8\sqrt{b\,x^2+a}\,a^2b\,x^2 - \sqrt{b\,x^2+a}\,a\,b^2x^4 + 2\sqrt{b\,x^2+a}\,b^3x^6 - 2\sqrt{b}\,b^3x^7}{35a^2x^7}$$

input
```
int((b*x^2+a)^(3/2)/x^8,x)
```

output
```
( - 5*sqrt(a + b*x**2)*a**3 - 8*sqrt(a + b*x**2)*a**2*b*x**2 - sqrt(a + b*
x**2)*a*b**2*x**4 + 2*sqrt(a + b*x**2)*b**3*x**6 - 2*sqrt(b)*b**3*x**7)/(3
5*a**2*x**7)
```

3.395 $\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx = -\frac{(a+bx^2)^{5/2}}{9ax^9} + \frac{4b(a+bx^2)^{5/2}}{63a^2x^7} - \frac{8b^2(a+bx^2)^{5/2}}{315a^3x^5}$$

output
```
-1/9*(b*x^2+a)^(5/2)/a/x^9+4/63*b*(b*x^2+a)^(5/2)/a^2/x^7-8/315*b^2*(b*x^2
+a)^(5/2)/a^3/x^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}} dx = \frac{(a+bx^2)^{5/2}(-35a^2+20abx^2-8b^2x^4)}{315a^3x^9}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^10,x]
```

output
```
((a + b*x^2)^(5/2)*(-35*a^2 + 20*a*b*x^2 - 8*b^2*x^4))/(315*a^3*x^9)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\int \frac{(bx^2+a)^{3/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^2+a)^{3/2}}{x^6}dx}{7a}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)}{9a}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

$$\Big\downarrow \text{242}$$

$$-\frac{4b\left(\frac{2b(a+bx^2)^{5/2}}{35a^2x^5}-\frac{(a+bx^2)^{5/2}}{7ax^7}\right)}{9a}-\frac{(a+bx^2)^{5/2}}{9ax^9}$$

input
```
Int[(a + b*x^2)^(3/2)/x^10,x]
```

output
```
-1/9*(a + b*x^2)^(5/2)/(a*x^9) - (4*b*(-1/7*(a + b*x^2)^(5/2)/(a*x^7) + (2
*b*(a + b*x^2)^(5/2))/(35*a^2*x^5)))/(9*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(8 b^{2} x^{4}-20 a b\,x^{2}+35 a^{2}\right)}{315 x^{9} a^{3}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(8 b^{2} x^{4}-20 a b\,x^{2}+35 a^{2}\right)}{315 x^{9} a^{3}}$	39
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}} \left(8 b^{2} x^{4}-20 a b\,x^{2}+35 a^{2}\right)}{315 x^{9} a^{3}}$	39
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9 a\,x^{9}}-\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 a\,x^{7}}+\frac{2 b \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 a^{2} x^{5}}\right)}{9 a}$	61
trager	$-\frac{\left(8 b^{4} x^{8}-4 a\,b^{3} x^{6}+3 a^{2} b^{2} x^{4}+50 a^{3} b\,x^{2}+35 a^{4}\right) \sqrt{b\,x^{2}+a}}{315 x^{9} a^{3}}$	61
risch	$-\frac{\left(8 b^{4} x^{8}-4 a\,b^{3} x^{6}+3 a^{2} b^{2} x^{4}+50 a^{3} b\,x^{2}+35 a^{4}\right) \sqrt{b\,x^{2}+a}}{315 x^{9} a^{3}}$	61

input
```
int((b*x^2+a)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(b*x^2+a)^(5/2)*(8*b^2*x^4-20*a*b*x^2+35*a^2)/x^9/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = -\frac{(8\,b^4x^8-4\,ab^3x^6+3\,a^2b^2x^4+50\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{315\,a^3x^9}$$

input
```
integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-1/315*(8*b^4*x^8 - 4*a*b^3*x^6 + 3*a^2*b^2*x^4 + 50*a^3*b*x^2 + 35*a^4)*s
qrt(b*x^2 + a)/(a^3*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. 2(61) = 122.

Time = 0.83 (sec) , antiderivative size = 420, normalized size of antiderivative = 6.18

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = -\frac{35a^6b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{120a^5b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{138a^4b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{52a^3b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{3a^2b^{\frac{17}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{12ab^{\frac{19}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}} - \frac{8b^{\frac{21}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{315a^5b^4x^8+630a^4b^5x^{10}+315a^3b^6x^{12}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**10,x)
```

output
```
-35*a**6*b**(9/2)*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5
*x**10 + 315*a**3*b**6*x**12) - 120*a**5*b**(11/2)*x**2*sqrt(a/(b*x**2) +
1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 138*
a**4*b**(13/2)*x**4*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b*
*5*x**10 + 315*a**3*b**6*x**12) - 52*a**3*b**(15/2)*x**6*sqrt(a/(b*x**2) +
 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 3*a
**2*b**(17/2)*x**8*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**
5*x**10 + 315*a**3*b**6*x**12) - 12*a*b**(19/2)*x**10*sqrt(a/(b*x**2) + 1)
/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10 + 315*a**3*b**6*x**12) - 8*b**(
21/2)*x**12*sqrt(a/(b*x**2) + 1)/(315*a**5*b**4*x**8 + 630*a**4*b**5*x**10
 + 315*a**3*b**6*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = -\frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{315\,a^3x^5}+\frac{4\,(bx^2+a)^{\frac{5}{2}}b}{63\,a^2x^7}-\frac{(bx^2+a)^{\frac{5}{2}}}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="maxima")
```

output
```
-8/315*(b*x^2 + a)^(5/2)*b^2/(a^3*x^5) + 4/63*(b*x^2 + a)^(5/2)*b/(a^2*x^7
) - 1/9*(b*x^2 + a)^(5/2)/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = \frac{16\left(210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{9}{2}}+441\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}{315}$$

input
```
integrate((b*x^2+a)^(3/2)/x^10,x, algorithm="giac")
```

output
```
16/315*(210*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(9/2) + 315*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a*b^(9/2) + 441*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(9
/2) + 126*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(9/2) + 36*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*a^4*b^(9/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(9/
2) + a^6*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = \frac{4\,b^3\,\sqrt{b\,x^2+a}}{315\,a^2\,x^3} - \frac{10\,b\,\sqrt{b\,x^2+a}}{63\,x^7} - \frac{b^2\,\sqrt{b\,x^2+a}}{105\,a\,x^5} - \frac{a\,\sqrt{b\,x^2+a}}{9\,x^9} - \frac{8\,b^4\,\sqrt{b\,x^2+a}}{315\,a^3\,x}$$

input
```
int((a + b*x^2)^(3/2)/x^10,x)
```

output
```
(4*b^3*(a + b*x^2)^(1/2))/(315*a^2*x^3) - (10*b*(a + b*x^2)^(1/2))/(63*x^7
) - (b^2*(a + b*x^2)^(1/2))/(105*a*x^5) - (a*(a + b*x^2)^(1/2))/(9*x^9) -
(8*b^4*(a + b*x^2)^(1/2))/(315*a^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.49

$$\int \frac{(a+bx^2)^{3/2}}{x^{10}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^4 - 50\sqrt{b\,x^2+a}\,a^3b\,x^2 - 3\sqrt{b\,x^2+a}\,a^2b^2x^4 + 4\sqrt{b\,x^2+a}\,a\,b^3x^6 - 8\sqrt{}}{315a^3x^9}$$

input
```
int((b*x^2+a)^(3/2)/x^10,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 50*sqrt(a + b*x**2)*a**3*b*x**2 - 3*sqrt(a
+ b*x**2)*a**2*b**2*x**4 + 4*sqrt(a + b*x**2)*a*b**3*x**6 - 8*sqrt(a + b*x
**2)*b**4*x**8 + 8*sqrt(b)*b**4*x**9)/(315*a**3*x**9)
```

3.396 $\int \frac{(a+bx^2)^{3/2}}{x^{12}} dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}} dx = -\frac{(a+bx^2)^{5/2}}{11ax^{11}} + \frac{2b(a+bx^2)^{5/2}}{33a^2x^9} - \frac{8b^2(a+bx^2)^{5/2}}{231a^3x^7} + \frac{16b^3(a+bx^2)^{5/2}}{1155a^4x^5}$$

output
```
-1/11*(b*x^2+a)^(5/2)/a/x^11+2/33*b*(b*x^2+a)^(5/2)/a^2/x^9-8/231*b^2*(b*x
^2+a)^(5/2)/a^3/x^7+16/1155*b^3*(b*x^2+a)^(5/2)/a^4/x^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}} dx = \frac{(a+bx^2)^{5/2} (-105a^3 + 70a^2bx^2 - 40ab^2x^4 + 16b^3x^6)}{1155a^4x^{11}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/x^12,x]
```

output
```
((a + b*x^2)^(5/2)*(-105*a^3 + 70*a^2*b*x^2 - 40*a*b^2*x^4 + 16*b^3*x^6))/
(1155*a^4*x^11)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{x^{12}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\int \frac{\left(bx^2+a\right)^{3/2}}{x^{10}}dx}{11a}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\int \frac{\left(bx^2+a\right)^{3/2}}{x^8}dx}{9a}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx}{7a}-\frac{\left(a+bx^2\right)^{5/2}}{7ax^7}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{5/2}}{35a^2x^5}-\frac{\left(a+bx^2\right)^{5/2}}{7ax^7}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{5/2}}{11ax^{11}}$$

input
```
Int[(a + b*x^2)^(3/2)/x^12,x]
```

output
```
-1/11*(a + b*x^2)^(5/2)/(a*x^11) - (6*b*(-1/9*(a + b*x^2)^(5/2)/(a*x^9) -
(4*b*(-1/7*(a + b*x^2)^(5/2)/(a*x^7) + (2*b*(a + b*x^2)^(5/2))/(35*a^2*x^5
)))/(9*a)))/(11*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-16 b^{3} x^{6}+40 a\,b^{2} x^{4}-70 a^{2} b\,x^{2}+105 a^{3}\right)}{1155 x^{11} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-16 b^{3} x^{6}+40 a\,b^{2} x^{4}-70 a^{2} b\,x^{2}+105 a^{3}\right)}{1155 x^{11} a^{4}}$	50
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}\left(-16 b^{3} x^{6}+40 a\,b^{2} x^{4}-70 a^{2} b\,x^{2}+105 a^{3}\right)}{1155 x^{11} a^{4}}$	50
trager	$-\frac{\left(-16 b^{5} x^{10}+8 a\,b^{4} x^{8}-6 a^{2} b^{3} x^{6}+5 a^{3} b^{2} x^{4}+140 a^{4} b\,x^{2}+105 a^{5}\right)\sqrt{b\,x^{2}+a}}{1155 x^{11} a^{4}}$	72
risch	$-\frac{\left(-16 b^{5} x^{10}+8 a\,b^{4} x^{8}-6 a^{2} b^{3} x^{6}+5 a^{3} b^{2} x^{4}+140 a^{4} b\,x^{2}+105 a^{5}\right)\sqrt{b\,x^{2}+a}}{1155 x^{11} a^{4}}$	72
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{11 a\,x^{11}}-\frac{6 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{9 a\,x^{9}}-\frac{4 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{7 a\,x^{7}}+\frac{2 b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{35 a^{2} x^{5}}\right)}{9 a}\right)}{11 a}$	85

input
```
int((b*x^2+a)^(3/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/1155*(b*x^2+a)^(5/2)*(-16*b^3*x^6+40*a*b^2*x^4-70*a^2*b*x^2+105*a^3)/x^
11/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = \frac{(16\,b^5x^{10}-8\,ab^4x^8+6\,a^2b^3x^6-5\,a^3b^2x^4-140\,a^4bx^2-105\,a^5)\sqrt{bx^2+a}}{1155\,a^4x^{11}}$$

input
```
integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="fricas")
```

output
```
1/1155*(16*b^5*x^10 - 8*a*b^4*x^8 + 6*a^2*b^3*x^6 - 5*a^3*b^2*x^4 - 140*a^
4*b*x^2 - 105*a^5)*sqrt(b*x^2 + a)/(a^4*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. 2(85) = 170.

Time = 1.08 (sec) , antiderivative size = 648, normalized size of antiderivative = 7.04

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = -\frac{105a^8b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} - \frac{455a^7b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} - \frac{740a^6b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} - \frac{534a^5b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} - \frac{145a^4b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} + \frac{5a^3b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} + \frac{30a^2b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} + \frac{40ab^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}} + \frac{16b^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{1155a^7b^9x^{10}+3465a^6b^{10}x^{12}+3465a^5b^{11}x^{14}+1155a^4b^{12}x^{16}}$$

input
```
integrate((b*x**2+a)**(3/2)/x**12,x)
```

output
```
-105*a**8*b**(19/2)*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6
*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 455*a**7*b
**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10
*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 740*a**6*b**(23/
2)*x**4*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12
 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 534*a**5*b**(25/2)*x**
6*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 346
5*a**5*b**11*x**14 + 1155*a**4*b**12*x**16) - 145*a**4*b**(27/2)*x**8*sqrt
(a/(b*x**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5
*b**11*x**14 + 1155*a**4*b**12*x**16) + 5*a**3*b**(29/2)*x**10*sqrt(a/(b*x
**2) + 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*
x**14 + 1155*a**4*b**12*x**16) + 30*a**2*b**(31/2)*x**12*sqrt(a/(b*x**2) +
 1)/(1155*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14
+ 1155*a**4*b**12*x**16) + 40*a*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(1155
*a**7*b**9*x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a*
*4*b**12*x**16) + 16*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(1155*a**7*b**9*
x**10 + 3465*a**6*b**10*x**12 + 3465*a**5*b**11*x**14 + 1155*a**4*b**12*x*
*16)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = \frac{16\,(bx^2+a)^{\frac{5}{2}}b^3}{1155\,a^4x^5} - \frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{231\,a^3x^7} + \frac{2\,(bx^2+a)^{\frac{5}{2}}b}{33\,a^2x^9} - \frac{(bx^2+a)^{\frac{5}{2}}}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="maxima")
```

output
```
16/1155*(b*x^2 + a)^(5/2)*b^3/(a^4*x^5) - 8/231*(b*x^2 + a)^(5/2)*b^2/(a^3
*x^7) + 2/33*(b*x^2 + a)^(5/2)*b/(a^2*x^9) - 1/11*(b*x^2 + a)^(5/2)/(a*x^1
1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(76) = 152$.

Time = 0.13 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.39

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = \frac{32\left(1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{11}{2}}+2079\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{11}{2}}+2541\left(\sqrt{b}x-\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(3/2)/x^12,x, algorithm="giac")
```

output
```
32/1155*(1155*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(11/2) + 2079*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
^2*b^(11/2) + 825*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(11/2) + 165*(sqrt
(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(11/2) - 55*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*a^5*b^(11/2) + 11*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(11/2) - a^7*b^
(11/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^11
```

Mupad [B] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = \frac{2\,b^3\,\sqrt{b\,x^2+a}}{385\,a^2\,x^5}-\frac{4\,b\,\sqrt{b\,x^2+a}}{33\,x^9} -\frac{b^2\,\sqrt{b\,x^2+a}}{231\,a\,x^7}-\frac{a\,\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{8\,b^4\,\sqrt{b\,x^2+a}}{1155\,a^3\,x^3}+\frac{16\,b^5\,\sqrt{b\,x^2+a}}{1155\,a^4\,x}$$

input
```
int((a + b*x^2)^(3/2)/x^12,x)
```

output
```
(2*b^3*(a + b*x^2)^(1/2))/(385*a^2*x^5) - (4*b*(a + b*x^2)^(1/2))/(33*x^9)
 - (b^2*(a + b*x^2)^(1/2))/(231*a*x^7) - (a*(a + b*x^2)^(1/2))/(11*x^11) -
 (8*b^4*(a + b*x^2)^(1/2))/(1155*a^3*x^3) + (16*b^5*(a + b*x^2)^(1/2))/(11
55*a^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{3/2}}{x^{12}}\,dx = \frac{-105\sqrt{b\,x^2+a}\,a^5-140\sqrt{b\,x^2+a}\,a^4b\,x^2-5\sqrt{b\,x^2+a}\,a^3b^2x^4+6\sqrt{b\,x^2+a}\,a^2b^3x^6-}{1155a^4x^{11}}$$

input
```
int((b*x^2+a)^(3/2)/x^12,x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**5 - 140*sqrt(a + b*x**2)*a**4*b*x**2 - 5*sqrt(
a + b*x**2)*a**3*b**2*x**4 + 6*sqrt(a + b*x**2)*a**2*b**3*x**6 - 8*sqrt(a
+ b*x**2)*a*b**4*x**8 + 16*sqrt(a + b*x**2)*b**5*x**10 - 16*sqrt(b)*b**5*x
**11)/(1155*a**4*x**11)
```

3.397 $\int x^7(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7(a+bx^2)^{5/2}\,dx=-\frac{a^3(a+bx^2)^{7/2}}{7b^4}+\frac{a^2(a+bx^2)^{9/2}}{3b^4}-\frac{3a(a+bx^2)^{11/2}}{11b^4}+\frac{(a+bx^2)^{13/2}}{13b^4}$$

output
```
-1/7*a^3*(b*x^2+a)^(7/2)/b^4+1/3*a^2*(b*x^2+a)^(9/2)/b^4-3/11*a*(b*x^2+a)^
(11/2)/b^4+1/13*(b*x^2+a)^(13/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7(a+bx^2)^{5/2}\,dx=\frac{(a+bx^2)^{7/2}(-16a^3+56a^2bx^2-126ab^2x^4+231b^3x^6)}{3003b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(5/2),x]
```

output
```
((a + b*x^2)^(7/2)*(-16*a^3 + 56*a^2*b*x^2 - 126*a*b^2*x^4 + 231*b^3*x^6))
/(3003*b^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^6 \left(bx^2+a\right)^{5/2}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{11/2}}{b^3}-\frac{3a\left(bx^2+a\right)^{9/2}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{7/2}}{b^3}-\frac{a^3\left(bx^2+a\right)^{5/2}}{b^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{2a^3\left(a+bx^2\right)^{7/2}}{7b^4}+\frac{2a^2\left(a+bx^2\right)^{9/2}}{3b^4}+\frac{2\left(a+bx^2\right)^{13/2}}{13b^4}-\frac{6a\left(a+bx^2\right)^{11/2}}{11b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(5/2),x]
```

output
```
((-2*a^3*(a + b*x^2)^(7/2))/(7*b^4) + (2*a^2*(a + b*x^2)^(9/2))/(3*b^4) -
(6*a*(a + b*x^2)^(11/2))/(11*b^4) + (2*(a + b*x^2)^(13/2))/(13*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-231 b^{3} x^{6}+126 a\,b^{2} x^{4}-56 a^{2} b\,x^{2}+16 a^{3}\right)}{3003 b^{4}}$	47
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-231 b^{3} x^{6}+126 a\,b^{2} x^{4}-56 a^{2} b\,x^{2}+16 a^{3}\right)}{3003 b^{4}}$	47
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-231 b^{3} x^{6}+126 a\,b^{2} x^{4}-56 a^{2} b\,x^{2}+16 a^{3}\right)}{3003 b^{4}}$	47
trager	$-\frac{\left(-231 b^{6} x^{12}-567 a\,b^{5} x^{10}-371 a^{2} b^{4} x^{8}-5 a^{3} x^{6} b^{3}+6 a^{4} b^{2} x^{4}-8 a^{5} b\,x^{2}+16 a^{6}\right) \sqrt{b\,x^{2}+a}}{3003 b^{4}}$	80
risch	$-\frac{\left(-231 b^{6} x^{12}-567 a\,b^{5} x^{10}-371 a^{2} b^{4} x^{8}-5 a^{3} x^{6} b^{3}+6 a^{4} b^{2} x^{4}-8 a^{5} b\,x^{2}+16 a^{6}\right) \sqrt{b\,x^{2}+a}}{3003 b^{4}}$	80
default	$\frac{x^{6} \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{13 b}-\frac{6 a \left(\frac{x^{4} \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11 b}-\frac{4 a \left(\frac{x^{2} \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9 b}-\frac{2 a \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63 b^{2}}\right)}{11 b}\right)}{13 b}$	82

input
```
int(x^7*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3003*(b*x^2+a)^(7/2)*(-231*b^3*x^6+126*a*b^2*x^4-56*a^2*b*x^2+16*a^3)/b
^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int x^7 (a+bx^2)^{5/2}\,dx = \frac{(231\,b^6x^{12}+567\,ab^5x^{10}+371\,a^2b^4x^8+5\,a^3b^3x^6-6\,a^4b^2x^4+8\,a^5bx^2-16\,a^6)\sqrt{bx^2+a}}{3003\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3003*(231*b^6*x^12 + 567*a*b^5*x^10 + 371*a^2*b^4*x^8 + 5*a^3*b^3*x^6 -
6*a^4*b^2*x^4 + 8*a^5*b*x^2 - 16*a^6)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(70) = 140$.

Time = 0.59 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.98

$$\int x^7 (a+bx^2)^{5/2}\,dx = \begin{cases} -\frac{16a^6\sqrt{a+bx^2}}{3003b^4}+\frac{8a^5x^2\sqrt{a+bx^2}}{3003b^3}-\frac{2a^4x^4\sqrt{a+bx^2}}{1001b^2}+\frac{5a^3x^6\sqrt{a+bx^2}}{3003b}+\frac{53a^2x^8\sqrt{a+bx^2}}{429}+\frac{27abx^{10}\sqrt{a+bx^2}}{143}+\frac{b^2 \cdots \\ \frac{a^{\frac{5}{2}}x^8}{8} \end{cases}$$

input
```
integrate(x**7*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-16*a**6*sqrt(a + b*x**2)/(3003*b**4) + 8*a**5*x**2*sqrt(a + b*
x**2)/(3003*b**3) - 2*a**4*x**4*sqrt(a + b*x**2)/(1001*b**2) + 5*a**3*x**6
*sqrt(a + b*x**2)/(3003*b) + 53*a**2*x**8*sqrt(a + b*x**2)/429 + 27*a*b*x*
*10*sqrt(a + b*x**2)/143 + b**2*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**
(5/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^7(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}x^6}{13\,b} - \frac{6\,(bx^2+a)^{\frac{7}{2}}ax^4}{143\,b^2} + \frac{8\,(bx^2+a)^{\frac{7}{2}}a^2x^2}{429\,b^3} - \frac{16\,(bx^2+a)^{\frac{7}{2}}a^3}{3003\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/13*(b*x^2 + a)^(7/2)*x^6/b - 6/143*(b*x^2 + a)^(7/2)*a*x^4/b^2 + 8/429*(
b*x^2 + a)^(7/2)*a^2*x^2/b^3 - 16/3003*(b*x^2 + a)^(7/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7(a+bx^2)^{5/2}\,dx = \frac{231\,(bx^2+a)^{\frac{13}{2}} - 819\,(bx^2+a)^{\frac{11}{2}}a + 1001\,(bx^2+a)^{\frac{9}{2}}a^2 - 429\,(bx^2+a)^{\frac{7}{2}}a^3}{3003\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3003*(231*(b*x^2 + a)^(13/2) - 819*(b*x^2 + a)^(11/2)*a + 1001*(b*x^2 +
a)^(9/2)*a^2 - 429*(b*x^2 + a)^(7/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94

$$\int x^7 (a+bx^2)^{5/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{53\,a^2\,x^8}{429}-\frac{16\,a^6}{3003\,b^4}+\frac{b^2\,x^{12}}{13}+\frac{5\,a^3\,x^6}{3003\,b}-\frac{2\,a^4\,x^4}{1001\,b^2}+\frac{8\,a^5\,x^2}{3003\,b^3}+\frac{27\,a\,b\,x^{10}}{143}\right)$$

input
```
int(x^7*(a + b*x^2)^(5/2),x)
```

output
```
(a + b*x^2)^(1/2)*((53*a^2*x^8)/429 - (16*a^6)/(3003*b^4) + (b^2*x^12)/13
+ (5*a^3*x^6)/(3003*b) - (2*a^4*x^4)/(1001*b^2) + (8*a^5*x^2)/(3003*b^3) +
 (27*a*b*x^10)/143)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

$$\int x^7 (a+bx^2)^{5/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(231b^6x^{12}+567a\,b^5x^{10}+371a^2b^4x^8+5a^3b^3x^6-6a^4b^2x^4+8a^5b\,x^2-16a^6)}{3003b^4}$$

input
```
int(x^7*(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**6 + 8*a**5*b*x**2 - 6*a**4*b**2*x**4 + 5*a**3*
b**3*x**6 + 371*a**2*b**4*x**8 + 567*a*b**5*x**10 + 231*b**6*x**12))/(3003
*b**4)
```

3.398 $\int x^5(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5\left(a+bx^2\right)^{5/2}\,dx=\frac{a^2\left(a+bx^2\right)^{7/2}}{7b^3}-\frac{2a\left(a+bx^2\right)^{9/2}}{9b^3}+\frac{\left(a+bx^2\right)^{11/2}}{11b^3}$$

output
```
1/7*a^2*(b*x^2+a)^(7/2)/b^3-2/9*a*(b*x^2+a)^(9/2)/b^3+1/11*(b*x^2+a)^(11/2
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}\left(8a^2-28abx^2+63b^2x^4\right)}{693b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(5/2),x]
```

output
```
((a + b*x^2)^(7/2)*(8*a^2 - 28*a*b*x^2 + 63*b^2*x^4))/(693*b^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^4 \left(bx^2+a\right)^{5/2} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{9/2}}{b^2}-\frac{2a\left(bx^2+a\right)^{7/2}}{b^2}+\frac{a^2\left(bx^2+a\right)^{5/2}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{2a^2\left(a+bx^2\right)^{7/2}}{7b^3}+\frac{2\left(a+bx^2\right)^{11/2}}{11b^3}-\frac{4a\left(a+bx^2\right)^{9/2}}{9b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(5/2),x]
```

output
```
((2*a^2*(a + b*x^2)^(7/2))/(7*b^3) - (4*a*(a + b*x^2)^(9/2))/(9*b^3) + (2*
(a + b*x^2)^(11/2))/(11*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{7}{2}}\left(63b^2x^4-28ab\,x^2+8a^2\right)}{693b^3}$	36
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{7}{2}}\left(63b^2x^4-28ab\,x^2+8a^2\right)}{693b^3}$	36
orering	$\frac{(b\,x^2+a)^{\frac{7}{2}}\left(63b^2x^4-28ab\,x^2+8a^2\right)}{693b^3}$	36
default	$\frac{x^4(b\,x^2+a)^{\frac{7}{2}}}{11b}-\frac{4a\left(\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^2+a)^{\frac{7}{2}}}{63b^2}\right)}{11b}$	58
trager	$\frac{\left(63b^5x^{10}+161a\,b^4x^8+113a^2b^3x^6+3a^3b^2x^4-4a^4b\,x^2+8a^5\right)\sqrt{b\,x^2+a}}{693b^3}$	69
risch	$\frac{\left(63b^5x^{10}+161a\,b^4x^8+113a^2b^3x^6+3a^3b^2x^4-4a^4b\,x^2+8a^5\right)\sqrt{b\,x^2+a}}{693b^3}$	69

input
```
int(x^5*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/693*(b*x^2+a)^(7/2)*(63*b^2*x^4-28*a*b*x^2+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int x^5(a+bx^2)^{5/2}\,dx=\frac{(63\,b^5x^{10}+161\,ab^4x^8+113\,a^2b^3x^6+3\,a^3b^2x^4-4\,a^4bx^2+8\,a^5)\sqrt{bx^2+a}}{693\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/693*(63*b^5*x^10 + 161*a*b^4*x^8 + 113*a^2*b^3*x^6 + 3*a^3*b^2*x^4 - 4*a
^4*b*x^2 + 8*a^5)*sqrt(b*x^2 + a)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(51) = 102$.

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.25

$$\int x^5(a+bx^2)^{5/2}\,dx = \begin{cases} \frac{8a^5\sqrt{a+bx^2}}{693b^3} - \frac{4a^4x^2\sqrt{a+bx^2}}{693b^2} + \frac{a^3x^4\sqrt{a+bx^2}}{231b} + \frac{113a^2x^6\sqrt{a+bx^2}}{693} + \frac{23abx^8\sqrt{a+bx^2}}{99} + \frac{b^2x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \neq \\ \frac{a^{\frac{5}{2}}x^6}{6} & \text{otherw} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((8*a**5*sqrt(a + b*x**2)/(693*b**3) - 4*a**4*x**2*sqrt(a + b*x**
2)/(693*b**2) + a**3*x**4*sqrt(a + b*x**2)/(231*b) + 113*a**2*x**6*sqrt(a
+ b*x**2)/693 + 23*a*b*x**8*sqrt(a + b*x**2)/99 + b**2*x**10*sqrt(a + b*x*
*2)/11, Ne(b, 0)), (a**(5/2)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}x^4}{11\,b} - \frac{4\,(bx^2+a)^{\frac{7}{2}}ax^2}{99\,b^2} + \frac{8\,(bx^2+a)^{\frac{7}{2}}a^2}{693\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/11*(b*x^2 + a)^(7/2)*x^4/b - 4/99*(b*x^2 + a)^(7/2)*a*x^2/b^2 + 8/693*(b
*x^2 + a)^(7/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 (a+bx^2)^{5/2}\,dx = \frac{63\,(bx^2+a)^{\frac{11}{2}} - 154\,(bx^2+a)^{\frac{9}{2}}a + 99\,(bx^2+a)^{\frac{7}{2}}a^2}{693\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/693*(63*(b*x^2 + a)^(11/2) - 154*(b*x^2 + a)^(9/2)*a + 99*(b*x^2 + a)^(7
/2)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int x^5 (a+bx^2)^{5/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,a^5}{693\,b^3}+\frac{113\,a^2\,x^6}{693}+\frac{b^2\,x^{10}}{11}+\frac{a^3\,x^4}{231\,b}-\frac{4\,a^4\,x^2}{693\,b^2}+\frac{23\,a\,b\,x^8}{99}\right)$$

input
```
int(x^5*(a + b*x^2)^(5/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*a^5)/(693*b^3) + (113*a^2*x^6)/693 + (b^2*x^10)/11 +
 (a^3*x^4)/(231*b) - (4*a^4*x^2)/(693*b^2) + (23*a*b*x^8)/99)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int x^5 (a+bx^2)^{5/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(63b^5x^{10}+161a\,b^4x^8+113a^2b^3x^6+3a^3b^2x^4-4a^4b\,x^2+8a^5)}{693b^3}$$

input
```
int(x^5*(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**5 - 4*a**4*b*x**2 + 3*a**3*b**2*x**4 + 113*a**2*b*
*3*x**6 + 161*a*b**4*x**8 + 63*b**5*x**10))/(693*b**3)
```

3.399 $\int x^3(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3(a+bx^2)^{5/2}\,dx=-\frac{a(a+bx^2)^{7/2}}{7b^2}+\frac{(a+bx^2)^{9/2}}{9b^2}$$

output
```
-1/7*a*(b*x^2+a)^(7/2)/b^2+1/9*(b*x^2+a)^(9/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3(a+bx^2)^{5/2}\,dx=\frac{(a+bx^2)^{7/2}\,(-2a+7bx^2)}{63b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(5/2),x]
```

output
```
((a + b*x^2)^(7/2)*(-2*a + 7*b*x^2))/(63*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2 \left(bx^2+a\right)^{5/2}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{7/2}}{b}-\frac{a\left(bx^2+a\right)^{5/2}}{b}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{9/2}}{9b^2}-\frac{2a\left(a+bx^2\right)^{7/2}}{7b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(5/2),x]
```

output
```
((-2*a*(a + b*x^2)^(7/2))/(7*b^2) + (2*(a + b*x^2)^(9/2))/(9*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-7b\,x^2+2a)}{63b^2}$	25
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-7b\,x^2+2a)}{63b^2}$	25
orering	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-7b\,x^2+2a)}{63b^2}$	25
default	$\frac{x^2(b\,x^2+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^2+a)^{\frac{7}{2}}}{63b^2}$	34
trager	$-\frac{(-7b^4x^8-19a\,b^3x^6-15a^2b^2x^4-a^3b\,x^2+2a^4)\sqrt{b\,x^2+a}}{63b^2}$	58
risch	$-\frac{(-7b^4x^8-19a\,b^3x^6-15a^2b^2x^4-a^3b\,x^2+2a^4)\sqrt{b\,x^2+a}}{63b^2}$	58

input
```
int(x^3*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/63*(b*x^2+a)^(7/2)*(-7*b*x^2+2*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int x^3(a+bx^2)^{5/2}\,dx=\frac{(7\,b^4x^8+19\,ab^3x^6+15\,a^2b^2x^4+a^3bx^2-2\,a^4)\sqrt{bx^2+a}}{63\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/63*(7*b^4*x^8 + 19*a*b^3*x^6 + 15*a^2*b^2*x^4 + a^3*b*x^2 - 2*a^4)*sqrt(
b*x^2 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.87

$$\int x^3(a+bx^2)^{5/2}\,dx = \begin{cases} -\frac{2a^4\sqrt{a+bx^2}}{63b^2} + \frac{a^3x^2\sqrt{a+bx^2}}{63b} + \frac{5a^2x^4\sqrt{a+bx^2}}{21} + \frac{19abx^6\sqrt{a+bx^2}}{63} + \frac{b^2x^8\sqrt{a+bx^2}}{9} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-2*a**4*sqrt(a + b*x**2)/(63*b**2) + a**3*x**2*sqrt(a + b*x**2)
/(63*b) + 5*a**2*x**4*sqrt(a + b*x**2)/21 + 19*a*b*x**6*sqrt(a + b*x**2)/6
3 + b**2*x**8*sqrt(a + b*x**2)/9, Ne(b, 0)), (a**(5/2)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}x^2}{9\,b} - \frac{2\,(bx^2+a)^{\frac{7}{2}}a}{63\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*x^2/b - 2/63*(b*x^2 + a)^(7/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a+bx^2)^{5/2} \, dx = \frac{7\,(bx^2+a)^{\frac{9}{2}} - 9\,(bx^2+a)^{\frac{7}{2}} a}{63\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/63*(7*(b*x^2 + a)^(9/2) - 9*(b*x^2 + a)^(7/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.39

$$\int x^3 (a+bx^2)^{5/2} \, dx = \sqrt{b\,x^2+a}\,\left(\frac{5\,a^2\,x^4}{21} - \frac{2\,a^4}{63\,b^2} + \frac{b^2\,x^8}{9} + \frac{a^3\,x^2}{63\,b} + \frac{19\,a\,b\,x^6}{63}\right)$$

input
```
int(x^3*(a + b*x^2)^(5/2),x)
```

output
```
(a + b*x^2)^(1/2)*((5*a^2*x^4)/21 - (2*a^4)/(63*b^2) + (b^2*x^8)/9 + (a^3*
x^2)/(63*b) + (19*a*b*x^6)/63)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int x^3 (a+bx^2)^{5/2} \, dx = \frac{\sqrt{b\,x^2+a}\,(7b^4x^8 + 19a\,b^3x^6 + 15a^2b^2x^4 + a^3b\,x^2 - 2a^4)}{63b^2}$$

input
```
int(x^3*(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a**4 + a**3*b*x**2 + 15*a**2*b**2*x**4 + 19*a*b**3
*x**6 + 7*b**4*x**8))/(63*b**2)
```

3.400 $\int x(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

output
```
1/7*(b*x^2+a)^(7/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(a+bx^2\right)^{7/2}}{7b}$$

input
```
Integrate[x*(a + b*x^2)^(5/2),x]
```

output
```
(a + b*x^2)^(7/2)/(7*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^{5/2}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{(a+bx^2)^{7/2}}{7b}$$

input
```
Int[x*(a + b*x^2)^(5/2),x]
```

output
```
(a + b*x^2)^(7/2)/(7*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
default	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
orering	$\frac{(b\,x^2+a)^{\frac{7}{2}}}{7b}$	15
trager	$\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7b}$	44
risch	$\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7b}$	44

input
```
int(x*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*(b*x^2+a)^(7/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int x(a+bx^2)^{5/2}\,dx = \frac{(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{bx^2+a}}{7\,b}$$

input
```
integrate(x*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(12) = 24$.

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 4.72

$$\int x(a+bx^2)^{5/2}\,dx = \begin{cases} \frac{a^3\sqrt{a+bx^2}}{7b} + \frac{3a^2x^2\sqrt{a+bx^2}}{7} + \frac{3abx^4\sqrt{a+bx^2}}{7} + \frac{b^2x^6\sqrt{a+bx^2}}{7} & \text{for } b \neq 0 \\ \frac{a^{\frac{5}{2}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((a**3*sqrt(a + b*x**2)/(7*b) + 3*a**2*x**2*sqrt(a + b*x**2)/7 +
3*a*b*x**4*sqrt(a + b*x**2)/7 + b**2*x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (
a**(5/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate(x*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(7/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}}{7\,b}$$

input
```
integrate(x*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/7*(b*x^2 + a)^(7/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{5/2}\,dx=\frac{(b\,x^2+a)^{7/2}}{7\,b}$$

input
```
int(x*(a + b*x^2)^(5/2),x)
```

output
```
(a + b*x^2)^(7/2)/(7*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x(a+bx^2)^{5/2}\,dx=\frac{\sqrt{b\,x^2+a}\,(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)}{7b}$$

input
```
int(x*(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6))/(7*b
)
```

3.401 $\int \frac{(a+bx^2)^{5/2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{(a+bx^2)^{5/2}}{x} dx = a^2\sqrt{a+bx^2} + \frac{1}{3}a(a+bx^2)^{3/2} + \frac{1}{5}(a+bx^2)^{5/2} - a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
a^2*(b*x^2+a)^(1/2)+1/3*a*(b*x^2+a)^(3/2)+1/5*(b*x^2+a)^(5/2)-a^(5/2)*arct
anh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{5/2}}{x} dx = \frac{1}{15}\sqrt{a+bx^2}(23a^2+11abx^2+3b^2x^4) - a^{5/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x,x]
```

output
```
(Sqrt[a + b*x^2]*(23*a^2 + 11*a*b*x^2 + 3*b^2*x^4))/15 - a^(5/2)*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx^2+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x,x]
```

output
```
((2*(a + b*x^2)^(5/2))/5 + a*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^
2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

method	result	size
pseudoelliptic	$-a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}+11 a b\,x^{2}+23 a^{2}\right)}{15}$	53
default	$\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)$	67

input
```
int((b*x^2+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-a^(5/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+1/15*(b*x^2+a)^(1/2)*(3*b^2*x^4+
11*a*b*x^2+23*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = \left[\frac{1}{2}\,a^{\frac{5}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + \frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a},\ \sqrt{-a}a^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) + \frac{1}{15}\left(3\,b^2x^4+11\,abx^2+23\,a^2\right)\sqrt{bx^2+a}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/2*a^(5/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/15*(3
*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2 + a), sqrt(-a)*a^2*arctan(sqrt(
b*x^2 + a)*sqrt(-a)/a) + 1/15*(3*b^2*x^4 + 11*a*b*x^2 + 23*a^2)*sqrt(b*x^2
 + a)]
```

Sympy [A] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = \frac{23a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{15} + \frac{a^{\frac{5}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{5}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right) + \frac{11a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{15} + \frac{\sqrt{a}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{5}$$

input
```
integrate((b*x**2+a)**(5/2)/x,x)
```

output
```
23*a**(5/2)*sqrt(1 + b*x**2/a)/15 + a**(5/2)*log(b*x**2/a)/2 - a**(5/2)*lo
g(sqrt(1 + b*x**2/a) + 1) + 11*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)/15 + sqr
t(a)*b**2*x**4*sqrt(1 + b*x**2/a)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = -a^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a + \sqrt{bx^2+a}\,a^2$$

input
```
integrate((b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

output
```
-a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2) + 1/3*(b*x^
2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = \frac{a^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}} + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a + \sqrt{bx^2+a}a^2$$

input
```
integrate((b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

output
```
a^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/5*(b*x^2 + a)^(5/2) + 1/
3*(b*x^2 + a)^(3/2)*a + sqrt(b*x^2 + a)*a^2
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = \frac{a\,(b\,x^2+a)^{3/2}}{3} + \frac{(b\,x^2+a)^{5/2}}{5} + a^2\,\sqrt{b\,x^2+a} + a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int((a + b*x^2)^(5/2)/x,x)
```

output
```
a^(5/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(3/2))/3
+ (a + b*x^2)^(5/2)/5 + a^2*(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^{5/2}}{x}\,dx = \frac{23\sqrt{b\,x^2+a}\,a^2}{15} + \frac{11\sqrt{b\,x^2+a}\,ab\,x^2}{15} + \frac{\sqrt{b\,x^2+a}\,b^2x^4}{5} + \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2 - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2$$

input
```
int((b*x^2+a)^(5/2)/x,x)
```

output
```
(23*sqrt(a + b*x**2)*a**2 + 11*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a + b*x*
*2)*b**2*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sq
rt(a))*a**2 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt
(a))*a**2)/15
```

3.402 $\int \frac{(a+bx^2)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{(a+bx^2)^{5/2}}{x^3} dx = 2ab\sqrt{a+bx^2} - \frac{a^2\sqrt{a+bx^2}}{2x^2} + \frac{1}{3}b(a+bx^2)^{3/2} - \frac{5}{2}a^{3/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
2*a*b*(b*x^2+a)^(1/2)-1/2*a^2*(b*x^2+a)^(1/2)/x^2+1/3*b*(b*x^2+a)^(3/2)-5/
2*a^(3/2)*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{5/2}}{x^3} dx = \frac{\sqrt{a+bx^2}(-3a^2+14abx^2+2b^2x^4)}{6x^2} - \frac{5}{2}a^{3/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2 + 14*a*b*x^2 + 2*b^2*x^4))/(6*x^2) - (5*a^(3/2)*b
*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{2}b\int \frac{\left(bx^2+a\right)^{3/2}}{x^2}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{5}{2}b\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x^3,x]
```

output
```
(-((a + b*x^2)^(5/2)/x^2) + (5*b*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a +
b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$-\frac{5\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)a^2 b\,x^2+\frac{\sqrt{a}\,\sqrt{b\,x^2+a}\,\left(-2b^2x^4-14ab\,x^2+3a^2\right)}{3}}{2\sqrt{a}\,x^2}$	68
risch	$-\frac{a^2\sqrt{b\,x^2+a}}{2x^2}-\frac{5b\,a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2}+\frac{b^2x^2\sqrt{b\,x^2+a}}{3}+\frac{7ab\sqrt{b\,x^2+a}}{3}$	78
default	$-\frac{\left(b\,x^2+a\right)^{\frac{7}{2}}}{2a\,x^2}+\frac{5b\left(\frac{\left(b\,x^2+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)\right)}{2a}$	91

input
```
int((b*x^2+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(1/2)*(5*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^2*b*x^2+1/3*a^(1/2)*(b*
x^2+a)^(1/2)*(-2*b^2*x^4-14*a*b*x^2+3*a^2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = \left[\frac{15\,a^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\left(2\,b^2x^4+14\,abx^2-3\,a^2\right)\sqrt{bx^2+a}}{12\,x^2}, \frac{15\sqrt{-ac}}{}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/12*(15*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(2*b^2*x^4 + 14*a*b*x^2 - 3*a^2)*sqrt(b*x^2 + a))/x^2, 1/6*(15*sqrt(
-a)*a*b*x^2*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (2*b^2*x^4 + 14*a*b*x^2 -
 3*a^2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = -\frac{a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2} + \frac{7a^{\frac{3}{2}}b\sqrt{1+\frac{bx^2}{a}}}{3} + \frac{5a^{\frac{3}{2}}b\log\left(\frac{bx^2}{a}\right)}{4} - \frac{5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2} + \frac{\sqrt{a}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{3}$$

input
```
integrate((b*x**2+a)**(5/2)/x**3,x)
```

output
```
-a**(5/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 7*a**(3/2)*b*sqrt(1 + b*x**2/a)/3
+ 5*a**(3/2)*b*log(b*x**2/a)/4 - 5*a**(3/2)*b*log(sqrt(1 + b*x**2/a) + 1)/
2 + sqrt(a)*b**2*x**2*sqrt(1 + b*x**2/a)/3
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = -\frac{5}{2}\,a^{\frac{3}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{5}{6}\,(bx^2+a)^{\frac{3}{2}}b + \frac{(bx^2+a)^{\frac{5}{2}}b}{2\,a} + \frac{5}{2}\,\sqrt{bx^2+a}ab - \frac{(bx^2+a)^{\frac{7}{2}}}{2\,ax^2}$$

input
```
integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
-5/2*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 5/6*(b*x^2 + a)^(3/2)*b + 1
/2*(b*x^2 + a)^(5/2)*b/a + 5/2*sqrt(b*x^2 + a)*a*b - 1/2*(b*x^2 + a)^(7/2)
/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = \frac{1}{6}\left(\frac{15\,a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,(bx^2+a)^{\frac{3}{2}} + 12\,\sqrt{bx^2+a}a - \frac{3\,\sqrt{bx^2+a}a^2}{bx^2}\right)b$$

input
```
integrate((b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/6*(15*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 2*(b*x^2 + a)^(3/2
) + 12*sqrt(b*x^2 + a)*a - 3*sqrt(b*x^2 + a)*a^2/(b*x^2))*b
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = \frac{b\,(b\,x^2+a)^{3/2}}{3} - \frac{a^2\,\sqrt{b\,x^2+a}}{2\,x^2} + 2\,a\,b\,\sqrt{b\,x^2+a} + \frac{a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

input
```
int((a + b*x^2)^(5/2)/x^3,x)
```

output
```
(b*(a + b*x^2)^(3/2))/3 - (a^2*(a + b*x^2)^(1/2))/(2*x^2) + (a^(3/2)*b*ata
n(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 + 2*a*b*(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^{5/2}}{x^3}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^2+14\sqrt{b\,x^2+a}\,ab\,x^2+2\sqrt{b\,x^2+a}\,b^2x^4+15\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{6x^2}$$

input
```
int((b*x^2+a)^(5/2)/x^3,x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2 + 14*sqrt(a + b*x**2)*a*b*x**2 + 2*sqrt(a + b*
x**2)*b**2*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/
sqrt(a))*a*b*x**2 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x
)/sqrt(a))*a*b*x**2)/(6*x**2)
```

3.403 $\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = b^2\sqrt{a+bx^2} - \frac{a^2\sqrt{a+bx^2}}{4x^4} - \frac{9ab\sqrt{a+bx^2}}{8x^2} - \frac{15}{8}\sqrt{a}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
b^2*(b*x^2+a)^(1/2)-1/4*a^2*(b*x^2+a)^(1/2)/x^4-9/8*a*b*(b*x^2+a)^(1/2)/x^
2-15/8*a^(1/2)*b^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = \frac{\sqrt{a+bx^2}(-2a^2-9abx^2+8b^2x^4)}{8x^4} - \frac{15}{8}\sqrt{a}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a^2 - 9*a*b*x^2 + 8*b^2*x^4))/(8*x^4) - (15*Sqrt[a]*b
^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/8
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{5/2}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{4}b\int \frac{\left(bx^2+a\right)^{3/2}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{2x^4}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^4}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{\left(a+bx^2\right)^{3/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{2x^4}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(5/2)/x^4 + (5*b*(-((a + b*x^2)^(3/2)/x^2) + (3*b*(2*Sqr
t[a + b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$-\frac{\frac{15\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) a\,b^{2} x^{4}}{2}+\left(-4 b^{2} x^{4}+\frac{9}{2} a b\,x^{2}+a^{2}\right) \sqrt{a}\,\sqrt{b\,x^{2}+a}}{4 \sqrt{a}\,x^{4}}$	65
risch	$-\frac{a\sqrt{b\,x^{2}+a}\,\left(9 b\,x^{2}+2 a\right)}{8 x^{4}}-\frac{15 \sqrt{a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right) b^{2}}{8}+b^{2} \sqrt{b\,x^{2}+a}$	71
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4 a\,x^{4}}+\frac{3 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2 a\,x^{2}}+\frac{5 b \left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a \left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a \left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)}{2 a}\right)}{4 a}$	115

input
```
int((b*x^2+a)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^(1/2)*(15/2*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a*b^2*x^4+(-4*b^2*x^4+
9/2*a*b*x^2+a^2)*a^(1/2)*(b*x^2+a)^(1/2))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = \left[\frac{15\,\sqrt{a}b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,b^2x^4-9\,abx^2-2\,a^2)\sqrt{bx^2+a}}{16\,x^4}, \frac{15\,\sqrt{-a}}{}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/16*(15*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(8*b^2*x^4 - 9*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^4, 1/8*(15*sqrt
(-a)*b^2*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (8*b^2*x^4 - 9*a*b*x^2 -
 2*a^2)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = -\frac{15\sqrt{a}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8} - \frac{a^3}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{11a^2\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{ab^{\frac{3}{2}}}{8x\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{5}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(5/2)/x**5,x)
```

output
```
-15*sqrt(a)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**3/(4*sqrt(b)*x**5*sqrt(
a/(b*x**2) + 1)) - 11*a**2*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) - a*b**(3
/2)/(8*x*sqrt(a/(b*x**2) + 1)) + b**(5/2)*x/sqrt(a/(b*x**2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = -\frac{15}{8}\sqrt{a}b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{15}{8}\sqrt{bx^2+a}b^2 + \frac{3\,(bx^2+a)^{\frac{5}{2}}b^2}{8\,a^2} + \frac{5\,(bx^2+a)^{\frac{3}{2}}b^2}{8\,a} - \frac{3\,(bx^2+a)^{\frac{7}{2}}b}{8\,a^2x^2} - \frac{(bx^2+a)^{\frac{7}{2}}}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
-15/8*sqrt(a)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 15/8*sqrt(b*x^2 + a)*b^2
 + 3/8*(b*x^2 + a)^(5/2)*b^2/a^2 + 5/8*(b*x^2 + a)^(3/2)*b^2/a - 3/8*(b*x^
2 + a)^(7/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(7/2)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = \frac{\frac{15\,ab^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 8\sqrt{bx^2+a}b^3 - \frac{9\,(bx^2+a)^{\frac{3}{2}}ab^3 - 7\sqrt{bx^2+a}a^2b^3}{b^2x^4}}{8\,b}$$

input
```
integrate((b*x^2+a)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/8*(15*a*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*sqrt(b*x^2 + a
)*b^3 - (9*(b*x^2 + a)^(3/2)*a*b^3 - 7*sqrt(b*x^2 + a)*a^2*b^3)/(b^2*x^4))
/b
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = b^2\sqrt{b\,x^2+a} - \frac{9\,a\,(b\,x^2+a)^{3/2}}{8\,x^4} + \frac{7\,a^2\sqrt{b\,x^2+a}}{8\,x^4} + \frac{\sqrt{a}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,15\mathrm{i}}{8}$$

input
```
int((a + b*x^2)^(5/2)/x^5,x)
```

output
```
b^2*(a + b*x^2)^(1/2) + (a^(1/2)*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*
15i)/8 - (9*a*(a + b*x^2)^(3/2))/(8*x^4) + (7*a^2*(a + b*x^2)^(1/2))/(8*x^
4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^{5/2}}{x^5}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2 - 9\sqrt{b\,x^2+a}\,ab\,x^2 + 8\sqrt{b\,x^2+a}\,b^2x^4 + 15\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right) b}{8x^4}$$

input
```
int((b*x^2+a)^(5/2)/x^5,x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2 - 9*sqrt(a + b*x**2)*a*b*x**2 + 8*sqrt(a + b*x
**2)*b**2*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/s
qrt(a))*b**2*x**4 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x
)/sqrt(a))*b**2*x**4)/(8*x**4)
```

3.404 $\int \frac{(a+bx^2)^{5/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{(a+bx^2)^{5/2}}{x^7} dx = -\frac{a^2\sqrt{a+bx^2}}{6x^6} - \frac{13ab\sqrt{a+bx^2}}{24x^4} - \frac{11b^2\sqrt{a+bx^2}}{16x^2} - \frac{5b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

output
```
-1/6*a^2*(b*x^2+a)^(1/2)/x^6-13/24*a*b*(b*x^2+a)^(1/2)/x^4-11/16*b^2*(b*x^
2+a)^(1/2)/x^2-5/16*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{5/2}}{x^7} dx = \frac{\sqrt{a+bx^2}(-8a^2-26abx^2-33b^2x^4)}{48x^6} - \frac{5b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16\sqrt{a}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2 - 26*a*b*x^2 - 33*b^2*x^4))/(48*x^6) - (5*b^3*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}}{x^8}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{6}b\int \frac{(bx^2+a)^{3/2}}{x^6}dx^2-\frac{(a+bx^2)^{5/2}}{3x^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{3}{4}b\int \frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x^7,x]
```

output
```
(-1/3*(a + b*x^2)^(5/2)/x^6 + (5*b*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(S
qrt[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/6
)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(33 b^{2} x^{4}+26 a b\,x^{2}+8 a^{2}\right)}{48 x^{6}}-\frac{5 b^{3} \ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{16 \sqrt{a}}$
pseudoelliptic	$\frac{-15\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b^{3} x^{6}-33 b^{2} x^{4} \sqrt{b\,x^{2}+a}\,\sqrt{a}-26 a^{\frac{3}{2}} b\,x^{2} \sqrt{b\,x^{2}+a}-8 a^{\frac{5}{2}} \sqrt{b\,x^{2}+a}}{48 x^{6} \sqrt{a}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{6 a\,x^{6}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4 a\,x^{4}}+\frac{3 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2 a\,x^{2}}+\frac{5 b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right.\right.\right.\right.}{2 a}\right.}{4 a}\right.}{6 a}$

input
```
int((b*x^2+a)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(b*x^2+a)^(1/2)*(33*b^2*x^4+26*a*b*x^2+8*a^2)/x^6-5/16*b^3/a^(1/2)*l
n((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.73

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = \left[\frac{15\,\sqrt{a} b^3 x^6 \log\left(-\frac{bx^2 - 2\,\sqrt{bx^2+a}\sqrt{a} + 2\,a}{x^2}\right) - 2\,(33\,ab^2x^4 + 26\,a^2bx^2 + 8\,a^3)\sqrt{bx^2+a}}{96\,ax^6}, \frac{15\,\sqrt{\ldots}}{\ldots}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(15*sqrt(a)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) - 2*(33*a*b^2*x^4 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6), 1/
48*(15*sqrt(-a)*b^3*x^6*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (33*a*b^2*x^4
 + 26*a^2*b*x^2 + 8*a^3)*sqrt(b*x^2 + a))/(a*x^6)]
```

Sympy [A] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{6x^5} - \frac{13ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{24x^3} - \frac{11b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{16x} - \frac{5b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(5/2)/x**7,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(6*x**5) - 13*a*b**(3/2)*sqrt(a/(b*x**2
) + 1)/(24*x**3) - 11*b**(5/2)*sqrt(a/(b*x**2) + 1)/(16*x) - 5*b**3*asinh(
sqrt(a)/(sqrt(b)*x))/(16*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = -\frac{5\,b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,\sqrt{a}} + \frac{(bx^2+a)^{\frac{5}{2}}b^3}{16\,a^3} + \frac{5\,(bx^2+a)^{\frac{3}{2}}b^3}{48\,a^2} + \frac{5\,\sqrt{bx^2+a}b^3}{16\,a} - \frac{(bx^2+a)^{\frac{7}{2}}b^2}{16\,a^3x^2} - \frac{(bx^2+a)^{\frac{7}{2}}b}{24\,a^2x^4} - \frac{(bx^2+a)^{\frac{7}{2}}}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
-5/16*b^3*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/16*(b*x^2 + a)^(5/2)*b
^3/a^3 + 5/48*(b*x^2 + a)^(3/2)*b^3/a^2 + 5/16*sqrt(b*x^2 + a)*b^3/a - 1/1
6*(b*x^2 + a)^(7/2)*b^2/(a^3*x^2) - 1/24*(b*x^2 + a)^(7/2)*b/(a^2*x^4) - 1
/6*(b*x^2 + a)^(7/2)/(a*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = \frac{1}{48}\,b^3\left(\frac{15\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{33\,(bx^2+a)^{\frac{5}{2}} - 40\,(bx^2+a)^{\frac{3}{2}}a + 15\,\sqrt{bx^2+a}a^2}{b^3x^6}\right)$$

input
```
integrate((b*x^2+a)^(5/2)/x^7,x, algorithm="giac")
```

output
```
1/48*b^3*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (33*(b*x^2 + a)^(
5/2) - 40*(b*x^2 + a)^(3/2)*a + 15*sqrt(b*x^2 + a)*a^2)/(b^3*x^6))
```

Mupad [B] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = \frac{5\,a\,(b\,x^2+a)^{3/2}}{6\,x^6} - \frac{11\,(b\,x^2+a)^{5/2}}{16\,x^6} - \frac{5\,a^2\,\sqrt{b\,x^2+a}}{16\,x^6} + \frac{b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,\sqrt{a}}$$

input
```
int((a + b*x^2)^(5/2)/x^7,x)
```

output
```
(b^3*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(1/2)) - (11*(a + b*x^
2)^(5/2))/(16*x^6) + (5*a*(a + b*x^2)^(3/2))/(6*x^6) - (5*a^2*(a + b*x^2)^
(1/2))/(16*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^{5/2}}{x^7}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^3-26\sqrt{b\,x^2+a}\,a^2b\,x^2-33\sqrt{b\,x^2+a}\,a\,b^2x^4+15\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{}}{\sqrt{a}}\right.}{48a\,x^6}$$

input
```
int((b*x^2+a)^(5/2)/x^7,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**3 - 26*sqrt(a + b*x**2)*a**2*b*x**2 - 33*sqrt(a
+ b*x**2)*a*b**2*x**4 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*b**3*x**6 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sq
rt(b)*x)/sqrt(a))*b**3*x**6)/(48*a*x**6)
```

3.405 $\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = -\frac{a^2\sqrt{a+bx^2}}{8x^8} - \frac{17ab\sqrt{a+bx^2}}{48x^6} - \frac{59b^2\sqrt{a+bx^2}}{192x^4} - \frac{5b^3\sqrt{a+bx^2}}{128ax^2} + \frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

output
```
-1/8*a^2*(b*x^2+a)^(1/2)/x^8-17/48*a*b*(b*x^2+a)^(1/2)/x^6-59/192*b^2*(b*x
^2+a)^(1/2)/x^4-5/128*b^3*(b*x^2+a)^(1/2)/a/x^2+5/128*b^4*arctanh((b*x^2+a
)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \frac{\sqrt{a+bx^2}(-48a^3-136a^2bx^2-118ab^2x^4-15b^3x^6)}{384ax^8} + \frac{5b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^9,x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^3 - 136*a^2*b*x^2 - 118*a*b^2*x^4 - 15*b^3*x^6))/(
384*a*x^8) + (5*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(128*a^(3/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^9}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}}{x^{10}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{8}b\int \frac{(bx^2+a)^{3/2}}{x^8}dx^2-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{bx^2+a}}{x^6}dx^2-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x^9,x]
```

output
```
(-1/4*(a + b*x^2)^(5/2)/x^8 + (5*b*(-1/3*(a + b*x^2)^(3/2)/x^6 + (b*(-1/2*
Sqrt[a + b*x^2]/x^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/a^(3/2)))/4))/2))/8)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$-\frac{5\left(-\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b^{4} x^{8}+\sqrt{b\,x^{2}+a}\,\left(\sqrt{a}\,b^{3} x^{6}+\frac{118 a^{\frac{3}{2}} b^{2} x^{4}}{15}+\frac{136 a^{\frac{5}{2}} b\,x^{2}}{15}+\frac{16 a^{\frac{7}{2}}}{5}\right)\right)}{128 a^{\frac{3}{2}} x^{8}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(15 b^{3} x^{6}+118 a\,b^{2} x^{4}+136 a^{2} b\,x^{2}+48 a^{3}\right)}{384 x^{8} a}+\frac{5 b^{4} \ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{128 a^{\frac{3}{2}}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8 a\,x^{8}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{6 a\,x^{6}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4 a\,x^{4}}+\frac{3 b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2 a\,x^{2}}+\frac{5 b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\right.\right.\right.}{2 a}\right.}{4 a}\right.}{6 a}\right.}{8 a}$

input
```
int((b*x^2+a)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-5/128/a^(3/2)*(-arctanh((b*x^2+a)^(1/2)/a^(1/2))*b^4*x^8+(b*x^2+a)^(1/2)*
(a^(1/2)*b^3*x^6+118/15*a^(3/2)*b^2*x^4+136/15*a^(5/2)*b*x^2+16/5*a^(7/2))
)/x^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \left[\frac{15\sqrt{a}b^4x^8\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right) - 2\left(15ab^3x^6 + 118a^2b^2x^4 + 136a^3bx^2 + 48a^4\right.}{768a^2x^8}\right.$$

$$\left. -\frac{15\sqrt{-a}b^4x^8\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) + (15ab^3x^6 + 118a^2b^2x^4 + 136a^3bx^2 + 48a^4)\sqrt{bx^2+a}}{384a^2x^8}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
[1/768*(15*sqrt(a)*b^4*x^8*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) - 2*(15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt(b*
x^2 + a))/(a^2*x^8), -1/384*(15*sqrt(-a)*b^4*x^8*arctan(sqrt(b*x^2 + a)*sq
rt(-a)/a) + (15*a*b^3*x^6 + 118*a^2*b^2*x^4 + 136*a^3*b*x^2 + 48*a^4)*sqrt
(b*x^2 + a))/(a^2*x^8)]
```

Sympy [A] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = -\frac{a^3}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{23a^2\sqrt{b}}{48x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{127ab^{\frac{3}{2}}}{192x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{133b^{\frac{5}{2}}}{384x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{7}{2}}}{128ax\sqrt{\frac{a}{bx^2}+1}} + \frac{5b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(5/2)/x**9,x)
```

output
```
-a**3/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - 23*a**2*sqrt(b)/(48*x**7*sqr
t(a/(b*x**2) + 1)) - 127*a*b**(3/2)/(192*x**5*sqrt(a/(b*x**2) + 1)) - 133*
b**(5/2)/(384*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(7/2)/(128*a*x*sqrt(a/(b*x
**2) + 1)) + 5*b**4*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \frac{5\,b^4\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{3}{2}}} - \frac{(bx^2+a)^{\frac{5}{2}}b^4}{128\,a^4} - \frac{5\,(bx^2+a)^{\frac{3}{2}}b^4}{384\,a^3} - \frac{5\,\sqrt{bx^2+a}b^4}{128\,a^2} + \frac{(bx^2+a)^{\frac{7}{2}}b^3}{128\,a^4x^2} + \frac{(bx^2+a)^{\frac{7}{2}}b^2}{192\,a^3x^4} + \frac{(bx^2+a)^{\frac{7}{2}}b}{48\,a^2x^6} - \frac{(bx^2+a)^{\frac{7}{2}}}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
5/128*b^4*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/128*(b*x^2 + a)^(5/2)*
b^4/a^4 - 5/384*(b*x^2 + a)^(3/2)*b^4/a^3 - 5/128*sqrt(b*x^2 + a)*b^4/a^2
+ 1/128*(b*x^2 + a)^(7/2)*b^3/(a^4*x^2) + 1/192*(b*x^2 + a)^(7/2)*b^2/(a^3
*x^4) + 1/48*(b*x^2 + a)^(7/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(7/2)/(a*x^8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = -\frac{\frac{15\,b^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{15\,(bx^2+a)^{\frac{7}{2}}b^5+73\,(bx^2+a)^{\frac{5}{2}}ab^5-55\,(bx^2+a)^{\frac{3}{2}}a^2b^5+15\,\sqrt{bx^2+a}a^3b^5}{ab^4x^8}}{384\,b}$$

input
```
integrate((b*x^2+a)^(5/2)/x^9,x, algorithm="giac")
```

output
```
-1/384*(15*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (15*(b*x^2
+ a)^(7/2)*b^5 + 73*(b*x^2 + a)^(5/2)*a*b^5 - 55*(b*x^2 + a)^(3/2)*a^2*b^5
 + 15*sqrt(b*x^2 + a)*a^3*b^5)/(a*b^4*x^8))/b
```

Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \frac{55\,a\,(b\,x^2+a)^{3/2}}{384\,x^8} - \frac{73\,(b\,x^2+a)^{5/2}}{384\,x^8} - \frac{5\,a^2\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{5\,(b\,x^2+a)^{7/2}}{128\,a\,x^8} - \frac{b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{128\,a^{3/2}}$$

input
```
int((a + b*x^2)^(5/2)/x^9,x)
```

output
```
(55*a*(a + b*x^2)^(3/2))/(384*x^8) - (b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1
/2))*5i)/(128*a^(3/2)) - (73*(a + b*x^2)^(5/2))/(384*x^8) - (5*a^2*(a + b*
x^2)^(1/2))/(128*x^8) - (5*(a + b*x^2)^(7/2))/(128*a*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^{5/2}}{x^9}\,dx = \frac{-48\sqrt{b\,x^2+a}\,a^4 - 136\sqrt{b\,x^2+a}\,a^3b\,x^2 - 118\sqrt{b\,x^2+a}\,a^2b^2x^4 - 15\sqrt{b\,x^2+a}\,a\,b^3x^6}{384a^2x^8}$$

input
```
int((b*x^2+a)^(5/2)/x^9,x)
```

output
```
( - 48*sqrt(a + b*x**2)*a**4 - 136*sqrt(a + b*x**2)*a**3*b*x**2 - 118*sqrt
(a + b*x**2)*a**2*b**2*x**4 - 15*sqrt(a + b*x**2)*a*b**3*x**6 - 15*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8 + 15*sqrt
(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**4*x**8)/(384*
a**2*x**8)
```

3.406 $\int \frac{(a+bx^2)^{5/2}}{x^{11}} dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}} dx = -\frac{a^2\sqrt{a+bx^2}}{10x^{10}} - \frac{21ab\sqrt{a+bx^2}}{80x^8} - \frac{31b^2\sqrt{a+bx^2}}{160x^6} - \frac{b^3\sqrt{a+bx^2}}{128ax^4} + \frac{3b^4\sqrt{a+bx^2}}{256a^2x^2} - \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

output
```
-1/10*a^2*(b*x^2+a)^(1/2)/x^10-21/80*a*b*(b*x^2+a)^(1/2)/x^8-31/160*b^2*(b
*x^2+a)^(1/2)/x^6-1/128*b^3*(b*x^2+a)^(1/2)/a/x^4+3/256*b^4*(b*x^2+a)^(1/2
)/a^2/x^2-3/256*b^5*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}} dx = \frac{\sqrt{a+bx^2}(-128a^4-336a^3bx^2-248a^2b^2x^4-10ab^3x^6+15b^4x^8)}{1280a^2x^{10}} - \frac{3b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256a^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^11,x]
```

output
```
(Sqrt[a + b*x^2]*(-128*a^4 - 336*a^3*b*x^2 - 248*a^2*b^2*x^4 - 10*a*b^3*x^
6 + 15*b^4*x^8))/(1280*a^2*x^10) - (3*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(256*a^(5/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{5/2}}{x^{12}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}b\int \frac{(bx^2+a)^{3/2}}{x^{10}}dx^2-\frac{(a+bx^2)^{5/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\int \frac{\sqrt{bx^2+a}}{x^8}dx^2-\frac{(a+bx^2)^{3/2}}{4x^8}\right)-\frac{(a+bx^2)^{5/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right)-\frac{(a+bx^2)^{5/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)$$

$$\downarrow 52$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)$$

$$\downarrow 73$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)$$

$$\downarrow 221$$

$$\frac{1}{2}\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)$$

input
```
Int[(a + b*x^2)^(5/2)/x^11,x]
```

output
```
(-1/5*(a + b*x^2)^(5/2)/x^10 + (b*(-1/4*(a + b*x^2)^(3/2)/x^8 + (3*b*(-1/3
*Sqrt[a + b*x^2]/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqrt[a +
 b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a)))/
6))/8))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{\frac{15\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b^{5} x^{10}}{128}+\sqrt{b\,x^{2}+a}\,\left(-\frac{15\sqrt{a}\,b^{4} x^{8}}{128}+\frac{5a^{\frac{3}{2}} b^{3} x^{6}}{64}+\frac{31a^{\frac{5}{2}} b^{2} x^{4}}{16}+\frac{21a^{\frac{7}{2}} b\,x^{2}}{8}+a^{\frac{9}{2}}\right)}{10a^{\frac{5}{2}} x^{10}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15b^{4} x^{8}+10a\,b^{3} x^{6}+248a^{2} b^{2} x^{4}+336a^{3} b\,x^{2}+128a^{4}\right)}{1280x^{10} a^{2}}-\frac{3b^{5} \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{256a^{\frac{5}{2}}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10a\,x^{10}}-\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8a\,x^{8}}-\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{6a\,x^{6}}+\frac{b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{4a\,x^{4}}+\frac{3b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}+\frac{5b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+\cdots\right)\right)}{\cdots}\right)}{4a}\right)}{6a}\right)}{8a}\right)}{10a}$

input
```
int((b*x^2+a)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/10*(15/128*arctanh((b*x^2+a)^(1/2)/a^(1/2))*b^5*x^10+(b*x^2+a)^(1/2)*(-
15/128*a^(1/2)*b^4*x^8+5/64*a^(3/2)*b^3*x^6+31/16*a^(5/2)*b^2*x^4+21/8*a^(
7/2)*b*x^2+a^(9/2)))/a^(5/2)/x^10
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \left[\frac{15\sqrt{a}b^5x^{10}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(15\,ab^4x^8-10\,a^2b^3x^6-248\,a^3b^2x^4-336\right.}{2560\,a^3x^{10}}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
[1/2560*(15*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3*b^2*x^4 - 336*a^4*b*x^
2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10), 1/1280*(15*sqrt(-a)*b^5*x^10*arc
tan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (15*a*b^4*x^8 - 10*a^2*b^3*x^6 - 248*a^3
*b^2*x^4 - 336*a^4*b*x^2 - 128*a^5)*sqrt(b*x^2 + a))/(a^3*x^10)]
```

Sympy [A] (verification not implemented)

Time = 28.66 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = -\frac{a^3}{10\sqrt{b}x^{11}\sqrt{\frac{a}{bx^2}+1}} - \frac{29a^2\sqrt{b}}{80x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{73ab^{\frac{3}{2}}}{160x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{129b^{\frac{5}{2}}}{640x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{7}{2}}}{256ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{9}{2}}}{256a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{256a^{\frac{5}{2}}}$$

input
```
integrate((b*x**2+a)**(5/2)/x**11,x)
```

output
```
-a**3/(10*sqrt(b)*x**11*sqrt(a/(b*x**2) + 1)) - 29*a**2*sqrt(b)/(80*x**9*s
qrt(a/(b*x**2) + 1)) - 73*a*b**(3/2)/(160*x**7*sqrt(a/(b*x**2) + 1)) - 129
*b**(5/2)/(640*x**5*sqrt(a/(b*x**2) + 1)) + b**(7/2)/(256*a*x**3*sqrt(a/(b
*x**2) + 1)) + 3*b**(9/2)/(256*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**5*asinh
(sqrt(a)/(sqrt(b)*x))/(256*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = -\frac{3\,b^5\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,a^{\frac{5}{2}}} + \frac{3\,(bx^2+a)^{\frac{5}{2}}b^5}{1280\,a^5} + \frac{(bx^2+a)^{\frac{3}{2}}b^5}{256\,a^4} + \frac{3\,\sqrt{bx^2+a}b^5}{256\,a^3} - \frac{3\,(bx^2+a)^{\frac{7}{2}}b^4}{1280\,a^5x^2} - \frac{(bx^2+a)^{\frac{7}{2}}b^3}{640\,a^4x^4} - \frac{(bx^2+a)^{\frac{7}{2}}b^2}{160\,a^3x^6} + \frac{3\,(bx^2+a)^{\frac{7}{2}}b}{80\,a^2x^8} - \frac{(bx^2+a)^{\frac{7}{2}}}{10\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
-3/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/1280*(b*x^2 + a)^(5/2
)*b^5/a^5 + 1/256*(b*x^2 + a)^(3/2)*b^5/a^4 + 3/256*sqrt(b*x^2 + a)*b^5/a^
3 - 3/1280*(b*x^2 + a)^(7/2)*b^4/(a^5*x^2) - 1/640*(b*x^2 + a)^(7/2)*b^3/(
a^4*x^4) - 1/160*(b*x^2 + a)^(7/2)*b^2/(a^3*x^6) + 3/80*(b*x^2 + a)^(7/2)*
b/(a^2*x^8) - 1/10*(b*x^2 + a)^(7/2)/(a*x^10)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \frac{1}{1280}\,b^5\left(\frac{15\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{15\,(bx^2+a)^{\frac{9}{2}} - 70\,(bx^2+a)^{\frac{7}{2}}a - 128\,(bx^2+a)^{\frac{5}{2}}a^2 +}{a^2b^5x^{10}}\right.$$

input
```
integrate((b*x^2+a)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/1280*b^5*(15*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (15*(b*x^
2 + a)^(9/2) - 70*(b*x^2 + a)^(7/2)*a - 128*(b*x^2 + a)^(5/2)*a^2 + 70*(b*
x^2 + a)^(3/2)*a^3 - 15*sqrt(b*x^2 + a)*a^4)/(a^2*b^5*x^10))
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \frac{7\,a\,(b\,x^2+a)^{3/2}}{128\,x^{10}} - \frac{(b\,x^2+a)^{5/2}}{10\,x^{10}} - \frac{3\,a^2\,\sqrt{b\,x^2+a}}{256\,x^{10}} - \frac{7\,(b\,x^2+a)^{7/2}}{128\,a\,x^{10}} + \frac{3\,(b\,x^2+a)^{9/2}}{256\,a^2\,x^{10}} + \frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,3\mathrm{i}}{256\,a^{5/2}}$$

input
```
int((a + b*x^2)^(5/2)/x^11,x)
```

output
```
(b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*3i)/(256*a^(5/2)) - (a + b*x^2)^
(5/2)/(10*x^10) + (7*a*(a + b*x^2)^(3/2))/(128*x^10) - (3*a^2*(a + b*x^2)^
(1/2))/(256*x^10) - (7*(a + b*x^2)^(7/2))/(128*a*x^10) + (3*(a + b*x^2)^(9
/2))/(256*a^2*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{5/2}}{x^{11}}\,dx = \frac{-128\sqrt{b\,x^2+a}\,a^5 - 336\sqrt{b\,x^2+a}\,a^4b\,x^2 - 248\sqrt{b\,x^2+a}\,a^3b^2x^4 - 10\sqrt{b\,x^2+a}\,a^2b^3x^6}{1}$$

input
```
int((b*x^2+a)^(5/2)/x^11,x)
```

output
```
( - 128*sqrt(a + b*x**2)*a**5 - 336*sqrt(a + b*x**2)*a**4*b*x**2 - 248*sqr
t(a + b*x**2)*a**3*b**2*x**4 - 10*sqrt(a + b*x**2)*a**2*b**3*x**6 + 15*sqr
t(a + b*x**2)*a*b**4*x**8 + 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + s
qrt(b)*x)/sqrt(a))*b**5*x**10 - 15*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a)
 + sqrt(b)*x)/sqrt(a))*b**5*x**10)/(1280*a**3*x**10)
```

3.407 $\int x^4(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int x^4\left(a+bx^2\right)^{5/2}\,dx=-\frac{3a^4x\sqrt{a+bx^2}}{256b^2}+\frac{a^3x^3\sqrt{a+bx^2}}{128b}+\frac{1}{32}a^2x^5\sqrt{a+bx^2}+\frac{1}{16}ax^5\left(a+bx^2\right)^{3/2}+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}+\frac{3a^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}$$

output
```
-3/256*a^4*x*(b*x^2+a)^(1/2)/b^2+1/128*a^3*x^3*(b*x^2+a)^(1/2)/b+1/32*a^2*
x^5*(b*x^2+a)^(1/2)+1/16*a*x^5*(b*x^2+a)^(3/2)+1/10*x^5*(b*x^2+a)^(5/2)+3/
256*a^5*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int x^4(a+bx^2)^{5/2}\,dx=\frac{\sqrt{a+bx^2}(-15a^4x+10a^3bx^3+248a^2b^2x^5+336ab^3x^7+128b^4x^9)}{1280b^2}+\frac{3a^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

input
```
Integrate[x^4*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-15*a^4*x + 10*a^3*b*x^3 + 248*a^2*b^2*x^5 + 336*a*b^3*x
^7 + 128*b^4*x^9))/(1280*b^2) + (3*a^5*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqr
t[a + b*x^2])])/(128*b^(5/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {248, 248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} a \int x^4 \left(bx^2+a\right)^{3/2} dx + \frac{1}{10} x^5 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} a \left(\frac{3}{8} a \int x^4 \sqrt{bx^2+a} dx + \frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}\right) + \frac{1}{10} x^5 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2} a \left(\frac{3}{8} a \left(\frac{1}{6} a \int \frac{x^4}{\sqrt{bx^2+a}} dx + \frac{1}{6} x^5 \sqrt{a+bx^2}\right) + \frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}\right) + \frac{1}{10} x^5 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2} a \left(\frac{3}{8} a \left(\frac{1}{6} a \left(\frac{x^3 \sqrt{a+bx^2}}{4b} - \frac{3a \int \frac{x^2}{\sqrt{bx^2+a}} dx}{4b}\right) + \frac{1}{6} x^5 \sqrt{a+bx^2}\right) + \frac{1}{8} x^5 \left(a+bx^2\right)^{3/2}\right) + \frac{1}{10} x^5 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

$\downarrow$ 224

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

$\downarrow$ 219

$$\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}$$

input
```
Int[x^4*(a + b*x^2)^(5/2),x]
```

output
```
(x^5*(a + b*x^2)^(5/2))/10 + (a*((x^5*(a + b*x^2)^(3/2))/8 + (3*a*((x^5*Sq
rt[a + b*x^2])/6 + (a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x
^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))
)/6))/8))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

method	result	si
risch	$-\frac{x\left(-128b^4x^8-336a\,b^3x^6-248a^2b^2x^4-10a^3b\,x^2+15a^4\right)\sqrt{b\,x^2+a}}{1280b^2}+\frac{3a^5\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{256b^{\frac{5}{2}}}$	84
pseudoelliptic	$\frac{\frac{3\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)a^5}{256}-\frac{3x\left(-\frac{128b^{\frac{9}{2}}x^8}{15}-\frac{112a\,b^{\frac{7}{2}}x^6}{5}-\frac{248a^2b^{\frac{5}{2}}x^4}{15}-\frac{2a^3b^{\frac{3}{2}}x^2}{3}+a^4\sqrt{b}\right)\sqrt{b\,x^2+a}}{256}}{b^{\frac{5}{2}}}$	87
default	$\frac{x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)}{10b}$	11

input
```
int(x^4*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1280*x*(-128*b^4*x^8-336*a*b^3*x^6-248*a^2*b^2*x^4-10*a^3*b*x^2+15*a^4)
*(b*x^2+a)^(1/2)/b^2+3/256*a^5/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.40

$$\int x^4\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{15\,a^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(128\,b^5x^9+336\,ab^4x^7+248\,a^2b^3x^5+10\,a\right.}{2560\,b^3}\right.$$
$$\left.-\frac{15\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(128\,b^5x^9+336\,ab^4x^7+248\,a^2b^3x^5+10\,a^3b^2x^3-15\,a^4bx)\sqrt{bx^2+a}}{1280\,b^3}\right]$$

input
```
integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/2560*(15*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^2*x^3 - 15*a^4
*b*x)*sqrt(b*x^2 + a))/b^3, -1/1280*(15*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (128*b^5*x^9 + 336*a*b^4*x^7 + 248*a^2*b^3*x^5 + 10*a^3*b^
2*x^3 - 15*a^4*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 29.56 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.29

$$\int x^4(a+bx^2)^{5/2}\,dx = -\frac{3a^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{a^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}} + \frac{129a^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}} + \frac{73a^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}} + \frac{29\sqrt{a}b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^5\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}} + \frac{b^3x^{11}}{10\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4*(b*x**2+a)**(5/2),x)
```

output
```
-3*a**(9/2)*x/(256*b**2*sqrt(1 + b*x**2/a)) - a**(7/2)*x**3/(256*b*sqrt(1
+ b*x**2/a)) + 129*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*a**(3/2)*b*
x**7/(160*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a
)) + 3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + b**3*x**11/(10*sqrt(
a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\int x^4(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}x^3}{10\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}}ax}{80\,b^2} + \frac{(bx^2+a)^{\frac{5}{2}}a^2x}{160\,b^2} + \frac{(bx^2+a)^{\frac{3}{2}}a^3x}{128\,b^2} + \frac{3\sqrt{bx^2+a}a^4x}{256\,b^2} + \frac{3\,a^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*x^3/b - 3/80*(b*x^2 + a)^(7/2)*a*x/b^2 + 1/160*(b*x
^2 + a)^(5/2)*a^2*x/b^2 + 1/128*(b*x^2 + a)^(3/2)*a^3*x/b^2 + 3/256*sqrt(b
*x^2 + a)*a^4*x/b^2 + 3/256*a^5*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.67

$$\int x^4(a+bx^2)^{5/2}\,dx = -\frac{3\,a^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}} + \frac{1}{1280}\left(2\left(4\left(2\left(8\,b^2x^2+21\,ab\right)x^2+31\,a^2\right)x^2+\frac{5\,a^3}{b}\right)x^2-\frac{15\,a^4}{b^2}\right)\sqrt{bx^2+a}x$$

input
```
integrate(x^4*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-3/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/1280*(2*(4*(
2*(8*b^2*x^2 + 21*a*b)*x^2 + 31*a^2)*x^2 + 5*a^3/b)*x^2 - 15*a^4/b^2)*sqrt
(b*x^2 + a)*x
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{5/2}\,dx = \int x^4\,\left(b\,x^2+a\right)^{5/2}\,dx$$

input
```
int(x^4*(a + b*x^2)^(5/2),x)
```

output
```
int(x^4*(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int x^4(a+bx^2)^{5/2}\,dx = \frac{-15\sqrt{b\,x^2+a}\,a^4bx+10\sqrt{b\,x^2+a}\,a^3b^2x^3+248\sqrt{b\,x^2+a}\,a^2b^3x^5+336\sqrt{b\,x^2+a}\,a\,b^4x^7+12}{1280b^3}$$

input
```
int(x^4*(b*x^2+a)^(5/2),x)
```

output
```
( - 15*sqrt(a + b*x**2)*a**4*b*x + 10*sqrt(a + b*x**2)*a**3*b**2*x**3 + 24
8*sqrt(a + b*x**2)*a**2*b**3*x**5 + 336*sqrt(a + b*x**2)*a*b**4*x**7 + 128
*sqrt(a + b*x**2)*b**5*x**9 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x
)/sqrt(a))*a**5)/(1280*b**3)
```

3.408 $\int x^2(a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int x^2\left(a+bx^2\right)^{5/2}\,dx=\frac{5a^3x\sqrt{a+bx^2}}{128b}+\frac{5}{64}a^2x^3\sqrt{a+bx^2}+\frac{5}{48}ax^3\left(a+bx^2\right)^{3/2}+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
5/128*a^3*x*(b*x^2+a)^(1/2)/b+5/64*a^2*x^3*(b*x^2+a)^(1/2)+5/48*a*x^3*(b*x
^2+a)^(3/2)+1/8*x^3*(b*x^2+a)^(5/2)-5/128*a^4*arctanh(b^(1/2)*x/(b*x^2+a)^
(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.83

$$\int x^2\left(a+bx^2\right)^{5/2}\,dx=\frac{x\sqrt{a+bx^2}(15a^3+118a^2bx^2+136ab^2x^4+48b^3x^6)}{384b}-\frac{5a^4\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{64b^{3/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^(5/2),x]
```

output
```
(x*Sqrt[a + b*x^2]*(15*a^3 + 118*a^2*b*x^2 + 136*a*b^2*x^4 + 48*b^3*x^6))/
(384*b) - (5*a^4*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(64*b^
(3/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{5/2} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{8} a \int x^2 \left(bx^2+a\right)^{3/2}dx+\frac{1}{8} x^3 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{8} a \left(\frac{1}{2} a \int x^2 \sqrt{bx^2+a}dx+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{8} a \left(\frac{1}{2} a \left(\frac{1}{4} a \int \frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{8} a \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x \sqrt{a+bx^2}}{2 b}-\frac{a \int \frac{1}{\sqrt{bx^2+a}}dx}{2 b}\right)+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{8} a \left(\frac{1}{2} a \left(\frac{1}{4} a \left(\frac{x \sqrt{a+bx^2}}{2 b}-\frac{a \int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2 b}\right)+\frac{1}{4} x^3 \sqrt{a+bx^2}\right)+\frac{1}{6} x^3 \left(a+bx^2\right)^{3/2}\right)+\frac{1}{8} x^3 \left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}$$

input
```
Int[x^2*(a + b*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*x^2)^(5/2))/8 + (5*a*((x^3*(a + b*x^2)^(3/2))/6 + (a*((x^3*Sqr
t[a + b*x^2])/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(2*b^(3/2))))/4))/2))/8
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{x\left(48b^{3}x^{6}+136a\,b^{2}x^{4}+118a^{2}b\,x^{2}+15a^{3}\right)\sqrt{b\,x^{2}+a}}{384b}-\frac{5a^{4}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}$	73
pseudoelliptic	$-\frac{5\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{4}-x\left(\frac{16b^{\frac{7}{2}}x^{6}}{5}+\frac{136a\,b^{\frac{5}{2}}x^{4}}{15}+\frac{118a^{2}b^{\frac{3}{2}}x^{2}}{15}+a^{3}\sqrt{b}\right)\sqrt{b\,x^{2}+a}\right)}{128b^{\frac{3}{2}}}$	76
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}$	90

input
```
int(x^2*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*x*(48*b^3*x^6+136*a*b^2*x^4+118*a^2*b*x^2+15*a^3)*(b*x^2+a)^(1/2)/b-
5/128/b^(3/2)*a^4*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.49

$$\int x^2(a+bx^2)^{5/2}\,dx = \left[\frac{15\,a^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,b^4x^7+136\,ab^3x^5+118\,a^2b^2x^3+15\,a^3}{768\,b^2}\right.$$

input
```
integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/768*(15*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(48*b^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 +
a))/b^2, 1/384*(15*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b
^4*x^7 + 136*a*b^3*x^5 + 118*a^2*b^2*x^3 + 15*a^3*b*x)*sqrt(b*x^2 + a))/b^
2]
```

Sympy [A] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

$$\int x^2(a+bx^2)^{5/2}\,dx = \frac{5a^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}} + \frac{133a^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}} + \frac{127a^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}} + \frac{23\sqrt{a}b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}} - \frac{5a^4\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}} + \frac{b^3x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2*(b*x**2+a)**(5/2),x)
```

output
```
5*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*a**(5/2)*x**3/(384*sqrt(1 +
b*x**2/a)) + 127*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*sqrt(a)*b**
2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(
3/2)) + b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.76

$$\int x^2(a+bx^2)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}x}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}ax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}a^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}a^3x}{128\,b} - \frac{5\,a^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(7/2)*x/b - 1/48*(b*x^2 + a)^(5/2)*a*x/b - 5/192*(b*x^2 +
a)^(3/2)*a^2*x/b - 5/128*sqrt(b*x^2 + a)*a^3*x/b - 5/128*a^4*arcsinh(b*x/s
qrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int x^2 (a+bx^2)^{5/2}\,dx = \frac{5\,a^4 \log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}} + \frac{1}{384}\left(2\left(4\left(6\,b^2x^2+17\,ab\right)x^2+59\,a^2\right)x^2+\frac{15\,a^3}{b}\right)\sqrt{bx^2+a}x$$

input
```
integrate(x^2*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
5/128*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/384*(2*(4*(6*
b^2*x^2 + 17*a*b)*x^2 + 59*a^2)*x^2 + 15*a^3/b)*sqrt(b*x^2 + a)*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^{5/2}\,dx = \int x^2\,\left(b\,x^2+a\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)^{5/2}\,dx = \frac{15\sqrt{b\,x^2+a}\,a^3bx+118\sqrt{b\,x^2+a}\,a^2b^2x^3+136\sqrt{b\,x^2+a}\,a\,b^3x^5+48\sqrt{b\,x^2+a}\,b^4x^7-15\sqrt{b}}{384b^2}$$

input
```
int(x^2*(b*x^2+a)^(5/2),x)
```

output
```
(15*sqrt(a + b*x**2)*a**3*b*x + 118*sqrt(a + b*x**2)*a**2*b**2*x**3 + 136*
sqrt(a + b*x**2)*a*b**3*x**5 + 48*sqrt(a + b*x**2)*b**4*x**7 - 15*sqrt(b)*
log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4)/(384*b**2)
```

3.409 $\int (a+bx^2)^{5/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int (a+bx^2)^{5/2}\,dx = \frac{5}{16}a^2x\sqrt{a+bx^2} + \frac{5}{24}ax(a+bx^2)^{3/2} + \frac{1}{6}x(a+bx^2)^{5/2} + \frac{5a^3\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/16*a^2*x*(b*x^2+a)^(1/2)+5/24*a*x*(b*x^2+a)^(3/2)+1/6*x*(b*x^2+a)^(5/2)+
5/16*a^3*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.85

$$\int (a+bx^2)^{5/2}\,dx = \frac{1}{48}\sqrt{a+bx^2}(33a^2x+26abx^3+8b^2x^5) - \frac{5a^3\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(33*a^2*x + 26*a*b*x^3 + 8*b^2*x^5))/48 - (5*a^3*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}$$

input
```
Int[(a + b*x^2)^(5/2),x]
```

output
```
(x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
+ b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{x \left(8 b^{2} x^{4}+26 a b \,x^{2}+33 a^{2}\right) \sqrt{b \,x^{2}+a}}{48}+\frac{5 a^{3} \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{16 \sqrt{b}}$	59
pseudoelliptic	$\frac{5 \,\operatorname{arctanh}\left(\frac{\sqrt{b \,x^{2}+a}}{x \sqrt{b}}\right) a^{3}}{16 \sqrt{b}}+\frac{11 x \left(\frac{8 b^{\frac{5}{2}} x^{4}}{33}+\frac{26 a \,b^{\frac{3}{2}} x^{2}}{33}+a^{2} \sqrt{b}\right) \sqrt{b \,x^{2}+a}}{16 \sqrt{b}}$	67
default	$\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x +\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}$	68

input
```
int((b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*x*(8*b^2*x^4+26*a*b*x^2+33*a^2)*(b*x^2+a)^(1/2)+5/16*a^3*ln(b^(1/2)*x
+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int (a+bx^2)^{5/2}\,dx = \left[\frac{15\,a^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{96\,b}, -\frac{15\,a^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3x^5+26\,ab^2x^3+33\,a^2bx)\sqrt{bx^2+a}}{48\,b}\right]$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/96*(15*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(8*b^3*x^5 + 26*a*b^2*x^3 + 33*a^2*b*x)*sqrt(b*x^2 + a))/b, -1/48*(15*a^3*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8*b^3*x^5 + 26*a*b^2*x^3 +
33*a^2*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.15

$$\int (a+bx^2)^{5/2}\,dx = \frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{24}+\frac{\sqrt{a}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(5/2),x)
```

output
```
11*a**(5/2)*x*sqrt(1 + b*x**2/a)/16 + 13*a**(3/2)*b*x**3*sqrt(1 + b*x**2/a
)/24 + sqrt(a)*b**2*x**5*sqrt(1 + b*x**2/a)/6 + 5*a**3*asinh(sqrt(b)*x/sqr
t(a))/(16*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.69

$$\int \left(a+bx^2\right)^{5/2}\,dx=\frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}x+\frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}ax+\frac{5}{16}\sqrt{bx^2+a}a^2x+\frac{5\,a^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*x + 5/24*(b*x^2 + a)^(3/2)*a*x + 5/16*sqrt(b*x^2 + a
)*a^2*x + 5/16*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

$$\int \left(a+bx^2\right)^{5/2}\,dx=-\frac{5\,a^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{48}\left(2\left(4\,b^2x^2+13\,ab\right)x^2+33\,a^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/48*(2*(4*b^2*
x^2 + 13*a*b)*x^2 + 33*a^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.44

$$\int \left(a+bx^2\right)^{5/2} dx = \frac{x\left(b\,x^2+a\right)^{5/2}{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2),x)
```

output
```
(x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \left(a+bx^2\right)^{5/2} dx = \frac{33\sqrt{b\,x^2+a}\,a^2bx+26\sqrt{b\,x^2+a}\,a\,b^2x^3+8\sqrt{b\,x^2+a}\,b^3x^5+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^3}{48b}$$

input
```
int((b*x^2+a)^(5/2),x)
```

output
```
(33*sqrt(a + b*x**2)*a**2*b*x + 26*sqrt(a + b*x**2)*a*b**2*x**3 + 8*sqrt(a
 + b*x**2)*b**3*x**5 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3)/(48*b)
```

3.410 $\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = -\frac{a^2\sqrt{a+bx^2}}{x}+\frac{9}{8}abx\sqrt{a+bx^2}+\frac{1}{4}b^2x^3\sqrt{a+bx^2}+\frac{15}{8}a^2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-a^2*(b*x^2+a)^(1/2)/x+9/8*a*b*x*(b*x^2+a)^(1/2)+1/4*b^2*x^3*(b*x^2+a)^(1/
2)+15/8*a^2*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-8a^2+9abx^2+2b^2x^4)}{8x}-\frac{15}{8}a^2\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2 + 9*a*b*x^2 + 2*b^2*x^4))/(8*x) - (15*a^2*Sqrt[b]
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$5b\int \left(bx^2+a\right)^{3/2}dx-\frac{\left(a+bx^2\right)^{5/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$5b\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}$$

$$\Big\downarrow \text{224}$$

$$5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}$$

$$\Big\downarrow \text{219}$$

$$5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}$$

input
```
Int[(a + b*x^2)^(5/2)/x^2,x]
```

output
```
-((a + b*x^2)^(5/2)/x) + 5*b*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a +
b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.68

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2b^{2}x^{4}-9ab\,x^{2}+8a^{2}\right)}{8x}+\frac{15a^{2}\sqrt{b}\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8}$	61
pseudoelliptic	$\frac{2b^{\frac{5}{2}}\sqrt{b\,x^{2}+a}\,x^{4}+9a\,b^{\frac{3}{2}}x^{2}\sqrt{b\,x^{2}+a}+15\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{2}bx-8\sqrt{b\,x^{2}+a}\,a^{2}\sqrt{b}}{8x\sqrt{b}}$	86
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}$	92

input
```
int((b*x^2+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(-2*b^2*x^4-9*a*b*x^2+8*a^2)/x+15/8*a^2*b^(1/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = \left[\frac{15\,a^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^4+9\,abx^2-8\,a^2)\sqrt{bx^2+a}}{16\,x},\right.$$
$$\left.-\frac{15\,a^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^4+9\,abx^2-8\,a^2)\sqrt{bx^2+a}}{8\,x}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/16*(15*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a))/x, -1/8*(15*a^2*sqrt(-b
)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^4 + 9*a*b*x^2 - 8*a^2)*s
qrt(b*x^2 + a))/x]
```

Sympy [A] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = -\frac{a^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} + \frac{a^{\frac{3}{2}}bx}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{11\sqrt{a}b^2x^3}{8\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8} + \frac{b^3x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(5/2)/x**2,x)
```

output
```
-a**(5/2)/(x*sqrt(1 + b*x**2/a)) + a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 1
1*sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*a**2*sqrt(b)*asinh(sqrt(b)
*x/sqrt(a))/8 + b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}bx + \frac{15}{8}\sqrt{bx^2+a}abx + \frac{15}{8}a^2\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{(bx^2+a)^{\frac{5}{2}}}{x}$$

input
```
integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5/4*(b*x^2 + a)^(3/2)*b*x + 15/8*sqrt(b*x^2 + a)*a*b*x + 15/8*a^2*sqrt(b)*
arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(5/2)/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = -\frac{15}{16}\,a^2\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\,a^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{8}\left(2\,b^2x^2+9\,ab\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

output
```
-15/16*a^2*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^3*sqrt(b)/((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/8*(2*b^2*x^2 + 9*a*b)*sqrt(b*x^2 +
 a)*x
```

Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.44

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = -\frac{(b\,x^2+a)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2)/x^2,x)
```

output
```
-((a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{5/2}}{x^2}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^2+9\sqrt{b\,x^2+a}\,ab\,x^2+2\sqrt{b\,x^2+a}\,b^2x^4+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2x -}{8x}$$

input
```
int((b*x^2+a)^(5/2)/x^2,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**2 + 9*sqrt(a + b*x**2)*a*b*x**2 + 2*sqrt(a + b*x
**2)*b**2*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
*2*x - 10*sqrt(b)*a**2*x)/(8*x)
```

3.411 $\int \frac{(a+bx^2)^{5/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{(a+bx^2)^{5/2}}{x^4} dx = -\frac{a^2\sqrt{a+bx^2}}{3x^3} - \frac{7ab\sqrt{a+bx^2}}{3x} + \frac{1}{2}b^2x\sqrt{a+bx^2} + \frac{5}{2}ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/3*a^2*(b*x^2+a)^(1/2)/x^3-7/3*a*b*(b*x^2+a)^(1/2)/x+1/2*b^2*x*(b*x^2+a)
^(1/2)+5/2*a*b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{5/2}}{x^4} dx = \frac{\sqrt{a+bx^2}(-2a^2-14abx^2+3b^2x^4)}{6x^3} + 5ab^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a^2 - 14*a*b*x^2 + 3*b^2*x^4))/(6*x^3) + 5*a*b^(3/2)*
ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {247, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^4}dx$$

$$\Big\downarrow \ 247$$

$$\frac{5}{3}b\int \frac{(bx^2+a)^{3/2}}{x^2}dx-\frac{(a+bx^2)^{5/2}}{3x^3}$$

$$\Big\downarrow \ 247$$

$$\frac{5}{3}b\left(3b\int \sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}$$

$$\Big\downarrow \ 224$$

$$\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(5/2)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(5/2)/x^3 + (5*b*(-((a + b*x^2)^(3/2)/x) + 3*b*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

method	result	s
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3b^{2}x^{4}+14ab\,x^{2}+2a^{2}\right)}{6x^{3}}+\frac{5a\,b^{\frac{3}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2}$	5
pseudoelliptic	$\frac{15b^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a\,x^{3}-\sqrt{b\,x^{2}+a}\,\left(-3b^{2}x^{4}+14ab\,x^{2}+2a^{2}\right)}{6x^{3}}$	6
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{3a\,x^{3}}+\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)}{3a}$	1

input
```
int((b*x^2+a)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*x^2+a)^(1/2)*(-3*b^2*x^4+14*a*b*x^2+2*a^2)/x^3+5/2*a*b^(3/2)*ln(b^
(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx=\left[\frac{15\,ab^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(3\,b^2x^4-14\,abx^2-2\,a^2)\sqrt{bx^2+a}}{12\,x^3}\right.$$
$$\left.-\frac{15\,a\sqrt{-b}bx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(3\,b^2x^4-14\,abx^2-2\,a^2)\sqrt{bx^2+a}}{6\,x^3}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/12*(15*a*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(3*b^2*x^4 - 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/x^3, -1/6*(15*a*sqrt(-
b)*b*x^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*b^2*x^4 - 14*a*b*x^2 - 2*
a^2)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3x^2} - \frac{7ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3} - \frac{5ab^{\frac{3}{2}}\log\left(\frac{a}{bx^2}\right)}{4} + \frac{5ab^{\frac{3}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2} + \frac{b^{\frac{5}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

input
```
integrate((b*x**2+a)**(5/2)/x**4,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*x**2) - 7*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/3 - 5*a*b**(3/2)*log(a/(b*x**2))/4 + 5*a*b**(3/2)*log(sqrt(a/(b*x**2
) + 1) + 1)/2 + b**(5/2)*x**2*sqrt(a/(b*x**2) + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = \frac{5}{2}\sqrt{bx^2+a}b^2x + \frac{5\,(bx^2+a)^{\frac{3}{2}}b^2x}{3\,a} + \frac{5}{2}\,ab^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{4\,(bx^2+a)^{\frac{5}{2}}b}{3\,ax} - \frac{(bx^2+a)^{\frac{7}{2}}}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
5/2*sqrt(b*x^2 + a)*b^2*x + 5/3*(b*x^2 + a)^(3/2)*b^2*x/a + 5/2*a*b^(3/2)*
arcsinh(b*x/sqrt(a*b)) - 4/3*(b*x^2 + a)^(5/2)*b/(a*x) - 1/3*(b*x^2 + a)^(
7/2)/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = \frac{1}{2}\sqrt{bx^2+a}b^2x - \frac{5}{4}ab^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(9\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^2b^{\frac{3}{2}} - 12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^3b^{\frac{3}{2}} + 7a^4b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(5/2)/x^4,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b^2*x - 5/4*a*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a)
)^2) + 2/3*(9*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*b^(3/2) - 12*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*a^3*b^(3/2) + 7*a^4*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2
 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = \int \frac{\left(b\,x^2+a\right)^{5/2}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(5/2)/x^4,x)
```

output
```
int((a + b*x^2)^(5/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{5/2}}{x^4}\,dx = \frac{-4\sqrt{b\,x^2+a}\,a^2-28\sqrt{b\,x^2+a}\,ab\,x^2+6\sqrt{b\,x^2+a}\,b^2x^4+30\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab\,x}{12x^3}$$

input
```
int((b*x^2+a)^(5/2)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x**2)*a**2 - 28*sqrt(a + b*x**2)*a*b*x**2 + 6*sqrt(a + b*
x**2)*b**2*x**4 + 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
*b*x**3 + 5*sqrt(b)*a*b*x**3)/(12*x**3)
```

3.412 $\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = -\frac{a^2\sqrt{a+bx^2}}{5x^5}-\frac{11ab\sqrt{a+bx^2}}{15x^3}-\frac{23b^2\sqrt{a+bx^2}}{15x}+b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/5*a^2*(b*x^2+a)^(1/2)/x^5-11/15*a*b*(b*x^2+a)^(1/2)/x^3-23/15*b^2*(b*x^
2+a)^(1/2)/x+b^(5/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = \frac{\sqrt{a+bx^2}(-3a^2-11abx^2-23b^2x^4)}{15x^5}-b^{5/2}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a^2 - 11*a*b*x^2 - 23*b^2*x^4))/(15*x^5) - b^(5/2)*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {247, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}}{x^6} \, dx$$

$$\Big\downarrow \text{247}$$

$$b\int \frac{\left(bx^2+a\right)^{3/2}}{x^4}dx-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{224}$$

$$b\left(b\left(b\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$b\left(b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{3/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^5}$$

input
```
Int[(a + b*x^2)^(5/2)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/2)/x^5 + b*(-1/3*(a + b*x^2)^(3/2)/x^3 + b*(-(Sqrt[a +
 b*x^2]/x) + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.65

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(23b^{2}x^{4}+11ab\,x^{2}+3a^{2})}{15x^{5}}+b^{\frac{5}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)$
pseudoelliptic	$\frac{15b^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{5}-\sqrt{b\,x^{2}+a}\,(23b^{2}x^{4}+11ab\,x^{2}+3a^{2})}{15x^{5}}$
default	$-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{5a\,x^{5}}+\frac{2b\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{3a\,x^{3}}+\frac{4b\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x\right.}}{2\sqrt{b}}\right.}{4}\right.}{6}\right.}{a}\right.}{3a}\right.}{5a}$

input
```
int((b*x^2+a)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(23*b^2*x^4+11*a*b*x^2+3*a^2)/x^5+b^(5/2)*ln(b^(1/2)
*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = \left[\frac{15\,b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(23\,b^2x^4+11\,abx^2+3\,a^2)\sqrt{bx^2+a}}{30\,x^5}, -\frac{15\,\sqrt{-b}b^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(23\,b^2x^4+11\,abx^2+3\,a^2)\sqrt{bx^2+a}}{15\,x^5}\right]$$

input
```
integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/30*(15*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
(23*b^2*x^4 + 11*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a))/x^5, -1/15*(15*sqrt(-b)
*b^2*x^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (23*b^2*x^4 + 11*a*b*x^2 + 3
*a^2)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{5x^4} - \frac{11ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{15x^2} - \frac{23b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{15} - \frac{b^{\frac{5}{2}}\log\left(\frac{a}{bx^2}\right)}{2} + b^{\frac{5}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

input
```
integrate((b*x**2+a)**(5/2)/x**6,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(5*x**4) - 11*a*b**(3/2)*sqrt(a/(b*x**2
) + 1)/(15*x**2) - 23*b**(5/2)*sqrt(a/(b*x**2) + 1)/15 - b**(5/2)*log(a/(b
*x**2))/2 + b**(5/2)*log(sqrt(a/(b*x**2) + 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{2}}b^3x}{3\,a^2} + \frac{\sqrt{bx^2+a}\,b^3x}{a} + b^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{8\,(bx^2+a)^{\frac{5}{2}}b^2}{15\,a^2x} - \frac{2\,(bx^2+a)^{\frac{7}{2}}b}{15\,a^2x^3} - \frac{(bx^2+a)^{\frac{7}{2}}}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
2/3*(b*x^2 + a)^(3/2)*b^3*x/a^2 + sqrt(b*x^2 + a)*b^3*x/a + b^(5/2)*arcsin
h(b*x/sqrt(a*b)) - 8/15*(b*x^2 + a)^(5/2)*b^2/(a^2*x) - 2/15*(b*x^2 + a)^(
7/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(7/2)/(a*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 0.14 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx = -\frac{1}{2}\,b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(45\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8ab^{\frac{5}{2}} - 90\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^2b^{\frac{5}{2}} + 140\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^3b^{\frac{5}{2}} - 70\left(\sqrt{b}x\right.\right.}{15\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

input
```
integrate((b*x^2+a)^(5/2)/x^6,x, algorithm="giac")
```

output
```
-1/2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/15*(45*(sqrt(b)*x -
sqrt(b*x^2 + a))^8*a*b^(5/2) - 90*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(5
/2) + 140*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(5/2) - 70*(sqrt(b)*x - sq
rt(b*x^2 + a))^2*a^4*b^(5/2) + 23*a^5*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx=\int \frac{(b\,x^2+a)^{5/2}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(5/2)/x^6,x)
```

output
```
int((a + b*x^2)^(5/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)^{5/2}}{x^6}\,dx=\frac{-3\sqrt{b\,x^2+a}\,a^2-11\sqrt{b\,x^2+a}\,ab\,x^2-23\sqrt{b\,x^2+a}\,b^2x^4+15\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x}{15x^5}$$

input
```
int((b*x^2+a)^(5/2)/x^6,x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2 - 11*sqrt(a + b*x**2)*a*b*x**2 - 23*sqrt(a + b
*x**2)*b**2*x**4 + 15*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*
b**2*x**5 + 5*sqrt(b)*b**2*x**5)/(15*x**5)
```

3.413 $\int \frac{(a+bx^2)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^2)^{5/2}}{x^8} dx = -\frac{(a+bx^2)^{7/2}}{7ax^7}$$

output
```
-1/7*(b*x^2+a)^(7/2)/a/x^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{5/2}}{x^8} dx = -\frac{(a+bx^2)^{7/2}}{7ax^7}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^8} \, dx$$

$$\Big\downarrow \; 242$$

$$-\frac{(a+bx^2)^{7/2}}{7ax^7}$$

input
```
Int[(a + b*x^2)^(5/2)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(7/2)/(a*x^7)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{7a\,x^7}$	18
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{7a\,x^7}$	18
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{7a\,x^7}$	18
orering	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{7a\,x^7}$	18
trager	$-\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7a\,x^7}$	47
risch	$-\frac{(b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3)\sqrt{b\,x^2+a}}{7a\,x^7}$	47

input
```
int((b*x^2+a)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/7*(b*x^2+a)^(7/2)/a/x^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = -\frac{(b^3x^6+3\,ab^2x^4+3\,a^2bx^2+a^3)\sqrt{bx^2+a}}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
-1/7*(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^3)*sqrt(b*x^2 + a)/(a*x^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(17) = 34$.

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.52

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6} - \frac{3ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^4} - \frac{3b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{7x^2} - \frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{7a}$$

input
```
integrate((b*x**2+a)**(5/2)/x**8,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 3*a*b**(3/2)*sqrt(a/(b*x**2)
 + 1)/(7*x**4) - 3*b**(5/2)*sqrt(a/(b*x**2) + 1)/(7*x**2) - b**(7/2)*sqrt(
a/(b*x**2) + 1)/(7*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = -\frac{(bx^2+a)^{\frac{7}{2}}}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
-1/7*(b*x^2 + a)^(7/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(17) = 34$.

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.38

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = \frac{2\left(7\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{7}{2}} + 35\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{7}{2}} + 21\left(\sqrt{b}x-\sqrt{bx^2+a}\right)\right.}{7\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

input
```
integrate((b*x^2+a)^(5/2)/x^8,x, algorithm="giac")
```

output
```
2/7*(7*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(7/2) + 35*(sqrt(b)*x - sqrt(b*x
^2 + a))^8*a^2*b^(7/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(7/2) +
a^6*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.38

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = -\frac{a^2\sqrt{b\,x^2+a}}{7\,x^7} - \frac{3\,b^2\sqrt{b\,x^2+a}}{7\,x^3} - \frac{b^3\sqrt{b\,x^2+a}}{7\,a\,x} - \frac{3\,a\,b\sqrt{b\,x^2+a}}{7\,x^5}$$

input
```
int((a + b*x^2)^(5/2)/x^8,x)
```

output
```
- (a^2*(a + b*x^2)^(1/2))/(7*x^7) - (3*b^2*(a + b*x^2)^(1/2))/(7*x^3) - (b
^3*(a + b*x^2)^(1/2))/(7*a*x) - (3*a*b*(a + b*x^2)^(1/2))/(7*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.90

$$\int \frac{(a+bx^2)^{5/2}}{x^8}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^3 - 3\sqrt{b\,x^2+a}\,a^2b\,x^2 - 3\sqrt{b\,x^2+a}\,a\,b^2x^4 - \sqrt{b\,x^2+a}\,b^3x^6 - \sqrt{b}\,b^3x^7}{7a\,x^7}$$

input
```
int((b*x^2+a)^(5/2)/x^8,x)
```

output
```
( - sqrt(a + b*x**2)*a**3 - 3*sqrt(a + b*x**2)*a**2*b*x**2 - 3*sqrt(a + b*
x**2)*a*b**2*x**4 - sqrt(a + b*x**2)*b**3*x**6 - sqrt(b)*b**3*x**7)/(7*a*x
**7)
```

3.414 $\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = -\frac{(a+bx^2)^{7/2}}{9ax^9} + \frac{2b(a+bx^2)^{7/2}}{63a^2x^7}$$

output
```
-1/9*(b*x^2+a)^(7/2)/a/x^9+2/63*b*(b*x^2+a)^(7/2)/a^2/x^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{(a+bx^2)^{7/2}(-7a+2bx^2)}{63a^2x^9}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^10,x]
```

output
```
((a + b*x^2)^(7/2)*(-7*a + 2*b*x^2))/(63*a^2*x^9)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}dx$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{2b\int \frac{(bx^2+a)^{5/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}$$

input
```
Int[(a + b*x^2)^(5/2)/x^10,x]
```

output
```
-1/9*(a + b*x^2)^(7/2)/(a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-2b\,x^2+7a)}{63x^9a^2}$	28
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-2b\,x^2+7a)}{63x^9a^2}$	28
orering	$-\frac{(b\,x^2+a)^{\frac{7}{2}}(-2b\,x^2+7a)}{63x^9a^2}$	28
default	$-\frac{(b\,x^2+a)^{\frac{7}{2}}}{9a\,x^9}+\frac{2b(b\,x^2+a)^{\frac{7}{2}}}{63a^2x^7}$	37
trager	$-\frac{(-2b^4x^8+a\,b^3x^6+15a^2b^2x^4+19a^3b\,x^2+7a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	60
risch	$-\frac{(-2b^4x^8+a\,b^3x^6+15a^2b^2x^4+19a^3b\,x^2+7a^4)\sqrt{b\,x^2+a}}{63x^9a^2}$	60

input
```
int((b*x^2+a)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/63*(b*x^2+a)^(7/2)*(-2*b*x^2+7*a)/x^9/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{(2\,b^4x^8-ab^3x^6-15\,a^2b^2x^4-19\,a^3bx^2-7\,a^4)\sqrt{bx^2+a}}{63\,a^2x^9}$$

input
```
integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
1/63*(2*b^4*x^8 - a*b^3*x^6 - 15*a^2*b^2*x^4 - 19*a^3*b*x^2 - 7*a^4)*sqrt(
b*x^2 + a)/(a^2*x^9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(37) = 74$.

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.75

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = -\frac{a^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8} - \frac{19ab^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6} - \frac{5b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{21x^4} - \frac{b^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{63ax^2} + \frac{2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{63a^2}$$

input
```
integrate((b*x**2+a)**(5/2)/x**10,x)
```

output
```
-a**2*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 19*a*b**(3/2)*sqrt(a/(b*x**2
) + 1)/(63*x**6) - 5*b**(5/2)*sqrt(a/(b*x**2) + 1)/(21*x**4) - b**(7/2)*sq
rt(a/(b*x**2) + 1)/(63*a*x**2) + 2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(63*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{2\,(bx^2+a)^{\frac{7}{2}}b}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{7}{2}}}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
2/63*(b*x^2 + a)^(7/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(7/2)/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 220, normalized size of antiderivative = 5.00

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{4\left(63\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{9}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{9}{2}}+315\left(\sqrt{b}x-\sqrt{bx^2+\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/x^10,x, algorithm="giac")
```

output
```
4/63*(63*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(9/2) + 105*(sqrt(b)*x - sqrt(
b*x^2 + a))^12*a*b^(9/2) + 315*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^2*b^(9/2
) + 189*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(9/2) + 189*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*a^4*b^(9/2) + 27*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(9/
2) + 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(9/2) - a^7*b^(9/2))/((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{2\,b^4\,\sqrt{b\,x^2+a}}{63\,a^2\,x} - \frac{5\,b^2\,\sqrt{b\,x^2+a}}{21\,x^5} - \frac{b^3\,\sqrt{b\,x^2+a}}{63\,a\,x^3} - \frac{a^2\,\sqrt{b\,x^2+a}}{9\,x^9} - \frac{19\,a\,b\,\sqrt{b\,x^2+a}}{63\,x^7}$$

input
```
int((a + b*x^2)^(5/2)/x^10,x)
```

output
```
(2*b^4*(a + b*x^2)^(1/2))/(63*a^2*x) - (5*b^2*(a + b*x^2)^(1/2))/(21*x^5)
- (b^3*(a + b*x^2)^(1/2))/(63*a*x^3) - (a^2*(a + b*x^2)^(1/2))/(9*x^9) - (
19*a*b*(a + b*x^2)^(1/2))/(63*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.30

$$\int \frac{(a+bx^2)^{5/2}}{x^{10}}\,dx = \frac{-7\sqrt{b\,x^2+a}\,a^4-19\sqrt{b\,x^2+a}\,a^3b\,x^2-15\sqrt{b\,x^2+a}\,a^2b^2x^4-\sqrt{b\,x^2+a}\,a\,b^3x^6+2\sqrt{b}}{63a^2x^9}$$

input
```
int((b*x^2+a)^(5/2)/x^10,x)
```

output
```
( - 7*sqrt(a + b*x**2)*a**4 - 19*sqrt(a + b*x**2)*a**3*b*x**2 - 15*sqrt(a
+ b*x**2)*a**2*b**2*x**4 - sqrt(a + b*x**2)*a*b**3*x**6 + 2*sqrt(a + b*x**
2)*b**4*x**8 - 2*sqrt(b)*b**4*x**9)/(63*a**2*x**9)
```

3.415 $\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = -\frac{(a+bx^2)^{7/2}}{11ax^{11}} + \frac{4b(a+bx^2)^{7/2}}{99a^2x^9} - \frac{8b^2(a+bx^2)^{7/2}}{693a^3x^7}$$

output
```
-1/11*(b*x^2+a)^(7/2)/a/x^11+4/99*b*(b*x^2+a)^(7/2)/a^2/x^9-8/693*b^2*(b*x
^2+a)^(7/2)/a^3/x^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = \frac{(a+bx^2)^{7/2}\,(-63a^2+28abx^2-8b^2x^4)}{693a^3x^{11}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^12,x]
```

output
```
((a + b*x^2)^(7/2)*(-63*a^2 + 28*a*b*x^2 - 8*b^2*x^4))/(693*a^3*x^11)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\int \frac{(bx^2+a)^{5/2}}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^2+a)^{5/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{7/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{4b\left(\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}$$

input
```
Int[(a + b*x^2)^(5/2)/x^12,x]
```

output
```
-1/11*(a + b*x^2)^(7/2)/(a*x^11) - (4*b*(-1/9*(a + b*x^2)^(7/2)/(a*x^9) +
(2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)))/(11*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(8 b^{2} x^{4}-28 a b\,x^{2}+63 a^{2}\right)}{693 x^{11} a^{3}}$	39
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(8 b^{2} x^{4}-28 a b\,x^{2}+63 a^{2}\right)}{693 x^{11} a^{3}}$	39
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(8 b^{2} x^{4}-28 a b\,x^{2}+63 a^{2}\right)}{693 x^{11} a^{3}}$	39
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11 a\,x^{11}}-\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9 a\,x^{9}}+\frac{2 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63 a^{2} x^{7}}\right)}{11 a}$	61
trager	$-\frac{\left(8 b^{5} x^{10}-4 a\,b^{4} x^{8}+3 a^{2} b^{3} x^{6}+113 a^{3} b^{2} x^{4}+161 a^{4} b\,x^{2}+63 a^{5}\right) \sqrt{b\,x^{2}+a}}{693 x^{11} a^{3}}$	72
risch	$-\frac{\left(8 b^{5} x^{10}-4 a\,b^{4} x^{8}+3 a^{2} b^{3} x^{6}+113 a^{3} b^{2} x^{4}+161 a^{4} b\,x^{2}+63 a^{5}\right) \sqrt{b\,x^{2}+a}}{693 x^{11} a^{3}}$	72

input
```
int((b*x^2+a)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/693*(b*x^2+a)^(7/2)*(8*b^2*x^4-28*a*b*x^2+63*a^2)/x^11/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = -\frac{(8\,b^5x^{10}-4\,ab^4x^8+3\,a^2b^3x^6+113\,a^3b^2x^4+161\,a^4bx^2+63\,a^5)\sqrt{bx^2+a}}{693\,a^3x^{11}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-1/693*(8*b^5*x^10 - 4*a*b^4*x^8 + 3*a^2*b^3*x^6 + 113*a^3*b^2*x^4 + 161*a
^4*b*x^2 + 63*a^5)*sqrt(b*x^2 + a)/(a^3*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. 2(61) = 122.

Time = 1.25 (sec) , antiderivative size = 481, normalized size of antiderivative = 7.07

$$\begin{aligned}\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = &-\frac{63a^7b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\cdot(693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12})} \\ &-\frac{287a^6b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{498a^5b^{\frac{13}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{390a^4b^{\frac{15}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{115a^3b^{\frac{17}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{3a^2b^{\frac{19}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{12ab^{\frac{21}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}} \\ &-\frac{8b^{\frac{23}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{693a^5b^4x^8+1386a^4b^5x^{10}+693a^3b^6x^{12}}\end{aligned}$$

input
```
integrate((b*x**2+a)**(5/2)/x**12,x)
```

output
```
-63*a**7*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**2*(693*a**5*b**4*x**8 + 1386*a*
*4*b**5*x**10 + 693*a**3*b**6*x**12)) - 287*a**6*b**(11/2)*sqrt(a/(b*x**2)
 + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12) -
498*a**5*b**(13/2)*x**2*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a*
*4*b**5*x**10 + 693*a**3*b**6*x**12) - 390*a**4*b**(15/2)*x**4*sqrt(a/(b*x
**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12
) - 115*a**3*b**(17/2)*x**6*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 138
6*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 3*a**2*b**(19/2)*x**8*sqrt(a/(b
*x**2) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**
12) - 12*a*b**(21/2)*x**10*sqrt(a/(b*x**2) + 1)/(693*a**5*b**4*x**8 + 1386
*a**4*b**5*x**10 + 693*a**3*b**6*x**12) - 8*b**(23/2)*x**12*sqrt(a/(b*x**2
) + 1)/(693*a**5*b**4*x**8 + 1386*a**4*b**5*x**10 + 693*a**3*b**6*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = -\frac{8\,(bx^2+a)^{\frac{7}{2}}b^2}{693\,a^3x^7} + \frac{4\,(bx^2+a)^{\frac{7}{2}}b}{99\,a^2x^9} - \frac{(bx^2+a)^{\frac{7}{2}}}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
-8/693*(b*x^2 + a)^(7/2)*b^2/(a^3*x^7) + 4/99*(b*x^2 + a)^(7/2)*b/(a^2*x^9
) - 1/11*(b*x^2 + a)^(7/2)/(a*x^11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = \frac{16\left(462\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}b^{\frac{11}{2}} + 1155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}ab^{\frac{11}{2}} + 2541\left(\sqrt{b}x-\sqrt{\right.\right.}{}}{}$$

input
```
integrate((b*x^2+a)^(5/2)/x^12,x, algorithm="giac")
```

output
```
16/693*(462*(sqrt(b)*x - sqrt(b*x^2 + a))^16*b^(11/2) + 1155*(sqrt(b)*x -
sqrt(b*x^2 + a))^14*a*b^(11/2) + 2541*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2
*b^(11/2) + 2079*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^3*b^(11/2) + 1485*(sqr
t(b)*x - sqrt(b*x^2 + a))^8*a^4*b^(11/2) + 297*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*a^5*b^(11/2) + 55*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(11/2) - 11*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^7*b^(11/2) + a^8*b^(11/2))/((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^11
```

Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.63

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = \frac{4\,b^4\,\sqrt{b\,x^2+a}}{693\,a^2\,x^3} - \frac{113\,b^2\,\sqrt{b\,x^2+a}}{693\,x^7} - \frac{b^3\,\sqrt{b\,x^2+a}}{231\,a\,x^5} - \frac{a^2\,\sqrt{b\,x^2+a}}{11\,x^{11}} - \frac{8\,b^5\,\sqrt{b\,x^2+a}}{693\,a^3\,x} - \frac{23\,a\,b\,\sqrt{b\,x^2+a}}{99\,x^9}$$

input
```
int((a + b*x^2)^(5/2)/x^12,x)
```

output
```
(4*b^4*(a + b*x^2)^(1/2))/(693*a^2*x^3) - (113*b^2*(a + b*x^2)^(1/2))/(693
*x^7) - (b^3*(a + b*x^2)^(1/2))/(231*a*x^5) - (a^2*(a + b*x^2)^(1/2))/(11*
x^11) - (8*b^5*(a + b*x^2)^(1/2))/(693*a^3*x) - (23*a*b*(a + b*x^2)^(1/2))
/(99*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^2)^{5/2}}{x^{12}}\,dx = \frac{-63\sqrt{b\,x^2+a}\,a^5 - 161\sqrt{b\,x^2+a}\,a^4b\,x^2 - 113\sqrt{b\,x^2+a}\,a^3b^2x^4 - 3\sqrt{b\,x^2+a}\,a^2b^3x^6 +}{693a^3x^{11}}$$

input
```
int((b*x^2+a)^(5/2)/x^12,x)
```

output
```
( - 63*sqrt(a + b*x**2)*a**5 - 161*sqrt(a + b*x**2)*a**4*b*x**2 - 113*sqrt
(a + b*x**2)*a**3*b**2*x**4 - 3*sqrt(a + b*x**2)*a**2*b**3*x**6 + 4*sqrt(a
 + b*x**2)*a*b**4*x**8 - 8*sqrt(a + b*x**2)*b**5*x**10 + 8*sqrt(b)*b**5*x*
*11)/(693*a**3*x**11)
```

3.416 $\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = -\frac{(a+bx^2)^{7/2}}{13ax^{13}}+\frac{6b(a+bx^2)^{7/2}}{143a^2x^{11}}-\frac{8b^2(a+bx^2)^{7/2}}{429a^3x^9}+\frac{16b^3(a+bx^2)^{7/2}}{3003a^4x^7}$$

output
```
-1/13*(b*x^2+a)^(7/2)/a/x^13+6/143*b*(b*x^2+a)^(7/2)/a^2/x^11-8/429*b^2*(b
*x^2+a)^(7/2)/a^3/x^9+16/3003*b^3*(b*x^2+a)^(7/2)/a^4/x^7
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{(a+bx^2)^{7/2}\left(-231a^3+126a^2bx^2-56ab^2x^4+16b^3x^6\right)}{3003a^4x^{13}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^14,x]
```

output
```
((a + b*x^2)^(7/2)*(-231*a^3 + 126*a^2*b*x^2 - 56*a*b^2*x^4 + 16*b^3*x^6))
/(3003*a^4*x^13)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\int \frac{(bx^2+a)^{5/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\int \frac{(bx^2+a)^{5/2}}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{(bx^2+a)^{5/2}}{x^8}dx}{9a}-\frac{(a+bx^2)^{7/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{2b(a+bx^2)^{7/2}}{63a^2x^7}-\frac{(a+bx^2)^{7/2}}{9ax^9}\right)}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}$$

input
```
Int[(a + b*x^2)^(5/2)/x^14,x]
```

output
```
-1/13*(a + b*x^2)^(7/2)/(a*x^13) - (6*b*(-1/11*(a + b*x^2)^(7/2)/(a*x^11)
- (4*b*(-1/9*(a + b*x^2)^(7/2)/(a*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x
^7)))/(11*a)))/(13*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-16 b^{3} x^{6}+56 a\,b^{2} x^{4}-126 a^{2} b\,x^{2}+231 a^{3}\right)}{3003 x^{13} a^{4}}$	50
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-16 b^{3} x^{6}+56 a\,b^{2} x^{4}-126 a^{2} b\,x^{2}+231 a^{3}\right)}{3003 x^{13} a^{4}}$	50
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \left(-16 b^{3} x^{6}+56 a\,b^{2} x^{4}-126 a^{2} b\,x^{2}+231 a^{3}\right)}{3003 x^{13} a^{4}}$	50
trager	$-\frac{\left(-16 b^{6} x^{12}+8 a\,b^{5} x^{10}-6 a^{2} b^{4} x^{8}+5 a^{3} b^{3} x^{6}+371 a^{4} b^{2} x^{4}+567 a^{5} b\,x^{2}+231 a^{6}\right) \sqrt{b\,x^{2}+a}}{3003 x^{13} a^{4}}$	83
risch	$-\frac{\left(-16 b^{6} x^{12}+8 a\,b^{5} x^{10}-6 a^{2} b^{4} x^{8}+5 a^{3} b^{3} x^{6}+371 a^{4} b^{2} x^{4}+567 a^{5} b\,x^{2}+231 a^{6}\right) \sqrt{b\,x^{2}+a}}{3003 x^{13} a^{4}}$	83
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{13 a\,x^{13}}-\frac{6 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11 a\,x^{11}}-\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9 a\,x^{9}}+\frac{2 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63 a^{2} x^{7}}\right)}{11 a}\right)}{13 a}$	85

input
```
int((b*x^2+a)^(5/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/3003*(b*x^2+a)^(7/2)*(-16*b^3*x^6+56*a*b^2*x^4-126*a^2*b*x^2+231*a^3)/x
^13/a^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{(16\,b^6x^{12} - 8\,ab^5x^{10} + 6\,a^2b^4x^8 - 5\,a^3b^3x^6 - 371\,a^4b^2x^4 - 567\,a^5bx^2 - 231\,a^6)\sqrt{bx^2 +}}{3003\,a^4x^{13}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="fricas")
```

output
```
1/3003*(16*b^6*x^12 - 8*a*b^5*x^10 + 6*a^2*b^4*x^8 - 5*a^3*b^3*x^6 - 371*a
^4*b^2*x^4 - 567*a^5*b*x^2 - 231*a^6)*sqrt(b*x^2 + a)/(a^4*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. 2(85) = 170.

Time = 1.42 (sec) , antiderivative size = 721, normalized size of antiderivative = 7.84

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = -\frac{231a^9b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} - \frac{1260a^8b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} - \frac{2765a^7b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} - \frac{3050a^6b^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} - \frac{1689a^5b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} - \frac{376a^4b^{\frac{29}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} + \frac{5a^3b^{\frac{31}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} + \frac{30a^2b^{\frac{33}{2}}x^{14}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} + \frac{40ab^{\frac{35}{2}}x^{16}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}} + \frac{16b^{\frac{37}{2}}x^{18}\sqrt{\frac{a}{bx^2}+1}}{3003a^7b^9x^{12}+9009a^6b^{10}x^{14}+9009a^5b^{11}x^{16}+3003a^4b^{12}x^{18}}$$

input

```
integrate((b*x**2+a)**(5/2)/x**14,x)
```

output
```
-231*a**9*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6
*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1260*a**8*
b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**1
0*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 2765*a**7*b**(2
3/2)*x**4*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**
14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 3050*a**6*b**(25/2)*
x**6*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 +
9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 1689*a**5*b**(27/2)*x**8*
sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*
a**5*b**11*x**16 + 3003*a**4*b**12*x**18) - 376*a**4*b**(29/2)*x**10*sqrt(
a/(b*x**2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*
b**11*x**16 + 3003*a**4*b**12*x**18) + 5*a**3*b**(31/2)*x**12*sqrt(a/(b*x*
*2) + 1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x
**16 + 3003*a**4*b**12*x**18) + 30*a**2*b**(33/2)*x**14*sqrt(a/(b*x**2) +
1)/(3003*a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 +
 3003*a**4*b**12*x**18) + 40*a*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(3003*
a**7*b**9*x**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**
4*b**12*x**18) + 16*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(3003*a**7*b**9*x
**12 + 9009*a**6*b**10*x**14 + 9009*a**5*b**11*x**16 + 3003*a**4*b**12*x**
18)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{16\,(bx^2+a)^{\frac{7}{2}}b^3}{3003\,a^4x^7} - \frac{8\,(bx^2+a)^{\frac{7}{2}}b^2}{429\,a^3x^9} + \frac{6\,(bx^2+a)^{\frac{7}{2}}b}{143\,a^2x^{11}} - \frac{(bx^2+a)^{\frac{7}{2}}}{13\,ax^{13}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="maxima")
```

output
```
16/3003*(b*x^2 + a)^(7/2)*b^3/(a^4*x^7) - 8/429*(b*x^2 + a)^(7/2)*b^2/(a^3
*x^9) + 6/143*(b*x^2 + a)^(7/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(7/2)/(a*x
^13)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{32\left(3003\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}b^{\frac{13}{2}}+9009\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{13}{2}}+18018\left(\sqrt{b}x-\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/x^14,x, algorithm="giac")
```

output
```
32/3003*(3003*(sqrt(b)*x - sqrt(b*x^2 + a))^18*b^(13/2) + 9009*(sqrt(b)*x
- sqrt(b*x^2 + a))^16*a*b^(13/2) + 18018*(sqrt(b)*x - sqrt(b*x^2 + a))^14*
a^2*b^(13/2) + 16302*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(13/2) + 10296
*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(13/2) + 2288*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*a^5*b^(13/2) + 286*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(13/2
) - 78*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(13/2) + 13*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^8*b^(13/2) - a^9*b^(13/2))/((sqrt(b)*x - sqrt(b*x^2 + a))
^2 - a)^13
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{2\,b^4\,\sqrt{b\,x^2+a}}{1001\,a^2\,x^5} - \frac{53\,b^2\,\sqrt{b\,x^2+a}}{429\,x^9} - \frac{5\,b^3\,\sqrt{b\,x^2+a}}{3003\,a\,x^7} - \frac{a^2\,\sqrt{b\,x^2+a}}{13\,x^{13}} - \frac{8\,b^5\,\sqrt{b\,x^2+a}}{3003\,a^3\,x^3} + \frac{16\,b^6\,\sqrt{b\,x^2+a}}{3003\,a^4\,x} - \frac{27\,a\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}$$

input
```
int((a + b*x^2)^(5/2)/x^14,x)
```

output
```
(2*b^4*(a + b*x^2)^(1/2))/(1001*a^2*x^5) - (53*b^2*(a + b*x^2)^(1/2))/(429
*x^9) - (5*b^3*(a + b*x^2)^(1/2))/(3003*a*x^7) - (a^2*(a + b*x^2)^(1/2))/(
13*x^13) - (8*b^5*(a + b*x^2)^(1/2))/(3003*a^3*x^3) + (16*b^6*(a + b*x^2)^
(1/2))/(3003*a^4*x) - (27*a*b*(a + b*x^2)^(1/2))/(143*x^11)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^{5/2}}{x^{14}}\,dx = \frac{-231\sqrt{b\,x^2+a}\,a^6-567\sqrt{b\,x^2+a}\,a^5b\,x^2-371\sqrt{b\,x^2+a}\,a^4b^2x^4-5\sqrt{b\,x^2+a}\,a^3b^3x^6}{3003a^4x^{13}}$$

input
```
int((b*x^2+a)^(5/2)/x^14,x)
```

output
```
( - 231*sqrt(a + b*x**2)*a**6 - 567*sqrt(a + b*x**2)*a**5*b*x**2 - 371*sqr
t(a + b*x**2)*a**4*b**2*x**4 - 5*sqrt(a + b*x**2)*a**3*b**3*x**6 + 6*sqrt(
a + b*x**2)*a**2*b**4*x**8 - 8*sqrt(a + b*x**2)*a*b**5*x**10 + 16*sqrt(a +
 b*x**2)*b**6*x**12 - 16*sqrt(b)*b**6*x**13)/(3003*a**4*x**13)
```

3.417 $\int \frac{(a+bx^2)^{5/2}}{x^{16}} dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}} dx = -\frac{(a+bx^2)^{7/2}}{15ax^{15}} + \frac{8b(a+bx^2)^{7/2}}{195a^2x^{13}} - \frac{16b^2(a+bx^2)^{7/2}}{715a^3x^{11}} + \frac{64b^3(a+bx^2)^{7/2}}{6435a^4x^9} - \frac{128b^4(a+bx^2)^{7/2}}{45045a^5x^7}$$

output
```
-1/15*(b*x^2+a)^(7/2)/a/x^15+8/195*b*(b*x^2+a)^(7/2)/a^2/x^13-16/715*b^2*(
b*x^2+a)^(7/2)/a^3/x^11+64/6435*b^3*(b*x^2+a)^(7/2)/a^4/x^9-128/45045*b^4*
(b*x^2+a)^(7/2)/a^5/x^7
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}} dx = \frac{(a+bx^2)^{7/2}(-3003a^4+1848a^3bx^2-1008a^2b^2x^4+448ab^3x^6-128b^4x^8)}{45045a^5x^{15}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^16,x]
```

output
```
((a + b*x^2)^(7/2)*(-3003*a^4 + 1848*a^3*b*x^2 - 1008*a^2*b^2*x^4 + 448*a*
b^3*x^6 - 128*b^4*x^8))/(45045*a^5*x^15)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\int \frac{(bx^2+a)^{5/2}}{x^{14}}dx}{15a}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\int \frac{(bx^2+a)^{5/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\int \frac{(bx^2+a)^{5/2}}{x^{10}}dx}{11a}-\frac{(a+bx^2)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+a\right)^{5/2}}{x^8}dx}{9a}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}$$

$\Big\downarrow$ 242

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{7/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}$$

input

```
Int[(a + b*x^2)^(5/2)/x^16,x]
```

output

```
-1/15*(a + b*x^2)^(7/2)/(a*x^15) - (8*b*(-1/13*(a + b*x^2)^(7/2)/(a*x^13)
- (6*b*(-1/11*(a + b*x^2)^(7/2)/(a*x^11) - (4*b*(-1/9*(a + b*x^2)^(7/2)/(a
*x^9) + (2*b*(a + b*x^2)^(7/2))/(63*a^2*x^7)))/(11*a)))/(13*a)))/(15*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(128b^{4}x^{8}-448a\,b^{3}x^{6}+1008a^{2}b^{2}x^{4}-1848a^{3}b\,x^{2}+3003a^{4}\right)}{45045x^{15}a^{5}}$	61
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(128b^{4}x^{8}-448a\,b^{3}x^{6}+1008a^{2}b^{2}x^{4}-1848a^{3}b\,x^{2}+3003a^{4}\right)}{45045x^{15}a^{5}}$	61
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}\left(128b^{4}x^{8}-448a\,b^{3}x^{6}+1008a^{2}b^{2}x^{4}-1848a^{3}b\,x^{2}+3003a^{4}\right)}{45045x^{15}a^{5}}$	61
trager	$-\frac{\left(128b^{7}x^{14}-64a\,b^{6}x^{12}+48a^{2}b^{5}x^{10}-40a^{3}b^{4}x^{8}+35a^{4}b^{3}x^{6}+4473a^{5}b^{2}x^{4}+7161a^{6}b\,x^{2}+3003a^{7}\right)\sqrt{b\,x^{2}+a}}{45045x^{15}a^{5}}$	94
risch	$-\frac{\left(128b^{7}x^{14}-64a\,b^{6}x^{12}+48a^{2}b^{5}x^{10}-40a^{3}b^{4}x^{8}+35a^{4}b^{3}x^{6}+4473a^{5}b^{2}x^{4}+7161a^{6}b\,x^{2}+3003a^{7}\right)\sqrt{b\,x^{2}+a}}{45045x^{15}a^{5}}$	94
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{15a\,x^{15}}-\frac{8b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{13a\,x^{13}}-\frac{6b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{11a\,x^{11}}-\frac{4b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9a\,x^{9}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63a^{2}x^{7}}\right)}{11a}\right)}{13a}\right)}{15a}$	109

input
```
int((b*x^2+a)^(5/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/45045*(b*x^2+a)^(7/2)*(128*b^4*x^8-448*a*b^3*x^6+1008*a^2*b^2*x^4-1848*
a^3*b*x^2+3003*a^4)/x^15/a^5
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = -\frac{(128\,b^7x^{14}-64\,ab^6x^{12}+48\,a^2b^5x^{10}-40\,a^3b^4x^8+35\,a^4b^3x^6+4473\,a^5b^2x^4+7161\,a^6bx^2+3003\,a^7)\sqrt{bx^2}}{45045\,a^5x^{15}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="fricas")
```

output
```
-1/45045*(128*b^7*x^14 - 64*a*b^6*x^12 + 48*a^2*b^5*x^10 - 40*a^3*b^4*x^8
+ 35*a^4*b^3*x^6 + 4473*a^5*b^2*x^4 + 7161*a^6*b*x^2 + 3003*a^7)*sqrt(b*x^
2 + a)/(a^5*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(109) = 218.

Time = 1.70 (sec) , antiderivative size = 1012, normalized size of antiderivative = 8.72

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(5/2)/x**16,x)
```

output
```
-3003*a**11*b**(33/2)*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 18018
0*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 4
5045*a**5*b**20*x**22) - 19173*a**10*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(
45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18
 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 51135*a**9*b**(37/2
)*x**4*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x*
*16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20
*x**22) - 72905*a**8*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16
*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b
**19*x**20 + 45045*a**5*b**20*x**22) - 58585*a**7*b**(41/2)*x**8*sqrt(a/(b
*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**
7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 25151*
a**6*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180
*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45
045*a**5*b**20*x**22) - 4501*a**5*b**(45/2)*x**12*sqrt(a/(b*x**2) + 1)/(45
045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16 + 270270*a**7*b**18*x**18 +
 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**22) - 35*a**4*b**(47/2)*x**
14*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x**14 + 180180*a**8*b**17*x**16
+ 270270*a**7*b**18*x**18 + 180180*a**6*b**19*x**20 + 45045*a**5*b**20*x**
22) - 280*a**3*b**(49/2)*x**16*sqrt(a/(b*x**2) + 1)/(45045*a**9*b**16*x...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = -\frac{128\,(bx^2+a)^{\frac{7}{2}}b^4}{45045\,a^5x^7} + \frac{64\,(bx^2+a)^{\frac{7}{2}}b^3}{6435\,a^4x^9} - \frac{16\,(bx^2+a)^{\frac{7}{2}}b^2}{715\,a^3x^{11}} + \frac{8\,(bx^2+a)^{\frac{7}{2}}b}{195\,a^2x^{13}} - \frac{(bx^2+a)^{\frac{7}{2}}}{15\,ax^{15}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="maxima")
```

output
```
-128/45045*(b*x^2 + a)^(7/2)*b^4/(a^5*x^7) + 64/6435*(b*x^2 + a)^(7/2)*b^3
/(a^4*x^9) - 16/715*(b*x^2 + a)^(7/2)*b^2/(a^3*x^11) + 8/195*(b*x^2 + a)^(
7/2)*b/(a^2*x^13) - 1/15*(b*x^2 + a)^(7/2)/(a*x^15)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(96) = 192$.

Time = 0.14 (sec) , antiderivative size = 300, normalized size of antiderivative = 2.59

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = \frac{256\left(18018\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{15}{2}}+60060\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{18}ab^{\frac{15}{2}}+115830\left(\sqrt{\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/x^16,x, algorithm="giac")
```

output
```
256/45045*(18018*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(15/2) + 60060*(sqrt(b
)*x - sqrt(b*x^2 + a))^18*a*b^(15/2) + 115830*(sqrt(b)*x - sqrt(b*x^2 + a)
)^16*a^2*b^(15/2) + 109395*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^3*b^(15/2) +
 65065*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^(15/2) + 15015*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^5*b^(15/2) + 1365*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^
6*b^(15/2) - 455*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^7*b^(15/2) + 105*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^8*b^(15/2) - 15*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a^9*b^(15/2) + a^10*b^(15/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^15
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = \frac{8\,b^4\,\sqrt{b\,x^2+a}}{9009\,a^2\,x^7}-\frac{71\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}-\frac{b^3\,\sqrt{b\,x^2+a}}{1287\,a\,x^9}-\frac{a^2\,\sqrt{b\,x^2+a}}{15\,x^{15}}-\frac{16\,b^5\,\sqrt{b\,x^2+a}}{15015\,a^3\,x^5}+\frac{64\,b^6\,\sqrt{b\,x^2+a}}{45045\,a^4\,x^3}-\frac{128\,b^7\,\sqrt{b\,x^2+a}}{45045\,a^5\,x}-\frac{31\,a\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}}$$

input
```
int((a + b*x^2)^(5/2)/x^16,x)
```

output
```
(8*b^4*(a + b*x^2)^(1/2))/(9009*a^2*x^7) - (71*b^2*(a + b*x^2)^(1/2))/(715
*x^11) - (b^3*(a + b*x^2)^(1/2))/(1287*a*x^9) - (a^2*(a + b*x^2)^(1/2))/(1
5*x^15) - (16*b^5*(a + b*x^2)^(1/2))/(15015*a^3*x^5) + (64*b^6*(a + b*x^2)
^(1/2))/(45045*a^4*x^3) - (128*b^7*(a + b*x^2)^(1/2))/(45045*a^5*x) - (31*
a*b*(a + b*x^2)^(1/2))/(195*x^13)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{5/2}}{x^{16}}\,dx = \frac{-3003\sqrt{b\,x^2+a}\,a^7-7161\sqrt{b\,x^2+a}\,a^6b\,x^2-4473\sqrt{b\,x^2+a}\,a^5b^2x^4-35\sqrt{b\,x^2+a}\,a^4$$

input
```
int((b*x^2+a)^(5/2)/x^16,x)
```

output
```
( - 3003*sqrt(a + b*x**2)*a**7 - 7161*sqrt(a + b*x**2)*a**6*b*x**2 - 4473*
sqrt(a + b*x**2)*a**5*b**2*x**4 - 35*sqrt(a + b*x**2)*a**4*b**3*x**6 + 40*
sqrt(a + b*x**2)*a**3*b**4*x**8 - 48*sqrt(a + b*x**2)*a**2*b**5*x**10 + 64
*sqrt(a + b*x**2)*a*b**6*x**12 - 128*sqrt(a + b*x**2)*b**7*x**14 + 128*sqr
t(b)*b**7*x**15)/(45045*a**5*x**15)
```

3.418 $\int \frac{(a+bx^2)^{5/2}}{x^{18}} dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}} dx = -\frac{(a+bx^2)^{7/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{7/2}}{51a^2x^{15}} - \frac{16b^2(a+bx^2)^{7/2}}{663a^3x^{13}} + \frac{32b^3(a+bx^2)^{7/2}}{2431a^4x^{11}} - \frac{128b^4(a+bx^2)^{7/2}}{21879a^5x^9} + \frac{256b^5(a+bx^2)^{7/2}}{153153a^6x^7}$$

output
```
-1/17*(b*x^2+a)^(7/2)/a/x^17+2/51*b*(b*x^2+a)^(7/2)/a^2/x^15-16/663*b^2*(b
*x^2+a)^(7/2)/a^3/x^13+32/2431*b^3*(b*x^2+a)^(7/2)/a^4/x^11-128/21879*b^4*
(b*x^2+a)^(7/2)/a^5/x^9+256/153153*b^5*(b*x^2+a)^(7/2)/a^6/x^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}} dx = \frac{(a+bx^2)^{7/2} (-9009a^5 + 6006a^4bx^2 - 3696a^3b^2x^4 + 2016a^2b^3x^6 - 896ab^4x^8 + 256b^5x}{153153a^6x^{17}}$$

input
```
Integrate[(a + b*x^2)^(5/2)/x^18,x]
```

output
```
((a + b*x^2)^(7/2)*(-9009*a^5 + 6006*a^4*b*x^2 - 3696*a^3*b^2*x^4 + 2016*a
^2*b^3*x^6 - 896*a*b^4*x^8 + 256*b^5*x^10))/(153153*a^6*x^17)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {245, 245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\int \frac{(bx^2+a)^{5/2}}{x^{16}}dx}{17a}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\int \frac{(bx^2+a)^{5/2}}{x^{14}}dx}{15a}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\int \frac{(bx^2+a)^{5/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{7/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{7/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int\frac{\left(bx^2+a\right)^{5/2}}{x^{10}}dx}{11a}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+a\right)^{5/2}}{x^8}dx}{9a}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{7/2}}{63a^2x^7}-\frac{\left(a+bx^2\right)^{7/2}}{9ax^9}\right)}{11a}-\frac{\left(a+bx^2\right)^{7/2}}{11ax^{11}}\right)}{13a}-\frac{\left(a+bx^2\right)^{7/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{7/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{7/2}}{17ax^{17}}$$

input
```
Int[(a + b*x^2)^(5/2)/x^18,x]
```

output
```
-1/17*(a + b*x^2)^(7/2)/(a*x^17) - (10*b*(-1/15*(a + b*x^2)^(7/2)/(a*x^15)
 - (8*b*(-1/13*(a + b*x^2)^(7/2)/(a*x^13) - (6*b*(-1/11*(a + b*x^2)^(7/2)/
(a*x^11) - (4*b*(-1/9*(a + b*x^2)^(7/2)/(a*x^9) + (2*b*(a + b*x^2)^(7/2))/
(63*a^2*x^7)))/(11*a)))/(13*a)))/(15*a)))/(17*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

method	result
gosper	$-\frac{(b x^2+a)^{\frac{7}{2}}(-256b^5x^{10}+896a b^4x^8-2016a^2b^3x^6+3696a^3b^2x^4-6006a^4b x^2+9009a^5)}{153153x^{17}a^6}$
pseudoelliptic	$-\frac{(b x^2+a)^{\frac{7}{2}}(-256b^5x^{10}+896a b^4x^8-2016a^2b^3x^6+3696a^3b^2x^4-6006a^4b x^2+9009a^5)}{153153x^{17}a^6}$
orering	$-\frac{(b x^2+a)^{\frac{7}{2}}(-256b^5x^{10}+896a b^4x^8-2016a^2b^3x^6+3696a^3b^2x^4-6006a^4b x^2+9009a^5)}{153153x^{17}a^6}$
trager	$-\frac{(-256b^8x^{16}+128a b^7x^{14}-96a^2b^6x^{12}+80a^3b^5x^{10}-70a^4b^4x^8+63a^5b^3x^6+12705a^6b^2x^4+21021a^7b x^2+9009a^8)\sqrt{b x^2+\cdots}}{153153x^{17}a^6}$
risch	$-\frac{(-256b^8x^{16}+128a b^7x^{14}-96a^2b^6x^{12}+80a^3b^5x^{10}-70a^4b^4x^8+63a^5b^3x^6+12705a^6b^2x^4+21021a^7b x^2+9009a^8)\sqrt{b x^2+\cdots}}{153153x^{17}a^6}$
default	$-\frac{(b x^2+a)^{\frac{7}{2}}}{17a x^{17}}-\frac{10b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{15a x^{15}}-\frac{8b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{13a x^{13}}-\frac{6b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{11a x^{11}}-\frac{4b\left(-\frac{(b x^2+a)^{\frac{7}{2}}}{9a x^{9}}+\frac{2b(b x^2+a)^{\frac{7}{2}}}{63a^2x^7}\right)}{11a}\right)}{13a}\right)}{15a}\right)}{17a}$

input
```
int((b*x^2+a)^(5/2)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/153153*(b*x^2+a)^(7/2)*(-256*b^5*x^10+896*a*b^4*x^8-2016*a^2*b^3*x^6+36
96*a^3*b^2*x^4-6006*a^4*b*x^2+9009*a^5)/x^17/a^6
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \frac{(256\,b^8x^{16} - 128\,ab^7x^{14} + 96\,a^2b^6x^{12} - 80\,a^3b^5x^{10} + 70\,a^4b^4x^8 - 63\,a^5b^3x^6 - 12705\,a^6}{153153\,a^6x^{17}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="fricas")
```

output
```
1/153153*(256*b^8*x^16 - 128*a*b^7*x^14 + 96*a^2*b^6*x^12 - 80*a^3*b^5*x^1
0 + 70*a^4*b^4*x^8 - 63*a^5*b^3*x^6 - 12705*a^6*b^2*x^4 - 21021*a^7*b*x^2
- 9009*a^8)*sqrt(b*x^2 + a)/(a^6*x^17)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1346 vs. 2(133) = 266.

Time = 2.12 (sec) , antiderivative size = 1346, normalized size of antiderivative = 9.61

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(5/2)/x**18,x)
```

output
```
-9009*a**13*b**(51/2)*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765
765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**2
2 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 66066*a**12*b**(5
3/2)*x**2*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b*
*26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a
**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 207900*a**11*b**(55/2)*x**4*s
qrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 +
 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x
**24 + 153153*a**6*b**30*x**26) - 363888*a**10*b**(57/2)*x**6*sqrt(a/(b*x*
*2) + 1)/(153153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a*
*9*b**27*x**20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 1531
53*a**6*b**30*x**26) - 382550*a**9*b**(59/2)*x**8*sqrt(a/(b*x**2) + 1)/(15
3153*a**11*b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**
20 + 1531530*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**3
0*x**26) - 241524*a**8*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(153153*a**11*
b**25*x**16 + 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 153153
0*a**8*b**28*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) -
84780*a**7*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(153153*a**11*b**25*x**16
+ 765765*a**10*b**26*x**18 + 1531530*a**9*b**27*x**20 + 1531530*a**8*b**28
*x**22 + 765765*a**7*b**29*x**24 + 153153*a**6*b**30*x**26) - 12768*a**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \frac{256\,(bx^2+a)^{\frac{7}{2}}b^5}{153153\,a^6x^7} - \frac{128\,(bx^2+a)^{\frac{7}{2}}b^4}{21879\,a^5x^9} + \frac{32\,(bx^2+a)^{\frac{7}{2}}b^3}{2431\,a^4x^{11}} - \frac{16\,(bx^2+a)^{\frac{7}{2}}b^2}{663\,a^3x^{13}} + \frac{2\,(bx^2+a)^{\frac{7}{2}}b}{51\,a^2x^{15}} - \frac{(bx^2+a)^{\frac{7}{2}}}{17\,ax^{17}}$$

input
```
integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="maxima")
```

output
```
256/153153*(b*x^2 + a)^(7/2)*b^5/(a^6*x^7) - 128/21879*(b*x^2 + a)^(7/2)*b
^4/(a^5*x^9) + 32/2431*(b*x^2 + a)^(7/2)*b^3/(a^4*x^11) - 16/663*(b*x^2 +
a)^(7/2)*b^2/(a^3*x^13) + 2/51*(b*x^2 + a)^(7/2)*b/(a^2*x^15) - 1/17*(b*x^
2 + a)^(7/2)/(a*x^17)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(116) = 232$.

Time = 0.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.34

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \frac{512\left(102102\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{17}{2}}+364650\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{17}{2}}+692835\left(\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(5/2)/x^18,x, algorithm="giac")
```

output
```
512/153153*(102102*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(17/2) + 364650*(sqr
t(b)*x - sqrt(b*x^2 + a))^20*a*b^(17/2) + 692835*(sqrt(b)*x - sqrt(b*x^2 +
 a))^18*a^2*b^(17/2) + 668525*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(17/2
) + 384098*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^4*b^(17/2) + 89726*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*a^5*b^(17/2) + 6188*(sqrt(b)*x - sqrt(b*x^2 + a))^
10*a^6*b^(17/2) - 2380*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(17/2) + 680*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^8*b^(17/2) - 136*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^9*b^(17/2) + 17*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(17/2) -
 a^11*b^(17/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^17
```

Mupad [B] (verification not implemented)

Time = 3.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \frac{10\,b^4\sqrt{b\,x^2+a}}{21879\,a^2\,x^9}-\frac{55\,b^2\sqrt{b\,x^2+a}}{663\,x^{13}} -\frac{b^3\sqrt{b\,x^2+a}}{2431\,a\,x^{11}}-\frac{a^2\sqrt{b\,x^2+a}}{17\,x^{17}}-\frac{80\,b^5\sqrt{b\,x^2+a}}{153153\,a^3\,x^7}+\frac{32\,b^6\sqrt{b\,x^2+a}}{51051\,a^4\,x^5} -\frac{128\,b^7\sqrt{b\,x^2+a}}{153153\,a^5\,x^3}+\frac{256\,b^8\sqrt{b\,x^2+a}}{153153\,a^6\,x}-\frac{7\,a\,b\sqrt{b\,x^2+a}}{51\,x^{15}}$$

input
```
int((a + b*x^2)^(5/2)/x^18,x)
```

output
```
(10*b^4*(a + b*x^2)^(1/2))/(21879*a^2*x^9) - (55*b^2*(a + b*x^2)^(1/2))/(6
63*x^13) - (b^3*(a + b*x^2)^(1/2))/(2431*a*x^11) - (a^2*(a + b*x^2)^(1/2))
/(17*x^17) - (80*b^5*(a + b*x^2)^(1/2))/(153153*a^3*x^7) + (32*b^6*(a + b*
x^2)^(1/2))/(51051*a^4*x^5) - (128*b^7*(a + b*x^2)^(1/2))/(153153*a^5*x^3)
 + (256*b^8*(a + b*x^2)^(1/2))/(153153*a^6*x) - (7*a*b*(a + b*x^2)^(1/2))/
(51*x^15)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)^{5/2}}{x^{18}}\,dx = \frac{-9009\sqrt{b\,x^2+a}\,a^8 - 21021\sqrt{b\,x^2+a}\,a^7b\,x^2 - 12705\sqrt{b\,x^2+a}\,a^6b^2x^4 - 63\sqrt{b\,x^2+a}$$

input
```
int((b*x^2+a)^(5/2)/x^18,x)
```

output
```
( - 9009*sqrt(a + b*x**2)*a**8 - 21021*sqrt(a + b*x**2)*a**7*b*x**2 - 1270
5*sqrt(a + b*x**2)*a**6*b**2*x**4 - 63*sqrt(a + b*x**2)*a**5*b**3*x**6 + 7
0*sqrt(a + b*x**2)*a**4*b**4*x**8 - 80*sqrt(a + b*x**2)*a**3*b**5*x**10 +
96*sqrt(a + b*x**2)*a**2*b**6*x**12 - 128*sqrt(a + b*x**2)*a*b**7*x**14 +
256*sqrt(a + b*x**2)*b**8*x**16 - 256*sqrt(b)*b**8*x**17)/(153153*a**6*x**
17)
```

3.419 $\int x^{15}(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 161

$$\int x^{15}(a+bx^2)^{9/2}\,dx = -\frac{a^7(a+bx^2)^{11/2}}{11b^8}+\frac{7a^6(a+bx^2)^{13/2}}{13b^8} -\frac{7a^5(a+bx^2)^{15/2}}{5b^8}+\frac{35a^4(a+bx^2)^{17/2}}{17b^8}-\frac{35a^3(a+bx^2)^{19/2}}{19b^8} +\frac{a^2(a+bx^2)^{21/2}}{b^8}-\frac{7a(a+bx^2)^{23/2}}{23b^8}+\frac{(a+bx^2)^{25/2}}{25b^8}$$

output

```
-1/11*a^7*(b*x^2+a)^(11/2)/b^8+7/13*a^6*(b*x^2+a)^(13/2)/b^8-7/5*a^5*(b*x^
2+a)^(15/2)/b^8+35/17*a^4*(b*x^2+a)^(17/2)/b^8-35/19*a^3*(b*x^2+a)^(19/2)/
b^8+a^2*(b*x^2+a)^(21/2)/b^8-7/23*a*(b*x^2+a)^(23/2)/b^8+1/25*(b*x^2+a)^(2
5/2)/b^8
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.58

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \frac{(a+bx^2)^{11/2}(-2048a^7+11264a^6bx^2-36608a^5b^2x^4+91520a^4b^3x^6-194480a^3b^4x^8+3695}{26558675b^8}$$

input
```
Integrate[x^15*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(-2048*a^7 + 11264*a^6*b*x^2 - 36608*a^5*b^2*x^4 + 915
20*a^4*b^3*x^6 - 194480*a^3*b^4*x^8 + 369512*a^2*b^5*x^10 - 646646*a*b^6*x
^12 + 1062347*b^7*x^14))/(26558675*b^8)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{15}(a+bx^2)^{9/2}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{14}(bx^2+a)^{9/2}\,dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^{23/2}}{b^7}-\frac{7a(bx^2+a)^{21/2}}{b^7}+\frac{21a^2(bx^2+a)^{19/2}}{b^7}-\frac{35a^3(bx^2+a)^{17/2}}{b^7}+\frac{35a^4(bx^2+a)^{15/2}}{b^7}-\frac{21a^5}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^7(a+bx^2)^{11/2}}{11b^8}+\frac{14a^6(a+bx^2)^{13/2}}{13b^8}-\frac{14a^5(a+bx^2)^{15/2}}{5b^8}+\frac{70a^4(a+bx^2)^{17/2}}{17b^8}-\frac{70a^3(a+bx^2)^{19/2}}{19b^8}+\right.$$

input
```
Int[x^15*(a + b*x^2)^(9/2),x]
```

output
```
((-2*a^7*(a + b*x^2)^(11/2))/(11*b^8) + (14*a^6*(a + b*x^2)^(13/2))/(13*b^
8) - (14*a^5*(a + b*x^2)^(15/2))/(5*b^8) + (70*a^4*(a + b*x^2)^(17/2))/(17
*b^8) - (70*a^3*(a + b*x^2)^(19/2))/(19*b^8) + (2*a^2*(a + b*x^2)^(21/2))/
b^8 - (14*a*(a + b*x^2)^(23/2))/(23*b^8) + (2*(a + b*x^2)^(25/2))/(25*b^8)
)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

method	result
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-1062347b^{7}x^{14}+646646a\,b^{6}x^{12}-369512a^{2}b^{5}x^{10}+194480a^{3}b^{4}x^{8}-91520a^{4}b^{3}x^{6}+36608a^{5}b^{2}x^{4}-11264a^{6}b\right.}{26558675b^{8}}$
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-1062347b^{7}x^{14}+646646a\,b^{6}x^{12}-369512a^{2}b^{5}x^{10}+194480a^{3}b^{4}x^{8}-91520a^{4}b^{3}x^{6}+36608a^{5}b^{2}x^{4}-11264a^{6}b\right.}{26558675b^{8}}$
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-1062347b^{7}x^{14}+646646a\,b^{6}x^{12}-369512a^{2}b^{5}x^{10}+194480a^{3}b^{4}x^{8}-91520a^{4}b^{3}x^{6}+36608a^{5}b^{2}x^{4}-11264a^{6}b\right.}{26558675b^{8}}$
trager	$-\frac{\left(-1062347b^{12}x^{24}-4665089a\,x^{22}b^{11}-7759752a^{2}x^{20}b^{10}-5810090a^{3}b^{9}x^{18}-1659515x^{16}a^{4}b^{8}-429x^{14}a^{5}b^{7}+462x^{12}a^{6}b^{6}\right.}{26558675b^{8}}$
risch	$-\frac{\left(-1062347b^{12}x^{24}-4665089a\,x^{22}b^{11}-7759752a^{2}x^{20}b^{10}-5810090a^{3}b^{9}x^{18}-1659515x^{16}a^{4}b^{8}-429x^{14}a^{5}b^{7}+462x^{12}a^{6}b^{6}\right.}{26558675b^{8}}$
	$14a\left(\frac{x^{12}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{23b}-\frac{12a\left(\frac{x^{10}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{21b}-\frac{10a\left(\frac{x^{8}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^{4}\left(b\,x^{2}+a\right)}{15b}\right.}{\cdots}\right.}{\cdots}\right.}{21b}\right.}{23b}\right.$

input
```
int(x^15*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/26558675*(b*x^2+a)^(11/2)*(-1062347*b^7*x^14+646646*a*b^6*x^12-369512*a
^2*b^5*x^10+194480*a^3*b^4*x^8-91520*a^4*b^3*x^6+36608*a^5*b^2*x^4-11264*a
^6*b*x^2+2048*a^7)/b^8
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \frac{(1062347\,b^{12}x^{24}+4665089\,ab^{11}x^{22}+7759752\,a^2b^{10}x^{20}+5810090\,a^3b^9x^{18}+1659515\,a^4b^8x^{16}$$

input
```
integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/26558675*(1062347*b^12*x^24 + 4665089*a*b^11*x^22 + 7759752*a^2*b^10*x^2
0 + 5810090*a^3*b^9*x^18 + 1659515*a^4*b^8*x^16 + 429*a^5*b^7*x^14 - 462*a
^6*b^6*x^12 + 504*a^7*b^5*x^10 - 560*a^8*b^4*x^8 + 640*a^9*b^3*x^6 - 768*a
^10*b^2*x^4 + 1024*a^11*b*x^2 - 2048*a^12)*sqrt(b*x^2 + a)/b^8
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(150) = 300.

Time = 2.64 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.87

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \begin{cases} -\frac{2048a^{12}\sqrt{a+bx^2}}{26558675b^8}+\frac{1024a^{11}x^2\sqrt{a+bx^2}}{26558675b^7}-\frac{768a^{10}x^4\sqrt{a+bx^2}}{26558675b^6}+\frac{128a^9x^6\sqrt{a+bx^2}}{5311735b^5}-\frac{112a^8x^8\sqrt{a+bx^2}}{5311735b^4}+\frac{504a^7x^{10}}{26558} \\ \frac{a^{\frac{9}{2}}x^{16}}{16} \end{cases}$$

input
```
integrate(x**15*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-2048*a**12*sqrt(a + b*x**2)/(26558675*b**8) + 1024*a**11*x**2*
sqrt(a + b*x**2)/(26558675*b**7) - 768*a**10*x**4*sqrt(a + b*x**2)/(265586
75*b**6) + 128*a**9*x**6*sqrt(a + b*x**2)/(5311735*b**5) - 112*a**8*x**8*s
qrt(a + b*x**2)/(5311735*b**4) + 504*a**7*x**10*sqrt(a + b*x**2)/(26558675
*b**3) - 42*a**6*x**12*sqrt(a + b*x**2)/(2414425*b**2) + 3*a**5*x**14*sqrt
(a + b*x**2)/(185725*b) + 2321*a**4*x**16*sqrt(a + b*x**2)/37145 + 478*a**
3*b*x**18*sqrt(a + b*x**2)/2185 + 168*a**2*b**2*x**20*sqrt(a + b*x**2)/575
 + 101*a*b**3*x**22*sqrt(a + b*x**2)/575 + b**4*x**24*sqrt(a + b*x**2)/25,
 Ne(b, 0)), (a**(9/2)*x**16/16, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.95

$$\int x^{15}\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{14}}{25\,b}-\frac{14\left(bx^2+a\right)^{\frac{11}{2}}ax^{12}}{575\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{11}{2}}a^2x^{10}}{575\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{11}{2}}a^3x^8}{2185\,b^4}+\frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^6}{37145\,b^5}-\frac{256\left(bx^2+a\right)^{\frac{11}{2}}a^5x^4}{185725\,b^6}+\frac{1024\left(bx^2+a\right)^{\frac{11}{2}}a^6x^2}{2414425\,b^7}-\frac{2048\left(bx^2+a\right)^{\frac{11}{2}}a^7}{26558675\,b^8}$$

input
```
integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/25*(b*x^2 + a)^(11/2)*x^14/b - 14/575*(b*x^2 + a)^(11/2)*a*x^12/b^2 + 8/
575*(b*x^2 + a)^(11/2)*a^2*x^10/b^3 - 16/2185*(b*x^2 + a)^(11/2)*a^3*x^8/b
^4 + 128/37145*(b*x^2 + a)^(11/2)*a^4*x^6/b^5 - 256/185725*(b*x^2 + a)^(11
/2)*a^5*x^4/b^6 + 1024/2414425*(b*x^2 + a)^(11/2)*a^6*x^2/b^7 - 2048/26558
675*(b*x^2 + a)^(11/2)*a^7/b^8
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.70

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \frac{1062347\,(bx^2+a)^{\frac{25}{2}} - 8083075\,(bx^2+a)^{\frac{23}{2}}a + 26558675\,(bx^2+a)^{\frac{21}{2}}a^2 - 48923875\,(bx^2+a}{}$$

input
```
integrate(x^15*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/26558675*(1062347*(b*x^2 + a)^(25/2) - 8083075*(b*x^2 + a)^(23/2)*a + 26
558675*(b*x^2 + a)^(21/2)*a^2 - 48923875*(b*x^2 + a)^(19/2)*a^3 + 54679625
*(b*x^2 + a)^(17/2)*a^4 - 37182145*(b*x^2 + a)^(15/2)*a^5 + 14300825*(b*x^
2 + a)^(13/2)*a^6 - 2414425*(b*x^2 + a)^(11/2)*a^7)/b^8
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.88

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{2321\,a^4\,x^{16}}{37145} - \frac{2048\,a^{12}}{26558675\,b^8} + \frac{b^4\,x^{24}}{25} + \frac{478\,a^3\,b\,x^{18}}{2185} + \frac{101\,a\,b^3\,x^{22}}{575} + \frac{3\,a^5\,x^{14}}{185725\,b} - \frac{42\,a^6\,x^{12}}{2414425\,b^2} + \frac{504\,a^7\,x^{10}}{26558675\,b^3} - \frac{112\,a^8\,x^8}{5311735\,b^4} + \frac{128\,a^9\,x^6}{5311735\,b^5} - \frac{768\,a^{10}\,x^4}{26558675\,b^6} + \frac{1024\,a^{11}\,x^2}{26558675\,b^7} + \frac{168\,a^2\,b^2\,x^{20}}{575}\right)$$

input
```
int(x^15*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((2321*a^4*x^16)/37145 - (2048*a^12)/(26558675*b^8) + (b
^4*x^24)/25 + (478*a^3*b*x^18)/2185 + (101*a*b^3*x^22)/575 + (3*a^5*x^14)/
(185725*b) - (42*a^6*x^12)/(2414425*b^2) + (504*a^7*x^10)/(26558675*b^3) -
 (112*a^8*x^8)/(5311735*b^4) + (128*a^9*x^6)/(5311735*b^5) - (768*a^10*x^4
)/(26558675*b^6) + (1024*a^11*x^2)/(26558675*b^7) + (168*a^2*b^2*x^20)/575
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.89

$$\int x^{15}(a+bx^2)^{9/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(1062347b^{12}x^{24}+4665089a\,b^{11}x^{22}+7759752a^2b^{10}x^{20}+5810090a^3b^9x^{18}+16595}{}$$

input
```
int(x^15*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2048*a**12 + 1024*a**11*b*x**2 - 768*a**10*b**2*x**4
 + 640*a**9*b**3*x**6 - 560*a**8*b**4*x**8 + 504*a**7*b**5*x**10 - 462*a**
6*b**6*x**12 + 429*a**5*b**7*x**14 + 1659515*a**4*b**8*x**16 + 5810090*a**
3*b**9*x**18 + 7759752*a**2*b**10*x**20 + 4665089*a*b**11*x**22 + 1062347*
b**12*x**24))/(26558675*b**8)
```

3.420 $\int x^{13}(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int x^{13}\left(a+bx^2\right)^{9/2}\,dx=\frac{a^6(a+bx^2)^{11/2}}{11b^7}-\frac{6a^5(a+bx^2)^{13/2}}{13b^7}+\frac{a^4(a+bx^2)^{15/2}}{b^7}-\frac{20a^3(a+bx^2)^{17/2}}{17b^7}+\frac{15a^2(a+bx^2)^{19/2}}{19b^7}-\frac{2a(a+bx^2)^{21/2}}{7b^7}+\frac{(a+bx^2)^{23/2}}{23b^7}$$

output
```
1/11*a^6*(b*x^2+a)^(11/2)/b^7-6/13*a^5*(b*x^2+a)^(13/2)/b^7+a^4*(b*x^2+a)^
(15/2)/b^7-20/17*a^3*(b*x^2+a)^(17/2)/b^7+15/19*a^2*(b*x^2+a)^(19/2)/b^7-2
/7*a*(b*x^2+a)^(21/2)/b^7+1/23*(b*x^2+a)^(23/2)/b^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

$$\int x^{13}(a+bx^2)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}\,(1024a^6-5632a^5bx^2+18304a^4b^2x^4-45760a^3b^3x^6+97240a^2b^4x^8-184756a}{7436429b^7}$$

input
```
Integrate[x^13*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(1024*a^6 - 5632*a^5*b*x^2 + 18304*a^4*b^2*x^4 - 45760
*a^3*b^3*x^6 + 97240*a^2*b^4*x^8 - 184756*a*b^5*x^10 + 323323*b^6*x^12))/(
7436429*b^7)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{13}\left(a+bx^2\right)^{9/2}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{12}\left(bx^2+a\right)^{9/2}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{21/2}}{b^6}-\frac{6a\left(bx^2+a\right)^{19/2}}{b^6}+\frac{15a^2\left(bx^2+a\right)^{17/2}}{b^6}-\frac{20a^3\left(bx^2+a\right)^{15/2}}{b^6}+\frac{15a^4\left(bx^2+a\right)^{13/2}}{b^6}-\frac{6a^5(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^6\left(a+bx^2\right)^{11/2}}{11b^7}-\frac{12a^5\left(a+bx^2\right)^{13/2}}{13b^7}+\frac{2a^4\left(a+bx^2\right)^{15/2}}{b^7}-\frac{40a^3\left(a+bx^2\right)^{17/2}}{17b^7}+\frac{30a^2\left(a+bx^2\right)^{19/2}}{19b^7}+\frac{2(a\right.$$

input
```
Int[x^13*(a + b*x^2)^(9/2),x]
```

output
```
((2*a^6*(a + b*x^2)^(11/2))/(11*b^7) - (12*a^5*(a + b*x^2)^(13/2))/(13*b^7
) + (2*a^4*(a + b*x^2)^(15/2))/b^7 - (40*a^3*(a + b*x^2)^(17/2))/(17*b^7)
+ (30*a^2*(a + b*x^2)^(19/2))/(19*b^7) - (4*a*(a + b*x^2)^(21/2))/(7*b^7)
+ (2*(a + b*x^2)^(23/2))/(23*b^7))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

method	result
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(323323b^{6}x^{12}-184756a\,b^{5}x^{10}+97240a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+18304a^{4}b^{2}x^{4}-5632a^{5}b\,x^{2}+1024a^{6}\right)}{7436429b^{7}}$
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(323323b^{6}x^{12}-184756a\,b^{5}x^{10}+97240a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+18304a^{4}b^{2}x^{4}-5632a^{5}b\,x^{2}+1024a^{6}\right)}{7436429b^{7}}$
orering	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(323323b^{6}x^{12}-184756a\,b^{5}x^{10}+97240a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+18304a^{4}b^{2}x^{4}-5632a^{5}b\,x^{2}+1024a^{6}\right)}{7436429b^{7}}$
trager	$\frac{\left(323323b^{11}x^{22}+1431859a\,b^{10}x^{20}+2406690a^{2}b^{9}x^{18}+1826110a^{3}b^{8}x^{16}+530959a^{4}b^{7}x^{14}+231a^{5}b^{6}x^{12}-252a^{6}b^{5}x^{10}+280a^{7}\cdots\right.}{7436429b^{7}}$
risch	$\frac{\left(323323b^{11}x^{22}+1431859a\,b^{10}x^{20}+2406690a^{2}b^{9}x^{18}+1826110a^{3}b^{8}x^{16}+530959a^{4}b^{7}x^{14}+231a^{5}b^{6}x^{12}-252a^{6}b^{5}x^{10}+280a^{7}\cdots\right.}{7436429b^{7}}$
default	$\frac{x^{12}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{23b}-\frac{12a\left(\frac{x^{10}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{21b}-\frac{10a\left(\frac{x^{8}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{1}{2}\cdots}}{13b}\cdots\right.}{\cdots}\right.}{17b}\cdots\right.}{19b}\cdots\right.}{21b}\cdots\right.}{23b}$

input
```
int(x^13*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7436429*(b*x^2+a)^(11/2)*(323323*b^6*x^12-184756*a*b^5*x^10+97240*a^2*b^
4*x^8-45760*a^3*b^3*x^6+18304*a^4*b^2*x^4-5632*a^5*b*x^2+1024*a^6)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int x^{13}(a+bx^2)^{9/2}\,dx = \frac{(323323\,b^{11}x^{22}+1431859\,ab^{10}x^{20}+2406690\,a^2b^9x^{18}+1826110\,a^3b^8x^{16}+530959\,a^4b^7x^{14}+\cdots}{7\cdots}$$

input
```
integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/7436429*(323323*b^11*x^22 + 1431859*a*b^10*x^20 + 2406690*a^2*b^9*x^18 +
 1826110*a^3*b^8*x^16 + 530959*a^4*b^7*x^14 + 231*a^5*b^6*x^12 - 252*a^6*b
^5*x^10 + 280*a^7*b^4*x^8 - 320*a^8*b^3*x^6 + 384*a^9*b^2*x^4 - 512*a^10*b
*x^2 + 1024*a^11)*sqrt(b*x^2 + a)/b^7
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. 2(129) = 258.

Time = 2.24 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.98

$$\int x^{13}(a+bx^2)^{9/2}\,dx = \begin{cases} \frac{1024a^{11}\sqrt{a+bx^2}}{7436429b^7} - \frac{512a^{10}x^2\sqrt{a+bx^2}}{7436429b^6} + \frac{384a^9x^4\sqrt{a+bx^2}}{7436429b^5} - \frac{320a^8x^6\sqrt{a+bx^2}}{7436429b^4} + \frac{40a^7x^8\sqrt{a+bx^2}}{1062347b^3} - \frac{36a^6x^{10}\sqrt{a+bx^2}}{1062347b^2} \cdots \\ \frac{a^{\frac{9}{2}}x^{14}}{14} \end{cases}$$

input
```
integrate(x**13*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((1024*a**11*sqrt(a + b*x**2)/(7436429*b**7) - 512*a**10*x**2*sqr
t(a + b*x**2)/(7436429*b**6) + 384*a**9*x**4*sqrt(a + b*x**2)/(7436429*b**
5) - 320*a**8*x**6*sqrt(a + b*x**2)/(7436429*b**4) + 40*a**7*x**8*sqrt(a +
 b*x**2)/(1062347*b**3) - 36*a**6*x**10*sqrt(a + b*x**2)/(1062347*b**2) +
3*a**5*x**12*sqrt(a + b*x**2)/(96577*b) + 3713*a**4*x**14*sqrt(a + b*x**2)
/52003 + 12770*a**3*b*x**16*sqrt(a + b*x**2)/52003 + 990*a**2*b**2*x**18*s
qrt(a + b*x**2)/3059 + 31*a*b**3*x**20*sqrt(a + b*x**2)/161 + b**4*x**22*s
qrt(a + b*x**2)/23, Ne(b, 0)), (a**(9/2)*x**14/14, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int x^{13}\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{12}}{23\,b}-\frac{4\left(bx^2+a\right)^{\frac{11}{2}}ax^{10}}{161\,b^2}+\frac{40\left(bx^2+a\right)^{\frac{11}{2}}a^2x^8}{3059\,b^3}-\frac{320\left(bx^2+a\right)^{\frac{11}{2}}a^3x^6}{52003\,b^4}+\frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^4}{52003\,b^5}-\frac{512\left(bx^2+a\right)^{\frac{11}{2}}a^5x^2}{676039\,b^6}+\frac{1024\left(bx^2+a\right)^{\frac{11}{2}}a^6}{7436429\,b^7}$$

input
```
integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/23*(b*x^2 + a)^(11/2)*x^12/b - 4/161*(b*x^2 + a)^(11/2)*a*x^10/b^2 + 40/
3059*(b*x^2 + a)^(11/2)*a^2*x^8/b^3 - 320/52003*(b*x^2 + a)^(11/2)*a^3*x^6
/b^4 + 128/52003*(b*x^2 + a)^(11/2)*a^4*x^4/b^5 - 512/676039*(b*x^2 + a)^(
11/2)*a^5*x^2/b^6 + 1024/7436429*(b*x^2 + a)^(11/2)*a^6/b^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int x^{13}\left(a+bx^2\right)^{9/2}\,dx=\frac{323323\left(bx^2+a\right)^{\frac{23}{2}}-2124694\left(bx^2+a\right)^{\frac{21}{2}}a+5870865\left(bx^2+a\right)^{\frac{19}{2}}a^2-8748740\left(bx^2+a\right)^{\frac{17}{2}}\ldots}{7436429\,b^7}$$

input
```
integrate(x^13*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/7436429*(323323*(b*x^2 + a)^(23/2) - 2124694*(b*x^2 + a)^(21/2)*a + 5870
865*(b*x^2 + a)^(19/2)*a^2 - 8748740*(b*x^2 + a)^(17/2)*a^3 + 7436429*(b*x
^2 + a)^(15/2)*a^4 - 3432198*(b*x^2 + a)^(13/2)*a^5 + 676039*(b*x^2 + a)^(
11/2)*a^6)/b^7
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.93

$$\int x^{13}(a+bx^2)^{9/2}\,dx=\sqrt{b\,x^2+a}\left(\frac{1024\,a^{11}}{7436429\,b^7}+\frac{3713\,a^4\,x^{14}}{52003}+\frac{b^4\,x^{22}}{23}+\frac{12770\,a^3\,b\,x^{16}}{52003}+\frac{31\,a\,b^3\,x^{20}}{161}+\frac{3\,a^5\,x^{12}}{96577\,b}-\frac{36\,a^6\,x^{10}}{1062347\,b^2}+\frac{40\,a^7\,x^8}{1062347\,b^3}-\frac{320\,a^8\,x^6}{7436429\,b^4}+\frac{384\,a^9\,x^4}{7436429\,b^5}-\frac{512\,a^{10}\,x^2}{7436429\,b^6}+\frac{990\,a^2\,b^2\,x^{18}}{3059}\right)$$

input
```
int(x^13*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((1024*a^11)/(7436429*b^7) + (3713*a^4*x^14)/52003 + (b^
4*x^22)/23 + (12770*a^3*b*x^16)/52003 + (31*a*b^3*x^20)/161 + (3*a^5*x^12)
/(96577*b) - (36*a^6*x^10)/(1062347*b^2) + (40*a^7*x^8)/(1062347*b^3) - (3
20*a^8*x^6)/(7436429*b^4) + (384*a^9*x^4)/(7436429*b^5) - (512*a^10*x^2)/(
7436429*b^6) + (990*a^2*b^2*x^18)/3059)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int x^{13}(a+bx^2)^{9/2}\,dx=\frac{\sqrt{b\,x^2+a}\,(323323b^{11}x^{22}+1431859a\,b^{10}x^{20}+2406690a^2b^9x^{18}+1826110a^3b^8x^{16}+530959a}{74}$$

input
```
int(x^13*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(1024*a**11 - 512*a**10*b*x**2 + 384*a**9*b**2*x**4 - 32
0*a**8*b**3*x**6 + 280*a**7*b**4*x**8 - 252*a**6*b**5*x**10 + 231*a**5*b**
6*x**12 + 530959*a**4*b**7*x**14 + 1826110*a**3*b**8*x**16 + 2406690*a**2*
b**9*x**18 + 1431859*a*b**10*x**20 + 323323*b**11*x**22))/(7436429*b**7)
```

3.421 $\int x^{11}(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int x^{11}\left(a+bx^2\right)^{9/2}\,dx=-\frac{a^5(a+bx^2)^{11/2}}{11b^6}+\frac{5a^4(a+bx^2)^{13/2}}{13b^6}-\frac{2a^3(a+bx^2)^{15/2}}{3b^6}+\frac{10a^2(a+bx^2)^{17/2}}{17b^6}-\frac{5a(a+bx^2)^{19/2}}{19b^6}+\frac{(a+bx^2)^{21/2}}{21b^6}$$

output
```
-1/11*a^5*(b*x^2+a)^(11/2)/b^6+5/13*a^4*(b*x^2+a)^(13/2)/b^6-2/3*a^3*(b*x^
2+a)^(15/2)/b^6+10/17*a^2*(b*x^2+a)^(17/2)/b^6-5/19*a*(b*x^2+a)^(19/2)/b^6
+1/21*(b*x^2+a)^(21/2)/b^6
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int x^{11}(a+bx^2)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}\left(-256a^5+1408a^4bx^2-4576a^3b^2x^4+11440a^2b^3x^6-24310ab^4x^8+46189b^5x^1\right.}{969969b^6}$$

input
```
Integrate[x^11*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(-256*a^5 + 1408*a^4*b*x^2 - 4576*a^3*b^2*x^4 + 11440*
a^2*b^3*x^6 - 24310*a*b^4*x^8 + 46189*b^5*x^10))/(969969*b^6)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+bx^2\right)^{9/2}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^{10}\left(bx^2+a\right)^{9/2}\,dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{19/2}}{b^5}-\frac{5a\left(bx^2+a\right)^{17/2}}{b^5}+\frac{10a^2\left(bx^2+a\right)^{15/2}}{b^5}-\frac{10a^3\left(bx^2+a\right)^{13/2}}{b^5}+\frac{5a^4\left(bx^2+a\right)^{11/2}}{b^5}-\frac{a^5\left(bx^2\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^5\left(a+bx^2\right)^{11/2}}{11b^6}+\frac{10a^4\left(a+bx^2\right)^{13/2}}{13b^6}-\frac{4a^3\left(a+bx^2\right)^{15/2}}{3b^6}+\frac{20a^2\left(a+bx^2\right)^{17/2}}{17b^6}+\frac{2\left(a+bx^2\right)^{21/2}}{21b^6}-\frac{10a\left(\right.}{}\right.$$

input
```
Int[x^11*(a + b*x^2)^(9/2),x]
```

output
```
((-2*a^5*(a + b*x^2)^(11/2))/(11*b^6) + (10*a^4*(a + b*x^2)^(13/2))/(13*b^
6) - (4*a^3*(a + b*x^2)^(15/2))/(3*b^6) + (20*a^2*(a + b*x^2)^(17/2))/(17*
b^6) - (10*a*(a + b*x^2)^(19/2))/(19*b^6) + (2*(a + b*x^2)^(21/2))/(21*b^6
))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-46189b^5x^{10}+24310a\,b^4x^8-11440a^2b^3x^6+4576a^3b^2x^4-1408a^4b\,x^2+256a^5)}{969969b^6}$
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-46189b^5x^{10}+24310a\,b^4x^8-11440a^2b^3x^6+4576a^3b^2x^4-1408a^4b\,x^2+256a^5)}{969969b^6}$
orering	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-46189b^5x^{10}+24310a\,b^4x^8-11440a^2b^3x^6+4576a^3b^2x^4-1408a^4b\,x^2+256a^5)}{969969b^6}$
trager	$-\frac{(-46189b^{10}x^{20}-206635a\,b^9x^{18}-351780a^2b^8x^{16}-271414a^3b^7x^{14}-80773a^4b^6x^{12}-63a^5b^5x^{10}+70a^6b^4x^8-80a^7b^3x^6+9}{969969b^6}$
risch	$-\frac{(-46189b^{10}x^{20}-206635a\,b^9x^{18}-351780a^2b^8x^{16}-271414a^3b^7x^{14}-80773a^4b^6x^{12}-63a^5b^5x^{10}+70a^6b^4x^8-80a^7b^3x^6+9}{969969b^6}$
default	$\frac{x^{10}(b\,x^2+a)^{\frac{11}{2}}}{21b}-\frac{10a\left(\frac{x^8\left(b\,x^2+a\right)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^6\left(b\,x^2+a\right)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^4\left(b\,x^2+a\right)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{11}{2}}}{13b}-\frac{2a\left(b\,x^2+a\right)^{\frac{11}{2}}}{143b^2}\right)}{15b}\right)}{17b}\right)}{19b}\right)}{21b}$

input
```
int(x^11*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/969969*(b*x^2+a)^(11/2)*(-46189*b^5*x^10+24310*a*b^4*x^8-11440*a^2*b^3*
x^6+4576*a^3*b^2*x^4-1408*a^4*b*x^2+256*a^5)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01

$$\int x^{11}(a+bx^2)^{9/2}\,dx = \frac{(46189\,b^{10}x^{20} + 206635\,ab^9x^{18} + 351780\,a^2b^8x^{16} + 271414\,a^3b^7x^{14} + 80773\,a^4b^6x^{12} + 63\,a^5b^5}{969969\,b^6}$$

input
```
integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/969969*(46189*b^10*x^20 + 206635*a*b^9*x^18 + 351780*a^2*b^8*x^16 + 2714
14*a^3*b^7*x^14 + 80773*a^4*b^6*x^12 + 63*a^5*b^5*x^10 - 70*a^6*b^4*x^8 +
80*a^7*b^3*x^6 - 96*a^8*b^2*x^4 + 128*a^9*b*x^2 - 256*a^10)*sqrt(b*x^2 + a
)/b^6
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(112) = 224.

Time = 1.93 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.07

$$\int x^{11}(a+bx^2)^{9/2}\,dx = \begin{cases} -\frac{256a^{10}\sqrt{a+bx^2}}{969969b^6} + \frac{128a^9x^2\sqrt{a+bx^2}}{969969b^5} - \frac{32a^8x^4\sqrt{a+bx^2}}{323323b^4} + \frac{80a^7x^6\sqrt{a+bx^2}}{969969b^3} - \frac{10a^6x^8\sqrt{a+bx^2}}{138567b^2} + \frac{3a^5x^{10}\sqrt{a+bx^2}}{46189b} \\ \frac{a^{\frac{9}{2}}x^{12}}{12} \end{cases}$$

input
```
integrate(x**11*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-256*a**10*sqrt(a + b*x**2)/(969969*b**6) + 128*a**9*x**2*sqrt(
a + b*x**2)/(969969*b**5) - 32*a**8*x**4*sqrt(a + b*x**2)/(323323*b**4) +
80*a**7*x**6*sqrt(a + b*x**2)/(969969*b**3) - 10*a**6*x**8*sqrt(a + b*x**2
)/(138567*b**2) + 3*a**5*x**10*sqrt(a + b*x**2)/(46189*b) + 1049*a**4*x**1
2*sqrt(a + b*x**2)/12597 + 1898*a**3*b*x**14*sqrt(a + b*x**2)/6783 + 820*a
**2*b**2*x**16*sqrt(a + b*x**2)/2261 + 85*a*b**3*x**18*sqrt(a + b*x**2)/39
9 + b**4*x**20*sqrt(a + b*x**2)/21, Ne(b, 0)), (a**(9/2)*x**12/12, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.93

$$\int x^{11}\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^{10}}{21\,b}-\frac{10\left(bx^2+a\right)^{\frac{11}{2}}ax^8}{399\,b^2}+\frac{80\left(bx^2+a\right)^{\frac{11}{2}}a^2x^6}{6783\,b^3}-\frac{32\left(bx^2+a\right)^{\frac{11}{2}}a^3x^4}{6783\,b^4}+\frac{128\left(bx^2+a\right)^{\frac{11}{2}}a^4x^2}{88179\,b^5}-\frac{256\left(bx^2+a\right)^{\frac{11}{2}}a^5}{969969\,b^6}$$

input
```
integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/21*(b*x^2 + a)^(11/2)*x^10/b - 10/399*(b*x^2 + a)^(11/2)*a*x^8/b^2 + 80/
6783*(b*x^2 + a)^(11/2)*a^2*x^6/b^3 - 32/6783*(b*x^2 + a)^(11/2)*a^3*x^4/b
^4 + 128/88179*(b*x^2 + a)^(11/2)*a^4*x^2/b^5 - 256/969969*(b*x^2 + a)^(11
/2)*a^5/b^6
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.70

$$\int x^{11}\left(a+bx^2\right)^{9/2}\,dx=\frac{46189\left(bx^2+a\right)^{\frac{21}{2}}-255255\left(bx^2+a\right)^{\frac{19}{2}}a+570570\left(bx^2+a\right)^{\frac{17}{2}}a^2-646646\left(bx^2+a\right)^{\frac{15}{2}}a^3+}{969969\,b^6}$$

input
```
integrate(x^11*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/969969*(46189*(b*x^2 + a)^(21/2) - 255255*(b*x^2 + a)^(19/2)*a + 570570*
(b*x^2 + a)^(17/2)*a^2 - 646646*(b*x^2 + a)^(15/2)*a^3 + 373065*(b*x^2 + a
)^(13/2)*a^4 - 88179*(b*x^2 + a)^(11/2)*a^5)/b^6
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^{11}(a+bx^2)^{9/2}\,dx=\sqrt{b\,x^2+a}\left(\frac{1049\,a^4\,x^{12}}{12597}-\frac{256\,a^{10}}{969969\,b^6}+\frac{b^4\,x^{20}}{21}+\frac{1898\,a^3\,b\,x^{14}}{6783}+\frac{85\,a\,b^3\,x^{18}}{399}+\frac{3\,a^5\,x^{10}}{46189\,b}-\frac{10\,a^6\,x^8}{138567\,b^2}+\frac{80\,a^7\,x^6}{969969\,b^3}-\frac{32\,a^8\,x^4}{323323\,b^4}+\frac{128\,a^9\,x^2}{969969\,b^5}+\frac{820\,a^2\,b^2\,x^{16}}{2261}\right)$$

input
```
int(x^11*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((1049*a^4*x^12)/12597 - (256*a^10)/(969969*b^6) + (b^4*
x^20)/21 + (1898*a^3*b*x^14)/6783 + (85*a*b^3*x^18)/399 + (3*a^5*x^10)/(46
189*b) - (10*a^6*x^8)/(138567*b^2) + (80*a^7*x^6)/(969969*b^3) - (32*a^8*x
^4)/(323323*b^4) + (128*a^9*x^2)/(969969*b^5) + (820*a^2*b^2*x^16)/2261)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00

$$\int x^{11}(a+bx^2)^{9/2}\,dx=\frac{\sqrt{b\,x^2+a}\,(46189b^{10}x^{20}+206635a\,b^9x^{18}+351780a^2b^8x^{16}+271414a^3b^7x^{14}+80773a^4b^6x^{12}}{969969b^6}$$

input
```
int(x^11*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 256*a**10 + 128*a**9*b*x**2 - 96*a**8*b**2*x**4 + 80
*a**7*b**3*x**6 - 70*a**6*b**4*x**8 + 63*a**5*b**5*x**10 + 80773*a**4*b**6
*x**12 + 271414*a**3*b**7*x**14 + 351780*a**2*b**8*x**16 + 206635*a*b**9*x
**18 + 46189*b**10*x**20))/(969969*b**6)
```

3.422 $\int x^9(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int x^9\left(a+bx^2\right)^{9/2}\,dx=\frac{a^4(a+bx^2)^{11/2}}{11b^5}-\frac{4a^3(a+bx^2)^{13/2}}{13b^5}+\frac{2a^2(a+bx^2)^{15/2}}{5b^5}-\frac{4a(a+bx^2)^{17/2}}{17b^5}+\frac{(a+bx^2)^{19/2}}{19b^5}$$

output
```
1/11*a^4*(b*x^2+a)^(11/2)/b^5-4/13*a^3*(b*x^2+a)^(13/2)/b^5+2/5*a^2*(b*x^2
+a)^(15/2)/b^5-4/17*a*(b*x^2+a)^(17/2)/b^5+1/19*(b*x^2+a)^(19/2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.60

$$\int x^9\left(a+bx^2\right)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}\left(128a^4-704a^3bx^2+2288a^2b^2x^4-5720ab^3x^6+12155b^4x^8\right)}{230945b^5}$$

input
```
Integrate[x^9*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(128*a^4 - 704*a^3*b*x^2 + 2288*a^2*b^2*x^4 - 5720*a*b
^3*x^6 + 12155*b^4*x^8))/(230945*b^5)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9 \left(a+bx^2\right)^{9/2} dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^8 \left(bx^2+a\right)^{9/2} dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{17/2}}{b^4}-\frac{4a\left(bx^2+a\right)^{15/2}}{b^4}+\frac{6a^2\left(bx^2+a\right)^{13/2}}{b^4}-\frac{4a^3\left(bx^2+a\right)^{11/2}}{b^4}+\frac{a^4\left(bx^2+a\right)^{9/2}}{b^4}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2a^4\left(a+bx^2\right)^{11/2}}{11b^5}-\frac{8a^3\left(a+bx^2\right)^{13/2}}{13b^5}+\frac{4a^2\left(a+bx^2\right)^{15/2}}{5b^5}+\frac{2\left(a+bx^2\right)^{19/2}}{19b^5}-\frac{8a\left(a+bx^2\right)^{17/2}}{17b^5}\right)$$

input
```
Int[x^9*(a + b*x^2)^(9/2),x]
```

output
```
((2*a^4*(a + b*x^2)^(11/2))/(11*b^5) - (8*a^3*(a + b*x^2)^(13/2))/(13*b^5)
 + (4*a^2*(a + b*x^2)^(15/2))/(5*b^5) - (8*a*(a + b*x^2)^(17/2))/(17*b^5)
+ (2*(a + b*x^2)^(19/2))/(19*b^5))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.57

method	result
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(12155b^{4}x^{8}-5720a\,b^{3}x^{6}+2288a^{2}b^{2}x^{4}-704a^{3}b\,x^{2}+128a^{4}\right)}{230945b^{5}}$
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(12155b^{4}x^{8}-5720a\,b^{3}x^{6}+2288a^{2}b^{2}x^{4}-704a^{3}b\,x^{2}+128a^{4}\right)}{230945b^{5}}$
orering	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(12155b^{4}x^{8}-5720a\,b^{3}x^{6}+2288a^{2}b^{2}x^{4}-704a^{3}b\,x^{2}+128a^{4}\right)}{230945b^{5}}$
default	$\frac{x^{8}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{19b}-\frac{8a\left(\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17b}-\frac{6a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15b}-\frac{4a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143b^{2}}\right)}{15b}\right)}{17b}\right)}{19b}$
trager	$\frac{(12155b^{9}x^{18}+55055a\,b^{8}x^{16}+95238a^{2}b^{7}x^{14}+75086a^{3}b^{6}x^{12}+23063a^{4}b^{5}x^{10}+35a^{5}b^{4}x^{8}-40a^{6}b^{3}x^{6}+48a^{7}b^{2}x^{4}-64a^{8}b\,x^{2}+}{230945b^{5}}$
risch	$\frac{(12155b^{9}x^{18}+55055a\,b^{8}x^{16}+95238a^{2}b^{7}x^{14}+75086a^{3}b^{6}x^{12}+23063a^{4}b^{5}x^{10}+35a^{5}b^{4}x^{8}-40a^{6}b^{3}x^{6}+48a^{7}b^{2}x^{4}-64a^{8}b\,x^{2}+}{230945b^{5}}$

input
```
int(x^9*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/230945*(b*x^2+a)^(11/2)*(12155*b^4*x^8-5720*a*b^3*x^6+2288*a^2*b^2*x^4-7
04*a^3*b*x^2+128*a^4)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11

$$\int x^9(a+bx^2)^{9/2}\,dx = \frac{(12155\,b^9x^{18} + 55055\,ab^8x^{16} + 95238\,a^2b^7x^{14} + 75086\,a^3b^6x^{12} + 23063\,a^4b^5x^{10} + 35\,a^5b^4x^8 -}{230945\,b^5}$$

input
```
integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/230945*(12155*b^9*x^18 + 55055*a*b^8*x^16 + 95238*a^2*b^7*x^14 + 75086*a
^3*b^6*x^12 + 23063*a^4*b^5*x^10 + 35*a^5*b^4*x^8 - 40*a^6*b^3*x^6 + 48*a^
7*b^2*x^4 - 64*a^8*b*x^2 + 128*a^9)*sqrt(b*x^2 + a)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. 2(92) = 184.

Time = 1.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.28

$$\int x^9(a+bx^2)^{9/2}\,dx = \begin{cases} \frac{128a^9\sqrt{a+bx^2}}{230945b^5} - \frac{64a^8x^2\sqrt{a+bx^2}}{230945b^4} + \frac{48a^7x^4\sqrt{a+bx^2}}{230945b^3} - \frac{8a^6x^6\sqrt{a+bx^2}}{46189b^2} + \frac{7a^5x^8\sqrt{a+bx^2}}{46189b} + \frac{23063a^4x^{10}\sqrt{a+bx^2}}{230945} + \\ \frac{a^{\frac{9}{2}}x^{10}}{10} \end{cases}$$

input
```
integrate(x**9*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((128*a**9*sqrt(a + b*x**2)/(230945*b**5) - 64*a**8*x**2*sqrt(a +
 b*x**2)/(230945*b**4) + 48*a**7*x**4*sqrt(a + b*x**2)/(230945*b**3) - 8*a
**6*x**6*sqrt(a + b*x**2)/(46189*b**2) + 7*a**5*x**8*sqrt(a + b*x**2)/(461
89*b) + 23063*a**4*x**10*sqrt(a + b*x**2)/230945 + 6826*a**3*b*x**12*sqrt(
a + b*x**2)/20995 + 666*a**2*b**2*x**14*sqrt(a + b*x**2)/1615 + 77*a*b**3*
x**16*sqrt(a + b*x**2)/323 + b**4*x**18*sqrt(a + b*x**2)/19, Ne(b, 0)), (a
**(9/2)*x**10/10, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92

$$\int x^9 (a + bx^2)^{9/2} \, dx = \frac{(bx^2 + a)^{\frac{11}{2}} x^8}{19 \, b} - \frac{8 \, (bx^2 + a)^{\frac{11}{2}} a x^6}{323 \, b^2} + \frac{16 \, (bx^2 + a)^{\frac{11}{2}} a^2 x^4}{1615 \, b^3} - \frac{64 \, (bx^2 + a)^{\frac{11}{2}} a^3 x^2}{20995 \, b^4} + \frac{128 \, (bx^2 + a)^{\frac{11}{2}} a^4}{230945 \, b^5}$$

input
```
integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/19*(b*x^2 + a)^(11/2)*x^8/b - 8/323*(b*x^2 + a)^(11/2)*a*x^6/b^2 + 16/16
15*(b*x^2 + a)^(11/2)*a^2*x^4/b^3 - 64/20995*(b*x^2 + a)^(11/2)*a^3*x^2/b^
4 + 128/230945*(b*x^2 + a)^(11/2)*a^4/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.70

$$\int x^9 (a + bx^2)^{9/2} \, dx = \frac{12155 \, (bx^2 + a)^{\frac{19}{2}} - 54340 \, (bx^2 + a)^{\frac{17}{2}} a + 92378 \, (bx^2 + a)^{\frac{15}{2}} a^2 - 71060 \, (bx^2 + a)^{\frac{13}{2}} a^3 + 209}{230945 \, b^5}$$

input
```
integrate(x^9*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/230945*(12155*(b*x^2 + a)^(19/2) - 54340*(b*x^2 + a)^(17/2)*a + 92378*(b
*x^2 + a)^(15/2)*a^2 - 71060*(b*x^2 + a)^(13/2)*a^3 + 20995*(b*x^2 + a)^(1
1/2)*a^4)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07

$$\int x^9 \left(a+bx^2\right)^{9/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{128\,a^9}{230945\,b^5}+\frac{23063\,a^4\,x^{10}}{230945}+\frac{b^4\,x^{18}}{19}+\frac{6826\,a^3\,b\,x^{12}}{20995}+\frac{77\,a\,b^3\,x^{16}}{323}+\frac{7\,a^5\,x^8}{46189\,b}-\frac{8\,a^6\,x^6}{46189\,b^2}+\frac{48\,a^7\,x^4}{230945\,b^3}-\frac{64\,a^8\,x^2}{230945\,b^4}+\frac{666\,a^2\,b^2\,x^{14}}{1615}\right)$$

input
```
int(x^9*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((128*a^9)/(230945*b^5) + (23063*a^4*x^10)/230945 + (b^4
*x^18)/19 + (6826*a^3*b*x^12)/20995 + (77*a*b^3*x^16)/323 + (7*a^5*x^8)/(4
6189*b) - (8*a^6*x^6)/(46189*b^2) + (48*a^7*x^4)/(230945*b^3) - (64*a^8*x^
2)/(230945*b^4) + (666*a^2*b^2*x^14)/1615)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10

$$\int x^9 (a+bx^2)^{9/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(12155b^9x^{18}+55055a\,b^8x^{16}+95238a^2b^7x^{14}+75086a^3b^6x^{12}+23063a^4b^5x^{10}+35}{230945b^5}$$

input
```
int(x^9*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(128*a**9 - 64*a**8*b*x**2 + 48*a**7*b**2*x**4 - 40*a**6
*b**3*x**6 + 35*a**5*b**4*x**8 + 23063*a**4*b**5*x**10 + 75086*a**3*b**6*x
**12 + 95238*a**2*b**7*x**14 + 55055*a*b**8*x**16 + 12155*b**9*x**18))/(23
0945*b**5)
```

3.423 $\int x^7(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7(a+bx^2)^{9/2}\,dx=-\frac{a^3(a+bx^2)^{11/2}}{11b^4}+\frac{3a^2(a+bx^2)^{13/2}}{13b^4}-\frac{a(a+bx^2)^{15/2}}{5b^4}+\frac{(a+bx^2)^{17/2}}{17b^4}$$

output
```
-1/11*a^3*(b*x^2+a)^(11/2)/b^4+3/13*a^2*(b*x^2+a)^(13/2)/b^4-1/5*a*(b*x^2+
a)^(15/2)/b^4+1/17*(b*x^2+a)^(17/2)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7(a+bx^2)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}(-16a^3+88a^2bx^2-286ab^2x^4+715b^3x^6)}{12155b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(-16*a^3 + 88*a^2*b*x^2 - 286*a*b^2*x^4 + 715*b^3*x^6)
)/(12155*b^4)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^{9/2} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^{9/2} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{15/2}}{b^3}-\frac{3a\left(bx^2+a\right)^{13/2}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{11/2}}{b^3}-\frac{a^3\left(bx^2+a\right)^{9/2}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^3\left(a+bx^2\right)^{11/2}}{11b^4}+\frac{6a^2\left(a+bx^2\right)^{13/2}}{13b^4}+\frac{2\left(a+bx^2\right)^{17/2}}{17b^4}-\frac{2a\left(a+bx^2\right)^{15/2}}{5b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(9/2),x]
```

output
```
((-2*a^3*(a + b*x^2)^(11/2))/(11*b^4) + (6*a^2*(a + b*x^2)^(13/2))/(13*b^4
) - (2*a*(a + b*x^2)^(15/2))/(5*b^4) + (2*(a + b*x^2)^(17/2))/(17*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-715 b^{3} x^{6}+286 a\,b^{2} x^{4}-88 a^{2} b\,x^{2}+16 a^{3}\right)}{12155 b^{4}}$
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-715 b^{3} x^{6}+286 a\,b^{2} x^{4}-88 a^{2} b\,x^{2}+16 a^{3}\right)}{12155 b^{4}}$
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-715 b^{3} x^{6}+286 a\,b^{2} x^{4}-88 a^{2} b\,x^{2}+16 a^{3}\right)}{12155 b^{4}}$
default	$\frac{x^{6}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17 b}-\frac{6 a\left(\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15 b}-\frac{4 a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13 b}-\frac{2 a\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143 b^{2}}\right)}{15 b}\right)}{17 b}$
trager	$-\frac{\left(-715 b^{8} x^{16}-3289 a\,b^{7} x^{14}-5808 a^{2} b^{6} x^{12}-4714 a^{3} b^{5} x^{10}-1515 a^{4} b^{4} x^{8}-5 a^{5} b^{3} x^{6}+6 a^{6} b^{2} x^{4}-8 a^{7} b\,x^{2}+16 a^{8}\right)\sqrt{b\,x^{2}+a}}{12155 b^{4}}$
risch	$-\frac{\left(-715 b^{8} x^{16}-3289 a\,b^{7} x^{14}-5808 a^{2} b^{6} x^{12}-4714 a^{3} b^{5} x^{10}-1515 a^{4} b^{4} x^{8}-5 a^{5} b^{3} x^{6}+6 a^{6} b^{2} x^{4}-8 a^{7} b\,x^{2}+16 a^{8}\right)\sqrt{b\,x^{2}+a}}{12155 b^{4}}$

input
```
int(x^7*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12155*(b*x^2+a)^(11/2)*(-715*b^3*x^6+286*a*b^2*x^4-88*a^2*b*x^2+16*a^3)
/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int x^7 (a+bx^2)^{9/2}\,dx = \frac{(715\,b^8x^{16}+3289\,ab^7x^{14}+5808\,a^2b^6x^{12}+4714\,a^3b^5x^{10}+1515\,a^4b^4x^8+5\,a^5b^3x^6-6\,a^6b^2x^4}{12155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/12155*(715*b^8*x^16 + 3289*a*b^7*x^14 + 5808*a^2*b^6*x^12 + 4714*a^3*b^5
*x^10 + 1515*a^4*b^4*x^8 + 5*a^5*b^3*x^6 - 6*a^6*b^2*x^4 + 8*a^7*b*x^2 - 1
6*a^8)*sqrt(b*x^2 + a)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(70) = 140$.

Time = 1.24 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.55

$$\int x^7 (a+bx^2)^{9/2}\,dx = \begin{cases} -\frac{16a^8\sqrt{a+bx^2}}{12155b^4}+\frac{8a^7x^2\sqrt{a+bx^2}}{12155b^3}-\frac{6a^6x^4\sqrt{a+bx^2}}{12155b^2}+\frac{a^5x^6\sqrt{a+bx^2}}{2431b}+\frac{303a^4x^8\sqrt{a+bx^2}}{2431}+\frac{4714a^3bx^{10}\sqrt{a+bx^2}}{12155}+ \\ \frac{a^{\frac{9}{2}}x^8}{8} \end{cases}$$

input
```
integrate(x**7*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-16*a**8*sqrt(a + b*x**2)/(12155*b**4) + 8*a**7*x**2*sqrt(a + b
*x**2)/(12155*b**3) - 6*a**6*x**4*sqrt(a + b*x**2)/(12155*b**2) + a**5*x**
6*sqrt(a + b*x**2)/(2431*b) + 303*a**4*x**8*sqrt(a + b*x**2)/2431 + 4714*a
**3*b*x**10*sqrt(a + b*x**2)/12155 + 528*a**2*b**2*x**12*sqrt(a + b*x**2)/
1105 + 23*a*b**3*x**14*sqrt(a + b*x**2)/85 + b**4*x**16*sqrt(a + b*x**2)/1
7, Ne(b, 0)), (a**(9/2)*x**8/8, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int x^7\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{11}{2}}x^6}{17\,b}-\frac{2\left(bx^2+a\right)^{\frac{11}{2}}ax^4}{85\,b^2}+\frac{8\left(bx^2+a\right)^{\frac{11}{2}}a^2x^2}{1105\,b^3}-\frac{16\left(bx^2+a\right)^{\frac{11}{2}}a^3}{12155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/17*(b*x^2 + a)^(11/2)*x^6/b - 2/85*(b*x^2 + a)^(11/2)*a*x^4/b^2 + 8/1105
*(b*x^2 + a)^(11/2)*a^2*x^2/b^3 - 16/12155*(b*x^2 + a)^(11/2)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7\left(a+bx^2\right)^{9/2}\,dx=\frac{715\left(bx^2+a\right)^{\frac{17}{2}}-2431\left(bx^2+a\right)^{\frac{15}{2}}a+2805\left(bx^2+a\right)^{\frac{13}{2}}a^2-1105\left(bx^2+a\right)^{\frac{11}{2}}a^3}{12155\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/12155*(715*(b*x^2 + a)^(17/2) - 2431*(b*x^2 + a)^(15/2)*a + 2805*(b*x^2
+ a)^(13/2)*a^2 - 1105*(b*x^2 + a)^(11/2)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int x^7 (a+bx^2)^{9/2}\,dx = \sqrt{b\,x^2+a}\left(\frac{303\,a^4\,x^8}{2431}-\frac{16\,a^8}{12155\,b^4}+\frac{b^4\,x^{16}}{17}+\frac{4714\,a^3\,b\,x^{10}}{12155}+\frac{23\,a\,b^3\,x^{14}}{85}+\frac{a^5\,x^6}{2431\,b}-\frac{6\,a^6\,x^4}{12155\,b^2}+\frac{8\,a^7\,x^2}{12155\,b^3}+\frac{528\,a^2\,b^2\,x^{12}}{1105}\right)$$

input
```
int(x^7*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((303*a^4*x^8)/2431 - (16*a^8)/(12155*b^4) + (b^4*x^16)/
17 + (4714*a^3*b*x^10)/12155 + (23*a*b^3*x^14)/85 + (a^5*x^6)/(2431*b) - (
6*a^6*x^4)/(12155*b^2) + (8*a^7*x^2)/(12155*b^3) + (528*a^2*b^2*x^12)/1105
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.25

$$\int x^7 (a+bx^2)^{9/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(715b^8x^{16}+3289a\,b^7x^{14}+5808a^2b^6x^{12}+4714a^3b^5x^{10}+1515a^4b^4x^8+5a^5b^3x^6-}{12155b^4}$$

input
```
int(x^7*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**8 + 8*a**7*b*x**2 - 6*a**6*b**2*x**4 + 5*a**5*
b**3*x**6 + 1515*a**4*b**4*x**8 + 4714*a**3*b**5*x**10 + 5808*a**2*b**6*x*
*12 + 3289*a*b**7*x**14 + 715*b**8*x**16))/(12155*b**4)
```

3.424 $\int x^5(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5(a+bx^2)^{9/2}\,dx=\frac{a^2(a+bx^2)^{11/2}}{11b^3}-\frac{2a(a+bx^2)^{13/2}}{13b^3}+\frac{(a+bx^2)^{15/2}}{15b^3}$$

output
```
1/11*a^2*(b*x^2+a)^(11/2)/b^3-2/13*a*(b*x^2+a)^(13/2)/b^3+1/15*(b*x^2+a)^(
15/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5(a+bx^2)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}(8a^2-44abx^2+143b^2x^4)}{2145b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(8*a^2 - 44*a*b*x^2 + 143*b^2*x^4))/(2145*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a+bx^2)^{9/2} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^4 (bx^2+a)^{9/2} \, dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^{13/2}}{b^2}-\frac{2a(bx^2+a)^{11/2}}{b^2}+\frac{a^2(bx^2+a)^{9/2}}{b^2}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2a^2(a+bx^2)^{11/2}}{11b^3}+\frac{2(a+bx^2)^{15/2}}{15b^3}-\frac{4a(a+bx^2)^{13/2}}{13b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(9/2),x]
```

output
```
((2*a^2*(a + b*x^2)^(11/2))/(11*b^3) - (4*a*(a + b*x^2)^(13/2))/(13*b^3) +
 (2*(a + b*x^2)^(15/2))/(15*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143 b^{2} x^{4}-44 a b\,x^{2}+8 a^{2}\right)}{2145 b^{3}}$	36
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143 b^{2} x^{4}-44 a b\,x^{2}+8 a^{2}\right)}{2145 b^{3}}$	36
orering	$\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(143 b^{2} x^{4}-44 a b\,x^{2}+8 a^{2}\right)}{2145 b^{3}}$	36
default	$\frac{x^{4}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15 b}-\frac{4 a\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13 b}-\frac{2 a\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143 b^{2}}\right)}{15 b}$	58
trager	$\frac{\left(143 b^{7} x^{14}+671 a\,b^{6} x^{12}+1218 a^{2} b^{5} x^{10}+1030 a^{3} b^{4} x^{8}+355 a^{4} b^{3} x^{6}+3 a^{5} b^{2} x^{4}-4 a^{6} b\,x^{2}+8 a^{7}\right)\sqrt{b\,x^{2}+a}}{2145 b^{3}}$	91
risch	$\frac{\left(143 b^{7} x^{14}+671 a\,b^{6} x^{12}+1218 a^{2} b^{5} x^{10}+1030 a^{3} b^{4} x^{8}+355 a^{4} b^{3} x^{6}+3 a^{5} b^{2} x^{4}-4 a^{6} b\,x^{2}+8 a^{7}\right)\sqrt{b\,x^{2}+a}}{2145 b^{3}}$	91

input
```
int(x^5*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/2145*(b*x^2+a)^(11/2)*(143*b^2*x^4-44*a*b*x^2+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

$$\int x^5 (a + b x^2)^{9/2}\, dx = \frac{(143\, b^7 x^{14} + 671\, a b^6 x^{12} + 1218\, a^2 b^5 x^{10} + 1030\, a^3 b^4 x^8 + 355\, a^4 b^3 x^6 + 3\, a^5 b^2 x^4 - 4\, a^6 b x^2 + 8}{2145\, b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/2145*(143*b^7*x^14 + 671*a*b^6*x^12 + 1218*a^2*b^5*x^10 + 1030*a^3*b^4*x
^8 + 355*a^4*b^3*x^6 + 3*a^5*b^2*x^4 - 4*a^6*b*x^2 + 8*a^7)*sqrt(b*x^2 + a
)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(51) = 102$.

Time = 1.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.05

$$\int x^5(a+bx^2)^{9/2}\,dx = \begin{cases} \frac{8a^7\sqrt{a+bx^2}}{2145b^3} - \frac{4a^6x^2\sqrt{a+bx^2}}{2145b^2} + \frac{a^5x^4\sqrt{a+bx^2}}{715b} + \frac{71a^4x^6\sqrt{a+bx^2}}{429} + \frac{206a^3bx^8\sqrt{a+bx^2}}{429} + \frac{406a^2b^2x^{10}\sqrt{a+bx^2}}{715} + 6 \\ \frac{a^{\frac{9}{2}}x^6}{6} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((8*a**7*sqrt(a + b*x**2)/(2145*b**3) - 4*a**6*x**2*sqrt(a + b*x*
*2)/(2145*b**2) + a**5*x**4*sqrt(a + b*x**2)/(715*b) + 71*a**4*x**6*sqrt(a
 + b*x**2)/429 + 206*a**3*b*x**8*sqrt(a + b*x**2)/429 + 406*a**2*b**2*x**1
0*sqrt(a + b*x**2)/715 + 61*a*b**3*x**12*sqrt(a + b*x**2)/195 + b**4*x**14
*sqrt(a + b*x**2)/15, Ne(b, 0)), (a**(9/2)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5(a+bx^2)^{9/2}\,dx = \frac{(bx^2+a)^{\frac{11}{2}}x^4}{15\,b} - \frac{4\,(bx^2+a)^{\frac{11}{2}}ax^2}{195\,b^2} + \frac{8\,(bx^2+a)^{\frac{11}{2}}a^2}{2145\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/15*(b*x^2 + a)^(11/2)*x^4/b - 4/195*(b*x^2 + a)^(11/2)*a*x^2/b^2 + 8/214
5*(b*x^2 + a)^(11/2)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 \left(a+bx^2\right)^{9/2} \, dx = \frac{143 \left(bx^2+a\right)^{\frac{15}{2}} - 330 \left(bx^2+a\right)^{\frac{13}{2}} a + 195 \left(bx^2+a\right)^{\frac{11}{2}} a^2}{2145 \, b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/2145*(143*(b*x^2 + a)^(15/2) - 330*(b*x^2 + a)^(13/2)*a + 195*(b*x^2 + a
)^(11/2)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^5 \left(a+bx^2\right)^{9/2} \, dx = \sqrt{b\,x^2+a}\left(\frac{8\,a^7}{2145\,b^3} + \frac{71\,a^4\,x^6}{429} + \frac{b^4\,x^{14}}{15} + \frac{206\,a^3\,b\,x^8}{429} + \frac{61\,a\,b^3\,x^{12}}{195} + \frac{a^5\,x^4}{715\,b} - \frac{4\,a^6\,x^2}{2145\,b^2} + \frac{406\,a^2\,b^2\,x^{10}}{715}\right)$$

input
```
int(x^5*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*a^7)/(2145*b^3) + (71*a^4*x^6)/429 + (b^4*x^14)/15 +
 (206*a^3*b*x^8)/429 + (61*a*b^3*x^12)/195 + (a^5*x^4)/(715*b) - (4*a^6*x^
2)/(2145*b^2) + (406*a^2*b^2*x^10)/715)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.51

$$\int x^5(a+bx^2)^{9/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(143b^7x^{14}+671a\,b^6x^{12}+1218a^2b^5x^{10}+1030a^3b^4x^8+355a^4b^3x^6+3a^5b^2x^4-4a^6}{2145b^3}$$

input
```
int(x^5*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**7 - 4*a**6*b*x**2 + 3*a**5*b**2*x**4 + 355*a**4*b*
*3*x**6 + 1030*a**3*b**4*x**8 + 1218*a**2*b**5*x**10 + 671*a*b**6*x**12 +
143*b**7*x**14))/(2145*b**3)
```

3.425 $\int x^3(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3(a+bx^2)^{9/2}\,dx=-\frac{a(a+bx^2)^{11/2}}{11b^2}+\frac{(a+bx^2)^{13/2}}{13b^2}$$

output
```
-1/11*a*(b*x^2+a)^(11/2)/b^2+1/13*(b*x^2+a)^(13/2)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3(a+bx^2)^{9/2}\,dx=\frac{(a+bx^2)^{11/2}(-2a+11bx^2)}{143b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(9/2),x]
```

output
```
((a + b*x^2)^(11/2)*(-2*a + 11*b*x^2))/(143*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^{9/2} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2} \int x^2 \left(bx^2+a\right)^{9/2}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{11/2}}{b}-\frac{a\left(bx^2+a\right)^{9/2}}{b}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2} \left(\frac{2\left(a+bx^2\right)^{13/2}}{13b^2}-\frac{2a\left(a+bx^2\right)^{11/2}}{11b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(9/2),x]
```

output
```
((-2*a*(a + b*x^2)^(11/2))/(11*b^2) + (2*(a + b*x^2)^(13/2))/(13*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-11 b\,x^{2}+2 a\right)}{143 b^{2}}$	25
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-11 b\,x^{2}+2 a\right)}{143 b^{2}}$	25
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-11 b\,x^{2}+2 a\right)}{143 b^{2}}$	25
default	$\frac{x^{2} \left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13 b}-\frac{2 a \left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143 b^{2}}$	34
trager	$-\frac{\left(-11 b^{6} x^{12}-53 a\,b^{5} x^{10}-100 a^{2} b^{4} x^{8}-90 a^{3} b^{3} x^{6}-35 a^{4} b^{2} x^{4}-a^{5} b\,x^{2}+2 a^{6}\right) \sqrt{b\,x^{2}+a}}{143 b^{2}}$	80
risch	$-\frac{\left(-11 b^{6} x^{12}-53 a\,b^{5} x^{10}-100 a^{2} b^{4} x^{8}-90 a^{3} b^{3} x^{6}-35 a^{4} b^{2} x^{4}-a^{5} b\,x^{2}+2 a^{6}\right) \sqrt{b\,x^{2}+a}}{143 b^{2}}$	80

input
```
int(x^3*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/143*(b*x^2+a)^(11/2)*(-11*b*x^2+2*a)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.05

$$\int x^3(a+bx^2)^{9/2}\,dx = \frac{(11\,b^6x^{12}+53\,ab^5x^{10}+100\,a^2b^4x^8+90\,a^3b^3x^6+35\,a^4b^2x^4+a^5bx^2-2\,a^6)\sqrt{bx^2+a}}{143\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/143*(11*b^6*x^12 + 53*a*b^5*x^10 + 100*a^2*b^4*x^8 + 90*a^3*b^3*x^6 + 35
*a^4*b^2*x^4 + a^5*b*x^2 - 2*a^6)*sqrt(b*x^2 + a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(31) = 62$.

Time = 0.87 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.11

$$\int x^3(a+bx^2)^{9/2}\,dx = \begin{cases} -\frac{2a^6\sqrt{a+bx^2}}{143b^2} + \frac{a^5x^2\sqrt{a+bx^2}}{143b} + \frac{35a^4x^4\sqrt{a+bx^2}}{143} + \frac{90a^3bx^6\sqrt{a+bx^2}}{143} + \frac{100a^2b^2x^8\sqrt{a+bx^2}}{143} + \frac{53ab^3x^{10}\sqrt{a+bx^2}}{143} \\ \frac{a^{\frac{9}{2}}x^4}{4} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-2*a**6*sqrt(a + b*x**2)/(143*b**2) + a**5*x**2*sqrt(a + b*x**2
)/(143*b) + 35*a**4*x**4*sqrt(a + b*x**2)/143 + 90*a**3*b*x**6*sqrt(a + b*
x**2)/143 + 100*a**2*b**2*x**8*sqrt(a + b*x**2)/143 + 53*a*b**3*x**10*sqrt
(a + b*x**2)/143 + b**4*x**12*sqrt(a + b*x**2)/13, Ne(b, 0)), (a**(9/2)*x*
*4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3(a+bx^2)^{9/2}\,dx = \frac{(bx^2+a)^{\frac{11}{2}}x^2}{13\,b} - \frac{2\,(bx^2+a)^{\frac{11}{2}}a}{143\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/13*(b*x^2 + a)^(11/2)*x^2/b - 2/143*(b*x^2 + a)^(11/2)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a+bx^2)^{9/2}\,dx = \frac{11\,(bx^2+a)^{\frac{13}{2}} - 13\,(bx^2+a)^{\frac{11}{2}}a}{143\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/143*(11*(b*x^2 + a)^(13/2) - 13*(b*x^2 + a)^(11/2)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a+bx^2)^{9/2}\,dx = -\frac{13\,a\,(b\,x^2+a)^{11/2} - 11\,(b\,x^2+a)^{13/2}}{143\,b^2}$$

input
```
int(x^3*(a + b*x^2)^(9/2),x)
```

output
```
-(13*a*(a + b*x^2)^(11/2) - 11*(a + b*x^2)^(13/2))/(143*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.03

$$\int x^3 (a+bx^2)^{9/2}\,dx = \frac{\sqrt{b\,x^2+a}\,(11b^6x^{12}+53a\,b^5x^{10}+100a^2b^4x^8+90a^3b^3x^6+35a^4b^2x^4+a^5b\,x^2-2a^6)}{143b^2}$$

input
```
int(x^3*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a**6 + a**5*b*x**2 + 35*a**4*b**2*x**4 + 90*a**3*b
**3*x**6 + 100*a**2*b**4*x**8 + 53*a*b**5*x**10 + 11*b**6*x**12))/(143*b**
2)
```

3.426 $\int x(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

output
```
1/11*(b*x^2+a)^(11/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)^{9/2}\,dx=\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

input
```
Integrate[x*(a + b*x^2)^(9/2),x]
```

output
```
(a + b*x^2)^(11/2)/(11*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{9/2}\,dx$$
$$\Big\downarrow \; 241$$
$$\frac{\left(a+bx^2\right)^{11/2}}{11b}$$

input
```
Int[x*(a + b*x^2)^(9/2),x]
```

output
```
(a + b*x^2)^(11/2)/(11*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{(b\,x^2+a)^{\frac{11}{2}}}{11b}$	15
derivativedivides	$\frac{(b\,x^2+a)^{\frac{11}{2}}}{11b}$	15
default	$\frac{(b\,x^2+a)^{\frac{11}{2}}}{11b}$	15
pseudoelliptic	$\frac{(b\,x^2+a)^{\frac{11}{2}}}{11b}$	15
orering	$\frac{(b\,x^2+a)^{\frac{11}{2}}}{11b}$	15
trager	$\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11b}$	66
risch	$\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11b}$	66

input
```
int(x*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/11*(b*x^2+a)^(11/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int x(a+bx^2)^{9/2}\,dx = \frac{(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{bx^2+a}}{11\,b}$$

input
```
integrate(x*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*x
^2 + a^5)*sqrt(b*x^2 + a)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(12) = 24$.

Time = 0.64 (sec) , antiderivative size = 133, normalized size of antiderivative = 7.39

$$\int x(a+bx^2)^{9/2}\,dx = \begin{cases} \frac{a^5\sqrt{a+bx^2}}{11b} + \frac{5a^4x^2\sqrt{a+bx^2}}{11} + \frac{10a^3bx^4\sqrt{a+bx^2}}{11} + \frac{10a^2b^2x^6\sqrt{a+bx^2}}{11} + \frac{5ab^3x^8\sqrt{a+bx^2}}{11} + \frac{b^4x^{10}\sqrt{a+bx^2}}{11} & \text{for } b \\ \frac{a^{\frac{9}{2}}x^2}{2} & \text{othe} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((a**5*sqrt(a + b*x**2)/(11*b) + 5*a**4*x**2*sqrt(a + b*x**2)/11
+ 10*a**3*b*x**4*sqrt(a + b*x**2)/11 + 10*a**2*b**2*x**6*sqrt(a + b*x**2)/
11 + 5*a*b**3*x**8*sqrt(a + b*x**2)/11 + b**4*x**10*sqrt(a + b*x**2)/11, N
e(b, 0)), (a**(9/2)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{9/2}\,dx = \frac{(bx^2+a)^{\frac{11}{2}}}{11\,b}$$

input
```
integrate(x*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/11*(b*x^2 + a)^(11/2)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{9/2}\,dx=\frac{(bx^2+a)^{\frac{11}{2}}}{11\,b}$$

input
```
integrate(x*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/11*(b*x^2 + a)^(11/2)/b
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{9/2}\,dx=\frac{(b\,x^2+a)^{11/2}}{11\,b}$$

input
```
int(x*(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(11/2)/(11*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 3.56

$$\int x(a+bx^2)^{9/2}\,dx=\frac{\sqrt{b\,x^2+a}\,(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)}{11b}$$

input
```
int(x*(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(a**5 + 5*a**4*b*x**2 + 10*a**3*b**2*x**4 + 10*a**2*b**3
*x**6 + 5*a*b**4*x**8 + b**5*x**10))/(11*b)
```

3.427 $\int \frac{(a+bx^2)^{9/2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{(a+bx^2)^{9/2}}{x} \, dx = a^4\sqrt{a+bx^2} + \frac{1}{3}a^3(a+bx^2)^{3/2} + \frac{1}{5}a^2(a+bx^2)^{5/2} + \frac{1}{7}a(a+bx^2)^{7/2} + \frac{1}{9}(a+bx^2)^{9/2} - a^{9/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
a^4*(b*x^2+a)^(1/2)+1/3*a^3*(b*x^2+a)^(3/2)+1/5*a^2*(b*x^2+a)^(5/2)+1/7*a*
(b*x^2+a)^(7/2)+1/9*(b*x^2+a)^(9/2)-a^(9/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{9/2}}{x} \, dx = \frac{1}{315}\sqrt{a+bx^2}(563a^4 + 506a^3bx^2 + 408a^2b^2x^4 + 185ab^3x^6 + 35b^4x^8) - a^{9/2}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x,x]
```

output
```
(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^
6 + 35*b^4*x^8))/315 - a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 60, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{(bx^2+a)^{7/2}}{x^2}dx^2+\frac{2}{9}(a+bx^2)^{9/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\int \frac{(bx^2+a)^{5/2}}{x^2}dx^2+\frac{2}{7}(a+bx^2)^{7/2}\right)+\frac{2}{9}(a+bx^2)^{9/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\left(a\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2}{7}(a+bx^2)^{7/2}\right)+\frac{2}{9}(a+bx^2)^{9/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\left(a\left(a\int \frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}(a+bx^2)^{3/2}\right)+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2}{7}(a+bx^2)^{7/2}\right)+\frac{2}{9}(a+bx^2)^{9/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\left(a\left(a\left(a\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}\right)+\frac{2}{9}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(a\left(a\left(a\left(a\left(\frac{2a\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(a\left(a\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}$$

input
```
Int[(a + b*x^2)^(9/2)/x,x]
```

output
```
((2*(a + b*x^2)^(9/2))/9 + a*((2*(a + b*x^2)^(7/2))/7 + a*((2*(a + b*x^2)^
(5/2))/5 + a*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[a]*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))))/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-a^{\frac{9}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{\sqrt{b\,x^{2}+a}\,\left(35 b^{4} x^{8}+185 a\,b^{3} x^{6}+408 a^{2} b^{2} x^{4}+506 a^{3} b\,x^{2}+563 a^{4}\right)}{315}$
default	$\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{9}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}}}{x}\right.\right.\right.\right.\right.$

input
```
int((b*x^2+a)^(9/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-a^(9/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+1/315*(b*x^2+a)^(1/2)*(35*b^4*x^
8+185*a*b^3*x^6+408*a^2*b^2*x^4+506*a^3*b*x^2+563*a^4)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \left[\frac{1}{2}\,a^{\frac{9}{2}}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right) + \frac{1}{315}\left(35\,b^4x^8+185\,ab^3x^6+408\,a^2b^2x^4+506\,a^3bx^2+563\,a^4\right)\sqrt{bx^2+a},\ \sqrt{-a}a^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) + \frac{1}{315}\left(35\,b^4x^8+185\,ab^3x^6+408\,a^2b^2x^4+506\,a^3bx^2+563\,a^4\right)\sqrt{bx^2+a}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x,x, algorithm="fricas")
```

output
```
[1/2*a^(9/2)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 1/315*(
35*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sq
rt(b*x^2 + a), sqrt(-a)*a^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + 1/315*(35
*b^4*x^8 + 185*a*b^3*x^6 + 408*a^2*b^2*x^4 + 506*a^3*b*x^2 + 563*a^4)*sqrt
(b*x^2 + a)]
```

Sympy [A] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.48

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{563a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{315} + \frac{a^{\frac{9}{2}}\log\left(\frac{bx^2}{a}\right)}{2} - a^{\frac{9}{2}}\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right) + \frac{506a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}{315} + \frac{136a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{105} + \frac{37a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{63} + \frac{\sqrt{a}b^4x^8\sqrt{1+\frac{bx^2}{a}}}{9}$$

input
```
integrate((b*x**2+a)**(9/2)/x,x)
```

output
```
563*a**(9/2)*sqrt(1 + b*x**2/a)/315 + a**(9/2)*log(b*x**2/a)/2 - a**(9/2)*
log(sqrt(1 + b*x**2/a) + 1) + 506*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)/315 +
 136*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a)/105 + 37*a**(3/2)*b**3*x**6*sqr
t(1 + b*x**2/a)/63 + sqrt(a)*b**4*x**8*sqrt(1 + b*x**2/a)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = -a^{\frac{9}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2 + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3 + \sqrt{bx^2+a}\,a^4$$

input
```
integrate((b*x^2+a)^(9/2)/x,x, algorithm="maxima")
```

output
```
-a^(9/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/9*(b*x^2 + a)^(9/2) + 1/7*(b*x^
2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*a^3 + s
qrt(b*x^2 + a)*a^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{a^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{9}\left(bx^2+a\right)^{\frac{9}{2}} + \frac{1}{7}\left(bx^2+a\right)^{\frac{7}{2}}a + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}a^2 + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}a^3 + \sqrt{bx^2+a}\,a^4$$

input
```
integrate((b*x^2+a)^(9/2)/x,x, algorithm="giac")
```

output
```
a^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/9*(b*x^2 + a)^(9/2) + 1/
7*(b*x^2 + a)^(7/2)*a + 1/5*(b*x^2 + a)^(5/2)*a^2 + 1/3*(b*x^2 + a)^(3/2)*
a^3 + sqrt(b*x^2 + a)*a^4
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{a\,(b\,x^2+a)^{7/2}}{7} + \frac{(b\,x^2+a)^{9/2}}{9} + a^4\,\sqrt{b\,x^2+a} + \frac{a^3\,(b\,x^2+a)^{3/2}}{3} + \frac{a^2\,(b\,x^2+a)^{5/2}}{5} + a^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int((a + b*x^2)^(9/2)/x,x)
```

output
```
a^(9/2)*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*1i + (a*(a + b*x^2)^(7/2))/7
+ (a + b*x^2)^(9/2)/9 + a^4*(a + b*x^2)^(1/2) + (a^3*(a + b*x^2)^(3/2))/3
+ (a^2*(a + b*x^2)^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^2)^{9/2}}{x}\,dx = \frac{563\sqrt{b\,x^2+a}\,a^4}{315} + \frac{506\sqrt{b\,x^2+a}\,a^3b\,x^2}{315} + \frac{136\sqrt{b\,x^2+a}\,a^2b^2x^4}{105} + \frac{37\sqrt{b\,x^2+a}\,a\,b^3x^6}{63} + \frac{\sqrt{b\,x^2+a}\,b^4x^8}{9} + \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^4 - \sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^4$$

input
```
int((b*x^2+a)^(9/2)/x,x)
```

output
```
(563*sqrt(a + b*x**2)*a**4 + 506*sqrt(a + b*x**2)*a**3*b*x**2 + 408*sqrt(a
 + b*x**2)*a**2*b**2*x**4 + 185*sqrt(a + b*x**2)*a*b**3*x**6 + 35*sqrt(a +
 b*x**2)*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*a**4 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*
x)/sqrt(a))*a**4)/315
```

3.428 $\int \frac{(a+bx^2)^{9/2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{(a+bx^2)^{9/2}}{x^3} dx = 4a^3b\sqrt{a+bx^2} - \frac{a^4\sqrt{a+bx^2}}{2x^2} + a^2b(a+bx^2)^{3/2} + \frac{2}{5}ab(a+bx^2)^{5/2} + \frac{1}{7}b(a+bx^2)^{7/2} - \frac{9}{2}a^{7/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
4*a^3*b*(b*x^2+a)^(1/2)-1/2*a^4*(b*x^2+a)^(1/2)/x^2+a^2*b*(b*x^2+a)^(3/2)+
2/5*a*b*(b*x^2+a)^(5/2)+1/7*b*(b*x^2+a)^(7/2)-9/2*a^(7/2)*b*arctanh((b*x^2
+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{9/2}}{x^3} dx = \frac{\sqrt{a+bx^2}(-35a^4+388a^3bx^2+156a^2b^2x^4+58ab^3x^6+10b^4x^8)}{70x^2} - \frac{9}{2}a^{7/2}b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^3,x]
```

output
```
(Sqrt[a + b*x^2]*(-35*a^4 + 388*a^3*b*x^2 + 156*a^2*b^2*x^4 + 58*a*b^3*x^6
 + 10*b^4*x^8))/(70*x^2) - (9*a^(7/2)*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {243, 51, 60, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{2}b\int \frac{(bx^2+a)^{7/2}}{x^2}dx^2-\frac{(a+bx^2)^{9/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\int \frac{(bx^2+a)^{5/2}}{x^2}dx^2+\frac{2}{7}(a+bx^2)^{7/2}\right)-\frac{(a+bx^2)^{9/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\left(a\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2+\frac{2}{5}(a+bx^2)^{5/2}\right)+\frac{2}{7}(a+bx^2)^{7/2}\right)-\frac{(a+bx^2)^{9/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\left(a\left(a\int\frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}\right)-\frac{\left(a+bx^2\right)^{9/2}}{x^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\left(a\left(a\left(a\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}\right)-$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7/2}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{9}{2}b\left(a\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)+\frac{2}{7}\left(a+bx^2\right)^{7}\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^3,x]
```

output
```
(-((a + b*x^2)^(9/2)/x^2) + (9*b*((2*(a + b*x^2)^(7/2))/7 + a*((2*(a + b*x
^2)^(5/2))/5 + a*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2*Sqrt[
a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$-\frac{9\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)a^{4}b\,x^{2}-\frac{2\left(\sqrt{a}\,b^{4}x^{8}+\frac{29a^{\frac{3}{2}}b^{3}x^{6}}{5}+\frac{78a^{\frac{5}{2}}b^{2}x^{4}}{5}+\frac{194a^{\frac{7}{2}}b\,x^{2}}{5}-\frac{7a^{\frac{9}{2}}}{2}\right)\sqrt{b\,x^{2}+a}}{63}\right)}{2\sqrt{a}\,x^{2}}$
risch	$-\frac{a^{4}\sqrt{b\,x^{2}+a}}{2x^{2}}-\frac{9b\,a^{\frac{7}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2}+\frac{b^{4}x^{6}\sqrt{b\,x^{2}+a}}{7}+\frac{29b^{3}a\,x^{4}\sqrt{b\,x^{2}+a}}{35}+\frac{78b^{2}a^{2}x^{2}\sqrt{b\,x^{2}+a}}{35}+\frac{194a^{3}}{}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{2a\,x^{2}}+\frac{9b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{9}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+}}{x}\right.\right.\right.\right.\right.}{2a}$

input
```
int((b*x^2+a)^(9/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-9/2*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^4*b*x^2-2/63*(a^(1/2)*b^4*x^8+29/
5*a^(3/2)*b^3*x^6+78/5*a^(5/2)*b^2*x^4+194/5*a^(7/2)*b*x^2-7/2*a^(9/2))*(b
*x^2+a)^(1/2))/a^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

$$\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx = \left[\frac{315\,a^{\frac{7}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(10\,b^4x^8+58\,ab^3x^6+156\,a^2b^2x^4+388\,a^3bx}{140\,x^2}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="fricas")
```

output
```
[1/140*(315*a^(7/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*(10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*b*x^2 - 35*
a^4)*sqrt(b*x^2 + a))/x^2, 1/70*(315*sqrt(-a)*a^3*b*x^2*arctan(sqrt(b*x^2
+ a)*sqrt(-a)/a) + (10*b^4*x^8 + 58*a*b^3*x^6 + 156*a^2*b^2*x^4 + 388*a^3*
b*x^2 - 35*a^4)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A] (verification not implemented)

Time = 10.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.44

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx &= -\frac{a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}}{2x^2}+\frac{194a^{\frac{7}{2}}b\sqrt{1+\frac{bx^2}{a}}}{35}\\ &+\frac{9a^{\frac{7}{2}}b\log\left(\frac{bx^2}{a}\right)}{4}-\frac{9a^{\frac{7}{2}}b\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2}\\ &+\frac{78a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{29a^{\frac{3}{2}}b^3x^4\sqrt{1+\frac{bx^2}{a}}}{35}+\frac{\sqrt{a}b^4x^6\sqrt{1+\frac{bx^2}{a}}}{7}\end{aligned}$$

input
```
integrate((b*x**2+a)**(9/2)/x**3,x)
```

output
```
-a**(9/2)*sqrt(1 + b*x**2/a)/(2*x**2) + 194*a**(7/2)*b*sqrt(1 + b*x**2/a)/
35 + 9*a**(7/2)*b*log(b*x**2/a)/4 - 9*a**(7/2)*b*log(sqrt(1 + b*x**2/a) +
1)/2 + 78*a**(5/2)*b**2*x**2*sqrt(1 + b*x**2/a)/35 + 29*a**(3/2)*b**3*x**4
*sqrt(1 + b*x**2/a)/35 + sqrt(a)*b**4*x**6*sqrt(1 + b*x**2/a)/7
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx = -\frac{9}{2}\,a^{\frac{7}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\frac{9}{14}\left(bx^2+a\right)^{\frac{7}{2}}b+\frac{\left(bx^2+a\right)^{\frac{9}{2}}b}{2\,a}+\frac{9}{10}\left(bx^2+a\right)^{\frac{5}{2}}ab+\frac{3}{2}\left(bx^2+a\right)^{\frac{3}{2}}a^2b+\frac{9}{2}\sqrt{bx^2+a}a^3b-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{2\,ax^2}$$

input
```
integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="maxima")
```

output
```
-9/2*a^(7/2)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 9/14*(b*x^2 + a)^(7/2)*b +
1/2*(b*x^2 + a)^(9/2)*b/a + 9/10*(b*x^2 + a)^(5/2)*a*b + 3/2*(b*x^2 + a)^(
3/2)*a^2*b + 9/2*sqrt(b*x^2 + a)*a^3*b - 1/2*(b*x^2 + a)^(11/2)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx = \frac{1}{70}\left(\frac{315\,a^4\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+10\left(bx^2+a\right)^{\frac{7}{2}}+28\left(bx^2+a\right)^{\frac{5}{2}}a+70\left(bx^2+a\right)^{\frac{3}{2}}a^2+\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^3,x, algorithm="giac")
```

output
```
1/70*(315*a^4*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 10*(b*x^2 + a)^(
7/2) + 28*(b*x^2 + a)^(5/2)*a + 70*(b*x^2 + a)^(3/2)*a^2 + 280*sqrt(b*x^2
+ a)*a^3 - 35*sqrt(b*x^2 + a)*a^4/(b*x^2))*b
```

Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx = \frac{b\,(b\,x^2+a)^{7/2}}{7} + 4\,a^3\,b\,\sqrt{b\,x^2+a} + a^2\,b\,\left(b\,x^2+a\right)^{3/2} - \frac{a^4\,\sqrt{b\,x^2+a}}{2\,x^2} + \frac{2\,a\,b\,(b\,x^2+a)^{5/2}}{5} + \frac{a^{7/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{2}$$

input
```
int((a + b*x^2)^(9/2)/x^3,x)
```

output
```
(b*(a + b*x^2)^(7/2))/7 + 4*a^3*b*(a + b*x^2)^(1/2) + a^2*b*(a + b*x^2)^(3
/2) - (a^4*(a + b*x^2)^(1/2))/(2*x^2) + (a^(7/2)*b*atan(((a + b*x^2)^(1/2)
*1i)/a^(1/2))*9i)/2 + (2*a*b*(a + b*x^2)^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^{9/2}}{x^3}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^4+388\sqrt{b\,x^2+a}\,a^3b\,x^2+156\sqrt{b\,x^2+a}\,a^2b^2x^4+58\sqrt{b\,x^2+a}\,a\,b^3x^6 \cdots}{}$$

input
```
int((b*x^2+a)^(9/2)/x^3,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 + 388*sqrt(a + b*x**2)*a**3*b*x**2 + 156*sqrt
(a + b*x**2)*a**2*b**2*x**4 + 58*sqrt(a + b*x**2)*a*b**3*x**6 + 10*sqrt(a
+ b*x**2)*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b
)*x)/sqrt(a))*a**3*b*x**2 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
sqrt(b)*x)/sqrt(a))*a**3*b*x**2)/(70*x**2)
```

3.429 $\int \frac{(a+bx^2)^{9/2}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 127

$$\int \frac{(a+bx^2)^{9/2}}{x^5} dx = 6a^2b^2\sqrt{a+bx^2} - \frac{a^4\sqrt{a+bx^2}}{4x^4} - \frac{17a^3b\sqrt{a+bx^2}}{8x^2} + ab^2(a+bx^2)^{3/2} + \frac{1}{5}b^2(a+bx^2)^{5/2} - \frac{63}{8}a^{5/2}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
6*a^2*b^2*(b*x^2+a)^(1/2)-1/4*a^4*(b*x^2+a)^(1/2)/x^4-17/8*a^3*b*(b*x^2+a)
^(1/2)/x^2+a*b^2*(b*x^2+a)^(3/2)+1/5*b^2*(b*x^2+a)^(5/2)-63/8*a^(5/2)*b^2*
arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{9/2}}{x^5} dx = \frac{\sqrt{a+bx^2}(-10a^4 - 85a^3bx^2 + 288a^2b^2x^4 + 56ab^3x^6 + 8b^4x^8)}{40x^4} - \frac{63}{8}a^{5/2}b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^5,x]
```

output
```
(Sqrt[a + b*x^2]*(-10*a^4 - 85*a^3*b*x^2 + 288*a^2*b^2*x^4 + 56*a*b^3*x^6
+ 8*b^4*x^8))/(40*x^4) - (63*a^(5/2)*b^2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 51, 51, 60, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{4}b\int \frac{(bx^2+a)^{7/2}}{x^4}dx^2-\frac{(a+bx^2)^{9/2}}{2x^4}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\int \frac{(bx^2+a)^{5/2}}{x^2}dx^2-\frac{(a+bx^2)^{7/2}}{x^2}\right)-\frac{(a+bx^2)^{9/2}}{2x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\left(a\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2+\frac{2}{5}(a+bx^2)^{5/2}\right)-\frac{(a+bx^2)^{7/2}}{x^2}\right)-\frac{(a+bx^2)^{9/2}}{2x^4}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\int\frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{9/2}}{2x^4}\right)$$

$\big\downarrow$ 60

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(a\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{x^2}\right)-$$

$\big\downarrow$ 73

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{x^2}$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{9}{4}b\left(\frac{7}{2}b\left(a\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)+\frac{2}{5}\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7}}{x^2}$$

input
```
Int[(a + b*x^2)^(9/2)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(9/2)/x^4 + (9*b*(-((a + b*x^2)^(7/2)/x^2) + (7*b*((2*(a
 + b*x^2)^(5/2))/5 + a*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^2] - 2
*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))))/2))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{63\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) a^{3} b^{2} x^{4}-\frac{8\left(\sqrt{a}\,b^{4} x^{8}+7 a^{\frac{3}{2}} b^{3} x^{6}+36 a^{\frac{5}{2}} b^{2} x^{4}-\frac{85 a^{\frac{7}{2}} b\,x^{2}}{8}-\frac{5 a^{\frac{9}{2}}}{4}\right) \sqrt{b\,x^{2}+a}}{315}\right)}{8 \sqrt{a}\,x^{4}}$
risch	$-\frac{a^{3} \sqrt{b\,x^{2}+a}\,(17 b\,x^{2}+2 a)}{8 x^{4}}-\frac{63 b^{2} a^{\frac{5}{2}} \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8}+\frac{b^{4} x^{4} \sqrt{b\,x^{2}+a}}{5}+\frac{7 b^{3} a\,x^{2} \sqrt{b\,x^{2}+a}}{5}+\frac{36 a^{2} b^{2} \sqrt{b\,x^{2}+a}}{5}$
default	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{4 a\,x^{4}}+\frac{7 b (-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{2 a\,x^{2}}+\frac{9 b (\frac{(b\,x^{2}+a)^{\frac{9}{2}}}{9}+a (\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{7}+a (\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{5}+a (\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a (\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln (}{2 a}}{4 a}$

input
```
int((b*x^2+a)^(9/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-63/8/a^(1/2)*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^3*b^2*x^4-8/315*(a^(1/2)
*b^4*x^8+7*a^(3/2)*b^3*x^6+36*a^(5/2)*b^2*x^4-85/8*a^(7/2)*b*x^2-5/4*a^(9/
2))*(b*x^2+a)^(1/2))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = \left[\frac{315\,a^{\frac{5}{2}}b^2x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,b^4x^8+56\,ab^3x^6+288\,a^2b^2x^4-85\,a^3bx^2}{80\,x^4}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="fricas")
```

output
```
[1/80*(315*a^(5/2)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b*x^2 - 10*a
^4)*sqrt(b*x^2 + a))/x^4, 1/40*(315*sqrt(-a)*a^2*b^2*x^4*arctan(sqrt(b*x^2
 + a)*sqrt(-a)/a) + (8*b^4*x^8 + 56*a*b^3*x^6 + 288*a^2*b^2*x^4 - 85*a^3*b
*x^2 - 10*a^4)*sqrt(b*x^2 + a))/x^4]
```

Sympy [A] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = -\frac{63a^{\frac{5}{2}}b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8}-\frac{a^5}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{19a^4\sqrt{b}}{8x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{203a^3b^{\frac{3}{2}}}{40x\sqrt{\frac{a}{bx^2}+1}}+\frac{43a^2b^{\frac{5}{2}}x}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{8ab^{\frac{7}{2}}x^3}{5\sqrt{\frac{a}{bx^2}+1}}+\frac{b^{\frac{9}{2}}x^5}{5\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**5,x)
```

output
```
-63*a**(5/2)*b**2*asinh(sqrt(a)/(sqrt(b)*x))/8 - a**5/(4*sqrt(b)*x**5*sqrt
(a/(b*x**2) + 1)) - 19*a**4*sqrt(b)/(8*x**3*sqrt(a/(b*x**2) + 1)) + 203*a*
*3*b**(3/2)/(40*x*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)*x/(5*sqrt(a/(b*
x**2) + 1)) + 8*a*b**(7/2)*x**3/(5*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x**5/(
5*sqrt(a/(b*x**2) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = -\frac{63}{8}\,a^{\frac{5}{2}}b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)+\frac{63}{40}\left(bx^2+a\right)^{\frac{5}{2}}b^2+\frac{7\left(bx^2+a\right)^{\frac{9}{2}}b^2}{8\,a^2}+\frac{9\left(bx^2+a\right)^{\frac{7}{2}}b^2}{8\,a} + \frac{21}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2+\frac{63}{8}\sqrt{bx^2+a}a^2b^2-\frac{7\left(bx^2+a\right)^{\frac{11}{2}}b}{8\,a^2x^2}-\frac{\left(bx^2+a\right)^{\frac{11}{2}}}{4\,ax^4}$$

input
```
integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="maxima")
```

output
```
-63/8*a^(5/2)*b^2*arcsinh(a/(sqrt(a*b)*abs(x))) + 63/40*(b*x^2 + a)^(5/2)*
b^2 + 7/8*(b*x^2 + a)^(9/2)*b^2/a^2 + 9/8*(b*x^2 + a)^(7/2)*b^2/a + 21/8*(
b*x^2 + a)^(3/2)*a*b^2 + 63/8*sqrt(b*x^2 + a)*a^2*b^2 - 7/8*(b*x^2 + a)^(1
1/2)*b/(a^2*x^2) - 1/4*(b*x^2 + a)^(11/2)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = \frac{\frac{315\,a^3b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+8\left(bx^2+a\right)^{\frac{5}{2}}b^3+40\left(bx^2+a\right)^{\frac{3}{2}}ab^3+240\sqrt{bx^2+a}a^2b^3-\frac{5\,(17}{}}{40\,b}$$

input
```
integrate((b*x^2+a)^(9/2)/x^5,x, algorithm="giac")
```

output
```
1/40*(315*a^3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 8*(b*x^2 + a
)^(5/2)*b^3 + 40*(b*x^2 + a)^(3/2)*a*b^3 + 240*sqrt(b*x^2 + a)*a^2*b^3 - 5
*(17*(b*x^2 + a)^(3/2)*a^3*b^3 - 15*sqrt(b*x^2 + a)*a^4*b^3)/(b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = \frac{\frac{15\,a^4\,b^2\,\sqrt{b\,x^2+a}}{8}-\frac{17\,a^3\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}{8}}{{\left(b\,x^2+a\right)}^2-2\,a\,\left(b\,x^2+a\right)+a^2}+\frac{b^2\,{\left(b\,x^2+a\right)}^{5/2}}{5}+a\,b^2\,{\left(b\,x^2+a\right)}^{3/2}+6\,a^2\,b^2\,\sqrt{b\,x^2+a}+\frac{a^{5/2}\,b^2\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,63{}\mathrm{i}}{8}$$

input
```
int((a + b*x^2)^(9/2)/x^5,x)
```

output
```
((15*a^4*b^2*(a + b*x^2)^(1/2))/8 - (17*a^3*b^2*(a + b*x^2)^(3/2))/8)/((a
+ b*x^2)^2 - 2*a*(a + b*x^2) + a^2) + (b^2*(a + b*x^2)^(5/2))/5 + (a^(5/2)
*b^2*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/8 + a*b^2*(a + b*x^2)^(3/2)
 + 6*a^2*b^2*(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{9/2}}{x^5}\,dx = \frac{-10\sqrt{b\,x^2+a}\,a^4-85\sqrt{b\,x^2+a}\,a^3b\,x^2+288\sqrt{b\,x^2+a}\,a^2b^2x^4+56\sqrt{b\,x^2+a}\,a\,b^3x^6+}{}$$

input
```
int((b*x^2+a)^(9/2)/x^5,x)
```

output
```
( - 10*sqrt(a + b*x**2)*a**4 - 85*sqrt(a + b*x**2)*a**3*b*x**2 + 288*sqrt(
a + b*x**2)*a**2*b**2*x**4 + 56*sqrt(a + b*x**2)*a*b**3*x**6 + 8*sqrt(a +
b*x**2)*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*
x)/sqrt(a))*a**2*b**2*x**4 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
 sqrt(b)*x)/sqrt(a))*a**2*b**2*x**4)/(40*x**4)
```

3.430 $\int \frac{(a+bx^2)^{9/2}}{x^7} dx$

Optimal result

Integrand size = 15, antiderivative size = 133

$$\int \frac{(a+bx^2)^{9/2}}{x^7} dx = 4ab^3\sqrt{a+bx^2} - \frac{a^4\sqrt{a+bx^2}}{6x^6} - \frac{25a^3b\sqrt{a+bx^2}}{24x^4} - \frac{55a^2b^2\sqrt{a+bx^2}}{16x^2} + \frac{1}{3}b^3(a+bx^2)^{3/2} - \frac{105}{16}a^{3/2}b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
4*a*b^3*(b*x^2+a)^(1/2)-1/6*a^4*(b*x^2+a)^(1/2)/x^6-25/24*a^3*b*(b*x^2+a)^
(1/2)/x^4-55/16*a^2*b^2*(b*x^2+a)^(1/2)/x^2+1/3*b^3*(b*x^2+a)^(3/2)-105/16
*a^(3/2)*b^3*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{9/2}}{x^7} dx = \frac{\sqrt{a+bx^2}(-8a^4 - 50a^3bx^2 - 165a^2b^2x^4 + 208ab^3x^6 + 16b^4x^8)}{48x^6} - \frac{105}{16}a^{3/2}b^3\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^7,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^4 - 50*a^3*b*x^2 - 165*a^2*b^2*x^4 + 208*a*b^3*x^6
+ 16*b^4*x^8))/(48*x^6) - (105*a^(3/2)*b^3*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/16
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 51, 51, 51, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^7}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^8}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{2}b\int \frac{(bx^2+a)^{7/2}}{x^6}dx^2-\frac{(a+bx^2)^{9/2}}{3x^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\int \frac{(bx^2+a)^{5/2}}{x^4}dx^2-\frac{(a+bx^2)^{7/2}}{2x^4}\right)-\frac{(a+bx^2)^{9/2}}{3x^6}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\int \frac{(bx^2+a)^{3/2}}{x^2}dx^2-\frac{(a+bx^2)^{5/2}}{x^2}\right)-\frac{(a+bx^2)^{7/2}}{2x^4}\right)-\frac{(a+bx^2)^{9/2}}{3x^6}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\int\frac{\sqrt{bx^2+a}}{x^2}dx^2+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{2x^4}\right)-\frac{\left(a+bx^2\right)^{9/2}}{3x^6}\right)$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(a\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{2x^4}\right)-\right.$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{2x^4}\right.\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{3}{2}b\left(\frac{7}{4}b\left(\frac{5}{2}b\left(a\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{2}{3}\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{7}}{2x^4}\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^7,x]
```

output
```
(-1/3*(a + b*x^2)^(9/2)/x^6 + (3*b*(-1/2*(a + b*x^2)^(7/2)/x^4 + (7*b*(-((
a + b*x^2)^(5/2)/x^2) + (5*b*((2*(a + b*x^2)^(3/2))/3 + a*(2*Sqrt[a + b*x^
2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])))/2))/4))/2)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{105\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)a^{2}b^{3}x^{6}-\frac{16\left(\sqrt{a}\,b^{4}x^{8}+13a^{\frac{3}{2}}b^{3}x^{6}-\frac{165a^{\frac{5}{2}}b^{2}x^{4}}{16}-\frac{25a^{\frac{7}{2}}b\,x^{2}}{8}-\frac{a^{\frac{9}{2}}}{2}\right)\sqrt{b\,x^{2}+a}}{315}\right)}{16\sqrt{a}\,x^{6}}$
risch	$-\frac{a^{2}\sqrt{b\,x^{2}+a}\,(165b^{2}x^{4}+50ab\,x^{2}+8a^{2})}{48x^{6}}-\frac{105b^{3}a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{16}+\frac{b^{4}x^{2}\sqrt{b\,x^{2}+a}}{3}+\frac{13a\,b^{3}\sqrt{b\,x^{2}+a}}{3}$
default	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{6a\,x^{6}}+\frac{5b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{4a\,x^{4}}+\frac{7b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{2a\,x^{2}}+\frac{9b\left(\frac{(b\,x^{2}+a)^{\frac{9}{2}}}{9}+a\left(\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{7}+a\left(\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+\cdots\right.\right.\right.\right.}{2a}\right.}{4a}\right.}{6a}$

input
```
int((b*x^2+a)^(9/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-105/16/a^(1/2)*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^2*b^3*x^6-16/315*(a^(1
/2)*b^4*x^8+13*a^(3/2)*b^3*x^6-165/16*a^(5/2)*b^2*x^4-25/8*a^(7/2)*b*x^2-1
/2*a^(9/2))*(b*x^2+a)^(1/2))/x^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = \left[\frac{315\,a^{\frac{3}{2}}b^3x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(16\,b^4x^8+208\,ab^3x^6-165\,a^2b^2x^4-50\,a^3b\cdots}{96\,x^6}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="fricas")
```

output
```
[1/96*(315*a^(3/2)*b^3*x^6*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/
x^2) + 2*(16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*b*x^2 - 8*
a^4)*sqrt(b*x^2 + a))/x^6, 1/48*(315*sqrt(-a)*a*b^3*x^6*arctan(sqrt(b*x^2
+ a)*sqrt(-a)/a) + (16*b^4*x^8 + 208*a*b^3*x^6 - 165*a^2*b^2*x^4 - 50*a^3*
b*x^2 - 8*a^4)*sqrt(b*x^2 + a))/x^6]
```

Sympy [A] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = -\frac{105a^{\frac{3}{2}}b^3\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16} - \frac{a^5}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{29a^4\sqrt{b}}{24x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{215a^3b^{\frac{3}{2}}}{48x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{43a^2b^{\frac{5}{2}}}{48x\sqrt{\frac{a}{bx^2}+1}} + \frac{14ab^{\frac{7}{2}}x}{3\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{9}{2}}x^3}{3\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**7,x)
```

output
```
-105*a**(3/2)*b**3*asinh(sqrt(a)/(sqrt(b)*x))/16 - a**5/(6*sqrt(b)*x**7*sq
rt(a/(b*x**2) + 1)) - 29*a**4*sqrt(b)/(24*x**5*sqrt(a/(b*x**2) + 1)) - 215
*a**3*b**(3/2)/(48*x**3*sqrt(a/(b*x**2) + 1)) + 43*a**2*b**(5/2)/(48*x*sqr
t(a/(b*x**2) + 1)) + 14*a*b**(7/2)*x/(3*sqrt(a/(b*x**2) + 1)) + b**(9/2)*x
**3/(3*sqrt(a/(b*x**2) + 1))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = -\frac{105}{16}\,a^{\frac{3}{2}}b^3\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{35}{16}\left(bx^2+a\right)^{\frac{3}{2}}b^3 + \frac{35\left(bx^2+a\right)^{\frac{9}{2}}b^3}{48\,a^3} + \frac{15\left(bx^2+a\right)^{\frac{7}{2}}b^3}{16\,a^2} + \frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3}{16\,a} + \frac{105}{16}\sqrt{bx^2+a}ab^3 - \frac{35\left(bx^2+a\right)^{\frac{11}{2}}b^2}{48\,a^3x^2} - \frac{5\left(bx^2+a\right)^{\frac{11}{2}}b}{24\,a^2x^4} - \frac{\left(bx^2+a\right)^{\frac{11}{2}}}{6\,ax^6}$$

input
```
integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="maxima")
```

output
```
-105/16*a^(3/2)*b^3*arcsinh(a/(sqrt(a*b)*abs(x))) + 35/16*(b*x^2 + a)^(3/2
)*b^3 + 35/48*(b*x^2 + a)^(9/2)*b^3/a^3 + 15/16*(b*x^2 + a)^(7/2)*b^3/a^2
+ 21/16*(b*x^2 + a)^(5/2)*b^3/a + 105/16*sqrt(b*x^2 + a)*a*b^3 - 35/48*(b*
x^2 + a)^(11/2)*b^2/(a^3*x^2) - 5/24*(b*x^2 + a)^(11/2)*b/(a^2*x^4) - 1/6*
(b*x^2 + a)^(11/2)/(a*x^6)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = \frac{1}{48}\left(\frac{315\,a^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 16\left(bx^2+a\right)^{\frac{3}{2}} + 192\sqrt{bx^2+a}\,a - \frac{165\left(bx^2+a\right)^{\frac{5}{2}}a^2 -\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(315*a^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 16*(b*x^2 + a)^(
3/2) + 192*sqrt(b*x^2 + a)*a - (165*(b*x^2 + a)^(5/2)*a^2 - 280*(b*x^2 + a
)^(3/2)*a^3 + 123*sqrt(b*x^2 + a)*a^4)/(b^3*x^6))*b^3
```

Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = \frac{\frac{41\,a^4\,b^3\,\sqrt{b\,x^2+a}}{16} - \frac{35\,a^3\,b^3\,{\left(b\,x^2+a\right)}^{3/2}}{6} + \frac{55\,a^2\,b^3\,{\left(b\,x^2+a\right)}^{5/2}}{16}}{3\,a\,{\left(b\,x^2+a\right)}^2 - 3\,a^2\,\left(b\,x^2+a\right) - {\left(b\,x^2+a\right)}^3 + a^3} + \frac{b^3\,{\left(b\,x^2+a\right)}^{3/2}}{3} + 4\,a\,b^3\,\sqrt{b\,x^2+a} + \frac{a^{3/2}\,b^3\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,105{}\mathrm{i}}{16}$$

input
```
int((a + b*x^2)^(9/2)/x^7,x)
```

output
```
((41*a^4*b^3*(a + b*x^2)^(1/2))/16 - (35*a^3*b^3*(a + b*x^2)^(3/2))/6 + (5
5*a^2*b^3*(a + b*x^2)^(5/2))/16)/(3*a*(a + b*x^2)^2 - 3*a^2*(a + b*x^2) -
(a + b*x^2)^3 + a^3) + (b^3*(a + b*x^2)^(3/2))/3 + (a^(3/2)*b^3*atan(((a +
 b*x^2)^(1/2)*1i)/a^(1/2))*105i)/16 + 4*a*b^3*(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^{9/2}}{x^7}\,dx = \frac{-8\sqrt{b\,x^2+a}\,a^4-50\sqrt{b\,x^2+a}\,a^3b\,x^2-165\sqrt{b\,x^2+a}\,a^2b^2x^4+208\sqrt{b\,x^2+a}\,a\,b^3x^6+}{}$$

input
```
int((b*x^2+a)^(9/2)/x^7,x)
```

output
```
( - 8*sqrt(a + b*x**2)*a**4 - 50*sqrt(a + b*x**2)*a**3*b*x**2 - 165*sqrt(a
 + b*x**2)*a**2*b**2*x**4 + 208*sqrt(a + b*x**2)*a*b**3*x**6 + 16*sqrt(a +
 b*x**2)*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*a*b**3*x**6 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**3*x**6)/(48*x**6)
```

3.431 $\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = b^4\sqrt{a+bx^2} - \frac{a^4\sqrt{a+bx^2}}{8x^8} - \frac{11a^3b\sqrt{a+bx^2}}{16x^6} - \frac{105a^2b^2\sqrt{a+bx^2}}{64x^4} - \frac{325ab^3\sqrt{a+bx^2}}{128x^2} - \frac{315}{128}\sqrt{a}b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
b^4*(b*x^2+a)^(1/2)-1/8*a^4*(b*x^2+a)^(1/2)/x^8-11/16*a^3*b*(b*x^2+a)^(1/2
)/x^6-105/64*a^2*b^2*(b*x^2+a)^(1/2)/x^4-325/128*a*b^3*(b*x^2+a)^(1/2)/x^2
-315/128*a^(1/2)*b^4*arctanh((b*x^2+a)^(1/2)/a^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = \frac{\sqrt{a+bx^2}(-16a^4 - 88a^3bx^2 - 210a^2b^2x^4 - 325ab^3x^6 + 128b^4x^8)}{128x^8} - \frac{315}{128}\sqrt{a}b^4\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^9,x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^4 - 88*a^3*b*x^2 - 210*a^2*b^2*x^4 - 325*a*b^3*x^6
 + 128*b^4*x^8))/(128*x^8) - (315*Sqrt[a]*b^4*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/128
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 51, 51, 51, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^9}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^{10}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{8}b\int \frac{(bx^2+a)^{7/2}}{x^8}dx^2-\frac{(a+bx^2)^{9/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\int \frac{(bx^2+a)^{5/2}}{x^6}dx^2-\frac{(a+bx^2)^{7/2}}{3x^6}\right)-\frac{(a+bx^2)^{9/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\int \frac{(bx^2+a)^{3/2}}{x^4}dx^2-\frac{(a+bx^2)^{5/2}}{2x^4}\right)-\frac{(a+bx^2)^{7/2}}{3x^6}\right)-\frac{(a+bx^2)^{9/2}}{4x^8}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\int\frac{\sqrt{bx^2+a}}{x^2}dx^2-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)-\frac{(a+bx^2)^{7/2}}{3x^6}\right)-\frac{(a+bx^2)^{9/2}}{4x^8}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)-\frac{(a+bx^2)^{7/2}}{3x^6}\right)-$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2a\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+2\sqrt{a+bx^2}\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)-\frac{(a+bx^2)^{7/2}}{3x^6}\right.\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{9}{8}b\left(\frac{7}{6}b\left(\frac{5}{4}b\left(\frac{3}{2}b\left(2\sqrt{a+bx^2}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{(a+bx^2)^{3/2}}{x^2}\right)-\frac{(a+bx^2)^{5/2}}{2x^4}\right)-\frac{(a+bx^2)^{7/2}}{3x^6}\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^9,x]
```

output
```
(-1/4*(a + b*x^2)^(9/2)/x^8 + (9*b*(-1/3*(a + b*x^2)^(7/2)/x^6 + (7*b*(-1/
2*(a + b*x^2)^(5/2)/x^4 + (5*b*(-((a + b*x^2)^(3/2)/x^2) + (3*b*(2*Sqrt[a
+ b*x^2] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2))/4))/6))/8)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{a\sqrt{b\,x^{2}+a}\,(325b^{3}x^{6}+210a\,b^{2}x^{4}+88a^{2}b\,x^{2}+16a^{3})}{128x^{8}}-\frac{315\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)b^{4}}{128}+b^{4}\sqrt{b\,x^{2}+a}$
pseudoelliptic	$\frac{-315\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)a\,b^{4}x^{8}+128b^{4}x^{8}\sqrt{a}\,\sqrt{b\,x^{2}+a}-325a^{\frac{3}{2}}b^{3}x^{6}\sqrt{b\,x^{2}+a}-210a^{\frac{5}{2}}b^{2}x^{4}\sqrt{b\,x^{2}+a}-88a^{\frac{7}{2}}b\,x^{2}\sqrt{b\,x^{2}+a}-}{128x^{8}\sqrt{a}}$
default	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{8a\,x^{8}}+\frac{3b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{6a\,x^{6}}+\frac{5b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{4a\,x^{4}}+\frac{7b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{2a\,x^{2}}+\frac{9b\left(\frac{(b\,x^{2}+a)^{\frac{9}{2}}}{9}+a\left(\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{7}+a\left(\frac{(b\,x^{2}+a)}{5}\right.\right.\right.}{}\right.}{}\right.}{6a}\right.}{8a}$

input
```
int((b*x^2+a)^(9/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/128*a*(b*x^2+a)^(1/2)*(325*b^3*x^6+210*a*b^2*x^4+88*a^2*b*x^2+16*a^3)/x
^8-315/128*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)*b^4+b^4*(b*x^2+a)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.42

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = \left[\frac{315\sqrt{a}b^4x^8\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(128b^4x^8-325ab^3x^6-210a^2b^2x^4-88a\right.}{256x^8}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="fricas")
```

output
```
[1/256*(315*sqrt(a)*b^4*x^8*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)
/x^2) + 2*(128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a^3*b*x^2 -
16*a^4)*sqrt(b*x^2 + a))/x^8, 1/128*(315*sqrt(-a)*b^4*x^8*arctan(sqrt(b*x^
2 + a)*sqrt(-a)/a) + (128*b^4*x^8 - 325*a*b^3*x^6 - 210*a^2*b^2*x^4 - 88*a
^3*b*x^2 - 16*a^4)*sqrt(b*x^2 + a))/x^8]
```

Sympy [A] (verification not implemented)

Time = 10.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = -\frac{315\sqrt{a}b^4\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128} - \frac{a^5}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{13a^4\sqrt{b}}{16x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{149a^3b^{\frac{3}{2}}}{64x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{535a^2b^{\frac{5}{2}}}{128x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{197ab^{\frac{7}{2}}}{128x\sqrt{\frac{a}{bx^2}+1}} + \frac{b^{\frac{9}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**9,x)
```

output
```
-315*sqrt(a)*b**4*asinh(sqrt(a)/(sqrt(b)*x))/128 - a**5/(8*sqrt(b)*x**9*sq
rt(a/(b*x**2) + 1)) - 13*a**4*sqrt(b)/(16*x**7*sqrt(a/(b*x**2) + 1)) - 149
*a**3*b**(3/2)/(64*x**5*sqrt(a/(b*x**2) + 1)) - 535*a**2*b**(5/2)/(128*x**
3*sqrt(a/(b*x**2) + 1)) - 197*a*b**(7/2)/(128*x*sqrt(a/(b*x**2) + 1)) + b*
*(9/2)*x/sqrt(a/(b*x**2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = -\frac{315}{128}\sqrt{a}b^4\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{315}{128}\sqrt{bx^2+a}b^4 + \frac{35\,(bx^2+a)^{\frac{9}{2}}b^4}{128\,a^4} + \frac{45\,(bx^2+a)^{\frac{7}{2}}b^4}{128\,a^3} + \frac{63\,(bx^2+a)^{\frac{5}{2}}b^4}{128\,a^2} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^4}{128\,a} - \frac{35\,(bx^2+a)^{\frac{11}{2}}b^3}{128\,a^4x^2} - \frac{5\,(bx^2+a)^{\frac{11}{2}}b^2}{64\,a^3x^4} - \frac{(bx^2+a)^{\frac{11}{2}}b}{16\,a^2x^6} - \frac{(bx^2+a)^{\frac{11}{2}}}{8\,ax^8}$$

input
```
integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="maxima")
```

output
```
-315/128*sqrt(a)*b^4*arcsinh(a/(sqrt(a*b)*abs(x))) + 315/128*sqrt(b*x^2 +
a)*b^4 + 35/128*(b*x^2 + a)^(9/2)*b^4/a^4 + 45/128*(b*x^2 + a)^(7/2)*b^4/a
^3 + 63/128*(b*x^2 + a)^(5/2)*b^4/a^2 + 105/128*(b*x^2 + a)^(3/2)*b^4/a -
35/128*(b*x^2 + a)^(11/2)*b^3/(a^4*x^2) - 5/64*(b*x^2 + a)^(11/2)*b^2/(a^3
*x^4) - 1/16*(b*x^2 + a)^(11/2)*b/(a^2*x^6) - 1/8*(b*x^2 + a)^(11/2)/(a*x^
8)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = \frac{\frac{315\,ab^5\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 128\,\sqrt{bx^2+a}b^5 - \frac{325\,(bx^2+a)^{\frac{7}{2}}ab^5 - 765\,(bx^2+a)^{\frac{5}{2}}a^2b^5 + 643\,(bx^2+a)^{\frac{3}{2}}a^3b^5 -}{b^4x^8}}{128\,b}$$

input
```
integrate((b*x^2+a)^(9/2)/x^9,x, algorithm="giac")
```

output
```
1/128*(315*a*b^5*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 128*sqrt(b*x^
2 + a)*b^5 - (325*(b*x^2 + a)^(7/2)*a*b^5 - 765*(b*x^2 + a)^(5/2)*a^2*b^5
+ 643*(b*x^2 + a)^(3/2)*a^3*b^5 - 187*sqrt(b*x^2 + a)*a^4*b^5)/(b^4*x^8))/
b
```

Mupad [B] (verification not implemented)

Time = 1.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = b^4\,\sqrt{b\,x^2+a} - \frac{325\,a\,(b\,x^2+a)^{7/2}}{128\,x^8} + \frac{187\,a^4\,\sqrt{b\,x^2+a}}{128\,x^8} - \frac{643\,a^3\,(b\,x^2+a)^{3/2}}{128\,x^8} + \frac{765\,a^2\,(b\,x^2+a)^{5/2}}{128\,x^8} + \frac{\sqrt{a}\,b^4\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,315\mathrm{i}}{128}$$

input
```
int((a + b*x^2)^(9/2)/x^9,x)
```

output
```
b^4*(a + b*x^2)^(1/2) + (a^(1/2)*b^4*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*
315i)/128 - (325*a*(a + b*x^2)^(7/2))/(128*x^8) + (187*a^4*(a + b*x^2)^(1/
2))/(128*x^8) - (643*a^3*(a + b*x^2)^(3/2))/(128*x^8) + (765*a^2*(a + b*x^
2)^(5/2))/(128*x^8)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^{9/2}}{x^9}\,dx = \frac{-16\sqrt{b\,x^2+a}\,a^4-88\sqrt{b\,x^2+a}\,a^3b\,x^2-210\sqrt{b\,x^2+a}\,a^2b^2x^4-325\sqrt{b\,x^2+a}\,a\,b^3x^6 +}{}$$

input
```
int((b*x^2+a)^(9/2)/x^9,x)
```

output
```
( - 16*sqrt(a + b*x**2)*a**4 - 88*sqrt(a + b*x**2)*a**3*b*x**2 - 210*sqrt(
a + b*x**2)*a**2*b**2*x**4 - 325*sqrt(a + b*x**2)*a*b**3*x**6 + 128*sqrt(a
 + b*x**2)*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*b**4*x**8 - 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + s
qrt(b)*x)/sqrt(a))*b**4*x**8)/(128*x**8)
```

3.432 $\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = -\frac{a^4\sqrt{a+bx^2}}{10x^{10}} - \frac{41a^3b\sqrt{a+bx^2}}{80x^8} - \frac{171a^2b^2\sqrt{a+bx^2}}{160x^6} - \frac{149ab^3\sqrt{a+bx^2}}{128x^4} - \frac{193b^4\sqrt{a+bx^2}}{256x^2} - \frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

output
```
-1/10*a^4*(b*x^2+a)^(1/2)/x^10-41/80*a^3*b*(b*x^2+a)^(1/2)/x^8-171/160*a^2
*b^2*(b*x^2+a)^(1/2)/x^6-149/128*a*b^3*(b*x^2+a)^(1/2)/x^4-193/256*b^4*(b*
x^2+a)^(1/2)/x^2-63/256*b^5*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \frac{\sqrt{a+bx^2}(-128a^4-656a^3bx^2-1368a^2b^2x^4-1490ab^3x^6-965b^4x^8)}{1280x^{10}} - \frac{63b^5\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{256\sqrt{a}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^11,x]
```

output
```
(Sqrt[a + b*x^2]*(-128*a^4 - 656*a^3*b*x^2 - 1368*a^2*b^2*x^4 - 1490*a*b^3
*x^6 - 965*b^4*x^8))/(1280*x^10) - (63*b^5*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(256*Sqrt[a])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 51, 51, 51, 51, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{11}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{9/2}}{x^{12}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{10}b\int \frac{\left(bx^2+a\right)^{7/2}}{x^{10}}dx^2-\frac{\left(a+bx^2\right)^{9/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\int \frac{\left(bx^2+a\right)^{5/2}}{x^8}dx^2-\frac{\left(a+bx^2\right)^{7/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{9/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\int \frac{\left(bx^2+a\right)^{3/2}}{x^6}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{7/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{9/2}}{5x^{10}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\int\frac{\sqrt{bx^2+a}}{x^4}dx^2-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)-\frac{(a+bx^2)^{7/2}}{4x^8}\right)-\frac{(a+bx^2)^{9/2}}{5x^{10}}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)-\frac{(a+bx^2)^{7/2}}{4x^8}\right)\right.$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)-\frac{(a+bx^2)^{7/2}}{4x^8}\right)\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{9}{10}b\left(\frac{7}{8}b\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{a+bx^2}}{x^2}\right)-\frac{(a+bx^2)^{3/2}}{2x^4}\right)-\frac{(a+bx^2)^{5/2}}{3x^6}\right)-\frac{(a+bx^2)^{7/2}}{4x^8}\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^11,x]
```

output
```
(-1/5*(a + b*x^2)^(9/2)/x^10 + (9*b*(-1/4*(a + b*x^2)^(7/2)/x^8 + (7*b*(-1
/3*(a + b*x^2)^(5/2)/x^6 + (5*b*(-1/2*(a + b*x^2)^(3/2)/x^4 + (3*b*(-(Sqrt
[a + b*x^2]/x^2) - (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/4))/6))/
8))/10)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.64

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(965b^4x^8+1490a\,b^3x^6+1368a^2b^2x^4+656a^3b\,x^2+128a^4)}{1280x^{10}}-\frac{63b^5\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{256\sqrt{a}}$
pseudoelliptic	$\frac{-315\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)b^5x^{10}-965b^4x^8\sqrt{a}\,\sqrt{b\,x^2+a}-1490a^{\frac{3}{2}}b^3x^6\sqrt{b\,x^2+a}-1368a^{\frac{5}{2}}b^2x^4\sqrt{b\,x^2+a}-656a^{\frac{7}{2}}b\,x^2\sqrt{b\,x^2+\cdots}}{1280x^{10}\sqrt{a}}$
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{10a\,x^{10}}+\frac{b\Bigg(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{8a\,x^8}+\frac{3b\Bigg(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{6a\,x^6}+\frac{5b\Bigg(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{4a\,x^4}+\frac{7b\Bigg(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{2a\,x^2}+\frac{9b\Big(\frac{(b\,x^2+a)^{\frac{9}{2}}}{9}+a\Big(\frac{(b\,x^2+\cdots}{7}\cdots}{\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots}{\cdots}\cdots}{10a}$

input
```
int((b*x^2+a)^(9/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/1280*(b*x^2+a)^(1/2)*(965*b^4*x^8+1490*a*b^3*x^6+1368*a^2*b^2*x^4+656*a
^3*b*x^2+128*a^4)/x^10-63/256*b^5/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2
))/x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \left[\frac{315\,\sqrt{a}b^5x^{10}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(965\,ab^4x^8+1490\,a^2b^3x^6+1368\,a^3b^2x^4\right.}{2560\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="fricas")
```

output
```
[1/2560*(315*sqrt(a)*b^5*x^10*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) - 2*(965*a*b^4*x^8 + 1490*a^2*b^3*x^6 + 1368*a^3*b^2*x^4 + 656*a^4
*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10), 1/1280*(315*sqrt(-a)*b^5*x^10
*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) - (965*a*b^4*x^8 + 1490*a^2*b^3*x^6 +
1368*a^3*b^2*x^4 + 656*a^4*b*x^2 + 128*a^5)*sqrt(b*x^2 + a))/(a*x^10)]
```

Sympy [A] (verification not implemented)

Time = 10.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{10x^9} - \frac{41a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{80x^7} - \frac{171a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{160x^5} - \frac{149ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{128x^3} - \frac{193b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{256x} - \frac{63b^5\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{256\sqrt{a}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**11,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(10*x**9) - 41*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(80*x**7) - 171*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(160*x**5) -
 149*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(128*x**3) - 193*b**(9/2)*sqrt(a/(b*x
**2) + 1)/(256*x) - 63*b**5*asinh(sqrt(a)/(sqrt(b)*x))/(256*sqrt(a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = -\frac{63\,b^5 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{256\,\sqrt{a}} + \frac{7\,(bx^2+a)^{\frac{9}{2}}b^5}{256\,a^5} + \frac{9\,(bx^2+a)^{\frac{7}{2}}b^5}{256\,a^4} + \frac{63\,(bx^2+a)^{\frac{5}{2}}b^5}{1280\,a^3} + \frac{21\,(bx^2+a)^{\frac{3}{2}}b^5}{256\,a^2} + \frac{63\,\sqrt{bx^2+a}b^5}{256\,a} - \frac{7\,(bx^2+a)^{\frac{11}{2}}b^4}{256\,a^5x^2} - \frac{(bx^2+a)^{\frac{11}{2}}b^3}{128\,a^4x^4} - \frac{(bx^2+a)^{\frac{11}{2}}b^2}{160\,a^3x^6} - \frac{(bx^2+a)^{\frac{11}{2}}b}{80\,a^2x^8} - \frac{(bx^2+a)^{\frac{11}{2}}}{10\,ax^{10}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="maxima")
```

output
```
-63/256*b^5*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 7/256*(b*x^2 + a)^(9/2
)*b^5/a^5 + 9/256*(b*x^2 + a)^(7/2)*b^5/a^4 + 63/1280*(b*x^2 + a)^(5/2)*b^
5/a^3 + 21/256*(b*x^2 + a)^(3/2)*b^5/a^2 + 63/256*sqrt(b*x^2 + a)*b^5/a -
7/256*(b*x^2 + a)^(11/2)*b^4/(a^5*x^2) - 1/128*(b*x^2 + a)^(11/2)*b^3/(a^4
*x^4) - 1/160*(b*x^2 + a)^(11/2)*b^2/(a^3*x^6) - 1/80*(b*x^2 + a)^(11/2)*b
/(a^2*x^8) - 1/10*(b*x^2 + a)^(11/2)/(a*x^10)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \frac{1}{1280}\,b^5 \left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{965\,(bx^2+a)^{\frac{9}{2}} - 2370\,(bx^2+a)^{\frac{7}{2}}a + 2688\,(bx^2+a}{b^5x^{10}}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^11,x, algorithm="giac")
```

output
```
1/1280*b^5*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) - (965*(b*x^2 +
a)^(9/2) - 2370*(b*x^2 + a)^(7/2)*a + 2688*(b*x^2 + a)^(5/2)*a^2 - 1470*(b
*x^2 + a)^(3/2)*a^3 + 315*sqrt(b*x^2 + a)*a^4)/(b^5*x^10))
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \frac{237\,a\,(b\,x^2+a)^{7/2}}{128\,x^{10}} - \frac{193\,(b\,x^2+a)^{9/2}}{256\,x^{10}} - \frac{63\,a^4\,\sqrt{b\,x^2+a}}{256\,x^{10}} + \frac{147\,a^3\,(b\,x^2+a)^{3/2}}{128\,x^{10}} - \frac{21\,a^2\,(b\,x^2+a)^{5/2}}{10\,x^{10}} + \frac{b^5\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,63\mathrm{i}}{256\,\sqrt{a}}$$

input
```
int((a + b*x^2)^(9/2)/x^11,x)
```

output
```
(b^5*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*63i)/(256*a^(1/2)) - (193*(a + b
*x^2)^(9/2))/(256*x^10) + (237*a*(a + b*x^2)^(7/2))/(128*x^10) - (63*a^4*(
a + b*x^2)^(1/2))/(256*x^10) + (147*a^3*(a + b*x^2)^(3/2))/(128*x^10) - (2
1*a^2*(a + b*x^2)^(5/2))/(10*x^10)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^{9/2}}{x^{11}}\,dx = \frac{-128\sqrt{b\,x^2+a}\,a^5 - 656\sqrt{b\,x^2+a}\,a^4b\,x^2 - 1368\sqrt{b\,x^2+a}\,a^3b^2x^4 - 1490\sqrt{b\,x^2+a}\,a^2}{}$$

input
```
int((b*x^2+a)^(9/2)/x^11,x)
```

output
```
( - 128*sqrt(a + b*x**2)*a**5 - 656*sqrt(a + b*x**2)*a**4*b*x**2 - 1368*sq
rt(a + b*x**2)*a**3*b**2*x**4 - 1490*sqrt(a + b*x**2)*a**2*b**3*x**6 - 965
*sqrt(a + b*x**2)*a*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a
) + sqrt(b)*x)/sqrt(a))*b**5*x**10 - 315*sqrt(a)*log((sqrt(a + b*x**2) + s
qrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**10)/(1280*a*x**10)
```

3.433 $\int \frac{(a+bx^2)^{9/2}}{x^{13}} dx$

Optimal result

Integrand size = 15, antiderivative size = 165

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}} dx = -\frac{a^4\sqrt{a+bx^2}}{12x^{12}} - \frac{49a^3b\sqrt{a+bx^2}}{120x^{10}} - \frac{253a^2b^2\sqrt{a+bx^2}}{320x^8} - \frac{1429ab^3\sqrt{a+bx^2}}{1920x^6} - \frac{491b^4\sqrt{a+bx^2}}{1536x^4} - \frac{21b^5\sqrt{a+bx^2}}{1024ax^2} + \frac{21b^6\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}$$

output
```
-1/12*a^4*(b*x^2+a)^(1/2)/x^12-49/120*a^3*b*(b*x^2+a)^(1/2)/x^10-253/320*a
^2*b^2*(b*x^2+a)^(1/2)/x^8-1429/1920*a*b^3*(b*x^2+a)^(1/2)/x^6-491/1536*b^
4*(b*x^2+a)^(1/2)/x^4-21/1024*b^5*(b*x^2+a)^(1/2)/a/x^2+21/1024*b^6*arctan
h((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.64

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}} dx = \frac{\sqrt{a+bx^2}(-1280a^5 - 6272a^4bx^2 - 12144a^3b^2x^4 - 11432a^2b^3x^6 - 4910ab^4x^8 - 315b^5}{15360ax^{12}} + \frac{21b^6\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{1024a^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^13,x]
```

output
```
(Sqrt[a + b*x^2]*(-1280*a^5 - 6272*a^4*b*x^2 - 12144*a^3*b^2*x^4 - 11432*a
^2*b^3*x^6 - 4910*a*b^4*x^8 - 315*b^5*x^10))/(15360*a*x^12) + (21*b^6*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(1024*a^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {243, 51, 51, 51, 51, 51, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{13}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{9/2}}{x^{14}}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\int \frac{\left(bx^2+a\right)^{7/2}}{x^{12}}dx^2-\frac{\left(a+bx^2\right)^{9/2}}{6x^{12}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\int \frac{\left(bx^2+a\right)^{5/2}}{x^{10}}dx^2-\frac{\left(a+bx^2\right)^{7/2}}{5x^{10}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{6x^{12}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\int \frac{\left(bx^2+a\right)^{3/2}}{x^8}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{7/2}}{5x^{10}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{6x^{12}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\int\frac{\sqrt{bx^2+a}}{x^6}dx^2-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)-\frac{(a+bx^2)^{7/2}}{5x^{10}}\right)-\frac{(a+bx^2)^{9/2}}{6x^{12}}\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int\frac{1}{x^4\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)-\frac{(a+bx^2)^{7/2}}{5x^{10}}\right)\right.$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right)-\right.\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right.\right.\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{3}{4}b\left(\frac{7}{10}b\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)-\frac{\sqrt{a+bx^2}}{2x^4}\right)-\frac{(a+bx^2)^{3/2}}{3x^6}\right)-\frac{(a+bx^2)^{5/2}}{4x^8}\right.\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^13,x]
```

output
```
(-1/6*(a + b*x^2)^(9/2)/x^12 + (3*b*(-1/5*(a + b*x^2)^(7/2)/x^10 + (7*b*(-
1/4*(a + b*x^2)^(5/2)/x^8 + (5*b*(-1/3*(a + b*x^2)^(3/2)/x^6 + (b*(-1/2*Sq
rt[a + b*x^2]/x^4 + (b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/a^(3/2)))/4))/2))/8))/10))/4)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{49\left(-\frac{45\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)b^6x^{12}}{896}+\sqrt{b\,x^2+a}\left(\frac{45\sqrt{a}\,b^5x^{10}}{896}+\frac{2455a^{\frac{3}{2}}b^4x^8}{3136}+\frac{1429a^{\frac{5}{2}}b^3x^6}{784}+\frac{759a^{\frac{7}{2}}b^2x^4}{392}+a^{\frac{9}{2}}b\,x^2+\frac{10a^{\frac{11}{2}}}{49}\right)\right)}{120a^{\frac{3}{2}}x^{12}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(315b^5x^{10}+4910a\,b^4x^8+11432a^2b^3x^6+12144a^3b^2x^4+6272a^4b\,x^2+1280a^5)}{15360x^{12}a}+\frac{21b^6\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{1024a^{\frac{3}{2}}}$
	$b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{10a\,x^{10}}+\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{8a\,x^8}+\frac{3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{6a\,x^6}+\frac{5b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{4a\,x^4}+\frac{7b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{2a\,x^2}+\frac{9b\left(\frac{\left(b\,x\right.}{}\right.}{}\right)}{}\right)}{}\right)}{}\right)}{}\right.$

input
```
int((b*x^2+a)^(9/2)/x^13,x,method=_RETURNVERBOSE)
```

output
```
-49/120*(-45/896*arctanh((b*x^2+a)^(1/2)/a^(1/2))*b^6*x^12+(b*x^2+a)^(1/2)
*(45/896*a^(1/2)*b^5*x^10+2455/3136*a^(3/2)*b^4*x^8+1429/784*a^(5/2)*b^3*x
^6+759/392*a^(7/2)*b^2*x^4+a^(9/2)*b*x^2+10/49*a^(11/2)))/a^(3/2)/x^12
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = \left[\frac{315\sqrt{a}b^6x^{12}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right) - 2\left(315\,ab^5x^{10} + 4910\,a^2b^4x^8 + 11432\,a^3b^3x\right.}{30720\,a^2x^{12}}\right.$$

$$-\frac{315\sqrt{-a}b^6x^{12}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right) + \left(315\,ab^5x^{10} + 4910\,a^2b^4x^8 + 11432\,a^3b^3x^6 + 12144\,a^4b^2x^4 + 6272\,a\right.}{15360\,a^2x^{12}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="fricas")
```

output
```
[1/30720*(315*sqrt(a)*b^6*x^12*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(315*a*b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 1214
4*a^4*b^2*x^4 + 6272*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12), -1
/15360*(315*sqrt(-a)*b^6*x^12*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (315*a*
b^5*x^10 + 4910*a^2*b^4*x^8 + 11432*a^3*b^3*x^6 + 12144*a^4*b^2*x^4 + 6272
*a^5*b*x^2 + 1280*a^6)*sqrt(b*x^2 + a))/(a^2*x^12)]
```

Sympy [A] (verification not implemented)

Time = 48.64 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = -\frac{a^5}{12\sqrt{b}x^{13}\sqrt{\frac{a}{bx^2}+1}} - \frac{59a^4\sqrt{b}}{120x^{11}\sqrt{\frac{a}{bx^2}+1}} - \frac{1151a^3b^{\frac{3}{2}}}{960x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{2947a^2b^{\frac{5}{2}}}{1920x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{8171ab^{\frac{7}{2}}}{7680x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{1045b^{\frac{9}{2}}}{3072x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{21b^{\frac{11}{2}}}{1024ax\sqrt{\frac{a}{bx^2}+1}} + \frac{21b^6\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{1024a^{\frac{3}{2}}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**13,x)
```

output
```
-a**5/(12*sqrt(b)*x**13*sqrt(a/(b*x**2) + 1)) - 59*a**4*sqrt(b)/(120*x**11
*sqrt(a/(b*x**2) + 1)) - 1151*a**3*b**(3/2)/(960*x**9*sqrt(a/(b*x**2) + 1)
) - 2947*a**2*b**(5/2)/(1920*x**7*sqrt(a/(b*x**2) + 1)) - 8171*a*b**(7/2)/
(7680*x**5*sqrt(a/(b*x**2) + 1)) - 1045*b**(9/2)/(3072*x**3*sqrt(a/(b*x**2
) + 1)) - 21*b**(11/2)/(1024*a*x*sqrt(a/(b*x**2) + 1)) + 21*b**6*asinh(sqr
t(a)/(sqrt(b)*x))/(1024*a**(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = \frac{21\,b^6\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{1024\,a^{\frac{3}{2}}} - \frac{7\,(bx^2+a)^{\frac{9}{2}}b^6}{3072\,a^6} - \frac{3\,(bx^2+a)^{\frac{7}{2}}b^6}{1024\,a^5} - \frac{21\,(bx^2+a)^{\frac{5}{2}}b^6}{5120\,a^4} - \frac{7\,(bx^2+a)^{\frac{3}{2}}b^6}{1024\,a^3} - \frac{21\,\sqrt{bx^2+a}b^6}{1024\,a^2} + \frac{7\,(bx^2+a)^{\frac{11}{2}}b^5}{3072\,a^6x^2} + \frac{(bx^2+a)^{\frac{11}{2}}b^4}{1536\,a^5x^4} + \frac{(bx^2+a)^{\frac{11}{2}}b^3}{1920\,a^4x^6} + \frac{(bx^2+a)^{\frac{11}{2}}b^2}{960\,a^3x^8} + \frac{(bx^2+a)^{\frac{11}{2}}b}{120\,a^2x^{10}} - \frac{(bx^2+a)^{\frac{11}{2}}}{12\,ax^{12}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="maxima")
```

output
```
21/1024*b^6*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 7/3072*(b*x^2 + a)^(9/
2)*b^6/a^6 - 3/1024*(b*x^2 + a)^(7/2)*b^6/a^5 - 21/5120*(b*x^2 + a)^(5/2)*
b^6/a^4 - 7/1024*(b*x^2 + a)^(3/2)*b^6/a^3 - 21/1024*sqrt(b*x^2 + a)*b^6/a
^2 + 7/3072*(b*x^2 + a)^(11/2)*b^5/(a^6*x^2) + 1/1536*(b*x^2 + a)^(11/2)*b
^4/(a^5*x^4) + 1/1920*(b*x^2 + a)^(11/2)*b^3/(a^4*x^6) + 1/960*(b*x^2 + a)
^(11/2)*b^2/(a^3*x^8) + 1/120*(b*x^2 + a)^(11/2)*b/(a^2*x^10) - 1/12*(b*x^
2 + a)^(11/2)/(a*x^12)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = -\frac{\frac{315\,b^7\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{315\,(bx^2+a)^{\frac{11}{2}}b^7+3335\,(bx^2+a)^{\frac{9}{2}}ab^7-5058\,(bx^2+a)^{\frac{7}{2}}a^2b^7+4158\,(bx^2+a)^{\frac{5}{2}}a^3b^7-1785\,(bx^2+a)^{\frac{3}{2}}a^4b^7+315\,\sqrt{b}\cdots}{ab^6x^{12}}}{15360\,b}$$

input
```
integrate((b*x^2+a)^(9/2)/x^13,x, algorithm="giac")
```

output
```
-1/15360*(315*b^7*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + (315*(b*
x^2 + a)^(11/2)*b^7 + 3335*(b*x^2 + a)^(9/2)*a*b^7 - 5058*(b*x^2 + a)^(7/2
)*a^2*b^7 + 4158*(b*x^2 + a)^(5/2)*a^3*b^7 - 1785*(b*x^2 + a)^(3/2)*a^4*b^
7 + 315*sqrt(b*x^2 + a)*a^5*b^7)/(a*b^6*x^12))/b
```

Mupad [B] (verification not implemented)

Time = 2.66 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.75

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx &= \frac{843\,a\,(b\,x^2+a)^{7/2}}{2560\,x^{12}} - \frac{667\,(b\,x^2+a)^{9/2}}{3072\,x^{12}} \\ &- \frac{21\,a^4\,\sqrt{b\,x^2+a}}{1024\,x^{12}} + \frac{119\,a^3\,(b\,x^2+a)^{3/2}}{1024\,x^{12}} - \frac{693\,a^2\,(b\,x^2+a)^{5/2}}{2560\,x^{12}} \\ &- \frac{21\,(b\,x^2+a)^{11/2}}{1024\,a\,x^{12}} - \frac{b^6\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,21\mathrm{i}}{1024\,a^{3/2}}\end{aligned}$$

input
```
int((a + b*x^2)^(9/2)/x^13,x)
```

output
```
(843*a*(a + b*x^2)^(7/2))/(2560*x^12) - (b^6*atan(((a + b*x^2)^(1/2)*1i)/a
^(1/2))*21i)/(1024*a^(3/2)) - (667*(a + b*x^2)^(9/2))/(3072*x^12) - (21*a^
4*(a + b*x^2)^(1/2))/(1024*x^12) + (119*a^3*(a + b*x^2)^(3/2))/(1024*x^12)
 - (693*a^2*(a + b*x^2)^(5/2))/(2560*x^12) - (21*(a + b*x^2)^(11/2))/(1024
*a*x^12)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^{9/2}}{x^{13}}\,dx = \frac{-1280\sqrt{b\,x^2+a}\,a^6-6272\sqrt{b\,x^2+a}\,a^5b\,x^2-12144\sqrt{b\,x^2+a}\,a^4b^2x^4-11432\sqrt{b\,x^2+}$$

input
```
int((b*x^2+a)^(9/2)/x^13,x)
```

output
```
( - 1280*sqrt(a + b*x**2)*a**6 - 6272*sqrt(a + b*x**2)*a**5*b*x**2 - 12144
*sqrt(a + b*x**2)*a**4*b**2*x**4 - 11432*sqrt(a + b*x**2)*a**3*b**3*x**6 -
 4910*sqrt(a + b*x**2)*a**2*b**4*x**8 - 315*sqrt(a + b*x**2)*a*b**5*x**10
- 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**6*x
**12 + 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b
**6*x**12)/(15360*a**2*x**12)
```

3.434 $\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 189

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = -\frac{a^4\sqrt{a+bx^2}}{14x^{14}} - \frac{19a^3b\sqrt{a+bx^2}}{56x^{12}} - \frac{351a^2b^2\sqrt{a+bx^2}}{560x^{10}} - \frac{2441ab^3\sqrt{a+bx^2}}{4480x^8} - \frac{253b^4\sqrt{a+bx^2}}{1280x^6} - \frac{3b^5\sqrt{a+bx^2}}{1024ax^4} + \frac{9b^6\sqrt{a+bx^2}}{2048a^2x^2} - \frac{9b^7\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}$$

output
```
-1/14*a^4*(b*x^2+a)^(1/2)/x^14-19/56*a^3*b*(b*x^2+a)^(1/2)/x^12-351/560*a^
2*b^2*(b*x^2+a)^(1/2)/x^10-2441/4480*a*b^3*(b*x^2+a)^(1/2)/x^8-253/1280*b^
4*(b*x^2+a)^(1/2)/x^6-3/1024*b^5*(b*x^2+a)^(1/2)/a/x^4+9/2048*b^6*(b*x^2+a
)^(1/2)/a^2/x^2-9/2048*b^7*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \frac{\sqrt{a+bx^2}(-5120a^6-24320a^5bx^2-44928a^4b^2x^4-39056a^3b^3x^6-14168a^2b^4x^8-210}{71680a^2x^{14}}$$
$$-\frac{9b^7\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2048a^{5/2}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^15,x]
```

output
```
(Sqrt[a + b*x^2]*(-5120*a^6 - 24320*a^5*b*x^2 - 44928*a^4*b^2*x^4 - 39056*
a^3*b^3*x^6 - 14168*a^2*b^4*x^8 - 210*a*b^5*x^10 + 315*b^6*x^12))/(71680*a
^2*x^14) - (9*b^7*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2048*a^(5/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {243, 51, 51, 51, 51, 51, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{9/2}}{x^{16}}dx^2$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{9}{14}b\int \frac{(bx^2+a)^{7/2}}{x^{14}}dx^2-\frac{(a+bx^2)^{9/2}}{7x^{14}}\right)$$

$$\Big\downarrow \; 51$$

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\int\frac{\left(bx^2+a\right)^{5/2}}{x^{12}}dx^2-\frac{\left(a+bx^2\right)^{7/2}}{6x^{12}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^{14}}\right)$$

$\big\downarrow$ 51

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\int\frac{\left(bx^2+a\right)^{3/2}}{x^{10}}dx^2-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)-\frac{\left(a+bx^2\right)^{7/2}}{6x^{12}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^{14}}\right)$$

$\big\downarrow$ 51

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\int\frac{\sqrt{bx^2+a}}{x^8}dx^2-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)-\frac{\left(a+bx^2\right)^{7/2}}{6x^{12}}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^{14}}\right)$$

$\big\downarrow$ 51

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int\frac{1}{x^6\sqrt{bx^2+a}}dx^2-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right)-\frac{\left(a+bx^2\right)^{7/2}}{6x^{12}}\right.\right.$$

$\big\downarrow$ 52

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int\frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)-\frac{\left(a+bx^2\right)^{5/2}}{5x^{10}}\right.\right.\right.$$

$\big\downarrow$ 52

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)\right.\right.\right.$$

$\big\downarrow$ 73

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{\left(a+bx^2\right)^{3/2}}{4x^8}\right)\right.\right.\right.$$

$\big\downarrow$ 221

$$\frac{1}{2}\left(\frac{9}{14}b\left(\frac{7}{12}b\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)-\frac{\sqrt{a+bx^2}}{3x^6}\right)-\frac{(a+bx^2)^{3/2}}{4x^8}\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*x^2)^(9/2)/x^15,x]
```

output
```
(-1/7*(a + b*x^2)^(9/2)/x^14 + (9*b*(-1/6*(a + b*x^2)^(7/2)/x^12 + (7*b*(-
1/5*(a + b*x^2)^(5/2)/x^10 + (b*(-1/4*(a + b*x^2)^(3/2)/x^8 + (3*b*(-1/3*S
qrt[a + b*x^2]/x^6 + (b*(-1/2*Sqrt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqrt[a + b
*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a)))/6)
)/8))/2))/12))/14)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{351\left(\frac{35\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)x^{14}b^7}{4992}+\sqrt{b\,x^2+a}\left(-\frac{35\sqrt{a}\,b^6x^{12}}{4992}+\frac{35a^{\frac{3}{2}}b^5x^{10}}{7488}+\frac{1771a^{\frac{5}{2}}b^4x^8}{5616}+\frac{2441a^{\frac{7}{2}}b^3x^6}{2808}+a^{\frac{9}{2}}b^2x^4+\frac{190a^{\frac{11}{2}}b\cdots}{351}\cdots\right.\right.}{560a^{\frac{5}{2}}x^{14}}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-315b^6x^{12}+210a\,b^5x^{10}+14168a^2b^4x^8+39056a^3b^3x^6+44928a^4b^2x^4+24320a^5b\,x^2+5120a^6)}{71680x^{14}a^2}-\frac{9b^7\ln\left(\frac{2a+\cdots}{\cdots}\right)}{20\cdots}$
	$\left(b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{10a\,x^{10}}+\frac{b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{8a\,x^8}+\frac{3b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{6a\,x^6}+\frac{5b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{4a\,x^4}+\frac{7b\left(-\frac{(b\cdots}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)\cdots\right.$

input
```
int((b*x^2+a)^(9/2)/x^15,x,method=_RETURNVERBOSE)
```

output
```
-351/560/a^(5/2)*(35/4992*arctanh((b*x^2+a)^(1/2)/a^(1/2))*x^14*b^7+(b*x^2
+a)^(1/2)*(-35/4992*a^(1/2)*b^6*x^12+35/7488*a^(3/2)*b^5*x^10+1771/5616*a^
(5/2)*b^4*x^8+2441/2808*a^(7/2)*b^3*x^6+a^(9/2)*b^2*x^4+190/351*a^(11/2)*b
*x^2+40/351*a^(13/2)))/x^14
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \left[\frac{315\sqrt{a}b^7x^{14}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\,(315\,ab^6x^{12}-210\,a^2b^5x^{10}-14168\,a^3b^4x^8}{143360\,a^3x^{14}}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="fricas")
```

output
```
[1/143360*(315*sqrt(a)*b^7*x^14*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) + 2*(315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 390
56*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^
2 + a))/(a^3*x^14), 1/71680*(315*sqrt(-a)*b^7*x^14*arctan(sqrt(b*x^2 + a)*
sqrt(-a)/a) + (315*a*b^6*x^12 - 210*a^2*b^5*x^10 - 14168*a^3*b^4*x^8 - 390
56*a^4*b^3*x^6 - 44928*a^5*b^2*x^4 - 24320*a^6*b*x^2 - 5120*a^7)*sqrt(b*x^
2 + a))/(a^3*x^14)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(9/2)/x**15,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = -\frac{9\,b^7\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2048\,a^{\frac{5}{2}}} + \frac{(bx^2+a)^{\frac{9}{2}}b^7}{2048\,a^7} + \frac{9\,(bx^2+a)^{\frac{7}{2}}b^7}{14336\,a^6} + \frac{9\,(bx^2+a)^{\frac{5}{2}}b^7}{10240\,a^5} + \frac{3\,(bx^2+a)^{\frac{3}{2}}b^7}{2048\,a^4} + \frac{9\,\sqrt{bx^2+a}b^7}{2048\,a^3} - \frac{(bx^2+a)^{\frac{11}{2}}b^6}{2048\,a^7x^2} - \frac{(bx^2+a)^{\frac{11}{2}}b^5}{7168\,a^6x^4} - \frac{(bx^2+a)^{\frac{11}{2}}b^4}{8960\,a^5x^6} - \frac{(bx^2+a)^{\frac{11}{2}}b^3}{4480\,a^4x^8} - \frac{(bx^2+a)^{\frac{11}{2}}b^2}{560\,a^3x^{10}} + \frac{(bx^2+a)^{\frac{11}{2}}b}{56\,a^2x^{12}} - \frac{(bx^2+a)^{\frac{11}{2}}}{14\,ax^{14}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="maxima")
```

output
```
-9/2048*b^7*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2048*(b*x^2 + a)^(9/
2)*b^7/a^7 + 9/14336*(b*x^2 + a)^(7/2)*b^7/a^6 + 9/10240*(b*x^2 + a)^(5/2)
*b^7/a^5 + 3/2048*(b*x^2 + a)^(3/2)*b^7/a^4 + 9/2048*sqrt(b*x^2 + a)*b^7/a
^3 - 1/2048*(b*x^2 + a)^(11/2)*b^6/(a^7*x^2) - 1/7168*(b*x^2 + a)^(11/2)*b
^5/(a^6*x^4) - 1/8960*(b*x^2 + a)^(11/2)*b^4/(a^5*x^6) - 1/4480*(b*x^2 + a
)^(11/2)*b^3/(a^4*x^8) - 1/560*(b*x^2 + a)^(11/2)*b^2/(a^3*x^10) + 1/56*(b
*x^2 + a)^(11/2)*b/(a^2*x^12) - 1/14*(b*x^2 + a)^(11/2)/(a*x^14)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \frac{1}{71680}b^7\left(\frac{315\,\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{315\,(bx^2+a)^{\frac{13}{2}} - 2100\,(bx^2+a)^{\frac{11}{2}}a - 8393\,(bx^2+\cdots}{}\right.$$

input
```
integrate((b*x^2+a)^(9/2)/x^15,x, algorithm="giac")
```

output
```
1/71680*b^7*(315*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (315*(b
*x^2 + a)^(13/2) - 2100*(b*x^2 + a)^(11/2)*a - 8393*(b*x^2 + a)^(9/2)*a^2
+ 9216*(b*x^2 + a)^(7/2)*a^3 - 5943*(b*x^2 + a)^(5/2)*a^4 + 2100*(b*x^2 +
a)^(3/2)*a^5 - 315*sqrt(b*x^2 + a)*a^6)/(a^2*b^7*x^14))
```

Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \frac{9\,a\,(b\,x^2+a)^{7/2}}{70\,x^{14}} - \frac{1199\,(b\,x^2+a)^{9/2}}{10240\,x^{14}} - \frac{9\,a^4\,\sqrt{b\,x^2+a}}{2048\,x^{14}} + \frac{15\,a^3\,(b\,x^2+a)^{3/2}}{512\,x^{14}} - \frac{849\,a^2\,(b\,x^2+a)^{5/2}}{10240\,x^{14}} - \frac{15\,(b\,x^2+a)^{11/2}}{512\,a\,x^{14}} + \frac{9\,(b\,x^2+a)^{13/2}}{2048\,a^2\,x^{14}} + \frac{b^7\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,9\mathrm{i}}{2048\,a^{5/2}}$$

input
```
int((a + b*x^2)^(9/2)/x^15,x)
```

output
```
(b^7*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*9i)/(2048*a^(5/2)) - (1199*(a +
b*x^2)^(9/2))/(10240*x^14) + (9*a*(a + b*x^2)^(7/2))/(70*x^14) - (9*a^4*(a
 + b*x^2)^(1/2))/(2048*x^14) + (15*a^3*(a + b*x^2)^(3/2))/(512*x^14) - (84
9*a^2*(a + b*x^2)^(5/2))/(10240*x^14) - (15*(a + b*x^2)^(11/2))/(512*a*x^1
4) + (9*(a + b*x^2)^(13/2))/(2048*a^2*x^14)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{9/2}}{x^{15}}\,dx = \frac{-5120\sqrt{b\,x^2+a}\,a^7 - 24320\sqrt{b\,x^2+a}\,a^6 b\,x^2 - 44928\sqrt{b\,x^2+a}\,a^5 b^2 x^4 - 39056\sqrt{b\,x^2+}}{}$$

input
```
int((b*x^2+a)^(9/2)/x^15,x)
```

output
```
( - 5120*sqrt(a + b*x**2)*a**7 - 24320*sqrt(a + b*x**2)*a**6*b*x**2 - 4492
8*sqrt(a + b*x**2)*a**5*b**2*x**4 - 39056*sqrt(a + b*x**2)*a**4*b**3*x**6
- 14168*sqrt(a + b*x**2)*a**3*b**4*x**8 - 210*sqrt(a + b*x**2)*a**2*b**5*x
**10 + 315*sqrt(a + b*x**2)*a*b**6*x**12 + 315*sqrt(a)*log((sqrt(a + b*x**
2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**7*x**14 - 315*sqrt(a)*log((sqrt(a +
b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**7*x**14)/(71680*a**3*x**14)
```

3.435 $\int x^6(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 202

$$\int x^6\left(a+bx^2\right)^{9/2}\,dx=\frac{45a^7x\sqrt{a+bx^2}}{32768b^3}-\frac{15a^6x^3\sqrt{a+bx^2}}{16384b^2}+\frac{3a^5x^5\sqrt{a+bx^2}}{4096b}+\frac{9a^4x^7\sqrt{a+bx^2}}{2048}+\frac{3}{256}a^3x^7\left(a+bx^2\right)^{3/2}+\frac{3}{128}a^2x^7\left(a+bx^2\right)^{5/2}+\frac{9}{224}ax^7\left(a+bx^2\right)^{7/2}+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}-\frac{45a^8\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{32768b^{7/2}}$$

output

```
45/32768*a^7*x*(b*x^2+a)^(1/2)/b^3-15/16384*a^6*x^3*(b*x^2+a)^(1/2)/b^2+3/
4096*a^5*x^5*(b*x^2+a)^(1/2)/b+9/2048*a^4*x^7*(b*x^2+a)^(1/2)+3/256*a^3*x^
7*(b*x^2+a)^(3/2)+3/128*a^2*x^7*(b*x^2+a)^(5/2)+9/224*a*x^7*(b*x^2+a)^(7/2
)+1/16*x^7*(b*x^2+a)^(9/2)-45/32768*a^8*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.68

$$\int x^6(a+bx^2)^{9/2}\,dx = \frac{\sqrt{a+bx^2}(315a^7x-210a^6bx^3+168a^5b^2x^5+32624a^4b^3x^7+98432a^3b^4x^9+119040a^2b^5x^{11}-}{229376b^3}$$
$$-\frac{45a^8\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{16384b^{7/2}}$$

input
```
Integrate[x^6*(a + b*x^2)^(9/2),x]
```

output
```
(Sqrt[a + b*x^2]*(315*a^7*x - 210*a^6*b*x^3 + 168*a^5*b^2*x^5 + 32624*a^4*
b^3*x^7 + 98432*a^3*b^4*x^9 + 119040*a^2*b^5*x^11 + 66560*a*b^6*x^13 + 143
36*b^7*x^15))/(229376*b^3) - (45*a^8*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[
a + b*x^2])])/(16384*b^(7/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {248, 248, 248, 248, 248, 262, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6(a+bx^2)^{9/2}\,dx$$
$$\Big\downarrow \text{248}$$
$$\frac{9}{16}a\int x^6(bx^2+a)^{7/2}dx+\frac{1}{16}x^7(a+bx^2)^{9/2}$$
$$\Big\downarrow \text{248}$$
$$\frac{9}{16}a\left(\frac{1}{2}a\int x^6(bx^2+a)^{5/2}dx+\frac{1}{14}x^7(a+bx^2)^{7/2}\right)+\frac{1}{16}x^7(a+bx^2)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\int x^6\left(bx^2+a\right)^{3/2}dx+\frac{1}{12}x^7\left(a+bx^2\right)^{5/2}\right)+\frac{1}{14}x^7\left(a+bx^2\right)^{7/2}\right)+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\int x^6\sqrt{bx^2+a}dx+\frac{1}{10}x^7\left(a+bx^2\right)^{3/2}\right)+\frac{1}{12}x^7\left(a+bx^2\right)^{5/2}\right)+\frac{1}{14}x^7\left(a+bx^2\right)^{7/2}\right)+\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\int\frac{x^6}{\sqrt{bx^2+a}}dx+\frac{1}{8}x^7\sqrt{a+bx^2}\right)+\frac{1}{10}x^7\left(a+bx^2\right)^{3/2}\right)+\frac{1}{12}x^7\left(a+bx^2\right)^{5/2}\right)+\frac{1}{14}x^7\right.$$
$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^5\sqrt{a+bx^2}}{6b}-\frac{5a\int\frac{x^4}{\sqrt{bx^2+a}}dx}{6b}\right)+\frac{1}{8}x^7\sqrt{a+bx^2}\right)+\frac{1}{10}x^7\left(a+bx^2\right)^{3/2}\right)+\frac{1}{12}x^7(\right.\right.$$
$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^5\sqrt{a+bx^2}}{6b}-\frac{5a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int\frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{6b}\right)+\frac{1}{8}x^7\sqrt{a+bx^2}\right)+\frac{1}{10}x^7\left(a+bx^2\right.\right.\right.\right.$$
$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^5\sqrt{a+bx^2}}{6b}-\frac{5a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{8}x^7\sqrt{a+bx^2}\right)+\right.\right.\right.\right.$$

$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$\big\downarrow$ 224

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^5\sqrt{a+bx^2}}{6b}-\frac{5a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{6b}\right)+\frac{1}{8}x^7\sqrt{a+bx^2}\right.\right.\right.\right.$$

$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

$\big\downarrow$ 219

$$\frac{9}{16}a\left(\frac{1}{2}a\left(\frac{5}{12}a\left(\frac{3}{10}a\left(\frac{1}{8}a\left(\frac{x^5\sqrt{a+bx^2}}{6b}-\frac{5a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)}{6b}\right)+\frac{1}{8}x^7\sqrt{a+bx^2}\right.\right.\right.\right.$$

$$\frac{1}{16}x^7\left(a+bx^2\right)^{9/2}$$

input
```
Int[x^6*(a + b*x^2)^(9/2),x]
```

output
```
(x^7*(a + b*x^2)^(9/2))/16 + (9*a*((x^7*(a + b*x^2)^(7/2))/14 + (a*((x^7*(
a + b*x^2)^(5/2))/12 + (5*a*((x^7*(a + b*x^2)^(3/2))/10 + (3*a*((x^7*Sqrt[
a + b*x^2])/8 + (a*((x^5*Sqrt[a + b*x^2])/(6*b) - (5*a*((x^3*Sqrt[a + b*x^
2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(2*b^(3/2))))/(4*b)))/(6*b)))/8))/10))/12))/2))/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

method	result
risch	$\frac{x\left(14336b^{7}x^{14}+66560a\,b^{6}x^{12}+119040a^{2}b^{5}x^{10}+98432a^{3}b^{4}x^{8}+32624a^{4}b^{3}x^{6}+168a^{5}b^{2}x^{4}-210a^{6}b\,x^{2}+315a^{7}\right)\sqrt{b\,x^{2}+a}}{229376b^{3}}-$
pseudoelliptic	$-\frac{45\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{8}-x\left(\frac{7936a^{2}b^{\frac{11}{2}}x^{10}}{21}+\frac{13312a\,b^{\frac{13}{2}}x^{12}}{63}+\frac{2048b^{\frac{15}{2}}x^{14}}{45}+a^{3}\left(\frac{98432b^{\frac{9}{2}}x^{8}}{315}+\frac{32624a\,b^{\frac{7}{2}}x^{6}}{315}+\frac{8a^{2}b^{\frac{5}{2}}x^{4}}{15}-\right.\right.\right.}{32768b^{\frac{7}{2}}}$
	$5a\left(\frac{x\left(b\,x^{2}\right.}{}\right.$
	$7a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\right.$
	$9a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\right.$
	$a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\right.$
	$3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{12b}-\frac{\ldots}{12b}\right.$

input
```
int(x^6*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/229376*x*(14336*b^7*x^14+66560*a*b^6*x^12+119040*a^2*b^5*x^10+98432*a^3*
b^4*x^8+32624*a^4*b^3*x^6+168*a^5*b^2*x^4-210*a^6*b*x^2+315*a^7)*(b*x^2+a)
^(1/2)/b^3-45/32768*a^8/b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.26

$$\int x^6(a+bx^2)^{9/2}\,dx = \left[\frac{315\,a^8\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(14336\,b^8x^{15}+66560\,ab^7x^{13}+119040\,a^2}{458752}\right.$$

input
```
integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/458752*(315*a^8*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11 + 98432*a^3*
b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^3 + 315*a^7*
b*x)*sqrt(b*x^2 + a))/b^4, 1/229376*(315*a^8*sqrt(-b)*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) + (14336*b^8*x^15 + 66560*a*b^7*x^13 + 119040*a^2*b^6*x^11
+ 98432*a^3*b^5*x^9 + 32624*a^4*b^4*x^7 + 168*a^5*b^3*x^5 - 210*a^6*b^2*x^
3 + 315*a^7*b*x)*sqrt(b*x^2 + a))/b^4]
```

Sympy [F(-1)]

Timed out.

$$\int x^6(a+bx^2)^{9/2}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.80

$$\int x^6 (a+bx^2)^{9/2}\, dx = \frac{(bx^2+a)^{\frac{11}{2}}x^5}{16\,b} - \frac{5\,(bx^2+a)^{\frac{11}{2}}ax^3}{224\,b^2} + \frac{5\,(bx^2+a)^{\frac{11}{2}}a^2x}{896\,b^3} - \frac{(bx^2+a)^{\frac{9}{2}}a^3x}{1792\,b^3} - \frac{9\,(bx^2+a)^{\frac{7}{2}}a^4x}{14336\,b^3} - \frac{3\,(bx^2+a)^{\frac{5}{2}}a^5x}{4096\,b^3} - \frac{15\,(bx^2+a)^{\frac{3}{2}}a^6x}{16384\,b^3} - \frac{45\,\sqrt{bx^2+a}a^7x}{32768\,b^3} - \frac{45\,a^8\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{32768\,b^{\frac{7}{2}}}$$

input
```
integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/16*(b*x^2 + a)^(11/2)*x^5/b - 5/224*(b*x^2 + a)^(11/2)*a*x^3/b^2 + 5/896
*(b*x^2 + a)^(11/2)*a^2*x/b^3 - 1/1792*(b*x^2 + a)^(9/2)*a^3*x/b^3 - 9/143
36*(b*x^2 + a)^(7/2)*a^4*x/b^3 - 3/4096*(b*x^2 + a)^(5/2)*a^5*x/b^3 - 15/1
6384*(b*x^2 + a)^(3/2)*a^6*x/b^3 - 45/32768*sqrt(b*x^2 + a)*a^7*x/b^3 - 45
/32768*a^8*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int x^6 (a+bx^2)^{9/2}\, dx = \frac{45\,a^8 \log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{32768\,b^{\frac{7}{2}}} + \frac{1}{229376}\left(\frac{315\,a^7}{b^3} - 2\left(\frac{105\,a^6}{b^2} - 4\left(\frac{21\,a^5}{b} + 2\left(2039\,a^4 + 8\left(769\,a^3b + 2\left(465\,a^2b^2 + 4\left(14\,b^4x^2 + 65\,ab^3\right)x^2\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^6*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
45/32768*a^8*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2) + 1/229376*(31
5*a^7/b^3 - 2*(105*a^6/b^2 - 4*(21*a^5/b + 2*(2039*a^4 + 8*(769*a^3*b + 2*
(465*a^2*b^2 + 4*(14*b^4*x^2 + 65*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*x^2)*sqr
t(b*x^2 + a)*x
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(a+bx^2\right)^{9/2}\,dx=\int x^6\,\left(b\,x^2+a\right)^{9/2}\,dx$$

input
```
int(x^6*(a + b*x^2)^(9/2),x)
```

output
```
int(x^6*(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int x^6 (a+bx^2)^{9/2}\,dx=\frac{315\sqrt{b\,x^2+a}\,a^7bx-210\sqrt{b\,x^2+a}\,a^6b^2x^3+168\sqrt{b\,x^2+a}\,a^5b^3x^5+32624\sqrt{b\,x^2+a}\,a^4b^4x^7}{}$$

input
```
int(x^6*(b*x^2+a)^(9/2),x)
```

output
```
(315*sqrt(a + b*x**2)*a**7*b*x - 210*sqrt(a + b*x**2)*a**6*b**2*x**3 + 168
*sqrt(a + b*x**2)*a**5*b**3*x**5 + 32624*sqrt(a + b*x**2)*a**4*b**4*x**7 +
 98432*sqrt(a + b*x**2)*a**3*b**5*x**9 + 119040*sqrt(a + b*x**2)*a**2*b**6
*x**11 + 66560*sqrt(a + b*x**2)*a*b**7*x**13 + 14336*sqrt(a + b*x**2)*b**8
*x**15 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**8)/(22
9376*b**4)
```

3.436 $\int x^4(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 178

$$\int x^4\left(a+bx^2\right)^{9/2}\,dx=-\frac{9a^6x\sqrt{a+bx^2}}{2048b^2}+\frac{3a^5x^3\sqrt{a+bx^2}}{1024b}+\frac{3}{256}a^4x^5\sqrt{a+bx^2}+\frac{3}{128}a^3x^5\left(a+bx^2\right)^{3/2}+\frac{3}{80}a^2x^5\left(a+bx^2\right)^{5/2}+\frac{3}{56}ax^5\left(a+bx^2\right)^{7/2}+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}+\frac{9a^7\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2048b^{5/2}}$$

output

```
-9/2048*a^6*x*(b*x^2+a)^(1/2)/b^2+3/1024*a^5*x^3*(b*x^2+a)^(1/2)/b+3/256*a
^4*x^5*(b*x^2+a)^(1/2)+3/128*a^3*x^5*(b*x^2+a)^(3/2)+3/80*a^2*x^5*(b*x^2+a
)^(5/2)+3/56*a*x^5*(b*x^2+a)^(7/2)+1/14*x^5*(b*x^2+a)^(9/2)+9/2048*a^7*arc
tanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.71

$$\int x^4(a+bx^2)^{9/2}\,dx = \frac{\sqrt{a+bx^2}(-315a^6x+210a^5bx^3+14168a^4b^2x^5+39056a^3b^3x^7+44928a^2b^4x^9+24320ab^5x^1}{71680b^2} + \frac{9a^7\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{1024b^{5/2}}$$

input
```
Integrate[x^4*(a + b*x^2)^(9/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-315*a^6*x + 210*a^5*b*x^3 + 14168*a^4*b^2*x^5 + 39056*a
^3*b^3*x^7 + 44928*a^2*b^4*x^9 + 24320*a*b^5*x^11 + 5120*b^6*x^13))/(71680
*b^2) + (9*a^7*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(1024*b^
(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {248, 248, 248, 248, 248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^2)^{9/2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{14}a\int x^4(bx^2+a)^{7/2}dx+\frac{1}{14}x^5(a+bx^2)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\int x^4(bx^2+a)^{5/2}dx+\frac{1}{12}x^5(a+bx^2)^{7/2}\right)+\frac{1}{14}x^5(a+bx^2)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\int x^4\left(bx^2+a\right)^{3/2}dx+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)+\frac{1}{12}x^5\left(a+bx^2\right)^{7/2}\right)+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\int x^4\sqrt{bx^2+a}dx+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)+\frac{1}{12}x^5\left(a+bx^2\right)^{7/2}\right)+\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\int\frac{x^4}{\sqrt{bx^2+a}}dx+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5\left(a+bx^2\right)^{5/2}\right)+\frac{1}{12}x^5(a$$

$$\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int\frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}\right)+\frac{1}{10}x^5(a+$$

$$\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5\left(a+bx^2\right)^{3/2}$$

$$\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{224}$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5(a+\right.\right.\right.\right.$$

$$\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \ 219$$

$$\frac{9}{14}a\left(\frac{7}{12}a\left(\frac{1}{2}a\left(\frac{3}{8}a\left(\frac{1}{6}a\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)+\frac{1}{6}x^5\sqrt{a+bx^2}\right)+\frac{1}{8}x^5(a+\right.\right.\right.\right.$$

$$\frac{1}{14}x^5\left(a+bx^2\right)^{9/2}$$

input
```
Int[x^4*(a + b*x^2)^(9/2),x]
```

output
```
(x^5*(a + b*x^2)^(9/2))/14 + (9*a*((x^5*(a + b*x^2)^(7/2))/12 + (7*a*((x^5
*(a + b*x^2)^(5/2))/10 + (a*((x^5*(a + b*x^2)^(3/2))/8 + (3*a*((x^5*Sqrt[a
 + b*x^2])/6 + (a*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])
/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/6)
)/8))/2))/12))/14
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{x\left(-5120b^{6}x^{12}-24320a\,b^{5}x^{10}-44928a^{2}b^{4}x^{8}-39056a^{3}b^{3}x^{6}-14168a^{4}b^{2}x^{4}-210a^{5}b\,x^{2}+315a^{6}\right)\sqrt{b\,x^{2}+a}}{71680b^{2}}+\frac{9a^{7}\ln\left(\sqrt{b}\right.}{20}$
pseudoelliptic	$\frac{\frac{9\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{7}}{2048}-\frac{9\left(-\frac{4864a\,b^{\frac{11}{2}}x^{10}}{63}-\frac{1024b^{\frac{13}{2}}x^{12}}{63}+a^{2}\left(-\frac{4992b^{\frac{9}{2}}x^{8}}{35}-\frac{39056a\,b^{\frac{7}{2}}x^{6}}{315}-\frac{2024a^{2}b^{\frac{5}{2}}x^{4}}{45}-\frac{2a^{3}b^{\frac{3}{2}}x^{2}}{3}+a^{4}\sqrt{b}\right)\right)}{2048}}{b^{\frac{5}{2}}}$
	$3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{12b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+}}{2}\right.}{}\right)}{6}\right)}{8}\right)}{10}\right)}{12b}\right.$

input
```
int(x^4*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/71680*x*(-5120*b^6*x^12-24320*a*b^5*x^10-44928*a^2*b^4*x^8-39056*a^3*b^
3*x^6-14168*a^4*b^2*x^4-210*a^5*b*x^2+315*a^6)*(b*x^2+a)^(1/2)/b^2+9/2048*
a^7/b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.31

$$\int x^4(a+bx^2)^{9/2}\,dx = \left[\frac{315\,a^7\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5}{143360\,b^3}\right.$$
$$-\frac{315\,a^7\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(5120\,b^7x^{13}+24320\,ab^6x^{11}+44928\,a^2b^5x^9+39056\,a^3b^4x^7+14168\,a^4b^3x^5}{71680\,b^3}$$

input
```
integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/143360*(315*a^7*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(5120*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4
*x^7 + 14168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))
/b^3, -1/71680*(315*a^7*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (512
0*b^7*x^13 + 24320*a*b^6*x^11 + 44928*a^2*b^5*x^9 + 39056*a^3*b^4*x^7 + 14
168*a^4*b^3*x^5 + 210*a^5*b^2*x^3 - 315*a^6*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{9/2} \, dx = \text{Timed out}$$

input
```
integrate(x**4*(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\int x^4 (a + bx^2)^{9/2} \, dx = \frac{(bx^2 + a)^{\frac{11}{2}} x^3}{14\, b} - \frac{(bx^2 + a)^{\frac{11}{2}} ax}{56\, b^2} + \frac{(bx^2 + a)^{\frac{9}{2}} a^2 x}{560\, b^2} + \frac{9\, (bx^2 + a)^{\frac{7}{2}} a^3 x}{4480\, b^2} + \frac{3\, (bx^2 + a)^{\frac{5}{2}} a^4 x}{1280\, b^2} + \frac{3\, (bx^2 + a)^{\frac{3}{2}} a^5 x}{1024\, b^2} + \frac{9\, \sqrt{bx^2 + a} a^6 x}{2048\, b^2} + \frac{9\, a^7 \operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2048\, b^{\frac{5}{2}}}$$

input
```
integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/14*(b*x^2 + a)^(11/2)*x^3/b - 1/56*(b*x^2 + a)^(11/2)*a*x/b^2 + 1/560*(b
*x^2 + a)^(9/2)*a^2*x/b^2 + 9/4480*(b*x^2 + a)^(7/2)*a^3*x/b^2 + 3/1280*(b
*x^2 + a)^(5/2)*a^4*x/b^2 + 3/1024*(b*x^2 + a)^(3/2)*a^5*x/b^2 + 9/2048*sq
rt(b*x^2 + a)*a^6*x/b^2 + 9/2048*a^7*arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.67

$$\int x^4(a+bx^2)^{9/2}\,dx = -\frac{9\,a^7\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2048\,b^{\frac{5}{2}}} - \frac{1}{71680}\left(\frac{315\,a^6}{b^2} - 2\left(\frac{105\,a^5}{b} + 4\left(1771\,a^4 + 2\left(2441\,a^3b + 8\left(351\,a^2b^2 + 10\left(4\,b^4x^2 + 19\,ab^3\right)x^2\right)x^2\right)x^2\right)x^2\right)\right.$$

input
```
integrate(x^4*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-9/2048*a^7*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/71680*(315*
a^6/b^2 - 2*(105*a^5/b + 4*(1771*a^4 + 2*(2441*a^3*b + 8*(351*a^2*b^2 + 10
*(4*b^4*x^2 + 19*a*b^3)*x^2)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{9/2}\,dx = \int x^4\,\left(b\,x^2+a\right)^{9/2}\,dx$$

input
```
int(x^4*(a + b*x^2)^(9/2),x)
```

output
```
int(x^4*(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

$$\int x^4(a+bx^2)^{9/2}\,dx = \frac{-315\sqrt{b\,x^2+a}\,a^6bx + 210\sqrt{b\,x^2+a}\,a^5b^2x^3 + 14168\sqrt{b\,x^2+a}\,a^4b^3x^5 + 39056\sqrt{b\,x^2+a}\,a^3b}{}$$

input
```
int(x^4*(b*x^2+a)^(9/2),x)
```

output
```
( - 315*sqrt(a + b*x**2)*a**6*b*x + 210*sqrt(a + b*x**2)*a**5*b**2*x**3 +
14168*sqrt(a + b*x**2)*a**4*b**3*x**5 + 39056*sqrt(a + b*x**2)*a**3*b**4*x
**7 + 44928*sqrt(a + b*x**2)*a**2*b**5*x**9 + 24320*sqrt(a + b*x**2)*a*b**
6*x**11 + 5120*sqrt(a + b*x**2)*b**7*x**13 + 315*sqrt(b)*log((sqrt(a + b*x
**2) + sqrt(b)*x)/sqrt(a))*a**7)/(71680*b**3)
```

3.437 $\int x^2(a+bx^2)^{9/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 154

$$\int x^2\left(a+bx^2\right)^{9/2}\,dx=\frac{21a^5x\sqrt{a+bx^2}}{1024b}+\frac{21}{512}a^4x^3\sqrt{a+bx^2}+\frac{7}{128}a^3x^3\left(a+bx^2\right)^{3/2}+\frac{21}{320}a^2x^3\left(a+bx^2\right)^{5/2}+\frac{3}{40}ax^3\left(a+bx^2\right)^{7/2}+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}-\frac{21a^6\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}}$$

output
```
21/1024*a^5*x*(b*x^2+a)^(1/2)/b+21/512*a^4*x^3*(b*x^2+a)^(1/2)+7/128*a^3*x
^3*(b*x^2+a)^(3/2)+21/320*a^2*x^3*(b*x^2+a)^(5/2)+3/40*a*x^3*(b*x^2+a)^(7/
2)+1/12*x^3*(b*x^2+a)^(9/2)-21/1024*a^6*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.75

$$\int x^2(a+bx^2)^{9/2}\,dx=\frac{\sqrt{a+bx^2}(315a^5x+4910a^4bx^3+11432a^3b^2x^5+12144a^2b^3x^7+6272ab^4x^9+1280b^5x^{11})}{15360b}-\frac{21a^6\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{512b^{3/2}}$$

input
```
Integrate[x^2*(a + b*x^2)^(9/2),x]
```

output
```
(Sqrt[a + b*x^2]*(315*a^5*x + 4910*a^4*b*x^3 + 11432*a^3*b^2*x^5 + 12144*a
^2*b^3*x^7 + 6272*a*b^4*x^9 + 1280*b^5*x^11))/(15360*b) - (21*a^6*ArcTanh[
(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(512*b^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {248, 248, 248, 248, 248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)^{9/2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{3}{4}a\int x^2(bx^2+a)^{7/2}dx+\frac{1}{12}x^3(a+bx^2)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{3}{4}a\left(\frac{7}{10}a\int x^2(bx^2+a)^{5/2}dx+\frac{1}{10}x^3(a+bx^2)^{7/2}\right)+\frac{1}{12}x^3(a+bx^2)^{9/2}$$

$$\Big\downarrow \text{248}$$

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\int x^2\left(bx^2+a\right)^{3/2}dx+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}\right)+\frac{1}{10}x^3\left(a+bx^2\right)^{7/2}\right)+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 248

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\left(\frac{1}{2}a\int x^2\sqrt{bx^2+a}dx+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}\right)+\frac{1}{10}x^3\left(a+bx^2\right)^{7/2}\right)+\frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 248

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\int\frac{x^2}{\sqrt{bx^2+a}}dx+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(a+bx^2\right)^{5/2}\right)+\frac{1}{10}x^3(a+ \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 262

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(a+bx^2\right)^{5} \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 224

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3(a \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 219

$$\frac{3}{4}a\left(\frac{7}{10}a\left(\frac{5}{8}a\left(\frac{1}{2}a\left(\frac{1}{4}a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)+\frac{1}{4}x^3\sqrt{a+bx^2}\right)+\frac{1}{6}x^3\left(a+bx^2\right)^{3/2}\right)+\frac{1}{8}x^3(a \frac{1}{12}x^3\left(a+bx^2\right)^{9/2}$$

input

```
Int[x^2*(a + b*x^2)^(9/2),x]
```

output
```
(x^3*(a + b*x^2)^(9/2))/12 + (3*a*((x^3*(a + b*x^2)^(7/2))/10 + (7*a*((x^3
*(a + b*x^2)^(5/2))/8 + (5*a*((x^3*(a + b*x^2)^(3/2))/6 + (a*((x^3*Sqrt[a
+ b*x^2])/4 + (a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[
a + b*x^2]])/(2*b^(3/2))))/4))/2))/8))/10))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.62

method	result
risch	$\frac{x\left(1280 b^{5} x^{10}+6272 a \,b^{4} x^{8}+12144 a^{2} b^{3} x^{6}+11432 a^{3} b^{2} x^{4}+4910 a^{4} b \,x^{2}+315 a^{5}\right) \sqrt{b \,x^{2}+a}}{15360 b}-\frac{21 a^{6} \ln \left(\sqrt{b}\, x+\sqrt{b \,x^{2}+a}\right)}{1024 b^{\frac{3}{2}}}$
default	$\frac{x \left(b \,x^{2}+a \right)^{\frac{11}{2}}}{12 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{9}{2}}}{10}+\frac{9 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{7}{2}}}{8}+\frac{7 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\, x+\sqrt{b \,x^{2}+ \cdots}\right.}{2 \sqrt{b}}\right.}{4}\right.}{6}\right.}{8}\right.}{10}\right.}{12 b}$
pseudoelliptic	$\frac{1280 \sqrt{b \,x^{2}+a}\, b^{\frac{11}{2}} x^{11}+6272 a \,b^{\frac{9}{2}} x^{9} \sqrt{b \,x^{2}+a}+12144 a^{2} b^{\frac{7}{2}} x^{7} \sqrt{b \,x^{2}+a}+11432 a^{3} b^{\frac{5}{2}} x^{5} \sqrt{b \,x^{2}+a}+4910 a^{4} b^{\frac{3}{2}} x^{3} \sqrt{b \,x^{2}+a}+315 \cdots}{15360 b^{\frac{3}{2}}}$

input
```
int(x^2*(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/15360*x*(1280*b^5*x^10+6272*a*b^4*x^8+12144*a^2*b^3*x^6+11432*a^3*b^2*x^
4+4910*a^4*b*x^2+315*a^5)*(b*x^2+a)^(1/2)/b-21/1024/b^(3/2)*a^6*ln(b^(1/2)
*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.37

$$\int x^2(a+bx^2)^{9/2}\,dx = \left[\frac{315\,a^6\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(1280\,b^6x^{11}+6272\,ab^5x^9+12144\,a^2b^4x^7}{30720\,b^2}\right.$$

input
```
integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/30720*(315*a^6*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(1280*b^6*x^11 + 6272*a*b^5*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^
5 + 4910*a^4*b^2*x^3 + 315*a^5*b*x)*sqrt(b*x^2 + a))/b^2, 1/15360*(315*a^6
*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (1280*b^6*x^11 + 6272*a*b^5
*x^9 + 12144*a^2*b^4*x^7 + 11432*a^3*b^3*x^5 + 4910*a^4*b^2*x^3 + 315*a^5*
b*x)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A] (verification not implemented)

Time = 48.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.32

$$\int x^2\left(a+bx^2\right)^{9/2}\,dx = \frac{21a^{\frac{11}{2}}x}{1024b\sqrt{1+\frac{bx^2}{a}}}+\frac{1045a^{\frac{9}{2}}x^3}{3072\sqrt{1+\frac{bx^2}{a}}}+\frac{8171a^{\frac{7}{2}}bx^5}{7680\sqrt{1+\frac{bx^2}{a}}}+\frac{2947a^{\frac{5}{2}}b^2x^7}{1920\sqrt{1+\frac{bx^2}{a}}}+\frac{1151a^{\frac{3}{2}}b^3x^9}{960\sqrt{1+\frac{bx^2}{a}}}+\frac{59\sqrt{a}b^4x^{11}}{120\sqrt{1+\frac{bx^2}{a}}}-\frac{21a^6\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{1024b^{\frac{3}{2}}}+\frac{b^5x^{13}}{12\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2*(b*x**2+a)**(9/2),x)
```

output
```
21*a**(11/2)*x/(1024*b*sqrt(1 + b*x**2/a)) + 1045*a**(9/2)*x**3/(3072*sqrt
(1 + b*x**2/a)) + 8171*a**(7/2)*b*x**5/(7680*sqrt(1 + b*x**2/a)) + 2947*a*
*(5/2)*b**2*x**7/(1920*sqrt(1 + b*x**2/a)) + 1151*a**(3/2)*b**3*x**9/(960*
sqrt(1 + b*x**2/a)) + 59*sqrt(a)*b**4*x**11/(120*sqrt(1 + b*x**2/a)) - 21*
a**6*asinh(sqrt(b)*x/sqrt(a))/(1024*b**(3/2)) + b**5*x**13/(12*sqrt(a)*sqr
t(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.79

$$\int x^2 (a+bx^2)^{9/2}\,dx = \frac{(bx^2+a)^{\frac{11}{2}}x}{12\,b} - \frac{(bx^2+a)^{\frac{9}{2}}ax}{120\,b} - \frac{3\,(bx^2+a)^{\frac{7}{2}}a^2x}{320\,b} - \frac{7\,(bx^2+a)^{\frac{5}{2}}a^3x}{640\,b} - \frac{7\,(bx^2+a)^{\frac{3}{2}}a^4x}{512\,b} - \frac{21\,\sqrt{bx^2+a}a^5x}{1024\,b} - \frac{21\,a^6\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{1024\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/12*(b*x^2 + a)^(11/2)*x/b - 1/120*(b*x^2 + a)^(9/2)*a*x/b - 3/320*(b*x^2
 + a)^(7/2)*a^2*x/b - 7/640*(b*x^2 + a)^(5/2)*a^3*x/b - 7/512*(b*x^2 + a)^
(3/2)*a^4*x/b - 21/1024*sqrt(b*x^2 + a)*a^5*x/b - 21/1024*a^6*arcsinh(b*x/
sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int x^2 (a+bx^2)^{9/2}\,dx = \frac{21\,a^6\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{1024\,b^{\frac{3}{2}}} + \frac{1}{15360}\left(\frac{315\,a^5}{b} + 2\,(2455\,a^4 + 4\,(1429\,a^3b + 2\,(759\,a^2b^2 + 8\,(10\,b^4x^2 + 49\,ab^3)x^2)x^2)x^2)x^2\right)\sqrt{bx^2+a}x$$

input
```
integrate(x^2*(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
21/1024*a^6*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/15360*(315*
a^5/b + 2*(2455*a^4 + 4*(1429*a^3*b + 2*(759*a^2*b^2 + 8*(10*b^4*x^2 + 49*
a*b^3)*x^2)*x^2)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^2\right)^{9/2}\,dx=\int x^2\,\left(b\,x^2+a\right)^{9/2}\,dx$$

input
```
int(x^2*(a + b*x^2)^(9/2),x)
```

output
```
int(x^2*(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.89

$$\int x^2 \left(a+bx^2\right)^{9/2}\,dx=\frac{315\sqrt{b\,x^2+a}\,a^5bx+4910\sqrt{b\,x^2+a}\,a^4b^2x^3+11432\sqrt{b\,x^2+a}\,a^3b^3x^5+12144\sqrt{b\,x^2+a}\,a^2b^4}{15360b^2}$$

input
```
int(x^2*(b*x^2+a)^(9/2),x)
```

output
```
(315*sqrt(a + b*x**2)*a**5*b*x + 4910*sqrt(a + b*x**2)*a**4*b**2*x**3 + 11
432*sqrt(a + b*x**2)*a**3*b**3*x**5 + 12144*sqrt(a + b*x**2)*a**2*b**4*x**
7 + 6272*sqrt(a + b*x**2)*a*b**5*x**9 + 1280*sqrt(a + b*x**2)*b**6*x**11 -
 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**6)/(15360*b**2
)
```

3.438 $\int (a+bx^2)^{9/2}\, dx$

Optimal result

Integrand size = 11, antiderivative size = 122

$$\int (a+bx^2)^{9/2}\, dx = \frac{63}{256}a^4x\sqrt{a+bx^2}+\frac{21}{128}a^3x(a+bx^2)^{3/2}+\frac{21}{160}a^2x(a+bx^2)^{5/2}+\frac{9}{80}ax(a+bx^2)^{7/2}+\frac{1}{10}x(a+bx^2)^{9/2}+\frac{63a^5\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}}$$

output
```
63/256*a^4*x*(b*x^2+a)^(1/2)+21/128*a^3*x*(b*x^2+a)^(3/2)+21/160*a^2*x*(b*
x^2+a)^(5/2)+9/80*a*x*(b*x^2+a)^(7/2)+1/10*x*(b*x^2+a)^(9/2)+63/256*a^5*ar
ctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int (a+bx^2)^{9/2}\, dx = \frac{\sqrt{a+bx^2}(965a^4x+1490a^3bx^3+1368a^2b^2x^5+656ab^3x^7+128b^4x^9)}{1280}-\frac{63a^5\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{256\sqrt{b}}$$

input
```
Integrate[(a + b*x^2)^(9/2),x]
```

output
```
(Sqrt[a + b*x^2]*(965*a^4*x + 1490*a^3*b*x^3 + 1368*a^2*b^2*x^5 + 656*a*b^
3*x^7 + 128*b^4*x^9))/1280 - (63*a^5*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
(256*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {211, 211, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{9/2} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{10}a\int \left(bx^2+a\right)^{7/2}dx+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{10}a\left(\frac{7}{8}a\int \left(bx^2+a\right)^{5/2}dx+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{10}a\left(\frac{7}{8}a\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{10}a\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{10}a\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 224

$$\frac{9}{10}a\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

$\Big\downarrow$ 219

$$\frac{9}{10}a\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right)+\frac{1}{10}x\left(a+bx^2\right)^{9/2}$$

input
```
Int[(a + b*x^2)^(9/2),x]
```

output
```
(x*(a + b*x^2)^(9/2))/10 + (9*a*((x*(a + b*x^2)^(7/2))/8 + (7*a*((x*(a + b
*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])
/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/8))/10
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{x\left(128b^{4}x^{8}+656a\,b^{3}x^{6}+1368a^{2}b^{2}x^{4}+1490a^{3}b\,x^{2}+965a^{4}\right)\sqrt{b\,x^{2}+a}}{1280}+\frac{63a^{5}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{256\sqrt{b}}$	81
default	$\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}$	100

input
```
int((b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/1280*x*(128*b^4*x^8+656*a*b^3*x^6+1368*a^2*b^2*x^4+1490*a^3*b*x^2+965*a^
4)*(b*x^2+a)^(1/2)+63/256*a^5*ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.56

$$\int (a+bx^2)^{9/2}\,dx = \left[\frac{315\,a^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(128\,b^5x^9+656\,ab^4x^7+1368\,a^2b^3x^5+1\cdots}{2560\,b}\right.$$
$$-\frac{315\,a^5\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(128\,b^5x^9+656\,ab^4x^7+1368\,a^2b^3x^5+1490\,a^3b^2x^3+965\,a^4bx)\sqrt{bx^2+a}}{1280\,b}$$

input
```
integrate((b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/2560*(315*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490*a^3*b^2*x^3 + 96
5*a^4*b*x)*sqrt(b*x^2 + a))/b, -1/1280*(315*a^5*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) - (128*b^5*x^9 + 656*a*b^4*x^7 + 1368*a^2*b^3*x^5 + 1490
*a^3*b^2*x^3 + 965*a^4*b*x)*sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 10.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.24

$$\int (a+bx^2)^{9/2}\,dx = \frac{193a^{\frac{9}{2}}x\sqrt{1+\frac{bx^2}{a}}}{256}+\frac{149a^{\frac{7}{2}}bx^3\sqrt{1+\frac{bx^2}{a}}}{128}+\frac{171a^{\frac{5}{2}}b^2x^5\sqrt{1+\frac{bx^2}{a}}}{160}+\frac{41a^{\frac{3}{2}}b^3x^7\sqrt{1+\frac{bx^2}{a}}}{80}+\frac{\sqrt{a}b^4x^9\sqrt{1+\frac{bx^2}{a}}}{10}+\frac{63a^5\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}}$$

input
```
integrate((b*x**2+a)**(9/2),x)
```

output
```
193*a**(9/2)*x*sqrt(1 + b*x**2/a)/256 + 149*a**(7/2)*b*x**3*sqrt(1 + b*x**
2/a)/128 + 171*a**(5/2)*b**2*x**5*sqrt(1 + b*x**2/a)/160 + 41*a**(3/2)*b**
3*x**7*sqrt(1 + b*x**2/a)/80 + sqrt(a)*b**4*x**9*sqrt(1 + b*x**2/a)/10 + 6
3*a**5*asinh(sqrt(b)*x/sqrt(a))/(256*sqrt(b))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int (a+bx^2)^{9/2}\,dx = \frac{1}{10}\left(bx^2+a\right)^{\frac{9}{2}}x + \frac{9}{80}\left(bx^2+a\right)^{\frac{7}{2}}ax + \frac{21}{160}\left(bx^2+a\right)^{\frac{5}{2}}a^2x + \frac{21}{128}\left(bx^2+a\right)^{\frac{3}{2}}a^3x + \frac{63}{256}\sqrt{bx^2+a}a^4x + \frac{63\,a^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\sqrt{b}}$$

input
```
integrate((b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(9/2)*x + 9/80*(b*x^2 + a)^(7/2)*a*x + 21/160*(b*x^2 + a)
^(5/2)*a^2*x + 21/128*(b*x^2 + a)^(3/2)*a^3*x + 63/256*sqrt(b*x^2 + a)*a^4
*x + 63/256*a^5*arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$\int (a+bx^2)^{9/2}\,dx = -\frac{63\,a^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\sqrt{b}} + \frac{1}{1280}\left(965\,a^4+2\left(745\,a^3b+4\left(171\,a^2b^2+2\left(8\,b^4x^2+41\,ab^3\right)x^2\right)x^2\right)x^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-63/256*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/1280*(965*a
^4 + 2*(745*a^3*b + 4*(171*a^2*b^2 + 2*(8*b^4*x^2 + 41*a*b^3)*x^2)*x^2)*x^
2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \left(a+bx^2\right)^{9/2}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{9/2}\,{{}}_2{\mathrm{F}}_1\left(-\frac{9}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{9/2}}$$

input
```
int((a + b*x^2)^(9/2),x)
```

output
```
(x*(a + b*x^2)^(9/2)*hypergeom([-9/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\int \left(a+bx^2\right)^{9/2}\,dx=\frac{965\sqrt{b\,x^2+a}\,a^4bx+1490\sqrt{b\,x^2+a}\,a^3b^2x^3+1368\sqrt{b\,x^2+a}\,a^2b^3x^5+656\sqrt{b\,x^2+a}\,a\,b^4x^7+}{1280b}$$

input
```
int((b*x^2+a)^(9/2),x)
```

output
```
(965*sqrt(a + b*x**2)*a**4*b*x + 1490*sqrt(a + b*x**2)*a**3*b**2*x**3 + 13
68*sqrt(a + b*x**2)*a**2*b**3*x**5 + 656*sqrt(a + b*x**2)*a*b**4*x**7 + 12
8*sqrt(a + b*x**2)*b**5*x**9 + 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)
*x)/sqrt(a))*a**5)/(1280*b)
```

3.439 $\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = -\frac{a^4\sqrt{a+bx^2}}{x}+\frac{325}{128}a^3bx\sqrt{a+bx^2}+\frac{105}{64}a^2b^2x^3\sqrt{a+bx^2}+\frac{11}{16}ab^3x^5\sqrt{a+bx^2}+\frac{1}{8}b^4x^7\sqrt{a+bx^2}+\frac{315}{128}a^4\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-a^4*(b*x^2+a)^(1/2)/x+325/128*a^3*b*x*(b*x^2+a)^(1/2)+105/64*a^2*b^2*x^3*
(b*x^2+a)^(1/2)+11/16*a*b^3*x^5*(b*x^2+a)^(1/2)+1/8*b^4*x^7*(b*x^2+a)^(1/2
)+315/128*a^4*b^(1/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = \frac{\sqrt{a+bx^2}(-128a^4+325a^3bx^2+210a^2b^2x^4+88ab^3x^6+16b^4x^8)}{128x}-\frac{315}{128}a^4\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-128*a^4 + 325*a^3*b*x^2 + 210*a^2*b^2*x^4 + 88*a*b^3*x^
6 + 16*b^4*x^8))/(128*x) - (315*a^4*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*
x^2]])/128
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 211, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^2} \, dx$$

$$\Big\downarrow \text{247}$$

$$9b\int (bx^2+a)^{7/2}dx - \frac{(a+bx^2)^{9/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$9b\left(\frac{7}{8}a\int (bx^2+a)^{5/2}dx + \frac{1}{8}x(a+bx^2)^{7/2}\right) - \frac{(a+bx^2)^{9/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$9b\left(\frac{7}{8}a\left(\frac{5}{6}a\int (bx^2+a)^{3/2}dx + \frac{1}{6}x(a+bx^2)^{5/2}\right) + \frac{1}{8}x(a+bx^2)^{7/2}\right) - \frac{(a+bx^2)^{9/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$9b\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx + \frac{1}{4}x(a+bx^2)^{3/2}\right) + \frac{1}{6}x(a+bx^2)^{5/2}\right) + \frac{1}{8}x(a+bx^2)^{7/2}\right) - \frac{(a+bx^2)^{9/2}}{x}$$

$$\Big\downarrow \text{211}$$

$$9b\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right)^{7/2}\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{x}$$

$$\Big\downarrow \text{224}$$

$$9b\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x(a\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{x}$$

$$\Big\downarrow \text{219}$$

$$9b\left(\frac{7}{8}a\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{1}{8}x\left(a+bx^2\right.\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{x}$$

input
```
Int[(a + b*x^2)^(9/2)/x^2,x]
```

output
```
-((a + b*x^2)^(9/2)/x) + 9*b*((x*(a + b*x^2)^(7/2))/8 + (7*a*((x*(a + b*x^
2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2
+ (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/8)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.60

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-16 b^{4} x^{8}-88 a\,b^{3} x^{6}-210 a^{2} b^{2} x^{4}-325 a^{3} b\,x^{2}+128 a^{4}\right)}{128 x}+\frac{315 a^{4} \sqrt{b}\,\ln \left(\sqrt{b}\,x +\sqrt{b\,x^{2}+a}\right)}{128}$
pseudoelliptic	$-\frac{-\frac{315\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x \sqrt{b}}\right) a^{4} b x}{128}+\sqrt{b\,x^{2}+a}\,\left(-\frac{b^{\frac{9}{2}} x^{8}}{8}-\frac{11 a\,b^{\frac{7}{2}} x^{6}}{16}-\frac{105 a^{2} b^{\frac{5}{2}} x^{4}}{64}-\frac{325 a^{3} b^{\frac{3}{2}} x^{2}}{128}+a^{4} \sqrt{b}\right)}{\sqrt{b}\,x}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{a x}+\frac{10 b \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a \left(\frac{x \left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(\sqrt{b}\,x +\sqrt{b}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6}\right)}{8}\right)}{10}\right)}{a}$

input
```
int((b*x^2+a)^(9/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128*(b*x^2+a)^(1/2)*(-16*b^4*x^8-88*a*b^3*x^6-210*a^2*b^2*x^4-325*a^3*b
*x^2+128*a^4)/x+315/128*a^4*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = \left[\frac{315\,a^4\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4}{256\,x}\right.$$

$$\left.-\frac{315\,a^4\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(16\,b^4x^8+88\,ab^3x^6+210\,a^2b^2x^4+325\,a^3bx^2-128\,a^4)\sqrt{bx^2+a}}{128\,x}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="fricas")
```

output
```
[1/256*(315*a^4*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 - 128*a^4
)*sqrt(b*x^2 + a))/x, -1/128*(315*a^4*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - (16*b^4*x^8 + 88*a*b^3*x^6 + 210*a^2*b^2*x^4 + 325*a^3*b*x^2 -
 128*a^4)*sqrt(b*x^2 + a))/x]
```

Sympy [A] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = -\frac{a^{\frac{9}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{197a^{\frac{7}{2}}bx}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{535a^{\frac{5}{2}}b^2x^3}{128\sqrt{1+\frac{bx^2}{a}}}+\frac{149a^{\frac{3}{2}}b^3x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{13\sqrt{a}b^4x^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{315a^4\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{128}+\frac{b^5x^9}{8\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**2,x)
```

output
```
-a**(9/2)/(x*sqrt(1 + b*x**2/a)) + 197*a**(7/2)*b*x/(128*sqrt(1 + b*x**2/a
)) + 535*a**(5/2)*b**2*x**3/(128*sqrt(1 + b*x**2/a)) + 149*a**(3/2)*b**3*x
**5/(64*sqrt(1 + b*x**2/a)) + 13*sqrt(a)*b**4*x**7/(16*sqrt(1 + b*x**2/a))
 + 315*a**4*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/128 + b**5*x**9/(8*sqrt(a)*sq
rt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = \frac{9}{8}\left(bx^2+a\right)^{\frac{7}{2}}bx + \frac{21}{16}\left(bx^2+a\right)^{\frac{5}{2}}abx + \frac{105}{64}\left(bx^2+a\right)^{\frac{3}{2}}a^2bx + \frac{315}{128}\sqrt{bx^2+a}a^3bx + \frac{315}{128}a^4\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{\left(bx^2+a\right)^{\frac{9}{2}}}{x}$$

input
```
integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="maxima")
```

output
```
9/8*(b*x^2 + a)^(7/2)*b*x + 21/16*(b*x^2 + a)^(5/2)*a*b*x + 105/64*(b*x^2
+ a)^(3/2)*a^2*b*x + 315/128*sqrt(b*x^2 + a)*a^3*b*x + 315/128*a^4*sqrt(b)
*arcsinh(b*x/sqrt(a*b)) - (b*x^2 + a)^(9/2)/x
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = -\frac{315}{256}a^4\sqrt{b}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\,a^5\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{128}\left(325\,a^3b+2\left(105\,a^2b^2+4\left(2\,b^4x^2+11\,ab^3\right)x^2\right)x^2\right)\sqrt{bx^2+a}x$$

input
```
integrate((b*x^2+a)^(9/2)/x^2,x, algorithm="giac")
```

output
```
-315/256*a^4*sqrt(b)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2*a^5*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/128*(325*a^3*b + 2*(105*a^2*b^2
+ 4*(2*b^4*x^2 + 11*a*b^3)*x^2)*x^2)*sqrt(b*x^2 + a)*x
```

Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = -\frac{(b\,x^2+a)^{9/2}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{9/2}}$$

input
```
int((a + b*x^2)^(9/2)/x^2,x)
```

output
```
-((a + b*x^2)^(9/2)*hypergeom([-9/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^{9/2}}{x^2}\,dx = \frac{-128\sqrt{b\,x^2+a}\,a^4+325\sqrt{b\,x^2+a}\,a^3b\,x^2+210\sqrt{b\,x^2+a}\,a^2b^2x^4+88\sqrt{b\,x^2+a}\,a\,b^3x^6}{128x}$$

input
```
int((b*x^2+a)^(9/2)/x^2,x)
```

output
```
( - 128*sqrt(a + b*x**2)*a**4 + 325*sqrt(a + b*x**2)*a**3*b*x**2 + 210*sqr
t(a + b*x**2)*a**2*b**2*x**4 + 88*sqrt(a + b*x**2)*a*b**3*x**6 + 16*sqrt(a
 + b*x**2)*b**4*x**8 + 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a**4*x - 189*sqrt(b)*a**4*x)/(128*x)
```

3.440 $\int \frac{(a+bx^2)^{9/2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{(a+bx^2)^{9/2}}{x^4} dx = -\frac{a^4\sqrt{a+bx^2}}{3x^3} - \frac{13a^3b\sqrt{a+bx^2}}{3x} + \frac{55}{16}a^2b^2x\sqrt{a+bx^2} + \frac{25}{24}ab^3x^3\sqrt{a+bx^2} + \frac{1}{6}b^4x^5\sqrt{a+bx^2} + \frac{105}{16}a^3b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output

```
-1/3*a^4*(b*x^2+a)^(1/2)/x^3-13/3*a^3*b*(b*x^2+a)^(1/2)/x+55/16*a^2*b^2*x*
(b*x^2+a)^(1/2)+25/24*a*b^3*x^3*(b*x^2+a)^(1/2)+1/6*b^4*x^5*(b*x^2+a)^(1/2
)+105/16*a^3*b^(3/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^{9/2}}{x^4} dx = \frac{\sqrt{a+bx^2}(-16a^4-208a^3bx^2+165a^2b^2x^4+50ab^3x^6+8b^4x^8)}{48x^3} - \frac{105}{16}a^3b^{3/2}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input

```
Integrate[(a + b*x^2)^(9/2)/x^4,x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^4 - 208*a^3*b*x^2 + 165*a^2*b^2*x^4 + 50*a*b^3*x^6
 + 8*b^4*x^8))/(48*x^3) - (105*a^3*b^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/16
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 247, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx$$

$$\Big\downarrow \text{247}$$

$$3b\int \frac{(bx^2+a)^{7/2}}{x^2}dx-\frac{(a+bx^2)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$3b\left(7b\int (bx^2+a)^{5/2}\,dx-\frac{(a+bx^2)^{7/2}}{x}\right)-\frac{(a+bx^2)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$3b\left(7b\left(\frac{5}{6}a\int (bx^2+a)^{3/2}\,dx+\frac{1}{6}x(a+bx^2)^{5/2}\right)-\frac{(a+bx^2)^{7/2}}{x}\right)-\frac{(a+bx^2)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$3b\left(7b\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)-\frac{(a+bx^2)^{7/2}}{x}\right)-\frac{(a+bx^2)^{9/2}}{3x^3}$$

$$\Big\downarrow \text{211}$$

$$3b\left(7b\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{x}\right)-\frac{\left(a+bx^2\right)^{9/2}}{3x^3}$$

$$\Big\downarrow \textbf{224}$$

$$3b\left(7b\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{(a+b}{}\right.\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{3x^3}$$

$$\Big\downarrow \textbf{219}$$

$$3b\left(7b\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{\left(a+bx^2\right)^{7/2}}{x}\right.\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(9/2)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(9/2)/x^3 + 3*b*(-((a + b*x^2)^(7/2)/x) + 7*b*((x*(a + b*
x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/
2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{\sqrt{b x^2+a}\left(-8 b^4 x^8-50 a b^3 x^6-165 a^2 b^2 x^4+208 a^3 b x^2+16 a^4\right)}{48 x^3}+\frac{105 a^3 b^{\frac{3}{2}} \ln\left(\sqrt{b}\, x+\sqrt{b x^2+a}\right)}{16}$
pseudoelliptic	$-\frac{-\frac{315 \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right) a^3 b^2 x^3}{16}+\sqrt{b x^2+a}\left(-\frac{b^{\frac{9}{2}} x^8}{2}-\frac{25 a b^{\frac{7}{2}} x^6}{8}-\frac{165 a^2 b^{\frac{5}{2}} x^4}{16}+13 a^3 b^{\frac{3}{2}} x^2+a^4\sqrt{b}\right)}{3\sqrt{b}\, x^3}$
	$8b\left(-\frac{\left(b x^2+a\right)^{\frac{11}{2}}}{a x}+\frac{10b\left(\frac{x\left(b x^2+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b x^2+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x\left(b x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x \cdots}}{2}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{8}\cdots\right.}{10}\cdots\right.}{a}\cdots\right.$

input
```
int((b*x^2+a)^(9/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(b*x^2+a)^(1/2)*(-8*b^4*x^8-50*a*b^3*x^6-165*a^2*b^2*x^4+208*a^3*b*x
^2+16*a^4)/x^3+105/16*a^3*b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx = \left[\frac{315\,a^3b^{\frac{3}{2}}x^3\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4}{96\,x^3}\right.$$

$$\left.-\frac{315\,a^3\sqrt{-b}bx^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^4x^8+50\,ab^3x^6+165\,a^2b^2x^4-208\,a^3bx^2-16\,a^4)\sqrt{bx^2+a}}{48\,x^3}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="fricas")
```

output
```
[1/96*(315*a^3*b^(3/2)*x^3*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^2 - 16*a^4)
*sqrt(b*x^2 + a))/x^3, -1/48*(315*a^3*sqrt(-b)*b*x^3*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (8*b^4*x^8 + 50*a*b^3*x^6 + 165*a^2*b^2*x^4 - 208*a^3*b*x^
2 - 16*a^4)*sqrt(b*x^2 + a))/x^3]
```

Sympy [A] (verification not implemented)

Time = 9.87 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx = -\frac{a^{\frac{9}{2}}}{3x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{14a^{\frac{7}{2}}b}{3x\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2x}{48\sqrt{1+\frac{bx^2}{a}}}$$

$$+\frac{215a^{\frac{3}{2}}b^3x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{29\sqrt{a}b^4x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{105a^3b^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{16}+\frac{b^5x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**4,x)
```

output
```
-a**(9/2)/(3*x**3*sqrt(1 + b*x**2/a)) - 14*a**(7/2)*b/(3*x*sqrt(1 + b*x**2
/a)) - 43*a**(5/2)*b**2*x/(48*sqrt(1 + b*x**2/a)) + 215*a**(3/2)*b**3*x**3
/(48*sqrt(1 + b*x**2/a)) + 29*sqrt(a)*b**4*x**5/(24*sqrt(1 + b*x**2/a)) +
105*a**3*b**(3/2)*asinh(sqrt(b)*x/sqrt(a))/16 + b**5*x**7/(6*sqrt(a)*sqrt(
1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx = \frac{7}{2}\left(bx^2+a\right)^{\frac{5}{2}}b^2x + \frac{3\left(bx^2+a\right)^{\frac{7}{2}}b^2x}{a} + \frac{35}{8}\left(bx^2+a\right)^{\frac{3}{2}}ab^2x + \frac{105}{16}\sqrt{bx^2+a}a^2b^2x + \frac{105}{16}a^3b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{8\left(bx^2+a\right)^{\frac{9}{2}}b}{3\,ax} - \frac{\left(bx^2+a\right)^{\frac{11}{2}}}{3\,ax^3}$$

input
```
integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="maxima")
```

output
```
7/2*(b*x^2 + a)^(5/2)*b^2*x + 3*(b*x^2 + a)^(7/2)*b^2*x/a + 35/8*(b*x^2 +
a)^(3/2)*a*b^2*x + 105/16*sqrt(b*x^2 + a)*a^2*b^2*x + 105/16*a^3*b^(3/2)*a
rcsinh(b*x/sqrt(a*b)) - 8/3*(b*x^2 + a)^(9/2)*b/(a*x) - 1/3*(b*x^2 + a)^(1
1/2)/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx = -\frac{105}{32}a^3b^{\frac{3}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{1}{48}\left(165\,a^2b^2+2\left(4\,b^4x^2+25\,ab^3\right)x^2\right)\sqrt{bx^2+a}x + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^4b^{\frac{3}{2}} - 24\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a^5b^{\frac{3}{2}} + 13\,a^6b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((b*x^2+a)^(9/2)/x^4,x, algorithm="giac")
```

output
```
-105/32*a^3*b^(3/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/48*(165*a^2*b
^2 + 2*(4*b^4*x^2 + 25*a*b^3)*x^2)*sqrt(b*x^2 + a)*x + 2/3*(15*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^4*b^(3/2) - 24*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*
b^(3/2) + 13*a^6*b^(3/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(9/2)/x^4,x)
```

output
```
int((a + b*x^2)^(9/2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^{9/2}}{x^4}\,dx=\frac{-128\sqrt{b\,x^2+a}\,a^4-1664\sqrt{b\,x^2+a}\,a^3b\,x^2+1320\sqrt{b\,x^2+a}\,a^2b^2x^4+400\sqrt{b\,x^2+a}\,a\,b}{384x^3}$$

input
```
int((b*x^2+a)^(9/2)/x^4,x)
```

output
```
( - 128*sqrt(a + b*x**2)*a**4 - 1664*sqrt(a + b*x**2)*a**3*b*x**2 + 1320*s
qrt(a + b*x**2)*a**2*b**2*x**4 + 400*sqrt(a + b*x**2)*a*b**3*x**6 + 64*sqr
t(a + b*x**2)*b**4*x**8 + 2520*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/
sqrt(a))*a**3*b*x**3 + 567*sqrt(b)*a**3*b*x**3)/(384*x**3)
```

3.441 $\int \frac{(a+bx^2)^{9/2}}{x^6} dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{(a+bx^2)^{9/2}}{x^6} dx = -\frac{a^4\sqrt{a+bx^2}}{5x^5} - \frac{7a^3b\sqrt{a+bx^2}}{5x^3} - \frac{36a^2b^2\sqrt{a+bx^2}}{5x} + \frac{17}{8}ab^3x\sqrt{a+bx^2} + \frac{1}{4}b^4x^3\sqrt{a+bx^2} + \frac{63}{8}a^2b^{5/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/5*a^4*(b*x^2+a)^(1/2)/x^5-7/5*a^3*b*(b*x^2+a)^(1/2)/x^3-36/5*a^2*b^2*(b
*x^2+a)^(1/2)/x+17/8*a*b^3*x*(b*x^2+a)^(1/2)+1/4*b^4*x^3*(b*x^2+a)^(1/2)+6
3/8*a^2*b^(5/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^2)^{9/2}}{x^6} dx = \frac{\sqrt{a+bx^2}(-8a^4-56a^3bx^2-288a^2b^2x^4+85ab^3x^6+10b^4x^8)}{40x^5} - \frac{63}{8}a^2b^{5/2}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^6,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^4 - 56*a^3*b*x^2 - 288*a^2*b^2*x^4 + 85*a*b^3*x^6 +
 10*b^4*x^8))/(40*x^5) - (63*a^2*b^(5/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/8
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 247, 247, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{5}b\int \frac{(bx^2+a)^{7/2}}{x^4}dx - \frac{(a+bx^2)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{5}b\left(\frac{7}{3}b\int \frac{(bx^2+a)^{5/2}}{x^2}dx - \frac{(a+bx^2)^{7/2}}{3x^3}\right) - \frac{(a+bx^2)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{5}b\left(\frac{7}{3}b\left(5b\int (bx^2+a)^{3/2}dx - \frac{(a+bx^2)^{5/2}}{x}\right) - \frac{(a+bx^2)^{7/2}}{3x^3}\right) - \frac{(a+bx^2)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{5}b\left(\frac{7}{3}b\left(5b\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx + \frac{1}{4}x(a+bx^2)^{3/2}\right) - \frac{(a+bx^2)^{5/2}}{x}\right) - \frac{(a+bx^2)^{7/2}}{3x^3}\right) - \frac{(a+bx^2)^{9/2}}{5x^5}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{5}b\left(\frac{7}{3}b\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)-\frac{\left(a+bx^2\right)^{7/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}$$

$$\Big\downarrow \; 224$$

$$\frac{9}{5}b\left(\frac{7}{3}b\left(5b\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)-\frac{\left(a+bx^2\right)^{7/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}$$

$$\Big\downarrow \; 219$$

$$\frac{9}{5}b\left(\frac{7}{3}b\left(5b\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{\left(a+bx^2\right)^{5/2}}{x}\right)-\frac{\left(a+bx^2\right)^{7/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{9/2}}{5x^5}$$

input
```
Int[(a + b*x^2)^(9/2)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(9/2)/x^5 + (9*b*(-1/3*(a + b*x^2)^(7/2)/x^3 + (7*b*(-((a
 + b*x^2)^(5/2)/x) + 5*b*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^
2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)))/3))/5
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-10 b^4 x^8-85 a\,b^3 x^6+288 a^2 b^2 x^4+56 a^3 b\,x^2+8 a^4\right)}{40 x^5}+\frac{63 a^2 b^{\frac{5}{2}} \ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{8}$
pseudoelliptic	$-\frac{-\frac{315\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right) a^2 b^3 x^5}{8}+\sqrt{b\,x^2+a}\,\left(-\frac{5 b^{\frac{9}{2}} x^8}{4}-\frac{85 a\,b^{\frac{7}{2}} x^6}{8}+36 a^2 b^{\frac{5}{2}} x^4+7 a^3 b^{\frac{3}{2}} x^2+a^4\sqrt{b}\right)}{5\sqrt{b}\,x^5}$
	$5a\left(\frac{x\left(b\right.}{}\right.$ … $7a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\right.$ … $9a\left(\frac{x\left(b\,x^2+a\right)^{\frac{7}{2}}}{8}+\right.$ … $10b\left(\frac{x\left(b\,x^2+a\right)^{\frac{9}{2}}}{10}+\right.$ … $8b\left(-\frac{\left(b\,x^2+a\right)^{\frac{11}{2}}}{a x}+\frac{\cdots}{a}\right.$ …

input
```
int((b*x^2+a)^(9/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/40*(b*x^2+a)^(1/2)*(-10*b^4*x^8-85*a*b^3*x^6+288*a^2*b^2*x^4+56*a^3*b*x
^2+8*a^4)/x^5+63/8*a^2*b^(5/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = \left[\frac{315\,a^2b^{\frac{5}{2}}x^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4}{80\,x^5}\right.$$
$$\left.-\frac{315\,a^2\sqrt{-b}b^2x^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(10\,b^4x^8+85\,ab^3x^6-288\,a^2b^2x^4-56\,a^3bx^2-8\,a^4)\sqrt{bx^2+a}}{40\,x^5}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="fricas")
```

output
```
[1/80*(315*a^2*b^(5/2)*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x^2 - 8*a^4)*
sqrt(b*x^2 + a))/x^5, -1/40*(315*a^2*sqrt(-b)*b^2*x^5*arctan(sqrt(-b)*x/sq
rt(b*x^2 + a)) - (10*b^4*x^8 + 85*a*b^3*x^6 - 288*a^2*b^2*x^4 - 56*a^3*b*x
^2 - 8*a^4)*sqrt(b*x^2 + a))/x^5]
```

Sympy [A] (verification not implemented)

Time = 10.41 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = -\frac{a^{\frac{9}{2}}}{5x^5\sqrt{1+\frac{bx^2}{a}}}-\frac{8a^{\frac{7}{2}}b}{5x^3\sqrt{1+\frac{bx^2}{a}}}-\frac{43a^{\frac{5}{2}}b^2}{5x\sqrt{1+\frac{bx^2}{a}}}$$
$$-\frac{203a^{\frac{3}{2}}b^3x}{40\sqrt{1+\frac{bx^2}{a}}}+\frac{19\sqrt{a}b^4x^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{63a^2b^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8}+\frac{b^5x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((b*x**2+a)**(9/2)/x**6,x)
```

output
```
-a**(9/2)/(5*x**5*sqrt(1 + b*x**2/a)) - 8*a**(7/2)*b/(5*x**3*sqrt(1 + b*x*
*2/a)) - 43*a**(5/2)*b**2/(5*x*sqrt(1 + b*x**2/a)) - 203*a**(3/2)*b**3*x/(
40*sqrt(1 + b*x**2/a)) + 19*sqrt(a)*b**4*x**3/(8*sqrt(1 + b*x**2/a)) + 63*
a**2*b**(5/2)*asinh(sqrt(b)*x/sqrt(a))/8 + b**5*x**5/(4*sqrt(a)*sqrt(1 + b
*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = \frac{21}{4}\left(bx^2+a\right)^{\frac{3}{2}}b^3x + \frac{18\left(bx^2+a\right)^{\frac{7}{2}}b^3x}{5\,a^2} + \frac{21\left(bx^2+a\right)^{\frac{5}{2}}b^3x}{5\,a} + \frac{63}{8}\sqrt{bx^2+a}\,ab^3x + \frac{63}{8}\,a^2b^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{16\left(bx^2+a\right)^{\frac{9}{2}}b^2}{5\,a^2x} - \frac{2\left(bx^2+a\right)^{\frac{11}{2}}b}{5\,a^2x^3} - \frac{\left(bx^2+a\right)^{\frac{11}{2}}}{5\,ax^5}$$

input
```
integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="maxima")
```

output
```
21/4*(b*x^2 + a)^(3/2)*b^3*x + 18/5*(b*x^2 + a)^(7/2)*b^3*x/a^2 + 21/5*(b*
x^2 + a)^(5/2)*b^3*x/a + 63/8*sqrt(b*x^2 + a)*a*b^3*x + 63/8*a^2*b^(5/2)*a
rcsinh(b*x/sqrt(a*b)) - 16/5*(b*x^2 + a)^(9/2)*b^2/(a^2*x) - 2/5*(b*x^2 +
a)^(11/2)*b/(a^2*x^3) - 1/5*(b*x^2 + a)^(11/2)/(a*x^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx =$$

$$-\frac{63}{16}\,a^2b^{\frac{5}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)+\frac{1}{8}\,(2\,b^4x^2+17\,ab^3)\sqrt{bx^2+a}x$$

$$+\frac{4\left(25\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8a^3b^{\frac{5}{2}}-75\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6a^4b^{\frac{5}{2}}+105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4a^5b^{\frac{5}{2}}-65\,(\sqrt{b}\right.}{5\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

input
```
integrate((b*x^2+a)^(9/2)/x^6,x, algorithm="giac")
```

output
```
-63/16*a^2*b^(5/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 1/8*(2*b^4*x^2 +
 17*a*b^3)*sqrt(b*x^2 + a)*x + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3
*b^(5/2) - 75*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(5/2) + 105*(sqrt(b)*x
 - sqrt(b*x^2 + a))^4*a^5*b^(5/2) - 65*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6
*b^(5/2) + 18*a^7*b^(5/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{9/2}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(9/2)/x^6,x)
```

output
```
int((a + b*x^2)^(9/2)/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^{9/2}}{x^6}\,dx = \frac{-32\sqrt{b\,x^2+a}\,a^4-224\sqrt{b\,x^2+a}\,a^3b\,x^2-1152\sqrt{b\,x^2+a}\,a^2b^2x^4+340\sqrt{b\,x^2+a}\,a\,b^3x}{160x^5}$$

input
```
int((b*x^2+a)^(9/2)/x^6,x)
```

output
```
( - 32*sqrt(a + b*x**2)*a**4 - 224*sqrt(a + b*x**2)*a**3*b*x**2 - 1152*sqr
t(a + b*x**2)*a**2*b**2*x**4 + 340*sqrt(a + b*x**2)*a*b**3*x**6 + 40*sqrt(
a + b*x**2)*b**4*x**8 + 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sq
rt(a))*a**2*b**2*x**5 + 651*sqrt(b)*a**2*b**2*x**5)/(160*x**5)
```

3.442 $\int \frac{(a+bx^2)^{9/2}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{(a+bx^2)^{9/2}}{x^8} dx = -\frac{a^4\sqrt{a+bx^2}}{7x^7} - \frac{29a^3b\sqrt{a+bx^2}}{35x^5} - \frac{78a^2b^2\sqrt{a+bx^2}}{35x^3} - \frac{194ab^3\sqrt{a+bx^2}}{35x} + \frac{1}{2}b^4x\sqrt{a+bx^2} + \frac{9}{2}ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/7*a^4*(b*x^2+a)^(1/2)/x^7-29/35*a^3*b*(b*x^2+a)^(1/2)/x^5-78/35*a^2*b^2
*(b*x^2+a)^(1/2)/x^3-194/35*a*b^3*(b*x^2+a)^(1/2)/x+1/2*b^4*x*(b*x^2+a)^(1
/2)+9/2*a*b^(7/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.72

$$\int \frac{(a+bx^2)^{9/2}}{x^8} dx = \frac{\sqrt{a+bx^2}(-10a^4 - 58a^3bx^2 - 156a^2b^2x^4 - 388ab^3x^6 + 35b^4x^8)}{70x^7} + 9ab^{7/2}\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^8,x]
```

output
```
(Sqrt[a + b*x^2]*(-10*a^4 - 58*a^3*b*x^2 - 156*a^2*b^2*x^4 - 388*a*b^3*x^6
 + 35*b^4*x^8))/(70*x^7) + 9*a*b^(7/2)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqr
t[a + b*x^2])]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 247, 247, 247, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^8}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{7}b\int \frac{(bx^2+a)^{7/2}}{x^6}dx-\frac{(a+bx^2)^{9/2}}{7x^7}$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{7}b\left(\frac{7}{5}b\int \frac{(bx^2+a)^{5/2}}{x^4}dx-\frac{(a+bx^2)^{7/2}}{5x^5}\right)-\frac{(a+bx^2)^{9/2}}{7x^7}$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{7}b\left(\frac{7}{5}b\left(\frac{5}{3}b\int \frac{(bx^2+a)^{3/2}}{x^2}dx-\frac{(a+bx^2)^{5/2}}{3x^3}\right)-\frac{(a+bx^2)^{7/2}}{5x^5}\right)-\frac{(a+bx^2)^{9/2}}{7x^7}$$

$$\Big\downarrow \text{247}$$

$$\frac{9}{7}b\left(\frac{7}{5}b\left(\frac{5}{3}b\left(3b\int \sqrt{bx^2+a}dx-\frac{(a+bx^2)^{3/2}}{x}\right)-\frac{(a+bx^2)^{5/2}}{3x^3}\right)-\frac{(a+bx^2)^{7/2}}{5x^5}\right)-\frac{(a+bx^2)^{9/2}}{7x^7}$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{7}b\left(\frac{7}{5}b\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7/2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}$$

$\Big\downarrow$ 224

$$\frac{9}{7}b\left(\frac{7}{5}b\left(\frac{5}{3}b\left(3b\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7}}{5x^5}\right.$$
$$\frac{\left(a+bx^2\right)^{9/2}}{7x^7}$$

$\Big\downarrow$ 219

$$\frac{9}{7}b\left(\frac{7}{5}b\left(\frac{5}{3}b\left(3b\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{3/2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7/2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{9/2}}{7x^7}$$

input
```
Int[(a + b*x^2)^(9/2)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(9/2)/x^7 + (9*b*(-1/5*(a + b*x^2)^(7/2)/x^5 + (7*b*(-1/3
*(a + b*x^2)^(5/2)/x^3 + (5*b*(-((a + b*x^2)^(3/2)/x) + 3*b*((x*Sqrt[a + b
*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))))/3))/5))
/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.59

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-35b^{4}x^{8}+388a\,b^{3}x^{6}+156a^{2}b^{2}x^{4}+58a^{3}b\,x^{2}+10a^{4}\right)}{70x^{7}}+\frac{9a\,b^{\frac{7}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2}$
pseudoelliptic	$-\frac{-\frac{63\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a\,b^{4}x^{7}}{2}+\sqrt{b\,x^{2}+a}\,\left(-\frac{7b^{\frac{9}{2}}x^{8}}{2}+\frac{194a\,b^{\frac{7}{2}}x^{6}}{5}+\frac{78a^{2}b^{\frac{5}{2}}x^{4}}{5}+\frac{29a^{3}b^{\frac{3}{2}}x^{2}}{5}+a^{4}\sqrt{b}\right)}{7\sqrt{b}\,x^{7}}$
	$8b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{a x}+\frac{10b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8}+\frac{7a\left(\frac{x(\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((b*x^2+a)^(9/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/70*(b*x^2+a)^(1/2)*(-35*b^4*x^8+388*a*b^3*x^6+156*a^2*b^2*x^4+58*a^3*b*
x^2+10*a^4)/x^7+9/2*a*b^(7/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = \left[\frac{315\,ab^{\frac{7}{2}}x^7\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4}{140\,x^7}\right.$$
$$\left.-\frac{315\,a\sqrt{-b}b^3x^7\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(35\,b^4x^8-388\,ab^3x^6-156\,a^2b^2x^4-58\,a^3bx^2-10\,a^4)\sqrt{bx^2+a}}{70\,x^7}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="fricas")
```

output
```
[1/140*(315*a*b^(7/2)*x^7*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x^2 - 10*a^4)
*sqrt(b*x^2 + a))/x^7, -1/70*(315*a*sqrt(-b)*b^3*x^7*arctan(sqrt(-b)*x/sqr
t(b*x^2 + a)) - (35*b^4*x^8 - 388*a*b^3*x^6 - 156*a^2*b^2*x^4 - 58*a^3*b*x
^2 - 10*a^4)*sqrt(b*x^2 + a))/x^7]
```

Sympy [A] (verification not implemented)

Time = 10.88 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{7x^6}-\frac{29a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^4}-\frac{78a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{35x^2}$$
$$-\frac{194ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{35}-\frac{9ab^{\frac{7}{2}}\log\left(\frac{a}{bx^2}\right)}{4}+\frac{9ab^{\frac{7}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)}{2}+\frac{b^{\frac{9}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2}$$

input
```
integrate((b*x**2+a)**(9/2)/x**8,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(7*x**6) - 29*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(35*x**4) - 78*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(35*x**2) - 19
4*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/35 - 9*a*b**(7/2)*log(a/(b*x**2))/4 + 9*
a*b**(7/2)*log(sqrt(a/(b*x**2) + 1) + 1)/2 + b**(9/2)*x**2*sqrt(a/(b*x**2)
 + 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.16

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = \frac{9}{2}\sqrt{bx^2+a}b^4x + \frac{72\,(bx^2+a)^{\frac{7}{2}}b^4x}{35\,a^3} + \frac{12\,(bx^2+a)^{\frac{5}{2}}b^4x}{5\,a^2} + \frac{3\,(bx^2+a)^{\frac{3}{2}}b^4x}{a} + \frac{9}{2}\,ab^{\frac{7}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{64\,(bx^2+a)^{\frac{9}{2}}b^3}{35\,a^3x} - \frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{35\,a^3x^3} - \frac{4\,(bx^2+a)^{\frac{11}{2}}b}{35\,a^2x^5} - \frac{(bx^2+a)^{\frac{11}{2}}}{7\,ax^7}$$

input
```
integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="maxima")
```

output
```
9/2*sqrt(b*x^2 + a)*b^4*x + 72/35*(b*x^2 + a)^(7/2)*b^4*x/a^3 + 12/5*(b*x^
2 + a)^(5/2)*b^4*x/a^2 + 3*(b*x^2 + a)^(3/2)*b^4*x/a + 9/2*a*b^(7/2)*arcsi
nh(b*x/sqrt(a*b)) - 64/35*(b*x^2 + a)^(9/2)*b^3/(a^3*x) - 8/35*(b*x^2 + a)
^(11/2)*b^2/(a^3*x^3) - 4/35*(b*x^2 + a)^(11/2)*b/(a^2*x^5) - 1/7*(b*x^2 +
 a)^(11/2)/(a*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(110) = 220.

Time = 0.14 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx = \frac{1}{2}\sqrt{bx^2+a}b^4x - \frac{9}{4}\,ab^{\frac{7}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{4\left(175\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^2b^{\frac{7}{2}} - 700\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^3b^{\frac{7}{2}} + 1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8a^4b^{\frac{7}{2}} - 18\cdots\right.}{35\left(\left(\sqrt{b}x-\sqrt{\cdots}\right.\right.}$$

input
```
integrate((b*x^2+a)^(9/2)/x^8,x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*b^4*x - 9/4*a*b^(7/2)*log((sqrt(b)*x - sqrt(b*x^2 + a)
)^2) + 4/35*(175*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^2*b^(7/2) - 700*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a)
)^8*a^4*b^(7/2) - 1820*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^5*b^(7/2) + 1337*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^6*b^(7/2) - 504*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^7*b^(7/2) + 97*a^8*b^(7/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{9/2}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(9/2)/x^8,x)
```

output
```
int((a + b*x^2)^(9/2)/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{9/2}}{x^8}\,dx=\frac{-10\sqrt{b\,x^2+a}\,a^4-58\sqrt{b\,x^2+a}\,a^3b\,x^2-156\sqrt{b\,x^2+a}\,a^2b^2x^4-388\sqrt{b\,x^2+a}\,a\,b^3x^6}{70x^7}$$

input
```
int((b*x^2+a)^(9/2)/x^8,x)
```

output
```
( - 10*sqrt(a + b*x**2)*a**4 - 58*sqrt(a + b*x**2)*a**3*b*x**2 - 156*sqrt(
a + b*x**2)*a**2*b**2*x**4 - 388*sqrt(a + b*x**2)*a*b**3*x**6 + 35*sqrt(a
+ b*x**2)*b**4*x**8 + 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a*b**3*x**7 + 213*sqrt(b)*a*b**3*x**7)/(70*x**7)
```

3.443 $\int \frac{(a+bx^2)^{9/2}}{x^{10}} dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}} dx = -\frac{a^4\sqrt{a+bx^2}}{9x^9} - \frac{37a^3b\sqrt{a+bx^2}}{63x^7} - \frac{136a^2b^2\sqrt{a+bx^2}}{105x^5} - \frac{506ab^3\sqrt{a+bx^2}}{315x^3} - \frac{563b^4\sqrt{a+bx^2}}{315x} + b^{9/2}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)$$

output
```
-1/9*a^4*(b*x^2+a)^(1/2)/x^9-37/63*a^3*b*(b*x^2+a)^(1/2)/x^7-136/105*a^2*b
^2*(b*x^2+a)^(1/2)/x^5-506/315*a*b^3*(b*x^2+a)^(1/2)/x^3-563/315*b^4*(b*x^
2+a)^(1/2)/x+b^(9/2)*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.66

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}} dx = \frac{\sqrt{a+bx^2}(-35a^4-185a^3bx^2-408a^2b^2x^4-506ab^3x^6-563b^4x^8)}{315x^9} - b^{9/2}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^10,x]
```

output
```
(Sqrt[a + b*x^2]*(-35*a^4 - 185*a^3*b*x^2 - 408*a^2*b^2*x^4 - 506*a*b^3*x^
6 - 563*b^4*x^8))/(315*x^9) - b^(9/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {247, 247, 247, 247, 247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}dx$$

$$\Big\downarrow \text{247}$$

$$b\int \frac{(bx^2+a)^{7/2}}{x^8}dx-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\int \frac{(bx^2+a)^{5/2}}{x^6}dx-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\left(b\int \frac{(bx^2+a)^{3/2}}{x^4}dx-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\left(b\left(b\int \frac{\sqrt{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$$\Big\downarrow \text{247}$$

$$b\left(b\left(b\left(b\left(b\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$\Big\downarrow$ 224

$$b\left(b\left(b\left(b\left(b\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

$\Big\downarrow$ 219

$$b\left(b\left(b\left(b\left(\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-\frac{\sqrt{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{3/2}}{3x^3}\right)-\frac{(a+bx^2)^{5/2}}{5x^5}\right)-\frac{(a+bx^2)^{7/2}}{7x^7}\right)-\frac{(a+bx^2)^{9/2}}{9x^9}$$

input
```
Int[(a + b*x^2)^(9/2)/x^10,x]
```

output
```
-1/9*(a + b*x^2)^(9/2)/x^9 + b*(-1/7*(a + b*x^2)^(7/2)/x^7 + b*(-1/5*(a +
b*x^2)^(5/2)/x^5 + b*(-1/3*(a + b*x^2)^(3/2)/x^3 + b*(-(Sqrt[a + b*x^2]/x)
 + Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]))))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.58

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(563b^{4}x^{8}+506a\,b^{3}x^{6}+408a^{2}b^{2}x^{4}+185a^{3}b\,x^{2}+35a^{4}\right)}{315x^{9}}+b^{\frac{9}{2}}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)$
pseudoelliptic	$\frac{315b^{\frac{9}{2}}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{9}-\sqrt{b\,x^{2}+a}\,\left(563b^{4}x^{8}+506a\,b^{3}x^{6}+408a^{2}b^{2}x^{4}+185a^{3}b\,x^{2}+35a^{4}\right)}{315x^{9}}$
	$\left(\left(\left(\left(8b\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{a x}+\frac{10b\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{9}{2}}}{10}+\frac{9a\left(\frac{x}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.\right.\right.\right.\right.$

input
```
int((b*x^2+a)^(9/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(b*x^2+a)^(1/2)*(563*b^4*x^8+506*a*b^3*x^6+408*a^2*b^2*x^4+185*a^3*
b*x^2+35*a^4)/x^9+b^(9/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.35

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = \left[\frac{315\,b^{\frac{9}{2}}x^9\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4}{630\,x^9}\right.$$
$$\left.-\frac{315\,\sqrt{-b}b^4x^9\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(563\,b^4x^8+506\,ab^3x^6+408\,a^2b^2x^4+185\,a^3bx^2+35\,a^4)\sqrt{bx^2+a}}{315\,x^9}\right]$$

input
```
integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="fricas")
```

output
```
[1/630*(315*b^(9/2)*x^9*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) -
2*(563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*x^2 + 35*a^4)
*sqrt(b*x^2 + a))/x^9, -1/315*(315*sqrt(-b)*b^4*x^9*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) + (563*b^4*x^8 + 506*a*b^3*x^6 + 408*a^2*b^2*x^4 + 185*a^3*b*
x^2 + 35*a^4)*sqrt(b*x^2 + a))/x^9]
```

Sympy [A] (verification not implemented)

Time = 11.79 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{9x^8}-\frac{37a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{63x^6}-\frac{136a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{105x^4}$$
$$-\frac{506ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{315x^2}-\frac{563b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{315}-\frac{b^{\frac{9}{2}}\log\left(\frac{a}{bx^2}\right)}{2}+b^{\frac{9}{2}}\log\left(\sqrt{\frac{a}{bx^2}+1}+1\right)$$

input
```
integrate((b*x**2+a)**(9/2)/x**10,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(9*x**8) - 37*a**3*b**(3/2)*sqrt(a/(b*x
**2) + 1)/(63*x**6) - 136*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(105*x**4) -
506*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(315*x**2) - 563*b**(9/2)*sqrt(a/(b*x*
*2) + 1)/315 - b**(9/2)*log(a/(b*x**2))/2 + b**(9/2)*log(sqrt(a/(b*x**2) +
 1) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = \frac{16\,(bx^2+a)^{\frac{7}{2}}b^5x}{35\,a^4} + \frac{8\,(bx^2+a)^{\frac{5}{2}}b^5x}{15\,a^3} + \frac{2\,(bx^2+a)^{\frac{3}{2}}b^5x}{3\,a^2} + \frac{\sqrt{bx^2+a}ab^5x}{a} + b^{\frac{9}{2}}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{128\,(bx^2+a)^{\frac{9}{2}}b^4}{315\,a^4x} - \frac{16\,(bx^2+a)^{\frac{11}{2}}b^3}{315\,a^4x^3} - \frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{315\,a^3x^5} - \frac{2\,(bx^2+a)^{\frac{11}{2}}b}{63\,a^2x^7} - \frac{(bx^2+a)^{\frac{11}{2}}}{9\,ax^9}$$

input
```
integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="maxima")
```

output
```
16/35*(b*x^2 + a)^(7/2)*b^5*x/a^4 + 8/15*(b*x^2 + a)^(5/2)*b^5*x/a^3 + 2/3
*(b*x^2 + a)^(3/2)*b^5*x/a^2 + sqrt(b*x^2 + a)*b^5*x/a + b^(9/2)*arcsinh(b
*x/sqrt(a*b)) - 128/315*(b*x^2 + a)^(9/2)*b^4/(a^4*x) - 16/315*(b*x^2 + a)
^(11/2)*b^3/(a^4*x^3) - 8/315*(b*x^2 + a)^(11/2)*b^2/(a^3*x^5) - 2/63*(b*x
^2 + a)^(11/2)*b/(a^2*x^7) - 1/9*(b*x^2 + a)^(11/2)/(a*x^9)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(110) = 220.

Time = 0.14 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.03

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx = -\frac{1}{2}\,b^{\frac{9}{2}}\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right) + \frac{2\left(1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}ab^{\frac{9}{2}} - 6300\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}a^2b^{\frac{9}{2}} + 21000\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}a^3b^{\frac{9}{2}} - \cdots\right.}{\cdots}$$

input
```
integrate((b*x^2+a)^(9/2)/x^10,x, algorithm="giac")
```

output
```
-1/2*b^(9/2)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2) + 2/315*(1575*(sqrt(b)*x
 - sqrt(b*x^2 + a))^16*a*b^(9/2) - 6300*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a
^2*b^(9/2) + 21000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^3*b^(9/2) - 31500*(s
qrt(b)*x - sqrt(b*x^2 + a))^10*a^4*b^(9/2) + 39438*(sqrt(b)*x - sqrt(b*x^2
 + a))^8*a^5*b^(9/2) - 26292*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^6*b^(9/2) +
 13968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^7*b^(9/2) - 3492*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*a^8*b^(9/2) + 563*a^9*b^(9/2))/((sqrt(b)*x - sqrt(b*x^2 +
a))^2 - a)^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx=\int \frac{(b\,x^2+a)^{9/2}}{x^{10}}\,dx$$

input
```
int((a + b*x^2)^(9/2)/x^10,x)
```

output
```
int((a + b*x^2)^(9/2)/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{9/2}}{x^{10}}\,dx=\frac{-35\sqrt{b\,x^2+a}\,a^4-185\sqrt{b\,x^2+a}\,a^3b\,x^2-408\sqrt{b\,x^2+a}\,a^2b^2x^4-506\sqrt{b\,x^2+a}\,a\,b^3x^6}{315x^9}$$

input
```
int((b*x^2+a)^(9/2)/x^10,x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 185*sqrt(a + b*x**2)*a**3*b*x**2 - 408*sqrt
(a + b*x**2)*a**2*b**2*x**4 - 506*sqrt(a + b*x**2)*a*b**3*x**6 - 563*sqrt(
a + b*x**2)*b**4*x**8 + 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqr
t(a))*b**4*x**9 + 213*sqrt(b)*b**4*x**9)/(315*x**9)
```

3.444 $\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

output
```
-1/11*(b*x^2+a)^(11/2)/a/x^11
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}} dx = -\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^12,x]
```

output
```
-1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx$$
$$\Big\downarrow \; 242$$
$$-\frac{(a+bx^2)^{11/2}}{11ax^{11}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^12,x]
```

output
```
-1/11*(a + b*x^2)^(11/2)/(a*x^11)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
orering	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{11a\,x^{11}}$	18
trager	$-\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11a\,x^{11}}$	69
risch	$-\frac{(b^5x^{10}+5a\,b^4x^8+10a^2b^3x^6+10a^3b^2x^4+5a^4b\,x^2+a^5)\sqrt{b\,x^2+a}}{11a\,x^{11}}$	69

input
```
int((b*x^2+a)^(9/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/11*(b*x^2+a)^(11/2)/a/x^11
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{(b^5x^{10}+5\,ab^4x^8+10\,a^2b^3x^6+10\,a^3b^2x^4+5\,a^4bx^2+a^5)\sqrt{bx^2+a}}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="fricas")
```

output
```
-1/11*(b^5*x^10 + 5*a*b^4*x^8 + 10*a^2*b^3*x^6 + 10*a^3*b^2*x^4 + 5*a^4*b*
x^2 + a^5)*sqrt(b*x^2 + a)/(a*x^11)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(17) = 34$.

Time = 1.27 (sec) , antiderivative size = 150, normalized size of antiderivative = 7.14

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{11x^{10}} - \frac{5a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^8} - \frac{10a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^6} - \frac{10ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^4} - \frac{5b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{11x^2} - \frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{11a}$$

input
```
integrate((b*x**2+a)**(9/2)/x**12,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(11*x**10) - 5*a**3*b**(3/2)*sqrt(a/(b*
x**2) + 1)/(11*x**8) - 10*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(11*x**6) - 1
0*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(11*x**4) - 5*b**(9/2)*sqrt(a/(b*x**2) +
 1)/(11*x**2) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(11*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{(bx^2+a)^{\frac{11}{2}}}{11\,ax^{11}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="maxima")
```

output
```
-1/11*(b*x^2 + a)^(11/2)/(a*x^11)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(17) = 34$.

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 7.95

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = \frac{2\left(11\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}b^{\frac{11}{2}}+165\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{16}a^2b^{\frac{11}{2}}+462\left(\sqrt{b}x-\sqrt{bx^2}\right.\right.}{11\left(\left(\sqrt{b}x-\sqrt{}\right.\right.}$$

input
```
integrate((b*x^2+a)^(9/2)/x^12,x, algorithm="giac")
```

output
```
2/11*(11*(sqrt(b)*x - sqrt(b*x^2 + a))^20*b^(11/2) + 165*(sqrt(b)*x - sqrt
(b*x^2 + a))^16*a^2*b^(11/2) + 462*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^4*b^
(11/2) + 330*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^6*b^(11/2) + 55*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^8*b^(11/2) + a^10*b^(11/2))/((sqrt(b)*x - sqrt(b*x^
2 + a))^2 - a)^11
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.29

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = -\frac{a^4\sqrt{b\,x^2+a}}{11\,x^{11}}-\frac{5\,b^4\sqrt{b\,x^2+a}}{11\,x^3}-\frac{10\,a\,b^3\sqrt{b\,x^2+a}}{11\,x^5}-\frac{5\,a^3\,b\sqrt{b\,x^2+a}}{11\,x^9}-\frac{b^5\sqrt{b\,x^2+a}}{11\,a\,x}-\frac{10\,a^2\,b^2\sqrt{b\,x^2+a}}{11\,x^7}$$

input
```
int((a + b*x^2)^(9/2)/x^12,x)
```

output
```
- (a^4*(a + b*x^2)^(1/2))/(11*x^11) - (5*b^4*(a + b*x^2)^(1/2))/(11*x^3) -
 (10*a*b^3*(a + b*x^2)^(1/2))/(11*x^5) - (5*a^3*b*(a + b*x^2)^(1/2))/(11*x
^9) - (b^5*(a + b*x^2)^(1/2))/(11*a*x) - (10*a^2*b^2*(a + b*x^2)^(1/2))/(1
1*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 5.71

$$\int \frac{(a+bx^2)^{9/2}}{x^{12}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^5-5\sqrt{b\,x^2+a}\,a^4b\,x^2-10\sqrt{b\,x^2+a}\,a^3b^2x^4-10\sqrt{b\,x^2+a}\,a^2b^3x^6-5\sqrt{}}{11a\,x^{11}}$$

input
```
int((b*x^2+a)^(9/2)/x^12,x)
```

output
```
( - sqrt(a + b*x**2)*a**5 - 5*sqrt(a + b*x**2)*a**4*b*x**2 - 10*sqrt(a + b
*x**2)*a**3*b**2*x**4 - 10*sqrt(a + b*x**2)*a**2*b**3*x**6 - 5*sqrt(a + b*
x**2)*a*b**4*x**8 - sqrt(a + b*x**2)*b**5*x**10 - sqrt(b)*b**5*x**11)/(11*
a*x**11)
```

3.445 $\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = -\frac{(a+bx^2)^{11/2}}{13ax^{13}} + \frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}$$

output
```
-1/13*(b*x^2+a)^(11/2)/a/x^13+2/143*b*(b*x^2+a)^(11/2)/a^2/x^11
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{(a+bx^2)^{11/2}(-11a+2bx^2)}{143a^2x^{13}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^14,x]
```

output
```
((a + b*x^2)^(11/2)*(-11*a + 2*b*x^2))/(143*a^2*x^13)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx$$

$$\Big\downarrow \ 245$$

$$-\frac{2b\int \frac{(bx^2+a)^{9/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

$$\Big\downarrow \ 242$$

$$\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^14,x]
```

output
```
-1/13*(a + b*x^2)^(11/2)/(a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11
)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-2b\,x^{2}+11a\right)}{143x^{13}a^{2}}$	28
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-2b\,x^{2}+11a\right)}{143x^{13}a^{2}}$	28
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}\left(-2b\,x^{2}+11a\right)}{143x^{13}a^{2}}$	28
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143a^{2}x^{11}}$	37
trager	$-\frac{\left(-2b^{6}x^{12}+a\,b^{5}x^{10}+35a^{2}b^{4}x^{8}+90a^{3}b^{3}x^{6}+100a^{4}b^{2}x^{4}+53a^{5}b\,x^{2}+11a^{6}\right)\sqrt{b\,x^{2}+a}}{143x^{13}a^{2}}$	82
risch	$-\frac{\left(-2b^{6}x^{12}+a\,b^{5}x^{10}+35a^{2}b^{4}x^{8}+90a^{3}b^{3}x^{6}+100a^{4}b^{2}x^{4}+53a^{5}b\,x^{2}+11a^{6}\right)\sqrt{b\,x^{2}+a}}{143x^{13}a^{2}}$	82

input
```
int((b*x^2+a)^(9/2)/x^14,x,method=_RETURNVERBOSE)
```

output
```
-1/143*(b*x^2+a)^(11/2)*(-2*b*x^2+11*a)/x^13/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{\left(2\,b^6x^{12}-ab^5x^{10}-35\,a^2b^4x^8-90\,a^3b^3x^6-100\,a^4b^2x^4-53\,a^5bx^2-11\,a^6\right)\sqrt{bx^2+a}}{143\,a^2x^{13}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="fricas")
```

output
```
1/143*(2*b^6*x^12 - a*b^5*x^10 - 35*a^2*b^4*x^8 - 90*a^3*b^3*x^6 - 100*a^4
*b^2*x^4 - 53*a^5*b*x^2 - 11*a^6)*sqrt(b*x^2 + a)/(a^2*x^13)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(37) = 74$.

Time = 1.50 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.98

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = -\frac{a^4\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{13x^{12}} - \frac{53a^3b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^{10}} - \frac{100a^2b^{\frac{5}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^8} - \frac{90ab^{\frac{7}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^6} - \frac{35b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{143x^4} - \frac{b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{143ax^2} + \frac{2b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{143a^2}$$

input
```
integrate((b*x**2+a)**(9/2)/x**14,x)
```

output
```
-a**4*sqrt(b)*sqrt(a/(b*x**2) + 1)/(13*x**12) - 53*a**3*b**(3/2)*sqrt(a/(b
*x**2) + 1)/(143*x**10) - 100*a**2*b**(5/2)*sqrt(a/(b*x**2) + 1)/(143*x**8
) - 90*a*b**(7/2)*sqrt(a/(b*x**2) + 1)/(143*x**6) - 35*b**(9/2)*sqrt(a/(b*
x**2) + 1)/(143*x**4) - b**(11/2)*sqrt(a/(b*x**2) + 1)/(143*a*x**2) + 2*b*
*(13/2)*sqrt(a/(b*x**2) + 1)/(143*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{2\,(bx^2+a)^{\frac{11}{2}}b}{143\,a^2x^{11}} - \frac{(bx^2+a)^{\frac{11}{2}}}{13\,ax^{13}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="maxima")
```

output
```
2/143*(b*x^2 + a)^(11/2)*b/(a^2*x^11) - 1/13*(b*x^2 + a)^(11/2)/(a*x^13)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 328, normalized size of antiderivative = 7.45

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{4\left(143\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}b^{\frac{13}{2}}+429\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{20}ab^{\frac{13}{2}}+2145\left(\sqrt{b}x-\sqrt{bx\right.\right.}}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^14,x, algorithm="giac")
```

output
```
4/143*(143*(sqrt(b)*x - sqrt(b*x^2 + a))^22*b^(13/2) + 429*(sqrt(b)*x - sq
rt(b*x^2 + a))^20*a*b^(13/2) + 2145*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^2*b
^(13/2) + 3003*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^3*b^(13/2) + 6006*(sqrt(
b)*x - sqrt(b*x^2 + a))^14*a^4*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a
))^12*a^5*b^(13/2) + 4290*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^6*b^(13/2) +
1430*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^7*b^(13/2) + 715*(sqrt(b)*x - sqrt(
b*x^2 + a))^6*a^8*b^(13/2) + 65*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^9*b^(13/
2) + 13*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^10*b^(13/2) - a^11*b^(13/2))/((s
qrt(b)*x - sqrt(b*x^2 + a))^2 - a)^13
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{2\,b^6\,\sqrt{b\,x^2+a}}{143\,a^2\,x}-\frac{35\,b^4\,\sqrt{b\,x^2+a}}{143\,x^5}-\frac{90\,a\,b^3\,\sqrt{b\,x^2+a}}{143\,x^7} - \frac{53\,a^3\,b\,\sqrt{b\,x^2+a}}{143\,x^{11}}-\frac{b^5\,\sqrt{b\,x^2+a}}{143\,a\,x^3}-\frac{a^4\,\sqrt{b\,x^2+a}}{13\,x^{13}}-\frac{100\,a^2\,b^2\,\sqrt{b\,x^2+a}}{143\,x^9}$$

input
```
int((a + b*x^2)^(9/2)/x^14,x)
```

output
```
(2*b^6*(a + b*x^2)^(1/2))/(143*a^2*x) - (35*b^4*(a + b*x^2)^(1/2))/(143*x^
5) - (90*a*b^3*(a + b*x^2)^(1/2))/(143*x^7) - (53*a^3*b*(a + b*x^2)^(1/2))
/(143*x^11) - (b^5*(a + b*x^2)^(1/2))/(143*a*x^3) - (a^4*(a + b*x^2)^(1/2)
)/(13*x^13) - (100*a^2*b^2*(a + b*x^2)^(1/2))/(143*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 3.16

$$\int \frac{(a+bx^2)^{9/2}}{x^{14}}\,dx = \frac{-11\sqrt{b\,x^2+a}\,a^6 - 53\sqrt{b\,x^2+a}\,a^5 b\,x^2 - 100\sqrt{b\,x^2+a}\,a^4 b^2 x^4 - 90\sqrt{b\,x^2+a}\,a^3 b^3 x^6 -}{143a^2x^{13}}$$

input
```
int((b*x^2+a)^(9/2)/x^14,x)
```

output
```
( - 11*sqrt(a + b*x**2)*a**6 - 53*sqrt(a + b*x**2)*a**5*b*x**2 - 100*sqrt(
a + b*x**2)*a**4*b**2*x**4 - 90*sqrt(a + b*x**2)*a**3*b**3*x**6 - 35*sqrt(
a + b*x**2)*a**2*b**4*x**8 - sqrt(a + b*x**2)*a*b**5*x**10 + 2*sqrt(a + b*
x**2)*b**6*x**12 - 2*sqrt(b)*b**6*x**13)/(143*a**2*x**13)
```

3.446 $\int \frac{(a+bx^2)^{9/2}}{x^{16}} dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}} dx = -\frac{(a+bx^2)^{11/2}}{15ax^{15}} + \frac{4b(a+bx^2)^{11/2}}{195a^2x^{13}} - \frac{8b^2(a+bx^2)^{11/2}}{2145a^3x^{11}}$$

output
```
-1/15*(b*x^2+a)^(11/2)/a/x^15+4/195*b*(b*x^2+a)^(11/2)/a^2/x^13-8/2145*b^2
*(b*x^2+a)^(11/2)/a^3/x^11
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}} dx = \frac{(a+bx^2)^{11/2}(-143a^2+44abx^2-8b^2x^4)}{2145a^3x^{15}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^16,x]
```

output
```
((a + b*x^2)^(11/2)*(-143*a^2 + 44*a*b*x^2 - 8*b^2*x^4))/(2145*a^3*x^15)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\int \frac{(bx^2+a)^{9/2}}{x^{14}}dx}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{4b\left(-\frac{2b\int \frac{(bx^2+a)^{9/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{4b\left(\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^16,x]
```

output
```
-1/15*(a + b*x^2)^(11/2)/(a*x^15) - (4*b*(-1/13*(a + b*x^2)^(11/2)/(a*x^13
) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)))/(15*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.57

method	result	size
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(8b^2x^4-44ab\,x^2+143a^2)}{2145x^{15}a^3}$	39
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(8b^2x^4-44ab\,x^2+143a^2)}{2145x^{15}a^3}$	39
orering	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(8b^2x^4-44ab\,x^2+143a^2)}{2145x^{15}a^3}$	39
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}$	61
trager	$-\frac{(8b^7x^{14}-4a\,b^6x^{12}+3a^2b^5x^{10}+355a^3b^4x^8+1030a^4b^3x^6+1218a^5b^2x^4+671a^6b\,x^2+143a^7)\sqrt{b\,x^2+a}}{2145x^{15}a^3}$	94
risch	$-\frac{(8b^7x^{14}-4a\,b^6x^{12}+3a^2b^5x^{10}+355a^3b^4x^8+1030a^4b^3x^6+1218a^5b^2x^4+671a^6b\,x^2+143a^7)\sqrt{b\,x^2+a}}{2145x^{15}a^3}$	94

input
```
int((b*x^2+a)^(9/2)/x^16,x,method=_RETURNVERBOSE)
```

output
```
-1/2145*(b*x^2+a)^(11/2)*(8*b^2*x^4-44*a*b*x^2+143*a^2)/x^15/a^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = -\frac{(8\,b^7x^{14} - 4\,ab^6x^{12} + 3\,a^2b^5x^{10} + 355\,a^3b^4x^8 + 1030\,a^4b^3x^6 + 1218\,a^5b^2x^4 + 671\,a^6bx^2 + 143\,a^7)\sqrt{bx^2+a}}{2145\,a^3x^{15}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="fricas")
```

output
```
-1/2145*(8*b^7*x^14 - 4*a*b^6*x^12 + 3*a^2*b^5*x^10 + 355*a^3*b^4*x^8 + 10
30*a^4*b^3*x^6 + 1218*a^5*b^2*x^4 + 671*a^6*b*x^2 + 143*a^7)*sqrt(b*x^2 +
a)/(a^3*x^15)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(61) = 122$.

Time = 1.94 (sec) , antiderivative size = 604, normalized size of antiderivative = 8.88

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = -\frac{143a^9b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^6\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})} - \frac{957a^8b^{\frac{11}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^4\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})} - \frac{2703a^7b^{\frac{13}{2}}\sqrt{\frac{a}{bx^2}+1}}{x^2\cdot(2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12})} - \frac{4137a^6b^{\frac{15}{2}}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{3633a^5b^{\frac{17}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{1743a^4b^{\frac{19}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{357a^3b^{\frac{21}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{3a^2b^{\frac{23}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{12ab^{\frac{25}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}} - \frac{8b^{\frac{27}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{2145a^5b^4x^8+4290a^4b^5x^{10}+2145a^3b^6x^{12}}$$

input

```
integrate((b*x**2+a)**(9/2)/x**16,x)
```

output
```
-143*a**9*b**(9/2)*sqrt(a/(b*x**2) + 1)/(x**6*(2145*a**5*b**4*x**8 + 4290*
a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 957*a**8*b**(11/2)*sqrt(a/(b*x*
*2) + 1)/(x**4*(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**
6*x**12)) - 2703*a**7*b**(13/2)*sqrt(a/(b*x**2) + 1)/(x**2*(2145*a**5*b**4
*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12)) - 4137*a**6*b**(15/2
)*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*
a**3*b**6*x**12) - 3633*a**5*b**(17/2)*x**2*sqrt(a/(b*x**2) + 1)/(2145*a**
5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 1743*a**4*b**
(19/2)*x**4*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**
10 + 2145*a**3*b**6*x**12) - 357*a**3*b**(21/2)*x**6*sqrt(a/(b*x**2) + 1)/
(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 3*a*
*2*b**(23/2)*x**8*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b*
*5*x**10 + 2145*a**3*b**6*x**12) - 12*a*b**(25/2)*x**10*sqrt(a/(b*x**2) +
1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**5*x**10 + 2145*a**3*b**6*x**12) - 8
*b**(27/2)*x**12*sqrt(a/(b*x**2) + 1)/(2145*a**5*b**4*x**8 + 4290*a**4*b**
5*x**10 + 2145*a**3*b**6*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = -\frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{2145\,a^3x^{11}} + \frac{4\,(bx^2+a)^{\frac{11}{2}}b}{195\,a^2x^{13}} - \frac{(bx^2+a)^{\frac{11}{2}}}{15\,ax^{15}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="maxima")
```

output
```
-8/2145*(b*x^2 + a)^(11/2)*b^2/(a^3*x^11) + 4/195*(b*x^2 + a)^(11/2)*b/(a^
2*x^13) - 1/15*(b*x^2 + a)^(11/2)/(a*x^15)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(56) = 112$.

Time = 0.15 (sec) , antiderivative size = 354, normalized size of antiderivative = 5.21

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = \frac{16\left(1430\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}b^{\frac{15}{2}}+6435\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{22}ab^{\frac{15}{2}}+24453\left(\sqrt{b}x-\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^16,x, algorithm="giac")
```

output
```
16/2145*(1430*(sqrt(b)*x - sqrt(b*x^2 + a))^24*b^(15/2) + 6435*(sqrt(b)*x
- sqrt(b*x^2 + a))^22*a*b^(15/2) + 24453*(sqrt(b)*x - sqrt(b*x^2 + a))^20*
a^2*b^(15/2) + 45045*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^3*b^(15/2) + 70785
*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^4*b^(15/2) + 64350*(sqrt(b)*x - sqrt(b
*x^2 + a))^14*a^5*b^(15/2) + 50050*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^6*b^
(15/2) + 21450*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^7*b^(15/2) + 7800*(sqrt(
b)*x - sqrt(b*x^2 + a))^8*a^8*b^(15/2) + 975*(sqrt(b)*x - sqrt(b*x^2 + a))
^6*a^9*b^(15/2) + 105*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^10*b^(15/2) - 15*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^11*b^(15/2) + a^12*b^(15/2))/((sqrt(b)*x
- sqrt(b*x^2 + a))^2 - a)^15
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.22

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = \frac{4\,b^6\,\sqrt{b\,x^2+a}}{2145\,a^2\,x^3} - \frac{71\,b^4\,\sqrt{b\,x^2+a}}{429\,x^7} - \frac{206\,a\,b^3\,\sqrt{b\,x^2+a}}{429\,x^9} - \frac{61\,a^3\,b\,\sqrt{b\,x^2+a}}{195\,x^{13}} - \frac{b^5\,\sqrt{b\,x^2+a}}{715\,a\,x^5} - \frac{a^4\,\sqrt{b\,x^2+a}}{15\,x^{15}} - \frac{8\,b^7\,\sqrt{b\,x^2+a}}{2145\,a^3\,x} - \frac{406\,a^2\,b^2\,\sqrt{b\,x^2+a}}{715\,x^{11}}$$

input
```
int((a + b*x^2)^(9/2)/x^16,x)
```

output
```
(4*b^6*(a + b*x^2)^(1/2))/(2145*a^2*x^3) - (71*b^4*(a + b*x^2)^(1/2))/(429
*x^7) - (206*a*b^3*(a + b*x^2)^(1/2))/(429*x^9) - (61*a^3*b*(a + b*x^2)^(1
/2))/(195*x^13) - (b^5*(a + b*x^2)^(1/2))/(715*a*x^5) - (a^4*(a + b*x^2)^(
1/2))/(15*x^15) - (8*b^7*(a + b*x^2)^(1/2))/(2145*a^3*x) - (406*a^2*b^2*(a
 + b*x^2)^(1/2))/(715*x^11)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.32

$$\int \frac{(a+bx^2)^{9/2}}{x^{16}}\,dx = \frac{-143\sqrt{b\,x^2+a}\,a^7 - 671\sqrt{b\,x^2+a}\,a^6b\,x^2 - 1218\sqrt{b\,x^2+a}\,a^5b^2x^4 - 1030\sqrt{b\,x^2+a}\,a^4$$

input
```
int((b*x^2+a)^(9/2)/x^16,x)
```

output
```
( - 143*sqrt(a + b*x**2)*a**7 - 671*sqrt(a + b*x**2)*a**6*b*x**2 - 1218*sq
rt(a + b*x**2)*a**5*b**2*x**4 - 1030*sqrt(a + b*x**2)*a**4*b**3*x**6 - 355
*sqrt(a + b*x**2)*a**3*b**4*x**8 - 3*sqrt(a + b*x**2)*a**2*b**5*x**10 + 4*
sqrt(a + b*x**2)*a*b**6*x**12 - 8*sqrt(a + b*x**2)*b**7*x**14 + 8*sqrt(b)*
b**7*x**15)/(2145*a**3*x**15)
```

3.447 $\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = -\frac{(a+bx^2)^{11/2}}{17ax^{17}} + \frac{2b(a+bx^2)^{11/2}}{85a^2x^{15}} - \frac{8b^2(a+bx^2)^{11/2}}{1105a^3x^{13}} + \frac{16b^3(a+bx^2)^{11/2}}{12155a^4x^{11}}$$

output
```
-1/17*(b*x^2+a)^(11/2)/a/x^17+2/85*b*(b*x^2+a)^(11/2)/a^2/x^15-8/1105*b^2*
(b*x^2+a)^(11/2)/a^3/x^13+16/12155*b^3*(b*x^2+a)^(11/2)/a^4/x^11
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{(a+bx^2)^{11/2}(-715a^3+286a^2bx^2-88ab^2x^4+16b^3x^6)}{12155a^4x^{17}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^18,x]
```

output
```
((a + b*x^2)^(11/2)*(-715*a^3 + 286*a^2*b*x^2 - 88*a*b^2*x^4 + 16*b^3*x^6)
)/(12155*a^4*x^17)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{9/2}}{x^{18}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\int \frac{\left(bx^2+a\right)^{9/2}}{x^{16}}dx}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\int \frac{\left(bx^2+a\right)^{9/2}}{x^{14}}dx}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{6b\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^2+a\right)^{9/2}}{x^{12}}dx}{13a}-\frac{\left(a+bx^2\right)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{6b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{11/2}}{143a^2x^{11}}-\frac{\left(a+bx^2\right)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^18,x]
```

output
```
-1/17*(a + b*x^2)^(11/2)/(a*x^17) - (6*b*(-1/15*(a + b*x^2)^(11/2)/(a*x^15
) - (4*b*(-1/13*(a + b*x^2)^(11/2)/(a*x^13) + (2*b*(a + b*x^2)^(11/2))/(14
3*a^2*x^11)))/(15*a)))/(17*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

method	result
gosper	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-16 b^{3} x^{6}+88 a\,b^{2} x^{4}-286 a^{2} b\,x^{2}+715 a^{3}\right)}{12155 x^{17} a^{4}}$
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-16 b^{3} x^{6}+88 a\,b^{2} x^{4}-286 a^{2} b\,x^{2}+715 a^{3}\right)}{12155 x^{17} a^{4}}$
orering	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}} \left(-16 b^{3} x^{6}+88 a\,b^{2} x^{4}-286 a^{2} b\,x^{2}+715 a^{3}\right)}{12155 x^{17} a^{4}}$
default	$-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{17 a\,x^{17}}-\frac{6 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{15 a\,x^{15}}-\frac{4 b \left(-\frac{\left(b\,x^{2}+a\right)^{\frac{11}{2}}}{13 a\,x^{13}}+\frac{2 b \left(b\,x^{2}+a\right)^{\frac{11}{2}}}{143 a^{2} x^{11}}\right)}{15 a}\right)}{17 a}$
trager	$-\frac{\left(-16 b^{8} x^{16}+8 a\,b^{7} x^{14}-6 a^{2} b^{6} x^{12}+5 a^{3} b^{5} x^{10}+1515 a^{4} b^{4} x^{8}+4714 a^{5} b^{3} x^{6}+5808 a^{6} b^{2} x^{4}+3289 a^{7} b\,x^{2}+715 a^{8}\right) \sqrt{b\,x^{2}+a}}{12155 x^{17} a^{4}}$
risch	$-\frac{\left(-16 b^{8} x^{16}+8 a\,b^{7} x^{14}-6 a^{2} b^{6} x^{12}+5 a^{3} b^{5} x^{10}+1515 a^{4} b^{4} x^{8}+4714 a^{5} b^{3} x^{6}+5808 a^{6} b^{2} x^{4}+3289 a^{7} b\,x^{2}+715 a^{8}\right) \sqrt{b\,x^{2}+a}}{12155 x^{17} a^{4}}$

input
```
int((b*x^2+a)^(9/2)/x^18,x,method=_RETURNVERBOSE)
```

output
```
-1/12155*(b*x^2+a)^(11/2)*(-16*b^3*x^6+88*a*b^2*x^4-286*a^2*b*x^2+715*a^3)
/x^17/a^4
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{(16\,b^8x^{16} - 8\,ab^7x^{14} + 6\,a^2b^6x^{12} - 5\,a^3b^5x^{10} - 1515\,a^4b^4x^8 - 4714\,a^5b^3x^6 - 5808\,a^6b^2}{12155\,a^4x^{17}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="fricas")
```

output
```
1/12155*(16*b^8*x^16 - 8*a*b^7*x^14 + 6*a^2*b^6*x^12 - 5*a^3*b^5*x^10 - 15
15*a^4*b^4*x^8 - 4714*a^5*b^3*x^6 - 5808*a^6*b^2*x^4 - 3289*a^7*b*x^2 - 71
5*a^8)*sqrt(b*x^2 + a)/(a^4*x^17)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(85) = 170$.

Time = 2.51 (sec) , antiderivative size = 867, normalized size of antiderivative = 9.42

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(9/2)/x**18,x)
```

output
```
-715*a**11*b**(19/2)*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a
**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 5434*
a**10*b**(21/2)*x**2*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a
**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 17820
*a**9*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a
**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 32720
*a**8*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a
**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 36370
*a**7*b**(27/2)*x**8*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a
**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 24500
*a**6*b**(29/2)*x**10*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*
a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 9268
*a**5*b**(31/2)*x**12*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*
a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) - 1520
*a**4*b**(33/2)*x**14*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*
a**6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 5*a*
*3*b**(35/2)*x**16*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**
6*b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 30*a**2
*b**(37/2)*x**18*sqrt(a/(b*x**2) + 1)/(12155*a**7*b**9*x**16 + 36465*a**6*
b**10*x**18 + 36465*a**5*b**11*x**20 + 12155*a**4*b**12*x**22) + 40*a*b...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{16\,(bx^2+a)^{\frac{11}{2}}b^3}{12155\,a^4x^{11}} - \frac{8\,(bx^2+a)^{\frac{11}{2}}b^2}{1105\,a^3x^{13}} + \frac{2\,(bx^2+a)^{\frac{11}{2}}b}{85\,a^2x^{15}} - \frac{(bx^2+a)^{\frac{11}{2}}}{17\,ax^{17}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="maxima")
```

output
```
16/12155*(b*x^2 + a)^(11/2)*b^3/(a^4*x^11) - 8/1105*(b*x^2 + a)^(11/2)*b^2
/(a^3*x^13) + 2/85*(b*x^2 + a)^(11/2)*b/(a^2*x^15) - 1/17*(b*x^2 + a)^(11/
2)/(a*x^17)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(76) = 152$.

Time = 0.14 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.15

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{32\left(12155\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}b^{\frac{17}{2}}+65637\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{24}ab^{\frac{17}{2}}+233376\left(\sqrt{b}\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^18,x, algorithm="giac")
```

output
```
32/12155*(12155*(sqrt(b)*x - sqrt(b*x^2 + a))^26*b^(17/2) + 65637*(sqrt(b)
*x - sqrt(b*x^2 + a))^24*a*b^(17/2) + 233376*(sqrt(b)*x - sqrt(b*x^2 + a))
^22*a^2*b^(17/2) + 466752*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^3*b^(17/2) +
692835*(sqrt(b)*x - sqrt(b*x^2 + a))^18*a^4*b^(17/2) + 668525*(sqrt(b)*x -
 sqrt(b*x^2 + a))^16*a^5*b^(17/2) + 486200*(sqrt(b)*x - sqrt(b*x^2 + a))^1
4*a^6*b^(17/2) + 221000*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^7*b^(17/2) + 71
825*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a^8*b^(17/2) + 9775*(sqrt(b)*x - sqrt
(b*x^2 + a))^8*a^9*b^(17/2) + 680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^10*b^(
17/2) - 136*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^11*b^(17/2) + 17*(sqrt(b)*x
- sqrt(b*x^2 + a))^2*a^12*b^(17/2) - a^13*b^(17/2))/((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^17
```

Mupad [B] (verification not implemented)

Time = 4.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{6\,b^6\,\sqrt{b\,x^2+a}}{12155\,a^2\,x^5} - \frac{303\,b^4\,\sqrt{b\,x^2+a}}{2431\,x^9} - \frac{4714\,a\,b^3\,\sqrt{b\,x^2+a}}{12155\,x^{11}} - \frac{23\,a^3\,b\,\sqrt{b\,x^2+a}}{85\,x^{15}} - \frac{b^5\,\sqrt{b\,x^2+a}}{2431\,a\,x^7} - \frac{a^4\,\sqrt{b\,x^2+a}}{17\,x^{17}} - \frac{8\,b^7\,\sqrt{b\,x^2+a}}{12155\,a^3\,x^3} + \frac{16\,b^8\,\sqrt{b\,x^2+a}}{12155\,a^4\,x} - \frac{528\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1105\,x^{13}}$$

input
```
int((a + b*x^2)^(9/2)/x^18,x)
```

output
```
(6*b^6*(a + b*x^2)^(1/2))/(12155*a^2*x^5) - (303*b^4*(a + b*x^2)^(1/2))/(2
431*x^9) - (4714*a*b^3*(a + b*x^2)^(1/2))/(12155*x^11) - (23*a^3*b*(a + b*
x^2)^(1/2))/(85*x^15) - (b^5*(a + b*x^2)^(1/2))/(2431*a*x^7) - (a^4*(a + b
*x^2)^(1/2))/(17*x^17) - (8*b^7*(a + b*x^2)^(1/2))/(12155*a^3*x^3) + (16*b
^8*(a + b*x^2)^(1/2))/(12155*a^4*x) - (528*a^2*b^2*(a + b*x^2)^(1/2))/(110
5*x^13)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.92

$$\int \frac{(a+bx^2)^{9/2}}{x^{18}}\,dx = \frac{-715\sqrt{b\,x^2+a}\,a^8 - 3289\sqrt{b\,x^2+a}\,a^7b\,x^2 - 5808\sqrt{b\,x^2+a}\,a^6b^2x^4 - 4714\sqrt{b\,x^2+a}\,a$$

input
```
int((b*x^2+a)^(9/2)/x^18,x)
```

output
```
( - 715*sqrt(a + b*x**2)*a**8 - 3289*sqrt(a + b*x**2)*a**7*b*x**2 - 5808*s
qrt(a + b*x**2)*a**6*b**2*x**4 - 4714*sqrt(a + b*x**2)*a**5*b**3*x**6 - 15
15*sqrt(a + b*x**2)*a**4*b**4*x**8 - 5*sqrt(a + b*x**2)*a**3*b**5*x**10 +
6*sqrt(a + b*x**2)*a**2*b**6*x**12 - 8*sqrt(a + b*x**2)*a*b**7*x**14 + 16*
sqrt(a + b*x**2)*b**8*x**16 - 16*sqrt(b)*b**8*x**17)/(12155*a**4*x**17)
```

3.448 $\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 116

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = -\frac{(a+bx^2)^{11/2}}{19ax^{19}} + \frac{8b(a+bx^2)^{11/2}}{323a^2x^{17}} - \frac{16b^2(a+bx^2)^{11/2}}{1615a^3x^{15}} + \frac{64b^3(a+bx^2)^{11/2}}{20995a^4x^{13}} - \frac{128b^4(a+bx^2)^{11/2}}{230945a^5x^{11}}$$

output
```
-1/19*(b*x^2+a)^(11/2)/a/x^19+8/323*b*(b*x^2+a)^(11/2)/a^2/x^17-16/1615*b^
2*(b*x^2+a)^(11/2)/a^3/x^15+64/20995*b^3*(b*x^2+a)^(11/2)/a^4/x^13-128/230
945*b^4*(b*x^2+a)^(11/2)/a^5/x^11
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = \frac{(a+bx^2)^{11/2}\left(-12155a^4+5720a^3bx^2-2288a^2b^2x^4+704ab^3x^6-128b^4x^8\right)}{230945a^5x^{19}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^20,x]
```

output
```
((a + b*x^2)^(11/2)*(-12155*a^4 + 5720*a^3*b*x^2 - 2288*a^2*b^2*x^4 + 704*
a*b^3*x^6 - 128*b^4*x^8))/(230945*a^5*x^19)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\int \frac{(bx^2+a)^{9/2}}{x^{18}}dx}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\int \frac{(bx^2+a)^{9/2}}{x^{16}}dx}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\int \frac{(bx^2+a)^{9/2}}{x^{14}}dx}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{(bx^2+a)^{9/2}}{x^{12}}dx}{13a}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

$\big\downarrow$ 242

$$-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^20,x]
```

output
```
-1/19*(a + b*x^2)^(11/2)/(a*x^19) - (8*b*(-1/17*(a + b*x^2)^(11/2)/(a*x^17
) - (6*b*(-1/15*(a + b*x^2)^(11/2)/(a*x^15) - (4*b*(-1/13*(a + b*x^2)^(11/
2)/(a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)))/(15*a)))/(17*a)))/
(19*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}\left(128b^4x^8-704a\,b^3x^6+2288a^2b^2x^4-5720a^3b\,x^2+12155a^4\right)}{230945x^{19}a^5}$
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{11}{2}}\left(128b^4x^8-704a\,b^3x^6+2288a^2b^2x^4-5720a^3b\,x^2+12155a^4\right)}{230945x^{19}a^5}$
orering	$-\frac{(b\,x^2+a)^{\frac{11}{2}}\left(128b^4x^8-704a\,b^3x^6+2288a^2b^2x^4-5720a^3b\,x^2+12155a^4\right)}{230945x^{19}a^5}$
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{19a\,x^{19}}-\frac{8b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}\right)}{17a}\right)}{19a}$
trager	$-\frac{(128b^9x^{18}-64a\,b^8x^{16}+48a^2b^7x^{14}-40a^3b^6x^{12}+35a^4b^5x^{10}+23063a^5b^4x^8+75086a^6b^3x^6+95238a^7b^2x^4+55055a^8b\,x^2+}{230945x^{19}a^5}$
risch	$-\frac{(128b^9x^{18}-64a\,b^8x^{16}+48a^2b^7x^{14}-40a^3b^6x^{12}+35a^4b^5x^{10}+23063a^5b^4x^8+75086a^6b^3x^6+95238a^7b^2x^4+55055a^8b\,x^2+}{230945x^{19}a^5}$

input
```
int((b*x^2+a)^(9/2)/x^20,x,method=_RETURNVERBOSE)
```

output
```
-1/230945*(b*x^2+a)^(11/2)*(128*b^4*x^8-704*a*b^3*x^6+2288*a^2*b^2*x^4-572
0*a^3*b*x^2+12155*a^4)/x^19/a^5
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx =$$
$$-\frac{(128\,b^9x^{18} - 64\,ab^8x^{16} + 48\,a^2b^7x^{14} - 40\,a^3b^6x^{12} + 35\,a^4b^5x^{10} + 23063\,a^5b^4x^8 + 75086\,a^6b^3x^6 + 95238\,a^7}{230945\,a^5x^{19}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="fricas")
```

output
```
-1/230945*(128*b^9*x^18 - 64*a*b^8*x^16 + 48*a^2*b^7*x^14 - 40*a^3*b^6*x^1
2 + 35*a^4*b^5*x^10 + 23063*a^5*b^4*x^8 + 75086*a^6*b^3*x^6 + 95238*a^7*b^
2*x^4 + 55055*a^8*b*x^2 + 12155*a^9)*sqrt(b*x^2 + a)/(a^5*x^19)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. 2(109) = 218.

Time = 3.22 (sec) , antiderivative size = 1182, normalized size of antiderivative = 10.19

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(9/2)/x**20,x)
```

output
```
-12155*a**13*b**(33/2)*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923
780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24
+ 230945*a**5*b**20*x**26) - 103675*a**12*b**(35/2)*x**2*sqrt(a/(b*x**2) +
 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**1
8*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 388388*a**1
1*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a*
*8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 2309
45*a**5*b**20*x**26) - 834988*a**10*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(2
30945*a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**2
2 + 923780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 1127210*a**9*b**(
41/2)*x**8*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**
17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**
5*b**20*x**26) - 978810*a**8*b**(43/2)*x**10*sqrt(a/(b*x**2) + 1)/(230945*
a**9*b**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 92
3780*a**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 534060*a**7*b**(45/2)*x
**12*sqrt(a/(b*x**2) + 1)/(230945*a**9*b**16*x**18 + 923780*a**8*b**17*x**
20 + 1385670*a**7*b**18*x**22 + 923780*a**6*b**19*x**24 + 230945*a**5*b**2
0*x**26) - 167436*a**6*b**(47/2)*x**14*sqrt(a/(b*x**2) + 1)/(230945*a**9*b
**16*x**18 + 923780*a**8*b**17*x**20 + 1385670*a**7*b**18*x**22 + 923780*a
**6*b**19*x**24 + 230945*a**5*b**20*x**26) - 23091*a**5*b**(49/2)*x**16...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = -\frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{230945\,a^5x^{11}} + \frac{64\,(bx^2+a)^{\frac{11}{2}}b^3}{20995\,a^4x^{13}} - \frac{16\,(bx^2+a)^{\frac{11}{2}}b^2}{1615\,a^3x^{15}} + \frac{8\,(bx^2+a)^{\frac{11}{2}}b}{323\,a^2x^{17}} - \frac{(bx^2+a)^{\frac{11}{2}}}{19\,ax^{19}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="maxima")
```

output
```
-128/230945*(b*x^2 + a)^(11/2)*b^4/(a^5*x^11) + 64/20995*(b*x^2 + a)^(11/2
)*b^3/(a^4*x^13) - 16/1615*(b*x^2 + a)^(11/2)*b^2/(a^3*x^15) + 8/323*(b*x^
2 + a)^(11/2)*b/(a^2*x^17) - 1/19*(b*x^2 + a)^(11/2)/(a*x^19)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(96) = 192$.

Time = 0.14 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.52

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = \frac{256\left(92378\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}b^{\frac{19}{2}}+554268\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{26}ab^{\frac{19}{2}}+1939938\left(\right.\right.}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^20,x, algorithm="giac")
```

output
```
256/230945*(92378*(sqrt(b)*x - sqrt(b*x^2 + a))^28*b^(19/2) + 554268*(sqrt
(b)*x - sqrt(b*x^2 + a))^26*a*b^(19/2) + 1939938*(sqrt(b)*x - sqrt(b*x^2 +
 a))^24*a^2*b^(19/2) + 4018443*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^3*b^(19/
2) + 5866003*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^4*b^(19/2) + 5773625*(sqrt
(b)*x - sqrt(b*x^2 + a))^18*a^5*b^(19/2) + 4094025*(sqrt(b)*x - sqrt(b*x^2
 + a))^16*a^6*b^(19/2) + 1889550*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^7*b^(1
9/2) + 581400*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^8*b^(19/2) + 80750*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*a^9*b^(19/2) + 3876*(sqrt(b)*x - sqrt(b*x^2 + a
))^8*a^10*b^(19/2) - 969*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^11*b^(19/2) + 1
71*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^12*b^(19/2) - 19*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*a^13*b^(19/2) + a^14*b^(19/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)^19
```

Mupad [B] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.65

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx &= \frac{8\,b^6\,\sqrt{b\,x^2+a}}{46189\,a^2\,x^7}-\frac{23063\,b^4\,\sqrt{b\,x^2+a}}{230945\,x^{11}}-\frac{6826\,a\,b^3\,\sqrt{b\,x^2+a}}{20995\,x^{13}}\\&\quad-\frac{77\,a^3\,b\,\sqrt{b\,x^2+a}}{323\,x^{17}}-\frac{7\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^9}-\frac{a^4\,\sqrt{b\,x^2+a}}{19\,x^{19}}-\frac{48\,b^7\,\sqrt{b\,x^2+a}}{230945\,a^3\,x^5}\\&\quad+\frac{64\,b^8\,\sqrt{b\,x^2+a}}{230945\,a^4\,x^3}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{230945\,a^5\,x}-\frac{666\,a^2\,b^2\,\sqrt{b\,x^2+a}}{1615\,x^{15}}\end{aligned}$$

input
```
int((a + b*x^2)^(9/2)/x^20,x)
```

output
```
(8*b^6*(a + b*x^2)^(1/2))/(46189*a^2*x^7) - (23063*b^4*(a + b*x^2)^(1/2))/
(230945*x^11) - (6826*a*b^3*(a + b*x^2)^(1/2))/(20995*x^13) - (77*a^3*b*(a
 + b*x^2)^(1/2))/(323*x^17) - (7*b^5*(a + b*x^2)^(1/2))/(46189*a*x^9) - (a
^4*(a + b*x^2)^(1/2))/(19*x^19) - (48*b^7*(a + b*x^2)^(1/2))/(230945*a^3*x
^5) + (64*b^8*(a + b*x^2)^(1/2))/(230945*a^4*x^3) - (128*b^9*(a + b*x^2)^(
1/2))/(230945*a^5*x) - (666*a^2*b^2*(a + b*x^2)^(1/2))/(1615*x^15)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)^{9/2}}{x^{20}}\,dx = \frac{-12155\sqrt{b\,x^2+a}\,a^9-55055\sqrt{b\,x^2+a}\,a^8b\,x^2-95238\sqrt{b\,x^2+a}\,a^7b^2x^4-75086\sqrt{b\,x^2}}{}$$

input
```
int((b*x^2+a)^(9/2)/x^20,x)
```

output
```
( - 12155*sqrt(a + b*x**2)*a**9 - 55055*sqrt(a + b*x**2)*a**8*b*x**2 - 952
38*sqrt(a + b*x**2)*a**7*b**2*x**4 - 75086*sqrt(a + b*x**2)*a**6*b**3*x**6
 - 23063*sqrt(a + b*x**2)*a**5*b**4*x**8 - 35*sqrt(a + b*x**2)*a**4*b**5*x
**10 + 40*sqrt(a + b*x**2)*a**3*b**6*x**12 - 48*sqrt(a + b*x**2)*a**2*b**7
*x**14 + 64*sqrt(a + b*x**2)*a*b**8*x**16 - 128*sqrt(a + b*x**2)*b**9*x**1
8 + 128*sqrt(b)*b**9*x**19)/(230945*a**5*x**19)
```

3.449 $\int \frac{(a+bx^2)^{9/2}}{x^{22}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}} \, dx = -\frac{(a+bx^2)^{11/2}}{21ax^{21}} + \frac{10b(a+bx^2)^{11/2}}{399a^2x^{19}} - \frac{80b^2(a+bx^2)^{11/2}}{6783a^3x^{17}} + \frac{32b^3(a+bx^2)^{11/2}}{6783a^4x^{15}} - \frac{128b^4(a+bx^2)^{11/2}}{88179a^5x^{13}} + \frac{256b^5(a+bx^2)^{11/2}}{969969a^6x^{11}}$$

output
```
-1/21*(b*x^2+a)^(11/2)/a/x^21+10/399*b*(b*x^2+a)^(11/2)/a^2/x^19-80/6783*b
^2*(b*x^2+a)^(11/2)/a^3/x^17+32/6783*b^3*(b*x^2+a)^(11/2)/a^4/x^15-128/881
79*b^4*(b*x^2+a)^(11/2)/a^5/x^13+256/969969*b^5*(b*x^2+a)^(11/2)/a^6/x^11
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}} \, dx = \frac{(a+bx^2)^{11/2} \left(-46189a^5 + 24310a^4bx^2 - 11440a^3b^2x^4 + 4576a^2b^3x^6 - 1408ab^4x^8 + 2\right.}{969969a^6x^{21}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^22,x]
```

output
```
((a + b*x^2)^(11/2)*(-46189*a^5 + 24310*a^4*b*x^2 - 11440*a^3*b^2*x^4 + 45
76*a^2*b^3*x^6 - 1408*a*b^4*x^8 + 256*b^5*x^10))/(969969*a^6*x^21)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {245, 245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\int \frac{(bx^2+a)^{9/2}}{x^{20}}dx}{21a}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\int \frac{(bx^2+a)^{9/2}}{x^{18}}dx}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\int \frac{(bx^2+a)^{9/2}}{x^{16}}dx}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int\frac{\left(bx^2+a\right)^{9/2}}{x^{14}}dx}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

$\big\downarrow$ 245

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+a\right)^{9/2}}{x^{12}}dx}{13a}-\frac{\left(a+bx^2\right)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}$$

$\big\downarrow$ 242

$$-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{2b(a+bx^2)^{11/2}}{143a^2x^{11}}-\frac{(a+bx^2)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{(a+bx^2)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{(a+bx^2)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{(a+bx^2)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^22,x]
```

output
```
-1/21*(a + b*x^2)^(11/2)/(a*x^21) - (10*b*(-1/19*(a + b*x^2)^(11/2)/(a*x^1
9) - (8*b*(-1/17*(a + b*x^2)^(11/2)/(a*x^17) - (6*b*(-1/15*(a + b*x^2)^(11
/2)/(a*x^15) - (4*b*(-1/13*(a + b*x^2)^(11/2)/(a*x^13) + (2*b*(a + b*x^2)^
(11/2))/(143*a^2*x^11)))/(15*a)))/(17*a)))/(19*a)))/(21*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 22.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

method	result
gosper	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-256b^5x^{10}+1408a\,b^4x^8-4576a^2b^3x^6+11440a^3b^2x^4-24310a^4b\,x^2+46189a^5)}{969969x^{21}a^6}$
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-256b^5x^{10}+1408a\,b^4x^8-4576a^2b^3x^6+11440a^3b^2x^4-24310a^4b\,x^2+46189a^5)}{969969x^{21}a^6}$
orering	$-\frac{(b\,x^2+a)^{\frac{11}{2}}(-256b^5x^{10}+1408a\,b^4x^8-4576a^2b^3x^6+11440a^3b^2x^4-24310a^4b\,x^2+46189a^5)}{969969x^{21}a^6}$
trager	$-\frac{(-256b^{10}x^{20}+128a\,b^9x^{18}-96a^2b^8x^{16}+80a^3b^7x^{14}-70a^4b^6x^{12}+63a^5b^5x^{10}+80773a^6b^4x^8+271414a^7b^3x^6+351780a^8b^2}{969969a^6x^{21}}$
risch	$-\frac{(-256b^{10}x^{20}+128a\,b^9x^{18}-96a^2b^8x^{16}+80a^3b^7x^{14}-70a^4b^6x^{12}+63a^5b^5x^{10}+80773a^6b^4x^8+271414a^7b^3x^6+351780a^8b^2}{969969a^6x^{21}}$
default	$-\frac{(b\,x^2+a)^{\frac{11}{2}}}{21a\,x^{21}}-\frac{10b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{19a\,x^{19}}-\frac{8b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^2+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(b\,x^2+a)^{\frac{11}{2}}}{143a^2x^{11}}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}$

input
```
int((b*x^2+a)^(9/2)/x^22,x,method=_RETURNVERBOSE)
```

output
```
-1/969969*(b*x^2+a)^(11/2)*(-256*b^5*x^10+1408*a*b^4*x^8-4576*a^2*b^3*x^6+
11440*a^3*b^2*x^4-24310*a^4*b*x^2+46189*a^5)/x^21/a^6
```

Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \frac{(256\,b^{10}x^{20} - 128\,ab^9x^{18} + 96\,a^2b^8x^{16} - 80\,a^3b^7x^{14} + 70\,a^4b^6x^{12} - 63\,a^5b^5x^{10} - 80773}{969969\,a^6}$$

input
```
integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="fricas")
```

output
```
1/969969*(256*b^10*x^20 - 128*a*b^9*x^18 + 96*a^2*b^8*x^16 - 80*a^3*b^7*x^
14 + 70*a^4*b^6*x^12 - 63*a^5*b^5*x^10 - 80773*a^6*b^4*x^8 - 271414*a^7*b^
3*x^6 - 351780*a^8*b^2*x^4 - 206635*a^9*b*x^2 - 46189*a^10)*sqrt(b*x^2 + a
)/(a^6*x^21)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1540 vs. 2(133) = 266.

Time = 3.76 (sec) , antiderivative size = 1540, normalized size of antiderivative = 11.00

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(9/2)/x**22,x)
```

output
```
-46189*a**15*b**(51/2)*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 48
49845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x*
*26 + 4849845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 437580*a**14*b
**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**
10*b**26*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 484
9845*a**7*b**29*x**28 + 969969*a**6*b**30*x**30) - 1846845*a**13*b**(55/2)
*x**4*sqrt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26
*x**22 + 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**
7*b**29*x**28 + 969969*a**6*b**30*x**30) - 4558554*a**12*b**(57/2)*x**6*sq
rt(a/(b*x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 +
 9699690*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*
x**28 + 969969*a**6*b**30*x**30) - 7252938*a**11*b**(59/2)*x**8*sqrt(a/(b*
x**2) + 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690
*a**9*b**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 +
969969*a**6*b**30*x**30) - 7715232*a**10*b**(61/2)*x**10*sqrt(a/(b*x**2) +
 1)/(969969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b
**27*x**24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*
a**6*b**30*x**30) - 5487650*a**9*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(969
969*a**11*b**25*x**20 + 4849845*a**10*b**26*x**22 + 9699690*a**9*b**27*x**
24 + 9699690*a**8*b**28*x**26 + 4849845*a**7*b**29*x**28 + 969969*a**6*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \frac{256\,(bx^2+a)^{\frac{11}{2}}b^5}{969969\,a^6x^{11}} - \frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{88179\,a^5x^{13}} + \frac{32\,(bx^2+a)^{\frac{11}{2}}b^3}{6783\,a^4x^{15}} - \frac{80\,(bx^2+a)^{\frac{11}{2}}b^2}{6783\,a^3x^{17}} + \frac{10\,(bx^2+a)^{\frac{11}{2}}b}{399\,a^2x^{19}} - \frac{(bx^2+a)^{\frac{11}{2}}}{21\,ax^{21}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="maxima")
```

output
```
256/969969*(b*x^2 + a)^(11/2)*b^5/(a^6*x^11) - 128/88179*(b*x^2 + a)^(11/2
)*b^4/(a^5*x^13) + 32/6783*(b*x^2 + a)^(11/2)*b^3/(a^4*x^15) - 80/6783*(b*
x^2 + a)^(11/2)*b^2/(a^3*x^17) + 10/399*(b*x^2 + a)^(11/2)*b/(a^2*x^19) -
1/21*(b*x^2 + a)^(11/2)/(a*x^21)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(116) = 232$.

Time = 0.15 (sec) , antiderivative size = 436, normalized size of antiderivative = 3.11

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \frac{512\left(646646\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{30}b^{\frac{21}{2}}+4157010\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{28}ab^{\frac{21}{2}}+145495\right.}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^22,x, algorithm="giac")
```

output
```
512/969969*(646646*(sqrt(b)*x - sqrt(b*x^2 + a))^30*b^(21/2) + 4157010*(sq
rt(b)*x - sqrt(b*x^2 + a))^28*a*b^(21/2) + 14549535*(sqrt(b)*x - sqrt(b*x^
2 + a))^26*a^2*b^(21/2) + 30715685*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^3*b^
(21/2) + 44618574*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^4*b^(21/2) + 44265858
*(sqrt(b)*x - sqrt(b*x^2 + a))^20*a^5*b^(21/2) + 31009615*(sqrt(b)*x - sqr
t(b*x^2 + a))^18*a^6*b^(21/2) + 14346045*(sqrt(b)*x - sqrt(b*x^2 + a))^16*
a^7*b^(21/2) + 4273290*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^8*b^(21/2) + 592
382*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a^9*b^(21/2) + 20349*(sqrt(b)*x - sqr
t(b*x^2 + a))^10*a^10*b^(21/2) - 5985*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^11
*b^(21/2) + 1330*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^12*b^(21/2) - 210*(sqrt
(b)*x - sqrt(b*x^2 + a))^4*a^13*b^(21/2) + 21*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*a^14*b^(21/2) - a^15*b^(21/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
1
```

Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \frac{10\,b^6\,\sqrt{b\,x^2+a}}{138567\,a^2\,x^9}-\frac{1049\,b^4\,\sqrt{b\,x^2+a}}{12597\,x^{13}}-\frac{1898\,a\,b^3\,\sqrt{b\,x^2+a}}{6783\,x^{15}}-\frac{85\,a^3\,b\,\sqrt{b\,x^2+a}}{399\,x^{19}}-\frac{3\,b^5\,\sqrt{b\,x^2+a}}{46189\,a\,x^{11}}-\frac{a^4\,\sqrt{b\,x^2+a}}{21\,x^{21}}-\frac{80\,b^7\,\sqrt{b\,x^2+a}}{969969\,a^3\,x^7}+\frac{32\,b^8\,\sqrt{b\,x^2+a}}{323323\,a^4\,x^5}-\frac{128\,b^9\,\sqrt{b\,x^2+a}}{969969\,a^5\,x^3}+\frac{256\,b^{10}\,\sqrt{b\,x^2+a}}{969969\,a^6\,x}-\frac{820\,a^2\,b^2\,\sqrt{b\,x^2+a}}{2261\,x^{17}}$$

input
```
int((a + b*x^2)^(9/2)/x^22,x)
```

output
```
(10*b^6*(a + b*x^2)^(1/2))/(138567*a^2*x^9) - (1049*b^4*(a + b*x^2)^(1/2))
/(12597*x^13) - (1898*a*b^3*(a + b*x^2)^(1/2))/(6783*x^15) - (85*a^3*b*(a
+ b*x^2)^(1/2))/(399*x^19) - (3*b^5*(a + b*x^2)^(1/2))/(46189*a*x^11) - (a
^4*(a + b*x^2)^(1/2))/(21*x^21) - (80*b^7*(a + b*x^2)^(1/2))/(969969*a^3*x
^7) + (32*b^8*(a + b*x^2)^(1/2))/(323323*a^4*x^5) - (128*b^9*(a + b*x^2)^(
1/2))/(969969*a^5*x^3) + (256*b^10*(a + b*x^2)^(1/2))/(969969*a^6*x) - (82
0*a^2*b^2*(a + b*x^2)^(1/2))/(2261*x^17)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^2)^{9/2}}{x^{22}}\,dx = \frac{-46189\sqrt{b\,x^2+a}\,a^{10} - 206635\sqrt{b\,x^2+a}\,a^9b\,x^2 - 351780\sqrt{b\,x^2+a}\,a^8b^2x^4 - 271414\sqrt{}$$

input
```
int((b*x^2+a)^(9/2)/x^22,x)
```

output
```
( - 46189*sqrt(a + b*x**2)*a**10 - 206635*sqrt(a + b*x**2)*a**9*b*x**2 - 3
51780*sqrt(a + b*x**2)*a**8*b**2*x**4 - 271414*sqrt(a + b*x**2)*a**7*b**3*
x**6 - 80773*sqrt(a + b*x**2)*a**6*b**4*x**8 - 63*sqrt(a + b*x**2)*a**5*b*
*5*x**10 + 70*sqrt(a + b*x**2)*a**4*b**6*x**12 - 80*sqrt(a + b*x**2)*a**3*
b**7*x**14 + 96*sqrt(a + b*x**2)*a**2*b**8*x**16 - 128*sqrt(a + b*x**2)*a*
b**9*x**18 + 256*sqrt(a + b*x**2)*b**10*x**20 - 256*sqrt(b)*b**10*x**21)/(
969969*a**6*x**21)
```

3.450 $\int \frac{(a+bx^2)^{9/2}}{x^{24}} dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}} dx = -\frac{(a+bx^2)^{11/2}}{23ax^{23}} + \frac{4b(a+bx^2)^{11/2}}{161a^2x^{21}} - \frac{40b^2(a+bx^2)^{11/2}}{3059a^3x^{19}} + \frac{320b^3(a+bx^2)^{11/2}}{52003a^4x^{17}} - \frac{128b^4(a+bx^2)^{11/2}}{52003a^5x^{15}} + \frac{512b^5(a+bx^2)^{11/2}}{676039a^6x^{13}} - \frac{1024b^6(a+bx^2)^{11/2}}{7436429a^7x^{11}}$$

output
```
-1/23*(b*x^2+a)^(11/2)/a/x^23+4/161*b*(b*x^2+a)^(11/2)/a^2/x^21-40/3059*b^
2*(b*x^2+a)^(11/2)/a^3/x^19+320/52003*b^3*(b*x^2+a)^(11/2)/a^4/x^17-128/52
003*b^4*(b*x^2+a)^(11/2)/a^5/x^15+512/676039*b^5*(b*x^2+a)^(11/2)/a^6/x^13
-1024/7436429*b^6*(b*x^2+a)^(11/2)/a^7/x^11
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = \frac{(a+bx^2)^{11/2}\,(-323323a^6+184756a^5bx^2-97240a^4b^2x^4+45760a^3b^3x^6-18304a^2b^4x}{7436429a^7x^{23}}$$

input
```
Integrate[(a + b*x^2)^(9/2)/x^24,x]
```

output
```
((a + b*x^2)^(11/2)*(-323323*a^6 + 184756*a^5*b*x^2 - 97240*a^4*b^2*x^4 +
45760*a^3*b^3*x^6 - 18304*a^2*b^4*x^8 + 5632*a*b^5*x^10 - 1024*b^6*x^12))/
(7436429*a^7*x^23)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {245, 245, 245, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}dx$$

$$\Big\downarrow \; 245$$

$$-\frac{12b\int \frac{(bx^2+a)^{9/2}}{x^{22}}dx}{23a}-\frac{(a+bx^2)^{11/2}}{23ax^{23}}$$

$$\Big\downarrow \; 245$$

$$-\frac{12b\left(-\frac{10b\int \frac{(bx^2+a)^{9/2}}{x^{20}}dx}{21a}-\frac{(a+bx^2)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{(a+bx^2)^{11/2}}{23ax^{23}}$$

$$\Big\downarrow \; 245$$

$$-\frac{12b\left(-\frac{10b\left(-\frac{8b\int\frac{\left(bx^2+a\right)^{9/2}}{x^{18}}dx}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

$$\Big\downarrow \ 245$$

$$-\frac{12b\left(-\frac{10b\left(-\frac{8b\left(-\frac{6b\int\frac{\left(bx^2+a\right)^{9/2}}{x^{16}}dx}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

$$\Big\downarrow \ 245$$

$$-\frac{12b\left(-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\int \frac{\left(bx^2+a\right)^{9/2}}{x^{14}}dx}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

$\Big\downarrow$ 245

$$-\frac{12b\left(-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{\left(bx^2+a\right)^{9/2}}{x^{12}}dx}{13a}-\frac{\left(a+bx^2\right)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{12b\left(-\frac{10b\left(-\frac{8b\left(-\frac{6b\left(-\frac{4b\left(\frac{2b\left(a+bx^2\right)^{11/2}}{143a^2x^{11}}-\frac{\left(a+bx^2\right)^{11/2}}{13ax^{13}}\right)}{15a}-\frac{\left(a+bx^2\right)^{11/2}}{15ax^{15}}\right)}{17a}-\frac{\left(a+bx^2\right)^{11/2}}{17ax^{17}}\right)}{19a}-\frac{\left(a+bx^2\right)^{11/2}}{19ax^{19}}\right)}{21a}-\frac{\left(a+bx^2\right)^{11/2}}{21ax^{21}}\right)}{23a}-\frac{\left(a+bx^2\right)^{11/2}}{23ax^{23}}$$

input
```
Int[(a + b*x^2)^(9/2)/x^24,x]
```

output
```
-1/23*(a + b*x^2)^(11/2)/(a*x^23) - (12*b*(-1/21*(a + b*x^2)^(11/2)/(a*x^2
1) - (10*b*(-1/19*(a + b*x^2)^(11/2)/(a*x^19) - (8*b*(-1/17*(a + b*x^2)^(1
1/2)/(a*x^17) - (6*b*(-1/15*(a + b*x^2)^(11/2)/(a*x^15) - (4*b*(-1/13*(a +
 b*x^2)^(11/2)/(a*x^13) + (2*b*(a + b*x^2)^(11/2))/(143*a^2*x^11)))/(15*a)
))/(17*a)))/(19*a)))/(21*a)))/(23*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 59.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.51

method	result
gosper	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}\left(1024b^{6}x^{12}-5632a\,b^{5}x^{10}+18304a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+97240a^{4}b^{2}x^{4}-184756a^{5}b\,x^{2}+323323a^{6}\right)}{7436429x^{23}a^{7}}$
pseudoelliptic	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}\left(1024b^{6}x^{12}-5632a\,b^{5}x^{10}+18304a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+97240a^{4}b^{2}x^{4}-184756a^{5}b\,x^{2}+323323a^{6}\right)}{7436429x^{23}a^{7}}$
orering	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}\left(1024b^{6}x^{12}-5632a\,b^{5}x^{10}+18304a^{2}b^{4}x^{8}-45760a^{3}b^{3}x^{6}+97240a^{4}b^{2}x^{4}-184756a^{5}b\,x^{2}+323323a^{6}\right)}{7436429x^{23}a^{7}}$
trager	$-\frac{(1024b^{11}x^{22}-512a\,b^{10}x^{20}+384a^{2}b^{9}x^{18}-320a^{3}b^{8}x^{16}+280a^{4}b^{7}x^{14}-252a^{5}b^{6}x^{12}+231a^{6}b^{5}x^{10}+530959a^{7}b^{4}x^{8}+182611\cdots}{7436429a^{7}x^{23}}$
risch	$-\frac{(1024b^{11}x^{22}-512a\,b^{10}x^{20}+384a^{2}b^{9}x^{18}-320a^{3}b^{8}x^{16}+280a^{4}b^{7}x^{14}-252a^{5}b^{6}x^{12}+231a^{6}b^{5}x^{10}+530959a^{7}b^{4}x^{8}+182611\cdots}{7436429a^{7}x^{23}}$
default	$-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{23a\,x^{23}}-\frac{12b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{21a\,x^{21}}-\frac{10b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{19a\,x^{19}}-\frac{8b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{17a\,x^{17}}-\frac{6b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{15a\,x^{15}}-\frac{4b\left(-\frac{(b\,x^{2}+a)^{\frac{11}{2}}}{13a\,x^{13}}+\frac{2b(\cdots}{\cdots}\right)}{15a}\right)}{17a}\right)}{19a}\right)}{21a}\right)}{23a}$

input
```
int((b*x^2+a)^(9/2)/x^24,x,method=_RETURNVERBOSE)
```

output
```
-1/7436429*(b*x^2+a)^(11/2)*(1024*b^6*x^12-5632*a*b^5*x^10+18304*a^2*b^4*x
^8-45760*a^3*b^3*x^6+97240*a^4*b^2*x^4-184756*a^5*b*x^2+323323*a^6)/x^23/a
^7
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx =$$
$$-\frac{(1024\,b^{11}x^{22} - 512\,ab^{10}x^{20} + 384\,a^2b^9x^{18} - 320\,a^3b^8x^{16} + 280\,a^4b^7x^{14} - 252\,a^5b^6x^{12} + 231\,a^6b^5x^{10} + 5309}{7436429\,a^7x^{23}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="fricas")
```

output
```
-1/7436429*(1024*b^11*x^22 - 512*a*b^10*x^20 + 384*a^2*b^9*x^18 - 320*a^3*
b^8*x^16 + 280*a^4*b^7*x^14 - 252*a^5*b^6*x^12 + 231*a^6*b^5*x^10 + 530959
*a^7*b^4*x^8 + 1826110*a^8*b^3*x^6 + 2406690*a^9*b^2*x^4 + 1431859*a^10*b*
x^2 + 323323*a^11)*sqrt(b*x^2 + a)/(a^7*x^23)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1950 vs. 2(156) = 312.

Time = 4.66 (sec) , antiderivative size = 1950, normalized size of antiderivative = 11.89

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = \text{Too large to display}$$

input
```
integrate((b*x**2+a)**(9/2)/x**24,x)
```

output
```
-323323*a**17*b**(73/2)*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 +
44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10
*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 74
36429*a**7*b**42*x**34) - 3371797*a**16*b**(75/2)*x**2*sqrt(a/(b*x**2) + 1
)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**1
1*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 +
 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 15847689*a**15*b*
*(77/2)*x**4*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a*
*12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**2
8 + 111546435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*
b**42*x**34) - 44210595*a**14*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(7436429
*a**13*b**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x*
*26 + 148728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*
a**8*b**41*x**32 + 7436429*a**7*b**42*x**34) - 81074994*a**13*b**(81/2)*x*
*8*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b**36*x**22 + 44618574*a**12*b**37*
x**24 + 111546435*a**11*b**38*x**26 + 148728580*a**10*b**39*x**28 + 111546
435*a**9*b**40*x**30 + 44618574*a**8*b**41*x**32 + 7436429*a**7*b**42*x**3
4) - 102129258*a**12*b**(83/2)*x**10*sqrt(a/(b*x**2) + 1)/(7436429*a**13*b
**36*x**22 + 44618574*a**12*b**37*x**24 + 111546435*a**11*b**38*x**26 + 14
8728580*a**10*b**39*x**28 + 111546435*a**9*b**40*x**30 + 44618574*a**8*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = -\frac{1024\,(bx^2+a)^{\frac{11}{2}}b^6}{7436429\,a^7x^{11}} + \frac{512\,(bx^2+a)^{\frac{11}{2}}b^5}{676039\,a^6x^{13}} - \frac{128\,(bx^2+a)^{\frac{11}{2}}b^4}{52003\,a^5x^{15}} + \frac{320\,(bx^2+a)^{\frac{11}{2}}b^3}{52003\,a^4x^{17}} - \frac{40\,(bx^2+a)^{\frac{11}{2}}b^2}{3059\,a^3x^{19}} + \frac{4\,(bx^2+a)^{\frac{11}{2}}b}{161\,a^2x^{21}} - \frac{(bx^2+a)^{\frac{11}{2}}}{23\,ax^{23}}$$

input
```
integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="maxima")
```

output
```
-1024/7436429*(b*x^2 + a)^(11/2)*b^6/(a^7*x^11) + 512/676039*(b*x^2 + a)^(
11/2)*b^5/(a^6*x^13) - 128/52003*(b*x^2 + a)^(11/2)*b^4/(a^5*x^15) + 320/5
2003*(b*x^2 + a)^(11/2)*b^3/(a^4*x^17) - 40/3059*(b*x^2 + a)^(11/2)*b^2/(a
^3*x^19) + 4/161*(b*x^2 + a)^(11/2)*b/(a^2*x^21) - 1/23*(b*x^2 + a)^(11/2)
/(a*x^23)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(136) = 272$.

Time = 0.14 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = \frac{2048\left(4249388\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{32}b^{\frac{23}{2}}+28683369\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{30}ab^{\frac{23}{2}}+100\right.}{}$$

input
```
integrate((b*x^2+a)^(9/2)/x^24,x, algorithm="giac")
```

output
```
2048/7436429*(4249388*(sqrt(b)*x - sqrt(b*x^2 + a))^32*b^(23/2) + 28683369
*(sqrt(b)*x - sqrt(b*x^2 + a))^30*a*b^(23/2) + 100922965*(sqrt(b)*x - sqrt
(b*x^2 + a))^28*a^2*b^(23/2) + 215656441*(sqrt(b)*x - sqrt(b*x^2 + a))^26*
a^3*b^(23/2) + 313006057*(sqrt(b)*x - sqrt(b*x^2 + a))^24*a^4*b^(23/2) + 3
11653979*(sqrt(b)*x - sqrt(b*x^2 + a))^22*a^5*b^(23/2) + 216800507*(sqrt(b
)*x - sqrt(b*x^2 + a))^20*a^6*b^(23/2) + 100105775*(sqrt(b)*x - sqrt(b*x^2
 + a))^18*a^7*b^(23/2) + 29173683*(sqrt(b)*x - sqrt(b*x^2 + a))^16*a^8*b^(
23/2) + 4004231*(sqrt(b)*x - sqrt(b*x^2 + a))^14*a^9*b^(23/2) + 100947*(sq
rt(b)*x - sqrt(b*x^2 + a))^12*a^10*b^(23/2) - 33649*(sqrt(b)*x - sqrt(b*x^
2 + a))^10*a^11*b^(23/2) + 8855*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^12*b^(23
/2) - 1771*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^13*b^(23/2) + 253*(sqrt(b)*x
- sqrt(b*x^2 + a))^4*a^14*b^(23/2) - 23*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^
15*b^(23/2) + a^16*b^(23/2))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^23
```

Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.41

$$\begin{aligned}\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx &= \frac{36\,b^6\,\sqrt{b\,x^2+a}}{1062347\,a^2\,x^{11}} - \frac{3713\,b^4\,\sqrt{b\,x^2+a}}{52003\,x^{15}} \\ &- \frac{12770\,a\,b^3\,\sqrt{b\,x^2+a}}{52003\,x^{17}} - \frac{31\,a^3\,b\,\sqrt{b\,x^2+a}}{161\,x^{21}} - \frac{3\,b^5\,\sqrt{b\,x^2+a}}{96577\,a\,x^{13}} \\ &- \frac{a^4\,\sqrt{b\,x^2+a}}{23\,x^{23}} - \frac{40\,b^7\,\sqrt{b\,x^2+a}}{1062347\,a^3\,x^9} + \frac{320\,b^8\,\sqrt{b\,x^2+a}}{7436429\,a^4\,x^7} - \frac{384\,b^9\,\sqrt{b\,x^2+a}}{7436429\,a^5\,x^5} \\ &+ \frac{512\,b^{10}\,\sqrt{b\,x^2+a}}{7436429\,a^6\,x^3} - \frac{1024\,b^{11}\,\sqrt{b\,x^2+a}}{7436429\,a^7\,x} - \frac{990\,a^2\,b^2\,\sqrt{b\,x^2+a}}{3059\,x^{19}}\end{aligned}$$

input
```
int((a + b*x^2)^(9/2)/x^24,x)
```

output
```
(36*b^6*(a + b*x^2)^(1/2))/(1062347*a^2*x^11) - (3713*b^4*(a + b*x^2)^(1/2
))/(52003*x^15) - (12770*a*b^3*(a + b*x^2)^(1/2))/(52003*x^17) - (31*a^3*b
*(a + b*x^2)^(1/2))/(161*x^21) - (3*b^5*(a + b*x^2)^(1/2))/(96577*a*x^13)
- (a^4*(a + b*x^2)^(1/2))/(23*x^23) - (40*b^7*(a + b*x^2)^(1/2))/(1062347*
a^3*x^9) + (320*b^8*(a + b*x^2)^(1/2))/(7436429*a^4*x^7) - (384*b^9*(a + b
*x^2)^(1/2))/(7436429*a^5*x^5) + (512*b^10*(a + b*x^2)^(1/2))/(7436429*a^6
*x^3) - (1024*b^11*(a + b*x^2)^(1/2))/(7436429*a^7*x) - (990*a^2*b^2*(a +
b*x^2)^(1/2))/(3059*x^19)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)^{9/2}}{x^{24}}\,dx = \frac{-323323\sqrt{b\,x^2+a}\,a^{11} - 1431859\sqrt{b\,x^2+a}\,a^{10}b\,x^2 - 2406690\sqrt{b\,x^2+a}\,a^9b^2x^4 - 182}{}$$

input
```
int((b*x^2+a)^(9/2)/x^24,x)
```

output
```
( - 323323*sqrt(a + b*x**2)*a**11 - 1431859*sqrt(a + b*x**2)*a**10*b*x**2
- 2406690*sqrt(a + b*x**2)*a**9*b**2*x**4 - 1826110*sqrt(a + b*x**2)*a**8*
b**3*x**6 - 530959*sqrt(a + b*x**2)*a**7*b**4*x**8 - 231*sqrt(a + b*x**2)*
a**6*b**5*x**10 + 252*sqrt(a + b*x**2)*a**5*b**6*x**12 - 280*sqrt(a + b*x*
*2)*a**4*b**7*x**14 + 320*sqrt(a + b*x**2)*a**3*b**8*x**16 - 384*sqrt(a +
b*x**2)*a**2*b**9*x**18 + 512*sqrt(a + b*x**2)*a*b**10*x**20 - 1024*sqrt(a
 + b*x**2)*b**11*x**22 + 1024*sqrt(b)*b**11*x**23)/(7436429*a**7*x**23)
```

3.451 $\int x^5\sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x^5\sqrt{9+4x^2}\,dx=\frac{27}{64}\left(9+4x^2\right)^{3/2}-\frac{9}{160}\left(9+4x^2\right)^{5/2}+\frac{1}{448}\left(9+4x^2\right)^{7/2}$$

output
```
27/64*(4*x^2+9)^(3/2)-9/160*(4*x^2+9)^(5/2)+1/448*(4*x^2+9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{9+4x^2}\,dx=\frac{1}{280}\left(9+4x^2\right)^{3/2}\left(27-18x^2+10x^4\right)$$

input
```
Integrate[x^5*Sqrt[9 + 4*x^2],x]
```

output
```
((9 + 4*x^2)^(3/2)*(27 - 18*x^2 + 10*x^4))/280
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4\sqrt{4x^2+9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{16}\left(4x^2+9\right)^{5/2}-\frac{9}{8}\left(4x^2+9\right)^{3/2}+\frac{81}{16}\sqrt{4x^2+9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{224}\left(4x^2+9\right)^{7/2}-\frac{9}{80}\left(4x^2+9\right)^{5/2}+\frac{27}{32}\left(4x^2+9\right)^{3/2}\right)$$

input
```
Int[x^5*Sqrt[9 + 4*x^2],x]
```

output
```
((27*(9 + 4*x^2)^(3/2))/32 - (9*(9 + 4*x^2)^(5/2))/80 + (9 + 4*x^2)^(7/2)/
224)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{(4x^2+9)^{\frac{3}{2}}(10x^4-18x^2+27)}{280}$	24
pseudoelliptic	$\frac{(4x^2+9)^{\frac{3}{2}}(10x^4-18x^2+27)}{280}$	24
orering	$\frac{(4x^2+9)^{\frac{3}{2}}(10x^4-18x^2+27)}{280}$	24
trager	$\left(\frac{1}{7}x^6+\frac{9}{140}x^4-\frac{27}{140}x^2+\frac{243}{280}\right)\sqrt{4x^2+9}$	28
risch	$\frac{(40x^6+18x^4-54x^2+243)\sqrt{4x^2+9}}{280}$	29
meijerg	$-\frac{2187\left(\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(\frac{4x^2}{9}+1\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4-\frac{16}{3}x^2+8\right)}{105}\right)}{256\sqrt{\pi}}$	38
default	$\frac{x^4(4x^2+9)^{\frac{3}{2}}}{28}-\frac{9x^2(4x^2+9)^{\frac{3}{2}}}{140}+\frac{27(4x^2+9)^{\frac{3}{2}}}{280}$	41

input
```
int(x^5*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*(4*x^2+9)^(3/2)*(10*x^4-18*x^2+27)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{9+4x^2}\,dx = \frac{1}{280}\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{4\,x^2+9}$$

input
```
integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^5\sqrt{9+4x^2}\,dx = \frac{x^6\sqrt{4x^2+9}}{7}+\frac{9x^4\sqrt{4x^2+9}}{140}-\frac{27x^2\sqrt{4x^2+9}}{140}+\frac{243\sqrt{4x^2+9}}{280}$$

input
```
integrate(x**5*(4*x**2+9)**(1/2),x)
```

output
```
x**6*sqrt(4*x**2 + 9)/7 + 9*x**4*sqrt(4*x**2 + 9)/140 - 27*x**2*sqrt(4*x**
2 + 9)/140 + 243*sqrt(4*x**2 + 9)/280
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x^5\sqrt{9+4x^2}\,dx = \frac{1}{28}\left(4\,x^2+9\right)^{\frac{3}{2}}x^4-\frac{9}{140}\left(4\,x^2+9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/28*(4*x^2 + 9)^(3/2)*x^4 - 9/140*(4*x^2 + 9)^(3/2)*x^2 + 27/280*(4*x^2 +
 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{9+4x^2}\,dx = \frac{1}{448}\left(4\,x^2+9\right)^{\frac{7}{2}} - \frac{9}{160}\left(4\,x^2+9\right)^{\frac{5}{2}} + \frac{27}{64}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/448*(4*x^2 + 9)^(7/2) - 9/160*(4*x^2 + 9)^(5/2) + 27/64*(4*x^2 + 9)^(3/2
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.54

$$\int x^5\sqrt{9+4x^2}\,dx = \sqrt{x^2+\frac{9}{4}}\left(\frac{2\,x^6}{7}+\frac{9\,x^4}{70}-\frac{27\,x^2}{70}+\frac{243}{140}\right)$$

input
```
int(x^5*(4*x^2 + 9)^(1/2),x)
```

output
```
(x^2 + 9/4)^(1/2)*((9*x^4)/70 - (27*x^2)/70 + (2*x^6)/7 + 243/140)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,(40x^6+18x^4-54x^2+243)}{280}$$

input
```
int(x^5*(4*x^2+9)^(1/2),x)
```

output
```
(sqrt(4*x**2 + 9)*(40*x**6 + 18*x**4 - 54*x**2 + 243))/280
```

3.452 $\int x^4\sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int x^4\sqrt{9+4x^2}\,dx=-\frac{81}{256}x\sqrt{9+4x^2}+\frac{3}{32}x^3\sqrt{9+4x^2}+\frac{1}{6}x^5\sqrt{9+4x^2}+\frac{729}{512}\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
-81/256*x*(4*x^2+9)^(1/2)+3/32*x^3*(4*x^2+9)^(1/2)+1/6*x^5*(4*x^2+9)^(1/2)
+729/512*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

$$\int x^4\sqrt{9+4x^2}\,dx=\frac{1}{768}x\sqrt{9+4x^2}\left(-243+72x^2+128x^4\right)-\frac{729}{512}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[x^4*Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*Log[-2*x + Sqrt[9
 + 4*x^2]])/512
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \; 248$$

$$\frac{3}{2}\int \frac{x^4}{\sqrt{4x^2+9}}dx+\frac{1}{6}\sqrt{4x^2+9}x^5$$

$$\Big\downarrow \; 262$$

$$\frac{3}{2}\left(\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\int \frac{x^2}{\sqrt{4x^2+9}}dx\right)+\frac{1}{6}\sqrt{4x^2+9}x^5$$

$$\Big\downarrow \; 262$$

$$\frac{3}{2}\left(\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{8}\int \frac{1}{\sqrt{4x^2+9}}dx\right)\right)+\frac{1}{6}\sqrt{4x^2+9}x^5$$

$$\Big\downarrow \; 222$$

$$\frac{3}{2}\left(\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\text{arcsinh}\left(\frac{2x}{3}\right)\right)\right)+\frac{1}{6}\sqrt{4x^2+9}x^5$$

input
```
Int[x^4*Sqrt[9 + 4*x^2],x]
```

output
```
(x^5*Sqrt[9 + 4*x^2])/6 + (3*((x^3*Sqrt[9 + 4*x^2])/16 - (27*((x*Sqrt[9 +
4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16))/16))/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{x\left(128x^4+72x^2-243\right)\sqrt{4x^2+9}}{768}+\frac{729\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{512}$	32
meijerg	$-\frac{729\left(\frac{\sqrt{\pi}\,x\left(-\frac{640}{81}x^4-\frac{40}{9}x^2+15\right)\sqrt{\frac{4x^2}{9}+1}}{90}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	43
trager	$\frac{x\left(128x^4+72x^2-243\right)\sqrt{4x^2+9}}{768}-\frac{729\ln\left(-\sqrt{4x^2+9}+2x\right)}{512}$	44
default	$\frac{x^3\left(4x^2+9\right)^{\frac{3}{2}}}{24}-\frac{9x\left(4x^2+9\right)^{\frac{3}{2}}}{128}+\frac{81x\sqrt{4x^2+9}}{256}+\frac{729\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{512}$	46
pseudoelliptic	$\frac{-531441\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)+531441\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)+\left(-124416x^5-69984x^3+236196x\right)\sqrt{4x^2+9}}{1024\left(2x+\sqrt{4x^2+9}\right)^3\left(-\sqrt{4x^2+9}+2x\right)^3}$	100

input
```
int(x^4*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/768*x*(128*x^4+72*x^2-243)*(4*x^2+9)^(1/2)+729/512*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.67

$$\int x^4\sqrt{9+4x^2}\,dx = \frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{4\,x^2+9}-\frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(4*x^2 + 9) - 729/512*log(-2*x + sqrt
(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 4.92 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int x^4\sqrt{9+4x^2}\,dx = \frac{2x^7}{3\sqrt{4x^2+9}}+\frac{15x^5}{8\sqrt{4x^2+9}}-\frac{27x^3}{64\sqrt{4x^2+9}}-\frac{729x}{256\sqrt{4x^2+9}}+\frac{729\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

input
```
integrate(x**4*(4*x**2+9)**(1/2),x)
```

output
```
2*x**7/(3*sqrt(4*x**2 + 9)) + 15*x**5/(8*sqrt(4*x**2 + 9)) - 27*x**3/(64*s
qrt(4*x**2 + 9)) - 729*x/(256*sqrt(4*x**2 + 9)) + 729*asinh(2*x/3)/512
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{9+4x^2}\,dx = \frac{1}{24}\left(4\,x^2+9\right)^{\frac{3}{2}}x^3 - \frac{9}{128}\left(4\,x^2+9\right)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{4\,x^2+9}x + \frac{729}{512}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(4*x^2 + 9)^(3/2)*x^3 - 9/128*(4*x^2 + 9)^(3/2)*x + 81/256*sqrt(4*x^2
 + 9)*x + 729/512*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{9+4x^2}\,dx = \frac{1}{768}\left(8\left(16\,x^2+9\right)x^2-243\right)\sqrt{4\,x^2+9}x - \frac{729}{512}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^4*(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/768*(8*(16*x^2 + 9)*x^2 - 243)*sqrt(4*x^2 + 9)*x - 729/512*log(-2*x + sq
rt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.48

$$\int x^4\sqrt{9+4x^2}\,dx=\frac{729\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{512}+\frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2\,x^5}{3}+\frac{3\,x^3}{8}-\frac{81\,x}{64}\right)}{2}$$

input
```
int(x^4*(4*x^2 + 9)^(1/2),x)
```

output
```
(729*asinh((2*x)/3))/512 + ((x^2 + 9/4)^(1/2)*((3*x^3)/8 - (81*x)/64 + (2*
x^5)/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^4\sqrt{9+4x^2}\,dx$$
$$=\frac{\sqrt{4x^2+9}\,x^5}{6}+\frac{3\sqrt{4x^2+9}\,x^3}{32}-\frac{81\sqrt{4x^2+9}\,x}{256}+\frac{729\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{512}$$

input
```
int(x^4*(4*x^2+9)^(1/2),x)
```

output
```
(256*sqrt(4*x**2 + 9)*x**5 + 144*sqrt(4*x**2 + 9)*x**3 - 486*sqrt(4*x**2 +
 9)*x + 2187*log((sqrt(4*x**2 + 9) + 2*x)/3))/1536
```

3.453 $\int x^3\sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{9+4x^2}\,dx = -\frac{3}{16}\left(9+4x^2\right)^{3/2}+\frac{1}{80}\left(9+4x^2\right)^{5/2}$$

output
```
-3/16*(4*x^2+9)^(3/2)+1/80*(4*x^2+9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{9+4x^2}\,dx = \frac{1}{40}\left(-3+2x^2\right)\left(9+4x^2\right)^{3/2}$$

input
```
Integrate[x^3*Sqrt[9 + 4*x^2],x]
```

output
```
((-3 + 2*x^2)*(9 + 4*x^2)^(3/2))/40
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2\sqrt{4x^2+9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{4}\left(4x^2+9\right)^{3/2}-\frac{9}{4}\sqrt{4x^2+9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{40}\left(4x^2+9\right)^{5/2}-\frac{3}{8}\left(4x^2+9\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[9 + 4*x^2],x]
```

output
```
((-3*(9 + 4*x^2)^(3/2))/8 + (9 + 4*x^2)^(5/2)/40)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{(4x^2+9)^{\frac{3}{2}}(2x^2-3)}{40}$	19
pseudoelliptic	$\frac{(4x^2+9)^{\frac{3}{2}}(2x^2-3)}{40}$	19
orering	$\frac{(4x^2+9)^{\frac{3}{2}}(2x^2-3)}{40}$	19
trager	$\left(\frac{1}{5}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{4x^2+9}$	23
risch	$\frac{(8x^4+6x^2-27)\sqrt{4x^2+9}}{40}$	24
default	$\frac{x^2(4x^2+9)^{\frac{3}{2}}}{20}-\frac{3(4x^2+9)^{\frac{3}{2}}}{40}$	27
meijerg	$-\frac{243\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{4x^2}{9}+1\right)^{\frac{3}{2}}\left(-\frac{4x^2}{3}+2\right)}{15}\right)}{64\sqrt{\pi}}$	33

input
```
int(x^3*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(4*x^2+9)^(3/2)*(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{9+4x^2}\,dx=\frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{4\,x^2+9}$$

input
```
integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(8*x^4 + 6*x^2 - 27)*sqrt(4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int x^3\sqrt{9+4x^2}\,dx=\frac{x^4\sqrt{4x^2+9}}{5}+\frac{3x^2\sqrt{4x^2+9}}{20}-\frac{27\sqrt{4x^2+9}}{40}$$

input
```
integrate(x**3*(4*x**2+9)**(1/2),x)
```

output
```
x**4*sqrt(4*x**2 + 9)/5 + 3*x**2*sqrt(4*x**2 + 9)/20 - 27*sqrt(4*x**2 + 9)
/40
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{9+4x^2}\,dx=\frac{1}{20}\left(4\,x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/20*(4*x^2 + 9)^(3/2)*x^2 - 3/40*(4*x^2 + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{9+4x^2}\,dx = \frac{1}{80}\left(4\,x^2+9\right)^{\frac{5}{2}} - \frac{3}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/80*(4*x^2 + 9)^(5/2) - 3/16*(4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

$$\int x^3\sqrt{9+4x^2}\,dx = \sqrt{x^2+\frac{9}{4}}\left(\frac{2\,x^4}{5}+\frac{3\,x^2}{10}-\frac{27}{20}\right)$$

input
```
int(x^3*(4*x^2 + 9)^(1/2),x)
```

output
```
(x^2 + 9/4)^(1/2)*((3*x^2)/10 + (2*x^4)/5 - 27/20)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,(8x^4+6x^2-27)}{40}$$

input
```
int(x^3*(4*x^2+9)^(1/2),x)
```

output
```
(sqrt(4*x**2 + 9)*(8*x**4 + 6*x**2 - 27))/40
```

3.454 $\int x^2\sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int x^2\sqrt{9+4x^2}\,dx=\frac{9}{32}x\sqrt{9+4x^2}+\frac{1}{4}x^3\sqrt{9+4x^2}-\frac{81}{64}\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
9/32*x*(4*x^2+9)^(1/2)+1/4*x^3*(4*x^2+9)^(1/2)-81/64*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int x^2\sqrt{9+4x^2}\,dx=\frac{1}{32}x\sqrt{9+4x^2}(9+8x^2)+\frac{81}{64}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[x^2*Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2]*(9 + 8*x^2))/32 + (81*Log[-2*x + Sqrt[9 + 4*x^2]])/64
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {248, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{4}\int \frac{x^2}{\sqrt{4x^2+9}}dx+\frac{1}{4}\sqrt{4x^2+9}x^3$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{4}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{8}\int \frac{1}{\sqrt{4x^2+9}}dx\right)+\frac{1}{4}\sqrt{4x^2+9}x^3$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{4}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\text{arcsinh}\left(\frac{2x}{3}\right)\right)+\frac{1}{4}\sqrt{4x^2+9}x^3$$

input
```
Int[x^2*Sqrt[9 + 4*x^2],x]
```

output
```
(x^3*Sqrt[9 + 4*x^2])/4 + (9*((x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])
/16))/4
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 248

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

method	result	size
risch	$\frac{x\left(8x^2+9\right)\sqrt{4x^2+9}}{32}-\frac{81\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{64}$	27
default	$\frac{x\left(4x^2+9\right)^{\frac{3}{2}}}{16}-\frac{9x\sqrt{4x^2+9}}{32}-\frac{81\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{64}$	32
meijerg	$-\frac{81\left(-\frac{\sqrt{\pi}\,x\left(\frac{8x^2}{3}+3\right)\sqrt{\frac{4x^2}{9}+1}}{9}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}}$	38
trager	$\frac{x\left(8x^2+9\right)\sqrt{4x^2+9}}{32}+\frac{81\ln\left(-\sqrt{4x^2+9}+2x\right)}{64}$	39
pseudoelliptic	$\frac{-6561\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)+6561\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)+\left(2592x^3+2916x\right)\sqrt{4x^2+9}}{128\left(2x+\sqrt{4x^2+9}\right)^2\left(-\sqrt{4x^2+9}+2x\right)^2}$	95

input

```
int(x^2*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/32*x*(8*x^2+9)*(4*x^2+9)^(1/2)-81/64*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{1}{32}\left(8\,x^3+9\,x\right)\sqrt{4\,x^2+9}+\frac{81}{64}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 + 9*x)*sqrt(4*x^2 + 9) + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.20

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{x^5}{\sqrt{4x^2+9}}+\frac{27x^3}{8\sqrt{4x^2+9}}+\frac{81x}{32\sqrt{4x^2+9}}-\frac{81\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{64}$$

input
```
integrate(x**2*(4*x**2+9)**(1/2),x)
```

output
```
x**5/sqrt(4*x**2 + 9) + 27*x**3/(8*sqrt(4*x**2 + 9)) + 81*x/(32*sqrt(4*x**
2 + 9)) - 81*asinh(2*x/3)/64
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{1}{16}\left(4\,x^2+9\right)^{\frac{3}{2}}x-\frac{9}{32}\,\sqrt{4\,x^2+9}x-\frac{81}{64}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*x^2 + 9)^(3/2)*x - 9/32*sqrt(4*x^2 + 9)*x - 81/64*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{1}{32}\left(8\,x^2+9\right)\sqrt{4\,x^2+9}x + \frac{81}{64}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^2*(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/32*(8*x^2 + 9)*sqrt(4*x^2 + 9)*x + 81/64*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.51

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{\left(x^3+\frac{9\,x}{8}\right)\,\sqrt{x^2+\frac{9}{4}}}{2} - \frac{81\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{64}$$

input
```
int(x^2*(4*x^2 + 9)^(1/2),x)
```

output
```
(((9*x)/8 + x^3)*(x^2 + 9/4)^(1/2))/2 - (81*asinh((2*x)/3))/64
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2\sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,x^3}{4} + \frac{9\sqrt{4x^2+9}\,x}{32} - \frac{81\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{64}$$

input
```
int(x^2*(4*x^2+9)^(1/2),x)
```

output
```
(16*sqrt(4*x**2 + 9)*x**3 + 18*sqrt(4*x**2 + 9)*x - 81*log((sqrt(4*x**2 +
9) + 2*x)/3))/64
```

3.455 $\int x\sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

output
```
1/12*(4*x^2+9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(9+4x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[9 + 4*x^2],x]
```

output
```
(9 + 4*x^2)^(3/2)/12
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \; 241$$

$$\frac{1}{12}\left(4x^2+9\right)^{3/2}$$

input
```
Int[x*Sqrt[9 + 4*x^2],x]
```

output
```
(9 + 4*x^2)^(3/2)/12
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
derivativedivides	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
default	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
risch	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
pseudoelliptic	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
orering	$\frac{\left(4x^2+9\right)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}+\frac{3}{4}\right)\sqrt{4x^2+9}$	18
meijerg	$-\frac{27\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{8x^2}{9}\right)\sqrt{\frac{4x^2}{9}+1}}{3}\right)}{16\sqrt{\pi}}$	33

input
```
int(x*(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*x^2+9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{9+4x^2}\,dx=\frac{1}{12}\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(4*x^2 + 9)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x\sqrt{9+4x^2}\,dx = \frac{x^2\sqrt{4x^2+9}}{3} + \frac{3\sqrt{4x^2+9}}{4}$$

input
```
integrate(x*(4*x**2+9)**(1/2),x)
```

output
```
x**2*sqrt(4*x**2 + 9)/3 + 3*sqrt(4*x**2 + 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{9+4x^2}\,dx = \frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(4*x^2 + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{9+4x^2}\,dx = \frac{1}{12}\left(4x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/12*(4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int x\sqrt{9+4x^2}\,dx = \frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{4\,x^2}{3}+3\right)}{2}$$

input
```
int(x*(4*x^2 + 9)^(1/2),x)
```

output
```
((x^2 + 9/4)^(1/2)*((4*x^2)/3 + 3))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x\sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,(4x^2+9)}{12}$$

input
```
int(x*(4*x^2+9)^(1/2),x)
```

output
```
(sqrt(4*x**2 + 9)*(4*x**2 + 9))/12
```

3.456 $\int \sqrt{9+4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}x\sqrt{9+4x^2}+\frac{9}{4}\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/2*x*(4*x^2+9)^(1/2)+9/4*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}x\sqrt{9+4x^2}-\frac{9}{4}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/2 - (9*Log[-2*x + Sqrt[9 + 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {211, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2+9}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{4x^2+9}}dx+\frac{1}{2}\sqrt{4x^2+9}x$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{4}\text{arcsinh}\left(\frac{2x}{3}\right)+\frac{1}{2}\sqrt{4x^2+9}x$$

input
```
Int[Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/2 + (9*ArcSinh[(2*x)/3])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}$	20
risch	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}$	20
trager	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\ln\left(2x+\sqrt{4x^2+9}\right)}{4}$	30
meijerg	$-\frac{9\left(-\frac{4\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{3}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	31
pseudoelliptic	$\frac{x\sqrt{4x^2+9}}{2}+\frac{9\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)}{8}-\frac{9\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{8}$	54

input
```
int((4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(4*x^2+9)^(1/2)+9/4*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sqrt{9+4x^2}\,dx=\frac{1}{2}\sqrt{4\,x^2+9}x-\frac{9}{4}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sqrt{9+4x^2}\,dx = \frac{x\sqrt{4x^2+9}}{2} + \frac{9\operatorname{asinh}\left(\frac{2x}{3}\right)}{4}$$

input
```
integrate((4*x**2+9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 + 9)/2 + 9*asinh(2*x/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2+9}x + \frac{9}{4}\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 + 9)*x + 9/4*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sqrt{9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2+9}x - \frac{9}{4}\log\left(-2x+\sqrt{4x^2+9}\right)$$

input
```
integrate((4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \sqrt{9+4x^2}\,dx = \frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{4} + x\,\sqrt{x^2+\frac{9}{4}}$$

input
```
int((4*x^2 + 9)^(1/2),x)
```

output
```
(9*asinh((2*x)/3))/4 + x*(x^2 + 9/4)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \sqrt{9+4x^2}\,dx = \frac{\sqrt{4x^2+9}\,x}{2} + \frac{9\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{4}$$

input
```
int((4*x^2+9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 + 9)*x + 9*log((sqrt(4*x**2 + 9) + 2*x)/3))/4
```

3.457 $\int \frac{\sqrt{9+4x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{9+4x^2}}{x} \, dx = \sqrt{9+4x^2} - 3\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
(4*x^2+9)^(1/2)-3*arctanh(1/3*(4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9+4x^2}}{x} \, dx = \sqrt{9+4x^2} - 3\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2]/x,x]
```

output
```
Sqrt[9 + 4*x^2] - 3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+9}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2+9}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(9\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2+2\sqrt{4x^2+9}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{9}{2}\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}+2\sqrt{4x^2+9}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(2\sqrt{4x^2+9}-6\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)\right)$$

input
```
Int[Sqrt[9 + 4*x^2]/x,x]
```

output
```
(2*Sqrt[9 + 4*x^2] - 6*ArcTanh[Sqrt[9 + 4*x^2]/3])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{4x^2+9}-3\,\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)$	25
trager	$\sqrt{4x^2+9}+3\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)$	29
pseudoelliptic	$\sqrt{4x^2+9}-\frac{3\ln\left(\sqrt{4x^2+9}+3\right)}{2}+\frac{3\ln\left(\sqrt{4x^2+9}-3\right)}{2}$	39
meijerg	$-\frac{3\left(-2(2+2\ln(x)-2\ln(3))\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	60

input
```
int((4*x^2+9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(4*x^2+9)^(1/2)-3*arctanh(3/(4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \sqrt{4\,x^2+9} - 3\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right) + 3\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(4*x^2 + 9) - 3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 3*log(-2*x + sqrt(4*
x^2 + 9) - 3)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \frac{2x}{\sqrt{1+\frac{9}{4x^2}}} - 3\operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9}{2x\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate((4*x**2+9)**(1/2)/x,x)
```

output
```
2*x/sqrt(1 + 9/(4*x**2)) - 3*asinh(3/(2*x)) + 9/(2*x*sqrt(1 + 9/(4*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \sqrt{4\,x^2+9} - 3\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(4*x^2 + 9) - 3*arcsinh(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \sqrt{4\,x^2+9} - \frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}+3\right) + \frac{3}{2}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(4*x^2 + 9) - 3/2*log(sqrt(4*x^2 + 9) + 3) + 3/2*log(sqrt(4*x^2 + 9) -
 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = 2\,\sqrt{x^2+\frac{9}{4}} - 3\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)$$

input
```
int((4*x^2 + 9)^(1/2)/x,x)
```

output
```
2*(x^2 + 9/4)^(1/2) - 3*atanh((2*(x^2 + 9/4)^(1/2))/3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{9+4x^2}}{x}\,dx = \sqrt{4x^2+9}+3\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)-3\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)$$

input
```
int((4*x^2+9)^(1/2)/x,x)
```

output
```
sqrt(4*x**2 + 9) + 3*log((sqrt(4*x**2 + 9) + 2*x - 3)/3) - 3*log((sqrt(4*x
**2 + 9) + 2*x + 3)/3)
```

3.458 $\int \frac{\sqrt{9+4x^2}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\sqrt{9+4x^2}}{x^2} \, dx = -\frac{\sqrt{9+4x^2}}{x} + 2\text{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
-(4*x^2+9)^(1/2)/x+2*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{9+4x^2}}{x^2} \, dx = -\frac{\sqrt{9+4x^2}}{x} - 2\log\left(-2x + \sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2]/x^2,x]
```

output
```
-(Sqrt[9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+9}}{x^2}\,dx$$

$$\Big\downarrow \; 247$$

$$4\int \frac{1}{\sqrt{4x^2+9}}dx - \frac{\sqrt{4x^2+9}}{x}$$

$$\Big\downarrow \; 222$$

$$2\text{arcsinh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2+9}}{x}$$

input
```
Int[Sqrt[9 + 4*x^2]/x^2,x]
```

output
```
-(Sqrt[9 + 4*x^2]/x) + 2*ArcSinh[(2*x)/3]
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\sqrt{4x^2+9}}{x}+2\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)$	22
trager	$-\frac{\sqrt{4x^2+9}}{x}-2\ln\left(\sqrt{4x^2+9}-2x\right)$	32
meijerg	$-\frac{\frac{6\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}}{x}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2\sqrt{\pi}}$	33
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{9x}+\frac{4x\sqrt{4x^2+9}}{9}+2\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)$	34
pseudoelliptic	$\frac{\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)x-\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)x-\sqrt{4x^2+9}}{x}$	58

input
```
int((4*x^2+9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(4*x^2+9)^(1/2)/x+2*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx=-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2+9}\right)+2\,x+\sqrt{4\,x^2+9}}{x}$$

input
```
integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(2*x*log(-2*x + sqrt(4*x^2 + 9)) + 2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx = 2\operatorname{asinh}\left(\frac{2x}{3}\right) - \frac{\sqrt{4x^2+9}}{x}$$

input
```
integrate((4*x**2+9)**(1/2)/x**2,x)
```

output
```
2*asinh(2*x/3) - sqrt(4*x**2 + 9)/x
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx = -\frac{\sqrt{4\,x^2+9}}{x} + 2\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-sqrt(4*x^2 + 9)/x + 2*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx = \frac{36}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9} - 2\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

output
```
36/((2*x - sqrt(4*x^2 + 9))^2 - 9) - 2*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx = 2\,\mathrm{asinh}\left(\frac{2\,x}{3}\right) - \frac{2\,\sqrt{x^2+\frac{9}{4}}}{x}$$

input
```
int((4*x^2 + 9)^(1/2)/x^2,x)
```

output
```
2*asinh((2*x)/3) - (2*(x^2 + 9/4)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{9+4x^2}}{x^2}\,dx = \frac{-\sqrt{4x^2+9}+2\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)x-2x}{x}$$

input
```
int((4*x^2+9)^(1/2)/x^2,x)
```

output
```
( - sqrt(4*x**2 + 9) + 2*log((sqrt(4*x**2 + 9) + 2*x)/3)*x - 2*x)/x
```

3.459 $\int \frac{\sqrt{9+4x^2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{9+4x^2}}{x^3} \, dx = -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
-1/2*(4*x^2+9)^(1/2)/x^2-2/3*arctanh(1/3*(4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9+4x^2}}{x^3} \, dx = -\frac{\sqrt{9+4x^2}}{2x^2} - \frac{2}{3}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2]/x^3,x]
```

output
```
-1/2*Sqrt[9 + 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+9}}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2+9}}{x^4}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2-\frac{\sqrt{4x^2+9}}{x^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}-\frac{\sqrt{4x^2+9}}{x^2}\right)$$

$$\Big\downarrow \text{ 220}$$

$$\frac{1}{2}\left(-\frac{4}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{x^2}\right)$$

input
```
Int[Sqrt[9 + 4*x^2]/x^3,x]
```

output
```
(-(Sqrt[9 + 4*x^2]/x^2) - (4*ArcTanh[Sqrt[9 + 4*x^2]/3])/3)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\sqrt{4x^2+9}}{2x^2}-\frac{2\,\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$	30
trager	$-\frac{\sqrt{4x^2+9}}{2x^2}+\frac{2\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)}{3}$	34
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{4x^2+9}}{9}-\frac{2\,\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{3}$	41
pseudoelliptic	$\frac{-2\ln\left(\sqrt{4x^2+9}+3\right)x^2+2\ln\left(\sqrt{4x^2+9}-3\right)x^2-3\sqrt{4x^2+9}}{6x^2}$	52
meijerg	$-\frac{\frac{9\sqrt{\pi}}{2x^2}-(-1+2\ln(x)-2\ln(3))\sqrt{\pi}-\frac{9\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)}{16x^2}+\frac{9\sqrt{\pi}\sqrt{\frac{4x^2}{9}+1}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)}{3\sqrt{\pi}}$	81

input
```
int((4*x^2+9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*x^2+9)^(1/2)/x^2-2/3*arctanh(3/(4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = -\frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)+3\,\sqrt{4\,x^2+9}}{6\,x^2}$$

input
```
integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/6*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2
+ 9) - 3) + 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3} - \frac{\sqrt{1+\frac{9}{4x^2}}}{x}$$

input
```
integrate((4*x**2+9)**(1/2)/x**3,x)
```

output
```
-2*asinh(3/(2*x))/3 - sqrt(1 + 9/(4*x**2))/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = \frac{2}{9}\sqrt{4\,x^2+9} - \frac{(4\,x^2+9)^{\frac{3}{2}}}{18\,x^2} - \frac{2}{3}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
2/9*sqrt(4*x^2 + 9) - 1/18*(4*x^2 + 9)^(3/2)/x^2 - 2/3*arcsinh(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = -\frac{\sqrt{4\,x^2+9}}{2\,x^2} - \frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}+3\right) + \frac{1}{3}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(4*x^2 + 9)/x^2 - 1/3*log(sqrt(4*x^2 + 9) + 3) + 1/3*log(sqrt(4*x
^2 + 9) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = -\frac{2\,\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3} - \frac{\sqrt{x^2+\frac{9}{4}}}{x^2}$$

input
```
int((4*x^2 + 9)^(1/2)/x^3,x)
```

output
```
- (2*atanh((2*(x^2 + 9/4)^(1/2))/3))/3 - (x^2 + 9/4)^(1/2)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{9+4x^2}}{x^3}\,dx = \frac{-3\sqrt{4x^2+9}+4\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)x^2-4\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)x^2}{6x^2}$$

input
```
int((4*x^2+9)^(1/2)/x^3,x)
```

output
```
( - 3*sqrt(4*x**2 + 9) + 4*log((sqrt(4*x**2 + 9) + 2*x - 3)/3)*x**2 - 4*lo
g((sqrt(4*x**2 + 9) + 2*x + 3)/3)*x**2)/(6*x**2)
```

3.460 $\int \frac{\sqrt{9+4x^2}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{9+4x^2}}{x^4} \, dx = -\frac{(9+4x^2)^{3/2}}{27x^3}$$

output
```
-1/27*(4*x^2+9)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9+4x^2}}{x^4} \, dx = -\frac{(9+4x^2)^{3/2}}{27x^3}$$

input
```
Integrate[Sqrt[9 + 4*x^2]/x^4,x]
```

output
```
-1/27*(9 + 4*x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{242\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+9}}{x^4}\,dx$$

$$\Big\downarrow \; 242$$

$$-\frac{\left(4x^2+9\right)^{3/2}}{27x^3}$$

input
```
Int[Sqrt[9 + 4*x^2]/x^4,x]
```

output
```
-1/27*(9 + 4*x^2)^(3/2)/x^3
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{\left(4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
default	$-\frac{\left(4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
trager	$-\frac{\left(4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
meijerg	$-\frac{\left(\frac{4x^{2}}{9}+1\right)^{\frac{3}{2}}}{x^{3}}$	15
pseudoelliptic	$-\frac{\left(4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
orering	$-\frac{\left(4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
risch	$-\frac{16x^{4}+72x^{2}+81}{27x^{3}\sqrt{4x^{2}+9}}$	27

input
```
int((4*x^2+9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27*(4*x^2+9)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{8\,x^3+\left(4\,x^2+9\right)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-1/27*(8*x^3 + (4*x^2 + 9)^(3/2))/x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{8\sqrt{1+\frac{9}{4x^2}}}{27} - \frac{2\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

input
```
integrate((4*x**2+9)**(1/2)/x**4,x)
```

output
```
-8*sqrt(1 + 9/(4*x**2))/27 - 2*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{(4\,x^2+9)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/27*(4*x^2 + 9)^(3/2)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = \frac{16\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^4+27\right)}{\left(\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9\right)^3}$$

input
```
integrate((4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

output
```
16*((2*x - sqrt(4*x^2 + 9))^4 + 27)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = -\frac{18\,\sqrt{x^2+\frac{9}{4}}+8\,x^2\,\sqrt{x^2+\frac{9}{4}}}{27\,x^3}$$

input
```
int((4*x^2 + 9)^(1/2)/x^4,x)
```

output
```
-(18*(x^2 + 9/4)^(1/2) + 8*x^2*(x^2 + 9/4)^(1/2))/(27*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{9+4x^2}}{x^4}\,dx = \frac{-4\sqrt{4x^2+9}\,x^2-9\sqrt{4x^2+9}-8x^3}{27x^3}$$

input
```
int((4*x^2+9)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(4*x**2 + 9)*x**2 - 9*sqrt(4*x**2 + 9) - 8*x**3)/(27*x**3)
```

3.461 $\int \frac{\sqrt{9+4x^2}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx=-\frac{\sqrt{9+4x^2}}{4x^4}-\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
-1/4*(4*x^2+9)^(1/2)/x^4-1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(1/3*(4*x^2+
9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx=\frac{(-9-2x^2)\sqrt{9+4x^2}}{36x^4}+\frac{2}{27}\operatorname{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[Sqrt[9 + 4*x^2]/x^5,x]
```

output
```
((-9 - 2*x^2)*Sqrt[9 + 4*x^2])/(36*x^4) + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/2
7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2+9}}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2+9}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\int \frac{1}{x^4\sqrt{4x^2+9}}dx^2-\frac{\sqrt{4x^2+9}}{2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2-\frac{\sqrt{4x^2+9}}{9x^2}-\frac{\sqrt{4x^2+9}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}-\frac{\sqrt{4x^2+9}}{9x^2}-\frac{\sqrt{4x^2+9}}{2x^4}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{9x^2}-\frac{\sqrt{4x^2+9}}{2x^4}\right)$$

input
```
Int[Sqrt[9 + 4*x^2]/x^5,x]
```

output
```
(-1/2*Sqrt[9 + 4*x^2]/x^4 - Sqrt[9 + 4*x^2]/(9*x^2) + (4*ArcTanh[Sqrt[9 +
4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result
trager	$-\frac{(2x^2+9)\sqrt{4x^2+9}}{36x^4}-\frac{2\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)}{27}$
risch	$-\frac{8x^4+54x^2+81}{36x^4\sqrt{4x^2+9}}+\frac{2\,\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$
default	$-\frac{(4x^2+9)^{\frac{3}{2}}}{36x^4}+\frac{(4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{4x^2+9}}{81}+\frac{2\,\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$
pseudoelliptic	$\frac{\frac{16\ln\left(\sqrt{4x^2+9}+3\right)x^4}{27}-\frac{16\ln\left(\sqrt{4x^2+9}-3\right)x^4}{27}-\frac{8x^2\sqrt{4x^2+9}}{9}-4\sqrt{4x^2+9}}{\left(\sqrt{4x^2+9}+3\right)^2\left(\sqrt{4x^2+9}-3\right)^2}$
meijerg	$-\frac{4\left(\frac{81\sqrt{\pi}}{16x^4}+\frac{9\sqrt{\pi}}{4x^2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{4}-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4+\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)\sqrt{\frac{4x^2}{9}+1}}{128x^4}-\frac{\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)}{2}\right)}{27\sqrt{\pi}}$

input
```
int((4*x^2+9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/36*(2*x^2+9)/x^4*(4*x^2+9)^(1/2)-2/27*ln(((4*x^2+9)^(1/2)-3)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = \frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}(2\,x^2+9)}{108\,x^4}$$

input
```
integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/108*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^2
 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 + 9))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = \frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{3}{4x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate((4*x**2+9)**(1/2)/x**5,x)
```

output
```
2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 3/(4*x**3*sqrt(1 + 9/
(4*x**2))) - 9/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = -\frac{2}{81}\sqrt{4x^2+9} + \frac{(4x^2+9)^{\frac{3}{2}}}{162x^2} - \frac{(4x^2+9)^{\frac{3}{2}}}{36x^4} + \frac{2}{27}\operatorname{arsinh}\left(\frac{3}{2|x|}\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/81*sqrt(4*x^2 + 9) + 1/162*(4*x^2 + 9)^(3/2)/x^2 - 1/36*(4*x^2 + 9)^(3/
2)/x^4 + 2/27*arcsinh(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = -\frac{(4x^2+9)^{\frac{3}{2}} + 9\sqrt{4x^2+9}}{72x^4} + \frac{1}{27}\log\left(\sqrt{4x^2+9}+3\right) - \frac{1}{27}\log\left(\sqrt{4x^2+9}-3\right)$$

input
```
integrate((4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/72*((4*x^2 + 9)^(3/2) + 9*sqrt(4*x^2 + 9))/x^4 + 1/27*log(sqrt(4*x^2 +
9) + 3) - 1/27*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27} + \frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2}{3\,x^2}-\frac{1}{x^4}\right)}{2} - \frac{4\sqrt{x^2+\frac{9}{4}}}{9\,x^2}$$

input
```
int((4*x^2 + 9)^(1/2)/x^5,x)
```

output
```
(2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 + ((x^2 + 9/4)^(1/2)*(2/(3*x^2) - 1/
x^4))/2 - (4*(x^2 + 9/4)^(1/2))/(9*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{9+4x^2}}{x^5}\,dx = \frac{-6\sqrt{4x^2+9}\,x^2-27\sqrt{4x^2+9}-8\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)x^4+8\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)x^4}{108x^4}$$

input
```
int((4*x^2+9)^(1/2)/x^5,x)
```

output
```
( - 6*sqrt(4*x**2 + 9)*x**2 - 27*sqrt(4*x**2 + 9) - 8*log((sqrt(4*x**2 + 9
) + 2*x - 3)/3)*x**4 + 8*log((sqrt(4*x**2 + 9) + 2*x + 3)/3)*x**4)/(108*x*
*4)
```

3.462 $\int x^5\sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x^5\sqrt{9-4x^2}\,dx=-\frac{27}{64}\left(9-4x^2\right)^{3/2}+\frac{9}{160}\left(9-4x^2\right)^{5/2}-\frac{1}{448}\left(9-4x^2\right)^{7/2}$$

output
```
-27/64*(-4*x^2+9)^(3/2)+9/160*(-4*x^2+9)^(5/2)-1/448*(-4*x^2+9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int x^5\sqrt{9-4x^2}\,dx=\frac{1}{280}\sqrt{9-4x^2}\left(-243-54x^2-18x^4+40x^6\right)$$

input
```
Integrate[x^5*Sqrt[9 - 4*x^2],x]
```

output
```
(Sqrt[9 - 4*x^2]*(-243 - 54*x^2 - 18*x^4 + 40*x^6))/280
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^4\sqrt{9-4x^2}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{1}{16}\left(9-4x^2\right)^{5/2}-\frac{9}{8}\left(9-4x^2\right)^{3/2}+\frac{81}{16}\sqrt{9-4x^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{1}{224}\left(9-4x^2\right)^{7/2}+\frac{9}{80}\left(9-4x^2\right)^{5/2}-\frac{27}{32}\left(9-4x^2\right)^{3/2}\right)$$

input
```
Int[x^5*Sqrt[9 - 4*x^2],x]
```

output
```
((-27*(9 - 4*x^2)^(3/2))/32 + (9*(9 - 4*x^2)^(5/2))/80 - (9 - 4*x^2)^(7/2)
/224)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{\left(-4x^{2}+9\right)^{\frac{3}{2}} \left(10x^{4}+18x^{2}+27\right)}{280}$	24
trager	$\left(\frac{1}{7}x^{6}-\frac{9}{140}x^{4}-\frac{27}{140}x^{2}-\frac{243}{280}\right) \sqrt{-4x^{2}+9}$	28
gosper	$\frac{(2x-3)(2x+3)\left(10x^{4}+18x^{2}+27\right) \sqrt{-4x^{2}+9}}{280}$	34
orering	$\frac{(2x-3)(2x+3)\left(10x^{4}+18x^{2}+27\right) \sqrt{-4x^{2}+9}}{280}$	34
risch	$-\frac{\left(40x^{6}-18x^{4}-54x^{2}-243\right) \left(4x^{2}-9\right)}{280\sqrt{-4x^{2}+9}}$	36
meijerg	$\frac{\frac{729\sqrt{\pi}}{280}-\frac{729\sqrt{\pi}\left(-\frac{4x^{2}}{9}+1\right)^{\frac{3}{2}} \left(\frac{80}{27}x^{4}+\frac{16}{3}x^{2}+8\right)}{2240}}{\sqrt{\pi}}$	38
default	$-\frac{x^{4}\left(-4x^{2}+9\right)^{\frac{3}{2}}}{28}-\frac{9x^{2}\left(-4x^{2}+9\right)^{\frac{3}{2}}}{140}-\frac{27\left(-4x^{2}+9\right)^{\frac{3}{2}}}{280}$	41

input
```
int(x^5*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(-4*x^2+9)^(3/2)*(10*x^4+18*x^2+27)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{9-4x^2}\,dx = \frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{-4\,x^2+9}$$

input
```
integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(-4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^5\sqrt{9-4x^2}\,dx = \frac{x^6\sqrt{9-4x^2}}{7} - \frac{9x^4\sqrt{9-4x^2}}{140} - \frac{27x^2\sqrt{9-4x^2}}{140} - \frac{243\sqrt{9-4x^2}}{280}$$

input
```
integrate(x**5*(-4*x**2+9)**(1/2),x)
```

output
```
x**6*sqrt(9 - 4*x**2)/7 - 9*x**4*sqrt(9 - 4*x**2)/140 - 27*x**2*sqrt(9 - 4
*x**2)/140 - 243*sqrt(9 - 4*x**2)/280
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x^5\sqrt{9-4x^2}\,dx = -\frac{1}{28}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^4 - \frac{9}{140}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^2 - \frac{27}{280}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/28*(-4*x^2 + 9)^(3/2)*x^4 - 9/140*(-4*x^2 + 9)^(3/2)*x^2 - 27/280*(-4*x
^2 + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int x^5\sqrt{9-4x^2}\,dx = \frac{1}{448}\left(4\,x^2-9\right)^3\sqrt{-4\,x^2+9} + \frac{9}{160}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9} - \frac{27}{64}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/448*(4*x^2 - 9)^3*sqrt(-4*x^2 + 9) + 9/160*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9
) - 27/64*(-4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{9-4x^2}\,dx = -\frac{\sqrt{\frac{9}{4}-x^2}\left(-\frac{4\,x^6}{7}+\frac{9\,x^4}{35}+\frac{27\,x^2}{35}+\frac{243}{70}\right)}{2}$$

input
```
int(x^5*(9 - 4*x^2)^(1/2),x)
```

output
```
-((9/4 - x^2)^(1/2)*((27*x^2)/35 + (9*x^4)/35 - (4*x^6)/7 + 243/70))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{9-4x^2}\,dx = \frac{\sqrt{-4x^2+9}\,(40x^6-18x^4-54x^2-243)}{280}$$

input
```
int(x^5*(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*(40*x**6 - 18*x**4 - 54*x**2 - 243))/280
```

3.463 $\int x^4\sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int x^4\sqrt{9-4x^2}\,dx=-\frac{81}{256}x\sqrt{9-4x^2}-\frac{3}{32}x^3\sqrt{9-4x^2}+\frac{1}{6}x^5\sqrt{9-4x^2}+\frac{729}{512}\arcsin\left(\frac{2x}{3}\right)$$

output
```
-81/256*x*(-4*x^2+9)^(1/2)-3/32*x^3*(-4*x^2+9)^(1/2)+1/6*x^5*(-4*x^2+9)^(1
/2)+729/512*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

$$\int x^4\sqrt{9-4x^2}\,dx=\frac{1}{768}x\sqrt{9-4x^2}(-243-72x^2+128x^4)+\frac{729}{256}\arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[x^4*Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*ArcTan[(2*x)/(-3
+ Sqrt[9 - 4*x^2])])/256
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{3}{2}\int \frac{x^4}{\sqrt{9-4x^2}}dx+\frac{1}{6}\sqrt{9-4x^2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{2}\left(\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}dx-\frac{1}{16}x^3\sqrt{9-4x^2}\right)+\frac{1}{6}\sqrt{9-4x^2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{2}\left(\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}dx-\frac{1}{8}x\sqrt{9-4x^2}\right)-\frac{1}{16}x^3\sqrt{9-4x^2}\right)+\frac{1}{6}\sqrt{9-4x^2}x^5$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{2}\left(\frac{27}{16}\left(\frac{9}{16}\arcsin\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}\right)-\frac{1}{16}x^3\sqrt{9-4x^2}\right)+\frac{1}{6}\sqrt{9-4x^2}x^5$$

input
```
Int[x^4*Sqrt[9 - 4*x^2],x]
```

output
```
(x^5*Sqrt[9 - 4*x^2])/6 + (3*(-1/16*(x^3*Sqrt[9 - 4*x^2]) + (27*(-1/8*(x*S
qrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3])/16))/16))/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{x(128x^4-72x^2-243)(4x^2-9)}{768\sqrt{-4x^2+9}}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}$	39
pseudoelliptic	$-\frac{729\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{512}+\frac{(128x^5-72x^3-243x)\sqrt{-4x^2+9}}{768}$	44
default	$-\frac{x^3(-4x^2+9)^{\frac{3}{2}}}{24}-\frac{9x(-4x^2+9)^{\frac{3}{2}}}{128}+\frac{81x\sqrt{-4x^2+9}}{256}+\frac{729\arcsin\left(\frac{2x}{3}\right)}{512}$	46
meijerg	$\frac{729i\left(\frac{i\sqrt{\pi}\,x\left(-\frac{640}{81}x^4+\frac{40}{9}x^2+15\right)\sqrt{-\frac{4x^2}{9}+1}}{90}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	46
trager	$\frac{x(128x^4-72x^2-243)\sqrt{-4x^2+9}}{768}-\frac{729\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{512}$	56

input
```
int(x^4*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768*x*(128*x^4-72*x^2-243)*(4*x^2-9)/(-4*x^2+9)^(1/2)+729/512*arcsin(2/
3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{9-4x^2}\,dx = \frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{-4\,x^2+9} - \frac{729}{256}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(-4*x^2 + 9) - 729/256*arctan(1/2*(sq
rt(-4*x^2 + 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.91 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int x^4\sqrt{9-4x^2}\,dx = \begin{cases} \frac{2ix^7}{3\sqrt{4x^2-9}} - \frac{15ix^5}{8\sqrt{4x^2-9}} - \frac{27ix^3}{64\sqrt{4x^2-9}} + \frac{729ix}{256\sqrt{4x^2-9}} - \frac{729i\operatorname{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } |x^2| > \frac{9}{4} \\ -\frac{2x^7}{3\sqrt{9-4x^2}} + \frac{15x^5}{8\sqrt{9-4x^2}} + \frac{27x^3}{64\sqrt{9-4x^2}} - \frac{729x}{256\sqrt{9-4x^2}} + \frac{729\operatorname{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((2*I*x**7/(3*sqrt(4*x**2 - 9)) - 15*I*x**5/(8*sqrt(4*x**2 - 9))
- 27*I*x**3/(64*sqrt(4*x**2 - 9)) + 729*I*x/(256*sqrt(4*x**2 - 9)) - 729*I
*acosh(2*x/3)/512, Abs(x**2) > 9/4), (-2*x**7/(3*sqrt(9 - 4*x**2)) + 15*x*
*5/(8*sqrt(9 - 4*x**2)) + 27*x**3/(64*sqrt(9 - 4*x**2)) - 729*x/(256*sqrt(
9 - 4*x**2)) + 729*asin(2*x/3)/512, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.71

$$\int x^4\sqrt{9-4x^2}\,dx = -\frac{1}{24}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^3 - \frac{9}{128}\left(-4\,x^2+9\right)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{-4\,x^2+9}x + \frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/24*(-4*x^2 + 9)^(3/2)*x^3 - 9/128*(-4*x^2 + 9)^(3/2)*x + 81/256*sqrt(-4
*x^2 + 9)*x + 729/512*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int x^4\sqrt{9-4x^2}\,dx = \frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{-4\,x^2+9}x + \frac{729}{512}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(-4*x^2 + 9)*x + 729/512*arcsin(2/3*x
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.51

$$\int x^4\sqrt{9-4x^2}\,dx = \frac{729\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{512} - \frac{\sqrt{\frac{9}{4}-x^2}\,\left(-\frac{2\,x^5}{3}+\frac{3\,x^3}{8}+\frac{81\,x}{64}\right)}{2}$$

input
```
int(x^4*(9 - 4*x^2)^(1/2),x)
```

output
```
(729*asin((2*x)/3))/512 - ((9/4 - x^2)^(1/2)*((81*x)/64 + (3*x^3)/8 - (2*x
^5)/3))/2
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int x^4\sqrt{9-4x^2}\,dx = \frac{729asin\left(\frac{2x}{3}\right)}{512} + \frac{\sqrt{-4x^2+9}\,x^5}{6} - \frac{3\sqrt{-4x^2+9}\,x^3}{32} - \frac{81\sqrt{-4x^2+9}\,x}{256}$$

input
```
int(x^4*(-4*x^2+9)^(1/2),x)
```

output
```
(2187*asin((2*x)/3) + 256*sqrt( - 4*x**2 + 9)*x**5 - 144*sqrt( - 4*x**2 +
9)*x**3 - 486*sqrt( - 4*x**2 + 9)*x)/1536
```

3.464 $\int x^3\sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{9-4x^2}\,dx=-\frac{3}{16}\left(9-4x^2\right)^{3/2}+\frac{1}{80}\left(9-4x^2\right)^{5/2}$$

output
```
-3/16*(-4*x^2+9)^(3/2)+1/80*(-4*x^2+9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{9-4x^2}\,dx=\frac{1}{40}\left(9-4x^2\right)^{3/2}\left(-3-2x^2\right)$$

input
```
Integrate[x^3*Sqrt[9 - 4*x^2],x]
```

output
```
((9 - 4*x^2)^(3/2)*(-3 - 2*x^2))/40
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^2\sqrt{9-4x^2}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{9}{4}\sqrt{9-4x^2}-\frac{1}{4}\left(9-4x^2\right)^{3/2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{40}\left(9-4x^2\right)^{5/2}-\frac{3}{8}\left(9-4x^2\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[9 - 4*x^2],x]
```

output
```
((-3*(9 - 4*x^2)^(3/2))/8 + (9 - 4*x^2)^(5/2)/40)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$-\frac{(2x^2+3)(-4x^2+9)^{\frac{3}{2}}}{40}$	19
trager	$\left(\frac{1}{5}x^4-\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2+9}$	23
default	$-\frac{x^2(-4x^2+9)^{\frac{3}{2}}}{20}-\frac{3(-4x^2+9)^{\frac{3}{2}}}{40}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+3)\sqrt{-4x^2+9}}{40}$	29
orering	$\frac{(2x-3)(2x+3)(2x^2+3)\sqrt{-4x^2+9}}{40}$	29
risch	$-\frac{(8x^4-6x^2-27)(4x^2-9)}{40\sqrt{-4x^2+9}}$	31
meijerg	$-\frac{243\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(-\frac{4x^2}{9}+1\right)^{\frac{3}{2}}\left(\frac{4x^2}{3}+2\right)}{15}\right)}{64\sqrt{\pi}}$	33

input
```
int(x^3*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(2*x^2+3)*(-4*x^2+9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{9-4x^2}\,dx = \frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{-4\,x^2+9}$$

input
```
integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(8*x^4 - 6*x^2 - 27)*sqrt(-4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int x^3\sqrt{9-4x^2}\,dx = \frac{x^4\sqrt{9-4x^2}}{5}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

input
```
integrate(x**3*(-4*x**2+9)**(1/2),x)
```

output
```
x**4*sqrt(9 - 4*x**2)/5 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**2)
/40
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{9-4x^2}\,dx = -\frac{1}{20}\left(-4\,x^2+9\right)^{\frac{3}{2}}x^2-\frac{3}{40}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*(-4*x^2 + 9)^(3/2)*x^2 - 3/40*(-4*x^2 + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int x^3\sqrt{9-4x^2}\,dx = \frac{1}{80}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9}-\frac{3}{16}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/80*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) - 3/16*(-4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{9-4x^2}\,dx = -\frac{\sqrt{\frac{9}{4}-x^2}\left(-\frac{4\,x^4}{5}+\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

input
```
int(x^3*(9 - 4*x^2)^(1/2),x)
```

output
```
-((9/4 - x^2)^(1/2)*((3*x^2)/5 - (4*x^4)/5 + 27/10))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{9-4x^2}\,dx = \frac{\sqrt{-4x^2+9}\,(8x^4-6x^2-27)}{40}$$

input
```
int(x^3*(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*(8*x**4 - 6*x**2 - 27))/40
```

3.465 $\int x^2\sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int x^2\sqrt{9-4x^2}\,dx=-\frac{9}{32}x\sqrt{9-4x^2}+\frac{1}{4}x^3\sqrt{9-4x^2}+\frac{81}{64}\arcsin\left(\frac{2x}{3}\right)$$

output
```
-9/32*x*(-4*x^2+9)^(1/2)+1/4*x^3*(-4*x^2+9)^(1/2)+81/64*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int x^2\sqrt{9-4x^2}\,dx=\frac{1}{32}x\sqrt{9-4x^2}(-9+8x^2)+\frac{81}{32}\arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[x^2*Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2]*(-9 + 8*x^2))/32 + (81*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^
2])])/32
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {248, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{9}{4}\int \frac{x^2}{\sqrt{9-4x^2}}dx+\frac{1}{4}\sqrt{9-4x^2}x^3$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{4}\left(\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}dx-\frac{1}{8}x\sqrt{9-4x^2}\right)+\frac{1}{4}\sqrt{9-4x^2}x^3$$

$$\Big\downarrow \text{223}$$

$$\frac{9}{4}\left(\frac{9}{16}\arcsin\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}\right)+\frac{1}{4}\sqrt{9-4x^2}x^3$$

input
```
Int[x^2*Sqrt[9 - 4*x^2],x]
```

output
```
(x^3*Sqrt[9 - 4*x^2])/4 + (9*(-1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3
])/16))/4
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
default	$-\frac{x\left(-4x^{2}+9\right)^{\frac{3}{2}}}{16}+\frac{9x\sqrt{-4x^{2}+9}}{32}+\frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$	32
risch	$-\frac{x\left(8x^{2}-9\right)\left(4x^{2}-9\right)}{32\sqrt{-4x^{2}+9}}+\frac{81\arcsin\left(\frac{2x}{3}\right)}{64}$	34
pseudoelliptic	$-\frac{81\arctan\left(\frac{\sqrt{-4x^{2}+9}}{2x}\right)}{64}+\frac{\left(8x^{3}-9x\right)\sqrt{-4x^{2}+9}}{32}$	39
meijerg	$-\frac{81i\left(-\frac{i\sqrt{\pi}\,x\left(-\frac{8x^{2}}{3}+3\right)\sqrt{-\frac{4x^{2}}{9}+1}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}}$	41
trager	$\frac{x\left(8x^{2}-9\right)\sqrt{-4x^{2}+9}}{32}-\frac{81\,\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-4x^{2}+9}+2x\right)}{64}$	51

input
```
int(x^2*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*x*(-4*x^2+9)^(3/2)+9/32*x*(-4*x^2+9)^(1/2)+81/64*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{1}{32}\left(8\,x^3-9\,x\right)\sqrt{-4\,x^2+9}-\frac{81}{32}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 - 9*x)*sqrt(-4*x^2 + 9) - 81/32*arctan(1/2*(sqrt(-4*x^2 + 9) -
 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

$$\int x^2\sqrt{9-4x^2}\,dx = \begin{cases} \frac{ix^5}{\sqrt{4x^2-9}}-\frac{27ix^3}{8\sqrt{4x^2-9}}+\frac{81ix}{32\sqrt{4x^2-9}}-\frac{81i\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for}\ |x^2|>\frac{9}{4} \\ -\frac{x^5}{\sqrt{9-4x^2}}+\frac{27x^3}{8\sqrt{9-4x^2}}-\frac{81x}{32\sqrt{9-4x^2}}+\frac{81\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((I*x**5/sqrt(4*x**2 - 9) - 27*I*x**3/(8*sqrt(4*x**2 - 9)) + 81*I
*x/(32*sqrt(4*x**2 - 9)) - 81*I*acosh(2*x/3)/64, Abs(x**2) > 9/4), (-x**5/
sqrt(9 - 4*x**2) + 27*x**3/(8*sqrt(9 - 4*x**2)) - 81*x/(32*sqrt(9 - 4*x**2
)) + 81*asin(2*x/3)/64, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{9-4x^2}\,dx = -\frac{1}{16}\left(-4\,x^2+9\right)^{\frac{3}{2}}x + \frac{9}{32}\,\sqrt{-4\,x^2+9}x + \frac{81}{64}\,\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(-4*x^2 + 9)^(3/2)*x + 9/32*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{1}{32}\left(8\,x^2-9\right)\sqrt{-4\,x^2+9}x + \frac{81}{64}\,\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/32*(8*x^2 - 9)*sqrt(-4*x^2 + 9)*x + 81/64*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{81\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{64} - \frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{9\,x}{8}-x^3\right)}{2}$$

input
```
int(x^2*(9 - 4*x^2)^(1/2),x)
```

output
```
(81*asin((2*x)/3))/64 - ((9/4 - x^2)^(1/2)*((9*x)/8 - x^3))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{9-4x^2}\,dx = \frac{81asin\left(\frac{2x}{3}\right)}{64} + \frac{\sqrt{-4x^2+9}\,x^3}{4} - \frac{9\sqrt{-4x^2+9}\,x}{32}$$

input

```
int(x^2*(-4*x^2+9)^(1/2),x)
```

output

```
(81*asin((2*x)/3) + 16*sqrt( - 4*x**2 + 9)*x**3 - 18*sqrt( - 4*x**2 + 9)*x
)/64
```

3.466 $\int x\sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

output
```
-1/12*(-4*x^2+9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\sqrt{9-4x^2}\,dx=-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[9 - 4*x^2],x]
```

output
```
-1/12*(9 - 4*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{9-4x^2}\,dx$$

$$\Big\downarrow \text{241}$$

$$-\frac{1}{12}\left(9-4x^2\right)^{3/2}$$

input
```
Int[x*Sqrt[9 - 4*x^2],x]
```

output
```
-1/12*(9 - 4*x^2)^(3/2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{12}$	12
default	$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{12}$	12
pseudoelliptic	$-\frac{\left(-4x^2+9\right)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}-\frac{3}{4}\right)\sqrt{-4x^2+9}$	18
risch	$-\frac{\left(4x^2-9\right)^2}{12\sqrt{-4x^2+9}}$	21
gosper	$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{12}$	22
orering	$\frac{(2x-3)(2x+3)\sqrt{-4x^2+9}}{12}$	22
meijerg	$\frac{\frac{9\sqrt{\pi}}{4}-\frac{9\sqrt{\pi}\left(2-\frac{8x^2}{9}\right)\sqrt{-\frac{4x^2}{9}+1}}{8}}{\sqrt{\pi}}$	33

input
```
int(x*(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-4*x^2+9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int x\sqrt{9-4x^2}\,dx=\frac{1}{12}\left(4\,x^2-9\right)\sqrt{-4\,x^2+9}$$

input
```
integrate(x*(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(4*x^2 - 9)*sqrt(-4*x^2 + 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(12) = 24$.

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x\sqrt{9-4x^2}\,dx = \frac{x^2\sqrt{9-4x^2}}{3} - \frac{3\sqrt{9-4x^2}}{4}$$

input
```
integrate(x*(-4*x**2+9)**(1/2),x)
```

output
```
x**2*sqrt(9 - 4*x**2)/3 - 3*sqrt(9 - 4*x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{9-4x^2}\,dx = -\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(-4*x^2 + 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{9-4x^2}\,dx = -\frac{1}{12}\left(-4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/12*(-4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int x\sqrt{9-4x^2}\,dx = \frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{4\,x^2}{3}-3\right)}{2}$$

input
```
int(x*(9 - 4*x^2)^(1/2),x)
```

output
```
((9/4 - x^2)^(1/2)*((4*x^2)/3 - 3))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x\sqrt{9-4x^2}\,dx = \frac{\sqrt{-4x^2+9}\,(4x^2-9)}{12}$$

input
```
int(x*(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*(4*x**2 - 9))/12
```

3.467 $\int \sqrt{9-4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \sqrt{9-4x^2}\,dx=\frac{1}{2}x\sqrt{9-4x^2}+\frac{9}{4}\arcsin\left(\frac{2x}{3}\right)$$

output
```
1/2*x*(-4*x^2+9)^(1/2)+9/4*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

$$\int \sqrt{9-4x^2}\,dx=\frac{1}{2}x\sqrt{9-4x^2}-\frac{9}{2}\arctan\left(\frac{\sqrt{9-4x^2}}{3+2x}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2])/2 - (9*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)])/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{9-4x^2}\,dx$$
$$\Big\downarrow \text{211}$$
$$\frac{9}{2}\int \frac{1}{\sqrt{9-4x^2}}dx+\frac{1}{2}\sqrt{9-4x^2}x$$
$$\Big\downarrow \text{223}$$
$$\frac{9}{4}\arcsin\left(\frac{2x}{3}\right)+\frac{1}{2}\sqrt{9-4x^2}x$$

input
```
Int[Sqrt[9 - 4*x^2],x]
```

output
```
(x*Sqrt[9 - 4*x^2])/2 + (9*ArcSin[(2*x)/3])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x\sqrt{-4x^2+9}}{2}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$	20
risch	$-\frac{(4x^2-9)x}{2\sqrt{-4x^2+9}}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{4}$	27
pseudoelliptic	$\frac{x\sqrt{-4x^2+9}}{2}-\frac{9\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{4}$	31
meijerg	$\frac{9i\left(-\frac{4i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	34
trager	$\frac{x\sqrt{-4x^2+9}}{2}-\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{4}$	44

input
```
int((-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-4*x^2+9)^(1/2)+9/4*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int\sqrt{9-4x^2}\,dx=\frac{1}{2}\sqrt{-4\,x^2+9}x-\frac{9}{2}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x - 9/2*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \sqrt{9-4x^2}\,dx = \frac{x\sqrt{9-4x^2}}{2} + \frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{4}$$

input
```
integrate((-4*x**2+9)**(1/2),x)
```

output
```
x*sqrt(9 - 4*x**2)/2 + 9*asin(2*x/3)/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \sqrt{9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{9-4x^2}\,dx = \frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{4} + x\,\sqrt{\frac{9}{4}-x^2}$$

input
```
int((9 - 4*x^2)^(1/2),x)
```

output
```
(9*asin((2*x)/3))/4 + x*(9/4 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \sqrt{9-4x^2}\,dx = \frac{9asin\left(\frac{2x}{3}\right)}{4} + \frac{\sqrt{-4x^2+9}\,x}{2}$$

input
```
int((-4*x^2+9)^(1/2),x)
```

output
```
(9*asin((2*x)/3) + 2*sqrt( - 4*x**2 + 9)*x)/4
```

3.468 $\int \frac{\sqrt{9-4x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{9-4x^2}}{x} \, dx = \sqrt{9-4x^2} - 3\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
(-4*x^2+9)^(1/2)-3*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-4x^2}}{x} \, dx = \sqrt{9-4x^2} - 3\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2]/x,x]
```

output
```
Sqrt[9 - 4*x^2] - 3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{9-4x^2}}{x} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{9-4x^2}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(9\int \frac{1}{x^2\sqrt{9-4x^2}}dx^2+2\sqrt{9-4x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\sqrt{9-4x^2}-\frac{9}{2}\int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2\sqrt{9-4x^2}-6\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)\right)$$

input
```
Int[Sqrt[9 - 4*x^2]/x,x]
```

output
```
(2*Sqrt[9 - 4*x^2] - 6*ArcTanh[Sqrt[9 - 4*x^2]/3])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{-4x^2+9}-3\,\operatorname{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)$	25
trager	$\sqrt{-4x^2+9}-3\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)$	29
pseudoelliptic	$\sqrt{-4x^2+9}-\frac{3\ln\left(\sqrt{-4x^2+9}+3\right)}{2}+\frac{3\ln\left(-3+\sqrt{-4x^2+9}\right)}{2}$	39
meijerg	$-\frac{3\left(-2(2+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-\frac{4x^2}{9}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	64

input
```
int((-4*x^2+9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-4*x^2+9)^(1/2)-3*arctanh(3/(-4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \sqrt{-4\,x^2+9}+3\,\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(-4*x^2 + 9) + 3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.50

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \begin{cases} i\sqrt{4x^2-9}-3\log{(x)}+\frac{3\log{(x^2)}}{2}+3i\operatorname{asin}{\left(\frac{3}{2x}\right)} & \text{for}\ |x^2|>\frac{9}{4} \\ \sqrt{9-4x^2}+\frac{3\log{(x^2)}}{2}-3\log{\left(\sqrt{1-\frac{4x^2}{9}}+1\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((-4*x**2+9)**(1/2)/x,x)
```

output
```
Piecewise((I*sqrt(4*x**2 - 9) - 3*log(x) + 3*log(x**2)/2 + 3*I*asin(3/(2*x
)), Abs(x**2) > 9/4), (sqrt(9 - 4*x**2) + 3*log(x**2)/2 - 3*log(sqrt(1 - 4
*x**2/9) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \sqrt{-4\,x^2+9} - 3\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(-4*x^2 + 9) - 3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \sqrt{-4\,x^2+9} - \frac{3}{2}\,\log\left(\sqrt{-4\,x^2+9}+3\right) + \frac{3}{2}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(-4*x^2 + 9) - 3/2*log(sqrt(-4*x^2 + 9) + 3) + 3/2*log(-sqrt(-4*x^2 +
9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = 3\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1} - \frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right) + 2\,\sqrt{\frac{9}{4}-x^2}$$

input
```
int((9 - 4*x^2)^(1/2)/x,x)
```

output
```
3*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2) + 2*(9/4 - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx = \sqrt{-4x^2+9}+3\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)-3$$

input
```
int((-4*x^2+9)^(1/2)/x,x)
```

output
```
sqrt( - 4*x**2 + 9) + 3*log(tan(asin((2*x)/3)/2)) - 3
```

3.469 $\int \frac{\sqrt{9-4x^2}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{\sqrt{9-4x^2}}{x^2} \, dx = -\frac{\sqrt{9-4x^2}}{x} - 2\arcsin\left(\frac{2x}{3}\right)$$

output
```
-(-4*x^2+9)^(1/2)/x-2*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{9-4x^2}}{x^2} \, dx = -\frac{\sqrt{9-4x^2}}{x} + 4\arctan\left(\frac{\sqrt{9-4x^2}}{3+2x}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2]/x^2,x]
```

output
```
-(Sqrt[9 - 4*x^2]/x) + 4*ArcTan[Sqrt[9 - 4*x^2]/(3 + 2*x)]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{9-4x^2}}{x^2} dx$$
$$\Big\downarrow \; 247$$
$$-4\int \frac{1}{\sqrt{9-4x^2}}dx - \frac{\sqrt{9-4x^2}}{x}$$
$$\Big\downarrow \; 223$$
$$-2\arcsin\left(\frac{2x}{3}\right) - \frac{\sqrt{9-4x^2}}{x}$$

input
```
Int[Sqrt[9 - 4*x^2]/x^2,x]
```

output
```
-(Sqrt[9 - 4*x^2]/x) - 2*ArcSin[(2*x)/3]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{4x^2-9}{x\sqrt{-4x^2+9}}-2\arcsin\left(\frac{2x}{3}\right)$	28
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{9x}-\frac{4x\sqrt{-4x^2+9}}{9}-2\arcsin\left(\frac{2x}{3}\right)$	34
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)x-\sqrt{-4x^2+9}}{x}$	35
meijerg	$-\frac{i\left(-\frac{6i\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{x}-4i\sqrt{\pi}\arcsin\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}}$	36
trager	$-\frac{\sqrt{-4x^2+9}}{x}+2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2+9}\right)$	44

input
```
int((-4*x^2+9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
(4*x^2-9)/x/(-4*x^2+9)^(1/2)-2*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx=\frac{4\,x\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)-\sqrt{-4\,x^2+9}}{x}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(4*x*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x) - sqrt(-4*x^2 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx = -2\operatorname{asin}\left(\frac{2x}{3}\right) - \frac{\sqrt{9-4x^2}}{x}$$

input
```
integrate((-4*x**2+9)**(1/2)/x**2,x)
```

output
```
-2*asin(2*x/3) - sqrt(9 - 4*x**2)/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx = -\frac{\sqrt{-4\,x^2+9}}{x} - 2\,\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-sqrt(-4*x^2 + 9)/x - 2*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx = \frac{2\,x}{\sqrt{-4\,x^2+9}-3} - \frac{\sqrt{-4\,x^2+9}-3}{2\,x} - 2\,\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*x/(sqrt(-4*x^2 + 9) - 3) - 1/2*(sqrt(-4*x^2 + 9) - 3)/x - 2*arcsin(2/3*x
)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx = -2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)-\frac{2\,\sqrt{\frac{9}{4}-x^2}}{x}$$

input
```
int((9 - 4*x^2)^(1/2)/x^2,x)
```

output
```
- 2*asin((2*x)/3) - (2*(9/4 - x^2)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{9-4x^2}}{x^2}\,dx = \frac{-2asin\left(\frac{2x}{3}\right)x-\sqrt{-4x^2+9}}{x}$$

input
```
int((-4*x^2+9)^(1/2)/x^2,x)
```

output
```
( - 2*asin((2*x)/3)*x - sqrt( - 4*x**2 + 9))/x
```

3.470 $\int \frac{\sqrt{9-4x^2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{9-4x^2}}{x^3} \, dx = -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
-1/2*(-4*x^2+9)^(1/2)/x^2+2/3*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-4x^2}}{x^3} \, dx = -\frac{\sqrt{9-4x^2}}{2x^2} + \frac{2}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2]/x^3,x]
```

output
```
-1/2*Sqrt[9 - 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 51, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{9-4x^2}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-2\int \frac{1}{x^2\sqrt{9-4x^2}}dx^2-\frac{\sqrt{9-4x^2}}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}-\frac{\sqrt{9-4x^2}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{4}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{x^2}\right)$$

input
```
Int[Sqrt[9 - 4*x^2]/x^3,x]
```

output
```
(-(Sqrt[9 - 4*x^2]/x^2) + (4*ArcTanh[Sqrt[9 - 4*x^2]/3])/3)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.87

method	result	size
trager	$-\frac{\sqrt{-4x^2+9}}{2x^2}+\frac{2\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)}{3}$	34
risch	$\frac{4x^2-9}{2x^2\sqrt{-4x^2+9}}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$	37
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{-4x^2+9}}{9}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{3}$	41
pseudoelliptic	$\frac{2\ln\left(\sqrt{-4x^2+9}+3\right)x^2-2\ln\left(-3+\sqrt{-4x^2+9}\right)x^2-3\sqrt{-4x^2+9}}{6x^2}$	52
meijerg	$\frac{-\frac{9\sqrt{\pi}}{2x^2}-(-1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}+\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{16x^2}-\frac{9\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{2x^2}+2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right)}{3\sqrt{\pi}}$	85

input
```
int((-4*x^2+9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-4*x^2+9)^(1/2)/x^2+2/3*ln(((-4*x^2+9)^(1/2)+3)/x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = -\frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)+3\,\sqrt{-4\,x^2+9}}{6\,x^2}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
-1/6*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) + 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = \begin{cases} \frac{2\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{3} + \frac{1}{x\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2i\,\mathrm{asin}\left(\frac{3}{2x}\right)}{3} - \frac{i}{x\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-4*x**2+9)**(1/2)/x**3,x)
```

output
```
Piecewise((2*acosh(3/(2*x))/3 + 1/(x*sqrt(-1 + 9/(4*x**2))) - 9/(4*x**3*sq
rt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*I*asin(3/(2*x))/3 - I/(x*sqr
t(1 - 9/(4*x**2))) + 9*I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = -\frac{2}{9}\sqrt{-4\,x^2+9} - \frac{(-4\,x^2+9)^{\frac{3}{2}}}{18\,x^2} + \frac{2}{3}\log\left(\frac{6\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/9*sqrt(-4*x^2 + 9) - 1/18*(-4*x^2 + 9)^(3/2)/x^2 + 2/3*log(6*sqrt(-4*x^
2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = -\frac{\sqrt{-4\,x^2+9}}{2\,x^2} + \frac{1}{3}\,\log\left(\sqrt{-4\,x^2+9}+3\right) - \frac{1}{3}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(-4*x^2 + 9)/x^2 + 1/3*log(sqrt(-4*x^2 + 9) + 3) - 1/3*log(-sqrt(
-4*x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = -\frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{3} - \frac{\sqrt{\frac{9}{4}-x^2}}{x^2}$$

input
```
int((9 - 4*x^2)^(1/2)/x^3,x)
```

output
```
- (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/3 - (9/4 - x^2)^(1/
2)/x^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{9-4x^2}}{x^3}\,dx = \frac{-3\sqrt{-4x^2+9} - 4\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)x^2}{6x^2}$$

input
```
int((-4*x^2+9)^(1/2)/x^3,x)
```

output
```
( - 3*sqrt( - 4*x**2 + 9) - 4*log(tan(asin((2*x)/3)/2))*x**2)/(6*x**2)
```

3.471 $\int \frac{\sqrt{9-4x^2}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{9-4x^2}}{x^4} \, dx = -\frac{(9-4x^2)^{3/2}}{27x^3}$$

output
```
-1/27*(-4*x^2+9)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-4x^2}}{x^4} \, dx = -\frac{(9-4x^2)^{3/2}}{27x^3}$$

input
```
Integrate[Sqrt[9 - 4*x^2]/x^4,x]
```

output
```
-1/27*(9 - 4*x^2)^(3/2)/x^3
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{9-4x^2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{(9-4x^2)^{3/2}}{27x^3}$$

input
```
Int[Sqrt[9 - 4*x^2]/x^4,x]
```

output
```
-1/27*(9 - 4*x^2)^(3/2)/x^3
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\left(-4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
meijerg	$-\frac{\left(-\frac{4x^{2}}{9}+1\right)^{\frac{3}{2}}}{x^{3}}$	15
pseudoelliptic	$-\frac{\left(-4x^{2}+9\right)^{\frac{3}{2}}}{27x^{3}}$	15
trager	$\frac{\left(4x^{2}-9\right)\sqrt{-4x^{2}+9}}{27x^{3}}$	22
gosper	$\frac{(2x-3)(2x+3)\sqrt{-4x^{2}+9}}{27x^{3}}$	25
orering	$\frac{(2x-3)(2x+3)\sqrt{-4x^{2}+9}}{27x^{3}}$	25
risch	$-\frac{16x^{4}-72x^{2}+81}{27x^{3}\sqrt{-4x^{2}+9}}$	27

input
```
int((-4*x^2+9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/27*(-4*x^2+9)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = \frac{(4\,x^2-9)\sqrt{-4\,x^2+9}}{27\,x^3}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/27*(4*x^2 - 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.22

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = \begin{cases} \frac{8\sqrt{-1+\frac{9}{4x^2}}}{27} - \frac{2\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for}\ \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{8i\sqrt{1-\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((-4*x**2+9)**(1/2)/x**4,x)
```

output
```
Piecewise((8*sqrt(-1 + 9/(4*x**2))/27 - 2*sqrt(-1 + 9/(4*x**2))/(3*x**2),
1/Abs(x**2) > 4/9), (8*I*sqrt(1 - 9/(4*x**2))/27 - 2*I*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = -\frac{(-4\,x^2+9)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-1/27*(-4*x^2 + 9)^(3/2)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.06

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = -\frac{2\,x^3\left(\frac{3\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}-4\right)}{27\left(\sqrt{-4\,x^2+9}-3\right)^3}+\frac{\sqrt{-4\,x^2+9}-3}{18\,x}-\frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{216\,x^3}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-2/27*x^3*(3*(sqrt(-4*x^2 + 9) - 3)^2/x^2 - 4)/(sqrt(-4*x^2 + 9) - 3)^3 +
1/18*(sqrt(-4*x^2 + 9) - 3)/x - 1/216*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = \frac{8\,x^2\,\sqrt{\frac{9}{4}-x^2}-18\,\sqrt{\frac{9}{4}-x^2}}{27\,x^3}$$

input
```
int((9 - 4*x^2)^(1/2)/x^4,x)
```

output
```
(8*x^2*(9/4 - x^2)^(1/2) - 18*(9/4 - x^2)^(1/2))/(27*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{9-4x^2}}{x^4}\,dx = \frac{\sqrt{-4x^2+9}\,(4x^2-9)}{27x^3}$$

input
```
int((-4*x^2+9)^(1/2)/x^4,x)
```

output
```
(sqrt( - 4*x**2 + 9)*(4*x**2 - 9))/(27*x**3)
```

3.472 $\int \frac{\sqrt{9-4x^2}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sqrt{9-4x^2}}{x^5} \, dx = -\frac{\sqrt{9-4x^2}}{4x^4} + \frac{\sqrt{9-4x^2}}{18x^2} + \frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
-1/4*(-4*x^2+9)^(1/2)/x^4+1/18*(-4*x^2+9)^(1/2)/x^2+2/27*arctanh(1/3*(-4*x
^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{9-4x^2}}{x^5} \, dx = \frac{\sqrt{9-4x^2}(-9+2x^2)}{36x^4} + \frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[Sqrt[9 - 4*x^2]/x^5,x]
```

output
```
(Sqrt[9 - 4*x^2]*(-9 + 2*x^2))/(36*x^4) + (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/2
7
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{9-4x^2}}{x^5} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{\sqrt{9-4x^2}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(-\int \frac{1}{x^4\sqrt{9-4x^2}}dx^2-\frac{\sqrt{9-4x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{2}{9} \int \frac{1}{x^2\sqrt{9-4x^2}}dx^2+\frac{\sqrt{9-4x^2}}{9x^2}-\frac{\sqrt{9-4x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2} \left(\frac{1}{9} \int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}+\frac{\sqrt{9-4x^2}}{9x^2}-\frac{\sqrt{9-4x^2}}{2x^4}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2} \left(\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)+\frac{\sqrt{9-4x^2}}{9x^2}-\frac{\sqrt{9-4x^2}}{2x^4}\right)$$

input
```
Int[Sqrt[9 - 4*x^2]/x^5,x]
```

output
```
(-1/2*Sqrt[9 - 4*x^2]/x^4 + Sqrt[9 - 4*x^2]/(9*x^2) + (4*ArcTanh[Sqrt[9 -
4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result
trager	$\frac{(2x^2-9)\sqrt{-4x^2+9}}{36x^4}+\frac{2\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)}{27}$
risch	$-\frac{8x^4-54x^2+81}{36x^4\sqrt{-4x^2+9}}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$
default	$-\frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4}-\frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{-4x^2+9}}{81}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$
pseudoelliptic	$\frac{\frac{16\ln\left(\sqrt{-4x^2+9}+3\right)x^4}{27}-\frac{16\ln\left(-3+\sqrt{-4x^2+9}\right)x^4}{27}+\frac{8x^2\sqrt{-4x^2+9}}{9}-4\sqrt{-4x^2+9}}{\left(\sqrt{-4x^2+9}+3\right)^2\left(-3+\sqrt{-4x^2+9}\right)^2}$
meijerg	$-\frac{4\left(\frac{81\sqrt{\pi}}{16x^4}-\frac{9\sqrt{\pi}}{4x^2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{4}-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4-\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{-\frac{4x^2}{9}+1}}{128x^4}-\frac{\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+}}{2}\right.}{2}\right.}{27\sqrt{\pi}}$

input
```
int((-4*x^2+9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(2*x^2-9)/x^4*(-4*x^2+9)^(1/2)+2/27*ln(((-4*x^2+9)^(1/2)+3)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx=-\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,(2\,x^2-9)\sqrt{-4\,x^2+9}}{108\,x^4}$$

input
```
integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-1/108*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 - 9)*sqrt(-4*x^2 +
9))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.17 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx = \begin{cases} \frac{2\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2i\operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((-4*x**2+9)**(1/2)/x**5,x)
```

output
```
Piecewise((2*acosh(3/(2*x))/27 - 1/(9*x*sqrt(-1 + 9/(4*x**2))) + 3/(4*x**3
*sqrt(-1 + 9/(4*x**2))) - 9/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) >
4/9), (-2*I*asin(3/(2*x))/27 + I/(9*x*sqrt(1 - 9/(4*x**2))) - 3*I/(4*x**3*
sqrt(1 - 9/(4*x**2))) + 9*I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx = -\frac{2}{81}\sqrt{-4x^2+9} - \frac{(-4x^2+9)^{\frac{3}{2}}}{162x^2} - \frac{(-4x^2+9)^{\frac{3}{2}}}{36x^4} + \frac{2}{27}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/81*sqrt(-4*x^2 + 9) - 1/162*(-4*x^2 + 9)^(3/2)/x^2 - 1/36*(-4*x^2 + 9)^
(3/2)/x^4 + 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx = -\frac{(-4\,x^2+9)^{\frac{3}{2}}+9\sqrt{-4\,x^2+9}}{72\,x^4} + \frac{1}{27}\log\left(\sqrt{-4\,x^2+9}+3\right) - \frac{1}{27}\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate((-4*x^2+9)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/72*((-4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 + 9))/x^4 + 1/27*log(sqrt(-4*x^2
 + 9) + 3) - 1/27*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx = \frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2} - \frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27} - \frac{\sqrt{\frac{9}{4}-x^2}}{2\,x^4}$$

input
```
int((9 - 4*x^2)^(1/2)/x^5,x)
```

output
```
(9/4 - x^2)^(1/2)/(9*x^2) - (2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2
))/2))/27 - (9/4 - x^2)^(1/2)/(2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{9-4x^2}}{x^5}\,dx = \frac{6\sqrt{-4x^2+9}\,x^2-27\sqrt{-4x^2+9}-8\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)x^4}{108x^4}$$

input
```
int((-4*x^2+9)^(1/2)/x^5,x)
```

output
```
(6*sqrt( - 4*x**2 + 9)*x**2 - 27*sqrt( - 4*x**2 + 9) - 8*log(tan(asin((2*x
)/3)/2))*x**4)/(108*x**4)
```

3.473 $\int x^5\sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x^5\sqrt{-9+4x^2}\,dx=\frac{27}{64}\left(-9+4x^2\right)^{3/2}+\frac{9}{160}\left(-9+4x^2\right)^{5/2}+\frac{1}{448}\left(-9+4x^2\right)^{7/2}$$

output
```
27/64*(4*x^2-9)^(3/2)+9/160*(4*x^2-9)^(5/2)+1/448*(4*x^2-9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{-9+4x^2}\,dx=\frac{1}{280}\left(-9+4x^2\right)^{3/2}\left(27+18x^2+10x^4\right)$$

input
```
Integrate[x^5*Sqrt[-9 + 4*x^2],x]
```

output
```
((-9 + 4*x^2)^(3/2)*(27 + 18*x^2 + 10*x^4))/280
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4\sqrt{4x^2-9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{16}(4x^2-9)^{5/2}+\frac{9}{8}(4x^2-9)^{3/2}+\frac{81}{16}\sqrt{4x^2-9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{224}(4x^2-9)^{7/2}+\frac{9}{80}(4x^2-9)^{5/2}+\frac{27}{32}(4x^2-9)^{3/2}\right)$$

input
```
Int[x^5*Sqrt[-9 + 4*x^2],x]
```

output
```
((27*(-9 + 4*x^2)^(3/2))/32 + (9*(-9 + 4*x^2)^(5/2))/80 + (-9 + 4*x^2)^(7/
2)/224)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$\frac{\left(4 x^{2}-9\right)^{\frac{3}{2}} \left(10 x^{4}+18 x^{2}+27\right)}{280}$	24
trager	$\left(\frac{1}{7} x^{6}-\frac{9}{140} x^{4}-\frac{27}{140} x^{2}-\frac{243}{280}\right) \sqrt{4 x^{2}-9}$	28
risch	$\frac{\left(40 x^{6}-18 x^{4}-54 x^{2}-243\right) \sqrt{4 x^{2}-9}}{280}$	29
gosper	$\frac{\left(2 x-3\right) \left(2 x+3\right) \left(10 x^{4}+18 x^{2}+27\right) \sqrt{4 x^{2}-9}}{280}$	34
orering	$\frac{\left(2 x-3\right) \left(2 x+3\right) \left(10 x^{4}+18 x^{2}+27\right) \sqrt{4 x^{2}-9}}{280}$	34
default	$\frac{x^{4} \left(4 x^{2}-9\right)^{\frac{3}{2}}}{28}+\frac{9 x^{2} \left(4 x^{2}-9\right)^{\frac{3}{2}}}{140}+\frac{27 \left(4 x^{2}-9\right)^{\frac{3}{2}}}{280}$	41
meijerg	$\frac{2187 \sqrt{\operatorname{signum}\left(-1+\frac{4 x^{2}}{9}\right)}\, \left(\frac{32 \sqrt{\pi}}{105}-\frac{4 \sqrt{\pi}\, \left(-\frac{4 x^{2}}{9}+1\right)^{\frac{3}{2}} \left(\frac{80}{27} x^{4}+\frac{16}{3} x^{2}+8\right)}{105}\right)}{256 \sqrt{\pi}\, \sqrt{-\operatorname{signum}\left(-1+\frac{4 x^{2}}{9}\right)}}$	60

input
```
int(x^5*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/280*(4*x^2-9)^(3/2)*(10*x^4+18*x^2+27)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{-9+4x^2}\,dx = \frac{1}{280}\left(40\,x^6-18\,x^4-54\,x^2-243\right)\sqrt{4\,x^2-9}$$

input
```
integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/280*(40*x^6 - 18*x^4 - 54*x^2 - 243)*sqrt(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int x^5\sqrt{-9+4x^2}\,dx = \frac{x^6\sqrt{4x^2-9}}{7}-\frac{9x^4\sqrt{4x^2-9}}{140}-\frac{27x^2\sqrt{4x^2-9}}{140}-\frac{243\sqrt{4x^2-9}}{280}$$

input
```
integrate(x**5*(4*x**2-9)**(1/2),x)
```

output
```
x**6*sqrt(4*x**2 - 9)/7 - 9*x**4*sqrt(4*x**2 - 9)/140 - 27*x**2*sqrt(4*x**
2 - 9)/140 - 243*sqrt(4*x**2 - 9)/280
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x^5\sqrt{-9+4x^2}\,dx = \frac{1}{28}\left(4\,x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\left(4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{27}{280}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/28*(4*x^2 - 9)^(3/2)*x^4 + 9/140*(4*x^2 - 9)^(3/2)*x^2 + 27/280*(4*x^2 -
 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{-9+4x^2}\,dx = \frac{1}{448}\,(4\,x^2-9)^{\frac{7}{2}} + \frac{9}{160}\,(4\,x^2-9)^{\frac{5}{2}} + \frac{27}{64}\,(4\,x^2-9)^{\frac{3}{2}}$$

input
```
integrate(x^5*(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/448*(4*x^2 - 9)^(7/2) + 9/160*(4*x^2 - 9)^(5/2) + 27/64*(4*x^2 - 9)^(3/2
)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{-9+4x^2}\,dx = -\sqrt{4\,x^2-9}\,\left(-\frac{x^6}{7}+\frac{9\,x^4}{140}+\frac{27\,x^2}{140}+\frac{243}{280}\right)$$

input
```
int(x^5*(4*x^2 - 9)^(1/2),x)
```

output
```
-(4*x^2 - 9)^(1/2)*((27*x^2)/140 + (9*x^4)/140 - x^6/7 + 243/280)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,(40x^6-18x^4-54x^2-243)}{280}$$

input
```
int(x^5*(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9)*(40*x**6 - 18*x**4 - 54*x**2 - 243))/280
```

3.474 $\int x^4\sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int x^4\sqrt{-9+4x^2}\,dx=-\frac{81}{256}x\sqrt{-9+4x^2}-\frac{3}{32}x^3\sqrt{-9+4x^2}+\frac{1}{6}x^5\sqrt{-9+4x^2}-\frac{729}{512}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
-81/256*x*(4*x^2-9)^(1/2)-3/32*x^3*(4*x^2-9)^(1/2)+1/6*x^5*(4*x^2-9)^(1/2)
-729/512*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int x^4\sqrt{-9+4x^2}\,dx=\frac{1}{768}x\sqrt{-9+4x^2}\left(-243-72x^2+128x^4\right)+\frac{729}{512}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[x^4*Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2]*(-243 - 72*x^2 + 128*x^4))/768 + (729*Log[-2*x + Sqrt[
-9 + 4*x^2]])/512
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6}x^5\sqrt{4x^2-9}-\frac{3}{2}\int \frac{x^4}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^5\sqrt{4x^2-9}-\frac{3}{2}\left(\frac{27}{16}\int \frac{x^2}{\sqrt{4x^2-9}}dx+\frac{1}{16}\sqrt{4x^2-9}x^3\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^5\sqrt{4x^2-9}-\frac{3}{2}\left(\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{\sqrt{4x^2-9}}dx+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{6}x^5\sqrt{4x^2-9}-\frac{3}{2}\left(\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}x^5\sqrt{4x^2-9}-\frac{3}{2}\left(\frac{27}{16}\left(\frac{9}{16}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3\right)$$

input
```
Int[x^4*Sqrt[-9 + 4*x^2],x]
```

output
```
(x^5*Sqrt[-9 + 4*x^2])/6 - (3*((x^3*Sqrt[-9 + 4*x^2])/16 + (27*((x*Sqrt[-9
 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16))/16))/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

method	result	size
trager	$\frac{x(128x^4-72x^2-243)\sqrt{4x^2-9}}{768}-\frac{729\ln\left(\sqrt{4x^2-9}+2x\right)}{512}$	42
risch	$\frac{x(128x^4-72x^2-243)\sqrt{4x^2-9}}{768}-\frac{729\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{1024}$	47
default	$\frac{x^3(4x^2-9)^{\frac{3}{2}}}{24}+\frac{9x(4x^2-9)^{\frac{3}{2}}}{128}+\frac{81x\sqrt{4x^2-9}}{256}-\frac{729\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{1024}$	61
meijerg	$\frac{729i\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{i\sqrt{\pi}\,x\left(-\frac{640}{81}x^4+\frac{40}{9}x^2+15\right)\sqrt{-\frac{4x^2}{9}+1}}{90}-\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	68
pseudoelliptic	$\frac{531441\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)-531441\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)+(124416x^5-69984x^3-236196x)\sqrt{4x^2-9}}{1024\left(\sqrt{4x^2-9}+2x\right)^3\left(-\sqrt{4x^2-9}+2x\right)^3}$	100

input
```
int(x^4*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/768*x*(128*x^4-72*x^2-243)*(4*x^2-9)^(1/2)-729/512*ln((4*x^2-9)^(1/2)+2*
x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int x^4\sqrt{-9+4x^2}\,dx=\frac{1}{768}\left(128\,x^5-72\,x^3-243\,x\right)\sqrt{4\,x^2-9}+\frac{729}{512}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/768*(128*x^5 - 72*x^3 - 243*x)*sqrt(4*x^2 - 9) + 729/512*log(-2*x + sqrt
(4*x^2 - 9))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.01 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.29

$$\int x^4\sqrt{-9+4x^2}\,dx = \begin{cases} \frac{2x^7}{3\sqrt{4x^2-9}} - \frac{15x^5}{8\sqrt{4x^2-9}} - \frac{27x^3}{64\sqrt{4x^2-9}} + \frac{729x}{256\sqrt{4x^2-9}} - \frac{729\,\mathrm{acosh}\left(\frac{2x}{3}\right)}{512} & \text{for } |x^2| > \frac{9}{4} \\ -\frac{2ix^7}{3\sqrt{9-4x^2}} + \frac{15ix^5}{8\sqrt{9-4x^2}} + \frac{27ix^3}{64\sqrt{9-4x^2}} - \frac{729ix}{256\sqrt{9-4x^2}} + \frac{729i\,\mathrm{asin}\left(\frac{2x}{3}\right)}{512} & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((2*x**7/(3*sqrt(4*x**2 - 9)) - 15*x**5/(8*sqrt(4*x**2 - 9)) - 27
*x**3/(64*sqrt(4*x**2 - 9)) + 729*x/(256*sqrt(4*x**2 - 9)) - 729*acosh(2*x
/3)/512, Abs(x**2) > 9/4), (-2*I*x**7/(3*sqrt(9 - 4*x**2)) + 15*I*x**5/(8*
sqrt(9 - 4*x**2)) + 27*I*x**3/(64*sqrt(9 - 4*x**2)) - 729*I*x/(256*sqrt(9
- 4*x**2)) + 729*I*asin(2*x/3)/512, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int x^4\sqrt{-9+4x^2}\,dx = \frac{1}{24}\left(4x^2-9\right)^{\frac{3}{2}}x^3 + \frac{9}{128}\left(4x^2-9\right)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{4x^2-9}\,x - \frac{729}{512}\log\left(8x+4\sqrt{4x^2-9}\right)$$

input
```
integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(4*x^2 - 9)^(3/2)*x^3 + 9/128*(4*x^2 - 9)^(3/2)*x + 81/256*sqrt(4*x^2
 - 9)*x - 729/512*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int x^4\sqrt{-9+4x^2}\,dx = \frac{1}{768}\left(8\left(16\,x^2-9\right)x^2-243\right)\sqrt{4\,x^2-9}x + \frac{729}{512}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

input
```
integrate(x^4*(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/768*(8*(16*x^2 - 9)*x^2 - 243)*sqrt(4*x^2 - 9)*x + 729/512*log(abs(-2*x
+ sqrt(4*x^2 - 9)))
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{-9+4x^2}\,dx = \int x^4\,\sqrt{4\,x^2-9}\,dx$$

input
```
int(x^4*(4*x^2 - 9)^(1/2),x)
```

output
```
int(x^4*(4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^4\sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,x^5}{6} - \frac{3\sqrt{4x^2-9}\,x^3}{32} - \frac{81\sqrt{4x^2-9}\,x}{256} - \frac{729\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{512}$$

input
```
int(x^4*(4*x^2-9)^(1/2),x)
```

output
```
(256*sqrt(4*x**2 - 9)*x**5 - 144*sqrt(4*x**2 - 9)*x**3 - 486*sqrt(4*x**2 -
 9)*x - 2187*log((sqrt(4*x**2 - 9) + 2*x)/3))/1536
```

3.475 $\int x^3\sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{-9+4x^2}\,dx=\frac{3}{16}\left(-9+4x^2\right)^{3/2}+\frac{1}{80}\left(-9+4x^2\right)^{5/2}$$

output
```
3/16*(4*x^2-9)^(3/2)+1/80*(4*x^2-9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{-9+4x^2}\,dx=\frac{1}{40}\left(3+2x^2\right)\left(-9+4x^2\right)^{3/2}$$

input
```
Integrate[x^3*Sqrt[-9 + 4*x^2],x]
```

output
```
((3 + 2*x^2)*(-9 + 4*x^2)^(3/2))/40
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2\sqrt{4x^2-9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{4}\left(4x^2-9\right)^{3/2}+\frac{9}{4}\sqrt{4x^2-9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{40}\left(4x^2-9\right)^{5/2}+\frac{3}{8}\left(4x^2-9\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[-9 + 4*x^2],x]
```

output
```
((3*(-9 + 4*x^2)^(3/2))/8 + (-9 + 4*x^2)^(5/2)/40)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
pseudoelliptic	$\frac{\left(4x^{2}-9\right)^{\frac{3}{2}}\left(2x^{2}+3\right)}{40}$	19
trager	$\left(\frac{1}{5}x^{4}-\frac{3}{20}x^{2}-\frac{27}{40}\right)\sqrt{4x^{2}-9}$	23
risch	$\frac{\left(8x^{4}-6x^{2}-27\right)\sqrt{4x^{2}-9}}{40}$	24
default	$\frac{x^{2}\left(4x^{2}-9\right)^{\frac{3}{2}}}{20}+\frac{3\left(4x^{2}-9\right)^{\frac{3}{2}}}{40}$	27
gosper	$\frac{\left(2x-3\right)\left(2x+3\right)\left(2x^{2}+3\right)\sqrt{4x^{2}-9}}{40}$	29
orering	$\frac{\left(2x-3\right)\left(2x+3\right)\left(2x^{2}+3\right)\sqrt{4x^{2}-9}}{40}$	29
meijerg	$-\frac{243\sqrt{\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(-\frac{4x^{2}}{9}+1\right)^{\frac{3}{2}}\left(\frac{4x^{2}}{3}+2\right)}{15}\right)}{64\sqrt{\pi}\sqrt{-\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}}$	55

input
```
int(x^3*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/40*(4*x^2-9)^(3/2)*(2*x^2+3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{-9+4x^2}\,dx = \frac{1}{40}\left(8\,x^4-6\,x^2-27\right)\sqrt{4\,x^2-9}$$

input
```
integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(8*x^4 - 6*x^2 - 27)*sqrt(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int x^3\sqrt{-9+4x^2}\,dx = \frac{x^4\sqrt{4x^2-9}}{5}-\frac{3x^2\sqrt{4x^2-9}}{20}-\frac{27\sqrt{4x^2-9}}{40}$$

input
```
integrate(x**3*(4*x**2-9)**(1/2),x)
```

output
```
x**4*sqrt(4*x**2 - 9)/5 - 3*x**2*sqrt(4*x**2 - 9)/20 - 27*sqrt(4*x**2 - 9)
/40
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{-9+4x^2}\,dx = \frac{1}{20}\left(4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/20*(4*x^2 - 9)^(3/2)*x^2 + 3/40*(4*x^2 - 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{-9+4x^2}\,dx = \frac{1}{80}\left(4\,x^2-9\right)^{\frac{5}{2}} + \frac{3}{16}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/80*(4*x^2 - 9)^(5/2) + 3/16*(4*x^2 - 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{-9+4x^2}\,dx = -\sqrt{4\,x^2-9}\left(-\frac{x^4}{5}+\frac{3\,x^2}{20}+\frac{27}{40}\right)$$

input
```
int(x^3*(4*x^2 - 9)^(1/2),x)
```

output
```
-(4*x^2 - 9)^(1/2)*((3*x^2)/20 - x^4/5 + 27/40)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,(8x^4-6x^2-27)}{40}$$

input
```
int(x^3*(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9)*(8*x**4 - 6*x**2 - 27))/40
```

3.476 $\int x^2\sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int x^2\sqrt{-9+4x^2}\,dx=-\frac{9}{32}x\sqrt{-9+4x^2}+\frac{1}{4}x^3\sqrt{-9+4x^2}-\frac{81}{64}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
-9/32*x*(4*x^2-9)^(1/2)+1/4*x^3*(4*x^2-9)^(1/2)-81/64*arctanh(2*x/(4*x^2-9
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int x^2\sqrt{-9+4x^2}\,dx=\frac{1}{32}x\sqrt{-9+4x^2}\left(-9+8x^2\right)+\frac{81}{64}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[x^2*Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2]*(-9 + 8*x^2))/32 + (81*Log[-2*x + Sqrt[-9 + 4*x^2]])/6
4
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{4}x^3\sqrt{4x^2-9}-\frac{9}{4}\int \frac{x^2}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}x^3\sqrt{4x^2-9}-\frac{9}{4}\left(\frac{9}{8}\int \frac{1}{\sqrt{4x^2-9}}dx+\frac{1}{8}\sqrt{4x^2-9}x\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}x^3\sqrt{4x^2-9}-\frac{9}{4}\left(\frac{9}{8}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}+\frac{1}{8}\sqrt{4x^2-9}x\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}x^3\sqrt{4x^2-9}-\frac{9}{4}\left(\frac{9}{16}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{8}\sqrt{4x^2-9}x\right)$$

input
```
Int[x^2*Sqrt[-9 + 4*x^2],x]
```

output
```
(x^3*Sqrt[-9 + 4*x^2])/4 - (9*((x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/S
qrt[-9 + 4*x^2]])/16))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

method	result	size
trager	$\frac{x(8x^2-9)\sqrt{4x^2-9}}{32}-\frac{81\ln\left(\sqrt{4x^2-9}+2x\right)}{64}$	37
risch	$\frac{x(8x^2-9)\sqrt{4x^2-9}}{32}-\frac{81\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{128}$	42
default	$\frac{x(4x^2-9)^{\frac{3}{2}}}{16}+\frac{9x\sqrt{4x^2-9}}{32}-\frac{81\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{128}$	47
meijerg	$-\frac{81i\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{i\sqrt{\pi}\,x\left(-\frac{8x^2}{3}+3\right)\sqrt{-\frac{4x^2}{9}+1}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{2}\right)}{32\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	63
pseudoelliptic	$\frac{6561\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)-6561\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)+(2592x^3-2916x)\sqrt{4x^2-9}}{128\left(\sqrt{4x^2-9}+2x\right)^2\left(-\sqrt{4x^2-9}+2x\right)^2}$	95

input
```
int(x^2*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/32*x*(8*x^2-9)*(4*x^2-9)^(1/2)-81/64*ln((4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{-9+4x^2}\,dx=\frac{1}{32}\left(8\,x^3-9\,x\right)\sqrt{4\,x^2-9}+\frac{81}{64}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 - 9*x)*sqrt(4*x^2 - 9) + 81/64*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.26

$$\int x^2\sqrt{-9+4x^2}\,dx = \begin{cases} \frac{x^5}{\sqrt{4x^2-9}} - \frac{27x^3}{8\sqrt{4x^2-9}} + \frac{81x}{32\sqrt{4x^2-9}} - \frac{81\operatorname{acosh}\left(\frac{2x}{3}\right)}{64} & \text{for } |x^2| > \frac{9}{4} \\ -\frac{ix^5}{\sqrt{9-4x^2}} + \frac{27ix^3}{8\sqrt{9-4x^2}} - \frac{81ix}{32\sqrt{9-4x^2}} + \frac{81i\operatorname{asin}\left(\frac{2x}{3}\right)}{64} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((x**5/sqrt(4*x**2 - 9) - 27*x**3/(8*sqrt(4*x**2 - 9)) + 81*x/(32
*sqrt(4*x**2 - 9)) - 81*acosh(2*x/3)/64, Abs(x**2) > 9/4), (-I*x**5/sqrt(9
 - 4*x**2) + 27*I*x**3/(8*sqrt(9 - 4*x**2)) - 81*I*x/(32*sqrt(9 - 4*x**2))
 + 81*I*asin(2*x/3)/64, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{-9+4x^2}\,dx = \frac{1}{16}\left(4x^2-9\right)^{\frac{3}{2}}x + \frac{9}{32}\sqrt{4x^2-9}x - \frac{81}{64}\log\left(8x+4\sqrt{4x^2-9}\right)$$

input
```
integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/16*(4*x^2 - 9)^(3/2)*x + 9/32*sqrt(4*x^2 - 9)*x - 81/64*log(8*x + 4*sqrt
(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int x^2\sqrt{-9+4x^2}\,dx = \frac{1}{32}\left(8x^2-9\right)\sqrt{4x^2-9}x + \frac{81}{64}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

input
```
integrate(x^2*(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/32*(8*x^2 - 9)*sqrt(4*x^2 - 9)*x + 81/64*log(abs(-2*x + sqrt(4*x^2 - 9))
)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{-9+4x^2}\,dx = \int x^2\sqrt{4x^2-9}\,dx$$

input
```
int(x^2*(4*x^2 - 9)^(1/2),x)
```

output
```
int(x^2*(4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^2\sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,x^3}{4} - \frac{9\sqrt{4x^2-9}\,x}{32} - \frac{81\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{64}$$

input
```
int(x^2*(4*x^2-9)^(1/2),x)
```

output
```
(16*sqrt(4*x**2 - 9)*x**3 - 18*sqrt(4*x**2 - 9)*x - 81*log((sqrt(4*x**2 -
9) + 2*x)/3))/64
```

3.477 $\int x\sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

output
```
1/12*(4*x^2-9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(-9+4x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[-9 + 4*x^2],x]
```

output
```
(-9 + 4*x^2)^(3/2)/12
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{12}\left(4x^2-9\right)^{3/2}$$

input
```
Int[x*Sqrt[-9 + 4*x^2],x]
```

output
```
(-9 + 4*x^2)^(3/2)/12
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
risch	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
pseudoelliptic	$\frac{(4x^2-9)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^2}{3}-\frac{3}{4}\right)\sqrt{4x^2-9}$	18
gosper	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{12}$	22
orering	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{12}$	22
meijerg	$\frac{27\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2-\frac{8x^2}{9}\right)\sqrt{-\frac{4x^2}{9}+1}}{3}\right)}{16\sqrt{\pi}\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	55

input
```
int(x*(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(4*x^2-9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9+4x^2}\,dx=\frac{1}{12}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(4*x^2 - 9)^(3/2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x\sqrt{-9+4x^2}\,dx = \frac{x^2\sqrt{4x^2-9}}{3} - \frac{3\sqrt{4x^2-9}}{4}$$

input
```
integrate(x*(4*x**2-9)**(1/2),x)
```

output
```
x**2*sqrt(4*x**2 - 9)/3 - 3*sqrt(4*x**2 - 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9+4x^2}\,dx = \frac{1}{12}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/12*(4*x^2 - 9)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9+4x^2}\,dx = \frac{1}{12}\left(4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/12*(4*x^2 - 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9+4x^2}\,dx = \frac{(4\,x^2-9)^{3/2}}{12}$$

input
```
int(x*(4*x^2 - 9)^(1/2),x)
```

output
```
(4*x^2 - 9)^(3/2)/12
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x\sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,(4x^2-9)}{12}$$

input
```
int(x*(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9)*(4*x**2 - 9))/12
```

3.478 $\int \sqrt{-9+4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \sqrt{-9+4x^2}\,dx=\frac{1}{2}x\sqrt{-9+4x^2}-\frac{9}{4}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
1/2*x*(4*x^2-9)^(1/2)-9/4*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \sqrt{-9+4x^2}\,dx=\frac{1}{2}x\sqrt{-9+4x^2}+\frac{9}{4}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/2 + (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2-9}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{2}\int \frac{1}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{2}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}x\sqrt{4x^2-9}-\frac{9}{4}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

input
```
Int[Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/2 - (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

method	result	size
trager	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4x^2-9}+2x\right)}{4}$	30
default	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
risch	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{8}$	35
pseudoelliptic	$\frac{x\sqrt{4x^2-9}}{2}-\frac{9\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)}{8}+\frac{9\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)}{8}$	54
meijerg	$\frac{9i\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{4i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-2i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	56

input
```
int((4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(4*x^2-9)^(1/2)-9/4*ln((4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int\sqrt{-9+4x^2}\,dx=\frac{1}{2}\sqrt{4\,x^2-9}x+\frac{9}{4}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{-9+4x^2}\,dx = \frac{x\sqrt{4x^2-9}}{2} - \frac{9\log\left(2x+\sqrt{4x^2-9}\right)}{4}$$

input
```
integrate((4*x**2-9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 - 9)/2 - 9*log(2*x + sqrt(4*x**2 - 9))/4
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2-9}x - \frac{9}{4}\log\left(8x+4\sqrt{4x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 - 9)*x - 9/4*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \sqrt{-9+4x^2}\,dx = \frac{1}{2}\sqrt{4x^2-9}x + \frac{9}{4}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

input
```
integrate((4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{-9+4x^2}\,dx = \frac{x\sqrt{4\,x^2-9}}{2} - \frac{9\,\ln\left(2\,x+\sqrt{4\,x^2-9}\right)}{4}$$

input
```
int((4*x^2 - 9)^(1/2),x)
```

output
```
(x*(4*x^2 - 9)^(1/2))/2 - (9*log(2*x + (4*x^2 - 9)^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \sqrt{-9+4x^2}\,dx = \frac{\sqrt{4x^2-9}\,x}{2} - \frac{9\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{4}$$

input
```
int((4*x^2-9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 - 9)*x - 9*log((sqrt(4*x**2 - 9) + 2*x)/3))/4
```

3.479 $\int \frac{\sqrt{-9+4x^2}}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{-9+4x^2}}{x} dx = \sqrt{-9+4x^2} - 3\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
(4*x^2-9)^(1/2)-3*arctan(1/3*(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9+4x^2}}{x} dx = \sqrt{-9+4x^2} - 3\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2]/x,x]
```

output
```
Sqrt[-9 + 4*x^2] - 3*ArcTan[Sqrt[-9 + 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-9}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2-9}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(2\sqrt{4x^2-9}-9\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(2\sqrt{4x^2-9}-\frac{9}{2}\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(2\sqrt{4x^2-9}-6\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)\right)$$

input
```
Int[Sqrt[-9 + 4*x^2]/x,x]
```

output
```
(2*Sqrt[-9 + 4*x^2] - 6*ArcTan[Sqrt[-9 + 4*x^2]/3])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{4x^2-9}+3\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)$	25
pseudoelliptic	$\sqrt{4x^2-9}-3\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right)$	25
trager	$\sqrt{4x^2-9}-3\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2+1\right)+\sqrt{4x^2-9}}{x}\right)$	42
meijerg	$-\frac{3\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-2(2+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{-\frac{4x^2}{9}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right)\right)}{4\sqrt{\pi}\,\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	86

input
```
int((4*x^2-9)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(4*x^2-9)^(1/2)+3*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx=\sqrt{4\,x^2-9}-6\,\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(4*x^2 - 9) - 6*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.67

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx=\begin{cases}\sqrt{4x^2-9}-3i\log{(x)}+\frac{3i\log{(x^2)}}{2}+3\operatorname{asin}{\left(\frac{3}{2x}\right)} & \text{for}\ |x^2|>\frac{9}{4}\\ i\sqrt{9-4x^2}+\frac{3i\log{(x^2)}}{2}-3i\log{\left(\sqrt{1-\frac{4x^2}{9}}+1\right)} & \text{otherwise}\end{cases}$$

input
```
integrate((4*x**2-9)**(1/2)/x,x)
```

output
```
Piecewise((sqrt(4*x**2 - 9) - 3*I*log(x) + 3*I*log(x**2)/2 + 3*asin(3/(2*x
)), Abs(x**2) > 9/4), (I*sqrt(9 - 4*x**2) + 3*I*log(x**2)/2 - 3*I*log(sqrt
(1 - 4*x**2/9) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx = \sqrt{4\,x^2-9}+3\,\arcsin\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(4*x^2 - 9) + 3*arcsin(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx = \sqrt{4\,x^2-9}-3\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(4*x^2 - 9) - 3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx = \sqrt{4\,x^2-9} - 3\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)$$

input
```
int((4*x^2 - 9)^(1/2)/x,x)
```

output
```
(4*x^2 - 9)^(1/2) - 3*atan((4*x^2 - 9)^(1/2)/3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{-9+4x^2}}{x}\,dx = -6atan\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)+\sqrt{4x^2-9}$$

input
```
int((4*x^2-9)^(1/2)/x,x)
```

output
```
- 6*atan((sqrt(4*x**2 - 9) + 2*x)/3) + sqrt(4*x**2 - 9)
```

3.480 $\int \frac{\sqrt{-9+4x^2}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\sqrt{-9+4x^2}}{x^2} dx = -\frac{\sqrt{-9+4x^2}}{x} + 2\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
-(4*x^2-9)^(1/2)/x+2*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{-9+4x^2}}{x^2} dx = -\frac{\sqrt{-9+4x^2}}{x} - 2\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2]/x^2,x]
```

output
```
-(Sqrt[-9 + 4*x^2]/x) - 2*Log[-2*x + Sqrt[-9 + 4*x^2]]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-9}}{x^2} \, dx$$

$$\Big\downarrow \text{247}$$

$$4\int \frac{1}{\sqrt{4x^2-9}}dx-\frac{\sqrt{4x^2-9}}{x}$$

$$\Big\downarrow \text{224}$$

$$4\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}-\frac{\sqrt{4x^2-9}}{x}$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)-\frac{\sqrt{4x^2-9}}{x}$$

input
```
Int[Sqrt[-9 + 4*x^2]/x^2,x]
```

output
```
-(Sqrt[-9 + 4*x^2]/x) + 2*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
trager	$-\frac{\sqrt{4x^2-9}}{x}+2\ln\left(\sqrt{4x^2-9}+2x\right)$	32
risch	$-\frac{\sqrt{4x^2-9}}{x}+\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}$	36
default	$\frac{\left(4x^2-9\right)^{\frac{3}{2}}}{9x}-\frac{4x\sqrt{4x^2-9}}{9}+\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}$	48
meijerg	$-\frac{i\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{6i\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{x}-4i\sqrt{\pi}\arcsin\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$	58
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)x-\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)x-\sqrt{4x^2-9}}{x}$	58

input
```
int((4*x^2-9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(4*x^2-9)^(1/2)/x+2*ln((4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int\frac{\sqrt{-9+4x^2}}{x^2}\,dx=-\frac{2\,x\log\left(-2\,x+\sqrt{4\,x^2-9}\right)+2\,x+\sqrt{4\,x^2-9}}{x}$$

input
```
integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
-(2*x*log(-2*x + sqrt(4*x^2 - 9)) + 2*x + sqrt(4*x^2 - 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = 2\log\left(2x+\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{x}$$

input
```
integrate((4*x**2-9)**(1/2)/x**2,x)
```

output
```
2*log(2*x + sqrt(4*x**2 - 9)) - sqrt(4*x**2 - 9)/x
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = -\frac{\sqrt{4\,x^2-9}}{x}+2\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-sqrt(4*x^2 - 9)/x + 2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = -\frac{36}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}-\log\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

output
```
-36/((2*x - sqrt(4*x^2 - 9))^2 + 9) - log((2*x - sqrt(4*x^2 - 9))^2)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = -\frac{\sqrt{4\,x^2-9}}{x} - \frac{2\,\mathrm{asin}\left(\frac{2\,x}{3}\right)\,\sqrt{4\,x^2-9}}{3\,\sqrt{1-\frac{4\,x^2}{9}}}$$

input
```
int((4*x^2 - 9)^(1/2)/x^2,x)
```

output
```
- (4*x^2 - 9)^(1/2)/x - (2*asin((2*x)/3)*(4*x^2 - 9)^(1/2))/(3*(1 - (4*x^2
)/9)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{-9+4x^2}}{x^2}\,dx = \frac{-\sqrt{4x^2-9}+2\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)x-2x}{x}$$

input
```
int((4*x^2-9)^(1/2)/x^2,x)
```

output
```
( - sqrt(4*x**2 - 9) + 2*log((sqrt(4*x**2 - 9) + 2*x)/3)*x - 2*x)/x
```

3.481 $\int \frac{\sqrt{-9+4x^2}}{x^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{-9+4x^2}}{x^3} \, dx = -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3} \arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
-1/2*(4*x^2-9)^(1/2)/x^2+2/3*arctan(1/3*(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9+4x^2}}{x^3} \, dx = -\frac{\sqrt{-9+4x^2}}{2x^2} + \frac{2}{3} \arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2]/x^3,x]
```

output
```
-1/2*Sqrt[-9 + 4*x^2]/x^2 + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 51, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-9}}{x^3} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2-9}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(2\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2-\frac{\sqrt{4x^2-9}}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}-\frac{\sqrt{4x^2-9}}{x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{4}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)-\frac{\sqrt{4x^2-9}}{x^2}\right)$$

input
```
Int[Sqrt[-9 + 4*x^2]/x^3,x]
```

output
```
(-(Sqrt[-9 + 4*x^2]/x^2) + (4*ArcTan[Sqrt[-9 + 4*x^2]/3])/3)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\sqrt{4x^2-9}}{2x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$
pseudoelliptic	$\frac{4\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right)x^2-3\sqrt{4x^2-9}}{6x^2}$
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{18x^2}-\frac{2\sqrt{4x^2-9}}{9}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{3}$
trager	$-\frac{\sqrt{4x^2-9}}{2x^2}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)-\sqrt{4x^2-9}}{x}\right)}{3}$
meijerg	$\frac{\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{9\sqrt{\pi}}{2x^2}-(-1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}+\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{16x^2}-\frac{9\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right.\right.}{3\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

input
```
int((4*x^2-9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(4*x^2-9)^(1/2)/x^2-2/3*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx=\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)-3\,\sqrt{4\,x^2-9}}{6\,x^2}$$

input
```
integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/6*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) - 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.49

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx = \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} + \frac{i}{x\sqrt{-1+\frac{9}{4x^2}}} - \frac{9i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{3} - \frac{1}{x\sqrt{1-\frac{9}{4x^2}}} + \frac{9}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((4*x**2-9)**(1/2)/x**3,x)
```

output
```
Piecewise((2*I*acosh(3/(2*x))/3 + I/(x*sqrt(-1 + 9/(4*x**2))) - 9*I/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*asin(3/(2*x))/3 - 1/(x*s
qrt(1 - 9/(4*x**2))) + 9/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx = -\frac{2}{9}\sqrt{4x^2-9} + \frac{(4x^2-9)^{\frac{3}{2}}}{18x^2} - \frac{2}{3}\arcsin\left(\frac{3}{2|x|}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
-2/9*sqrt(4*x^2 - 9) + 1/18*(4*x^2 - 9)^(3/2)/x^2 - 2/3*arcsin(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx = -\frac{\sqrt{4\,x^2-9}}{2\,x^2} + \frac{2}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(4*x^2 - 9)/x^2 + 2/3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{3} - \frac{\sqrt{4\,x^2-9}}{2\,x^2}$$

input
```
int((4*x^2 - 9)^(1/2)/x^3,x)
```

output
```
(2*atan((4*x^2 - 9)^(1/2)/3))/3 - (4*x^2 - 9)^(1/2)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{-9+4x^2}}{x^3}\,dx = \frac{8atan\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)x^2-3\sqrt{4x^2-9}}{6x^2}$$

input
```
int((4*x^2-9)^(1/2)/x^3,x)
```

output
```
(8*atan((sqrt(4*x**2 - 9) + 2*x)/3)*x**2 - 3*sqrt(4*x**2 - 9))/(6*x**2)
```

3.482 $\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx = \frac{(-9+4x^2)^{3/2}}{27x^3}$$

output
```
1/27*(4*x^2-9)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9+4x^2}}{x^4} \, dx = \frac{(-9+4x^2)^{3/2}}{27x^3}$$

input
```
Integrate[Sqrt[-9 + 4*x^2]/x^4,x]
```

output
```
(-9 + 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{242\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-9}}{x^4} \, dx$$

$$\Big\downarrow \ 242$$

$$\frac{\left(4x^2-9\right)^{3/2}}{27x^3}$$

input
```
Int[Sqrt[-9 + 4*x^2]/x^4,x]
```

output
```
(-9 + 4*x^2)^(3/2)/(27*x^3)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{27x^3}$	15
trager	$\frac{(4x^2-9)^{\frac{3}{2}}}{27x^3}$	15
pseudoelliptic	$\frac{(4x^2-9)^{\frac{3}{2}}}{27x^3}$	15
gosper	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{27x^3}$	25
orering	$\frac{(2x-3)(2x+3)\sqrt{4x^2-9}}{27x^3}$	25
risch	$\frac{16x^4-72x^2+81}{27x^3\sqrt{4x^2-9}}$	27
meijerg	$-\frac{\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{4x^2}{9}+1\right)^{\frac{3}{2}}}{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,x^3}$	37

input
```
int((4*x^2-9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/27*(4*x^2-9)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{8\,x^3+(4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/27*(8*x^3 + (4*x^2 - 9)^(3/2))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.22

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \begin{cases} \frac{8i\sqrt{-1+\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{3x^2} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{8\sqrt{1-\frac{9}{4x^2}}}{27} - \frac{2\sqrt{1-\frac{9}{4x^2}}}{3x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((4*x**2-9)**(1/2)/x**4,x)
```

output
```
Piecewise((8*I*sqrt(-1 + 9/(4*x**2))/27 - 2*I*sqrt(-1 + 9/(4*x**2))/(3*x**
2), 1/Abs(x**2) > 4/9), (8*sqrt(1 - 9/(4*x**2))/27 - 2*sqrt(1 - 9/(4*x**2)
)/(3*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{(4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/27*(4*x^2 - 9)^(3/2)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{16\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^4+27\right)}{\left(\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9\right)^3}$$

input
```
integrate((4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

output
```
16*((2*x - sqrt(4*x^2 - 9))^4 + 27)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{4\,x^2\,\sqrt{4\,x^2-9}-9\,\sqrt{4\,x^2-9}}{27\,x^3}$$

input
```
int((4*x^2 - 9)^(1/2)/x^4,x)
```

output
```
(4*x^2*(4*x^2 - 9)^(1/2) - 9*(4*x^2 - 9)^(1/2))/(27*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{-9+4x^2}}{x^4}\,dx = \frac{4\sqrt{4x^2-9}\,x^2-9\sqrt{4x^2-9}+8x^3}{27x^3}$$

input
```
int((4*x^2-9)^(1/2)/x^4,x)
```

output
```
(4*sqrt(4*x**2 - 9)*x**2 - 9*sqrt(4*x**2 - 9) + 8*x**3)/(27*x**3)
```

3.483 $\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = -\frac{\sqrt{-9+4x^2}}{4x^4} + \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
-1/4*(4*x^2-9)^(1/2)/x^4+1/18*(4*x^2-9)^(1/2)/x^2+2/27*arctan(1/3*(4*x^2-9
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \frac{(-9+2x^2)\sqrt{-9+4x^2}}{36x^4} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[Sqrt[-9 + 4*x^2]/x^5,x]
```

output
```
((-9 + 2*x^2)*Sqrt[-9 + 4*x^2])/(36*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/
27
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{4x^2-9}}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{4x^2-9}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\int \frac{1}{x^4\sqrt{4x^2-9}}dx^2-\frac{\sqrt{4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2+\frac{\sqrt{4x^2-9}}{9x^2}-\frac{\sqrt{4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{9}\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}+\frac{\sqrt{4x^2-9}}{9x^2}-\frac{\sqrt{4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{9x^2}-\frac{\sqrt{4x^2-9}}{2x^4}\right)$$

input
```
Int[Sqrt[-9 + 4*x^2]/x^5,x]
```

output
```
(-1/2*Sqrt[-9 + 4*x^2]/x^4 + Sqrt[-9 + 4*x^2]/(9*x^2) + (4*ArcTan[Sqrt[-9
+ 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
risch	$\frac{8x^4-54x^2+81}{36x^4\sqrt{4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
pseudoelliptic	$\frac{8\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right)x^4+6x^2\sqrt{4x^2-9}-27\sqrt{4x^2-9}}{108x^4}$
default	$\frac{(4x^2-9)^{\frac{3}{2}}}{36x^4}+\frac{(4x^2-9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{4x^2-9}}{81}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
trager	$\frac{(2x^2-9)\sqrt{4x^2-9}}{36x^4}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)-\sqrt{4x^2-9}}{x}\right)}{27}$
meijerg	$-\frac{4\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{81\sqrt{\pi}}{16x^4}-\frac{9\sqrt{\pi}}{4x^2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{4}-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4-\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{-\frac{4x^2}{9}+1}}{128x^4}\right.}{27\sqrt{\pi}\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

input
```
int((4*x^2-9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(8*x^4-54*x^2+81)/x^4/(4*x^2-9)^(1/2)-2/27*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}(2\,x^2-9)}{108\,x^4}$$

input
```
integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/108*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*
x^2 - 9))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.44

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{3i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{3}{4x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{9}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((4*x**2-9)**(1/2)/x**5,x)
```

output
```
Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + 3*I/(4*
x**3*sqrt(-1 + 9/(4*x**2))) - 9*I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x*
*2) > 4/9), (-2*asin(3/(2*x))/27 + 1/(9*x*sqrt(1 - 9/(4*x**2))) - 3/(4*x**
3*sqrt(1 - 9/(4*x**2))) + 9/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = -\frac{2}{81}\sqrt{4x^2-9} + \frac{(4x^2-9)^{\frac{3}{2}}}{162x^2} + \frac{(4x^2-9)^{\frac{3}{2}}}{36x^4} - \frac{2}{27}\arcsin\left(\frac{3}{2|x|}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/81*sqrt(4*x^2 - 9) + 1/162*(4*x^2 - 9)^(3/2)/x^2 + 1/36*(4*x^2 - 9)^(3/
2)/x^4 - 2/27*arcsin(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \frac{(4\,x^2-9)^{\frac{3}{2}} - 9\,\sqrt{4\,x^2-9}}{72\,x^4} + \frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate((4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/72*((4*x^2 - 9)^(3/2) - 9*sqrt(4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqrt(4*
x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27} - \frac{\frac{\sqrt{4\,x^2-9}}{8} - \frac{(4\,x^2-9)^{3/2}}{72}}{x^4}$$

input
```
int((4*x^2 - 9)^(1/2)/x^5,x)
```

output
```
(2*atan((4*x^2 - 9)^(1/2)/3))/27 - ((4*x^2 - 9)^(1/2)/8 - (4*x^2 - 9)^(3/2
)/72)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{-9+4x^2}}{x^5}\,dx = \frac{16atan\left(\frac{\sqrt{4x^2-9}}{3} + \frac{2x}{3}\right)x^4 + 6\sqrt{4x^2-9}\,x^2 - 27\sqrt{4x^2-9}}{108x^4}$$

input
```
int((4*x^2-9)^(1/2)/x^5,x)
```

output
```
(16*atan((sqrt(4*x**2 - 9) + 2*x)/3)*x**4 + 6*sqrt(4*x**2 - 9)*x**2 - 27*s
qrt(4*x**2 - 9))/(108*x**4)
```

3.484 $\int x^5\sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int x^5\sqrt{-9-4x^2}\,dx=-\frac{27}{64}\left(-9-4x^2\right)^{3/2}-\frac{9}{160}\left(-9-4x^2\right)^{5/2}-\frac{1}{448}\left(-9-4x^2\right)^{7/2}$$

output
```
-27/64*(-4*x^2-9)^(3/2)-9/160*(-4*x^2-9)^(5/2)-1/448*(-4*x^2-9)^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{-9-4x^2}\,dx=\frac{1}{280}\left(-9-4x^2\right)^{3/2}\left(-27+18x^2-10x^4\right)$$

input
```
Integrate[x^5*Sqrt[-9 - 4*x^2],x]
```

output
```
((-9 - 4*x^2)^(3/2)*(-27 + 18*x^2 - 10*x^4))/280
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^4\sqrt{-4x^2-9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{16}\left(-4x^2-9\right)^{5/2}+\frac{9}{8}\left(-4x^2-9\right)^{3/2}+\frac{81}{16}\sqrt{-4x^2-9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{224}\left(-4x^2-9\right)^{7/2}-\frac{9}{80}\left(-4x^2-9\right)^{5/2}-\frac{27}{32}\left(-4x^2-9\right)^{3/2}\right)$$

input
```
Int[x^5*Sqrt[-9 - 4*x^2],x]
```

output
```
((-27*(-9 - 4*x^2)^(3/2))/32 - (9*(-9 - 4*x^2)^(5/2))/80 - (-9 - 4*x^2)^(7
/2)/224)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{(-4x^2-9)^{\frac{3}{2}}(10x^4-18x^2+27)}{280}$	24
pseudoelliptic	$-\frac{(-4x^2-9)^{\frac{3}{2}}(10x^4-18x^2+27)}{280}$	24
trager	$\left(\frac{1}{7}x^6+\frac{9}{140}x^4-\frac{27}{140}x^2+\frac{243}{280}\right)\sqrt{-4x^2-9}$	28
orering	$\frac{(4x^2+9)(10x^4-18x^2+27)\sqrt{-4x^2-9}}{280}$	31
risch	$-\frac{(40x^6+18x^4-54x^2+243)(4x^2+9)}{280\sqrt{-4x^2-9}}$	36
meijerg	$-\frac{2187i\left(\frac{32\sqrt{\pi}}{105}-\frac{4\sqrt{\pi}\left(\frac{4x^2}{9}+1\right)^{\frac{3}{2}}\left(\frac{80}{27}x^4-\frac{16}{3}x^2+8\right)}{105}\right)}{256\sqrt{\pi}}$	39
default	$-\frac{x^4(-4x^2-9)^{\frac{3}{2}}}{28}+\frac{9x^2(-4x^2-9)^{\frac{3}{2}}}{140}-\frac{27(-4x^2-9)^{\frac{3}{2}}}{280}$	41

input
```
int(x^5*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/280*(-4*x^2-9)^(3/2)*(10*x^4-18*x^2+27)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.61

$$\int x^5\sqrt{-9-4x^2}\,dx = \frac{1}{280}\,\left(40\,x^6+18\,x^4-54\,x^2+243\right)\sqrt{-4\,x^2-9}$$

input
```
integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/280*(40*x^6 + 18*x^4 - 54*x^2 + 243)*sqrt(-4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int x^5\sqrt{-9-4x^2}\,dx = \frac{x^6\sqrt{-4x^2-9}}{7}+\frac{9x^4\sqrt{-4x^2-9}}{140}-\frac{27x^2\sqrt{-4x^2-9}}{140}+\frac{243\sqrt{-4x^2-9}}{280}$$

input
```
integrate(x**5*(-4*x**2-9)**(1/2),x)
```

output
```
x**6*sqrt(-4*x**2 - 9)/7 + 9*x**4*sqrt(-4*x**2 - 9)/140 - 27*x**2*sqrt(-4*
x**2 - 9)/140 + 243*sqrt(-4*x**2 - 9)/280
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int x^5\sqrt{-9-4x^2}\,dx = -\frac{1}{28}\,\left(-4\,x^2-9\right)^{\frac{3}{2}}x^4+\frac{9}{140}\,\left(-4\,x^2-9\right)^{\frac{3}{2}}x^2-\frac{27}{280}\,\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/28*(-4*x^2 - 9)^(3/2)*x^4 + 9/140*(-4*x^2 - 9)^(3/2)*x^2 - 27/280*(-4*x
^2 - 9)^(3/2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int x^5\sqrt{-9-4x^2}\,dx = \frac{1}{448}i\left(4\,x^2+9\right)^{\frac{7}{2}} - \frac{9}{160}i\left(4\,x^2+9\right)^{\frac{5}{2}} + \frac{27}{64}i\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^5*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/448*I*(4*x^2 + 9)^(7/2) - 9/160*I*(4*x^2 + 9)^(5/2) + 27/64*I*(4*x^2 + 9
)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{-9-4x^2}\,dx = \sqrt{-4\,x^2-9}\left(\frac{x^6}{7}+\frac{9\,x^4}{140}-\frac{27\,x^2}{140}+\frac{243}{280}\right)$$

input
```
int(x^5*(- 4*x^2 - 9)^(1/2),x)
```

output
```
(- 4*x^2 - 9)^(1/2)*((9*x^4)/140 - (27*x^2)/140 + x^6/7 + 243/280)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int x^5\sqrt{-9-4x^2}\,dx = \frac{\sqrt{-4x^2-9}\,(-40x^6-18x^4+54x^2-243)}{280}$$

input
```
int(x^5*(-4*x^2-9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 - 9)*( - 40*x**6 - 18*x**4 + 54*x**2 - 243))/280
```

3.485 $\int x^4\sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int x^4\sqrt{-9-4x^2}\,dx = -\frac{81}{256}x\sqrt{-9-4x^2}+\frac{3}{32}x^3\sqrt{-9-4x^2}+\frac{1}{6}x^5\sqrt{-9-4x^2}-\frac{729}{512}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
-81/256*x*(-4*x^2-9)^(1/2)+3/32*x^3*(-4*x^2-9)^(1/2)+1/6*x^5*(-4*x^2-9)^(1
/2)-729/512*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int x^4\sqrt{-9-4x^2}\,dx = \frac{1}{768}x\sqrt{-9-4x^2}\left(-243+72x^2+128x^4\right)-\frac{729}{512}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

input
```
Integrate[x^4*Sqrt[-9 - 4*x^2],x]
```

output
```
(x*Sqrt[-9 - 4*x^2]*(-243 + 72*x^2 + 128*x^4))/768 - (729*ArcTan[(2*x)/Sqr
t[-9 - 4*x^2]])/512
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 262, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6}x^5\sqrt{-4x^2-9}-\frac{3}{2}\int \frac{x^4}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^5\sqrt{-4x^2-9}-\frac{3}{2}\left(-\frac{27}{16}\int \frac{x^2}{\sqrt{-4x^2-9}}dx-\frac{1}{16}\sqrt{-4x^2-9}x^3\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^5\sqrt{-4x^2-9}-\frac{3}{2}\left(-\frac{27}{16}\left(-\frac{9}{8}\int \frac{1}{\sqrt{-4x^2-9}}dx-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{6}x^5\sqrt{-4x^2-9}-\frac{3}{2}\left(-\frac{27}{16}\left(-\frac{9}{8}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{6}x^5\sqrt{-4x^2-9}-\frac{3}{2}\left(-\frac{27}{16}\left(-\frac{9}{16}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3\right)$$

input
```
Int[x^4*Sqrt[-9 - 4*x^2],x]
```

output
```
(x^5*Sqrt[-9 - 4*x^2])/6 - (3*(-1/16*(x^3*Sqrt[-9 - 4*x^2]) - (27*(-1/8*(x
*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16))/16))/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

method	result	size
meijerg	$-\frac{729i\left(\frac{\sqrt{\pi}\,x\left(-\frac{640}{81}x^4-\frac{40}{9}x^2+15\right)\sqrt{\frac{4x^2}{9}+1}}{90}-\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{4}\right)}{128\sqrt{\pi}}$	44
pseudoelliptic	$\frac{729\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)}{512}+\frac{(128x^5+72x^3-243x)\sqrt{-4x^2-9}}{768}$	44
risch	$-\frac{x(128x^4+72x^2-243)(4x^2+9)}{768\sqrt{-4x^2-9}}-\frac{729\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{512}$	48
default	$-\frac{x^3(-4x^2-9)^{\frac{3}{2}}}{24}+\frac{9x(-4x^2-9)^{\frac{3}{2}}}{128}+\frac{81x\sqrt{-4x^2-9}}{256}-\frac{729\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{512}$	55
trager	$\frac{x(128x^4+72x^2-243)\sqrt{-4x^2-9}}{768}-\frac{729\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{512}$	55

input
```
int(x^4*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-729/128*I/Pi^(1/2)*(1/90*Pi^(1/2)*x*(-640/81*x^4-40/9*x^2+15)*(4/9*x^2+1)
^(1/2)-1/4*Pi^(1/2)*arcsinh(2/3*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int x^4\sqrt{-9-4x^2}\,dx = \frac{1}{768}\left(128\,x^5+72\,x^3-243\,x\right)\sqrt{-4\,x^2-9} - \frac{729}{1024}i\,\log\left(-\frac{4\left(2\,x+i\sqrt{-4\,x^2-9}\right)}{x}\right) + \frac{729}{1024}i\,\log\left(-\frac{4\left(2\,x-i\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input
```
integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/768*(128*x^5 + 72*x^3 - 243*x)*sqrt(-4*x^2 - 9) - 729/1024*I*log(-4*(2*x
 + I*sqrt(-4*x^2 - 9))/x) + 729/1024*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.15

$$\int x^4\sqrt{-9-4x^2}\,dx = \frac{2ix^7}{3\sqrt{4x^2+9}} + \frac{15ix^5}{8\sqrt{4x^2+9}} - \frac{27ix^3}{64\sqrt{4x^2+9}} - \frac{729ix}{256\sqrt{4x^2+9}} + \frac{729i\operatorname{asinh}\left(\frac{2x}{3}\right)}{512}$$

input
```
integrate(x**4*(-4*x**2-9)**(1/2),x)
```

output
```
2*I*x**7/(3*sqrt(4*x**2 + 9)) + 15*I*x**5/(8*sqrt(4*x**2 + 9)) - 27*I*x**3
/(64*sqrt(4*x**2 + 9)) - 729*I*x/(256*sqrt(4*x**2 + 9)) + 729*I*asinh(2*x/
3)/512
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^4\sqrt{-9-4x^2}\,dx = -\frac{1}{24}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^3 + \frac{9}{128}\left(-4\,x^2-9\right)^{\frac{3}{2}}x + \frac{81}{256}\sqrt{-4\,x^2-9}x + \frac{729}{512}i\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/24*(-4*x^2 - 9)^(3/2)*x^3 + 9/128*(-4*x^2 - 9)^(3/2)*x + 81/256*sqrt(-4
*x^2 - 9)*x + 729/512*I*arcsinh(2/3*x)
```

Giac [F]

$$\int x^4\sqrt{-9-4x^2}\,dx = \int \sqrt{-4\,x^2-9}x^4\,dx$$

input
```
integrate(x^4*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{-9-4x^2}\,dx = \int x^4\,\sqrt{-4\,x^2-9}\,dx$$

input
```
int(x^4*(- 4*x^2 - 9)^(1/2),x)
```

output
```
int(x^4*(- 4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62

$$\int x^4\sqrt{-9-4x^2}\,dx = \frac{729asinh\left(\frac{2x}{3}\right)i}{512} - \frac{\sqrt{-4x^2-9}\,x^5}{6} - \frac{3\sqrt{-4x^2-9}\,x^3}{32} + \frac{81\sqrt{-4x^2-9}\,x}{256}$$

input
```
int(x^4*(-4*x^2-9)^(1/2),x)
```

output
```
(2187*asinh((2*x)/3)*i - 256*sqrt( - 4*x**2 - 9)*x**5 - 144*sqrt( - 4*x**2
 - 9)*x**3 + 486*sqrt( - 4*x**2 - 9)*x)/1536
```

3.486 $\int x^3\sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int x^3\sqrt{-9-4x^2}\,dx=\frac{3}{16}\left(-9-4x^2\right)^{3/2}+\frac{1}{80}\left(-9-4x^2\right)^{5/2}$$

output
```
3/16*(-4*x^2-9)^(3/2)+1/80*(-4*x^2-9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{-9-4x^2}\,dx=\frac{1}{40}\left(-9-4x^2\right)^{3/2}\left(3-2x^2\right)$$

input
```
Integrate[x^3*Sqrt[-9 - 4*x^2],x]
```

output
```
((-9 - 4*x^2)^(3/2)*(3 - 2*x^2))/40
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^2\sqrt{-4x^2-9}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(-\frac{1}{4}\left(-4x^2-9\right)^{3/2}-\frac{9}{4}\sqrt{-4x^2-9}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{40}\left(-4x^2-9\right)^{5/2}+\frac{3}{8}\left(-4x^2-9\right)^{3/2}\right)$$

input
```
Int[x^3*Sqrt[-9 - 4*x^2],x]
```

output
```
((3*(-9 - 4*x^2)^(3/2))/8 + (-9 - 4*x^2)^(5/2)/40)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{(2x^2-3)(-4x^2-9)^{\frac{3}{2}}}{40}$	19
pseudoelliptic	$-\frac{(2x^2-3)(-4x^2-9)^{\frac{3}{2}}}{40}$	19
trager	$\left(\frac{1}{5}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2-9}$	23
orering	$\frac{(4x^2+9)(2x^2-3)\sqrt{-4x^2-9}}{40}$	26
default	$-\frac{x^2(-4x^2-9)^{\frac{3}{2}}}{20}+\frac{3(-4x^2-9)^{\frac{3}{2}}}{40}$	27
risch	$-\frac{(8x^4+6x^2-27)(4x^2+9)}{40\sqrt{-4x^2-9}}$	31
meijerg	$-\frac{243i\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{4x^2}{9}+1\right)^{\frac{3}{2}}\left(-\frac{4x^2}{3}+2\right)}{15}\right)}{64\sqrt{\pi}}$	34

input
```
int(x^3*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/40*(2*x^2-3)*(-4*x^2-9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{-9-4x^2}\,dx = \frac{1}{40}\left(8\,x^4+6\,x^2-27\right)\sqrt{-4\,x^2-9}$$

input
```
integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(8*x^4 + 6*x^2 - 27)*sqrt(-4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

$$\int x^3\sqrt{-9-4x^2}\,dx = \frac{x^4\sqrt{-4x^2-9}}{5}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

input
```
integrate(x**3*(-4*x**2-9)**(1/2),x)
```

output
```
x**4*sqrt(-4*x**2 - 9)/5 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2 -
 9)/40
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int x^3\sqrt{-9-4x^2}\,dx = -\frac{1}{20}\left(-4\,x^2-9\right)^{\frac{3}{2}}x^2+\frac{3}{40}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*(-4*x^2 - 9)^(3/2)*x^2 + 3/40*(-4*x^2 - 9)^(3/2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int x^3\sqrt{-9-4x^2}\,dx = \frac{1}{80}i\left(4\,x^2+9\right)^{\frac{5}{2}} - \frac{3}{16}i\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x^3*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/80*I*(4*x^2 + 9)^(5/2) - 3/16*I*(4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{-9-4x^2}\,dx = \sqrt{-4\,x^2-9}\left(\frac{x^4}{5}+\frac{3\,x^2}{20}-\frac{27}{40}\right)$$

input
```
int(x^3*(- 4*x^2 - 9)^(1/2),x)
```

output
```
(- 4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/5 - 27/40)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x^3\sqrt{-9-4x^2}\,dx = \frac{\sqrt{-4x^2-9}\,(-8x^4-6x^2+27)}{40}$$

input
```
int(x^3*(-4*x^2-9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 - 9)*( - 8*x**4 - 6*x**2 + 27))/40
```

3.487 $\int x^2\sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int x^2\sqrt{-9-4x^2}\,dx=\frac{9}{32}x\sqrt{-9-4x^2}+\frac{1}{4}x^3\sqrt{-9-4x^2}+\frac{81}{64}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
9/32*x*(-4*x^2-9)^(1/2)+1/4*x^3*(-4*x^2-9)^(1/2)+81/64*arctan(2*x/(-4*x^2-
9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int x^2\sqrt{-9-4x^2}\,dx=\frac{1}{64}\left(2x\sqrt{-9-4x^2}(9+8x^2)+81\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\right)$$

input
```
Integrate[x^2*Sqrt[-9 - 4*x^2],x]
```

output
```
(2*x*Sqrt[-9 - 4*x^2]*(9 + 8*x^2) + 81*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/64
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{4}x^3\sqrt{-4x^2-9}-\frac{9}{4}\int \frac{x^2}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{4}x^3\sqrt{-4x^2-9}-\frac{9}{4}\left(-\frac{9}{8}\int \frac{1}{\sqrt{-4x^2-9}}dx-\frac{1}{8}\sqrt{-4x^2-9}x\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{4}x^3\sqrt{-4x^2-9}-\frac{9}{4}\left(-\frac{9}{8}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}-\frac{1}{8}\sqrt{-4x^2-9}x\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{1}{4}x^3\sqrt{-4x^2-9}-\frac{9}{4}\left(-\frac{9}{16}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{8}\sqrt{-4x^2-9}x\right)$$

input
```
Int[x^2*Sqrt[-9 - 4*x^2],x]
```

output
```
(x^3*Sqrt[-9 - 4*x^2])/4 - (9*(-1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)
/Sqrt[-9 - 4*x^2]])/16))/4
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

method	result	size
meijerg	$-\frac{81 i \left(-\frac{\sqrt{\pi}\, x \left(\frac{8 x^{2}}{3}+3\right) \sqrt{\frac{4 x^{2}}{9}+1}}{9}+\frac{\sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{2 x}{3}\right)}{2}\right)}{32 \sqrt{\pi}}$	39
pseudoelliptic	$-\frac{81 \arctan\left(\frac{\sqrt{-4 x^{2}-9}}{2 x}\right)}{64}+\frac{\left(8 x^{3}+9 x \right) \sqrt{-4 x^{2}-9}}{32}$	39
default	$-\frac{x \left(-4 x^{2}-9\right)^{\frac{3}{2}}}{16}-\frac{9 x \sqrt{-4 x^{2}-9}}{32}+\frac{81 \arctan\left(\frac{2 x}{\sqrt{-4 x^{2}-9}}\right)}{64}$	41
risch	$-\frac{x \left(8 x^{2}+9\right) \left(4 x^{2}+9\right)}{32 \sqrt{-4 x^{2}-9}}+\frac{81 \arctan\left(\frac{2 x}{\sqrt{-4 x^{2}-9}}\right)}{64}$	43
trager	$\frac{x \left(8 x^{2}+9\right) \sqrt{-4 x^{2}-9}}{32}+\frac{81 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-4 x^{2}-9}+2 x \right)}{64}$	50

input
```
int(x^2*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-81/32*I/Pi^(1/2)*(-1/9*Pi^(1/2)*x*(8/3*x^2+3)*(4/9*x^2+1)^(1/2)+1/2*Pi^(1
/2)*arcsinh(2/3*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int x^2\sqrt{-9-4x^2}\,dx = \frac{1}{32}\left(8\,x^{3}+9\,x\right)\sqrt{-4\,x^{2}-9} + \frac{81}{128}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^{2}-9}\right)}{x}\right) - \frac{81}{128}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^{2}-9}\right)}{x}\right)$$

input
```
integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/32*(8*x^3 + 9*x)*sqrt(-4*x^2 - 9) + 81/128*I*log(-4*(2*x + I*sqrt(-4*x^2
 - 9))/x) - 81/128*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.95 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int x^2\sqrt{-9-4x^2}\,dx = \frac{ix^5}{\sqrt{4x^2+9}} + \frac{27ix^3}{8\sqrt{4x^2+9}} + \frac{81ix}{32\sqrt{4x^2+9}} - \frac{81i\operatorname{asinh}\left(\frac{2x}{3}\right)}{64}$$

input
```
integrate(x**2*(-4*x**2-9)**(1/2),x)
```

output
```
I*x**5/sqrt(4*x**2 + 9) + 27*I*x**3/(8*sqrt(4*x**2 + 9)) + 81*I*x/(32*sqrt
(4*x**2 + 9)) - 81*I*asinh(2*x/3)/64
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{-9-4x^2}\,dx = -\frac{1}{16}\left(-4x^2-9\right)^{\frac{3}{2}}x - \frac{9}{32}\sqrt{-4x^2-9}x - \frac{81}{64}i\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

input
```
integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*(-4*x^2 - 9)^(3/2)*x - 9/32*sqrt(-4*x^2 - 9)*x - 81/64*I*arcsinh(2/3
*x)
```

Giac [F]

$$\int x^2\sqrt{-9-4x^2}\,dx = \int \sqrt{-4x^2-9}x^2\,dx$$

input
```
integrate(x^2*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{-9-4x^2}\,dx=\int x^2\,\sqrt{-4\,x^2-9}\,dx$$

input
```
int(x^2*(- 4*x^2 - 9)^(1/2),x)
```

output
```
int(x^2*(- 4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int x^2\sqrt{-9-4x^2}\,dx=-\frac{81asinh\left(\frac{2x}{3}\right)i}{64}-\frac{\sqrt{-4x^2-9}\,x^3}{4}-\frac{9\sqrt{-4x^2-9}\,x}{32}$$

input
```
int(x^2*(-4*x^2-9)^(1/2),x)
```

output
```
( - 81*asinh((2*x)/3)*i - 16*sqrt( - 4*x**2 - 9)*x**3 - 18*sqrt( - 4*x**2
- 9)*x)/64
```

3.488 $\int x\sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int x\sqrt{-9-4x^2}\,dx=-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

output
```
-1/12*(-4*x^2-9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int x\sqrt{-9-4x^2}\,dx=-\frac{1}{12}\left(-9-4x^2\right)^{3/2}$$

input
```
Integrate[x*Sqrt[-9 - 4*x^2],x]
```

output
```
-1/12*(-9 - 4*x^2)^(3/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \text{241}$$

$$-\frac{1}{12}\left(-4x^2-9\right)^{3/2}$$

input
```
Int[x*Sqrt[-9 - 4*x^2],x]
```

output
```
-1/12*(-9 - 4*x^2)^(3/2)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{12}$	12
derivativedivides	$-\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{12}$	12
default	$-\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{12}$	12
pseudoelliptic	$-\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{12}$	12
trager	$\left(\frac{x^{2}}{3}+\frac{3}{4}\right)\sqrt{-4x^{2}-9}$	18
orering	$\frac{\left(4x^{2}+9\right)\sqrt{-4x^{2}-9}}{12}$	19
risch	$-\frac{\left(4x^{2}+9\right)^{2}}{12\sqrt{-4x^{2}-9}}$	21
meijerg	$-\frac{27i\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{8x^{2}}{9}\right)\sqrt{\frac{4x^{2}}{9}+1}}{3}\right)}{16\sqrt{\pi}}$	34

input
```
int(x*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-4*x^2-9)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int x\sqrt{-9-4x^2}\,dx = \frac{1}{12}\left(4\,x^{2}+9\right)\sqrt{-4\,x^{2}-9}$$

input
```
integrate(x*(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(4*x^2 + 9)*sqrt(-4*x^2 - 9)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int x\sqrt{-9-4x^2}\,dx = \frac{x^2\sqrt{-4x^2-9}}{3} + \frac{3\sqrt{-4x^2-9}}{4}$$

input
```
integrate(x*(-4*x**2-9)**(1/2),x)
```

output
```
x**2*sqrt(-4*x**2 - 9)/3 + 3*sqrt(-4*x**2 - 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9-4x^2}\,dx = -\frac{1}{12}\left(-4\,x^2-9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(-4*x^2 - 9)^(3/2)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9-4x^2}\,dx = \frac{1}{12}i\left(4\,x^2+9\right)^{\frac{3}{2}}$$

input
```
integrate(x*(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/12*I*(4*x^2 + 9)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int x\sqrt{-9-4x^2}\,dx = -\frac{{\left(-4\,x^2-9\right)}^{3/2}}{12}$$

input
```
int(x*(- 4*x^2 - 9)^(1/2),x)
```

output
```
-(- 4*x^2 - 9)^(3/2)/12
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int x\sqrt{-9-4x^2}\,dx = \frac{\sqrt{-4x^2-9}\,(-4x^2-9)}{12}$$

input
```
int(x*(-4*x^2-9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 - 9)*( - 4*x**2 - 9))/12
```

3.489 $\int \sqrt{-9-4x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \sqrt{-9-4x^2}\,dx=\frac{1}{2}x\sqrt{-9-4x^2}-\frac{9}{4}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
1/2*x*(-4*x^2-9)^(1/2)-9/4*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \sqrt{-9-4x^2}\,dx=\frac{1}{4}\left(2x\sqrt{-9-4x^2}-9\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\right)$$

input
```
Integrate[Sqrt[-9 - 4*x^2],x]
```

output
```
(2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-4x^2-9}\,dx$$

$$\Big\downarrow \ 211$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{2}\int \frac{1}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \ 224$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{2}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}$$

$$\Big\downarrow \ 216$$

$$\frac{1}{2}x\sqrt{-4x^2-9}-\frac{9}{4}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

input
```
Int[Sqrt[-9 - 4*x^2],x]
```

output
```
(x*Sqrt[-9 - 4*x^2])/2 - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/4
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))    Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x\sqrt{-4x^2-9}}{2}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$	29
pseudoelliptic	$\frac{x\sqrt{-4x^2-9}}{2}+\frac{9\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)}{4}$	31
meijerg	$-\frac{9i\left(-\frac{4\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{3}-2\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{8\sqrt{\pi}}$	32
risch	$-\frac{(4x^2+9)x}{2\sqrt{-4x^2-9}}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$	36
trager	$\frac{x\sqrt{-4x^2-9}}{2}+\frac{9\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{4}$	44

input
```
int((-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(-4*x^2-9)^(1/2)-9/4*arctan(2*x/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \sqrt{-9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-9}x-\frac{9}{8}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right)+\frac{9}{8}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-4*x^2 - 9)*x - 9/8*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + 9/8*
I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \sqrt{-9-4x^2}\,dx = \frac{x\sqrt{-4x^2-9}}{2} - \frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{4}$$

input
```
integrate((-4*x**2-9)**(1/2),x)
```

output
```
x*sqrt(-4*x**2 - 9)/2 - 9*atan(2*x/sqrt(-4*x**2 - 9))/4
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \sqrt{-9-4x^2}\,dx = \frac{1}{2}\sqrt{-4\,x^2-9}x + \frac{9}{4}i\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 - 9)*x + 9/4*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \sqrt{-9-4x^2}\,dx = \int \sqrt{-4\,x^2-9}\,dx$$

input
```
integrate((-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9), x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{-9-4x^2}\,dx = \frac{x\,\sqrt{-4\,x^2-9}}{2} - \frac{9\,\mathrm{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{4}$$

input
```
int((- 4*x^2 - 9)^(1/2),x)
```

output
```
(x*(- 4*x^2 - 9)^(1/2))/2 - (9*atan((2*x)/(- 4*x^2 - 9)^(1/2)))/4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \sqrt{-9-4x^2}\,dx = \frac{9asinh\left(\frac{2x}{3}\right)i}{4} - \frac{\sqrt{-4x^2-9}\,x}{2}$$

input
```
int((-4*x^2-9)^(1/2),x)
```

output
```
(9*asinh((2*x)/3)*i - 2*sqrt( - 4*x**2 - 9)*x)/4
```

3.490 $\int \frac{\sqrt{-9-4x^2}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{\sqrt{-9-4x^2}}{x} \, dx = \sqrt{-9-4x^2} - 3 \arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
(-4*x^2-9)^(1/2)-3*arctan(1/3*(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9-4x^2}}{x} \, dx = \sqrt{-9-4x^2} - 3 \arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[Sqrt[-9 - 4*x^2]/x,x]
```

output
```
Sqrt[-9 - 4*x^2] - 3*ArcTan[Sqrt[-9 - 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 60, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-4x^2-9}}{x} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{-4x^2-9}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(2\sqrt{-4x^2-9}-9\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{9}{2}\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}+2\sqrt{-4x^2-9}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(2\sqrt{-4x^2-9}-6\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)\right)$$

input
```
Int[Sqrt[-9 - 4*x^2]/x,x]
```

output
```
(2*Sqrt[-9 - 4*x^2] - 6*ArcTan[Sqrt[-9 - 4*x^2]/3])/2
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\sqrt{-4x^2-9}+3\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)$	25
pseudoelliptic	$\sqrt{-4x^2-9}-3\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)$	25
trager	$\sqrt{-4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-4x^2-9}}{x}\right)$	42
meijerg	$-\frac{3i\left(-2(2+2\ln(x)-2\ln(3))\sqrt{\pi}+4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)\right)}{4\sqrt{\pi}}$	61

input
```
int(1/x*(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-4*x^2-9)^(1/2)+3*arctan(3/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \sqrt{-4\,x^2-9}-\frac{3}{2}i\,\log\left(-\frac{6\left(i\,\sqrt{-4\,x^2-9}-3\right)}{x}\right)+\frac{3}{2}i\,\log\left(-\frac{6\left(-i\,\sqrt{-4\,x^2-9}-3\right)}{x}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x,x, algorithm="fricas")
```

output
```
sqrt(-4*x^2 - 9) - 3/2*I*log(-6*(I*sqrt(-4*x^2 - 9) - 3)/x) + 3/2*I*log(-6
*(-I*sqrt(-4*x^2 - 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \frac{2ix}{\sqrt{1+\frac{9}{4x^2}}} - 3i\operatorname{asinh}\left(\frac{3}{2x}\right) + \frac{9i}{2x\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate((-4*x**2-9)**(1/2)/x,x)
```

output
```
2*I*x/sqrt(1 + 9/(4*x**2)) - 3*I*asinh(3/(2*x)) + 9*I/(2*x*sqrt(1 + 9/(4*x
**2)))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \sqrt{-4\,x^2-9} + 3i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x,x, algorithm="maxima")
```

output
```
sqrt(-4*x^2 - 9) + 3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \sqrt{-4\,x^2-9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x,x, algorithm="giac")
```

output
```
sqrt(-4*x^2 - 9) - 3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = \sqrt{-4\,x^2-9} - 3\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)$$

input
```
int((- 4*x^2 - 9)^(1/2)/x,x)
```

output
```
(- 4*x^2 - 9)^(1/2) - 3*atan((- 4*x^2 - 9)^(1/2)/3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

$$\int \frac{\sqrt{-9-4x^2}}{x}\,dx = -\sqrt{-4x^2-9} + 3\log\left(\frac{\sqrt{-4x^2-9}\,i}{3} + \frac{2x}{3} - 1\right)i - 3\log\left(\frac{\sqrt{-4x^2-9}\,i}{3} + \frac{2x}{3} + 1\right)i$$

input
```
int((-4*x^2-9)^(1/2)/x,x)
```

output
```
- sqrt( - 4*x**2 - 9) + 3*log((sqrt( - 4*x**2 - 9)*i + 2*x - 3)/3)*i - 3*
log((sqrt( - 4*x**2 - 9)*i + 2*x + 3)/3)*i
```

3.491 $\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx = -\frac{\sqrt{-9-4x^2}}{x} - 2\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
-(-4*x^2-9)^(1/2)/x-2*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{-9-4x^2}}{x^2} \, dx = -\frac{\sqrt{-9-4x^2} + 2x\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)}{x}$$

input
```
Integrate[Sqrt[-9 - 4*x^2]/x^2,x]
```

output
```
-((Sqrt[-9 - 4*x^2] + 2*x*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/x)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-4x^2-9}}{x^2} \, dx$$

$$\Big\downarrow \text{247}$$

$$-4\int \frac{1}{\sqrt{-4x^2-9}}dx - \frac{\sqrt{-4x^2-9}}{x}$$

$$\Big\downarrow \text{224}$$

$$-4\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}} - \frac{\sqrt{-4x^2-9}}{x}$$

$$\Big\downarrow \text{216}$$

$$-2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right) - \frac{\sqrt{-4x^2-9}}{x}$$

input
```
Int[Sqrt[-9 - 4*x^2]/x^2,x]
```

output
```
-(Sqrt[-9 - 4*x^2]/x) - 2*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

method	result	size
meijerg	$-\frac{i\left(\frac{6\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}}{x}-4\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{2\sqrt{\pi}}$	34
pseudoelliptic	$\frac{2\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)x-\sqrt{-4x^2-9}}{x}$	35
risch	$\frac{4x^2+9}{x\sqrt{-4x^2-9}}-2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$	37
default	$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{9x}+\frac{4x\sqrt{-4x^2-9}}{9}-2\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$	43
trager	$-\frac{\sqrt{-4x^2-9}}{x}+2\,\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-4x^2-9}\right)$	44

input
```
int((-4*x^2-9)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/Pi^(1/2)*(6*Pi^(1/2)/x*(4/9*x^2+1)^(1/2)-4*Pi^(1/2)*arcsinh(2/3*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = \frac{-i\,x\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right)+i\,x\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)-\sqrt{-4\,x^2-9}}{x}$$

input
```
integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(-I*x*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + I*x*log(-4*(2*x - I*sqrt(-4*x
^2 - 9))/x) - sqrt(-4*x^2 - 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = -2\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{\sqrt{-4x^2-9}}{x}$$

input
```
integrate((-4*x**2-9)**(1/2)/x**2,x)
```

output
```
-2*atan(2*x/sqrt(-4*x**2 - 9)) - sqrt(-4*x**2 - 9)/x
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = -\frac{\sqrt{-4\,x^2-9}}{x} + 2i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-sqrt(-4*x^2 - 9)/x + 2*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = \int \frac{\sqrt{-4\,x^2-9}}{x^2}\,dx$$

input
```
integrate((-4*x^2-9)^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = -\frac{\sqrt{-4\,x^2-9}}{x} - \frac{\operatorname{asin}\left(\frac{x\,2\mathrm{i}}{3}\right)\,\sqrt{-4\,x^2-9}\,2\mathrm{i}}{3\,\sqrt{\frac{4\,x^2}{9}+1}}$$

input
```
int((- 4*x^2 - 9)^(1/2)/x^2,x)
```

output
```
- (- 4*x^2 - 9)^(1/2)/x - (asin((x*2i)/3)*(- 4*x^2 - 9)^(1/2)*2i)/(3*((4*x
^2)/9 + 1)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-9-4x^2}}{x^2}\,dx = \frac{2asinh\left(\frac{2x}{3}\right)ix + \sqrt{-4x^2-9} - 2ix}{x}$$

input
```
int((-4*x^2-9)^(1/2)/x^2,x)
```

output
```
(2*asinh((2*x)/3)*i*x + sqrt( - 4*x**2 - 9) - 2*i*x)/x
```

3.492 $\int \frac{\sqrt{-9-4x^2}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{\sqrt{-9-4x^2}}{x^3} dx = -\frac{\sqrt{-9-4x^2}}{2x^2} - \frac{2}{3}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
-1/2*(-4*x^2-9)^(1/2)/x^2-2/3*arctan(1/3*(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9-4x^2}}{x^3} dx = -\frac{\sqrt{-9-4x^2}}{2x^2} - \frac{2}{3}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[Sqrt[-9 - 4*x^2]/x^3,x]
```

output
```
-1/2*Sqrt[-9 - 4*x^2]/x^2 - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 51, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-4x^2-9}}{x^3}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{-4x^2-9}}{x^4}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-2\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2-\frac{\sqrt{-4x^2-9}}{x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}-\frac{\sqrt{-4x^2-9}}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{4}{3}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{x^2}\right)$$

input
```
Int[Sqrt[-9 - 4*x^2]/x^3,x]
```

output
```
(-(Sqrt[-9 - 4*x^2]/x^2) - (4*ArcTan[Sqrt[-9 - 4*x^2]/3])/3)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

method	result	size
pseudoelliptic	$\frac{-4\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)x^2-3\sqrt{-4x^2-9}}{6x^2}$	35
risch	$\frac{4x^2+9}{2x^2\sqrt{-4x^2-9}}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	37
default	$\frac{\left(-4x^2-9\right)^{\frac{3}{2}}}{18x^2}+\frac{2\sqrt{-4x^2-9}}{9}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	41
trager	$-\frac{\sqrt{-4x^2-9}}{2x^2}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{-4x^2-9}}{x}\right)}{3}$	47
meijerg	$-\frac{i\left(\frac{9\sqrt{\pi}}{2x^2}-(-1+2\ln(x)-2\ln(3))\sqrt{\pi}-\frac{9\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)}{16x^2}+\frac{9\sqrt{\pi}\sqrt{\frac{4x^2}{9}+1}}{2x^2}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)\right)}{3\sqrt{\pi}}$	82

input
```
int((-4*x^2-9)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-4*arctan(1/3*(-4*x^2-9)^(1/2))*x^2-3*(-4*x^2-9)^(1/2))/x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int\frac{\sqrt{-9-4x^2}}{x^3}\,dx=\frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}-3\right)}{3\,x}\right)-3\sqrt{-4\,x^2-9}}{6\,x^2}$$

input
```
integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
1/6*(-2*I*x^2*log(-4/3*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/3*(-I*
sqrt(-4*x^2 - 9) - 3)/x) - 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = -\frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{3} - \frac{i\sqrt{1+\frac{9}{4x^2}}}{x}$$

input
```
integrate((-4*x**2-9)**(1/2)/x**3,x)
```

output
```
-2*I*asinh(3/(2*x))/3 - I*sqrt(1 + 9/(4*x**2))/x
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = \frac{2}{9}\sqrt{-4\,x^2-9} + \frac{(-4\,x^2-9)^{\frac{3}{2}}}{18\,x^2} + \frac{2}{3}i\,\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
2/9*sqrt(-4*x^2 - 9) + 1/18*(-4*x^2 - 9)^(3/2)/x^2 + 2/3*I*log(6*sqrt(4*x^
2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = -\frac{\sqrt{-4\,x^2-9}}{2\,x^2} - \frac{2}{3}\arctan\left(\frac{1}{3}\sqrt{-4\,x^2-9}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-1/2*sqrt(-4*x^2 - 9)/x^2 - 2/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{3}-\frac{\sqrt{-4\,x^2-9}}{2\,x^2}$$

input
```
int((- 4*x^2 - 9)^(1/2)/x^3,x)
```

output
```
- (2*atan((- 4*x^2 - 9)^(1/2)/3))/3 - (- 4*x^2 - 9)^(1/2)/(2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \frac{\sqrt{-9-4x^2}}{x^3}\,dx = \frac{3\sqrt{-4x^2-9}+4\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}-1\right)i\,x^2-4\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}+1\right)i\,x^2}{6x^2}$$

input
```
int((-4*x^2-9)^(1/2)/x^3,x)
```

output
```
(3*sqrt( - 4*x**2 - 9) + 4*log((sqrt( - 4*x**2 - 9)*i + 2*x - 3)/3)*i*x**2
 - 4*log((sqrt( - 4*x**2 - 9)*i + 2*x + 3)/3)*i*x**2)/(6*x**2)
```

3.493 $\int \frac{\sqrt{-9-4x^2}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\sqrt{-9-4x^2}}{x^4} dx = \frac{(-9-4x^2)^{3/2}}{27x^3}$$

output
```
1/27*(-4*x^2-9)^(3/2)/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{-9-4x^2}}{x^4} dx = \frac{(-9-4x^2)^{3/2}}{27x^3}$$

input
```
Integrate[Sqrt[-9 - 4*x^2]/x^4,x]
```

output
```
(-9 - 4*x^2)^(3/2)/(27*x^3)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-4x^2-9}}{x^4} \, dx$$
$$\Big\downarrow \text{ 242}$$
$$\frac{\left(-4x^2-9\right)^{3/2}}{27x^3}$$

input
```
Int[Sqrt[-9 - 4*x^2]/x^4,x]
```

output
```
(-9 - 4*x^2)^(3/2)/(27*x^3)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{27x^{3}}$	15
default	$\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{27x^{3}}$	15
pseudoelliptic	$\frac{\left(-4x^{2}-9\right)^{\frac{3}{2}}}{27x^{3}}$	15
meijerg	$-\frac{i\left(\frac{4x^{2}}{9}+1\right)^{\frac{3}{2}}}{x^{3}}$	16
trager	$-\frac{\left(4x^{2}+9\right)\sqrt{-4x^{2}-9}}{27x^{3}}$	22
orering	$-\frac{\left(4x^{2}+9\right)\sqrt{-4x^{2}-9}}{27x^{3}}$	22
risch	$\frac{16x^{4}+72x^{2}+81}{27x^{3}\sqrt{-4x^{2}-9}}$	27

input
```
int((-4*x^2-9)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/27*(-4*x^2-9)^(3/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = \frac{\left(-4\,x^{2}-9\right)^{\frac{3}{2}}}{27\,x^{3}}$$

input
```
integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = -\frac{8i\sqrt{1+\frac{9}{4x^2}}}{27} - \frac{2i\sqrt{1+\frac{9}{4x^2}}}{3x^2}$$

input
```
integrate((-4*x**2-9)**(1/2)/x**4,x)
```

output
```
-8*I*sqrt(1 + 9/(4*x**2))/27 - 2*I*sqrt(1 + 9/(4*x**2))/(3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = \frac{(-4\,x^2-9)^{\frac{3}{2}}}{27\,x^3}$$

input
```
integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
1/27*(-4*x^2 - 9)^(3/2)/x^3
```

Giac [F]

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = \int \frac{\sqrt{-4\,x^2-9}}{x^4}\,dx$$

input
```
integrate((-4*x^2-9)^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(-4*x^2 - 9)/x^4, x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = -\frac{4\,x^2\,\sqrt{-4\,x^2-9}+9\,\sqrt{-4\,x^2-9}}{27\,x^3}$$

input
```
int((- 4*x^2 - 9)^(1/2)/x^4,x)
```

output
```
-(4*x^2*(- 4*x^2 - 9)^(1/2) + 9*(- 4*x^2 - 9)^(1/2))/(27*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{-9-4x^2}}{x^4}\,dx = \frac{4\sqrt{-4x^2-9}\,x^2+9\sqrt{-4x^2-9}-8i\,x^3}{27x^3}$$

input
```
int((-4*x^2-9)^(1/2)/x^4,x)
```

output
```
(4*sqrt( - 4*x**2 - 9)*x**2 + 9*sqrt( - 4*x**2 - 9) - 8*i*x**3)/(27*x**3)
```

3.494 $\int \frac{\sqrt{-9-4x^2}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{\sqrt{-9-4x^2}}{x^5} \, dx = -\frac{\sqrt{-9-4x^2}}{4x^4} - \frac{\sqrt{-9-4x^2}}{18x^2} + \frac{2}{27} \arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
-1/4*(-4*x^2-9)^(1/2)/x^4-1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(1/3*(-4*x^
2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{-9-4x^2}}{x^5} \, dx = \frac{\sqrt{-9-4x^2}(-9-2x^2)}{36x^4} + \frac{2}{27} \arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[Sqrt[-9 - 4*x^2]/x^5,x]
```

output
```
(Sqrt[-9 - 4*x^2]*(-9 - 2*x^2))/(36*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/
27
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 51, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-4x^2-9}}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt{-4x^2-9}}{x^6}dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(-\int \frac{1}{x^4\sqrt{-4x^2-9}}dx^2-\frac{\sqrt{-4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2-\frac{\sqrt{-4x^2-9}}{9x^2}-\frac{\sqrt{-4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}-\frac{\sqrt{-4x^2-9}}{9x^2}-\frac{\sqrt{-4x^2-9}}{2x^4}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{9x^2}-\frac{\sqrt{-4x^2-9}}{2x^4}\right)$$

input
```
Int[Sqrt[-9 - 4*x^2]/x^5,x]
```

output
```
(-1/2*Sqrt[-9 - 4*x^2]/x^4 - Sqrt[-9 - 4*x^2]/(9*x^2) + (4*ArcTan[Sqrt[-9
- 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
risch	$\frac{8x^4+54x^2+81}{36x^4\sqrt{-4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$
pseudoelliptic	$\frac{8\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)x^4-6x^2\sqrt{-4x^2-9}-27\sqrt{-4x^2-9}}{108x^4}$
trager	$-\frac{(2x^2+9)\sqrt{-4x^2-9}}{36x^4}-\frac{2\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\operatorname{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$
default	$\frac{(-4x^2-9)^{\frac{3}{2}}}{36x^4}-\frac{(-4x^2-9)^{\frac{3}{2}}}{162x^2}-\frac{2\sqrt{-4x^2-9}}{81}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$
meijerg	$-\frac{4i\left(\frac{81\sqrt{\pi}}{16x^4}+\frac{9\sqrt{\pi}}{4x^2}+\frac{\left(\frac{1}{2}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{4}-\frac{81\sqrt{\pi}\left(\frac{16}{81}x^4+\frac{32}{9}x^2+8\right)}{128x^4}+\frac{81\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)\sqrt{\frac{4x^2}{9}+1}}{128x^4}-\frac{\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)}{2}\right)}{27\sqrt{\pi}}$

input
```
int((-4*x^2-9)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(8*x^4+54*x^2+81)/x^4/(-4*x^2-9)^(1/2)-2/27*arctan(3/(-4*x^2-9)^(1/2)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = \frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{27\,x}\right)-3\,(2\,x^2+9)\sqrt{-4\,x^2-9}}{108\,x^4}$$

input
```
integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
1/108*(-4*I*x^4*log(-4/27*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/27*
(-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 + 9)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = \frac{2i\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{3i}{4x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{9i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate((-4*x**2-9)**(1/2)/x**5,x)
```

output
```
2*I*asinh(3/(2*x))/27 - I/(9*x*sqrt(1 + 9/(4*x**2))) - 3*I/(4*x**3*sqrt(1
+ 9/(4*x**2))) - 9*I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = -\frac{2}{81}\sqrt{-4x^2-9} - \frac{(-4x^2-9)^{\frac{3}{2}}}{162x^2} + \frac{(-4x^2-9)^{\frac{3}{2}}}{36x^4} - \frac{2}{27}i\log\left(\frac{6\sqrt{4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
-2/81*sqrt(-4*x^2 - 9) - 1/162*(-4*x^2 - 9)^(3/2)/x^2 + 1/36*(-4*x^2 - 9)^
(3/2)/x^4 - 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = -\frac{i\,(4\,x^2+9)^{\frac{3}{2}}+9\,\sqrt{-4\,x^2-9}}{72\,x^4}+\frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

input
```
integrate((-4*x^2-9)^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/72*(I*(4*x^2 + 9)^(3/2) + 9*sqrt(-4*x^2 - 9))/x^4 + 2/27*arctan(1/3*sqr
t(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27}-\frac{\frac{\sqrt{-4\,x^2-9}}{8}-\frac{{\left(-4\,x^2-9\right)}^{3/2}}{72}}{x^4}$$

input
```
int((- 4*x^2 - 9)^(1/2)/x^5,x)
```

output
```
(2*atan((- 4*x^2 - 9)^(1/2)/3))/27 - ((- 4*x^2 - 9)^(1/2)/8 - (- 4*x^2 - 9
)^(3/2)/72)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt{-9-4x^2}}{x^5}\,dx = \frac{6\sqrt{-4x^2-9}\,x^2+27\sqrt{-4x^2-9}-8\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}-1\right)i\,x^4+8\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}+1\right)i\,x^4}{108x^4}$$

input
```
int((-4*x^2-9)^(1/2)/x^5,x)
```

output
```
(6*sqrt( - 4*x**2 - 9)*x**2 + 27*sqrt( - 4*x**2 - 9) - 8*log((sqrt( - 4*x*
*2 - 9)*i + 2*x - 3)/3)*i*x**4 + 8*log((sqrt( - 4*x**2 - 9)*i + 2*x + 3)/3
)*i*x**4)/(108*x**4)
```

3.495 $\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{a^2\sqrt{a+bx^2}}{b^3} - \frac{2a(a+bx^2)^{3/2}}{3b^3} + \frac{(a+bx^2)^{5/2}}{5b^3}$$

output
```
a^2*(b*x^2+a)^(1/2)/b^3-2/3*a*(b*x^2+a)^(3/2)/b^3+1/5*(b*x^2+a)^(5/2)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(8a^2-4abx^2+3b^2x^4)}{15b^3}$$

input
```
Integrate[x^5/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2 - 4*a*b*x^2 + 3*b^2*x^4))/(15*b^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2\sqrt{bx^2+a}}-\frac{2\sqrt{bx^2+a}a}{b^2}+\frac{\left(bx^2+a\right)^{3/2}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^2\sqrt{a+bx^2}}{b^3}+\frac{2\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{4a\left(a+bx^2\right)^{3/2}}{3b^3}\right)$$

input
```
Int[x^5/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*Sqrt[a + b*x^2])/b^3 - (4*a*(a + b*x^2)^(3/2))/(3*b^3) + (2*(a + b
*x^2)^(5/2))/(5*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}-4 a b\,x^{2}+8 a^{2}\right)}{15 b^{3}}$	36
trager	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}-4 a b\,x^{2}+8 a^{2}\right)}{15 b^{3}}$	36
risch	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}-4 a b\,x^{2}+8 a^{2}\right)}{15 b^{3}}$	36
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}-4 a b\,x^{2}+8 a^{2}\right)}{15 b^{3}}$	36
orering	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 b^{2} x^{4}-4 a b\,x^{2}+8 a^{2}\right)}{15 b^{3}}$	36
default	$\frac{x^{4} \sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a \left(\frac{x^{2} \sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a \sqrt{b\,x^{2}+a}}{3 b^{2}}\right)}{5 b}$	58

input
```
int(x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(b*x^2+a)^(1/2)*(3*b^2*x^4-4*a*b*x^2+8*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx=\frac{(3\,b^2x^4-4\,abx^2+8\,a^2)\sqrt{bx^2+a}}{15\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/15*(3*b^2*x^4 - 4*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/b^3
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{8a^2\sqrt{a+bx^2}}{15b^3} - \frac{4ax^2\sqrt{a+bx^2}}{15b^2} + \frac{x^4\sqrt{a+bx^2}}{5b} & \text{for } b \neq 0 \\ \frac{x^6}{6\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((8*a**2*sqrt(a + b*x**2)/(15*b**3) - 4*a*x**2*sqrt(a + b*x**2)/(
15*b**2) + x**4*sqrt(a + b*x**2)/(5*b), Ne(b, 0)), (x**6/(6*sqrt(a)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}\,x^4}{5\,b} - \frac{4\,\sqrt{bx^2+a}\,ax^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}\,a^2}{15\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*x^4/b - 4/15*sqrt(b*x^2 + a)*a*x^2/b^2 + 8/15*sqrt(b*x
^2 + a)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}a^2}{b^3} + \frac{3\,(bx^2+a)^{\frac{5}{2}} - 10\,(bx^2+a)^{\frac{3}{2}}a}{15\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)*a^2/b^3 + 1/15*(3*(b*x^2 + a)^(5/2) - 10*(b*x^2 + a)^(3/2)
*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\,\left(\frac{8\,a^2}{15\,b^3} + \frac{x^4}{5\,b} - \frac{4\,a\,x^2}{15\,b^2}\right)$$

input
```
int(x^5/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((8*a^2)/(15*b^3) + x^4/(5*b) - (4*a*x^2)/(15*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(3b^2x^4 - 4ab\,x^2 + 8a^2)}{15b^3}$$

input
```
int(x^5/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**2 - 4*a*b*x**2 + 3*b**2*x**4))/(15*b**3)
```

3.496 $\int \frac{x^4}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx=-\frac{3ax\sqrt{a+bx^2}}{8b^2}+\frac{x^3\sqrt{a+bx^2}}{4b}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
-3/8*a*x*(b*x^2+a)^(1/2)/b^2+1/4*x^3*(b*x^2+a)^(1/2)/b+3/8*a^2*arctanh(b^(
1/2)*x/(b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-3ax+2bx^3)}{8b^2}+\frac{3a^2\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{5/2}}$$

input
```
Integrate[x^4/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-3*a*x + 2*b*x^3))/(8*b^2) + (3*a^2*ArcTanh[(Sqrt[b]*x)/
(-Sqrt[a] + Sqrt[a + b*x^2])])/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}$$

input
```
Int[x^4/Sqrt[a + b*x^2],x]
```

output
```
(x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

method	result	size
risch	$-\frac{x\left(-2b\,x^{2}+3a\right)\sqrt{b\,x^{2}+a}}{8b^{2}}+\frac{3a^{2}\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}$	51
pseudoelliptic	$\frac{2\sqrt{b\,x^{2}+a}\,b^{\frac{3}{2}}x^{3}-3ax\sqrt{b\,x^{2}+a}\,\sqrt{b}+3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a^{2}}{8b^{\frac{5}{2}}}$	62
default	$\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	63

input
```
int(x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-2*b*x^2+3*a)*(b*x^2+a)^(1/2)/b^2+3/8*a^2/b^(5/2)*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.70

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \left[\frac{3\,a^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{16\,b^3},\ -\frac{3\,a^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,b^2x^3-3\,abx)\sqrt{bx^2+a}}{8\,b^3}\right]$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3, -1/8*(3*a^2*sqrt(-b)*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - (2*b^2*x^3 - 3*a*b*x)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.30

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = -\frac{3a^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{x^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/2),x)
```

output
```
-3*a**(3/2)*x/(8*b**2*sqrt(1 + b*x**2/a)) - sqrt(a)*x**3/(8*b*sqrt(1 + b*x
**2/a)) + 3*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) + x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}x^3}{4\,b} - \frac{3\,\sqrt{bx^2+a}ax}{8\,b^2} + \frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*x^3/b - 3/8*sqrt(b*x^2 + a)*a*x/b^2 + 3/8*a^2*arcsinh(
b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \frac{1}{8}\,\sqrt{bx^2+a}x\left(\frac{2\,x^2}{b} - \frac{3\,a}{b^2}\right) - \frac{3\,a^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*x*(2*x^2/b - 3*a/b^2) - 3/8*a^2*log(abs(-sqrt(b)*x + s
qrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \int \frac{x^4}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(1/2),x)
```

output
```
int(x^4/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

$$\int \frac{x^4}{\sqrt{a+bx^2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2}{8b^3}$$

input
```
int(x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2)/(8*b**3)
```

3.497 $\int \frac{x^3}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^3}{\sqrt{a+bx^2}} \, dx = -\frac{a\sqrt{a+bx^2}}{b^2} + \frac{(a+bx^2)^{3/2}}{3b^2}$$

output
```
-a*(b*x^2+a)^(1/2)/b^2+1/3*(b*x^2+a)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\sqrt{a+bx^2}} \, dx = \frac{(-2a+bx^2)\sqrt{a+bx^2}}{3b^2}$$

input
```
Integrate[x^3/Sqrt[a + b*x^2],x]
```

output
```
((-2*a + b*x^2)*Sqrt[a + b*x^2])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{\sqrt{bx^2+a}}{b}-\frac{a}{b\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx^2}}{b^2}\right)$$

input
```
Int[x^3/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*Sqrt[a + b*x^2])/b^2 + (2*(a + b*x^2)^(3/2))/(3*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{2}-2a\right)\sqrt{b\,x^{2}+a}}{3b^{2}}$	24
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2a\right)}{3b^{2}}$	25
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2a\right)}{3b^{2}}$	25
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2a\right)}{3b^{2}}$	25
orering	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-b\,x^{2}+2a\right)}{3b^{2}}$	25
default	$\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}$	34

input
```
int(x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2-2*a)*(b*x^2+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.64

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}(bx^2-2\,a)}{3\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*x^2 + a)*(b*x^2 - 2*a)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{2a\sqrt{a+bx^2}}{3b^2} + \frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b \neq 0 \\ \frac{x^4}{4\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), N
e(b, 0)), (x**4/(4*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}x^2}{3\,b} - \frac{2\,\sqrt{bx^2+a}a}{3\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*x^2/b - 2/3*sqrt(b*x^2 + a)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}}{3\,b^2} - \frac{\sqrt{bx^2+a}a}{b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3*(b*x^2 + a)^(3/2)/b^2 - sqrt(b*x^2 + a)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2}$$

input
```
int(x^3/(a + b*x^2)^(1/2),x)
```

output
```
-((a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(b\,x^2-2a)}{3b^2}$$

input
```
int(x^3/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a + b*x**2))/(3*b**2)
```

3.498 $\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx=\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*x*(b*x^2+a)^(1/2)/b-1/2*a*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx=\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[x^2/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*
x^2])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{262}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}$$
$$\Big\downarrow \text{224}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}$$
$$\Big\downarrow \text{219}$$
$$\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

input
```
Int[x^2/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^
(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	39
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	39
pseudoelliptic	$-\frac{-\sqrt{b\,x^{2}+a}\,x\sqrt{b}+\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a}{2b^{\frac{3}{2}}}$	41

input
```
int(x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \left[\frac{2\,\sqrt{bx^2+a}bx+a\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{4\,b^2}, \frac{\sqrt{bx^2+a}bx+a\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{2\,b^2}\right]$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*sqrt(b*x^2 + a)*b*x + a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*s
qrt(b)*x - a))/b^2, 1/2*(sqrt(b*x^2 + a)*b*x + a*sqrt(-b)*arctan(sqrt(-b)*
x/sqrt(b*x^2 + a)))/b^2]
```

Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a}x\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{a\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}x}{2\,b} - \frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*x/b - 1/2*a*arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}x}{2\,b} + \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x/b + 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(
3/2)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{x^3}{3\sqrt{a}} & \text{if } b=0 \\ \frac{x\sqrt{b\,x^2+a}}{2\,b} - \frac{a\,\ln\left(2\sqrt{b}\,x+2\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} & \text{if } b\neq 0 \end{cases}$$

input
```
int(x^2/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, x^3/(3*a^(1/2)), b ~= 0, (x*(a + b*x^2)^(1/2))/(2*b) - (
a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx - \sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{2b^2}$$

input
```
int(x^2/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x - sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a)/(2*b**2)
```

3.499 $\int \frac{x}{\sqrt{a+bx^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{\sqrt{a+bx^2}} \, dx = \frac{\sqrt{a+bx^2}}{b}$$

output
```
(b*x^2+a)^(1/2)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{a+bx^2}} \, dx = \frac{\sqrt{a+bx^2}}{b}$$

input
```
Integrate[x/Sqrt[a + b*x^2],x]
```

output
```
Sqrt[a + b*x^2]/b
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 241$$

$$\frac{\sqrt{a+bx^2}}{b}$$

input
```
Int[x/Sqrt[a + b*x^2],x]
```

output
```
Sqrt[a + b*x^2]/b
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
derivativedivides	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
default	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
trager	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
risch	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
pseudoelliptic	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14
orering	$\frac{\sqrt{b\,x^{2}+a}}{b}$	14

input
```
int(x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}}{b}$$

input
```
integrate(x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b*x^2 + a)/b
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}}{b}$$

input
```
int(x/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}}{b}$$

input
```
int(x/(b*x^2+a)^(1/2),x)
```

output
```
sqrt(a + b*x**2)/b
```

3.500 $\int \frac{1}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Integrate[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-\frac{bx^2}{a+bx^2}}d\frac{x}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

input
```
Int[1/Sqrt[a + b*x^2],x]
```

output
```
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/Sqrt[b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$	21
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{\sqrt{b}}$	22

input
```
int(1/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(b^(1/2)*x+(b*x^2+a)^(1/2))/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,\sqrt{b}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b}\right]$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)/sqrt(b), -sqrt(-b)*ar
ctan(sqrt(-b)*x/sqrt(b*x^2 + a))/b]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x**2+a)**(1/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/sqrt(b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(b*x/sqrt(a*b))/sqrt(b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.48

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}x - \frac{a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}}$$

input
```
integrate(1/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*x - 1/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(
b)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int(1/(a + b*x^2)^(1/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)}{b}$$

input
```
int(1/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)))/b
```

3.501 $\int \frac{1}{x\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[1/(x*Sqrt[a + b*x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \; 73$$

$$\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}$$

$$\Big\downarrow \; 221$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[1/(x*Sqrt[a + b*x^2]),x]
```

output
```
-(ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/Sqrt[a])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{\sqrt{a}}$	20
default	$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$	29

input
```
int(1/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = \left[\frac{\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\sqrt{a}}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)}{a}\right]$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)/sqrt(a), sqrt(-a)
*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)/a]
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = -\frac{\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x**2+a)**(1/2),x)
```

output
```
-asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int(1/(x*(a + b*x^2)^(1/2)),x)
```

output
```
-atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.12

$$\int \frac{1}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a}\left(\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)-\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{a}$$

input
```
int(1/x/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a)*(log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)) - log((sqr
t(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))))/a
```

3.502 $\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

output
```
-(b*x^2+a)^(1/2)/a/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{ax}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-(Sqrt[a + b*x^2]/(a*x))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{242\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 242$$
$$-\frac{\sqrt{a+bx^2}}{ax}$$

input
```
Int[1/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-(Sqrt[a + b*x^2]/(a*x))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
default	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
trager	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
risch	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
pseudoelliptic	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18
orering	$-\frac{\sqrt{b\,x^{2}+a}}{a x}$	18

input
```
int(1/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)^(1/2)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x^2 + a)/(a*x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}}{ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(b*x^2 + a)/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(b)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}}{a\,x}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
-(a + b*x^2)^(1/2)/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2\sqrt{a+bx^2}}\,dx = \frac{-\sqrt{b\,x^2+a}-\sqrt{b}\,x}{ax}$$

input
```
int(1/x^2/(b*x^2+a)^(1/2),x)
```

output
```
( - (sqrt(a + b*x**2) + sqrt(b)*x))/(a*x)
```

3.503 $\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 50

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{2ax^2}+\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*Sqrt[a + b*x^2]/(a*x^2) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(
3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)$$

input

```
Int[1/(x^3*Sqrt[a + b*x^2]),x]
```

output

```
(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b\,x^{2}-\sqrt{a}\,\sqrt{b\,x^{2}+a}}{2 a^{\frac{3}{2}} x^{2}}$	43
default	$-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}$	48
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}$	48

input
```
int(1/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*b*x^2-a^(1/2)*(b*x^2+a)^(1/2))/a^(3/
2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = \left[\frac{\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}a}{4a^2x^2}, -\frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\sqrt{bx^2+a}a}{2a^2x^2}\right]$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
2*sqrt(b*x^2 + a)*a)/(a^2*x^2), -1/2*(sqrt(-a)*b*x^2*arctan(sqrt(b*x^2 + a
)*sqrt(-a)/a) + sqrt(b*x^2 + a)*a)/(a^2*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate(1/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a*
*(3/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = \frac{b\,\text{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}}{2\,ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - 1/2*sqrt(b*x^2 + a)/(a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = -\frac{1}{2}\,b\left(\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx^2+a}}{abx^2}\right)$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*b*(arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x^2 + a)/(a
*b*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx = \frac{b\,\text{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (a + b*x^2)^(1/2)/(2*a*
x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.58

$$\int \frac{1}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \frac{-\sqrt{b\,x^2+a}\,a-\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2}{2a^2x^2}$$

input
```
int(1/x^3/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a - sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)
*x)/sqrt(a))*b*x**2 + sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*b*x**2)/(2*a**2*x**2)
```

3.504 $\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{3ax^3}+\frac{2b\sqrt{a+bx^2}}{3a^2x}$$

output
```
-1/3*(b*x^2+a)^(1/2)/a/x^3+2/3*b*(b*x^2+a)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-a+2bx^2)}{3a^2x^3}$$

input
```
Integrate[1/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-a + 2*b*x^2))/(3*a^2*x^3)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{245}$$
$$-\frac{2b\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}$$
$$\Big\downarrow \text{242}$$
$$\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*Sqrt[a + b*x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
pseudoelliptic	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
orering	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-2 b\,x^{2}+a \right)}{3 a^{2} x^{3}}$	26
default	$-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b \sqrt{b\,x^{2}+a}}{3 a^{2} x}$	37

input
```
int(1/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)^(1/2)*(-2*b*x^2+a)/a^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx=\frac{(2\,bx^2-a)\sqrt{bx^2+a}}{3\,a^2x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^2 - a)*sqrt(b*x^2 + a)/(a^2*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{3ax^2} + \frac{2b^{\frac{3}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/2),x)
```

output
```
-sqrt(b)*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*sqrt(a/(b*x**2) + 1)
/(3*a**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{bx^2+a}b}{3a^2x} - \frac{\sqrt{bx^2+a}}{3ax^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
2/3*sqrt(b*x^2 + a)*b/(a^2*x) - 1/3*sqrt(b*x^2 + a)/(a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = \frac{4\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
4/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*b^(3/2)/((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}$$

input
```
int(1/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4\sqrt{a+bx^2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a+2\sqrt{b\,x^2+a}\,b\,x^2-2\sqrt{b}\,b\,x^3}{3a^2x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(1/2),x)
```

output
```
( - sqrt(a + b*x**2)*a + 2*sqrt(a + b*x**2)*b*x**2 - 2*sqrt(b)*b*x**3)/(3*
a**2*x**3)
```

3.505 $\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}}{4ax^4}+\frac{3b\sqrt{a+bx^2}}{8a^2x^2}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/4*(b*x^2+a)^(1/2)/a/x^4+3/8*b*(b*x^2+a)^(1/2)/a^2/x^2-3/8*b^2*arctanh((
b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2a+3bx^2)}{8a^2x^4}-\frac{3b^2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[1/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a + 3*b*x^2))/(8*a^2*x^4) - (3*b^2*ArcTanh[Sqrt[a + b
*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(-\frac{3b\int \frac{1}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(-\frac{3b\left(-\frac{b\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(-\frac{3b\left(-\frac{\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{a}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx^2}}{ax^2}\right)}{4a}-\frac{\sqrt{a+bx^2}}{2ax^4}\right)$$

input
```
Int[1/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(-1/2*Sqrt[a + b*x^2]/(a*x^4) - (3*b*(-(Sqrt[a + b*x^2]/(a*x^2)) + (b*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(4*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-3b\,x^{2}+2a\right)}{8a^{2}x^{4}}-\frac{3b^{2}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{8a^{\frac{5}{2}}}$	60
pseudoelliptic	$\frac{-3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)b^{2}x^{4}+3b\,x^{2}\sqrt{b\,x^{2}+a}\,\sqrt{a}-2a^{\frac{3}{2}}\sqrt{b\,x^{2}+a}}{8a^{\frac{5}{2}}x^{4}}$	64
default	$-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}$	72

input
```
int(1/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(-3*b*x^2+2*a)/a^2/x^4-3/8*b^2/a^(5/2)*ln((2*a+2*a^(1
/2)*(b*x^2+a)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.86

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx$$
$$= \left[\frac{3\sqrt{a}b^2x^4\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,abx^2-2\,a^2)\sqrt{bx^2+a}}{16\,a^3x^4}, \frac{3\sqrt{-a}b^2x^4\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(3}{8\,a^3x^4}\right.$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*(3*sqrt(a)*b^2*x^4*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^
2) + 2*(3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 + a))/(a^3*x^4), 1/8*(3*sqrt(-a)*b^2
*x^4*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*a*b*x^2 - 2*a^2)*sqrt(b*x^2 +
 a))/(a^3*x^4)]
```

Sympy [A] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = -\frac{1}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}}{8ax^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{3}{2}}}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{3b^2\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate(1/x**5/(b*x**2+a)**(1/2),x)
```

output
```
-1/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)/(8*a*x**3*sqrt(a/(b*x**
2) + 1)) + 3*b**(3/2)/(8*a**2*x*sqrt(a/(b*x**2) + 1)) - 3*b**2*asinh(sqrt(
a)/(sqrt(b)*x))/(8*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = -\frac{3\,b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{3\sqrt{bx^2+a}b}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}}{4\,ax^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 3/8*sqrt(b*x^2 + a)*b/(a^
2*x^2) - 1/4*sqrt(b*x^2 + a)/(a*x^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = \frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,(bx^2+a)^{\frac{3}{2}}b^3 - 5\,\sqrt{bx^2+a}ab^3}{a^2b^2x^4}}{8\,b}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(3*b^3*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a
)^(3/2)*b^3 - 5*sqrt(b*x^2 + a)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = \frac{3\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4} - \frac{5\,\sqrt{b\,x^2+a}}{8\,a\,x^4} - \frac{3\,b^2\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

input
```
int(1/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
(3*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (5*(a + b*x^2)^(1/2))/(8*a*x^4) - (3*b
^2*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^5\sqrt{a+bx^2}}\,dx = \frac{-2\sqrt{b\,x^2+a}\,a^2 + 3\sqrt{b\,x^2+a}\,ab\,x^2 + 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^4 - 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b^2x^4}{8a^3x^4}$$

input
```
int(1/x^5/(b*x^2+a)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x**2)*a**2 + 3*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 - 3*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4)/(8*a**3*x**
4)
```

3.506 $\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = -\frac{a^2}{b^3\sqrt{a+bx^2}} - \frac{2a\sqrt{a+bx^2}}{b^3} + \frac{(a+bx^2)^{3/2}}{3b^3}$$

output
```
-a^2/b^3/(b*x^2+a)^(1/2)-2*a*(b*x^2+a)^(1/2)/b^3+1/3*(b*x^2+a)^(3/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = \frac{-8a^2-4abx^2+b^2x^4}{3b^3\sqrt{a+bx^2}}$$

input
```
Integrate[x^5/(a + b*x^2)^(3/2),x]
```

output
```
(-8*a^2 - 4*a*b*x^2 + b^2*x^4)/(3*b^3*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2 (bx^2+a)^{3/2}}-\frac{2a}{b^2\sqrt{bx^2+a}}+\frac{\sqrt{bx^2+a}}{b^2}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{2a^2}{b^3\sqrt{a+bx^2}}-\frac{4a\sqrt{a+bx^2}}{b^3}+\frac{2(a+bx^2)^{3/2}}{3b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^(3/2),x]
```

output
```
((-2*a^2)/(b^3*Sqrt[a + b*x^2]) - (4*a*Sqrt[a + b*x^2])/b^3 + (2*(a + b*x^
2)^(3/2))/(3*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{b^{2} x^{4}-4 a b \,x^{2}-8 a^{2}}{3 \sqrt{b \,x^{2}+a}\, b^{3}}$	35
gosper	$-\frac{-b^{2} x^{4}+4 a b \,x^{2}+8 a^{2}}{3 \sqrt{b \,x^{2}+a}\, b^{3}}$	36
trager	$-\frac{-b^{2} x^{4}+4 a b \,x^{2}+8 a^{2}}{3 \sqrt{b \,x^{2}+a}\, b^{3}}$	36
orering	$-\frac{-b^{2} x^{4}+4 a b \,x^{2}+8 a^{2}}{3 \sqrt{b \,x^{2}+a}\, b^{3}}$	36
risch	$-\frac{\left(-b \,x^{2}+5 a \right) \sqrt{b \,x^{2}+a}}{3 b^{3}}-\frac{a^{2}}{b^{3} \sqrt{b \,x^{2}+a}}$	43
default	$\frac{x^{4}}{3 \sqrt{b \,x^{2}+a}\, b}-\frac{4 a \left(\frac{x^{2}}{b \sqrt{b \,x^{2}+a}}+\frac{2 a}{b^{2} \sqrt{b \,x^{2}+a}}\right)}{3 b}$	57

input
```
int(x^5/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b^2*x^4-4*a*b*x^2-8*a^2)/(b*x^2+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = \frac{(b^2x^4-4\,abx^2-8\,a^2)\sqrt{bx^2+a}}{3\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(b^2*x^4 - 4*a*b*x^2 - 8*a^2)*sqrt(b*x^2 + a)/(b^4*x^2 + a*b^3)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{8a^2}{3b^3\sqrt{a+bx^2}} - \frac{4ax^2}{3b^2\sqrt{a+bx^2}} + \frac{x^4}{3b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-8*a**2/(3*b**3*sqrt(a + b*x**2)) - 4*a*x**2/(3*b**2*sqrt(a + b
*x**2)) + x**4/(3*b*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(3/2)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = \frac{x^4}{3\sqrt{bx^2+a}b} - \frac{4\,ax^2}{3\sqrt{bx^2+a}b^2} - \frac{8\,a^2}{3\sqrt{bx^2+a}b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/3*x^4/(sqrt(b*x^2 + a)*b) - 4/3*a*x^2/(sqrt(b*x^2 + a)*b^2) - 8/3*a^2/(s
qrt(b*x^2 + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{3\,a^2}{\sqrt{bx^2+a}b} - \frac{(bx^2+a)^{\frac{3}{2}}b^2-6\sqrt{bx^2+a}ab^2}{b^3}}{3\,b^2}$$

input
```
integrate(x^5/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/3*(3*a^2/(sqrt(b*x^2 + a)*b) - ((b*x^2 + a)^(3/2)*b^2 - 6*sqrt(b*x^2 +
a)*a*b^2)/b^3)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = -\frac{6\,a\,(b\,x^2+a)-(b\,x^2+a)^2+3\,a^2}{3\,b^3\,\sqrt{b\,x^2+a}}$$

input
```
int(x^5/(a + b*x^2)^(3/2),x)
```

output
```
-(6*a*(a + b*x^2) - (a + b*x^2)^2 + 3*a^2)/(3*b^3*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{x^5}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(b^2x^4-4ab\,x^2-8a^2)}{3b^3\,(b\,x^2+a)}$$

input
```
int(x^5/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 8*a**2 - 4*a*b*x**2 + b**2*x**4))/(3*b**3*(a + b*x**
2))
```

3.507 $\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = -\frac{x^3}{b\sqrt{a+bx^2}}+\frac{3x\sqrt{a+bx^2}}{2b^2}-\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-x^3/b/(b*x^2+a)^(1/2)+3/2*x*(b*x^2+a)^(1/2)/b^2-3/2*a*arctanh(b^(1/2)*x/(
b*x^2+a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \frac{3ax+bx^3}{2b^2\sqrt{a+bx^2}}-\frac{3a\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(3/2),x]
```

output
```
(3*a*x + b*x^3)/(2*b^2*Sqrt[a + b*x^2]) - (3*a*ArcTanh[(Sqrt[b]*x)/(-Sqrt[
a] + Sqrt[a + b*x^2])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/2}}dx$$
$$\Big\downarrow \text{252}$$
$$\frac{3\int \frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{262}$$
$$\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{224}$$
$$\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{219}$$
$$\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}$$

input
```
Int[x^4/(a + b*x^2)^(3/2),x]
```

output
```
-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2b^{2}}+\frac{ax}{b^{2}\sqrt{b\,x^{2}+a}}-\frac{3a\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{5}{2}}}$	54
pseudoelliptic	$\frac{b^{\frac{3}{2}}x^{3}-3\sqrt{b\,x^{2}+a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)a+3xa\sqrt{b}}{2b^{\frac{5}{2}}\sqrt{b\,x^{2}+a}}$	59
default	$\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}$	61

input
```
int(x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*(b*x^2+a)^(1/2)/b^2+a/b^2*x/(b*x^2+a)^(1/2)-3/2*a/b^(5/2)*ln(b^(1/2)
*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.34

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \left[\frac{3\,(abx^2+a^2)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(b^2x^3+3\,abx)\sqrt{bx^2+a}}{4\,(b^4x^2+ab^3)}, \ 3$$

input
```
integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
 - a) + 2*(b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3), 1/2*(3*(
a*b*x^2 + a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (b^2*x^3 + 3*
a*b*x)*sqrt(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [A] (verification not implemented)

Time = 1.84 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4/(b*x**2+a)**(3/2),x)
```

output
```
3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*
b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \frac{x^3}{2\sqrt{bx^2+ab}} + \frac{3\,ax}{2\sqrt{bx^2+ab^2}} - \frac{3\,a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*x^3/(sqrt(b*x^2 + a)*b) + 3/2*a*x/(sqrt(b*x^2 + a)*b^2) - 3/2*a*arcsin
h(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \frac{x\left(\frac{x^2}{b}+\frac{3\,a}{b^2}\right)}{2\sqrt{bx^2+a}} + \frac{3\,a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*x*(x^2/b + 3*a/b^2)/sqrt(b*x^2 + a) + 3/2*a*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(3/2),x)
```

output
```
int(x^4/(a + b*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(a+bx^2)^{3/2}}\,dx = \frac{12\sqrt{b\,x^2+a}\,abx+4\sqrt{b\,x^2+a}\,b^2x^3-12\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2-12\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+}{\sqrt{a}}\right.}{8b^3\,(b\,x^2+a)}$$

input
```
int(x^4/(b*x^2+a)^(3/2),x)
```

output
```
(12*sqrt(a + b*x**2)*a*b*x + 4*sqrt(a + b*x**2)*b**2*x**3 - 12*sqrt(b)*log
((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2 - 12*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a*b*x**2 + 9*sqrt(b)*a**2 + 9*sqrt(b)*a*b*x**
2)/(8*b**3*(a + b*x**2))
```

3.508 $\int \frac{x^3}{(a+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{x^3}{(a+bx^2)^{3/2}} \, dx = \frac{a}{b^2\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^2}$$

output
```
a/b^2/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{(a+bx^2)^{3/2}} \, dx = \frac{2a+bx^2}{b^2\sqrt{a+bx^2}}$$

input
```
Integrate[x^3/(a + b*x^2)^(3/2),x]
```

output
```
(2*a + b*x^2)/(b^2*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx$$

$$\big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{3/2}}dx^2$$

$$\big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{b\sqrt{bx^2+a}}-\frac{a}{b\,(bx^2+a)^{3/2}}\right)dx^2$$

$$\big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}}{b^2}\right)$$

input
```
Int[x^3/(a + b*x^2)^(3/2),x]
```

output
```
((2*a)/(b^2*Sqrt[a + b*x^2]) + (2*Sqrt[a + b*x^2])/b^2)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{b\,x^2+2a}{\sqrt{b\,x^2+a}\,b^2}$	23
trager	$\frac{b\,x^2+2a}{\sqrt{b\,x^2+a}\,b^2}$	23
pseudoelliptic	$\frac{b\,x^2+2a}{\sqrt{b\,x^2+a}\,b^2}$	23
orering	$\frac{b\,x^2+2a}{\sqrt{b\,x^2+a}\,b^2}$	23
risch	$\frac{a}{b^2\sqrt{b\,x^2+a}}+\frac{\sqrt{b\,x^2+a}}{b^2}$	29
default	$\frac{x^2}{b\sqrt{b\,x^2+a}}+\frac{2a}{b^2\sqrt{b\,x^2+a}}$	33

input
```
int(x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+2*a)/(b*x^2+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \frac{(bx^2+2\,a)\sqrt{bx^2+a}}{b^3x^2+ab^2}$$

input
```
integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^3*x^2 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
0)), (x**4/(4*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \frac{x^2}{\sqrt{bx^2+a}b} + \frac{2\,a}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x^2/(sqrt(b*x^2 + a)*b) + 2*a/(sqrt(b*x^2 + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{\sqrt{bx^2+a}}{b}+\frac{a}{\sqrt{bx^2+ab}}}{b}$$

input
```
integrate(x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(b*x^2 + a)/b + a/(sqrt(b*x^2 + a)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \frac{b\,x^2+2\,a}{b^2\,\sqrt{b\,x^2+a}}$$

input
```
int(x^3/(a + b*x^2)^(3/2),x)
```

output
```
(2*a + b*x^2)/(b^2*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(b\,x^2+2a)}{b^2\,(b\,x^2+a)}$$

input
```
int(x^3/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(2*a + b*x**2))/(b**2*(a + b*x**2))
```

3.509 $\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = -\frac{x}{b\sqrt{a+bx^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
-x/b/(b*x^2+a)^(1/2)+arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = -\frac{x}{b\sqrt{a+bx^2}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(3/2),x]
```

output
```
-(x/(b*Sqrt[a + b*x^2])) + (2*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x
^2])])/b^(3/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(3/2),x]
```

output
```
-(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]/b^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}$	37
pseudoelliptic	$-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)}{b^{\frac{3}{2}}}$	38

input
```
int(x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.02

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = \left[-\frac{2\,\sqrt{bx^2+a}bx-(bx^2+a)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)}{2\,(b^3x^2+ab^2)},\right.$$
$$\left.-\frac{\sqrt{bx^2+a}bx+(bx^2+a)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)}{b^3x^2+ab^2}\right]$$

input
```
integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(2*sqrt(b*x^2 + a)*b*x - (b*x^2 + a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a))/(b^3*x^2 + a*b^2), -(sqrt(b*x^2 + a)*b*x + (b*x^
2 + a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)))/(b^3*x^2 + a*b^2)]
```

Sympy [A] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = \frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2/(b*x**2+a)**(3/2),x)
```

output
```
asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = -\frac{x}{\sqrt{bx^2+a}b} + \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-x/(sqrt(b*x^2 + a)*b) + arcsinh(b*x/sqrt(a*b))/b^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = -\frac{x}{\sqrt{bx^2+a}\,b} - \frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-x/(sqrt(b*x^2 + a)*b) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = \frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int(x^2/(a + b*x^2)^(3/2),x)
```

output
```
log(b^(1/2)*x + (a + b*x^2)^(1/2))/b^(3/2) - x/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.05

$$\int \frac{x^2}{(a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,bx+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a+\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2-\sqrt{b}\,a-\sqrt{b}\,b\,x}{b^2\left(b\,x^2+a\right)}$$

input
```
int(x^2/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*b*x + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt
(a))*a + sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*b*x**2 - sqrt
(b)*a - sqrt(b)*b*x**2)/(b**2*(a + b*x**2))
```

3.510 $\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{1}{b\sqrt{a+bx^2}}$$

output
```
-1/b/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{1}{b\sqrt{a+bx^2}}$$

input
```
Integrate[x/(a + b*x^2)^(3/2),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2]))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{241}$$

$$-\frac{1}{b\sqrt{a+bx^2}}$$

input
```
Int[x/(a + b*x^2)^(3/2),x]
```

output
```
-(1/(b*Sqrt[a + b*x^2]))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
derivativedivides	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
default	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
trager	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
pseudoelliptic	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15
orering	$-\frac{1}{b\sqrt{b\,x^{2}+a}}$	15

input
```
int(x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{\sqrt{bx^2+a}}{b^2x^2+ab}$$

input
```
integrate(x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-sqrt(b*x^2 + a)/(b^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = \begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{1}{\sqrt{bx^2+a}b}$$

input
```
integrate(x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-1/(sqrt(b*x^2 + a)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{1}{\sqrt{bx^2+a}b}$$

input
```
integrate(x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-1/(sqrt(b*x^2 + a)*b)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{1}{b\sqrt{b\,x^2+a}}$$

input
```
int(x/(a + b*x^2)^(3/2),x)
```

output
```
-1/(b*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{x}{(a+bx^2)^{3/2}}\,dx = -\frac{\sqrt{b\,x^2+a}}{b\,(b\,x^2+a)}$$

input
```
int(x/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2))/(b*(a + b*x**2))
```

3.511 $\int \frac{1}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+bx^2}}$$

output
```
x/a/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx$$
$$\Big\downarrow \ 208$$
$$\frac{x}{a\sqrt{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-3/2),x]
```

output
```
x/(a*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
default	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
trager	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
pseudoelliptic	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15
orering	$\frac{x}{a\sqrt{b\,x^{2}+a}}$	15

input
```
int(1/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x/a/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{bx^2+a}\,x}{abx^2+a^2}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x/(a*b*x^2 + a^2)
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(3/2),x)
```

output
```
x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{x}{a\sqrt{b\,x^2+a}}$$

input
```
int(1/(a + b*x^2)^(3/2),x)
```

output
```
x/(a*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,bx+\sqrt{b}\,a+\sqrt{b}\,b\,x^2}{ab\,(b\,x^2+a)}$$

input
```
int(1/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*b*x + sqrt(b)*a + sqrt(b)*b*x**2)/(a*b*(a + b*x**2))
```

3.512 $\int \frac{1}{x(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 41

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{1}{a\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
1/a/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{1}{a\sqrt{a+bx^2}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(3/2)),x]
```

output
```
1/(a*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(3/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2(bx^2+a)^{3/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}$	34
default	$\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}$	43

input
```
int(1/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.15

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{(bx^2+a)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\sqrt{bx^2+a}a}{2\left(a^2bx^2+a^3\right)}, \frac{(bx^2+a)\sqrt{-a}\arctan\left(\frac{\sqrt{ba}}{}\right.}{a^2bx^2+}\right.$$

input
```
integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((b*x^2 + a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x
^2) + 2*sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3), ((b*x^2 + a)*sqrt(-a)*arctan
(sqrt(b*x^2 + a)*sqrt(-a)/a) + sqrt(b*x^2 + a)*a)/(a^2*b*x^2 + a^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. 2(34) = 68.

Time = 0.98 (sec) , antiderivative size = 184, normalized size of antiderivative = 4.49

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}$$

input
```
integrate(1/x/(b*x**2+a)**(3/2),x)
```

output
```
2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*x*
*2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) + 1
)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{1}{x(a+bx^2)^{3/2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{1}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 1/(sqrt(b*x^2 + a)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{x(a+bx^2)^{3/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{1}{\sqrt{bx^2+a}a}$$

input
```
integrate(1/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 1/(sqrt(b*x^2 + a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+bx^2)^{3/2}}\,dx = \frac{1}{a\sqrt{b\,x^2+a}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int(1/(x*(a + b*x^2)^(3/2)),x)
```

output
```
1/(a*(a + b*x^2)^(1/2)) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.32

$$\int \frac{1}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,a+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a+\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)b\,x^2-\sqrt{a}\log\left(\right.}{a^2\left(b\,x^2+a\right)}$$

input
```
int(1/x/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*a + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)
/sqrt(a))*a + sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a)
)*b*x**2 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a
 - sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b*x**2)/(
a**2*(a + b*x**2))
```

3.513 $\int \frac{1}{x^2(a+bx^2)^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} dx = \frac{1}{ax\sqrt{a+bx^2}} - \frac{2\sqrt{a+bx^2}}{a^2x}$$

output
```
1/a/x/(b*x^2+a)^(1/2)-2*(b*x^2+a)^(1/2)/a^2/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} dx = \frac{-a-2bx^2}{a^2x\sqrt{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-a - 2*b*x^2)/(a^2*x*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 245$$

$$-\frac{2b\int \frac{1}{(bx^2+a)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
-(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x)/(a^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
trager	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
pseudoelliptic	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
orering	$-\frac{2b\,x^{2}+a}{x\sqrt{b\,x^{2}+a}\,a^{2}}$	26
default	$-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}$	35
risch	$-\frac{\sqrt{b\,x^{2}+a}}{a^{2}x}-\frac{bx}{a^{2}\sqrt{b\,x^{2}+a}}$	35

input
```
int(1/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(2*b*x^2+a)/x/(b*x^2+a)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a+bx^2\right)^{3/2}}\,dx=-\frac{(2\,bx^2+a)\sqrt{bx^2+a}}{a^2bx^3+a^3x}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-(2*b*x^2 + a)*sqrt(b*x^2 + a)/(a^2*b*x^3 + a^3*x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} \, dx = -\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/2),x)
```

output
```
-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2)
 + 1))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} \, dx = -\frac{2\,bx}{\sqrt{bx^2+a}a^2} - \frac{1}{\sqrt{bx^2+a}ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*b*x/(sqrt(b*x^2 + a)*a^2) - 1/(sqrt(b*x^2 + a)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} \, dx = -\frac{bx}{\sqrt{bx^2+a}a^2} + \frac{2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-b*x/(sqrt(b*x^2 + a)*a^2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 -
 a)*a)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} \, dx = -\frac{\sqrt{b\,x^2+a}\,\left(\frac{1}{a}+\frac{2\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

input
```
int(1/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
-((a + b*x^2)^(1/2)*(1/a + (2*b*x^2)/a^2))/(a*x + b*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.47

$$\int \frac{1}{x^2 (a+bx^2)^{3/2}} \, dx = \frac{-\sqrt{b\,x^2+a}\,a-2\sqrt{b\,x^2+a}\,b\,x^2-2\sqrt{b}\,ax-2\sqrt{b}\,b\,x^3}{a^2x\,(b\,x^2+a)}$$

input
```
int(1/x^2/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a - 2*sqrt(a + b*x**2)*b*x**2 - 2*sqrt(b)*a*x - 2*sqr
t(b)*b*x**3)/(a**2*x*(a + b*x**2))
```

3.514 $\int \frac{1}{x^3(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx = -\frac{3b}{2a^2\sqrt{a+bx^2}} - \frac{1}{2ax^2\sqrt{a+bx^2}} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
-3/2*b/a^2/(b*x^2+a)^(1/2)-1/2/a/x^2/(b*x^2+a)^(1/2)+3/2*b*arctanh((b*x^2+
a)^(1/2)/a^(1/2))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx = \frac{-a-3bx^2}{2a^2x^2\sqrt{a+bx^2}} + \frac{3b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-a - 3*b*x^2)/(2*a^2*x^2*Sqrt[a + b*x^2]) + (3*b*ArcTanh[Sqrt[a + b*x^2]/
Sqrt[a]])/(2*a^(5/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 52, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^2\right)^{3/2}} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(bx^2+a\right)^{3/2}}dx^2$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2} \left(-\frac{3b \int \frac{1}{x^2 (bx^2+a)^{3/2}}dx^2}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2} \left(-\frac{3b \left(\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2} \left(-\frac{3b \left(\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2} \left(-\frac{3b \left(\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}\right)}{2a}-\frac{1}{ax^2\sqrt{a+bx^2}}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(1/(a*x^2*Sqrt[a + b*x^2])) - (3*b*(2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[S
qrt[a + b*x^2]/Sqrt[a]])/a^(3/2)))/(2*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
pseudoelliptic	$\frac{b\left(-\frac{\sqrt{b\,x^{2}+a}}{2x^{2}b}+\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\sqrt{a}}-\frac{1}{\sqrt{b\,x^{2}+a}}\right)}{a^{2}}$	54
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2a^{2}x^{2}}-\frac{b}{a^{2}\sqrt{b\,x^{2}+a}}+\frac{3b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{5}{2}}}$	63
default	$-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}$	67

input
```
int(1/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b/a^2*(-1/2*(b*x^2+a)^(1/2)/x^2/b+3/2*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(
1/2)-1/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.52

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx = \left[\frac{3\,(b^2x^4+abx^2)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,abx^2+a^2)\sqrt{bx^2+a}}{4\,(a^3bx^4+a^4x^2)},\right.$$
$$\left.-\frac{3\,(b^2x^4+abx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(3\,abx^2+a^2)\sqrt{bx^2+a}}{2\,(a^3bx^4+a^4x^2)}\right]$$

input
```
integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(b^2*x^4 + a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a
) + 2*a)/x^2) - 2*(3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)
, -1/2*(3*(b^2*x^4 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a)
+ (3*a*b*x^2 + a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}} \, dx = -\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}$$

input
```
integrate(1/x**3/(b*x**2+a)**(3/2),x)
```

output
```
-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/(b
*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}} \, dx = \frac{3\, b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\, a^{\frac{5}{2}}} - \frac{3\, b}{2\, \sqrt{bx^2+a}a^2} - \frac{1}{2\, \sqrt{bx^2+a}ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
3/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) - 3/2*b/(sqrt(b*x^2 + a)*a^2)
- 1/2/(sqrt(b*x^2 + a)*a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}} \, dx = -\frac{3\, b \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\, \sqrt{-a}a^2} - \frac{3\,(bx^2+a)b - 2\, ab}{2\left((bx^2+a)^{\frac{3}{2}} - \sqrt{bx^2+a}a\right)a^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) - 1/2*(3*(b*x^2 + a
)*b - 2*a*b)/(((b*x^2 + a)^(3/2) - sqrt(b*x^2 + a)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx = \frac{3\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{1}{2\,a\,x^2\,\sqrt{b\,x^2+a}} - \frac{3\,b}{2\,a^2\,\sqrt{b\,x^2+a}}$$

input
```
int(1/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
(3*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - 1/(2*a*x^2*(a + b*x^2
)^(1/2)) - (3*b)/(2*a^2*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.49

$$\int \frac{1}{x^3 (a+bx^2)^{3/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^2 - 3\sqrt{b\,x^2+a}\,ab\,x^2 - 3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)ab\,x^2 - 3\sqrt{a}\log\Big(\frac{\sqrt{}}{}}{2a^3x^2}$$

input
```
int(1/x^3/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2 - 3*sqrt(a + b*x**2)*a*b*x**2 - 3*sqrt(a)*log((s
qrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*x**2 - 3*sqrt(a)*log((
sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 + 3*sqrt(a)*log
((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*x**2 + 3*sqrt(a)*lo
g((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4)/(2*a**3*x**
2*(a + b*x**2))
```

3.515 $\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx = \frac{1}{ax^3\sqrt{a+bx^2}} - \frac{4\sqrt{a+bx^2}}{3a^2x^3} + \frac{8b\sqrt{a+bx^2}}{3a^3x}$$

output
```
1/a/x^3/(b*x^2+a)^(1/2)-4/3*(b*x^2+a)^(1/2)/a^2/x^3+8/3*b*(b*x^2+a)^(1/2)/
a^3/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4(a+bx^2)^{3/2}}\,dx = \frac{-a^2+4abx^2+8b^2x^4}{3a^3x^3\sqrt{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
(-a^2 + 4*a*b*x^2 + 8*b^2*x^4)/(3*a^3*x^3*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {245, 245, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{4b \int \frac{1}{x^2(bx^2+a)^{3/2}}dx}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{4b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{4b\left(-\frac{2bx}{a^2\sqrt{a+bx^2}}-\frac{1}{ax\sqrt{a+bx^2}}\right)}{3a}-\frac{1}{3ax^3\sqrt{a+bx^2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(3/2)),x]
```

output
```
-1/3*1/(a*x^3*Sqrt[a + b*x^2]) - (4*b*(-(1/(a*x*Sqrt[a + b*x^2])) - (2*b*x
)/(a^2*Sqrt[a + b*x^2])))/(3*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{-8b^{2}x^{4}-4ab\,x^{2}+a^{2}}{3x^{3}\sqrt{b\,x^{2}+a}\,a^{3}}$	37
trager	$-\frac{-8b^{2}x^{4}-4ab\,x^{2}+a^{2}}{3x^{3}\sqrt{b\,x^{2}+a}\,a^{3}}$	37
pseudoelliptic	$-\frac{-8b^{2}x^{4}-4ab\,x^{2}+a^{2}}{3x^{3}\sqrt{b\,x^{2}+a}\,a^{3}}$	37
orering	$-\frac{-8b^{2}x^{4}-4ab\,x^{2}+a^{2}}{3x^{3}\sqrt{b\,x^{2}+a}\,a^{3}}$	37
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-5b\,x^{2}+a\right)}{3a^{3}x^{3}}+\frac{x\,b^{2}}{\sqrt{b\,x^{2}+a}\,a^{3}}$	44
default	$-\frac{1}{3a\,x^{3}\sqrt{b\,x^{2}+a}}-\frac{4b\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)}{3a}$	59

input
```
int(1/x^4/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-8*b^2*x^4-4*a*b*x^2+a^2)/x^3/(b*x^2+a)^(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}}\,dx = \frac{(8\,b^2x^4+4\,abx^2-a^2)\sqrt{bx^2+a}}{3\,(a^3bx^5+a^4x^3)}$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(8*b^2*x^4 + 4*a*b*x^2 - a^2)*sqrt(b*x^2 + a)/(a^3*b*x^5 + a^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(56) = 112.

Time = 0.74 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.76

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}}\,dx = -\frac{a^3b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{3a^2b^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{12ab^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6} + \frac{8b^{\frac{15}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4x^2+6a^4b^5x^4+3a^3b^6x^6}$$

input
```
integrate(1/x**4/(b*x**2+a)**(3/2),x)
```

output
```
-a**3*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 +
 3*a**3*b**6*x**6) + 3*a**2*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b*
*4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) + 12*a*b**(13/2)*x**4*sqrt(
a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**4 + 3*a**3*b**6*x**6) +
 8*b**(15/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4*x**2 + 6*a**4*b**5*x**
4 + 3*a**3*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}} \, dx = \frac{8 \, b^2 x}{3 \, \sqrt{bx^2+a} a^3} + \frac{4 \, b}{3 \, \sqrt{bx^2+a} a^2 x} - \frac{1}{3 \, \sqrt{bx^2+a} a x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
8/3*b^2*x/(sqrt(b*x^2 + a)*a^3) + 4/3*b/(sqrt(b*x^2 + a)*a^2*x) - 1/3/(sqr
t(b*x^2 + a)*a*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(52) = 104.

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.71

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}} \, dx = \frac{b^2 x}{\sqrt{bx^2+a} a^3} - \frac{2 \left(3 \left(\sqrt{b} x - \sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}} - 12 \left(\sqrt{b} x - \sqrt{bx^2+a}\right)^2 a b^{\frac{3}{2}} + 5 \, a^2 b^{\frac{3}{2}}\right)}{3 \left(\left(\sqrt{b} x - \sqrt{bx^2+a}\right)^2 - a\right)^3 a^2}$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
b^2*x/(sqrt(b*x^2 + a)*a^3) - 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/
2) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 5*a^2*b^(3/2))/(((sqrt
(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.61

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}} \, dx = \frac{-a^2 + 4\,a\,b\,x^2 + 8\,b^2\,x^4}{3\,a^3\,x^3\,\sqrt{b\,x^2+a}}$$

input
```
int(1/(x^4*(a + b*x^2)^(3/2)),x)
```

output
```
(8*b^2*x^4 - a^2 + 4*a*b*x^2)/(3*a^3*x^3*(a + b*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.31

$$\int \frac{1}{x^4 (a+bx^2)^{3/2}} \, dx = \frac{-\sqrt{b\,x^2+a}\,a^2 + 4\sqrt{b\,x^2+a}\,ab\,x^2 + 8\sqrt{b\,x^2+a}\,b^2x^4 - 8\sqrt{b}\,ab\,x^3 - 8\sqrt{b}\,b^2x^5}{3a^3x^3\,(b\,x^2+a)}$$

input
```
int(1/x^4/(b*x^2+a)^(3/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**2 + 4*sqrt(a + b*x**2)*a*b*x**2 + 8*sqrt(a + b*x**
2)*b**2*x**4 - 8*sqrt(b)*a*b*x**3 - 8*sqrt(b)*b**2*x**5)/(3*a**3*x**3*(a +
 b*x**2))
```

3.516 $\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx=-\frac{x^5}{3b(a+bx^2)^{3/2}}-\frac{5x^3}{3b^2\sqrt{a+bx^2}}+\frac{5x\sqrt{a+bx^2}}{2b^3}-\frac{5a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{7/2}}$$

output
```
-1/3*x^5/b/(b*x^2+a)^(3/2)-5/3*x^3/b^2/(b*x^2+a)^(1/2)+5/2*x*(b*x^2+a)^(1/
2)/b^3-5/2*a*arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx=\frac{15a^2x+20abx^3+3b^2x^5}{6b^3(a+bx^2)^{3/2}}-\frac{5a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(5/2),x]
```

output
```
(15*a^2*x + 20*a*b*x^3 + 3*b^2*x^5)/(6*b^3*(a + b*x^2)^(3/2)) - (5*a*ArcTa
nh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\int \frac{x^4}{(bx^2+a)^{3/2}}dx}{3b}-\frac{x^5}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[x^6/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*x^5/(b*(a + b*x^2)^(3/2)) + (5*(-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*S
qrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2
))))/b))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{5\left(\left(b x^2+a\right)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{x\sqrt{b}}\right) a-\frac{b^{\frac{5}{2}} x^5}{5}-\frac{4 x^3 a b^{\frac{3}{2}}}{3}-\sqrt{b}\, a^2 x\right)}{2\left(b x^2+a\right)^{\frac{3}{2}} b^{\frac{7}{2}}}$
default	$\frac{x^5}{2b\left(b x^2+a\right)^{\frac{3}{2}}}-\frac{5a\left(-\frac{x^3}{3b\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b x^2+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{b^{\frac{3}{2}}}}{b}\right)}{2b}$
risch	$\frac{x\sqrt{b x^2+a}}{2b^3}-\frac{5a\ln\left(\sqrt{b}\,x+\sqrt{b x^2+a}\right)}{2b^{\frac{7}{2}}}-\frac{a^2\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}}{12b^4\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)^2}+\frac{7a\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(x-\right.}}{6b^4\left(x-\frac{\sqrt{-ab}}{b}\right)}$

input
```
int(x^6/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5/2/(b*x^2+a)^(3/2)/b^(7/2)*((b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^
(1/2))*a-1/5*b^(5/2)*x^5-4/3*x^3*a*b^(3/2)-b^(1/2)*a^2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.49

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = \left[\frac{15\,(ab^2x^4+2\,a^2bx^2+a^3)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(3\,b^3x^5+20\,ab}{12\,(b^6x^4+2\,ab^5x^2+a^2b^4)}\right.$$

input
```
integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) + 2*(3*b^3*x^5 + 20*a*b^2*x^3 + 15*a^2*b*x)*sqrt(b
*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4), 1/6*(15*(a*b^2*x^4 + 2*a^2*b
*x^2 + a^3)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*b^3*x^5 + 20*
a*b^2*x^3 + 15*a^2*b*x)*sqrt(b*x^2 + a))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(83) = 166$.

Time = 2.86 (sec) , antiderivative size = 367, normalized size of antiderivative = 4.03

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = -\frac{15a^{\frac{81}{2}}b^{22}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{15a^{\frac{79}{2}}b^{23}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{15a^{40}b^{\frac{45}{2}}x}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{20a^{39}b^{\frac{47}{2}}x^3}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{38}b^{\frac{49}{2}}x^5}{6a^{\frac{79}{2}}b^{\frac{51}{2}}\sqrt{1+\frac{bx^2}{a}}+6a^{\frac{77}{2}}b^{\frac{53}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**6/(b*x**2+a)**(5/2),x)
```

output
```
-15*a**(81/2)*b**22*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(6*a**(79/
2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x*
*2/a)) - 15*a**(79/2)*b**23*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a
))/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*
sqrt(1 + b*x**2/a)) + 15*a**40*b**(45/2)*x/(6*a**(79/2)*b**(51/2)*sqrt(1 +
 b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a)) + 20*a**39*b**
(47/2)*x**3/(6*a**(79/2)*b**(51/2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53
/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**38*b**(49/2)*x**5/(6*a**(79/2)*b**(51/
2)*sqrt(1 + b*x**2/a) + 6*a**(77/2)*b**(53/2)*x**2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = \frac{x^5}{2\,(bx^2+a)^{\frac{3}{2}}b} + \frac{5\,ax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{6\,b} + \frac{5\,ax}{6\,\sqrt{bx^2+a}b^3} - \frac{5\,a\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{7}{2}}}$$

input
```
integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/2*x^5/((b*x^2 + a)^(3/2)*b) + 5/6*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a
/((b*x^2 + a)^(3/2)*b^2))/b + 5/6*a*x/(sqrt(b*x^2 + a)*b^3) - 5/2*a*arcsin
h(b*x/sqrt(a*b))/b^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = \frac{\left(x^2\left(\frac{3x^2}{b}+\frac{20a}{b^2}\right)+\frac{15a^2}{b^3}\right)x}{6(bx^2+a)^{\frac{3}{2}}}+\frac{5a\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{7}{2}}}$$

input
```
integrate(x^6/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/6*(x^2*(3*x^2/b + 20*a/b^2) + 15*a^2/b^3)*x/(b*x^2 + a)^(3/2) + 5/2*a*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(5/2),x)
```

output
```
int(x^6/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.08

$$\int \frac{x^6}{(a+bx^2)^{5/2}}\,dx = \frac{30\sqrt{b\,x^2+a}\,a^2bx+40\sqrt{b\,x^2+a}\,a\,b^2x^3+6\sqrt{b\,x^2+a}\,b^3x^5-30\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a}{12b}$$

input
```
int(x^6/(b*x^2+a)^(5/2),x)
```

output
```
(30*sqrt(a + b*x**2)*a**2*b*x + 40*sqrt(a + b*x**2)*a*b**2*x**3 + 6*sqrt(a
 + b*x**2)*b**3*x**5 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(
a))*a**3 - 60*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b*x
**2 - 30*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**2*x**4 -
 5*sqrt(b)*a**3 - 10*sqrt(b)*a**2*b*x**2 - 5*sqrt(b)*a*b**2*x**4)/(12*b**4
*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.517 $\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = -\frac{a^2}{3b^3(a+bx^2)^{3/2}} + \frac{2a}{b^3\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^3}$$

output
```
-1/3*a^2/b^3/(b*x^2+a)^(3/2)+2*a/b^3/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{8a^2+12abx^2+3b^2x^4}{3b^3(a+bx^2)^{3/2}}$$

input
```
Integrate[x^5/(a + b*x^2)^(5/2),x]
```

output
```
(8*a^2 + 12*a*b*x^2 + 3*b^2*x^4)/(3*b^3*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^{5/2}}-\frac{2a}{b^2(bx^2+a)^{3/2}}+\frac{1}{b^2\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2}{3b^3(a+bx^2)^{3/2}}+\frac{4a}{b^3\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}}{b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^(5/2),x]
```

output
```
((-2*a^2)/(3*b^3*(a + b*x^2)^(3/2)) + (4*a)/(b^3*Sqrt[a + b*x^2]) + (2*Sqr
t[a + b*x^2])/b^3)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{3b^{2}x^{4}+12ab\,x^{2}+8a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	36
trager	$\frac{3b^{2}x^{4}+12ab\,x^{2}+8a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	36
pseudoelliptic	$\frac{3b^{2}x^{4}+12ab\,x^{2}+8a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	36
orering	$\frac{3b^{2}x^{4}+12ab\,x^{2}+8a^{2}}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}b^{3}}$	36
default	$\frac{x^{4}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{4a\left(-\frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}b}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)}{b}$	57
risch	$\frac{\sqrt{b\,x^{2}+a}}{b^{3}}+\frac{\sqrt{b\,x^{2}+a}\,\left(6b\,x^{2}+5a\right)a}{3b^{3}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}$	60

input
```
int(x^5/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(3*b^2*x^4+12*a*b*x^2+8*a^2)/(b*x^2+a)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{(3\,b^2x^4+12\,abx^2+8\,a^2)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(3*b^2*x^4 + 12*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^5*x^4 + 2*a*b^4*x^
2 + a^2*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(48) = 96.

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.56

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \begin{cases} \frac{8a^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{12abx^2}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} + \frac{3b^2x^4}{3ab^3\sqrt{a+bx^2}+3b^4x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^6}{6a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((8*a**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2
)) + 12*a*b*x**2/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2)
) + 3*b**2*x**4/(3*a*b**3*sqrt(a + b*x**2) + 3*b**4*x**2*sqrt(a + b*x**2))
, Ne(b, 0)), (x**6/(6*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{x^4}{(bx^2+a)^{\frac{3}{2}}b} + \frac{4\,ax^2}{(bx^2+a)^{\frac{3}{2}}b^2} + \frac{8\,a^2}{3\,(bx^2+a)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
x^4/((b*x^2 + a)^(3/2)*b) + 4*a*x^2/((b*x^2 + a)^(3/2)*b^2) + 8/3*a^2/((b*
x^2 + a)^(3/2)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{\frac{3\sqrt{bx^2+a}}{b} + \frac{6\,(bx^2+a)a-a^2}{(bx^2+a)^{\frac{3}{2}}b}}{3\,b^2}$$

input
```
integrate(x^5/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(3*sqrt(b*x^2 + a)/b + (6*(b*x^2 + a)*a - a^2)/((b*x^2 + a)^(3/2)*b))/
b^2
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.70

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{2\,a\,(b\,x^2+a) + (b\,x^2+a)^2 - \frac{a^2}{3}}{b^3\,(b\,x^2+a)^{3/2}}$$

input
```
int(x^5/(a + b*x^2)^(5/2),x)
```

output
```
(2*a*(a + b*x^2) + (a + b*x^2)^2 - a^2/3)/(b^3*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(3b^2x^4+12ab\,x^2+8a^2)}{3b^3\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^5/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*(8*a**2 + 12*a*b*x**2 + 3*b**2*x**4))/(3*b**3*(a**2 + 2*
a*b*x**2 + b**2*x**4))
```

3.518 $\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx=-\frac{x^3}{3b(a+bx^2)^{3/2}}-\frac{x}{b^2\sqrt{a+bx^2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
-1/3*x^3/b/(b*x^2+a)^(3/2)-x/b^2/(b*x^2+a)^(1/2)+arctanh(b^(1/2)*x/(b*x^2+
a)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.03

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx=\frac{-3ax-4bx^3}{3b^2(a+bx^2)^{3/2}}+\frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(5/2),x]
```

output
```
(-3*a*x - 4*b*x^3)/(3*b^2*(a + b*x^2)^(3/2)) + (2*ArcTanh[(Sqrt[b]*x)/(-Sq
rt[a] + Sqrt[a + b*x^2])])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {252, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{5/2}} dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\int \frac{x^2}{(bx^2+a)^{3/2}}dx}{b} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b} - \frac{x}{b\sqrt{a+bx^2}}}{b} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b} - \frac{x}{b\sqrt{a+bx^2}}}{b} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}} - \frac{x}{b\sqrt{a+bx^2}}}{b} - \frac{x^3}{3b(a+bx^2)^{3/2}}$$

input
```
Int[x^4/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*x^3/(b*(a + b*x^2)^(3/2)) + (-(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt
[b]*x)/Sqrt[a + b*x^2]]/b^(3/2))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
pseudoelliptic	$\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\frac{4 b^{\frac{3}{2}} x^{3}}{3}-x a\sqrt{b}}{b^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	57
default	$-\frac{x^{3}}{3 b \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}$	59

input
```
int(x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-4/3*b^(3/2)*x^3-x*a*b^
(1/2))/b^(5/2)/(b*x^2+a)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.11

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(4\,b^2x^3+3\,abx)\sqrt{}}{6\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right.$$
$$\left.-\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(4\,b^2x^3+3\,abx)\sqrt{bx^2+a}}{3\,(b^5x^4+2\,ab^4x^2+a^2b^3)}\right]$$

input
```
integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) - 2*(4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*
a*b^4*x^2 + a^2*b^3), -1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*b^2*x^3 + 3*a*b*x)*sqrt(b*x^2 + a))/(b^5*
x^4 + 2*a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(54) = 108$.

Time = 1.63 (sec) , antiderivative size = 303, normalized size of antiderivative = 4.73

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = \frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$
$$-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$
$$-\frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4/(b*x**2+a)**(5/2),x)
```

output
```
3*a**(39/2)*b**11*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)
*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2
/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/
(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x
**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2
)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x
**2*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)-\frac{x}{3\,\sqrt{bx^2+a}b^2}+\frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - 1/3*x
/(sqrt(b*x^2 + a)*b^2) + arcsinh(b*x/sqrt(a*b))/b^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = -\frac{x\left(\frac{4\,x^2}{b}+\frac{3\,a}{b^2}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*(4*x^2/b + 3*a/b^2)/(b*x^2 + a)^(3/2) - log(abs(-sqrt(b)*x + sqrt(b
*x^2 + a)))/b^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(5/2),x)
```

output
```
int(x^4/(a + b*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int \frac{x^4}{(a+bx^2)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,abx-4\sqrt{b\,x^2+a}\,b^2x^3+3\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2+6\sqrt{b}\log\left(\frac{\sqrt{b\,x^2+a}+\sqrt{\ }}{\sqrt{a}}\right.}{3b^3\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int(x^4/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a*b*x - 4*sqrt(a + b*x**2)*b**2*x**3 + 3*sqrt(b)*lo
g((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2 + 6*sqrt(b)*log((sqrt(a + b
*x**2) + sqrt(b)*x)/sqrt(a))*a*b*x**2 + 3*sqrt(b)*log((sqrt(a + b*x**2) +
sqrt(b)*x)/sqrt(a))*b**2*x**4)/(3*b**3*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.519 $\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = \frac{a}{3b^2(a+bx^2)^{3/2}} - \frac{1}{b^2\sqrt{a+bx^2}}$$

output
```
1/3*a/b^2/(b*x^2+a)^(3/2)-1/b^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = \frac{-2a-3bx^2}{3b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[x^3/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a - 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int \frac{x^2}{(bx^2+a)^{5/2}} dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2} \int \left(\frac{1}{b (bx^2+a)^{3/2}} - \frac{a}{b (bx^2+a)^{5/2}} \right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{2a}{3b^2 (a+bx^2)^{3/2}} - \frac{2}{b^2 \sqrt{a+bx^2}} \right)$$

input
```
Int[x^3/(a + b*x^2)^(5/2),x]
```

output
```
((2*a)/(3*b^2*(a + b*x^2)^(3/2)) - 2/(b^2*Sqrt[a + b*x^2]))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

method	result	size
gosper	$-\frac{3b\,x^{2}+2a}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}$	25
trager	$-\frac{3b\,x^{2}+2a}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}$	25
pseudoelliptic	$\frac{-3b\,x^{2}-2a}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}$	25
orering	$-\frac{3b\,x^{2}+2a}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} b^{2}}$	25
default	$-\frac{x^{2}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}} b}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	34

input
```
int(x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(3*b*x^2+2*a)/(b*x^2+a)^(3/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = -\frac{(3\,bx^2+2\,a)\sqrt{bx^2+a}}{3\,(b^4x^4+2\,ab^3x^2+a^2b^2)}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(3*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(31) = 62$.

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.56

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = \begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2))
 - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{2\,a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-x^2/((b*x^2 + a)^(3/2)*b) - 2/3*a/((b*x^2 + a)^(3/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = -\frac{3\,bx^2+2\,a}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(3*b*x^2 + 2*a)/((b*x^2 + a)^(3/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = -\frac{3\,b\,x^2+2\,a}{3\,b^2\,(b\,x^2+a)^{3/2}}$$

input
```
int(x^3/(a + b*x^2)^(5/2),x)
```

output
```
-(2*a + 3*b*x^2)/(3*b^2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int \frac{x^3}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-3b\,x^2-2a)}{3b^2\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^3/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a - 3*b*x**2))/(3*b**2*(a**2 + 2*a*b*x**2 + b**2*x
**4))
```

3.520 $\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{x^3}{3a\,(a+bx^2)^{3/2}}$$

output
```
1/3*x^3/a/(b*x^2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{x^3}{3a\,(a+bx^2)^{3/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(5/2),x]
```

output
```
x^3/(3*a*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx$$
$$\Big\downarrow \ 242$$
$$\frac{x^3}{3a\,(a+bx^2)^{3/2}}$$

input
```
Int[x^2/(a + b*x^2)^(5/2),x]
```

output
```
x^3/(3*a*(a + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
gosper	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
trager	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
pseudoelliptic	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
orering	$\frac{x^{3}}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	18
default	$-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}$	54

input
```
int(x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3/a/(b*x^2+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{bx^2+a}x^3}{3\left(ab^2x^4+2\,a^2bx^2+a^3\right)}$$

input
```
integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(b*x^2 + a)*x^3/(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(15) = 30$.

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.10

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{x^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**2/(b*x**2+a)**(5/2),x)
```

output
```
x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = -\frac{x}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{x}{3\sqrt{bx^2+a}ab}$$

input
```
integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*x/((b*x^2 + a)^(3/2)*b) + 1/3*x/(sqrt(b*x^2 + a)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{x^3}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x^3/((b*x^2 + a)^(3/2)*a)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{x^3}{3\,a\,(b\,x^2+a)^{3/2}}$$

input
```
int(x^2/(a + b*x^2)^(5/2),x)
```

output
```
x^3/(3*a*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{x^2}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b^2x^3+\sqrt{b}\,a^2+2\sqrt{b}\,ab\,x^2+\sqrt{b}\,b^2x^4}{3a\,b^2\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x^2/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(a + b*x**2)*b**2*x**3 + sqrt(b)*a**2 + 2*sqrt(b)*a*b*x**2 + sqrt(b)*
b**2*x**4)/(3*a*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.521 $\int \frac{x}{\left(a+bx^2\right)^{5/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}} dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

output
```
-1/3/b/(b*x^2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^2\right)^{5/2}} dx = -\frac{1}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[x/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*1/(b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{3b(a+bx^2)^{3/2}}$$

input
```
Int[x/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*1/(b*(a + b*x^2)^(3/2))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
derivativedivides	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
default	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
trager	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
pseudoelliptic	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15
orering	$-\frac{1}{3b(b\,x^2+a)^{\frac{3}{2}}}$	15

input
```
int(x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/b/(b*x^2+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{\sqrt{bx^2+a}}{3\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

input
```
integrate(x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(b*x^2 + a)/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(15) = 30$.

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(5/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3/((b*x^2 + a)^(3/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3/((b*x^2 + a)^(3/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int(x/(a + b*x^2)^(5/2),x)
```

output
```
-1/(3*b*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{x}{(a+bx^2)^{5/2}}\,dx = -\frac{\sqrt{b\,x^2+a}}{3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(x/(b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2))/(3*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.522 $\int \frac{1}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{x}{3a\,(a+bx^2)^{3/2}}+\frac{2x}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx=\frac{3ax+2bx^3}{3a^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(-5/2),x]
```

output
```
(3*a*x + 2*b*x^3)/(3*a^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 209$$

$$\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\,(a+bx^2)^{3/2}}$$

input
```
Int[(a + b*x^2)^(-5/2),x]
```

output
```
x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
gosper	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
trager	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
pseudoelliptic	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
orering	$\frac{x\left(2b\,x^{2}+3a\right)}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}}$	26
default	$\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}$	32

input
```
int(1/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x*(2*b*x^2+3*a)/(b*x^2+a)^(3/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx = \frac{\left(2\,bx^3+3\,ax\right)\sqrt{bx^2+a}}{3\,\left(a^2b^2x^4+2\,a^3bx^2+a^4\right)}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*b*x^3 + 3*a*x)*sqrt(b*x^2 + a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(32) = 64$.

Time = 0.52 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.44

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(1/(b*x**2+a)**(5/2),x)
```

output
```
3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a
)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x}{3\,\sqrt{bx^2+a}a^2} + \frac{x}{3\,(bx^2+a)^{\frac{3}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*x/(sqrt(b*x^2 + a)*a^2) + 1/3*x/((b*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{x\left(\frac{2\,bx^2}{a^2}+\frac{3}{a}\right)}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(2*b*x^2/a^2 + 3/a)/(b*x^2 + a)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{2\,x\,(b\,x^2+a)+a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}$$

input
```
int(1/(a + b*x^2)^(5/2),x)
```

output
```
(2*x*(a + b*x^2) + a*x)/(3*a^2*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a+bx^2)^{5/2}}\,dx = \frac{3\sqrt{b\,x^2+a}\,abx+2\sqrt{b\,x^2+a}\,b^2x^3-2\sqrt{b}\,a^2-4\sqrt{b}\,ab\,x^2-2\sqrt{b}\,b^2x^4}{3a^2b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/(b*x^2+a)^(5/2),x)
```

output
```
(3*sqrt(a + b*x**2)*a*b*x + 2*sqrt(a + b*x**2)*b**2*x**3 - 2*sqrt(b)*a**2
- 4*sqrt(b)*a*b*x**2 - 2*sqrt(b)*b**2*x**4)/(3*a**2*b*(a**2 + 2*a*b*x**2 +
 b**2*x**4))
```

3.523 $\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{1}{3a\left(a+bx^2\right)^{3/2}} + \frac{1}{a^2\sqrt{a+bx^2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
1/3/a/(b*x^2+a)^(3/2)+1/a^2/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2
))/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{4a+3bx^2}{3a^2\left(a+bx^2\right)^{3/2}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(4*a + 3*b*x^2)/(3*a^2*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a
]]/a^(5/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2 (bx^2+a)^{5/2}}dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2(bx^2+a)^{3/2}}dx^2}{a}+\frac{2}{3a (a+bx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a (a+bx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a (a+bx^2)^{3/2}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a (a+bx^2)^{3/2}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(2/(3*a*(a + b*x^2)^(3/2)) + (2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/a^(3/2))/a)/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

method	result	size
pseudoelliptic	$-\frac{(b\,x^2+a)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-b\,x^2\sqrt{a}-\frac{4a^{\frac{3}{2}}}{3}}{a^{\frac{5}{2}}(b\,x^2+a)^{\frac{3}{2}}}$	54
default	$\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}$	62

input
```
int(1/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-((b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))-b*x^2*a^(1/2)-4/3*a^(3/
2))/a^(5/2)/(b*x^2+a)^(3/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(47) = 94.

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.39

$$\int \frac{1}{x\,(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,abx^2+4\,a^2)\sqrt{bx^2+a}}{6\,(a^3b^2x^4+2\,a^4bx^2+a^5)}, \ldots\right.$$

input
```
integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^
4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(-a)*arctan
(sqrt(b*x^2 + a)*sqrt(-a)/a) + (3*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a))/(a^3*b
^2*x^4 + 2*a^4*b*x^2 + a^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(51) = 102$.

Time = 1.59 (sec) , antiderivative size = 740, normalized size of antiderivative = 12.54

$$\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x**2+a)**(5/2),x)
```

output
```
8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2
)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) +
 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*
a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x
**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(
15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(1
3/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2
*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*
a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2
/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.76

$$\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{1}{\sqrt{bx^2+a}a^2} + \frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

input
```
integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/(sqrt(b*x^2 + a)*a^2) + 1/3/((b
*x^2 + a)^(3/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{1}{x(a+bx^2)^{5/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3\,bx^2+4\,a}{3\,(bx^2+a)^{\frac{3}{2}}a^2}$$

input
```
integrate(1/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + 1/3*(3*b*x^2 + 4*a)/((b*
x^2 + a)^(3/2)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+bx^2)^{5/2}}\,dx = \frac{\frac{b\,x^2+a}{a^2}+\frac{1}{3\,a}}{(b\,x^2+a)^{3/2}} - \frac{\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int(1/(x*(a + b*x^2)^(5/2)),x)
```

output
```
((a + b*x^2)/a^2 + 1/(3*a))/(a + b*x^2)^(3/2) - atanh((a + b*x^2)^(1/2)/a^
(1/2))/a^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.03

$$\int \frac{1}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{4\sqrt{b\,x^2+a}\,a^2+3\sqrt{b\,x^2+a}\,ab\,x^2+3\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}+\sqrt{b}\,x}{\sqrt{a}}\right)a^2+6\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}}{}\right.}{}$$

input
```
int(1/x/(b*x^2+a)^(5/2),x)
```

output
```
(4*sqrt(a + b*x**2)*a**2 + 3*sqrt(a + b*x**2)*a*b*x**2 + 3*sqrt(a)*log((sq
rt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2 + 6*sqrt(a)*log((sqrt(
a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*x**2 + 3*sqrt(a)*log((sqrt
(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4 - 3*sqrt(a)*log((sq
rt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2 - 6*sqrt(a)*log((sqrt(
a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b*x**2 - 3*sqrt(a)*log((sqrt
(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**2*x**4)/(3*a**3*(a**2 + 2*
a*b*x**2 + b**2*x**4))
```

3.524 $\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx = \frac{1}{3ax(a+bx^2)^{3/2}} + \frac{4}{3a^2x\sqrt{a+bx^2}} - \frac{8\sqrt{a+bx^2}}{3a^3x}$$

output
```
1/3/a/x/(b*x^2+a)^(3/2)+4/3/a^2/x/(b*x^2+a)^(1/2)-8/3*(b*x^2+a)^(1/2)/a^3/
x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2(a+bx^2)^{5/2}}\,dx = \frac{-3a^2-12abx^2-8b^2x^4}{3a^3x(a+bx^2)^{3/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^2 - 12*a*b*x^2 - 8*b^2*x^4)/(3*a^3*x*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{4b \int \frac{1}{(bx^2+a)^{5/2}}dx}{a} - \frac{1}{ax (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 209}$$

$$-\frac{4b \left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a} + \frac{x}{3a(a+bx^2)^{3/2}} \right)}{a} - \frac{1}{ax (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{4b \left(\frac{2x}{3a^2\sqrt{a+bx^2}} + \frac{x}{3a(a+bx^2)^{3/2}} \right)}{a} - \frac{1}{ax (a+bx^2)^{3/2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
-(1/(a*x*(a + b*x^2)^(3/2))) - (4*b*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*
a^2*Sqrt[a + b*x^2])))/a
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$-\frac{\frac{8}{3} b^{2} x^{4}+4 a b \,x^{2}+a^{2}}{\left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3} x}$	37
gosper	$-\frac{8 b^{2} x^{4}+12 a b \,x^{2}+3 a^{2}}{3 x \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3}}$	39
trager	$-\frac{8 b^{2} x^{4}+12 a b \,x^{2}+3 a^{2}}{3 x \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3}}$	39
orering	$-\frac{8 b^{2} x^{4}+12 a b \,x^{2}+3 a^{2}}{3 x \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{3}}$	39
default	$-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{4 b \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{a}$	56
risch	$-\frac{\sqrt{b \,x^{2}+a}}{a^{3} x}-\frac{\sqrt{b \,x^{2}+a}\, x \left(5 b \,x^{2}+6 a \right) b}{3 a^{3} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}$	65

input
```
int(1/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x^2+a)^(3/2)*(8/3*b^2*x^4+4*a*b*x^2+a^2)/a^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a + bx^2)^{5/2}}\, dx = -\frac{(8\, b^2x^4 + 12\, abx^2 + 3\, a^2)\sqrt{bx^2 + a}}{3\, (a^3b^2x^5 + 2\, a^4bx^3 + a^5x)}$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/3*(8*b^2*x^4 + 12*a*b*x^2 + 3*a^2)*sqrt(b*x^2 + a)/(a^3*b^2*x^5 + 2*a^4
*b*x^3 + a^5*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(53) = 106.

Time = 0.76 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.58

$$\int \frac{1}{x^2 (a + bx^2)^{5/2}}\, dx = -\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4} - \frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2} + 1}}{3a^5b^4 + 6a^4b^5x^2 + 3a^3b^6x^4}$$

input
```
integrate(1/x**2/(b*x**2+a)**(5/2),x)
```

output
```
-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*
a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2 (a + bx^2)^{5/2}}\, dx = -\frac{8\, bx}{3\, \sqrt{bx^2 + a}a^3} - \frac{4\, bx}{3\, (bx^2 + a)^{\frac{3}{2}} a^2} - \frac{1}{(bx^2 + a)^{\frac{3}{2}} ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-8/3*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*b*x/((b*x^2 + a)^(3/2)*a^2) - 1/((b*x
^2 + a)^(3/2)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a + bx^2)^{5/2}}\, dx = -\frac{x\left(\frac{5\, b^2 x^2}{a^3} + \frac{6\, b}{a^2}\right)}{3\, (bx^2 + a)^{\frac{3}{2}}} + \frac{2\, \sqrt{b}}{\left(\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^2 - a\right)a^2}$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*x*(5*b^2*x^2/a^3 + 6*b/a^2)/(b*x^2 + a)^(3/2) + 2*sqrt(b)/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2 (a + bx^2)^{5/2}}\, dx = \frac{4\, a\, (b\, x^2 + a) - 8\, (b\, x^2 + a)^2 + a^2}{3\, a^3\, x\, (b\, x^2 + a)^{3/2}}$$

input
```
int(1/(x^2*(a + b*x^2)^(5/2)),x)
```

output
```
(4*a*(a + b*x^2) - 8*(a + b*x^2)^2 + a^2)/(3*a^3*x*(a + b*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.56

$$\int \frac{1}{x^2 (a+bx^2)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^2-12\sqrt{b\,x^2+a}\,ab\,x^2-8\sqrt{b\,x^2+a}\,b^2x^4+8\sqrt{b}\,a^2x+16\sqrt{b}\,ab\,x^3+}{3a^3x\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/x^2/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**2 - 12*sqrt(a + b*x**2)*a*b*x**2 - 8*sqrt(a + b*
x**2)*b**2*x**4 + 8*sqrt(b)*a**2*x + 16*sqrt(b)*a*b*x**3 + 8*sqrt(b)*b**2*
x**5)/(3*a**3*x*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.525 $\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx = -\frac{5b}{6a^2(a+bx^2)^{3/2}} - \frac{1}{2ax^2(a+bx^2)^{3/2}} - \frac{5b}{2a^3\sqrt{a+bx^2}} + \frac{5b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
-5/6*b/a^2/(b*x^2+a)^(3/2)-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a^3/(b*x^2+a)^(
1/2)+5/2*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3(a+bx^2)^{5/2}}\,dx = \frac{-3a^2-20abx^2-15b^2x^4}{6a^3x^2(a+bx^2)^{3/2}} + \frac{5b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^2 - 20*a*b*x^2 - 15*b^2*x^4)/(6*a^3*x^2*(a + b*x^2)^(3/2)) + (5*b*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 52, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^{5/2}} dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{5b \int \frac{1}{x^2 (bx^2+a)^{5/2}} dx^2}{2a} - \frac{1}{ax^2 (a+bx^2)^{3/2}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{5b \left(\frac{\int \frac{1}{x^2 \left(bx^2+a\right)^{3/2}} dx^2}{a} + \frac{2}{3a(a+bx^2)^{3/2}} \right)}{2a} - \frac{1}{ax^2 (a+bx^2)^{3/2}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{5b \left(\frac{\frac{\int \frac{1}{x^2 \sqrt{bx^2+a}} dx^2}{a} + \frac{2}{a\sqrt{a+bx^2}}}{a} + \frac{2}{3a(a+bx^2)^{3/2}} \right)}{2a} - \frac{1}{ax^2 (a+bx^2)^{3/2}} \right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{5b\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{5b\left(\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a(a+bx^2)^{3/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{3/2}}\right)$$

input

```
Int[1/(x^3*(a + b*x^2)^(5/2)),x]
```

output

```
(-(1/(a*x^2*(a + b*x^2)^(3/2))) - (5*b*(2/(3*a*(a + b*x^2)^(3/2)) + (2/(a*
Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/a))/(2*a)
)/2
```

Defintions of rubi rules used

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{\frac{5\left(b\,x^{2}+a\right)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right) b\,x^{2}}{2}-\frac{5\sqrt{a}\,b^{2} x^{4}}{2}-\frac{10 b\,x^{2} a^{\frac{3}{2}}}{3}-\frac{a^{\frac{5}{2}}}{2}}{x^{2} a^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{3}{2}}}$
default	$-\frac{1}{2 a\,x^{2} \left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5 b \left(\frac{1}{3 a \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2 a}$
risch	$-\frac{\sqrt{b\,x^{2}+a}}{2 a^{3} x^{2}}+\frac{5 b \ln\left(\frac{2 a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{7}{2}}}-\frac{13 b \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} b+2\sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}}{12 a^{3} \sqrt{-a b}\,\left(x-\frac{\sqrt{-a b}}{b}\right)}+\frac{13 b \sqrt{\left(x+\frac{\sqrt{-a b}}{b}\right)^{2} b-2\sqrt{-a b}\,(}}{12 a^{3} \sqrt{-a b}\,\left(x+\frac{\sqrt{-a}}{b}\right.}$

input
```
int(1/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/2/(b*x^2+a)^(3/2)/a^(7/2)*((b*x^2+a)^(3/2)*arctanh((b*x^2+a)^(1/2)/a^(1/
2))*b*x^2-a^(1/2)*b^2*x^4-4/3*b*x^2*a^(3/2)-1/5*a^(5/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.77

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = \left[\frac{15\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(15\,ab^2x^4+20\,a^2bx^2}{12\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}\right.$$
$$\left.-\frac{15\,(b^3x^6+2\,ab^2x^4+a^2bx^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+(15\,ab^2x^4+20\,a^2bx^2+3\,a^3)\sqrt{bx^2+a}}{6\,(a^4b^2x^6+2\,a^5bx^4+a^6x^2)}\right]$$

input
```
integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt
(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*
sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(b^3*x^6
+ 2*a*b^2*x^4 + a^2*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (
15*a*b^2*x^4 + 20*a^2*b*x^2 + 3*a^3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5
*b*x^4 + a^6*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(82) = 164.

Time = 2.91 (sec) , antiderivative size = 864, normalized size of antiderivative = 9.82

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x**2+a)**(5/2),x)
```

output
```
-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 + b
*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x
**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**
8) + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a
**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a*
*15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4
+ 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*lo
g(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x
**6 + 12*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a)
+ 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/
2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2
)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**
3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**
4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*
log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2
*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = \frac{5\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} - \frac{5\,b}{2\,\sqrt{bx^2+a}a^3} - \frac{5\,b}{6\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{1}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
5/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*b/(sqrt(b*x^2 + a)*a^3)
- 5/6*b/((b*x^2 + a)^(3/2)*a^2) - 1/2/((b*x^2 + a)^(3/2)*a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = -\frac{5\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a^3} - \frac{6\,(bx^2+a)b+ab}{3\,(bx^2+a)^{\frac{3}{2}}a^3} - \frac{\sqrt{bx^2+a}}{2\,a^3x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^3) - 1/3*(6*(b*x^2 + a
)*b + a*b)/((b*x^2 + a)^(3/2)*a^3) - 1/2*sqrt(b*x^2 + a)/(a^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = \frac{5\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}} - \frac{1}{2\,a\,x^2\,(b\,x^2+a)^{3/2}} - \frac{10\,b}{3\,a^2\,(b\,x^2+a)^{3/2}} - \frac{5\,b^2\,x^2}{2\,a^3\,(b\,x^2+a)^{3/2}}$$

input
```
int(1/(x^3*(a + b*x^2)^(5/2)),x)
```

output
```
(5*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - 1/(2*a*x^2*(a + b*x^2
)^(3/2)) - (10*b)/(3*a^2*(a + b*x^2)^(3/2)) - (5*b^2*x^2)/(2*a^3*(a + b*x^
2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 272, normalized size of antiderivative = 3.09

$$\int \frac{1}{x^3 (a+bx^2)^{5/2}}\,dx = \frac{-3\sqrt{b\,x^2+a}\,a^3-20\sqrt{b\,x^2+a}\,a^2b\,x^2-15\sqrt{b\,x^2+a}\,a\,b^2x^4-15\sqrt{a}\log\left(\frac{\sqrt{b\,x^2+a}-\sqrt{a}}{\sqrt{a}}\right.}{}$$

input
```
int(1/x^3/(b*x^2+a)^(5/2),x)
```

output
```
( - 3*sqrt(a + b*x**2)*a**3 - 20*sqrt(a + b*x**2)*a**2*b*x**2 - 15*sqrt(a
+ b*x**2)*a*b**2*x**4 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(
b)*x)/sqrt(a))*a**2*b*x**2 - 30*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) +
sqrt(b)*x)/sqrt(a))*a*b**2*x**4 - 15*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**3*x**6 + 15*sqrt(a)*log((sqrt(a + b*x**2) + sq
rt(a) + sqrt(b)*x)/sqrt(a))*a**2*b*x**2 + 30*sqrt(a)*log((sqrt(a + b*x**2)
 + sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**2*x**4 + 15*sqrt(a)*log((sqrt(a + b*
x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*b**3*x**6)/(6*a**4*x**2*(a**2 + 2*a*
b*x**2 + b**2*x**4))
```

3.526 $\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 84

$$\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx=\frac{1}{3ax^3(a+bx^2)^{3/2}}+\frac{2}{a^2x^3\sqrt{a+bx^2}}-\frac{8\sqrt{a+bx^2}}{3a^3x^3}+\frac{16b\sqrt{a+bx^2}}{3a^4x}$$

output
```
1/3/a/x^3/(b*x^2+a)^(3/2)+2/a^2/x^3/(b*x^2+a)^(1/2)-8/3*(b*x^2+a)^(1/2)/a^
3/x^3+16/3*b*(b*x^2+a)^(1/2)/a^4/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4(a+bx^2)^{5/2}}\,dx=\frac{-a^3+6a^2bx^2+24ab^2x^4+16b^3x^6}{3a^4x^3(a+bx^2)^{3/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(5/2)),x]
```

output
```
(-a^3 + 6*a^2*b*x^2 + 24*a*b^2*x^4 + 16*b^3*x^6)/(3*a^4*x^3*(a + b*x^2)^(3
/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {245, 245, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}} dx$$
$$\Big\downarrow \text{245}$$
$$-\frac{2b\int \frac{1}{x^2(bx^2+a)^{5/2}}dx}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}$$
$$\Big\downarrow \text{245}$$
$$-\frac{2b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}$$
$$\Big\downarrow \text{209}$$
$$-\frac{2b\left(-\frac{4b\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}$$
$$\Big\downarrow \text{208}$$
$$-\frac{2b\left(-\frac{4b\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{3/2}}\right)}{a}-\frac{1}{3ax^3(a+bx^2)^{3/2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*1/(a*x^3*(a + b*x^2)^(3/2)) - (2*b*(-(1/(a*x*(a + b*x^2)^(3/2))) - (4
*b*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/a))/a
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{\left(2 b \,x^{2}+a \right) \left(-8 b^{2} x^{4}-8 a b \,x^{2}+a^{2}\right)}{3 \left(b \,x^{2}+a \right)^{\frac{3}{2}} x^{3} a^{4}}$	45
gosper	$-\frac{-16 b^{3} x^{6}-24 a \,b^{2} x^{4}-6 a^{2} b \,x^{2}+a^{3}}{3 x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{4}}$	48
trager	$-\frac{-16 b^{3} x^{6}-24 a \,b^{2} x^{4}-6 a^{2} b \,x^{2}+a^{3}}{3 x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{4}}$	48
orering	$-\frac{-16 b^{3} x^{6}-24 a \,b^{2} x^{4}-6 a^{2} b \,x^{2}+a^{3}}{3 x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}} a^{4}}$	48
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(-8 b \,x^{2}+a \right)}{3 a^{4} x^{3}}+\frac{\sqrt{b \,x^{2}+a}\, x \left(8 b \,x^{2}+9 a \right) b^{2}}{3 a^{4} \left(b^{2} x^{4}+2 a b \,x^{2}+a^{2}\right)}$	75
default	$-\frac{1}{3 a \,x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{2 b \left(-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}-\frac{4 b \left(\frac{x}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b \,x^{2}+a}}\right)}{a}\right)}{a}$	80

input
```
int(1/x^4/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2*b*x^2+a)*(-8*b^2*x^4-8*a*b*x^2+a^2)/(b*x^2+a)^(3/2)/x^3/a^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}}\,dx = \frac{(16\,b^3x^6+24\,ab^2x^4+6\,a^2bx^2-a^3)\sqrt{bx^2+a}}{3\,(a^4b^2x^7+2\,a^5bx^5+a^6x^3)}$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(16*b^3*x^6 + 24*a*b^2*x^4 + 6*a^2*b*x^2 - a^3)*sqrt(b*x^2 + a)/(a^4*b
^2*x^7 + 2*a^5*b*x^5 + a^6*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(76) = 152.

Time = 1.01 (sec) , antiderivative size = 354, normalized size of antiderivative = 4.21

$$\begin{aligned}\int \frac{1}{x^4 (a+bx^2)^{5/2}}\,dx = &-\frac{a^4b^{\frac{19}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\\ &+\frac{5a^3b^{\frac{21}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\\ &+\frac{30a^2b^{\frac{23}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\\ &+\frac{40ab^{\frac{25}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\\ &+\frac{16b^{\frac{27}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{3a^7b^9x^2+9a^6b^{10}x^4+9a^5b^{11}x^6+3a^4b^{12}x^8}\end{aligned}$$

input
```
integrate(1/x**4/(b*x**2+a)**(5/2),x)
```

output
```
-a**4*b**(19/2)*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4
 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 5*a**3*b**(21/2)*x**2*sqrt(a/(
b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3
*a**4*b**12*x**8) + 30*a**2*b**(23/2)*x**4*sqrt(a/(b*x**2) + 1)/(3*a**7*b*
*9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 40*
a*b**(25/2)*x**6*sqrt(a/(b*x**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**
4 + 9*a**5*b**11*x**6 + 3*a**4*b**12*x**8) + 16*b**(27/2)*x**8*sqrt(a/(b*x
**2) + 1)/(3*a**7*b**9*x**2 + 9*a**6*b**10*x**4 + 9*a**5*b**11*x**6 + 3*a*
*4*b**12*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}} \, dx = \frac{16 \, b^2 x}{3 \sqrt{bx^2+a} a^4} + \frac{8 \, b^2 x}{3 \, (bx^2+a)^{\frac{3}{2}} a^3} + \frac{2 \, b}{(bx^2+a)^{\frac{3}{2}} a^2 x} - \frac{1}{3 \, (bx^2+a)^{\frac{3}{2}} a x^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
16/3*b^2*x/(sqrt(b*x^2 + a)*a^4) + 8/3*b^2*x/((b*x^2 + a)^(3/2)*a^3) + 2*b
/((b*x^2 + a)^(3/2)*a^2*x) - 1/3/((b*x^2 + a)^(3/2)*a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.44

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}} \, dx = \frac{x\left(\frac{8 \, b^3 x^2}{a^4} + \frac{9 \, b^2}{a^3}\right)}{3 \, (bx^2+a)^{\frac{3}{2}}} - \frac{4 \left(3 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}} - 9 \left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 a b^{\frac{3}{2}} + 4 \, a^2 b^{\frac{3}{2}}\right)}{3 \left(\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^2 - a\right)^3 a^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*x*(8*b^3*x^2/a^4 + 9*b^2/a^3)/(b*x^2 + a)^(3/2) - 4/3*(3*(sqrt(b)*x -
sqrt(b*x^2 + a))^4*b^(3/2) - 9*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) +
 4*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^3*a^3)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}}\,dx = -\frac{6\,a^2\,(b\,x^2+a)-24\,a\,(b\,x^2+a)^2+16\,(b\,x^2+a)^3+a^3}{(b\,x^2+a)^{3/2}\,\left(\frac{3\,a^5\,x}{b}-\frac{3\,a^4\,x\,(b\,x^2+a)}{b}\right)}$$

input
```
int(1/(x^4*(a + b*x^2)^(5/2)),x)
```

output
```
-(6*a^2*(a + b*x^2) - 24*a*(a + b*x^2)^2 + 16*(a + b*x^2)^3 + a^3)/((a + b
*x^2)^(3/2)*((3*a^5*x)/b - (3*a^4*x*(a + b*x^2))/b))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.48

$$\int \frac{1}{x^4 (a+bx^2)^{5/2}}\,dx = \frac{-\sqrt{b\,x^2+a}\,a^3+6\sqrt{b\,x^2+a}\,a^2b\,x^2+24\sqrt{b\,x^2+a}\,a\,b^2x^4+16\sqrt{b\,x^2+a}\,b^3x^6-16\sqrt{}}{3a^4x^3\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int(1/x^4/(b*x^2+a)^(5/2),x)
```

output
```
( - sqrt(a + b*x**2)*a**3 + 6*sqrt(a + b*x**2)*a**2*b*x**2 + 24*sqrt(a + b
*x**2)*a*b**2*x**4 + 16*sqrt(a + b*x**2)*b**3*x**6 - 16*sqrt(b)*a**2*b*x**
3 - 32*sqrt(b)*a*b**2*x**5 - 16*sqrt(b)*b**3*x**7)/(3*a**4*x**3*(a**2 + 2*
a*b*x**2 + b**2*x**4))
```

3.527 $\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 131

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = -\frac{x^9}{7b(a+bx^2)^{7/2}} - \frac{9x^7}{35b^2(a+bx^2)^{5/2}} - \frac{3x^5}{5b^3(a+bx^2)^{3/2}} - \frac{3x^3}{b^4\sqrt{a+bx^2}} + \frac{9x\sqrt{a+bx^2}}{2b^5} - \frac{9a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

output
```
-1/7*x^9/b/(b*x^2+a)^(7/2)-9/35*x^7/b^2/(b*x^2+a)^(5/2)-3/5*x^5/b^3/(b*x^2
+a)^(3/2)-3*x^3/b^4/(b*x^2+a)^(1/2)+9/2*x*(b*x^2+a)^(1/2)/b^5-9/2*a*arctan
h(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \frac{315a^4x + 1050a^3bx^3 + 1218a^2b^2x^5 + 528ab^3x^7 + 35b^4x^9}{70b^5(a+bx^2)^{7/2}} - \frac{9a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

input
```
Integrate[x^10/(a + b*x^2)^(9/2),x]
```

output
```
(315*a^4*x + 1050*a^3*b*x^3 + 1218*a^2*b^2*x^5 + 528*a*b^3*x^7 + 35*b^4*x^
9)/(70*b^5*(a + b*x^2)^(7/2)) - (9*a*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[
a + b*x^2])])/b^(11/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {252, 252, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{9\int \frac{x^8}{(bx^2+a)^{7/2}}dx}{7b}-\frac{x^9}{7b\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 252$$

$$\frac{9\left(\frac{7\int \frac{x^6}{\left(bx^2+a\right)^{5/2}}dx}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 252$$

$$\frac{9\left(\frac{7\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^{3/2}}dx}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 252$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\int \frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{9\left(\frac{7\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)}{5b}-\frac{x^7}{5b(a+bx^2)^{5/2}}\right)}{7b}-\frac{x^9}{7b\left(a+bx^2\right)^{7/2}}$$

input
```
Int[x^10/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*x^9/(b*(a + b*x^2)^(7/2)) + (9*(-1/5*x^7/(b*(a + b*x^2)^(5/2)) + (7*(
-1/3*x^5/(b*(a + b*x^2)^(3/2)) + (5*(-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*S
qrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2
))))/b))/(3*b)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{9\left(\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)\left(b\,x^{2}+a\right)^{\frac{7}{2}} a-\left(\left(\frac{b^{\frac{9}{2}} x^{8}}{9}+\frac{176 a\,b^{\frac{7}{2}} x^{6}}{105}+\frac{58 a^{2} b^{\frac{5}{2}} x^{4}}{15}+\frac{10 a^{3} b^{\frac{3}{2}} x^{2}}{3}+a^{4} \sqrt{b}\right) x\right)}{2\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{\frac{11}{2}}}$
default	$\frac{x^{9}}{2 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{9 a \left(-\frac{x^{7}}{7 b \left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 b \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3 b \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)}{2 b}$
risch	$\frac{x\sqrt{b\,x^{2}+a}}{2 b^{5}}-\frac{53 a^{2} \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} b+2\sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}}{560 b^{7} \left(x-\frac{\sqrt{-a b}}{b}\right)^{3}}-\frac{571 a^{2} \sqrt{\left(x-\frac{\sqrt{-a b}}{b}\right)^{2} b+2\sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)}}{1120 b^{6} \sqrt{-a b}\left(x-\frac{\sqrt{-a b}}{b}\right)^{2}}+\frac{97 a\sqrt{\left(x-\right.}}{}$

input
```
int(x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-9/2/(b*x^2+a)^(7/2)/b^(11/2)*(arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*(b*x^2+a
)^(7/2)*a-(1/9*b^(9/2)*x^8+176/105*a*b^(7/2)*x^6+58/15*a^2*b^(5/2)*x^4+10/
3*a^3*b^(3/2)*x^2+a^4*b^(1/2))*x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.74

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \left[\frac{315\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-\right.}{140\,(b^{10}x^8+4\,ab^9x^6+6\,a^2}\right.$$

input
```
integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5
)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(35*b^5*x^9
+ 528*a*b^4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(
b*x^2 + a))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*
b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 +
 a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (35*b^5*x^9 + 528*a*b^
4*x^7 + 1218*a^2*b^3*x^5 + 1050*a^3*b^2*x^3 + 315*a^4*b*x)*sqrt(b*x^2 + a)
)/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7*x^2 + a^4*b^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3181 vs. $2(122) = 244$.

Time = 8.55 (sec) , antiderivative size = 3181, normalized size of antiderivative = 24.28

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**10/(b*x**2+a)**(9/2),x)
```

output

```
-315*a**(311/2)*b**66*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(
309/2)*b**(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt
(1 + b*x**2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400
*a**(303/2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2
)*x**8*sqrt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**
2/a) + 70*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 1890*a**(309/2
)*b**67*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b*
*(143/2)*sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x*
*2/a) + 1050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/
2)*b**(149/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sq
rt(1 + b*x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70
*a**(297/2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 4725*a**(307/2)*b**68*x
**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*
sqrt(1 + b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1
050*a**(305/2)*b**(147/2)*x**4*sqrt(1 + b*x**2/a) + 1400*a**(303/2)*b**(14
9/2)*x**6*sqrt(1 + b*x**2/a) + 1050*a**(301/2)*b**(151/2)*x**8*sqrt(1 + b*
x**2/a) + 420*a**(299/2)*b**(153/2)*x**10*sqrt(1 + b*x**2/a) + 70*a**(297/
2)*b**(155/2)*x**12*sqrt(1 + b*x**2/a)) - 6300*a**(305/2)*b**69*x**6*sqrt(
1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(70*a**(309/2)*b**(143/2)*sqrt(1 +
b*x**2/a) + 420*a**(307/2)*b**(145/2)*x**2*sqrt(1 + b*x**2/a) + 1050*a*...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(105) = 210$.

Time = 0.06 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.18

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \frac{x^9}{2\,(bx^2+a)^{\frac{7}{2}}b} + \frac{9\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)ax}{70\,b} + \frac{3\,ax\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b} + \frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2} + \frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{10\,b^2} + \frac{3\,ax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{2\,b^3} + \frac{9\,a^2x^3}{2\,(bx^2+a)^{\frac{5}{2}}b^4} - \frac{417\,ax}{70\,\sqrt{bx^2+a}b^5} - \frac{51\,a^2x}{70\,(bx^2+a)^{\frac{3}{2}}b^5} + \frac{261\,a^3x}{70\,(bx^2+a)^{\frac{5}{2}}b^5} - \frac{9\,a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/2*x^9/((b*x^2 + a)^(7/2)*b) + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*
x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*a*x/b + 3/10*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 + 3/
2*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 + 9/
2*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - 417/70*a*x/(sqrt(b*x^2 + a)*b^5) - 51/
70*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*a^3*x/((b*x^2 + a)^(5/2)*b^5) -
9/2*a*arcsinh(b*x/sqrt(a*b))/b^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(x^2\left(\frac{35\,x^2}{b} + \frac{528\,a}{b^2}\right) + \frac{1218\,a^2}{b^3}\right)x^2 + \frac{1050\,a^3}{b^4}\right)x^2 + \frac{315\,a^4}{b^5}\right)x}{70\,(bx^2+a)^{\frac{7}{2}}} + \frac{9\,a\log\left(\left|-\sqrt{b}x + \sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
 + 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^
2 + a)))/b^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^{10}}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int(x^10/(a + b*x^2)^(9/2),x)
```

output
```
int(x^10/(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.59

$$\int \frac{x^{10}}{(a+bx^2)^{9/2}}\,dx = \frac{315\sqrt{b\,x^2+a}\,a^4bx+1050\sqrt{b\,x^2+a}\,a^3b^2x^3+1218\sqrt{b\,x^2+a}\,a^2b^3x^5+528\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(x^10/(b*x^2+a)^(9/2),x)
```

output

```
(315*sqrt(a + b*x**2)*a**4*b*x + 1050*sqrt(a + b*x**2)*a**3*b**2*x**3 + 12
18*sqrt(a + b*x**2)*a**2*b**3*x**5 + 528*sqrt(a + b*x**2)*a*b**4*x**7 + 35
*sqrt(a + b*x**2)*b**5*x**9 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*
x)/sqrt(a))*a**5 - 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a)
)*a**4*b*x**2 - 1890*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**3*b**2*x**4 - 1260*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a
**2*b**3*x**6 - 315*sqrt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*
b**4*x**8 - 213*sqrt(b)*a**5 - 852*sqrt(b)*a**4*b*x**2 - 1278*sqrt(b)*a**3
*b**2*x**4 - 852*sqrt(b)*a**2*b**3*x**6 - 213*sqrt(b)*a*b**4*x**8)/(70*b**
6*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.528 $\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = -\frac{a^4}{7b^5(a+bx^2)^{7/2}} + \frac{4a^3}{5b^5(a+bx^2)^{5/2}} - \frac{2a^2}{b^5(a+bx^2)^{3/2}} + \frac{4a}{b^5\sqrt{a+bx^2}} + \frac{\sqrt{a+bx^2}}{b^5}$$

output
```
-1/7*a^4/b^5/(b*x^2+a)^(7/2)+4/5*a^3/b^5/(b*x^2+a)^(5/2)-2*a^2/b^5/(b*x^2+
a)^(3/2)+4*a/b^5/(b*x^2+a)^(1/2)+(b*x^2+a)^(1/2)/b^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8}{35b^5(a+bx^2)^{7/2}}$$

input
```
Integrate[x^9/(a + b*x^2)^(9/2),x]
```

output
```
(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^4*x^8)/(
35*b^5*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^9}{(a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^8}{(bx^2+a)^{9/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^4}{b^4(bx^2+a)^{9/2}}-\frac{4a^3}{b^4(bx^2+a)^{7/2}}+\frac{6a^2}{b^4(bx^2+a)^{5/2}}-\frac{4a}{b^4(bx^2+a)^{3/2}}+\frac{1}{b^4\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^4}{7b^5(a+bx^2)^{7/2}}+\frac{8a^3}{5b^5(a+bx^2)^{5/2}}-\frac{4a^2}{b^5(a+bx^2)^{3/2}}+\frac{8a}{b^5\sqrt{a+bx^2}}+\frac{2\sqrt{a+bx^2}}{b^5}\right)$$

input
```
Int[x^9/(a + b*x^2)^(9/2),x]
```

output
```
((-2*a^4)/(7*b^5*(a + b*x^2)^(7/2)) + (8*a^3)/(5*b^5*(a + b*x^2)^(5/2)) -
(4*a^2)/(b^5*(a + b*x^2)^(3/2)) + (8*a)/(b^5*Sqrt[a + b*x^2]) + (2*Sqrt[a
+ b*x^2])/b^5)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{5}}$	58
trager	$\frac{35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{5}}$	58
pseudoelliptic	$\frac{35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{5}}$	58
orering	$\frac{35b^{4}x^{8}+280a\,b^{3}x^{6}+560a^{2}b^{2}x^{4}+448a^{3}b\,x^{2}+128a^{4}}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}b^{5}}$	58
risch	$\frac{\sqrt{b\,x^{2}+a}}{b^{5}}+\frac{\sqrt{b\,x^{2}+a}\,\left(140b^{3}x^{6}+350a\,b^{2}x^{4}+308a^{2}b\,x^{2}+93a^{3}\right)a}{35b^{5}\left(b^{4}x^{8}+4a\,b^{3}x^{6}+6a^{2}b^{2}x^{4}+4a^{3}b\,x^{2}+a^{4}\right)}$	104
default	$\frac{x^{8}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{8a\left(-\frac{x^{6}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)}{b}$	105

input
```
int(x^9/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*a^4)/(b*x
^2+a)^(7/2)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.09

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{(35\,b^4x^8 + 280\,ab^3x^6 + 560\,a^2b^2x^4 + 448\,a^3bx^2 + 128\,a^4)\sqrt{bx^2+a}}{35\,(b^9x^8 + 4\,ab^8x^6 + 6\,a^2b^7x^4 + 4\,a^3b^6x^2 + a^4b^5)}$$

input
```
integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a
^4)*sqrt(b*x^2 + a)/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2
 + a^4*b^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(87) = 174.

Time = 0.78 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.83

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \begin{cases} \frac{128a^4}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}+105ab^7x^4\sqrt{a+bx^2}+35b^8x^6\sqrt{a+bx^2}} + \frac{448a}{35a^3b^5\sqrt{a+bx^2}+105a^2b^6x^2\sqrt{a+bx^2}} \\ \frac{x^{10}}{10a^{\frac{9}{2}}} \end{cases}$$

input
```
integrate(x**9/(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((128*a**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*x**2*sq
rt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sqrt(a +
b*x**2)) + 448*a**3*b*x**2/(35*a**3*b**5*sqrt(a + b*x**2) + 105*a**2*b**6*
x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b**8*x**6*sq
rt(a + b*x**2)) + 560*a**2*b**2*x**4/(35*a**3*b**5*sqrt(a + b*x**2) + 105*
a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2) + 35*b*
*8*x**6*sqrt(a + b*x**2)) + 280*a*b**3*x**6/(35*a**3*b**5*sqrt(a + b*x**2)
 + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x**2)
+ 35*b**8*x**6*sqrt(a + b*x**2)) + 35*b**4*x**8/(35*a**3*b**5*sqrt(a + b*x
**2) + 105*a**2*b**6*x**2*sqrt(a + b*x**2) + 105*a*b**7*x**4*sqrt(a + b*x*
*2) + 35*b**8*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**10/(10*a**(9/2)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{x^8}{(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,ax^6}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{16\,a^2x^4}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{64\,a^3x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^4} + \frac{128\,a^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5}$$

input
```
integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
x^8/((b*x^2 + a)^(7/2)*b) + 8*a*x^6/((b*x^2 + a)^(7/2)*b^2) + 16*a^2*x^4/(
(b*x^2 + a)^(7/2)*b^3) + 64/5*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) + 128/35*a^4
/((b*x^2 + a)^(7/2)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{35\sqrt{bx^2+a}}{b} + \frac{140\,(bx^2+a)^3a-70\,(bx^2+a)^2a^2+28\,(bx^2+a)a^3-5\,a^4}{(bx^2+a)^{\frac{7}{2}}b}}{35\,b^4}$$

input
```
integrate(x^9/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*(35*sqrt(b*x^2 + a)/b + (140*(b*x^2 + a)^3*a - 70*(b*x^2 + a)^2*a^2 +
 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b))/b^4
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.85

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}}{b^5}+\frac{4\,a}{b^5\,\sqrt{b\,x^2+a}} - \frac{2\,a^2}{b^5\,(b\,x^2+a)^{3/2}}+\frac{4\,a^3}{5\,b^5\,(b\,x^2+a)^{5/2}}-\frac{a^4}{7\,b^5\,(b\,x^2+a)^{7/2}}$$

input
```
int(x^9/(a + b*x^2)^(9/2),x)
```

output
```
(a + b*x^2)^(1/2)/b^5 + (4*a)/(b^5*(a + b*x^2)^(1/2)) - (2*a^2)/(b^5*(a +
b*x^2)^(3/2)) + (4*a^3)/(5*b^5*(a + b*x^2)^(5/2)) - a^4/(7*b^5*(a + b*x^2)
^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04

$$\int \frac{x^9}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(35b^4x^8+280a\,b^3x^6+560a^2b^2x^4+448a^3b\,x^2+128a^4)}{35b^5\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^9/(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*(128*a**4 + 448*a**3*b*x**2 + 560*a**2*b**2*x**4 + 280*a
*b**3*x**6 + 35*b**4*x**8))/(35*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x
**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.529 $\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = -\frac{x^7}{7b(a+bx^2)^{7/2}} - \frac{x^5}{5b^2(a+bx^2)^{5/2}} - \frac{x^3}{3b^3(a+bx^2)^{3/2}} - \frac{x}{b^4\sqrt{a+bx^2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
-1/7*x^7/b/(b*x^2+a)^(7/2)-1/5*x^5/b^2/(b*x^2+a)^(5/2)-1/3*x^3/b^3/(b*x^2+
a)^(3/2)-x/b^4/(b*x^2+a)^(1/2)+arctanh(b^(1/2)*x/(b*x^2+a)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = -\frac{x(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6)}{105b^4(a+bx^2)^{7/2}} + \frac{2\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

input
```
Integrate[x^8/(a + b*x^2)^(9/2),x]
```

output
```
-1/105*(x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^6))/(b^4*(a
 + b*x^2)^(7/2)) + (2*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b
^(9/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {252, 252, 252, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{\int \frac{x^6}{(bx^2+a)^{7/2}}dx}{b}-\frac{x^7}{7b(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\int \frac{x^4}{\left(bx^2+a\right)^{5/2}}dx}{b}-\frac{x^5}{5b(a+bx^2)^{5/2}}}{b}-\frac{x^7}{7b(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\frac{\int \frac{x^2}{\left(bx^2+a\right)^{3/2}}dx}{b}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}}{b}-\frac{x^5}{5b(a+bx^2)^{5/2}}}{b}-\frac{x^7}{7b(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x}{b\sqrt{a+bx^2}}}{b}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}}{b}-\frac{x^5}{5b(a+bx^2)^{5/2}}}{b}-\frac{x^7}{7b(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x}{b\sqrt{a+bx^2}}}{b}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}}{b}-\frac{x^5}{5b(a+bx^2)^{5/2}}}{b}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}}{b}-\frac{x^3}{3b\left(a+bx^2\right)^{3/2}}}{b}-\frac{x^5}{5b(a+bx^2)^{5/2}}}{b}-\frac{x^7}{7b\left(a+bx^2\right)^{7/2}}$$

input
```
Int[x^8/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*x^7/(b*(a + b*x^2)^(7/2)) + (-1/5*x^5/(b*(a + b*x^2)^(5/2)) + (-1/3*x
^3/(b*(a + b*x^2)^(3/2)) + (-(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]]/b^(3/2))/b)/b)/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

method	result	size
pseudoelliptic	$\frac{(b\,x^{2}+a)^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\frac{176x^{7}b^{\frac{7}{2}}}{105}-\frac{58b^{\frac{5}{2}}a\,x^{5}}{15}-\frac{10b^{\frac{3}{2}}a^{2}x^{3}}{3}-\sqrt{b}\,a^{3}x}{b^{\frac{9}{2}}(b\,x^{2}+a)^{\frac{7}{2}}}$	79
default	$-\frac{x^{7}}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(\sqrt{b}\,x+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}$	103

input
```
int(x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^(9/2)*((b*x^2+a)^(7/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-176/105*x^7*
b^(7/2)-58/15*b^(5/2)*a*x^5-10/3*b^(3/2)*a^2*x^3-b^(1/2)*a^3*x)/(b*x^2+a)^
(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.12

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right.}{210\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a}\right.$$

$$-\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(176\,b^4x^7+406\,ab^3x^5+350\,a^2b^2x}{105\,(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5)}$$

input
```
integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sq
rt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(176*b^4*x^7 + 4
06*a*b^3*x^5 + 350*a^2*b^2*x^3 + 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 +
4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(b^4*x
^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) + (176*b^4*x^7 + 406*a*b^3*x^5 + 350*a^2*b^2*x^3 +
 105*a^3*b*x)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*
a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. $2(92) = 184$.

Time = 4.78 (sec) , antiderivative size = 2980, normalized size of antiderivative = 28.11

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**8/(b*x**2+a)**(9/2),x)
```

output
```
105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(
205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(
1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*
a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2
/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)
*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b*
*(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**
2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqr
t(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105
*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x
**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*
sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1
575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(10
5/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*
x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt
(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 +
 b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(86) = 172$.

Time = 0.06 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.41

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)x - \frac{x\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{ax^3}{(bx^2+a)^{\frac{5}{2}}b^3} + \frac{139\,x}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,ax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,a^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*x - 1/
15*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/
((b*x^2 + a)^(3/2)*b^2))/b^2 - a*x^3/((b*x^2 + a)^(5/2)*b^3) + 139/105*x/(
sqrt(b*x^2 + a)*b^4) + 17/105*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*a^2*x/((
b*x^2 + a)^(5/2)*b^4) + arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.74

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/
(b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^8}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int(x^8/(a + b*x^2)^(9/2),x)
```

output
```
int(x^8/(a + b*x^2)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.96

$$\int \frac{x^8}{(a+bx^2)^{9/2}}\,dx = \frac{-105\sqrt{b\,x^2+a}\,a^3bx - 350\sqrt{b\,x^2+a}\,a^2b^2x^3 - 406\sqrt{b\,x^2+a}\,a\,b^3x^5 - 176\sqrt{b\,x^2+a}\,b^4}{}$$

input
```
int(x^8/(b*x^2+a)^(9/2),x)
```

output
```
( - 105*sqrt(a + b*x**2)*a**3*b*x - 350*sqrt(a + b*x**2)*a**2*b**2*x**3 -
406*sqrt(a + b*x**2)*a*b**3*x**5 - 176*sqrt(a + b*x**2)*b**4*x**7 + 105*sq
rt(b)*log((sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**4 + 420*sqrt(b)*log((
sqrt(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**3*b*x**2 + 630*sqrt(b)*log((sqrt
(a + b*x**2) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x**4 + 420*sqrt(b)*log((sqrt(
a + b*x**2) + sqrt(b)*x)/sqrt(a))*a*b**3*x**6 + 105*sqrt(b)*log((sqrt(a +
b*x**2) + sqrt(b)*x)/sqrt(a))*b**4*x**8 + 56*sqrt(b)*a**4 + 224*sqrt(b)*a*
*3*b*x**2 + 336*sqrt(b)*a**2*b**2*x**4 + 224*sqrt(b)*a*b**3*x**6 + 56*sqrt
(b)*b**4*x**8)/(105*b**5*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b*
*3*x**6 + b**4*x**8))
```

3.530 $\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = \frac{a^3}{7b^4(a+bx^2)^{7/2}} - \frac{3a^2}{5b^4(a+bx^2)^{5/2}} + \frac{a}{b^4(a+bx^2)^{3/2}} - \frac{1}{b^4\sqrt{a+bx^2}}$$

output
```
1/7*a^3/b^4/(b*x^2+a)^(7/2)-3/5*a^2/b^4/(b*x^2+a)^(5/2)+a/b^4/(b*x^2+a)^(3
/2)-1/b^4/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = \frac{-16a^3-56a^2bx^2-70ab^2x^4-35b^3x^6}{35b^4(a+bx^2)^{7/2}}$$

input
```
Integrate[x^7/(a + b*x^2)^(9/2),x]
```

output
```
(-16*a^3 - 56*a^2*b*x^2 - 70*a*b^2*x^4 - 35*b^3*x^6)/(35*b^4*(a + b*x^2)^(
7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^{9/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3\left(bx^2+a\right)^{9/2}}+\frac{3a^2}{b^3\left(bx^2+a\right)^{7/2}}-\frac{3a}{b^3\left(bx^2+a\right)^{5/2}}+\frac{1}{b^3\left(bx^2+a\right)^{3/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2a^3}{7b^4\left(a+bx^2\right)^{7/2}}-\frac{6a^2}{5b^4\left(a+bx^2\right)^{5/2}}+\frac{2a}{b^4\left(a+bx^2\right)^{3/2}}-\frac{2}{b^4\sqrt{a+bx^2}}\right)$$

input
```
Int[x^7/(a + b*x^2)^(9/2),x]
```

output
```
((2*a^3)/(7*b^4*(a + b*x^2)^(7/2)) - (6*a^2)/(5*b^4*(a + b*x^2)^(5/2)) + (
2*a)/(b^4*(a + b*x^2)^(3/2)) - 2/(b^4*Sqrt[a + b*x^2]))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
trager	$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
pseudoelliptic	$\frac{-35b^3x^6-70a\,b^2x^4-56a^2b\,x^2-16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
orering	$-\frac{35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3}{35(b\,x^2+a)^{\frac{7}{2}}b^4}$	47
default	$-\frac{x^6}{b(b\,x^2+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^4}{3b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}$	82

input
```
int(x^7/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)/(b*x^2+a)^(7/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = -\frac{(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3)\sqrt{bx^2+a}}{35\,(b^8x^8+4\,ab^7x^6+6\,a^2b^6x^4+4\,a^3b^5x^2+a^4b^4)}$$

input
```
integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^2 + a)/
(b^8*x^8 + 4*a*b^7*x^6 + 6*a^2*b^6*x^4 + 4*a^3*b^5*x^2 + a^4*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. 2(68) = 136.

Time = 0.77 (sec) , antiderivative size = 364, normalized size of antiderivative = 4.85

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = \begin{cases} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{5}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+} \\ \frac{x^8}{8a^{\frac{9}{2}}} \end{cases}$$

input
```
integrate(x**7/(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sq
rt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a +
b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x
**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqr
t(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*
b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x*
*6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a
**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**
7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = -\frac{x^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{2\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,a^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16\,a^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-x^6/((b*x^2 + a)^(7/2)*b) - 2*a*x^4/((b*x^2 + a)^(7/2)*b^2) - 8/5*a^2*x^2
/((b*x^2 + a)^(7/2)*b^3) - 16/35*a^3/((b*x^2 + a)^(7/2)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = -\frac{35\,(bx^2+a)^3 - 35\,(bx^2+a)^2a + 21\,(bx^2+a)a^2 - 5\,a^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 - 5*a^3)
/((b*x^2 + a)^(7/2)*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = \frac{a}{b^4\,(b\,x^2+a)^{3/2}} - \frac{1}{b^4\,\sqrt{b\,x^2+a}} - \frac{3\,a^2}{5\,b^4\,(b\,x^2+a)^{5/2}} + \frac{a^3}{7\,b^4\,(b\,x^2+a)^{7/2}}$$

input
```
int(x^7/(a + b*x^2)^(9/2),x)
```

output
```
a/(b^4*(a + b*x^2)^(3/2)) - 1/(b^4*(a + b*x^2)^(1/2)) - (3*a^2)/(5*b^4*(a
+ b*x^2)^(5/2)) + a^3/(7*b^4*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.16

$$\int \frac{x^7}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-35b^3x^6-70a\,b^2x^4-56a^2b\,x^2-16a^3)}{35b^4\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^7/(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 16*a**3 - 56*a**2*b*x**2 - 70*a*b**2*x**4 - 35*b**3*
x**6))/(35*b**4*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 +
 b**4*x**8))
```

3.531 $\int \frac{x^6}{(a+bx^2)^{9/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{x^6}{(a+bx^2)^{9/2}} \, dx = \frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

output
```
1/7*x^7/a/(b*x^2+a)^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^2)^{9/2}} \, dx = \frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(9/2),x]
```

output
```
x^7/(7*a*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{242\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{242}$$

$$\frac{x^7}{7a\,(a+bx^2)^{7/2}}$$

input
```
Int[x^6/(a + b*x^2)^(9/2),x]
```

output
```
x^7/(7*a*(a + b*x^2)^(7/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result
gosper	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
trager	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
pseudoelliptic	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
orering	$\frac{x^7}{7a(b\,x^2+a)^{\frac{7}{2}}}$
default	$-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\ldots}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)}{2b}$

input
```
int(x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7/a/(b*x^2+a)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(17) = 34$.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.81

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{bx^2+a}\,x^7}{7\,(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5)}$$

input
```
integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/7*sqrt(b*x^2 + a)*x^7/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4
*b*x^2 + a^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(15) = 30$.

Time = 0.71 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.52

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = \frac{x^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**6/(b*x**2+a)**(9/2),x)
```

output
```
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a
) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(17) = 34$.

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.90

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = -\frac{x^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,ax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{x}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{x}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,ax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,a^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 1/14*
x/((b*x^2 + a)^(3/2)*b^3) + 1/7*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*a*x/((b*x
^2 + a)^(5/2)*b^3) - 15/56*a^2*x/((b*x^2 + a)^(7/2)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = \frac{x^7}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = \frac{x}{7\,a\,b^3\,\sqrt{b\,x^2+a}} - \frac{3\,x}{7\,b^3\,(b\,x^2+a)^{3/2}} - \frac{a^2\,x}{7\,b^3\,(b\,x^2+a)^{7/2}} + \frac{3\,a\,x}{7\,b^3\,(b\,x^2+a)^{5/2}}$$

input
```
int(x^6/(a + b*x^2)^(9/2),x)
```

output
```
x/(7*a*b^3*(a + b*x^2)^(1/2)) - (3*x)/(7*b^3*(a + b*x^2)^(3/2)) - (a^2*x)/
(7*b^3*(a + b*x^2)^(7/2)) + (3*a*x)/(7*b^3*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 5.52

$$\int \frac{x^6}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,b^4x^7 + \sqrt{b}\,a^4 + 4\sqrt{b}\,a^3b\,x^2 + 6\sqrt{b}\,a^2b^2x^4 + 4\sqrt{b}\,a\,b^3x^6 + \sqrt{b}\,b^4x^8}{7a\,b^4\,(b^4x^8 + 4a\,b^3x^6 + 6a^2b^2x^4 + 4a^3b\,x^2 + a^4)}$$

input
```
int(x^6/(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*b**4*x**7 + sqrt(b)*a**4 + 4*sqrt(b)*a**3*b*x**2 + 6*sqr
t(b)*a**2*b**2*x**4 + 4*sqrt(b)*a*b**3*x**6 + sqrt(b)*b**4*x**8)/(7*a*b**4
*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.532 $\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = -\frac{a^2}{7b^3(a+bx^2)^{7/2}} + \frac{2a}{5b^3(a+bx^2)^{5/2}} - \frac{1}{3b^3(a+bx^2)^{3/2}}$$

output
```
-1/7*a^2/b^3/(b*x^2+a)^(7/2)+2/5*a/b^3/(b*x^2+a)^(5/2)-1/3/b^3/(b*x^2+a)^(
3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^2-28abx^2-35b^2x^4}{105b^3(a+bx^2)^{7/2}}$$

input
```
Integrate[x^5/(a + b*x^2)^(9/2),x]
```

output
```
(-8*a^2 - 28*a*b*x^2 - 35*b^2*x^4)/(105*b^3*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{9/2}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2\left(bx^2+a\right)^{9/2}}-\frac{2a}{b^2\left(bx^2+a\right)^{7/2}}+\frac{1}{b^2\left(bx^2+a\right)^{5/2}}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{2a^2}{7b^3\left(a+bx^2\right)^{7/2}}+\frac{4a}{5b^3\left(a+bx^2\right)^{5/2}}-\frac{2}{3b^3\left(a+bx^2\right)^{3/2}}\right)$$

input
```
Int[x^5/(a + b*x^2)^(9/2),x]
```

output
```
((-2*a^2)/(7*b^3*(a + b*x^2)^(7/2)) + (4*a)/(5*b^3*(a + b*x^2)^(5/2)) - 2/
(3*b^3*(a + b*x^2)^(3/2)))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{35 b^{2} x^{4}+28 a b \,x^{2}+8 a^{2}}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} b^{3}}$	36
trager	$-\frac{35 b^{2} x^{4}+28 a b \,x^{2}+8 a^{2}}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} b^{3}}$	36
pseudoelliptic	$\frac{-35 b^{2} x^{4}-28 a b \,x^{2}-8 a^{2}}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} b^{3}}$	36
orering	$-\frac{35 b^{2} x^{4}+28 a b \,x^{2}+8 a^{2}}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} b^{3}}$	36
default	$-\frac{x^{4}}{3 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{4 a \left(-\frac{x^{2}}{5 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{2 a}{35 b^{2} \left(b \,x^{2}+a \right)^{\frac{7}{2}}}\right)}{3 b}$	58

input
```
int(x^5/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(35*b^2*x^4+28*a*b*x^2+8*a^2)/(b*x^2+a)^(7/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = -\frac{(35\,b^2x^4+28\,abx^2+8\,a^2)\sqrt{bx^2+a}}{105\,(b^7x^8+4\,ab^6x^6+6\,a^2b^5x^4+4\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^2 + a)/(b^7*x^8 + 4*a*b^
6*x^6 + 6*a^2*b^5*x^4 + 4*a^3*b^4*x^2 + a^4*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(53) = 106.

Time = 0.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.61

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = \begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}$$

input
```
integrate(x**5/(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sq
rt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a +
 b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x*
*2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqr
t(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b
**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x*
*6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = -\frac{x^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,ax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/3*x^4/((b*x^2 + a)^(7/2)*b) - 4/15*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 8/10
5*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = -\frac{35\,(bx^2+a)^2 - 42\,(bx^2+a)a + 15\,a^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/((b*x^2 + a)^(7/2)*b
^3)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = -\frac{35\,(b\,x^2+a)^2 - 42\,a\,(b\,x^2+a) + 15\,a^2}{105\,b^3\,(b\,x^2+a)^{7/2}}$$

input
```
int(x^5/(a + b*x^2)^(9/2),x)
```

output
```
-(35*(a + b*x^2)^2 - 42*a*(a + b*x^2) + 15*a^2)/(105*b^3*(a + b*x^2)^(7/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{x^5}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-35b^2x^4-28ab\,x^2-8a^2)}{105b^3\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^5/(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 8*a**2 - 28*a*b*x**2 - 35*b**2*x**4))/(105*b**3*(a**
4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.533 $\int \frac{x^4}{(a+bx^2)^{9/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 43

$$\int \frac{x^4}{(a+bx^2)^{9/2}} \, dx = \frac{x^5}{7a(a+bx^2)^{7/2}} + \frac{2x^5}{35a^2(a+bx^2)^{5/2}}$$

output
```
1/7*x^5/a/(b*x^2+a)^(7/2)+2/35*x^5/a^2/(b*x^2+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.72

$$\int \frac{x^4}{(a+bx^2)^{9/2}} \, dx = \frac{7ax^5 + 2bx^7}{35a^2(a+bx^2)^{7/2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(9/2),x]
```

output
```
(7*a*x^5 + 2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \ 245$$

$$\frac{2b\int \frac{x^6}{(bx^2+a)^{9/2}}dx}{5a}+\frac{x^5}{5a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 242$$

$$\frac{2bx^7}{35a^2(a+bx^2)^{7/2}}+\frac{x^5}{5a(a+bx^2)^{7/2}}$$

input
```
Int[x^4/(a + b*x^2)^(9/2),x]
```

output
```
x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7)/(35*a^2*(a + b*x^2)^(7/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{x^{5}\left(2b\,x^{2}+7a\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}$	28
trager	$\frac{x^{5}\left(2b\,x^{2}+7a\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}$	28
pseudoelliptic	$\frac{x^{5}\left(2b\,x^{2}+7a\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}$	28
orering	$\frac{x^{5}\left(2b\,x^{2}+7a\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}}$	28
default	$-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}$	120

input
```
int(x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x^5*(2*b*x^2+7*a)/(b*x^2+a)^(7/2)/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(35) = 70$.

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = \frac{(2\,bx^7+7\,ax^5)\sqrt{bx^2+a}}{35\,(a^2b^4x^8+4\,a^3b^3x^6+6\,a^4b^2x^4+4\,a^5bx^2+a^6)}$$

input
```
integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(2*b*x^7 + 7*a*x^5)*sqrt(b*x^2 + a)/(a^2*b^4*x^8 + 4*a^3*b^3*x^6 + 6*
a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(36) = 72$.

Time = 0.80 (sec) , antiderivative size = 199, normalized size of antiderivative = 4.63

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = \frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4/(b*x**2+a)**(9/2),x)
```

output
```
7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b
*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x*
*6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.98

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = -\frac{x^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,x}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,x}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{x}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,ax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*x^3/((b*x^2 + a)^(7/2)*b) + 3/140*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*x/
(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*a*x/((
b*x^2 + a)^(7/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = \frac{x^5\left(\frac{2\,bx^2}{a^2}+\frac{7}{a}\right)}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = \frac{2\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}} - \frac{8\,x}{35\,b^2\,\left(b\,x^2+a\right)^{5/2}} + \frac{x}{35\,a\,b^2\,\left(b\,x^2+a\right)^{3/2}} + \frac{a\,x}{7\,b^2\,\left(b\,x^2+a\right)^{7/2}}$$

input
```
int(x^4/(a + b*x^2)^(9/2),x)
```

output
```
(2*x)/(35*a^2*b^2*(a + b*x^2)^(1/2)) - (8*x)/(35*b^2*(a + b*x^2)^(5/2)) +
x/(35*a*b^2*(a + b*x^2)^(3/2)) + (a*x)/(7*b^2*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.16

$$\int \frac{x^4}{(a+bx^2)^{9/2}}\,dx = \frac{7\sqrt{b\,x^2+a}\,a\,b^3x^5+2\sqrt{b\,x^2+a}\,b^4x^7-2\sqrt{b}\,a^4-8\sqrt{b}\,a^3b\,x^2-12\sqrt{b}\,a^2b^2x^4-8\sqrt{b}\,a\,b^3}{35a^2b^3\left(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4\right)}$$

input
```
int(x^4/(b*x^2+a)^(9/2),x)
```

output
```
(7*sqrt(a + b*x**2)*a*b**3*x**5 + 2*sqrt(a + b*x**2)*b**4*x**7 - 2*sqrt(b)
*a**4 - 8*sqrt(b)*a**3*b*x**2 - 12*sqrt(b)*a**2*b**2*x**4 - 8*sqrt(b)*a*b*
*3*x**6 - 2*sqrt(b)*b**4*x**8)/(35*a**2*b**3*(a**4 + 4*a**3*b*x**2 + 6*a**
2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.534 $\int \frac{x^3}{(a+bx^2)^{9/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^3}{(a+bx^2)^{9/2}} dx = \frac{a}{7b^2(a+bx^2)^{7/2}} - \frac{1}{5b^2(a+bx^2)^{5/2}}$$

output
```
1/7*a/b^2/(b*x^2+a)^(7/2)-1/5/b^2/(b*x^2+a)^(5/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{(a+bx^2)^{9/2}} dx = \frac{-2a-7bx^2}{35b^2(a+bx^2)^{7/2}}$$

input
```
Integrate[x^3/(a + b*x^2)^(9/2),x]
```

output
```
(-2*a - 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{9/2}} \, dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{9/2}}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2}\int \left(\frac{1}{b(bx^2+a)^{7/2}}-\frac{a}{b(bx^2+a)^{9/2}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{2a}{7b^2(a+bx^2)^{7/2}}-\frac{2}{5b^2(a+bx^2)^{5/2}}\right)$$

input
```
Int[x^3/(a + b*x^2)^(9/2),x]
```

output
```
((2*a)/(7*b^2*(a + b*x^2)^(7/2)) - 2/(5*b^2*(a + b*x^2)^(5/2)))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{7b\,x^{2}+2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}$	25
trager	$-\frac{7b\,x^{2}+2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}$	25
pseudoelliptic	$\frac{-7b\,x^{2}-2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}$	25
orering	$-\frac{7b\,x^{2}+2a}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}} b^{2}}$	25
default	$-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}$	34

input
```
int(x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/35*(7*b*x^2+2*a)/(b*x^2+a)^(7/2)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(30) = 60$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = -\frac{(7\,bx^2+2\,a)\sqrt{bx^2+a}}{35\,(b^6x^8+4\,ab^5x^6+6\,a^2b^4x^4+4\,a^3b^3x^2+a^4b^2)}$$

input
```
integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(7*b*x^2 + 2*a)*sqrt(b*x^2 + a)/(b^6*x^8 + 4*a*b^5*x^6 + 6*a^2*b^4*x
^4 + 4*a^3*b^3*x^2 + a^4*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(32) = 64$.

Time = 0.76 (sec) , antiderivative size = 180, normalized size of antiderivative = 4.74

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = \begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+b}} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
 + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x*
*2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
 + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = -\frac{x^2}{5\,(bx^2+a)^{\frac{7}{2}}b} - \frac{2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/5*x^2/((b*x^2 + a)^(7/2)*b) - 2/35*a/((b*x^2 + a)^(7/2)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = -\frac{7\,bx^2+2\,a}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = -\frac{7\,b\,x^2+2\,a}{35\,b^2\,(b\,x^2+a)^{7/2}}$$

input
```
int(x^3/(a + b*x^2)^(9/2),x)
```

output
```
-(2*a + 7*b*x^2)/(35*b^2*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{x^3}{(a+bx^2)^{9/2}}\,dx = \frac{\sqrt{b\,x^2+a}\,(-7b\,x^2-2a)}{35b^2\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x^3/(b*x^2+a)^(9/2),x)
```

output
```
(sqrt(a + b*x**2)*( - 2*a - 7*b*x**2))/(35*b**2*(a**4 + 4*a**3*b*x**2 + 6*
a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.535 $\int \frac{x^2}{(a+bx^2)^{9/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 64

$$\int \frac{x^2}{(a+bx^2)^{9/2}} \, dx = \frac{x^3}{7a(a+bx^2)^{7/2}} + \frac{4x^3}{35a^2(a+bx^2)^{5/2}} + \frac{8x^3}{105a^3(a+bx^2)^{3/2}}$$

output
```
1/7*x^3/a/(b*x^2+a)^(7/2)+4/35*x^3/a^2/(b*x^2+a)^(5/2)+8/105*x^3/a^3/(b*x^
2+a)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.66

$$\int \frac{x^2}{(a+bx^2)^{9/2}} \, dx = \frac{35a^2x^3 + 28abx^5 + 8b^2x^7}{105a^3(a+bx^2)^{7/2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(9/2),x]
```

output
```
(35*a^2*x^3 + 28*a*b*x^5 + 8*b^2*x^7)/(105*a^3*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{ 245}$$

$$\frac{4b\int \frac{x^4}{(bx^2+a)^{9/2}}dx}{3a}+\frac{x^3}{3a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 245}$$

$$\frac{4b\left(\frac{2b\int \frac{x^6}{(bx^2+a)^{9/2}}dx}{5a}+\frac{x^5}{5a(a+bx^2)^{7/2}}\right)}{3a}+\frac{x^3}{3a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{4b\left(\frac{2bx^7}{35a^2(a+bx^2)^{7/2}}+\frac{x^5}{5a(a+bx^2)^{7/2}}\right)}{3a}+\frac{x^3}{3a\,(a+bx^2)^{7/2}}$$

input
```
Int[x^2/(a + b*x^2)^(9/2),x]
```

output
```
x^3/(3*a*(a + b*x^2)^(7/2)) + (4*b*(x^5/(5*a*(a + b*x^2)^(7/2)) + (2*b*x^7
)/(35*a^2*(a + b*x^2)^(7/2))))/(3*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{x^{3}\left(8 b^{2} x^{4}+28 a b \,x^{2}+35 a^{2}\right)}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{3}}$	39
trager	$\frac{x^{3}\left(8 b^{2} x^{4}+28 a b \,x^{2}+35 a^{2}\right)}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{3}}$	39
pseudoelliptic	$\frac{x^{3}\left(8 b^{2} x^{4}+28 a b \,x^{2}+35 a^{2}\right)}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{3}}$	39
orering	$\frac{x^{3}\left(8 b^{2} x^{4}+28 a b \,x^{2}+35 a^{2}\right)}{105 \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{3}}$	39
default	$-\frac{x}{6 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{a \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)}{6 b}$	96

input
```
int(x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*x^3*(8*b^2*x^4+28*a*b*x^2+35*a^2)/(b*x^2+a)^(7/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,b^2x^7+28\,abx^5+35\,a^2x^3)\sqrt{bx^2+a}}{105\,(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7)}$$

input
```
integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*b^2*x^7 + 28*a*b*x^5 + 35*a^2*x^3)*sqrt(b*x^2 + a)/(a^3*b^4*x^8 +
 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 517 vs. 2(56) = 112.

Time = 1.00 (sec) , antiderivative size = 517, normalized size of antiderivative = 8.08

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1}}$$
$$+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1}}$$
$$+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1}}$$

input
```
integrate(x**2/(b*x**2+a)**(9/2),x)
```

output
```
35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt
(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a
**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b
*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(
17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) +
 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b
**4*x**8*sqrt(1 + b*x**2/a))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = -\frac{x}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,x}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,x}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{x}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

input
```
integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/7*x/((b*x^2 + a)^(7/2)*b) + 8/105*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*x/(
(b*x^2 + a)^(3/2)*a^2*b) + 1/35*x/((b*x^2 + a)^(5/2)*a*b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{8\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} - \frac{x}{7\,b\,(b\,x^2+a)^{7/2}} + \frac{4\,x}{105\,a^2\,b\,(b\,x^2+a)^{3/2}} + \frac{x}{35\,a\,b\,(b\,x^2+a)^{5/2}}$$

input
```
int(x^2/(a + b*x^2)^(9/2),x)
```

output
```
(8*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - x/(7*b*(a + b*x^2)^(7/2)) + (4*x)/(1
05*a^2*b*(a + b*x^2)^(3/2)) + x/(35*a*b*(a + b*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.42

$$\int \frac{x^2}{(a+bx^2)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^2b^2x^3 + 28\sqrt{b\,x^2+a}\,a\,b^3x^5 + 8\sqrt{b\,x^2+a}\,b^4x^7 - 8\sqrt{b}\,a^4 - 32\sqrt{b}\,a^3b\,x^2 - \cdots}{105a^3b^2\left(b^4x^8 + 4a\,b^3x^6 + 6a^2b^2x^4 + 4a^3b\,x^2 + a \cdots\right.}$$

input
```
int(x^2/(b*x^2+a)^(9/2),x)
```

output
```
(35*sqrt(a + b*x**2)*a**2*b**2*x**3 + 28*sqrt(a + b*x**2)*a*b**3*x**5 + 8*
sqrt(a + b*x**2)*b**4*x**7 - 8*sqrt(b)*a**4 - 32*sqrt(b)*a**3*b*x**2 - 48*
sqrt(b)*a**2*b**2*x**4 - 32*sqrt(b)*a*b**3*x**6 - 8*sqrt(b)*b**4*x**8)/(10
5*a**3*b**2*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**
4*x**8))
```

3.536 $\int \frac{x}{(a+bx^2)^{9/2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{(a+bx^2)^{9/2}} \, dx = -\frac{1}{7b\,(a+bx^2)^{7/2}}$$

output
```
-1/7/b/(b*x^2+a)^(7/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)^{9/2}} \, dx = -\frac{1}{7b\,(a+bx^2)^{7/2}}$$

input
```
Integrate[x/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*1/(b*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{7b\,(a+bx^2)^{7/2}}$$

input
```
Int[x/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*1/(b*(a + b*x^2)^(7/2))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15
derivativedivides	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15
default	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15
trager	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15
pseudoelliptic	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15
orering	$-\frac{1}{7b(b\,x^{2}+a)^{\frac{7}{2}}}$	15

input
```
int(x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7/b/(b*x^2+a)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(14) = 28.

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 3.17

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = -\frac{\sqrt{bx^2+a}}{7\,(b^5x^8+4\,ab^4x^6+6\,a^2b^3x^4+4\,a^3b^2x^2+a^4b)}$$

input
```
integrate(x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/7*sqrt(b*x^2 + a)/(b^5*x^8 + 4*a*b^4*x^6 + 6*a^2*b^3*x^4 + 4*a^3*b^2*x^
2 + a^4*b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(15) = 30$.

Time = 0.74 (sec) , antiderivative size = 90, normalized size of antiderivative = 5.00

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = \begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(9/2),x)
```

output
```
Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x*
*2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/7/((b*x^2 + a)^(7/2)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate(x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/7/((b*x^2 + a)^(7/2)*b)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{7\,b\,(b\,x^2+a)^{7/2}}$$

input
```
int(x/(a + b*x^2)^(9/2),x)
```

output
```
-1/(7*b*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.06

$$\int \frac{x}{(a+bx^2)^{9/2}}\,dx = -\frac{\sqrt{b\,x^2+a}}{7b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+4a^3b\,x^2+a^4)}$$

input
```
int(x/(b*x^2+a)^(9/2),x)
```

output
```
( - sqrt(a + b*x**2))/(7*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*
b**3*x**6 + b**4*x**8))
```

3.537 $\int \frac{1}{(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 77

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{x}{7a(a+bx^2)^{7/2}} + \frac{6x}{35a^2(a+bx^2)^{5/2}} + \frac{8x}{35a^3(a+bx^2)^{3/2}} + \frac{16x}{35a^4\sqrt{a+bx^2}}$$

output
```
1/7*x/a/(b*x^2+a)^(7/2)+6/35*x/a^2/(b*x^2+a)^(5/2)+8/35*x/a^3/(b*x^2+a)^(3
/2)+16/35*x/a^4/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{35a^3x + 70a^2bx^3 + 56ab^2x^5 + 16b^3x^7}{35a^4(a+bx^2)^{7/2}}$$

input
```
Integrate[(a + b*x^2)^(-9/2),x]
```

output
```
(35*a^3*x + 70*a^2*b*x^3 + 56*a*b^2*x^5 + 16*b^3*x^7)/(35*a^4*(a + b*x^2)^
(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{209}$$

$$\frac{6\int \frac{1}{(bx^2+a)^{7/2}}dx}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\int \frac{1}{(bx^2+a)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}$$

input
```
Int[(a + b*x^2)^(-9/2),x]
```

output
```
x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a
+ b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

method	result	size
gosper	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
trager	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
pseudoelliptic	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
orering	$\frac{x\left(16b^{3}x^{6}+56a\,b^{2}x^{4}+70a^{2}b\,x^{2}+35a^{3}\right)}{35\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$	48
default	$\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}$	74

input
```
int(1/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*x*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/(b*x^2+a)^(7/2)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{(16\,b^3x^7+56\,ab^2x^5+70\,a^2bx^3+35\,a^3x)\sqrt{bx^2+a}}{35\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/35*(16*b^3*x^7 + 56*a*b^2*x^5 + 70*a^2*b*x^3 + 35*a^3*x)*sqrt(b*x^2 + a)
/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1265 vs. 2(70) = 140.

Time = 1.18 (sec) , antiderivative size = 1265, normalized size of antiderivative = 16.43

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*x**2+a)**(9/2),x)
```

output
```
35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b
**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqr
t(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210
*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*
x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*
x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35
*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37
/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**
(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x
**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 42
9*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*
sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(
31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x*
*2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x*
*12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*s
qrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a*...
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{16\,x}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,x}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,x}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{x}{7\,(bx^2+a)^{\frac{7}{2}}a}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*x/(sqrt(b*x^2 + a)*a^4) + 8/35*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*x/((
b*x^2 + a)^(5/2)*a^2) + 1/7*x/((b*x^2 + a)^(7/2)*a)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(1/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^
2 + a)^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{16\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,x}{35\,a^3\,(b\,x^2+a)^{3/2}} + \frac{6\,x}{35\,a^2\,(b\,x^2+a)^{5/2}}+\frac{x}{7\,a\,(b\,x^2+a)^{7/2}}$$

input
```
int(1/(a + b*x^2)^(9/2),x)
```

output
```
(16*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*
x)/(35*a^2*(a + b*x^2)^(5/2)) + x/(7*a*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a+bx^2)^{9/2}}\,dx = \frac{35\sqrt{b\,x^2+a}\,a^3bx+70\sqrt{b\,x^2+a}\,a^2b^2x^3+56\sqrt{b\,x^2+a}\,a\,b^3x^5+16\sqrt{b\,x^2+a}\,b^4x^7-16}{35a^4b\,(b^4x^8+4a\,b^3x^6+6a^2b^2x^4+}$$

input
```
int(1/(b*x^2+a)^(9/2),x)
```

output
```
(35*sqrt(a + b*x**2)*a**3*b*x + 70*sqrt(a + b*x**2)*a**2*b**2*x**3 + 56*sq
rt(a + b*x**2)*a*b**3*x**5 + 16*sqrt(a + b*x**2)*b**4*x**7 - 16*sqrt(b)*a*
*4 - 64*sqrt(b)*a**3*b*x**2 - 96*sqrt(b)*a**2*b**2*x**4 - 64*sqrt(b)*a*b**
3*x**6 - 16*sqrt(b)*b**4*x**8)/(35*a**4*b*(a**4 + 4*a**3*b*x**2 + 6*a**2*b
**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.538 $\int \frac{1}{x(a+bx^2)^{9/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 95

$$\int \frac{1}{x(a+bx^2)^{9/2}} dx = \frac{1}{7a(a+bx^2)^{7/2}} + \frac{1}{5a^2(a+bx^2)^{5/2}} + \frac{1}{3a^3(a+bx^2)^{3/2}} + \frac{1}{a^4\sqrt{a+bx^2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
1/7/a/(b*x^2+a)^(7/2)+1/5/a^2/(b*x^2+a)^(5/2)+1/3/a^3/(b*x^2+a)^(3/2)+1/a^
4/(b*x^2+a)^(1/2)-arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

$$\int \frac{1}{x(a+bx^2)^{9/2}} dx = \frac{176a^3 + 406a^2bx^2 + 350ab^2x^4 + 105b^3x^6}{105a^4(a+bx^2)^{7/2}} - \frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(9/2)),x]
```

output
```
(176*a^3 + 406*a^2*b*x^2 + 350*a*b^2*x^4 + 105*b^3*x^6)/(105*a^4*(a + b*x^
2)^(7/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/a^(9/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 61, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2 (bx^2+a)^{9/2}}dx^2$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{x^2(bx^2+a)^{7/2}}dx^2}{a}+\frac{2}{7a (a+bx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{x^2\left(bx^2+a\right)^{5/2}}dx^2}{a}+\frac{2}{5a(a+bx^2)^{5/2}}}{a}+\frac{2}{7a (a+bx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\int \frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a(a+bx^2)^{5/2}}}{a}+\frac{2}{7a (a+bx^2)^{7/2}}\right)$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\frac{\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a(a+bx^2)^{5/2}}}{a}+\frac{2}{7a\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a(a+bx^2)^{5/2}}}{a}+\frac{2}{7a\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a(a+bx^2)^{5/2}}}{a}+\frac{2}{7a\left(a+bx^2\right)^{7/2}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(9/2)),x]
```

output
```
(2/(7*a*(a + b*x^2)^(7/2)) + (2/(5*a*(a + b*x^2)^(5/2)) + (2/(3*a*(a + b*x
^2)^(3/2)) + (2/(a*Sqrt[a + b*x^2]) - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(3/2))/a)/a)/a)/2
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.80

method	result	size
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)-\sqrt{a}\,b^{3} x^{6}-\frac{10 a^{\frac{3}{2}} b^{2} x^{4}}{3}-\frac{58 a^{\frac{5}{2}} b\,x^{2}}{15}-\frac{176 a^{\frac{7}{2}}}{105}}{a^{\frac{9}{2}} \left(b\,x^{2}+a\right)^{\frac{7}{2}}}$	76
default	$\frac{1}{7 a \left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{1}{5 a \left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3 a \left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a \sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2 a+2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}$	100

input
```
int(1/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-((b*x^2+a)^(7/2)*arctanh((b*x^2+a)^(1/2)/a^(1/2))-a^(1/2)*b^3*x^6-10/3*a^
(3/2)*b^2*x^4-58/15*a^(5/2)*b*x^2-176/105*a^(7/2))/a^(9/2)/(b*x^2+a)^(7/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(75) = 150$.

Time = 0.09 (sec) , antiderivative size = 332, normalized size of antiderivative = 3.49

$$\int \frac{1}{x\,(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(1}{210\,(a^5b^4x^8+4\,a^6b^3x^6+6\,a^7b^2x^4+4\,a}\right.$$

input
```
integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sq
rt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(105*a*b^3*x
^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*
x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4 + 4*a^8*b*x^2 + a^9), 1/105*(105*(b^4*
x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-a)*arctan(sqr
t(b*x^2 + a)*sqrt(-a)/a) + (105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^
2 + 176*a^4)*sqrt(b*x^2 + a))/(a^5*b^4*x^8 + 4*a^6*b^3*x^6 + 6*a^7*b^2*x^4
 + 4*a^8*b*x^2 + a^9)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5250 vs. $2(85) = 170$.

Time = 4.13 (sec) , antiderivative size = 5250, normalized size of antiderivative = 55.26

$$\int \frac{1}{x\,(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(b*x**2+a)**(9/2),x)
```

output
```
352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x*
*8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100
*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 +
 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*
b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100
*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1
+ b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b
**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a
**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x*
*14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2
)*b**10*x**20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 210
0*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6
+ 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)
*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 210
0*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*lo
g(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x
**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.77

$$\int \frac{1}{x\left(a+bx^2\right)^{9/2}}\,dx = -\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{1}{\sqrt{bx^2+a}a^4} + \frac{1}{3\left(bx^2+a\right)^{\frac{3}{2}}a^3} + \frac{1}{5\left(bx^2+a\right)^{\frac{5}{2}}a^2} + \frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

input
```
integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 1/(sqrt(b*x^2 + a)*a^4) + 1/3/((b
*x^2 + a)^(3/2)*a^3) + 1/5/((b*x^2 + a)^(5/2)*a^2) + 1/7/((b*x^2 + a)^(7/2
)*a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(a+bx^2)^{9/2}}\,dx = \frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\,(bx^2+a)^3+35\,(bx^2+a)^2a+21\,(bx^2+a)a^2+15\,a^3}{105\,(bx^2+a)^{\frac{7}{2}}a^4}$$

input
```
integrate(1/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + 1/105*(105*(b*x^2 + a)^3
 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2)*a^
4)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \frac{1}{x\,(a+bx^2)^{9/2}}\,dx = \frac{\frac{b\,x^2+a}{5\,a^2}+\frac{1}{7\,a}+\frac{(b\,x^2+a)^2}{3\,a^3}+\frac{(b\,x^2+a)^3}{a^4}}{(b\,x^2+a)^{7/2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
int(1/(x*(a + b*x^2)^(9/2)),x)
```

output
```
((a + b*x^2)/(5*a^2) + 1/(7*a) + (a + b*x^2)^2/(3*a^3) + (a + b*x^2)^3/a^4
)/(a + b*x^2)^(7/2) - atanh((a + b*x^2)^(1/2)/a^(1/2))/a^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 438, normalized size of antiderivative = 4.61

$$\int \frac{1}{x\,(a+bx^2)^{9/2}}\,dx = \frac{176\sqrt{b\,x^2+a}\,a^4+406\sqrt{b\,x^2+a}\,a^3b\,x^2+350\sqrt{b\,x^2+a}\,a^2b^2x^4+105\sqrt{b\,x^2+a}\,a\,b^3x}{}$$

input
```
int(1/x/(b*x^2+a)^(9/2),x)
```

output
```
(176*sqrt(a + b*x**2)*a**4 + 406*sqrt(a + b*x**2)*a**3*b*x**2 + 350*sqrt(a
 + b*x**2)*a**2*b**2*x**4 + 105*sqrt(a + b*x**2)*a*b**3*x**6 + 105*sqrt(a)
*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4 + 420*sqrt(a)*
log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b*x**2 + 630*sq
rt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**2*x**4
 + 420*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**
3*x**6 + 105*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))
*b**4*x**8 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt
(a))*a**4 - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**3*b*x**2 - 630*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)
/sqrt(a))*a**2*b**2*x**4 - 420*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + s
qrt(b)*x)/sqrt(a))*a*b**3*x**6 - 105*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(
a) + sqrt(b)*x)/sqrt(a))*b**4*x**8)/(105*a**5*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.539 $\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 106

$$\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx = \frac{1}{7ax(a+bx^2)^{7/2}}+\frac{8}{35a^2x(a+bx^2)^{5/2}}+\frac{16}{35a^3x(a+bx^2)^{3/2}}+\frac{64}{35a^4x\sqrt{a+bx^2}}-\frac{128\sqrt{a+bx^2}}{35a^5x}$$

output
```
1/7/a/x/(b*x^2+a)^(7/2)+8/35/a^2/x/(b*x^2+a)^(5/2)+16/35/a^3/x/(b*x^2+a)^(
3/2)+64/35/a^4/x/(b*x^2+a)^(1/2)-128/35*(b*x^2+a)^(1/2)/a^5/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.60

$$\int \frac{1}{x^2(a+bx^2)^{9/2}}\,dx = \frac{-35a^4-280a^3bx^2-560a^2b^2x^4-448ab^3x^6-128b^4x^8}{35a^5x(a+bx^2)^{7/2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-35*a^4 - 280*a^3*b*x^2 - 560*a^2*b^2*x^4 - 448*a*b^3*x^6 - 128*b^4*x^8)/
(35*a^5*x*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{8b \int \frac{1}{(bx^2+a)^{9/2}}dx}{a}-\frac{1}{ax (a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$-\frac{8b \left(\frac{6\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}\right)}{a}-\frac{1}{ax (a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$-\frac{8b \left(\frac{6\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}\right)}{a}-\frac{1}{ax (a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a(a+bx^2)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
-(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*(x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*
a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a
 + b*x^2])))/(5*a)))/(7*a)))/a
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{\frac{128}{35} b^{4} x^{8}+\frac{64}{5} a \,b^{3} x^{6}+16 a^{2} b^{2} x^{4}+8 a^{3} b \,x^{2}+a^{4}}{\left(b \,x^{2}+a \right)^{\frac{7}{2}} x \,a^{5}}$	59
gosper	$-\frac{128 b^{4} x^{8}+448 a \,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b \,x^{2}+35 a^{4}}{35 x \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{5}}$	61
trager	$-\frac{128 b^{4} x^{8}+448 a \,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b \,x^{2}+35 a^{4}}{35 x \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{5}}$	61
orering	$-\frac{128 b^{4} x^{8}+448 a \,b^{3} x^{6}+560 a^{2} b^{2} x^{4}+280 a^{3} b \,x^{2}+35 a^{4}}{35 x \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{5}}$	61
default	$-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{8 b \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)}{a}$	98
risch	$-\frac{\sqrt{b \,x^{2}+a}}{a^{5} x}-\frac{\sqrt{b \,x^{2}+a}\, x \left(93 b^{3} x^{6}+308 a \,b^{2} x^{4}+350 a^{2} b \,x^{2}+140 a^{3}\right) b}{35 \left(b^{4} x^{8}+4 a \,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b \,x^{2}+a^{4}\right) a^{5}}$	109

input
```
int(1/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-(128/35*b^4*x^8+64/5*a*b^3*x^6+16*a^2*b^2*x^4+8*a^3*b*x^2+a^4)/(b*x^2+a)^
(7/2)/x/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\, dx = -\frac{(128\, b^4x^8 + 448\, ab^3x^6 + 560\, a^2b^2x^4 + 280\, a^3bx^2 + 35\, a^4)\sqrt{bx^2+a}}{35\,(a^5b^4x^9 + 4\, a^6b^3x^7 + 6\, a^7b^2x^5 + 4\, a^8bx^3 + a^9x)}$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*
a^4)*sqrt(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*
b*x^3 + a^9*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(90) = 180.

Time = 1.59 (sec) , antiderivative size = 400, normalized size of antiderivative = 3.77

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\, dx = -\frac{35a^4b^{\frac{33}{2}}\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16} + 140a^8b^{17}x^2 + 210a^7b^{18}x^4 + 140a^6b^{19}x^6 + 35a^5b^{20}x^8} - \frac{280a^3b^{\frac{35}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16} + 140a^8b^{17}x^2 + 210a^7b^{18}x^4 + 140a^6b^{19}x^6 + 35a^5b^{20}x^8} - \frac{560a^2b^{\frac{37}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16} + 140a^8b^{17}x^2 + 210a^7b^{18}x^4 + 140a^6b^{19}x^6 + 35a^5b^{20}x^8} - \frac{448ab^{\frac{39}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16} + 140a^8b^{17}x^2 + 210a^7b^{18}x^4 + 140a^6b^{19}x^6 + 35a^5b^{20}x^8} - \frac{128b^{\frac{41}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{35a^9b^{16} + 140a^8b^{17}x^2 + 210a^7b^{18}x^4 + 140a^6b^{19}x^6 + 35a^5b^{20}x^8}$$

input
```
integrate(1/x**2/(b*x**2+a)**(9/2),x)
```

output
```
-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x*
*2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280
*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 5
60*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**1
7*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x*
*2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\,dx = -\frac{128\,bx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,bx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,bx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,bx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{1}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-128/35*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*b*x/((b*x^2 + a)^(3/2)*a^4) - 48
/35*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*b*x/((b*x^2 + a)^(7/2)*a^2) - 1/((b*
x^2 + a)^(7/2)*a*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)
*x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^
4)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}} dx = -\frac{\frac{1}{a^4} + \frac{128 b x^2}{35 a^5}}{x \sqrt{b x^2 + a}} - \frac{29 b x}{35 a^4 (b x^2 + a)^{3/2}} - \frac{13 b x}{35 a^3 (b x^2 + a)^{5/2}} - \frac{b x}{7 a^2 (b x^2 + a)^{7/2}}$$

input
```
int(1/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
- (1/a^4 + (128*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (29*b*x)/(35*a^4*
(a + b*x^2)^(3/2)) - (13*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (b*x)/(7*a^2*(a
 + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.75

$$\int \frac{1}{x^2 (a+bx^2)^{9/2}} dx = \frac{-35\sqrt{b x^2 + a}\, a^4 - 280\sqrt{b x^2 + a}\, a^3 b x^2 - 560\sqrt{b x^2 + a}\, a^2 b^2 x^4 - 448\sqrt{b x^2 + a}\, a b^3}{35 a^5 x (b^4 x^8 + 4}$$

input
```
int(1/x^2/(b*x^2+a)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**4 - 280*sqrt(a + b*x**2)*a**3*b*x**2 - 560*sqrt
(a + b*x**2)*a**2*b**2*x**4 - 448*sqrt(a + b*x**2)*a*b**3*x**6 - 128*sqrt(
a + b*x**2)*b**4*x**8 + 128*sqrt(b)*a**4*x + 512*sqrt(b)*a**3*b*x**3 + 768
*sqrt(b)*a**2*b**2*x**5 + 512*sqrt(b)*a*b**3*x**7 + 128*sqrt(b)*b**4*x**9)
/(35*a**5*x*(a**4 + 4*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**
4*x**8))
```

3.540 $\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx = -\frac{9b}{14a^2(a+bx^2)^{7/2}} - \frac{1}{2ax^2(a+bx^2)^{7/2}} - \frac{9b}{10a^3(a+bx^2)^{5/2}} - \frac{3b}{2a^4(a+bx^2)^{3/2}} - \frac{9b}{2a^5\sqrt{a+bx^2}} + \frac{9b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

output
```
-9/14*b/a^2/(b*x^2+a)^(7/2)-1/2/a/x^2/(b*x^2+a)^(7/2)-9/10*b/a^3/(b*x^2+a)
^(5/2)-3/2*b/a^4/(b*x^2+a)^(3/2)-9/2*b/a^5/(b*x^2+a)^(1/2)+9/2*b*arctanh((
b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3(a+bx^2)^{9/2}}\,dx = \frac{-35a^4-528a^3bx^2-1218a^2b^2x^4-1050ab^3x^6-315b^4x^8}{70a^5x^2(a+bx^2)^{7/2}} + \frac{9b\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
(-35*a^4 - 528*a^3*b*x^2 - 1218*a^2*b^2*x^4 - 1050*a*b^3*x^6 - 315*b^4*x^8
)/(70*a^5*x^2*(a + b*x^2)^(7/2)) + (9*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/
(2*a^(11/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 52, 61, 61, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a + bx^2)^{9/2}} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2 + a)^{9/2}} dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{9b \int \frac{1}{x^2 (bx^2+a)^{9/2}} dx^2}{2a} - \frac{1}{ax^2 (a + bx^2)^{7/2}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{9b \left(\frac{\int \frac{1}{x^2 \left(bx^2+a\right)^{7/2}} dx^2}{a} + \frac{2}{7a(a+bx^2)^{7/2}} \right)}{2a} - \frac{1}{ax^2 (a + bx^2)^{7/2}} \right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{9b\left(\frac{\frac{\int\frac{1}{x^2\left(bx^2+a\right)^{5/2}}dx^2}{a}+\frac{2}{5a\left(a+bx^2\right)^{5/2}}}{a}+\frac{2}{7a(a+bx^2)^{7/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(-\frac{9b\left(\frac{\frac{\frac{\int\frac{1}{x^2\left(bx^2+a\right)^{3/2}}dx^2}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a\left(a+bx^2\right)^{5/2}}}{a}+\frac{2}{7a(a+bx^2)^{7/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(-\frac{9b\left(\frac{\frac{\frac{\frac{\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2}{a}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a\left(a+bx^2\right)^{5/2}}}{a}+\frac{2}{7a(a+bx^2)^{7/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(-\frac{9b\left(\frac{\frac{\frac{\frac{2\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{2}{a\sqrt{a+bx^2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a\left(a+bx^2\right)^{5/2}}}{a}+\frac{2}{7a(a+bx^2)^{7/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{7/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-\frac{9b\left(\frac{\frac{\frac{\frac{2}{a\sqrt{a+bx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{2}{3a\left(a+bx^2\right)^{3/2}}}{a}+\frac{2}{5a\left(a+bx^2\right)^{5/2}}}{a}+\frac{2}{7a(a+bx^2)^{7/2}}\right)}{2a}-\frac{1}{ax^2\left(a+bx^2\right)^{7/2}}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
(-(1/(a*x^2*(a + b*x^2)^(7/2))) - (9*b*(2/(7*a*(a + b*x^2)^(7/2)) + (2/(5*
a*(a + b*x^2)^(5/2)) + (2/(3*a*(a + b*x^2)^(3/2)) + (2/(a*Sqrt[a + b*x^2])
 - (2*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/a)/a)/a))/(2*a))/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$\frac{\frac{9\,\mathrm{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right)\left(b x^2+a\right)^{\frac{7}{2}} b x^2}{2}-\frac{9\sqrt{a}\,b^4 x^8}{2}-15 a^{\frac{3}{2}} b^3 x^6-\frac{87 a^{\frac{5}{2}} b^2 x^4}{5}-\frac{264 a^{\frac{7}{2}} b x^2}{35}-\frac{a^{\frac{9}{2}}}{2}}{x^2 a^{\frac{11}{2}}\left(b x^2+a\right)^{\frac{7}{2}}}$
default	$-\frac{1}{2a x^2\left(b x^2+a\right)^{\frac{7}{2}}}-\frac{9b\left(\frac{1}{7a\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)}{2a}$
risch	$-\frac{\sqrt{b x^2+a}}{2a^5 x^2}+\frac{9b\ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{2a^{\frac{11}{2}}}-\frac{2629 b\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2 b+2\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}}{1120 a^5\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{2629 b\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2 b-2\sqrt{\cdots}}}{1120 a^5\sqrt{-ab}\left(x+\cdots\right.}$

input
```
int(1/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
9/2*(arctanh((b*x^2+a)^(1/2)/a^(1/2))*(b*x^2+a)^(7/2)*b*x^2-a^(1/2)*b^4*x^
8-10/3*a^(3/2)*b^3*x^6-58/15*a^(5/2)*b^2*x^4-176/105*a^(7/2)*b*x^2-1/9*a^(
9/2))/(b*x^2+a)^(7/2)/a^(11/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.98

$$\int \frac{1}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{315\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{140\left(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8\cdots\right.}\right.$$

$$-\frac{315\left(b^5x^{10}+4\,ab^4x^8+6\,a^2b^3x^6+4\,a^3b^2x^4+a^4bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^2+a}\sqrt{-a}}{a}\right)+\left(315\,ab^4x^8+1050\,a^2b^3x^{\cdots}\right.}{70\left(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2\right)}$$

input
```
integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/140*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a^3*b^2*x^4 + a^4*
b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(31
5*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5
)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b
*x^4 + a^10*x^2), -1/70*(315*(b^5*x^10 + 4*a*b^4*x^8 + 6*a^2*b^3*x^6 + 4*a
^3*b^2*x^4 + a^4*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^2 + a)*sqrt(-a)/a) + (315
*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2 + 35*a^5)
*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*
x^4 + a^10*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5540 vs. $2(119) = 238$.

Time = 7.61 (sec) , antiderivative size = 5540, normalized size of antiderivative = 43.97

$$\int \frac{1}{x^3 (a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/x**3/(b*x**2+a)**(9/2),x)
```

output
```
-70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4
 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2
)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**
9*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a
)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**
6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97
/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**
10*x**22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**
(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 +
29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*
b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400
*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(
sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 63
00*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**
4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*
a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**
20 + 140*a**(87/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^3 (a+bx^2)^{9/2}}\,dx = \frac{9\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{11}{2}}} - \frac{9\,b}{2\,\sqrt{bx^2+a}a^5} - \frac{3\,b}{2\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{9\,b}{10\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{9\,b}{14\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{1}{2\,(bx^2+a)^{\frac{7}{2}}ax^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
9/2*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(11/2) - 9/2*b/(sqrt(b*x^2 + a)*a^5)
 - 3/2*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*b/((b*x^2 + a)^(5/2)*a^3) - 9/14*b
/((b*x^2 + a)^(7/2)*a^2) - 1/2/((b*x^2 + a)^(7/2)*a*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^3 (a+bx^2)^{9/2}}\,dx = -\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a^5} - \frac{\sqrt{bx^2+a}}{2\,a^5x^2} - \frac{140\,(bx^2+a)^3b + 35\,(bx^2+a)^2ab + 14\,(bx^2+a)a^2b + 5\,a^3b}{35\,(bx^2+a)^{\frac{7}{2}}a^5}$$

input
```
integrate(1/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 +
a)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^
2 + a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3 (a+bx^2)^{9/2}}\,dx = \frac{9\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}} - \frac{\frac{b}{7\,a} + \frac{3\,b\,(b\,x^2+a)^2}{5\,a^3} + \frac{3\,b\,(b\,x^2+a)^3}{a^4} - \frac{9\,b\,(b\,x^2+a)^4}{2\,a^5} + \frac{9\,b\,(b\,x^2+a)}{35\,a^2}}{a\,(b\,x^2+a)^{7/2} - (b\,x^2+a)^{9/2}}$$

input
```
int(1/(x^3*(a + b*x^2)^(9/2)),x)
```

output
```
(9*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (b/(7*a) + (3*b*(a +
 b*x^2)^2)/(5*a^3) + (3*b*(a + b*x^2)^3)/a^4 - (9*b*(a + b*x^2)^4)/(2*a^5)
 + (9*b*(a + b*x^2))/(35*a^2))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.75

$$\int \frac{1}{x^3 (a+bx^2)^{9/2}}\,dx = \frac{-35\sqrt{b\,x^2+a}\,a^5-528\sqrt{b\,x^2+a}\,a^4b\,x^2-1218\sqrt{b\,x^2+a}\,a^3b^2x^4-1050\sqrt{b\,x^2+a}\,a}{}$$

input
```
int(1/x^3/(b*x^2+a)^(9/2),x)
```

output
```
( - 35*sqrt(a + b*x**2)*a**5 - 528*sqrt(a + b*x**2)*a**4*b*x**2 - 1218*sqr
t(a + b*x**2)*a**3*b**2*x**4 - 1050*sqrt(a + b*x**2)*a**2*b**3*x**6 - 315*
sqrt(a + b*x**2)*a*b**4*x**8 - 315*sqrt(a)*log((sqrt(a + b*x**2) - sqrt(a)
 + sqrt(b)*x)/sqrt(a))*a**4*b*x**2 - 1260*sqrt(a)*log((sqrt(a + b*x**2) -
sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b**2*x**4 - 1890*sqrt(a)*log((sqrt(a +
b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a**2*b**3*x**6 - 1260*sqrt(a)*log(
(sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*a*b**4*x**8 - 315*sqrt(a
)*log((sqrt(a + b*x**2) - sqrt(a) + sqrt(b)*x)/sqrt(a))*b**5*x**10 + 315*s
qrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**4*b*x**2 +
 1260*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(a))*a**3*b
**2*x**4 + 1890*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b)*x)/sqrt(
a))*a**2*b**3*x**6 + 1260*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) + sqrt(b
)*x)/sqrt(a))*a*b**4*x**8 + 315*sqrt(a)*log((sqrt(a + b*x**2) + sqrt(a) +
sqrt(b)*x)/sqrt(a))*b**5*x**10)/(70*a**6*x**2*(a**4 + 4*a**3*b*x**2 + 6*a*
*2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.541 $\int \frac{1}{x^4(a+bx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 128

$$\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{1}{7ax^3\left(a+bx^2\right)^{7/2}}+\frac{2}{7a^2x^3\left(a+bx^2\right)^{5/2}}+\frac{16}{21a^3x^3\left(a+bx^2\right)^{3/2}}+\frac{32}{7a^4x^3\sqrt{a+bx^2}}-\frac{128\sqrt{a+bx^2}}{21a^5x^3}+\frac{256b\sqrt{a+bx^2}}{21a^6x}$$

output
```
1/7/a/x^3/(b*x^2+a)^(7/2)+2/7/a^2/x^3/(b*x^2+a)^(5/2)+16/21/a^3/x^3/(b*x^2
+a)^(3/2)+32/7/a^4/x^3/(b*x^2+a)^(1/2)-128/21*(b*x^2+a)^(1/2)/a^5/x^3+256/
21*b*(b*x^2+a)^(1/2)/a^6/x
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10}}{21a^6x^3\left(a+bx^2\right)^{7/2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^3*x^6 + 896*a*b^4*x^
8 + 256*b^5*x^10)/(21*a^6*x^3*(a + b*x^2)^(7/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {245, 245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \ 245$$

$$-\frac{10b \int \frac{1}{x^2 (bx^2+a)^{9/2}}dx}{3a}-\frac{1}{3ax^3 (a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 245$$

$$-\frac{10b \left(-\frac{8b \int \frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{1}{3ax^3 (a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{10b \left(-\frac{8b \left(\frac{6 \int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{1}{3ax^3 (a+bx^2)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 209}$$

$$-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 208}$$

$$-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax(a+bx^2)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
-1/3*1/(a*x^3*(a + b*x^2)^(7/2)) - (10*b*(-(1/(a*x*(a + b*x^2)^(7/2))) - (
8*b*(x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*
a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)))/a))
/(3*a)
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{-\frac{256}{7} b^{5} x^{10}-128 a \,b^{4} x^{8}-160 a^{2} b^{3} x^{6}-80 a^{3} b^{2} x^{4}-10 a^{4} b \,x^{2}+a^{5}}{3 \left(b \,x^{2}+a \right)^{\frac{7}{2}} x^{3} a^{6}}$
gosper	$-\frac{-256 b^{5} x^{10}-896 a \,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b \,x^{2}+7 a^{5}}{21 x^{3} \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{6}}$
trager	$-\frac{-256 b^{5} x^{10}-896 a \,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b \,x^{2}+7 a^{5}}{21 x^{3} \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{6}}$
orering	$-\frac{-256 b^{5} x^{10}-896 a \,b^{4} x^{8}-1120 a^{2} b^{3} x^{6}-560 a^{3} b^{2} x^{4}-70 a^{4} b \,x^{2}+7 a^{5}}{21 x^{3} \left(b \,x^{2}+a \right)^{\frac{7}{2}} a^{6}}$
risch	$-\frac{\sqrt{b \,x^{2}+a}\, \left(-14 b \,x^{2}+a \right)}{3 a^{6} x^{3}}+\frac{\sqrt{b \,x^{2}+a}\, x \left(158 b^{3} x^{6}+511 a \,b^{2} x^{4}+560 a^{2} b \,x^{2}+210 a^{3}\right) b^{2}}{21 a^{6} \left(b^{4} x^{8}+4 a \,b^{3} x^{6}+6 a^{2} b^{2} x^{4}+4 a^{3} b \,x^{2}+a^{4}\right)}$
default	$-\frac{1}{3 a \,x^{3} \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{10 b \left(-\frac{1}{a x \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{8 b \left(\frac{x}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{8 x}{15 a^{2} \sqrt{b \,x^{2}+a}}\right)}{7 a}}{a}\right)}{a}\right)}{3 a}$

input
```
int(1/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x^2+a)^(7/2)*(-256/7*b^5*x^10-128*a*b^4*x^8-160*a^2*b^3*x^6-80*a^3
*b^2*x^4-10*a^4*b*x^2+a^5)/x^3/a^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = \frac{(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5)\sqrt{bx^2+a}}{21\,(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3)}$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 +
70*a^4*b*x^2 - 7*a^5)*sqrt(b*x^2 + a)/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^
8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(119) = 238.

Time = 1.93 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.22

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = -\frac{7a^6b^{\frac{51}{2}}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{63a^5b^{\frac{53}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{630a^4b^{\frac{55}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{1680a^3b^{\frac{57}{2}}x^6\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{2016a^2b^{\frac{59}{2}}x^8\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{1152ab^{\frac{61}{2}}x^{10}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}} + \frac{256b^{\frac{63}{2}}x^{12}\sqrt{\frac{a}{bx^2}+1}}{21a^{11}b^{25}x^2+105a^{10}b^{26}x^4+210a^9b^{27}x^6+210a^8b^{28}x^8+105a^7b^{29}x^{10}+21a^6b^{30}x^{12}}$$

input
```
integrate(1/x**4/(b*x**2+a)**(9/2),x)
```

output
```
-7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b*
*26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**1
0 + 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21
*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*
b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55
/2)*x**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4
+ 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a*
*6*b**30*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*
b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x
**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x*
*8*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*
a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**
30*x**12) + 1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x*
*2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 10
5*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*
x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x*
*6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = \frac{256\,b^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,b^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,b^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,b^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} + \frac{10\,b}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{1}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
256/21*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*b^2*x/((b*x^2 + a)^(3/2)*a^5)
+ 32/7*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*b^2*x/((b*x^2 + a)^(7/2)*a^3)
 + 10/3*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3/((b*x^2 + a)^(7/2)*a*x^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\,(bx^2+a)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4 b^{\frac{3}{2}} - 15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 ab^{\frac{3}{2}} + 7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3 a^5}$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^
3)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) -
15*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = \frac{\frac{128\,b}{21\,a^5}+\frac{256\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}} - \frac{\frac{1}{3\,a^2}+\frac{19\,b\,x^2}{21\,a^3}}{x^3\,(b\,x^2+a)^{5/2}} - \frac{32\,b}{21\,a^4\,x\,(b\,x^2+a)^{3/2}} + \frac{b^2\,x}{7\,a^3\,(b\,x^2+a)^{7/2}}$$

input
```
int(1/(x^4*(a + b*x^2)^(9/2)),x)
```

output
```
((128*b)/(21*a^5) + (256*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (1/(3*
a^2) + (19*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - (32*b)/(21*a^4*x*(a
+ b*x^2)^(3/2)) + (b^2*x)/(7*a^3*(a + b*x^2)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.64

$$\int \frac{1}{x^4 (a+bx^2)^{9/2}}\,dx = \frac{-7\sqrt{b\,x^2+a}\,a^5+70\sqrt{b\,x^2+a}\,a^4b\,x^2+560\sqrt{b\,x^2+a}\,a^3b^2x^4+1120\sqrt{b\,x^2+a}\,a^2b^3}{2}$$

input
```
int(1/x^4/(b*x^2+a)^(9/2),x)
```

output
```
( - 7*sqrt(a + b*x**2)*a**5 + 70*sqrt(a + b*x**2)*a**4*b*x**2 + 560*sqrt(a
 + b*x**2)*a**3*b**2*x**4 + 1120*sqrt(a + b*x**2)*a**2*b**3*x**6 + 896*sqr
t(a + b*x**2)*a*b**4*x**8 + 256*sqrt(a + b*x**2)*b**5*x**10 - 256*sqrt(b)*
a**4*b*x**3 - 1024*sqrt(b)*a**3*b**2*x**5 - 1536*sqrt(b)*a**2*b**3*x**7 -
1024*sqrt(b)*a*b**4*x**9 - 256*sqrt(b)*b**5*x**11)/(21*a**6*x**3*(a**4 + 4
*a**3*b*x**2 + 6*a**2*b**2*x**4 + 4*a*b**3*x**6 + b**4*x**8))
```

3.542 $\int \frac{x^5}{\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{81}{64}\sqrt{9+4x^2} - \frac{3}{32}\left(9+4x^2\right)^{3/2} + \frac{1}{320}\left(9+4x^2\right)^{5/2}$$

output
```
81/64*(4*x^2+9)^(1/2)-3/32*(4*x^2+9)^(3/2)+1/320*(4*x^2+9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{1}{40}\sqrt{9+4x^2}\left(27-6x^2+2x^4\right)$$

input
```
Integrate[x^5/Sqrt[9 + 4*x^2],x]
```

output
```
(Sqrt[9 + 4*x^2]*(27 - 6*x^2 + 2*x^4))/40
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{4x^2+9}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{4x^2+9}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{1}{16}\left(4x^2+9\right)^{3/2}-\frac{9}{8}\sqrt{4x^2+9}+\frac{81}{16\sqrt{4x^2+9}}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{1}{160}\left(4x^2+9\right)^{5/2}-\frac{3}{16}\left(4x^2+9\right)^{3/2}+\frac{81}{32}\sqrt{4x^2+9}\right)$$

input
```
Int[x^5/Sqrt[9 + 4*x^2],x]
```

output
```
((81*Sqrt[9 + 4*x^2])/32 - (3*(9 + 4*x^2)^(3/2))/16 + (9 + 4*x^2)^(5/2)/16
0)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\sqrt{4x^2+9}\left(\frac{1}{20}x^4-\frac{3}{20}x^2+\frac{27}{40}\right)$	23
gosper	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
risch	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
pseudoelliptic	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
orering	$\frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$	24
meijerg	$\frac{-\frac{81\sqrt{\pi}}{40}+\frac{81\sqrt{\pi}\left(\frac{32}{27}x^4-\frac{32}{9}x^2+16\right)\sqrt{\frac{4x^2}{9}+1}}{640}}{\sqrt{\pi}}$	38
default	$\frac{x^4\sqrt{4x^2+9}}{20}-\frac{3x^2\sqrt{4x^2+9}}{20}+\frac{27\sqrt{4x^2+9}}{40}$	41

input
```
int(x^5/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4*x^2+9)^(1/2)*(1/20*x^4-3/20*x^2+27/40)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{1}{40}\left(2\,x^4-6\,x^2+27\right)\sqrt{4\,x^2+9}$$

input
```
integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(2*x^4 - 6*x^2 + 27)*sqrt(4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{x^4\sqrt{4x^2+9}}{20} - \frac{3x^2\sqrt{4x^2+9}}{20} + \frac{27\sqrt{4x^2+9}}{40}$$

input
```
integrate(x**5/(4*x**2+9)**(1/2),x)
```

output
```
x**4*sqrt(4*x**2 + 9)/20 - 3*x**2*sqrt(4*x**2 + 9)/20 + 27*sqrt(4*x**2 + 9
)/40
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{1}{20}\,\sqrt{4\,x^2+9}x^4 - \frac{3}{20}\,\sqrt{4\,x^2+9}x^2 + \frac{27}{40}\,\sqrt{4\,x^2+9}$$

input
```
integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(4*x^2 + 9)*x^4 - 3/20*sqrt(4*x^2 + 9)*x^2 + 27/40*sqrt(4*x^2 + 9
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{1}{320}\left(4\,x^2+9\right)^{\frac{5}{2}} - \frac{3}{32}\left(4\,x^2+9\right)^{\frac{3}{2}} + \frac{81}{64}\sqrt{4\,x^2+9}$$

input
```
integrate(x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/320*(4*x^2 + 9)^(5/2) - 3/32*(4*x^2 + 9)^(3/2) + 81/64*sqrt(4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.46

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{x^4}{5}-\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

input
```
int(x^5/(4*x^2 + 9)^(1/2),x)
```

output
```
((x^2 + 9/4)^(1/2)*(x^4/5 - (3*x^2)/5 + 27/10))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}\,(2x^4-6x^2+27)}{40}$$

input
```
int(x^5/(4*x^2+9)^(1/2),x)
```

output
```
(sqrt(4*x**2 + 9)*(2*x**4 - 6*x**2 + 27))/40
```

3.543 $\int \frac{x^4}{\sqrt{9+4x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^4}{\sqrt{9+4x^2}} \, dx = -\frac{27}{128} x \sqrt{9+4x^2} + \frac{1}{16} x^3 \sqrt{9+4x^2} + \frac{243}{256} \operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
-27/128*x*(4*x^2+9)^(1/2)+1/16*x^3*(4*x^2+9)^(1/2)+243/256*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\sqrt{9+4x^2}} \, dx = \frac{1}{128} x \sqrt{9+4x^2} \left(-27+8x^2\right) - \frac{243}{256} \log \left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[x^4/Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2]*(-27 + 8*x^2))/128 - (243*Log[-2*x + Sqrt[9 + 4*x^2]])/
256
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\int \frac{x^2}{\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{8}\int \frac{1}{\sqrt{4x^2+9}}dx\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{16}x^3\sqrt{4x^2+9}-\frac{27}{16}\left(\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\text{arcsinh}\left(\frac{2x}{3}\right)\right)$$

input
```
Int[x^4/Sqrt[9 + 4*x^2],x]
```

output
```
(x^3*Sqrt[9 + 4*x^2])/16 - (27*((x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3
])/16))/16
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

method	result	size
risch	$\frac{x(8x^2-27)\sqrt{4x^2+9}}{128}+\frac{243\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{256}$	27
default	$-\frac{27x\sqrt{4x^2+9}}{128}+\frac{x^3\sqrt{4x^2+9}}{16}+\frac{243\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{256}$	34
meijerg	$\frac{-\frac{27\sqrt{\pi}\,x\left(-\frac{40x^2}{9}+15\right)\sqrt{\frac{4x^2}{9}+1}}{640}+\frac{243\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{256}}{\sqrt{\pi}}$	38
trager	$\frac{x(8x^2-27)\sqrt{4x^2+9}}{128}-\frac{243\ln\left(-\sqrt{4x^2+9}+2x\right)}{256}$	39
pseudoelliptic	$\frac{19683\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)-19683\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)+(2592x^3-8748x)\sqrt{4x^2+9}}{512\left(2x+\sqrt{4x^2+9}\right)^2\left(-\sqrt{4x^2+9}+2x\right)^2}$	95

input
```
int(x^4/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*x*(8*x^2-27)*(4*x^2+9)^(1/2)+243/256*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{1}{128}\left(8\,x^3-27\,x\right)\sqrt{4\,x^2+9}-\frac{243}{256}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*x^3 - 27*x)*sqrt(4*x^2 + 9) - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{x^3\sqrt{4x^2+9}}{16} - \frac{27x\sqrt{4x^2+9}}{128} + \frac{243\,\mathrm{asinh}\left(\frac{2x}{3}\right)}{256}$$

input
```
integrate(x**4/(4*x**2+9)**(1/2),x)
```

output
```
x**3*sqrt(4*x**2 + 9)/16 - 27*x*sqrt(4*x**2 + 9)/128 + 243*asinh(2*x/3)/25
6
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{1}{16}\sqrt{4\,x^2+9}x^3 - \frac{27}{128}\sqrt{4\,x^2+9}x + \frac{243}{256}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(4*x^2 + 9)*x^3 - 27/128*sqrt(4*x^2 + 9)*x + 243/256*arcsinh(2/3*
x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{1}{128}\left(8\,x^2-27\right)\sqrt{4\,x^2+9}x - \frac{243}{256}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/128*(8*x^2 - 27)*sqrt(4*x^2 + 9)*x - 243/256*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.56

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{243\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{256} - \frac{\sqrt{x^2+\frac{9}{4}}\left(\frac{27\,x}{32}-\frac{x^3}{4}\right)}{2}$$

input
```
int(x^4/(4*x^2 + 9)^(1/2),x)
```

output
```
(243*asinh((2*x)/3))/256 - ((x^2 + 9/4)^(1/2)*((27*x)/32 - x^3/4))/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int \frac{x^4}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}\,x^3}{16} - \frac{27\sqrt{4x^2+9}\,x}{128} + \frac{243\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{256}$$

input
```
int(x^4/(4*x^2+9)^(1/2),x)
```

output
```
(16*sqrt(4*x**2 + 9)*x**3 - 54*sqrt(4*x**2 + 9)*x + 243*log((sqrt(4*x**2 +
 9) + 2*x)/3))/256
```

3.544 $\int \frac{x^3}{\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = -\frac{9}{16}\sqrt{9+4x^2}+\frac{1}{48}\left(9+4x^2\right)^{3/2}$$

output
```
-9/16*(4*x^2+9)^(1/2)+1/48*(4*x^2+9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{1}{24}\left(-9+2x^2\right)\sqrt{9+4x^2}$$

input
```
Integrate[x^3/Sqrt[9 + 4*x^2],x]
```

output
```
((-9 + 2*x^2)*Sqrt[9 + 4*x^2])/24
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{4x^2+9}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{4}\sqrt{4x^2+9}-\frac{9}{4\sqrt{4x^2+9}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{24}\left(4x^2+9\right)^{3/2}-\frac{9}{8}\sqrt{4x^2+9}\right)$$

input
```
Int[x^3/Sqrt[9 + 4*x^2],x]
```

output
```
((-9*Sqrt[9 + 4*x^2])/8 + (9 + 4*x^2)^(3/2)/24)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
trager	$\sqrt{4x^2+9}\left(\frac{x^2}{12}-\frac{3}{8}\right)$	18
gosper	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
risch	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
pseudoelliptic	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
orering	$\frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$	19
default	$\frac{x^2\sqrt{4x^2+9}}{12}-\frac{3\sqrt{4x^2+9}}{8}$	27
meijerg	$\frac{\frac{9\sqrt{\pi}}{8}-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{\frac{4x^2}{9}+1}}{64}}{\sqrt{\pi}}$	33

input
```
int(x^3/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(4*x^2+9)^(1/2)*(1/12*x^2-3/8)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{1}{24}\,\sqrt{4\,x^2+9}\big(2\,x^2-9\big)$$

input
```
integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(4*x^2 + 9)*(2*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{x^2\sqrt{4x^2+9}}{12} - \frac{3\sqrt{4x^2+9}}{8}$$

input
```
integrate(x**3/(4*x**2+9)**(1/2),x)
```

output
```
x**2*sqrt(4*x**2 + 9)/12 - 3*sqrt(4*x**2 + 9)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{1}{12}\,\sqrt{4\,x^2+9}x^2 - \frac{3}{8}\,\sqrt{4\,x^2+9}$$

input
```
integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(4*x^2 + 9)*x^2 - 3/8*sqrt(4*x^2 + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{1}{48}\left(4\,x^2+9\right)^{\frac{3}{2}} - \frac{9}{16}\sqrt{4\,x^2+9}$$

input
```
integrate(x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/48*(4*x^2 + 9)^(3/2) - 9/16*sqrt(4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.48

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \sqrt{x^2+\frac{9}{4}}\left(\frac{x^2}{6}-\frac{3}{4}\right)$$

input
```
int(x^3/(4*x^2 + 9)^(1/2),x)
```

output
```
(x^2 + 9/4)^(1/2)*(x^2/6 - 3/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}\,(2x^2-9)}{24}$$

input
```
int(x^3/(4*x^2+9)^(1/2),x)
```

output
```
(sqrt(4*x**2 + 9)*(2*x**2 - 9))/24
```

3.545 $\int \frac{x^2}{\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx=\frac{1}{8}x\sqrt{9+4x^2}-\frac{9}{16}\operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/8*x*(4*x^2+9)^(1/2)-9/16*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx=\frac{1}{8}x\sqrt{9+4x^2}+\frac{9}{16}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[x^2/Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/8 + (9*Log[-2*x + Sqrt[9 + 4*x^2]])/16
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {262, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4x^2+9}}dx$$
$$\Big\downarrow \text{262}$$
$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{8}\int \frac{1}{\sqrt{4x^2+9}}dx$$
$$\Big\downarrow \text{222}$$
$$\frac{1}{8}x\sqrt{4x^2+9}-\frac{9}{16}\text{arcsinh}\left(\frac{2x}{3}\right)$$

input
```
Int[x^2/Sqrt[9 + 4*x^2],x]
```

output
```
(x*Sqrt[9 + 4*x^2])/8 - (9*ArcSinh[(2*x)/3])/16
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{16}$	20
risch	$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{16}$	20
trager	$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\ln\left(2x+\sqrt{4x^2+9}\right)}{16}$	30
meijerg	$\frac{\frac{3\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{8}-\frac{9\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{2x}{3}\right)}{16}}{\sqrt{\pi}}$	31
pseudoelliptic	$\frac{x\sqrt{4x^2+9}}{8}-\frac{9\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)}{32}+\frac{9\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{32}$	54

input
```
int(x^2/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(4*x^2+9)^(1/2)-9/16*arcsinh(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx=\frac{1}{8}\sqrt{4\,x^2+9}x+\frac{9}{16}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \frac{x\sqrt{4x^2+9}}{8} - \frac{9\operatorname{asinh}\left(\frac{2x}{3}\right)}{16}$$

input
```
integrate(x**2/(4*x**2+9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 + 9)/8 - 9*asinh(2*x/3)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \frac{1}{8}\sqrt{4x^2+9}x - \frac{9}{16}\operatorname{arsinh}\left(\frac{2}{3}x\right)$$

input
```
integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(4*x^2 + 9)*x - 9/16*arcsinh(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \frac{1}{8}\sqrt{4x^2+9}x + \frac{9}{16}\log\left(-2x+\sqrt{4x^2+9}\right)$$

input
```
integrate(x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(4*x^2 + 9)*x + 9/16*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \frac{x\,\sqrt{x^2+\frac{9}{4}}}{4} - \frac{9\,\mathrm{asinh}\left(\frac{2\,x}{3}\right)}{16}$$

input
```
int(x^2/(4*x^2 + 9)^(1/2),x)
```

output
```
(x*(x^2 + 9/4)^(1/2))/4 - (9*asinh((2*x)/3))/16
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int \frac{x^2}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}\,x}{8} - \frac{9\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{16}$$

input
```
int(x^2/(4*x^2+9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 + 9)*x - 9*log((sqrt(4*x**2 + 9) + 2*x)/3))/16
```

3.546 $\int \frac{x}{\sqrt{9+4x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{\sqrt{9+4x^2}} \, dx = \frac{1}{4}\sqrt{9+4x^2}$$

output
```
1/4*(4*x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{9+4x^2}} \, dx = \frac{1}{4}\sqrt{9+4x^2}$$

input
```
Integrate[x/Sqrt[9 + 4*x^2],x]
```

output
```
Sqrt[9 + 4*x^2]/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x^2+9}} \, dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{1}{4}\sqrt{4x^2+9}$$

input
```
Int[x/Sqrt[9 + 4*x^2],x]
```

output
```
Sqrt[9 + 4*x^2]/4
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$\frac{\sqrt{4x^2+9}}{4}$	12
derivativedivides	$\frac{\sqrt{4x^2+9}}{4}$	12
default	$\frac{\sqrt{4x^2+9}}{4}$	12
trager	$\frac{\sqrt{4x^2+9}}{4}$	12
risch	$\frac{\sqrt{4x^2+9}}{4}$	12
pseudoelliptic	$\frac{\sqrt{4x^2+9}}{4}$	12
orering	$\frac{\sqrt{4x^2+9}}{4}$	12
meijerg	$\frac{-\frac{3\sqrt{\pi}}{4}+\frac{3\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}}{4}}{\sqrt{\pi}}$	26

input
```
int(x/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\,\sqrt{4\,x^2+9}$$

input
```
integrate(x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}}{4}$$

input
```
integrate(x/(4*x**2+9)**(1/2),x)
```

output
```
sqrt(4*x**2 + 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\,\sqrt{4\,x^2+9}$$

input
```
integrate(x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(4*x^2 + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{1}{4}\,\sqrt{4\,x^2+9}$$

input
```
integrate(x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{x^2+\frac{9}{4}}}{2}$$

input
```
int(x/(4*x^2 + 9)^(1/2),x)
```

output
```
(x^2 + 9/4)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4x^2+9}}{4}$$

input
```
int(x/(4*x^2+9)^(1/2),x)
```

output
```
sqrt(4*x**2 + 9)/4
```

3.547 $\int \frac{1}{\sqrt{9+4x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9+4x^2}} \, dx = \frac{1}{2}\operatorname{arcsinh}\left(\frac{2x}{3}\right)$$

output
```
1/2*arcsinh(2/3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{9+4x^2}} \, dx = -\frac{1}{2}\log\left(-2x+\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/Sqrt[9 + 4*x^2],x]
```

output
```
-1/2*Log[-2*x + Sqrt[9 + 4*x^2]]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2+9}}\,dx$$
$$\Big\downarrow \text{222}$$
$$\frac{1}{2}\text{arcsinh}\left(\frac{2x}{3}\right)$$

input
```
Int[1/Sqrt[9 + 4*x^2],x]
```

output
```
ArcSinh[(2*x)/3]/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	7
trager	$\frac{\ln\left(2x+\sqrt{4x^2+9}\right)}{2}$	17
pseudoelliptic	$\frac{\ln\left(\frac{2x+\sqrt{4x^2+9}}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{4x^2+9}-2x}{x}\right)}{4}$	42

input
```
int(1/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx=-\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

input
```
integrate(1/(4*x**2+9)**(1/2),x)
```

output
```
asinh(2*x/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsinh(2/3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.90

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{1}{2}\,\sqrt{4\,x^2+9}x - \frac{9}{4}\,\log\left(-2\,x+\sqrt{4\,x^2+9}\right)$$

input
```
integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 + 9)*x - 9/4*log(-2*x + sqrt(4*x^2 + 9))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{2\,x}{3}\right)}{2}$$

input
```
int(1/(4*x^2 + 9)^(1/2),x)
```

output
```
asinh((2*x)/3)/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{1}{\sqrt{9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right)}{2}$$

input
```
int(1/(4*x^2+9)^(1/2),x)
```

output
```
log((sqrt(4*x**2 + 9) + 2*x)/3)/2
```

3.548 $\int \frac{1}{x\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
-1/3*arctanh(1/3*(4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/(x*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{4}\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)$$

input
```
Int[1/(x*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/3*ArcTanh[Sqrt[9 + 4*x^2]/3]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{4x^{2}+9}}\right)}{3}$	15
trager	$-\frac{\ln\left(\frac{\sqrt{4x^{2}+9}+3}{x}\right)}{3}$	19
pseudoelliptic	$-\frac{\ln\left(\sqrt{4x^{2}+9}+3\right)}{6}+\frac{\ln\left(\sqrt{4x^{2}+9}-3\right)}{6}$	30
meijerg	$\frac{\left(2\ln\left(x\right)-2\ln\left(3\right)\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^{2}}{9}+1}}{2}\right)}{6\sqrt{\pi}}$	39

input
```
int(1/x/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctanh(3/(4*x^2+9)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)+\frac{1}{3}\,\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/3*log(-2*x + sqrt(4*x^2 + 9) + 3) + 1/3*log(-2*x + sqrt(4*x^2 + 9) - 3)
```

Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{\operatorname{asinh}\left(\frac{3}{2x}\right)}{3}$$

input
```
integrate(1/x/(4*x**2+9)**(1/2),x)
```

output
```
-asinh(3/(2*x))/3
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{1}{3}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*arcsinh(3/2/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{1}{6}\log\left(\sqrt{4\,x^2+9}+3\right)+\frac{1}{6}\log\left(\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate(1/x/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/6*log(sqrt(4*x^2 + 9) + 3) + 1/6*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = -\frac{\operatorname{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{3}$$

input
```
int(1/(x*(4*x^2 + 9)^(1/2)),x)
```

output
```
-atanh((2*(x^2 + 9/4)^(1/2))/3)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)}{3}-\frac{\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)}{3}$$

input
```
int(1/x/(4*x^2+9)^(1/2),x)
```

output
```
(log((sqrt(4*x**2 + 9) + 2*x - 3)/3) - log((sqrt(4*x**2 + 9) + 2*x + 3)/3)
)/3
```

3.549 $\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

output
```
-1/9*(4*x^2+9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{9x}$$

input
```
Integrate[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/9*Sqrt[9 + 4*x^2]/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{4x^2+9}}\,dx$$

$$\Big\downarrow \; 242$$

$$-\frac{\sqrt{4x^2+9}}{9x}$$

input
```
Int[1/(x^2*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/9*Sqrt[9 + 4*x^2]/x
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{\sqrt{4x^2+9}}{9x}$	15
default	$-\frac{\sqrt{4x^2+9}}{9x}$	15
trager	$-\frac{\sqrt{4x^2+9}}{9x}$	15
meijerg	$-\frac{\sqrt{\frac{4x^2}{9}+1}}{3x}$	15
risch	$-\frac{\sqrt{4x^2+9}}{9x}$	15
pseudoelliptic	$-\frac{\sqrt{4x^2+9}}{9x}$	15
orering	$-\frac{\sqrt{4x^2+9}}{9x}$	15

input
```
int(1/x^2/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(4*x^2+9)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{2\,x+\sqrt{4\,x^2+9}}{9\,x}$$

input
```
integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*(2*x + sqrt(4*x^2 + 9))/x
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{2\sqrt{1+\frac{9}{4x^2}}}{9}$$

input
```
integrate(1/x**2/(4*x**2+9)**(1/2),x)
```

output
```
-2*sqrt(1 + 9/(4*x**2))/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{4\,x^2+9}}{9\,x}$$

input
```
integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*sqrt(4*x^2 + 9)/x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = \frac{4}{\left(2\,x-\sqrt{4\,x^2+9}\right)^2-9}$$

input
```
integrate(1/x^2/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
4/((2*x - sqrt(4*x^2 + 9))^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = -\frac{2\sqrt{x^2+\frac{9}{4}}}{9\,x}$$

input
```
int(1/(x^2*(4*x^2 + 9)^(1/2)),x)
```

output
```
-(2*(x^2 + 9/4)^(1/2))/(9*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{9+4x^2}}\,dx = \frac{-\sqrt{4x^2+9}-2x}{9x}$$

input
```
int(1/x^2/(4*x^2+9)^(1/2),x)
```

output
```
( - sqrt(4*x**2 + 9) - 2*x)/(9*x)
```

3.550 $\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
-1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(1/3*(4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{18x^2}+\frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/18*Sqrt[9 + 4*x^2]/x^2 + (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/27
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{4x^2+9}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2-\frac{\sqrt{4x^2+9}}{9x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}-\frac{\sqrt{4x^2+9}}{9x^2}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)-\frac{\sqrt{4x^2+9}}{9x^2}\right)$$

input
```
Int[1/(x^3*Sqrt[9 + 4*x^2]),x]
```

output
```
(-1/9*Sqrt[9 + 4*x^2]/x^2 + (4*ArcTanh[Sqrt[9 + 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$	30
risch	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{27}$	30
trager	$-\frac{\sqrt{4x^2+9}}{18x^2}+\frac{2\ln\left(\frac{\sqrt{4x^2+9}+3}{x}\right)}{27}$	34
pseudoelliptic	$\frac{2\ln\left(\sqrt{4x^2+9}+3\right)x^2-2\ln\left(\sqrt{4x^2+9}-3\right)x^2-3\sqrt{4x^2+9}}{54x^2}$	52
meijerg	$\frac{-\frac{\sqrt{\pi}}{6x^2}-\frac{(1+2\ln(x)-2\ln(3))\sqrt{\pi}}{27}+\frac{\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)}{48x^2}-\frac{\sqrt{\pi}\sqrt{\frac{4x^2}{9}+1}}{6x^2}+\frac{2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)}{27}}{\sqrt{\pi}}$	80

input
```
int(1/x^3/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(4*x^2+9)^(1/2)/x^2+2/27*arctanh(3/(4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = \frac{4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-4\,x^2\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}}{54\,x^2}$$

input
```
integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(4*x^2*log(-2*x + sqrt(4*x^2 + 9) + 3) - 4*x^2*log(-2*x + sqrt(4*x^2
+ 9) - 3) - 3*sqrt(4*x^2 + 9))/x^2
```

Sympy [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = \frac{2\operatorname{asinh}\left(\frac{3}{2x}\right)}{27} - \frac{1}{9x\sqrt{1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate(1/x**3/(4*x**2+9)**(1/2),x)
```

output
```
2*asinh(3/(2*x))/27 - 1/(9*x*sqrt(1 + 9/(4*x**2))) - 1/(4*x**3*sqrt(1 + 9/
(4*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{4\,x^2+9}}{18\,x^2} + \frac{2}{27}\,\operatorname{arsinh}\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/18*sqrt(4*x^2 + 9)/x^2 + 2/27*arcsinh(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{4\,x^2+9}}{18\,x^2} + \frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}+3\right) - \frac{1}{27}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate(1/x^3/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/18*sqrt(4*x^2 + 9)/x^2 + 1/27*log(sqrt(4*x^2 + 9) + 3) - 1/27*log(sqrt(
4*x^2 + 9) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{2\sqrt{x^2+\frac{9}{4}}}{3}\right)}{27} - \frac{\sqrt{x^2+\frac{9}{4}}}{9\,x^2}$$

input
```
int(1/(x^3*(4*x^2 + 9)^(1/2)),x)
```

output
```
(2*atanh((2*(x^2 + 9/4)^(1/2))/3))/27 - (x^2 + 9/4)^(1/2)/(9*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \frac{1}{x^3\sqrt{9+4x^2}}\,dx = \frac{-3\sqrt{4x^2+9} - 4\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)x^2 + 4\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)x^2}{54x^2}$$

input
```
int(1/x^3/(4*x^2+9)^(1/2),x)
```

output
```
( - 3*sqrt(4*x**2 + 9) - 4*log((sqrt(4*x**2 + 9) + 2*x - 3)/3)*x**2 + 4*lo
g((sqrt(4*x**2 + 9) + 2*x + 3)/3)*x**2)/(54*x**2)
```

3.551 $\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx=-\frac{\sqrt{9+4x^2}}{27x^3}+\frac{8\sqrt{9+4x^2}}{243x}$$

output
```
-1/27*(4*x^2+9)^(1/2)/x^3+8/243*(4*x^2+9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx=\frac{\sqrt{9+4x^2}(-9+8x^2)}{243x^3}$$

input
```
Integrate[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

output
```
(Sqrt[9 + 4*x^2]*(-9 + 8*x^2))/(243*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \ 245$$

$$-\frac{8}{27}\int \frac{1}{x^2\sqrt{4x^2+9}}dx-\frac{\sqrt{4x^2+9}}{27x^3}$$

$$\Big\downarrow \ 242$$

$$\frac{8\sqrt{4x^2+9}}{243x}-\frac{\sqrt{4x^2+9}}{27x^3}$$

input
```
Int[1/(x^4*Sqrt[9 + 4*x^2]),x]
```

output
```
-1/27*Sqrt[9 + 4*x^2]/x^3 + (8*Sqrt[9 + 4*x^2])/(243*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{\sqrt{4x^{2}+9}\,(8x^{2}-9)}{243x^{3}}$	22
trager	$\frac{\sqrt{4x^{2}+9}\,(8x^{2}-9)}{243x^{3}}$	22
meijerg	$-\frac{\left(1-\frac{8x^{2}}{9}\right)\sqrt{\frac{4x^{2}}{9}+1}}{9x^{3}}$	22
pseudoelliptic	$\frac{\sqrt{4x^{2}+9}\,(8x^{2}-9)}{243x^{3}}$	22
orering	$\frac{\sqrt{4x^{2}+9}\,(8x^{2}-9)}{243x^{3}}$	22
risch	$\frac{32x^{4}+36x^{2}-81}{243x^{3}\sqrt{4x^{2}+9}}$	27
default	$-\frac{\sqrt{4x^{2}+9}}{27x^{3}}+\frac{8\sqrt{4x^{2}+9}}{243x}$	30

input
```
int(1/x^4/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*(4*x^2+9)^(1/2)*(8*x^2-9)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx=\frac{16\,x^3+(8\,x^2-9)\sqrt{4\,x^2+9}}{243\,x^3}$$

input
```
integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/243*(16*x^3 + (8*x^2 - 9)*sqrt(4*x^2 + 9))/x^3
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx = \frac{16\sqrt{1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

input
```
integrate(1/x**4/(4*x**2+9)**(1/2),x)
```

output
```
16*sqrt(1 + 9/(4*x**2))/243 - 2*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx = \frac{8\sqrt{4x^2+9}}{243x} - \frac{\sqrt{4x^2+9}}{27x^3}$$

input
```
integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
8/243*sqrt(4*x^2 + 9)/x - 1/27*sqrt(4*x^2 + 9)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx = \frac{32\left(\left(2x-\sqrt{4x^2+9}\right)^2-3\right)}{\left(\left(2x-\sqrt{4x^2+9}\right)^2-9\right)^3}$$

input
```
integrate(1/x^4/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
32*((2*x - sqrt(4*x^2 + 9))^2 - 3)/((2*x - sqrt(4*x^2 + 9))^2 - 9)^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx = \sqrt{x^2+\frac{9}{4}}\left(\frac{16}{243\,x}-\frac{2}{27\,x^3}\right)$$

input
```
int(1/(x^4*(4*x^2 + 9)^(1/2)),x)
```

output
```
(x^2 + 9/4)^(1/2)*(16/(243*x) - 2/(27*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\sqrt{9+4x^2}}\,dx = \frac{8\sqrt{4x^2+9}\,x^2-9\sqrt{4x^2+9}-16x^3}{243x^3}$$

input
```
int(1/x^4/(4*x^2+9)^(1/2),x)
```

output
```
(8*sqrt(4*x**2 + 9)*x**2 - 9*sqrt(4*x**2 + 9) - 16*x**3)/(243*x**3)
```

3.552 $\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = -\frac{\sqrt{9+4x^2}}{36x^4}+\frac{\sqrt{9+4x^2}}{54x^2}-\frac{2}{81}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

output
```
-1/36*(4*x^2+9)^(1/2)/x^4+1/54*(4*x^2+9)^(1/2)/x^2-2/81*arctanh(1/3*(4*x^2
+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = \frac{(-3+2x^2)\sqrt{9+4x^2}}{108x^4}-\frac{2}{81}\text{arctanh}\left(\frac{1}{3}\sqrt{9+4x^2}\right)$$

input
```
Integrate[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

output
```
((-3 + 2*x^2)*Sqrt[9 + 4*x^2])/(108*x^4) - (2*ArcTanh[Sqrt[9 + 4*x^2]/3])/
81
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 52, 52, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{4x^2+9}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{4x^2+9}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{1}{3}\int \frac{1}{x^4\sqrt{4x^2+9}}dx^2-\frac{\sqrt{4x^2+9}}{18x^4}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2+9}}dx^2+\frac{\sqrt{4x^2+9}}{9x^2}\right)-\frac{\sqrt{4x^2+9}}{18x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{9}\int \frac{1}{\frac{x^4}{4}-\frac{9}{4}}d\sqrt{4x^2+9}+\frac{\sqrt{4x^2+9}}{9x^2}\right)-\frac{\sqrt{4x^2+9}}{18x^4}\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{\sqrt{4x^2+9}}{9x^2}-\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{4x^2+9}\right)\right)-\frac{\sqrt{4x^2+9}}{18x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[9 + 4*x^2]),x]
```

output
```
(-1/18*Sqrt[9 + 4*x^2]/x^4 + (Sqrt[9 + 4*x^2]/(9*x^2) - (4*ArcTanh[Sqrt[9
+ 4*x^2]/3])/27)/3)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result	size
trager	$\frac{(2x^2-3)\sqrt{4x^2+9}}{108x^4}+\frac{2\ln\left(\frac{\sqrt{4x^2+9}-3}{x}\right)}{81}$	41
risch	$\frac{8x^4+6x^2-27}{108x^4\sqrt{4x^2+9}}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}$	42
default	$-\frac{\sqrt{4x^2+9}}{36x^4}+\frac{\sqrt{4x^2+9}}{54x^2}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{4x^2+9}}\right)}{81}$	44
pseudoelliptic	$\frac{-\frac{16\ln\left(\sqrt{4x^2+9}+3\right)x^4}{81}+\frac{16\ln\left(\sqrt{4x^2+9}-3\right)x^4}{81}+\frac{8x^2\sqrt{4x^2+9}}{27}-\frac{4\sqrt{4x^2+9}}{9}}{\left(\sqrt{4x^2+9}+3\right)^2\left(\sqrt{4x^2+9}-3\right)^2}$	89
meijerg	$\frac{-\frac{\sqrt{\pi}}{12x^4}+\frac{\sqrt{\pi}}{27x^2}+\frac{\left(\frac{7}{6}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{81}+\frac{\sqrt{\pi}\left(-\frac{112}{81}x^4-\frac{32}{9}x^2+8\right)}{96x^4}-\frac{\sqrt{\pi}\left(-\frac{16x^2}{3}+8\right)\sqrt{\frac{4x^2}{9}+1}}{96x^4}-\frac{2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)}{81}}{\sqrt{\pi}}$	101

input
```
int(1/x^5/(4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/108*(2*x^2-3)/x^4*(4*x^2+9)^(1/2)+2/81*ln(((4*x^2+9)^(1/2)-3)/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = -\frac{8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}+3\right)-8\,x^4\log\left(-2\,x+\sqrt{4\,x^2+9}-3\right)-3\,\sqrt{4\,x^2+9}(2\,x^2-3)}{324\,x^4}$$

input
```
integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/324*(8*x^4*log(-2*x + sqrt(4*x^2 + 9) + 3) - 8*x^4*log(-2*x + sqrt(4*x^
2 + 9) - 3) - 3*sqrt(4*x^2 + 9)*(2*x^2 - 3))/x^4
```

Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = -\frac{2\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1+\frac{9}{4x^2}}} + \frac{1}{36x^3\sqrt{1+\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate(1/x**5/(4*x**2+9)**(1/2),x)
```

output
```
-2*asinh(3/(2*x))/81 + 1/(27*x*sqrt(1 + 9/(4*x**2))) + 1/(36*x**3*sqrt(1 +
 9/(4*x**2))) - 1/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = \frac{\sqrt{4\,x^2+9}}{54\,x^2} - \frac{\sqrt{4\,x^2+9}}{36\,x^4} - \frac{2}{81}\,\mathrm{arsinh}\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/54*sqrt(4*x^2 + 9)/x^2 - 1/36*sqrt(4*x^2 + 9)/x^4 - 2/81*arcsinh(3/2/abs
(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = \frac{(4\,x^2+9)^{\frac{3}{2}} - 15\,\sqrt{4\,x^2+9}}{216\,x^4} - \frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}+3\right) + \frac{1}{81}\,\log\left(\sqrt{4\,x^2+9}-3\right)$$

input
```
integrate(1/x^5/(4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/216*((4*x^2 + 9)^(3/2) - 15*sqrt(4*x^2 + 9))/x^4 - 1/81*log(sqrt(4*x^2 +
 9) + 3) + 1/81*log(sqrt(4*x^2 + 9) - 3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = \frac{\sqrt{x^2+\frac{9}{4}}\,\left(\frac{2}{27\,x^2}-\frac{1}{9\,x^4}\right)}{2}-\frac{2\,\mathrm{atanh}\left(\frac{2\,\sqrt{x^2+\frac{9}{4}}}{3}\right)}{81}$$

input
```
int(1/(x^5*(4*x^2 + 9)^(1/2)),x)
```

output
```
((x^2 + 9/4)^(1/2)*(2/(27*x^2) - 1/(9*x^4)))/2 - (2*atanh((2*(x^2 + 9/4)^(
1/2))/3))/81
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^5\sqrt{9+4x^2}}\,dx = \frac{6\sqrt{4x^2+9}\,x^2-9\sqrt{4x^2+9}+8\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}-1\right)x^4-8\log\left(\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}+1\right)x^4}{324x^4}$$

input
```
int(1/x^5/(4*x^2+9)^(1/2),x)
```

output
```
(6*sqrt(4*x**2 + 9)*x**2 - 9*sqrt(4*x**2 + 9) + 8*log((sqrt(4*x**2 + 9) +
2*x - 3)/3)*x**4 - 8*log((sqrt(4*x**2 + 9) + 2*x + 3)/3)*x**4)/(324*x**4)
```

3.553 $\int \frac{x^5}{\sqrt{9-4x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^5}{\sqrt{9-4x^2}} \, dx = -\frac{81}{64}\sqrt{9-4x^2} + \frac{3}{32}\left(9-4x^2\right)^{3/2} - \frac{1}{320}\left(9-4x^2\right)^{5/2}$$

output
```
-81/64*(-4*x^2+9)^(1/2)+3/32*(-4*x^2+9)^(3/2)-1/320*(-4*x^2+9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt{9-4x^2}} \, dx = \frac{1}{40}\sqrt{9-4x^2}\left(-27-6x^2-2x^4\right)$$

input
```
Integrate[x^5/Sqrt[9 - 4*x^2],x]
```

output
```
(Sqrt[9 - 4*x^2]*(-27 - 6*x^2 - 2*x^4))/40
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{9-4x^2}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{1}{16}\left(9-4x^2\right)^{3/2}-\frac{9}{8}\sqrt{9-4x^2}+\frac{81}{16\sqrt{9-4x^2}}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{1}{160}\left(9-4x^2\right)^{5/2}+\frac{3}{16}\left(9-4x^2\right)^{3/2}-\frac{81}{32}\sqrt{9-4x^2}\right)$$

input
```
Int[x^5/Sqrt[9 - 4*x^2],x]
```

output
```
((-81*Sqrt[9 - 4*x^2])/32 + (3*(9 - 4*x^2)^(3/2))/16 - (9 - 4*x^2)^(5/2)/1
60)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(-\frac{1}{20}x^{4}-\frac{3}{20}x^{2}-\frac{27}{40}\right)\sqrt{-4x^{2}+9}$	23
pseudoelliptic	$-\frac{\sqrt{-4x^{2}+9}\,\left(2x^{4}+6x^{2}+27\right)}{40}$	24
risch	$\frac{\left(2x^{4}+6x^{2}+27\right)\left(4x^{2}-9\right)}{40\sqrt{-4x^{2}+9}}$	31
gosper	$\frac{(2x-3)(2x+3)\left(2x^{4}+6x^{2}+27\right)}{40\sqrt{-4x^{2}+9}}$	34
orering	$\frac{(2x-3)(2x+3)\left(2x^{4}+6x^{2}+27\right)}{40\sqrt{-4x^{2}+9}}$	34
meijerg	$-\frac{243\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\,\left(\frac{32}{27}x^{4}+\frac{32}{9}x^{2}+16\right)\sqrt{-\frac{4x^{2}}{9}+1}}{15}\right)}{128\sqrt{\pi}}$	38
default	$-\frac{x^{4}\sqrt{-4x^{2}+9}}{20}-\frac{3x^{2}\sqrt{-4x^{2}+9}}{20}-\frac{27\sqrt{-4x^{2}+9}}{40}$	41

input
```
int(x^5/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/20*x^4-3/20*x^2-27/40)*(-4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = -\frac{1}{40}\left(2\,x^4+6\,x^2+27\right)\sqrt{-4\,x^2+9}$$

input
```
integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/40*(2*x^4 + 6*x^2 + 27)*sqrt(-4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = -\frac{x^4\sqrt{9-4x^2}}{20}-\frac{3x^2\sqrt{9-4x^2}}{20}-\frac{27\sqrt{9-4x^2}}{40}$$

input
```
integrate(x**5/(-4*x**2+9)**(1/2),x)
```

output
```
-x**4*sqrt(9 - 4*x**2)/20 - 3*x**2*sqrt(9 - 4*x**2)/20 - 27*sqrt(9 - 4*x**
2)/40
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = -\frac{1}{20}\,\sqrt{-4\,x^2+9}x^4-\frac{3}{20}\,\sqrt{-4\,x^2+9}x^2-\frac{27}{40}\,\sqrt{-4\,x^2+9}$$

input
```
integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(-4*x^2 + 9)*x^4 - 3/20*sqrt(-4*x^2 + 9)*x^2 - 27/40*sqrt(-4*x^2
 + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = -\frac{1}{320}\left(4\,x^2-9\right)^2\sqrt{-4\,x^2+9}+\frac{3}{32}\left(-4\,x^2+9\right)^{\frac{3}{2}}-\frac{81}{64}\sqrt{-4\,x^2+9}$$

input
```
integrate(x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/320*(4*x^2 - 9)^2*sqrt(-4*x^2 + 9) + 3/32*(-4*x^2 + 9)^(3/2) - 81/64*sq
rt(-4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{\frac{9}{4}-x^2}\left(\frac{x^4}{5}+\frac{3\,x^2}{5}+\frac{27}{10}\right)}{2}$$

input
```
int(x^5/(9 - 4*x^2)^(1/2),x)
```

output
```
-((9/4 - x^2)^(1/2)*((3*x^2)/5 + x^4/5 + 27/10))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{9-4x^2}}\,dx = \frac{\sqrt{-4x^2+9}\,(-2x^4-6x^2-27)}{40}$$

input
```
int(x^5/(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*( - 2*x**4 - 6*x**2 - 27))/40
```

3.554 $\int \frac{x^4}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 45

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{27}{128}x\sqrt{9-4x^2} - \frac{1}{16}x^3\sqrt{9-4x^2} + \frac{243}{256}\arcsin\left(\frac{2x}{3}\right)$$

output
```
-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)+243/256*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{1}{128}x\sqrt{9-4x^2}(27+8x^2) + \frac{243}{128}\arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[x^4/Sqrt[9 - 4*x^2],x]
```

output
```
-1/128*(x*Sqrt[9 - 4*x^2]*(27 + 8*x^2)) + (243*ArcTan[(2*x)/(-3 + Sqrt[9 -
 4*x^2])])/128
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{9-4x^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{27}{16}\int \frac{x^2}{\sqrt{9-4x^2}}dx-\frac{1}{16}x^3\sqrt{9-4x^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}dx-\frac{1}{8}x\sqrt{9-4x^2}\right)-\frac{1}{16}x^3\sqrt{9-4x^2}$$

$$\Big\downarrow \text{223}$$

$$\frac{27}{16}\left(\frac{9}{16}\arcsin\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}\right)-\frac{1}{16}x^3\sqrt{9-4x^2}$$

input
```
Int[x^4/Sqrt[9 - 4*x^2],x]
```

output
```
-1/16*(x^3*Sqrt[9 - 4*x^2]) + (27*(-1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2
*x)/3])/16))/16
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

method	result	size
default	$-\frac{27x\sqrt{-4x^2+9}}{128}-\frac{x^3\sqrt{-4x^2+9}}{16}+\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}$	34
risch	$\frac{x\left(8x^2+27\right)\left(4x^2-9\right)}{128\sqrt{-4x^2+9}}+\frac{243\arcsin\left(\frac{2x}{3}\right)}{256}$	34
pseudoelliptic	$-\frac{243\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{256}+\frac{\left(-8x^3-27x\right)\sqrt{-4x^2+9}}{128}$	39
meijerg	$-\frac{81i\left(-\frac{i\sqrt{\pi}\,x\left(\frac{40x^2}{9}+15\right)\sqrt{-\frac{4x^2}{9}+1}}{30}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{64\sqrt{\pi}}$	41
trager	$-\frac{x\left(8x^2+27\right)\sqrt{-4x^2+9}}{128}-\frac{243\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{256}$	51

input
```
int(x^4/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-27/128*x*(-4*x^2+9)^(1/2)-1/16*x^3*(-4*x^2+9)^(1/2)+243/256*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{1}{128}\left(8\,x^3+27\,x\right)\sqrt{-4\,x^2+9}-\frac{243}{128}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/128*(8*x^3 + 27*x)*sqrt(-4*x^2 + 9) - 243/128*arctan(1/2*(sqrt(-4*x^2 +
 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{x^3\sqrt{9-4x^2}}{16} - \frac{27x\sqrt{9-4x^2}}{128} + \frac{243\operatorname{asin}\left(\frac{2x}{3}\right)}{256}$$

input
```
integrate(x**4/(-4*x**2+9)**(1/2),x)
```

output
```
-x**3*sqrt(9 - 4*x**2)/16 - 27*x*sqrt(9 - 4*x**2)/128 + 243*asin(2*x/3)/25
6
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.73

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{1}{16}\,\sqrt{-4\,x^2+9}x^3 - \frac{27}{128}\,\sqrt{-4\,x^2+9}x + \frac{243}{256}\,\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*sqrt(-4*x^2 + 9)*x^3 - 27/128*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/
3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.58

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = -\frac{1}{128}\left(8\,x^2+27\right)\sqrt{-4\,x^2+9}x + \frac{243}{256}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/128*(8*x^2 + 27)*sqrt(-4*x^2 + 9)*x + 243/256*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = \frac{243\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{256} - \frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{x^3}{4}+\frac{27\,x}{32}\right)}{2}$$

input
```
int(x^4/(9 - 4*x^2)^(1/2),x)
```

output
```
(243*asin((2*x)/3))/256 - ((9/4 - x^2)^(1/2)*((27*x)/32 + x^3/4))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{\sqrt{9-4x^2}}\,dx = \frac{243 asin\left(\frac{2x}{3}\right)}{256} - \frac{\sqrt{-4x^2+9}\,x^3}{16} - \frac{27\sqrt{-4x^2+9}\,x}{128}$$

input
```
int(x^4/(-4*x^2+9)^(1/2),x)
```

output
```
(243*asin((2*x)/3) - 16*sqrt( - 4*x**2 + 9)*x**3 - 54*sqrt( - 4*x**2 + 9)*
x)/256
```

3.555 $\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = -\frac{9}{16}\sqrt{9-4x^2}+\frac{1}{48}\left(9-4x^2\right)^{3/2}$$

output
```
-9/16*(-4*x^2+9)^(1/2)+1/48*(-4*x^2+9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = \frac{1}{24}\sqrt{9-4x^2}\left(-9-2x^2\right)$$

input
```
Integrate[x^3/Sqrt[9 - 4*x^2],x]
```

output
```
(Sqrt[9 - 4*x^2]*(-9 - 2*x^2))/24
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{9-4x^2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{9}{4\sqrt{9-4x^2}}-\frac{1}{4}\sqrt{9-4x^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{24}\left(9-4x^2\right)^{3/2}-\frac{9}{8}\sqrt{9-4x^2}\right)$$

input
```
Int[x^3/Sqrt[9 - 4*x^2],x]
```

output
```
((-9*Sqrt[9 - 4*x^2])/8 + (9 - 4*x^2)^(3/2)/24)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
trager	$\left(-\frac{x^2}{12}-\frac{3}{8}\right)\sqrt{-4x^2+9}$	18
pseudoelliptic	$-\frac{(2x^2+9)\sqrt{-4x^2+9}}{24}$	19
risch	$\frac{(2x^2+9)(4x^2-9)}{24\sqrt{-4x^2+9}}$	26
default	$-\frac{x^2\sqrt{-4x^2+9}}{12}-\frac{3\sqrt{-4x^2+9}}{8}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{-4x^2+9}}$	29
orering	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{-4x^2+9}}$	29
meijerg	$\frac{\frac{9\sqrt{\pi}}{8}-\frac{9\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)\sqrt{-\frac{4x^2}{9}+1}}{64}}{\sqrt{\pi}}$	33

input
```
int(x^3/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/12*x^2-3/8)*(-4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = -\frac{1}{24}\left(2\,x^2+9\right)\sqrt{-4\,x^2+9}$$

input
```
integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(2*x^2 + 9)*sqrt(-4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = -\frac{x^2\sqrt{9-4x^2}}{12} - \frac{3\sqrt{9-4x^2}}{8}$$

input
```
integrate(x**3/(-4*x**2+9)**(1/2),x)
```

output
```
-x**2*sqrt(9 - 4*x**2)/12 - 3*sqrt(9 - 4*x**2)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = -\frac{1}{12}\sqrt{-4\,x^2+9}x^2 - \frac{3}{8}\sqrt{-4\,x^2+9}$$

input
```
integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(-4*x^2 + 9)*x^2 - 3/8*sqrt(-4*x^2 + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = \frac{1}{48}\left(-4\,x^2+9\right)^{\frac{3}{2}} - \frac{9}{16}\sqrt{-4\,x^2+9}$$

input
```
integrate(x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/48*(-4*x^2 + 9)^(3/2) - 9/16*sqrt(-4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{\frac{9}{4}-x^2}\left(\frac{x^2}{3}+\frac{3}{2}\right)}{2}$$

input
```
int(x^3/(9 - 4*x^2)^(1/2),x)
```

output
```
-((9/4 - x^2)^(1/2)*(x^2/3 + 3/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt{9-4x^2}}\,dx = \frac{\sqrt{-4x^2+9}\,(-2x^2-9)}{24}$$

input
```
int(x^3/(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*( - 2*x**2 - 9))/24
```

3.556 $\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{16}\arcsin\left(\frac{2x}{3}\right)$$

output
```
-1/8*x*(-4*x^2+9)^(1/2)+9/16*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = -\frac{1}{8}x\sqrt{9-4x^2}+\frac{9}{8}\arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[x^2/Sqrt[9 - 4*x^2],x]
```

output
```
-1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])])/8
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx$$
$$\Big\downarrow \text{ 262}$$
$$\frac{9}{8}\int \frac{1}{\sqrt{9-4x^2}}dx-\frac{1}{8}x\sqrt{9-4x^2}$$
$$\Big\downarrow \text{ 223}$$
$$\frac{9}{16}\arcsin\left(\frac{2x}{3}\right)-\frac{1}{8}x\sqrt{9-4x^2}$$

input
```
Int[x^2/Sqrt[9 - 4*x^2],x]
```

output
```
-1/8*(x*Sqrt[9 - 4*x^2]) + (9*ArcSin[(2*x)/3])/16
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{x\sqrt{-4x^2+9}}{8}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}$	20
risch	$\frac{(4x^2-9)x}{8\sqrt{-4x^2+9}}+\frac{9\arcsin\left(\frac{2x}{3}\right)}{16}$	27
pseudoelliptic	$-\frac{x\sqrt{-4x^2+9}}{8}-\frac{9\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{16}$	31
meijerg	$\frac{9i\left(\frac{2i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}}$	34
trager	$-\frac{x\sqrt{-4x^2+9}}{8}-\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{16}$	44

input
```
int(x^2/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-4*x^2+9)^(1/2)+9/16*arcsin(2/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx=-\frac{1}{8}\sqrt{-4\,x^2+9}x-\frac{9}{8}\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-4*x^2 + 9)*x - 9/8*arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = -\frac{x\sqrt{9-4x^2}}{8} + \frac{9\,\mathrm{asin}\left(\frac{2x}{3}\right)}{16}$$

input
```
integrate(x**2/(-4*x**2+9)**(1/2),x)
```

output
```
-x*sqrt(9 - 4*x**2)/8 + 9*asin(2*x/3)/16
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2+9}x + \frac{9}{16}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2+9}x + \frac{9}{16}\arcsin\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/8*sqrt(-4*x^2 + 9)*x + 9/16*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = \frac{9\,\mathrm{asin}\left(\frac{2\,x}{3}\right)}{16} - \frac{x\,\sqrt{\frac{9}{4}-x^2}}{4}$$

input
```
int(x^2/(9 - 4*x^2)^(1/2),x)
```

output
```
(9*asin((2*x)/3))/16 - (x*(9/4 - x^2)^(1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

$$\int \frac{x^2}{\sqrt{9-4x^2}}\,dx = \frac{9asin\left(\frac{2x}{3}\right)}{16} - \frac{\sqrt{-4x^2+9}\,x}{8}$$

input
```
int(x^2/(-4*x^2+9)^(1/2),x)
```

output
```
(9*asin((2*x)/3) - 2*sqrt( - 4*x**2 + 9)*x)/16
```

3.557 $\int \frac{x}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{9-4x^2}$$

output
```
-1/4*(-4*x^2+9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{9-4x^2}$$

input
```
Integrate[x/Sqrt[9 - 4*x^2],x]
```

output
```
-1/4*Sqrt[9 - 4*x^2]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{4}\sqrt{9-4x^2}$$

input
```
Int[x/Sqrt[9 - 4*x^2],x]
```

output
```
-1/4*Sqrt[9 - 4*x^2]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\sqrt{-4x^2+9}}{4}$	12
default	$-\frac{\sqrt{-4x^2+9}}{4}$	12
trager	$-\frac{\sqrt{-4x^2+9}}{4}$	12
pseudoelliptic	$-\frac{\sqrt{-4x^2+9}}{4}$	12
risch	$\frac{4x^2-9}{4\sqrt{-4x^2+9}}$	19
gosper	$\frac{(2x-3)(2x+3)}{4\sqrt{-4x^2+9}}$	22
orering	$\frac{(2x-3)(2x+3)}{4\sqrt{-4x^2+9}}$	22
meijerg	$-\frac{3\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-\frac{4x^2}{9}+1}\right)}{8\sqrt{\pi}}$	26

input
```
int(x/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\,\sqrt{-4\,x^2+9}$$

input
```
integrate(x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-4*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{4}$$

input
```
integrate(x/(-4*x**2+9)**(1/2),x)
```

output
```
-sqrt(9 - 4*x**2)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-4x^2+9}$$

input
```
integrate(x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-4*x^2 + 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-4x^2+9}$$

input
```
integrate(x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-4*x^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{\frac{9}{4}-x^2}}{2}$$

input
```
int(x/(9 - 4*x^2)^(1/2),x)
```

output
```
-(9/4 - x^2)^(1/2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4x^2+9}}{4}$$

input
```
int(x/(-4*x^2+9)^(1/2),x)
```

output
```
( - sqrt( - 4*x**2 + 9))/4
```

3.558 $\int \frac{1}{\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2x}{3}\right)$$

output
```
1/2*arcsin(2/3*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \arctan\left(\frac{2x}{-3+\sqrt{9-4x^2}}\right)$$

input
```
Integrate[1/Sqrt[9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/(-3 + Sqrt[9 - 4*x^2])]
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{9-4x^2}} \, dx$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2} \arcsin\left(\frac{2x}{3}\right)$$

input
```
Int[1/Sqrt[9 - 4*x^2],x]
```

output
```
ArcSin[(2*x)/3]/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
meijerg	$\frac{\arcsin\left(\frac{2x}{3}\right)}{2}$	7
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-4x^2+9}}{2x}\right)}{2}$	18
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2+9}+2x\right)}{2}$	30

input
```
int(1/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(2/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = -\arctan\left(\frac{\sqrt{-4\,x^2+9}-3}{2\,x}\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(1/2*(sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{2x}{3}\right)}{2}$$

input
```
integrate(1/(-4*x**2+9)**(1/2),x)
```

output
```
asin(2*x/3)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsin(2/3*x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(6) = 12.

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{1}{2}\sqrt{-4x^2+9}x + \frac{9}{4}\arcsin\left(\frac{2}{3}x\right)$$

input
```
integrate(1/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-4*x^2 + 9)*x + 9/4*arcsin(2/3*x)
```

Mupad [B] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{2\,x}{3}\right)}{2}$$

input
```
int(1/(9 - 4*x^2)^(1/2),x)
```

output
```
asin((2*x)/3)/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx = \frac{asin\left(\frac{2x}{3}\right)}{2}$$

input
```
int(1/(-4*x^2+9)^(1/2),x)
```

output
```
asin((2*x)/3)/2
```

3.559 $\int \frac{1}{x\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
-1/3*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[1/(x*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{9-4x^2}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{9-4x^2}}dx^2$$
$$\Big\downarrow \text{73}$$
$$-\frac{1}{4}\int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}$$
$$\Big\downarrow \text{219}$$
$$-\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Int[1/(x*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/3*ArcTanh[Sqrt[9 - 4*x^2]/3]
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\operatorname{arctanh}\left(\frac{3}{\sqrt{-4x^{2}+9}}\right)}{3}$	15
trager	$\frac{\ln\left(\frac{-3+\sqrt{-4x^{2}+9}}{x}\right)}{3}$	19
pseudoelliptic	$-\frac{\ln\left(\sqrt{-4x^{2}+9}+3\right)}{6}+\frac{\ln\left(-3+\sqrt{-4x^{2}+9}\right)}{6}$	30
meijerg	$\frac{\left(2\ln\left(x\right)-2\ln\left(3\right)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^{2}}{9}+1}}{2}\right)}{6\sqrt{\pi}}$	43

input
```
int(1/x/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctanh(3/(-4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \frac{1}{3}\,\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)$$

input
```
integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/3*log((sqrt(-4*x^2 + 9) - 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \begin{cases} -\frac{\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{i\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((-acosh(3/(2*x))/3, 1/Abs(x**2) > 4/9), (I*asin(3/(2*x))/3, True
))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = -\frac{1}{3}\log\left(\frac{6\sqrt{-4x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = -\frac{1}{6}\log\left(\sqrt{-4\,x^2+9}+3\right)+\frac{1}{6}\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate(1/x/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/6*log(sqrt(-4*x^2 + 9) + 3) + 1/6*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \frac{\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\sqrt{\frac{1}{x^2}}}{2}\right)}{3}$$

input
```
int(1/(x*(9 - 4*x^2)^(1/2)),x)
```

output
```
log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2)/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.50

$$\int \frac{1}{x\sqrt{9-4x^2}}\,dx = \frac{\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)}{3}$$

input
```
int(1/x/(-4*x^2+9)^(1/2),x)
```

output
```
log(tan(asin((2*x)/3)/2))/3
```

3.560 $\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx=-\frac{\sqrt{9-4x^2}}{9x}$$

output
```
-1/9*(-4*x^2+9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx=-\frac{\sqrt{9-4x^2}}{9x}$$

input
```
Integrate[1/(x^2*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/9*Sqrt[9 - 4*x^2]/x
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx$$
$$\Big\downarrow \; 242$$
$$-\frac{\sqrt{9-4x^2}}{9x}$$

input
```
Int[1/(x^2*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/9*Sqrt[9 - 4*x^2]/x
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$-\frac{\sqrt{-4x^2+9}}{9x}$	15
trager	$-\frac{\sqrt{-4x^2+9}}{9x}$	15
meijerg	$-\frac{\sqrt{-\frac{4x^2}{9}+1}}{3x}$	15
pseudoelliptic	$-\frac{\sqrt{-4x^2+9}}{9x}$	15
risch	$\frac{4x^2-9}{9x\sqrt{-4x^2+9}}$	22
gosper	$\frac{(2x-3)(2x+3)}{9x\sqrt{-4x^2+9}}$	25
orering	$\frac{(2x-3)(2x+3)}{9x\sqrt{-4x^2+9}}$	25

input
```
int(1/x^2/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(-4*x^2+9)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

input
```
integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/9*sqrt(-4*x^2 + 9)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = \begin{cases} -\frac{i\sqrt{4x^2-9}}{9x} & \text{for } |x^2| > \frac{9}{4} \\ -\frac{\sqrt{9-4x^2}}{9x} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((-I*sqrt(4*x**2 - 9)/(9*x), Abs(x**2) > 9/4), (-sqrt(9 - 4*x**2)
/(9*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2+9}}{9\,x}$$

input
```
integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/9*sqrt(-4*x^2 + 9)/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.83

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = \frac{2\,x}{9\left(\sqrt{-4\,x^2+9}-3\right)} - \frac{\sqrt{-4\,x^2+9}-3}{18\,x}$$

input
```
integrate(1/x^2/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
2/9*x/(sqrt(-4*x^2 + 9) - 3) - 1/18*(sqrt(-4*x^2 + 9) - 3)/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{2\sqrt{\frac{9}{4}-x^2}}{9\,x}$$

input
```
int(1/(x^2*(9 - 4*x^2)^(1/2)),x)
```

output
```
-(2*(9/4 - x^2)^(1/2))/(9*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^2\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4x^2+9}}{9x}$$

input
```
int(1/x^2/(-4*x^2+9)^(1/2),x)
```

output
```
( - sqrt( - 4*x**2 + 9))/(9*x)
```

3.561 $\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx=-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
-1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(1/3*(-4*x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx=-\frac{\sqrt{9-4x^2}}{18x^2}-\frac{2}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/18*Sqrt[9 - 4*x^2]/x^2 - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/27
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{9-4x^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{9-4x^2}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{9-4x^2}}dx^2-\frac{\sqrt{9-4x^2}}{9x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(-\frac{1}{9}\int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}-\frac{\sqrt{9-4x^2}}{9x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{9x^2}\right)$$

input
```
Int[1/(x^3*Sqrt[9 - 4*x^2]),x]
```

output
```
(-1/9*Sqrt[9 - 4*x^2]/x^2 - (4*ArcTanh[Sqrt[9 - 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{\sqrt{-4x^2+9}}{18x^2}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$	30
trager	$-\frac{\sqrt{-4x^2+9}}{18x^2}-\frac{2\ln\left(\frac{\sqrt{-4x^2+9}+3}{x}\right)}{27}$	34
risch	$\frac{4x^2-9}{18x^2\sqrt{-4x^2+9}}-\frac{2\,\mathrm{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{27}$	37
pseudoelliptic	$\frac{-2\ln\left(\sqrt{-4x^2+9}+3\right)x^2+2\ln\left(-3+\sqrt{-4x^2+9}\right)x^2-3\sqrt{-4x^2+9}}{54x^2}$	52
meijerg	$-\frac{2\left(\frac{9\sqrt{\pi}}{4x^2}-\frac{(1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}}{2}-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{32x^2}+\frac{9\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right)\right)}{27\sqrt{\pi}}$	84

input
```
int(1/x^3/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*(-4*x^2+9)^(1/2)/x^2-2/27*arctanh(3/(-4*x^2+9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = \frac{4\,x^2\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,\sqrt{-4\,x^2+9}}{54\,x^2}$$

input
```
integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(4*x^2*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*sqrt(-4*x^2 + 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = \begin{cases} -\frac{2\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{2i\,\mathrm{asin}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((-2*acosh(3/(2*x))/27 + 1/(9*x*sqrt(-1 + 9/(4*x**2))) - 1/(4*x**
3*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (2*I*asin(3/(2*x))/27 - I/(9
*x*sqrt(1 - 9/(4*x**2))) + I/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2+9}}{18\,x^2} - \frac{2}{27}\,\log\left(\frac{6\,\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/18*sqrt(-4*x^2 + 9)/x^2 - 2/27*log(6*sqrt(-4*x^2 + 9)/abs(x) + 18/abs(x
))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2+9}}{18\,x^2} - \frac{1}{27}\log\left(\sqrt{-4\,x^2+9}+3\right) + \frac{1}{27}\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate(1/x^3/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
-1/18*sqrt(-4*x^2 + 9)/x^2 - 1/27*log(sqrt(-4*x^2 + 9) + 3) + 1/27*log(-sq
rt(-4*x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = \frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\frac{3\,\sqrt{\frac{1}{x^2}}}{2}\right)}{27} - \frac{\sqrt{\frac{9}{4}-x^2}}{9\,x^2}$$

input
```
int(1/(x^3*(9 - 4*x^2)^(1/2)),x)
```

output
```
(2*log((9/(4*x^2) - 1)^(1/2) - (3*(1/x^2)^(1/2))/2))/27 - (9/4 - x^2)^(1/2
)/(9*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{9-4x^2}}\,dx = \frac{-3\sqrt{-4x^2+9}+4\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)x^2}{54x^2}$$

input
```
int(1/x^3/(-4*x^2+9)^(1/2),x)
```

output
```
( - 3*sqrt( - 4*x**2 + 9) + 4*log(tan(asin((2*x)/3)/2))*x**2)/(54*x**2)
```

3.562 $\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{27x^3}-\frac{8\sqrt{9-4x^2}}{243x}$$

output
```
-1/27*(-4*x^2+9)^(1/2)/x^3-8/243*(-4*x^2+9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = \frac{(-9-8x^2)\sqrt{9-4x^2}}{243x^3}$$

input
```
Integrate[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

output
```
((-9 - 8*x^2)*Sqrt[9 - 4*x^2])/(243*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{9-4x^2}}dx$$

$$\Big\downarrow \; 245$$

$$\frac{8}{27}\int \frac{1}{x^2\sqrt{9-4x^2}}dx-\frac{\sqrt{9-4x^2}}{27x^3}$$

$$\Big\downarrow \; 242$$

$$-\frac{8\sqrt{9-4x^2}}{243x}-\frac{\sqrt{9-4x^2}}{27x^3}$$

input
```
Int[1/(x^4*Sqrt[9 - 4*x^2]),x]
```

output
```
-1/27*Sqrt[9 - 4*x^2]/x^3 - (8*Sqrt[9 - 4*x^2])/(243*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
trager	$-\frac{(8x^2+9)\sqrt{-4x^2+9}}{243x^3}$	22
meijerg	$-\frac{\left(1+\frac{8x^2}{9}\right)\sqrt{-\frac{4x^2}{9}+1}}{9x^3}$	22
pseudoelliptic	$-\frac{(8x^2+9)\sqrt{-4x^2+9}}{243x^3}$	22
risch	$\frac{32x^4-36x^2-81}{243x^3\sqrt{-4x^2+9}}$	27
default	$-\frac{\sqrt{-4x^2+9}}{27x^3}-\frac{8\sqrt{-4x^2+9}}{243x}$	30
gosper	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{-4x^2+9}}$	32
orering	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{-4x^2+9}}$	32

input
```
int(1/x^4/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/243*(8*x^2+9)/x^3*(-4*x^2+9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx=-\frac{(8\,x^2+9)\sqrt{-4\,x^2+9}}{243\,x^3}$$

input
```
integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
-1/243*(8*x^2 + 9)*sqrt(-4*x^2 + 9)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.16

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = \begin{cases} -\frac{16\sqrt{-1+\frac{9}{4x^2}}}{243} - \frac{2\sqrt{-1+\frac{9}{4x^2}}}{27x^2} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{16i\sqrt{1-\frac{9}{4x^2}}}{243} - \frac{2i\sqrt{1-\frac{9}{4x^2}}}{27x^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((-16*sqrt(-1 + 9/(4*x**2))/243 - 2*sqrt(-1 + 9/(4*x**2))/(27*x**
2), 1/Abs(x**2) > 4/9), (-16*I*sqrt(1 - 9/(4*x**2))/243 - 2*I*sqrt(1 - 9/(
4*x**2))/(27*x**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = -\frac{8\sqrt{-4x^2+9}}{243x} - \frac{\sqrt{-4x^2+9}}{27x^3}$$

input
```
integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-8/243*sqrt(-4*x^2 + 9)/x - 1/27*sqrt(-4*x^2 + 9)/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.97

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = \frac{2\,x^3\left(\frac{9\,\left(\sqrt{-4\,x^2+9}-3\right)^2}{x^2}+4\right)}{243\,\left(\sqrt{-4\,x^2+9}-3\right)^3} - \frac{\sqrt{-4\,x^2+9}-3}{54\,x} - \frac{\left(\sqrt{-4\,x^2+9}-3\right)^3}{1944\,x^3}$$

input
```
integrate(1/x^4/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
2/243*x^3*(9*(sqrt(-4*x^2 + 9) - 3)^2/x^2 + 4)/(sqrt(-4*x^2 + 9) - 3)^3 -
1/54*(sqrt(-4*x^2 + 9) - 3)/x - 1/1944*(sqrt(-4*x^2 + 9) - 3)^3/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = -\sqrt{\frac{9}{4}-x^2}\left(\frac{16}{243\,x}+\frac{2}{27\,x^3}\right)$$

input
```
int(1/(x^4*(9 - 4*x^2)^(1/2)),x)
```

output
```
-(9/4 - x^2)^(1/2)*(16/(243*x) + 2/(27*x^3))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^4\sqrt{9-4x^2}}\,dx = \frac{\sqrt{-4x^2+9}\,(-8x^2-9)}{243x^3}$$

input
```
int(1/x^4/(-4*x^2+9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 + 9)*( - 8*x**2 - 9))/(243*x**3)
```

3.563 $\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{9-4x^2}}{36x^4} - \frac{\sqrt{9-4x^2}}{54x^2} - \frac{2}{81}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

output
```
-1/36*(-4*x^2+9)^(1/2)/x^4-1/54*(-4*x^2+9)^(1/2)/x^2-2/81*arctanh(1/3*(-4*
x^2+9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = \frac{\sqrt{9-4x^2}(-3-2x^2)}{108x^4} - \frac{2}{81}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)$$

input
```
Integrate[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

output
```
(Sqrt[9 - 4*x^2]*(-3 - 2*x^2))/(108*x^4) - (2*ArcTanh[Sqrt[9 - 4*x^2]/3])/
81
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 52, 52, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{9-4x^2}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^6\sqrt{9-4x^2}}dx^2$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^4\sqrt{9-4x^2}}dx^2-\frac{\sqrt{9-4x^2}}{18x^4}\right)$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{9-4x^2}}dx^2-\frac{\sqrt{9-4x^2}}{9x^2}\right)-\frac{\sqrt{9-4x^2}}{18x^4}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{9}\int \frac{1}{\frac{9}{4}-\frac{x^4}{4}}d\sqrt{9-4x^2}-\frac{\sqrt{9-4x^2}}{9x^2}\right)-\frac{\sqrt{9-4x^2}}{18x^4}\right)$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{4}{27}\text{arctanh}\left(\frac{1}{3}\sqrt{9-4x^2}\right)-\frac{\sqrt{9-4x^2}}{9x^2}\right)-\frac{\sqrt{9-4x^2}}{18x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[9 - 4*x^2]),x]
```

output
```
(-1/18*Sqrt[9 - 4*x^2]/x^4 + (-1/9*Sqrt[9 - 4*x^2]/x^2 - (4*ArcTanh[Sqrt[9
 - 4*x^2]/3])/27)/3)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

method	result
trager	$-\frac{(2x^2+3)\sqrt{-4x^2+9}}{108x^4}+\frac{2\ln\left(\frac{-3+\sqrt{-4x^2+9}}{x}\right)}{81}$
risch	$\frac{8x^4-6x^2-27}{108x^4\sqrt{-4x^2+9}}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}$
default	$-\frac{\sqrt{-4x^2+9}}{36x^4}-\frac{\sqrt{-4x^2+9}}{54x^2}-\frac{2\,\text{arctanh}\left(\frac{3}{\sqrt{-4x^2+9}}\right)}{81}$
pseudoelliptic	$\frac{-\frac{16\ln\left(\sqrt{-4x^2+9}+3\right)x^4}{81}+\frac{16\ln\left(-3+\sqrt{-4x^2+9}\right)x^4}{81}-\frac{8x^2\sqrt{-4x^2+9}}{27}-\frac{4\sqrt{-4x^2+9}}{9}}{\left(\sqrt{-4x^2+9}+3\right)^2\left(-3+\sqrt{-4x^2+9}\right)^2}$
meijerg	$\frac{-\frac{\sqrt{\pi}}{12x^4}-\frac{\sqrt{\pi}}{27x^2}+\frac{\left(\frac{7}{6}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{81}+\frac{\sqrt{\pi}\left(-\frac{112}{81}x^4+\frac{32}{9}x^2+8\right)}{96x^4}-\frac{\sqrt{\pi}\left(\frac{16x^2}{3}+8\right)\sqrt{-\frac{4x^2}{9}+1}}{96x^4}-\frac{2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right)}{81}}{\sqrt{\pi}}$

input
```
int(1/x^5/(-4*x^2+9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/108*(2*x^2+3)/x^4*(-4*x^2+9)^(1/2)+2/81*ln((-3+(-4*x^2+9)^(1/2))/x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx=\frac{8\,x^4\log\left(\frac{\sqrt{-4\,x^2+9}-3}{x}\right)-3\,(2\,x^2+3)\sqrt{-4\,x^2+9}}{324\,x^4}$$

input
```
integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="fricas")
```

output
```
1/324*(8*x^4*log((sqrt(-4*x^2 + 9) - 3)/x) - 3*(2*x^2 + 3)*sqrt(-4*x^2 + 9
))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.75 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = \begin{cases} -\frac{2\,\mathrm{acosh}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{36x^3\sqrt{-1+\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{2i\,\mathrm{asin}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{36x^3\sqrt{1-\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(-4*x**2+9)**(1/2),x)
```

output
```
Piecewise((-2*acosh(3/(2*x))/81 + 1/(27*x*sqrt(-1 + 9/(4*x**2))) - 1/(36*x
**3*sqrt(-1 + 9/(4*x**2))) - 1/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2)
 > 4/9), (2*I*asin(3/(2*x))/81 - I/(27*x*sqrt(1 - 9/(4*x**2))) + I/(36*x**
3*sqrt(1 - 9/(4*x**2))) + I/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2+9}}{54\,x^2} - \frac{\sqrt{-4\,x^2+9}}{36\,x^4} - \frac{2}{81}\log\left(\frac{6\sqrt{-4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="maxima")
```

output
```
-1/54*sqrt(-4*x^2 + 9)/x^2 - 1/36*sqrt(-4*x^2 + 9)/x^4 - 2/81*log(6*sqrt(-
4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = \frac{(-4\,x^2+9)^{\frac{3}{2}} - 15\,\sqrt{-4\,x^2+9}}{216\,x^4} - \frac{1}{81}\,\log\left(\sqrt{-4\,x^2+9}+3\right) + \frac{1}{81}\,\log\left(-\sqrt{-4\,x^2+9}+3\right)$$

input
```
integrate(1/x^5/(-4*x^2+9)^(1/2),x, algorithm="giac")
```

output
```
1/216*((-4*x^2 + 9)^(3/2) - 15*sqrt(-4*x^2 + 9))/x^4 - 1/81*log(sqrt(-4*x^
2 + 9) + 3) + 1/81*log(-sqrt(-4*x^2 + 9) + 3)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = \frac{2\,\ln\left(\sqrt{\frac{9}{4\,x^2}-1}-\sqrt{\frac{9}{4\,x^2}}\right)}{81} - \frac{\sqrt{\frac{9}{4}-x^2}\,\left(\frac{2}{27\,x^2}+\frac{1}{9\,x^4}\right)}{2}$$

input
```
int(1/(x^5*(9 - 4*x^2)^(1/2)),x)
```

output
```
(2*log((9/(4*x^2) - 1)^(1/2) - (9/(4*x^2))^(1/2)))/81 - ((9/4 - x^2)^(1/2)
*(2/(27*x^2) + 1/(9*x^4)))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\sqrt{9-4x^2}}\,dx = \frac{-6\sqrt{-4x^2+9}\,x^2 - 9\sqrt{-4x^2+9} + 8\log\left(\tan\left(\frac{asin\left(\frac{2x}{3}\right)}{2}\right)\right)x^4}{324x^4}$$

input
```
int(1/x^5/(-4*x^2+9)^(1/2),x)
```

output
```
( - 6*sqrt( - 4*x**2 + 9)*x**2 - 9*sqrt( - 4*x**2 + 9) + 8*log(tan(asin((2
*x)/3)/2))*x**4)/(324*x**4)
```

3.564 $\int \frac{x^5}{\sqrt{-9+4x^2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^5}{\sqrt{-9+4x^2}} dx = \frac{81}{64}\sqrt{-9+4x^2} + \frac{3}{32}\left(-9+4x^2\right)^{3/2} + \frac{1}{320}\left(-9+4x^2\right)^{5/2}$$

output
```
81/64*(4*x^2-9)^(1/2)+3/32*(4*x^2-9)^(3/2)+1/320*(4*x^2-9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt{-9+4x^2}} dx = \frac{1}{40}\sqrt{-9+4x^2}\left(27+6x^2+2x^4\right)$$

input
```
Integrate[x^5/Sqrt[-9 + 4*x^2],x]
```

output
```
(Sqrt[-9 + 4*x^2]*(27 + 6*x^2 + 2*x^4))/40
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{4x^2-9}}dx$$
$$\Big\downarrow \ 243$$
$$\frac{1}{2}\int \frac{x^4}{\sqrt{4x^2-9}}dx^2$$
$$\Big\downarrow \ 53$$
$$\frac{1}{2}\int \left(\frac{1}{16}\left(4x^2-9\right)^{3/2}+\frac{9}{8}\sqrt{4x^2-9}+\frac{81}{16\sqrt{4x^2-9}}\right)dx^2$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\left(\frac{1}{160}\left(4x^2-9\right)^{5/2}+\frac{3}{16}\left(4x^2-9\right)^{3/2}+\frac{81}{32}\sqrt{4x^2-9}\right)$$

input
```
Int[x^5/Sqrt[-9 + 4*x^2],x]
```

output
```
((81*Sqrt[-9 + 4*x^2])/32 + (3*(-9 + 4*x^2)^(3/2))/16 + (-9 + 4*x^2)^(5/2)
/160)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(\frac{1}{20}x^{4}+\frac{3}{20}x^{2}+\frac{27}{40}\right)\sqrt{4x^{2}-9}$	23
risch	$\frac{\left(2x^{4}+6x^{2}+27\right)\sqrt{4x^{2}-9}}{40}$	24
pseudoelliptic	$\frac{\left(2x^{4}+6x^{2}+27\right)\sqrt{4x^{2}-9}}{40}$	24
gosper	$\frac{\left(2x-3\right)\left(2x+3\right)\left(2x^{4}+6x^{2}+27\right)}{40\sqrt{4x^{2}-9}}$	34
orering	$\frac{\left(2x-3\right)\left(2x+3\right)\left(2x^{4}+6x^{2}+27\right)}{40\sqrt{4x^{2}-9}}$	34
default	$\frac{x^{4}\sqrt{4x^{2}-9}}{20}+\frac{3x^{2}\sqrt{4x^{2}-9}}{20}+\frac{27\sqrt{4x^{2}-9}}{40}$	41
meijerg	$-\frac{243\sqrt{-\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{32}{27}x^{4}+\frac{32}{9}x^{2}+16\right)\sqrt{-\frac{4x^{2}}{9}+1}}{15}\right)}{128\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}}$	60

input
```
int(x^5/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/20*x^4+3/20*x^2+27/40)*(4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \frac{1}{40}\left(2\,x^4+6\,x^2+27\right)\sqrt{4\,x^2-9}$$

input
```
integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/40*(2*x^4 + 6*x^2 + 27)*sqrt(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \frac{x^4\sqrt{4x^2-9}}{20} + \frac{3x^2\sqrt{4x^2-9}}{20} + \frac{27\sqrt{4x^2-9}}{40}$$

input
```
integrate(x**5/(4*x**2-9)**(1/2),x)
```

output
```
x**4*sqrt(4*x**2 - 9)/20 + 3*x**2*sqrt(4*x**2 - 9)/20 + 27*sqrt(4*x**2 - 9
)/40
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \frac{1}{20}\sqrt{4\,x^2-9}x^4 + \frac{3}{20}\sqrt{4\,x^2-9}x^2 + \frac{27}{40}\sqrt{4\,x^2-9}$$

input
```
integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(4*x^2 - 9)*x^4 + 3/20*sqrt(4*x^2 - 9)*x^2 + 27/40*sqrt(4*x^2 - 9
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \frac{1}{320}\left(4\,x^2-9\right)^{\frac{5}{2}} + \frac{3}{32}\left(4\,x^2-9\right)^{\frac{3}{2}} + \frac{81}{64}\sqrt{4\,x^2-9}$$

input
```
integrate(x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/320*(4*x^2 - 9)^(5/2) + 3/32*(4*x^2 - 9)^(3/2) + 81/64*sqrt(4*x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \sqrt{4\,x^2-9}\left(\frac{x^4}{20} + \frac{3\,x^2}{20} + \frac{27}{40}\right)$$

input
```
int(x^5/(4*x^2 - 9)^(1/2),x)
```

output
```
(4*x^2 - 9)^(1/2)*((3*x^2)/20 + x^4/20 + 27/40)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}\,(2x^4+6x^2+27)}{40}$$

input
```
int(x^5/(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9)*(2*x**4 + 6*x**2 + 27))/40
```

3.565 $\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx=\frac{27}{128}x\sqrt{-9+4x^2}+\frac{1}{16}x^3\sqrt{-9+4x^2}+\frac{243}{256}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
27/128*x*(4*x^2-9)^(1/2)+1/16*x^3*(4*x^2-9)^(1/2)+243/256*arctanh(2*x/(4*x
^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx=\frac{1}{128}x\sqrt{-9+4x^2}\left(27+8x^2\right)-\frac{243}{256}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[x^4/Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2]*(27 + 8*x^2))/128 - (243*Log[-2*x + Sqrt[-9 + 4*x^2]])
/256
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{27}{16}\int \frac{x^2}{\sqrt{4x^2-9}}dx+\frac{1}{16}\sqrt{4x^2-9}x^3$$

$$\Big\downarrow \text{262}$$

$$\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{\sqrt{4x^2-9}}dx+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3$$

$$\Big\downarrow \text{224}$$

$$\frac{27}{16}\left(\frac{9}{8}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3$$

$$\Big\downarrow \text{219}$$

$$\frac{27}{16}\left(\frac{9}{16}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{8}\sqrt{4x^2-9}x\right)+\frac{1}{16}\sqrt{4x^2-9}x^3$$

input
```
Int[x^4/Sqrt[-9 + 4*x^2],x]
```

output
```
(x^3*Sqrt[-9 + 4*x^2])/16 + (27*((x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)
/Sqrt[-9 + 4*x^2]])/16))/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

method	result	size
trager	$\frac{x\left(8x^2+27\right)\sqrt{4x^2-9}}{128}-\frac{243\ln\left(-\sqrt{4x^2-9}+2x\right)}{256}$	39
risch	$\frac{x\left(8x^2+27\right)\sqrt{4x^2-9}}{128}+\frac{243\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{512}$	42
default	$\frac{x^3\sqrt{4x^2-9}}{16}+\frac{27x\sqrt{4x^2-9}}{128}+\frac{243\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{512}$	49
meijerg	$-\frac{81i\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{i\sqrt{\pi}\,x\left(\frac{40x^2}{9}+15\right)\sqrt{-\frac{4x^2}{9}+1}}{30}+\frac{3i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)}{4}\right)}{64\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	63
pseudoelliptic	$\frac{-19683\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)+19683\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)+\left(2592x^3+8748x\right)\sqrt{4x^2-9}}{512\left(\sqrt{4x^2-9}+2x\right)^2\left(-\sqrt{4x^2-9}+2x\right)^2}$	95

input
```
int(x^4/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/128*x*(8*x^2+27)*(4*x^2-9)^(1/2)-243/256*ln(-(4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{1}{128}\left(8\,x^3+27\,x\right)\sqrt{4\,x^2-9}-\frac{243}{256}\,\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/128*(8*x^3 + 27*x)*sqrt(4*x^2 - 9) - 243/256*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{x^3\sqrt{4x^2-9}}{16}+\frac{27x\sqrt{4x^2-9}}{128}+\frac{243\log\left(2x+\sqrt{4x^2-9}\right)}{256}$$

input
```
integrate(x**4/(4*x**2-9)**(1/2),x)
```

output
```
x**3*sqrt(4*x**2 - 9)/16 + 27*x*sqrt(4*x**2 - 9)/128 + 243*log(2*x + sqrt(
4*x**2 - 9))/256
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{1}{16}\,\sqrt{4\,x^2-9}x^3+\frac{27}{128}\,\sqrt{4\,x^2-9}x+\frac{243}{256}\,\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(4*x^2 - 9)*x^3 + 27/128*sqrt(4*x^2 - 9)*x + 243/256*log(8*x + 4*
sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{1}{128}\left(8\,x^2+27\right)\sqrt{4\,x^2-9}x - \frac{243}{256}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

input
```
integrate(x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/128*(8*x^2 + 27)*sqrt(4*x^2 - 9)*x - 243/256*log(abs(-2*x + sqrt(4*x^2 -
 9)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \int \frac{x^4}{\sqrt{4\,x^2-9}}\,dx$$

input
```
int(x^4/(4*x^2 - 9)^(1/2),x)
```

output
```
int(x^4/(4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}\,x^3}{16} + \frac{27\sqrt{4x^2-9}\,x}{128} + \frac{243\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{256}$$

input
```
int(x^4/(4*x^2-9)^(1/2),x)
```

output
```
(16*sqrt(4*x**2 - 9)*x**3 + 54*sqrt(4*x**2 - 9)*x + 243*log((sqrt(4*x**2 -
 9) + 2*x)/3))/256
```

3.566 $\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{9}{16}\sqrt{-9+4x^2}+\frac{1}{48}\left(-9+4x^2\right)^{3/2}$$

output
```
9/16*(4*x^2-9)^(1/2)+1/48*(4*x^2-9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{1}{24}\left(9+2x^2\right)\sqrt{-9+4x^2}$$

input
```
Integrate[x^3/Sqrt[-9 + 4*x^2],x]
```

output
```
((9 + 2*x^2)*Sqrt[-9 + 4*x^2])/24
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{4x^2-9}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{1}{4}\sqrt{4x^2-9}+\frac{9}{4\sqrt{4x^2-9}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{1}{24}\left(4x^2-9\right)^{3/2}+\frac{9}{8}\sqrt{4x^2-9}\right)$$

input
```
Int[x^3/Sqrt[-9 + 4*x^2],x]
```

output
```
((9*Sqrt[-9 + 4*x^2])/8 + (-9 + 4*x^2)^(3/2)/24)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
trager	$\left(\frac{x^2}{12}+\frac{3}{8}\right)\sqrt{4x^2-9}$	18
risch	$\frac{(2x^2+9)\sqrt{4x^2-9}}{24}$	19
pseudoelliptic	$\frac{(2x^2+9)\sqrt{4x^2-9}}{24}$	19
default	$\frac{x^2\sqrt{4x^2-9}}{12}+\frac{3\sqrt{4x^2-9}}{8}$	27
gosper	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{4x^2-9}}$	29
orering	$\frac{(2x-3)(2x+3)(2x^2+9)}{24\sqrt{4x^2-9}}$	29
meijerg	$\frac{27\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)\sqrt{-\frac{4x^2}{9}+1}}{6}\right)}{32\sqrt{\pi}\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	55

input
```
int(x^3/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(1/12*x^2+3/8)*(4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{1}{24}\sqrt{4\,x^2-9}\left(2\,x^2+9\right)$$

input
```
integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/24*sqrt(4*x^2 - 9)*(2*x^2 + 9)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{x^2\sqrt{4x^2-9}}{12} + \frac{3\sqrt{4x^2-9}}{8}$$

input
```
integrate(x**3/(4*x**2-9)**(1/2),x)
```

output
```
x**2*sqrt(4*x**2 - 9)/12 + 3*sqrt(4*x**2 - 9)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{1}{12}\sqrt{4\,x^2-9}x^2 + \frac{3}{8}\sqrt{4\,x^2-9}$$

input
```
integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/12*sqrt(4*x^2 - 9)*x^2 + 3/8*sqrt(4*x^2 - 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{1}{48}\left(4\,x^2-9\right)^{\frac{3}{2}} + \frac{9}{16}\sqrt{4\,x^2-9}$$

input
```
integrate(x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/48*(4*x^2 - 9)^(3/2) + 9/16*sqrt(4*x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{\left(2\,x^2+9\right)\,\sqrt{4\,x^2-9}}{24}$$

input
```
int(x^3/(4*x^2 - 9)^(1/2),x)
```

output
```
((2*x^2 + 9)*(4*x^2 - 9)^(1/2))/24
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}\,(2x^2+9)}{24}$$

input
```
int(x^3/(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9)*(2*x**2 + 9))/24
```

3.567 $\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}x\sqrt{-9+4x^2}+\frac{9}{16}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
1/8*x*(4*x^2-9)^(1/2)+9/16*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}x\sqrt{-9+4x^2}-\frac{9}{16}\log\left(-2x+\sqrt{-9+4x^2}\right)$$

input
```
Integrate[x^2/Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/8 - (9*Log[-2*x + Sqrt[-9 + 4*x^2]])/16
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{8}\int \frac{1}{\sqrt{4x^2-9}}dx+\frac{1}{8}\sqrt{4x^2-9}x$$

$$\Big\downarrow \text{224}$$

$$\frac{9}{8}\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}+\frac{1}{8}\sqrt{4x^2-9}x$$

$$\Big\downarrow \text{219}$$

$$\frac{9}{16}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)+\frac{1}{8}\sqrt{4x^2-9}x$$

input
```
Int[x^2/Sqrt[-9 + 4*x^2],x]
```

output
```
(x*Sqrt[-9 + 4*x^2])/8 + (9*ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]])/16
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

method	result	size
trager	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(\sqrt{4x^2-9}+2x\right)}{16}$	30
default	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{32}$	35
risch	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{32}$	35
pseudoelliptic	$\frac{x\sqrt{4x^2-9}}{8}+\frac{9\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)}{32}-\frac{9\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)}{32}$	54
meijerg	$\frac{9i\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{2i\sqrt{\pi}\,x\sqrt{-\frac{4x^2}{9}+1}}{3}-i\sqrt{\pi}\,\arcsin\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	56

input
```
int(x^2/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(4*x^2-9)^(1/2)+9/16*ln((4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}\sqrt{4\,x^2-9}x-\frac{9}{16}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(4*x^2 - 9)*x - 9/16*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{x\sqrt{4x^2-9}}{8} + \frac{9\log\left(2x+\sqrt{4x^2-9}\right)}{16}$$

input
```
integrate(x**2/(4*x**2-9)**(1/2),x)
```

output
```
x*sqrt(4*x**2 - 9)/8 + 9*log(2*x + sqrt(4*x**2 - 9))/16
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}\sqrt{4x^2-9}x + \frac{9}{16}\log\left(8x+4\sqrt{4x^2-9}\right)$$

input
```
integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(4*x^2 - 9)*x + 9/16*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{1}{8}\sqrt{4x^2-9}x - \frac{9}{16}\log\left(\left|-2x+\sqrt{4x^2-9}\right|\right)$$

input
```
integrate(x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(4*x^2 - 9)*x - 9/16*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{9\,\ln\left(x+\frac{\sqrt{4\,x^2-9}}{2}\right)}{16} + \frac{x\,\sqrt{4\,x^2-9}}{8}$$

input
```
int(x^2/(4*x^2 - 9)^(1/2),x)
```

output
```
(9*log(x + (4*x^2 - 9)^(1/2)/2))/16 + (x*(4*x^2 - 9)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

$$\int \frac{x^2}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}\,x}{8} + \frac{9\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{16}$$

input
```
int(x^2/(4*x^2-9)^(1/2),x)
```

output
```
(2*sqrt(4*x**2 - 9)*x + 9*log((sqrt(4*x**2 - 9) + 2*x)/3))/16
```

3.568 $\int \frac{x}{\sqrt{-9+4x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{\sqrt{-9+4x^2}} \, dx = \frac{1}{4}\sqrt{-9+4x^2}$$

output
```
1/4*(4*x^2-9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-9+4x^2}} \, dx = \frac{1}{4}\sqrt{-9+4x^2}$$

input
```
Integrate[x/Sqrt[-9 + 4*x^2],x]
```

output
```
Sqrt[-9 + 4*x^2]/4
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{4x^2-9}}\,dx$$

$$\Big\downarrow \; 241$$

$$\frac{1}{4}\sqrt{4x^2-9}$$

input
```
Int[x/Sqrt[-9 + 4*x^2],x]
```

output
```
Sqrt[-9 + 4*x^2]/4
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\sqrt{4x^2-9}}{4}$	12
default	$\frac{\sqrt{4x^2-9}}{4}$	12
trager	$\frac{\sqrt{4x^2-9}}{4}$	12
risch	$\frac{\sqrt{4x^2-9}}{4}$	12
pseudoelliptic	$\frac{\sqrt{4x^2-9}}{4}$	12
gosper	$\frac{(2x-3)(2x+3)}{4\sqrt{4x^2-9}}$	22
orering	$\frac{(2x-3)(2x+3)}{4\sqrt{4x^2-9}}$	22
meijerg	$-\frac{3\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-\frac{4x^2}{9}+1}\right)}{8\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	48

input
```
int(x/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\,\sqrt{4\,x^2-9}$$

input
```
integrate(x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}}{4}$$

input
```
integrate(x/(4*x**2-9)**(1/2),x)
```

output
```
sqrt(4*x**2 - 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\sqrt{4x^2-9}$$

input
```
integrate(x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(4*x^2 - 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{1}{4}\sqrt{4x^2-9}$$

input
```
integrate(x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(4*x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{4}$$

input
```
int(x/(4*x^2 - 9)^(1/2),x)
```

output
```
(4*x^2 - 9)^(1/2)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}}{4}$$

input
```
int(x/(4*x^2-9)^(1/2),x)
```

output
```
sqrt(4*x**2 - 9)/4
```

3.569 $\int \frac{1}{\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx=\frac{1}{2}\text{arctanh}\left(\frac{2x}{\sqrt{-9+4x^2}}\right)$$

output
```
1/2*arctanh(2*x/(4*x^2-9)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(19) = 38.

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx=-\frac{1}{4}\log\left(1-\frac{2x}{\sqrt{-9+4x^2}}\right)+\frac{1}{4}\log\left(1+\frac{2x}{\sqrt{-9+4x^2}}\right)$$

input
```
Integrate[1/Sqrt[-9 + 4*x^2],x]
```

output
```
-1/4*Log[1 - (2*x)/Sqrt[-9 + 4*x^2]] + Log[1 + (2*x)/Sqrt[-9 + 4*x^2]]/4
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2-9}}dx$$
$$\Big\downarrow \text{224}$$
$$\int \frac{1}{1-\frac{4x^2}{4x^2-9}}d\frac{x}{\sqrt{4x^2-9}}$$
$$\Big\downarrow \text{219}$$
$$\frac{1}{2}\text{arctanh}\left(\frac{2x}{\sqrt{4x^2-9}}\right)$$

input
```
Int[1/Sqrt[-9 + 4*x^2],x]
```

output
```
ArcTanh[(2*x)/Sqrt[-9 + 4*x^2]]/2
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

method	result	size
trager	$-\frac{\ln\left(-\sqrt{4x^2-9}+2x\right)}{2}$	19
default	$\frac{\ln\left(\sqrt{4}\,x+\sqrt{4x^2-9}\right)\sqrt{4}}{4}$	22
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\arcsin\left(\frac{2x}{3}\right)}{2\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}}$	29
pseudoelliptic	$\frac{\ln\left(\frac{\sqrt{4x^2-9}+2x}{x}\right)}{4}-\frac{\ln\left(\frac{\sqrt{4x^2-9}-2x}{x}\right)}{4}$	42

input
```
int(1/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(-(4*x^2-9)^(1/2)+2*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = -\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 - 9))
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\log\left(2x+\sqrt{4x^2-9}\right)}{2}$$

input
```
integrate(1/(4*x**2-9)**(1/2),x)
```

output
```
log(2*x + sqrt(4*x**2 - 9))/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\log\left(8\,x+4\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(8*x + 4*sqrt(4*x^2 - 9))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{1}{2}\sqrt{4\,x^2-9}x + \frac{9}{4}\log\left(\left|-2\,x+\sqrt{4\,x^2-9}\right|\right)$$

input
```
integrate(1/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(4*x^2 - 9)*x + 9/4*log(abs(-2*x + sqrt(4*x^2 - 9)))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\ln\left(2\,x+\sqrt{4\,x^2-9}\right)}{2}$$

input
```
int(1/(4*x^2 - 9)^(1/2),x)
```

output
```
log(2*x + (4*x^2 - 9)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-9+4x^2}}\,dx = \frac{\log\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{2}$$

input
```
int(1/(4*x^2-9)^(1/2),x)
```

output
```
log((sqrt(4*x**2 - 9) + 2*x)/3)/2
```

3.570 $\int \frac{1}{x\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
1/3*arctan(1/3*(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[1/(x*Sqrt[-9 + 4*x^2]),x]
```

output
```
ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{4x^2-9}}dx$$
$$\Big\downarrow \text{ 243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2$$
$$\Big\downarrow \text{ 73}$$
$$\frac{1}{4}\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}$$
$$\Big\downarrow \text{ 216}$$
$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)$$

input
```
Int[1/(x*Sqrt[-9 + 4*x^2]),x]
```

output
```
ArcTan[Sqrt[-9 + 4*x^2]/3]/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\arctan\left(\frac{3}{\sqrt{4x^{2}-9}}\right)}{3}$	15
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{4x^{2}-9}}{3}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-3\operatorname{RootOf}\left(\_Z^{2}+1\right)+\sqrt{4x^{2}-9}}{x}\right)}{3}$	32
meijerg	$\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}\left(\left(2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^{2}}{9}+1}}{2}\right)\right)}{6\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{4x^{2}}{9}\right)}}$	65

input
```
int(1/x/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{2}{3}\arctan\left(-\frac{2}{3}\,x + \frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
2/3*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \begin{cases} \frac{i\,\operatorname{acosh}\left(\frac{3}{2x}\right)}{3} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{\operatorname{asin}\left(\frac{3}{2x}\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x/(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((I*acosh(3/(2*x))/3, 1/Abs(x**2) > 4/9), (-asin(3/(2*x))/3, True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = -\frac{1}{3}\arcsin\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*arcsin(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{1}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/x/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/3*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{\ln\left(\frac{\sqrt{4\,x^2-9}+3\mathrm{i}}{x}\right)\,1\mathrm{i}}{3}$$

input
```
int(1/(x*(4*x^2 - 9)^(1/2)),x)
```

output
```
(log(((4*x^2 - 9)^(1/2) + 3i)/x)*1i)/3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{-9+4x^2}}\,dx = \frac{2atan\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)}{3}$$

input
```
int(1/x/(4*x^2-9)^(1/2),x)
```

output
```
(2*atan((sqrt(4*x**2 - 9) + 2*x)/3))/3
```

3.571 $\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{9x}$$

output
```
1/9*(4*x^2-9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{9x}$$

input
```
Integrate[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

output
```
Sqrt[-9 + 4*x^2]/(9*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{4x^2-9}}\,dx$$

$$\Big\downarrow \text{242}$$

$$\frac{\sqrt{4x^2-9}}{9x}$$

input
```
Int[1/(x^2*Sqrt[-9 + 4*x^2]),x]
```

output
```
Sqrt[-9 + 4*x^2]/(9*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\sqrt{4x^2-9}}{9x}$	15
trager	$\frac{\sqrt{4x^2-9}}{9x}$	15
risch	$\frac{\sqrt{4x^2-9}}{9x}$	15
pseudoelliptic	$\frac{\sqrt{4x^2-9}}{9x}$	15
gosper	$\frac{(2x-3)(2x+3)}{9x\sqrt{4x^2-9}}$	25
orering	$\frac{(2x-3)(2x+3)}{9x\sqrt{4x^2-9}}$	25
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,\sqrt{-\frac{4x^2}{9}+1}}{3\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,x}$	37

input
```
int(1/x^2/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(4*x^2-9)^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{2\,x+\sqrt{4\,x^2-9}}{9\,x}$$

input
```
integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/9*(2*x + sqrt(4*x^2 - 9))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \begin{cases} \frac{2i\sqrt{-1+\frac{9}{4x^2}}}{9} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ \frac{2\sqrt{1-\frac{9}{4x^2}}}{9} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**2/(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((2*I*sqrt(-1 + 9/(4*x**2))/9, 1/Abs(x**2) > 4/9), (2*sqrt(1 - 9/
(4*x**2))/9, True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{9\,x}$$

input
```
integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(4*x^2 - 9)/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{4}{\left(2\,x-\sqrt{4\,x^2-9}\right)^2+9}$$

input
```
integrate(1/x^2/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
4/((2*x - sqrt(4*x^2 - 9))^2 + 9)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{9\,x}$$

input
```
int(1/(x^2*(4*x^2 - 9)^(1/2)),x)
```

output
```
(4*x^2 - 9)^(1/2)/(9*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x^2\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4x^2-9}+2x}{9x}$$

input
```
int(1/x^2/(4*x^2-9)^(1/2),x)
```

output
```
(sqrt(4*x**2 - 9) + 2*x)/(9*x)
```

3.572 $\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
1/18*(4*x^2-9)^(1/2)/x^2+2/27*arctan(1/3*(4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{18x^2} + \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

output
```
Sqrt[-9 + 4*x^2]/(18*x^2) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/27
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {243, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{4x^2-9}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2+\frac{\sqrt{4x^2-9}}{9x^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{9}\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}+\frac{\sqrt{4x^2-9}}{9x^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{9x^2}\right)$$

input
```
Int[1/(x^3*Sqrt[-9 + 4*x^2]),x]
```

output
```
(Sqrt[-9 + 4*x^2]/(9*x^2) + (4*ArcTan[Sqrt[-9 + 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result
default	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
risch	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{27}$
pseudoelliptic	$\frac{4\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right)x^2+3\sqrt{4x^2-9}}{54x^2}$
trager	$\frac{\sqrt{4x^2-9}}{18x^2}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-3\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{4x^2-9}}{x}\right)}{27}$
meijerg	$-\frac{2\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(\frac{9\sqrt{\pi}}{4x^2}-\frac{(1+2\ln(x)-2\ln(3)+i\pi)\sqrt{\pi}}{2}-\frac{9\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)}{32x^2}+\frac{9\sqrt{\pi}\sqrt{-\frac{4x^2}{9}+1}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{-\frac{4x^2}{9}+1}}{2}\right.\right.}{27\sqrt{\pi}\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

input
```
int(1/x^3/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(4*x^2-9)^(1/2)/x^2-2/27*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx=\frac{8\,x^2\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}}{54\,x^2}$$

input
```
integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(8*x^2*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{27} - \frac{i}{9x\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{27} + \frac{1}{9x\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{4x^3\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**3/(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((2*I*acosh(3/(2*x))/27 - I/(9*x*sqrt(-1 + 9/(4*x**2))) + I/(4*x*
*3*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2) > 4/9), (-2*asin(3/(2*x))/27 + 1/(9
*x*sqrt(1 - 9/(4*x**2))) - 1/(4*x**3*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.62

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{18\,x^2} - \frac{2}{27}\arcsin\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/18*sqrt(4*x^2 - 9)/x^2 - 2/27*arcsin(3/2/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{18\,x^2} + \frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/x^3/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/18*sqrt(4*x^2 - 9)/x^2 + 2/27*arctan(1/3*sqrt(4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{27} + \frac{\sqrt{4\,x^2-9}}{18\,x^2}$$

input
```
int(1/(x^3*(4*x^2 - 9)^(1/2)),x)
```

output
```
(2*atan((4*x^2 - 9)^(1/2)/3))/27 + (4*x^2 - 9)^(1/2)/(18*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\sqrt{-9+4x^2}}\,dx = \frac{8atan\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)x^2+3\sqrt{4x^2-9}}{54x^2}$$

input
```
int(1/x^3/(4*x^2-9)^(1/2),x)
```

output
```
(8*atan((sqrt(4*x**2 - 9) + 2*x)/3)*x**2 + 3*sqrt(4*x**2 - 9))/(54*x**2)
```

3.573 $\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{27x^3} + \frac{8\sqrt{-9+4x^2}}{243x}$$

output
```
1/27*(4*x^2-9)^(1/2)/x^3+8/243*(4*x^2-9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}(9+8x^2)}{243x^3}$$

input
```
Integrate[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

output
```
(Sqrt[-9 + 4*x^2]*(9 + 8*x^2))/(243*x^3)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{4x^2-9}}dx$$

$$\Big\downarrow \ 245$$

$$\frac{8}{27}\int \frac{1}{x^2\sqrt{4x^2-9}}dx+\frac{\sqrt{4x^2-9}}{27x^3}$$

$$\Big\downarrow \ 242$$

$$\frac{8\sqrt{4x^2-9}}{243x}+\frac{\sqrt{4x^2-9}}{27x^3}$$

input
```
Int[1/(x^4*Sqrt[-9 + 4*x^2]),x]
```

output
```
Sqrt[-9 + 4*x^2]/(27*x^3) + (8*Sqrt[-9 + 4*x^2])/(243*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
trager	$\frac{(8x^2+9)\sqrt{4x^2-9}}{243x^3}$	22
pseudoelliptic	$\frac{(8x^2+9)\sqrt{4x^2-9}}{243x^3}$	22
risch	$\frac{32x^4-36x^2-81}{243x^3\sqrt{4x^2-9}}$	27
default	$\frac{\sqrt{4x^2-9}}{27x^3}+\frac{8\sqrt{4x^2-9}}{243x}$	30
gosper	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{4x^2-9}}$	32
orering	$\frac{(2x-3)(2x+3)(8x^2+9)}{243x^3\sqrt{4x^2-9}}$	32
meijerg	$-\frac{\sqrt{-\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\left(1+\frac{8x^2}{9}\right)\sqrt{-\frac{4x^2}{9}+1}}{9\sqrt{\operatorname{signum}\left(-1+\frac{4x^2}{9}\right)}\,x^3}$	44

input
```
int(1/x^4/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/243*(8*x^2+9)/x^3*(4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{16\,x^3+(8\,x^2+9)\sqrt{4\,x^2-9}}{243\,x^3}$$

input
```
integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/243*(16*x^3 + (8*x^2 + 9)*sqrt(4*x^2 - 9))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.84

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \begin{cases} \frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3} & \text{for } |x^2| > \frac{9}{4} \\ \frac{8i\sqrt{9-4x^2}}{243x} + \frac{i\sqrt{9-4x^2}}{27x^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**4/(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((8*sqrt(4*x**2 - 9)/(243*x) + sqrt(4*x**2 - 9)/(27*x**3), Abs(x*
*2) > 9/4), (8*I*sqrt(9 - 4*x**2)/(243*x) + I*sqrt(9 - 4*x**2)/(27*x**3),
True))
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{8\sqrt{4x^2-9}}{243x} + \frac{\sqrt{4x^2-9}}{27x^3}$$

input
```
integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
8/243*sqrt(4*x^2 - 9)/x + 1/27*sqrt(4*x^2 - 9)/x^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{32\left(\left(2x-\sqrt{4x^2-9}\right)^2+3\right)}{\left(\left(2x-\sqrt{4x^2-9}\right)^2+9\right)^3}$$

input
```
integrate(1/x^4/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
32*((2*x - sqrt(4*x^2 - 9))^2 + 3)/((2*x - sqrt(4*x^2 - 9))^2 + 9)^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{8\,x^2\,\sqrt{4\,x^2-9}+9\,\sqrt{4\,x^2-9}}{243\,x^3}$$

input
```
int(1/(x^4*(4*x^2 - 9)^(1/2)),x)
```

output
```
(8*x^2*(4*x^2 - 9)^(1/2) + 9*(4*x^2 - 9)^(1/2))/(243*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\sqrt{-9+4x^2}}\,dx = \frac{8\sqrt{4x^2-9}\,x^2+9\sqrt{4x^2-9}-16x^3}{243x^3}$$

input
```
int(1/x^4/(4*x^2-9)^(1/2),x)
```

output
```
(8*sqrt(4*x**2 - 9)*x**2 + 9*sqrt(4*x**2 - 9) - 16*x**3)/(243*x**3)
```

3.574 $\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{-9+4x^2}}{36x^4} + \frac{\sqrt{-9+4x^2}}{54x^2} + \frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

output
```
1/36*(4*x^2-9)^(1/2)/x^4+1/54*(4*x^2-9)^(1/2)/x^2+2/81*arctan(1/3*(4*x^2-9
)^(1/2))
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{(3+2x^2)\sqrt{-9+4x^2}}{108x^4} + \frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-9+4x^2}\right)$$

input
```
Integrate[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

output
```
((3 + 2*x^2)*Sqrt[-9 + 4*x^2])/(108*x^4) + (2*ArcTan[Sqrt[-9 + 4*x^2]/3])/
81
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {243, 52, 52, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{4x^2-9}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^6\sqrt{4x^2-9}}dx^2$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{2}\left(\frac{1}{3}\int \frac{1}{x^4\sqrt{4x^2-9}}dx^2+\frac{\sqrt{4x^2-9}}{18x^4}\right)$$
$$\Big\downarrow \text{52}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{4x^2-9}}dx^2+\frac{\sqrt{4x^2-9}}{9x^2}\right)+\frac{\sqrt{4x^2-9}}{18x^4}\right)$$
$$\Big\downarrow \text{73}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{1}{9}\int \frac{1}{\frac{x^4}{4}+\frac{9}{4}}d\sqrt{4x^2-9}+\frac{\sqrt{4x^2-9}}{9x^2}\right)+\frac{\sqrt{4x^2-9}}{18x^4}\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{4x^2-9}\right)+\frac{\sqrt{4x^2-9}}{9x^2}\right)+\frac{\sqrt{4x^2-9}}{18x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[-9 + 4*x^2]),x]
```

output
```
(Sqrt[-9 + 4*x^2]/(18*x^4) + (Sqrt[-9 + 4*x^2]/(9*x^2) + (4*ArcTan[Sqrt[-9
 + 4*x^2]/3])/27)/3)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
risch	$\frac{8x^4-6x^2-27}{108x^4\sqrt{4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}$
default	$\frac{\sqrt{4x^2-9}}{36x^4}+\frac{\sqrt{4x^2-9}}{54x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{4x^2-9}}\right)}{81}$
pseudoelliptic	$\frac{8\arctan\left(\frac{\sqrt{4x^2-9}}{3}\right)x^4+6x^2\sqrt{4x^2-9}+9\sqrt{4x^2-9}}{324x^4}$
trager	$\frac{(2x^2+3)\sqrt{4x^2-9}}{108x^4}+\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)+\sqrt{4x^2-9}}{x}\right)}{81}$
meijerg	$\frac{8\sqrt{-\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}\left(-\frac{81\sqrt{\pi}}{32x^4}-\frac{9\sqrt{\pi}}{8x^2}+\frac{3\left(\frac{7}{6}+2\ln(x)-2\ln(3)+i\pi\right)\sqrt{\pi}}{8}+\frac{81\sqrt{\pi}\left(-\frac{112}{81}x^4+\frac{32}{9}x^2+8\right)}{256x^4}-\frac{81\sqrt{\pi}\left(\frac{16x^2}{3}+8\right)\sqrt{-\frac{4x^2}{9}}}{256x^4}\right.}{243\sqrt{\pi}\sqrt{\mathrm{signum}\left(-1+\frac{4x^2}{9}\right)}}$

input
```
int(1/x^5/(4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/108*(8*x^4-6*x^2-27)/x^4/(4*x^2-9)^(1/2)-2/81*arctan(3/(4*x^2-9)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{16\,x^4\arctan\left(-\frac{2}{3}\,x+\frac{1}{3}\,\sqrt{4\,x^2-9}\right)+3\,\sqrt{4\,x^2-9}(2\,x^2+3)}{324\,x^4}$$

input
```
integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/324*(16*x^4*arctan(-2/3*x + 1/3*sqrt(4*x^2 - 9)) + 3*sqrt(4*x^2 - 9)*(2*
x^2 + 3))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx$$
$$= \begin{cases} \frac{2i\operatorname{acosh}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{36x^3\sqrt{-1+\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{-1+\frac{9}{4x^2}}} & \text{for } \frac{1}{|x^2|} > \frac{4}{9} \\ -\frac{2\operatorname{asin}\left(\frac{3}{2x}\right)}{81} + \frac{1}{27x\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{36x^3\sqrt{1-\frac{9}{4x^2}}} - \frac{1}{8x^5\sqrt{1-\frac{9}{4x^2}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**5/(4*x**2-9)**(1/2),x)
```

output
```
Piecewise((2*I*acosh(3/(2*x))/81 - I/(27*x*sqrt(-1 + 9/(4*x**2))) + I/(36*
x**3*sqrt(-1 + 9/(4*x**2))) + I/(8*x**5*sqrt(-1 + 9/(4*x**2))), 1/Abs(x**2
) > 4/9), (-2*asin(3/(2*x))/81 + 1/(27*x*sqrt(1 - 9/(4*x**2))) - 1/(36*x**
3*sqrt(1 - 9/(4*x**2))) - 1/(8*x**5*sqrt(1 - 9/(4*x**2))), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{\sqrt{4\,x^2-9}}{54\,x^2} + \frac{\sqrt{4\,x^2-9}}{36\,x^4} - \frac{2}{81}\arcsin\left(\frac{3}{2\,|x|}\right)$$

input
```
integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/54*sqrt(4*x^2 - 9)/x^2 + 1/36*sqrt(4*x^2 - 9)/x^4 - 2/81*arcsin(3/2/abs(
x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{(4\,x^2-9)^{\frac{3}{2}}+15\,\sqrt{4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{4\,x^2-9}\right)$$

input
```
integrate(1/x^5/(4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/216*((4*x^2 - 9)^(3/2) + 15*sqrt(4*x^2 - 9))/x^4 + 2/81*arctan(1/3*sqrt(
4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{4\,x^2-9}}{3}\right)}{81}+\frac{\frac{10\,\sqrt{4\,x^2-9}}{9}+\frac{2\,(4\,x^2-9)^{3/2}}{27}}{72\,x^2+(4\,x^2-9)^2-81}$$

input
```
int(1/(x^5*(4*x^2 - 9)^(1/2)),x)
```

output
```
(2*atan((4*x^2 - 9)^(1/2)/3))/81 + ((10*(4*x^2 - 9)^(1/2))/9 + (2*(4*x^2 -
 9)^(3/2))/27)/(72*x^2 + (4*x^2 - 9)^2 - 81)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^5\sqrt{-9+4x^2}}\,dx = \frac{16atan\left(\frac{\sqrt{4x^2-9}}{3}+\frac{2x}{3}\right)x^4+6\sqrt{4x^2-9}\,x^2+9\sqrt{4x^2-9}}{324x^4}$$

input
```
int(1/x^5/(4*x^2-9)^(1/2),x)
```

output
```
(16*atan((sqrt(4*x**2 - 9) + 2*x)/3)*x**4 + 6*sqrt(4*x**2 - 9)*x**2 + 9*sq
rt(4*x**2 - 9))/(324*x**4)
```

3.575 $\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\frac{81}{64}\sqrt{-9-4x^2}-\frac{3}{32}\left(-9-4x^2\right)^{3/2}-\frac{1}{320}\left(-9-4x^2\right)^{5/2}$$

output
```
-81/64*(-4*x^2-9)^(1/2)-3/32*(-4*x^2-9)^(3/2)-1/320*(-4*x^2-9)^(5/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = \frac{1}{40}\sqrt{-9-4x^2}\left(-27+6x^2-2x^4\right)$$

input
```
Integrate[x^5/Sqrt[-9 - 4*x^2],x]
```

output
```
(Sqrt[-9 - 4*x^2]*(-27 + 6*x^2 - 2*x^4))/40
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{-4x^2-9}} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int \frac{x^4}{\sqrt{-4x^2-9}} dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2} \int \left(\frac{1}{16} \left(-4x^2-9\right)^{3/2} + \frac{9}{8} \sqrt{-4x^2-9} + \frac{81}{16\sqrt{-4x^2-9}}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2} \left(-\frac{1}{160} \left(-4x^2-9\right)^{5/2} - \frac{3}{16} \left(-4x^2-9\right)^{3/2} - \frac{81}{32} \sqrt{-4x^2-9}\right)$$

input
```
Int[x^5/Sqrt[-9 - 4*x^2],x]
```

output
```
((-81*Sqrt[-9 - 4*x^2])/32 - (3*(-9 - 4*x^2)^(3/2))/16 - (-9 - 4*x^2)^(5/2
)/160)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

method	result	size
trager	$\left(-\frac{1}{20}x^4+\frac{3}{20}x^2-\frac{27}{40}\right)\sqrt{-4x^2-9}$	23
gosper	$-\frac{\sqrt{-4x^2-9}\,(2x^4-6x^2+27)}{40}$	24
pseudoelliptic	$-\frac{\sqrt{-4x^2-9}\,(2x^4-6x^2+27)}{40}$	24
risch	$\frac{(4x^2+9)(2x^4-6x^2+27)}{40\sqrt{-4x^2-9}}$	31
orering	$\frac{(4x^2+9)(2x^4-6x^2+27)}{40\sqrt{-4x^2-9}}$	31
meijerg	$-\frac{243i\left(-\frac{16\sqrt{\pi}}{15}+\frac{\sqrt{\pi}\left(\frac{32}{27}x^4-\frac{32}{9}x^2+16\right)\sqrt{\frac{4x^2}{9}+1}}{15}\right)}{128\sqrt{\pi}}$	39
default	$-\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$	41

input
```
int(x^5/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/20*x^4+3/20*x^2-27/40)*(-4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{40}\left(2\,x^4-6\,x^2+27\right)\sqrt{-4\,x^2-9}$$

input
```
integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/40*(2*x^4 - 6*x^2 + 27)*sqrt(-4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\frac{x^4\sqrt{-4x^2-9}}{20}+\frac{3x^2\sqrt{-4x^2-9}}{20}-\frac{27\sqrt{-4x^2-9}}{40}$$

input
```
integrate(x**5/(-4*x**2-9)**(1/2),x)
```

output
```
-x**4*sqrt(-4*x**2 - 9)/20 + 3*x**2*sqrt(-4*x**2 - 9)/20 - 27*sqrt(-4*x**2
 - 9)/40
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{20}\sqrt{-4\,x^2-9}x^4+\frac{3}{20}\sqrt{-4\,x^2-9}x^2-\frac{27}{40}\sqrt{-4\,x^2-9}$$

input
```
integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(-4*x^2 - 9)*x^4 + 3/20*sqrt(-4*x^2 - 9)*x^2 - 27/40*sqrt(-4*x^2
 - 9)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{320}i\left(4x^2+9\right)^{\frac{5}{2}} + \frac{3}{32}i\left(4x^2+9\right)^{\frac{3}{2}} - \frac{81}{64}\sqrt{-4x^2-9}$$

input
```
integrate(x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
-1/320*I*(4*x^2 + 9)^(5/2) + 3/32*I*(4*x^2 + 9)^(3/2) - 81/64*sqrt(-4*x^2
- 9)
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = -\sqrt{-4x^2-9}\left(\frac{x^4}{20}-\frac{3x^2}{20}+\frac{27}{40}\right)$$

input
```
int(x^5/(- 4*x^2 - 9)^(1/2),x)
```

output
```
-(- 4*x^2 - 9)^(1/2)*(x^4/20 - (3*x^2)/20 + 27/40)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.48

$$\int \frac{x^5}{\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4x^2-9}\,(2x^4-6x^2+27)}{40}$$

input
```
int(x^5/(-4*x^2-9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 - 9)*(2*x**4 - 6*x**2 + 27))/40
```

3.576 $\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = \frac{27}{128}x\sqrt{-9-4x^2} - \frac{1}{16}x^3\sqrt{-9-4x^2} + \frac{243}{256}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
27/128*x*(-4*x^2-9)^(1/2)-1/16*x^3*(-4*x^2-9)^(1/2)+243/256*arctan(2*x/(-4
*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = \frac{1}{256}\left(2x\left(27-8x^2\right)\sqrt{-9-4x^2} + 243\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\right)$$

input
```
Integrate[x^4/Sqrt[-9 - 4*x^2],x]
```

output
```
(2*x*(27 - 8*x^2)*Sqrt[-9 - 4*x^2] + 243*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/2
56
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {262, 262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{27}{16}\int \frac{x^2}{\sqrt{-4x^2-9}}dx-\frac{1}{16}\sqrt{-4x^2-9}x^3$$

$$\Big\downarrow \text{262}$$

$$-\frac{27}{16}\left(-\frac{9}{8}\int \frac{1}{\sqrt{-4x^2-9}}dx-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3$$

$$\Big\downarrow \text{224}$$

$$-\frac{27}{16}\left(-\frac{9}{8}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3$$

$$\Big\downarrow \text{216}$$

$$-\frac{27}{16}\left(-\frac{9}{16}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{8}\sqrt{-4x^2-9}x\right)-\frac{1}{16}\sqrt{-4x^2-9}x^3$$

input
```
Int[x^4/Sqrt[-9 - 4*x^2],x]
```

output
```
-1/16*(x^3*Sqrt[-9 - 4*x^2]) - (27*(-1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[
(2*x)/Sqrt[-9 - 4*x^2]])/16))/16
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

method	result	size
meijerg	$-\frac{81 i \left(-\frac{\sqrt{\pi}\, x \left(-\frac{40 x^{2}}{9}+15\right) \sqrt{\frac{4 x^{2}}{9}+1}}{30}+\frac{3 \sqrt{\pi}\, \operatorname{arcsinh}\left(\frac{2 x}{3}\right)}{4}\right)}{64 \sqrt{\pi}}$	39
pseudoelliptic	$-\frac{243 \arctan\left(\frac{\sqrt{-4 x^{2}-9}}{2 x}\right)}{256}+\frac{\left(-8 x^{3}+27 x \right) \sqrt{-4 x^{2}-9}}{128}$	39
default	$\frac{27 x \sqrt{-4 x^{2}-9}}{128}-\frac{x^{3} \sqrt{-4 x^{2}-9}}{16}+\frac{243 \arctan\left(\frac{2 x}{\sqrt{-4 x^{2}-9}}\right)}{256}$	43
risch	$\frac{x \left(8 x^{2}-27\right) \left(4 x^{2}+9\right)}{128 \sqrt{-4 x^{2}-9}}+\frac{243 \arctan\left(\frac{2 x}{\sqrt{-4 x^{2}-9}}\right)}{256}$	43
trager	$-\frac{x \left(8 x^{2}-27\right) \sqrt{-4 x^{2}-9}}{128}-\frac{243 \operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-4 x^{2}-9}+2 x \right)}{256}$	51

input
```
int(x^4/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-81/64*I/Pi^(1/2)*(-1/30*Pi^(1/2)*x*(-40/9*x^2+15)*(4/9*x^2+1)^(1/2)+3/4*P
i^(1/2)*arcsinh(2/3*x))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{128}\left(8\,x^3-27\,x\right)\sqrt{-4\,x^2-9} + \frac{243}{512}i\,\log\left(-\frac{4\left(2\,x+i\,\sqrt{-4\,x^2-9}\right)}{x}\right) - \frac{243}{512}i\,\log\left(-\frac{4\left(2\,x-i\,\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input
```
integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/128*(8*x^3 - 27*x)*sqrt(-4*x^2 - 9) + 243/512*I*log(-4*(2*x + I*sqrt(-4
*x^2 - 9))/x) - 243/512*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = -\frac{x^3\sqrt{-4x^2-9}}{16} + \frac{27x\sqrt{-4x^2-9}}{128} + \frac{243\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{256}$$

input
```
integrate(x**4/(-4*x**2-9)**(1/2),x)
```

output
```
-x**3*sqrt(-4*x**2 - 9)/16 + 27*x*sqrt(-4*x**2 - 9)/128 + 243*atan(2*x/sqr
t(-4*x**2 - 9))/256
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.61

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{16}\sqrt{-4\,x^2-9}x^3+\frac{27}{128}\sqrt{-4\,x^2-9}x-\frac{243}{256}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/16*sqrt(-4*x^2 - 9)*x^3 + 27/128*sqrt(-4*x^2 - 9)*x - 243/256*I*arcsinh
(2/3*x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = \int \frac{x^4}{\sqrt{-4\,x^2-9}}\,dx$$

input
```
integrate(x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt(-4*x^2 - 9), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = \int \frac{x^4}{\sqrt{-4\,x^2-9}}\,dx$$

input
```
int(x^4/(- 4*x^2 - 9)^(1/2),x)
```

output
```
int(x^4/(- 4*x^2 - 9)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{x^4}{\sqrt{-9-4x^2}}\,dx = -\frac{243 asinh\left(\frac{2x}{3}\right) i}{256} + \frac{\sqrt{-4x^2-9}\,x^3}{16} - \frac{27\sqrt{-4x^2-9}\,x}{128}$$

input
```
int(x^4/(-4*x^2-9)^(1/2),x)
```

output
```
( - 243*asinh((2*x)/3)*i + 16*sqrt( - 4*x**2 - 9)*x**3 - 54*sqrt( - 4*x**2
 - 9)*x)/256
```

3.577 $\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 31

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx=\frac{9}{16}\sqrt{-9-4x^2}+\frac{1}{48}\left(-9-4x^2\right)^{3/2}$$

output
```
9/16*(-4*x^2-9)^(1/2)+1/48*(-4*x^2-9)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx=\frac{1}{24}\sqrt{-9-4x^2}\left(9-2x^2\right)$$

input
```
Integrate[x^3/Sqrt[-9 - 4*x^2],x]
```

output
```
(Sqrt[-9 - 4*x^2]*(9 - 2*x^2))/24
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{-4x^2-9}}dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{x^2}{\sqrt{-4x^2-9}}dx^2$$
$$\Big\downarrow \text{53}$$
$$\frac{1}{2}\int \left(-\frac{1}{4}\sqrt{-4x^2-9}-\frac{9}{4\sqrt{-4x^2-9}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{1}{24}\left(-4x^2-9\right)^{3/2}+\frac{9}{8}\sqrt{-4x^2-9}\right)$$

input
```
Int[x^3/Sqrt[-9 - 4*x^2],x]
```

output
```
((9*Sqrt[-9 - 4*x^2])/8 + (-9 - 4*x^2)^(3/2)/24)/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result	size
trager	$\left(-\frac{x^2}{12}+\frac{3}{8}\right)\sqrt{-4x^2-9}$	18
gosper	$-\frac{(2x^2-9)\sqrt{-4x^2-9}}{24}$	19
pseudoelliptic	$-\frac{(2x^2-9)\sqrt{-4x^2-9}}{24}$	19
risch	$\frac{(4x^2+9)(2x^2-9)}{24\sqrt{-4x^2-9}}$	26
orering	$\frac{(4x^2+9)(2x^2-9)}{24\sqrt{-4x^2-9}}$	26
default	$-\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$	27
meijerg	$-\frac{27i\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{16x^2}{9}+8\right)\sqrt{\frac{4x^2}{9}+1}}{6}\right)}{32\sqrt{\pi}}$	34

input
```
int(x^3/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/12*x^2+3/8)*(-4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{24}\,\left(2\,x^2-9\right)\sqrt{-4\,x^2-9}$$

input
```
integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/24*(2*x^2 - 9)*sqrt(-4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = -\frac{x^2\sqrt{-4x^2-9}}{12}+\frac{3\sqrt{-4x^2-9}}{8}$$

input
```
integrate(x**3/(-4*x**2-9)**(1/2),x)
```

output
```
-x**2*sqrt(-4*x**2 - 9)/12 + 3*sqrt(-4*x**2 - 9)/8
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{12}\,\sqrt{-4\,x^2-9}x^2+\frac{3}{8}\,\sqrt{-4\,x^2-9}$$

input
```
integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/12*sqrt(-4*x^2 - 9)*x^2 + 3/8*sqrt(-4*x^2 - 9)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{48}i\left(4\,x^2+9\right)^{\frac{3}{2}} + \frac{9}{16}\sqrt{-4\,x^2-9}$$

input
```
integrate(x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
-1/48*I*(4*x^2 + 9)^(3/2) + 9/16*sqrt(-4*x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = -\frac{(2\,x^2-9)\,\sqrt{-4\,x^2-9}}{24}$$

input
```
int(x^3/(- 4*x^2 - 9)^(1/2),x)
```

output
```
-((2*x^2 - 9)*(- 4*x^2 - 9)^(1/2))/24
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{x^3}{\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4x^2-9}\,(2x^2-9)}{24}$$

input
```
int(x^3/(-4*x^2-9)^(1/2),x)
```

output
```
(sqrt( - 4*x**2 - 9)*(2*x**2 - 9))/24
```

3.578 $\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 36

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx=-\frac{1}{8}x\sqrt{-9-4x^2}-\frac{9}{16}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
-1/8*x*(-4*x^2-9)^(1/2)-9/16*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx=\frac{1}{16}\left(-2x\sqrt{-9-4x^2}-9\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)\right)$$

input
```
Integrate[x^2/Sqrt[-9 - 4*x^2],x]
```

output
```
(-2*x*Sqrt[-9 - 4*x^2] - 9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{9}{8}\int \frac{1}{\sqrt{-4x^2-9}}dx-\frac{1}{8}\sqrt{-4x^2-9}x$$

$$\Big\downarrow \text{224}$$

$$-\frac{9}{8}\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}-\frac{1}{8}\sqrt{-4x^2-9}x$$

$$\Big\downarrow \text{216}$$

$$-\frac{9}{16}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)-\frac{1}{8}\sqrt{-4x^2-9}x$$

input
```
Int[x^2/Sqrt[-9 - 4*x^2],x]
```

output
```
-1/8*(x*Sqrt[-9 - 4*x^2]) - (9*ArcTan[(2*x)/Sqrt[-9 - 4*x^2]])/16
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{x\sqrt{-4x^2-9}}{8}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$	29
pseudoelliptic	$-\frac{x\sqrt{-4x^2-9}}{8}+\frac{9\arctan\left(\frac{\sqrt{-4x^2-9}}{2x}\right)}{16}$	31
meijerg	$-\frac{9i\left(\frac{2\sqrt{\pi}\,x\sqrt{\frac{4x^2}{9}+1}}{3}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{2x}{3}\right)\right)}{16\sqrt{\pi}}$	32
risch	$\frac{(4x^2+9)x}{8\sqrt{-4x^2-9}}-\frac{9\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$	36
trager	$-\frac{x\sqrt{-4x^2-9}}{8}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-4x^2-9}+2x\right)}{16}$	44

input
```
int(x^2/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*x*(-4*x^2-9)^(1/2)-9/16*arctan(2*x/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2-9}x - \frac{9}{32}i\,\log\left(-\frac{4\left(2\,x+i\sqrt{-4\,x^2-9}\right)}{x}\right) + \frac{9}{32}i\,\log\left(-\frac{4\left(2\,x-i\sqrt{-4\,x^2-9}\right)}{x}\right)$$

input
```
integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/8*sqrt(-4*x^2 - 9)*x - 9/32*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) + 9/
32*I*log(-4*(2*x - I*sqrt(-4*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = -\frac{x\sqrt{-4x^2-9}}{8} - \frac{9\,\mathrm{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{16}$$

input
```
integrate(x**2/(-4*x**2-9)**(1/2),x)
```

output
```
-x*sqrt(-4*x**2 - 9)/8 - 9*atan(2*x/sqrt(-4*x**2 - 9))/16
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{8}\sqrt{-4\,x^2-9}x + \frac{9}{16}i\ \text{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(-4*x^2 - 9)*x + 9/16*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = \int \frac{x^2}{\sqrt{-4\,x^2-9}}\,dx$$

input
```
integrate(x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(-4*x^2 - 9), x)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = -\frac{x\,\sqrt{-4\,x^2-9}}{8} + \frac{\ln\left(x-\frac{\sqrt{-4\,x^2-9}\,1\mathrm{i}}{2}\right)\,9\mathrm{i}}{16}$$

input
```
int(x^2/(- 4*x^2 - 9)^(1/2),x)
```

output
```
(log(x - ((- 4*x^2 - 9)^(1/2)*1i)/2)*9i)/16 - (x*(- 4*x^2 - 9)^(1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\sqrt{-9-4x^2}}\,dx = \frac{9asinh\left(\frac{2x}{3}\right)i}{16} + \frac{\sqrt{-4x^2-9}\,x}{8}$$

input
```
int(x^2/(-4*x^2-9)^(1/2),x)
```

output
```
(9*asinh((2*x)/3)*i + 2*sqrt( - 4*x**2 - 9)*x)/16
```

3.579 $\int \frac{x}{\sqrt{-9-4x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{x}{\sqrt{-9-4x^2}} \, dx = -\frac{1}{4}\sqrt{-9-4x^2}$$

output
```
-1/4*(-4*x^2-9)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{-9-4x^2}} \, dx = -\frac{1}{4}\sqrt{-9-4x^2}$$

input
```
Integrate[x/Sqrt[-9 - 4*x^2],x]
```

output
```
-1/4*Sqrt[-9 - 4*x^2]
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-4x^2-9}}\,dx$$

$$\Big\downarrow \; 241$$

$$-\frac{1}{4}\sqrt{-4x^2-9}$$

input
```
Int[x/Sqrt[-9 - 4*x^2],x]
```

output
```
-1/4*Sqrt[-9 - 4*x^2]
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{\sqrt{-4x^2-9}}{4}$	12
derivativedivides	$-\frac{\sqrt{-4x^2-9}}{4}$	12
default	$-\frac{\sqrt{-4x^2-9}}{4}$	12
trager	$-\frac{\sqrt{-4x^2-9}}{4}$	12
pseudoelliptic	$-\frac{\sqrt{-4x^2-9}}{4}$	12
risch	$\frac{4x^2+9}{4\sqrt{-4x^2-9}}$	19
orering	$\frac{4x^2+9}{4\sqrt{-4x^2-9}}$	19
meijerg	$-\frac{3i\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{\frac{4x^2}{9}+1}\right)}{8\sqrt{\pi}}$	27

input
```
int(x/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-4\,x^2-9}$$

input
```
integrate(x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-4*x^2 - 9)
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{\sqrt{-4x^2-9}}{4}$$

input
```
integrate(x/(-4*x**2-9)**(1/2),x)
```

output
```
-sqrt(-4*x**2 - 9)/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-4\,x^2-9}$$

input
```
integrate(x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/4*sqrt(-4*x^2 - 9)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{4}\sqrt{-4\,x^2-9}$$

input
```
integrate(x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-4*x^2 - 9)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2-9}}{4}$$

input
```
int(x/(- 4*x^2 - 9)^(1/2),x)
```

output
```
-(- 4*x^2 - 9)^(1/2)/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4x^2-9}}{4}$$

input
```
int(x/(-4*x^2-9)^(1/2),x)
```

output
```
sqrt( - 4*x**2 - 9)/4
```

3.580 $\int \frac{1}{\sqrt{-9-4x^2}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{1}{\sqrt{-9-4x^2}} \, dx = \frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

output
```
1/2*arctan(2*x/(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-9-4x^2}} \, dx = \frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-9-4x^2}}\right)$$

input
```
Integrate[1/Sqrt[-9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {224, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-4x^2-9}}dx$$
$$\Big\downarrow \text{224}$$
$$\int \frac{1}{\frac{4x^2}{-4x^2-9}+1}d\frac{x}{\sqrt{-4x^2-9}}$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}\arctan\left(\frac{2x}{\sqrt{-4x^2-9}}\right)$$

input
```
Int[1/Sqrt[-9 - 4*x^2],x]
```

output
```
ArcTan[(2*x)/Sqrt[-9 - 4*x^2]]/2
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

method	result	size
meijerg	$-\frac{i \operatorname{arcsinh}\left(\frac{2x}{3}\right)}{2}$	8
default	$\frac{\arctan\left(\frac{2x}{\sqrt{-4x^{2}-9}}\right)}{2}$	16
pseudoelliptic	$-\frac{\arctan\left(\frac{\sqrt{-4x^{2}-9}}{2x}\right)}{2}$	18
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right) \ln\left(-\operatorname{RootOf}\left(\_Z^{2}+1\right) \sqrt{-4x^{2}-9}+2x\right)}{2}$	31

input
```
int(1/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*arcsinh(2/3*x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{1}{4} i \log\left(-\frac{4\,(2\,x + i\,\sqrt{-4\,x^{2} - 9})}{x}\right) - \frac{1}{4} i \log\left(-\frac{4\,(2\,x - i\,\sqrt{-4\,x^{2} - 9})}{x}\right)$$

input
```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/4*I*log(-4*(2*x + I*sqrt(-4*x^2 - 9))/x) - 1/4*I*log(-4*(2*x - I*sqrt(-4
*x^2 - 9))/x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{2x}{\sqrt{-4x^2-9}}\right)}{2}$$

input
```
integrate(1/(-4*x**2-9)**(1/2),x)
```

output
```
atan(2*x/sqrt(-4*x**2 - 9))/2
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.32

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = -\frac{1}{2}i\,\operatorname{arsinh}\left(\frac{2}{3}\,x\right)$$

input
```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*I*arcsinh(2/3*x)
```

Giac [F]

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \int \frac{1}{\sqrt{-4\,x^2-9}}\,dx$$

input
```
integrate(1/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(-4*x^2 - 9), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = \frac{\operatorname{atan}\left(\frac{2\,x}{\sqrt{-4\,x^2-9}}\right)}{2}$$

input
```
int(1/(- 4*x^2 - 9)^(1/2),x)
```

output
```
atan((2*x)/(- 4*x^2 - 9)^(1/2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{-9-4x^2}}\,dx = -\frac{asinh\left(\frac{2x}{3}\right) i}{2}$$

input
```
int(1/(-4*x^2-9)^(1/2),x)
```

output
```
( - asinh((2*x)/3)*i)/2
```

3.581 $\int \frac{1}{x\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
1/3*arctan(1/3*(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[1/(x*Sqrt[-9 - 4*x^2]),x]
```

output
```
ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-4x^2-9}}\,dx$$
$$\Big\downarrow \text{243}$$
$$\frac{1}{2}\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2$$
$$\Big\downarrow \text{73}$$
$$-\frac{1}{4}\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}$$
$$\Big\downarrow \text{217}$$
$$\frac{1}{3}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)$$

input
```
Int[1/(x*Sqrt[-9 - 4*x^2]),x]
```

output
```
ArcTan[Sqrt[-9 - 4*x^2]/3]/3
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{3}$	15
pseudoelliptic	$\frac{\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)}{3}$	15
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\operatorname{RootOf}\left(\_Z^2+1\right)}{x}\right)}{3}$	32
meijerg	$-\frac{i\left((2\ln(x)-2\ln(3))\sqrt{\pi}-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)\right)}{6\sqrt{\pi}}$	40

input
```
int(1/x/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(3/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = -\frac{1}{6}i\log\left(-\frac{2\left(i\sqrt{-4x^2-9}+3\right)}{3x}\right) + \frac{1}{6}i\log\left(-\frac{2\left(-i\sqrt{-4x^2-9}+3\right)}{3x}\right)$$

input
```
integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*I*log(-2/3*(I*sqrt(-4*x^2 - 9) + 3)/x) + 1/6*I*log(-2/3*(-I*sqrt(-4*x
^2 - 9) + 3)/x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.40

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{3}$$

input
```
integrate(1/x/(-4*x**2-9)**(1/2),x)
```

output
```
I*asinh(3/(2*x))/3
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = -\frac{1}{3}i\,\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|}+\frac{18}{|x|}\right)$$

input
```
integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{1}{3}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

input
```
integrate(1/x/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/3*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{3}$$

input
```
int(1/(x*(- 4*x^2 - 9)^(1/2)),x)
```

output
```
atan((- 4*x^2 - 9)^(1/2)/3)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\sqrt{-9-4x^2}}\,dx = \frac{i\left(-\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}-1\right)+\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}+1\right)\right)}{3}$$

input
```
int(1/x/(-4*x^2-9)^(1/2),x)
```

output
```
(i*( - log((sqrt( - 4*x**2 - 9)*i + 2*x - 3)/3) + log((sqrt( - 4*x**2 - 9)
*i + 2*x + 3)/3)))/3
```

3.582 $\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx=\frac{\sqrt{-9-4x^2}}{9x}$$

output
```
1/9*(-4*x^2-9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx=\frac{\sqrt{-9-4x^2}}{9x}$$

input
```
Integrate[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

output
```
Sqrt[-9 - 4*x^2]/(9*x)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{-4x^2-9}}\,dx$$

$$\Big\downarrow \ 242$$

$$\frac{\sqrt{-4x^2-9}}{9x}$$

input
```
Int[1/(x^2*Sqrt[-9 - 4*x^2]),x]
```

output
```
Sqrt[-9 - 4*x^2]/(9*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{\sqrt{-4x^{2}-9}}{9x}$	15
default	$\frac{\sqrt{-4x^{2}-9}}{9x}$	15
trager	$\frac{\sqrt{-4x^{2}-9}}{9x}$	15
pseudoelliptic	$\frac{\sqrt{-4x^{2}-9}}{9x}$	15
meijerg	$\frac{i\sqrt{\frac{4x^{2}}{9}+1}}{3x}$	16
risch	$-\frac{4x^{2}+9}{9x\sqrt{-4x^{2}-9}}$	22
orering	$-\frac{4x^{2}+9}{9x\sqrt{-4x^{2}-9}}$	22

input
```
int(1/x^2/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9/x*(-4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx=\frac{\sqrt{-4\,x^2-9}}{9\,x}$$

input
```
integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(-4*x^2 - 9)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{2i\sqrt{1+\frac{9}{4x^2}}}{9}$$

input
```
integrate(1/x**2/(-4*x**2-9)**(1/2),x)
```

output
```
2*I*sqrt(1 + 9/(4*x**2))/9
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4\,x^2-9}}{9\,x}$$

input
```
integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(-4*x^2 - 9)/x
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \int \frac{1}{\sqrt{-4\,x^2-9}x^2}\,dx$$

input
```
integrate(1/x^2/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-4*x^2 - 9)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4\,x^2-9}}{9\,x}$$

input
```
int(1/(x^2*(- 4*x^2 - 9)^(1/2)),x)
```

output
```
(- 4*x^2 - 9)^(1/2)/(9*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\sqrt{-9-4x^2}}\,dx = \frac{-\sqrt{-4x^2-9}+2ix}{9x}$$

input
```
int(1/x^2/(-4*x^2-9)^(1/2),x)
```

output
```
( - sqrt( - 4*x**2 - 9) + 2*i*x)/(9*x)
```

3.583 $\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 39

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
1/18*(-4*x^2-9)^(1/2)/x^2-2/27*arctan(1/3*(-4*x^2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{18x^2} - \frac{2}{27}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

output
```
Sqrt[-9 - 4*x^2]/(18*x^2) - (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/27
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {243, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \textbf{243}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{-4x^2-9}}dx^2$$

$$\Big\downarrow \textbf{52}$$

$$\frac{1}{2}\left(\frac{\sqrt{-4x^2-9}}{9x^2}-\frac{2}{9}\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{1}{9}\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}+\frac{\sqrt{-4x^2-9}}{9x^2}\right)$$

$$\Big\downarrow \textbf{217}$$

$$\frac{1}{2}\left(\frac{\sqrt{-4x^2-9}}{9x^2}-\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)\right)$$

input
```
Int[1/(x^3*Sqrt[-9 - 4*x^2]),x]
```

output
```
(Sqrt[-9 - 4*x^2]/(9*x^2) - (4*ArcTan[Sqrt[-9 - 4*x^2]/3])/27)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$	30
pseudoelliptic	$\frac{-4\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)x^2+3\sqrt{-4x^2-9}}{54x^2}$	35
risch	$-\frac{4x^2+9}{18x^2\sqrt{-4x^2-9}}+\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{27}$	37
trager	$\frac{\sqrt{-4x^2-9}}{18x^2}+\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)}{27}$	47
meijerg	$-\frac{2i\left(-\frac{9\sqrt{\pi}}{4x^2}-\frac{(1+2\ln(x)-2\ln(3))\sqrt{\pi}}{2}+\frac{9\sqrt{\pi}\left(\frac{16x^2}{9}+8\right)}{32x^2}-\frac{9\sqrt{\pi}\sqrt{\frac{4x^2}{9}+1}}{4x^2}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+1}}{2}\right)\right)}{27\sqrt{\pi}}$	81

input
```
int(1/x^3/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(-4*x^2-9)^(1/2)/x^2+2/27*arctan(3/(-4*x^2-9)^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{-2i\,x^2\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+2i\,x^2\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}-3\right)}{27\,x}\right)+3\sqrt{-4\,x^2-9}}{54\,x^2}$$

input
```
integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/54*(-2*I*x^2*log(-4/27*(I*sqrt(-4*x^2 - 9) - 3)/x) + 2*I*x^2*log(-4/27*(
-I*sqrt(-4*x^2 - 9) - 3)/x) + 3*sqrt(-4*x^2 - 9))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = -\frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{27} + \frac{i}{9x\sqrt{1+\frac{9}{4x^2}}} + \frac{i}{4x^3\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate(1/x**3/(-4*x**2-9)**(1/2),x)
```

output
```
-2*I*asinh(3/(2*x))/27 + I/(9*x*sqrt(1 + 9/(4*x**2))) + I/(4*x**3*sqrt(1 +
 9/(4*x**2)))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4\,x^2-9}}{18\,x^2} + \frac{2}{27}i\,\log\left(\frac{6\,\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
1/18*sqrt(-4*x^2 - 9)/x^2 + 2/27*I*log(6*sqrt(4*x^2 + 9)/abs(x) + 18/abs(x
))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4\,x^2-9}}{18\,x^2} - \frac{2}{27}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

input
```
integrate(1/x^3/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/18*sqrt(-4*x^2 - 9)/x^2 - 2/27*arctan(1/3*sqrt(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-4\,x^2-9}}{18\,x^2} - \frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{27}$$

input
```
int(1/(x^3*(- 4*x^2 - 9)^(1/2)),x)
```

output
```
(- 4*x^2 - 9)^(1/2)/(18*x^2) - (2*atan((- 4*x^2 - 9)^(1/2)/3))/27
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.59

$$\int \frac{1}{x^3\sqrt{-9-4x^2}}\,dx = \frac{-3\sqrt{-4x^2-9}+4\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}-1\right)i\,x^2-4\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}+1\right)i\,x^2}{54x^2}$$

input
```
int(1/x^3/(-4*x^2-9)^(1/2),x)
```

output

```
( - 3*sqrt( - 4*x**2 - 9) + 4*log((sqrt( - 4*x**2 - 9)*i + 2*x - 3)/3)*i*x
**2 - 4*log((sqrt( - 4*x**2 - 9)*i + 2*x + 3)/3)*i*x**2)/(54*x**2)
```

3.584 $\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{27x^3} - \frac{8\sqrt{-9-4x^2}}{243x}$$

output
```
1/27*(-4*x^2-9)^(1/2)/x^3-8/243*(-4*x^2-9)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = \frac{(9-8x^2)\sqrt{-9-4x^2}}{243x^3}$$

input
```
Integrate[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

output
```
((9 - 8*x^2)*Sqrt[-9 - 4*x^2])/(243*x^3)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{245}$$

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8}{27}\int \frac{1}{x^2\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{242}$$

$$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$$

input
```
Int[1/(x^4*Sqrt[-9 - 4*x^2]),x]
```

output
```
Sqrt[-9 - 4*x^2]/(27*x^3) - (8*Sqrt[-9 - 4*x^2])/(243*x)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$	22
trager	$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$	22
pseudoelliptic	$-\frac{(8x^2-9)\sqrt{-4x^2-9}}{243x^3}$	22
meijerg	$\frac{i\left(1-\frac{8x^2}{9}\right)\sqrt{\frac{4x^2}{9}+1}}{9x^3}$	23
risch	$\frac{32x^4+36x^2-81}{243x^3\sqrt{-4x^2-9}}$	27
orering	$\frac{(4x^2+9)(8x^2-9)}{243x^3\sqrt{-4x^2-9}}$	29
default	$\frac{\sqrt{-4x^2-9}}{27x^3}-\frac{8\sqrt{-4x^2-9}}{243x}$	30

input
```
int(1/x^4/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/243/x^3*(8*x^2-9)*(-4*x^2-9)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = -\frac{(8\,x^2-9)\sqrt{-4\,x^2-9}}{243\,x^3}$$

input
```
integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
-1/243*(8*x^2 - 9)*sqrt(-4*x^2 - 9)/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = -\frac{16i\sqrt{1+\frac{9}{4x^2}}}{243}+\frac{2i\sqrt{1+\frac{9}{4x^2}}}{27x^2}$$

input
```
integrate(1/x**4/(-4*x**2-9)**(1/2),x)
```

output
```
-16*I*sqrt(1 + 9/(4*x**2))/243 + 2*I*sqrt(1 + 9/(4*x**2))/(27*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = -\frac{8\sqrt{-4\,x^2-9}}{243\,x}+\frac{\sqrt{-4\,x^2-9}}{27\,x^3}$$

input
```
integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-8/243*sqrt(-4*x^2 - 9)/x + 1/27*sqrt(-4*x^2 - 9)/x^3
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = \int \frac{1}{\sqrt{-4\,x^2-9}x^4}\,dx$$

input
```
integrate(1/x^4/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-4*x^2 - 9)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = -\frac{8\,x^2\,\sqrt{-4\,x^2-9}-9\,\sqrt{-4\,x^2-9}}{243\,x^3}$$

input
```
int(1/(x^4*(- 4*x^2 - 9)^(1/2)),x)
```

output
```
-(8*x^2*(- 4*x^2 - 9)^(1/2) - 9*(- 4*x^2 - 9)^(1/2))/(243*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^4\sqrt{-9-4x^2}}\,dx = \frac{8\sqrt{-4x^2-9}\,x^2-9\sqrt{-4x^2-9}+16i\,x^3}{243x^3}$$

input
```
int(1/x^4/(-4*x^2-9)^(1/2),x)
```

output
```
(8*sqrt( - 4*x**2 - 9)*x**2 - 9*sqrt( - 4*x**2 - 9) + 16*i*x**3)/(243*x**3
)
```

3.585 $\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 57

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}}{36x^4} - \frac{\sqrt{-9-4x^2}}{54x^2} + \frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

output
```
1/36*(-4*x^2-9)^(1/2)/x^4-1/54*(-4*x^2-9)^(1/2)/x^2+2/81*arctan(1/3*(-4*x^
2-9)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{\sqrt{-9-4x^2}(3-2x^2)}{108x^4} + \frac{2}{81}\arctan\left(\frac{1}{3}\sqrt{-9-4x^2}\right)$$

input
```
Integrate[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

output
```
(Sqrt[-9 - 4*x^2]*(3 - 2*x^2))/(108*x^4) + (2*ArcTan[Sqrt[-9 - 4*x^2]/3])/
81
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {243, 52, 52, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{-4x^2-9}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{-4x^2-9}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{\sqrt{-4x^2-9}}{18x^4}-\frac{1}{3}\int \frac{1}{x^4\sqrt{-4x^2-9}}dx^2\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{2}{9}\int \frac{1}{x^2\sqrt{-4x^2-9}}dx^2-\frac{\sqrt{-4x^2-9}}{9x^2}\right)+\frac{\sqrt{-4x^2-9}}{18x^4}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(-\frac{1}{9}\int \frac{1}{-\frac{x^4}{4}-\frac{9}{4}}d\sqrt{-4x^2-9}-\frac{\sqrt{-4x^2-9}}{9x^2}\right)+\frac{\sqrt{-4x^2-9}}{18x^4}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}\left(\frac{4}{27}\arctan\left(\frac{1}{3}\sqrt{-4x^2-9}\right)-\frac{\sqrt{-4x^2-9}}{9x^2}\right)+\frac{\sqrt{-4x^2-9}}{18x^4}\right)$$

input
```
Int[1/(x^5*Sqrt[-9 - 4*x^2]),x]
```

output
```
(Sqrt[-9 - 4*x^2]/(18*x^4) + (-1/9*Sqrt[-9 - 4*x^2]/x^2 + (4*ArcTan[Sqrt[-
9 - 4*x^2]/3])/27)/3)/2
```

Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
risch	$\frac{8x^4+6x^2-27}{108x^4\sqrt{-4x^2-9}}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}$
default	$\frac{\sqrt{-4x^2-9}}{36x^4}-\frac{\sqrt{-4x^2-9}}{54x^2}-\frac{2\arctan\left(\frac{3}{\sqrt{-4x^2-9}}\right)}{81}$
pseudoelliptic	$\frac{8\arctan\left(\frac{\sqrt{-4x^2-9}}{3}\right)x^4-6x^2\sqrt{-4x^2-9}+9\sqrt{-4x^2-9}}{324x^4}$
trager	$-\frac{(2x^2-3)\sqrt{-4x^2-9}}{108x^4}-\frac{2\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{\sqrt{-4x^2-9}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)}{81}$
meijerg	$-\frac{8i\left(-\frac{81\sqrt{\pi}}{32x^4}+\frac{9\sqrt{\pi}}{8x^2}+\frac{3\left(\frac{7}{6}+2\ln(x)-2\ln(3)\right)\sqrt{\pi}}{8}+\frac{81\sqrt{\pi}\left(-\frac{112}{81}x^4-\frac{32}{9}x^2+8\right)}{256x^4}-\frac{81\sqrt{\pi}\left(-\frac{16x^2}{3}+8\right)\sqrt{\frac{4x^2}{9}+1}}{256x^4}-\frac{3\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{\frac{4x^2}{9}+}}{2}\right.}{4}\right.}{243\sqrt{\pi}}$

input
```
int(1/x^5/(-4*x^2-9)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/108*(8*x^4+6*x^2-27)/x^4/(-4*x^2-9)^(1/2)-2/81*arctan(3/(-4*x^2-9)^(1/2)
)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{-4i\,x^4\log\left(-\frac{4\left(i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)+4i\,x^4\log\left(-\frac{4\left(-i\sqrt{-4\,x^2-9}+3\right)}{81\,x}\right)-3\left(2\,x^2-3\right)\sqrt{-4\,x^2-9}}{324\,x^4}$$

input
```
integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="fricas")
```

output
```
1/324*(-4*I*x^4*log(-4/81*(I*sqrt(-4*x^2 - 9) + 3)/x) + 4*I*x^4*log(-4/81*
(-I*sqrt(-4*x^2 - 9) + 3)/x) - 3*(2*x^2 - 3)*sqrt(-4*x^2 - 9))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.86 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{2i\,\mathrm{asinh}\left(\frac{3}{2x}\right)}{81} - \frac{i}{27x\sqrt{1+\frac{9}{4x^2}}} - \frac{i}{36x^3\sqrt{1+\frac{9}{4x^2}}} + \frac{i}{8x^5\sqrt{1+\frac{9}{4x^2}}}$$

input
```
integrate(1/x**5/(-4*x**2-9)**(1/2),x)
```

output
```
2*I*asinh(3/(2*x))/81 - I/(27*x*sqrt(1 + 9/(4*x**2))) - I/(36*x**3*sqrt(1
+ 9/(4*x**2))) + I/(8*x**5*sqrt(1 + 9/(4*x**2)))
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = -\frac{\sqrt{-4\,x^2-9}}{54\,x^2} + \frac{\sqrt{-4\,x^2-9}}{36\,x^4} - \frac{2}{81}i\,\log\left(\frac{6\sqrt{4\,x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

input
```
integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="maxima")
```

output
```
-1/54*sqrt(-4*x^2 - 9)/x^2 + 1/36*sqrt(-4*x^2 - 9)/x^4 - 2/81*I*log(6*sqrt
(4*x^2 + 9)/abs(x) + 18/abs(x))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{-i\,(4\,x^2+9)^{\frac{3}{2}}+15\,\sqrt{-4\,x^2-9}}{216\,x^4}+\frac{2}{81}\,\arctan\left(\frac{1}{3}\,\sqrt{-4\,x^2-9}\right)$$

input
```
integrate(1/x^5/(-4*x^2-9)^(1/2),x, algorithm="giac")
```

output
```
1/216*(-I*(4*x^2 + 9)^(3/2) + 15*sqrt(-4*x^2 - 9))/x^4 + 2/81*arctan(1/3*s
qrt(-4*x^2 - 9))
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\sqrt{-4\,x^2-9}}{3}\right)}{81}-\frac{\frac{10\,\sqrt{-4\,x^2-9}}{9}+\frac{2\,{\left(-4\,x^2-9\right)}^{3/2}}{27}}{72\,x^2-{\left(4\,x^2+9\right)}^2+81}$$

input
```
int(1/(x^5*(- 4*x^2 - 9)^(1/2)),x)
```

output
```
(2*atan((- 4*x^2 - 9)^(1/2)/3))/81 - ((10*(- 4*x^2 - 9)^(1/2))/9 + (2*(- 4
*x^2 - 9)^(3/2))/27)/(72*x^2 - (4*x^2 + 9)^2 + 81)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.32

$$\int \frac{1}{x^5\sqrt{-9-4x^2}}\,dx = \frac{6\sqrt{-4x^2-9}\,x^2-9\sqrt{-4x^2-9}-8\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}-1\right)i\,x^4+8\log\left(\frac{\sqrt{-4x^2-9}\,i}{3}+\frac{2x}{3}+1\right)i\,x^4}{324x^4}$$

input
```
int(1/x^5/(-4*x^2-9)^(1/2),x)
```

output
```
(6*sqrt( - 4*x**2 - 9)*x**2 - 9*sqrt( - 4*x**2 - 9) - 8*log((sqrt( - 4*x**
2 - 9)*i + 2*x - 3)/3)*i*x**4 + 8*log((sqrt( - 4*x**2 - 9)*i + 2*x + 3)/3)
*i*x**4)/(324*x**4)
```

3.586 $\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = -\frac{20a^2c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{4ac(cx)^{5/2}\sqrt{a+bx^2}}{77b} + \frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c} + \frac{10a^{11/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}$$

output
```
-20/231*a^2*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2+4/77*a*c*(c*x)^(5/2)*(b*x^
2+a)^(1/2)/b+2/11*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c+10/231*a^(11/4)*c^(7/2)*(a
^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(9/4)/(b*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.56

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \frac{2c^3\sqrt{cx}\sqrt{a+bx^2}\left(\sqrt{1+\frac{bx^2}{a}}(-5a^2+2abx^2+7b^2x^4)+5a^2\,\text{Hypergeometric2F1}\left(\right.\right.}{77b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2
+ 7*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77
*b^2*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{11}a\int \frac{(cx)^{7/2}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{7b}\right)$$

$$\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}$$

input
```
Int[(c*x)^(7/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*(c*x)^(9/2)*Sqrt[a + b*x^2])/(11*c) + (2*a*((2*c*(c*x)^(5/2)*Sqrt[a + b
*x^2])/(7*b) - (5*a*c^2*((2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*
Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sq
rt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1
/2])/(3*b^(5/4)*Sqrt[a + b*x^2]))))/(7*b)))/11
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

method	result
default	$\frac{2c^3\sqrt{cx}\left(21b^4x^7+5\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^3+27a\,b^3x^5-4a^2b^2x^3-10a^3bx\right)}{231x\sqrt{b\,x^2+a}\,b^3}$
risch	$-\frac{2(-21b^2x^4-6ab\,x^2+10a^2)x\sqrt{b\,x^2+a}\,c^4}{231b^2\sqrt{cx}}+\frac{10a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{231b^3\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2c^3x^4\sqrt{bc\,x^3+acx}}{11}+\frac{4a\,c^3x^2\sqrt{bc\,x^3+acx}}{77b}-\frac{20a^2c^3\sqrt{bc\,x^3+acx}}{231b^2}+\frac{10a^3c^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-}}{231b^3\sqrt{bc\,x^3+ac}}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/231*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(21*b^4*x^7+5*2^(1/2)*((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(
-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*(-a*b)^(1/2)*a^3+27*a*b^3*x^5-4*a^2*b^2*x^3-10*a^3*b*x)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.41

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx=\frac{2\left(10\,\sqrt{bc}a^3c^3\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+\left(21\,b^3c^3x^4+6\,ab^2c^3x^2-10\,a^2bc^3\right)\right.}{231\,b^3}$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/231*(10*sqrt(b*c)*a^3*c^3*weierstrassPInverse(-4*a/b, 0, x) + (21*b^3*c^
3*x^4 + 6*a*b^2*c^3*x^2 - 10*a^2*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.25

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/4))
```

Maxima [F]

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

Giac [F]

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx=\int (c\,x)^{7/2}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(7/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(7/2)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{7/2}\sqrt{a+bx^2}\,dx=\frac{2\sqrt{c}\,c^3\Big(-10\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2+6\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2+21\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4+5\Big(\int}{231b^2}$$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(c)*c**3*( - 10*sqrt(x)*sqrt(a + b*x**2)*a**2 + 6*sqrt(x)*sqrt(a +
b*x**2)*a*b*x**2 + 21*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 + 5*int((sqrt(x)*
sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**3))/(231*b**2)
```

3.587 $\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx=\frac{4ac\sqrt{cx}\sqrt{a+bx^2}}{21b}+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}-\frac{2a^{7/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{5/4}\sqrt{a+bx^2}}$$

output
```
4/21*a*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b+2/7*(c*x)^(5/2)*(b*x^2+a)^(1/2)/c-2
/21*a^(7/4)*c^(3/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^
(1/2))/b^(5/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.95 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.56

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx=\frac{2c\sqrt{cx}\sqrt{a+bx^2}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{7b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hyperg
eometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{7}a\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}$$

input
```
Int[(c*x)^(3/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*(c*x)^(5/2)*Sqrt[a + b*x^2])/(7*c) + (2*a*((2*c*Sqrt[c*x]*Sqrt[a + b*x^
2])/(3*b) - (a^(3/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2
*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/
(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/7
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.90

method	result
default	$-\frac{2c\sqrt{cx}\left(\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2-3b^3x^5-5a\,b^2x^3-2a^2bx\right)}{21x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2(3b\,x^2+2a)x\sqrt{b\,x^2+a}\,c^2}{21b\sqrt{cx}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)c^2\sqrt{cx(b\,x^2}}{21b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2c\,x^2\sqrt{bc\,x^3+acx}}{7}+\frac{4ac\sqrt{bc\,x^3+acx}}{21b}-\frac{2a^2c^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-}}{b}\right.}{\sqrt{-a}}}\right.}{21b^2\sqrt{bc\,x^3+acx}}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*((-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a^2-3*b^3*x^5-5*a*b^2*x^3-2*a^2*b*x)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx = -\frac{2\left(2\sqrt{bc}a^2c\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(3b^2cx^2+2abc)\sqrt{bx^2+a}\sqrt{cx}\right)}{21b^2}$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/21*(2*sqrt(b*c)*a^2*c*weierstrassPInverse(-4*a/b, 0, x) - (3*b^2*c*x^2
+ 2*a*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(9/4))
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx=\int (c\,x)^{3/2}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(3/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/2)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{3/2}\sqrt{a+bx^2}\,dx=\frac{2\sqrt{c}\,c\left(2\sqrt{x}\,\sqrt{b\,x^2+a}\,a+3\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2-\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2\right)}{21b}$$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(c)*c*(2*sqrt(x)*sqrt(a + b*x**2)*a + 3*sqrt(x)*sqrt(a + b*x**2)*b*
x**2 - int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2))/(21*b)
```

3.588 $\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c} + \frac{2a^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

output

```
2/3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+2/3*a^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+
a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/
2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

output
```
(2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/(Sqr
t[c*x]*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{2}{3}a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4a\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3c}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2a^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}$$

input
```
Int[Sqrt[a + b*x^2]/Sqrt[c*x],x]
```

output
```
(2*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*c) + (2*a^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)
*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[
(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*c^(3/2)*Sqrt[a +
b*x^2])
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a}{3}+\frac{2b^{2}x^{3}}{3}+\frac{2abx}{3}}{\sqrt{b\,x^{2}+a}\,\sqrt{cx}\,b}$	119
risch	$\frac{2x\sqrt{b\,x^{2}+a}}{3\sqrt{cx}}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx\left(b\,x^{2}+a\right)}}{3b\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	156
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\,\left(\frac{2\sqrt{bc\,x^{3}+acx}}{3c}+\frac{2a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b\sqrt{bc\,x^{3}+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	158

input
```
int((b*x^2+a)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a+b^2*x^3+
a*b*x)/(c*x)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.33

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\left(2\sqrt{bc}a\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\sqrt{cx}b\right)}{3\,bc}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(b*c)*a*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*sqr
t(c*x)*b)/(b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(1/2),x)
```

output
```
sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{c\,x}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a\right)}{3c}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(1/2),x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**2))/(a*x
 + b*x**3),x)*a))/(3*c)
```

3.589 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} \, dx = -\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}c^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*(b*x^2+a)^(1/2)/c/(c*x)^(3/2)+2/3*b^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2
+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(1/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}} \, dx = -\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(3
*(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {247, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3c^2}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3c^3}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(5/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(3*c*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c
*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(7/2)*Sqrt[a
 + b*x^2])
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x}{3}-\frac{2b\,x^{2}}{3}-\frac{2a}{3}}{\sqrt{b\,x^{2}+a}\,x\,c^{2}\sqrt{cx}}$	120
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{3x\,c^{2}\sqrt{cx}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^{2}+a)}}{3\sqrt{bc\,x^{3}+acx}\,c^{2}\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	160
elliptic	$\frac{\sqrt{cx(b\,x^{2}+a)}\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{3c^{3}x^{2}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3c^{2}\sqrt{bc\,x^{3}+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	160

input
```
int((b*x^2+a)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x-b*x^2-
a)/c^2/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx=\frac{2\left(2\sqrt{bc}x^2\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\sqrt{bx^2+a}\sqrt{cx}\right)}{3\,c^3x^2}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(b*c)*x^2*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^2 + a)*s
qrt(c*x))/(c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(5/2),x)
```

output
```
sqrt(a)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{5/2}} \,d x$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/2}}\,dx=-\frac{2\sqrt{c}\,\left(\sqrt{b\,x^{2}+a}+\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{2}+a}}{b\,x^{5}+a\,x^{3}}d x \right) a x \right)}{\sqrt{x}\,c^{3} x}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(5/2),x)
```

output
```
( - 2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(
a*x**3 + b*x**5),x)*a*x))/(sqrt(x)*c**3*x)
```

3.590 $\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}} - \frac{4b\sqrt{a+bx^2}}{21ac^3(cx)^{3/2}} - \frac{2b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21a^{5/4}c^{9/2}\sqrt{a+bx^2}}$$

output
```
-2/7*(b*x^2+a)^(1/2)/c/(c*x)^(7/2)-4/21*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(3/2
)-2/21*b^(7/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)
*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2)
)/a^(5/4)/c^(9/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(9/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-7/4, -1/2, -3/4, -((b*x^2)/a)])/(
7*(c*x)^(9/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {247, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\left(-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(9/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(7*c*(c*x)^(7/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(3*a*c
*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2
)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(
1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(7/2)*Sqrt[a + b*x^2])))/(7*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88

method	result
default	$-\frac{2\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^{3}+2b^{2}x^{4}+5ab\,x^{2}+3a^{2}\right)}{21\sqrt{b\,x^{2}+a}\,x^{3}c^{4}\sqrt{cx}\,a}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,(2b\,x^{2}+3a)}{21x^{3}a\,c^{4}\sqrt{cx}}-\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^{2}+a)}}{21a\sqrt{bc\,x^{3}+acx}\,c^{4}\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^{2}+a)}\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{7c^{5}x^{4}}-\frac{4b\sqrt{bc\,x^{3}+acx}}{21a\,c^{5}x^{2}}-\frac{2b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21a\,c^{4}\sqrt{bc\,x^{3}+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/21/(b*x^2+a)^(1/2)/x^3*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^
3+2*b^2*x^4+5*a*b*x^2+3*a^2)/c^4/(c*x)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2\left(2\sqrt{bc}bx^4\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(2bx^2+3a)\sqrt{bx^2+a}\sqrt{cx}\right)}{21ac^5x^4}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
-2/21*(2*sqrt(b*c)*b*x^4*weierstrassPInverse(-4*a/b, 0, x) + (2*b*x^2 + 3*
a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.74 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, -\frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(9/2),x)
```

output
```
sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(9/2)*x**(7/2)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx=\int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{(c\,x)}^{9/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(9/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{9/2}}\,dx=-\frac{2\sqrt{c}\left(\sqrt{b\,x^2+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^7+a\,x^5}dx\right)a\,x^3\right)}{5\sqrt{x}\,c^5x^3}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(9/2),x)
```

output
```
( - 2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(
a*x**5 + b*x**7),x)*a*x**3))/(5*sqrt(x)*c**5*x**3)
```

3.591 $\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx=-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}-\frac{4b\sqrt{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{20b^2\sqrt{a+bx^2}}{231a^2c^5(cx)^{3/2}}+\frac{10b^{11/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231a^{9/4}c^{13/2}\sqrt{a+bx^2}}$$

output
```
-2/11*(b*x^2+a)^(1/2)/c/(c*x)^(11/2)-4/77*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(7
/2)+20/231*b^2*(b*x^2+a)^(1/2)/a^2/c^5/(c*x)^(3/2)+10/231*b^(11/4)*(a^(1/2
)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(9/4)/c^(13/2)/(b*
x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.30

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx=-\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{2},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(13/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((b*x^2)/a)])/
(11*(c*x)^(13/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {247, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{(cx)^{9/2}\sqrt{bx^2+a}}dx}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{5b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{5b\left(-\frac{b\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\left(-\frac{5b\left(-\frac{2b\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2b\left(-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(13/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(11*c*(c*x)^(11/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(7*a
*c*(c*x)^(7/2)) - (5*b*((-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4
)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*
c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(
3*a^(5/4)*c^(7/2)*Sqrt[a + b*x^2])))/(7*a*c^2)))/(11*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.81

method	result
default	$\frac{\frac{10\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b^2x^5}{231}+\frac{20b^3x^6}{231}+\frac{8a\,b^2x^4}{231}-\frac{18a^2b\,x^2}{77}-\frac{2a^3}{11}}{\sqrt{b\,x^2+a}\,x^5c^6\sqrt{cx}\,a^2}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-10b^2x^4+6ab\,x^2+21a^2)}{231x^5a^2c^6\sqrt{cx}}+\frac{10b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{231a^2\sqrt{bc\,x^3+acx}\,c^6\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{11c^7x^6}-\frac{4b\sqrt{bc\,x^3+acx}}{77a\,c^7x^4}+\frac{20b^2\sqrt{bc\,x^3+acx}}{231a^2c^7x^2}+\frac{10b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\right.}{231a^2c^6\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
2/231/(b*x^2+a)^(1/2)/x^5*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Elli
pticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b^
2*x^5+10*b^3*x^6+4*a*b^2*x^4-27*a^2*b*x^2-21*a^3)/c^6/(c*x)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx=\frac{2\left(10\sqrt{bc}b^2x^6\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(10\,b^2x^4-6\,abx^2-21\,a^2)\sqrt{bx^2+a}\sqrt{cx}\right.}{231\,a^2c^7x^6}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(13/2),x, algorithm="fricas")
```

output
```
2/231*(10*sqrt(b*c)*b^2*x^6*weierstrassPInverse(-4*a/b, 0, x) + (10*b^2*x^
4 - 6*a*b*x^2 - 21*a^2)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*c^7*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 157.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{11}{4}, -\frac{1}{2} \\ -\frac{7}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{13}{2}}x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(13/2),x)
```

output
```
sqrt(a)*gamma(-11/4)*hyper((-11/4, -1/2), (-7/4,), b*x**2*exp_polar(I*pi)/
a)/(2*c**(13/2)*x**(11/2)*gamma(-7/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(13/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx=\int \frac{\sqrt{b\,x^2+a}}{{\left(c\,x\right)}^{13/2}} \,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(13/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{13/2}}\,dx=-\frac{2\sqrt{c}\,\left(\sqrt{b\,x^{2}+a}+\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^{2}+a}}{b\,x^{9}+a\,x^{7}}d x \right) a\,x^{5}\right)}{9\sqrt{x}\,c^{7} x^{5}}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(13/2),x)
```

output
```
( - 2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(
a*x**7 + b*x**9),x)*a*x**5))/(9*sqrt(x)*c**7*x**5)
```

3.592 $\int (cx)^{9/2}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = -\frac{28a^2c^3(cx)^{3/2}\sqrt{a+bx^2}}{585b^2} + \frac{4ac(cx)^{7/2}\sqrt{a+bx^2}}{117b} + \frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c} + \frac{28a^3c^4\sqrt{cx}\sqrt{a+bx^2}}{195b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{28a^{13/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{195b^{11/4}\sqrt{a+bx^2}} + \frac{14a^{13/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{195b^{11/4}\sqrt{a+bx^2}}$$

output
```
-28/585*a^2*c^3*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b^2+4/117*a*c*(c*x)^(7/2)*(b*x
^2+a)^(1/2)/b+2/13*(c*x)^(11/2)*(b*x^2+a)^(1/2)/c+28/195*a^3*c^4*(c*x)^(1/
2)*(b*x^2+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x)-28/195*a^(13/4)*c^(9/2)*(a^
(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b^(11/4)/(b*x^2+a
)^(1/2)+14/195*a^(13/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(
1/2)),1/2*2^(1/2))/b^(11/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.31

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \frac{2c^3(cx)^{3/2}\sqrt{a+bx^2}\left(\sqrt{1+\frac{bx^2}{a}}(-7a^2+2abx^2+9b^2x^4)+7a^2\,\text{Hypergeometric2F1}\right.}{117b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(9/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*c^3*(c*x)^(3/2)*Sqrt[a + b*x^2]*(Sqrt[1 + (b*x^2)/a]*(-7*a^2 + 2*a*b*x^
2 + 9*b^2*x^4) + 7*a^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(
117*b^2*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {248, 262, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{13}a\int \frac{(cx)^{9/2}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\int \frac{(cx)^{5/2}}{\sqrt{bx^2+a}}dx}{9b}\right)+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ac^2\int\frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}\right)}{9b}\right)+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$\downarrow$ 266

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}\right)}{9b}\right)+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$\downarrow$ 834

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{9b}\right)+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$\downarrow$ 27

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{9b}\right)+\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$\downarrow$ 761

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\right.}{5b}\right.}{9b}\right.$$

$$\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

$\downarrow$ 1510

$$\frac{2}{13}a\left(\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\right.}{}\right)}{9b}\right)$$

$$\frac{2(cx)^{11/2}\sqrt{a+bx^2}}{13c}$$

input
```
Int[(c*x)^(9/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*(c*x)^(11/2)*Sqrt[a + b*x^2])/(13*c) + (2*a*((2*c*(c*x)^(7/2)*Sqrt[a +
b*x^2])/(9*b) - (7*a*c^2*((2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c
*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/
4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c +
 Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c
+ Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*Ellip
ticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqr
t[a + b*x^2])))/(5*b)))/(9*b)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2c^{4}\sqrt{cx}\left(-45b^{4}x^{8}-55a\,b^{3}x^{6}+21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{4}-42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\cdots}\right.}{585x\sqrt{b\,x^{2}+a}\,b^{3}}$
risch	$-\frac{2x^{2}\left(-45b^{2}x^{4}-10ab\,x^{2}+14a^{2}\right)\sqrt{b\,x^{2}+a}\,c^{5}}{585b^{2}\sqrt{cx}}+\frac{14a^{3}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\cdots}\right.}{b}\cdots\right.}{195b^{3}\sqrt{bc\,x^{3}+acx}\,\cdots}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{2c^{4}x^{5}\sqrt{bc\,x^{3}+acx}}{13}+\frac{4a\,c^{4}x^{3}\sqrt{bc\,x^{3}+acx}}{117b}-\frac{28a^{2}c^{4}x\sqrt{bc\,x^{3}+acx}}{585b^{2}}+\frac{14a^{3}c^{5}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{\cdots}}{\cdots}\right.}{cx\sqrt{b\,x^{2}+a}}$

input

```
int((c*x)^(9/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-2/585*c^4*(c*x)^(1/2)*(-45*b^4*x^8-55*a*b^3*x^6+21*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a^4-42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4+4*a^2*b^2*x^4+14*a^3*b*x^
2)/x/(b*x^2+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.26

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx =$$
$$-\frac{2\left(42\sqrt{bc}a^3c^4\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\left(45\,b^3c^4x^5+10\,ab^2c^4x^3-14\,a\right.}{585\,b^3}$$

input
```
integrate((c*x)^(9/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/585*(42*sqrt(b*c)*a^3*c^4*weierstrassZeta(-4*a/b, 0, weierstrassPInvers
e(-4*a/b, 0, x)) - (45*b^3*c^4*x^5 + 10*a*b^2*c^4*x^3 - 14*a^2*b*c^4*x)*sq
rt(b*x^2 + a)*sqrt(c*x))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.14

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(9/2)*x**(11/2)*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), b*x**2
*exp_polar(I*pi)/a)/(2*gamma(15/4))
```

Maxima [F]

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{9}{2}}\,dx$$

input
```
integrate((c*x)^(9/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(9/2), x)
```

Giac [F]

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{9}{2}}\,dx$$

input
```
integrate((c*x)^(9/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \int (c\,x)^{9/2}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(9/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(9/2)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{9/2}\sqrt{a+bx^2}\,dx = \frac{2\sqrt{c}\,c^4\Big(-14\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2x+10\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^3+45\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^5+2}{585b^2}$$

input
```
int((c*x)^(9/2)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(c)*c**4*( - 14*sqrt(x)*sqrt(a + b*x**2)*a**2*x + 10*sqrt(x)*sqrt(a
 + b*x**2)*a*b*x**3 + 45*sqrt(x)*sqrt(a + b*x**2)*b**2*x**5 + 21*int((sqrt
(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**3))/(585*b**2)
```

3.593 $\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 301

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{4ac(cx)^{3/2}\sqrt{a+bx^2}}{45b} + \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c} - \frac{4a^2c^2\sqrt{cx}\sqrt{a+bx^2}}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{4a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}} - \frac{2a^{9/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{7/4}\sqrt{a+bx^2}}$$

output
```
4/45*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b+2/9*(c*x)^(7/2)*(b*x^2+a)^(1/2)/c-4
/15*a^2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x)+4/15*a
^(9/4)*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)
*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))
/b^(7/4)/(b*x^2+a)^(1/2)-2/15*a^(9/4)*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+
a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/
2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(7/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.28

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}} - a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hype
rgeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {248, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{9}a\int \frac{(cx)^{5/2}}{\sqrt{bx^2+a}}dx + \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{3ac^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}\right) + \frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$\downarrow$ 834

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$\downarrow$ 27

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$\downarrow$ 761

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

$\downarrow$ 1510

$$\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\cdots}}{\cdots}\right.}{5b}\right.$$

$$\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}$$

input
```
Int[(c*x)^(5/2)*Sqrt[a + b*x^2],x]
```

output
```
(2*(c*x)^(7/2)*Sqrt[a + b*x^2])/(9*c) + (2*a*((2*c*(c*x)^(3/2)*Sqrt[a + b*
x^2])/(5*b) - (6*a*c*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sq
rt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c
^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x]
)/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)
*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + S
qrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b)))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

method	result
default	$-\frac{2c^2\sqrt{cx}\left(-5b^3x^6+6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\right.}{45x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2x^2(5b\,x^2+2a)\sqrt{b\,x^2+a}\,c^3}{45b\sqrt{cx}}-\frac{2a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2c^2x^3\sqrt{bc\,x^3+acx}}{9}+\frac{4a\,c^2x\sqrt{bc\,x^3+acx}}{45b}-\frac{2a^2c^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{Ellipti}}{}\right.}{15b^2\sqrt{bc\,x^3}}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(5/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-5*b^3*x^6+6*((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*a^3-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-7*a*b^2*x^4-2*a^2*b*
x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.24

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{2\left(6\sqrt{bc}a^2c^2\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(5b^2c^2x^3\right.}{45b^2}$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/45*(6*sqrt(b*c)*a^2*c^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-
4*a/b, 0, x)) + (5*b^2*c^2*x^3 + 2*a*b*c^2*x)*sqrt(b*x^2 + a)*sqrt(c*x))/b
^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.15

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(11/4))
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx=\int (c\,x)^{5/2}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(5/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/2)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{5/2}\sqrt{a+bx^2}\,dx = \frac{2\sqrt{c}\,c^2\left(2\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+5\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3-3\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2\right)}{45b}$$

input
```
int((c*x)^(5/2)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(c)*c**2*(2*sqrt(x)*sqrt(a + b*x**2)*a*x + 5*sqrt(x)*sqrt(a + b*x**
2)*b*x**3 - 3*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/(45*b)
```

3.594 $\int \sqrt{cx}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 269

$$\begin{aligned}\int \sqrt{cx}\sqrt{a+bx^2}\,dx &= \frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}+\frac{4a\sqrt{cx}\sqrt{a+bx^2}}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}\\ &\quad-\frac{4a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}\\ &\quad+\frac{2a^{5/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{3/4}\sqrt{a+bx^2}}\end{aligned}$$

output

```
2/5*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+4/5*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2
)/(a^(1/2)+b^(1/2)*x)-4/5*a^(5/4)*c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(
a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2))),1/2*2^(1/2))/b^(3/4)/(b*x^2+a)^(1/2)+2/5*a^(5/4)*c^(1/2)*(a
^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(
2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(3/4)/(b*x^2+
a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.21

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{2x\sqrt{cx}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)
/a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{2}{5}a\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4a\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{4a\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

$\big\downarrow$ 27

$$\frac{4a\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

$\big\downarrow$ 761

$$\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

$\big\downarrow$ 1510

$$\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{5c}$$

$$\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}$$

input
```
Int[Sqrt[c*x]*Sqrt[a + b*x^2],x]
```

output
```
(2*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*c) + (4*a*(-((-((c^2*Sqrt[c*x]*Sqrt[a +
 b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b
]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*A
rcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^
2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b
*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*
x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.76

method	result
default	$\frac{2\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{Elliptic}}{5\sqrt{b\,x^2+a}\,bx}$
risch	$\frac{2x^2\sqrt{b\,x^2+a}\,c}{5\sqrt{cx}}+\frac{2a\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}}{}\right.}{5b\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2x\sqrt{bc\,x^3+acx}}{5}+\frac{2ac\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right.}{5b\sqrt{bc\,x^3+acx}}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1
/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))*a^2+b^2*x^4+a*b*x^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{2\left(\sqrt{bx^2+a}\sqrt{cx}bx - 2\sqrt{bc}a\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)\right)}{5b}$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^2 + a)*sqrt(c*x)*b*x - 2*sqrt(b*c)*a*weierstrassZeta(-4*a/b,
 0, weierstrassPInverse(-4*a/b, 0, x)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), b*x**2*exp_
polar(I*pi)/a)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{c\,x}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(1/2)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\sqrt{a+bx^2}\,dx = \frac{2\sqrt{c}\left(\sqrt{x}\,\sqrt{b\,x^2+a}\,x+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a\right)}{5}$$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(1/2),x)
```

output
```
(2*sqrt(c)*(sqrt(x)*sqrt(a + b*x**2)*x + int((sqrt(x)*sqrt(a + b*x**2))/(a
 + b*x**2),x)*a))/5
```

3.595 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 263

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}} + \frac{4\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}} + \frac{2\sqrt[4]{a}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{c^{3/2}\sqrt{a+bx^2}}$$

output
```
-2*(b*x^2+a)^(1/2)/c/(c*x)^(1/2)+4*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2
/(a^(1/2)+b^(1/2)*x)-4*a^(1/4)*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(
1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4
)/c^(1/2))),1/2*2^(1/2))/c^(3/2)/(b*x^2+a)^(1/2)+2*a^(1/4)*b^(1/4)*(a^(1/2
)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arc
tan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/c^(3/2)/(b*x^2+a)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = -\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/((
c*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {247, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{2b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{c^2}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{4b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 27$$

$$\frac{4b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 761$$

$$\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 1510$$

$$\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(3/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(c*Sqrt[c*x]) + (4*b*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*
x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c
*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcT
an[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2])
)/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^
2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])
/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.74

method	result
default	$\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{b}{}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}\,c}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{c\sqrt{cx}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\right.}{}\right.}{\sqrt{bc\,x^{3}+acx}\,c\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\left(-\frac{2\left(x^{2}bc+ac\right)}{c^{2}\sqrt{x\left(x^{2}bc+ac\right)}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{c\sqrt{bc\,x^{3}+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/
2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1
/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-b*x^2
-a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\left(2\sqrt{bc}x\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\sqrt{cx}\right)}{c^2x}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*sqrt(b*c)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0
, x)) + sqrt(b*x^2 + a)*sqrt(c*x))/(c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(3/2),x)
```

output
```
sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{(c\,x)}^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left(\sqrt{b\,x^2+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)a\right)}{\sqrt{x}\,c^2}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(3/2),x)
```

output
```
(2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x
**2 + b*x**4),x)*a))/(sqrt(x)*c**2)
```

3.596 $\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 303

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{4b\sqrt{a+bx^2}}{5ac^3\sqrt{cx}} + \frac{4b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5ac^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{4b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}} + \frac{2b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{3/4}c^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*(b*x^2+a)^(1/2)/c/(c*x)^(5/2)-4/5*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(1/2)
+4/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/c^4/(a^(1/2)+b^(1/2)*x)-4/5*b^(
5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE
(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(3/4)/c
^(7/2)/(b*x^2+a)^(1/2)+2/5*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)
+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)),1/2*2^(1/2))/a^(3/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/(
5*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {247, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$\downarrow$ 834

$$\frac{2b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$\downarrow$ 27

$$\frac{2b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$\downarrow$ 761

$$\frac{2b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

$\downarrow$ 1510

$$\frac{2b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ac^3}\right.}{5c^2}$$

$$\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(7/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(5*c*(c*x)^(5/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(a*c*S
qrt[c*x]) + (2*b*(-((-(((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b
]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x
^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqr
t[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[
b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*c^3)))/(5*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.72

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-}}{\sqrt{-a}}}\right.}{5}}{x^2\sqrt{b\,x^2+a}\,c^3\sqrt{cx}\,a}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(2b\,x^2+a)}{5x^2a\,c^3\sqrt{cx}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-a}}{}\right.}{5a\sqrt{bc\,x^3+acx}\,c^3\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{5c^4x^3}-\frac{4\left(x^2bc+ac\right)b}{5a\,c^4\sqrt{x\left(x^2bc+ac\right)}}+\frac{2b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{(x+}{}}\right.}{b}\right.}{5a\,c^3\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/x^2*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)
^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a*b*x^2-2*b^2*x^4-3*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.21

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\left(2\sqrt{bc}bx^3\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(2bx^2+a\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{5ac^4x^3}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(2*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*
a/b, 0, x)) + (2*b*x^2 + a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(7/2),x)
```

output
```
sqrt(a)*gamma(-5/4)*hyper((-5/4, -1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(c\,x)^{7/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(7/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{c}\left(\sqrt{b\,x^2+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx\right)a\,x^2\right)}{3\sqrt{x}\,c^4x^2}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(7/2),x)
```

output
```
( - 2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(
a*x**4 + b*x**6),x)*a*x**2))/(3*sqrt(x)*c**4*x**2)
```

3.597 $\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}} dx$

Optimal result

Integrand size = 19, antiderivative size = 334

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}} dx = -\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}} - \frac{4b\sqrt{a+bx^2}}{45ac^3(cx)^{5/2}} + \frac{4b^2\sqrt{a+bx^2}}{15a^2c^5\sqrt{cx}} - \frac{4b^{5/2}\sqrt{cx}\sqrt{a+bx^2}}{15a^2c^6\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{7/4}c^{11/2}\sqrt{a+bx^2}} - \frac{2b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15a^{7/4}c^{11/2}\sqrt{a+bx^2}}$$

output
```
-2/9*(b*x^2+a)^(1/2)/c/(c*x)^(9/2)-4/45*b*(b*x^2+a)^(1/2)/a/c^3/(c*x)^(5/2
)+4/15*b^2*(b*x^2+a)^(1/2)/a^2/c^5/(c*x)^(1/2)-4/15*b^(5/2)*(c*x)^(1/2)*(b
*x^2+a)^(1/2)/a^2/c^6/(a^(1/2)+b^(1/2)*x)+4/15*b^(9/4)*(a^(1/2)+b^(1/2)*x)
*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c
*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(7/4)/c^(11/2)/(b*x^2+a)^(1/2)-
2/15*b^(9/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*I
nverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/
a^(7/4)/c^(11/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{2x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{1}{2},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(11/2),x]
```

output
```
(-2*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/4, -1/2, -5/4, -((b*x^2)/a)])/(
9*(c*x)^(11/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {247, 264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{(cx)^{7/2}\sqrt{bx^2+a}}dx}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{3b\left(\frac{b\int\frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\left(-\frac{3b\left(\frac{2b\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \; 834$$

$$\frac{2b\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2b\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2b\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{9c^2}-\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

$\downarrow$ 1510

$$2b\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ac^3}}{5ac^2}\right.$$

$$9c^2$$

$$\frac{2\sqrt{a+bx^2}}{9c(cx)^{9/2}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(11/2),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(9*c*(c*x)^(9/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(5*a*c
*(c*x)^(5/2)) - (3*b*((-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-((c
^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c
]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*
c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(
b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]
*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x
^2])))/(a*c^3)))/(5*a*c^2)))/(9*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

method	result
default	$-\frac{2\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^4-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{Ellip}}{45x^4\sqrt{b\,x^2+a}\,c^5\sqrt{cx}\,a^2}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-6b^2x^4+2ab\,x^2+5a^2)}{45x^4a^2c^5\sqrt{cx}}-\frac{2b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{b}\right.}{15a^2\sqrt{bc\,x^3+acx}\,c^5\sqrt{cx}\,\sqrt{b\,x^2}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{9c^6x^5}-\frac{4b\sqrt{bc\,x^3+acx}}{45a\,c^6x^3}+\frac{4\left(x^2bc+ac\right)b^2}{15a^2c^6\sqrt{x\left(x^2bc+ac\right)}}-\frac{2b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-}}{}\right.}{15}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45/x^4*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^4-3*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/
(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a*b^2*x^4-6*b^3*x^6-4*a*b^2*x^4+7*a^2*b*x^2+5*a^3)/(b*x^2+a)^(1
/2)/c^5/(c*x)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = \frac{2\left(6\sqrt{bc}b^2x^5\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(6b^2x^4-2abx\right.\right.}{45a^2c^6x^5}$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
2/45*(6*sqrt(b*c)*b^2*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-
4*a/b, 0, x)) + (6*b^2*x^4 - 2*a*b*x^2 - 5*a^2)*sqrt(b*x^2 + a)*sqrt(c*x))
/(a^2*c^6*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 47.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{4}, -\frac{1}{2} \\ -\frac{5}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{11}{2}}x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(11/2),x)
```

output
```
sqrt(a)*gamma(-9/4)*hyper((-9/4, -1/2), (-5/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(11/2)*x**(9/2)*gamma(-5/4))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{{(c\,x)}^{11/2}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(11/2),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{2\sqrt{c}\left(\sqrt{b\,x^2+a}+\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^8+a\,x^6}dx\right)a\,x^4\right)}{7\sqrt{x}\,c^6x^4}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(11/2),x)
```

output
```
( - 2*sqrt(c)*(sqrt(a + b*x**2) + sqrt(x)*int((sqrt(x)*sqrt(a + b*x**2))/(
a*x**6 + b*x**8),x)*a*x**4))/(7*sqrt(x)*c**6*x**4)
```

3.598 $\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 212

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx = -\frac{8a^3c^3\sqrt{cx}\sqrt{a+bx^2}}{231b^2} + \frac{8a^2c(cx)^{5/2}\sqrt{a+bx^2}}{385b} + \frac{4a(cx)^{9/2}\sqrt{a+bx^2}}{55c} + \frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c} + \frac{4a^{15/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231b^{9/4}\sqrt{a+bx^2}}$$

output
```
-8/231*a^3*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2+8/385*a^2*c*(c*x)^(5/2)*(b*
x^2+a)^(1/2)/b+4/55*a*(c*x)^(9/2)*(b*x^2+a)^(1/2)/c+2/15*(c*x)^(9/2)*(b*x^
2+a)^(3/2)/c+4/231*a^(15/4)*c^(7/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2
)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)),1/2*2^(1/2))/b^(9/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.48

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx = \frac{2c^3\sqrt{cx}\sqrt{a+bx^2}\left(-\left(\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}}\right)+5a^3 \text{Hypergeometric2F1}\left(-\frac{3}{2}, \ldots\right.\right.}{165b^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

output
```
(2*c^3*Sqrt[c*x]*Sqrt[a + b*x^2]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1
+ (b*x^2)/a]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(1
65*b^2*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {248, 248, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \; 248$$

$$\frac{2}{5}a\int (cx)^{7/2}\sqrt{bx^2+a}dx+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \; 248$$

$$\frac{2}{5}a\left(\frac{2}{11}a\int \frac{(cx)^{7/2}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}\right)+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\int\frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}\right)+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}\right)+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)}{7b}\right)+\frac{2(cx)^{9/2}\sqrt{a+bx^2}}{11c}\right)+\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}a\left(\frac{2}{11}a\left(\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right.}{7b}\right.\right.$$

$$\frac{2(cx)^{9/2}\left(a+bx^2\right)^{3/2}}{15c}$$

input
```
Int[(c*x)^(7/2)*(a + b*x^2)^(3/2),x]
```

output
```
(2*(c*x)^(9/2)*(a + b*x^2)^(3/2))/(15*c) + (2*a*((2*(c*x)^(9/2)*Sqrt[a + b
*x^2])/(11*c) + (2*a*((2*c*(c*x)^(5/2)*Sqrt[a + b*x^2])/(7*b) - (5*a*c^2*(
(2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt
[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a +
b*x^2])))/(7*b)))/11))/5
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

method	result
default	$\frac{2c^3\sqrt{cx}\left(77b^5x^9+196a\,b^4x^7+10\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^4+131a^2b^3x^5-8a^3b^2}{1155x\sqrt{b\,x^2+a}\,b^3}$
risch	$-\frac{2(-77b^3x^6-119a\,b^2x^4-12a^2b\,x^2+20a^3)x\sqrt{b\,x^2+a}\,c^4}{1155b^2\sqrt{cx}}+\frac{4a^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\right.}{231b^3\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2b\,c^3x^6\sqrt{bc\,x^3+acx}}{15}+\frac{34a\,c^3x^4\sqrt{bc\,x^3+acx}}{165}+\frac{8a^2c^3x^2\sqrt{bc\,x^3+acx}}{385b}-\frac{8a^3c^3\sqrt{bc\,x^3+acx}}{231b^2}+\frac{4a^4c^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}}{}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(77*b^5*x^9+196*a*b^4*x^7+10*(-a*
b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4+131*a^2*b^3*x^5-8*a^3*b^2*x^3-2
0*a^4*b*x)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.42

$$\int (cx)^{7/2}\left(a+bx^2\right)^{3/2}\,dx=\frac{2\left(20\sqrt{bc}a^4c^3\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(77\,b^4c^3x^6+119\,ab^3c^3x^4+12\,a^2b^2c^3x^2-20\right.\right.}{1155\,b^3}$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(20*sqrt(b*c)*a^4*c^3*weierstrassPInverse(-4*a/b, 0, x) + (77*b^4*c
^3*x^6 + 119*a*b^3*c^3*x^4 + 12*a^2*b^2*c^3*x^2 - 20*a^3*b*c^3)*sqrt(b*x^2
 + a)*sqrt(c*x))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.22

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} \, dx = \frac{a^{\frac{3}{2}} c^{\frac{7}{2}} x^{\frac{9}{2}} \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{9}{4} \\ \frac{13}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-3/2, 9/4), (13/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(13/4))
```

Maxima [F]

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}} \, dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

Giac [F]

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{7}{2}} dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx = \int (c\,x)^{7/2} \left(b\,x^2+a\right)^{3/2} dx$$

input
```
int((c*x)^(7/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(7/2)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{7/2} \left(a+bx^2\right)^{3/2} dx = \frac{2\sqrt{c}\,c^3\left(-20\sqrt{x}\,\sqrt{b\,x^2+a}\,a^3+12\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2b\,x^2+119\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,b^2x^4+77\sqrt{x}\,\sqrt{\right.}{1155b^2}$$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(c)*c**3*( - 20*sqrt(x)*sqrt(a + b*x**2)*a**3 + 12*sqrt(x)*sqrt(a +
 b*x**2)*a**2*b*x**2 + 119*sqrt(x)*sqrt(a + b*x**2)*a*b**2*x**4 + 77*sqrt(
x)*sqrt(a + b*x**2)*b**3*x**6 + 10*int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b
*x**3),x)*a**4))/(1155*b**2)
```

3.599 $\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx$

Optimal result

Integrand size = 19, antiderivative size = 181

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \frac{8a^2c\sqrt{cx}\sqrt{a+bx^2}}{77b} + \frac{12a(cx)^{5/2}\sqrt{a+bx^2}}{77c} + \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c} - \frac{4a^{11/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{77b^{5/4}\sqrt{a+bx^2}}$$

output
```
8/77*a^2*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b+12/77*a*(c*x)^(5/2)*(b*x^2+a)^(1/
2)/c+2/11*(c*x)^(5/2)*(b*x^2+a)^(3/2)/c-4/77*a^(11/4)*c^(3/2)*(a^(1/2)+b^(
1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(5/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \frac{2c\sqrt{cx}\sqrt{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt{1+\frac{bx^2}{a}} - a^2 \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{11b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

output
```
(2*c*Sqrt[c*x]*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hy
pergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 248, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{11}a\int (cx)^{3/2}\sqrt{bx^2+a}dx + \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx + \frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}\right) + \frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{262}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}\right)+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)+\frac{2(cx)^{5/2}\sqrt{a+bx^2}}{7c}\right)+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{11}a\left(\frac{2}{7}a\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)+\frac{2(c\right)+\frac{2(cx)^{5/2}\left(a+bx^2\right)^{3/2}}{11c}$$

input

```
Int[(c*x)^(3/2)*(a + b*x^2)^(3/2),x]
```

output

```
(2*(c*x)^(5/2)*(a + b*x^2)^(3/2))/(11*c) + (6*a*((2*(c*x)^(5/2)*Sqrt[a + b
*x^2])/(7*c) + (2*a*((2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*Sqrt
[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b
]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])
/(3*b^(5/4)*Sqrt[a + b*x^2])))/7))/11
```

Defintions of rubi rules used

rule 248

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

method	result
default	$-\frac{2c\sqrt{cx}\left(-7b^4x^7+2\sqrt{2}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a^3-20a\,b^3x^5-17a^2b^2x^3-4a^3bx\right)}{77x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2(7b^2x^4+13ab\,x^2+4a^2)x\sqrt{b\,x^2+a}\,c^2}{77b\sqrt{cx}}-\frac{4a^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)c}{77b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2bc\,x^4\sqrt{bc\,x^3+acx}}{11}+\frac{26ac\,x^2\sqrt{bc\,x^3+acx}}{77}+\frac{8a^2c\sqrt{bc\,x^3+acx}}{77b}-\frac{4a^3c^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}}{77b^2\sqrt{bc\,x^3+acx}}\right)}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-7*b^4*x^7+2*2^(1/2)*((b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*(-a*b)^(1/2)*a^3-20*a*b^3*x^5-17*a^2*b^2*x^3-4*a^3*b*x)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.38

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = -\frac{2\left(4\sqrt{bc}a^3c\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-\left(7b^3cx^4+13ab^2cx^2+4a^2bc\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{77b^2}$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/77*(4*sqrt(b*c)*a^3*c*weierstrassPInverse(-4*a/b, 0, x) - (7*b^3*c*x^4
+ 13*a*b^2*c*x^2 + 4*a^2*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.25

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \frac{a^{\frac{3}{2}}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-3/2, 5/4), (9/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(9/4))
```

Maxima [F]

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{3}{2}} dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \int (c\,x)^{3/2} \left(b\,x^2+a\right)^{3/2} dx$$

input
```
int((c*x)^(3/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(3/2)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{3/2} \left(a+bx^2\right)^{3/2} dx = \frac{2\sqrt{c}\, c\left(4\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2+13\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2+7\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4-2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right.\right.}{77b}$$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(c)*c*(4*sqrt(x)*sqrt(a + b*x**2)*a**2 + 13*sqrt(x)*sqrt(a + b*x**2
)*a*b*x**2 + 7*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 - 2*int((sqrt(x)*sqrt(a
+ b*x**2))/(a*x + b*x**3),x)*a**3))/(77*b)
```

3.600 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{4a\sqrt{cx}\sqrt{a+bx^2}}{7c} + \frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c} + \frac{4a^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

output
```
4/7*a*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c+2/7*(c*x)^(1/2)*(b*x^2+a)^(3/2)/c+4/7*
a^(7/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(1/
4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

output
```
(2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/(S
qrt[c*x]*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {248, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{7}a\int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}dx+\frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c}$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{7}a\left(\frac{2}{3}a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)+\frac{2\sqrt{cx}(a+bx^2)^{3/2}}{7c}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}a\left(\frac{4a\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3c}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)+\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{7}a\left(\frac{2a^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)+$$
$$\frac{2\sqrt{cx}\left(a+bx^2\right)^{3/2}}{7c}$$

input
```
Int[(a + b*x^2)^(3/2)/Sqrt[c*x],x]
```

output
```
(2*Sqrt[c*x]*(a + b*x^2)^(3/2))/(7*c) + (6*a*((2*Sqrt[c*x]*Sqrt[a + b*x^2]
)/(3*c) + (2*a^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(S
qrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)
*Sqrt[c])], 1/2])/(3*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2])))/7
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{4\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2}{7}+\frac{2b^3x^5}{7}+\frac{8ab^2x^3}{7}+\frac{6a^2bx}{7}}{\sqrt{bx^2+a}\,b\sqrt{cx}}$
risch	$\frac{2(bx^2+3a)x\sqrt{bx^2+a}}{7\sqrt{cx}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(bx^2+a)}}{7b\sqrt{bcx^3+acx}\sqrt{cx}\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{cx(bx^2+a)}\left(\frac{2bx^2\sqrt{bcx^3+acx}}{7c}+\frac{6a\sqrt{bcx^3+acx}}{7c}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7b\sqrt{bcx^3+acx}}\right)}{\sqrt{cx}\sqrt{bx^2+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/(b*x^2+a)^(1/2)*(2*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2+b^3*
x^5+4*a*b^2*x^3+3*a^2*b*x)/b/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx=\frac{2\left(4\sqrt{bc}a^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(b^2x^2+3ab)\sqrt{bx^2+a}\sqrt{cx}\right)}{7bc}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
2/7*(4*sqrt(b*c)*a^2*weierstrassPInverse(-4*a/b, 0, x) + (b^2*x^2 + 3*a*b)
*sqrt(b*x^2 + a)*sqrt(c*x))/(b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{a^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ \frac{5}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(1/2),x)
```

output
```
a**(3/2)*sqrt(x)*gamma(1/4)*hyper((-3/2, 1/4), (5/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{\sqrt{c\,x}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(1/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{c}\left(3\sqrt{x}\,\sqrt{b\,x^2+a}\,a+\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2+2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2\right)}{7c}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(1/2),x)
```

output
```
(2*sqrt(c)*(3*sqrt(x)*sqrt(a + b*x**2)*a + sqrt(x)*sqrt(a + b*x**2)*b*x**2
 + 2*int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2))/(7*c)
```

3.601 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{3c(cx)^{3/2}}+\frac{2b\sqrt{cx}\sqrt{a+bx^2}}{3c^3} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3c^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*a*(b*x^2+a)^(1/2)/c/(c*x)^(3/2)+2/3*b*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^3
+4/3*a^(3/4)*b^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)
^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2
^(1/2))/c^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{1+\frac{bx^2}{a}}}$$

input

```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

output

```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])/
(3*(c*x)^(5/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {247, 248, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx$$

$$\Big\downarrow \; 247$$

$$\frac{2b\int \frac{\sqrt{bx^2+a}}{\sqrt{cx}}dx}{c^2}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \; 248$$

$$\frac{2b\left(\frac{2}{3}a\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)}{c^2}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2b\left(\frac{4a\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3c}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)}{c^2}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2b\left(\frac{2a^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{2\sqrt{cx}\sqrt{a+bx^2}}{3c}\right)}{c^2}-\frac{2(a+bx^2)^{3/2}}{3c(cx)^{3/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(5/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(3*c*(c*x)^(3/2)) + (2*b*((2*Sqrt[c*x]*Sqrt[a + b*x
^2])/(3*c) + (2*a^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)
/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(3*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2])))/c^2
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.82

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,ax}{3}+\frac{2b^2x^4}{3}-\frac{2a^2}{3}}{\sqrt{b\,x^2+a}\,x\,c^2\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-b\,x^2+a)}{3x\,c^2\sqrt{cx}}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^2+a)}}{3\sqrt{bc\,x^3+acx}\,c^2\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2a\sqrt{bc\,x^3+acx}}{3c^3x^2}+\frac{2b\sqrt{bc\,x^3+acx}}{3c^3}+\frac{4a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{3c^2\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/(b*x^2+a)^(1/2)/x*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Elliptic
F(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a*x+b^
2*x^4-a^2)/c^2/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\frac{2\left(4\sqrt{bc}ax^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}(bx^2-a)\sqrt{cx}\right)}{3\,c^3x^2}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(4*sqrt(b*c)*a*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)
*(b*x^2 - a)*sqrt(c*x))/(c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(5/2),x)
```

output
```
a**(3/2)*gamma(-3/4)*hyper((-3/2, -3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(5/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/2}}\,dx=\frac{2\sqrt{c}\left(-5\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2-6\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx\right)a^2x\right)}{3\sqrt{x}\,c^3x}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 5*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 - 6*sqrt(x)*
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x)*a**2*x))/(3*sqrt(x)*c*
*3*x)
```

3.602 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{7c(cx)^{7/2}} - \frac{6b\sqrt{a+bx^2}}{7c^3(cx)^{3/2}} + \frac{4b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{7\sqrt[4]{a}c^{9/2}\sqrt{a+bx^2}}$$

output
```
-2/7*a*(b*x^2+a)^(1/2)/c/(c*x)^(7/2)-6/7*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(3/2)
+4/7*b^(7/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*I
nverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/
a^(1/4)/c^(9/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.37

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{3}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)])
/(7*(c*x)^(9/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {247, 247, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{6b\int \frac{\sqrt{bx^2+a}}{(cx)^{5/2}}dx}{7c^2}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{6b\left(\frac{2b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3c^2}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}\right)}{7c^2}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{6b\left(\frac{4b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3c^3}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}\right)}{7c^2}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{6b\left(\frac{2b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3\sqrt[4]{a}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3c(cx)^{3/2}}\right)}{7c^2}-\frac{2(a+bx^2)^{3/2}}{7c(cx)^{7/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(9/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(7*c*(c*x)^(7/2)) + (6*b*((-2*Sqrt[a + b*x^2])/(3*c
*(c*x)^(3/2)) + (2*b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x
^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(3*a^(1/4)*c^(7/2)*Sqrt[a + b*x^2])))/(7*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

method	result
default	$\frac{\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^3}{7}-\frac{6b^2x^4}{7}-\frac{8ab\,x^2}{7}-\frac{2a^2}{7}}{\sqrt{b\,x^2+a}\,x^3c^4\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(3b\,x^2+a)}{7x^3c^4\sqrt{cx}}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^2+a)}}{7\sqrt{bc\,x^3+acx}\,c^4\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2a\sqrt{bc\,x^3+acx}}{7c^5x^4}-\frac{6b\sqrt{bc\,x^3+acx}}{7c^5x^2}+\frac{4b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{7c^4\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/7/(b*x^2+a)^(1/2)/x^3*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)
*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Ellipt
icF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x^
3-3*b^2*x^4-4*a*b*x^2-a^2)/c^4/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \frac{2\left(4\sqrt{bc}bx^4\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)-(3\,bx^2+a)\sqrt{bx^2+a}\sqrt{cx}\right)}{7\,c^5x^4}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
2/7*(4*sqrt(b*c)*b*x^4*weierstrassPInverse(-4*a/b, 0, x) - (3*b*x^2 + a)*s
qrt(b*x^2 + a)*sqrt(c*x))/(c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.78 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{3}{2}\\ -\frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(9/2),x)
```

output
```
a**(3/2)*gamma(-7/4)*hyper((-7/4, -3/2), (-3/4,), b*x**2*exp_polar(I*pi)/a
)/(2*c**(9/2)*x**(7/2)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx=\int \frac{\left(b\,x^2+a\right)^{3/2}}{\left(c\,x\right)^{9/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(9/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{9/2}}\,dx=\frac{2\sqrt{c}\left(\sqrt{b\,x^2+a}\,a-5\sqrt{b\,x^2+a}\,b\,x^2+6\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^7+a\,x^5}dx\right)a^2x^3\right)}{5\sqrt{x}\,c^5x^3}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(9/2),x)
```

output
```
(2*sqrt(c)*(sqrt(a + b*x**2)*a - 5*sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*int
((sqrt(x)*sqrt(a + b*x**2))/(a*x**5 + b*x**7),x)*a**2*x**3))/(5*sqrt(x)*c*
*5*x**3)
```

3.603 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 184

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}} \, dx = -\frac{2a\sqrt{a+bx^2}}{11c(cx)^{11/2}} - \frac{26b\sqrt{a+bx^2}}{77c^3(cx)^{7/2}} - \frac{8b^2\sqrt{a+bx^2}}{77ac^5(cx)^{3/2}} - \frac{4b^{11/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{77a^{5/4}c^{13/2}\sqrt{a+bx^2}}$$

output

```
-2/11*a*(b*x^2+a)^(1/2)/c/(c*x)^(11/2)-26/77*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(
7/2)-8/77*b^2*(b*x^2+a)^(1/2)/a/c^5/(c*x)^(3/2)-4/77*b^(11/4)*(a^(1/2)+b^(
1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(5/4)/c^(13/2)/(b*x^2+a
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{3}{2},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(13/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)]
)/(11*(c*x)^(13/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 247, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{6b\int \frac{\sqrt{bx^2+a}}{(cx)^{9/2}}dx}{11c^2}-\frac{2(a+bx^2)^{3/2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{6b\left(\frac{2b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}\right)}{11c^2}-\frac{2(a+bx^2)^{3/2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{6b\left(\frac{2b\left(-\frac{b\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}\right)}{11c^2}-\frac{2(a+bx^2)^{3/2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{6b\left(\frac{2b\left(-\frac{2b\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}\right)}{11c^2}-\frac{2(a+bx^2)^{3/2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{6b\left(\frac{2b\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{a+bx^2}}{7c(cx)^{7/2}}\right)}{11c^2}-\frac{2(a+bx^2)^{3/2}}{11c(cx)^{11/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(13/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(11*c*(c*x)^(11/2)) + (6*b*((-2*Sqrt[a + b*x^2])/(7
*c*(c*x)^(7/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4
)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*
c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(
3*a^(5/4)*c^(7/2)*Sqrt[a + b*x^2])))/(7*c^2)))/(11*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

method	result
default	$-\frac{2\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b^2x^5+4b^3x^6+17a\,b^2x^4+20a^2b\,x^2+7a^3\right)}{77\sqrt{b\,x^2+a}\,x^5a\,c^6\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(4b^2x^4+13ab\,x^2+7a^2)}{77x^5a\,c^6\sqrt{cx}}-\frac{4b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{\cdots}}{77a\sqrt{bc\,x^3+acx}\,c^6\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2a\sqrt{bc\,x^3+acx}}{11c^7x^6}-\frac{26b\sqrt{bc\,x^3+acx}}{77c^7x^4}-\frac{8b^2\sqrt{bc\,x^3+acx}}{77a\,c^7x^2}-\frac{4b^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\cdots\right)}{77a\,c^6\sqrt{bc\,x^3+acx}}\cdots\right)}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/77/(b*x^2+a)^(1/2)/x^5*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/
2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Elli
pticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b^
2*x^5+4*b^3*x^6+17*a*b^2*x^4+20*a^2*b*x^2+7*a^3)/a/c^6/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = -\frac{2\left(4\sqrt{bc}b^2x^6\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(4\,b^2x^4+13\,abx^2+7\,a^2)\sqrt{bx^2+a}\sqrt{cx}\right)}{77\,ac^7x^6}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(13/2),x, algorithm="fricas")
```

output
```
-2/77*(4*sqrt(b*c)*b^2*x^6*weierstrassPInverse(-4*a/b, 0, x) + (4*b^2*x^4
+ 13*a*b*x^2 + 7*a^2)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^7*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 150.91 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.29

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{11}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{11}{4}, -\frac{3}{2} \\ -\frac{7}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{13}{2}}x^{\frac{11}{2}}\Gamma\left(-\frac{7}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(13/2),x)
```

output
```
a**(3/2)*gamma(-11/4)*hyper((-11/4, -3/2), (-7/4,), b*x**2*exp_polar(I*pi)
/a)/(2*c**(13/2)*x**(11/2)*gamma(-7/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(13/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(13/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{13/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(13/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{13/2}}\,dx = \frac{2\sqrt{c}\left(-\sqrt{b\,x^2+a}\,a-3\sqrt{b\,x^2+a}\,b\,x^2+2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^9+a\,x^7}dx\right)a^2x^5\right)}{15\sqrt{x}\,c^7x^5}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(13/2),x)
```

output
```
(2*sqrt(c)*( - sqrt(a + b*x**2)*a - 3*sqrt(a + b*x**2)*b*x**2 + 2*sqrt(x)*
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**7 + b*x**9),x)*a**2*x**5))/(15*sqrt(x
)*c**7*x**5)
```

3.604 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 215

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{15c(cx)^{15/2}} - \frac{34b\sqrt{a+bx^2}}{165c^3(cx)^{11/2}} - \frac{8b^2\sqrt{a+bx^2}}{385ac^5(cx)^{7/2}} + \frac{8b^3\sqrt{a+bx^2}}{231a^2c^7(cx)^{3/2}} + \frac{4b^{15/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{231a^{9/4}c^{17/2}\sqrt{a+bx^2}}$$

output
```
-2/15*a*(b*x^2+a)^(1/2)/c/(c*x)^(15/2)-34/165*b*(b*x^2+a)^(1/2)/c^3/(c*x)^
(11/2)-8/385*b^2*(b*x^2+a)^(1/2)/a/c^5/(c*x)^(7/2)+8/231*b^3*(b*x^2+a)^(1/
2)/a^2/c^7/(c*x)^(3/2)+4/231*b^(15/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1
/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/
4)/c^(1/2)),1/2*2^(1/2))/a^(9/4)/c^(17/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.27

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{15}{4},-\frac{3}{2},-\frac{11}{4},-\frac{bx^2}{a}\right)}{15(cx)^{17/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(17/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((b*x^2)/a)
])/(15*(c*x)^(17/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {247, 247, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{\sqrt{bx^2+a}}{(cx)^{13/2}}dx}{5c^2}-\frac{2(a+bx^2)^{3/2}}{15c(cx)^{15/2}}$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\left(\frac{2b\int \frac{1}{(cx)^{9/2}\sqrt{bx^2+a}}dx}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}\right)}{5c^2}-\frac{2(a+bx^2)^{3/2}}{15c(cx)^{15/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(\frac{2b\left(-\frac{5b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{15c(cx)^{15/2}}$$

$\Big\downarrow$ 264

$$\frac{2b\left(\frac{2b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{15c(cx)^{15/2}}$$

$\Big\downarrow$ 266

$$\frac{2b\left(\frac{2b\left(-\frac{5b\left(-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{15c(cx)^{15/2}}$$

$\Big\downarrow$ 761

$$\frac{2b\left(\frac{2b\left(-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt{a+bx^2}}{11c(cx)^{11/2}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{15c(cx)^{15/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(17/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(15*c*(c*x)^(15/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(1
1*c*(c*x)^(11/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(7*a*c*(c*x)^(7/2)) - (5*b*
((-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a]*c + Sqrt[b]*
c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*Arc
Tan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(7/2)*Sqrt[
a + b*x^2]))/(7*a*c^2)))/(11*c^2)))/(5*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 2.75 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.75

method	result
default	$\frac{\frac{4\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) b^{3} x^{7}}{231}+\frac{8 b^{4} x^{8}}{231}+\frac{16 a\, b^{3} x^{6}}{1155}-\frac{262 a^{2} b^{2} x^{4}}{1155}-\frac{56 a^{3} b\, x^{2}}{165}-\frac{2 a^{4}}{15}}{\sqrt{b\, x^{2}+a}\, x^{7} a^{2} c^{8} \sqrt{c x}}$
risch	$-\frac{2 \sqrt{b\, x^{2}+a}\, \left(-20 b^{3} x^{6}+12 a\, b^{2} x^{4}+119 a^{2} b\, x^{2}+77 a^{3}\right)}{1155 x^{7} a^{2} c^{8} \sqrt{c x}}+\frac{4 b^{3} \sqrt{-ab}\, \sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\, \sqrt{-\frac{bx}{\sqrt{-ab}}}\, \operatorname{EllipticF}\left(\sqrt{\frac{(x+\cdots}{\cdots}}\cdots\right.}{231 a^{2} \sqrt{b c\, x^{3}+a c x}\, c^{8} \sqrt{c x}\, \sqrt{b\, x^{2}+a}\cdots}$
elliptic	$\frac{\sqrt{c x \left(b\, x^{2}+a\right)}\, \left(-\frac{2 a \sqrt{b c\, x^{3}+a c x}}{15 c^{9} x^{8}}-\frac{34 b \sqrt{b c\, x^{3}+a c x}}{165 c^{9} x^{6}}-\frac{8 b^{2} \sqrt{b c\, x^{3}+a c x}}{385 a\, c^{9} x^{4}}+\frac{8 b^{3} \sqrt{b c\, x^{3}+a c x}}{231 a^{2} c^{9} x^{2}}+\frac{4 b^{3} \sqrt{-ab}\, \sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\, \sqrt{-\frac{2 \left(x-\frac{\sqrt{-ab}}{b}\right) b}{\sqrt{-ab}}}\, \sqrt{-\cdots}}{231 a^{2} c^{8} \sqrt{b c\, x^{3}\cdots}}\cdots\right.}{\sqrt{c x}\, \sqrt{b\, x^{2}+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155/(b*x^2+a)^(1/2)/x^7*(10*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2
)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
b^3*x^7+20*b^4*x^8+8*a*b^3*x^6-131*a^2*b^2*x^4-196*a^3*b*x^2-77*a^4)/a^2/c
^8/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.38

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \frac{2\left(20\sqrt{bc}\,b^3x^8\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right) + \left(20\,b^3x^6 - 12\,ab^2x^4 - 119\,a^2bx^2 - 77 \cdots\right.\right.}{1155\,a^2c^9x^8}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(17/2),x, algorithm="fricas")
```

output
```
2/1155*(20*sqrt(b*c)*b^3*x^8*weierstrassPInverse(-4*a/b, 0, x) + (20*b^3*x
^6 - 12*a*b^2*x^4 - 119*a^2*b*x^2 - 77*a^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^
2*c^9*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{17}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(17/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(17/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{17}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(17/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(17/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{17/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(17/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(17/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{17/2}}\,dx = \frac{2\sqrt{c}\left(-7\sqrt{b\,x^2+a}\,a-13\sqrt{b\,x^2+a}\,b\,x^2+6\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^{11}+a\,x^9}dx\right)a^2x^7\right)}{117\sqrt{x}\,c^9x^7}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(17/2),x)
```

output
```
(2*sqrt(c)*( - 7*sqrt(a + b*x**2)*a - 13*sqrt(a + b*x**2)*b*x**2 + 6*sqrt(
x)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**9 + b*x**11),x)*a**2*x**7))/(117*s
qrt(x)*c**9*x**7)
```

3.605 $\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 329

$$\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx = \frac{8a^2c(cx)^{3/2}\sqrt{a+bx^2}}{195b} + \frac{4a(cx)^{7/2}\sqrt{a+bx^2}}{39c} - \frac{8a^3c^2\sqrt{cx}\sqrt{a+bx^2}}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c} + \frac{8a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}} - \frac{4a^{13/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{65b^{7/4}\sqrt{a+bx^2}}$$

output
```
8/195*a^2*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b+4/39*a*(c*x)^(7/2)*(b*x^2+a)^(1/
2)/c-8/65*a^3*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)/(a^(1/2)+b^(1/2)*x)+
2/13*(c*x)^(7/2)*(b*x^2+a)^(3/2)/c+8/65*a^(13/4)*c^(5/2)*(a^(1/2)+b^(1/2)*
x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b^(7/4)/(b*x^2+a)^(1/2)-4/65*a^
(13/4)*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)
*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2)
)/b^(7/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.27

$$\int (cx)^{5/2} (a+bx^2)^{3/2}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}}-a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

output
```
(2*c*(c*x)^(3/2)*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*
Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a
])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {248, 248, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2} (a+bx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{13}a\int (cx)^{5/2}\sqrt{bx^2+a}dx+\frac{2(cx)^{7/2}(a+bx^2)^{3/2}}{13c}$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\int \frac{(cx)^{5/2}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}\right)+\frac{2(cx)^{7/2}(a+bx^2)^{3/2}}{13c}$$

$$\Big\downarrow \text{262}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ac^2\int\frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}\right)+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}\right)+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{834}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}\right)+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{27}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)+\frac{2(cx)^{7/2}\sqrt{a+bx^2}}{9c}\right)+\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{761}$$

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{}}{\sqrt{b}}\right.}{5b}\right.\right.$$

$$\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

$\downarrow$ 1510

$$\frac{6}{13}a\left(\frac{2}{9}a\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{}\right.}{}\right.}{5b}\right.\right.$$

$$\frac{2(cx)^{7/2}\left(a+bx^2\right)^{3/2}}{13c}$$

input
```
Int[(c*x)^(5/2)*(a + b*x^2)^(3/2),x]
```

output
```
(2*(c*x)^(7/2)*(a + b*x^2)^(3/2))/(13*c) + (6*a*((2*(c*x)^(7/2)*Sqrt[a + b
*x^2])/(9*c) + (2*a*((2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c*(-((
-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sq
rt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt
[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2
])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqr
t[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[
2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a +
 b*x^2]))/(5*b)))/9))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.71

method	result
default	$-\frac{2c^2\sqrt{cx}\left(-15b^4x^8-40a\,b^3x^6+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^4-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\cdots}\right.}{195x\sqrt{b\,x^2+a}\,b^2}$
risch	$\frac{2x^2(15b^2x^4+25ab\,x^2+4a^2)\sqrt{b\,x^2+a}\,c^3}{195b\sqrt{cx}}-\frac{4a^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-\cdots}}{b}\right)}{\sqrt{-a\cdots}}}\cdots\right.}{b}\cdots\right.}{65b^2\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\cdots}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{2b\,c^2x^5\sqrt{bc\,x^3+acx}}{13}+\frac{10a\,c^2x^3\sqrt{bc\,x^3+acx}}{39}+\frac{8a^2c^2x\sqrt{bc\,x^3+acx}}{195b}-\frac{4a^3c^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{\cdots}}{\cdots}\right.}{cx\sqrt{b\,x^2+a}}$

input

```
int((c*x)^(5/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-2/195*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(-15*b^4*x^8-40*a*b^3*x^6+12*
((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/
2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^4-29*a
^2*b^2*x^4-4*a^3*b*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.26

$$\int (cx)^{5/2} (a+bx^2)^{3/2} \, dx = \frac{2 \left(12 \sqrt{bc} a^3 c^2 \mathrm{weierstrassZeta}\left(-\frac{4a}{b}, 0, \mathrm{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right)\right) + (15 b^3 c^2 x^5 + 25 ab\right.}{195 b^2}$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
2/195*(12*sqrt(b*c)*a^3*c^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse
(-4*a/b, 0, x)) + (15*b^3*c^2*x^5 + 25*a*b^2*c^2*x^3 + 4*a^2*b*c^2*x)*sqrt
(b*x^2 + a)*sqrt(c*x))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.41 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.14

$$\int (cx)^{5/2} (a+bx^2)^{3/2} \, dx = \frac{a^{\frac{3}{2}} c^{\frac{5}{2}} x^{\frac{7}{2}} \Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-3/2, 7/4), (11/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(11/4))
```

Maxima [F]

$$\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}} \, dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}} \, dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2} \left(a+bx^2\right)^{3/2} \, dx = \int (c\,x)^{5/2}\,\left(b\,x^2+a\right)^{3/2} \,d x$$

input
```
int((c*x)^(5/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(5/2)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{5/2}\left(a+bx^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\,c^2\left(4\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2x+25\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^3+15\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^5-6\left(\int\frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}\right.\right.}{195b}$$

input
```
int((c*x)^(5/2)*(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(c)*c**2*(4*sqrt(x)*sqrt(a + b*x**2)*a**2*x + 25*sqrt(x)*sqrt(a + b
*x**2)*a*b*x**3 + 15*sqrt(x)*sqrt(a + b*x**2)*b**2*x**5 - 6*int((sqrt(x)*s
qrt(a + b*x**2))/(a + b*x**2),x)*a**3))/(195*b)
```

3.606 $\int \sqrt{cx}(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 297

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{4a(cx)^{3/2}\sqrt{a+bx^2}}{15c}+\frac{8a^2\sqrt{cx}\sqrt{a+bx^2}}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)}+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}-\frac{8a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}+\frac{4a^{9/4}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{3/4}\sqrt{a+bx^2}}$$

output
```
4/15*a*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c+8/15*a^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/
b^(1/2)/(a^(1/2)+b^(1/2)*x)+2/9*(c*x)^(3/2)*(b*x^2+a)^(3/2)/c-8/15*a^(9/4)
*c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Ellip
ticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b^(3/
4)/(b*x^2+a)^(1/2)+4/15*a^(9/4)*c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^
(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2)),1/2*2^(1/2))/b^(3/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.19

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{2ax\sqrt{cx}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

output
```
(2*a*x*Sqrt[c*x]*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^
2)/a)])/(3*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {248, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{2}{3}a\int \sqrt{cx}\sqrt{bx^2+a}dx+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{2}{3}a\left(\frac{2}{5}a\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{3}a\left(\frac{4a\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$\big\downarrow$ 834

$$\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$\big\downarrow$ 27

$$\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$\big\downarrow$ 761

$$\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)+\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

$\big\downarrow$ 1510

$$\frac{2}{3}a\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5c}\right.$$

$$\frac{2(cx)^{3/2}\left(a+bx^2\right)^{3/2}}{9c}$$

input
```
Int[Sqrt[c*x]*(a + b*x^2)^(3/2),x]
```

output
```
(2*(c*x)^(3/2)*(a + b*x^2)^(3/2))/(9*c) + (2*a*((2*(c*x)^(3/2)*Sqrt[a + b*
x^2])/(5*c) + (4*a*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt
[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2
*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/
(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*S
qrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqr
t[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*c)))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\sqrt{cx}\left(5b^3x^6+12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3-6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\right.}{45\sqrt{bx^2+a}\,bx}$
risch	$\frac{2x^2(5bx^2+11a)\sqrt{bx^2+a}\,c}{45\sqrt{cx}}+\frac{4a^2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{15b\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{bx^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(bx^2+a)}\left(\frac{2bx^3\sqrt{bc\,x^3+acx}}{9}+\frac{22ax\sqrt{bc\,x^3+acx}}{45}+\frac{4a^2c\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\right.}{}\right.}{15b\sqrt{bc\,x^3+ac}}\right.}{cx\sqrt{bx^2+a}}$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b*(5*b^3*x^6+12*((b*x+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)
^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2))*a^3-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(
1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3+16*a*b^2*x^4+11*a^2*b*x^2)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.21

$$\int\sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=$$

$$-\frac{2\left(12\sqrt{bc}a^2\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\left(5b^2x^3+11abx\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{45b}$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-2/45*(12*sqrt(b*c)*a^2*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*
a/b, 0, x)) - (5*b^2*x^3 + 11*a*b*x)*sqrt(b*x^2 + a)*sqrt(c*x))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.15

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{a^{\frac{3}{2}}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-3/2, 3/4), (7/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\int \sqrt{c\,x}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((c*x)^(1/2)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(1/2)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{2\sqrt{c}\left(11\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+5\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3+6\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2\right)}{45}$$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(3/2),x)
```

output
```
(2*sqrt(c)*(11*sqrt(x)*sqrt(a + b*x**2)*a*x + 5*sqrt(x)*sqrt(a + b*x**2)*b
*x**3 + 6*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/45
```

3.607 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 297

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{c\sqrt{cx}}+\frac{2b(cx)^{3/2}\sqrt{a+bx^2}}{5c^3}+\frac{24a\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{5c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{24a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}} + \frac{12a^{5/4}\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{3/2}\sqrt{a+bx^2}}$$

output
```
-2*a*(b*x^2+a)^(1/2)/c/(c*x)^(1/2)+2/5*b*(c*x)^(3/2)*(b*x^2+a)^(1/2)/c^3+2
4/5*a*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^2/(a^(1/2)+b^(1/2)*x)-24/5*a^(
5/4)*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/c
^(3/2)/(b*x^2+a)^(1/2)+12/5*a^(5/4)*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)
/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)),1/2*2^(1/2))/c^(3/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.19

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])/
((c*x)^(3/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {247, 248, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{6b\int \sqrt{cx}\sqrt{bx^2+a}dx}{c^2}-\frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{248}$$

$$\frac{6b\left(\frac{2}{5}a\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)}{c^2}-\frac{2(a+bx^2)^{3/2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{6b\left(\frac{4a\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)}{c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 834

$$\frac{6b\left(\frac{4a\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)}{c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 27

$$\frac{6b\left(\frac{4a\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)}{c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 761

$$\frac{6b\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5c}+\frac{2(cx)^{3/2}\sqrt{a+bx^2}}{5c}\right)}{c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 1510

$$6b\left(\frac{4a\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}\cdots}{\sqrt{b}}\right)}{5c}\cdots\right)$$

$$\overline{\qquad\qquad c^2\qquad\qquad}$$

$$\frac{2\left(a+bx^2\right)^{3/2}}{c\sqrt{cx}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(3/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(c*Sqrt[c*x]) + (6*b*((2*(c*x)^(3/2)*Sqrt[a + b*x^2
])/(5*c) + (4*a*(-((-(((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]
*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^
2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^
(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt
[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b
]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])
/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*c)))/c^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.70

method	result
default	$\frac{\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\cdots\right.}{5}\cdots}{\sqrt{cx}\,\sqrt{b\,x^2+a}\,c}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-b\,x^2+5a)}{5c\sqrt{cx}}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right.}{5\sqrt{bc\,x^3+acx}\,c\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\left(x^2bc+ac\right)a}{c^2\sqrt{x\left(x^2bc+ac\right)}}+\frac{2bx\sqrt{bc\,x^3+acx}}{5c^2}+\frac{12a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\cdots\right.}{\cdots}}\cdots\right.}{b}\cdots\right.}{5c\sqrt{bc\,x^3+acx}}\cdots\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2
)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
a^2+b^2*x^4-4*a*b*x^2-5*a^2)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.20

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx =$$
$$-\frac{2\left(12\sqrt{bc}ax\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\sqrt{bx^2+a}(bx^2-5a)\sqrt{cx}\right)}{5c^2x}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/5*(12*sqrt(b*c)*a*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a
/b, 0, x)) - sqrt(b*x^2 + a)*(b*x^2 - 5*a)*sqrt(c*x))/(c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.16

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(3/2),x)
```

output
```
a**(3/2)*gamma(-1/4)*hyper((-3/2, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(3/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\left(7\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2+6\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)a^2\right)}{5\sqrt{x}\,c^2}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(3/2),x)
```

output
```
(2*sqrt(c)*(7*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*int
((sqrt(x)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x)*a**2))/(5*sqrt(x)*c**2)
```

3.608 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 298

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{5c(cx)^{5/2}} - \frac{14b\sqrt{a+bx^2}}{5c^3\sqrt{cx}} + \frac{24b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5c^4\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{24\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}} + \frac{12\sqrt[4]{a}b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5c^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*a*(b*x^2+a)^(1/2)/c/(c*x)^(5/2)-14/5*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2
)+24/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/c^4/(a^(1/2)+b^(1/2)*x)-24/5*a^
(1/4)*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*
EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/
c^(7/2)/(b*x^2+a)^(1/2)+12/5*a^(1/4)*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a
)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2
)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/c^(7/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.19

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = -\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)])
/(5*(c*x)^(7/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {247, 247, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx$$

$$\Big\downarrow \ 247$$

$$\frac{6b\int \frac{\sqrt{bx^2+a}}{(cx)^{3/2}}dx}{5c^2}-\frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \ 247$$

$$\frac{6b\left(\frac{2b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{c^2}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}\right)}{5c^2}-\frac{2(a+bx^2)^{3/2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \ 266$$

$$\frac{6b\left(\frac{4b\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}$$

$\big\downarrow$ 834

$$\frac{6b\left(\frac{4b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}$$

$\big\downarrow$ 27

$$\frac{6b\left(\frac{4b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}$$

$\big\downarrow$ 761

$$\frac{6b\left(\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c^3}-\frac{2\sqrt{a+bx^2}}{c\sqrt{cx}}\right)}{5c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}$$

$\big\downarrow$ 1510

$$6b\left(\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{c^3}\right.$$

$$\overline{\qquad\qquad 5c^2 \qquad\qquad}$$

$$\frac{2\left(a+bx^2\right)^{3/2}}{5c(cx)^{5/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(7/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(5*c*(c*x)^(5/2)) + (6*b*((-2*Sqrt[a + b*x^2])/(c*S
qrt[c*x]) + (4*b*(-((-(-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b
]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x
^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqr
t[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[
b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2]
)/(2*b^(3/4)*Sqrt[a + b*x^2])))/c^3))/(5*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.72

method	result
default	$\frac{\frac{24\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2}{5}-\frac{12\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\cdots}{\cdots}}\cdots\right.}{5}\cdots}{x^2\sqrt{b\,x^2+a}\,c^3\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(7b\,x^2+a)}{5x^2c^3\sqrt{cx}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-\cdots}}{\cdots}\cdots\right.}{5\sqrt{bc\,x^3+acx}\,c^3\sqrt{cx}\,\sqrt{b\,x^2+a}}\cdots$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2a\sqrt{bc\,x^3+acx}}{5c^4x^3}-\frac{14\left(x^2bc+ac\right)b}{5c^4\sqrt{x\left(x^2bc+ac\right)}}+\frac{12b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x-\cdots\right.}{\cdots}}\cdots\right.}{b}\cdots\right.}{5c^3\sqrt{bc\,x^3+acx}}\cdots\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}\cdots$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5/x^2*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-6*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a*b*x^2-7*b^2*x^4-8*a*b*x^2-a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.20

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx =$$

$$-\frac{2\left(12\sqrt{bc}bx^3\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(7bx^2+a)\sqrt{bx^2+a}\sqrt{cx}\right)}{5c^4x^3}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(12*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4
*a/b, 0, x)) + (7*b*x^2 + a)*sqrt(b*x^2 + a)*sqrt(c*x))/(c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\ -\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(7/2),x)
```

output
```
a**(3/2)*gamma(-5/4)*hyper((-3/2, -5/4), (-1/4,), b*x**2*exp_polar(I*pi)/a
)/(2*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=\int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{7/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(7/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{7/2}}\,dx=\frac{2\sqrt{c}\left(-\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2-2\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx\right)a^2x^2\right)}{\sqrt{x}\,c^4x^2}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(7/2),x)
```

output
```
(2*sqrt(c)*( - sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 - 2*sqrt(x)*in
t((sqrt(x)*sqrt(a + b*x**2))/(a*x**4 + b*x**6),x)*a**2*x**2))/(sqrt(x)*c**
4*x**2)
```

3.609 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 332

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2a\sqrt{a+bx^2}}{9c(cx)^{9/2}}-\frac{22b\sqrt{a+bx^2}}{45c^3(cx)^{5/2}}-\frac{8b^2\sqrt{a+bx^2}}{15ac^5\sqrt{cx}}+\frac{8b^{5/2}\sqrt{cx}\sqrt{a+bx^2}}{15ac^6\left(\sqrt{a}+\sqrt{b}x\right)}-\frac{8b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}+\frac{4b^{9/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15a^{3/4}c^{11/2}\sqrt{a+bx^2}}$$

output
```
-2/9*a*(b*x^2+a)^(1/2)/c/(c*x)^(9/2)-22/45*b*(b*x^2+a)^(1/2)/c^3/(c*x)^(5/
2)-8/15*b^2*(b*x^2+a)^(1/2)/a/c^5/(c*x)^(1/2)+8/15*b^(5/2)*(c*x)^(1/2)*(b*
x^2+a)^(1/2)/a/c^6/(a^(1/2)+b^(1/2)*x)-8/15*b^(9/4)*(a^(1/2)+b^(1/2)*x)*((
b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)
^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(3/4)/c^(11/2)/(b*x^2+a)^(1/2)+4/1
5*b^(9/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(
3/4)/c^(11/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.17

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx=-\frac{2ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},-\frac{3}{2},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

output
```
(-2*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)])
/(9*(c*x)^(11/2)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {247, 247, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx$$

$$\Big\downarrow \ 247$$

$$\frac{2b\int \frac{\sqrt{bx^2+a}}{(cx)^{7/2}}dx}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \ 247$$

$$\frac{2b\left(\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$$\Big\downarrow \ 264$$

$$\frac{2b\left(\frac{2b\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$\Big\downarrow$ 266

$$\frac{2b\left(\frac{2b\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$\Big\downarrow$ 834

$$\frac{2b\left(\frac{2b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$\Big\downarrow$ 27

$$\frac{2b\left(\frac{2b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

$\Big\downarrow$ 761

$$\frac{2b\left(\frac{2b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{a+bx^2}}{5c(cx)^{5/2}}\right)}{3c^2}-\frac{2\left(a+bx^2\right)^{3/2}}{9c(cx)^{9/2}}$$

$\downarrow$ 1510

$$2b\left(\frac{2b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{ac^3}}{5c^2}}{3c^2}$$

$$\frac{2(a+bx^2)^{3/2}}{9c(cx)^{9/2}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(11/2),x]
```

output
```
(-2*(a + b*x^2)^(3/2))/(9*c*(c*x)^(9/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(5*c
*(c*x)^(5/2)) + (2*b*((-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-(((c
^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c
]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*
c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(
b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]
*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x
^2])))/(a*c^3)))/(5*c^2)))/(3*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

method	result
default	$\frac{\frac{8\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^{2}x^{4}}{15}-\frac{4\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{}}{\sqrt{-}}}\right.}{15}}{x^{4}\sqrt{b\,x^{2}+a}\,a\,c^{5}\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}\,\left(12b^{2}x^{4}+11ab\,x^{2}+5a^{2}\right)}{45x^{4}a\,c^{5}\sqrt{cx}}+\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{15a\sqrt{bc\,x^{3}+acx}\,c^{5}\sqrt{cx}\,\sqrt{b\,x^{2}}}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\,\left(-\frac{2a\sqrt{bc\,x^{3}+acx}}{9c^{6}x^{5}}-\frac{22b\sqrt{bc\,x^{3}+acx}}{45c^{6}x^{3}}-\frac{8\left(x^{2}bc+ac\right)b^{2}}{15a\,c^{6}\sqrt{x\left(x^{2}bc+ac\right)}}+\frac{4b^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{}}{}\right.}{}\right.}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/45/x^4*(12*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)
^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*
b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b^2*x^4-6*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/
(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a*b^2*x^4-12*b^3*x^6-23*a*b^2*x^4-16*a^2*b*x^2-5*a^3)/(b*x^2+a)
^(1/2)/a/c^5/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx =$$
$$-\frac{2\left(12\sqrt{bc}b^2x^5\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(12b^2x^4+11abx^2+5a^2)\sqrt{bx}\right.}{45ac^6x^5}$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
-2/45*(12*sqrt(b*c)*b^2*x^5*weierstrassZeta(-4*a/b, 0, weierstrassPInverse
(-4*a/b, 0, x)) + (12*b^2*x^4 + 11*a*b*x^2 + 5*a^2)*sqrt(b*x^2 + a)*sqrt(c
*x))/(a*c^6*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 46.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.16

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{9}{4},-\frac{3}{2}\\-\frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{11}{2}}x^{\frac{9}{2}}\Gamma\left(-\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(11/2),x)
```

output
```
a**(3/2)*gamma(-9/4)*hyper((-9/4, -3/2), (-5/4,), b*x**2*exp_polar(I*pi)/a
)/(2*c**(11/2)*x**(9/2)*gamma(-5/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{11/2}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(11/2),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{11/2}}\,dx = \frac{2\sqrt{c}\left(-\sqrt{b\,x^2+a}\,a-7\sqrt{b\,x^2+a}\,b\,x^2+6\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^8+a\,x^6}dx\right)a^2x^4\right)}{21\sqrt{x}\,c^6x^4}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(11/2),x)
```

output
```
(2*sqrt(c)*( - sqrt(a + b*x**2)*a - 7*sqrt(a + b*x**2)*b*x**2 + 6*sqrt(x)*
int((sqrt(x)*sqrt(a + b*x**2))/(a*x**6 + b*x**8),x)*a**2*x**4))/(21*sqrt(x
)*c**6*x**4)
```

3.610 $\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx=-\frac{2}{7}c\sqrt{cx}\sqrt{3a-2ax^2}+\frac{2(cx)^{5/2}\sqrt{3a-2ax^2}}{7c}+\frac{6^{3/4}ac^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{7\sqrt{3a-2ax^2}}$$

output
```
-2/7*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)+2/7*(c*x)^(5/2)*(-2*a*x^2+3*a)^(1/
2)/c+1/7*6^(3/4)*a*c^(3/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*
(c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.74 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx=\frac{c\sqrt{cx}\sqrt{a(3-2x^2)}\left(-(3-2x^2)^{3/2}+3\sqrt{3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{2x^2}{3}\right)\right)}{7\sqrt{3-2x^2}}$$

input
```
Integrate[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(c*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hypergeom
etric2F1[-1/2, 1/4, 5/4, (2*x^2)/3]))/(7*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {248, 262, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3a-2ax^2}(cx)^{3/2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{7}a\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}dx+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}$$

$$\Big\downarrow \text{262}$$

$$\frac{6}{7}a\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}$$

$$\Big\downarrow \text{266}$$

$$\frac{6}{7}a\left(c\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}$$

$$\Big\downarrow \text{765}$$

$$\frac{6}{7}a\left(\frac{c\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{\sqrt{3}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}$$

$$\Big\downarrow \text{762}$$

$$\frac{6}{7}a\left(\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{5/2}}{7c}$$

input
```
Int[(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(2*(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2])/(7*c) + (6*a*(-1/3*(c*Sqrt[c*x]*Sqrt[3
*a - 2*a*x^2])/a + (c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*
Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[3*a - 2*a*x^2])))/7
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.13

method	result
default	$-\frac{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(-8x^5+\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{}}}{6}\right.\right.}{14x(2x^2-3)}$
risch	$-\frac{2(x^2-1)x(2x^2-3)a\,c^2}{7\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)a\,c^2\sqrt{-cxa(2}}{126\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(\frac{2c\,x^2\sqrt{-2ac\,x^3+3acx}}{7}-\frac{2c\sqrt{-2ac\,x^3+3acx}}{7}+\frac{a\,c^2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}}}{126\sqrt{-2ac\,x^3+3a}}\right.}{cxa(2x^2-3)}$

input
```
int((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/14*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(-8*x^5+((2*x+3^(1/2)*2^(1/2))*3^
(1/2)*2^(1/2))^(1/2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1
/2)*2^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*
3^(1/2)*2^(1/2))^(1/2),1/2*2^(1/2))+20*x^3-12*x)/x/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.38

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx = -\frac{3}{7}\sqrt{2}\sqrt{-ac}c\text{weierstrassPInverse}(6,0,x)+\frac{2}{7}\sqrt{-2\,ax^2+3\,a}(cx^2-c)\sqrt{cx}$$

input
```
integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-3/7*sqrt(2)*sqrt(-a*c)*c*weierstrassPInverse(6, 0, x) + 2/7*sqrt(-2*a*x^2
 + 3*a)*(c*x^2 - c)*sqrt(c*x)
```

Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.45

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx = \frac{\sqrt{3}\sqrt{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)*(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*sqrt(a)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), 2*
x**2*exp_polar(2*I*pi)/3)/(2*gamma(9/4))
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx = \int \sqrt{-2\,ax^2+3\,a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx = \int \sqrt{-2\,ax^2+3\,a}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx = \int (c\,x)^{3/2}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

input
```
int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{3/2}\sqrt{3a-2ax^2}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(2\sqrt{x}\,\sqrt{-2x^2+3}\,x^2-2\sqrt{x}\,\sqrt{-2x^2+3}-3\left(\int\frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^3-3x}dx\right)\right)}{7}$$

input
```
int((c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*(2*sqrt(x)*sqrt( - 2*x**2 + 3)*x**2 - 2*sqrt(x)*sqrt( -
 2*x**2 + 3) - 3*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**3 - 3*x),x)))/7
```

3.611 $\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\sqrt{3a-2ax^2}}{3c} + \frac{2\,2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{3a-2ax^2}}$$

output
```
2/3*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)/c+2/3*2^(3/4)*a*(-2*x^2+3)^(1/2)*Elli
pticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(3/4)/c^(1/2)/(-2*a*x^2
+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\frac{2x\sqrt{a\left(9-6x^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{5}{4},\frac{2x^2}{3}\right)}{\sqrt{cx}\sqrt{3-2x^2}}$$

input
```
Integrate[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

output
```
(2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, (2*x^2)/3])/(Sq
rt[c*x]*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {248, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx$$

$$\Big\downarrow \text{248}$$

$$2a\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

$$\Big\downarrow \text{266}$$

$$\frac{4a\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{c}+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

$$\Big\downarrow \text{765}$$

$$\frac{4a\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{\sqrt{3}c\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\,2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}\sqrt{cx}}{3c}$$

input
```
Int[Sqrt[3*a - 2*a*x^2]/Sqrt[c*x],x]
```

output
```
(2*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/(3*c) + (2*2^(3/4)*a*Sqrt[3 - 2*x^2]*Ell
ipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3^(1/4)*Sqrt[c]*Sqrt
[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)-4x^3\right.}{3\sqrt{cx}\,(2x^2-3)}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(\frac{2\sqrt{-2ac\,x^3+3acx}}{3c}+\frac{a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{27\sqrt{-2ac\,x^3+3acx}}\right)}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
risch	$-\frac{2x(2x^2-3)a}{3\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)a\sqrt{-cxa(2x^2-3)}}{27\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-a*(2*x^2-3))^(1/2)*(((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(
(-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/2)*E
llipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)
,1/2*2^(1/2))-4*x^3+6*x)/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=-\frac{2\left(3\sqrt{2}\sqrt{-ac}\text{weierstrassPInverse}(6,0,x)-\sqrt{-2\,ax^2+3\,a}\sqrt{cx}\right)}{3\,c}$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) - sqrt(-2*a*x^2 +
3*a)*sqrt(c*x))/c
```

Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\frac{\sqrt{3}\sqrt{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(1/2),x)
```

output
```
sqrt(3)*sqrt(a)*sqrt(x)*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(2*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{-2\,ax^2+3\,a}}{\sqrt{cx}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\int \frac{\sqrt{3\,a-2\,a\,x^2}}{\sqrt{c\,x}}\,dx$$

input
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2),x)
```

output
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3a-2ax^2}}{\sqrt{cx}}\,dx=\frac{2\sqrt{c}\,\sqrt{a}\,\left(\sqrt{x}\,\sqrt{-2x^2+3}-3\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^3-3x}dx\right)\right)}{3c}$$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*(sqrt(x)*sqrt( - 2*x**2 + 3) - 3*int((sqrt(x)*sqrt( - 2
*x**2 + 3))/(2*x**3 - 3*x),x)))/(3*c)
```

3.612 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}} - \frac{4\,2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}$$

output
```
-2/3*(-2*a*x^2+3*a)^(1/2)/c/(c*x)^(3/2)-4/9*2^(3/4)*a*(-2*x^2+3)^(1/2)*Ell
ipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(3/4)/c^(5/2)/(-2*a*x^
2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = -\frac{2x\sqrt{a(3-2x^2)}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{4},\frac{2x^2}{3}\right)}{(cx)^{5/2}\sqrt{9-6x^2}}$$

input
```
Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

output
```
(-2*x*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, (2*x^2)/3])/(
(c*x)^(5/2)*Sqrt[9 - 6*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {247, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{4a\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{3c^2}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{8a\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{3c^3}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{765}$$

$$-\frac{8a\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{3\sqrt{3}c^3\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$-\frac{4\,2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3c(cx)^{3/2}}$$

input
```
Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(5/2),x]
```

output
```
(-2*Sqrt[3*a - 2*a*x^2])/(3*c*(c*x)^(3/2)) - (4*2^(3/4)*a*Sqrt[3 - 2*x^2]*
EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*3^(1/4)*c^(5/2)
*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.33

method	result
default	$\frac{2\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)x-6x^2-}{9x\,c^2\sqrt{cx}\,(2x^2-3)}$
risch	$\frac{2(2x^2-3)a}{3x\,c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}}-\frac{2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)a\sqrt{-cxa(2x^2-}}{81\sqrt{-2ac\,x^3+3acx}\,c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{2\sqrt{-2ac\,x^3+3acx}}{3c^3x^2}-\frac{2a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\text{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3}\right.}{81c^2\sqrt{-2ac\,x^3+3acx}}\right.}{\sqrt{cx}\,a(2x^2-3)}$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(-a*(2*x^2-3))^(1/2)*((((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*((
-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/2)*El
lipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2),
1/2*2^(1/2))*x-6*x^2+9)/x/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = \frac{2\left(2\sqrt{2}\sqrt{-ac}x^2\text{weierstrassPInverse}(6,0,x)-\sqrt{-2\,ax^2+3\,a}\sqrt{cx}\right)}{3\,c^3x^2}$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(2)*sqrt(-a*c)*x^2*weierstrassPInverse(6, 0, x) - sqrt(-2*a*x^2
 + 3*a)*sqrt(c*x))/(c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{2}i\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{3}{2x^2}\right)}{2c^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(5/2),x)
```

output
```
sqrt(2)*I*sqrt(a)*gamma(-1/4)*hyper((-1/2, 1/4), (5/4,), 3/(2*x**2))/(2*c*
*(5/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx=\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx=\int \frac{\sqrt{3\,a-2\,a\,x^2}}{(c\,x)^{5/2}}\,dx$$

input
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2),x)
```

output
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{5/2}}\,dx=\frac{2\sqrt{c}\,\sqrt{a}\left(-\sqrt{-2x^2+3}+3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^5-3x^3}dx\right)x\right)}{\sqrt{x}\,c^3x}$$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(5/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*( - sqrt( - 2*x**2 + 3) + 3*sqrt(x)*int((sqrt(x)*sqrt(
- 2*x**2 + 3))/(2*x**5 - 3*x**3),x)*x))/(sqrt(x)*c**3*x)
```

3.613 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}+\frac{8\sqrt{3a-2ax^2}}{63c^3(cx)^{3/2}} - \frac{8\,2^{3/4}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{63\sqrt[4]{3}c^{9/2}\sqrt{3a-2ax^2}}$$

output
```
-2/7*(-2*a*x^2+3*a)^(1/2)/c/(c*x)^(7/2)+8/63*(-2*a*x^2+3*a)^(1/2)/c^3/(c*x
)^(3/2)-8/189*2^(3/4)*a*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*
x)^(1/2)/c^(1/2),I)*3^(3/4)/c^(9/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt{a\left(9-6x^2\right)}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{2},-\frac{3}{4},\frac{2x^2}{3}\right)}{7(cx)^{9/2}\sqrt{3-2x^2}}$$

input
```
Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(9/2),x]
```

output
```
(-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-7/4, -1/2, -3/4, (2*x^2)/3])/
(7*(c*x)^(9/2)*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {247, 264, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{4a\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}dx}{7c^2}-\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{4a\left(\frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{9c^2}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{4a\left(\frac{4\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{9c^3}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}$$

$\Big\downarrow$ 765

$$-\frac{4a\left(\frac{4\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{9\sqrt{3}c^3\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}$$

$\Big\downarrow$ 762

$$-\frac{4a\left(\frac{2\ 2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt{3a-2ax^2}}{7c(cx)^{7/2}}$$

input
```
Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(9/2),x]
```

output
```
(-2*Sqrt[3*a - 2*a*x^2])/(7*c*(c*x)^(7/2)) - (4*a*((-2*Sqrt[3*a - 2*a*x^2]
)/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)
^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*Sqrt[3*a - 2*a*x^2])))
/(7*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

method	result
default	$\frac{2\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)x^3+24\right.}{189x^3c^4\sqrt{cx}\,(2x^2-3)}$
risch	$-\frac{2(8x^4-30x^2+27)a}{63x^3c^4\sqrt{cx}\,\sqrt{-a(2x^2-3)}}-\frac{4\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)a\sqrt{-cxa(}}{1701\sqrt{-2ac\,x^3+3acx}\,c^4\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{2\sqrt{-2ac\,x^3+3acx}}{7c^5x^4}+\frac{8\sqrt{-2ac\,x^3+3acx}}{63c^5x^2}-\frac{4a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \mathrm{EllipticF}}{1701c^4\sqrt{-2ac\,x^3+3acx}}\right.}{\sqrt{cx}\,a(2x^2-3)}$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/189*(-a*(2*x^2-3))^(1/2)*(2*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2
)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/2
)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1
/2),1/2*2^(1/2))*x^3+24*x^4-90*x^2+81)/x^3/c^4/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = \frac{2\left(4\sqrt{2}\sqrt{-ac}x^4\text{weierstrassPInverse}(6,0,x)+\sqrt{-2\,ax^2+3\,a}\sqrt{cx}(4\,x^2-9)\right)}{63\,c^5x^4}$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
2/63*(4*sqrt(2)*sqrt(-a*c)*x^4*weierstrassPInverse(6, 0, x) + sqrt(-2*a*x^
2 + 3*a)*sqrt(c*x)*(4*x^2 - 9))/(c^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 16.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{3}\sqrt{a}\Gamma\left(-\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{7}{4},-\frac{1}{2}\\-\frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(9/2),x)
```

output
```
sqrt(3)*sqrt(a)*gamma(-7/4)*hyper((-7/4, -1/2), (-3/4,), 2*x**2*exp_polar(
2*I*pi)/3)/(2*c**(9/2)*x**(7/2)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{9/2}}\,dx = \int \frac{\sqrt{3\,a-2\,a\,x^2}}{(c\,x)^{9/2}}\,dx$$

input
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(9/2),x)
```

output
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3a-2a x^2}}{(cx)^{9/2}}\,dx=\frac{2\sqrt{c}\,\sqrt{a}\,\left(-\sqrt{-2x^2+3}+3\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^7-3x^5}dx\right)x^3\right)}{5\sqrt{x}\,c^5x^3}$$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(9/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*( - sqrt( - 2*x**2 + 3) + 3*sqrt(x)*int((sqrt(x)*sqrt(
- 2*x**2 + 3))/(2*x**7 - 3*x**5),x)*x**3))/(5*sqrt(x)*c**5*x**3)
```

3.614 $\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 181

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = -\frac{2}{15}c(cx)^{3/2}\sqrt{3a-2ax^2}+\frac{2(cx)^{7/2}\sqrt{3a-2ax^2}}{9c} + \frac{3\sqrt[4]{6}ac^{5/2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{3a-2ax^2}} - \frac{3\sqrt[4]{6}ac^{5/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{5\sqrt{3a-2ax^2}}$$

output
```
-2/15*c*(c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2)+2/9*(c*x)^(7/2)*(-2*a*x^2+3*a)^(1
/2)/c+3/5*6^(1/4)*a*c^(5/2)*(-2*x^2+3)^(1/2)*EllipticE(1/3*2^(1/4)*3^(3/4)
*(c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3*a)^(1/2)-3/5*6^(1/4)*a*c^(5/2)*(-2*x^2
+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3
*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.41

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx=\frac{c(cx)^{3/2}\sqrt{a(3-2x^2)}\left(-(3-2x^2)^{3/2}+3\sqrt{3}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{2x^2}{3}\right)\right)}{9\sqrt{3-2x^2}}$$

input
```
Integrate[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(c*(c*x)^(3/2)*Sqrt[a*(3 - 2*x^2)]*(-(3 - 2*x^2)^(3/2) + 3*Sqrt[3]*Hyperge
ometric2F1[-1/2, 3/4, 7/4, (2*x^2)/3]))/(9*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {248, 262, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3a-2ax^2}(cx)^{5/2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{3}a\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}dx+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{3}a\left(\frac{9}{10}c^2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow \text{261}$$

$$\frac{2}{3}a\left(\frac{9c^2\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{10\sqrt{x}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow 260$$

$$\frac{2}{3}a\left(\frac{3\sqrt{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow 27$$

$$\frac{2}{3}a\left(\frac{9c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow 259$$

$$\frac{2}{3}a\left(-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

$$\Big\downarrow 327$$

$$\frac{2}{3}a\left(-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)+\frac{2\sqrt{3a-2ax^2}(cx)^{7/2}}{9c}$$

input
```
Int[(c*x)^(5/2)*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(2*(c*x)^(7/2)*Sqrt[3*a - 2*a*x^2])/(9*c) + (2*a*(-1/5*(c*(c*x)^(3/2)*Sqrt
[3*a - 2*a*x^2])/a - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[Ar
cSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sqrt[x]*Sqrt[3*a - 2*a*x
^2])))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{2x^2(5x^2-3)(2x^2-3)c^3a}{45\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\cdots}{\cdots}\right.}{90\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}\cdots}$
elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(\frac{2c^2x^3\sqrt{-2ac\,x^3+3acx}}{9}-\frac{2c^2x\sqrt{-2ac\,x^3+3acx}}{15}+\frac{c^3a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\cdots}}{\cdots}\right.}{cxa(2x^2-3)\cdots}$
default	$\frac{c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(80x^6+18\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\cdots\right.}\cdots\right.}{\cdots}$

input
```
int((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x^2*(5*x^2-3)*(2*x^2-3)*c^3*a/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)+1/90*
6^(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2
))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*Ellipti
cE(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*El
lipticF(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))*c^3*a*(-
c*x*a*(2*x^2-3))^(1/2)/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.31

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = \frac{3}{5}\sqrt{2}\sqrt{-ac}c^2\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x)) + \frac{2}{45}\left(5\,c^2x^3-3\,c^2x\right)\sqrt{-2\,ax^2+3\,a}\sqrt{cx}$$

input
```
integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
3/5*sqrt(2)*sqrt(-a*c)*c^2*weierstrassZeta(6, 0, weierstrassPInverse(6, 0,
 x)) + 2/45*(5*c^2*x^3 - 3*c^2*x)*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)
```

Sympy [A] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.29

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = \frac{\sqrt{3}\sqrt{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)*(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*sqrt(a)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), 2
*x**2*exp_polar(2*I*pi)/3)/(2*gamma(11/4))
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx=\int \sqrt{-2\,ax^2+3\,a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx=\int \sqrt{-2\,ax^2+3\,a}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx=\int (c\,x)^{5/2}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

input
```
int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{5/2}\sqrt{3a-2ax^2}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(10\sqrt{x}\,\sqrt{-2x^2+3}\,x^3-6\sqrt{x}\,\sqrt{-2x^2+3}\,x-27\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^2-3}dx\right)\right)}{45}$$

input
```
int((c*x)^(5/2)*(-2*a*x^2+3*a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(10*sqrt(x)*sqrt( - 2*x**2 + 3)*x**3 - 6*sqrt(x)*sqr
t( - 2*x**2 + 3)*x - 27*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**2 - 3),x))
)/45
```

3.615 $\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx = \frac{2(cx)^{3/2}\sqrt{3a-2ax^2}}{5c} + \frac{6\sqrt[4]{6}a\sqrt{c}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{5\sqrt{3a-2ax^2}} - \frac{6\sqrt[4]{6}a\sqrt{c}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{5\sqrt{3a-2ax^2}}$$

output
```
2/5*(c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2)/c+6/5*6^(1/4)*a*c^(1/2)*(-2*x^2+3)^(1
/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3*a)^(1
/2)-6/5*6^(1/4)*a*c^(1/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(
c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.57 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.33

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx = \frac{2x\sqrt{cx}\sqrt{a(3-2x^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{2x^2}{3}\right)}{\sqrt{9-6x^2}}$$

input
```
Integrate[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, (2*x^
2)/3])/Sqrt[9 - 6*x^2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {248, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3a-2ax^2}\sqrt{cx}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{6}{5}a\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx+\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}$$

$$\Big\downarrow \text{261}$$

$$\frac{6a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{5\sqrt{x}}+\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}$$

$$\Big\downarrow \text{260}$$

$$\frac{2\sqrt{3}a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{5\sqrt{x}\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}$$

$$\Big\downarrow \ 27$$

$$\frac{6a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{5\sqrt{x}\sqrt{3a-2ax^2}}+\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}$$

$$\Big\downarrow \ 259$$

$$\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \ 327$$

$$\frac{2\sqrt{3a-2ax^2}(cx)^{3/2}}{5c}-\frac{6\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\sqrt{x}\sqrt{3a-2ax^2}}$$

input
```
Int[Sqrt[c*x]*Sqrt[3*a - 2*a*x^2],x]
```

output
```
(2*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/(5*c) - (6*6^(1/4)*a*Sqrt[c*x]*Sqrt[3
- 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*Sqrt[x]*Sqr
t[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{2x^2(2x^2-3)ac}{5\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\,\text{Ellip}}{}\right.}{45\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(\frac{2x\sqrt{-2ac\,x^3+3acx}}{5}+\frac{ac\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{(}}{}\right.\right.}{45\sqrt{-2ac\,x^3+3a}}\right.}{cxa(2x^2-3)}$
default	$\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}}\right.}{10x(2}$

input
```
int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x^2*(2*x^2-3)*a*c/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)+1/45*6^(1/2)*3^(1/
2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*
6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2
)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3
^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))*a*c*(-c*x*a*(2*x^2-3)
)^(1/2)/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx = \frac{2}{5}\sqrt{-2\,ax^2+3\,a}\sqrt{cx}x + \frac{6}{5}\sqrt{2}\sqrt{-ac}\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))$$

input
```
integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*x + 6/5*sqrt(2)*sqrt(-a*c)*weierstrassZ
eta(6, 0, weierstrassPInverse(6, 0, x))
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx = \frac{\sqrt{3}\sqrt{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)*(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*sqrt(a)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), 2*x
**2*exp_polar(2*I*pi)/3)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx=\int \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx=\int \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx=\int \sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}\,dx$$

input
```
int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{cx}\sqrt{3a-2ax^2}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\left(\sqrt{x}\,\sqrt{-2x^2+3}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^2-3}dx\right)\right)}{5}$$

input
```
int((c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*(sqrt(x)*sqrt( - 2*x**2 + 3)*x - 3*int((sqrt(x)*sqrt( -
 2*x**2 + 3))/(2*x**2 - 3),x)))/5
```

3.616 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 149

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}} - \frac{4\sqrt[4]{6}a\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{c^{3/2}\sqrt{3a-2ax^2}} + \frac{4\sqrt[4]{6}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{c^{3/2}\sqrt{3a-2ax^2}}$$

output
```
-2*(-2*a*x^2+3*a)^(1/2)/c/(c*x)^(1/2)-4*6^(1/4)*a*(-2*x^2+3)^(1/2)*Ellipti
cE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)/c^(3/2)/(-2*a*x^2+3*a)^(1/2)
+4*6^(1/4)*a*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^
(1/2),I)/c^(3/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = -\frac{2x\sqrt{a(9-6x^2)}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{3}{4},\frac{2x^2}{3}\right)}{(cx)^{3/2}\sqrt{3-2x^2}}$$

input
```
Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

output
```
(-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, (2*x^2)/3])/(
(c*x)^(3/2)*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {247, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{4a\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{c^2}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{261}$$

$$-\frac{4a\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{c^2\sqrt{x}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{260}$$

$$-\frac{4a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{\sqrt{3}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 259}$$

$$\frac{4\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{4\sqrt[4]{6}a\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{c\sqrt{cx}}$$

input

```
Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(3/2),x]
```

output

```
(-2*Sqrt[3*a - 2*a*x^2])/(c*Sqrt[c*x]) + (4*6^(1/4)*a*Sqrt[c*x]*Sqrt[3 - 2
*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(c^2*Sqrt[x]*Sqrt
[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

method	result
risch	$\frac{2(2x^2-3)a}{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}}-\frac{2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\ \text{Ellipti}}{}\right.}{27\sqrt{-2ac\,x^3+3acx}\,c\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{2\left(-2ac\,x^2+3ac\right)}{c^2\sqrt{x\left(-2ac\,x^2+3ac\right)}}-\frac{2a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{(}}{}\right.\right.}{27c\sqrt{-2ac\,x^3+3ac}}\right.}{\sqrt{cx}\,a(2x^2-3)}$
default	$-\frac{\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{}}\right.}{3c\sqrt{cx}\,(}$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*x^2-3)/c*a/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)-2/27*6^(1/2)*3^(1/2)*((x+
1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)
*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1
/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*
((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))/c*a*(-c*x*a*(2*x^2-3))^(1/2)
/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = -\frac{2\left(2\sqrt{2}\sqrt{-ac}x\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x)) + \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\right)}{c^2x}$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*(2*sqrt(2)*sqrt(-a*c)*x*weierstrassZeta(6, 0, weierstrassPInverse(6, 0,
 x)) + sqrt(-2*a*x^2 + 3*a)*sqrt(c*x))/(c^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{3}\sqrt{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(3/2),x)
```

output
```
sqrt(3)*sqrt(a)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), 2*x**2*exp_polar(2
*I*pi)/3)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{3/2}}\,dx = \int \frac{\sqrt{3\,a-2\,a\,x^2}}{(c\,x)^{3/2}}\,dx$$

input
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2),x)
```

output
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3a-2a x^2}}{(cx)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{a}\left(\sqrt{-2x^2+3}-3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^4-3x^2}dx\right)\right)}{\sqrt{x}\,c^2}$$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(3/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*(sqrt( - 2*x**2 + 3) - 3*sqrt(x)*int((sqrt(x)*sqrt( - 2
*x**2 + 3))/(2*x**4 - 3*x**2),x)))/(sqrt(x)*c**2)
```

3.617 $\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 193

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}+\frac{8\sqrt{3a-2ax^2}}{15c^3\sqrt{cx}} + \frac{8\sqrt[4]{2}a\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{5\ 3^{3/4}c^{7/2}\sqrt{3a-2ax^2}} - \frac{8\sqrt[4]{2}a\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{5\ 3^{3/4}c^{7/2}\sqrt{3a-2ax^2}}$$

output
```
-2/5*(-2*a*x^2+3*a)^(1/2)/c/(c*x)^(5/2)+8/15*(-2*a*x^2+3*a)^(1/2)/c^3/(c*x
)^(1/2)+8/15*2^(1/4)*a*(-2*x^2+3)^(1/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x
)^(1/2)/c^(1/2),I)*3^(1/4)/c^(7/2)/(-2*a*x^2+3*a)^(1/2)-8/15*2^(1/4)*a*(-2
*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(1/4)
/c^(7/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.27

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx = -\frac{2x\sqrt{a(9-6x^2)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},-\frac{1}{4},\frac{2x^2}{3}\right)}{5(cx)^{7/2}\sqrt{3-2x^2}}$$

input
```
Integrate[Sqrt[3*a - 2*a*x^2]/(c*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[a*(9 - 6*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, (2*x^2)/3])/
(5*(c*x)^(7/2)*Sqrt[3 - 2*x^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {247, 264, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 247}$$

$$-\frac{4a\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}dx}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{4a\left(-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{3c^2}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$$\Big\downarrow \text{ 261}$$

$$-\frac{4a\left(-\frac{2\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{3c^2\sqrt{x}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$\Big\downarrow$ 260

$$-\frac{4a\left(-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{3\sqrt{3}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{4a\left(-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$\Big\downarrow$ 259

$$-\frac{4a\left(\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

$\Big\downarrow$ 327

$$-\frac{4a\left(\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}\right)}{5c^2}-\frac{2\sqrt{3a-2ax^2}}{5c(cx)^{5/2}}$$

input
```
Int[Sqrt[3*a - 2*a*x^2]/(c*x)^(7/2),x]
```

output
```
(-2*Sqrt[3*a - 2*a*x^2])/(5*c*(c*x)^(5/2)) - (4*a*((-2*Sqrt[3*a - 2*a*x^2]
)/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSi
n[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^
2]))/(5*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{2(8x^4-18x^2+9)a}{15x^2c^3\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{4\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\right.}{405\sqrt{-2ac\,x^3+3acx}\,c^3\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{2\sqrt{-2ac\,x^3+3acx}}{5c^4x^3}+\frac{-\frac{16}{15}ac\,x^2+\frac{8}{5}ac}{c^4\sqrt{x\left(-2ac\,x^2+3ac\right)}}+\frac{4a\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\right.}{}\right.}{\sqrt{cx}\,a(2x^2-3)}$
default	$\frac{2\sqrt{-a(2x^2-3)}\left(2\sqrt{2}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\right.}{45x^2c}$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(8*x^4-18*x^2+9)/x^2/c^3*a/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)+4/405*6^
(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))
^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE
(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*Elli
pticF(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))/c^3*a*(-c*
x*a*(2*x^2-3))^(1/2)/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx = \frac{2\left(4\sqrt{2}\sqrt{-ac}x^3\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x)) + \sqrt{-2\,ax^2+3\,a}\sqrt{\right.}{15\,c^4x^3}$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(4*sqrt(2)*sqrt(-a*c)*x^3*weierstrassZeta(6, 0, weierstrassPInverse(6
, 0, x)) + sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*(4*x^2 - 3))/(c^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 4.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx = \frac{\sqrt{2}i\sqrt{a}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| \frac{3}{2x^2}\right)}{2c^{\frac{7}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate((-2*a*x**2+3*a)**(1/2)/(c*x)**(7/2),x)
```

output
```
sqrt(2)*I*sqrt(a)*gamma(-3/4)*hyper((-1/2, 3/4), (7/4,), 3/(2*x**2))/(2*c*
*(7/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx = \int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx=\int \frac{\sqrt{-2\,ax^2+3\,a}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-2*a*x^2+3*a)^(1/2)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-2*a*x^2 + 3*a)/(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx=\int \frac{\sqrt{3\,a-2\,a\,x^2}}{(c\,x)^{7/2}}\,dx$$

input
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(7/2),x)
```

output
```
int((3*a - 2*a*x^2)^(1/2)/(c*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{3a-2ax^2}}{(cx)^{7/2}}\,dx=\frac{2\sqrt{c}\,\sqrt{a}\left(-\sqrt{-2x^2+3}+3\sqrt{x}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^6-3x^4}dx\right)x^2\right)}{3\sqrt{x}\,c^4x^2}$$

input
```
int((-2*a*x^2+3*a)^(1/2)/(c*x)^(7/2),x)
```

output
```
(2*sqrt(c)*sqrt(a)*( - sqrt( - 2*x**2 + 3) + 3*sqrt(x)*int((sqrt(x)*sqrt(
- 2*x**2 + 3))/(2*x**6 - 3*x**4),x)*x**2))/(3*sqrt(x)*c**4*x**2)
```

3.618 $\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = -\frac{10ac^3\sqrt{cx}\sqrt{a+bx^2}}{21b^2}+\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} + \frac{5a^{7/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21b^{9/4}\sqrt{a+bx^2}}$$

output
```
-10/21*a*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2+2/7*c*(c*x)^(5/2)*(b*x^2+a)^(
1/2)/b+5/21*a^(7/4)*c^(7/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2
)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
),1/2*2^(1/2))/b^(9/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c^3\sqrt{cx}\left(-5a^2-2abx^2+3b^2x^4+5a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{21b^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c^3*Sqrt[c*x]*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/
a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[a + b*x^2
])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx}{7b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b}-\frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)}{7b}$$

$$\Big\downarrow \text{761}$$

$$\frac{2c(cx)^{5/2}\sqrt{a+bx^2}}{7b} - \frac{5ac^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b} - \frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{7b}$$

input
```
Int[(c*x)^(7/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c*(c*x)^(5/2)*Sqrt[a + b*x^2])/(7*b) - (5*a*c^2*((2*c*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(3*b) - (a^(3/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 +
 b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/(7*b)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

method	result
default	$\frac{c^{3}\sqrt{cx}\left(5\sqrt{-ab}\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{2}+6b^{3}x^{5}-4a\,b^{2}x^{3}-10a^{2}bx\right)}{21x\sqrt{b\,x^{2}+a}\,b^{3}}$
risch	$-\frac{2\left(-3b\,x^{2}+5a\right)x\sqrt{b\,x^{2}+a}\,c^{4}}{21b^{2}\sqrt{cx}}+\frac{5a^{2}\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)c^{4}\sqrt{cx(}}{21b^{3}\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^{2}+a)}\left(\frac{2c^{3}x^{2}\sqrt{bc\,x^{3}+acx}}{7b}-\frac{10c^{3}a\sqrt{bc\,x^{3}+acx}}{21b^{2}}+\frac{5c^{4}a^{2}\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\left(\sqrt{\frac{(x+}{\sqrt{}}}\right.}{21b^{3}\sqrt{bc\,x^{3}+acx}}\right.}{cx\sqrt{b\,x^{2}+a}}$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*c^3/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(5*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/
(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a^2+6*b^3*x^5-4*a*b^2*x^3-10*a^2*b*x)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.40

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{2\left(5\sqrt{bc}a^2c^3\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(3b^2c^3x^2-5abc^3\right)\sqrt{bx^2+a}\sqrt{cx}\right)}{21\,b^3}$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*sqrt(b*c)*a^2*c^3*weierstrassPInverse(-4*a/b, 0, x) + (3*b^2*c^3*x
^2 - 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

output
```
c**(7/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{7/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(7/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\,c^3\left(-10\sqrt{x}\,\sqrt{b\,x^2+a}\,a+6\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2+5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a^2\right)}{21b^2}$$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*c**3*( - 10*sqrt(x)*sqrt(a + b*x**2)*a + 6*sqrt(x)*sqrt(a + b*x**
2)*b*x**2 + 5*int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x)*a**2))/(21*
b**2)
```

3.619 $\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx=\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

output

```
2/3*c*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b-1/3*a^(3/4)*c^(3/2)*(a^(1/2)+b^(1/2)*x
)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(5/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.54

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx=\frac{2c\sqrt{cx}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c*Sqrt[c*x]*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1
/2, 5/4, -((b*x^2)/a)]))/(3*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(3/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt
[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2
*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a +
b*x^2])
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{c\sqrt{cx}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a-2b^{2}x^{3}-2abx\right)}{3x\sqrt{b\,x^{2}+a}\,b^{2}}$	125
risch	$\frac{2x\sqrt{b\,x^{2}+a}\,c^{2}}{3b\sqrt{cx}}-\frac{a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)c^{2}\sqrt{cx\left(b\,x^{2}+a\right)}}{3b^{2}\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	165
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^{2}+a\right)}\left(\frac{2c\sqrt{bc\,x^{3}+acx}}{3b}-\frac{a\,c^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3b^{2}\sqrt{bc\,x^{3}+acx}}\right)}{cx\sqrt{b\,x^{2}+a}}$	168

input
```
int((c*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1
/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(
1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)
^(1/2)*a-2*b^2*x^3-2*a*b*x)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.32

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = -\frac{2\left(\sqrt{bc}ac\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\sqrt{bx^2+a}\sqrt{cx}bc\right)}{3\,b^2}$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*c)*a*c*weierstrassPInverse(-4*a/b, 0, x) - sqrt(b*x^2 + a)*sq
rt(c*x)*b*c)/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{3/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\,c\left(2\sqrt{x}\,\sqrt{b\,x^2+a}-\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)a\right)}{3b}$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*c*(2*sqrt(x)*sqrt(a + b*x**2) - int((sqrt(x)*sqrt(a + b*x**2))/(a
*x + b*x**3),x)*a))/(3*b)
```

3.620 $\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx=\frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

output
```
(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiA
M(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(1/4)/b^(1/
4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx=\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

output
```
(2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/(
Sqrt[c*x]*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{266, 761\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}$$

input
```
Int[1/(Sqrt[c*x]*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(a
^(1/4)*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{b\,x^{2}+a}\,b\sqrt{cx}}$	104
elliptic	$\frac{\sqrt{cx(b\,x^{2}+a)}\,\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}\,b\sqrt{bc\,x^{3}+acx}}$	136

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(1/2)*(-a*b)^(1/2)*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(
1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*El
lipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))/b/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.23

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \frac{2\,\sqrt{bc}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)}{bc}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x)/(b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x + b*x**3),x))/c
```

3.621 $\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}} - \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{5/2}\sqrt{a+bx^2}}$$

output
```
-2/3*(b*x^2+a)^(1/2)/a/c/(c*x)^(3/2)-1/3*b^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*x
^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^
(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(5/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.43

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])
/(3*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 264$$

$$-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \ 266$$

$$-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \ 761$$

$$-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c
*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(7/2)*Sqrt[a
 + b*x^2])
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x+2b\,x^{2}+2a}{3\sqrt{b\,x^{2}+a}\,xa\,c^{2}\sqrt{cx}}$	123
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{3ax\,c^{2}\sqrt{cx}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^{2}+a)}}{3a\sqrt{bc\,x^{3}+acx}\,c^{2}\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	166
elliptic	$\frac{\sqrt{cx(b\,x^{2}+a)}\left(-\frac{2\sqrt{bc\,x^{3}+acx}}{3a\,c^{3}x^{2}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3a\,c^{2}\sqrt{bc\,x^{3}+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	166

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(b*x^2+a)^(1/2)/x*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((
-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+2*b*x
^2+2*a)/a/c^2/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.35

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = -\frac{2\left(\sqrt{bc}x^2\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\sqrt{cx}\right)}{3\,ac^3x^2}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(b*c)*x^2*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)*sq
rt(c*x))/(a*c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.37

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx\right)}{c^3}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x))/c**3
```

3.622 $\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 158

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}+\frac{10b\sqrt{a+bx^2}}{21a^2c^3(cx)^{3/2}} + \frac{5b^{7/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{21a^{9/4}c^{9/2}\sqrt{a+bx^2}}$$

output
```
-2/7*(b*x^2+a)^(1/2)/a/c/(c*x)^(7/2)+10/21*b*(b*x^2+a)^(1/2)/a^2/c^3/(c*x)
^(3/2)+5/21*b^(7/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^
(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^
(1/2))/a^(9/4)/c^(9/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{1}{2},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(9/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((b*x^2)/a)]
)/(7*(c*x)^(9/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{5b\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{5b\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{5b\left(-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{761}$$

$$-\frac{5b\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt{a+bx^2}}{7ac(cx)^{7/2}}$$

input
```
Int[1/((c*x)^(9/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(7*a*c*(c*x)^(7/2)) - (5*b*((-2*Sqrt[a + b*x^2])/(3*a
*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x
^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*c^(7/2)*Sqrt[a + b*x^2])))/(7*a*c^2)
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

method	result
default	$\frac{5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^3+10b^2x^4+4ab\,x^2-6a^2}{21\sqrt{b\,x^2+a}\,x^3a^2c^4\sqrt{cx}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-5b\,x^2+3a)}{21a^2x^3c^4\sqrt{cx}}+\frac{5b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{cx(b\,x^2+a}}{21a^2\sqrt{bc\,x^3+acx}\,c^4\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{7a\,c^5x^4}+\frac{10b\sqrt{bc\,x^3+acx}}{21a^2c^5x^2}+\frac{5b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21a^2c^4\sqrt{bc\,x^3+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/21/(b*x^2+a)^(1/2)/x^3*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Ellip
ticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*b*x
^3+10*b^2*x^4+4*a*b*x^2-6*a^2)/a^2/c^4/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx=\frac{2\left(5\sqrt{bc}bx^4\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+(5\,bx^2-3\,a)\sqrt{bx^2+a}\sqrt{cx}\right)}{21\,a^2c^5x^4}$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/21*(5*sqrt(b*c)*b*x^4*weierstrassPInverse(-4*a/b, 0, x) + (5*b*x^2 - 3*a
)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 21.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.32

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{7}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{7}{4}, \frac{1}{2} \\ -\frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{9}{2}}x^{\frac{7}{2}}\Gamma\left(-\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-7/4)*hyper((-7/4, 1/2), (-3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*c**(9/2)*x**(7/2)*gamma(-3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{9/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^7+a\,x^5}dx\right)}{c^5}$$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**5 + b*x**7),x))/c**5
```

3.623 $\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 304

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = -\frac{14ac^3(cx)^{3/2}\sqrt{a+bx^2}}{45b^2} + \frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b} + \frac{14a^2c^4\sqrt{cx}\sqrt{a+bx^2}}{15b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{14a^{9/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}} + \frac{7a^{9/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{15b^{11/4}\sqrt{a+bx^2}}$$

output
```
-14/45*a*c^3*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b^2+2/9*c*(c*x)^(7/2)*(b*x^2+a)^(
1/2)/b+14/15*a^2*c^4*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*
x)-14/15*a^(9/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x
)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/
2*2^(1/2))/b^(11/4)/(b*x^2+a)^(1/2)+7/15*a^(9/4)*c^(9/2)*(a^(1/2)+b^(1/2)*
x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(11/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.29

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c^3(cx)^{3/2}\left(-7a^2-2abx^2+5b^2x^4+7a^2\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(9/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c^3*(c*x)^(3/2)*(-7*a^2 - 2*a*b*x^2 + 5*b^2*x^4 + 7*a^2*Sqrt[1 + (b*x^2
)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)]))/(45*b^2*Sqrt[a + b*x
^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {262, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\int \frac{(cx)^{5/2}}{\sqrt{bx^2+a}}dx}{9b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ac^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}\right)}{9b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}\right)}{9b}$$

$\Big\downarrow$ 834

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$\Big\downarrow$ 27

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$\Big\downarrow$ 761

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b}-\frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{9b}$$

$\Big\downarrow$ 1510

$$\frac{2c(cx)^{7/2}\sqrt{a+bx^2}}{9b} - \frac{7ac^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}} - \frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{E}}{\sqrt[4]{b}\sqrt{a+bx^2}}\right)}{5b}\right)}{9b}$$

input
```
Int[(c*x)^(9/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c*(c*x)^(7/2)*Sqrt[a + b*x^2])/(9*b) - (7*a*c^2*((2*c*(c*x)^(3/2)*Sqrt[
a + b*x^2])/(5*b) - (6*a*c*(-((-(((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*
c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2
 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a
^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]
*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[
c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b)))/(9*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.73

method	result
default	$\frac{c^4\sqrt{cx}\left(10b^3x^6+42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^3-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\cdots}\right.}{45x\sqrt{b\,x^2+a}\,b^3}$
risch	$-\frac{2x^2\left(-5b\,x^2+7a\right)\sqrt{b\,x^2+a}\,c^5}{45b^2\sqrt{cx}}+\frac{7a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\cdots\right)}{b}\cdots\right)}{15b^3\sqrt{bc\,x^3+acx}\,\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx\left(b\,x^2+a\right)}\left(\frac{2c^4x^3\sqrt{bc\,x^3+acx}}{9b}-\frac{14c^4ax\sqrt{bc\,x^3+acx}}{45b^2}+\frac{7c^5a^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{Ellipt}\cdots}{\cdots}\right)}{15b^3\sqrt{bc\,x^3\cdots}}\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/45*c^4/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^3*(10*b^3*x^6+42*((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(
-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
,1/2*2^(1/2))*a^3-21*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^3-4*a*b^2*x^4-14*a^2*
b*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.24

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = -\frac{2\left(21\sqrt{bc}\,a^2c^4\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)-\left(5\,b^2c^4x^3-7\,abc^4x\right)\sqrt{bx^2+a}\sqrt{\cdots}\right.}{45\,b^3}$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2/45*(21*sqrt(b*c)*a^2*c^4*weierstrassZeta(-4*a/b, 0, weierstrassPInverse
(-4*a/b, 0, x)) - (5*b^2*c^4*x^3 - 7*a*b*c^4*x)*sqrt(b*x^2 + a)*sqrt(c*x))
/b^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 30.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**2+a)**(1/2),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/4)*hyper((1/2, 11/4), (15/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*sqrt(a)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{9/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\,c^4\left(-14\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+10\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3+21\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2\right)}{45b^2}$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*c**4*( - 14*sqrt(x)*sqrt(a + b*x**2)*a*x + 10*sqrt(x)*sqrt(a + b*
x**2)*b*x**3 + 21*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/(4
5*b**2)
```

3.624 $\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 273

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b} - \frac{6ac^2\sqrt{cx}\sqrt{a+bx^2}}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{6a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}} - \frac{3a^{5/4}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5b^{7/4}\sqrt{a+bx^2}}$$

output
```
2/5*c*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b-6/5*a*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/
b^(3/2)/(a^(1/2)+b^(1/2)*x)+6/5*a^(5/4)*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^
2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/
2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b^(7/4)/(b*x^2+a)^(1/2)-3/5*a^(5/4)*c^(5
/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJac
obiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(7/4)/(
b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.25

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2c(cx)^{3/2}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

output
```
(2*c*(c*x)^(3/2)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2,
 3/4, 7/4, -((b*x^2)/a)]))/(5*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ac^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}$$

$$\Big\downarrow \text{266}$$

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}$$

$$\Big\downarrow \text{834}$$

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}$$

$\big\downarrow$ 27

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}$$

$\big\downarrow$ 761

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}$$

$\big\downarrow$ 1510

$$\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{5b}$$

input

```
Int[(c*x)^(5/2)/Sqrt[a + b*x^2],x]
```

output

```
(2*c*(c*x)^(3/2)*Sqrt[a + b*x^2])/(5*b) - (6*a*c*(-((-((c^2*Sqrt[c*x]*Sqrt
[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sq
rt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE
[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a +
b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2
 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqr
t[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.77

method	result
default	$-\frac{c^{2}\sqrt{cx}\,\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^{2}-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{El}\cdots\right.}{5x\sqrt{b\,x^{2}+a}\,b^{2}}$
risch	$\frac{2x^{2}\sqrt{b\,x^{2}+a}\,c^{3}}{5b\sqrt{cx}}-\frac{3a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{Elliptic}\cdots}{\cdots}\right.}{5b^{2}\sqrt{bc\,x^{3}+acx}\,\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^{2}+a)}\,\left(\frac{2c^{2}x\sqrt{bc\,x^{3}+acx}}{5b}-\frac{3c^{3}a\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\cdots\right)}{b}\cdots\right.}{5b^{2}\sqrt{bc\,x^{3}+acx}}\cdots\right.}{cx\sqrt{b\,x^{2}+a}}$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*c^2/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)/b^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(
1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1
/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)
)*a^2-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-2*b^2*x^4-2*a*b*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.20

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{2\left(\sqrt{bx^2+a}\sqrt{cx}bc^2x + 3\sqrt{bc}ac^2\text{weierstrassZeta}\left(-\frac{4a}{b}, 0, \text{weierstrassPInverse}\left(-\frac{4a}{b}, 0, x\right.\right.\right.}{5b^2}$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(sqrt(b*x^2 + a)*sqrt(c*x)*b*c^2*x + 3*sqrt(b*c)*a*c^2*weierstrassZeta
(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/b^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.16

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(1/2),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{5/2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\,c^2\left(2\sqrt{x}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a\right)}{5b}$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(x)*sqrt(a + b*x**2)*x - 3*int((sqrt(x)*sqrt(a + b*x*
*2))/(a + b*x**2),x)*a))/(5*b)
```

3.625 $\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 236

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \frac{2\sqrt{cx}\sqrt{a+bx^2}}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}} + \frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{b^{3/4}\sqrt{a+bx^2}}$$

output
```
2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(1/2)/(a^(1/2)+b^(1/2)*x)-2*a^(1/4)*c^(1/2
)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b^(3/4)/(b*x
^2+a)^(1/2)+a^(1/4)*c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2
)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
),1/2*2^(1/2))/b^(3/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx=\frac{2x\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x
^2)/a)])/(3*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{834}$$

$$\frac{2\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{c}$$

$$\Big\downarrow \text{1510}$$

$$\frac{2\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{c}$$

input
```
Int[Sqrt[c*x]/Sqrt[a + b*x^2],x]
```

output
```
(2*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(
1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c
 + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c]
)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*
c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*S
qrt[a + b*x^2])))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

method	result
default	$\frac{\sqrt{cx}\, a\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(2\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{b\,x^{2}+a}\,bx}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^{2}+a)}\,\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\left(\right.}{}\right.}{x\sqrt{b\,x^{2}+a}\,b\sqrt{bc\,x^{3}+acx}}$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x)^(1/2)/(b*x^2+a)^(1/2)*a/b*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*(
2*EllipticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-EllipticF
(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b}$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x))
/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(1/2),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c\,x}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{\sqrt{a+bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a + b*x**2),x)
```

3.626 $\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 268

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}+\frac{2\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{ac^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{2\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}} + \frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{a^{3/4}c^{3/2}\sqrt{a+bx^2}}$$

output
```
-2*(b*x^2+a)^(1/2)/a/c/(c*x)^(1/2)+2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a
/c^2/(a^(1/2)+b^(1/2)*x)-2*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)
+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2))),1/2*2^(1/2))/a^(3/4)/c^(3/2)/(b*x^2+a)^(1/2)+b^(1/4)*(a^(1/2)+b^(1
/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(3/4)/c^(3/2)/(b*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.20

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])
/((c*x)^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

$\Big\downarrow$ 27

$$\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

$\Big\downarrow$ 761

$$\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

$\Big\downarrow$ 1510

$$\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ac^3}$$

$$\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-((c^2*Sqrt[c*x]*Sqrt[a +
b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]
*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*Ar
cTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2
]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*
c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x
])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx}{\cdots}}\cdots\right.}{\sqrt{b\,x^{2}+a}\,c\sqrt{cx}\,a}$
risch	$-\frac{2\sqrt{b\,x^{2}+a}}{ac\sqrt{cx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\operatorname{EllipticF}\left(\cdots\right)}{\cdots}\cdots\right)}{a\sqrt{b c\,x^{3}+acx}\,c\sqrt{cx}\,\sqrt{b\,x^{2}+a}\cdots}$
elliptic	$\frac{\sqrt{cx\left(b\,x^{2}+a\right)}\left(-\frac{2\left(x^{2}bc+ac\right)}{a\,c^{2}\sqrt{x\left(x^{2}bc+ac\right)}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{ac\sqrt{b c\,x^{3}+acx}}\cdots\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$

input

```
int(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)
*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2
)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-2*b*x^2
-2*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.19

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = -\frac{2\left(\sqrt{bc}x\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\sqrt{bx^2+a}\sqrt{cx}\right)}{ac^2x}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(b*c)*x*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0,
x)) + sqrt(b*x^2 + a)*sqrt(c*x))/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x))/c**2
```

3.627 $\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 306

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}+\frac{6b\sqrt{a+bx^2}}{5a^2c^3\sqrt{cx}}-\frac{6b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^2c^4\left(\sqrt{a}+\sqrt{b}x\right)}$$
$$+\frac{6b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$
$$-\frac{3b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{5a^{7/4}c^{7/2}\sqrt{a+bx^2}}$$

output
```
-2/5*(b*x^2+a)^(1/2)/a/c/(c*x)^(5/2)+6/5*b*(b*x^2+a)^(1/2)/a^2/c^3/(c*x)^(
1/2)-6/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^4/(a^(1/2)+b^(1/2)*x)+6
/5*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Ell
ipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(
7/4)/c^(7/2)/(b*x^2+a)^(1/2)-3/5*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a
^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2)),1/2*2^(1/2))/a^(7/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.18

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)]
)/(5*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 264$$

$$-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \ 264$$

$$-\frac{3b\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \ 266$$

$$-\frac{3b\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$\Big\downarrow$ 834

$$-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$\Big\downarrow$ 27

$$-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$\Big\downarrow$ 761

$$-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

$\Big\downarrow$ 1510

$$-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ac^3}\right.}{5ac^2}$$

$$\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*Sqrt[a + b*x^2]),x]
```

output
```
(-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) - (3*b*((-2*Sqrt[a + b*x^2])/(a*c
*Sqrt[c*x]) + (2*b*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt
[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2
*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/
(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*S
qrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqr
t[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/
2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*c^3)))/(5*a*c^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.72

method	result
default	$-\frac{6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{Elliptic}\cdots}{5x^2\sqrt{b\,x^2+a}\,c^3\sqrt{cx}\,a^2}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,(-3b\,x^2+a)}{5a^2x^2c^3\sqrt{cx}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right.}{5a^2\sqrt{bc\,x^3+acx}\,c^3\sqrt{cx}\,\sqrt{b\,x^2+a}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{5a\,c^4x^3}+\frac{6\left(x^2bc+ac\right)b}{5a^2c^4\sqrt{x\left(x^2bc+ac\right)}}-\frac{3b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\cdots}\right)}{b}\cdots\right.}{5a^2c^3\sqrt{bc\,x^3+acx}}\cdots\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5/x^2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-3*((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a*b*x^2-6*b^2*x^4-4*a*b*x^2+2*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^(1/2)
/a^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.21

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx=\frac{2\left(3\sqrt{bc}bx^3\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(3bx^2-\cdots\right.\right.}{5a^2c^4x^3}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
2/5*(3*sqrt(b*c)*b*x^3*weierstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a
/b, 0, x)) + (3*b*x^2 - a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.72 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.17

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ -\frac{1}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 1/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{7/2}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx\right)}{c^4}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a*x**4 + b*x**6),x))/c**4
```

3.628 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}+\frac{5c^3\sqrt{cx}\sqrt{a+bx^2}}{3b^2} - \frac{5a^{3/4}c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{6b^{9/4}\sqrt{a+bx^2}}$$

output
```
-c*(c*x)^(5/2)/b/(b*x^2+a)^(1/2)+5/3*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^2-5
/6*a^(3/4)*c^(7/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(
1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(
1/2))/b^(9/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.48

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c^3\sqrt{cx}\left(5a+2bx^2-5a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

output
```
(c^3*Sqrt[c*x]*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[
1/4, 1/2, 5/4, -((b*x^2)/a)]))/(3*b^2*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {252, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5c^2\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(7/2)/(a + b*x^2)^(3/2),x]
```

output
```
-((c*(c*x)^(5/2))/(b*Sqrt[a + b*x^2])) + (5*c^2*((2*c*Sqrt[c*x]*Sqrt[a + b
*x^2])/(3*b) - (a^(3/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*
c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x
])/(a^(1/4)*Sqrt[c])], 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/(2*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.84

method	result
default	$-\frac{c^3\sqrt{cx}\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,a-4b^2x^3-10abx\right)}{6x\sqrt{bx^2+a}\,b^3}$
elliptic	$\frac{\sqrt{cx}\sqrt{cx(bx^2+a)}\left(\frac{c^4xa}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{2c^3\sqrt{bcx^3+acx}}{3b^2}-\frac{5ac^4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)\ldots}{\sqrt{-ab}}}\ldots\right.}{6b^3\sqrt{bcx^3+acx}}\ldots\right.}{cx\sqrt{bx^2+a}}$
risch	$\frac{2x\sqrt{bx^2+a}\,c^4}{3b^2\sqrt{cx}}-\frac{a\left(\frac{4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{bcx^3+acx}}-3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{\sqrt{-\ldots}}{\ldots}\ldots\right.\right.}{3b^2\sqrt{cx}\sqrt{bx^2+\ldots}}$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*c^3/x*(c*x)^(1/2)*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Ellipti
cF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*a-4*b
^2*x^3-10*a*b*x)/(b*x^2+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{5\,(abc^3x^2+a^2c^3)\sqrt{bc}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(2\,b^2c^3x^2+5\,abc^3)\sqrt{bx^2+a}\sqrt{cx}}{3\,(b^4x^2+ab^3)}$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(a*b*c^3*x^2 + a^2*c^3)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x
) - (2*b^2*c^3*x^2 + 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(b^4*x^2 + a*b^
3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.29

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

output
```
c**(7/2)*x***(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{7/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(7/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,c^3\left(10\sqrt{x}\,\sqrt{b\,x^2+a}\,a+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^2-5\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)a^3-5\left(\int \frac{\sqrt{}}{b^2x^5}\right.\right.}{3b^2\,(b\,x^2+a)}$$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*c**3*(10*sqrt(x)*sqrt(a + b*x**2)*a + 2*sqrt(x)*sqrt(a + b*x**2)*
b*x**2 - 5*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5
),x)*a**3 - 5*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x
**5),x)*a**2*b*x**2))/(3*b**2*(a + b*x**2))
```

3.629 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 125

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}$$

output
```
-c*(c*x)^(1/2)/b/(b*x^2+a)^(1/2)+1/2*c^(3/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a
)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2
)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(1/4)/b^(5/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c\sqrt{cx}\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

output
```
(c*Sqrt[c*x]*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(
(b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{2b}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{b}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(3/2)/(a + b*x^2)^(3/2),x]
```

output
```
-((c*Sqrt[c*x])/(b*Sqrt[a + b*x^2])) + (Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*
Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(
b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b
*x^2])
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

method	result	siz
default	$\frac{c\sqrt{cx}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}-2bx\right)}{2x\sqrt{b\,x^{2}+a}\,b^{2}}$	11
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^{2}+a)}\left(-\frac{c^{2}x}{b\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}+\frac{c^{2}\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2b^{2}\sqrt{bc\,x^{3}+acx}}\right)}{cx\sqrt{b\,x^{2}+a}}$	17

input
```
int((c*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c/x*(c*x)^(1/2)*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*
x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((
b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)-2*b*x)/(b*
x^2+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{\sqrt{bx^2+a}\sqrt{cx}bc - (bcx^2+ac)\sqrt{bc}\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)}{b^3x^2+ab^2}$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*x^2 + a)*sqrt(c*x)*b*c - (b*c*x^2 + a*c)*sqrt(b*c)*weierstrassPIn
verse(-4*a/b, 0, x))/(b^3*x^2 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,c\left(-2\sqrt{x}\,\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)a^2+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)ab\,x^2\right)}{b\,(b\,x^2+a)}$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**2))
/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**2 + int((sqrt(x)*sqrt(a + b*x**2)
)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a*b*x**2))/(b*(a + b*x**2))
```

3.630 $\int \frac{1}{\sqrt{cx}(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{cx}}{ac\sqrt{a+bx^2}} + \frac{\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

output
```
(c*x)^(1/2)/a/c/(b*x^2+a)^(1/2)+1/2*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2
)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2)),1/2*2^(1/2))/a^(5/4)/b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.87 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{x + x\sqrt{1+\frac{bx^2}{a}}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{a\sqrt{cx}\sqrt{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

output
```
(x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])
/(a*Sqrt[c*x]*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 761$$

$$\frac{\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/2)),x]
```

output
```
Sqrt[c*x]/(a*c*Sqrt[a + b*x^2]) + ((Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 +
 b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*a^(5/4)*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2]
)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}+2bx}{2\sqrt{b\,x^{2}+a}\,ba\sqrt{cx}}$	114
elliptic	$\frac{\sqrt{cx(b\,x^{2}+a)}\left(\frac{x}{a\sqrt{\left(x^{2}+\frac{a}{b}\right)bcx}}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2ab\sqrt{bc\,x^{3}+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^{2}+a}}$	162

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/
(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)+2*b*x)/(b*x^2+a)^(1/2)/b/a
/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.46

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{(bx^2+a)\sqrt{bc}\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\sqrt{bx^2+a}\sqrt{cx}b}{ab^2cx^2+a^2bc}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
((b*x^2 + a)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x) + sqrt(b*x^2 + a)
*sqrt(c*x)*b)/(a*b^2*c*x^2 + a^2*b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(3/2)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5),
x))/c
```

3.631 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx = \frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}} - \frac{5\sqrt{a+bx^2}}{3a^2c(cx)^{3/2}} - \frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{6a^{9/4}c^{5/2}\sqrt{a+bx^2}}$$

output
```
1/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-5/3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(3/2)-5/
6*b^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Inve
rseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(
9/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3a(cx)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])
/(3*a*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {253, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{5\left(-\frac{b\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x]
```

output
```
1/(a*c*(c*x)^(3/2)*Sqrt[a + b*x^2]) + (5*((-2*Sqrt[a + b*x^2])/(3*a*c*(c*x
)^(3/2)) - (b^(3/4)*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sq
rt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*
Sqrt[c])], 1/2])/(3*a^(5/4)*c^(7/2)*Sqrt[a + b*x^2])))/(2*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result
default	$-\frac{5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,x+10b\,x^2+4a}{6x\sqrt{b\,x^2+a}\,a^2c^2\sqrt{cx}}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{bx}{c^2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^3+acx}}{3a^2c^3x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6a^2c^2\sqrt{bc\,x^3+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
risch	$-\frac{2\sqrt{b\,x^2+a}}{3a^2x\,c^2\sqrt{cx}}-\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{bc\,x^3+acx}}+3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\cdots\right)\right)}{3a^2c^2\sqrt{cx}\,\sqrt{b\,x^2+\cdots}}$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/x*(5*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*x+10*b*x^2+4*a)/(b*x^
2+a)^(1/2)/a^2/c^2/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{5\left(bx^4+ax^2\right)\sqrt{bc}\,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)+\left(5\,bx^2+2\,a\right)\sqrt{bx^2+a}\sqrt{cx}}{3\left(a^2bc^3x^4+a^3c^3x^2\right)}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(5*(b*x^4 + a*x^2)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0, x) + (5*b
*x^2 + 2*a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b*c^3*x^4 + a^3*c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.93 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^2)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right)\,{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{1}{4} \end{matrix}\middle|\, \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx=\int \frac{1}{(c\,x)^{5/2}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^7+2ab\,x^5+a^2x^3}dx\right)}{c^3}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**3 + 2*a*b*x**5 + b**2*x**
7),x))/c**3
```

3.632 $\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 299

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}} + \frac{7c^3(cx)^{3/2}\sqrt{a+bx^2}}{5b^2} - \frac{21ac^4\sqrt{cx}\sqrt{a+bx^2}}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{21a^{5/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{a+bx^2}} - \frac{21a^{5/4}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{10b^{11/4}\sqrt{a+bx^2}}$$

output
```
-c*(c*x)^(7/2)/b/(b*x^2+a)^(1/2)+7/5*c^3*(c*x)^(3/2)*(b*x^2+a)^(1/2)/b^2-2
1/5*a*c^4*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x)+21/5*a^(
5/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b
^(11/4)/(b*x^2+a)^(1/2)-21/10*a^(5/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+
a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/
2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(11/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.24

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \frac{2c^3(cx)^{3/2}\left(-7a+bx^2+7a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(9/2)/(a + b*x^2)^(3/2),x]
```

output
```
(2*c^3*(c*x)^(3/2)*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2
F1[3/4, 3/2, 7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {252, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{7c^2\int \frac{(cx)^{5/2}}{\sqrt{bx^2+a}}dx}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{7c^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{3ac^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{5b}\right)}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{7c^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{5b}\right)}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 834

$$\frac{7c^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{7c^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 761

$$\frac{7c^2\left(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{5b}\right)}{2b}-\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

$\Big\downarrow$ 1510

$$\frac{7c^2\Bigg(\frac{2c(cx)^{3/2}\sqrt{a+bx^2}}{5b}-\frac{6ac\Bigg(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,E\Big(}{\sqrt[4]{b}\sqrt{a+bx^2}}}{5b}}{2b}$$

$$\frac{c(cx)^{7/2}}{b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(9/2)/(a + b*x^2)^(3/2),x]
```

output
```
-((c*(c*x)^(7/2))/(b*Sqrt[a + b*x^2])) + (7*c^2*((2*c*(c*x)^(3/2)*Sqrt[a +
 b*x^2])/(5*b) - (6*a*c*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c +
 Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 +
b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1
/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c
+ Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])
], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(5*b)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.70

method	result
default	$-\frac{c^4\sqrt{cx}\left(42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a^2-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\cdots\right.}{10x\sqrt{b\,x^2+a}\,b^3}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{c^5x^2a}{b^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{2c^4x\sqrt{bc\,x^3+acx}}{5b^2}-\frac{21a\,c^5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\cdots}{\cdots}\cdots\right.}{10b^3\sqrt{bc\,x^3+ac\cdots}}\cdots\right.}{cx\sqrt{b\,x^2+a}}$
risch	$\frac{2x^2\sqrt{b\,x^2+a}\,c^5}{5b^2\sqrt{cx}}-\frac{a\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\operatorname{EllipticF}\cdots}{\cdots}\right.}{b\sqrt{bc\,x^3+acx}}\cdots\right.}{\cdots}$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10*c^4/x*(c*x)^(1/2)*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2
)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Ellip
ticE(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a^2-21*((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^
(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))*a^2-4*b^2*x^4-14*a*b*x^2)/(b*x^2+a)^(1/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.31

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \frac{21\left(abc^4x^2+a^2c^4\right)\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+(2b}{5\left(b^4x^2+ab^3\right)}$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(a*b*c^4*x^2 + a^2*c^4)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weier
strassPInverse(-4*a/b, 0, x)) + (2*b^2*c^4*x^3 + 7*a*b*c^4*x)*sqrt(b*x^2 +
 a)*sqrt(c*x))/(b^4*x^2 + a*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\\frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**2+a)**(3/2),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/4)*hyper((3/2, 11/4), (15/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(3/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{9/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,c^4\left(-14\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3+21\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^3+21\left(\int \right.\right.}{5b^2\,(b\,x^2+a)}$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*c**4*( - 14*sqrt(x)*sqrt(a + b*x**2)*a*x + 2*sqrt(x)*sqrt(a + b*x
**2)*b*x**3 + 21*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*
x**4),x)*a**3 + 21*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**
2*x**4),x)*a**2*b*x**2))/(5*b**2*(a + b*x**2))
```

3.633 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 266

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}+\frac{3c^2\sqrt{cx}\sqrt{a+bx^2}}{b^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{b^{7/4}\sqrt{a+bx^2}} + \frac{3\sqrt[4]{a}c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{7/4}\sqrt{a+bx^2}}$$

output
```
-c*(c*x)^(3/2)/b/(b*x^2+a)^(1/2)+3*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(3/2)
/(a^(1/2)+b^(1/2)*x)-3*a^(1/4)*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(
1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4
)/c^(1/2))),1/2*2^(1/2))/b^(7/4)/(b*x^2+a)^(1/2)+3/2*a^(1/4)*c^(5/2)*(a^(1
/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*a
rctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(7/4)/(b*x^2+a)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.23

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = -\frac{2c(cx)^{3/2}\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

output
```
(-2*c*(c*x)^(3/2)*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/
4, -((b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {252, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{3c^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{2b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3c\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 834}$$

$$\frac{3c\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

$\big\downarrow$ 27

$$\frac{3c\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

$\big\downarrow$ 761

$$\frac{3c\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

$\big\downarrow$ 1510

$$\frac{3c\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right)}{b}$$

$$\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^2)^(3/2),x]
```

output
```
-((c*(c*x)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*c*(-((-((c^2*Sqrt[c*x]*Sqrt[a
+ b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[
b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*
ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x
^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 +
b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c
*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

method	result
default	$\frac{c^2\sqrt{cx}\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{Ellipt}\cdots\right.}{2x\sqrt{b\,x^2+a}\,b^2}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(-\frac{c^3x^2}{b\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{3c^3\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\cdots\right.}{b}\cdots\right.}{2b^2\sqrt{bc\,x^3+acx}}\cdots\right.}{cx\sqrt{b\,x^2+a}}$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*c^2/x*(c*x)^(1/2)*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*(
(-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*Elliptic
E(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1
/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*
(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.29

$$\int \frac{(cx)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\sqrt{bx^2+a}\sqrt{cx}bc^2x+3\left(bc^2x^2+ac^2\right)\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{b^3x^2+ab^2}$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(b*x^2 + a)*sqrt(c*x)*b*c^2*x + 3*(b*c^2*x^2 + a*c^2)*sqrt(b*c)*weie
rstrassZeta(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/(b^3*x^2 + a*b^
2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.17

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(3/2),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,c^2\left(2\sqrt{x}\,\sqrt{b\,x^2+a}\,x-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^2-3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)ab\,x^2\right)}{b\,(b\,x^2+a)}$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(x)*sqrt(a + b*x**2)*x - 3*int((sqrt(x)*sqrt(a + b*x*
*2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2 - 3*int((sqrt(x)*sqrt(a + b*x
**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a*b*x**2))/(b*(a + b*x**2))
```

3.634 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 266

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{a+bx^2}} - \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{a+bx^2}}$$

output
```
(c*x)^(3/2)/a/c/(b*x^2+a)^(1/2)-(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(1/2)/(a^(
1/2)+b^(1/2)*x)+c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)
^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2
*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)-1/2*c^(1/2)*(a^(1/2)+b^(1/2)*x)*
((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(3/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.22

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{2x\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x
^2)/a)])/(3*a*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{2a}$$

$$\Big\downarrow \; 266$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}} - \frac{\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}$$

$$\Big\downarrow \; 834$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-$$
$$\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-}{\sqrt{b}}}{ac}$$

input
```
Int[Sqrt[c*x]/(a + b*x^2)^(3/2),x]
```

output
```
(c*x)^(3/2)/(a*c*Sqrt[a + b*x^2]) - (-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/
(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sq
rt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^
(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt
[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)
/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1
/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2]))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.74

method	result
default	$-\frac{\sqrt{cx}\left(2\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{Ellipti}\right)}{2\sqrt{bx^2+a}\,bxa}$
elliptic	$\frac{\sqrt{cx}\sqrt{cx(bx^2+a)}\left(\frac{cx^2}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right)}{2ab\sqrt{bcx^3+acx}}\right)}{cx\sqrt{bx^2+a}}$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x)^(1/2)*(2*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b
*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-((b*x+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a
*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2
^(1/2))*a-2*b*x^2)/(b*x^2+a)^(1/2)/b/x/a
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.24

$$\int\frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx=\frac{\sqrt{bx^2+a}\sqrt{cx}bx+(bx^2+a)\sqrt{bc}\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,\right)\right)}{ab^2x^2+a^2b}$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(sqrt(b*x^2 + a)*sqrt(c*x)*b*x + (b*x^2 + a)*sqrt(b*c)*weierstrassZeta(-4*
a/b, 0, weierstrassPInverse(-4*a/b, 0, x)))/(a*b^2*x^2 + a^2*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(3/2),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.635 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 296

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/2}}\,dx = \frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}} - \frac{3\sqrt{a+bx^2}}{a^2c\sqrt{cx}} + \frac{3\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{a^2c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{a^{7/4}c^{3/2}\sqrt{a+bx^2}} + \frac{3\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{7/4}c^{3/2}\sqrt{a+bx^2}}$$

output
```
1/a/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-3*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(1/2)+3*b^
(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/c^2/(a^(1/2)+b^(1/2)*x)-3*b^(1/4)*(a
^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*
arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(7/4)/c^(3/2)/
(b*x^2+a)^(1/2)+3/2*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2
)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
),1/2*2^(1/2))/a^(7/4)/c^(3/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.19

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx=-\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])
/(a*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {253, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{3\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{1510}$$

$$3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}-\cdots}{\sqrt{b}}\cdots\right)}{ac^3}\cdots\right.$$

$$\overline{\qquad\qquad\qquad\qquad 2a \qquad\qquad\qquad\qquad}$$

$$\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x]
```

output
```
1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^2]) + (3*((-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x
]) + (2*b*(-((-(-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x))
 + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sq
rt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*
Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(S
qrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^
(3/4)*Sqrt[a + b*x^2])))/(a*c^3)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.67

method	result
default	$\frac{6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{b}{}}\right.}{2\sqrt{bx^2+a}\,c\sqrt{cx}\,a^2}$
elliptic	$\frac{\sqrt{cx(bx^2+a)}\left(-\frac{bx^2}{ca^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\left(x^2bc+ac\right)}{a^2c^2\sqrt{x\left(x^2bc+ac\right)}}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\right.\right.}{b}\right.}{2a^2c\sqrt{bcx^3+acx}}\right.}{\sqrt{cx}\,\sqrt{bx^2+a}}$
risch	$-\frac{2\sqrt{bx^2+a}}{a^2c\sqrt{cx}}+\frac{b^2\left(\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\text{EllipticF}\left(\right.}{}\right.}{b^2\sqrt{bcx^3+acx}}\right.}{}$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(6*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2)
)/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-3*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x
)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a-6
*b*x^2-4*a)/(b*x^2+a)^(1/2)/c/(c*x)^(1/2)/a^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.28

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = -\frac{3\left(bx^3+ax\right)\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(3bx^2+2a\right)\sqrt{bx^2+a}\sqrt{cx}}{a^2bc^2x^3+a^3c^2x}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
-(3*(b*x^3 + a*x)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstrassPInverse
(-4*a/b, 0, x)) + (3*b*x^2 + 2*a)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b*c^2*x^
3 + a^3*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.16

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{3/2}\,{\left(b\,x^2+a\right)}^{3/2}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2} (a+bx^2)^{3/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{x}\, \sqrt{b\, x^2+a}}{b^2 x^6+2 a b\, x^4+a^2 x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**2 + 2*a*b*x**4 + b**2*x**
6),x))/c**2
```

3.636 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 331

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx = \frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{5a^2c(cx)^{5/2}} + \frac{21b\sqrt{a+bx^2}}{5a^3c^3\sqrt{cx}} - \frac{21b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{5a^3c^4\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{5a^{11/4}c^{7/2}\sqrt{a+bx^2}} - \frac{21b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{10a^{11/4}c^{7/2}\sqrt{a+bx^2}}$$

output
```
1/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)-7/5*(b*x^2+a)^(1/2)/a^2/c/(c*x)^(5/2)+21
/5*b*(b*x^2+a)^(1/2)/a^3/c^3/(c*x)^(1/2)-21/5*b^(3/2)*(c*x)^(1/2)*(b*x^2+a
)^(1/2)/a^3/c^4/(a^(1/2)+b^(1/2)*x)+21/5*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x
^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(11/4)/c^(7/2)/(b*x^2+a)^(1/2)-21/10*
b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*Invers
eJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(11
/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.18

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{3}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)]
)/(5*a*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {253, 264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{(cx)^{7/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(-\frac{3b\left(\frac{b\int\frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$\downarrow$ 266

$$\frac{7\left(-\frac{3b\left(\frac{2b\int\frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$\downarrow$ 834

$$\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$\downarrow$ 761

$$\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+$$

$$\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{ac^3}\right.}{5ac^2}\right.}{2a}$$

$$\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x]
```

output
```
1/(a*c*(c*x)^(5/2)*Sqrt[a + b*x^2]) + (7*((-2*Sqrt[a + b*x^2])/(5*a*c*(c*x
)^(5/2)) - (3*b*((-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-(((c^2*Sq
rt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sq
rt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/
4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)
*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[
(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2]))
)/(a*c^3)))/(5*a*c^2)))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 2.81 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.66

method	result
default	$-\frac{42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\;\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\;\text{Ellipt}}{10x^2\sqrt{b\,x^2+a}\,c^3\sqrt{cx}\,a^3}$
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{5a^2c^4x^3}+\frac{16\left(x^2bc+ac\right)b}{5a^3c^4\sqrt{x\left(x^2bc+ac\right)}}+\frac{b^2x^2}{c^3a^3\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{21b\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2}{}\right.}{}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-8b\,x^2+a\right)}{5a^3x^2c^3\sqrt{cx}}-\frac{b^2\left(\frac{8\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\right.}{b\sqrt{bc\,x^3+acx}}\right.}{}$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/10/x^2*(42*((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)*EllipticE(((b*x+(-a
*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*a*b*x^2-21*((b*x+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-b/
(-a*b)^(1/2)*x)^(1/2)*EllipticF(((b*x+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/
2*2^(1/2))*a*b*x^2-42*b^2*x^4-28*a*b*x^2+4*a^2)/(b*x^2+a)^(1/2)/c^3/(c*x)^
(1/2)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.31

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{21\left(b^2x^5+abx^3\right)\sqrt{bc}\,\mathrm{weierstrassZeta}\left(-\frac{4a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)}{5\left(a^3bc^4x^5+a^4c^4x^3\right)}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
1/5*(21*(b^2*x^5 + a*b*x^3)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, weierstra
ssPInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 14*a*b*x^2 - 2*a^2)*sqrt(b*x^2 +
 a)*sqrt(c*x))/(a^3*b*c^4*x^5 + a^4*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.15

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 3/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3
/2)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{7/2}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^2x^8+2ab\,x^6+a^2x^4}dx\right)}{c^4}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**2*x**4 + 2*a*b*x**6 + b**2*x**
8),x))/c**4
```

3.637 $\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 183

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c(cx)^{9/2}}{3b(a+bx^2)^{3/2}} - \frac{3c^3(cx)^{5/2}}{2b^2\sqrt{a+bx^2}} + \frac{5c^5\sqrt{cx}\sqrt{a+bx^2}}{2b^3} - \frac{5a^{3/4}c^{11/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4b^{13/4}\sqrt{a+bx^2}}$$

output
```
-1/3*c*(c*x)^(9/2)/b/(b*x^2+a)^(3/2)-3/2*c^3*(c*x)^(5/2)/b^2/(b*x^2+a)^(1/
2)+5/2*c^5*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^3-5/4*a^(3/4)*c^(11/2)*(a^(1/2)+b
^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan
(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(13/4)/(b*x^2+a)^(1/2
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.50

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^5\sqrt{cx}\left(15a^2+21abx^2+4b^2x^4-15a(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4}\right.\right.}{6b^3\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(c*x)^(11/2)/(a + b*x^2)^(5/2),x]
```

output
```
(c^5*Sqrt[c*x]*(15*a^2 + 21*a*b*x^2 + 4*b^2*x^4 - 15*a*(a + b*x^2)*Sqrt[1
+ (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^3*(a +
b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {252, 252, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{3c^2\int \frac{(cx)^{7/2}}{(bx^2+a)^{3/2}}dx}{2b}-\frac{c(cx)^{9/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 252$$

$$\frac{3c^2\left(\frac{5c^2\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}dx}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{9/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{3c^2\left(\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{ac^2\int\frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{3b}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{9/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3c^2\left(\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{2ac\int\frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3b}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{9/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{3c^2\left(\frac{5c^2\left(\frac{2c\sqrt{cx}\sqrt{a+bx^2}}{3b}-\frac{a^{3/4}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3b^{5/4}\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{5/2}}{b\sqrt{a+bx^2}}\right)}{2b}-\frac{c(cx)^{9/2}}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c*x)^(11/2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(c*(c*x)^(9/2))/(b*(a + b*x^2)^(3/2)) + (3*c^2*(-((c*(c*x)^(5/2))/(b*
Sqrt[a + b*x^2])) + (5*c^2*((2*c*Sqrt[c*x]*Sqrt[a + b*x^2])/(3*b) - (a^(3/
4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c +
 Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])]
, 1/2])/(3*b^(5/4)*Sqrt[a + b*x^2])))/(2*b)))/(2*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(-\frac{a^2c^5\sqrt{bc\,x^3+acx}}{3b^5\left(x^2+\frac{a}{b}\right)^2}+\frac{13c^6xa}{6b^3\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{2c^5\sqrt{bc\,x^3+acx}}{3b^3}-\frac{5a\,c^6\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\cdots}{4b^4\sqrt{bc\,x^3+acx}}\cdots\right.}{cx\sqrt{b\,x^2+a}}$
default	$-\frac{\left(15\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-ab}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2+15\sqrt{-ab}\,\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\cdots\right.}{12x\,b^4(b\,x^2+a)^{\frac{3}{2}}}$
risch	$\frac{2x\sqrt{b\,x^2+a}\,c^6}{3b^3\sqrt{cx}}-\frac{a\left(\frac{7\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{bc\,x^3+acx}}-9a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{\sqrt{-\cdots}}{\cdots}\right.\right.}{\cdots}$

input
```
int((c*x)^(11/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(-1/3*a^2*c^5/b^5*
(b*c*x^3+a*c*x)^(1/2)/(x^2+a/b)^2+13/6/b^3*c^6*x*a/((x^2+a/b)*b*c*x)^(1/2)
+2/3/b^3*c^5*(b*c*x^3+a*c*x)^(1/2)-5/4*a*c^6/b^4*(-a*b)^(1/2)*((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(
1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*c*x)^(1/2)*EllipticF(((x+1/b*(-a
*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.68

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{15\left(ab^2c^5x^4+2\,a^2bc^5x^2+a^3c^5\right)\sqrt{bc}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-\left(4\,b^3c^5x^4+21\,ab^2c^5x^2+15\,a^2bc^5\right)\sqrt{\cdots}}{6\left(b^6x^4+2\,ab^5x^2+a^2b^4\right)}$$

input
```
integrate((c*x)^(11/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(15*(a*b^2*c^5*x^4 + 2*a^2*b*c^5*x^2 + a^3*c^5)*sqrt(b*c)*weierstrass
PInverse(-4*a/b, 0, x) - (4*b^3*c^5*x^4 + 21*a*b^2*c^5*x^2 + 15*a^2*b*c^5)
*sqrt(b*x^2 + a)*sqrt(c*x))/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 102.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.24

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{11}{2}}x^{\frac{13}{2}}\Gamma\left(\frac{13}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{13}{4}\\ \frac{17}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{17}{4}\right)}$$

input
```
integrate((c*x)**(11/2)/(b*x**2+a)**(5/2),x)
```

output
```
c**(11/2)*x**(13/2)*gamma(13/4)*hyper((5/2, 13/4), (17/4,), b*x**2*exp_pol
ar(I*pi)/a)/(2*a**(5/2)*gamma(17/4))
```

Maxima [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(11/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{11}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(11/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(11/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{11/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(11/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(11/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{11/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c^5\left(18\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2+18\sqrt{x}\,\sqrt{b\,x^2+a}\,ab\,x^2+2\sqrt{x}\,\sqrt{b\,x^2+a}\,b^2x^4-9\left(\int \frac{}{b^3x^7+3}\right.\right.}{3b}$$

input
```
int((c*x)^(11/2)/(b*x^2+a)^(5/2),x)
```

output

```
(sqrt(c)*c**5*(18*sqrt(x)*sqrt(a + b*x**2)*a**2 + 18*sqrt(x)*sqrt(a + b*x*
*2)*a*b*x**2 + 2*sqrt(x)*sqrt(a + b*x**2)*b**2*x**4 - 9*int((sqrt(x)*sqrt(
a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**5
- 18*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**
5 + b**3*x**7),x)*a**4*b*x**2 - 9*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x +
 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**3*b**2*x**4))/(3*b**3*(a
**2 + 2*a*b*x**2 + b**2*x**4))
```

3.638 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}} - \frac{5c^3\sqrt{cx}}{6b^2\sqrt{a+bx^2}} + \frac{5c^{7/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12\sqrt[4]{a}b^{9/4}\sqrt{a+bx^2}}$$

output
```
-1/3*c*(c*x)^(5/2)/b/(b*x^2+a)^(3/2)-5/6*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/
2)+5/12*c^(7/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2
)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2
))/a^(1/4)/b^(9/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^3\sqrt{cx}\left(-5a-7bx^2+5(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{6b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

output
```
(c^3*Sqrt[c*x]*(-5*a - 7*b*x^2 + 5*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {252, 252, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{5c^2\int \frac{(cx)^{3/2}}{(bx^2+a)^{3/2}}dx}{6b}-\frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{252}$$

$$\frac{5c^2\left(\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{2b}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{5/2}}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5c^2\left(\frac{c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{b}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5c^2\left(\frac{\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2\sqrt[4]{a}b^{5/4}\sqrt{a+bx^2}}-\frac{c\sqrt{cx}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{5/2}}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c*x)^(7/2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(c*(c*x)^(5/2))/(b*(a + b*x^2)^(3/2)) + (5*c^2*(-((c*Sqrt[c*x])/(b*Sq
rt[a + b*x^2])) + (Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x
^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a
^(1/4)*Sqrt[c])], 1/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a + b*x^2])))/(6*b)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 1.75 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.32

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{a\,c^3\sqrt{bc\,x^3+acx}}{3b^4\left(x^2+\frac{a}{b}\right)^2}-\frac{7c^4x}{6b^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{5c^4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{12b^3\sqrt{bc\,x^3+acx}}}{cx\sqrt{b\,x^2+a}}$
default	$\frac{\left(5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^2+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{Elli}\right.}{12x\,b^3\left(b\,x^2+a\right)^{\frac{3}{2}}}$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(1/3*a*c^3/b^4*(b*
c*x^3+a*c*x)^(1/2)/(x^2+a/b)^2-7/6/b^2*c^4*x/((x^2+a/b)*b*c*x)^(1/2)+5/12*
c^4/b^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/
b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a
*c*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1
/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.70

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \frac{5\,(b^2c^3x^4+2\,abc^3x^2+a^2c^3)\sqrt{bc}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)-(7\,b^2c^3x^2+5\,abc^3)\sqrt{}}{6\,(b^5x^4+2\,ab^4x^2+a^2b^3)}$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(5*(b^2*c^3*x^4 + 2*a*b*c^3*x^2 + a^2*c^3)*sqrt(b*c)*weierstrassPInver
se(-4*a/b, 0, x) - (7*b^2*c^3*x^2 + 5*a*b*c^3)*sqrt(b*x^2 + a)*sqrt(c*x))/
(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{4},\frac{5}{2}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

output
```
c**(7/2)*x**(9/2)*gamma(9/4)*hyper((9/4, 5/2), (13/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{7/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(7/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c^3\left(-2\sqrt{x}\,\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx\right)a^3+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}\right.\right.}{b^2\,(b\,x^2+a)}$$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*c**3*( - 2*sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**
2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**3 + int((sq
rt(x)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**
7),x)*a**2*b*x**2))/(b**2*(a + b*x**2))
```

3.639 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c\sqrt{cx}}{3b\,(a+bx^2)^{3/2}}+\frac{c\sqrt{cx}}{6ab\sqrt{a+bx^2}} + \frac{c^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{5/4}b^{5/4}\sqrt{a+bx^2}}$$

output
```
-1/3*c*(c*x)^(1/2)/b/(b*x^2+a)^(3/2)+1/6*c*(c*x)^(1/2)/a/b/(b*x^2+a)^(1/2)
+1/12*c^(3/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*
InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))
/a^(5/4)/b^(5/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c\sqrt{cx}\left(-a+bx^2+(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{6ab\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

output
```
(c*Sqrt[c*x]*(-a + b*x^2 + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2
F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(6*a*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {252, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c^2\int \frac{1}{\sqrt{cx}(bx^2+a)^{3/2}}dx}{6b} - \frac{c\sqrt{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{c^2\left(\frac{\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6b} - \frac{c\sqrt{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{c^2\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6b}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{c^2\left(\frac{\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6b}-\frac{c\sqrt{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

input

```
Int[(c*x)^(3/2)/(a + b*x^2)^(5/2),x]
```

output

```
-1/3*(c*Sqrt[c*x])/(b*(a + b*x^2)^(3/2)) + (c^2*(Sqrt[c*x]/(a*c*Sqrt[a + b
*x^2]) + ((Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c +
Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])],
 1/2])/(2*a^(5/4)*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2])))/(6*b)
```

Defintions of rubi rules used

rule 252

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(-\frac{c\sqrt{bc\,x^3+acx}}{3b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{c^2x}{6ba\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{c^2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)}{\sqrt{-ab}}}\right.}{12b^2a\sqrt{bc\,x^3+acx}}\right.}{cx\sqrt{b\,x^2+a}}$
default	$\frac{\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^2+\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{Ellipti}\right.}{12xa\,b^2(b\,x^2+a)^{\frac{3}{2}}}$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(-1/3*c/b^3*(b*c*x
^3+a*c*x)^(1/2)/(x^2+a/b)^2+1/6/b*c^2*x/a/((x^2+a/b)*b*c*x)^(1/2)+1/12/b^2
/a*c^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b
*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*
c*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.63

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{(b^2cx^4+2\,abcx^2+a^2c)\sqrt{bc}\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(b^2cx^2-abc)\sqrt{bx^2+a}\sqrt{}}{6\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*((b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*sqrt(b*c)*weierstrassPInverse(-4*a/
b, 0, x) + (b^2*c*x^2 - a*b*c)*sqrt(b*x^2 + a)*sqrt(c*x))/(a*b^4*x^4 + 2*a
^2*b^3*x^2 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/2), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c\left(-2\sqrt{x}\,\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx\right)a^3+2\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}\right.\right.}{5b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(x)*sqrt(a + b*x**2) + int((sqrt(x)*sqrt(a + b*x**2))
/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**3 + 2*int((sqr
t(x)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7
),x)*a**2*b*x**2 + int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3
+ 3*a*b**2*x**5 + b**3*x**7),x)*a*b**2*x**4))/(5*b*(a**2 + 2*a*b*x**2 + b*
*2*x**4))
```

3.640 $\int \frac{1}{\sqrt{cx}(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{cx}}{3ac\,(a+bx^2)^{3/2}} + \frac{5\sqrt{cx}}{6a^2c\sqrt{a+bx^2}} + \frac{5\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{12a^{9/4}\sqrt[4]{b}\sqrt{c}\sqrt{a+bx^2}}$$

output
```
1/3*(c*x)^(1/2)/a/c/(b*x^2+a)^(3/2)+5/6*(c*x)^(1/2)/a^2/c/(b*x^2+a)^(1/2)+
5/12*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJa
cobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(9/4)/
b^(1/4)/c^(1/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{7ax+5bx^3+5x(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\frac{bx^2}{a}\right)}{6a^2\sqrt{cx}\,(a+bx^2)^{3/2}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

output
```
(7*a*x + 5*b*x^3 + 5*x*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/4, 1/2, 5/4, -((b*x^2)/a)])/(6*a^2*Sqrt[c*x]*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {253, 253, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5\int \frac{1}{\sqrt{cx}(bx^2+a)^{3/2}}dx}{6a}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{cx}\sqrt{bx^2+a}}dx}{2a}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{5\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{5\left(\frac{\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}c^{3/2}\sqrt{a+bx^2}}+\frac{\sqrt{cx}}{ac\sqrt{a+bx^2}}\right)}{6a}+\frac{\sqrt{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/2)),x]
```

output
```
Sqrt[c*x]/(3*a*c*(a + b*x^2)^(3/2)) + (5*(Sqrt[c*x]/(a*c*Sqrt[a + b*x^2])
+ ((Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]
*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/
(2*a^(5/4)*b^(1/4)*c^(3/2)*Sqrt[a + b*x^2])))/(6*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.26

method	result
elliptic	$\frac{\sqrt{cx(b\,x^2+a)}\left(\frac{\sqrt{bc\,x^3+acx}}{3ac\,b^2\left(x^2+\frac{a}{b}\right)^2}+\frac{5x}{6a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{12a^2b\sqrt{bc\,x^3+acx}}\right)}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
default	$\frac{5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^2+5\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{Ellipt}}{12\sqrt{cx}\,a^2b(b\,x^2+a)^{\frac{3}{2}}}$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^2+a))^(1/2)/(c*x)^(1/2)/(b*x^2+a)^(1/2)*(1/3/a/c/b^2*(b*c*x^3+a*
c*x)^(1/2)/(x^2+a/b)^2+5/6*x/a^2/((x^2+a/b)*b*c*x)^(1/2)+5/12/a^2/b*(-a*b)
^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2)
)*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*c*x)^(1/2)*El
lipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{5\,(b^2x^4+2\,abx^2+a^2)\sqrt{bc}\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)+(5\,b^2x^2+7\,ab)\sqrt{bx^2}}{6\,(a^2b^3cx^4+2\,a^3b^2cx^2+a^4bc)}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(5*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*c)*weierstrassPInverse(-4*a/b, 0
, x) + (5*b^2*x^2 + 7*a*b)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^2*b^3*c*x^4 + 2*a
^3*b^2*c*x^2 + a^4*b*c)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 5/2), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(5/2)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2
*x**5 + b**3*x**7),x))/c
```

3.641 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 185

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}}\,dx = \frac{1}{3ac(cx)^{3/2}(a+bx^2)^{3/2}}+\frac{3}{2a^2c(cx)^{3/2}\sqrt{a+bx^2}} -\frac{5\sqrt{a+bx^2}}{2a^3c(cx)^{3/2}}-\frac{5b^{3/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{13/4}c^{5/2}\sqrt{a+bx^2}}$$

output
```
1/3/a/c/(c*x)^(3/2)/(b*x^2+a)^(3/2)+3/2/a^2/c/(c*x)^(3/2)/(b*x^2+a)^(1/2)-
5/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(3/2)-5/4*b^(3/4)*(a^(1/2)+b^(1/2)*x)*((b*
x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)
^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(13/4)/c^(5/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.32

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{1}{4},-\frac{bx^2}{a}\right)}{3a^2(cx)^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 5/2, 1/4, -((b*x^2)/a)])
/(3*a^2*(c*x)^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {253, 253, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{3 \int \frac{1}{(cx)^{5/2} (bx^2+a)^{3/2}}dx}{2a}+\frac{1}{3ac(cx)^{3/2} (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{3 \left(\frac{5 \int \frac{1}{(cx)^{5/2} \sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac(cx)^{3/2} \sqrt{a+bx^2}}\right)}{2a}+\frac{1}{3ac(cx)^{3/2} (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{3 \left(\frac{5 \left(-\frac{b \int \frac{1}{\sqrt{cx} \sqrt{bx^2+a}}dx}{3ac^2}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2} \sqrt{a+bx^2}}\right)}{2a}+\frac{1}{3ac(cx)^{3/2} (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{3\left(\frac{5\left(-\frac{2b\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}\right)}{2a}+\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \ 761$$

$$\frac{3\left(\frac{5\left(-\frac{b^{3/4}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{3a^{5/4}c^{7/2}\sqrt{a+bx^2}}-\frac{2\sqrt{a+bx^2}}{3ac(cx)^{3/2}}\right)}{2a}+\frac{1}{ac(cx)^{3/2}\sqrt{a+bx^2}}\right)}{2a}+\frac{1}{3ac(cx)^{3/2}\left(a+bx^2\right)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x]
```

output
```
1/(3*a*c*(c*x)^(3/2)*(a + b*x^2)^(3/2)) + (3*(1/(a*c*(c*x)^(3/2)*Sqrt[a +
b*x^2]) + (5*((-2*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/2)) - (b^(3/4)*(Sqrt[a]
*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(3*a^(5/4)*
c^(7/2)*Sqrt[a + b*x^2])))/(2*a)))/(2*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 4.12 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

method	result
elliptic	$\frac{\sqrt{cx(bx^2+a)}\left(-\frac{\sqrt{bc\,x^3+acx}}{3a^2c^3b\left(x^2+\frac{a}{b}\right)^2}-\frac{11bx}{6c^2a^3\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\sqrt{bc\,x^3+acx}}{3a^3c^3x^2}-\frac{5\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}}{4a^3c^2\sqrt{bc\,x^3+acx}}\right.}{\sqrt{cx}\sqrt{bx^2+a}}$
default	$-\frac{15\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)\sqrt{-ab}\,b\,x^3+15\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}}{12x\,c^2\sqrt{cx}\,a^3(bx^2+a)^{\frac{3}{2}}}$
risch	$-\frac{2\sqrt{bx^2+a}}{3a^3x\,c^2\sqrt{cx}}-\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b\sqrt{bc\,x^3+acx}}+3a\left(\frac{x}{a\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{\sqrt{-a}}{}\right.\right.}{}$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^2+a))^(1/2)/(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-1/3/a^2/c^3/b*(b*c*x^3
+a*c*x)^(1/2)/(x^2+a/b)^2-11/6*b/c^2*x/a^3/((x^2+a/b)*b*c*x)^(1/2)-2/3/a^3
/c^3*(b*c*x^3+a*c*x)^(1/2)/x^2-5/4/a^3/c^2*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/
2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(
-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*c*x)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1
/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{5/2}} \, dx = -\frac{15 \left(b^2 x^6 + 2 \, abx^4 + a^2 x^2\right) \sqrt{bc} \text{weierstrassPInverse}\left(-\frac{4 \, a}{b}, 0, x\right) + \left(15 \, b^2 x^4 + 21 \, abx^2 + 4 \, a^2\right) \sqrt{bx^2 + a} \sqrt{cx}}{6 \left(a^3 b^2 c^3 x^6 + 2 \, a^4 b c^3 x^4 + a^5 c^3 x^2\right)}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(15*(b^2*x^6 + 2*a*b*x^4 + a^2*x^2)*sqrt(b*c)*weierstrassPInverse(-4*
a/b, 0, x) + (15*b^2*x^4 + 21*a*b*x^2 + 4*a^2)*sqrt(b*x^2 + a)*sqrt(c*x))/
(a^3*b^2*c^3*x^6 + 2*a^4*b*c^3*x^4 + a^5*c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.68 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.26

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{5/2}} \, dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}} c^{\frac{5}{2}} x^{\frac{3}{2}} \Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 5/2), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/
2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{2}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{5/2}\,{\left(b\,x^2+a\right)}^{5/2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2} (a + bx^2)^{5/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^9+3a\,b^2x^7+3a^2b\,x^5+a^3x^3} dx \right)}{c^3}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x**3 + 3*a**2*b*x**5 + 3*a*b
**2*x**7 + b**3*x**9),x))/c**3
```

3.642 $\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 300

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c(cx)^{7/2}}{3b(a+bx^2)^{3/2}} - \frac{7c^3(cx)^{3/2}}{6b^2\sqrt{a+bx^2}} + \frac{7c^4\sqrt{cx}\sqrt{a+bx^2}}{2b^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{7\sqrt[4]{a}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2b^{11/4}\sqrt{a+bx^2}} + \frac{7\sqrt[4]{a}c^{9/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4b^{11/4}\sqrt{a+bx^2}}$$

output
```
-1/3*c*(c*x)^(7/2)/b/(b*x^2+a)^(3/2)-7/6*c^3*(c*x)^(3/2)/b^2/(b*x^2+a)^(1/
2)+7/2*c^4*(c*x)^(1/2)*(b*x^2+a)^(1/2)/b^(5/2)/(a^(1/2)+b^(1/2)*x)-7/2*a^(
1/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*E
llipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/b
^(11/4)/(b*x^2+a)^(1/2)+7/4*a^(1/4)*c^(9/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)
/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)
/a^(1/4)/c^(1/2)),1/2*2^(1/2))/b^(11/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{2c^3(cx)^{3/2}\left(-7a-3bx^2+7(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3b^2(a+bx^2)^{3/2}}$$

input
```
Integrate[(c*x)^(9/2)/(a + b*x^2)^(5/2),x]
```

output
```
(-2*c^3*(c*x)^(3/2)*(-7*a - 3*b*x^2 + 7*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hy
pergeometric2F1[3/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {252, 252, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \ 252$$

$$\frac{7c^2\int \frac{(cx)^{5/2}}{(bx^2+a)^{3/2}}dx}{6b} - \frac{c(cx)^{7/2}}{3b(a+bx^2)^{3/2}}$$

$$\Big\downarrow \ 252$$

$$\frac{7c^2\left(\frac{3c^2\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{2b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{7/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{7c^2\left(\frac{3c\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{7/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{7c^2\left(\frac{3c\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{7/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{7c^2\left(\frac{3c\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{7/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{7c^2\left(\frac{3c\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{b}-\frac{c(cx)^{3/2}}{b\sqrt{a+bx^2}}\right)}{6b}-\frac{c(cx)^{7/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{7c^2\left(\frac{3c\left(\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a}c+\sqrt{b}cx)\sqrt{\frac{ac^2+bc^2x^2}{(\sqrt{a}c+\sqrt{b}cx)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a}c+\sqrt{b}cx)\sqrt{\frac{ac^2+bc^2x^2}{(\sqrt{a}c+\sqrt{b}cx)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\right.}{b}\right.}{6b}$$

$$\frac{c(cx)^{7/2}}{3b(a+bx^2)^{3/2}}$$

input
```
Int[(c*x)^(9/2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(c*(c*x)^(7/2))/(b*(a + b*x^2)^(3/2)) + (7*c^2*(-((c*(c*x)^(3/2))/(b*
Sqrt[a + b*x^2])) + (3*c*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c
+ Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 +
 b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[
c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(
1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c
 + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c]
)], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/b))/(6*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(\frac{a\,c^4x\sqrt{bc\,x^3+acx}}{3b^4\left(x^2+\frac{a}{b}\right)^2}-\frac{3c^5x^2}{2b^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}+\frac{7c^5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}(\cdots}{\cdots}\right.}{4b^3\sqrt{bc\,x^3+ac\cdots}}\right.}{cx\sqrt{b\,x^2+a}}$
default	$\frac{\left(42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \mathrm{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \mathrm{Ellipti}\cdots\right.}{\cdots}$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(1/3*a*c^4/b^4*x*(
b*c*x^3+a*c*x)^(1/2)/(x^2+a/b)^2-3/2/b^2*c^5*x^2/((x^2+a/b)*b*c*x)^(1/2)+7
/4*c^5/b^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x
-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^
3+a*c*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)
^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2)
)*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.39

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{21\left(b^2c^4x^4+2\,abc^4x^2+a^2c^4\right)\sqrt{bc}\,\text{weierstrassZeta}\left(-\frac{4a}{b},0,\text{weierstrassPInverse}\left(-\frac{4a}{b},0,x\right)\right)+\left(9\,b^2c^4x^3+\right.}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(21*(b^2*c^4*x^4 + 2*a*b*c^4*x^2 + a^2*c^4)*sqrt(b*c)*weierstrassZeta
(-4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (9*b^2*c^4*x^3 + 7*a*b*c^
4*x)*sqrt(b*x^2 + a)*sqrt(c*x))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 39.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**2+a)**(5/2),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/4)*hyper((5/2, 11/4), (15/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(5/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{9/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c^4\left(14\sqrt{x}\,\sqrt{b\,x^2+a}\,ax+6\sqrt{x}\,\sqrt{b\,x^2+a}\,b\,x^3-21\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)a^4 - \cdots}{3b^2\left(b^2x^4+2ab \cdots\right.}$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*c**4*(14*sqrt(x)*sqrt(a + b*x**2)*a*x + 6*sqrt(x)*sqrt(a + b*x**2
)*b*x**3 - 21*int((sqrt(x)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b
**2*x**4 + b**3*x**6),x)*a**4 - 42*int((sqrt(x)*sqrt(a + b*x**2))/(a**3 +
3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**3*b*x**2 - 21*int((sqrt(x
)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*
a**2*b**2*x**4))/(3*b**2*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.643 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 304

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = -\frac{c(cx)^{3/2}}{3b(a+bx^2)^{3/2}}+\frac{c(cx)^{3/2}}{2ab\sqrt{a+bx^2}}-\frac{c^2\sqrt{cx}\sqrt{a+bx^2}}{2ab^{3/2}\left(\sqrt{a}+\sqrt{b}x\right)}$$
$$+\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$
$$-\frac{c^{5/2}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{3/4}b^{7/4}\sqrt{a+bx^2}}$$

output
```
-1/3*c*(c*x)^(3/2)/b/(b*x^2+a)^(3/2)+1/2*c*(c*x)^(3/2)/a/b/(b*x^2+a)^(1/2)
-1/2*c^2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a/b^(3/2)/(a^(1/2)+b^(1/2)*x)+1/2*c^(
5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE
(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(3/4)/b
^(7/4)/(b*x^2+a)^(1/2)-1/4*c^(5/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)
+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2)),1/2*2^(1/2))/a^(3/4)/b^(7/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.24

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = \frac{2c(cx)^{3/2}\left(-a+(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

output
```
(2*c*(c*x)^(3/2)*(-a + (a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3
/4, 5/2, 7/4, -((b*x^2)/a)]))/(3*a*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {252, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c^2\int \frac{\sqrt{cx}}{(bx^2+a)^{3/2}}dx}{2b}-\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{2a}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{c^2\left(\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ac}\right)}{2b}-\frac{c(cx)^{3/2}}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(c*(c*x)^(3/2))/(b*(a + b*x^2)^(3/2)) + (c^2*((c*x)^(3/2)/(a*c*Sqrt[a
 + b*x^2]) - (-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*
x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/
(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/
4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]
*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c
*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2
*b^(3/4)*Sqrt[a + b*x^2]))/(a*c)))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.87

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(b\,x^2+a)}\left(-\frac{c^2 x\sqrt{bc\,x^3+acx}}{3b^3\left(x^2+\frac{a}{b}\right)^2}+\frac{c^3x^2}{2ba\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c^3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\cdots\right.}{\cdots}\right.}{4b^2a\sqrt{bc\,x^3+a\cdots}}\right.}{cx\sqrt{b\,x^2+a}}$
default	$-\frac{\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \text{Ellipti}\cdots\right.}{\cdots}$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(-1/3*c^2/b^3*x*(b
*c*x^3+a*c*x)^(1/2)/(x^2+a/b)^2+1/2/b*c^3*x^2/a/((x^2+a/b)*b*c*x)^(1/2)-1/
4/b^2/a*c^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(
x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x
^3+a*c*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2
))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.39

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = \frac{3\,(b^2c^2x^4+2\,abc^2x^2+a^2c^2)\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,\right.\right.}{6\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)}$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(b^2*c^2*x^4 + 2*a*b*c^2*x^2 + a^2*c^2)*sqrt(b*c)*weierstrassZeta(-
4*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (3*b^2*c^2*x^3 + a*b*c^2*x)
*sqrt(b*x^2 + a)*sqrt(c*x))/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.14

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(5/2),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((7/4, 5/2), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,c^2\left(-2\sqrt{x}\,\sqrt{b\,x^2+a}\,x+3\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)a^3+6\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}\right.\right.}{3b\,(b^2x^4+2ab\,x^2+a^2)}$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*c**2*( - 2*sqrt(x)*sqrt(a + b*x**2)*x + 3*int((sqrt(x)*sqrt(a + b
*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**3 + 6*int
((sqrt(x)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x
**6),x)*a**2*b*x**2 + 3*int((sqrt(x)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x*
*2 + 3*a*b**2*x**4 + b**3*x**6),x)*a*b**2*x**4))/(3*b*(a**2 + 2*a*b*x**2 +
 b**2*x**4))
```

3.644 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 302

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{(cx)^{3/2}}{3ac\,(a+bx^2)^{3/2}} + \frac{(cx)^{3/2}}{2a^2c\sqrt{a+bx^2}} - \frac{\sqrt{cx}\sqrt{a+bx^2}}{2a^2\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)} + \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{7/4}b^{3/4}\sqrt{a+bx^2}} - \frac{\sqrt{c}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{7/4}b^{3/4}\sqrt{a+bx^2}}$$

output
```
1/3*(c*x)^(3/2)/a/c/(b*x^2+a)^(3/2)+1/2*(c*x)^(3/2)/a^2/c/(b*x^2+a)^(1/2)-
1/2*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^2/b^(1/2)/(a^(1/2)+b^(1/2)*x)+1/2*c^(1/2
)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(si
n(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(7/4)/b^(3
/4)/(b*x^2+a)^(1/2)-1/4*c^(1/2)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^
(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(
1/2)),1/2*2^(1/2))/a^(7/4)/b^(3/4)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.20

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{2x\sqrt{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{2},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a^2\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 5/2, 7/4, -((b*x
^2)/a)])/(3*a^2*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {253, 253, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{\int \frac{\sqrt{cx}}{(bx^2+a)^{3/2}}dx}{2a}+\frac{(cx)^{3/2}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{2a}}{2a}+\frac{(cx)^{3/2}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac}}{2a}+\frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}}{2a}+\frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}}{2a}+\frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}}{ac}}{2a}+\frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}$$

$\Big\downarrow$ 1510

$$\frac{\frac{(cx)^{3/2}}{ac\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)\right.}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ac}}{2a}$$

$$\frac{(cx)^{3/2}}{3ac\left(a+bx^2\right)^{3/2}}$$

input
```
Int[Sqrt[c*x]/(a + b*x^2)^(5/2),x]
```

output
```
(c*x)^(3/2)/(3*a*c*(a + b*x^2)^(3/2)) + ((c*x)^(3/2)/(a*c*Sqrt[a + b*x^2])
 - (-((-(-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(
1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c
 + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c]
)], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*
c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*S
qrt[a + b*x^2]))/(a*c))/(2*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.85

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{cx(bx^2+a)}\left(\frac{x\sqrt{bc\,x^3+acx}}{3a\,b^2\left(x^2+\frac{a}{b}\right)^2}+\frac{c\,x^2}{2a^2\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{c\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \text{EllipticE}\left(\sqrt{\frac{(\ldots}{}}\right.}{b}\right.}{4a^2b\sqrt{bc\,x^3+acx}}\right)}{cx\sqrt{b\,x^2+a}}$
default	$-\frac{\left(6\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-3\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\ \text{Ellipti}\ldots\right.}{}$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(b*x^2+a)^(1/2)*(c*x*(b*x^2+a))^(1/2)*(1/3/a/b^2*x*(b*c*
x^3+a*c*x)^(1/2)/(x^2+a/b)^2+1/2*c*x^2/a^2/((x^2+a/b)*b*c*x)^(1/2)-1/4/a^2
*c/b*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(
-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*c*
x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-
a*b)^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{3\,(b^2x^4+2\,abx^2+a^2)\sqrt{bc}\,\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+}{6\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(b^2*x^4 + 2*a*b*x^2 + a^2)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0, we
ierstrassPInverse(-4*a/b, 0, x)) + (3*b^2*x^3 + 5*a*b*x)*sqrt(b*x^2 + a)*s
qrt(c*x))/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(5/2),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/2), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x*
*4 + b**3*x**6),x)
```

3.645 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 333

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = \frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}} + \frac{7}{6a^2c\sqrt{cx}\sqrt{a+bx^2}} - \frac{7\sqrt{a+bx^2}}{2a^3c\sqrt{cx}} + \frac{7\sqrt{b}\sqrt{cx}\sqrt{a+bx^2}}{2a^3c^2\left(\sqrt{a}+\sqrt{b}x\right)} - \frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{2a^{11/4}c^{3/2}\sqrt{a+bx^2}} + \frac{7\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{4a^{11/4}c^{3/2}\sqrt{a+bx^2}}$$

output
```
1/3/a/c/(c*x)^(1/2)/(b*x^2+a)^(3/2)+7/6/a^2/c/(c*x)^(1/2)/(b*x^2+a)^(1/2)-
7/2*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(1/2)+7/2*b^(1/2)*(c*x)^(1/2)*(b*x^2+a)^(1
/2)/a^3/c^2/(a^(1/2)+b^(1/2)*x)-7/2*b^(1/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)
/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2))),1/2*2^(1/2))/a^(11/4)/c^(3/2)/(b*x^2+a)^(1/2)+7/4*b^(1/4)
*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobi
AM(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(11/4)/c^(
3/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.17

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{2},\frac{3}{4},-\frac{bx^2}{a}\right)}{a^2(cx)^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 5/2, 3/4, -((b*x^2)/a)])
/(a^2*(c*x)^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {253, 253, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{(cx)^{3/2}(bx^2+a)^{3/2}}dx}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{7\left(\frac{3\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{3\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 266

$$\frac{7\left(\frac{3\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 834

$$\frac{7\left(\frac{3\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{7\left(\frac{3\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

$\Big\downarrow$ 761

$$\frac{7\left(\frac{3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{2a}+\frac{1}{ac\sqrt{cx}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac\sqrt{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1510$$

$$\frac{7\left(\frac{3\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a}c+\sqrt{b}cx)\sqrt{\frac{ac^2+bc^2x^2}{(\sqrt{a}c+\sqrt{b}cx)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}(\sqrt{a}c+\sqrt{b}cx)\sqrt{\frac{ac^2+bc^2x^2}{(\sqrt{a}c+\sqrt{b}cx)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}}{ac^3}}{2a}}{6a}$$

$$\frac{1}{3ac\sqrt{cx}\,(a+bx^2)^{3/2}}$$

input
```
Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x]
```

output
```
1/(3*a*c*Sqrt[c*x]*(a + b*x^2)^(3/2)) + (7*(1/(a*c*Sqrt[c*x]*Sqrt[a + b*x^
2]) + (3*((-2*Sqrt[a + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-(((c^2*Sqrt[c*x]
*Sqrt[a + b*x^2])/(Sqrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c
 + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*Elli
pticE[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt
[a + b*x^2]))/Sqrt[b]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(
a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*c^
3)))/(2*a)))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 3.79 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{\sqrt{cx(bx^2+a)}\left(-\frac{x\sqrt{bc\,x^3+acx}}{3a^2c^2b\left(x^2+\frac{a}{b}\right)^2}-\frac{3b\,x^2}{2c\,a^3\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{2\left(x^2bc+ac\right)}{a^3c^2\sqrt{x\left(x^2bc+ac\right)}}+\frac{7\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\cdots\right.}{\cdots}\right.}{\sqrt{cx}\,\sqrt{b\,x^2+a}}$
default	$\frac{42\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)ab\,x^2-21\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{2}\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\operatorname{EllipticF}\cdots}{\cdots}$
risch	$-\frac{2\sqrt{b\,x^2+a}}{a^3c\sqrt{cx}}+\frac{b\left(\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\operatorname{EllipticF}\left(\cdots\right.}{\cdots}\right.}{b\sqrt{bc\,x^3+acx}}\right.}{\cdots}$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^2+a))^(1/2)/(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-1/3/a^2/c^2/b*x*(b*c*x
^3+a*c*x)^(1/2)/(x^2+a/b)^2-3/2*b/c*x^2/a^3/((x^2+a/b)*b*c*x)^(1/2)-2*(b*c
*x^2+a*c)/a^3/c^2/(x*(b*c*x^2+a*c))^(1/2)+7/4/a^3/c*(-a*b)^(1/2)*((x+1/b*(
-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2)
)^(1/2)*(-b/(-a*b)^(1/2)*x)^(1/2)/(b*c*x^3+a*c*x)^(1/2)*(-2/b*(-a*b)^(1/2)
*EllipticE(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-
a*b)^(1/2)*EllipticF(((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = -\frac{21\left(b^2x^5+2\,abx^3+a^2x\right)\sqrt{bc}\mathrm{weierstrassZeta}\left(-\frac{4\,a}{b},0,\mathrm{weierstrassPInverse}\left(-\frac{4\,a}{b},0,x\right)\right)+\left(21\,b^2x^4+35\,ab\right.}{6\left(a^3b^2c^2x^5+2\,a^4bc^2x^3+a^5c^2x\right)}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/6*(21*(b^2*x^5 + 2*a*b*x^3 + a^2*x)*sqrt(b*c)*weierstrassZeta(-4*a/b, 0
, weierstrassPInverse(-4*a/b, 0, x)) + (21*b^2*x^4 + 35*a*b*x^2 + 12*a^2)*
sqrt(b*x^2 + a)*sqrt(c*x))/(a^3*b^2*c^2*x^5 + 2*a^4*b*c^2*x^3 + a^5*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.14

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx = \frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/2),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 5/2), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/
2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2} (a+bx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^8+3a\,b^2x^6+3a^2b\,x^4+a^3x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x**2 + 3*a**2*b*x**4 + 3*a*b
**2*x**6 + b**3*x**8),x))/c**2
```

3.646 $\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 362

$$\begin{aligned}\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/2}}\,dx &= \frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}+\frac{11}{6a^2c(cx)^{5/2}\sqrt{a+bx^2}}\\ &-\frac{77\sqrt{a+bx^2}}{30a^3c(cx)^{5/2}}+\frac{77b\sqrt{a+bx^2}}{10a^4c^3\sqrt{cx}}-\frac{77b^{3/2}\sqrt{cx}\sqrt{a+bx^2}}{10a^4c^4\left(\sqrt{a}+\sqrt{b}x\right)}\\ &+\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)|\frac{1}{2}\right)}{10a^{15/4}c^{7/2}\sqrt{a+bx^2}}\\ &-\frac{77b^{5/4}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{20a^{15/4}c^{7/2}\sqrt{a+bx^2}}\end{aligned}$$

output
```
1/3/a/c/(c*x)^(5/2)/(b*x^2+a)^(3/2)+11/6/a^2/c/(c*x)^(5/2)/(b*x^2+a)^(1/2)
-77/30*(b*x^2+a)^(1/2)/a^3/c/(c*x)^(5/2)+77/10*b*(b*x^2+a)^(1/2)/a^4/c^3/(
c*x)^(1/2)-77/10*b^(3/2)*(c*x)^(1/2)*(b*x^2+a)^(1/2)/a^4/c^4/(a^(1/2)+b^(1
/2)*x)+77/10*b^(5/4)*(a^(1/2)+b^(1/2)*x)*((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)
^(1/2)*EllipticE(sin(2*arctan(b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))),1/2*2^
(1/2))/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)-77/20*b^(5/4)*(a^(1/2)+b^(1/2)*x)*
((b*x^2+a)/(a^(1/2)+b^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2)),1/2*2^(1/2))/a^(15/4)/c^(7/2)/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.16

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{5/2}} \, dx = -\frac{2x\sqrt{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{2},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5a^2(cx)^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

output
```
(-2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 5/2, -1/4, -((b*x^2)/a)]
)/(5*a^2*(c*x)^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {253, 253, 264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11 \int \frac{1}{(cx)^{7/2}(bx^2+a)^{3/2}}dx}{6a}+\frac{1}{3ac(cx)^{5/2} (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{11 \left(\frac{7 \int \frac{1}{(cx)^{7/2}\sqrt{bx^2+a}}dx}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2} (a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{7\left(-\frac{3b\int \frac{1}{(cx)^{3/2}\sqrt{bx^2+a}}dx}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{2b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\sqrt{a}c\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{a}c\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt{a}c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}-\frac{\int \frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{1}{ac(cx)^{5/2}\sqrt{a+bx^2}}\right)}{6a}+\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 761

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\int\frac{\sqrt{a}c-\sqrt{b}cx}{\sqrt{bx^2+a}}d\sqrt{cx}}{\sqrt{b}}\right)}{ac^3}-\frac{2\sqrt{a+bx^2}}{ac\sqrt{cx}}\right)}{5ac^2}-\frac{2\sqrt{a+bx^2}}{5ac(cx)^{5/2}}\right)}{2a}+\frac{}{ac(}\right.}{6a}$$

$$\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

$\downarrow$ 1510

$$\frac{11\left(\frac{7\left(-\frac{3b\left(\frac{2b\left(\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,\mathrm{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right),\frac{1}{2}\right)}{2b^{3/4}\sqrt{a+bx^2}}-\frac{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{a}c+\sqrt{b}cx\right)\sqrt{\frac{ac^2+bc^2x^2}{\left(\sqrt{a}c+\sqrt{b}cx\right)^2}}\,E\left(2\arctan\left(\frac{\sqrt[4]{b}\sqrt{c\cdots}}{\sqrt[4]{a}\sqrt{\cdots}}\right)\cdots\right)}{\sqrt[4]{b}\sqrt{a+bx^2}}}{\sqrt{b}}\cdots\right)}{ac^3}\cdots\right)}{5ac^2}\cdots\right)}{2a}\cdots\right)}{6a}\cdots$$

$$\frac{1}{3ac(cx)^{5/2}\left(a+bx^2\right)^{3/2}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x]
```

output
```
1/(3*a*c*(c*x)^(5/2)*(a + b*x^2)^(3/2)) + (11*(1/(a*c*(c*x)^(5/2)*Sqrt[a +
 b*x^2]) + (7*((-2*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/2)) - (3*b*((-2*Sqrt[a
 + b*x^2])/(a*c*Sqrt[c*x]) + (2*b*(-((-((c^2*Sqrt[c*x]*Sqrt[a + b*x^2])/(S
qrt[a]*c + Sqrt[b]*c*x)) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt
[(a*c^2 + b*c^2*x^2)/(Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticE[2*ArcTan[(b^(1
/4)*Sqrt[c*x])/(a^(1/4)*Sqrt[c])], 1/2])/(b^(1/4)*Sqrt[a + b*x^2]))/Sqrt[b
]) + (a^(1/4)*Sqrt[c]*(Sqrt[a]*c + Sqrt[b]*c*x)*Sqrt[(a*c^2 + b*c^2*x^2)/(
Sqrt[a]*c + Sqrt[b]*c*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[c*x])/(a^(1/4
)*Sqrt[c])], 1/2])/(2*b^(3/4)*Sqrt[a + b*x^2])))/(a*c^3)))/(5*a*c^2)))/(2*
a)))/(6*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.86

method	result
elliptic	$\frac{\sqrt{cx(bx^2+a)}\left(-\frac{2\sqrt{bc\,x^3+acx}}{5a^3c^4x^3}+\frac{26\left(x^2bc+ac\right)b}{5a^4c^4\sqrt{x\left(x^2bc+ac\right)}}+\frac{x\sqrt{bc\,x^3+acx}}{3a^3c^4\left(x^2+\frac{a}{b}\right)^2}+\frac{5b^2x^2}{2c^3a^4\sqrt{\left(x^2+\frac{a}{b}\right)bcx}}-\frac{77b\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)\cdots}{\sqrt{-ab}}}\cdots}{\cdots}\cdots\right)}{\sqrt{cx}\,\sqrt{b\,x^2+a}\cdots}$
default	$-\frac{462\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticE}\left(\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)a\,b^2x^4-231\sqrt{\frac{bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-bx+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{El}\cdots}{\cdots}$
risch	$-\frac{2\sqrt{b\,x^2+a}\,\left(-13b\,x^2+a\right)}{5a^4x^2c^3\sqrt{cx}}-\frac{b^2\left(\frac{13\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\cdots\right)}{b\sqrt{bc\,x^3+acx}}\cdots\right)}{\cdots}$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x*(b*x^2+a))^(1/2)/(c*x)^(1/2)/(b*x^2+a)^(1/2)*(-2/5/a^3/c^4*(b*c*x^3+a
*c*x)^(1/2)/x^3+26/5*(b*c*x^2+a*c)*b/a^4/c^4/(x*(b*c*x^2+a*c))^(1/2)+1/3/a
^3/c^4*x*(b*c*x^3+a*c*x)^(1/2)/(x^2+a/b)^2+5/2*b^2/c^3*x^2/a^4/((x^2+a/b)*
b*c*x)^(1/2)-77/20*b/a^4/c^3*(-a*b)^(1/2)*((x+1/b*(-a*b)^(1/2))*b/(-a*b)^(
1/2))^(1/2)*(-2*(x-1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2)*(-b/(-a*b)^(1/2
)*x)^(1/2)/(b*c*x^3+a*c*x)^(1/2)*(-2/b*(-a*b)^(1/2)*EllipticE(((x+1/b*(-a*
b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+1/b*(-a*b)^(1/2)*EllipticF(((
x+1/b*(-a*b)^(1/2))*b/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.38

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \frac{231\,(b^3x^7+2\,ab^2x^5+a^2bx^3)\sqrt{bc}\text{weierstrassZeta}\left(-\frac{4\,a}{b},0,\text{weierstrassPInverse}(\right.}{30\,(a^4b^2c^4x^7+2\,a^5bc}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/30*(231*(b^3*x^7 + 2*a*b^2*x^5 + a^2*b*x^3)*sqrt(b*c)*weierstrassZeta(-4
*a/b, 0, weierstrassPInverse(-4*a/b, 0, x)) + (231*b^3*x^6 + 385*a*b^2*x^4
 + 132*a^2*b*x^2 - 12*a^3)*sqrt(b*x^2 + a)*sqrt(c*x))/(a^4*b^2*c^4*x^7 + 2
*a^5*b*c^4*x^5 + a^6*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 24.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.14

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \frac{\Gamma\left(-\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{5}{2}\\-\frac{1}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{7}{2}}x^{\frac{5}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/2),x)
```

output
```
gamma(-5/4)*hyper((-5/4, 5/2), (-1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5
/2)*c**(7/2)*x**(5/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(c\,x)^{7/2}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{5/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{x}\,\sqrt{b\,x^2+a}}{b^3x^{10}+3a\,b^2x^8+3a^2b\,x^6+a^3x^4} dx\right)}{c^4}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*sqrt(a + b*x**2))/(a**3*x**4 + 3*a**2*b*x**6 + 3*a*b
**2*x**8 + b**3*x**10),x))/c**4
```

3.647 $\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 89

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = -\frac{c\sqrt{cx}\sqrt{3a-2ax^2}}{3a} + \frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{3a-2ax^2}}$$

output
```
-1/3*c*(c*x)^(1/2)*(-2*a*x^2+3*a)^(1/2)/a+1/6*c^(3/2)*(-2*x^2+3)^(1/2)*Ell
ipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*6^(3/4)/(-2*a*x^2+3*a)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = \frac{c\sqrt{cx}\left(-3+2x^2+\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^2}{3}\right)\right)}{3\sqrt{a\left(3-2x^2\right)}}$$

input
```
Integrate[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

output
```
(c*Sqrt[c*x]*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4
, (2*x^2)/3]))/(3*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {262, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

$$\Big\downarrow \text{266}$$

$$c\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

$$\Big\downarrow \text{765}$$

$$\frac{c\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{\sqrt{3}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

$$\downarrow \text{762}$$

$$\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}$$

input
```
Int[(c*x)^(3/2)/Sqrt[3*a - 2*a*x^2],x]
```

output
```
-1/3*(c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2])/a + (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipt
icF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[3*a - 2*a*
x^2])
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

method	result
default	$-\frac{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\right.}{12xa(2x^2-3)}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{c\sqrt{-2ac\,x^3+3acx}}{3a}+\frac{c^2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{108\sqrt{-2ac\,x^3+3acx}}\right)}{cx\sqrt{-a(2x^2-3)}}$
risch	$\frac{x(2x^2-3)c^2}{3\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)c^2\sqrt{-cxa(2x^2-3)}}{108\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$

input
```
int((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2
^(1/2))^(1/2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(
1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)
*2^(1/2))^(1/2),1/2*2^(1/2))+8*x^3-12*x)/x/a/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx=-\frac{3\,\sqrt{2}\sqrt{-ac}c\mathrm{weierstrassPInverse}(6,0,x)+2\,\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c}{6\,a}$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*(3*sqrt(2)*sqrt(-a*c)*c*weierstrassPInverse(6, 0, x) + 2*sqrt(-2*a*x^
2 + 3*a)*sqrt(c*x)*c)/a
```

Sympy [A] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(6*sqrt(a)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx=\int \frac{{\left(c\,x\right)}^{3/2}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/2)/(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(-2\sqrt{x}\,\sqrt{-2x^2+3}-3\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^3-3x}dx\right)\right)}{6a}$$

input
```
int((c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*( - 2*sqrt(x)*sqrt( - 2*x**2 + 3) - 3*int((sqrt(x)*sqrt
( - 2*x**2 + 3))/(2*x**3 - 3*x),x)))/(6*a)
```

3.648 $\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \frac{2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{3a-2ax^2}}$$

output
```
1/3*2^(3/4)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(
1/2),I)*3^(3/4)/c^(1/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \frac{2x\sqrt{3-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^2}{3}\right)}{\sqrt{3}\sqrt{cx}\sqrt{a\,(3-2x^2)}}$$

input
```
Integrate[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3])/(Sqrt[3]
*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3a-2ax^2}\sqrt{cx}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{765}$$

$$\frac{2\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{\sqrt{3}c\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{3}\sqrt{c}\sqrt{3a-2ax^2}}$$

input
```
Int[1/(Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]]
, -1])/(3^(1/4)*Sqrt[c]*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(51) = 102$.

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.78

method	result	size
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\,\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{54\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{-2ac\,x^3+3acx}}$	114
default	$-\frac{\sqrt{-a(2x^2-3)}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)}{6\sqrt{cx}\,a(2x^2-3)}$	117

input
```
int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/54*(-c*x*a*(2*x^2-3))^(1/2)/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*6^(1/2)*3^(
1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-
3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/
2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.33

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = -\frac{\sqrt{2}\sqrt{-ac}\,\text{weierstrassPInverse}(6,0,x)}{ac}$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x)/(a*c)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), 2*x**2*exp_polar(2*I*
pi)/3)/(6*sqrt(a)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^3-3x}dx\right)}{ac}$$

input
```
int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**3 - 3*x),x))/(
a*c)
```

3.649 $\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}} + \frac{2\,2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}$$

output

```
-2/9*(-2*a*x^2+3*a)^(1/2)/a/c/(c*x)^(3/2)+2/27*2^(3/4)*(-2*x^2+3)^(1/2)*El
lipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(3/4)/c^(5/2)/(-2*a*x
^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2x\sqrt{3-2x^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{1}{4},\frac{2x^2}{3}\right)}{3(cx)^{5/2}\sqrt{a(9-6x^2)}}$$

input
```
Integrate[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-3/4, 1/2, 1/4, (2*x^2)/3])/(3*(c*
x)^(5/2)*Sqrt[a*(9 - 6*x^2)])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {264, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3a-2ax^2}(cx)^{5/2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{9c^2}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{9c^3}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{4\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{9\sqrt{3}c^3\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{2\ 2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(-2*Sqrt[3*a - 2*a*x^2])/(9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*
EllipticF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)
*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

method	result
default	$-\frac{\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)x+12x\cdots\right.}{27xa\,c^2\sqrt{cx}\,(2x^2-3)}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(-\frac{2\sqrt{-2ac\,x^3+3acx}}{9a\,c^3x^2}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{243c^2\sqrt{-2ac\,x^3+3acx}}\right)}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
risch	$\frac{\frac{4x^2}{9}-\frac{2}{3}}{x\,c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)\sqrt{-cxa(2x^2-3)}}{243\sqrt{-2ac\,x^3+3acx}\,c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$

input
```
int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(-a*(2*x^2-3))^(1/2)*(((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*
((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/2)*
EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2
),1/2*2^(1/2))*x+12*x^2-18)/x/a/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\left(\sqrt{2}\sqrt{-ac}x^2\text{weierstrassPInverse}(6,0,x)+\sqrt{-2\,ax^2+3\,a}\sqrt{cx}\right)}{9\,ac^3x^2}$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-2/9*(sqrt(2)*sqrt(-a*c)*x^2*weierstrassPInverse(6, 0, x) + sqrt(-2*a*x^2
+ 3*a)*sqrt(c*x))/(a*c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{1}{2} \\ \frac{1}{4} \end{matrix}\middle| \frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)
/(6*sqrt(a)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^5-3x^3}dx\right)}{a\,c^3}$$

input
```
int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**5 - 3*x**3),x)
)/(a*c**3)
```

3.650 $\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 164

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = -\frac{c(cx)^{3/2}\sqrt{3a-2ax^2}}{5a} + \frac{9\sqrt[4]{3}c^{5/2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{5\,2^{3/4}\sqrt{3a-2ax^2}} - \frac{9\sqrt[4]{3}c^{5/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{5\,2^{3/4}\sqrt{3a-2ax^2}}$$

output
```
-1/5*c*(c*x)^(3/2)*(-2*a*x^2+3*a)^(1/2)/a+9/10*3^(1/4)*c^(5/2)*(-2*x^2+3)^
(1/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*2^(1/4)/(-2*a*x
^2+3*a)^(1/2)-9/10*3^(1/4)*c^(5/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*
3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*2^(1/4)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.37

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx=\frac{c(cx)^{3/2}\left(-3+2x^2+\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{2x^2}{3}\right)\right)}{5\sqrt{a(3-2x^2)}}$$

input
```
Integrate[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

output
```
(c*(c*x)^(3/2)*(-3 + 2*x^2 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/2, 3/4, 7
/4, (2*x^2)/3]))/(5*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {262, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{9}{10}c^2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

$$\Big\downarrow \text{261}$$

$$\frac{9c^2\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{10\sqrt{x}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

$$\Big\downarrow \text{260}$$

$$\frac{3\sqrt{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{9c^2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

$$\Big\downarrow \textbf{259}$$

$$-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

$$\Big\downarrow \textbf{327}$$

$$-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}$$

input
```
Int[(c*x)^(5/2)/Sqrt[3*a - 2*a*x^2],x]
```

output
```
-1/5*(c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2])/a - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt
[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*
Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(-\frac{c^2x\sqrt{-2ac\,x^3+3acx}}{5a}+\frac{c^3\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{}}{2}\right.\right.}{60\sqrt{-2ac\,x^3+3acx}}\right.}{cx\sqrt{-a(2x^2-3)}}$
risch	$\frac{x^2(2x^2-3)c^3}{5\sqrt{cx}\,\sqrt{-a(2x^2-3)}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\,\mathrm{Elliptic}}{}\right.}{60\sqrt{-2ac\,x^3+3acx}\,\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
default	$\frac{c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(6\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)}\right.}{40x}$

input

```
int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
1/c/x*(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(-c*x*a*(2*x^2-3))^(1/2)*(-1/5*c^2/
a*x*(-2*a*c*x^3+3*a*c*x)^(1/2)+1/60*c^3*6^(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6
^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*
a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^
(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*6^(1/2
))*6^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.30

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = -\frac{2\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c^2x-9\sqrt{2}\sqrt{-ac}c^2\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))}{10\,a}$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*(2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x - 9*sqrt(2)*sqrt(-a*c)*c^2*w
eierstrassZeta(6, 0, weierstrassPInverse(6, 0, x)))/a
```

Sympy [A] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\right|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), 2*x**2*exp
_polar(2*I*pi)/3)/(6*sqrt(a)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/sqrt(-2*a*x^2 + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{(c\,x)^{5/2}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/2)/(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(-2\sqrt{x}\,\sqrt{-2x^2+3}\,x-9\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^2-3}dx\right)\right)}{10a}$$

input
```
int((c*x)^(5/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*( - 2*sqrt(x)*sqrt( - 2*x**2 + 3)*x - 9*int((sqrt(x)
*sqrt( - 2*x**2 + 3))/(2*x**2 - 3),x)))/(10*a)
```

3.651 $\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt[4]{6}\sqrt{c}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{\sqrt{3a-2ax^2}} - \frac{\sqrt[4]{6}\sqrt{c}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt{3a-2ax^2}}$$

output

```
6^(1/4)*c^(1/2)*(-2*x^2+3)^(1/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)
/c^(1/2),I)/(-2*a*x^2+3*a)^(1/2)-6^(1/4)*c^(1/2)*(-2*x^2+3)^(1/2)*Elliptic
F(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{2x\sqrt{cx}\sqrt{3-2x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\frac{2x^2}{3}\right)}{3\sqrt{a(9-6x^2)}}$$

input
```
Integrate[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

output
```
(2*x*Sqrt[c*x]*Sqrt[3 - 2*x^2]*Hypergeometric2F1[1/2, 3/4, 7/4, (2*x^2)/3]
)/(3*Sqrt[a*(9 - 6*x^2)])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.56, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx$$

$$\Big\downarrow \text{ 261}$$

$$\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{ 260}$$

$$\frac{\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{\sqrt{3}\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \; 259$$

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \; 327$$

$$-\frac{\sqrt[4]{6}\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{\sqrt{x}\sqrt{3a-2ax^2}}$$

input
```
Int[Sqrt[c*x]/Sqrt[3*a - 2*a*x^2],x]
```

output
```
-((6^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/
Sqrt[6]], 2])/(Sqrt[x]*Sqrt[3*a - 2*a*x^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.27

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\,\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\,\mathrm{EllipticF}\left(\right.}{}\right.}{54x\sqrt{-a(2x^2-3)}\,\sqrt{-2ac\,x^3+3acx}}$
default	$\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\left(2\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\right.\right.}{12xa(2x^2-3)}$

input
```
int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/54/x*(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(-c*x*a*(2*x^2-3))^(1/2)*6^(1/2)*3
^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*
(-3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^
(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1
/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.17

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{2}\sqrt{-ac}\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))}{a}$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(2)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x))/a
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(6*sqrt(a)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{\sqrt{cx}}{\sqrt{-2\,ax^2+3\,a}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/sqrt(-2*a*x^2 + 3*a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = \int \frac{\sqrt{c\,x}}{\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/2)/(3*a - 2*a*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^2-3}dx\right)}{a}$$

input
```
int((c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**2 - 3),x))/a
```

3.652 $\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}} - \frac{2\sqrt[4]{2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3^{3/4}c^{3/2}\sqrt{3a-2ax^2}} + \frac{2\sqrt[4]{2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3^{3/4}c^{3/2}\sqrt{3a-2ax^2}}$$

output
```
-2/3*(-2*a*x^2+3*a)^(1/2)/a/c/(c*x)^(1/2)-2/3*2^(1/4)*(-2*x^2+3)^(1/2)*Ell
ipticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(1/4)/c^(3/2)/(-2*a*x^
2+3*a)^(1/2)+2/3*2^(1/4)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c
*x)^(1/2)/c^(1/2),I)*3^(1/4)/c^(3/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.31

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2x\sqrt{3-2x^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{2x^2}{3}\right)}{(cx)^{3/2}\sqrt{a(9-6x^2)}}$$

input
```
Integrate[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(-2*x*Sqrt[3 - 2*x^2]*Hypergeometric2F1[-1/4, 1/2, 3/4, (2*x^2)/3])/((c*x)
^(3/2)*Sqrt[a*(9 - 6*x^2)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {264, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3a-2ax^2}(cx)^{3/2}}dx$$

$$\Big\downarrow \ 264$$

$$-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{3c^2}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

$$\Big\downarrow \ 261$$

$$-\frac{2\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{3c^2\sqrt{x}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

$$\Big\downarrow \ 260$$

$$-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{3\sqrt{3}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

$$\Big\downarrow \text{259}$$

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]),x]
```

output
```
(-2*Sqrt[3*a - 2*a*x^2])/(3*a*c*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 -
 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sq
rt[x]*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.12

method	result
risch	$\frac{\frac{4x^2}{3}-2}{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)+\frac{\sqrt{6}\ \mathrm{Elliptic}}{}\right.}{81\sqrt{-2ac\,x^3+3acx}\,c\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(-\frac{2\left(-2ac\,x^2+3ac\right)}{3a\,c^2\sqrt{x\left(-2ac\,x^2+3ac\right)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)\right.}{81c\sqrt{-2ac\,x^3+3acx}}\right.}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
default	$-\frac{\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{}}\right.}{18c\sqrt{cx}\,}$

input
```
int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(2*x^2-3)/c/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)-1/81*6^(1/2)*3^(1/2)*((x+
1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)
*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1
/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*
((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2)))/c*(-c*x*a*(2*x^2-3))^(1/2)/(
c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.30

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = -\frac{2\left(\sqrt{2}\sqrt{-ac}x\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x)) + \sqrt{-2\,ax^2+3\,a}\sqrt{cx}\right)}{3\,ac^2x}$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(sqrt(2)*sqrt(-a*c)*x*weierstrassZeta(6, 0, weierstrassPInverse(6, 0,
 x)) + sqrt(-2*a*x^2 + 3*a)*sqrt(c*x))/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.33

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = \frac{\sqrt{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{6\sqrt{a}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(1/2),x)
```

output
```
sqrt(3)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)
/(6*sqrt(a)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{\sqrt{-2\,ax^2+3\,a}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-2*a*x^2 + 3*a)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = \int \frac{1}{{\left(c\,x\right)}^{3/2}\,\sqrt{3\,a-2\,a\,x^2}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}\,dx = -\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{2x^4-3x^2}dx\right)}{a\,c^2}$$

input
```
int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2),x)
```

output
```
( - sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(2*x**4 - 3*x**2),x)
)/(a*c**2)
```

3.653 $\int \frac{(cx)^{7/2}}{\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 126

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx=\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}+\frac{5c^3\sqrt{cx}\sqrt{3a-2ax^2}}{12a^2}-\frac{5c^{7/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{4\sqrt[4]{6}a\sqrt{3a-2ax^2}}$$

output
```
1/2*c*(c*x)^(5/2)/a/(-2*a*x^2+3*a)^(1/2)+5/12*c^3*(c*x)^(1/2)*(-2*a*x^2+3*
a)^(1/2)/a^2-5/24*c^(7/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(
c*x)^(1/2)/c^(1/2),I)*6^(3/4)/a/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.53

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = -\frac{c^3\sqrt{cx}\left(-15+4x^2+5\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^2}{3}\right)\right)}{12a\sqrt{a(3-2x^2)}}$$

input
```
Integrate[(c*x)^(7/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
-1/12*(c^3*Sqrt[c*x]*(-15 + 4*x^2 + 5*Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/
4, 1/2, 5/4, (2*x^2)/3]))/(a*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {252, 262, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}-\frac{5c^2\int \frac{(cx)^{3/2}}{\sqrt{3a-2ax^2}}dx}{4a}$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}-\frac{5c^2\left(\frac{1}{2}c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)}{4a}$$

$$\Big\downarrow \text{266}$$

$$\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}-\frac{5c^2\left(c\int\frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)}{4a}$$

$\Big\downarrow$ 765

$$\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}-\frac{5c^2\left(\frac{c\sqrt{3-2x^2}\int\frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{\sqrt{3}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)}{4a}$$

$\Big\downarrow$ 762

$$\frac{c(cx)^{5/2}}{2a\sqrt{3a-2ax^2}}-\frac{5c^2\left(\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{\sqrt[4]{6}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}\sqrt{cx}}{3a}\right)}{4a}$$

input
```
Int[(c*x)^(7/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*(c*x)^(5/2))/(2*a*Sqrt[3*a - 2*a*x^2]) - (5*c^2*(-1/3*(c*Sqrt[c*x]*Sqrt
[3*a - 2*a*x^2])/a + (c^(3/2)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1/4
)*Sqrt[c*x])/Sqrt[c]], -1])/(6^(1/4)*Sqrt[3*a - 2*a*x^2])))/(4*a)
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

method	result
default	$\frac{c^3\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(5\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\ldots\right.}{48x\,a^2(2x^2-3)}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(\frac{3c^4x}{4a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}+\frac{c^3\sqrt{-2ac\,x^3+3acx}}{6a^2}-\frac{5c^4\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\operatorname{EllipticF}\left(\ldots\right.}{432a\sqrt{-2ac\,x^3+3acx}}\right.}{cx\sqrt{-a(2x^2-3)}}$

input
```
int((c*x)^(7/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*c^3*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(5*((2*x+3^(1/2)*2^(1/2))*3^(1/2
)*2^(1/2))^(1/2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*
2^(1/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1
/2)*2^(1/2))^(1/2),1/2*2^(1/2))+16*x^3-60*x)/x/a^2/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.64

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{15\sqrt{2}(2\,c^3x^2-3\,c^3)\sqrt{-ac}\text{weierstrassPInverse}(6,0,x)+2\,(4\,c^3x^2-15\,c^3)\sqrt{-2\,ax^2}}{24\,(2\,a^2x^2-3\,a^2)}$$

input
```
integrate((c*x)^(7/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(15*sqrt(2)*(2*c^3*x^2 - 3*c^3)*sqrt(-a*c)*weierstrassPInverse(6, 0,
x) + 2*(4*c^3*x^2 - 15*c^3)*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x))/(2*a^2*x^2 - 3
*a^2)
```

Sympy [A] (verification not implemented)

Time = 12.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((3/2, 9/4), (13/4,), 2*x**2*exp
_polar(2*I*pi)/3)/(18*a**(3/2)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(c\,x)^{7/2}}{(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

input
```
int((c*x)^(7/2)/(3*a - 2*a*x^2)^(3/2),x)
```

output
```
int((c*x)^(7/2)/(3*a - 2*a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^3\left(4\sqrt{x}\,\sqrt{-2x^2+3}\,x^2-30\sqrt{x}\,\sqrt{-2x^2+3}-90\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^5-12x^3+9x}dx\right)x^2+135}{12a^2\,(2x^2-3)}$$

input

```
int((c*x)^(7/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output

```
(sqrt(c)*sqrt(a)*c**3*(4*sqrt(x)*sqrt( - 2*x**2 + 3)*x**2 - 30*sqrt(x)*sqr
t( - 2*x**2 + 3) - 90*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**5 - 12*x**3
+ 9*x),x)*x**2 + 135*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**5 - 12*x**3 +
 9*x),x)))/(12*a**2*(2*x**2 - 3))
```

3.654 $\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 95

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx=\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{2\sqrt[4]{6}a\sqrt{3a-2ax^2}}$$

output
```
1/2*c*(c*x)^(1/2)/a/(-2*a*x^2+3*a)^(1/2)-1/12*c^(3/2)*(-2*x^2+3)^(1/2)*Ell
ipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*6^(3/4)/a/(-2*a*x^2+3*a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.85 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx=-\frac{c\sqrt{cx}\left(-3+\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^2}{3}\right)\right)}{6a\sqrt{a(3-2x^2)}}$$

input
```
Integrate[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
-1/6*(c*Sqrt[c*x]*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (
2*x^2)/3]))/(a*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {252, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{4a}$$

$$\Big\downarrow \text{266}$$

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{2a}$$

$$\Big\downarrow \text{765}$$

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{2\sqrt{3}a\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{c\sqrt{cx}}{2a\sqrt{3a-2ax^2}}-\frac{c^{3/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{2\sqrt[4]{6}a\sqrt{3a-2ax^2}}$$

input
```
Int[(c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*Sqrt[c*x])/(2*a*Sqrt[3*a - 2*a*x^2]) - (c^(3/2)*Sqrt[3 - 2*x^2]*Ellipti
cF[ArcSin[((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(2*6^(1/4)*a*Sqrt[3*a - 2
*a*x^2])
```

Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.33

method	result
default	$\frac{c\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)-1\right.}{24x\,a^2(2x^2-3)}$
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(\frac{c^2x}{2a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{c^2\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\ \mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{216a\sqrt{-2ac\,x^3+3acx}}\right)}{cx\sqrt{-a(2x^2-3)}}$

input
```
int((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*c*(c*x)^(1/2)*(-a*(2*x^2-3))^(1/2)*(((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^
(1/2))^(1/2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1
/2)*x)^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*
2^(1/2))^(1/2),1/2*2^(1/2))-12*x)/x/a^2/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{2}(2\,cx^2-3\,c)\sqrt{-ac}\mathrm{weierstrassPInverse}(6,0,x)-2\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c}{4\,(2\,a^2x^2-3\,a^2)}$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*(2*c*x^2 - 3*c)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) - 2*s
qrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c)/(2*a^2*x^2 - 3*a^2)
```

Sympy [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.54

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 3/2), (9/4,), 2*x**2*exp_
polar(2*I*pi)/3)/(18*a**(3/2)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx=\int \frac{{\left(c\,x\right)}^{3/2}}{{\left(3\,a-2\,a\,x^2\right)}^{3/2}} \,d x$$

input
```
int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2),x)
```

output
```
int((c*x)^(3/2)/(3*a - 2*a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(3a-2ax^2)^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\,c\left(\sqrt{-2x^{2}+3}\,\left(\int \frac{\sqrt{x}\,\sqrt{-2x^{2}+3}}{2x^{3}-3x}d x \right)+2\sqrt{x}\right)}{4\sqrt{-2x^{2}+3}\,a^{2}}$$

input
```
int((c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c*(sqrt( - 2*x**2 + 3)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/
(2*x**3 - 3*x),x) + 2*sqrt(x)))/(4*sqrt( - 2*x**2 + 3)*a**2)
```

3.655 $\int \frac{1}{\sqrt{cx}\left(3a-2ax^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 97

$$\int \frac{1}{\sqrt{cx}\left(3a-2ax^2\right)^{3/2}}\,dx=\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}+\frac{\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{6}a\sqrt{c}\sqrt{3a-2ax^2}}$$

output
```
1/3*(c*x)^(1/2)/a/c/(-2*a*x^2+3*a)^(1/2)+1/18*(-2*x^2+3)^(1/2)*EllipticF(1
/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*6^(3/4)/a/c^(1/2)/(-2*a*x^2+3*a)
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.88 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.61

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \frac{x\left(3+\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},\frac{2x^2}{3}\right)\right)}{9a\sqrt{cx}\sqrt{a\,(3-2x^2)}}$$

input
```
Integrate[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
(x*(3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, (2*x^2)/3]))/(9*a
*Sqrt[c*x]*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {253, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3a-2ax^2)^{3/2}\sqrt{cx}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{6a}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{3ac}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{3\sqrt{3}ac\sqrt{3a-2ax^2}}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3\sqrt[4]{6}a\sqrt{c}\sqrt{3a-2ax^2}}+\frac{\sqrt{cx}}{3ac\sqrt{3a-2ax^2}}$$

input
```
Int[1/(Sqrt[c*x]*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
Sqrt[c*x]/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[3 - 2*x^2]*EllipticF[ArcSin[
((2/3)^(1/4)*Sqrt[c*x])/Sqrt[c]], -1])/(3*6^(1/4)*a*Sqrt[c]*Sqrt[3*a - 2*a
*x^2])
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{-a(2x^2-3)}\left(\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)+12x\right)}{36a^2\sqrt{cx}\,(2x^2-3)}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(\frac{x}{3a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}+\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\operatorname{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{324a\sqrt{-2ac\,x^3+3acx}}\right)}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$

input
```
int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(-a*(2*x^2-3))^(1/2)*(((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*
((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/2)*
EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2
),1/2*2^(1/2))+12*x)/a^2/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.63

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{-ac}(2\,x^2-3)\text{weierstrassPInverse}(6,0,x)+2\,\sqrt{-2\,ax^2+3\,a}\sqrt{cx}}{6\,(2\,a^2cx^2-3\,a^2c)}$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(2)*sqrt(-a*c)*(2*x^2 - 3)*weierstrassPInverse(6, 0, x) + 2*sqrt
(-2*a*x^2 + 3*a)*sqrt(c*x))/(2*a^2*c*x^2 - 3*a^2*c)
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\sqrt{c}\,\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*sqrt(x)*gamma(1/4)*hyper((1/4, 3/2), (5/4,), 2*x**2*exp_polar(2*I*
pi)/3)/(18*a**(3/2)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{(-2\,ax^2+3\,a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{(-2\,ax^2+3\,a)^{\frac{3}{2}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(1/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^5-12x^3+9x}dx\right)}{a^2c}$$

input
```
int(1/(c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**5 - 12*x**3 + 9*x
),x))/(a**2*c)
```

3.656 $\int \frac{1}{(cx)^{5/2}(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 133

$$\int \frac{1}{(cx)^{5/2}(3a-2ax^2)^{3/2}}\,dx=\frac{1}{3ac(cx)^{3/2}\sqrt{3a-2ax^2}}-\frac{5\sqrt{3a-2ax^2}}{27a^2c(cx)^{3/2}}+\frac{5\,2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{27\sqrt[4]{3}ac^{5/2}\sqrt{3a-2ax^2}}$$

output
```
1/3/a/c/(c*x)^(3/2)/(-2*a*x^2+3*a)^(1/2)-5/27*(-2*a*x^2+3*a)^(1/2)/a^2/c/(
c*x)^(3/2)+5/81*2^(3/4)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*
x)^(1/2)/c^(1/2),I)*3^(3/4)/a/c^(5/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{5/2}\,(3a-2ax^2)^{3/2}}\,dx = -\frac{2x(3-2x^2)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{2},\frac{1}{4},\frac{2x^2}{3}\right)}{9\sqrt{3}(cx)^{5/2}\,(a\,(3-2x^2))^{3/2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
(-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-3/4, 3/2, 1/4, (2*x^2)/3])/(9*S
qrt[3]*(c*x)^(5/2)*(a*(3 - 2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {253, 264, 266, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3a-2ax^2)^{3/2}\,(cx)^{5/2}}\,dx$$

$$\Big\downarrow \ 253$$

$$\frac{5\int \frac{1}{(cx)^{5/2}\sqrt{3a-2ax^2}}dx}{6a}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

$$\Big\downarrow \ 264$$

$$\frac{5\left(\frac{2\int \frac{1}{\sqrt{cx}\sqrt{3a-2ax^2}}dx}{9c^2}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{6a}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

$$\Big\downarrow \ 266$$

$$\frac{5\left(\frac{4\int \frac{1}{\sqrt{3a-2ax^2}}d\sqrt{cx}}{9c^3}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{6a}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

$$\Big\downarrow \text{765}$$

$$\frac{5\left(\frac{4\sqrt{3-2x^2}\int \frac{1}{\sqrt{1-\frac{2x^2}{3}}}d\sqrt{cx}}{9\sqrt{3}c^3\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{6a}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

$$\Big\downarrow \text{762}$$

$$\frac{5\left(\frac{2\ 2^{3/4}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{9\sqrt[4]{3}c^{5/2}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{9ac(cx)^{3/2}}\right)}{6a}+\frac{1}{3ac\sqrt{3a-2ax^2}(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
1/(3*a*c*(c*x)^(3/2)*Sqrt[3*a - 2*a*x^2]) + (5*((-2*Sqrt[3*a - 2*a*x^2])/(
9*a*c*(c*x)^(3/2)) + (2*2^(3/4)*Sqrt[3 - 2*x^2]*EllipticF[ArcSin[((2/3)^(1
/4)*Sqrt[c*x])/Sqrt[c]], -1])/(9*3^(1/4)*c^(5/2)*Sqrt[3*a - 2*a*x^2])))/(6
*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] && !GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sqrt{-a(2x^2-3)}\left(5\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)x+60}{162x\,a^2c^2\sqrt{cx}\,(2x^2-3)}$
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(\frac{2x}{9a\,c^2\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{2\sqrt{-2ac\,x^3+3acx}}{27a^2c^3x^2}+\frac{5\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\;\mathrm{EllipticF}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)}}{3}\right.}{1458a\,c^2\sqrt{-2ac\,x^3+3acx}}\right.}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$

input
```
int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/162*(-a*(2*x^2-3))^(1/2)*(5*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/
2)*((-2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(1/2)*(-3^(1/2)*2^(1/2)*x)^(1/
2)*EllipticF(1/6*3^(1/2)*2^(1/2)*((2*x+3^(1/2)*2^(1/2))*3^(1/2)*2^(1/2))^(
1/2),1/2*2^(1/2))*x+60*x^2-36)/x/a^2/c^2/(c*x)^(1/2)/(2*x^2-3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx = -\frac{5\sqrt{2}(2x^4-3x^2)\sqrt{-ac}\text{weierstrassPInverse}(6,0,x)+2\sqrt{-2ax^2+3a}\sqrt{cx}(5x^2-3)}{27\left(2a^2c^3x^4-3a^2c^3x^2\right)}$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/27*(5*sqrt(2)*(2*x^4 - 3*x^2)*sqrt(-a*c)*weierstrassPInverse(6, 0, x) +
 2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*(5*x^2 - 3))/(2*a^2*c^3*x^4 - 3*a^2*c^3*
x^2)
```

Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.41

$$\int \frac{1}{(cx)^{5/2}\left(3a-2ax^2\right)^{3/2}}\,dx = \frac{\sqrt{3}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), 2*x**2*exp_polar(2*I*pi)/3)
/(18*a**(3/2)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\,(3a-2ax^2)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(5/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/2} (3a - 2ax^2)^{3/2}} \, dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^7-12x^5+9x^3}dx\right)}{a^2c^3}$$

input
```
int(1/(c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**7 - 12*x**5 + 9*x
**3),x))/(a**2*c**3)
```

3.657 $\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 201

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}+\frac{7c^3(cx)^{3/2}\sqrt{3a-2ax^2}}{20a^2} - \frac{63\sqrt[4]{3}c^{9/2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{20\ 2^{3/4}a\sqrt{3a-2ax^2}} + \frac{63\sqrt[4]{3}c^{9/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{20\ 2^{3/4}a\sqrt{3a-2ax^2}}$$

output

```
1/2*c*(c*x)^(7/2)/a/(-2*a*x^2+3*a)^(1/2)+7/20*c^3*(c*x)^(3/2)*(-2*a*x^2+3*
a)^(1/2)/a^2-63/40*3^(1/4)*c^(9/2)*(-2*x^2+3)^(1/2)*EllipticE(1/3*2^(1/4)*
3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*2^(1/4)/a/(-2*a*x^2+3*a)^(1/2)+63/40*3^(1/4
)*c^(9/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/
2),I)*2^(1/4)/a/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.33

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{c^3(cx)^{3/2}\left(-21-2x^2+7\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{2x^2}{3}\right)\right)}{10a\sqrt{a(3-2x^2)}}$$

input
```
Integrate[(c*x)^(9/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c^3*(c*x)^(3/2)*(-21 - 2*x^2 + 7*Sqrt[9 - 6*x^2]*Hypergeometric2F1[3/4, 3
/2, 7/4, (2*x^2)/3]))/(10*a*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {252, 262, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\int \frac{(cx)^{5/2}}{\sqrt{3a-2ax^2}}dx}{4a}$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(\frac{9}{10}c^2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

$$\Big\downarrow \text{261}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(\frac{9c^2\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{10\sqrt{x}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

$$\Big\downarrow \text{260}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(\frac{3\sqrt{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(\frac{9c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{10\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

$$\Big\downarrow \text{259}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

$$\Big\downarrow \text{327}$$

$$\frac{c(cx)^{7/2}}{2a\sqrt{3a-2ax^2}}-\frac{7c^2\left(-\frac{9\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{5\ 2^{3/4}\sqrt{x}\sqrt{3a-2ax^2}}-\frac{c\sqrt{3a-2ax^2}(cx)^{3/2}}{5a}\right)}{4a}$$

input
```
Int[(c*x)^(9/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*(c*x)^(7/2))/(2*a*Sqrt[3*a - 2*a*x^2]) - (7*c^2*(-1/5*(c*(c*x)^(3/2)*Sq
rt[3*a - 2*a*x^2])/a - (9*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[
ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(5*2^(3/4)*Sqrt[x]*Sqrt[3*a - 2*a
*x^2])))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.04

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(\frac{3c^5x^2}{4a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}+\frac{c^4x\sqrt{-2ac\,x^3+3acx}}{10a^2}-\frac{7c^5\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \mathrm{EllipticE}\left(\right.\right.}{240a\sqrt{-2ac}}\right.}{cx\sqrt{-a(2x^2-3)}}$
default	$-\frac{c^4\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(42\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\right.}\right.}{1}$

input
```
int((c*x)^(9/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(-c*x*a*(2*x^2-3))^(1/2)*(3/4/a*c^5
*x^2/(-2*(x^2-3/2)*a*c*x)^(1/2)+1/10/a^2*c^4*x*(-2*a*c*x^3+3*a*c*x)^(1/2)-
7/240*c^5/a*6^(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x-1/2*6^(
1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(
1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2))+1/
2*6^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(1/2
))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.42

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx =$$
$$-\frac{63\sqrt{2}(2c^4x^2-3c^4)\sqrt{-ac}\mathrm{weierstrassZeta}(6,0,\mathrm{weierstrassPInverse}(6,0,x))-2(4c^4x^3-21c^4x)\sqrt{-2a}}{40(2a^2x^2-3a^2)}$$

input
```
integrate((c*x)^(9/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/40*(63*sqrt(2)*(2*c^4*x^2 - 3*c^4)*sqrt(-a*c)*weierstrassZeta(6, 0, wei
erstrassPInverse(6, 0, x)) - 2*(4*c^4*x^3 - 21*c^4*x)*sqrt(-2*a*x^2 + 3*a)
*sqrt(c*x))/(2*a^2*x^2 - 3*a^2)
```

Sympy [A] (verification not implemented)

Time = 36.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.25

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*c**(9/2)*x**(11/2)*gamma(11/4)*hyper((3/2, 11/4), (15/4,), 2*x**2*
exp_polar(2*I*pi)/3)/(18*a**(3/2)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(c\,x)^{9/2}}{(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

input
```
int((c*x)^(9/2)/(3*a - 2*a*x^2)^(3/2),x)
```

output
```
int((c*x)^(9/2)/(3*a - 2*a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^4\left(4\sqrt{x}\,\sqrt{-2x^2+3}\,x^3+42\sqrt{x}\,\sqrt{-2x^2+3}\,x+378\left(\int\frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^4-12x^2+9}dx\right)x^2-\right.}{20a^2\,(2x^2-3)}$$

input
```
int((c*x)^(9/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**4*(4*sqrt(x)*sqrt( - 2*x**2 + 3)*x**3 + 42*sqrt(x)*sqr
t( - 2*x**2 + 3)*x + 378*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**4 - 12*x*
*2 + 9),x)*x**2 - 567*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**4 - 12*x**2
+ 9),x)))/(20*a**2*(2*x**2 - 3))
```

3.658 $\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}} - \frac{3\sqrt[4]{3}c^{5/2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{2\,2^{3/4}a\sqrt{3a-2ax^2}} + \frac{3\sqrt[4]{3}c^{5/2}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{2\,2^{3/4}a\sqrt{3a-2ax^2}}$$

output
```
1/2*c*(c*x)^(3/2)/a/(-2*a*x^2+3*a)^(1/2)-3/4*3^(1/4)*c^(5/2)*(-2*x^2+3)^(1
/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*2^(1/4)/a/(-2*a*x
^2+3*a)^(1/2)+3/4*3^(1/4)*c^(5/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3
^(3/4)*(c*x)^(1/2)/c^(1/2),I)*2^(1/4)/a/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.35

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{c(cx)^{3/2}\left(-3+\sqrt{9-6x^2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{2x^2}{3}\right)\right)}{3a\sqrt{a(3-2x^2)}}$$

input
```
Integrate[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*(c*x)^(3/2)*(-3 + Sqrt[9 - 6*x^2]*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x
^2)/3]))/(3*a*Sqrt[a*(3 - 2*x^2)])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {252, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 252$$

$$\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}-\frac{3c^2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{4a}$$

$$\Big\downarrow \ 261$$

$$\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}-\frac{3c^2\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{4a\sqrt{x}}$$

$$\Big\downarrow \ 260$$

$$\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}-\frac{\sqrt{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{4a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}-\frac{3c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{4a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{259}$$

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{2\ 2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{3\sqrt[4]{3}c^2\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{2\ 2^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{c(cx)^{3/2}}{2a\sqrt{3a-2ax^2}}$$

input
```
Int[(c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*(c*x)^(3/2))/(2*a*Sqrt[3*a - 2*a*x^2]) + (3*3^(1/4)*c^2*Sqrt[c*x]*Sqrt[
3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(2*2^(3/4)*a
*Sqrt[x]*Sqrt[3*a - 2*a*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\Bigg(\frac{c^3x^2}{2a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{c^3\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\,\Bigg(-\sqrt{6}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)}{72a\sqrt{-2ac\,x^3+3acx}}}{cx\sqrt{-a(2x^2-3)}}$
default	$-\frac{c^2\sqrt{cx}\,\sqrt{-a(2x^2-3)}\,\Bigg(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\Big(2x+\sqrt{3}\,\sqrt{2}}}{16x\,a^2}$

input
```
int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(-c*x*a*(2*x^2-3))^(1/2)*(1/2/a*c^3
*x^2/(-2*(x^2-3/2)*a*c*x)^(1/2)-1/72*c^3/a*6^(1/2)*3^(1/2)*((x+1/2*6^(1/2)
)*6^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(
-2*a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*6^(1/2))
*6^(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*6^(
1/2))*6^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.44

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = -\frac{2\sqrt{-2\,ax^2+3\,a}\sqrt{cx}c^2x+3\sqrt{2}(2\,c^2x^2-3\,c^2)\sqrt{-ac}\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))}{4\,(2\,a^2x^2-3\,a^2)}$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*c^2*x + 3*sqrt(2)*(2*c^2*x^2 - 3*c^
2)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPInverse(6, 0, x)))/(2*a^2*
x^2 - 3*a^2)
```

Sympy [A] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.30

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/2, 7/4), (11/4,), 2*x**2*exp
_polar(2*I*pi)/3)/(18*a**(3/2)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{(c\,x)^{5/2}}{(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

input
```
int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2),x)
```

output
```
int((c*x)^(5/2)/(3*a - 2*a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\,c^2\left(2\sqrt{x}\,\sqrt{-2x^2+3}\,x+18\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^4-12x^2+9}dx\right)x^2-27\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^4-12x^2+9}dx\right)\right)}{2a^2\,(2x^2-3)}$$

input
```
int((c*x)^(5/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*c**2*(2*sqrt(x)*sqrt( - 2*x**2 + 3)*x + 18*int((sqrt(x)*s
qrt( - 2*x**2 + 3))/(4*x**4 - 12*x**2 + 9),x)*x**2 - 27*int((sqrt(x)*sqrt(
 - 2*x**2 + 3))/(4*x**4 - 12*x**2 + 9),x)))/(2*a**2*(2*x**2 - 3))
```

3.659 $\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}} - \frac{\sqrt{c}\sqrt{3-2x^2}\,E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{6^{3/4}a\sqrt{3a-2ax^2}} + \frac{\sqrt{c}\sqrt{3-2x^2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{6^{3/4}a\sqrt{3a-2ax^2}}$$

output
```
1/3*(c*x)^(3/2)/a/c/(-2*a*x^2+3*a)^(1/2)-1/6*c^(1/2)*(-2*x^2+3)^(1/2)*Elli
pticE(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*6^(1/4)/a/(-2*a*x^2+3*a)^
(1/2)+1/6*c^(1/2)*(-2*x^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/
2)/c^(1/2),I)*6^(1/4)/a/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \frac{2x\sqrt{cx}(3-2x^2)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\frac{2x^2}{3}\right)}{9\sqrt{3}\,(a\,(3-2x^2))^{3/2}}$$

input
```
Integrate[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(2*x*Sqrt[c*x]*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[3/4, 3/2, 7/4, (2*x^2)/
3])/(9*Sqrt[3]*(a*(3 - 2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.64, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {253, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{6a}$$

$$\Big\downarrow \text{261}$$

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{6a\sqrt{x}}$$

$$\Big\downarrow \text{260}$$

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{6\sqrt{3}a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}-\frac{\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{6a\sqrt{x}\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{259}$$

$$\frac{\sqrt{3-2x^2}\sqrt{cx}\int\frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{6^{3/4}a\sqrt{x}\sqrt{3a-2ax^2}}+\frac{(cx)^{3/2}}{3ac\sqrt{3a-2ax^2}}$$

input
```
Int[Sqrt[c*x]/(3*a - 2*a*x^2)^(3/2),x]
```

output
```
(c*x)^(3/2)/(3*a*c*Sqrt[3*a - 2*a*x^2]) + (Sqrt[c*x]*Sqrt[3 - 2*x^2]*Ellip
ticE[ArcSin[Sqrt[3 - Sqrt[6]*x]/Sqrt[6]], 2])/(6^(3/4)*a*Sqrt[x]*Sqrt[3*a
 - 2*a*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16

method	result
elliptic	$\frac{\sqrt{cx}\,\sqrt{-cxa(2x^2-3)}\left(\frac{c\,x^2}{3a\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{c\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}}{3},\frac{\sqrt{2}}{2}\right)-\cdots\right.}{324a\sqrt{-2ac\,x^3+3acx}\cdots}\cdots\right.}{cx\sqrt{-a(2x^2-3)}\cdots}$
default	$-\frac{\sqrt{cx}\,\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\,\mathrm{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\cdots}\right.}{72a^2x(\cdots}$

input
```
int((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c/x*(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(-c*x*a*(2*x^2-3))^(1/2)*(1/3/a*c*x
^2/(-2*(x^2-3/2)*a*c*x)^(1/2)-1/324/a*c*6^(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6
^(1/2))^(1/2)*(-6*(x-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*
a*c*x^3+3*a*c*x)^(1/2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^
(1/2))^(1/2),1/2*2^(1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*6^(1/2
))*6^(1/2))^(1/2),1/2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{-ac}(2\,x^2-3)\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))+2\,\sqrt{-2\,ax^2+3\,a}\sqrt{cx}x}{6\,(2\,a^2x^2-3\,a^2)}$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/6*(sqrt(2)*sqrt(-a*c)*(2*x^2 - 3)*weierstrassZeta(6, 0, weierstrassPInv
erse(6, 0, x)) + 2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*x)/(2*a^2*x^2 - 3*a^2)
```

Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.32

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \frac{\sqrt{3}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/2), (7/4,), 2*x**2*exp_p
olar(2*I*pi)/3)/(18*a**(3/2)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{\sqrt{cx}}{(-2\,ax^2+3\,a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(-2*a*x^2 + 3*a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(3a-2ax^2)^{3/2}}\,dx = \int \frac{\sqrt{c\,x}}{(3\,a-2\,a\,x^2)^{3/2}}\,dx$$

input
```
int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2),x)
```

output
```
int((c*x)^(1/2)/(3*a - 2*a*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(3a-2a x^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^4-12x^2+9}dx\right)}{a^2}$$

input
```
int((c*x)^(1/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**4 - 12*x**2 + 9),
x))/a**2
```

3.660 $\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 198

$$\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx=\frac{1}{3ac\sqrt{cx}\sqrt{3a-2ax^2}}-\frac{\sqrt{3a-2ax^2}}{3a^2c\sqrt{cx}}-\frac{\sqrt[4]{2}\sqrt{3-2x^2}E\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right)\middle|-1\right)}{3^{3/4}ac^{3/2}\sqrt{3a-2ax^2}}+\frac{\sqrt[4]{2}\sqrt{3-2x^2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{\frac{2}{3}}\sqrt{cx}}{\sqrt{c}}\right),-1\right)}{3^{3/4}ac^{3/2}\sqrt{3a-2ax^2}}$$

output
```
1/3/a/c/(c*x)^(1/2)/(-2*a*x^2+3*a)^(1/2)-1/3*(-2*a*x^2+3*a)^(1/2)/a^2/c/(c
*x)^(1/2)-1/3*2^(1/4)*(-2*x^2+3)^(1/2)*EllipticE(1/3*2^(1/4)*3^(3/4)*(c*x)
^(1/2)/c^(1/2),I)*3^(1/4)/a/c^(3/2)/(-2*a*x^2+3*a)^(1/2)+1/3*2^(1/4)*(-2*x
^2+3)^(1/2)*EllipticF(1/3*2^(1/4)*3^(3/4)*(c*x)^(1/2)/c^(1/2),I)*3^(1/4)/a
/c^(3/2)/(-2*a*x^2+3*a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.29

$$\int \frac{1}{(cx)^{3/2}(3a-2ax^2)^{3/2}}\,dx=-\frac{2x(3-2x^2)^{3/2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{3}{4},\frac{2x^2}{3}\right)}{3\sqrt{3}(cx)^{3/2}\left(a\left(3-2x^2\right)\right)^{3/2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
(-2*x*(3 - 2*x^2)^(3/2)*Hypergeometric2F1[-1/4, 3/2, 3/4, (2*x^2)/3])/(3*S
qrt[3]*(c*x)^(3/2)*(a*(3 - 2*x^2))^(3/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.74, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {253, 264, 261, 260, 27, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3a-2ax^2)^{3/2}(cx)^{3/2}}\,dx$$

$$\Big\downarrow \ 253$$

$$\frac{\int \frac{1}{(cx)^{3/2}\sqrt{3a-2ax^2}}dx}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \ 264$$

$$\frac{-\frac{2\int \frac{\sqrt{cx}}{\sqrt{3a-2ax^2}}dx}{3c^2}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \ 261$$

$$\frac{-\frac{2\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3a-2ax^2}}dx}{3c^2\sqrt{x}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \text{260}$$

$$\frac{-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{3}\sqrt{x}}{\sqrt{3-2x^2}}dx}{3\sqrt{3}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{2\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{3-2x^2}}dx}{3c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \text{259}$$

$$\frac{\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}\int \frac{\sqrt{\frac{1}{3}\left(\sqrt{6}x-3\right)+1}}{\sqrt{\frac{1}{6}\left(\sqrt{6}x-3\right)+1}}d\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

$$\Big\downarrow \text{327}$$

$$\frac{\frac{2\sqrt[4]{2}\sqrt{3-2x^2}\sqrt{cx}E\left(\arcsin\left(\frac{\sqrt{3-\sqrt{6}x}}{\sqrt{6}}\right)\middle|2\right)}{3^{3/4}c^2\sqrt{x}\sqrt{3a-2ax^2}}-\frac{2\sqrt{3a-2ax^2}}{3ac\sqrt{cx}}}{2a}+\frac{1}{3ac\sqrt{3a-2ax^2}\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x]
```

output
```
1/(3*a*c*Sqrt[c*x]*Sqrt[3*a - 2*a*x^2]) + ((-2*Sqrt[3*a - 2*a*x^2])/(3*a*c
*Sqrt[c*x]) + (2*2^(1/4)*Sqrt[c*x]*Sqrt[3 - 2*x^2]*EllipticE[ArcSin[Sqrt[3
 - Sqrt[6]*x]/Sqrt[6]], 2])/(3^(3/4)*c^2*Sqrt[x]*Sqrt[3*a - 2*a*x^2]))/(2*
a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 260
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[1 + b*(x^2/a
)]/Sqrt[a + b*x^2]   Int[Sqrt[x]/Sqrt[1 + b*(x^2/a)], x], x] /; FreeQ[{a, b
}, x] && GtQ[-b/a, 0] &&  !GtQ[a, 0]
```

rule 261
```
Int[Sqrt[(c_)*(x_)]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[Sqrt[c*x]/
Sqrt[x]   Int[Sqrt[x]/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ
[-b/a, 0]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.09

method	result
elliptic	$\frac{\sqrt{-cxa(2x^2-3)}\left(\frac{2x^2}{9ac\sqrt{-2\left(x^2-\frac{3}{2}\right)acx}}-\frac{2\left(-2ac\,x^2+3ac\right)}{9a^2c^2\sqrt{x\left(-2ac\,x^2+3ac\right)}}-\frac{\sqrt{6}\,\sqrt{3}\,\sqrt{\left(x+\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-6\left(x-\frac{\sqrt{6}}{2}\right)\sqrt{6}}\,\sqrt{-3\sqrt{6}\,x}\left(-\sqrt{6}\ \text{EllipticE}\left(\frac{\sqrt{3}}{}\right.\right.}{162ac\sqrt{-2ac\,x}}\right.}{\sqrt{cx}\,\sqrt{-a(2x^2-3)}}$
default	$-\frac{\sqrt{-a(2x^2-3)}\left(2\sqrt{\left(-2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}\,\sqrt{3}\,\sqrt{-\sqrt{3}\,\sqrt{2}\,x}\ \text{EllipticE}\left(\frac{\sqrt{3}\,\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}\,\sqrt{2}}}{6},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\sqrt{\left(2x+\sqrt{3}\,\sqrt{2}\right)\sqrt{3}}\right.}{36a^2c\sqrt{c}}$

input
```
int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-c*x*a*(2*x^2-3))^(1/2)/(c*x)^(1/2)/(-a*(2*x^2-3))^(1/2)*(2/9/a/c*x^2/(-2
*(x^2-3/2)*a*c*x)^(1/2)-2/9*(-2*a*c*x^2+3*a*c)/a^2/c^2/(x*(-2*a*c*x^2+3*a*
c))^(1/2)-1/162/a/c*6^(1/2)*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2)*(-6*(x
-1/2*6^(1/2))*6^(1/2))^(1/2)*(-3*6^(1/2)*x)^(1/2)/(-2*a*c*x^3+3*a*c*x)^(1/
2)*(-6^(1/2)*EllipticE(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/2*2^(
1/2))+1/2*6^(1/2)*EllipticF(1/3*3^(1/2)*((x+1/2*6^(1/2))*6^(1/2))^(1/2),1/
2*2^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.38

$$\int \frac{1}{(cx)^{3/2}\,(3a-2ax^2)^{3/2}}\,dx = -\frac{\sqrt{2}(2\,x^3-3\,x)\sqrt{-ac}\text{weierstrassZeta}(6,0,\text{weierstrassPInverse}(6,0,x))+2\,\sqrt{-2\,ax^2+3\,a}\sqrt{cx}(x^2-1)}{3\,(2\,a^2c^2x^3-3\,a^2c^2x)}$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(sqrt(2)*(2*x^3 - 3*x)*sqrt(-a*c)*weierstrassZeta(6, 0, weierstrassPI
nverse(6, 0, x)) + 2*sqrt(-2*a*x^2 + 3*a)*sqrt(c*x)*(x^2 - 1))/(2*a^2*c^2*
x^3 - 3*a^2*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.27

$$\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx=\frac{\sqrt{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{2x^2e^{2i\pi}}{3}\right)}{18a^{\frac{3}{2}}c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(-2*a*x**2+3*a)**(3/2),x)
```

output
```
sqrt(3)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), 2*x**2*exp_polar(2*I*pi)/3)
/(18*a**(3/2)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx=\int \frac{1}{\left(-2\,ax^2+3\,a\right)^{\frac{3}{2}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((-2*a*x^2 + 3*a)^(3/2)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx=\int \frac{1}{\left(c\,x\right)^{3/2}\left(3\,a-2\,a\,x^2\right)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(3/2)*(3*a - 2*a*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\left(3a-2ax^2\right)^{3/2}}\,dx=\frac{\sqrt{c}\,\sqrt{a}\left(\int \frac{\sqrt{x}\,\sqrt{-2x^2+3}}{4x^6-12x^4+9x^2}dx\right)}{a^2c^2}$$

input
```
int(1/(c*x)^(3/2)/(-2*a*x^2+3*a)^(3/2),x)
```

output
```
(sqrt(c)*sqrt(a)*int((sqrt(x)*sqrt( - 2*x**2 + 3))/(4*x**6 - 12*x**4 + 9*x
**2),x))/(a**2*c**2)
```

3.661 $\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = -2E\left(\arcsin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\middle|2\right)$$

output
```
-2*EllipticE(1/2*(1-x)^(1/2)*2^(1/2),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = \frac{2}{3}x^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)$$

input
```
Integrate[Sqrt[x]/Sqrt[1 - x^2],x]
```

output
```
(2*x^(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, x^2])/3
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \ 259$$

$$-2\int \frac{\sqrt{x}}{\sqrt{\frac{x-1}{2}+1}}d\frac{\sqrt{1-x}}{\sqrt{2}}$$

$$\Big\downarrow \ 327$$

$$-2E\left(\arcsin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\middle|2\right)$$

input
```
Int[Sqrt[x]/Sqrt[1 - x^2],x]
```

output
```
-2*EllipticE[ArcSin[Sqrt[1 - x]/Sqrt[2]], 2]
```

Defintions of rubi rules used

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
meijerg	$\frac{2x^{\frac{3}{2}}\,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],x^{2}\right)}{3}$	15
default	$\frac{\left(-2\,\text{EllipticE}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)\right)\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}}{\sqrt{-x^{2}+1}\,\sqrt{x}}$	56
elliptic	$\frac{\sqrt{-x\left(x^{2}-1\right)}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\,\left(-2\,\text{EllipticE}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)+\text{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x}\,\sqrt{-x^{2}+1}\,\sqrt{-x^{3}+x}}$	75

input
```
int(x^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*hypergeom([1/2,3/4],[7/4],x^2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate(x^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2*I*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = \frac{x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle| x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(2*I*pi))/(2*g
amma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{x}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate(x^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x)/sqrt(-x^2 + 1), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = \mathrm{E}\left(\mathrm{asin}\left(\frac{\sqrt{2}\,\sqrt{x+1}}{2}\right)\middle|\,2\right)\,2\mathrm{i}$$

input
```
int(x^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
ellipticE(asin((2^(1/2)*(x + 1)^(1/2))/2), 2)*2i
```

Reduce [F]

$$\int \frac{\sqrt{x}}{\sqrt{1-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^2+1}}{x^2-1}dx\right)$$

input
```
int(x^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - x**2 + 1))/(x**2 - 1),x)
```

3.662 $\int \sqrt{\frac{x}{1-x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{\frac{x}{1-x^2}}\,dx=-\frac{2\sqrt{\frac{x}{1-x^2}}\sqrt{1-x^2}E\left(\arcsin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{x}}$$

output
```
-2*(x/(-x^2+1))^(1/2)*(-x^2+1)^(1/2)*EllipticE(1/2*(1-x)^(1/2)*2^(1/2),2^(
1/2))/x^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \sqrt{\frac{x}{1-x^2}}\,dx=\frac{2}{3}x\sqrt{1-x^2}\sqrt{-\frac{x}{-1+x^2}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)$$

input
```
Integrate[Sqrt[x/(1 - x^2)],x]
```

output
```
(2*x*Sqrt[1 - x^2]*Sqrt[-(x/(-1 + x^2))]*Hypergeometric2F1[1/2, 3/4, 7/4,
x^2])/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7270, 259, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{1-x^2}}\,dx$$

$$\Big\downarrow \text{ 7270}$$

$$\frac{\sqrt{\frac{x}{1-x^2}}\sqrt{1-x^2}\int \frac{\sqrt{x}}{\sqrt{1-x^2}}dx}{\sqrt{x}}$$

$$\Big\downarrow \text{ 259}$$

$$-\frac{2\sqrt{\frac{x}{1-x^2}}\sqrt{1-x^2}\int \frac{\sqrt{x}}{\sqrt{\frac{x-1}{2}+1}}d\frac{\sqrt{1-x}}{\sqrt{2}}}{\sqrt{x}}$$

$$\Big\downarrow \text{ 327}$$

$$-\frac{2\sqrt{\frac{x}{1-x^2}}\sqrt{1-x^2}E\left(\arcsin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\middle|2\right)}{\sqrt{x}}$$

input
```
Int[Sqrt[x/(1 - x^2)],x]
```

output
```
(-2*Sqrt[x/(1 - x^2)]*Sqrt[1 - x^2]*EllipticE[ArcSin[Sqrt[1 - x]/Sqrt[2]],
 2])/Sqrt[x]
```

Defintions of rubi rules used

rule 259
```
Int[Sqrt[x_]/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[-2/(Sqrt[a]*(-b/a
)^(3/4))   Subst[Int[Sqrt[1 - 2*x^2]/Sqrt[1 - x^2], x], x, Sqrt[1 - Sqrt[-b
/a]*x]/Sqrt[2]], x] /; FreeQ[{a, b}, x] && GtQ[-b/a, 0] && GtQ[a, 0]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !Free
Q[v, x] && !FreeQ[w, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

method	result	size
elliptic	$\frac{\sqrt{-\frac{x}{x^2-1}}\,\sqrt{x(x^2-1)}\,\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\left(-2\,\mathrm{EllipticE}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)\right)}{x\sqrt{x^3-x}}$	77
default	$-\frac{\sqrt{-\frac{x}{x^2-1}}\,(x^2-1)\sqrt{1+x}\,\sqrt{2-2x}\,\sqrt{-x}\left(2\,\mathrm{EllipticE}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)-\mathrm{EllipticF}\left(\sqrt{1+x},\frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x(x^2-1)}\,\sqrt{x^3-x}}$	82

input
```
int((x/(-x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-1/(x^2-1)*x)^(1/2)*(x*(x^2-1))^(1/2)/x*(1+x)^(1/2)*(2-2*x)^(1/2)*(-x)^(1
/2)/(x^3-x)^(1/2)*(-2*EllipticE((1+x)^(1/2),1/2*2^(1/2))+EllipticF((1+x)^(
1/2),1/2*2^(1/2)))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.18

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = -2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate((x/(-x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, x))
```

Sympy [F]

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = \int \sqrt{\frac{x}{1-x^2}}\,dx$$

input
```
integrate((x/(-x**2+1))**(1/2),x)
```

output
```
Integral(sqrt(x/(1 - x**2)), x)
```

Maxima [F]

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = \int \sqrt{-\frac{x}{x^2-1}}\,dx$$

input
```
integrate((x/(-x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x/(x^2 - 1)), x)
```

Giac [F]

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = \int \sqrt{-\frac{x}{x^2-1}}\,dx$$

input
```
integrate((x/(-x^2+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x/(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = \int \sqrt{-\frac{x}{x^2-1}}\,dx$$

input
```
int((-x/(x^2 - 1))^(1/2),x)
```

output
```
int((-x/(x^2 - 1))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{x}{1-x^2}}\,dx = \left(\int \frac{\sqrt{x}\,\sqrt{x^2-1}}{x^2-1}dx\right)i$$

input
```
int((x/(-x^2+1))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(x**2 - 1))/(x**2 - 1),x)*i
```

3.663 $\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 21

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}\sqrt{x}\right),-1\right)}{\sqrt{a}}$$

output
```
2*EllipticF(a^(1/2)*x^(1/2),I)/a^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = 2\sqrt{x}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},a^2x^2\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

output
```
2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, a^2*x^2]
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {266, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx$$

$$\Big\downarrow \ 266$$

$$2\int \frac{1}{\sqrt{1-a^2x^2}}d\sqrt{x}$$

$$\Big\downarrow \ 762$$

$$\frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}\sqrt{x}\right),-1\right)}{\sqrt{a}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[1 - a^2*x^2]),x]
```

output
```
(2*EllipticF[ArcSin[Sqrt[a]*Sqrt[x]], -1])/Sqrt[a]
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
meijerg	$2\sqrt{x}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],a^{2}x^{2}\right)$	19
default	$\frac{\sqrt{ax+1}\,\sqrt{-2ax+2}\,\sqrt{-ax}\ \mathrm{EllipticF}\left(\sqrt{ax+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{-a^{2}x^{2}+1}\,\sqrt{x}\,a}$	54
elliptic	$\frac{\sqrt{-x(a^{2}x^{2}-1)}\,\sqrt{a\left(x+\frac{1}{a}\right)}\,\sqrt{-2a\left(x-\frac{1}{a}\right)}\,\sqrt{-ax}\ \mathrm{EllipticF}\left(\sqrt{a\left(x+\frac{1}{a}\right)},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\,\sqrt{-a^{2}x^{2}+1}\,a\sqrt{-a^{2}x^{3}+x}}$	88

input
```
int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*hypergeom([1/4,1/2],[5/4],a^2*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = -\frac{2\sqrt{-a^2}\mathrm{weierstrassPInverse}\left(\frac{4}{a^2},0,x\right)}{a^2}$$

input
```
integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-a^2)*weierstrassPInverse(4/a^2, 0, x)/a^2
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{5}{4} \end{matrix}\middle| a^2x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(1/2)/(-a**2*x**2+1)**(1/2),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**2*exp_polar(2*I*pi))/
(2*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{-a^2x^2+1}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-a^2*x^2 + 1)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{1-a^2\,x^2}}\,dx$$

input
```
int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(1 - a^2*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1-a^2x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-a^2x^2+1}}{a^2x^3-x}dx\right)$$

input
```
int(1/x^(1/2)/(-a^2*x^2+1)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - a**2*x**2 + 1))/(a**2*x**3 - x),x)
```

3.664 $\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \frac{\left(1+\sqrt{a}x\right)\sqrt{\frac{1+ax^2}{\left(1+\sqrt{a}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\sqrt[4]{a}\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{1+ax^2}}$$

output
```
(1+a^(1/2)*x)*((a*x^2+1)/(1+a^(1/2)*x)^2)^(1/2)*InverseJacobiAM(2*arctan(a
^(1/4)*x^(1/2)),1/2*2^(1/2))/a^(1/4)/(a*x^2+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = 2\sqrt{x}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-ax^2\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

output
```
2*Sqrt[x]*Hypergeometric2F1[1/4, 1/2, 5/4, -(a*x^2)]
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+1}}dx$$

$$\Big\downarrow \text{266}$$

$$2\int \frac{1}{\sqrt{ax^2+1}}d\sqrt{x}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(\sqrt{a}x+1\right)\sqrt{\frac{ax^2+1}{\left(\sqrt{a}x+1\right)^2}}\text{EllipticF}\left(2\arctan\left(\sqrt[4]{a}\sqrt{x}\right),\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax^2+1}}$$

input
```
Int[1/(Sqrt[x]*Sqrt[1 + a*x^2]),x]
```

output
```
((1 + Sqrt[a]*x)*Sqrt[(1 + a*x^2)/(1 + Sqrt[a]*x)^2]*EllipticF[2*ArcTan[a^
(1/4)*Sqrt[x]], 1/2])/(a^(1/4)*Sqrt[1 + a*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.27

method	result	size
meijerg	$2\sqrt{x}\ \text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-a\,x^{2}\right)$	18
default	$-\frac{\sqrt{-\sqrt{-a}\,x+1}\,\sqrt{2}\,\sqrt{\sqrt{-a}\,x+1}\,\sqrt{\sqrt{-a}\,x}\ \text{EllipticF}\left(\sqrt{-\sqrt{-a}\,x+1},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\,\sqrt{a\,x^{2}+1}\,\sqrt{-a}}$	73
elliptic	$-\frac{\sqrt{x(a\,x^{2}+1)}\,\sqrt{-\left(x-\frac{1}{\sqrt{-a}}\right)\sqrt{-a}}\,\sqrt{2}\,\sqrt{\left(x+\frac{1}{\sqrt{-a}}\right)\sqrt{-a}}\,\sqrt{\sqrt{-a}\,x}\ \text{EllipticF}\left(\sqrt{-\left(x-\frac{1}{\sqrt{-a}}\right)\sqrt{-a}},\frac{\sqrt{2}}{2}\right)}{\sqrt{x}\,\sqrt{a\,x^{2}+1}\,\sqrt{-a}\,\sqrt{a\,x^{3}+x}}$	109

input
```
int(1/x^(1/2)/(a*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*x^(1/2)*hypergeom([1/4,1/2],[5/4],-a*x^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.19

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \frac{2\,\text{weierstrassPInverse}\left(-\frac{4}{a},0,x\right)}{\sqrt{a}}$$

input
```
integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(-4/a, 0, x)/sqrt(a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|ax^2e^{i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/x**(1/2)/(a*x**2+1)**(1/2),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a*x**2*exp_polar(I*pi))/(2*ga
mma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \int \frac{1}{\sqrt{ax^2+1}\sqrt{x}}\,dx$$

input
```
integrate(1/x^(1/2)/(a*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*x^2 + 1)*sqrt(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \int \frac{1}{\sqrt{x}\,\sqrt{a\,x^2+1}}\,dx$$

input
```
int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^(1/2)*(a*x^2 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{x}\sqrt{1+ax^2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{a\,x^2+1}}{a\,x^3+x}dx$$

input
```
int(1/x^(1/2)/(a*x^2+1)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(a*x**2 + 1))/(a*x**3 + x),x)
```

3.665 $\int (cx)^{5/4}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 537

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx=\frac{16ac\sqrt[4]{cx}\sqrt{a+bx^2}}{65b}+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$-\frac{8a^2c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{65\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$+\frac{8a^2c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{65\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
16/65*a*c*(c*x)^(1/4)*(b*x^2+a)^(1/2)/b+4/13*(c*x)^(9/4)*(b*x^2+a)^(1/2)/c
-8/65*a^2*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(
1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*
x)^(1/2))+8/65*a^2*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a
^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))
^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2
*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(
1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^
(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.16

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}\left((a+bx^2)\sqrt{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{9}{8},-\frac{bx^2}{a}\right)\right)}{13b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(5/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*c*(c*x)^(1/4)*Sqrt[a + b*x^2]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hype
rgeometric2F1[-1/2, 1/8, 9/8, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {248, 262, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \; 248$$

$$\frac{4}{13}a\int \frac{(cx)^{5/4}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \; 262$$

$$\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{ac^2\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \; 266$$

$$\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \; 767$$

$$\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \; 27$$

$$\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

$$\Big\downarrow \; 2422$$

$$\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right.\right.}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}\right.}{}\right.$$

$$\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}$$

input
```
Int[(c*x)^(5/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*(c*x)^(9/4)*Sqrt[a + b*x^2])/(13*c) + (4*a*((4*c*(c*x)^(1/4)*Sqrt[a + b
*x^2])/(5*b) - (4*a*c*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sq
rt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/
4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c
+ Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1
/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[
a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*S
qrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c
] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[A
rcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sq
rt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])
])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*
x]))))/(5*b)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int (cx)^{\frac{5}{4}} \sqrt{b\,x^{2}+a}dx$$

input
```
int((c*x)^(5/4)*(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(5/4)*(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)*c*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.94 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{8}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(5/4)*x**(9/4)*gamma(9/8)*hyper((-1/2, 9/8), (17/8,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(17/8))
```

Maxima [F]

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(5/4), x)
```

Giac [F]

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \int (c\,x)^{5/4}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(5/4)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/4)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{5/4}\sqrt{a+bx^2}\,dx = \frac{4c^{\frac{5}{4}}\left(4x^{\frac{1}{4}}\sqrt{b\,x^2+a}\,a+5x^{\frac{9}{4}}\sqrt{b\,x^2+a}\,b-\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a+x^{\frac{11}{4}}b}dx\right)a^2\right)}{65b}$$

input
```
int((c*x)^(5/4)*(b*x^2+a)^(1/2),x)
```

output
```
(4*c**(1/4)*c*(4*x**(1/4)*sqrt(a + b*x**2)*a + 5*x**(1/4)*sqrt(a + b*x**2)
*b*x**2 - int(sqrt(a + b*x**2)/(x**(3/4)*a + x**(3/4)*b*x**2),x)*a**2))/(6
5*b)
```

3.666 $\int (cx)^{3/4}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1045

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx = \text{Too large to display}$$

output
```
16/33*a*c*(b*x^2+a)^(1/2)/b/(c*x)^(1/4)+4/11*(c*x)^(7/4)*(b*x^2+a)^(1/2)/c
+8/33*(2+2^(1/2))^(1/2)*a^(7/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^
(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^
(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*
b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/
2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b
^(1/4)*(c*x)^(1/2))+8/33*(2+2^(1/2))^(1/2)*a^(7/4)*c^(1/2)*(c*x)^(3/4)*(-(
b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^
2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2
)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2)
)/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/2)/(b*x^2+a)^(1/2)
/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-8/33*a^(7/4)*c^(1/2)*(c*x)^(3/4)*(-
(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^
2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/
2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2
))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(1
/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-8/33*a^(7/4)*c^(
1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b
^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF
(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.05

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx=\frac{4x(cx)^{3/4}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*x*(c*x)^(3/4)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 7/8, 15/8, -((b*x
^2)/a)])/(7*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {248, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{4}{11}a\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{16a\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{11c}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{16a\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{11c\sqrt{a+bx^2}}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{16a(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{77c\sqrt{a+bx^2}}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}$$

input
```
Int[(c*x)^(3/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*(c*x)^(7/4)*Sqrt[a + b*x^2])/(11*c) + (16*a*(c*x)^(7/4)*Sqrt[1 + (b*x^2
)/a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((b*x^2)/a)])/(77*c*Sqrt[a + b*x^2
])
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{3}{4}} \sqrt{b\,x^{2}+a}dx$$

input
```
int((c*x)^(3/4)*(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(3/4)*(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.40 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.04

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx=\frac{\sqrt{a}c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{8}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(3/4)*x**(7/4)*gamma(7/8)*hyper((-1/2, 7/8), (15/8,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(15/8))
```

Maxima [F]

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(3/4), x)
```

Giac [F]

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx = \int (c\,x)^{3/4}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(3/4)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/4)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^{3/4}\sqrt{a+bx^2}\,dx = \frac{4c^{\frac{3}{4}}\left(x^{\frac{7}{4}}\sqrt{b\,x^2+a}+\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a\right)}{11}$$

input
```
int((c*x)^(3/4)*(b*x^2+a)^(1/2),x)
```

output
```
(4*c**(3/4)*(x**(3/4)*sqrt(a + b*x**2)*x + int((x**(3/4)*sqrt(a + b*x**2))
/(a + b*x**2),x)*a))/11
```

3.667 $\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 998

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \text{Too large to display}$$

output
```
4/9*(c*x)^(5/4)*(b*x^2+a)^(1/2)/c-8/9*(2+2^(1/2))^(1/2)*a^(3/2)*(c*x)^(3/4
)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/
2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c
^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/4)/(b*x^2+a)^
(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+8/9*(2+2^(1/2))^(1/2)*a^(3/2)*
(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4
)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*
(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/4)/
(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))+8/9*a^(3/2)*(c*x)^(3
/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(
1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)
*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
)/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-8/9*a^(3/2
)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1
/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/
2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2
)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \frac{4x\sqrt[4]{cx}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(1/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*x*(c*x)^(1/4)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 5/8, 13/8, -((b*x
^2)/a)])/(5*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {248, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{4}{9}a\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}$$

$$\Big\downarrow \text{266}$$

$$\frac{16a\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{9c}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}$$

$$\Big\downarrow \text{889}$$

$$\frac{16a\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{9c\sqrt{a+bx^2}}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}$$

$$\Big\downarrow \text{888}$$

$$\frac{16a(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{45c\sqrt{a+bx^2}}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}$$

input
```
Int[(c*x)^(1/4)*Sqrt[a + b*x^2],x]
```

output
```
(4*(c*x)^(5/4)*Sqrt[a + b*x^2])/(9*c) + (16*a*(c*x)^(5/4)*Sqrt[1 + (b*x^2)
/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -((b*x^2)/a)])/(45*c*Sqrt[a + b*x^2]
)
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{1}{4}} \sqrt{b\,x^{2}+a}dx$$

input
```
int((c*x)^(1/4)*(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(1/4)*(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.05

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \frac{\sqrt{a}\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{8}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**(1/4)*x**(5/4)*gamma(5/8)*hyper((-1/2, 5/8), (13/8,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(13/8))
```

Maxima [F]

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \int \sqrt{bx^2+a}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \int (c\,x)^{1/4}\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^(1/4)*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/4)*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt[4]{cx}\sqrt{a+bx^2}\,dx = \frac{4c^{\frac{1}{4}}\left(x^{\frac{5}{4}}\sqrt{b\,x^2+a}+\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a\right)}{9}$$

input
```
int((c*x)^(1/4)*(b*x^2+a)^(1/2),x)
```

output
```
(4*c**(1/4)*(x**(1/4)*sqrt(a + b*x**2)*x + int((x**(1/4)*sqrt(a + b*x**2))
/(a + b*x**2),x)*a))/9
```

3.668 $\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 512

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

$$-\frac{8a^{5/4}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2\right.}{7\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{8a^{5/4}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2\right.}{7\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
4/7*(c*x)^(3/4)*(b*x^2+a)^(1/2)/c-8/7*a^(5/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1
/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1
/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^
(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*
x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c^(1/2)/(b*x^2+a)^
(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-8/7*a^(5/4)*(c*x)^(3/4)*(-(b*x
^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a
^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(
2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b
^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c^(1/2)/
(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \frac{4x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{11}{8},-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(1/4),x]
```

output
```
(4*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 3/8, 11/8, -((b*x^2)/a)])/(3*
(c*x)^(1/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{4}{7}a\int \frac{1}{\sqrt[4]{cx}\sqrt{bx^2+a}}dx+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{16a\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{ 838}$$

$$\frac{16a\left(\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{16a\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

$$\Big\downarrow \text{ 2422}$$

$$\frac{16a\left(-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\right.}{7c}$$

$$\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(1/4),x]
```

output
```
(4*(c*x)^(3/4)*Sqrt[a + b*x^2])/(7*c) + (16*a*(-1/2*(a^(1/4)*Sqrt[c]*(c*x)
^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*
Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*Ellipt
icF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(
1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 -
Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*S
qrt[c*x])) - (a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt
[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/
4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c +
 Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/
4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a
+ b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{(cx)^{\frac{1}{4}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(1/4),x)
```

output
```
int((b*x^2+a)^(1/2)/(c*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/4),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \frac{\sqrt{a}x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{8}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{c}\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(1/4),x)
```

output
```
sqrt(a)*x**(3/4)*gamma(3/8)*hyper((-1/2, 3/8), (11/8,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(1/4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(1/4),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\left(c\,x\right)^{1/4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(1/4),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt[4]{cx}}\,dx = \frac{\frac{4x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{7}+\frac{4\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{1}{4}}a+x^{\frac{9}{4}}b}dx\right)a}{7}}{c^{\frac{1}{4}}}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(1/4),x)
```

output
```
(4*(x**(3/4)*sqrt(a + b*x**2) + int(sqrt(a + b*x**2)/(x**(1/4)*a + x**(1/4
)*b*x**2),x)*a))/(7*c**(1/4))
```

3.669 $\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 510

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

$$+\frac{8a\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{5\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{8a\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{5\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
4/5*(c*x)^(1/4)*(b*x^2+a)^(1/2)/c+8/5*a*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^
(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^
(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+
2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(
c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c/(b*x^2+a)^(1/2
)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-8/5*a*b^(1/4)*(c*x)^(3/4)*(-(b*x^2
+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(
1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^
(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(
1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c/(b*x^2+a
)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx=\frac{4x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{9}{8},-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(3/4),x]
```

output
```
(4*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, 1/8, 9/8, -((b*x^2)/a)])/((c*
x)^(3/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {248, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx$$

$$\Big\downarrow \; 248$$

$$\frac{4}{5}a\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \; 266$$

$$\frac{16a\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \; 767$$

$$\frac{16a\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \; 27$$

$$\frac{16a\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

$$\Big\downarrow \; 2422$$

$$\frac{16a\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(\dots}{\dots}\right)}{5c}$$

$$\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(3/4),x]
```

output
```
(4*(c*x)^(1/4)*Sqrt[a + b*x^2])/(5*c) + (16*a*((b^(1/4)*(c*x)^(3/4)*Sqrt[-
((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(
1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sq
rt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*
Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2
*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x]))
- (b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))
]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*
Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*
x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x
])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*S
qrt[c] - b^(1/4)*Sqrt[c*x]))))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\left(c x\right)^{\frac{3}{4}}}d x$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(3/4),x)
```

output
```
int((b*x^2+a)^(1/2)/(c*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/4),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \frac{\sqrt{a}\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right){}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{8} \\ \frac{9}{8} \end{matrix}\middle| \frac{bx^{2}e^{i\pi}}{a}\right)}{2c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(3/4),x)
```

output
```
sqrt(a)*x**(1/4)*gamma(1/8)*hyper((-1/2, 1/8), (9/8,), b*x**2*exp_polar(I*
pi)/a)/(2*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(3/4),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(c\,x)^{3/4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(3/4),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{3/4}}\,dx = \frac{\frac{4x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{5}+\frac{4\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a+x^{\frac{11}{4}}b}dx\right)a}{5}}{c^{\frac{3}{4}}}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(3/4),x)
```

output
```
(4*(x**(1/4)*sqrt(a + b*x**2) + int(sqrt(a + b*x**2)/(x**(3/4)*a + x**(3/4
)*b*x**2),x)*a))/(5*c**(3/4))
```

3.670 $\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 1018

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}} dx = \text{Too large to display}$$

output
```
4/3*(b*x^2+a)^(1/2)/c/(c*x)^(1/4)+8/3*(2+2^(1/2))^(1/2)*a^(3/4)*b^(1/2)*(c
*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(
c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a
^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^
(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/c^(3/2)/(b
*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+8/3*(2+2^(1/2))^(1/2)*
a^(3/4)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4
)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2
)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))
^(1/2))/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-8/3*
a^(3/4)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)
*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)
*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1
/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))
^(1/2))/(2+2^(1/2))^(1/2)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)
*(c*x)^(1/2))-8/3*a^(3/4)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/
x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)
/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2
)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.05

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = -\frac{4x\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{7}{8},-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[Sqrt[a + b*x^2]/(c*x)^(5/4),x]
```

output
```
(-4*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, -1/8, 7/8, -((b*x^2)/a)])/((
c*x)^(5/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {247, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}} \, dx$$

$$\Big\downarrow \text{247}$$

$$\frac{4b\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{c^2}-\frac{4\sqrt{a+bx^2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{16b\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{c^3}-\frac{4\sqrt{a+bx^2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{889}$$

$$\frac{16b\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{c^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{888}$$

$$\frac{16b(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7c^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{c\sqrt[4]{cx}}$$

input
```
Int[Sqrt[a + b*x^2]/(c*x)^(5/4),x]
```

output
```
(-4*Sqrt[a + b*x^2])/(c*(c*x)^(1/4)) + (16*b*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/
a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((b*x^2)/a)])/(7*c^3*Sqrt[a + b*x^2]
)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{(cx)^{\frac{5}{4}}}dx$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(5/4),x)
```

output
```
int((b*x^2+a)^(1/2)/(c*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/4),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.05

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \frac{\sqrt{a}\Gamma\left(-\frac{1}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{8}\\ \frac{7}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate((b*x**2+a)**(1/2)/(c*x)**(5/4),x)
```

output
```
sqrt(a)*gamma(-1/8)*hyper((-1/2, -1/8), (7/8,), b*x**2*exp_polar(I*pi)/a)/
(2*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(5/4), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \int \frac{\sqrt{bx^2+a}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/2)/(c*x)^(5/4),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)/(c*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{(c\,x)^{5/4}}\,dx$$

input
```
int((a + b*x^2)^(1/2)/(c*x)^(5/4),x)
```

output
```
int((a + b*x^2)^(1/2)/(c*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{a+bx^2}}{(cx)^{5/4}}\,dx = \frac{\frac{4\sqrt{b\,x^2+a}}{3}+\frac{4x^{\frac{1}{4}}\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{5}{4}}a+x^{\frac{13}{4}}b}dx\right)a}{3}}{x^{\frac{1}{4}}c^{\frac{5}{4}}}$$

input
```
int((b*x^2+a)^(1/2)/(c*x)^(5/4),x)
```

output
```
(4*(sqrt(a + b*x**2) + x**(1/4)*int(sqrt(a + b*x**2)/(x**(1/4)*a*x + x**(1
/4)*b*x**3),x)*a))/(3*x**(1/4)*c**(1/4)*c)
```

3.671 $\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 565

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \frac{64a^2c\sqrt[4]{cx}\sqrt{a+bx^2}}{455b}$$

$$+ \frac{16a(cx)^{9/4}\sqrt{a+bx^2}}{91c} + \frac{4(cx)^{9/4}\left(a+bx^2\right)^{3/2}}{21c}$$

$$- \frac{32a^3c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right), -2(1}{455\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$+ \frac{32a^3c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right), -2(1}{455\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
64/455*a^2*c*(c*x)^(1/4)*(b*x^2+a)^(1/2)/b+16/91*a*(c*x)^(9/4)*(b*x^2+a)^(
1/2)/c+4/21*(c*x)^(9/4)*(b*x^2+a)^(3/2)/c-32/455*a^3*c*(c*x)^(3/4)*(-(b*x^
2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(
1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2
^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^
(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(
b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+32/455*a^3*c*(c*x)^(3
/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^
(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)
*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
)/b^(3/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.16

$$\int (cx)^{5/4} (a+bx^2)^{3/2} \, dx = \frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}\left((a+bx^2)^2\sqrt{1+\frac{bx^2}{a}} - a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},\frac{9}{8},-\frac{bx^2}{a}\right)\right)}{21b\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(5/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*c*(c*x)^(1/4)*Sqrt[a + b*x^2]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*
Hypergeometric2F1[-3/2, 1/8, 9/8, -((b*x^2)/a)]))/(21*b*Sqrt[1 + (b*x^2)/a
])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {248, 248, 262, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \ 248$$

$$\frac{4}{7}a \int (cx)^{5/4}\sqrt{bx^2+a}dx+\frac{4(cx)^{9/4} \left(a+bx^2\right)^{3/2}}{21c}$$

$$\Big\downarrow \ 248$$

$$\frac{4}{7}a \left(\frac{4}{13}a \int \frac{(cx)^{5/4}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}\right)+\frac{4(cx)^{9/4} \left(a+bx^2\right)^{3/2}}{21c}$$

$$\Big\downarrow \ 262$$

$$\frac{4}{7}a \left(\frac{4}{13}a \left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{ac^2 \int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}\right)+\frac{4(cx)^{9/4} \left(a+bx^2\right)^{3/2}}{21c}$$

$$\Big\downarrow \ 266$$

$$\frac{4}{7}a \left(\frac{4}{13}a \left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac \int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}\right)+\frac{4(cx)^{9/4} \left(a+bx^2\right)^{3/2}}{21c}$$

$$\Big\downarrow \ 767$$

$$\frac{4}{7}a \left(\frac{4}{13}a \left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac \left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}\right)+\frac{4(cx)^{9/4} \left(a+bx^2\right)^{3/2}}{21c}$$

$\downarrow$ 27

$$\frac{4}{7}a\left(\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{\int\frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int\frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{5b}\right)+\frac{4(cx)^{9/4}\sqrt{a+bx^2}}{13c}\right)+$$

$$\frac{4(cx)^{9/4}\left(a+bx^2\right)^{3/2}}{21c}$$

$\downarrow$ 2422

$$\frac{4}{7}a\left(\frac{4}{13}a\left(\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{\cdots}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\cdots}}\right.\right.}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}\cdots\right)}{\cdots}\right.\right.$$

$$\frac{4(cx)^{9/4}\left(a+bx^2\right)^{3/2}}{21c}$$

input

```
Int[(c*x)^(5/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*(c*x)^(9/4)*(a + b*x^2)^(3/2))/(21*c) + (4*a*((4*(c*x)^(9/4)*Sqrt[a + b
*x^2])/(13*c) + (4*a*((4*c*(c*x)^(1/4)*Sqrt[a + b*x^2])/(5*b) - (4*a*c*((b
^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sq
rt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c
*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2
*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/
2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[
c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)
/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/
(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[
a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)
*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*S
qrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/(5*b)))/13))/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int (cx)^{\frac{5}{4}} \left(b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^(5/4)*(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(5/4)*(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*c*x^3 + a*c*x)*sqrt(b*x^2 + a)*(c*x)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \frac{a^{\frac{3}{2}} c^{\frac{5}{4}} x^{\frac{9}{4}} \Gamma\left(\frac{9}{8}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{9}{8} \\ \frac{17}{8} \end{matrix}\middle| \frac{bx^{2} e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(5/4)*x**(9/4)*gamma(9/8)*hyper((-3/2, 9/8), (17/8,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(17/8))
```

Maxima [F]

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{5}{4}} \, dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(5/4), x)
```

Giac [F]

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{5}{4}} \, dx$$

input
```
integrate((c*x)^(5/4)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \int (c\,x)^{5/4} \left(b\,x^2+a\right)^{3/2} \, dx$$

input
```
int((c*x)^(5/4)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(5/4)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{5/4} \left(a+bx^2\right)^{3/2} \, dx = \frac{4c^{\frac{5}{4}} \left(48x^{\frac{1}{4}}\sqrt{b\,x^2+a}\,a^2+125x^{\frac{9}{4}}\sqrt{b\,x^2+a}\,ab+65x^{\frac{17}{4}}\sqrt{b\,x^2+a}\,b^2-12\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a+x^{\frac{11}{4}}b}dx\right)a^3\right)}{1365b}$$

input
```
int((c*x)^(5/4)*(b*x^2+a)^(3/2),x)
```

output
```
(4*c**(1/4)*c*(48*x**(1/4)*sqrt(a + b*x**2)*a**2 + 125*x**(1/4)*sqrt(a + b
*x**2)*a*b*x**2 + 65*x**(1/4)*sqrt(a + b*x**2)*b**2*x**4 - 12*int(sqrt(a +
 b*x**2)/(x**(3/4)*a + x**(3/4)*b*x**2),x)*a**3))/(1365*b)
```

3.672 $\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1073

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx = \text{Too large to display}$$

output
```
64/209*a^2*c*(b*x^2+a)^(1/2)/b/(c*x)^(1/4)+48/209*a*(c*x)^(7/4)*(b*x^2+a)^
(1/2)/c+4/19*(c*x)^(7/4)*(b*x^2+a)^(3/2)/c+32/209*(2+2^(1/2))^(1/2)*a^(11/
4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1
/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Elli
pticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2
))/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+32/209*(2
+2^(1/2))^(1/2)*a^(11/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x
)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/
(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)
*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/
2),(-2+2*2^(1/2))^(1/2))/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*
(c*x)^(1/2))-32/209*a^(11/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/
2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/
2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(
1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(1/2)/(b*x^2+a)^(1/2)/(a^
(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-32/209*a^(11/4)*c^(1/2)*(c*x)^(3/4)*(-(
b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^
2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.05

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx = \frac{4ax(cx)^{3/4}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*a*x*(c*x)^(3/4)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 7/8, 15/8, -((b
*x^2)/a)])/(7*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 248, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{12}{19} a \int (cx)^{3/4} \sqrt{bx^2+a}dx+\frac{4 (cx)^{7/4} \left(a+bx^2\right)^{3/2}}{19 c}$$

$$\Big\downarrow \text{248}$$

$$\frac{12}{19} a \left(\frac{4}{11} a \int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx+\frac{4 (cx)^{7/4} \sqrt{a+bx^2}}{11 c}\right)+\frac{4 (cx)^{7/4} \left(a+bx^2\right)^{3/2}}{19 c}$$

$$\Big\downarrow \text{266}$$

$$\frac{12}{19} a \left(\frac{16 a \int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{11 c}+\frac{4 (cx)^{7/4} \sqrt{a+bx^2}}{11 c}\right)+\frac{4 (cx)^{7/4} \left(a+bx^2\right)^{3/2}}{19 c}$$

$$\Big\downarrow \text{889}$$

$$\frac{12}{19} a \left(\frac{16 a \sqrt{\frac{bx^2}{a}+1} \int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{11 c \sqrt{a+bx^2}}+\frac{4 (cx)^{7/4} \sqrt{a+bx^2}}{11 c}\right)+\frac{4 (cx)^{7/4} \left(a+bx^2\right)^{3/2}}{19 c}$$

$$\Big\downarrow \text{888}$$

$$\frac{12}{19} a \left(\frac{16 a (cx)^{7/4} \sqrt{\frac{bx^2}{a}+1} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{77 c \sqrt{a+bx^2}}+\frac{4 (cx)^{7/4} \sqrt{a+bx^2}}{11 c}\right)+\frac{4 (cx)^{7/4} \left(a+bx^2\right)^{3/2}}{19 c}$$

input
```
Int[(c*x)^(3/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*(c*x)^(7/4)*(a + b*x^2)^(3/2))/(19*c) + (12*a*((4*(c*x)^(7/4)*Sqrt[a +
b*x^2])/(11*c) + (16*a*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1
/2, 7/8, 15/8, -((b*x^2)/a)])/(77*c*Sqrt[a + b*x^2])))/19
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{3}{4}} \left(b\,x^{2}+a\right)^{\frac{3}{2}}d x$$

input
```
int((c*x)^(3/4)*(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(3/4)*(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{3}{4}}\, dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.72 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.04

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} dx = \frac{a^{\frac{3}{2}} c^{\frac{3}{4}} x^{\frac{7}{4}} \Gamma\left(\frac{7}{8}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{7}{8} \\ \frac{15}{8} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(3/4)*x**(7/4)*gamma(7/8)*hyper((-3/2, 7/8), (15/8,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(15/8))
```

Maxima [F]

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{3}{4}} \, dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(3/4), x)
```

Giac [F]

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^{\frac{3}{4}} \, dx$$

input
```
integrate((c*x)^(3/4)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} \, dx = \int (c\,x)^{3/4}\,\left(b\,x^2+a\right)^{3/2} \,d x$$

input
```
int((c*x)^(3/4)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(3/4)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^{3/4} \left(a+bx^2\right)^{3/2} dx = \frac{4c^{\frac{3}{4}} \left(23x^{\frac{7}{4}}\sqrt{b\,x^2+a}\,a+11x^{\frac{15}{4}}\sqrt{b\,x^2+a}\,b+12\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2\right)}{209}$$

input
```
int((c*x)^(3/4)*(b*x^2+a)^(3/2),x)
```

output
```
(4*c**(3/4)*(23*x**(3/4)*sqrt(a + b*x**2)*a*x + 11*x**(3/4)*sqrt(a + b*x**
2)*b*x**3 + 12*int((x**(3/4)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/209
```

3.673 $\int \sqrt[4]{cx}(a+bx^2)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1024

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx = \text{Too large to display}$$

output
```
16/51*a*(c*x)^(5/4)*(b*x^2+a)^(1/2)/c+4/17*(c*x)^(5/4)*(b*x^2+a)^(3/2)/c-3
2/51*(2+2^(1/2))^(1/2)*a^(5/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^
(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*
x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x
/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2)
,(-2+2*2^(1/2))^(1/2))/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c
*x)^(1/2))+32/51*(2+2^(1/2))^(1/2)*a^(5/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)
/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4
)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/
2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^
(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2
)-b^(1/4)*(c*x)^(1/2))+32/51*a^(5/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/
2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/
2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(
1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a^
(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-32/51*a^(5/2)*(c*x)^(3/4)*(-(b*x^2+a)/a
^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/
b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)
+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.06

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{4ax\sqrt[4]{cx}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(1/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*a*x*(c*x)^(1/4)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 5/8, 13/8, -((b
*x^2)/a)])/(5*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.24 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {248, 248, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{12}{17}a\int \sqrt[4]{cx}\sqrt{bx^2+a}dx+\frac{4(cx)^{5/4}\left(a+bx^2\right)^{3/2}}{17c}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{12}{17}a\left(\frac{4}{9}a\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}\right)+\frac{4(cx)^{5/4}\left(a+bx^2\right)^{3/2}}{17c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{12}{17}a\left(\frac{16a\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{9c}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}\right)+\frac{4(cx)^{5/4}\left(a+bx^2\right)^{3/2}}{17c}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{12}{17}a\left(\frac{16a\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{9c\sqrt{a+bx^2}}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}\right)+\frac{4(cx)^{5/4}\left(a+bx^2\right)^{3/2}}{17c}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{12}{17}a\left(\frac{16a(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{45c\sqrt{a+bx^2}}+\frac{4(cx)^{5/4}\sqrt{a+bx^2}}{9c}\right)+\frac{4(cx)^{5/4}\left(a+bx^2\right)^{3/2}}{17c}$$

input
```
Int[(c*x)^(1/4)*(a + b*x^2)^(3/2),x]
```

output
```
(4*(c*x)^(5/4)*(a + b*x^2)^(3/2))/(17*c) + (12*a*((4*(c*x)^(5/4)*Sqrt[a +
b*x^2])/(9*c) + (16*a*(c*x)^(5/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
2, 5/8, 13/8, -((b*x^2)/a)])/(45*c*Sqrt[a + b*x^2])))/17
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{1}{4}} \left(b\,x^{2}+a\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^(1/4)*(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(1/4)*(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.04

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx=\frac{a^{\frac{3}{2}}\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},\frac{5}{8}\\\frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**(1/4)*x**(5/4)*gamma(5/8)*hyper((-3/2, 5/8), (13/8,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(13/8))
```

Maxima [F]

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx = \int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x)^(1/4)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx = \int (c\,x)^{1/4}\,\left(b\,x^2+a\right)^{3/2}\,dx$$

input
```
int((c*x)^(1/4)*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(1/4)*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \sqrt[4]{cx}\left(a+bx^2\right)^{3/2}\,dx = \frac{4c^{\frac{1}{4}}\left(7x^{\frac{5}{4}}\sqrt{b\,x^2+a}\,a+3x^{\frac{13}{4}}\sqrt{b\,x^2+a}\,b+4\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a^2\right)}{51}$$

input
```
int((c*x)^(1/4)*(b*x^2+a)^(3/2),x)
```

output
```
(4*c**(1/4)*(7*x**(1/4)*sqrt(a + b*x**2)*a*x + 3*x**(1/4)*sqrt(a + b*x**2)
*b*x**3 + 4*int((x**(1/4)*sqrt(a + b*x**2))/(a + b*x**2),x)*a**2))/51
```

3.674 $\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 538

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \frac{16a(cx)^{3/4}\sqrt{a+bx^2}}{35c} + \frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$-\frac{32a^{9/4}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(\cdots}{35\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{32a^{9/4}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(\cdots}{35\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
16/35*a*(c*x)^(3/4)*(b*x^2+a)^(1/2)/c+4/15*(c*x)^(3/4)*(b*x^2+a)^(3/2)/c-3
2/35*a^(9/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^
(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*El
lipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1
/2))/(2+2^(1/2))^(1/2)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c
*x)^(1/2))-32/35*a^(9/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*
(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1
/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/
2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2
*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2
)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.11

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \frac{4ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},\frac{11}{8},-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(1/4),x]
```

output
```
(4*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 3/8, 11/8, -((b*x^2)/a)])/(
3*(c*x)^(1/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {248, 248, 266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{\sqrt[4]{cx}}dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{4}{5}a\int \frac{\sqrt{bx^2+a}}{\sqrt[4]{cx}}dx+\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{4}{5}a\left(\frac{4}{7}a\int \frac{1}{\sqrt[4]{cx}\sqrt{bx^2+a}}dx+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}\right)+\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4}{5}a\left(\frac{16a\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}\right)+\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{ 838}$$

$$\frac{4}{5}a\left(\frac{16a\left(\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}\right)+\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{5}a\left(\frac{16a\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{7c}+\frac{4(cx)^{3/4}\sqrt{a+bx^2}}{7c}\right)+\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

$$\Big\downarrow \text{ 2422}$$

$$\frac{4}{5}a\left(\frac{16a\left(-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right.}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}\right.}{7}\right.$$

$$\frac{4(cx)^{3/4}\left(a+bx^2\right)^{3/2}}{15c}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(1/4),x]
```

output
```
(4*(c*x)^(3/4)*(a + b*x^2)^(3/2))/(15*c) + (4*a*((4*(c*x)^(3/4)*Sqrt[a + b
*x^2])/(7*c) + (16*a*(-1/2*(a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*
c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*
x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2
]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(
a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt
[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (a^(1/4)*Sqr
t[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt
[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c
*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*
a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2]
, -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c]
 - b^(1/4)*Sqrt[c*x]))))/(7*c)))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(cx)^{\frac{1}{4}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(1/4),x)
```

output
```
int((b*x^2+a)^(3/2)/(c*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^(3/4)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \frac{a^{\frac{3}{2}}x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{8} \\ \frac{11}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{c}\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(1/4),x)
```

output
```
a**(3/2)*x**(3/4)*gamma(3/8)*hyper((-3/2, 3/8), (11/8,), b*x**2*exp_polar(
I*pi)/a)/(2*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(1/4), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{1/4}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(1/4),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(1/4), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt[4]{cx}}\,dx = \frac{\frac{76x^{\frac{3}{4}}\sqrt{b\,x^2+a}\,a}{105}+\frac{4x^{\frac{11}{4}}\sqrt{b\,x^2+a}\,b}{15}+\frac{16\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{1}{4}}a+x^{\frac{9}{4}}b}dx\right)a^2}{35}}{c^{\frac{1}{4}}}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(1/4),x)
```

output
```
(4*(19*x**(3/4)*sqrt(a + b*x**2)*a + 7*x**(3/4)*sqrt(a + b*x**2)*b*x**2 +
12*int(sqrt(a + b*x**2)/(x**(1/4)*a + x**(1/4)*b*x**2),x)*a**2))/(105*c**(
1/4))
```

3.675 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 540

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx = \frac{48a\sqrt[4]{cx}\sqrt{a+bx^2}}{65c} + \frac{4\sqrt[4]{cx}(a+bx^2)^{3/2}}{13c}$$

$$+ \frac{96a^2\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2\right.}{65\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$- \frac{96a^2\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2\right.}{65\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
48/65*a*(c*x)^(1/4)*(b*x^2+a)^(1/2)/c+4/13*(c*x)^(1/4)*(b*x^2+a)^(3/2)/c+9
6/65*a^2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4
)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2
)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(
1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2)
)^(1/2))/(2+2^(1/2))^(1/2)/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x
)^(1/2))-96/65*a^2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2
)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^
(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(
1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2
+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^
(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx = \frac{4ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},\frac{9}{8},-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(3/4),x]
```

output
```
(4*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, 1/8, 9/8, -((b*x^2)/a)])/((
c*x)^(3/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {248, 248, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}}{(cx)^{3/4}}dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{12}{13}a\int \frac{\sqrt{bx^2+a}}{(cx)^{3/4}}dx+\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{12}{13}a\left(\frac{4}{5}a\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}\right)+\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{12}{13}a\left(\frac{16a\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}\right)+\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{ 767}$$

$$\frac{12}{13}a\left(\frac{16a\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}\right)+\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{12}{13}a\left(\frac{16a\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{5c}+\frac{4\sqrt[4]{cx}\sqrt{a+bx^2}}{5c}\right)+\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

$$\Big\downarrow \text{ 2422}$$

$$\frac{12}{13}a\left(\frac{16a\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)} -\right.}{5c}\right.$$

$$\frac{4\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}{13c}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(3/4),x]
```

output
```
(4*(c*x)^(1/4)*(a + b*x^2)^(3/2))/(13*c) + (12*a*((4*(c*x)^(1/4)*Sqrt[a +
b*x^2])/(5*c) + (16*a*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sq
rt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/
4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c
+ Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1
/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[
a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*S
qrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c
] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[A
rcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sq
rt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])
])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*
x]))))/(5*c)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{4}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(3/4),x)
```

output
```
int((b*x^2+a)^(3/2)/(c*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^(1/4)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx=\frac{a^{\frac{3}{2}}\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{8}\\\frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(3/4),x)
```

output
```
a**(3/2)*x**(1/4)*gamma(1/8)*hyper((-3/2, 1/8), (9/8,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(3/4), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{3/4}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(3/4),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(3/4), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{3/4}}\,dx = \frac{\frac{68x^{\frac{1}{4}}\sqrt{b\,x^2+a}\,a}{65}+\frac{4x^{\frac{9}{4}}\sqrt{b\,x^2+a}\,b}{13}+\frac{48\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a+x^{\frac{11}{4}}b}dx\right)a^2}{65}}{c^{\frac{3}{4}}}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(3/4),x)
```

output
```
(4*(17*x**(1/4)*sqrt(a + b*x**2)*a + 5*x**(1/4)*sqrt(a + b*x**2)*b*x**2 +
12*int(sqrt(a + b*x**2)/(x**(3/4)*a + x**(3/4)*b*x**2),x)*a**2))/(65*c**(3
/4))
```

3.676 $\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1045

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \text{Too large to display}$$

output
```
20/11*a*(b*x^2+a)^(1/2)/c/(c*x)^(1/4)+4/11*b*(c*x)^(7/4)*(b*x^2+a)^(1/2)/c
^3+32/11*(2+2^(1/2))^(1/2)*a^(7/4)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)
/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)
/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/
2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^
(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2
)+b^(1/4)*(c*x)^(1/2))+32/11*(2+2^(1/2))^(1/2)*a^(7/4)*b^(1/2)*(c*x)^(3/4)
*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/
2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^
(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(
1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/c^(3/2)/(b*x^2+a)^(
1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-32/11*a^(7/4)*b^(1/2)*(c*x)^(3/
4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1
/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*
c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c
^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-32/11*a^(7/
4)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(
1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ell
ipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = -\frac{4ax\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},\frac{7}{8},-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(3/2)/(c*x)^(5/4),x]
```

output
```
(-4*a*x*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, -1/8, 7/8, -((b*x^2)/a)])/
((c*x)^(5/4)*Sqrt[1 + (b*x^2)/a])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {247, 248, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{12b\int (cx)^{3/4}\sqrt{bx^2+a}dx}{c^2}-\frac{4(a+bx^2)^{3/2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{248}$$

$$\frac{12b\left(\frac{4}{11}a\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}\right)}{c^2}-\frac{4(a+bx^2)^{3/2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{12b\left(\frac{16a\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{11c}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}\right)}{c^2}-\frac{4(a+bx^2)^{3/2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{889}$$

$$\frac{12b\left(\frac{16a\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{11c\sqrt{a+bx^2}}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}\right)}{c^2}-\frac{4(a+bx^2)^{3/2}}{c\sqrt[4]{cx}}$$

$$\Big\downarrow \text{888}$$

$$\frac{12b\left(\frac{16a(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{77c\sqrt{a+bx^2}}+\frac{4(cx)^{7/4}\sqrt{a+bx^2}}{11c}\right)}{c^2}-\frac{4(a+bx^2)^{3/2}}{c\sqrt[4]{cx}}$$

input
```
Int[(a + b*x^2)^(3/2)/(c*x)^(5/4),x]
```

output
```
(-4*(a + b*x^2)^(3/2))/(c*(c*x)^(1/4)) + (12*b*((4*(c*x)^(7/4)*Sqrt[a + b*
x^2])/(11*c) + (16*a*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2
, 7/8, 15/8, -((b*x^2)/a)])/(77*c*Sqrt[a + b*x^2])))/c^2
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{4}}}dx$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(5/4),x)
```

output
```
int((b*x^2+a)^(3/2)/(c*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^(3/4)/(c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \frac{a^{\frac{3}{2}}\Gamma\left(-\frac{1}{8}\right)\,{}_2F_1\left(\begin{matrix} -\frac{3}{2},-\frac{1}{8} \\ \frac{7}{8} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate((b*x**2+a)**(3/2)/(c*x)**(5/4),x)
```

output
```
a**(3/2)*gamma(-1/8)*hyper((-3/2, -1/8), (7/8,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(5/4), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate((b*x^2+a)^(3/2)/(c*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)/(c*x)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \int \frac{(b\,x^2+a)^{3/2}}{(c\,x)^{5/4}}\,dx$$

input
```
int((a + b*x^2)^(3/2)/(c*x)^(5/4),x)
```

output
```
int((a + b*x^2)^(3/2)/(c*x)^(5/4), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/2}}{(cx)^{5/4}}\,dx = \frac{\frac{20\sqrt{b\,x^2+a}\,a}{11}+\frac{4\sqrt{b\,x^2+a}\,b\,x^2}{11}+\frac{16x^{\frac{1}{4}}\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{5}{4}}a+x^{\frac{13}{4}}b}dx\right)a^2}{11}}{x^{\frac{1}{4}}c^{\frac{5}{4}}}$$

input
```
int((b*x^2+a)^(3/2)/(c*x)^(5/4),x)
```

output
```
(4*(5*sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2 + 4*x**(1/4)*int(sqrt(a
 + b*x**2)/(x**(1/4)*a*x + x**(1/4)*b*x**3),x)*a**2))/(11*x**(1/4)*c**(1/4
)*c)
```

3.677 $\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 507

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}$$

$$-\frac{2ac(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-}{5\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$+\frac{2ac(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-}{5\sqrt{2+\sqrt{2}}b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
4/5*c*(c*x)^(1/4)*(b*x^2+a)^(1/2)/b-2/5*a*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2
)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4
)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1
/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)
^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2+a)^(1
/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+2/5*a*c*(c*x)^(3/4)*(-(b*x^2+a)/
a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)
/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2
)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)
/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2
+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.14

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \frac{4c\sqrt[4]{cx}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},-\frac{bx^2}{a}\right)\right)}{5b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*c*(c*x)^(1/4)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/8,
 1/2, 9/8, -((b*x^2)/a)]))/(5*b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {262, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx$$

$\Big\downarrow$ 262

$$\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{ac^2\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{5b}$$

$\Big\downarrow$ 266

$$\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5b}$$

$\Big\downarrow$ 767

$$\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{5b}$$

$\Big\downarrow$ 27

$$\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-\frac{4ac\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{5b}$$

$\Big\downarrow$ 2422

$$\frac{4c\sqrt[4]{cx}\sqrt{a+bx^2}}{5b}-$$

$$\frac{4ac\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(c}{}\right.}{5b}$$

input
```
Int[(c*x)^(5/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*c*(c*x)^(1/4)*Sqrt[a + b*x^2])/(5*b) - (4*a*c*((b^(1/4)*(c*x)^(3/4)*Sqr
t[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] +
b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin
[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[
c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])
/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x]
)) - (b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*
x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[
c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]
*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[
c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4
)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/(5*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{4}}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/4)*c*x/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{9}{8}\\ \frac{17}{8}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)/(b*x**2+a)**(1/2),x)
```

output
```
c**(5/4)*x**(9/4)*gamma(9/8)*hyper((1/2, 9/8), (17/8,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(17/8))
```

Maxima [F]

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/4)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/4)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx=\int \frac{(c\,x)^{5/4}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(5/4)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(5/4)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/4}}{\sqrt{a+bx^2}}\,dx=\frac{c^{\frac{5}{4}}\left(4x^{\frac{1}{4}}\sqrt{b\,x^2+a}-\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a+x^{\frac{11}{4}}b}dx\right)a\right)}{5b}$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*c*(4*x**(1/4)*sqrt(a + b*x**2) - int(sqrt(a + b*x**2)/(x**(3/4)*
a + x**(3/4)*b*x**2),x)*a))/(5*b)
```

3.678 $\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 483

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx =$$

$$-\frac{2\sqrt[4]{a}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2}{\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{2\sqrt[4]{a}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(}{\sqrt{2+\sqrt{2}}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
-2*a^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1
/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Elli
pticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(
c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2
))/(2+2^(1/2))^(1/2)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x
)^(1/2))-2*a^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(
1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(
1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1
/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{11}{8},-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(1/4)*Sqrt[a + b*x^2]),x]
```

output
```
(4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/8, 1/2, 11/8, -((b*x^2)/a)])/
(3*(c*x)^(1/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx$$

$\downarrow$ 266

$$\frac{4\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{c}$$

$\downarrow$ 838

$$\frac{4\left(\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{c}$$

$\downarrow$ 27

$$\frac{4\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{c}$$

$\downarrow$ 2422

$$\frac{4\left(-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{}\cdots}{\cdots}\right)}{c}$$

input
```
Int[1/((c*x)^(1/4)*Sqrt[a + b*x^2]),x]
```

output
```
(4*(-1/2*(a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*
Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(
1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt
[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sq
rt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^
2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (a^(1/4)*Sqrt[c]*(c*x)^(3/4)*S
qrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c
] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[A
rcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sq
rt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])
])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*
x]))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{4}} \sqrt{b\,x^{2}+a}}dx$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{8},\frac{1}{2}\\ \frac{11}{8}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt{a}\sqrt[4]{c}\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/2),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((3/8, 1/2), (11/8,), b*x**2*exp_polar(I*pi)/a)/(
2*sqrt(a)*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{1/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{1}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^3+ax}dx\right)}{\sqrt{c}}$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a*x + b*x**3),x))/sqrt(c)
```

3.679 $\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 479

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \frac{2\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\ldots}{\ldots}\right.}{\sqrt[4]{b}\ldots}}\right.\right.}{\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{2\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1\ldots\right.}{\sqrt{2+\sqrt{2}}c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/
2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellip
ticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c
*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2)
)/(2+2^(1/2))^(1/2)/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2)
)-2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^
(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*El
lipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2)/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/
2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.11

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/4)*Sqrt[a + b*x^2]),x]
```

output
```
(4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/8, 1/2, 9/8, -((b*x^2)/a)])/(
(c*x)^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 266$$

$$\frac{4\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{c}$$

$$\Big\downarrow \; 767$$

$$\frac{4\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{c}$$

$$\Big\downarrow \; 27$$

$$\frac{4\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{c}$$

$$\Big\downarrow \; 2422$$

$$\frac{4\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(cx)^{3}\cdots}{\cdots}\right.}{c}$$

input
```
Int[1/((c*x)^(3/4)*Sqrt[a + b*x^2]),x]
```

output
```
(4*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x
))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*
Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c
*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*
x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)
*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^
2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*
x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]
*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a
^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt
[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} \sqrt{b\,x^{2}+a}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \frac{\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{2}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x**2+a)**(1/2),x)
```

output
```
x**(1/4)*gamma(1/8)*hyper((1/8, 1/2), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2
*sqrt(a)*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt{a+bx^2}}\,dx = \frac{\int \frac{x^{\frac{5}{4}}\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx}{c^{\frac{1}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(3/4)*int((x**(5/4)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x))/(sqrt(c)*c
)
```

3.680 $\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 517

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = -\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

$$+\frac{2b(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{bx}}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\cdots}{5\sqrt{2+\sqrt{2}}a^{3/4}c^{5/2}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$+\frac{2b(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{bx}}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{bx}}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\cdots}{5\sqrt{2+\sqrt{2}}a^{3/4}c^{5/2}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
-4/5*(b*x^2+a)^(1/2)/a/c/(c*x)^(5/4)+2/5*b*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)
/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)
/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/
2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^
(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/c^(5/2)/(b*x^
2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+2/5*b*(c*x)^(3/4)*(-(b*x^
2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^
(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2
^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^
(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/c
^(5/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.11

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = -\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{8},-\frac{bx^2}{a}\right)}{5(cx)^{9/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(9/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/8, 1/2, 3/8, -((b*x^2)/a)])
/(5*(c*x)^(9/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {264, 266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \; 264$$

$$-\frac{b\int \frac{1}{\sqrt[4]{cx}\sqrt{bx^2+a}}dx}{5ac^2}-\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

$$\Big\downarrow \; 266$$

$$-\frac{4b\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{5ac^3}-\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

$$\Big\downarrow \; 838$$

$$-\frac{4b\left(\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{5ac^3}-\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

$$\Big\downarrow \; 27$$

$$-\frac{4b\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}\right)}{5ac^3}-\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

$$\Big\downarrow \; 2422$$

$$-\frac{4b\left(-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\right.}{5ac^3}$$

$$\frac{4\sqrt{a+bx^2}}{5ac(cx)^{5/4}}$$

input
```
Int[1/((c*x)^(9/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*Sqrt[a + b*x^2])/(5*a*c*(c*x)^(5/4)) - (4*b*(-1/2*(a^(1/4)*Sqrt[c]*(c*
x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4
)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*Elli
pticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b
^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1
- Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)
*Sqrt[c*x])) - (a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sq
rt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(
1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c
 + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(
1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[
a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/(5*a*c^3)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{9}{4}} \sqrt{b\,x^2+a}}dx$$

input
```
int(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.90 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{5}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{8}, \frac{1}{2} \\ \frac{3}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{9}{4}}x^{\frac{5}{4}}\Gamma\left(\frac{3}{8}\right)}$$

input
```
integrate(1/(c*x)**(9/4)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-5/8)*hyper((-5/8, 1/2), (3/8,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(9/4)*x**(5/4)*gamma(3/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(9/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(9/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{9/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(9/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(9/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{9/4}\sqrt{a+bx^2}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx}{c^{\frac{7}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x))/(sqrt(c)*c
**2)
```

3.681 $\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1011

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
4/9*c*(c*x)^(5/4)*(b*x^2+a)^(1/2)/b+10/9*(2+2^(1/2))^(1/2)*a^(3/2)*c^2*(c*
x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c
*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^
(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(
1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(5/4)/(b*
x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-10/9*(2+2^(1/2))^(1/2)*
a^(3/2)*c^2*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^
(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*El
lipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*
(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/
2))/b^(5/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-10/9*a^(
3/2)*c^2*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2
)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellipt
icF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*
x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/b^(5/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^
(1/2))+10/9*a^(3/2)*c^2*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(
-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/
2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2
)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.07

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \frac{4c(cx)^{5/4}\left(a+bx^2-a\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)\right)}{9b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(9/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*c*(c*x)^(5/4)*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2,
 5/8, 13/8, -((b*x^2)/a)]))/(9*b*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {262, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4c(cx)^{5/4}\sqrt{a+bx^2}}{9b}-\frac{5ac^2\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx}{9b}$$

$$\Big\downarrow \text{266}$$

$$\frac{4c(cx)^{5/4}\sqrt{a+bx^2}}{9b}-\frac{20ac\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{9b}$$

$$\Big\downarrow \text{889}$$

$$\frac{4c(cx)^{5/4}\sqrt{a+bx^2}}{9b}-\frac{20ac\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{9b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4c(cx)^{5/4}\sqrt{a+bx^2}}{9b}-\frac{4ac(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{9b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(9/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*c*(c*x)^(5/4)*Sqrt[a + b*x^2])/(9*b) - (4*a*c*(c*x)^(5/4)*Sqrt[1 + (b*x
^2)/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -((b*x^2)/a)])/(9*b*Sqrt[a + b*x^
2])
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{9}{4}}}{\sqrt{b\,x^2+a}}dx$$

input
```
int((c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/4)*c^2*x^2/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 49.75 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{9}{4}}x^{\frac{13}{4}}\Gamma\left(\frac{13}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{13}{8}\\ \frac{21}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{21}{8}\right)}$$

input
```
integrate((c*x)**(9/4)/(b*x**2+a)**(1/2),x)
```

output
```
c**(9/4)*x**(13/4)*gamma(13/8)*hyper((1/2, 13/8), (21/8,), b*x**2*exp_pola
r(I*pi)/a)/(2*sqrt(a)*gamma(21/8))
```

Maxima [F]

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/4)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(9/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/4)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{9/4}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(9/4)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(9/4)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/4}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{9}{4}}\left(4x^{\frac{5}{4}}\sqrt{b\,x^2+a}-5\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)a\right)}{9b}$$

input
```
int((c*x)^(9/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*c**2*(4*x**(1/4)*sqrt(a + b*x**2)*x - 5*int((x**(1/4)*sqrt(a + b
*x**2))/(a + b*x**2),x)*a))/(9*b)
```

3.682 $\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1019

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
4/3*c*(b*x^2+a)^(1/2)/b/(c*x)^(1/4)+2/3*(2+2^(1/2))^(1/2)*a^(3/4)*c^(1/2)*
(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)
*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(
-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/
a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/2)/
(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+2/3*(2+2^(1/2))^(1/2
)*a^(3/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1
/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1
/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2
))^(1/2))/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-2/
3*a^(3/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/
4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/
2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2)/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/
4)*(c*x)^(1/2))-2/3*a^(3/4)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2
)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/
2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1
/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.05

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \frac{4x(cx)^{3/4}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*x*(c*x)^(3/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((
b*x^2)/a)])/(7*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{4\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{c}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{4\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{c\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{4(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7c\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(3/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((b*
x^2)/a)])/(7*c*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{4}}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^(3/4)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{8}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)/(b*x**2+a)**(1/2),x)
```

output
```
c**(3/4)*x**(7/4)*gamma(7/8)*hyper((1/2, 7/8), (15/8,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(15/8))
```

Maxima [F]

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/4)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/4)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{3/4}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(3/4)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(3/4)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/4}}{\sqrt{a+bx^2}}\,dx = c^{\frac{3}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(1/2),x)
```

output
```
c**(3/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a + b*x**2),x)
```

3.683 $\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 965

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
-2*(2+2^(1/2))^(1/2)*a^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1
/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)
^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a
^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(
-2+2*2^(1/2))^(1/2))/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x
)^(1/2))+2*(2+2^(1/2))^(1/2)*a^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/
2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1
/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(
1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))
^(1/2),(-2+2*2^(1/2))^(1/2))/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1
/4)*(c*x)^(1/2))+2*a^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2
)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(
1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(
1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2
+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1
/2)+b^(1/4)*(c*x)^(1/2))-2*a^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)
/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2
)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/
2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(
1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \frac{4x\sqrt[4]{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(1/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*x*(c*x)^(1/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -((
b*x^2)/a])/(5*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.06, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{4\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{c}$$

$$\Big\downarrow \text{889}$$

$$\frac{4\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{c\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5c\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(1/4)/Sqrt[a + b*x^2],x]
```

output
```
(4*(c*x)^(5/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -((b*
x^2)/a)])/(5*c*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{4}}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/4)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.05

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{8}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)/(b*x**2+a)**(1/2),x)
```

output
```
c**(1/4)*x**(5/4)*gamma(5/8)*hyper((1/2, 5/8), (13/8,), b*x**2*exp_polar(I
*pi)/a)/(2*sqrt(a)*gamma(13/8))
```

Maxima [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/4)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/4)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^{1/4}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^(1/4)/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^(1/4)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt{a+bx^2}}\,dx = c^{\frac{1}{4}}\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(1/2),x)
```

output
```
c**(1/4)*int((x**(1/4)*sqrt(a + b*x**2))/(a + b*x**2),x)
```

3.684 $\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 985

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
2*(2+2^(1/2))^(1/2)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/
2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^
(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^
(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-
2+2*2^(1/2))^(1/2))/a^(1/4)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/
4)*(c*x)^(1/2))+2*(2+2^(1/2))^(1/2)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2
)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/
4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1
/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)
^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/
4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-2*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/
b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/
c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2
)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(
1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/4)/c^(3/2)/(b*x^2
+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-2*b^(1/2)*(c*x)^(3/4)*(-(b
*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2
/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)
*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))
/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = -\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{7}{8},-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/8, 1/2, 7/8, -((b*x^2)/a)])
/((c*x)^(5/4)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = $\{264, 266, 889, 888\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{3b\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{12b\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac^3}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}$$

$$\Big\downarrow \text{889}$$

$$\frac{12b\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}$$

$$\Big\downarrow \text{888}$$

$$\frac{12b(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}$$

input
```
Int[1/((c*x)^(5/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*Sqrt[a + b*x^2])/(a*c*(c*x)^(1/4)) + (12*b*(c*x)^(7/4)*Sqrt[1 + (b*x^2
)/a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((b*x^2)/a)])/(7*a*c^3*Sqrt[a + b*
x^2])
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{4}} \sqrt{b\,x^{2}+a}}dx$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{1}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{1}{2}\\ \frac{7}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate(1/(c*x)**(5/4)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-1/8)*hyper((-1/8, 1/2), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{5/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt{a+bx^2}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx}{c^{\frac{3}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x))/(sqrt(c)*c
)
```

3.685 $\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1013

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
-4/3*(b*x^2+a)^(1/2)/a/c/(c*x)^(3/4)-2/3*(2+2^(1/2))^(1/2)*b^(3/4)*(c*x)^(
3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^
(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4
)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)
/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/2)/c^2/(b*
x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+2/3*(2+2^(1/2))^(1/2)*b
^(3/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)
-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellipti
cE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)
^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a
^(1/2)/c^2/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))+2/3*b^(3/
4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1
/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/
2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/
2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/a^(1/2)/c^2/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1
/2))-2/3*b^(3/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/
4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/
2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(
1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx = -\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{5}{8},-\frac{bx^2}{a}\right)}{3(cx)^{7/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/8, 1/2, 5/8, -((b*x^2)/a)])
/(3*(c*x)^(7/4)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {264, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{b\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx}{3ac^2}-\frac{4\sqrt{a+bx^2}}{3ac(cx)^{3/4}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{3ac^3}-\frac{4\sqrt{a+bx^2}}{3ac(cx)^{3/4}}$$

$$\Big\downarrow \text{889}$$

$$\frac{4b\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{3ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{3ac(cx)^{3/4}}$$

$$\Big\downarrow \text{888}$$

$$\frac{4b(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{15ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{3ac(cx)^{3/4}}$$

input
```
Int[1/((c*x)^(7/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*Sqrt[a + b*x^2])/(3*a*c*(c*x)^(3/4)) + (4*b*(c*x)^(5/4)*Sqrt[1 + (b*x^
2)/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -((b*x^2)/a)])/(15*a*c^3*Sqrt[a +
b*x^2])
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{7}{4}} \sqrt{b\,x^{2}+a}}dx$$

input
```
int(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{3}{8}\right){}_2F_1\left(\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{5}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{7}{4}}x^{\frac{3}{4}}\Gamma\left(\frac{5}{8}\right)}$$

input
```
integrate(1/(c*x)**(7/4)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-3/8)*hyper((-3/8, 1/2), (5/8,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(a
)*c**(7/4)*x**(3/4)*gamma(5/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx=\int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(7/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx=\int \frac{1}{(c\,x)^{7/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(7/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(7/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{7/4}\sqrt{a+bx^2}}\,dx=\frac{\int \frac{x^{\frac{5}{4}}\sqrt{b\,x^2+a}}{b\,x^5+a\,x^3}dx}{c^{\frac{5}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(7/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(3/4)*int((x**(5/4)*sqrt(a + b*x**2))/(a*x**3 + b*x**5),x))/(sqrt(c)*c
**2)
```

3.686 $\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1021

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

output
```
-4/9*(b*x^2+a)^(1/2)/a/c/(c*x)^(9/4)-10/9*(2+2^(1/2))^(1/2)*b^(3/2)*(c*x)^
(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)
^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/
4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4
)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/c^(7/2
)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-10/9*(2+2^(1/2))^(
1/2)*b^(3/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c
^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*E
llipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)
*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1
/2))/a^(5/4)/c^(7/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
+10/9*b^(3/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c
^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*E
llipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/a^(5/4)/c^(7/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b
^(1/4)*(c*x)^(1/2))+10/9*b^(3/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x
)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/
(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)
*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = -\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{8},\frac{1}{2},-\frac{1}{8},-\frac{bx^2}{a}\right)}{9(cx)^{13/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(13/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-9/8, 1/2, -1/8, -((b*x^2)/a)]
)/(9*(c*x)^(13/4)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {264, 264, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{5b\int \frac{1}{(cx)^{5/4}\sqrt{bx^2+a}}dx}{9ac^2}-\frac{4\sqrt{a+bx^2}}{9ac(cx)^{9/4}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{5b\left(\frac{3b\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{9ac^2}-\frac{4\sqrt{a+bx^2}}{9ac(cx)^{9/4}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{5b\left(\frac{12b\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac^3}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{9ac^2}-\frac{4\sqrt{a+bx^2}}{9ac(cx)^{9/4}}$$

$$\Big\downarrow \text{ 889}$$

$$-\frac{5b\left(\frac{12b\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{9ac^2}-\frac{4\sqrt{a+bx^2}}{9ac(cx)^{9/4}}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{5b\left(\frac{12b(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{9ac^2}-\frac{4\sqrt{a+bx^2}}{9ac(cx)^{9/4}}$$

input
```
Int[1/((c*x)^(13/4)*Sqrt[a + b*x^2]),x]
```

output
```
(-4*Sqrt[a + b*x^2])/(9*a*c*(c*x)^(9/4)) - (5*b*((-4*Sqrt[a + b*x^2])/(a*c
*(c*x)^(1/4)) + (12*b*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/
2, 7/8, 15/8, -((b*x^2)/a)])/(7*a*c^3*Sqrt[a + b*x^2])))/(9*a*c^2)
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{13}{4}} \sqrt{b\,x^{2}+a}}dx$$

input
```
int(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x)
```

output
```
int(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b*c^4*x^6 + a*c^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 98.79 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{9}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{9}{8}, \frac{1}{2} \\ -\frac{1}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}c^{\frac{13}{4}}x^{\frac{9}{4}}\Gamma\left(-\frac{1}{8}\right)}$$

input
```
integrate(1/(c*x)**(13/4)/(b*x**2+a)**(1/2),x)
```

output
```
gamma(-9/8)*hyper((-9/8, 1/2), (-1/8,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt(
a)*c**(13/4)*x**(9/4)*gamma(-1/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(13/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\,(cx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*(c*x)^(13/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{13/4}\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/((c*x)^(13/4)*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/((c*x)^(13/4)*(a + b*x^2)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{13/4}\sqrt{a+bx^2}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b\,x^6+a\,x^4}dx}{c^{\frac{11}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(13/4)/(b*x^2+a)^(1/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a*x**4 + b*x**6),x))/(sqrt(c)*c
**3)
```

3.687 $\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 503

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = -\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

$$+\frac{c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{\ldots}}{2\sqrt{2+\sqrt{2}}\,b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{\ldots}}{2\sqrt{2+\sqrt{2}}\,b^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
-c*(c*x)^(1/4)/b/(b*x^2+a)^(1/2)+1/2*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(
1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(
1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b
^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2
))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2+a)^(1/2)/(
a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-1/2*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)
/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4
)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/
2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^
(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^(3/4)/(b*x^2+a)^(1/
2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.12

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \frac{c\sqrt[4]{cx}\left(-1+\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},-\frac{bx^2}{a}\right)\right)}{b\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/4)/(a + b*x^2)^(3/2),x]
```

output
```
(c*(c*x)^(1/4)*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/8, 1/2, 9/8,
-((b*x^2)/a)]))/(b*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {252, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx$$

$\downarrow$ 252

$$\frac{c^2\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{4b}-\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

$\downarrow$ 266

$$\frac{c\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{b}-\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

$\downarrow$ 767

$$\frac{c\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{b}-\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{c\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{b}-\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

$\downarrow$ 2422

$$\frac{c\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(cx)^{3}\cdots}{\cdots}\right.}{b}$$

$$\frac{c\sqrt[4]{cx}}{b\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(5/4)/(a + b*x^2)^(3/2),x]
```

output
```
-((c*(c*x)^(1/4))/(b*Sqrt[a + b*x^2])) + (c*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((
a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/
4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt
[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sq
rt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*S
qrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) -
(b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*
Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sq
rt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x
+ 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])
]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqr
t[c] - b^(1/4)*Sqrt[c*x]))))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.99 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right)\,{}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{3}{2}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)/(b*x**2+a)**(3/2),x)
```

output
```
c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 3/2), (17/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(17/8))
```

Maxima [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{5/4}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(5/4)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(5/4)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{5}{4}}\left(-4x^{\frac{1}{4}}\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a^2+2x^{\frac{11}{4}}ab+x^{\frac{19}{4}}b^2}dx\right)a^2+\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a^2+2x^{\frac{11}{4}}ab+x^{\frac{19}{4}}b^2}dx\right)ab\,x^2\right)}{3b\,(b\,x^2+a)}$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

output
```
(c**(1/4)*c*( - 4*x**(1/4)*sqrt(a + b*x**2) + int(sqrt(a + b*x**2)/(x**(3/
4)*a**2 + 2*x**(3/4)*a*b*x**2 + x**(3/4)*b**2*x**4),x)*a**2 + int(sqrt(a +
 b*x**2)/(x**(3/4)*a**2 + 2*x**(3/4)*a*b*x**2 + x**(3/4)*b**2*x**4),x)*a*b
*x**2))/(3*b*(a + b*x**2))
```

3.688 $\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1045

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}} \, dx = \text{Too large to display}$$

output
```
(c*x)^(7/4)/a/c/(b*x^2+a)^(1/2)-c*(b*x^2+a)^(1/2)/a/b/(c*x)^(1/4)-1/2*(2+2
^(1/2))^(1/2)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a
^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))
^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-
2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^
(1/2))^(1/2))/a^(1/4)/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*
x)^(1/2))-1/2*(2+2^(1/2))^(1/2)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^
(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c
^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*
b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/
2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c
^(1/2)-b^(1/4)*(c*x)^(1/2))+1/2*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^
(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^
(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*
b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/
2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/4)/b^(1/2)/(b*x^2+a
)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+1/2*c^(1/2)*(c*x)^(3/4)*(-(b
*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2
/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)
*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.06

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \frac{4x(cx)^{3/4}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{15}{8},-\frac{bx^2}{a}\right)}{7a\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/4)/(a + b*x^2)^(3/2),x]
```

output
```
(4*x*(c*x)^(3/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[7/8, 3/2, 15/8, -((
b*x^2)/a)])/(7*a*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {253, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{4a}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(3/4)/(a + b*x^2)^(3/2),x]
```

output
```
(c*x)^(7/4)/(a*c*Sqrt[a + b*x^2]) - (3*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hyp
ergeometric2F1[1/2, 7/8, 15/8, -((b*x^2)/a)])/(7*a*c*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right) {}_2F_1\left(\begin{matrix} \frac{7}{8}, \frac{3}{2} \\ \frac{15}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)/(b*x**2+a)**(3/2),x)
```

output
```
c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 3/2), (15/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(15/8))
```

Maxima [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{3/4}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(3/4)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(3/4)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{3/2}}\,dx = c^{\frac{3}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

output
```
c**(3/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x
)
```

3.689 $\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 998

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

output
```
(c*x)^(5/4)/a/c/(b*x^2+a)^(1/2)+1/2*(2+2^(1/2))^(1/2)*(c*x)^(3/4)*(-(b*x^2
+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1
/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^
(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(
1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/2)/b^(1/4)/(b*x^2+a)^(1
/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-1/2*(2+2^(1/2))^(1/2)*(c*x)^(3/4
)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1
/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c
^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/2)/b^(1/4)/(b
*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-1/2*(c*x)^(3/4)*(-(b*x
^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^
(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(
2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b
^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/
b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+1/2*(c*x)^(3
/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^
(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)
*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/
c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.06

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{4x\sqrt[4]{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{13}{8},-\frac{bx^2}{a}\right)}{5a\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(1/4)/(a + b*x^2)^(3/2),x]
```

output
```
(4*x*(c*x)^(1/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/8, 3/2, 13/8, -((
b*x^2)/a)])/(5*a*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {253, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx}{4a}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5ac\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^(1/4)/(a + b*x^2)^(3/2),x]
```

output
```
(c*x)^(5/4)/(a*c*Sqrt[a + b*x^2]) - ((c*x)^(5/4)*Sqrt[1 + (b*x^2)/a]*Hyper
geometric2F1[1/2, 5/8, 13/8, -((b*x^2)/a)])/(5*a*c*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^{2}+a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{3}{2}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)/(b*x**2+a)**(3/2),x)
```

output
```
c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 3/2), (13/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/2)*gamma(13/8))
```

Maxima [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = \int \frac{(c\,x)^{1/4}}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((c*x)^(1/4)/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^(1/4)/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{3/2}}\,dx = c^{\frac{1}{4}}\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b^2x^4+2ab\,x^2+a^2}dx\right)$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

output
```
c**(1/4)*int((x**(1/4)*sqrt(a + b*x**2))/(a**2 + 2*a*b*x**2 + b**2*x**4),x
)
```

3.690 $\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 512

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

$$-\frac{(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right), -2(1-\sqrt{}\right.}{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right), -2(1-\sqrt{}\right.}{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
(c*x)^(3/4)/a/c/(b*x^2+a)^(1/2)-1/2*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2
)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2
)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1
/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^
(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/c^(1/2)/(b*x^2+a)^(1
/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-1/2*(c*x)^(3/4)*(-(b*x^2+a)/a^(1
/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(
1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^
(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*
x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/c^(1/2)/(b
*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.12

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{11}{8},-\frac{bx^2}{a}\right)}{3a\sqrt[4]{cx}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(3/2)),x]
```

output
```
(4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/8, 3/2, 11/8, -((b*x^2)/a)])/
(3*a*(c*x)^(1/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {253, 266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}dx$$

$\big\downarrow$ 253

$$\frac{\int \frac{1}{\sqrt[4]{cx}\sqrt{bx^2+a}}dx}{4a}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

$\big\downarrow$ 266

$$\frac{\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

$\big\downarrow$ 838

$$\frac{\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

$\big\downarrow$ 27

$$\frac{\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

$\big\downarrow$ 2422

$$\frac{-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{a}\sqrt{c}\ldots}{\ldots}}{ac}$$

$$\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(1/4)*(a + b*x^2)^(3/2)),x]
```

output
```
(c*x)^(3/4)/(a*c*Sqrt[a + b*x^2]) + (-1/2*(a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqr
t[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] +
b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin
[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[
c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])
/(Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x]))
 - (a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b
]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)
*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*S
qrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]
*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(
a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{4}} (b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

output
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx=\frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{3}{2}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/(c*x)**(1/4)/(b*x**2+a)**(3/2),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((3/8, 3/2), (11/8,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(3/2)*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{1/4}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{3/2}}\,dx = \frac{c^{\frac{1}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b^2x^5+2ab\,x^3+a^2x}dx\right)}{\sqrt{c}}$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(3/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**2*x + 2*a*b*x**3 + b**2*x**5
),x))/sqrt(c)
```

3.691 $\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 514

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}\,dx = \frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$$+ \frac{3\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\right.}{2\sqrt{2+\sqrt{2}}ac\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$- \frac{3\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\right.}{2\sqrt{2+\sqrt{2}}ac\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
(c*x)^(1/4)/a/c/(b*x^2+a)^(1/2)+3/2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2
)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4
)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1
/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)
^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a/c/(b*x^2+a)^(1/2)/
(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-3/2*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/
a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)
/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2
)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)
/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a/c/(b*x^2+a)^
(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.12

$$\int \frac{1}{(cx)^{3/4} (a+bx^2)^{3/2}} \, dx = \frac{x + 3x\sqrt{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(\frac{1}{8}, \frac{1}{2}, \frac{9}{8}, -\frac{bx^2}{a}\right)}{a(cx)^{3/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(3/2)),x]
```

output
```
(x + 3*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/8, 1/2, 9/8, -((b*x^2)/a)
])/(a*(c*x)^(3/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {253, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}dx$$

$\Big\downarrow$ 253

$$\frac{3\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{4a}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$\Big\downarrow$ 266

$$\frac{3\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$\Big\downarrow$ 767

$$\frac{3\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

$\Big\downarrow$ 2422

$$\frac{3\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(cx)^{3}\cdots}{\cdots}\right.}{ac}$$

$$\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(3/4)*(a + b*x^2)^(3/2)),x]
```

output
```
(c*x)^(1/4)/(a*c*Sqrt[a + b*x^2]) + (3*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2
 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sq
rt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((S
qrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*
x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2
 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1
/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[
-((a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*
x]))]*EllipticF[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a
^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2],
 -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c]
- b^(1/4)*Sqrt[c*x]))))/(a*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4} (a+bx^2)^{3/2}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}} (cx)^{\frac{3}{4}}} \, dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}\,dx=\frac{\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{3}{2}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x**2+a)**(3/2),x)
```

output
```
x**(1/4)*gamma(1/8)*hyper((1/8, 3/2), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(3/2)*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{2}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{3/2}}\,dx = \frac{\int \frac{x^{\frac{5}{4}}\sqrt{b\,x^2+a}}{b^2x^6+2ab\,x^4+a^2x^2}dx}{c^{\frac{1}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(3/2),x)
```

output
```
(c**(3/4)*int((x**(5/4)*sqrt(a + b*x**2))/(a**2*x**2 + 2*a*b*x**4 + b**2*x
**6),x))/(sqrt(c)*c)
```

3.692 $\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1018

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}\,dx = \text{Too large to display}$$

output
```
1/a/c/(c*x)^(1/4)/(b*x^2+a)^(1/2)+5/2*(2+2^(1/2))^(1/2)*b^(1/2)*(c*x)^(3/4
)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/
2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c
^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^
(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/c^(3/2)/(b
*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+5/2*(2+2^(1/2))^(1/2)*
b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2
)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellipt
icE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/
a^(5/4)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-5/2*
b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)
+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellipti
cF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2)/a^(5/4)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)
*(c*x)^(1/2))-5/2*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)
*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(
1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1
/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.06

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{3/2}}\,dx=-\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{3}{2},\frac{7}{8},-\frac{bx^2}{a}\right)}{a(cx)^{5/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(3/2)),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/8, 3/2, 7/8, -((b*x^2)/a)])
/(a*(c*x)^(5/4)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {253, 264, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{1}{(cx)^{5/4}\sqrt{bx^2+a}}dx}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5\left(\frac{3b\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{5\left(\frac{12b\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac^3}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{5\left(\frac{12b\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{5\left(\frac{12b(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}$$

input
```
Int[1/((c*x)^(5/4)*(a + b*x^2)^(3/2)),x]
```

output
```
1/(a*c*(c*x)^(1/4)*Sqrt[a + b*x^2]) + (5*((-4*Sqrt[a + b*x^2])/(a*c*(c*x)^
(1/4)) + (12*b*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 7/8,
 15/8, -((b*x^2)/a)])/(7*a*c^3*Sqrt[a + b*x^2])))/(4*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{4}} (b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

output
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/4}\,(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^
2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{5/4}\,(a+bx^2)^{3/2}}\,dx = \frac{\Gamma\left(-\frac{1}{8}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{3}{2} \\ \frac{7}{8} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate(1/(c*x)**(5/4)/(b*x**2+a)**(3/2),x)
```

output
```
gamma(-1/8)*hyper((-1/8, 3/2), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
2)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{2}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/2)*(c*x)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x)^{5/4}\,(b\,x^2+a)^{3/2}}\,dx$$

input
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(3/2)),x)
```

output
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/4} (a+bx^2)^{3/2}} \, dx = \frac{\int \frac{x^{\frac{3}{4}} \sqrt{b\,x^2+a}}{b^2 x^6+2ab\,x^4+a^2 x^2} dx}{c^{\frac{3}{4}} \sqrt{c}}$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(3/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**2*x**2 + 2*a*b*x**4 + b**2*x
**6),x))/(sqrt(c)*c)
```

3.693 $\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 540

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = -\frac{c\sqrt[4]{cx}}{3b(a+bx^2)^{3/2}}+\frac{c\sqrt[4]{cx}}{12ab\sqrt{a+bx^2}}$$

$$+\frac{c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{\ldots}\right.}{8\sqrt{2+\sqrt{2}}ab^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{c(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{\ldots}\right.}{8\sqrt{2+\sqrt{2}}ab^{3/4}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
-1/3*c*(c*x)^(1/4)/b/(b*x^2+a)^(3/2)+1/12*c*(c*x)^(1/4)/a/b/(b*x^2+a)^(1/2
)+1/8*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)
+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*Ellipti
cF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x
)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2)/a/b^(3/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)
^(1/2))-1/8*c*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*
c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*
EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/a/b^(3/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)
*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.15

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \frac{c\sqrt[4]{cx}\left(-3a+bx^2+3(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},-\frac{bx^2}{a}\right)\right)}{12ab\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(c*x)^(5/4)/(a + b*x^2)^(5/2),x]
```

output
```
(c*(c*x)^(1/4)*(-3*a + b*x^2 + 3*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeom
etric2F1[1/8, 1/2, 9/8, -((b*x^2)/a)]))/(12*a*b*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {252, 253, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{c^2\int \frac{1}{(cx)^{3/4}(bx^2+a)^{3/2}}dx}{12b}-\frac{c\sqrt[4]{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$\frac{c^2\left(\frac{3\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{4a}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12b}-\frac{c\sqrt[4]{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{c^2\left(\frac{3\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12b}-\frac{c\sqrt[4]{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 767$$

$$\frac{c^2\left(\frac{3\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12b}-\frac{c\sqrt[4]{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2\left(\frac{3\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12b}-\frac{c\sqrt[4]{cx}}{3b\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 2422$$

$$\frac{c^2\left(\frac{3\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\cdots}}{\cdots}\right)}{ac}\cdots\right)}{12b}$$

$$\frac{c\sqrt[4]{cx}}{3b\left(a+bx^2\right)^{3/2}}$$

input

```
Int[(c*x)^(5/4)/(a + b*x^2)^(5/2),x]
```

output

```
-1/3*(c*(c*x)^(1/4))/(b*(a + b*x^2)^(3/2)) + (c^2*((c*x)^(1/4)/(a*c*Sqrt[a
 + b*x^2]) + (3*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*
Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(
1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt
[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sq
rt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*
x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*Sqrt[-(
(a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^
(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[
Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*
Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*
Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))
/(a*c)))/(12*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)*c*x/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*
x^2 + a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.86 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{9}{8},\frac{5}{2}\\ \frac{17}{8}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)/(b*x**2+a)**(5/2),x)
```

output
```
c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 5/2), (17/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(17/8))
```

Maxima [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{5/4}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(5/4)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(5/4)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{5}{4}}\left(-4x^{\frac{1}{4}}\sqrt{b\,x^2+a}+\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a^3+3x^{\frac{11}{4}}a^2b+3x^{\frac{19}{4}}a\,b^2+x^{\frac{27}{4}}b^3}dx\right)a^3+2\left(\int \frac{\sqrt{b\,x^2+a}}{x^{\frac{3}{4}}a^3+3x^{\frac{11}{4}}a^2b+3x^{\frac{19}{4}}a\,b\right.\right.}{11b\left(b^2x^4+2ab\,x^2+a^2\right)}$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

output
```
(c**(1/4)*c*( - 4*x**(1/4)*sqrt(a + b*x**2) + int(sqrt(a + b*x**2)/(x**(3/
4)*a**3 + 3*x**(3/4)*a**2*b*x**2 + 3*x**(3/4)*a*b**2*x**4 + x**(3/4)*b**3*
x**6),x)*a**3 + 2*int(sqrt(a + b*x**2)/(x**(3/4)*a**3 + 3*x**(3/4)*a**2*b*
x**2 + 3*x**(3/4)*a*b**2*x**4 + x**(3/4)*b**3*x**6),x)*a**2*b*x**2 + int(s
qrt(a + b*x**2)/(x**(3/4)*a**3 + 3*x**(3/4)*a**2*b*x**2 + 3*x**(3/4)*a*b**
2*x**4 + x**(3/4)*b**3*x**6),x)*a*b**2*x**4))/(11*b*(a**2 + 2*a*b*x**2 + b
**2*x**4))
```

3.694 $\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1078

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3*(c*x)^(7/4)/a/c/(b*x^2+a)^(3/2)+5/12*(c*x)^(7/4)/a^2/c/(b*x^2+a)^(1/2)
-5/12*c*(b*x^2+a)^(1/2)/a^2/b/(c*x)^(1/4)-5/24*(2+2^(1/2))^(1/2)*c^(1/2)*(
c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*
(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-
a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a
^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/b
^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-5/24*(2+2^(1/
2))^(1/2)*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1
/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1
/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^
(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2
))^(1/2))/a^(5/4)/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(
1/2))+5/24*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1
/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1
/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b
^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/4)/b^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1
/2)+b^(1/4)*(c*x)^(1/2))+5/24*c^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1
/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(
1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.05

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx = \frac{4x(cx)^{3/4}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{15}{8},-\frac{bx^2}{a}\right)}{7a^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/4)/(a + b*x^2)^(5/2),x]
```

output
```
(4*x*(c*x)^(3/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[7/8, 5/2, 15/8, -((
b*x^2)/a)])/(7*a^2*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {253, 253, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\int \frac{(cx)^{3/4}}{(bx^2+a)^{3/2}}dx}{12a}+\frac{(cx)^{7/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5\left(\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{4a}\right)}{12a}+\frac{(cx)^{7/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{5\left(\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}\right)}{12a}+\frac{(cx)^{7/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 889}$$

$$\frac{5\left(\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{(cx)^{7/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{5\left(\frac{(cx)^{7/4}}{ac\sqrt{a+bx^2}}-\frac{3(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac\sqrt{a+bx^2}}\right)}{12a}+\frac{(cx)^{7/4}}{3ac\,(a+bx^2)^{3/2}}$$

input
```
Int[(c*x)^(3/4)/(a + b*x^2)^(5/2),x]
```

output
```
(c*x)^(7/4)/(3*a*c*(a + b*x^2)^(3/2)) + (5*((c*x)^(7/4)/(a*c*Sqrt[a + b*x^
2]) - (3*(c*x)^(7/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 7/8, 15/8,
 -((b*x^2)/a)])/(7*a*c*Sqrt[a + b*x^2])))/(12*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^{2}+a)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2
+ a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx = \frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right)\,{}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{5}{2}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)/(b*x**2+a)**(5/2),x)
```

output
```
c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 5/2), (15/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(15/8))
```

Maxima [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx=\int \frac{(c\,x)^{3/4}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(3/4)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(3/4)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{5/2}}\,dx = c^{\frac{3}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

output
```
c**(3/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*
x**4 + b**3*x**6),x)
```

3.695 $\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1029

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3*(c*x)^(5/4)/a/c/(b*x^2+a)^(3/2)+7/12*(c*x)^(5/4)/a^2/c/(b*x^2+a)^(1/2)
+7/24*(2+2^(1/2))^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(
(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2
))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2
)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*
2^(1/2))^(1/2))/a^(3/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(
c*x)^(1/2))-7/24*(2+2^(1/2))^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)
/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2
)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/
2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(
1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2
)-b^(1/4)*(c*x)^(1/2))-7/24*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/
2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^
(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^
(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-
2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/2)/b^(1/4)/(b*x^2+a)^(1/2)/(a^(
1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))+7/24*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(
1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^
(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b
^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.06

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{4x\sqrt[4]{cx}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{13}{8},-\frac{bx^2}{a}\right)}{5a^2\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^(1/4)/(a + b*x^2)^(5/2),x]
```

output
```
(4*x*(c*x)^(1/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/8, 5/2, 13/8, -((
b*x^2)/a)])/(5*a^2*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {253, 253, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{\sqrt[4]{cx}}{(bx^2+a)^{3/2}}dx}{12a}+\frac{(cx)^{5/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{7\left(\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{\sqrt[4]{cx}}{\sqrt{bx^2+a}}dx}{4a}\right)}{12a}+\frac{(cx)^{5/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{7\left(\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\int \frac{cx}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}\right)}{12a}+\frac{(cx)^{5/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{7\left(\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{cx}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{(cx)^{5/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{7\left(\frac{(cx)^{5/4}}{ac\sqrt{a+bx^2}}-\frac{(cx)^{5/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5ac\sqrt{a+bx^2}}\right)}{12a}+\frac{(cx)^{5/4}}{3ac\,(a+bx^2)^{3/2}}$$

input
```
Int[(c*x)^(1/4)/(a + b*x^2)^(5/2),x]
```

output
```
(c*x)^(5/4)/(3*a*c*(a + b*x^2)^(3/2)) + (7*((c*x)^(5/4)/(a*c*Sqrt[a + b*x^
2]) - ((c*x)^(5/4)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 5/8, 13/8, -
((b*x^2)/a)])/(5*a*c*Sqrt[a + b*x^2])))/(12*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2
+ a^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.04

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{5}{2}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)/(b*x**2+a)**(5/2),x)
```

output
```
c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 5/2), (13/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/2)*gamma(13/8))
```

Maxima [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^{1/4}}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^(1/4)/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^(1/4)/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{5/2}}\,dx = c^{\frac{1}{4}}\left(\int \frac{x^{\frac{1}{4}}\sqrt{b\,x^2+a}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

output
```
c**(1/4)*int((x**(1/4)*sqrt(a + b*x**2))/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*
x**4 + b**3*x**6),x)
```

3.696 $\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 543

$$\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{5/2}}\,dx = \frac{(cx)^{3/4}}{3ac\left(a+bx^2\right)^{3/2}}+\frac{3(cx)^{3/4}}{4a^2c\sqrt{a+bx^2}}$$

$$-\frac{3(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{}}{8\sqrt{2+\sqrt{2}}a^{7/4}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$-\frac{3(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1-\sqrt{}}{8\sqrt{2+\sqrt{2}}a^{7/4}\sqrt{c}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
1/3*(c*x)^(3/4)/a/c/(b*x^2+a)^(3/2)+3/4*(c*x)^(3/4)/a^2/c/(b*x^2+a)^(1/2)-
3/8*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(
1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1
/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1
/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2)/a^(7/4)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*
x)^(1/2))-3/8*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*
c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*
EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4
)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/a^(7/4)/c^(1/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b
^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.11

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{11}{8},-\frac{bx^2}{a}\right)}{3a^2\sqrt[4]{cx}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(5/2)),x]
```

output
```
(4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/8, 5/2, 11/8, -((b*x^2)/a)])/
(3*a^2*(c*x)^(1/4)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 582, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {253, 253, 266, 838, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{3\int \frac{1}{\sqrt[4]{cx}(bx^2+a)^{3/2}}dx}{4a}+\frac{(cx)^{3/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 253$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt[4]{cx}\sqrt{bx^2+a}}dx}{4a}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}\right)}{4a}+\frac{(cx)^{3/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{3\left(\frac{\int \frac{\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}\right)}{4a}+\frac{(cx)^{3/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 838$$

$$\frac{3\left(\frac{\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\sqrt[4]{a}\sqrt{c}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}\right)}{4a}+\frac{(cx)^{3/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}-\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{b}}}{ac}+\frac{(cx)^{3/4}}{ac\sqrt{a+bx^2}}\right)}{4a}+\frac{(cx)^{3/4}}{3ac\,(a+bx^2)^{3/2}}$$

$$\Big\downarrow \; 2422$$

$$\frac{3\left(\frac{-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{a}\sqrt{c}(cx)^{3/4}\cdots}{\cdots}}{ac}\cdots\right)}{4a}$$

$$\frac{(cx)^{3/4}}{3ac\left(a+bx^2\right)^{3/2}}$$

input

```
Int[1/((c*x)^(1/4)*(a + b*x^2)^(5/2)),x]
```

output

```
(c*x)^(3/4)/(3*a*c*(a + b*x^2)^(3/2)) + (3*((c*x)^(3/4)/(a*c*Sqrt[a + b*x^
2]) + (-1/2*(a^(1/4)*Sqrt[c]*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[
a]*Sqrt[b]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*
b^(1/4)*Sqrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + S
qrt[2]*Sqrt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)
*Sqrt[c]*Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*Sqrt[a + b
*x^2]*(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (a^(1/4)*Sqrt[c]*(c*x)^(3/4
)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqr
t[c] - b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*Elliptic
F[ArcSin[Sqrt[(Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)
*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[
2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt
[c*x]))))/(a*c)))/(4*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{4}}\,(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

output
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^3*c*x^7 + 3*a*b^2*c*x^5 + 3*a^2*b*
c*x^3 + a^3*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{5}{2}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/(c*x)**(1/4)/(b*x**2+a)**(5/2),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((3/8, 5/2), (11/8,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(5/2)*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{2}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx=\int \frac{1}{(c\,x)^{1/4}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{5/2}}\,dx=\frac{c^{\frac{1}{4}}\left(\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx\right)}{\sqrt{c}}$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(5/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**3*x + 3*a**2*b*x**3 + 3*a*b*
*2*x**5 + b**3*x**7),x))/sqrt(c)
```

3.697 $\int \frac{1}{(cx)^{3/4}(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 545

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{5/2}}\,dx = \frac{\sqrt[4]{cx}}{3ac(a+bx^2)^{3/2}} + \frac{11\sqrt[4]{cx}}{12a^2c\sqrt{a+bx^2}}$$

$$+ \frac{11\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{8\sqrt{2+\sqrt{2}}a^2c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}$$

$$- \frac{11\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{a+bx^2}{\sqrt{a}\sqrt{b}x}}\sqrt{-\frac{\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\sqrt{c}\left(\sqrt{2}+\frac{\sqrt{2}\sqrt{b}x}{\sqrt{a}}+\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}}\right)}{\sqrt[4]{b}\sqrt{cx}}}\right),-2(1}{8\sqrt{2+\sqrt{2}}a^2c\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}\right)}$$

output
```
1/3*(c*x)^(1/4)/a/c/(b*x^2+a)^(3/2)+11/12*(c*x)^(1/4)/a^2/c/(b*x^2+a)^(1/2
)+11/8*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*
c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*
EllipticF(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/
4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^
(1/2))/(2+2^(1/2))^(1/2)/a^2/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c
*x)^(1/2))-11/8*b^(1/4)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(
-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/
2))^(1/2)*EllipticF(1/2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2
)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^2/c/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)-b
^(1/4)*(c*x)^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{1}{(cx)^{3/4} (a+bx^2)^{5/2}} dx = \frac{15ax + 11bx^3 + 33x(a+bx^2)\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{9}{8},-\frac{bx^2}{a}\right)}{12a^2(cx)^{3/4}(a+bx^2)^{3/2}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(5/2)),x]
```

output
```
(15*a*x + 11*b*x^3 + 33*x*(a + b*x^2)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F
1[1/8, 1/2, 9/8, -((b*x^2)/a)])/(12*a^2*(c*x)^(3/4)*(a + b*x^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {253, 253, 266, 767, 27, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11\int \frac{1}{(cx)^{3/4}(bx^2+a)^{3/2}}dx}{12a}+\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{11\left(\frac{3\int \frac{1}{(cx)^{3/4}\sqrt{bx^2+a}}dx}{4a}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{11\left(\frac{3\int \frac{1}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{767}$$

$$\frac{11\left(\frac{3\left(\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}+\frac{1}{2}\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a}\sqrt{c}\sqrt{bx^2+a}}d\sqrt[4]{cx}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{11\left(\frac{3\left(\frac{\int \frac{\sqrt[4]{a}\sqrt{c}-\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}+\frac{\int \frac{\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{2\sqrt[4]{a}\sqrt{c}}\right)}{ac}+\frac{\sqrt[4]{cx}}{ac\sqrt{a+bx^2}}\right)}{12a}+\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{2422}$$

$$11\left(\frac{3\left(\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{-\frac{ac^2+bc^2x^2}{\sqrt{a}\sqrt{b}c^2x}}\sqrt{\frac{\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)^2}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}\sqrt{b}xc+\sqrt{2}\sqrt{a}c-2\sqrt[4]{a}\sqrt[4]{b}\sqrt{cx}\sqrt{c}}{\sqrt[4]{a}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\sqrt{a+bx^2}\left(\sqrt[4]{a}\sqrt{c}+\sqrt[4]{b}\sqrt{cx}\right)}-\frac{\sqrt[4]{b}(cx)^{3/4}\sqrt{\cdots}}{\cdots}\right)}{ac}\cdots\right)}{12a}$$

$$\frac{\sqrt[4]{cx}}{3ac\left(a+bx^2\right)^{3/2}}$$

input
```
Int[1/((c*x)^(3/4)*(a + b*x^2)^(5/2)),x]
```

output
```
(c*x)^(1/4)/(3*a*c*(a + b*x^2)^(3/2)) + (11*((c*x)^(1/4)/(a*c*Sqrt[a + b*x
^2]) + (3*((b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b
]*c^2*x))]*Sqrt[(a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*S
qrt[c]*Sqrt[c*x])]*EllipticF[ArcSin[Sqrt[-((Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sq
rt[b]*c*x - 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*
Sqrt[c*x]))]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2 + Sqrt[2]]*Sqrt[a + b*x^2]*(
a^(1/4)*Sqrt[c] + b^(1/4)*Sqrt[c*x])) - (b^(1/4)*(c*x)^(3/4)*Sqrt[-((a*c^2
 + b*c^2*x^2)/(Sqrt[a]*Sqrt[b]*c^2*x))]*Sqrt[-((a^(1/4)*Sqrt[c] - b^(1/4)*
Sqrt[c*x])^2/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x]))]*EllipticF[ArcSin[Sqrt[(
Sqrt[2]*Sqrt[a]*c + Sqrt[2]*Sqrt[b]*c*x + 2*a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c
*x])/(a^(1/4)*b^(1/4)*Sqrt[c]*Sqrt[c*x])]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2
 + Sqrt[2]]*Sqrt[a + b*x^2]*(a^(1/4)*Sqrt[c] - b^(1/4)*Sqrt[c*x]))))/(a*c)
))/(12*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 767
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/2   Int[(1 - Rt[b/a, 4
]*x^2)/Sqrt[a + b*x^8], x], x] + Simp[1/2   Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a
 + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(1/4)/(b^3*c*x^7 + 3*a*b^2*c*x^5 + 3*a^2*b*
c*x^3 + a^3*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.08

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{5/2}}\,dx = \frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{5}{2}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x**2+a)**(5/2),x)
```

output
```
x**(1/4)*gamma(1/8)*hyper((1/8, 5/2), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(5/2)*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(b\,x^2+a)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}\,(a+bx^2)^{5/2}}\,dx = \frac{\int \frac{x^{\frac{5}{4}}\sqrt{b\,x^2+a}}{b^3x^8+3a\,b^2x^6+3a^2b\,x^4+a^3x^2}dx}{c^{\frac{1}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(5/2),x)
```

output
```
(c**(3/4)*int((x**(5/4)*sqrt(a + b*x**2))/(a**3*x**2 + 3*a**2*b*x**4 + 3*a
*b**2*x**6 + b**3*x**8),x))/(sqrt(c)*c)
```

3.698 $\int \frac{1}{(cx)^{5/4}(a+bx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 1049

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

output
```
1/3/a/c/(c*x)^(1/4)/(b*x^2+a)^(3/2)+13/12/a^2/c/(c*x)^(1/4)/(b*x^2+a)^(1/2
)+65/24*(2+2^(1/2))^(1/2)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/
x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/
(c*x)^(1/2))^(1/2)*EllipticE(1/2*(-a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2
)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(1/4)/(c*x)^(1/2))^(1
/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)
+b^(1/4)*(c*x)^(1/2))+65/24*(2+2^(1/2))^(1/2)*b^(1/2)*(c*x)^(3/4)*(-(b*x^2
+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))^2/a^(
1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticE(1/2*(a^(1/4)*c^(1/2)*(2^
(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b^(
1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/c^(3/2)/(b*x^2+a)^(1
/2)/(a^(1/4)*c^(1/2)-b^(1/4)*(c*x)^(1/2))-65/24*b^(1/2)*(c*x)^(3/4)*(-(b*x
^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*((a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))^2/a^
(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/2*(-a^(1/4)*c^(1/2)*(
2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)-2*b^(1/4)*(c*x)^(1/2)/a^(1/4)/c^(1/2))/b
^(1/4)/(c*x)^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(9/4)/
c^(3/2)/(b*x^2+a)^(1/2)/(a^(1/4)*c^(1/2)+b^(1/4)*(c*x)^(1/2))-65/24*b^(1/2
)*(c*x)^(3/4)*(-(b*x^2+a)/a^(1/2)/b^(1/2)/x)^(1/2)*(-(a^(1/4)*c^(1/2)-b^(1
/4)*(c*x)^(1/2))^2/a^(1/4)/b^(1/4)/c^(1/2)/(c*x)^(1/2))^(1/2)*EllipticF(1/
2*(a^(1/4)*c^(1/2)*(2^(1/2)+2^(1/2)*b^(1/2)*x/a^(1/2)+2*b^(1/4)*(c*x)^(...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{5/2}}\,dx = -\frac{4x\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{8},-\frac{bx^2}{a}\right)}{a^2(cx)^{5/4}\sqrt{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(5/2)),x]
```

output
```
(-4*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/8, 5/2, 7/8, -((b*x^2)/a)])
/(a^2*(c*x)^(5/4)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {253, 253, 264, 266, 889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{13\int \frac{1}{(cx)^{5/4}(bx^2+a)^{3/2}}dx}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{253}$$

$$\frac{13\left(\frac{5\int \frac{1}{(cx)^{5/4}\sqrt{bx^2+a}}dx}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}\right)}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{13\left(\frac{5\left(\frac{3b\int \frac{(cx)^{3/4}}{\sqrt{bx^2+a}}dx}{ac^2}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}\right)}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{13\left(\frac{5\left(\frac{12b\int \frac{(cx)^{3/2}}{\sqrt{bx^2+a}}d\sqrt[4]{cx}}{ac^3}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}\right)}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{889}$$

$$\frac{13\left(\frac{5\left(\frac{12b\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/2}}{\sqrt{\frac{bx^2}{a}+1}}d\sqrt[4]{cx}}{ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}\right)}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{888}$$

$$\frac{13\left(\frac{5\left(\frac{12b(cx)^{7/4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac^3\sqrt{a+bx^2}}-\frac{4\sqrt{a+bx^2}}{ac\sqrt[4]{cx}}\right)}{4a}+\frac{1}{ac\sqrt[4]{cx}\sqrt{a+bx^2}}\right)}{12a}+\frac{1}{3ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/2}}$$

input
```
Int[1/((c*x)^(5/4)*(a + b*x^2)^(5/2)),x]
```

output
```
1/(3*a*c*(c*x)^(1/4)*(a + b*x^2)^(3/2)) + (13*(1/(a*c*(c*x)^(1/4)*Sqrt[a +
 b*x^2]) + (5*((-4*Sqrt[a + b*x^2])/(a*c*(c*x)^(1/4)) + (12*b*(c*x)^(7/4)*
Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 7/8, 15/8, -((b*x^2)/a)])/(7*a*
c^3*Sqrt[a + b*x^2])))/(4*a)))/(12*a)
```

Defintions of rubi rules used

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{4}} (b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

output
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/4} (a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}} (cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^(3/4)/(b^3*c^2*x^8 + 3*a*b^2*c^2*x^6 + 3*a^
2*b*c^2*x^4 + a^3*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.88 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(cx)^{5/4}\,(a+bx^2)^{5/2}}\,dx = \frac{\Gamma\left(-\frac{1}{8}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{5}{2} \\ \frac{7}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate(1/(c*x)**(5/4)/(b*x**2+a)**(5/2),x)
```

output
```
gamma(-1/8)*hyper((-1/8, 5/2), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/
2)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/4}\,(a+bx^2)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{2}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/2)*(c*x)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{5/2}}\,dx = \int \frac{1}{(c\,x)^{5/4}\,\left(b\,x^2+a\right)^{5/2}}\,dx$$

input
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(5/2)),x)
```

output
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{5/2}}\,dx = \frac{\int \frac{x^{\frac{3}{4}}\sqrt{b\,x^2+a}}{b^3x^8+3a\,b^2x^6+3a^2b\,x^4+a^3x^2}dx}{c^{\frac{3}{4}}\sqrt{c}}$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(5/2),x)
```

output
```
(c**(1/4)*int((x**(3/4)*sqrt(a + b*x**2))/(a**3*x**2 + 3*a**2*b*x**4 + 3*a
*b**2*x**6 + b**3*x**8),x))/(sqrt(c)*c)
```

3.699 $\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 239

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = -\frac{2\sqrt{\frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}}x^{3/4}\sqrt{-\frac{1+x^2}{x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}-2\sqrt{x}+\sqrt{2}x}{\sqrt{x}}}\right),-2\left(1-\sqrt{2}\right)\right)}{\sqrt{2+\sqrt{2}}\left(1+\sqrt{x}\right)\sqrt{1+x^2}} - \frac{2\sqrt{-\frac{\left(1-\sqrt{x}\right)^2}{\sqrt{x}}}x^{3/4}\sqrt{-\frac{1+x^2}{x}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt{2}+2\sqrt{x}+\sqrt{2}x}{\sqrt{x}}}\right),-2\left(1-\sqrt{2}\right)\right)}{\sqrt{2+\sqrt{2}}\left(1-\sqrt{x}\right)\sqrt{1+x^2}}$$

output
```
-2*((1+x^(1/2))^2/x^(1/2))^(1/2)*x^(3/4)*(-(x^2+1)/x)^(1/2)*EllipticF(1/2*
(-(2^(1/2)-2*x^(1/2)+x*2^(1/2))/x^(1/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/(1+x^(1/2))/(x^2+1)^(1/2)-2*(-(1-x^(1/2))^2/x^(1/2))^(1/2)*x^
(3/4)*(-(x^2+1)/x)^(1/2)*EllipticF(1/2*((2^(1/2)+2*x^(1/2)+x*2^(1/2))/x^(1
/2))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/(1-x^(1/2))/(x^2+1)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \frac{4}{3}x^{3/4}\,\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{11}{8},-x^2\right)$$

input
```
Integrate[1/(x^(1/4)*Sqrt[1 + x^2]),x]
```

output
```
(4*x^(3/4)*Hypergeometric2F1[3/8, 1/2, 11/8, -x^2])/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {266, 838, 2422}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{x}\sqrt{x^2+1}}\,dx$$

$$\Big\downarrow \text{266}$$

$$4\int \frac{\sqrt{x}}{\sqrt{x^2+1}}\,d\sqrt[4]{x}$$

$$\Big\downarrow \text{838}$$

$$4\left(\frac{1}{2}\int \frac{\sqrt{x}+1}{\sqrt{x^2+1}}\,d\sqrt[4]{x}-\frac{1}{2}\int \frac{1-\sqrt{x}}{\sqrt{x^2+1}}\,d\sqrt[4]{x}\right)$$

$$\Big\downarrow \text{2422}$$

$$4\left(-\frac{\sqrt{\frac{(\sqrt{x}+1)^2}{\sqrt{x}}}\,x^{3/4}\sqrt{-\frac{x^2+1}{x}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt{2}x-2\sqrt{x}+\sqrt{2}}{\sqrt{x}}}\right),-2\left(1-\sqrt{2}\right)\right)}{2\sqrt{2+\sqrt{2}}\left(\sqrt{x}+1\right)\sqrt{x^2+1}}-\frac{\sqrt{-\frac{(1-\sqrt{x})^2}{\sqrt{x}}}\,x^{3/4}\sqrt{-\frac{x^2+1}{x}}\cdots}{\cdots}\right.$$

input
```
Int[1/(x^(1/4)*Sqrt[1 + x^2]),x]
```

output
```
4*(-1/2*(Sqrt[(1 + Sqrt[x])^2/Sqrt[x]]*x^(3/4)*Sqrt[-((1 + x^2)/x)]*Ellipt
icF[ArcSin[Sqrt[-((Sqrt[2] - 2*Sqrt[x] + Sqrt[2]*x)/Sqrt[x])]/2], -2*(1 -
Sqrt[2])])/(Sqrt[2 + Sqrt[2]]*(1 + Sqrt[x])*Sqrt[1 + x^2]) - (Sqrt[-((1 -
Sqrt[x])^2/Sqrt[x])]*x^(3/4)*Sqrt[-((1 + x^2)/x)]*EllipticF[ArcSin[Sqrt[(S
qrt[2] + 2*Sqrt[x] + Sqrt[2]*x)/Sqrt[x]]/2], -2*(1 - Sqrt[2])])/(2*Sqrt[2
+ Sqrt[2]]*(1 - Sqrt[x])*Sqrt[1 + x^2]))
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 838
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[1/(2*Rt[b/a, 4])
Int[(1 + Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] - Simp[1/(2*Rt[b/a, 4])
Int[(1 - Rt[b/a, 4]*x^2)/Sqrt[a + b*x^8], x], x] /; FreeQ[{a, b}, x]
```

rule 2422
```
Int[((c_) + (d_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^8], x_Symbol] :> Simp[(-c)
*d*x^3*Sqrt[-(c - d*x^2)^2/(c*d*x^2)]*(Sqrt[(-d^2)*((a + b*x^8)/(b*c^2*x^4)
)]/(Sqrt[2 + Sqrt[2]]*(c - d*x^2)*Sqrt[a + b*x^8]))*EllipticF[ArcSin[(1/2)*
Sqrt[(Sqrt[2]*c^2 + 2*c*d*x^2 + Sqrt[2]*d^2*x^4)/(c*d*x^2)]], -2*(1 - Sqrt[
2])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^4 - a*d^4, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.07

method	result	size
meijerg	$\frac{4x^{\frac{3}{4}}\,\text{hypergeom}\left(\left[\frac{3}{8},\frac{1}{2}\right],\left[\frac{11}{8}\right],-x^{2}\right)}{3}$	17

input
```
int(1/x^(1/4)/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*x^(3/4)*hypergeom([3/8,1/2],[11/8],-x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^2 + 1)*x^(3/4)/(x^3 + x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.13

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{3}{8}, \frac{1}{2} \\ \frac{11}{8} \end{matrix}\middle| x^2e^{i\pi}\right)}{2\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/x**(1/4)/(x**2+1)**(1/2),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((3/8, 1/2), (11/8,), x**2*exp_polar(I*pi))/(2*ga
mma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(x^2 + 1)*x^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \int \frac{1}{\sqrt{x^2+1}x^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/x^(1/4)/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(x^2 + 1)*x^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \int \frac{1}{x^{1/4}\sqrt{x^2+1}}\,dx$$

input
```
int(1/(x^(1/4)*(x^2 + 1)^(1/2)),x)
```

output
```
int(1/(x^(1/4)*(x^2 + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{x}\sqrt{1+x^2}}\,dx = \int \frac{x^{\frac{1}{4}}\sqrt{x^2+1}}{\sqrt{x}\,x^2+\sqrt{x}}dx$$

input
```
int(1/x^(1/4)/(x^2+1)^(1/2),x)
```

output
```
int((x**(1/4)*sqrt(x**2 + 1))/(sqrt(x)*x**2 + sqrt(x)),x)
```

3.700 $\int (cx)^m \left(a+bx^2\right)^{3/2}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int (cx)^m \left(a+bx^2\right)^{3/2}\, dx = \frac{a(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

output
```
a*(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],-b*x
^2/a)/c/(1+m)/(1+b*x^2/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\int (cx)^m (a+bx^2)^{3/2}\, dx = \frac{ax(cx)^m\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^(3/2),x]
```

output
```
(a*x*(c*x)^m*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, 1 + (1 + m
)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2}(cx)^m\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{a+bx^2}\int (cx)^m\left(\frac{bx^2}{a}+1\right)^{3/2}dx}{\sqrt{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{a\sqrt{a+bx^2}(cx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

input
```
Int[(c*x)^m*(a + b*x^2)^(3/2),x]
```

output
```
(a*(c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-3/2, (1 + m)/2, (3 + m
)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[1 + (b*x^2)/a])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^2+a\right)^{\frac{3}{2}}dx$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2}\,dx=\int \left(bx^2+a\right)^{\frac{3}{2}}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/2)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int (cx)^m \left(a+bx^2\right)^{3/2} dx = \frac{a^{\frac{3}{2}} c^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(3/2),x)
```

output
```
a**(3/2)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-3/2, m/2 + 1/2), (m/2 +
3/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} dx = \int \left(bx^2+a\right)^{\frac{3}{2}} (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/2)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \, dx = \int (c\,x)^m \, \left(b\,x^2+a\right)^{3/2} \, dx$$

input
```
int((c*x)^m*(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^2\right)^{3/2} \, dx = c^m \left(\left(\int x^m \sqrt{b\,x^2+a}\,x^2 dx\right) b + \left(\int x^m \sqrt{b\,x^2+a} dx\right) a\right)$$

input
```
int((c*x)^m*(b*x^2+a)^(3/2),x)
```

output
```
c**m*(int(x**m*sqrt(a + b*x**2)*x**2,x)*b + int(x**m*sqrt(a + b*x**2),x)*a
)
```

3.701 $\int (cx)^m \sqrt{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int (cx)^m \sqrt{a+bx^2}\,dx = \frac{(cx)^{1+m}\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

output
```
(c*x)^(1+m)*(b*x^2+a)^(1/2)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2
/a)/c/(1+m)/(1+b*x^2/a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int (cx)^m \sqrt{a+bx^2}\,dx = \frac{x(cx)^m\sqrt{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{(1+m)\sqrt{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^m*Sqrt[a + b*x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, 1 + (1 + m)/
2, -((b*x^2)/a)])/((1 + m)*Sqrt[1 + (b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(cx)^m\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt{a+bx^2}\int (cx)^m\sqrt{\frac{bx^2}{a}+1}dx}{\sqrt{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{a+bx^2}(cx)^{m+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{\frac{bx^2}{a}+1}}$$

input
```
Int[(c*x)^m*Sqrt[a + b*x^2],x]
```

output
```
((c*x)^(1 + m)*Sqrt[a + b*x^2]*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/
2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[1 + (b*x^2)/a])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \sqrt{b\,x^2+a}dx$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^m*(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int (cx)^m\sqrt{a+bx^2}\,dx=\int \sqrt{bx^2+a}(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int (cx)^m\sqrt{a+bx^2}\,dx=\frac{\sqrt{a}c^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m*(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-1/2, m/2 + 1/2), (m/2 + 3
/2,), b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int (cx)^m \sqrt{a+bx^2}\, dx = \int \sqrt{bx^2+a}(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \sqrt{a+bx^2}\, dx = \int \sqrt{bx^2+a}(cx)^m\, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^2 + a)*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m\sqrt{a+bx^2}\,dx=\int (c\,x)^m\,\sqrt{b\,x^2+a}\,dx$$

input
```
int((c*x)^m*(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^m*(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int (cx)^m\sqrt{a+bx^2}\,dx=c^m\left(\int x^m\sqrt{b\,x^2+a}dx\right)$$

input
```
int((c*x)^m*(b*x^2+a)^(1/2),x)
```

output
```
c**m*int(x**m*sqrt(a + b*x**2),x)
```

3.702 $\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx=\frac{(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c(1+m)\sqrt{a+bx^2}}$$

output
```
(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^
2/a)/c/(1+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx=\frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^m/Sqrt[a + b*x^2],x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 +
m)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^m/Sqrt[a + b*x^2],x]
```

output
```
((c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(c*(1 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{\sqrt{b\,x^2+a}}dx$$

input
```
int((c*x)^m/(b*x^2+a)^(1/2),x)
```

output
```
int((c*x)^m/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x)^m/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = \frac{c^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\\frac{m}{2}+\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m/(b*x**2+a)**(1/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(cx)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{(c\,x)^m}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c*x)^m/(a + b*x^2)^(1/2),x)
```

output
```
int((c*x)^m/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{\sqrt{a+bx^2}}\,dx = c^m\left(\int \frac{x^m}{\sqrt{b\,x^2+a}}dx\right)$$

input
```
int((c*x)^m/(b*x^2+a)^(1/2),x)
```

output
```
c**m*int(x**m/sqrt(a + b*x**2),x)
```

3.703 $\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \frac{(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ac(1+m)\sqrt{a+bx^2}}$$

output
```
(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^
2/a)/a/c/(1+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^m/(a + b*x^2)^(3/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, 1 + (1 +
m)/2, -((b*x^2)/a)])/(a*(1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^m}{\left(\frac{bx^2}{a}+1\right)^{3/2}}dx}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 278$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^m/(a + b*x^2)^(3/2),x]
```

output
```
((c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(a*c*(1 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^2+a)^{\frac{3}{2}}}dx$$

input
```
int((c*x)^m/(b*x^2+a)^(3/2),x)
```

output
```
int((c*x)^m/(b*x^2+a)^(3/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \frac{c^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m/(b*x**2+a)**(3/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((3/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*a**(3/2)*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x^2 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx=\int \frac{{(c\,x)}^m}{{(b\,x^2+a)}^{3/2}} \,d x$$

input
```
int((c*x)^m/(a + b*x^2)^(3/2),x)
```

output
```
int((c*x)^m/(a + b*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{3/2}}\,dx=c^m \left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}d x \right)$$

input
```
int((c*x)^m/(b*x^2+a)^(3/2),x)
```

output
```
c**m*int(x**m/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)
```

3.704 $\int \frac{(cx)^m}{(a+bx^2)^{5/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 71

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}} dx = \frac{(cx)^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{a^2c(1+m)\sqrt{a+bx^2}}$$

output
```
(c*x)^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^
2/a)/a^2/c/(1+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}} dx = \frac{x(cx)^m\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a^2(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(c*x)^m/(a + b*x^2)^(5/2),x]
```

output
```
(x*(c*x)^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, 1 + (1 +
m)/2, -((b*x^2)/a)])/(a^2*(1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{(cx)^m}{\left(\frac{bx^2}{a}+1\right)^{5/2}}dx}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 278$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}(cx)^{m+1}\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{a^2c(m+1)\sqrt{a+bx^2}}$$

input
```
Int[(c*x)^m/(a + b*x^2)^(5/2),x]
```

output
```
((c*x)^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[5/2, (1 + m)/2, (3 +
m)/2, -((b*x^2)/a)])/(a^2*c*(1 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^m}{(b\,x^2+a)^{\frac{5}{2}}}dx$$

input
```
int((c*x)^m/(b*x^2+a)^(5/2),x)
```

output
```
int((c*x)^m/(b*x^2+a)^(5/2),x)
```

Fricas [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*x^2 + a)*(c*x)^m/(b^3*x^6 + 3*a*b^2*x^4 + 3*a^2*b*x^2 + a^
3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.79

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = \frac{c^m x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{5}{2}, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{2}}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m/(b*x**2+a)**(5/2),x)
```

output
```
c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((5/2, m/2 + 1/2), (m/2 + 3/2,), b*x
**2*exp_polar(I*pi)/a)/(2*a**(5/2)*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x)^m/(b*x^2 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = \int \frac{(cx)^m}{(bx^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x)^m/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x)^m/(b*x^2 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = \int \frac{(c\,x)^m}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((c*x)^m/(a + b*x^2)^(5/2),x)
```

output
```
int((c*x)^m/(a + b*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(cx)^m}{(a+bx^2)^{5/2}}\,dx = c^m\left(\int \frac{x^m}{\sqrt{b\,x^2+a}\,a^2+2\sqrt{b\,x^2+a}\,ab\,x^2+\sqrt{b\,x^2+a}\,b^2x^4}dx\right)$$

input
```
int((c*x)^m/(b*x^2+a)^(5/2),x)
```

output
```
c**m*int(x**m/(sqrt(a + b*x**2)*a**2 + 2*sqrt(a + b*x**2)*a*b*x**2 + sqrt(
a + b*x**2)*b**2*x**4),x)
```

3.705 $\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx=\frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},\frac{5+m}{2},-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}$$

output
```
x^(3+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],-b*x^2/a)
/(3+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx=\frac{x^{3+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+m}{2},1+\frac{3+m}{2},-\frac{bx^2}{a}\right)}{(3+m)\sqrt{a+bx^2}}$$

input
```
Integrate[x^(2 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, 1 + (3 +
m)/2, -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+2}}{\sqrt{a+bx^2}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{x^{m+2}}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+3}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},-\frac{bx^2}{a}\right)}{(m+3)\sqrt{a+bx^2}}$$

input
```
Int[x^(2 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^(3 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2
, -((b*x^2)/a)])/((3 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2+m}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx = \frac{x^{m+3}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{3}{2}\\ \frac{m}{2}+\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate(x**(2+m)/(b*x**2+a)**(1/2),x)
```

output
```
x**(m + 3)*gamma(m/2 + 3/2)*hyper((1/2, m/2 + 3/2), (m/2 + 5/2,), b*x**2*e
xp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 5/2))
```

Maxima [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m+2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m+2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^(m + 2)/(a + b*x^2)^(1/2),x)
```

output
```
int(x^(m + 2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{2+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^m x^2}{\sqrt{b\,x^2+a}}dx$$

input
```
int(x^(2+m)/(b*x^2+a)^(1/2),x)
```

output
```
int((x**m*x**2)/sqrt(a + b*x**2),x)
```

3.706 $\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx = \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}$$

output
```
x^(2+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],-b*x^2/a)/(2+
m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx = \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},1+\frac{2+m}{2},-\frac{bx^2}{a}\right)}{(2+m)\sqrt{a+bx^2}}$$

input
```
Integrate[x^(1 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, 1 + (2 +
m)/2, -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+1}}{\sqrt{a+bx^2}} \, dx$$
$$\Big\downarrow \text{ 279}$$
$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{x^{m+1}}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \text{ 278}$$
$$\frac{x^{m+2}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{(m+2)\sqrt{a+bx^2}}$$

input
```
Int[x^(1 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2
, -((b*x^2)/a)])/((2 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{1+m}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.73

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx=\frac{x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\\frac{m}{2}+2\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate(x**(1+m)/(b*x**2+a)**(1/2),x)
```

output
```
x**(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_pol
ar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 2))
```

Maxima [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m+1}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^(m + 1)/(a + b*x^2)^(1/2),x)
```

output
```
int(x^(m + 1)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{1+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m x}{\sqrt{b\,x^2+a}}dx$$

input
```
int(x^(1+m)/(b*x^2+a)^(1/2),x)
```

output
```
int((x**m*x)/sqrt(a + b*x**2),x)
```

3.707 $\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

output
```
x^(1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)
/(1+m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx=\frac{x^{1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[x^m/Sqrt[a + b*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, 1 + (1 +
m)/2, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{x^m}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^{m+1}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{(m+1)\sqrt{a+bx^2}}$$

input
```
Int[x^m/Sqrt[a + b*x^2],x]
```

output
```
(x^(1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, -((b*x^2)/a)])/((1 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^m}{\sqrt{b\,x^2+a}}dx$$

input
```
int(x^m/(b*x^2+a)^(1/2),x)
```

output
```
int(x^m/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \frac{x^{m+1}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+\frac{1}{2}\\ \frac{m}{2}+\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate(x**m/(b*x**2+a)**(1/2),x)
```

output
```
x**(m + 1)*gamma(m/2 + 1/2)*hyper((1/2, m/2 + 1/2), (m/2 + 3/2,), b*x**2*e
xp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^m/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^m/(a + b*x^2)^(1/2),x)
```

output
```
int(x^m/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^m}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{b\,x^2+a}}dx$$

input
```
int(x^m/(b*x^2+a)^(1/2),x)
```

output
```
int(x**m/sqrt(a + b*x**2),x)
```

3.708 $\int \frac{x^{-1+m}}{\sqrt{a+bx^2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}} dx = \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{2+m}{2},-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

output
```
x^m*(1+b*x^2/a)^(1/2)*hypergeom([1/2, 1/2*m],[1+1/2*m],-b*x^2/a)/m/(b*x^2+
a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}} dx = \frac{x^m\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},1+\frac{m}{2},-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

input
```
Integrate[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, 1 + m/2, -((b*x^2)/a)
])/(m*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}}{\sqrt{a+bx^2}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{x^{m-1}}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m+2}{2},-\frac{bx^2}{a}\right)}{m\sqrt{a+bx^2}}$$

input
```
Int[x^(-1 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/
a)])/(m*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{m-1}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int(x^(m-1)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^(m-1)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx = \frac{a^{\frac{m}{2}}a^{-\frac{m}{2}-\frac{1}{2}}x^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}\\\frac{m}{2}+1\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+1\right)}$$

input
```
integrate(x**(-1+m)/(b*x**2+a)**(1/2),x)
```

output
```
a**(m/2)*a**(-m/2 - 1/2)*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b*x
**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 1))
```

Maxima [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m-1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-1+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^(m - 1)/(a + b*x^2)^(1/2),x)
```

output
```
int(x^(m - 1)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-1+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{b\,x^2+a}\,x}dx$$

input
```
int(x^(-1+m)/(b*x^2+a)^(1/2),x)
```

output
```
int(x**m/(sqrt(a + b*x**2)*x),x)
```

3.709 $\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 66

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = -\frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+m),\frac{1+m}{2},-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

output
```
-x^(-1+m)*(1+b*x^2/a)^(1/2)*hypergeom([1/2, -1/2+1/2*m],[1/2+1/2*m],-b*x^2
/a)/(1-m)/(b*x^2+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \frac{x^{-1+m}\sqrt{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+m),1+\frac{1}{2}(-1+m),-\frac{bx^2}{a}\right)}{(-1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

output
```
(x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, 1 + (-1
 + m)/2, -((b*x^2)/a)])/((-1 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-2}}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\sqrt{\frac{bx^2}{a}+1}\int \frac{x^{m-2}}{\sqrt{\frac{bx^2}{a}+1}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{x^{m-1}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},-\frac{bx^2}{a}\right)}{(1-m)\sqrt{a+bx^2}}$$

input
```
Int[x^(-2 + m)/Sqrt[a + b*x^2],x]
```

output
```
-((x^(-1 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (-1 + m)/2, (1 +
m)/2, -((b*x^2)/a)])/((1 - m)*Sqrt[a + b*x^2]))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{-2+m}}{\sqrt{b\,x^{2}+a}}dx$$

input
```
int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

output
```
int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

Fricas [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx=\int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \frac{x^{m-1}\Gamma\left(\frac{m}{2}-\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{m}{2}-\frac{1}{2} \\ \frac{m}{2}+\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}$$

input
```
integrate(x**(-2+m)/(b*x**2+a)**(1/2),x)
```

output
```
x**(m - 1)*gamma(m/2 - 1/2)*hyper((1/2, m/2 - 1/2), (m/2 + 1/2,), b*x**2*e
xp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 1/2))
```

Maxima [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m-2}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(-2+m)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^{m-2}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(x^(m - 2)/(a + b*x^2)^(1/2),x)
```

output
```
int(x^(m - 2)/(a + b*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{x^{-2+m}}{\sqrt{a+bx^2}}\,dx = \int \frac{x^m}{\sqrt{b\,x^2+a}\,x^2}dx$$

input
```
int(x^(-2+m)/(b*x^2+a)^(1/2),x)
```

output
```
int(x**m/(sqrt(a + b*x**2)*x**2),x)
```

3.710 $\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 17

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx = x^{2+m}\sqrt{a+bx^2}$$

output
```
x^(2+m)*(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 6.12

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\left(a(4+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)+b(3+m)x^2\,\text{Hypergeometric2F1}\left(\right.\right.}{(4+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

output
```
(x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2
, (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/
2, (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{356\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m+1}(a(m+2)+b(m+3)x^2)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{356}$$
$$x^{m+2}\sqrt{a+bx^2}$$

input
```
Int[(x^(1 + m)*(a*(2 + m) + b*(3 + m)*x^2))/Sqrt[a + b*x^2],x]
```

output
```
x^(2 + m)*Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
gosper	$x^{2+m}\sqrt{b\,x^{2}+a}$	16
orering	$\frac{x\sqrt{b\,x^{2}+a}\,x^{1+m}\left(a(2+m)+b(3+m)x^{2}\right)}{bm\,x^{2}+3b\,x^{2}+am+2a}$	52

input
```
int(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x^(2+m)*(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx=\sqrt{bx^2+a}xx^{m+1}$$

input
```
integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="fric
as")
```

output
```
sqrt(b*x^2 + a)*x*x^(m + 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.65 (sec) , antiderivative size = 196, normalized size of antiderivative = 11.53

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a}mx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{\sqrt{a}x^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\Gamma\left(\frac{m}{2}+2\right)} + \frac{bmx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)} + \frac{3bx^{m+4}\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate(x**(1+m)*(a*(2+m)+b*(3+m)*x**2)/(b*x**2+a)**(1/2),x)
```

output
```
sqrt(a)*m*x**(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + sqrt(a)*x**(m + 2)*gamma(m/2 +
1)*hyper((1/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/gamma(m/2 +
 2) + b*m*x**(m + 4)*gamma(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x*
*2*exp_polar(I*pi)/a)/(2*sqrt(a)*gamma(m/2 + 3)) + 3*b*x**(m + 4)*gamma(m/
2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*sqrt
(a)*gamma(m/2 + 3))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx = \sqrt{bx^2+a}x^2x^m$$

input
```
integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="maxi
ma")
```

output
```
sqrt(b*x^2 + a)*x^2*x^m
```

Giac [F]

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx=\int \frac{(b(m+3)x^2+a(m+2))x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x, algorithm="giac
")
```

output
```
integrate((b*(m + 3)*x^2 + a*(m + 2))*x^(m + 1)/sqrt(b*x^2 + a), x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.41

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx=\frac{x^{m+1}\,(b\,x^3+a\,x)}{\sqrt{b\,x^2+a}}$$

input
```
int((x^(m + 1)*(a*(m + 2) + b*x^2*(m + 3)))/(a + b*x^2)^(1/2),x)
```

output
```
(x^(m + 1)*(a*x + b*x^3))/(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \frac{x^{1+m}(a(2+m)+b(3+m)x^2)}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{b\,x^2+a}\left(2\sqrt{b}\,\sqrt{b\,x^2+a}\,x+2b\,x^2\right)^m x^2}{b^{\frac{m}{2}}\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^m 2^m}$$

input
```
int(x^(1+m)*(a*(2+m)+b*(3+m)*x^2)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(2*sqrt(b)*sqrt(a + b*x**2)*x + 2*b*x**2)**m*x**2)/(b**(
m/2)*(sqrt(a + b*x**2) + sqrt(b)*x)**m*2**m)
```

3.711 $\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx$

Optimal result

Integrand size = 43, antiderivative size = 17

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx = x^{2+m}\sqrt{a+bx^2}$$

output
```
x^(2+m)*(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 104, normalized size of antiderivative = 6.12

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right) dx = \frac{x^{2+m}\sqrt{1+\frac{bx^2}{a}}\left(a(4+m)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)+b(3+m)x^2\,\text{Hypergeometric2F1}\left(\right.\right.}{(4+m)\sqrt{a+bx^2}}$$

input
```
Integrate[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sq
rt[a + b*x^2],x]
```

output
```
(x^(2 + m)*Sqrt[1 + (b*x^2)/a]*(a*(4 + m)*Hypergeometric2F1[1/2, (2 + m)/2
, (4 + m)/2, -((b*x^2)/a)] + b*(3 + m)*x^2*Hypergeometric2F1[1/2, (4 + m)/
2, (6 + m)/2, -((b*x^2)/a)]))/((4 + m)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 7.47, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{a(m+2)x^{m+1}}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+3}}{\sqrt{a+bx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{ax^{m+2}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}+\frac{b(m+3)x^{m+4}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+4}{2},\frac{m+6}{2},-\frac{bx^2}{a}\right)}{(m+4)\sqrt{a+bx^2}}$$

input
```
Int[(a*(2 + m)*x^(1 + m))/Sqrt[a + b*x^2] + (b*(3 + m)*x^(3 + m))/Sqrt[a +
 b*x^2],x]
```

output
```
(a*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)
/2, -((b*x^2)/a)])/Sqrt[a + b*x^2] + (b*(3 + m)*x^(4 + m)*Sqrt[1 + (b*x^2)
/a]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, -((b*x^2)/a)])/((4 + m)*S
qrt[a + b*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(15) = 30$.

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 4.12

method	result	size
orering	$\frac{x\left(b\,x^{2}+a\right)\left(\frac{a(2+m)x^{1+m}}{\sqrt{b\,x^{2}+a}}+\frac{b(3+m)x^{3+m}}{\sqrt{b\,x^{2}+a}}\right)}{bm\,x^{2}+3b\,x^{2}+am+2a}$	70

input
```
int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x,meth
od=_RETURNVERBOSE)
```

output
```
x*(b*x^2+a)/(b*m*x^2+3*b*x^2+a*m+2*a)*(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(
3+m)*x^(3+m)/(b*x^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx=\frac{\sqrt{bx^2+a}x^{m+3}}{x}$$

input
```
integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),
x, algorithm="fricas")
```

output
```
sqrt(b*x^2 + a)*x^(m + 3)/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 102, normalized size of antiderivative = 6.00

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx = \frac{\sqrt{a}x^{m+2}(m+2)\,\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+2\right)} + \frac{bx^{m+4}(m+3)\,\Gamma\left(\frac{m}{2}+2\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{m}{2}+2\\ \frac{m}{2}+3\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{a}\Gamma\left(\frac{m}{2}+3\right)}$$

input
```
integrate(a*(2+m)*x**(1+m)/(b*x**2+a)**(1/2)+b*(3+m)*x**(3+m)/(b*x**2+a)**
(1/2),x)
```

output
```
sqrt(a)*x**(m + 2)*(m + 2)*gamma(m/2 + 1)*hyper((1/2, m/2 + 1), (m/2 + 2,)
, b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 2)) + b*x**(m + 4)*(m + 3)*gamm
a(m/2 + 2)*hyper((1/2, m/2 + 2), (m/2 + 3,), b*x**2*exp_polar(I*pi)/a)/(2*
sqrt(a)*gamma(m/2 + 3))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx = \sqrt{bx^2+a}x^2x^m$$

input
```
integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),
x, algorithm="maxima")
```

output
```
sqrt(b*x^2 + a)*x^2*x^m
```

Giac [F]

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx=\int \frac{b(m+3)x^{m+3}}{\sqrt{bx^2+a}}+\frac{a(m+2)x^{m+1}}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),
x, algorithm="giac")
```

output
```
integrate(b*(m + 3)*x^(m + 3)/sqrt(b*x^2 + a) + a*(m + 2)*x^(m + 1)/sqrt(b
*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx=\int \frac{a\,x^{m+1}\,(m+2)}{\sqrt{b\,x^2+a}}+\frac{b\,x^{m+3}\,(m+3)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b
*x^2)^(1/2),x)
```

output
```
int((a*x^(m + 1)*(m + 2))/(a + b*x^2)^(1/2) + (b*x^(m + 3)*(m + 3))/(a + b
*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.71

$$\int \left(\frac{a(2+m)x^{1+m}}{\sqrt{a+bx^2}}+\frac{b(3+m)x^{3+m}}{\sqrt{a+bx^2}}\right)\,dx=\frac{\sqrt{b\,x^2+a}\,\left(2\sqrt{b}\,\sqrt{b\,x^2+a}\,x+2b\,x^2\right)^m x^2}{b^{\frac{m}{2}}\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^m 2^m}$$

input
```
int(a*(2+m)*x^(1+m)/(b*x^2+a)^(1/2)+b*(3+m)*x^(3+m)/(b*x^2+a)^(1/2),x)
```

output
```
(sqrt(a + b*x**2)*(2*sqrt(b)*sqrt(a + b*x**2)*x + 2*b*x**2)**m*x**2)/(b**(
m/2)*(sqrt(a + b*x**2) + sqrt(b)*x)**m*2**m)
```

3.712 $\int \frac{x^{-1+m}\left(am+b(-1+m)x^2\right)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 15

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{x^m}{\sqrt{a+bx^2}}$$

output
```
x^m/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.87

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{x^m\sqrt{1+\frac{bx^2}{a}}\left(a(2+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{2},\frac{2+m}{2},-\frac{bx^2}{a}\right)+b(-\right.}{a(2+m)\sqrt{a+bx}}$$

input
```
Integrate[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
(x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
 -((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {356}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{m-1}\left(am+b(m-1)x^2\right)}{\left(a+bx^2\right)^{3/2}}dx$$
$$\Big\downarrow \text{356}$$
$$\frac{x^m}{\sqrt{a+bx^2}}$$

input
```
Int[(x^(-1 + m)*(a*m + b*(-1 + m)*x^2))/(a + b*x^2)^(3/2),x]
```

output
```
x^m/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 356
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] /;
 FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) -
b*c*(m + 2*p + 3), 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{x^{m}}{\sqrt{b\,x^{2}+a}}$	14
orering	$\frac{x\,x^{m-1}\left(a m+b(m-1)x^{2}\right)}{\sqrt{b\,x^{2}+a}\,\left(b m\,x^{2}-b\,x^{2}+a m\right)}$	47

input
```
int(x^(m-1)*(a*m+b*(m-1)*x^2)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
x^m/(b*x^2+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{xx^{m-1}}{\sqrt{bx^2+a}}$$

input
```
integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="fricas
")
```

output
```
x*x^(m - 1)/sqrt(b*x^2 + a)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 107, normalized size of antiderivative = 7.13

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{aa^{\frac{m}{2}}a^{-\frac{m}{2}-\frac{3}{2}}mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+1\right)} + \frac{bx^{m+2}(m-1)\,\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate(x**(-1+m)*(a*m+b*(-1+m)*x**2)/(b*x**2+a)**(3/2),x)
```

output
```
a*a**(m/2)*a**(-m/2 - 3/2)*m*x**m*gamma(m/2)*hyper((3/2, m/2), (m/2 + 1,),
 b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 1)) + b*x**(m + 2)*(m - 1)*gamma
(m/2 + 1)*hyper((3/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a
**(3/2)*gamma(m/2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx = \frac{x^m}{\sqrt{bx^2+a}}$$

input
```
integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="maxima
")
```

output
```
x^m/sqrt(b*x^2 + a)
```

Giac [F]

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx=\int \frac{(b(m-1)x^2+am)x^{m-1}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*(m - 1)*x^2 + a*m)*x^(m - 1)/(b*x^2 + a)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx=\frac{x^m}{\sqrt{b\,x^2+a}}$$

input
```
int((x^(m - 1)*(a*m + b*x^2*(m - 1)))/(a + b*x^2)^(3/2),x)
```

output
```
x^m/(a + b*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 206, normalized size of antiderivative = 13.73

$$\int \frac{x^{-1+m}(am+b(-1+m)x^2)}{(a+bx^2)^{3/2}}\,dx=\frac{\sqrt{b\,x^2+a}\left(a^{\frac{m}{2}}\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+b\,x^2\right)^m\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^m 2^m m}{b^{\frac{m}{2}}\left(\sqrt{a}\,\sqrt{}\right.}$$

input
```
int(x^(-1+m)*(a*m+b*(-1+m)*x^2)/(b*x^2+a)^(3/2),x)
```

output
```
(sqrt(a + b*x**2)*(a**(m/2)*(sqrt(b)*sqrt(a + b*x**2)*x + b*x**2)**m*(sqrt
(a + b*x**2) + sqrt(b)*x)**m*2**m*m - a**(m/2)*(sqrt(b)*sqrt(a + b*x**2)*x
 + b*x**2)**m*(sqrt(a + b*x**2) + sqrt(b)*x)**m*2**m + (2*sqrt(b)*sqrt(a +
 b*x**2)*x + 2*b*x**2)**m*(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x)**
m))/(b**(m/2)*(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x)**m*(sqrt(a +
b*x**2) + sqrt(b)*x)**m*2**m*m*(a + b*x**2))
```

3.713 $\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) \, dx$

Optimal result

Integrand size = 38, antiderivative size = 15

$$\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) \, dx=\frac{x^m}{\sqrt{a+bx^2}}$$

output
```
x^m/(b*x^2+a)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.87

$$\int \left(-\frac{bx^{1+m}}{\left(a+bx^2\right)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right) \, dx=\frac{x^m\sqrt{1+\frac{bx^2}{a}}\left(a(2+m)\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m}{2},\frac{2+m}{2},-\frac{bx^2}{a}\right)+b(-1+m)x^2\,\text{Hyperg}\right.}{a(2+m)\sqrt{a+bx^2}}$$

input
```
Integrate[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x
^2],x]
```

output
```
(x^m*Sqrt[1 + (b*x^2)/a]*(a*(2 + m)*Hypergeometric2F1[3/2, m/2, (2 + m)/2,
 -((b*x^2)/a)] + b*(-1 + m)*x^2*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)]))/(a*(2 + m)*Sqrt[a + b*x^2])
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 8.20, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{mx^{m-1}}{\sqrt{a+bx^2}}-\frac{bx^{m+1}}{(a+bx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^m\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m}{2},\frac{m+2}{2},-\frac{bx^2}{a}\right)}{\sqrt{a+bx^2}}-\frac{bx^{m+2}\sqrt{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{a(m+2)\sqrt{a+bx^2}}$$

input
```
Int[-((b*x^(1 + m))/(a + b*x^2)^(3/2)) + (m*x^(-1 + m))/Sqrt[a + b*x^2],x]
```

output
```
(x^m*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, -((b*x^2)/
a)])/Sqrt[a + b*x^2] - (b*x^(2 + m)*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[
3/2, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*(2 + m)*Sqrt[a + b*x^2])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(13) = 26.

Time = 0.46 (sec) , antiderivative size = 62, normalized size of antiderivative = 4.13

method	result	size
orering	$\frac{x\left(b\,x^{2}+a\right)\left(-\frac{b\,x^{1+m}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{m\,x^{m-1}}{\sqrt{b\,x^{2}+a}}\right)}{bm\,x^{2}-b\,x^{2}+am}$	62

input
```
int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x^2+a)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
x*(b*x^2+a)/(b*m*x^2-b*x^2+a*m)*(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(m-1)/(b*x
^2+a)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)\,dx=\frac{\sqrt{bx^2+a}x^{m+1}}{bx^3+ax}$$

input
```
integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorit
hm="fricas")
```

output
```
sqrt(b*x^2 + a)*x^(m + 1)/(b*x^3 + a*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 6.80

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx = \frac{a^{\frac{m}{2}}a^{-\frac{m}{2}-\frac{1}{2}}mx^m\Gamma\left(\frac{m}{2}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{m}{2}\\ \frac{m}{2}+1\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+1\right)} - \frac{bx^{m+2}\Gamma\left(\frac{m}{2}+1\right){}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{m}{2}+1\\ \frac{m}{2}+2\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{2}}\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate(-b*x**(1+m)/(b*x**2+a)**(3/2)+m*x**(-1+m)/(b*x**2+a)**(1/2),x)
```

output
```
a**(m/2)*a**(-m/2 - 1/2)*m*x**m*gamma(m/2)*hyper((1/2, m/2), (m/2 + 1,), b
*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 1)) - b*x**(m + 2)*gamma(m/2 + 1)*
hyper((3/2, m/2 + 1), (m/2 + 2,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/2)*ga
mma(m/2 + 2))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx = \frac{x^m}{\sqrt{bx^2+a}}$$

input
```
integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

output
```
x^m/sqrt(b*x^2 + a)
```

Giac [F]

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx=\int \frac{mx^{m-1}}{\sqrt{bx^2+a}}-\frac{bx^{m+1}}{(bx^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(m*x^(m - 1)/sqrt(b*x^2 + a) - b*x^(m + 1)/(b*x^2 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx=-\int \frac{b\,x^{m+1}}{(b\,x^2+a)^{3/2}}-\frac{m\,x^{m-1}}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((m*x^(m - 1))/(a + b*x^2)^(1/2) - (b*x^(m + 1))/(a + b*x^2)^(3/2),x)
```

output
```
-int((b*x^(m + 1))/(a + b*x^2)^(3/2) - (m*x^(m - 1))/(a + b*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 13.73

$$\int \left(-\frac{bx^{1+m}}{(a+bx^2)^{3/2}}+\frac{mx^{-1+m}}{\sqrt{a+bx^2}}\right)dx=\frac{\sqrt{b\,x^2+a}\left(a^{\frac{m}{2}}\left(\sqrt{b}\sqrt{b\,x^2+a}\,x+b\,x^2\right)^m\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^m 2^m m-a^{\frac{m}{2}}\left(\sqrt{b}\sqrt{b\,x^2-\cdots}\right.\right.}{b^{\frac{m}{2}}\left(\sqrt{a}\sqrt{b\,x^2+a}+\sqrt{b}\sqrt{\cdots}\right.}$$

input
```
int(-b*x^(1+m)/(b*x^2+a)^(3/2)+m*x^(-1+m)/(b*x^2+a)^(1/2),x)
```

output

```
(sqrt(a + b*x**2)*(a**(m/2)*(sqrt(b)*sqrt(a + b*x**2)*x + b*x**2)**m*(sqrt
(a + b*x**2) + sqrt(b)*x)**m*2**m*m - a**(m/2)*(sqrt(b)*sqrt(a + b*x**2)*x
 + b*x**2)**m*(sqrt(a + b*x**2) + sqrt(b)*x)**m*2**m + (2*sqrt(b)*sqrt(a +
 b*x**2)*x + 2*b*x**2)**m*(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x)**
m))/(b**(m/2)*(sqrt(a)*sqrt(a + b*x**2) + sqrt(b)*sqrt(a)*x)**m*(sqrt(a +
b*x**2) + sqrt(b)*x)**m*2**m*m*(a + b*x**2))
```

3.714 $\int x^7 \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = -\frac{3a^3(a+bx^2)^{4/3}}{8b^4}+\frac{9a^2(a+bx^2)^{7/3}}{14b^4}-\frac{9a(a+bx^2)^{10/3}}{20b^4}+\frac{3(a+bx^2)^{13/3}}{26b^4}$$

output
```
-3/8*a^3*(b*x^2+a)^(4/3)/b^4+9/14*a^2*(b*x^2+a)^(7/3)/b^4-9/20*a*(b*x^2+a)
^(10/3)/b^4+3/26*(b*x^2+a)^(13/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \frac{3(a+bx^2)^{4/3}(-81a^3+108a^2bx^2-126ab^2x^4+140b^3x^6)}{3640b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(4/3)*(-81*a^3 + 108*a^2*b*x^2 - 126*a*b^2*x^4 + 140*b^3*x^
6))/(3640*b^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int x^6 \sqrt[3]{bx^2+a}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{10/3}}{b^3}-\frac{3a\left(bx^2+a\right)^{7/3}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{4/3}}{b^3}-\frac{a^3\sqrt[3]{bx^2+a}}{b^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(-\frac{3a^3\left(a+bx^2\right)^{4/3}}{4b^4}+\frac{9a^2\left(a+bx^2\right)^{7/3}}{7b^4}+\frac{3\left(a+bx^2\right)^{13/3}}{13b^4}-\frac{9a\left(a+bx^2\right)^{10/3}}{10b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(1/3),x]
```

output
```
((-3*a^3*(a + b*x^2)^(4/3))/(4*b^4) + (9*a^2*(a + b*x^2)^(7/3))/(7*b^4) -
(9*a*(a + b*x^2)^(10/3))/(10*b^4) + (3*(a + b*x^2)^(13/3))/(13*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{6}+126 a\,b^{2} x^{4}-108 a^{2} b\,x^{2}+81 a^{3}\right)}{3640 b^{4}}$	47
pseudoelliptic	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{6}+126 a\,b^{2} x^{4}-108 a^{2} b\,x^{2}+81 a^{3}\right)}{3640 b^{4}}$	47
orering	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}} \left(-140 b^{3} x^{6}+126 a\,b^{2} x^{4}-108 a^{2} b\,x^{2}+81 a^{3}\right)}{3640 b^{4}}$	47
trager	$-\frac{3\left(-140 b^{4} x^{8}-14 a\,b^{3} x^{6}+18 a^{2} b^{2} x^{4}-27 a^{3} b\,x^{2}+81 a^{4}\right) \left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3640 b^{4}}$	58
risch	$-\frac{3\left(-140 b^{4} x^{8}-14 a\,b^{3} x^{6}+18 a^{2} b^{2} x^{4}-27 a^{3} b\,x^{2}+81 a^{4}\right) \left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3640 b^{4}}$	58

input
```
int(x^7*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/3640*(b*x^2+a)^(4/3)*(-140*b^3*x^6+126*a*b^2*x^4-108*a^2*b*x^2+81*a^3)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \frac{3\,(140\,b^4x^8 + 14\,ab^3x^6 - 18\,a^2b^2x^4 + 27\,a^3bx^2 - 81\,a^4)(bx^2+a)^{\frac{1}{3}}}{3640\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/3640*(140*b^4*x^8 + 14*a*b^3*x^6 - 18*a^2*b^2*x^4 + 27*a^3*b*x^2 - 81*a^
4)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(75) = 150$.

Time = 1.60 (sec) , antiderivative size = 1795, normalized size of antiderivative = 22.44

$$\int x^7 \sqrt[3]{a + bx^2}\, dx = \text{Too large to display}$$

input
```
integrate(x**7*(b*x**2+a)**(1/3),x)
```

output
```
-243*a**(73/3)*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x
**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**
8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 243*a**(73/3)/(3640
*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*
b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b*
*10*x**12) - 1377*a**(70/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 +
54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) +
 1458*a**(70/3)*b*x**2/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a*
*18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**1
5*b**9*x**10 + 3640*a**14*b**10*x**12) - 3213*a**(67/3)*b**2*x**4*(1 + b*x
**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18*b**6*
x**4 + 72800*a**17*b**7*x**6 + 54600*a**16*b**8*x**8 + 21840*a**15*b**9*x*
*10 + 3640*a**14*b**10*x**12) + 3645*a**(67/3)*b**2*x**4/(3640*a**20*b**4
+ 21840*a**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 +
54600*a**16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) -
 3927*a**(64/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(3640*a**20*b**4 + 21840*a
**19*b**5*x**2 + 54600*a**18*b**6*x**4 + 72800*a**17*b**7*x**6 + 54600*a**
16*b**8*x**8 + 21840*a**15*b**9*x**10 + 3640*a**14*b**10*x**12) + 4860*a**
(64/3)*b**3*x**6/(3640*a**20*b**4 + 21840*a**19*b**5*x**2 + 54600*a**18...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{13}{3}}}{26\,b^4} - \frac{9\,(bx^2+a)^{\frac{10}{3}}a}{20\,b^4} + \frac{9\,(bx^2+a)^{\frac{7}{3}}a^2}{14\,b^4} - \frac{3\,(bx^2+a)^{\frac{4}{3}}a^3}{8\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/26*(b*x^2 + a)^(13/3)/b^4 - 9/20*(b*x^2 + a)^(10/3)*a/b^4 + 9/14*(b*x^2
+ a)^(7/3)*a^2/b^4 - 3/8*(b*x^2 + a)^(4/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(140\,(bx^2+a)^{\frac{13}{3}} - 546\,(bx^2+a)^{\frac{10}{3}}a + 780\,(bx^2+a)^{\frac{7}{3}}a^2 - 455\,(bx^2+a)^{\frac{4}{3}}a^3\right)}{3640\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/3640*(140*(b*x^2 + a)^(13/3) - 546*(b*x^2 + a)^(10/3)*a + 780*(b*x^2 + a
)^(7/3)*a^2 - 455*(b*x^2 + a)^(4/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \left(b\,x^2+a\right)^{1/3}\left(\frac{3\,x^8}{26} - \frac{243\,a^4}{3640\,b^4} + \frac{3\,a\,x^6}{260\,b} - \frac{27\,a^2\,x^4}{1820\,b^2} + \frac{81\,a^3\,x^2}{3640\,b^3}\right)$$

input
```
int(x^7*(a + b*x^2)^(1/3),x)
```

output
```
(a + b*x^2)^(1/3)*((3*x^8)/26 - (243*a^4)/(3640*b^4) + (3*a*x^6)/(260*b) -
 (27*a^2*x^4)/(1820*b^2) + (81*a^3*x^2)/(3640*b^3))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05

$$\int x^7 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}\left(140b^4x^8+14a\,b^3x^6-18a^2b^2x^4+27a^3b\,x^2-81a^4\right)}{3640\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b^4}$$

input
```
int(x^7*(b*x^2+a)^(1/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*( - 81*a**4 + 27*a**3*
b*x**2 - 18*a**2*b**2*x**4 + 14*a*b**3*x**6 + 140*b**4*x**8))/(3640*(sqrt(
a + b*x**2) + sqrt(b)*x)**(2/3)*b**4)
```

3.715 $\int x^5 \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \frac{3a^2(a+bx^2)^{4/3}}{8b^3} - \frac{3a(a+bx^2)^{7/3}}{7b^3} + \frac{3(a+bx^2)^{10/3}}{20b^3}$$

output
```
3/8*a^2*(b*x^2+a)^(4/3)/b^3-3/7*a*(b*x^2+a)^(7/3)/b^3+3/20*(b*x^2+a)^(10/3
)/b^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \frac{3(a+bx^2)^{4/3}(9a^2-12abx^2+14b^2x^4)}{280b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(4/3)*(9*a^2 - 12*a*b*x^2 + 14*b^2*x^4))/(280*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \sqrt[3]{a+bx^2} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2} \int x^4 \sqrt[3]{bx^2+a} dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2} \int \left(\frac{(bx^2+a)^{7/3}}{b^2} - \frac{2a(bx^2+a)^{4/3}}{b^2} + \frac{a^2 \sqrt[3]{bx^2+a}}{b^2}\right) dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{3a^2(a+bx^2)^{4/3}}{4b^3} + \frac{3(a+bx^2)^{10/3}}{10b^3} - \frac{6a(a+bx^2)^{7/3}}{7b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(1/3),x]
```

output
```
((3*a^2*(a + b*x^2)^(4/3))/(4*b^3) - (6*a*(a + b*x^2)^(7/3))/(7*b^3) + (3*
(a + b*x^2)^(10/3))/(10*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{4}{3}}(14b^2x^4-12ab\,x^2+9a^2)}{280b^3}$	36
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{4}{3}}(14b^2x^4-12ab\,x^2+9a^2)}{280b^3}$	36
orering	$\frac{3(b\,x^2+a)^{\frac{4}{3}}(14b^2x^4-12ab\,x^2+9a^2)}{280b^3}$	36
trager	$\frac{3(14b^3x^6+2a\,b^2x^4-3a^2b\,x^2+9a^3)(b\,x^2+a)^{\frac{1}{3}}}{280b^3}$	47
risch	$\frac{3(14b^3x^6+2a\,b^2x^4-3a^2b\,x^2+9a^3)(b\,x^2+a)^{\frac{1}{3}}}{280b^3}$	47

input
```
int(x^5*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/280*(b*x^2+a)^(4/3)*(14*b^2*x^4-12*a*b*x^2+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^5\sqrt[3]{a+bx^2}\,dx=\frac{3\,(14\,b^3x^6+2\,ab^2x^4-3\,a^2bx^2+9\,a^3)(bx^2+a)^{\frac{1}{3}}}{280\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/280*(14*b^3*x^6 + 2*a*b^2*x^4 - 3*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(1/3)/b
^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(54) = 108$.

Time = 1.10 (sec) , antiderivative size = 700, normalized size of antiderivative = 11.86

$$\int x^5\sqrt[3]{a+bx^2}\,dx = \frac{27a^{\frac{34}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} - \frac{27a^{\frac{34}{3}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{72a^{\frac{31}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} - \frac{81a^{\frac{31}{3}}bx^2}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{60a^{\frac{28}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} - \frac{81a^{\frac{28}{3}}b^2x^4}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{60a^{\frac{25}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} - \frac{27a^{\frac{25}{3}}b^3x^6}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{135a^{\frac{22}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{132a^{\frac{19}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6} + \frac{42a^{\frac{16}{3}}b^6x^{12}\sqrt[3]{1+\frac{bx^2}{a}}}{280a^8b^3+840a^7b^4x^2+840a^6b^5x^4+280a^5b^6x^6}$$

input
```
integrate(x**5*(b*x**2+a)**(1/3),x)
```

output
```
27*a**(34/3)*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 8
40*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(34/3)/(280*a**8*b**3 + 84
0*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 72*a**(31/3)
*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a*
*6*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(31/3)*b*x**2/(280*a**8*b**3 +
840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60*a**(28/
3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 8
40*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 81*a**(28/3)*b**2*x**4/(280*a**8
*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 60
*a**(25/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*
x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) - 27*a**(25/3)*b**3*x**6/(
280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x*
*6) + 135*a**(22/3)*b**4*x**8*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a
**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 132*a**(19/3)*b
**5*x**10*(1 + b*x**2/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*
a**6*b**5*x**4 + 280*a**5*b**6*x**6) + 42*a**(16/3)*b**6*x**12*(1 + b*x**2
/a)**(1/3)/(280*a**8*b**3 + 840*a**7*b**4*x**2 + 840*a**6*b**5*x**4 + 280*
a**5*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{10}{3}}}{20\,b^3} - \frac{3\,(bx^2+a)^{\frac{7}{3}}a}{7\,b^3} + \frac{3\,(bx^2+a)^{\frac{4}{3}}a^2}{8\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/20*(b*x^2 + a)^(10/3)/b^3 - 3/7*(b*x^2 + a)^(7/3)*a/b^3 + 3/8*(b*x^2 + a
)^(4/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(14\,(bx^2+a)^{\frac{10}{3}} - 40\,(bx^2+a)^{\frac{7}{3}}a + 35\,(bx^2+a)^{\frac{4}{3}}a^2\right)}{280\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/280*(14*(b*x^2 + a)^(10/3) - 40*(b*x^2 + a)^(7/3)*a + 35*(b*x^2 + a)^(4/
3)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \left(b\,x^2+a\right)^{1/3}\left(\frac{3\,x^6}{20}+\frac{27\,a^3}{280\,b^3}+\frac{3\,a\,x^4}{140\,b}-\frac{9\,a^2\,x^2}{280\,b^2}\right)$$

input
```
int(x^5*(a + b*x^2)^(1/3),x)
```

output
```
(a + b*x^2)^(1/3)*((3*x^6)/20 + (27*a^3)/(280*b^3) + (3*a*x^4)/(140*b) - (
9*a^2*x^2)/(280*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int x^5 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}\left(14b^3x^6+2a\,b^2x^4-3a^2b\,x^2+9a^3\right)}{280\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b^3}$$

input
```
int(x^5*(b*x^2+a)^(1/3),x)
```

output

```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*(9*a**3 - 3*a**2*b*x**
2 + 2*a*b**2*x**4 + 14*b**3*x**6))/(280*(sqrt(a + b*x**2) + sqrt(b)*x)**(2
/3)*b**3)
```

3.716 $\int x^3 \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = -\frac{3a(a+bx^2)^{4/3}}{8b^2} + \frac{3(a+bx^2)^{7/3}}{14b^2}$$

output
```
-3/8*a*(b*x^2+a)^(4/3)/b^2+3/14*(b*x^2+a)^(7/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = \frac{3\sqrt[3]{a+bx^2}(-3a^2+abx^2+4b^2x^4)}{56b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(1/3)*(-3*a^2 + a*b*x^2 + 4*b^2*x^4))/(56*b^2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sqrt[3]{a+bx^2} \, dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^2 \sqrt[3]{bx^2+a}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{4/3}}{b}-\frac{a\sqrt[3]{bx^2+a}}{b}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{3\left(a+bx^2\right)^{7/3}}{7b^2}-\frac{3a\left(a+bx^2\right)^{4/3}}{4b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(1/3),x]
```

output
```
((-3*a*(a + b*x^2)^(4/3))/(4*b^2) + (3*(a + b*x^2)^(7/3))/(7*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$	25
pseudoelliptic	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$	25
orering	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-4b\,x^{2}+3a\right)}{56b^{2}}$	25
trager	$-\frac{3\left(-4b^{2}x^{4}-ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{56b^{2}}$	36
risch	$-\frac{3\left(-4b^{2}x^{4}-ab\,x^{2}+3a^{2}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{56b^{2}}$	36

input
```
int(x^3*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/56*(b*x^2+a)^(4/3)*(-4*b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int x^3\sqrt[3]{a+bx^2}\,dx=\frac{3\,(4\,b^2x^4+abx^2-3\,a^2)(bx^2+a)^{\frac{1}{3}}}{56\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/56*(4*b^2*x^4 + a*b*x^2 - 3*a^2)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(34) = 68.

Time = 0.71 (sec) , antiderivative size = 223, normalized size of antiderivative = 5.87

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = -\frac{9a^{\frac{13}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{9a^{\frac{13}{3}}}{56a^2b^2+56ab^3x^2} - \frac{6a^{\frac{10}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{9a^{\frac{10}{3}}bx^2}{56a^2b^2+56ab^3x^2} + \frac{15a^{\frac{7}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2} + \frac{12a^{\frac{4}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{56a^2b^2+56ab^3x^2}$$

input
```
integrate(x**3*(b*x**2+a)**(1/3),x)
```

output
```
-9*a**(13/3)*(1 + b*x**2/a)**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**
(13/3)/(56*a**2*b**2 + 56*a*b**3*x**2) - 6*a**(10/3)*b*x**2*(1 + b*x**2/a)
**(1/3)/(56*a**2*b**2 + 56*a*b**3*x**2) + 9*a**(10/3)*b*x**2/(56*a**2*b**2
 + 56*a*b**3*x**2) + 15*a**(7/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(56*a**2*
b**2 + 56*a*b**3*x**2) + 12*a**(4/3)*b**3*x**6*(1 + b*x**2/a)**(1/3)/(56*a
**2*b**2 + 56*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{7}{3}}}{14\,b^2} - \frac{3\,(bx^2+a)^{\frac{4}{3}}a}{8\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/14*(b*x^2 + a)^(7/3)/b^2 - 3/8*(b*x^2 + a)^(4/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(4\,(bx^2+a)^{\frac{7}{3}} - 7\,(bx^2+a)^{\frac{4}{3}}a\right)}{56\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/56*(4*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^(4/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = \left(b\,x^2+a\right)^{1/3}\left(\frac{3\,x^4}{14} - \frac{9\,a^2}{56\,b^2} + \frac{3\,a\,x^2}{56\,b}\right)$$

input
```
int(x^3*(a + b*x^2)^(1/3),x)
```

output
```
(a + b*x^2)^(1/3)*((3*x^4)/14 - (9*a^2)/(56*b^2) + (3*a*x^2)/(56*b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.61

$$\int x^3 \sqrt[3]{a+bx^2}\,dx = \frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x + a + b\,x^2\right)^{\frac{2}{3}}\left(4b^2x^4 + ab\,x^2 - 3a^2\right)}{56\left(\sqrt{b\,x^2+a} + \sqrt{b}\,x\right)^{\frac{2}{3}} b^2}$$

input
```
int(x^3*(b*x^2+a)^(1/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*( - 3*a**2 + a*b*x**2
+ 4*b**2*x**4))/(56*(sqrt(a + b*x**2) + sqrt(b)*x)**(2/3)*b**2)
```

3.717 $\int x\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3(a+bx^2)^{4/3}}{8b}$$

output
```
3/8*(b*x^2+a)^(4/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3(a+bx^2)^{4/3}}{8b}$$

input
```
Integrate[x*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(4/3))/(8*b)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{3(a+bx^2)^{4/3}}{8b}$$

input
```
Int[x*(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(4/3))/(8*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
derivativedivides	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
default	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
trager	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
risch	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
pseudoelliptic	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15
orering	$\frac{3(b x^2+a)^{\frac{4}{3}}}{8b}$	15

input
```
int(x*(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/8*(b*x^2+a)^(4/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/8*(b*x^2 + a)^(4/3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x\sqrt[3]{a+bx^2}\,dx = \begin{cases} \frac{3a\sqrt[3]{a+bx^2}}{8b} + \frac{3x^2\sqrt[3]{a+bx^2}}{8} & \text{for } b \neq 0 \\ \frac{\sqrt[3]{a}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(1/3),x)
```

output
```
Piecewise((3*a*(a + b*x**2)**(1/3)/(8*b) + 3*x**2*(a + b*x**2)**(1/3)/8, N
e(b, 0)), (a**(1/3)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/8*(b*x^2 + a)^(4/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt[3]{a+bx^2}\,dx = \frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b}$$

input
```
integrate(x*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/8*(b*x^2 + a)^(4/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3\,{\left(b\,x^2+a\right)}^{4/3}}{8\,b}$$

input
```
int(x*(a + b*x^2)^(1/3),x)
```

output
```
(3*(a + b*x^2)^(4/3))/(8*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int x\sqrt[3]{a+bx^2}\,dx=\frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}\left(b\,x^2+a\right)}{8\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b}$$

input
```
int(x*(b*x^2+a)^(1/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*(a + b*x**2))/(8*(sqrt
(a + b*x**2) + sqrt(b)*x)**(2/3)*b)
```

3.718 $\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = \frac{3}{2}\sqrt[3]{a+bx^2} - \frac{1}{2}\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}\sqrt[3]{a}\log(x) + \frac{3}{4}\sqrt[3]{a}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

output
```
3/2*(b*x^2+a)^(1/3)-1/2*3^(1/2)*a^(1/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1
/3))*3^(1/2)/a^(1/3))-1/2*a^(1/3)*ln(x)+3/4*a^(1/3)*ln(a^(1/3)-(b*x^2+a)^(
1/3))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx=\frac{1}{4}\left(6\sqrt[3]{a+bx^2}-2\sqrt{3}\sqrt[3]{a}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\sqrt[3]{a}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\sqrt[3]{a}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)$$

input
```
Integrate[(a + b*x^2)^(1/3)/x,x]
```

output
```
(6*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a^(1/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))
/a^(1/3))/Sqrt[3]] + 2*a^(1/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - a^(1/3)
*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {243, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{bx^2+a}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2\left(bx^2+a\right)^{2/3}}dx^2+3\sqrt[3]{a+bx^2}\right)$$

$\Big\downarrow$ 69

$$\frac{1}{2}\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(a\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(a\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)$$

input
```
Int[(a + b*x^2)^(1/3)/x,x]
```

output
```
(3*(a + b*x^2)^(1/3) + a*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(2/3)) - Log[x^2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*
x^2)^(1/3)])/(2*a^(2/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{1}{3}}}{2}+\frac{a^{\frac{1}{3}}\ln\left((b\,x^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2}-\frac{a^{\frac{1}{3}}\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}(b\,x^2+a)^{\frac{1}{3}}+(b\,x^2+a)^{\frac{2}{3}}\right)}{4}-\frac{a^{\frac{1}{3}}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,(b\,x^2+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right.}{2}$

input
```
int((b*x^2+a)^(1/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x^2+a)^(1/3)+1/2*a^(1/3)*ln((b*x^2+a)^(1/3)-a^(1/3))-1/4*a^(1/3)*ln
(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))-1/2*a^(1/3)*3^(1/2)*arct
an(2/3*3^(1/2)/a^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3\,a}\right) - \frac{1}{4}a^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}a^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}(bx^2+a)^{\frac{1}{3}}$$

input
```
integrate((b*x^2+a)^(1/3)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*a^(1/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqr
t(3)*a)/a) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 +
a)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = -\frac{\sqrt[3]{b}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/x,x)
```

output
```
-b**(1/3)*x**(2/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}a^{\frac{1}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{3}{2}\left(bx^2+a\right)^{\frac{1}{3}}$$

input
```
integrate((b*x^2+a)^(1/3)/x,x, algorithm="maxima")
```

output
```
-1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/2*a^(1/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2 + a)^(
1/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}\,a^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}\,a^{\frac{1}{3}}\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}$$

input

```
integrate((b*x^2+a)^(1/3)/x,x, algorithm="giac")
```

output

```
-1/2*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/4*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/2*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/2*(b*x^2 +
 a)^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = \frac{a^{1/3}\,\ln\left(\frac{9\,a\,{\left(b\,x^2+a\right)}^{1/3}}{4}-\frac{9\,a^{4/3}}{4}\right)}{2}+\frac{3\,{\left(b\,x^2+a\right)}^{1/3}}{2} - \frac{a^{1/3}\,\ln\left(\frac{9\,a^{4/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2}+\frac{9\,a\,{\left(b\,x^2+a\right)}^{1/3}}{2}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2} + a^{1/3}\,\ln\left(\frac{9\,a\,{\left(b\,x^2+a\right)}^{1/3}}{2}-9\,a^{4/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1{}\mathrm{i}}{4}\right)$$

input

```
int((a + b*x^2)^(1/3)/x,x)
```

output
```
(a^(1/3)*log((9*a*(a + b*x^2)^(1/3))/4 - (9*a^(4/3))/4))/2 + (3*(a + b*x^2
)^(1/3))/2 - (a^(1/3)*log((9*a^(4/3)*((3^(1/2)*1i)/2 + 1/2))/2 + (9*a*(a +
 b*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(1/3)*log((9*a*(a + b*x^2)
^(1/3))/2 - 9*a^(4/3)*((3^(1/2)*1i)/4 - 1/4))*((3^(1/2)*1i)/4 - 1/4)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}}{2} + \left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)a$$

input
```
int((b*x^2+a)^(1/3)/x,x)
```

output
```
(3*(a + b*x**2)**(1/3) + 2*int((a + b*x**2)**(1/3)/(a*x + b*x**3),x)*a)/2
```

3.719 $\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx=-\frac{\sqrt[3]{a+bx^2}}{2x^2}-\frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{2/3}}-\frac{b\log(x)}{6a^{2/3}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

output
```
-1/2*(b*x^2+a)^(1/3)/x^2-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(2/3)-1/6*b*ln(x)/a^(2/3)+1/4*b*ln(a^(1/3)-(b*x^2+a
)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx=-\frac{6a^{2/3}\sqrt[3]{a+bx^2}+2\sqrt{3}bx^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)+bx^2\log\left(a^{2/3}+\cdots\right.}{12a^{2/3}x^2}$$

input
```
Integrate[(a + b*x^2)^(1/3)/x^3,x]
```

output
```
-1/12*(6*a^(2/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b
*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)]
 + b*x^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(a^
(2/3)*x^2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{\sqrt[3]{bx^2+a}}{x^4} dx^2$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2} \left(\frac{1}{3} b \int \frac{1}{x^2 \left(bx^2+a\right)^{2/3}} dx^2 - \frac{\sqrt[3]{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2} \left(\frac{1}{3} b \left(-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2a^{2/3}} - \frac{3 \int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}} - \frac{\log\left(x^2\right)}{2a^{2/3}}\right) - \frac{\sqrt[3]{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)$$

input
```
Int[(a + b*x^2)^(1/3)/x^3,x]
```

output
```
(-((a + b*x^2)^(1/3)/x^2) + (b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (
a + b*x^2)^(1/3)])/(2*a^(2/3))))/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{-b\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\,\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)x^{2}+b\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^{2}-\frac{b\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)x^{2}}{2}-3\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{6a^{\frac{2}{3}}x^{2}}$

input
```
int((b*x^2+a)^(1/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^(2/3)*(-b*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(
1/2))*x^2+b*ln((b*x^2+a)^(1/3)-a^(1/3))*x^2-1/2*b*ln(a^(2/3)+a^(1/3)*(b*x^
2+a)^(1/3)+(b*x^2+a)^(2/3))*x^2-3*(b*x^2+a)^(1/3)*a^(2/3))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx = -\frac{6\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}abx^2\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^2+a)^{\frac{1}{3}}\,(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+(a^2)^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx\right.}{12\,a^2x^2}$$

input
```
integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="fricas")
```

output
```
-1/12*(6*sqrt(1/3)*(a^2)^(1/6)*a*b*x^2*arctan(sqrt(1/3)*(a^2)^(1/6)*((a^2)
^(1/3)*a + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*b*x^2*log((
b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a
^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 6*(b*x^2 + a)^(1/
3)*a^2)/(a^2*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx = -\frac{\sqrt[3]{b}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{1}{3},\frac{2}{3} \\ \frac{5}{3} \end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/x**3,x)
```

output
```
-b**(1/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2)
)/(2*x**(4/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx = &-\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{2}{3}}} \\ &-\frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12\,a^{\frac{2}{3}}} \\ &+\frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6\,a^{\frac{2}{3}}}-\frac{(bx^2+a)^{\frac{1}{3}}}{2\,x^2}\end{aligned}$$

input
```
integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(2/3) - 1/12*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2
/3))/a^(2/3) + 1/6*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/2*(b*x^2
 + a)^(1/3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3} dx = -\frac{1}{12} b \left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} + \frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}} + \left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}} - \frac{2\log\left(\left|(bx^2+a\right.\right.}{a^{\frac{2}{3}}}\right.$$

input
```
integrate((b*x^2+a)^(1/3)/x^3,x, algorithm="giac")
```

output
```
-1/12*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(2/3) + log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(2/3) - 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) + 6*(b*x^2 + a
)^(1/3)/(b*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3} dx = \frac{b\ln\left(\frac{3b\left(b\,x^2+a\right)^{1/3}}{2} - \frac{3\,a^{1/3}\,b}{2}\right)}{6\,a^{2/3}} - \frac{\left(b\,x^2+a\right)^{1/3}}{2\,x^2} - \frac{\ln\left(\frac{3\,a^{1/3}\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4} + \frac{3\,b\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{2/3}} - \frac{\ln\left(\frac{3\,a^{1/3}\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4} + \frac{3\,b\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{2/3}}$$

input
```
int((a + b*x^2)^(1/3)/x^3,x)
```

output
```
(b*log((3*b*(a + b*x^2)^(1/3))/2 - (3*a^(1/3)*b)/2))/(6*a^(2/3)) - (a + b*
x^2)^(1/3)/(2*x^2) - (log((3*a^(1/3)*(b - 3^(1/2)*b*1i))/4 + (3*b*(a + b*x
^2)^(1/3))/2)*(b - 3^(1/2)*b*1i))/(12*a^(2/3)) - (log((3*a^(1/3)*(b + 3^(1
/2)*b*1i))/4 + (3*b*(a + b*x^2)^(1/3))/2)*(b + 3^(1/2)*b*1i))/(12*a^(2/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^3}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}+2\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)b\,x^2}{6x^2}$$

input
```
int((b*x^2+a)^(1/3)/x^3,x)
```

output
```
( - 3*(a + b*x**2)**(1/3) + 2*int((a + b*x**2)**(1/3)/(a*x + b*x**3),x)*b*
x**2)/(6*x**2)
```

3.720 $\int \frac{\sqrt[3]{a+bx^2}}{x^5} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5} \, dx = -\frac{\sqrt[3]{a+bx^2}}{4x^4} - \frac{b\sqrt[3]{a+bx^2}}{12ax^2} + \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}} + \frac{b^2 \log(x)}{18a^{5/3}} - \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{5/3}}$$

output
```
-1/4*(b*x^2+a)^(1/3)/x^4-1/12*b*(b*x^2+a)^(1/3)/a/x^2+1/18*b^2*arctan(1/3*
(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/18*b^2*ln(x
)/a^(5/3)-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{(-3a-bx^2)\sqrt[3]{a+bx^2}}{12ax^4} + \frac{b^2\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{5/3}} - \frac{b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{18a^{5/3}} + \frac{b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{5/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/x^5,x]
```

output
```
((-3*a - b*x^2)*(a + b*x^2)^(1/3))/(12*a*x^4) + (b^2*ArcTan[1/Sqrt[3] + (2
*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(5/3)) - (b^2*Log[-a^
(1/3) + (a + b*x^2)^(1/3)])/(18*a^(5/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a +
 b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(5/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 51, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\sqrt[3]{bx^2+a}}{x^6}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{1}{6}b\int \frac{1}{x^4\left(bx^2+a\right)^{2/3}}dx^2-\frac{\sqrt[3]{a+bx^2}}{2x^4}\right)$$

$\Big\downarrow$ 52

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{2b\int \frac{1}{x^2(bx^2+a)^{2/3}}dx^2}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)-\frac{\sqrt[3]{a+bx^2}}{2x^4}\right)$$

$\Big\downarrow$ 69

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{2b\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)\right)$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{2b\left(-\frac{3\int\frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)-\frac{\sqrt[3]{a}}{2}\right.$$

$\Big\downarrow$ 1082

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{2b\left(\frac{3\int\frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)-\frac{\sqrt[3]{a+bx^2}}{2x^4}\right)$$

$\Big\downarrow$ 217

$$\frac{1}{2}\left(\frac{1}{6}b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)-\frac{\sqrt[3]{a+bx^2}}{2x^4}\right)$$

input
```
Int[(a + b*x^2)^(1/3)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(1/3)/x^4 + (b*(-((a + b*x^2)^(1/3)/(a*x^2)) - (2*b*(-((
Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Lo
g[x^2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3))))/(3
*a)))/6)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{2b^{2}\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^{4}-2b^{2}\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^{4}+b^{2}\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)x^{4}-3b\,x}{36a^{\frac{5}{3}}x^{4}}$

input
```
int((b*x^2+a)^(1/3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(2*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3)
)*x^4-2*b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4+b^2*ln(a^(2/3)+a^(1/3)*(b*x^2+
a)^(1/3)+(b*x^2+a)^(2/3))*x^4-3*b*x^2*a^(2/3)*(b*x^2+a)^(1/3)-9*(b*x^2+a)^
(1/3)*a^(5/3))/a^(5/3)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{6\sqrt{\frac{1}{3}}ab^2x^4\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}b^2x^4\log\left(\left(bx^2+a\right)^{\frac{2}{3}}\right.}{}$$

input
```
integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="fricas")
```

output
```
1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^
(1/3)*a - 2*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a
^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(
1/3)*(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (-a^
2)^(2/3)) - 3*(a^2*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^3*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = -\frac{\sqrt[3]{b}\,\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{10}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/x**5,x)
```

output
```
-b**(1/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2)
)/(2*x**(10/3)*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{5}{3}}} + \frac{b^2 \log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{36\,a^{\frac{5}{3}}} - \frac{b^2 \log\left((bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{18\,a^{\frac{5}{3}}} - \frac{(bx^2+a)^{\frac{4}{3}}b^2 + 2\,(bx^2+a)^{\frac{1}{3}}ab^2}{12\,\left((bx^2+a)^2a - 2\,(bx^2+a)a^2 + a^3\right)}$$

input
```
integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="maxima")
```

output
```
1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(5/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(5/3) - 1/1
2*((b*x^2 + a)^(4/3)*b^2 + 2*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a - 2
*(b*x^2 + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{\frac{2\sqrt{3}b^3 \arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}} + \frac{b^3 \log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}} - \frac{2\,b^3 \log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{5}{3}}} - \frac{3\left((bx^2+a)^{\frac{4}{3}}b^3+2\,(\right.}{ab^2x^4}}{36\,b}$$

input
```
integrate((b*x^2+a)^(1/3)/x^5,x, algorithm="giac")
```

output
```
1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(5/3) + b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3))/a^(5/3) - 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(
(b*x^2 + a)^(4/3)*b^3 + 2*(b*x^2 + a)^(1/3)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{b^2\ \ln\left(\frac{b^2}{2\,{\left(-a\right)}^{2/3}}-\frac{b^2\,{\left(b\,x^2+a\right)}^{1/3}}{2\,a}\right)}{18\,{\left(-a\right)}^{5/3}} - \frac{\ln\left(\frac{b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}}{4\,{\left(-a\right)}^{2/3}}+\frac{b^2\,{\left(b\,x^2+a\right)}^{1/3}}{2\,a}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{36\,{\left(-a\right)}^{5/3}} - \frac{\frac{b^2\,{\left(b\,x^2+a\right)}^{1/3}}{3}+\frac{b^2\,{\left(b\,x^2+a\right)}^{4/3}}{6\,a}}{2\,{\left(b\,x^2+a\right)}^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2} + \frac{b^2\ \ln\left(\frac{b^2\,{\left(b\,x^2+a\right)}^{1/3}}{2\,a}-\frac{b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,{\left(-a\right)}^{2/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,{\left(-a\right)}^{5/3}}$$

input
```
int((a + b*x^2)^(1/3)/x^5,x)
```

output
```
(b^2*log(b^2/(2*(-a)^(2/3)) - (b^2*(a + b*x^2)^(1/3))/(2*a)))/(18*(-a)^(5/
3)) - (log((3^(1/2)*b^2*1i + b^2)/(4*(-a)^(2/3)) + (b^2*(a + b*x^2)^(1/3))
/(2*a))*(3^(1/2)*b^2*1i + b^2))/(36*(-a)^(5/3)) - ((b^2*(a + b*x^2)^(1/3))
/3 + (b^2*(a + b*x^2)^(4/3))/(6*a))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2
*a^2) + (b^2*log((b^2*(a + b*x^2)^(1/3))/(2*a) - (b^2*((3^(1/2)*1i)/2 - 1/
2))/(2*(-a)^(2/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*(-a)^(5/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^5}\,dx = \frac{-9(b\,x^2+a)^{\frac{1}{3}}\,a-3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2-4\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)b^2x^4}{36a\,x^4}$$

input
```
int((b*x^2+a)^(1/3)/x^5,x)
```

output
```
( - 9*(a + b*x**2)**(1/3)*a - 3*(a + b*x**2)**(1/3)*b*x**2 - 4*int((a + b*
x**2)**(1/3)/(a*x + b*x**3),x)*b**2*x**4)/(36*a*x**4)
```

3.721 $\int x^4 \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 314

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = -\frac{54a^2x\sqrt[3]{a+bx^2}}{935b^2}+\frac{6ax^3\sqrt[3]{a+bx^2}}{187b}+\frac{3}{17}x^5\sqrt[3]{a+bx^2} - \frac{54\ 3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),\ldots\right)}{935b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-54/935*a^2*x*(b*x^2+a)^(1/3)/b^2+6/187*a*x^3*(b*x^2+a)^(1/3)/b+3/17*x^5*(
b*x^2+a)^(1/3)-54/935*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^3*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.30

$$\int x^4\sqrt[3]{a+bx^2}\,dx$$
$$=\frac{3x\sqrt[3]{a+bx^2}\left(\sqrt[3]{1+\frac{bx^2}{a}}(-9a^2+2abx^2+11b^2x^4)+9a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{187b^2\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-9*a^2 + 2*a*b*x^2 + 11*b^2
*x^4) + 9*a^2*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)]))/(187*b^2*(
1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {248, 262, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt[3]{a+bx^2}\,dx$$
$$\Big\downarrow \; 248$$
$$\frac{2}{17}a\int \frac{x^4}{(bx^2+a)^{2/3}}dx+\frac{3}{17}x^5\sqrt[3]{a+bx^2}$$
$$\Big\downarrow \; 262$$

$$\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\int \frac{x^2}{(bx^2+a)^{2/3}}dx}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx}{5b}\right)}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}\right)}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{}}{\left(1-\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{11b}\right.$$

$$\frac{3}{17}x^5\sqrt[3]{a+bx^2}$$

input
```
Int[x^4*(a + b*x^2)^(1/3),x]
```

output
```
(3*x^5*(a + b*x^2)^(1/3))/17 + (2*a*((3*x^3*(a + b*x^2)^(1/3))/(11*b) - (9
*a*((3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a
+ b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(11*b)
))/17
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}+a\right)^{\frac{1}{3}}d x$$

input
```
int(x^4*(b*x^2+a)^(1/3),x)
```

output
```
int(x^4*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \frac{\sqrt[3]{a}x^5 {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*x**5*hyper((-1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \int x^4 \left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/3),x)
```

output
```
int(x^4*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int x^4 \sqrt[3]{a+bx^2}\,dx = \frac{-\frac{54(b\,x^2+a)^{\frac{1}{3}}a^2x}{935} + \frac{6(b\,x^2+a)^{\frac{1}{3}}ab\,x^3}{187} + \frac{3(b\,x^2+a)^{\frac{1}{3}}b^2x^5}{17} + \frac{54\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^3}{935}}{b^2}$$

input
```
int(x^4*(b*x^2+a)^(1/3),x)
```

output
```
(3*( - 18*(a + b*x**2)**(1/3)*a**2*x + 10*(a + b*x**2)**(1/3)*a*b*x**3 + 5
5*(a + b*x**2)**(1/3)*b**2*x**5 + 18*int((a + b*x**2)**(1/3)/(a + b*x**2),
x)*a**3))/(935*b**2)
```

3.722 $\int x^2 \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \frac{6ax\sqrt[3]{a+bx^2}}{55b} + \frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

$$+ \frac{6\ 3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ }}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\ }}\right.\right.}{55b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
6/55*a*x*(b*x^2+a)^(1/3)/b+3/11*x^3*(b*x^2+a)^(1/3)+6/55*3^(3/4)*(1/2*6^(1
/2)-1/2*2^(1/2))*a^2*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)
^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ell
ipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.40 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int x^2\sqrt[3]{a+bx^2}\,dx=\frac{3x\sqrt[3]{a+bx^2}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{11b}$$

input
```
Integrate[x^2*(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*(a + b*x^2 - (a*Hypergeometric2F1[-1/3, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(11*b)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {248, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \ 248$$

$$\frac{2}{11}a\int \frac{x^2}{(bx^2+a)^{2/3}}dx+\frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \ 262$$

$$\frac{2}{11}a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int \frac{1}{(bx^2+a)^{2/3}}dx}{5b}\right)+\frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \ 234$$

$$\frac{2}{11}a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}\right)+\frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

$\downarrow$ 760

$$\frac{2}{11}a\left(\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2}}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{3}{11}x^3\sqrt[3]{a+bx^2}$$

input
```
Int[x^2*(a + b*x^2)^(1/3),x]
```

output
```
(3*x^3*(a + b*x^2)^(1/3))/11 + (2*a*((3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^
(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^
(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a +
 b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2
*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])))/11
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a\right)^{\frac{1}{3}}dx$$

input
```
int(x^2*(b*x^2+a)^(1/3),x)
```

output
```
int(x^2*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \frac{\sqrt[3]{a}x^3{}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*x**3*hyper((-1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \int x^2 \left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/3),x)
```

output
```
int(x^2*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int x^2 \sqrt[3]{a+bx^2}\,dx = \frac{\frac{6(b\,x^2+a)^{\frac{1}{3}}a x}{55}+\frac{3(b\,x^2+a)^{\frac{1}{3}}b\,x^3}{11}-\frac{6\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^2}{55}}{b}$$

input
```
int(x^2*(b*x^2+a)^(1/3),x)
```

output
```
(3*(2*(a + b*x**2)**(1/3)*a*x + 5*(a + b*x**2)**(1/3)*b*x**3 - 2*int((a +
b*x**2)**(1/3)/(a + b*x**2),x)*a**2))/(55*b)
```

3.723 $\int \sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 266

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3}{5}x\sqrt[3]{a+bx^2} - \frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/5*x*(b*x^2+a)^(1/3)-2/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(a^(1/3)-(b*
x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(
1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \sqrt[3]{a+bx^2}\,dx=\frac{x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3),x]
```

output
```
(x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{2}{5}a\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right)\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input
```
Int[(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a +
 b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 +
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(
1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{3}}dx$$

input
```
int((b*x^2+a)^(1/3),x)
```

output
```
int((b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^{2}+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \sqrt[3]{a+bx^2}\,dx = \sqrt[3]{a}x{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)$$

input
```
integrate((b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*x*hyper((-1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{x\,\left(b\,x^2+a\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/3}}$$

input
```
int((a + b*x^2)^(1/3),x)
```

output
```
(x*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/3)
```

Reduce [F]

$$\int \sqrt[3]{a+bx^2}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,x}{5} + \frac{2\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a}{5}$$

input
```
int((b*x^2+a)^(1/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*x + 2*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a)/5
```

3.724 $\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 260

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx^2}}{x} - \frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \mathrm{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\cdots\right)}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-(b*x^2+a)^(1/3)/x-2/3*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))
*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.43 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.19

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{3},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/x^2,x]
```

output
```
-(((a + b*x^2)^(1/3)*Hypergeometric2F1[-1/2, -1/3, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(1/3)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2}{3}b\int \frac{1}{(bx^2+a)^{2/3}}dx-\frac{\sqrt[3]{a+bx^2}}{x}$$

$$\Big\downarrow \text{234}$$

$$\frac{\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{x}-\frac{\sqrt[3]{a+bx^2}}{x}$$

$$\Big\downarrow \text{760}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-7\right.}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$\frac{\sqrt[3]{a+bx^2}}{x}$$

input
```
Int[(a + b*x^2)^(1/3)/x^2,x]
```

output
```
-((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((
1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{3}}}{x^{2}}dx$$

input
```
int((b*x^2+a)^(1/3)/x^2,x)
```

output
```
int((b*x^2+a)^(1/3)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[3]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{3} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(1/3)/x**2,x)
```

output
```
-a**(1/3)*hyper((-1/2, -1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = -\frac{3\,{\left(b\,x^2+a\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{1}{6};\ \frac{7}{6};\ -\frac{a}{b\,x^2}\right)}{x\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/3}}$$

input
```
int((a + b*x^2)^(1/3)/x^2,x)
```

output
```
-(3*(a + b*x^2)^(1/3)*hypergeom([-1/3, 1/6], 7/6, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(1/3))
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^2}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}-2\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((b*x^2+a)^(1/3)/x^2,x)
```

output
```
( - 3*(a + b*x**2)**(1/3) - 2*int((a + b*x**2)**(1/3)/(a*x**2 + b*x**4),x)
*a*x)/x
```

3.725 $\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3x^3} - \frac{2b\sqrt[3]{a+bx^2}}{9ax} + \frac{2\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{9\sqrt[4]{3}\,ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(1/3)/x^3-2/9*b*(b*x^2+a)^(1/3)/a/x+2/27*(1/2*6^(1/2)-1/2*2
^(1/2))*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x
^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*
I-I*3^(1/2))*3^(3/4)/a/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*
a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -1/3, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {247, 264, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2}{9}b\int \frac{1}{x^2\left(bx^2+a\right)^{2/3}}dx-\frac{\sqrt[3]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{9}b\left(-\frac{b\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax}\right)-\frac{\sqrt[3]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{234}$$

$$\frac{2}{9}b\left(-\frac{\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\sqrt[3]{a+bx^2}}{ax}\right)-\frac{\sqrt[3]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{760}$$

$$\frac{2}{9}b\left(\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),\cdot\right.}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{\sqrt[3]{a+bx^2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(1/3)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(1/3)/x^3 + (2*b*(-((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 -
Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]
*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3
])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2)])))/9
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{3}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(1/3)/x^4,x)
```

output
```
int((b*x^2+a)^(1/3)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[3]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(1/3)/x**4,x)
```

output
```
-a**(1/3)*hyper((-3/2, -1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = \int \frac{{\left(b\,x^2+a\right)}^{1/3}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(1/3)/x^4,x)
```

output
```
int((a + b*x^2)^(1/3)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{x^4}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}-2\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^6+a\,x^4}dx\right)a\,x^3}{7x^3}$$

input
```
int((b*x^2+a)^(1/3)/x^4,x)
```

output
```
( - 3*(a + b*x**2)**(1/3) - 2*int((a + b*x**2)**(1/3)/(a*x**4 + b*x**6),x)
*a*x**3)/(7*x**3)
```

3.726 $\int x^7(a+bx^2)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7\left(a+bx^2\right)^{2/3}\,dx=-\frac{3a^3(a+bx^2)^{5/3}}{10b^4}+\frac{9a^2(a+bx^2)^{8/3}}{16b^4}-\frac{9a(a+bx^2)^{11/3}}{22b^4}+\frac{3(a+bx^2)^{14/3}}{28b^4}$$

output
```
-3/10*a^3*(b*x^2+a)^(5/3)/b^4+9/16*a^2*(b*x^2+a)^(8/3)/b^4-9/22*a*(b*x^2+a
)^(11/3)/b^4+3/28*(b*x^2+a)^(14/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7\left(a+bx^2\right)^{2/3}\,dx=\frac{3(a+bx^2)^{5/3}\left(-81a^3+135a^2bx^2-180ab^2x^4+220b^3x^6\right)}{6160b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(5/3)*(-81*a^3 + 135*a^2*b*x^2 - 180*a*b^2*x^4 + 220*b^3*x^
6))/(6160*b^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^{2/3} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^{2/3} dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{11/3}}{b^3}-\frac{3a\left(bx^2+a\right)^{8/3}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{5/3}}{b^3}-\frac{a^3\left(bx^2+a\right)^{2/3}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(-\frac{3a^3\left(a+bx^2\right)^{5/3}}{5b^4}+\frac{9a^2\left(a+bx^2\right)^{8/3}}{8b^4}+\frac{3\left(a+bx^2\right)^{14/3}}{14b^4}-\frac{9a\left(a+bx^2\right)^{11/3}}{11b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(2/3),x]
```

output
```
((-3*a^3*(a + b*x^2)^(5/3))/(5*b^4) + (9*a^2*(a + b*x^2)^(8/3))/(8*b^4) -
(9*a*(a + b*x^2)^(11/3))/(11*b^4) + (3*(a + b*x^2)^(14/3))/(14*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-220 b^{3} x^{6}+180 a\,b^{2} x^{4}-135 a^{2} b\,x^{2}+81 a^{3}\right)}{6160 b^{4}}$	47
pseudoelliptic	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-220 b^{3} x^{6}+180 a\,b^{2} x^{4}-135 a^{2} b\,x^{2}+81 a^{3}\right)}{6160 b^{4}}$	47
orering	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(-220 b^{3} x^{6}+180 a\,b^{2} x^{4}-135 a^{2} b\,x^{2}+81 a^{3}\right)}{6160 b^{4}}$	47
trager	$-\frac{3\left(-220 b^{4} x^{8}-40 a\,b^{3} x^{6}+45 a^{2} b^{2} x^{4}-54 a^{3} b\,x^{2}+81 a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{6160 b^{4}}$	58
risch	$-\frac{3\left(-220 b^{4} x^{8}-40 a\,b^{3} x^{6}+45 a^{2} b^{2} x^{4}-54 a^{3} b\,x^{2}+81 a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{6160 b^{4}}$	58

input
```
int(x^7*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/6160*(b*x^2+a)^(5/3)*(-220*b^3*x^6+180*a*b^2*x^4-135*a^2*b*x^2+81*a^3)/
b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7 \left(a+bx^2\right)^{2/3} \, dx = \frac{3 \,{\left(220 \, b^{4} x^{8} + 40 \, a b^{3} x^{6} - 45 \, a^{2} b^{2} x^{4} + 54 \, a^{3} b x^{2} - 81 \, a^{4}\right)} {\left(b x^{2} + a\right)}^{\frac{2}{3}}}{6160 \, b^{4}}$$

input
```
integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/6160*(220*b^4*x^8 + 40*a*b^3*x^6 - 45*a^2*b^2*x^4 + 54*a^3*b*x^2 - 81*a^
4)*(b*x^2 + a)^(2/3)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1795 vs. $2(75) = 150$.

Time = 1.66 (sec) , antiderivative size = 1795, normalized size of antiderivative = 22.44

$$\int x^7 (a + bx^2)^{2/3} \, dx = \text{Too large to display}$$

input
```
integrate(x**7*(b*x**2+a)**(2/3),x)
```

output
```
-243*a**(74/3)*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x
**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x*
*8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 243*a**(74/3)/(616
0*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**1
7*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*
b**10*x**12) - 1296*a**(71/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**
4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6
 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12
) + 1458*a**(71/3)*b*x**2/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400
*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*
a**15*b**9*x**10 + 6160*a**14*b**10*x**12) - 2808*a**(68/3)*b**2*x**4*(1 +
 b*x**2/a)**(2/3)/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b
**6*x**4 + 123200*a**17*b**7*x**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b*
*9*x**10 + 6160*a**14*b**10*x**12) + 3645*a**(68/3)*b**2*x**4/(6160*a**20*
b**4 + 36960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x
**6 + 92400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x*
*12) - 3120*a**(65/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(6160*a**20*b**4 + 3
6960*a**19*b**5*x**2 + 92400*a**18*b**6*x**4 + 123200*a**17*b**7*x**6 + 92
400*a**16*b**8*x**8 + 36960*a**15*b**9*x**10 + 6160*a**14*b**10*x**12) + 4
860*a**(65/3)*b**3*x**6/(6160*a**20*b**4 + 36960*a**19*b**5*x**2 + 9240...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^7 (a+bx^2)^{2/3} \, dx = \frac{3 (bx^2+a)^{\frac{14}{3}}}{28 b^4} - \frac{9 (bx^2+a)^{\frac{11}{3}} a}{22 b^4} + \frac{9 (bx^2+a)^{\frac{8}{3}} a^2}{16 b^4} - \frac{3 (bx^2+a)^{\frac{5}{3}} a^3}{10 b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/28*(b*x^2 + a)^(14/3)/b^4 - 9/22*(b*x^2 + a)^(11/3)*a/b^4 + 9/16*(b*x^2
+ a)^(8/3)*a^2/b^4 - 3/10*(b*x^2 + a)^(5/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7 (a+bx^2)^{2/3} \, dx = \frac{3 \left(220 (bx^2+a)^{\frac{14}{3}} - 840 (bx^2+a)^{\frac{11}{3}} a + 1155 (bx^2+a)^{\frac{8}{3}} a^2 - 616 (bx^2+a)^{\frac{5}{3}} a^3\right)}{6160 b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/6160*(220*(b*x^2 + a)^(14/3) - 840*(b*x^2 + a)^(11/3)*a + 1155*(b*x^2 +
a)^(8/3)*a^2 - 616*(b*x^2 + a)^(5/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int x^7 (a+bx^2)^{2/3} \, dx = (b\,x^2+a)^{2/3} \left(\frac{3\,x^8}{28} - \frac{243\,a^4}{6160\,b^4} + \frac{3\,a\,x^6}{154\,b} - \frac{27\,a^2\,x^4}{1232\,b^2} + \frac{81\,a^3\,x^2}{3080\,b^3}\right)$$

input
```
int(x^7*(a + b*x^2)^(2/3),x)
```

output
```
(a + b*x^2)^(2/3)*((3*x^8)/28 - (243*a^4)/(6160*b^4) + (3*a*x^6)/(154*b) -
 (27*a^2*x^4)/(1232*b^2) + (81*a^3*x^2)/(3080*b^3))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.15

$$\int x^7(a+bx^2)^{2/3}\,dx = \frac{3\sqrt{b\,x^2+a}\left(\sqrt{b}\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{1}{3}}(220b^4x^8+40a\,b^3x^6-45a^2b^2x^4+54a^3b\,x^2-81a}{6160\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{1}{3}}b^4}$$

input
```
int(x^7*(b*x^2+a)^(2/3),x)
```

output
```
(3*a**(7/6)*sqrt(a + b*x**2)*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(1
/3)*( - 81*a**4 + 54*a**3*b*x**2 - 45*a**2*b**2*x**4 + 40*a*b**3*x**6 + 22
0*b**4*x**8))/(6160*a**(1/6)*(sqrt(a + b*x**2) + sqrt(b)*x)**(1/3)*a*b**4)
```

3.727 $\int x^5(a+bx^2)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5(a+bx^2)^{2/3}\,dx=\frac{3a^2(a+bx^2)^{5/3}}{10b^3}-\frac{3a(a+bx^2)^{8/3}}{8b^3}+\frac{3(a+bx^2)^{11/3}}{22b^3}$$

output
```
3/10*a^2*(b*x^2+a)^(5/3)/b^3-3/8*a*(b*x^2+a)^(8/3)/b^3+3/22*(b*x^2+a)^(11/
3)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5(a+bx^2)^{2/3}\,dx=\frac{3(a+bx^2)^{5/3}(9a^2-15abx^2+20b^2x^4)}{440b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(5/3)*(9*a^2 - 15*a*b*x^2 + 20*b^2*x^4))/(440*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+bx^2\right)^{2/3} dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^4 \left(bx^2+a\right)^{2/3} dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{8/3}}{b^2}-\frac{2a\left(bx^2+a\right)^{5/3}}{b^2}+\frac{a^2\left(bx^2+a\right)^{2/3}}{b^2}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{3a^2\left(a+bx^2\right)^{5/3}}{5b^3}+\frac{3\left(a+bx^2\right)^{11/3}}{11b^3}-\frac{3a\left(a+bx^2\right)^{8/3}}{4b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(2/3),x]
```

output
```
((3*a^2*(a + b*x^2)^(5/3))/(5*b^3) - (3*a*(a + b*x^2)^(8/3))/(4*b^3) + (3*
(a + b*x^2)^(11/3))/(11*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(20b^{2}x^{4}-15ab\,x^{2}+9a^{2}\right)}{440b^{3}}$	36
pseudoelliptic	$\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(20b^{2}x^{4}-15ab\,x^{2}+9a^{2}\right)}{440b^{3}}$	36
orering	$\frac{3\left(b\,x^{2}+a\right)^{\frac{5}{3}}\left(20b^{2}x^{4}-15ab\,x^{2}+9a^{2}\right)}{440b^{3}}$	36
trager	$\frac{3\left(20b^{3}x^{6}+5a\,b^{2}x^{4}-6a^{2}b\,x^{2}+9a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{440b^{3}}$	47
risch	$\frac{3\left(20b^{3}x^{6}+5a\,b^{2}x^{4}-6a^{2}b\,x^{2}+9a^{3}\right)\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{440b^{3}}$	47

input
```
int(x^5*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/440*(b*x^2+a)^(5/3)*(20*b^2*x^4-15*a*b*x^2+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int x^5 \left(a+bx^2\right)^{2/3}\,dx = \frac{3\,(20\,b^3x^6+5\,ab^2x^4-6\,a^2bx^2+9\,a^3)(bx^2+a)^{\frac{2}{3}}}{440\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/440*(20*b^3*x^6 + 5*a*b^2*x^4 - 6*a^2*b*x^2 + 9*a^3)*(b*x^2 + a)^(2/3)/b
^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(54) = 108$.

Time = 1.27 (sec) , antiderivative size = 700, normalized size of antiderivative = 11.86

$$\int x^5 (a+bx^2)^{2/3}\,dx = \frac{27a^{\frac{35}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} - \frac{27a^{\frac{35}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{63a^{\frac{32}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} - \frac{81a^{\frac{32}{3}}bx^2}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{42a^{\frac{29}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} - \frac{81a^{\frac{29}{3}}b^2x^4}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{78a^{\frac{26}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} - \frac{27a^{\frac{26}{3}}b^3x^6}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{207a^{\frac{23}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{195a^{\frac{20}{3}}b^5x^{10}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6} + \frac{60a^{\frac{17}{3}}b^6x^{12}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{440a^8b^3+1320a^7b^4x^2+1320a^6b^5x^4+440a^5b^6x^6}$$

input
```
integrate(x**5*(b*x**2+a)**(2/3),x)
```

output
```
27*a**(35/3)*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 +
1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(35/3)/(440*a**8*b**3 +
1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 63*a**(3
2/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1
320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(32/3)*b*x**2/(440*a**8*b
**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 42
*a**(29/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4
*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 81*a**(29/3)*b**2*x**4
/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**
6*x**6) + 78*a**(26/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 13
20*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) - 27*a**(26/
3)*b**3*x**6/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 +
440*a**5*b**6*x**6) + 207*a**(23/3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(440*a
**8*b**3 + 1320*a**7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6)
 + 195*a**(20/3)*b**5*x**10*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a*
*7*b**4*x**2 + 1320*a**6*b**5*x**4 + 440*a**5*b**6*x**6) + 60*a**(17/3)*b*
*6*x**12*(1 + b*x**2/a)**(2/3)/(440*a**8*b**3 + 1320*a**7*b**4*x**2 + 1320
*a**6*b**5*x**4 + 440*a**5*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^5 (a + bx^2)^{2/3} \, dx = \frac{3 \, (bx^2 + a)^{\frac{11}{3}}}{22 \, b^3} - \frac{3 \, (bx^2 + a)^{\frac{8}{3}} a}{8 \, b^3} + \frac{3 \, (bx^2 + a)^{\frac{5}{3}} a^2}{10 \, b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/22*(b*x^2 + a)^(11/3)/b^3 - 3/8*(b*x^2 + a)^(8/3)*a/b^3 + 3/10*(b*x^2 +
a)^(5/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 (a + bx^2)^{2/3} \, dx = \frac{3 \left(20 \, (bx^2 + a)^{\frac{11}{3}} - 55 \, (bx^2 + a)^{\frac{8}{3}} a + 44 \, (bx^2 + a)^{\frac{5}{3}} a^2\right)}{440 \, b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/440*(20*(b*x^2 + a)^(11/3) - 55*(b*x^2 + a)^(8/3)*a + 44*(b*x^2 + a)^(5/
3)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^5 (a + bx^2)^{2/3} \, dx = \left(b \, x^2 + a\right)^{2/3} \left(\frac{3 \, x^6}{22} + \frac{27 \, a^3}{440 \, b^3} + \frac{3 \, a \, x^4}{88 \, b} - \frac{9 \, a^2 \, x^2}{220 \, b^2}\right)$$

input
```
int(x^5*(a + b*x^2)^(2/3),x)
```

output
```
(a + b*x^2)^(2/3)*((3*x^6)/22 + (27*a^3)/(440*b^3) + (3*a*x^4)/(88*b) - (9
*a^2*x^2)/(220*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int x^5 (a + bx^2)^{2/3} \, dx = \frac{3\sqrt{b \, x^2 + a} \left(\sqrt{b} \, \sqrt{b \, x^2 + a} \, x + a + b \, x^2\right)^{\frac{1}{3}} (20b^3 x^6 + 5a \, b^2 x^4 - 6a^2 b \, x^2 + 9a^3)}{440 \left(\sqrt{b \, x^2 + a} + \sqrt{b} \, x\right)^{\frac{1}{3}} b^3}$$

input
```
int(x^5*(b*x^2+a)^(2/3),x)
```

output
```
(3*a**(7/6)*sqrt(a + b*x**2)*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(1
/3)*(9*a**3 - 6*a**2*b*x**2 + 5*a*b**2*x**4 + 20*b**3*x**6))/(440*a**(1/6)
*(sqrt(a + b*x**2) + sqrt(b)*x)**(1/3)*a*b**3)
```

3.728 $\int x^3(a+bx^2)^{2/3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3\left(a+bx^2\right)^{2/3} \, dx = -\frac{3a(a+bx^2)^{5/3}}{10b^2}+\frac{3(a+bx^2)^{8/3}}{16b^2}$$

output
```
-3/10*a*(b*x^2+a)^(5/3)/b^2+3/16*(b*x^2+a)^(8/3)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int x^3\left(a+bx^2\right)^{2/3} \, dx = \frac{3(a+bx^2)^{2/3}\left(-3a^2+2abx^2+5b^2x^4\right)}{80b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(2/3)*(-3*a^2 + 2*a*b*x^2 + 5*b^2*x^4))/(80*b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^{2/3}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int x^2\left(bx^2+a\right)^{2/3}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{5/3}}{b}-\frac{a\left(bx^2+a\right)^{2/3}}{b}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{3\left(a+bx^2\right)^{8/3}}{8b^2}-\frac{3a\left(a+bx^2\right)^{5/3}}{5b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(2/3),x]
```

output
```
((-3*a*(a + b*x^2)^(5/3))/(5*b^2) + (3*(a + b*x^2)^(8/3))/(8*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3(b\,x^{2}+a)^{\frac{5}{3}}(-5b\,x^{2}+3a)}{80b^{2}}$	25
pseudoelliptic	$-\frac{3(b\,x^{2}+a)^{\frac{5}{3}}(-5b\,x^{2}+3a)}{80b^{2}}$	25
orering	$-\frac{3(b\,x^{2}+a)^{\frac{5}{3}}(-5b\,x^{2}+3a)}{80b^{2}}$	25
trager	$-\frac{3(-5b^{2}x^{4}-2ab\,x^{2}+3a^{2})(b\,x^{2}+a)^{\frac{2}{3}}}{80b^{2}}$	36
risch	$-\frac{3(-5b^{2}x^{4}-2ab\,x^{2}+3a^{2})(b\,x^{2}+a)^{\frac{2}{3}}}{80b^{2}}$	36

input
```
int(x^3*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/80*(b*x^2+a)^(5/3)*(-5*b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^2)^{2/3}\,dx=\frac{3\,(5\,b^2x^4+2\,abx^2-3\,a^2)(bx^2+a)^{\frac{2}{3}}}{80\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/80*(5*b^2*x^4 + 2*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.74

$$\int x^3 (a + bx^2)^{2/3} \, dx = \begin{cases} -\frac{9a^2(a+bx^2)^{\frac{2}{3}}}{80b^2} + \frac{3ax^2(a+bx^2)^{\frac{2}{3}}}{40b} + \frac{3x^4(a+bx^2)^{\frac{2}{3}}}{16} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(2/3),x)
```

output
```
Piecewise((-9*a**2*(a + b*x**2)**(2/3)/(80*b**2) + 3*a*x**2*(a + b*x**2)**
(2/3)/(40*b) + 3*x**4*(a + b*x**2)**(2/3)/16, Ne(b, 0)), (a**(2/3)*x**4/4,
 True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^3 (a + bx^2)^{2/3} \, dx = \frac{3 (bx^2 + a)^{\frac{8}{3}}}{16 b^2} - \frac{3 (bx^2 + a)^{\frac{5}{3}} a}{10 b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/16*(b*x^2 + a)^(8/3)/b^2 - 3/10*(b*x^2 + a)^(5/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a + bx^2)^{2/3} \, dx = \frac{3 \left(5 (bx^2 + a)^{\frac{8}{3}} - 8 (bx^2 + a)^{\frac{5}{3}} a\right)}{80 \, b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/80*(5*(b*x^2 + a)^(8/3) - 8*(b*x^2 + a)^(5/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int x^3 (a + bx^2)^{2/3} \, dx = (b \, x^2 + a)^{2/3} \left(\frac{3 \, x^4}{16} - \frac{9 \, a^2}{80 \, b^2} + \frac{3 \, a \, x^2}{40 \, b}\right)$$

input
```
int(x^3*(a + b*x^2)^(2/3),x)
```

output
```
(a + b*x^2)^(2/3)*((3*x^4)/16 - (9*a^2)/(80*b^2) + (3*a*x^2)/(40*b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int x^3 (a + bx^2)^{2/3} \, dx = \frac{3\sqrt{b \, x^2 + a} \left(\sqrt{b} \, \sqrt{b \, x^2 + a} \, x + a + b \, x^2\right)^{\frac{1}{3}} (5b^2x^4 + 2ab \, x^2 - 3a^2)}{80 \left(\sqrt{b \, x^2 + a} + \sqrt{b} \, x\right)^{\frac{1}{3}} b^2}$$

input
```
int(x^3*(b*x^2+a)^(2/3),x)
```

output
```
(3*a**(7/6)*sqrt(a + b*x**2)*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(1
/3)*( - 3*a**2 + 2*a*b*x**2 + 5*b**2*x**4))/(80*a**(1/6)*(sqrt(a + b*x**2)
 + sqrt(b)*x)**(1/3)*a*b**2)
```

3.729 $\int x(a+bx^2)^{2/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3(a+bx^2)^{5/3}}{10b}$$

output
```
3/10*(b*x^2+a)^(5/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3(a+bx^2)^{5/3}}{10b}$$

input
```
Integrate[x*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(5/3))/(10*b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^{2/3} dx$$

$$\Big\downarrow \; 241$$

$$\frac{3(a+bx^2)^{5/3}}{10b}$$

input
```
Int[x*(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(5/3))/(10*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
derivativedivides	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
default	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
trager	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
risch	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
pseudoelliptic	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15
orering	$\frac{3(b x^2+a)^{\frac{5}{3}}}{10b}$	15

input
```
int(x*(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/10*(b*x^2+a)^(5/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\left(a+bx^2\right)^{2/3}\,dx=\frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

input
```
integrate(x*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(14) = 28$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x(a+bx^2)^{2/3}\,dx = \begin{cases} \frac{3a(a+bx^2)^{\frac{2}{3}}}{10b} + \frac{3x^2(a+bx^2)^{\frac{2}{3}}}{10} & \text{for } b \neq 0 \\ \frac{a^{\frac{2}{3}}x^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**(2/3),x)
```

output
```
Piecewise((3*a*(a + b*x**2)**(2/3)/(10*b) + 3*x**2*(a + b*x**2)**(2/3)/10,
 Ne(b, 0)), (a**(2/3)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{2/3}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

input
```
integrate(x*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{2/3}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b}$$

input
```
integrate(x*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{2/3}\,dx=\frac{3\,(b\,x^2+a)^{5/3}}{10\,b}$$

input
```
int(x*(a + b*x^2)^(2/3),x)
```

output
```
(3*(a + b*x^2)^(5/3))/(10*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.11

$$\int x(a+bx^2)^{2/3}\,dx=\frac{3\sqrt{b\,x^2+a}\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{1}{3}}(b\,x^2+a)}{10\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{1}{3}}b}$$

input
```
int(x*(b*x^2+a)^(2/3),x)
```

output
```
(3*a**(7/6)*sqrt(a + b*x**2)*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(1
/3)*(a + b*x**2))/(10*a**(1/6)*(sqrt(a + b*x**2) + sqrt(b)*x)**(1/3)*a*b)
```

3.730 $\int \frac{(a+bx^2)^{2/3}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{(a+bx^2)^{2/3}}{x} \, dx = \frac{3}{4}(a+bx^2)^{2/3} + \frac{1}{2}\sqrt{3}a^{2/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right) - \frac{1}{2}a^{2/3}\log(x) + \frac{3}{4}a^{2/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

output
```
3/4*(b*x^2+a)^(2/3)+1/2*3^(1/2)*a^(2/3)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1
/3))*3^(1/2)/a^(1/3))-1/2*a^(2/3)*ln(x)+3/4*a^(2/3)*ln(a^(1/3)-(b*x^2+a)^(
1/3))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^2)^{2/3}}{x} \, dx = \frac{1}{4}\left(3(a+bx^2)^{2/3} + 2\sqrt{3}a^{2/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2a^{2/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - a^{2/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+b\right.\right.$$

input
```
Integrate[(a + b*x^2)^(2/3)/x,x]
```

output
```
(3*(a + b*x^2)^(2/3) + 2*Sqrt[3]*a^(2/3)*ArcTan[(1 + (2*(a + b*x^2)^(1/3))
/a^(1/3))/Sqrt[3]] + 2*a^(2/3)*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - a^(2/3)
*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/4
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 60, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{2/3}}{x}dx$$

$$\Big\downarrow 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{2/3}}{x^2}dx^2$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left(a\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx^2+\frac{3}{2}(a+bx^2)^{2/3}\right)$$

$$\Big\downarrow 67$$

$$\frac{1}{2}\left(a\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx^2)^{2/3}\right)$$

$$\Big\downarrow 16$$

$$\frac{1}{2}\left(a\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)+\frac{3}{2}(a+bx^2)^{2/3}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(a\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^2\right)^{2/3}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(a\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)+\frac{3}{2}\left(a+bx^2\right)^{2/3}\right)$$

input
```
Int[(a + b*x^2)^(2/3)/x,x]
```

output
```
((3*(a + b*x^2)^(2/3))/2 + a*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a
^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b
*x^2)^(1/3)])/(2*a^(1/3))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$\frac{3\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{4}+\frac{a^{\frac{2}{3}}\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2}-\frac{a^{\frac{2}{3}}\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)}{4}+\frac{a^{\frac{2}{3}}\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}\right.}{2}$

input
```
int((b*x^2+a)^(2/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3/4*(b*x^2+a)^(2/3)+1/2*a^(2/3)*ln((b*x^2+a)^(1/3)-a^(1/3))-1/4*a^(2/3)*ln
(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))+1/2*a^(2/3)*3^(1/2)*arct
an(2/3*3^(1/2)/a^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = \frac{1}{2}\sqrt{3}(a^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}a+2\sqrt{3}(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{1}{3}}}{3a}\right) - \frac{1}{4}(a^2)^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right) + \frac{1}{2}(a^2)^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{1}{3}}a-(a^2)^{\frac{2}{3}}\right)+\frac{3}{4}(bx^2+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^2+a)^(2/3)/x,x, algorithm="fricas")
```

output
```
1/2*sqrt(3)*(a^2)^(1/3)*arctan(1/3*(sqrt(3)*a + 2*sqrt(3)*(b*x^2 + a)^(1/3
)*(a^2)^(1/3))/a) - 1/4*(a^2)^(1/3)*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*
a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) + 1/2*(a^2)^(1/3)*log((b*x^2 + a)^(1/3)
*a - (a^2)^(2/3)) + 3/4*(b*x^2 + a)^(2/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.46

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = -\frac{b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(-\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},-\frac{2}{3}\\ \frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{1}{3}\right)}$$

input
```
integrate((b*x**2+a)**(2/3)/x,x)
```

output
```
-b**(2/3)*x**(4/3)*gamma(-2/3)*hyper((-2/3, -2/3), (1/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = \frac{1}{2}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}\,a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}\,a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{4}\,(bx^2+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^2+a)^(2/3)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 1/2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/4*(b*x^2 + a)^(2
/3)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = \frac{1}{2}\sqrt{3}a^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}\,a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}\,a^{\frac{2}{3}}\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)+\frac{3}{4}\,(bx^2+a)^{\frac{2}{3}}$$

input
```
integrate((b*x^2+a)^(2/3)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3)) - 1/4*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 1/2*a^(2/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/4*(b*x^2 +
a)^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = \frac{3\,(b\,x^2+a)^{2/3}}{4} + \frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{4} - \frac{9\,a^{7/3}}{4}\right)}{2} - \frac{a^{2/3}\,\ln\left(\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{4} - \frac{9\,a^{7/3}\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2}{4}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2} + a^{2/3}\,\ln\left(\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{4} - 9\,a^{7/3}\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)^2\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

input
```
int((a + b*x^2)^(2/3)/x,x)
```

output
```
(3*(a + b*x^2)^(2/3))/4 + (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^
(7/3))/4))/2 - (a^(2/3)*log((9*a^2*(a + b*x^2)^(1/3))/4 - (9*a^(7/3)*((3^(
1/2)*1i)/2 + 1/2)^2)/4)*((3^(1/2)*1i)/2 + 1/2))/2 + a^(2/3)*log((9*a^2*(a
+ b*x^2)^(1/3))/4 - 9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4)^2)*((3^(1/2)*1i)/4 -
1/4)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{2/3}}{x}\,dx = \frac{3(b\,x^2+a)^{\frac{2}{3}}}{4} + \left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{b\,x^3+ax}dx\right)a$$

input
```
int((b*x^2+a)^(2/3)/x,x)
```

output
```
(3*(a + b*x**2)**(2/3) + 4*int((a + b*x**2)**(2/3)/(a*x + b*x**3),x)*a)/4
```

3.731 $\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = -\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} - \frac{b\log(x)}{3\sqrt[3]{a}}+\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}$$

output
```
-1/2*(b*x^2+a)^(2/3)/x^2+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1
/2)/a^(1/3))*3^(1/2)/a^(1/3)-1/3*b*ln(x)/a^(1/3)+1/2*b*ln(a^(1/3)-(b*x^2+a
)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = -\frac{(a+bx^2)^{2/3}}{2x^2}+\frac{b\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\sqrt[3]{a}} + \frac{b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{3\sqrt[3]{a}} - \frac{b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{6\sqrt[3]{a}}$$

input
```
Integrate[(a + b*x^2)^(2/3)/x^3,x]
```

output
```
-1/2*(a + b*x^2)^(2/3)/x^2 + (b*ArcTan[1/Sqrt[3] + (2*(a + b*x^2)^(1/3))/(
Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)) + (b*Log[-a^(1/3) + (a + b*x^2)^(1/3)
])/(3*a^(1/3)) - (b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^
(2/3)])/(6*a^(1/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 51, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{2/3}}{x^4}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{2}{3}b\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx^2-\frac{\left(a+bx^2\right)^{2/3}}{x^2}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^2\right)^{2/3}}{x^2}\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^2\right)^{2/3}}{x^2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(-\frac{3\int\frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^2\right)^{2/3}}{x^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)-\frac{\left(a+bx^2\right)^{2/3}}{x^2}\right)$$

input
```
Int[(a + b*x^2)^(2/3)/x^3,x]
```

output
```
(-((a + b*x^2)^(2/3)/x^2) + (2*b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(2*a^(1/3))))/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

method	result
pseudoelliptic	$\frac{2b\sqrt{3}\,\arctan\left(\frac{2\sqrt{3}\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)x^{2}+2b\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^{2}-b\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)x^{2}-3(b\,x^{2}+a}{6x^{2}a^{\frac{1}{3}}}$

input
```
int((b*x^2+a)^(2/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(1/2))*x
^2+2*b*ln((b*x^2+a)^(1/3)-a^(1/3))*x^2-b*ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3
)+(b*x^2+a)^(2/3))*x^2-3*(b*x^2+a)^(2/3)*a^(1/3))/x^2/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.79

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)}{}\right.$$

input
```
integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="fricas")
```

output
```
[1/6*(3*sqrt(1/3)*a*b*x^2*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(2*(
b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3))
 - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*b*x^2*log((b*x^2 + a)
^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b*x^2*log((b*x^2
 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*a)/(a*x^2), 1/6*(6*sqrt(1/3)*
a^(2/3)*b*x^2*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)) -
a^(2/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
 + 2*a^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(b*x^2 + a)^(2/3)*
a)/(a*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = -\frac{b^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**2+a)**(2/3)/x**3,x)
```

output
```
-b**(2/3)*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2)
)/(2*x**(2/3)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = \frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{3\,a^{\frac{1}{3}}} - \frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{6\,a^{\frac{1}{3}}} + \frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}} - \frac{(bx^2+a)^{\frac{2}{3}}}{2\,x^2}$$

input
```
integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(1/3) - 1/6*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3
))/a^(1/3) + 1/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3) - 1/2*(b*x^2 +
 a)^(2/3)/x^2
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = \frac{1}{6}\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{1}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{1}{3}}} + \frac{2}{}\right.$$

input
```
integrate((b*x^2+a)^(2/3)/x^3,x, algorithm="giac")
```

output
```
1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(1/3) - log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^
(1/3) + 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3) - 3*(b*x^2 + a)^(2
/3)/(b*x^2))*b
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.31

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = \frac{b\ln\left(a^{1/3}\,b^2-b^2\,(b\,x^2+a)^{1/3}\right)}{3\,a^{1/3}} - \frac{(b\,x^2+a)^{2/3}}{2\,x^2} - \frac{\ln\left(\frac{a^{1/3}\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4}-b^2\,(b\,x^2+a)^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{1/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{4}-b^2\,(b\,x^2+a)^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{1/3}}$$

input
```
int((a + b*x^2)^(2/3)/x^3,x)
```

output
```
(b*log(a^(1/3)*b^2 - b^2*(a + b*x^2)^(1/3)))/(3*a^(1/3)) - (a + b*x^2)^(2/
3)/(2*x^2) - (log((a^(1/3)*(b - 3^(1/2)*b*1i)^2)/4 - b^2*(a + b*x^2)^(1/3)
)*(b - 3^(1/2)*b*1i))/(6*a^(1/3)) - (log((a^(1/3)*(b + 3^(1/2)*b*1i)^2)/4
- b^2*(a + b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(6*a^(1/3))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^3}\,dx = \frac{-3(b\,x^2+a)^{\frac{2}{3}}+4\left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{b\,x^3+ax}dx\right)b\,x^2}{6x^2}$$

input
```
int((b*x^2+a)^(2/3)/x^3,x)
```

output
```
( - 3*(a + b*x**2)**(2/3) + 4*int((a + b*x**2)**(2/3)/(a*x + b*x**3),x)*b*
x**2)/(6*x**2)
```

3.732 $\int \frac{(a+bx^2)^{2/3}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{(a+bx^2)^{2/3}}{x^5} dx = -\frac{(a+bx^2)^{2/3}}{4x^4} - \frac{b(a+bx^2)^{2/3}}{6ax^2} - \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}} + \frac{b^2 \log(x)}{18a^{4/3}} - \frac{b^2 \log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{4/3}}$$

output
```
-1/4*(b*x^2+a)^(2/3)/x^4-1/6*b*(b*x^2+a)^(2/3)/a/x^2-1/18*b^2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(4/3)+1/18*b^2*ln(x)
/a^(4/3)-1/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^{2/3}}{x^5} dx = \frac{(-3a-2bx^2)(a+bx^2)^{2/3}}{12ax^4} - \frac{b^2 \arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{4/3}} - \frac{b^2 \log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{18a^{4/3}} + \frac{b^2 \log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{36a^{4/3}}$$

input
```
Integrate[(a + b*x^2)^(2/3)/x^5,x]
```

output
```
((-3*a - 2*b*x^2)*(a + b*x^2)^(2/3))/(12*a*x^4) - (b^2*ArcTan[1/Sqrt[3] +
(2*(a + b*x^2)^(1/3))/(Sqrt[3]*a^(1/3))])/(6*Sqrt[3]*a^(4/3)) - (b^2*Log[-
a^(1/3) + (a + b*x^2)^(1/3)])/(18*a^(4/3)) + (b^2*Log[a^(2/3) + a^(1/3)*(a
 + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(4/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 51, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{2/3}}{x^5} dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{2/3}}{x^6}dx^2$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}\left(\frac{1}{3}b\int \frac{1}{x^4\sqrt[3]{bx^2+a}}dx^2-\frac{(a+bx^2)^{2/3}}{2x^4}\right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{b\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx^2}{3a}-\frac{(a+bx^2)^{2/3}}{ax^2}\right)-\frac{(a+bx^2)^{2/3}}{2x^4}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right.\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)-\frac{(a\ldots\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{b\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)-\frac{\left(a+bx^2\right)^{2/3}}{2x^4}\ldots\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(\frac{1}{3}b\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{(a+bx^2)^{2/3}}{ax^2}\right)-\frac{(a+bx^2)^{2/3}}{2x^4}\right)$$

input
```
Int[(a + b*x^2)^(2/3)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(2/3)/x^4 + (b*(-((a + b*x^2)^(2/3)/(a*x^2)) - (b*((Sqrt
[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2
]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(1/3))))/(3*a)))
/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$\frac{-2b^2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^2+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^4-2b^2\ln\left(\left(b\,x^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^4+b^2\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+\left(b\,x^2+a\right)^{\frac{2}{3}}\right)x^4-6b}{36a^{\frac{4}{3}}x^4}$

input
```
int((b*x^2+a)^(2/3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(-2*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3
))*x^4-2*b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4+b^2*ln(a^(2/3)+a^(1/3)*(b*x^2
+a)^(1/3)+(b*x^2+a)^(2/3))*x^4-6*b*x^2*(b*x^2+a)^(2/3)*a^(1/3)-9*(b*x^2+a)
^(2/3)*a^(4/3))/a^(4/3)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.41

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)}{}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}a^{\frac{2}{3}}b^2x^4\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{a^{\frac{1}{3}}}\right)-a^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)+2\,a^{\frac{2}{3}}b^2x^4\,\mathrm{lo}}{36\,a^2x^4}$$

input
```
integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="fricas")
```

output
```
[1/36*(3*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 - 3*sqrt(1/3)*(
2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/
3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) + a^(2/3)*b^2*x^4*log((b*x^2
 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 2*a^(2/3)*b^2*x^4*log
((b*x^2 + a)^(1/3) - a^(1/3)) - 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(
a^2*x^4), -1/36*(6*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2 +
a)^(1/3) + a^(1/3))/a^(1/3)) - a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) + (b*
x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3
) - a^(1/3)) + 3*(2*a*b*x^2 + 3*a^2)*(b*x^2 + a)^(2/3))/(a^2*x^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.31

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = -\frac{b^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)\,{}_2F_1\left(\begin{matrix} -\frac{2}{3},\frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate((b*x**2+a)**(2/3)/x**5,x)
```

output
```
-b**(2/3)*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2)
)/(2*x**(8/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = -\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{18\,a^{\frac{4}{3}}} + \frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{36\,a^{\frac{4}{3}}} - \frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{18\,a^{\frac{4}{3}}} - \frac{2\,(bx^2+a)^{\frac{5}{3}}b^2+(bx^2+a)^{\frac{2}{3}}ab^2}{12\,\left((bx^2+a)^2a-2\,(bx^2+a)a^2+a^3\right)}$$

input
```
integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="maxima")
```

output
```
-1/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(4/3) + 1/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(4/3) - 1/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) - 1/
12*(2*(b*x^2 + a)^(5/3)*b^2 + (b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a -
2*(b*x^2 + a)*a^2 + a^3)
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = -\frac{\frac{2\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}}-\frac{b^3\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}}+\frac{2\,b^3\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{4}{3}}}+\frac{3\left(2\,(bx^2+a)^{\frac{5}{3}}b^3+(b\right.}{ab^2x^4}}{36\,b}$$

input
```
integrate((b*x^2+a)^(2/3)/x^5,x, algorithm="giac")
```

output
```
-1/36*(2*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(4/3) - b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(4/3) + 2*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3*
(2*(b*x^2 + a)^(5/3)*b^3 + (b*x^2 + a)^(2/3)*a*b^3)/(a*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = \frac{(-1)^{1/3}\,b^2\,\ln\left((b\,x^2+a)^{1/3}-(-1)^{2/3}\,a^{1/3}\right)}{18\,a^{4/3}} - \frac{\frac{b^2\,(b\,x^2+a)^{2/3}}{6}+\frac{b^2\,(b\,x^2+a)^{5/3}}{3\,a}}{2\,(b\,x^2+a)^2-4\,a\,(b\,x^2+a)+2\,a^2} + \frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(b\,x^2+a)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2}{36\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18\,a^{4/3}} - \frac{(-1)^{1/3}\,b^2\,\ln\left(\frac{b^4\,(b\,x^2+a)^{1/3}}{36\,a^2}-\frac{(-1)^{2/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)^2}{36\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1i}{2}\right)}{18\,a^{4/3}}$$

input
```
int((a + b*x^2)^(2/3)/x^5,x)
```

output
```
((-1)^(1/3)*b^2*log((a + b*x^2)^(1/3) - (-1)^(2/3)*a^(1/3)))/(18*a^(4/3))
- ((b^2*(a + b*x^2)^(2/3))/6 + (b^2*(a + b*x^2)^(5/3))/(3*a))/(2*(a + b*x^
2)^2 - 4*a*(a + b*x^2) + 2*a^2) + ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(1/
3))/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(36*a^(5/3)))*((3
^(1/2)*1i)/2 - 1/2))/(18*a^(4/3)) - ((-1)^(1/3)*b^2*log((b^4*(a + b*x^2)^(
1/3))/(36*a^2) - ((-1)^(2/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)/(36*a^(5/3)))*(
(3^(1/2)*1i)/2 + 1/2))/(18*a^(4/3))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^5}\,dx = \frac{-9(b\,x^2+a)^{\frac{2}{3}}\,a-6(b\,x^2+a)^{\frac{2}{3}}\,b\,x^2-4\left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{b\,x^3+ax}dx\right)b^2x^4}{36a\,x^4}$$

input
```
int((b*x^2+a)^(2/3)/x^5,x)
```

output
```
( - 9*(a + b*x**2)**(2/3)*a - 6*(a + b*x**2)**(2/3)*b*x**2 - 4*int((a + b*
x**2)**(2/3)/(a*x + b*x**3),x)*b**2*x**4)/(36*a*x**4)
```

3.733 $\int x^4(a+bx^2)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 601

$$\int x^4(a+bx^2)^{2/3}\,dx=-\frac{108a^2x(a+bx^2)^{2/3}}{1729b^2}+\frac{12ax^3(a+bx^2)^{2/3}}{247b}$$
$$+\frac{3}{19}x^5(a+bx^2)^{2/3}-\frac{324a^3x}{1729b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}+\frac{162\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{10/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\cdots}{((\cdots}}}{1729b^3\cdots}$$

output
```
-108/1729*a^2*x*(b*x^2+a)^(2/3)/b^2+12/247*a*x^3*(b*x^2+a)^(2/3)/b+3/19*x^
5*(b*x^2+a)^(2/3)-324/1729*a^3*x/b^2/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
+162/1729*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(10/3)*(a^(1/3)-(b*x^2+a)^(1
/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-108
/1729*2^(1/2)*3^(3/4)*a^(10/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)
*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)
^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3
)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3)
)/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.16

$$\int x^4(a+bx^2)^{2/3}\,dx = \frac{3x(a+bx^2)^{2/3}\left(\left(1+\frac{bx^2}{a}\right)^{2/3}(-9a^2+4abx^2+13b^2x^4)+9a^2\,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2}\right.\right.}{247b^2\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[x^4*(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(2/3)*((1 + (b*x^2)/a)^(2/3)*(-9*a^2 + 4*a*b*x^2 + 13*b^2
*x^4) + 9*a^2*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)]))/(247*b^2*(
1 + (b*x^2)/a)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {248, 262, 262, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^2)^{2/3}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{4}{19}a\int \frac{x^4}{\sqrt[3]{bx^2+a}}dx+\frac{3}{19}x^5(a+bx^2)^{2/3}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{19}a\left(\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{9a\int \frac{x^2}{\sqrt[3]{bx^2+a}}dx}{13b}\right)+\frac{3}{19}x^5(a+bx^2)^{2/3}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{19}a\left(\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{3a\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}\right)}{13b}\right)+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{233}$$

$$\frac{4}{19}a\left(\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}\right)}{13b}\right)+\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{833}$$

$$\frac{4}{19}a\left(\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2}\right.}{14b^2x}\right.}{13b}\right.$$

$$\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{760}$$

$$\frac{4}{19}a\left(\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{}\right.}{}\right.}{}\right.$$

$$\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}$$

$\downarrow$ 2418

$$\frac{4}{19}a\left(\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}}\cdots\right)}{\cdots}\right)}{\cdots}\right)$$

$$\frac{3}{19}x^5\left(a+bx^2\right)^{2/3}$$

input
```
Int[x^4*(a + b*x^2)^(2/3),x]
```

output
```
(3*x^5*(a + b*x^2)^(2/3))/19 + (4*a*((3*x^3*(a + b*x^2)^(2/3))/(13*b) - (9
*a*((3*x*(a + b*x^2)^(2/3))/(7*b) - (9*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1
/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Elli
pticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)
*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1
 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3]
)*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b
*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(14*b^2*x
)))/(13*b)))/19
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^4 (b x^2 + a)^{\frac{2}{3}} dx$$

input
```
int(x^4*(b*x^2+a)^(2/3),x)
```

output
```
int(x^4*(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{2/3} \, dx = \int \left(bx^2+a\right)^{\frac{2}{3}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int x^4 \left(a+bx^2\right)^{2/3} \, dx = \frac{a^{\frac{2}{3}} x^5 {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{bx^2 e^{i\pi}}{a}} \right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(2/3),x)
```

output
```
a**(2/3)*x**5*hyper((-2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 \left(a+bx^2\right)^{2/3} \, dx = \int \left(bx^2+a\right)^{\frac{2}{3}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a+bx^2\right)^{2/3} dx = \int x^4 \left(b\,x^2+a\right)^{2/3} dx$$

input
```
int(x^4*(a + b*x^2)^(2/3),x)
```

output
```
int(x^4*(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int x^4 \left(a+bx^2\right)^{2/3} dx = \frac{-\frac{108\left(b\,x^2+a\right)^{\frac{2}{3}} a^2 x}{1729}+\frac{12\left(b\,x^2+a\right)^{\frac{2}{3}} a b\,x^3}{247}+\frac{3\left(b\,x^2+a\right)^{\frac{2}{3}} b^2 x^5}{19}+\frac{108\left(\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}}dx\right) a^3}{1729}}{b^2}$$

input
```
int(x^4*(b*x^2+a)^(2/3),x)
```

output
```
(3*( - 36*(a + b*x**2)**(2/3)*a**2*x + 28*(a + b*x**2)**(2/3)*a*b*x**3 + 9
1*(a + b*x**2)**(2/3)*b**2*x**5 + 36*int((a + b*x**2)**(2/3)/(a + b*x**2),
x)*a**3))/(1729*b**2)
```

3.734 $\int x^2(a+bx^2)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 577

$$\int x^2\left(a+bx^2\right)^{2/3}\,dx=\frac{12ax\left(a+bx^2\right)^{2/3}}{91b}$$

$$+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}+\frac{36a^2x}{91b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{18\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{12\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{91b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
12/91*a*x*(b*x^2+a)^(2/3)/b+3/13*x^3*(b*x^2+a)^(2/3)+36/91*a^2*x/b/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))-18/91*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(
7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)
^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/
2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3
^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))^2)^(1/2)+12/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1
/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int x^2 \left(a+bx^2\right)^{2/3} dx = \frac{3x(a+bx^2)^{2/3}\left(a+bx^2-\frac{a \,\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}\right)}{13b}$$

input
```
Integrate[x^2*(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(2/3)*(a + b*x^2 - (a*Hypergeometric2F1[-2/3, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(2/3)))/(13*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {248, 262, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{2/3} \, dx$$

$$\Big\downarrow \; 248$$

$$\frac{4}{13}a\int \frac{x^2}{\sqrt[3]{bx^2+a}}dx+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \; 262$$

$$\frac{4}{13}a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{3a\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}\right)+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \; 233$$

$$\frac{4}{13}a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}\right)+\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \; 833$$

$$\frac{4}{13}a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{14b^2x}\right)+$$

$$\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \; 760$$

$$\frac{4}{13}a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{}\right)}{14b^2x}\right)$$

$$\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

$\downarrow$ 2418

$$\frac{4}{13}a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right)}{}\right)$$

$$\frac{3}{13}x^3\left(a+bx^2\right)^{2/3}$$

input
```
Int[x^2*(a + b*x^2)^(2/3),x]
```

output
```
(3*x^3*(a + b*x^2)^(2/3))/13 + (4*a*((3*x*(a + b*x^2)^(2/3))/(7*b) - (9*a*
Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))
+ (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a
^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[
3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3
])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^
2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^
2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt
[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*
Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))^2)])))/(14*b^2*x)))/13
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^2 \left(b\,x^2+a\right)^{\frac{2}{3}}dx$$

input
```
int(x^2*(b*x^2+a)^(2/3),x)
```

output
```
int(x^2*(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int x^2 \left(a+b x^2\right)^{2/3} dx = \int (b x^2 + a)^{\frac{2}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int x^2 \left(a+b x^2\right)^{2/3} dx = \frac{a^{\frac{2}{3}} x^3 {}_{2}F_{1}\left(\begin{matrix} -\frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{b x^2 e^{i\pi}}{a} \right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(2/3),x)
```

output
```
a**(2/3)*x**3*hyper((-2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \left(a+b x^2\right)^{2/3} dx = \int (b x^2 + a)^{\frac{2}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3)*x^2, x)
```

Giac [F]

$$\int x^2 \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^2\right)^{2/3} dx = \int x^2 \left(b\,x^2+a\right)^{2/3} dx$$

input
```
int(x^2*(a + b*x^2)^(2/3),x)
```

output
```
int(x^2*(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int x^2 \left(a+bx^2\right)^{2/3} dx = \frac{\frac{12\left(b\,x^2+a\right)^{\frac{2}{3}} a x}{91} + \frac{3\left(b\,x^2+a\right)^{\frac{2}{3}} b\,x^3}{13} - \frac{12\left(\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}} dx\right) a^2}{91}}{b}$$

input
```
int(x^2*(b*x^2+a)^(2/3),x)
```

output
```
(3*(4*(a + b*x**2)**(2/3)*a*x + 7*(a + b*x**2)**(2/3)*b*x**3 - 4*int((a +
b*x**2)**(2/3)/(a + b*x**2),x)*a**2))/(91*b)
```

3.735 $\int (a+bx^2)^{2/3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 550

$$\int (a+bx^2)^{2/3} \, dx = \frac{3}{7}x(a+bx^2)^{2/3} - \frac{12ax}{7\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+ \frac{6\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$- \frac{4\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{7bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/7*x*(b*x^2+a)^(2/3)-12*a*x/(7*(1-3^(1/2))*a^(1/3)-7*(b*x^2+a)^(1/3))+6/7
*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(
2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^
2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-4/7*2^(1/2)*3^(
3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(
b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF((
(1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
,2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int (a+bx^2)^{2/3} \, dx = \frac{x(a+bx^2)^{2/3} \operatorname{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(2/3),x]
```

output
```
(x*(a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(2/3)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {211, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{2/3}\,dx$$

$$\Big\downarrow \text{ 211}$$

$$\frac{4}{7}a\int \frac{1}{\sqrt[3]{bx^2+a}}dx+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{6a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{6a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{7bx}+\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{6a\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+b}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx}}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\right.}{7bx}$$

$$\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{6a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2-}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{3}{7}x\left(a+bx^2\right)^{2/3}$$

input
```
Int[(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(2/3))/7 + (6*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
 b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Ar
cSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]) -
 (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*
Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[
3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 +
4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1
/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]))))/(7*b*x)
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{2}{3}}d x$$

input
```
int((b*x^2+a)^(2/3),x)
```

output
```
int((b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{2/3}\,dx=\int \left(bx^{2}+a\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.05

$$\int \left(a+bx^2\right)^{2/3} dx = a^{\frac{2}{3}} x\,{}_2F_1\left(\begin{matrix} -\frac{2}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(2/3),x)
```

output
```
a**(2/3)*x*hyper((-2/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{2/3} dx = \int \left(bx^2+a\right)^{\frac{2}{3}} dx$$

input
```
integrate((b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \left(a+bx^2\right)^{2/3}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{2/3}}$$

input
```
int((a + b*x^2)^(2/3),x)
```

output
```
(x*(a + b*x^2)^(2/3)*hypergeom([-2/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(2/3)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{2/3}\,dx=\frac{3(b\,x^2+a)^{\frac{2}{3}}\,x}{7}+\frac{4\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}}dx\right)a}{7}$$

input
```
int((b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(2/3)*x + 4*int((a + b*x**2)**(2/3)/(a + b*x**2),x)*a)/7
```

3.736 $\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 538

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = -\frac{(a+bx^2)^{2/3}}{x} - \frac{4bx}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}$$
$$+\frac{2\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|\cdots}{x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{4\sqrt{2}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-\cdots}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(2/3)/x-4*b*x/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+2*3^(1/4)*(
1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/
3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^
2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1
/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/
((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-4/3*2^(1/2)*a^(1/3)*(a^(1/3
)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-
(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3
/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.31 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = -\frac{(a+bx^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(2/3)/x^2,x]
```

output
```
-(((a + b*x^2)^(2/3)*Hypergeometric2F1[-2/3, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(2/3)))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^2}dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{4}{3}b\int \frac{1}{\sqrt[3]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{2/3}}{x}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{2\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{x}-\frac{\left(a+bx^2\right)^{2/3}}{x}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{2\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{x}-\frac{\left(a+bx^2\right)^{2/3}}{x}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{2\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}\right.}{x}-\frac{\left(a+bx^2\right)^{2/3}}{x}$$

$$\Big\downarrow \text{ 2418}$$

$$\frac{2\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

$$\frac{\left(a+bx^2\right)^{2/3}}{x}$$

input
```
Int[(a + b*x^2)^(2/3)/x^2,x]
```

output
```
-((a + b*x^2)^(2/3)/x) + (2*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3)
- (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^
2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[
((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a +
b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (
a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*S
qrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))
/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/x
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{x^2}dx$$

input
```
int((b*x^2+a)^(2/3)/x^2,x)
```

output
```
int((b*x^2+a)^(2/3)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = -\frac{a^{\frac{2}{3}}{}_2F_1\left(\begin{matrix} -\frac{2}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(2/3)/x**2,x)
```

output
```
-a**(2/3)*hyper((-2/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{2}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = \frac{3\,(b\,x^2+a)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},-\frac{1}{6};\ \frac{5}{6};\ -\frac{a}{b\,x^2}\right)}{x\,\left(\frac{a}{b\,x^2}+1\right)^{2/3}}$$

input
```
int((a + b*x^2)^(2/3)/x^2,x)
```

output
```
(3*(a + b*x^2)^(2/3)*hypergeom([-2/3, -1/6], 5/6, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(2/3))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^2}\,dx = \frac{3(b\,x^2+a)^{\frac{2}{3}}+4\left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((b*x^2+a)^(2/3)/x^2,x)
```

output
```
(3*(a + b*x**2)**(2/3) + 4*int((a + b*x**2)**(2/3)/(a*x**2 + b*x**4),x)*a*
x)/x
```

3.737 $\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 575

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = -\frac{(a+bx^2)^{2/3}}{3x^3} - \frac{4b(a+bx^2)^{2/3}}{9ax} - \frac{4b^2x}{9a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7+}{3\,3^{3/4}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{4\sqrt{2}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7}{9\sqrt[4]{3}\,a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(2/3)/x^3-4/9*b*(b*x^2+a)^(2/3)/a/x-4/9*b^2*x/a/((1-3^(1/2)
)*a^(1/3)-(b*x^2+a)^(1/3))+2/9*(1/2*6^(1/2)+1/2*2^(1/2))*b*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a
)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/a^(2
/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3))^2)^(1/2)-4/27*2^(1/2)*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*
(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^
(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x
^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = -\frac{(a+bx^2)^{2/3}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{2}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(2/3)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(2/3)*Hypergeometric2F1[-3/2, -2/3, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {247, 264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{2/3}}{x^4}dx$$

$\big\downarrow$ 247

$$\frac{4}{9}b\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

$\big\downarrow$ 264

$$\frac{4}{9}b\left(\frac{b\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)-\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

$\big\downarrow$ 233

$$\frac{4}{9}b\left(\frac{\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)-\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

$\big\downarrow$ 833

$$\frac{4}{9}b\left(\frac{\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)-\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

$\big\downarrow$ 760

$$\frac{4}{9}b\left(\frac{\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{\right.}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{\right.}}}}\right.}{2ax}\right.$$

$$\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

$\big\downarrow$ 2418

$$\frac{4}{9}b\left(\frac{\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.$$

$$\frac{\left(a+bx^2\right)^{2/3}}{3x^3}$$

input

```
Int[(a + b*x^2)^(2/3)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(2/3)/x^3 + (4*b*(-((a + b*x^2)^(2/3)/(a*x)) + (Sqrt[b*x^
2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4
)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
 + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqr
t[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipt
icF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a
^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2)])))/(2*a*x)))/9
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{x^4}dx$$

input
```
int((b*x^2+a)^(2/3)/x^4,x)
```

output
```
int((b*x^2+a)^(2/3)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{2}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.06

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = -\frac{a^{\frac{2}{3}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{2}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(2/3)/x**4,x)
```

output
```
-a**(2/3)*hyper((-3/2, -2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{2}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(2/3)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{2}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(2/3)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(2/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx=\int \frac{(b\,x^2+a)^{2/3}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(2/3)/x^4,x)
```

output
```
int((a + b*x^2)^(2/3)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{2/3}}{x^4}\,dx=\frac{-3(b\,x^2+a)^{\frac{2}{3}}-4\left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{b\,x^6+a\,x^4}dx\right)a\,x^3}{5x^3}$$

input
```
int((b*x^2+a)^(2/3)/x^4,x)
```

output
```
( - 3*(a + b*x**2)**(2/3) - 4*int((a + b*x**2)**(2/3)/(a*x**4 + b*x**6),x)
*a*x**3)/(5*x**3)
```

3.738 $\int x^7(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int x^7(a+bx^2)^{4/3}\,dx = -\frac{3a^3(a+bx^2)^{7/3}}{14b^4} + \frac{9a^2(a+bx^2)^{10/3}}{20b^4} - \frac{9a(a+bx^2)^{13/3}}{26b^4} + \frac{3(a+bx^2)^{16/3}}{32b^4}$$

output
```
-3/14*a^3*(b*x^2+a)^(7/3)/b^4+9/20*a^2*(b*x^2+a)^(10/3)/b^4-9/26*a*(b*x^2+
a)^(13/3)/b^4+3/32*(b*x^2+a)^(16/3)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int x^7(a+bx^2)^{4/3}\,dx = \frac{3(a+bx^2)^{7/3}\left(-81a^3+189a^2bx^2-315ab^2x^4+455b^3x^6\right)}{14560b^4}$$

input
```
Integrate[x^7*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)^(7/3)*(-81*a^3 + 189*a^2*b*x^2 - 315*a*b^2*x^4 + 455*b^3*x^
6))/(14560*b^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^{4/3} dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int x^6 \left(bx^2+a\right)^{4/3} dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{13/3}}{b^3}-\frac{3a\left(bx^2+a\right)^{10/3}}{b^3}+\frac{3a^2\left(bx^2+a\right)^{7/3}}{b^3}-\frac{a^3\left(bx^2+a\right)^{4/3}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{3a^3\left(a+bx^2\right)^{7/3}}{7b^4}+\frac{9a^2\left(a+bx^2\right)^{10/3}}{10b^4}+\frac{3\left(a+bx^2\right)^{16/3}}{16b^4}-\frac{9a\left(a+bx^2\right)^{13/3}}{13b^4}\right)$$

input
```
Int[x^7*(a + b*x^2)^(4/3),x]
```

output
```
((-3*a^3*(a + b*x^2)^(7/3))/(7*b^4) + (9*a^2*(a + b*x^2)^(10/3))/(10*b^4)
- (9*a*(a + b*x^2)^(13/3))/(13*b^4) + (3*(a + b*x^2)^(16/3))/(16*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}\left(-455b^{3}x^{6}+315a\,b^{2}x^{4}-189a^{2}b\,x^{2}+81a^{3}\right)}{14560b^{4}}$	47
pseudoelliptic	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}\left(-455b^{3}x^{6}+315a\,b^{2}x^{4}-189a^{2}b\,x^{2}+81a^{3}\right)}{14560b^{4}}$	47
orering	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}\left(-455b^{3}x^{6}+315a\,b^{2}x^{4}-189a^{2}b\,x^{2}+81a^{3}\right)}{14560b^{4}}$	47
trager	$-\frac{3\left(-455b^{5}x^{10}-595a\,b^{4}x^{8}-14a^{2}b^{3}x^{6}+18a^{3}b^{2}x^{4}-27a^{4}b\,x^{2}+81a^{5}\right)(b\,x^{2}+a)^{\frac{1}{3}}}{14560b^{4}}$	69
risch	$-\frac{3\left(-455b^{5}x^{10}-595a\,b^{4}x^{8}-14a^{2}b^{3}x^{6}+18a^{3}b^{2}x^{4}-27a^{4}b\,x^{2}+81a^{5}\right)(b\,x^{2}+a)^{\frac{1}{3}}}{14560b^{4}}$	69

input
```
int(x^7*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14560*(b*x^2+a)^(7/3)*(-455*b^3*x^6+315*a*b^2*x^4-189*a^2*b*x^2+81*a^3)
/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x^{7}(a+bx^{2})^{4/3}\,dx=\frac{3\,(455\,b^{5}x^{10}+595\,ab^{4}x^{8}+14\,a^{2}b^{3}x^{6}-18\,a^{3}b^{2}x^{4}+27\,a^{4}bx^{2}-81\,a^{5})(bx^{2}+a)^{\frac{1}{3}}}{14560\,b^{4}}$$

input
```
integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/14560*(455*b^5*x^10 + 595*a*b^4*x^8 + 14*a^2*b^3*x^6 - 18*a^3*b^2*x^4 +
27*a^4*b*x^2 - 81*a^5)*(b*x^2 + a)^(1/3)/b^4
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70

$$\int x^7(a+bx^2)^{4/3}\,dx = \begin{cases} -\frac{243a^5\sqrt[3]{a+bx^2}}{14560b^4} + \frac{81a^4x^2\sqrt[3]{a+bx^2}}{14560b^3} - \frac{27a^3x^4\sqrt[3]{a+bx^2}}{7280b^2} + \frac{3a^2x^6\sqrt[3]{a+bx^2}}{1040b} + \frac{51ax^8\sqrt[3]{a+bx^2}}{416} + \cdots \\ \frac{a^{\frac{4}{3}}x^8}{8} \end{cases}$$

input
```
integrate(x**7*(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((-243*a**5*(a + b*x**2)**(1/3)/(14560*b**4) + 81*a**4*x**2*(a +
b*x**2)**(1/3)/(14560*b**3) - 27*a**3*x**4*(a + b*x**2)**(1/3)/(7280*b**2)
 + 3*a**2*x**6*(a + b*x**2)**(1/3)/(1040*b) + 51*a*x**8*(a + b*x**2)**(1/3
)/416 + 3*b*x**10*(a + b*x**2)**(1/3)/32, Ne(b, 0)), (a**(4/3)*x**8/8, Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^7(a+bx^2)^{4/3}\,dx = \frac{3\,(bx^2+a)^{\frac{16}{3}}}{32\,b^4} - \frac{9\,(bx^2+a)^{\frac{13}{3}}a}{26\,b^4} + \frac{9\,(bx^2+a)^{\frac{10}{3}}a^2}{20\,b^4} - \frac{3\,(bx^2+a)^{\frac{7}{3}}a^3}{14\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/32*(b*x^2 + a)^(16/3)/b^4 - 9/26*(b*x^2 + a)^(13/3)*a/b^4 + 9/20*(b*x^2
+ a)^(10/3)*a^2/b^4 - 3/14*(b*x^2 + a)^(7/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int x^7(a+bx^2)^{4/3}\,dx = \frac{3\left(455\,(bx^2+a)^{\frac{16}{3}}-1680\,(bx^2+a)^{\frac{13}{3}}a+2184\,(bx^2+a)^{\frac{10}{3}}a^2-1040\,(bx^2+a)^{\frac{7}{3}}a^3\right)}{14560\,b^4}$$

input
```
integrate(x^7*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/14560*(455*(b*x^2 + a)^(16/3) - 1680*(b*x^2 + a)^(13/3)*a + 2184*(b*x^2
+ a)^(10/3)*a^2 - 1040*(b*x^2 + a)^(7/3)*a^3)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int x^7(a+bx^2)^{4/3}\,dx = (b\,x^2+a)^{1/3}\left(\frac{51\,a\,x^8}{416}+\frac{3\,b\,x^{10}}{32}-\frac{243\,a^5}{14560\,b^4}+\frac{3\,a^2\,x^6}{1040\,b}-\frac{27\,a^3\,x^4}{7280\,b^2}+\frac{81\,a^4\,x^2}{14560\,b^3}\right)$$

input
```
int(x^7*(a + b*x^2)^(4/3),x)
```

output
```
(a + b*x^2)^(1/3)*((51*a*x^8)/416 + (3*b*x^10)/32 - (243*a^5)/(14560*b^4)
+ (3*a^2*x^6)/(1040*b) - (27*a^3*x^4)/(7280*b^2) + (81*a^4*x^2)/(14560*b^3
))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.19

$$\int x^7(a+bx^2)^{4/3}\,dx = \frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}(455b^5x^{10}+595a\,b^4x^8+14a^2b^3x^6-18a^3b^2x^4+27a^4b\,x^2-8}{14560\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b^4}$$

input
```
int(x^7*(b*x^2+a)^(4/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*( - 81*a**5 + 27*a**4*
b*x**2 - 18*a**3*b**2*x**4 + 14*a**2*b**3*x**6 + 595*a*b**4*x**8 + 455*b**
5*x**10))/(14560*(sqrt(a + b*x**2) + sqrt(b)*x)**(2/3)*b**4)
```

3.739 $\int x^5(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int x^5(a+bx^2)^{4/3}\,dx=\frac{3a^2(a+bx^2)^{7/3}}{14b^3}-\frac{3a(a+bx^2)^{10/3}}{10b^3}+\frac{3(a+bx^2)^{13/3}}{26b^3}$$

output
```
3/14*a^2*(b*x^2+a)^(7/3)/b^3-3/10*a*(b*x^2+a)^(10/3)/b^3+3/26*(b*x^2+a)^(1
3/3)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int x^5(a+bx^2)^{4/3}\,dx=\frac{3(a+bx^2)^{7/3}\,(9a^2-21abx^2+35b^2x^4)}{910b^3}$$

input
```
Integrate[x^5*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)^(7/3)*(9*a^2 - 21*a*b*x^2 + 35*b^2*x^4))/(910*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+bx^2\right)^{4/3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int x^4\left(bx^2+a\right)^{4/3}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{\left(bx^2+a\right)^{10/3}}{b^2}-\frac{2a\left(bx^2+a\right)^{7/3}}{b^2}+\frac{a^2\left(bx^2+a\right)^{4/3}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{3a^2\left(a+bx^2\right)^{7/3}}{7b^3}+\frac{3\left(a+bx^2\right)^{13/3}}{13b^3}-\frac{3a\left(a+bx^2\right)^{10/3}}{5b^3}\right)$$

input
```
Int[x^5*(a + b*x^2)^(4/3),x]
```

output
```
((3*a^2*(a + b*x^2)^(7/3))/(7*b^3) - (3*a*(a + b*x^2)^(10/3))/(5*b^3) + (3
*(a + b*x^2)^(13/3))/(13*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$	36
pseudoelliptic	$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$	36
orering	$\frac{3\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(35b^{2}x^{4}-21ab\,x^{2}+9a^{2}\right)}{910b^{3}}$	36
trager	$\frac{3\left(35b^{4}x^{8}+49a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}-3a^{3}b\,x^{2}+9a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{910b^{3}}$	58
risch	$\frac{3\left(35b^{4}x^{8}+49a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}-3a^{3}b\,x^{2}+9a^{4}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{910b^{3}}$	58

input
```
int(x^5*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/910*(b*x^2+a)^(7/3)*(35*b^2*x^4-21*a*b*x^2+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int x^5\left(a+bx^2\right)^{4/3}\,dx=\frac{3\,(35\,b^4x^8+49\,ab^3x^6+2\,a^2b^2x^4-3\,a^3bx^2+9\,a^4)(bx^2+a)^{\frac{1}{3}}}{910\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/910*(35*b^4*x^8 + 49*a*b^3*x^6 + 2*a^2*b^2*x^4 - 3*a^3*b*x^2 + 9*a^4)*(b
*x^2 + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.90

$$\int x^5(a+bx^2)^{4/3}\,dx = \begin{cases} \frac{27a^4\sqrt[3]{a+bx^2}}{910b^3} - \frac{9a^3x^2\sqrt[3]{a+bx^2}}{910b^2} + \frac{3a^2x^4\sqrt[3]{a+bx^2}}{455b} + \frac{21ax^6\sqrt[3]{a+bx^2}}{130} + \frac{3bx^8\sqrt[3]{a+bx^2}}{26} & \text{for } b \\ \frac{a^{\frac{4}{3}}x^6}{6} & \text{othe} \end{cases}$$

input
```
integrate(x**5*(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((27*a**4*(a + b*x**2)**(1/3)/(910*b**3) - 9*a**3*x**2*(a + b*x**
2)**(1/3)/(910*b**2) + 3*a**2*x**4*(a + b*x**2)**(1/3)/(455*b) + 21*a*x**6
*(a + b*x**2)**(1/3)/130 + 3*b*x**8*(a + b*x**2)**(1/3)/26, Ne(b, 0)), (a*
*(4/3)*x**6/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int x^5(a+bx^2)^{4/3}\,dx = \frac{3(bx^2+a)^{\frac{13}{3}}}{26\,b^3} - \frac{3(bx^2+a)^{\frac{10}{3}}a}{10\,b^3} + \frac{3(bx^2+a)^{\frac{7}{3}}a^2}{14\,b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/26*(b*x^2 + a)^(13/3)/b^3 - 3/10*(b*x^2 + a)^(10/3)*a/b^3 + 3/14*(b*x^2
+ a)^(7/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

$$\int x^5 (a+bx^2)^{4/3}\, dx = \frac{3 \left(35 \, (bx^2+a)^{\frac{13}{3}} - 91 \, (bx^2+a)^{\frac{10}{3}} a + 65 \, (bx^2+a)^{\frac{7}{3}} a^2\right)}{910 \, b^3}$$

input
```
integrate(x^5*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/910*(35*(b*x^2 + a)^(13/3) - 91*(b*x^2 + a)^(10/3)*a + 65*(b*x^2 + a)^(7
/3)*a^2)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int x^5 (a+bx^2)^{4/3}\, dx = \left(b\,x^2+a\right)^{1/3} \left(\frac{21\,a\,x^6}{130} + \frac{3\,b\,x^8}{26} + \frac{27\,a^4}{910\,b^3} + \frac{3\,a^2\,x^4}{455\,b} - \frac{9\,a^3\,x^2}{910\,b^2}\right)$$

input
```
int(x^5*(a + b*x^2)^(4/3),x)
```

output
```
(a + b*x^2)^(1/3)*((21*a*x^6)/130 + (3*b*x^8)/26 + (27*a^4)/(910*b^3) + (3
*a^2*x^4)/(455*b) - (9*a^3*x^2)/(910*b^2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.42

$$\int x^5 (a+bx^2)^{4/3}\, dx = \frac{3 \left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x + a + b\,x^2\right)^{\frac{2}{3}} \left(35b^4x^8 + 49a\,b^3x^6 + 2a^2b^2x^4 - 3a^3b\,x^2 + 9a^4\right)}{910 \left(\sqrt{b\,x^2+a} + \sqrt{b}\,x\right)^{\frac{2}{3}} b^3}$$

input
```
int(x^5*(b*x^2+a)^(4/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*(9*a**4 - 3*a**3*b*x**
2 + 2*a**2*b**2*x**4 + 49*a*b**3*x**6 + 35*b**4*x**8))/(910*(sqrt(a + b*x*
*2) + sqrt(b)*x)**(2/3)*b**3)
```

3.740 $\int x^3(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int x^3(a+bx^2)^{4/3}\,dx = -\frac{3a(a+bx^2)^{7/3}}{14b^2} + \frac{3(a+bx^2)^{10/3}}{20b^2}$$

output
```
-3/14*a*(b*x^2+a)^(7/3)/b^2+3/20*(b*x^2+a)^(10/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int x^3(a+bx^2)^{4/3}\,dx = \frac{3(a+bx^2)^{7/3}(-3a+7bx^2)}{140b^2}$$

input
```
Integrate[x^3*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)^(7/3)*(-3*a + 7*b*x^2))/(140*b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^{4/3} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^2 \left(bx^2+a\right)^{4/3} \, dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(\frac{\left(bx^2+a\right)^{7/3}}{b}-\frac{a\left(bx^2+a\right)^{4/3}}{b}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{3\left(a+bx^2\right)^{10/3}}{10b^2}-\frac{3a\left(a+bx^2\right)^{7/3}}{7b^2}\right)$$

input
```
Int[x^3*(a + b*x^2)^(4/3),x]
```

output
```
((-3*a*(a + b*x^2)^(7/3))/(7*b^2) + (3*(a + b*x^2)^(10/3))/(10*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}(-7b\,x^{2}+3a)}{140b^{2}}$	25
pseudoelliptic	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}(-7b\,x^{2}+3a)}{140b^{2}}$	25
orering	$-\frac{3(b\,x^{2}+a)^{\frac{7}{3}}(-7b\,x^{2}+3a)}{140b^{2}}$	25
trager	$-\frac{3(-7b^{3}x^{6}-11a\,b^{2}x^{4}-a^{2}b\,x^{2}+3a^{3})(b\,x^{2}+a)^{\frac{1}{3}}}{140b^{2}}$	47
risch	$-\frac{3(-7b^{3}x^{6}-11a\,b^{2}x^{4}-a^{2}b\,x^{2}+3a^{3})(b\,x^{2}+a)^{\frac{1}{3}}}{140b^{2}}$	47

input
```
int(x^3*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/140*(b*x^2+a)^(7/3)*(-7*b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int x^3(a+bx^2)^{4/3}\,dx=\frac{3\,(7\,b^3x^6+11\,ab^2x^4+a^2bx^2-3\,a^3)(bx^2+a)^{\frac{1}{3}}}{140\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/140*(7*b^3*x^6 + 11*a*b^2*x^4 + a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(34) = 68$.

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.32

$$\int x^3(a+bx^2)^{4/3}\,dx = \begin{cases} -\frac{9a^3\sqrt[3]{a+bx^2}}{140b^2} + \frac{3a^2x^2\sqrt[3]{a+bx^2}}{140b} + \frac{33ax^4\sqrt[3]{a+bx^2}}{140} + \frac{3bx^6\sqrt[3]{a+bx^2}}{20} & \text{for } b \neq 0 \\ \frac{a^{\frac{4}{3}}x^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((-9*a**3*(a + b*x**2)**(1/3)/(140*b**2) + 3*a**2*x**2*(a + b*x**
2)**(1/3)/(140*b) + 33*a*x**4*(a + b*x**2)**(1/3)/140 + 3*b*x**6*(a + b*x*
*2)**(1/3)/20, Ne(b, 0)), (a**(4/3)*x**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int x^3(a+bx^2)^{4/3}\,dx = \frac{3\,(bx^2+a)^{\frac{10}{3}}}{20\,b^2} - \frac{3\,(bx^2+a)^{\frac{7}{3}}a}{14\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/20*(b*x^2 + a)^(10/3)/b^2 - 3/14*(b*x^2 + a)^(7/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.76

$$\int x^3 (a+bx^2)^{4/3}\,dx = \frac{3\left(7\,(bx^2+a)^{\frac{10}{3}} - 10\,(bx^2+a)^{\frac{7}{3}}a\right)}{140\,b^2}$$

input
```
integrate(x^3*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/140*(7*(b*x^2 + a)^(10/3) - 10*(b*x^2 + a)^(7/3)*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int x^3 (a+bx^2)^{4/3}\,dx = (b\,x^2+a)^{1/3}\left(\frac{33\,a\,x^4}{140} + \frac{3\,b\,x^6}{20} - \frac{9\,a^3}{140\,b^2} + \frac{3\,a^2\,x^2}{140\,b}\right)$$

input
```
int(x^3*(a + b*x^2)^(4/3),x)
```

output
```
(a + b*x^2)^(1/3)*((33*a*x^4)/140 + (3*b*x^6)/20 - (9*a^3)/(140*b^2) + (3*
a^2*x^2)/(140*b))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int x^3 (a+bx^2)^{4/3}\,dx = \frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}(7b^3x^6+11a\,b^2x^4+a^2b\,x^2-3a^3)}{140\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b^2}$$

input
```
int(x^3*(b*x^2+a)^(4/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*( - 3*a**3 + a**2*b*x*
*2 + 11*a*b**2*x**4 + 7*b**3*x**6))/(140*(sqrt(a + b*x**2) + sqrt(b)*x)**(
2/3)*b**2)
```

3.741 $\int x(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int x(a+bx^2)^{4/3}\,dx=\frac{3(a+bx^2)^{7/3}}{14b}$$

output
```
3/14*(b*x^2+a)^(7/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x(a+bx^2)^{4/3}\,dx=\frac{3(a+bx^2)^{7/3}}{14b}$$

input
```
Integrate[x*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)^(7/3))/(14*b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{4/3} dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{3\left(a+bx^2\right)^{7/3}}{14b}$$

input
```
Int[x*(a + b*x^2)^(4/3),x]
```

output
```
(3*(a + b*x^2)^(7/3))/(14*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{7}{3}}}{14b}$	15
derivativedivides	$\frac{3(b\,x^2+a)^{\frac{7}{3}}}{14b}$	15
default	$\frac{3(b\,x^2+a)^{\frac{7}{3}}}{14b}$	15
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{7}{3}}}{14b}$	15
orering	$\frac{3(b\,x^2+a)^{\frac{7}{3}}}{14b}$	15
trager	$\frac{3(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)^{\frac{1}{3}}}{14b}$	33
risch	$\frac{3(b^2x^4+2ab\,x^2+a^2)(b\,x^2+a)^{\frac{1}{3}}}{14b}$	33

input
```
int(x*(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/14*(b*x^2+a)^(7/3)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(14) = 28.

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int x(a+bx^2)^{4/3}\,dx = \frac{3\,(b^2x^4+2\,abx^2+a^2)(bx^2+a)^{\frac{1}{3}}}{14\,b}$$

input
```
integrate(x*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(14) = 28$.

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\begin{cases}\frac{3a^2\sqrt[3]{a+bx^2}}{14b}+\frac{3ax^2\sqrt[3]{a+bx^2}}{7}+\frac{3bx^4\sqrt[3]{a+bx^2}}{14} & \text{for } b\neq 0\\ \frac{a^{\frac{4}{3}}x^2}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((3*a**2*(a + b*x**2)**(1/3)/(14*b) + 3*a*x**2*(a + b*x**2)**(1/3
)/7 + 3*b*x**4*(a + b*x**2)**(1/3)/14, Ne(b, 0)), (a**(4/3)*x**2/2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

input
```
integrate(x*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/14*(b*x^2 + a)^(7/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x\left(a+bx^2\right)^{4/3}\,dx=\frac{3\left(bx^2+a\right)^{\frac{7}{3}}}{14\,b}$$

input
```
integrate(x*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/14*(b*x^2 + a)^(7/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int x(a+bx^2)^{4/3}\,dx=\frac{3\,(b\,x^2+a)^{7/3}}{14\,b}$$

input
```
int(x*(a + b*x^2)^(4/3),x)
```

output
```
(3*(a + b*x^2)^(7/3))/(14*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int x(a+bx^2)^{4/3}\,dx=\frac{3\left(\sqrt{b}\,\sqrt{b\,x^2+a}\,x+a+b\,x^2\right)^{\frac{2}{3}}(b^2x^4+2ab\,x^2+a^2)}{14\left(\sqrt{b\,x^2+a}+\sqrt{b}\,x\right)^{\frac{2}{3}}b}$$

input
```
int(x*(b*x^2+a)^(4/3),x)
```

output
```
(3*(sqrt(b)*sqrt(a + b*x**2)*x + a + b*x**2)**(2/3)*(a**2 + 2*a*b*x**2 + b
**2*x**4))/(14*(sqrt(a + b*x**2) + sqrt(b)*x)**(2/3)*b)
```

3.742 $\int \frac{(a+bx^2)^{4/3}}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = \frac{3}{2}a\sqrt[3]{a+bx^2}+\frac{3}{8}(a+bx^2)^{4/3} -\frac{1}{2}\sqrt{3}a^{4/3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)-\frac{1}{2}a^{4/3}\log(x)+\frac{3}{4}a^{4/3}\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

output
```
3/2*a*(b*x^2+a)^(1/3)+3/8*(b*x^2+a)^(4/3)-1/2*3^(1/2)*a^(4/3)*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))-1/2*a^(4/3)*ln(x)+3/4*a^(4/3)*
ln(a^(1/3)-(b*x^2+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = \frac{1}{8}\left(3\sqrt[3]{a+bx^2}(5a+bx^2) -4\sqrt{3}a^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+4a^{4/3}\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-2a^{4/3}\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+\right.\right.$$

input
```
Integrate[(a + b*x^2)^(4/3)/x,x]
```

output
```
(3*(a + b*x^2)^(1/3)*(5*a + b*x^2) - 4*Sqrt[3]*a^(4/3)*ArcTan[(1 + (2*(a +
 b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(4/3)*Log[-a^(1/3) + (a + b*x^2)^(1
/3)] - 2*a^(4/3)*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3)])/8
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 60, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^{4/3}}{x^2}dx^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\int \frac{\sqrt[3]{bx^2+a}}{x^2}dx^2+\frac{3}{4}(a+bx^2)^{4/3}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(a\left(a\int \frac{1}{x^2(bx^2+a)^{2/3}}dx^2+3\sqrt[3]{a+bx^2}\right)+\frac{3}{4}(a+bx^2)^{4/3}\right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2}\left(a\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)\right.$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(a\left(a\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)+\frac{3}{4}(\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(a\left(a\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)+\frac{3}{4}\left(a+bx^2\right)\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(a\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)+\frac{3}{4}\left(a+bx^2\right.\right.$$

input
```
Int[(a + b*x^2)^(4/3)/x,x]
```

output
```
((3*(a + b*x^2)^(4/3))/4 + a*(3*(a + b*x^2)^(1/3) + a*(-((Sqrt[3]*ArcTan[(
1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^2]/(2*a^(2/3
)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3)))))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(b\,x^{2}+5a\right)}{8}+\frac{\left(-\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+\right.\right.}{2}\right.}{2}$

input
```
int((b*x^2+a)^(4/3)/x,x,method=_RETURNVERBOSE)
```

output
```
3/8*(b*x^2+a)^(1/3)*(b*x^2+5*a)+1/2*(-3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2
+a)^(1/3))*3^(1/2)/a^(1/3))+ln((b*x^2+a)^(1/3)-a^(1/3))-1/2*ln(a^(2/3)+a^(
1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3)))*a^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{2\sqrt{3}(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right) - \frac{1}{4}a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{3}{8}(bx^2+5a)(bx^2+a)^{\frac{1}{3}}$$

input
```
integrate((b*x^2+a)^(4/3)/x,x, algorithm="fricas")
```

output
```
-1/2*sqrt(3)*a^(4/3)*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2/3) + sqr
t(3)*a)/a) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 +
5*a)*(b*x^2 + a)^(1/3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = -\frac{b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{4}{3}\\-\frac{1}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/x,x)
```

output
```
-b**(4/3)*x**(8/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), a*exp_polar(I*
pi)/(b*x**2))/(2*gamma(-1/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) + \frac{3}{8}\,(bx^2+a)^{\frac{4}{3}} + \frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}a$$

input
```
integrate((b*x^2+a)^(4/3)/x,x, algorithm="maxima")
```

output
```
-1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/2*a^(4/3)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/8*(b*x^2 + a)^(
4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = -\frac{1}{2}\sqrt{3}a^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right) - \frac{1}{4}a^{\frac{4}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right) + \frac{1}{2}a^{\frac{4}{3}}\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right) + \frac{3}{8}\,(bx^2+a)^{\frac{4}{3}} + \frac{3}{2}\,(bx^2+a)^{\frac{1}{3}}a$$

input
```
integrate((b*x^2+a)^(4/3)/x,x, algorithm="giac")
```

output
```
-1/2*sqrt(3)*a^(4/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3)) - 1/4*a^(4/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3)) + 1/2*a^(4/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) + 3/8*(b*x^2 +
 a)^(4/3) + 3/2*(b*x^2 + a)^(1/3)*a
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = \frac{3\,a\,(b\,x^2+a)^{1/3}}{2} + \frac{3\,(b\,x^2+a)^{4/3}}{8} + \frac{a^{4/3}\,\ln\left(\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{2}-\frac{9\,a^{7/3}}{2}\right)}{2} - \frac{a^{4/3}\,\ln\left(\frac{9\,a^{7/3}\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2}+\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{2}\right)\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{2} + a^{4/3}\,\ln\left(9\,a^{7/3}\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)-\frac{9\,a^2\,(b\,x^2+a)^{1/3}}{2}\right)\left(-\frac{1}{4}+\frac{\sqrt{3}\,1\mathrm{i}}{4}\right)$$

input
```
int((a + b*x^2)^(4/3)/x,x)
```

output
```
(3*a*(a + b*x^2)^(1/3))/2 + (3*(a + b*x^2)^(4/3))/8 + (a^(4/3)*log((9*a^2*
(a + b*x^2)^(1/3))/2 - (9*a^(7/3))/2))/2 - (a^(4/3)*log((9*a^(7/3)*((3^(1/
2)*1i)/2 + 1/2))/2 + (9*a^2*(a + b*x^2)^(1/3))/2)*((3^(1/2)*1i)/2 + 1/2))/
2 + a^(4/3)*log(9*a^(7/3)*((3^(1/2)*1i)/4 - 1/4) - (9*a^2*(a + b*x^2)^(1/3
))/2)*((3^(1/2)*1i)/4 - 1/4)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{x}\,dx = \frac{15(b\,x^2+a)^{\frac{1}{3}}\,a}{8}+\frac{3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}{8}+\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)a^2$$

input
```
int((b*x^2+a)^(4/3)/x,x)
```

output
```
(15*(a + b*x**2)**(1/3)*a + 3*(a + b*x**2)**(1/3)*b*x**2 + 8*int((a + b*x*
*2)**(1/3)/(a*x + b*x**3),x)*a**2)/8
```

3.743 $\int \frac{(a+bx^2)^{4/3}}{x^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 119

$$\int \frac{(a+bx^2)^{4/3}}{x^3} dx = \frac{3}{2}b\sqrt[3]{a+bx^2} - \frac{a\sqrt[3]{a+bx^2}}{2x^2} - \frac{2\sqrt[3]{a}b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}} - \frac{2}{3}\sqrt[3]{a}b\log(x) + \sqrt[3]{a}b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)$$

output
```
3/2*b*(b*x^2+a)^(1/3)-1/2*a*(b*x^2+a)^(1/3)/x^2-2/3*a^(1/3)*b*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)-2/3*a^(1/3)*b*ln(x)+a^
(1/3)*b*ln(a^(1/3)-(b*x^2+a)^(1/3))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx = \frac{1}{6}\left(-\frac{3(a-3bx^2)\sqrt[3]{a+bx^2}}{x^2} - 4\sqrt{3}\sqrt[3]{a}b\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4\sqrt[3]{a}b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - 2\sqrt[3]{a}b\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)\right)$$

input
```
Integrate[(a + b*x^2)^(4/3)/x^3,x]
```

output
```
((-3*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/x^2 - 4*Sqrt[3]*a^(1/3)*b*ArcTan[(1
+ (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*a^(1/3)*b*Log[-a^(1/3) + (a
+ b*x^2)^(1/3)] - 2*a^(1/3)*b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3)])/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {243, 51, 60, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{x^3}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{4/3}}{x^4}dx^2$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{4}{3}b\int \frac{\sqrt[3]{bx^2+a}}{x^2}dx^2-\frac{\left(a+bx^2\right)^{4/3}}{x^2}\right)$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}\left(\frac{4}{3}b\left(a\int \frac{1}{x^2\left(bx^2+a\right)^{2/3}}dx^2+3\sqrt[3]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{4/3}}{x^2}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{2}\left(\frac{4}{3}b\left(a\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right.\right.$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}\left(\frac{4}{3}b\left(a\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)-\right.$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{4}{3}b\left(a\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)-\frac{\left(a+bx^2\right)}{x^2}\right.$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{4}{3}b\left(a\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)+3\sqrt[3]{a+bx^2}\right)-\frac{\left(a+bx^2\right)}{x^2}\right.$$

input
```
Int[(a + b*x^2)^(4/3)/x^3,x]
```

output
```
(-((a + b*x^2)^(4/3)/x^2) + (4*b*(3*(a + b*x^2)^(1/3) + a*(-((Sqrt[3]*ArcT
an[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^2]/(2*a^
(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3)))))/3)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{(9b x^2-3a)(b x^2+a)^{\frac{1}{3}}-2b x^2 a^{\frac{1}{3}}\left(2\arctan\left(\frac{2\sqrt{3}\,(b x^2+a)^{\frac{1}{3}}}{3a^{\frac{1}{3}}}+\frac{\sqrt{3}}{3}\right)\sqrt{3}+\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}(b x^2+a)^{\frac{1}{3}}+(b x^2+a)^{\frac{2}{3}}\right)-2\ln\left((b x^2\right.\right.}{6x^2}$

input
```
int((b*x^2+a)^(4/3)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*((9*b*x^2-3*a)*(b*x^2+a)^(1/3)-2*b*x^2*a^(1/3)*(2*arctan(2/3*3^(1/2)/a
^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(1/2))*3^(1/2)+ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))-2*ln((b*x^2+a)^(1/3)-a^(1/3))))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx =$$

$$-\frac{4\sqrt{3}a^{\frac{1}{3}}bx^2\arctan\left(\frac{2\sqrt{3}(bx^2+a)^{\frac{1}{3}}a^{\frac{2}{3}}+\sqrt{3}a}{3a}\right)+2a^{\frac{1}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4a^{\frac{1}{3}}bx^2\log\left(\right.}{6x^2}$$

input
```
integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="fricas")
```

output
```
-1/6*(4*sqrt(3)*a^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(b*x^2 + a)^(1/3)*a^(2
/3) + sqrt(3)*a)/a) + 2*a^(1/3)*b*x^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^
(1/3)*a^(1/3) + a^(2/3)) - 4*a^(1/3)*b*x^2*log((b*x^2 + a)^(1/3) - a^(1/3)
) - 3*(3*b*x^2 - a)*(b*x^2 + a)^(1/3))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.39

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx = -\frac{b^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},-\frac{1}{3}\\ \frac{2}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/x**3,x)
```

output
```
-b**(4/3)*x**(2/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), a*exp_polar(I*p
i)/(b*x**2))/(2*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx &= -\frac{2}{3}\sqrt{3}a^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)\\ &\quad-\frac{1}{3}a^{\frac{1}{3}}b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)\\ &\quad+\frac{2}{3}a^{\frac{1}{3}}b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)+\frac{3}{2}(bx^2+a)^{\frac{1}{3}}b-\frac{(bx^2+a)^{\frac{1}{3}}a}{2\,x^2}\end{aligned}$$

input
```
integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*a^(1/3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/
a^(1/3)) - 1/3*a^(1/3)*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
 + a^(2/3)) + 2/3*a^(1/3)*b*log((b*x^2 + a)^(1/3) - a^(1/3)) + 3/2*(b*x^2
+ a)^(1/3)*b - 1/2*(b*x^2 + a)^(1/3)*a/x^2
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx =$$

$$-\frac{1}{6}\left(4\sqrt{3}a^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)+2\,a^{\frac{1}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-4\,a^{\frac{1}{3}}\log\right.$$

input
```
integrate((b*x^2+a)^(4/3)/x^3,x, algorithm="giac")
```

output
```
-1/6*(4*sqrt(3)*a^(1/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))
/a^(1/3)) + 2*a^(1/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3)) - 4*a^(1/3)*log(abs((b*x^2 + a)^(1/3) - a^(1/3))) - 9*(b*x^2 + a)
^(1/3) + 3*(b*x^2 + a)^(1/3)*a/(b*x^2))*b
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx = \frac{3\,b\,(b\,x^2+a)^{1/3}}{2}-\frac{a\,(b\,x^2+a)^{1/3}}{2\,x^2}$$
$$+\frac{2\,a^{1/3}\,b\,\ln\left(6\,a^{4/3}\,b-6\,a\,b\,(b\,x^2+a)^{1/3}\right)}{3}$$
$$+\frac{a^{1/3}\,b\,\ln\left(6\,a\,b\,(b\,x^2+a)^{1/3}-3\,a^{4/3}\,b\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$
$$-\frac{a^{1/3}\,b\,\ln\left(3\,a^{4/3}\,b\,\left(1+\sqrt{3}\,1\mathrm{i}\right)+6\,a\,b\,(b\,x^2+a)^{1/3}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{3}$$

input
```
int((a + b*x^2)^(4/3)/x^3,x)
```

output
```
(3*b*(a + b*x^2)^(1/3))/2 - (a*(a + b*x^2)^(1/3))/(2*x^2) + (2*a^(1/3)*b*l
og(6*a^(4/3)*b - 6*a*b*(a + b*x^2)^(1/3)))/3 + (a^(1/3)*b*log(6*a*b*(a + b
*x^2)^(1/3) - 3*a^(4/3)*b*(3^(1/2)*1i - 1))*(3^(1/2)*1i - 1))/3 - (a^(1/3)
*b*log(3*a^(4/3)*b*(3^(1/2)*1i + 1) + 6*a*b*(a + b*x^2)^(1/3))*(3^(1/2)*1i
 + 1))/3
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^3}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}\,a+9(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+8\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)ab\,x^2}{6x^2}$$

input
```
int((b*x^2+a)^(4/3)/x^3,x)
```

output
```
( - 3*(a + b*x**2)**(1/3)*a + 9*(a + b*x**2)**(1/3)*b*x**2 + 8*int((a + b*
x**2)**(1/3)/(a*x + b*x**3),x)*a*b*x**2)/(6*x**2)
```

3.744 $\int \frac{(a+bx^2)^{4/3}}{x^5} dx$

Optimal result

Integrand size = 15, antiderivative size = 133

$$\int \frac{(a+bx^2)^{4/3}}{x^5} dx = -\frac{a\sqrt[3]{a+bx^2}}{4x^4} - \frac{7b\sqrt[3]{a+bx^2}}{12x^2} - \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{2/3}} - \frac{b^2\log(x)}{9a^{2/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{2/3}}$$

output
```
-1/4*a*(b*x^2+a)^(1/3)/x^4-7/12*b*(b*x^2+a)^(1/3)/x^2-1/9*b^2*arctan(1/3*(
a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(2/3)-1/9*b^2*ln(x)/
a^(2/3)+1/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = \frac{1}{36}\left(-\frac{3\sqrt[3]{a+bx^2}(3a+7bx^2)}{x^4}-\frac{4\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}}+\frac{4b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)}{a^{2/3}}-\frac{2b^2\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{a^{2/3}}\right)$$

input
```
Integrate[(a + b*x^2)^(4/3)/x^5,x]
```

output
```
((-3*(a + b*x^2)^(1/3)*(3*a + 7*b*x^2))/x^4 - (4*Sqrt[3]*b^2*ArcTan[(1 + (
2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3) + (4*b^2*Log[-a^(1/3) + (a
 + b*x^2)^(1/3)])/a^(2/3) - (2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
 + (a + b*x^2)^(2/3)])/a^(2/3))/36
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {243, 51, 51, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}\int \frac{\left(bx^2+a\right)^{4/3}}{x^6}dx^2$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{2}{3}b\int \frac{\sqrt[3]{bx^2+a}}{x^4}dx^2-\frac{\left(a+bx^2\right)^{4/3}}{2x^4}\right)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{1}{3}b\int \frac{1}{x^2\left(bx^2+a\right)^{2/3}}dx^2-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)^{4/3}}{2x^4}\right)$$

$$\Big\downarrow \ 69$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}$$

$$\Big\downarrow \ 16$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)-$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{1}{3}b\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)-\frac{\left(a+bx^2\right)}{2x^4}$$

$$\Big\downarrow \ 217$$

$$\frac{1}{2}\left(\frac{2}{3}b\left(\frac{1}{3}b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)-\frac{\sqrt[3]{a+bx^2}}{x^2}\right)-\frac{(a+bx^2}{2x^4}$$

input
```
Int[(a + b*x^2)^(4/3)/x^5,x]
```

output
```
(-1/2*(a + b*x^2)^(4/3)/x^4 + (2*b*(-((a + b*x^2)^(1/3)/x^2) + (b*(-((Sqrt
[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^
2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3))))/3))/3)
/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.02

method	result
pseudoelliptic	$\frac{-4b^2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^2+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^4+4b^2\ln\left(\left(b\,x^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^4-2b^2\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+\left(b\,x^2+a\right)^{\frac{2}{3}}\right)x^4-}{36x^4a^{\frac{2}{3}}}$

input
```
int((b*x^2+a)^(4/3)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/36*(-4*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3
))*x^4+4*b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4-2*b^2*ln(a^(2/3)+a^(1/3)*(b*x
^2+a)^(1/3)+(b*x^2+a)^(2/3))*x^4-21*b*x^2*a^(2/3)*(b*x^2+a)^(1/3)-9*(b*x^2
+a)^(1/3)*a^(5/3))/x^4/a^(2/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = -\frac{12\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+2\,(a^2)^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(\right.}{36\,a^2x^4}$$

input
```
integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="fricas")
```

output
```
-1/36*(12*sqrt(1/3)*(a^2)^(1/6)*a*b^2*x^4*arctan(sqrt(1/3)*(a^2)^(1/6)*((a
^2)^(1/3)*a + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 2*(a^2)^(2/3)*b^2*x^
4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3))
 - 4*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) + 3*(7*a^2
*b*x^2 + 3*a^3)*(b*x^2 + a)^(1/3))/(a^2*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = -\frac{b^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/x**5,x)
```

output
```
-b**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2)
)/(2*x**(4/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = -\frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{2}{3}}} - \frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{2}{3}}} + \frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{2}{3}}} - \frac{7\,(bx^2+a)^{\frac{4}{3}}b^2-4\,(bx^2+a)^{\frac{1}{3}}ab^2}{12\,\left((bx^2+a)^2-2\,(bx^2+a)a+a^2\right)}$$

input
```
integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="maxima")
```

output
```
-1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(2/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(2/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3) - 1/12
*(7*(b*x^2 + a)^(4/3)*b^2 - 4*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2 - 2*
(b*x^2 + a)*a + a^2)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = -\frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{2}{3}}} + \frac{2\,b^3\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{2}{3}}} - \frac{4\,b^3\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{2}{3}}} + \frac{3\left(7\,(bx^2+a)^{\frac{4}{3}}b^3-\right.}{b^2x}}{36\,b}$$

input
```
integrate((b*x^2+a)^(4/3)/x^5,x, algorithm="giac")
```

output
```
-1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^
(1/3))/a^(2/3) + 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(2/3) - 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3) +
3*(7*(b*x^2 + a)^(4/3)*b^3 - 4*(b*x^2 + a)^(1/3)*a*b^3)/(b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = \frac{b^2\,\ln\left(b^2\,(b\,x^2+a)^{1/3}-a^{1/3}\,b^2\right)}{9\,a^{2/3}} - \frac{\ln\left(\frac{a^{1/3}\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{2}+b^2\,(b\,x^2+a)^{1/3}\right)\,\left(b^2+\sqrt{3}\,b^2\,1\mathrm{i}\right)}{18\,a^{2/3}} - \frac{\frac{7\,b^2\,(b\,x^2+a)^{4/3}}{6}-\frac{2\,a\,b^2\,(b\,x^2+a)^{1/3}}{3}}{2\,(b\,x^2+a)^2-4\,a\,(b\,x^2+a)+2\,a^2} + \frac{b^2\,\ln\left(b^2\,(b\,x^2+a)^{1/3}-a^{1/3}\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{2/3}}$$

input
```
int((a + b*x^2)^(4/3)/x^5,x)
```

output
```
(b^2*log(b^2*(a + b*x^2)^(1/3) - a^(1/3)*b^2))/(9*a^(2/3)) - (log((a^(1/3)
*(3^(1/2)*b^2*1i + b^2))/2 + b^2*(a + b*x^2)^(1/3))*(3^(1/2)*b^2*1i + b^2)
)/(18*a^(2/3)) - ((7*b^2*(a + b*x^2)^(4/3))/6 - (2*a*b^2*(a + b*x^2)^(1/3)
)/3)/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log(b^2*(a + b*x^2
)^(1/3) - a^(1/3)*b^2*((3^(1/2)*1i)/2 - 1/2))*((3^(1/2)*1i)/2 - 1/2))/(9*a
^(2/3))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^5}\,dx = \frac{-9(b\,x^2+a)^{\frac{1}{3}}\,a-21(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+8\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^3+ax}dx\right)b^2x^4}{36x^4}$$

input
```
int((b*x^2+a)^(4/3)/x^5,x)
```

output
```
( - 9*(a + b*x**2)**(1/3)*a - 21*(a + b*x**2)**(1/3)*b*x**2 + 8*int((a + b
*x**2)**(1/3)/(a*x + b*x**3),x)*b**2*x**4)/(36*x**4)
```

3.745 $\int x^4(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 335

$$\int x^4\left(a+bx^2\right)^{4/3}\,dx=-\frac{432a^3x\sqrt[3]{a+bx^2}}{21505b^2}+\frac{48a^2x^3\sqrt[3]{a+bx^2}}{4301b}+\frac{24}{391}ax^5\sqrt[3]{a+bx^2}+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}-\frac{432\ 3^{3/4}\sqrt{2-\sqrt{3}}a^4\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}}\right.\right.}{21505b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-432/21505*a^3*x*(b*x^2+a)^(1/3)/b^2+48/4301*a^2*x^3*(b*x^2+a)^(1/3)/b+24/
391*a*x^5*(b*x^2+a)^(1/3)+3/23*x^5*(b*x^2+a)^(4/3)-432/21505*3^(3/4)*(1/2*
6^(1/2)-1/2*2^(1/2))*a^4*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^
2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-
3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int x^4(a+bx^2)^{4/3}\,dx=\frac{3x\sqrt[3]{a+bx^2}\left(-\left((9a-17bx^2)(a+bx^2)^2\right)+\frac{9a^3\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{391b^2}$$

input
```
Integrate[x^4*(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*(-((9*a - 17*b*x^2)*(a + b*x^2)^2) + (9*a^3*Hyperge
ometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(391*b^2
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {248, 248, 262, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+bx^2)^{4/3}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{8}{23}a\int x^4\sqrt[3]{bx^2+a}dx+\frac{3}{23}x^5(a+bx^2)^{4/3}$$

$$\Big\downarrow \text{248}$$

$$\frac{8}{23}a\left(\frac{2}{17}a\int\frac{x^4}{(bx^2+a)^{2/3}}dx+\frac{3}{17}x^5\sqrt[3]{a+bx^2}\right)+\frac{3}{23}x^5(a+bx^2)^{4/3}$$

$$\Big\downarrow \text{262}$$

$$\frac{8}{23}a\left(\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\int\frac{x^2}{(bx^2+a)^{2/3}}dx}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}\right)+\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{262}$$

$$\frac{8}{23}a\left(\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int\frac{1}{(bx^2+a)^{2/3}}dx}{5b}\right)}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}\right)+$$

$$\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{234}$$

$$\frac{8}{23}a\left(\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}\right)}{11b}\right)+\frac{3}{17}x^5\sqrt[3]{a+bx^2}\right)+$$

$$\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{760}$$

$$\frac{8}{23}a\left(\frac{2}{17}a\left(\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(\ldots}{(\ldots}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\ldots}{11b}\right.\right.$$

$$\frac{3}{23}x^5\left(a+bx^2\right)^{4/3}$$

input
```
Int[x^4*(a + b*x^2)^(4/3),x]
```

output
```
(3*x^5*(a + b*x^2)^(4/3))/23 + (8*a*((3*x^5*(a + b*x^2)^(1/3))/17 + (2*a*(
(3*x^3*(a + b*x^2)^(1/3))/(11*b) - (9*a*((3*x*(a + b*x^2)^(1/3))/(5*b) + (
3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5
*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2])))/(11*b)))/17))/23
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{\frac{4}{3}}d x$$

input
```
int(x^4*(b*x^2+a)^(4/3),x)
```

output
```
int(x^4*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{4/3} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{4}{3}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^4 \left(a+bx^2\right)^{4/3} \, dx = \frac{a^{\frac{4}{3}} x^{5} {{}_{2}F_{1}\left(\begin{matrix} - \frac{4}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*x**5*hyper((-4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a + bx^2)^{4/3} \, dx = \int (bx^2 + a)^{\frac{4}{3}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2)^{4/3} \, dx = \int (bx^2 + a)^{\frac{4}{3}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{4/3} \, dx = \int x^4 \left(b\,x^2 + a\right)^{4/3} \, dx$$

input
```
int(x^4*(a + b*x^2)^(4/3),x)
```

output
```
int(x^4*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int x^4\left(a+bx^2\right)^{4/3}\,dx=\frac{-\frac{432\left(b\,x^2+a\right)^{\frac{1}{3}}a^3x}{21505}+\frac{48\left(b\,x^2+a\right)^{\frac{1}{3}}a^2b\,x^3}{4301}+\frac{75\left(b\,x^2+a\right)^{\frac{1}{3}}a\,b^2x^5}{391}+\frac{3\left(b\,x^2+a\right)^{\frac{1}{3}}b^3x^7}{23}+\frac{432\left(\int\frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right)a^4}{21505}}{b^2}$$

input

```
int(x^4*(b*x^2+a)^(4/3),x)
```

output

```
(3*( - 144*(a + b*x**2)**(1/3)*a**3*x + 80*(a + b*x**2)**(1/3)*a**2*b*x**3
 + 1375*(a + b*x**2)**(1/3)*a*b**2*x**5 + 935*(a + b*x**2)**(1/3)*b**3*x**
7 + 144*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*a**4))/(21505*b**2)
```

3.746 $\int x^2(a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 311

$$\int x^2\left(a+bx^2\right)^{4/3}\,dx=\frac{48a^2x\sqrt[3]{a+bx^2}}{935b}+\frac{24}{187}ax^3\sqrt[3]{a+bx^2}+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$
$$+\frac{48\,3^{3/4}\sqrt{2-\sqrt{3}}a^3\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{935b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
48/935*a^2*x*(b*x^2+a)^(1/3)/b+24/187*a*x^3*(b*x^2+a)^(1/3)+3/17*x^3*(b*x^
2+a)^(4/3)+48/935*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a^3*(a^(1/3)-(b*x^2+a)
^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(
1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.87 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int x^2 \left(a+bx^2\right)^{4/3} \, dx = \frac{3x\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2 - \frac{a^2 \text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}\right)}{17b}$$

input
```
Integrate[x^2*(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-4/3, 1/2,
3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/3)))/(17*b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 248, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{4/3} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{8}{17}a\int x^2\sqrt[3]{bx^2+a}dx+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{248}$$

$$\frac{8}{17}a\left(\frac{2}{11}a\int \frac{x^2}{\left(bx^2+a\right)^{2/3}}dx+\frac{3}{11}x^3\sqrt[3]{a+bx^2}\right)+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{262}$$

$$\frac{8}{17}a\left(\frac{2}{11}a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int \frac{1}{(bx^2+a)^{2/3}}dx}{5b}\right)+\frac{3}{11}x^3\sqrt[3]{a+bx^2}\right)+\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \ 234$$

$$\frac{8}{17}a\left(\frac{2}{11}a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}\right)+\frac{3}{11}x^3\sqrt[3]{a+bx^2}\right)+$$
$$\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \ 760$$

$$\frac{8}{17}a\left(\frac{2}{11}a\left(\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.\right.$$
$$\frac{3}{17}x^3\left(a+bx^2\right)^{4/3}$$

input
```
Int[x^2*(a + b*x^2)^(4/3),x]
```

output
```
(3*x^3*(a + b*x^2)^(4/3))/17 + (8*a*((3*x^3*(a + b*x^2)^(1/3))/11 + (2*a*(
(3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((
1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/11))/17
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^2 \left(b\,x^2+a\right)^{\frac{4}{3}}dx$$

input
```
int(x^2*(b*x^2+a)^(4/3),x)
```

output
```
int(x^2*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int x^2 (a + bx^2)^{4/3} \, dx = \int (bx^2 + a)^{\frac{4}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^2 (a + bx^2)^{4/3} \, dx = \frac{a^{\frac{4}{3}} x^3 {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*x**3*hyper((-4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a + bx^2)^{4/3} \, dx = \int (bx^2 + a)^{\frac{4}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^{4/3} \, dx = \int (bx^2 + a)^{\frac{4}{3}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^{4/3} \, dx = \int x^2 \left(b\,x^2 + a\right)^{4/3} \, dx$$

input
```
int(x^2*(a + b*x^2)^(4/3),x)
```

output
```
int(x^2*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int x^2 (a+bx^2)^{4/3} \, dx = \frac{\frac{48(b\,x^2+a)^{\frac{1}{3}} a^2 x}{935} + \frac{57(b\,x^2+a)^{\frac{1}{3}} a b\,x^3}{187} + \frac{3(b\,x^2+a)^{\frac{1}{3}} b^2 x^5}{17} - \frac{48\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}} dx\right) a^3}{935}}{b}$$

input
```
int(x^2*(b*x^2+a)^(4/3),x)
```

output
```
(3*(16*(a + b*x**2)**(1/3)*a**2*x + 95*(a + b*x**2)**(1/3)*a*b*x**3 + 55*(
a + b*x**2)**(1/3)*b**2*x**5 - 16*int((a + b*x**2)**(1/3)/(a + b*x**2),x)*
a**3))/(935*b)
```

3.747 $\int (a+bx^2)^{4/3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 285

$$\int (a+bx^2)^{4/3}\,dx = \frac{24}{55}ax\sqrt[3]{a+bx^2}+\frac{3}{11}x(a+bx^2)^{4/3} - \frac{16\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\ldots}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}\ldots}\right.\right.}{55bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
24/55*a*x*(b*x^2+a)^(1/3)+3/11*x*(b*x^2+a)^(4/3)-16/55*3^(3/4)*(1/2*6^(1/2
)-1/2*2^(1/2))*a^2*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ellip
ticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^
(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.16

$$\int \left(a+bx^2\right)^{4/3}\,dx=\frac{ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3),x]
```

output
```
(a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/3)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {211, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{4/3}\,dx$$

$$\Big\downarrow \; 211$$

$$\frac{8}{11}a\int \sqrt[3]{bx^2+a}dx+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \; 211$$

$$\frac{8}{11}a\left(\frac{2}{5}a\int \frac{1}{\left(bx^2+a\right)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \; 234$$

$$\frac{8}{11}a\left(\frac{3a\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)+\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

$$\Big\downarrow \text{760}$$

$$\frac{8}{11}a\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(}{(}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{3}{11}x\left(a+bx^2\right)^{4/3}$$

input
```
Int[(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(a + b*x^2)^(4/3))/11 + (8*a*((3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*
Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*
(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/(
(1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])))/11
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{4}{3}}d x$$

input
```
int((b*x^2+a)^(4/3),x)
```

output
```
int((b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{4/3} \, dx=\int \left(bx^{2}+a\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \left(a+bx^2\right)^{4/3} \, dx=a^{\frac{4}{3}}x\,{}_2F_1\left(\begin{matrix} -\frac{4}{3},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)$$

input
```
integrate((b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*x*hyper((-4/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} dx$$

input
```
integrate((b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \left(a+bx^2\right)^{4/3} dx = \frac{x\left(b\,x^2+a\right)^{4/3}{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{4/3}}$$

input
```
int((a + b*x^2)^(4/3),x)
```

output
```
(x*(a + b*x^2)^(4/3)*hypergeom([-4/3, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(4/3)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{4/3} dx = \frac{39(b\,x^2+a)^{\frac{1}{3}}\,ax}{55} + \frac{3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^3}{11} + \frac{16\left(\int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^2}{55}$$

input
```
int((b*x^2+a)^(4/3),x)
```

output
```
(39*(a + b*x**2)**(1/3)*a*x + 15*(a + b*x**2)**(1/3)*b*x**3 + 16*int((a +
b*x**2)**(1/3)/(a + b*x**2),x)*a**2)/55
```

3.748 $\int \frac{(a+bx^2)^{4/3}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 280

$$\int \frac{(a+bx^2)^{4/3}}{x^2} \, dx = \frac{8}{5} bx \sqrt[3]{a+bx^2} - \frac{(a+bx^2)^{4/3}}{x} - \frac{16\sqrt{2-\sqrt{3}}\, a \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}} \operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{5\sqrt[4]{3}\, x \sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
8/5*b*x*(b*x^2+a)^(1/3)-(b*x^2+a)^(4/3)/x-16/15*(1/2*6^(1/2)-1/2*2^(1/2))*
a*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2
/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1
/2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b
*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.91 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx=-\frac{a\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/x^2,x]
```

output
```
-((a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/2, 1/2, -((b*x^2)/a)])/(
x*(1 + (b*x^2)/a)^(1/3)))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 211, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{8}{3}b\int \sqrt[3]{bx^2+a}dx-\frac{(a+bx^2)^{4/3}}{x}$$

$$\Big\downarrow \text{211}$$

$$\frac{8}{3}b\left(\frac{2}{5}a\int \frac{1}{(bx^2+a)^{2/3}}dx+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)-\frac{(a+bx^2)^{4/3}}{x}$$

$$\Big\downarrow \text{234}$$

$$\frac{8}{3}b\left(\frac{3a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{5bx}+\frac{3}{5}x\sqrt[3]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{4/3}}{x}$$

$$\Big\downarrow \text{760}$$

$$\frac{8}{3}b\left(\frac{3}{5}x\sqrt[3]{a+bx^2}-\frac{2\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\right.}{\left(1-\right.}\right.\right.}{5bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{\left(a+bx^2\right)^{4/3}}{x}$$

input
```
Int[(a + b*x^2)^(4/3)/x^2,x]
```

output
```
-((a + b*x^2)^(4/3)/x) + (8*b*((3*x*(a + b*x^2)^(1/3))/5 - (2*3^(3/4)*Sqrt
[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a +
 b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1
/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b*x*Sqrt[-((a
^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))^2])))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{4}{3}}}{x^{2}}dx$$

input
```
int((b*x^2+a)^(4/3)/x^2,x)
```

output
```
int((b*x^2+a)^(4/3)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx=\int \frac{(bx^2+a)^{\frac{4}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)/x^2, x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx = -\frac{a^{\frac{4}{3}}\,{}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(4/3)/x**2,x)
```

output
```
-a**(4/3)*hyper((-4/3, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx = \frac{3\,(b\,x^2+a)^{4/3}\,{}_2F_1\left(-\frac{4}{3},-\frac{5}{6};\ \frac{1}{6};\ -\frac{a}{b\,x^2}\right)}{5\,x\,\left(\frac{a}{b\,x^2}+1\right)^{4/3}}$$

input
```
int((a + b*x^2)^(4/3)/x^2,x)
```

output
```
(3*(a + b*x^2)^(4/3)*hypergeom([-4/3, -5/6], 1/6, -a/(b*x^2)))/(5*x*(a/(b*
x^2) + 1)^(4/3))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^2}\,dx = \frac{-21(b\,x^2+a)^{\frac{1}{3}}\,a+3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2-16\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^4+a\,x^2}dx\right)a^2x}{5x}$$

input
```
int((b*x^2+a)^(4/3)/x^2,x)
```

output
```
( - 21*(a + b*x**2)**(1/3)*a + 3*(a + b*x**2)**(1/3)*b*x**2 - 16*int((a +
b*x**2)**(1/3)/(a*x**2 + b*x**4),x)*a**2*x)/(5*x)
```

3.749 $\int \frac{(a+bx^2)^{4/3}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 284

$$\int \frac{(a+bx^2)^{4/3}}{x^4} dx = -\frac{8b\sqrt[3]{a+bx^2}}{9x} - \frac{(a+bx^2)^{4/3}}{3x^3} - \frac{16\sqrt{2-\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{9\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-8/9*b*(b*x^2+a)^(1/3)/x-1/3*(b*x^2+a)^(4/3)/x^3-16/27*(1/2*6^(1/2)-1/2*2^
(1/2))*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I
-I*3^(1/2))*3^(3/4)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = -\frac{a\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{4}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/x^4,x]
```

output
```
-1/3*(a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-3/2, -4/3, -1/2, -((b*x^2)/a)
])/(x^3*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {247, 247, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{8}{9}b\int \frac{\sqrt[3]{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{4/3}}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$\frac{8}{9}b\left(\frac{2}{3}b\int \frac{1}{(bx^2+a)^{2/3}}dx-\frac{\sqrt[3]{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{4/3}}{3x^3}$$

$$\Big\downarrow \text{234}$$

$$\frac{8}{9}b\left(\frac{\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{x}-\frac{\sqrt[3]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{4/3}}{3x^3}$$

$\big\downarrow$ 760

$$\frac{8}{9}b\left(-\frac{2\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{\left(a+bx^2\right)^{4/3}}{3x^3}$$

input
```
Int[(a + b*x^2)^(4/3)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(4/3)/x^3 + (8*b*(-((a + b*x^2)^(1/3)/x) - (2*Sqrt[2 - Sq
rt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(
1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*E
llipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*Sqrt[-((a^(1/3
)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2)])))/9
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{x^4}dx$$

input
```
int((b*x^2+a)^(4/3)/x^4,x)
```

output
```
int((b*x^2+a)^(4/3)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)/x^4, x)
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = -\frac{a^{\frac{4}{3}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{4}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(4/3)/x**4,x)
```

output
```
-a**(4/3)*hyper((-3/2, -4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(4/3)/x^4,x)
```

output
```
int((a + b*x^2)^(4/3)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{x^4}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,a-21(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+16\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{b\,x^6+a\,x^4}dx\right)a^2x^3}{7x^3}$$

input
```
int((b*x^2+a)^(4/3)/x^4,x)
```

output
```
(3*(a + b*x**2)**(1/3)*a - 21*(a + b*x**2)**(1/3)*b*x**2 + 16*int((a + b*x
**2)**(1/3)/(a*x**4 + b*x**6),x)*a**2*x**3)/(7*x**3)
```

3.750 $\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx=-\frac{3a^3(a+bx^2)^{2/3}}{4b^4}+\frac{9a^2(a+bx^2)^{5/3}}{10b^4}-\frac{9a(a+bx^2)^{8/3}}{16b^4}+\frac{3(a+bx^2)^{11/3}}{22b^4}$$

output
```
-3/4*a^3*(b*x^2+a)^(2/3)/b^4+9/10*a^2*(b*x^2+a)^(5/3)/b^4-9/16*a*(b*x^2+a)
^(8/3)/b^4+3/22*(b*x^2+a)^(11/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx=\frac{3(a+bx^2)^{2/3}(-81a^3+54a^2bx^2-45ab^2x^4+40b^3x^6)}{880b^4}$$

input
```
Integrate[x^7/(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(2/3)*(-81*a^3 + 54*a^2*b*x^2 - 45*a*b^2*x^4 + 40*b^3*x^6))
/(880*b^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^6}{\sqrt[3]{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3\sqrt[3]{bx^2+a}}+\frac{3\left(bx^2+a\right)^{2/3}a^2}{b^3}-\frac{3\left(bx^2+a\right)^{5/3}a}{b^3}+\frac{\left(bx^2+a\right)^{8/3}}{b^3}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{3a^3\left(a+bx^2\right)^{2/3}}{2b^4}+\frac{9a^2\left(a+bx^2\right)^{5/3}}{5b^4}+\frac{3\left(a+bx^2\right)^{11/3}}{11b^4}-\frac{9a\left(a+bx^2\right)^{8/3}}{8b^4}\right)$$

input
```
Int[x^7/(a + b*x^2)^(1/3),x]
```

output
```
((-3*a^3*(a + b*x^2)^(2/3))/(2*b^4) + (9*a^2*(a + b*x^2)^(5/3))/(5*b^4) -
(9*a*(a + b*x^2)^(8/3))/(8*b^4) + (3*(a + b*x^2)^(11/3))/(11*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{3(b\,x^{2}+a)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
trager	$-\frac{3(b\,x^{2}+a)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
risch	$-\frac{3(b\,x^{2}+a)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
pseudoelliptic	$-\frac{3(b\,x^{2}+a)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47
orering	$-\frac{3(b\,x^{2}+a)^{\frac{2}{3}}\left(-40b^{3}x^{6}+45a\,b^{2}x^{4}-54a^{2}b\,x^{2}+81a^{3}\right)}{880b^{4}}$	47

input
```
int(x^7/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/880*(b*x^2+a)^(2/3)*(-40*b^3*x^6+45*a*b^2*x^4-54*a^2*b*x^2+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(40\,b^3x^6 - 45\,ab^2x^4 + 54\,a^2bx^2 - 81\,a^3)(bx^2+a)^{\frac{2}{3}}}{880\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/880*(40*b^3*x^6 - 45*a*b^2*x^4 + 54*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(2/3
)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(75) = 150.

Time = 1.51 (sec) , antiderivative size = 1690, normalized size of antiderivative = 21.12

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(b*x**2+a)**(1/3),x)
```

output
```
-243*a**(71/3)*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**
2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8
+ 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 243*a**(71/3)/(880*a**2
0*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x
**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**1
2) - 1296*a**(68/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a*
*19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**1
6*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) + 1458*a**(68
/3)*b*x**2/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4
+ 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 +
880*a**14*b**10*x**12) - 2808*a**(65/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(8
80*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17
*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*b**9*x**10 + 880*a**14*b**
10*x**12) + 3645*a**(65/3)*b**2*x**4/(880*a**20*b**4 + 5280*a**19*b**5*x**
2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8
+ 5280*a**15*b**9*x**10 + 880*a**14*b**10*x**12) - 3120*a**(62/3)*b**3*x**
6*(1 + b*x**2/a)**(2/3)/(880*a**20*b**4 + 5280*a**19*b**5*x**2 + 13200*a**
18*b**6*x**4 + 17600*a**17*b**7*x**6 + 13200*a**16*b**8*x**8 + 5280*a**15*
b**9*x**10 + 880*a**14*b**10*x**12) + 4860*a**(62/3)*b**3*x**6/(880*a**20*
b**4 + 5280*a**19*b**5*x**2 + 13200*a**18*b**6*x**4 + 17600*a**17*b**7*...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{11}{3}}}{22\,b^4} - \frac{9\,(bx^2+a)^{\frac{8}{3}}a}{16\,b^4} + \frac{9\,(bx^2+a)^{\frac{5}{3}}a^2}{10\,b^4} - \frac{3\,(bx^2+a)^{\frac{2}{3}}a^3}{4\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/22*(b*x^2 + a)^(11/3)/b^4 - 9/16*(b*x^2 + a)^(8/3)*a/b^4 + 9/10*(b*x^2 +
 a)^(5/3)*a^2/b^4 - 3/4*(b*x^2 + a)^(2/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = -\frac{3\,(bx^2+a)^{\frac{2}{3}}a^3}{4\,b^4} + \frac{3\left(40\,(bx^2+a)^{\frac{11}{3}} - 165\,(bx^2+a)^{\frac{8}{3}}a + 264\,(bx^2+a)^{\frac{5}{3}}a^2\right)}{880\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
-3/4*(b*x^2 + a)^(2/3)*a^3/b^4 + 3/880*(40*(b*x^2 + a)^(11/3) - 165*(b*x^2
 + a)^(8/3)*a + 264*(b*x^2 + a)^(5/3)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = -\left(b\,x^2+a\right)^{2/3}\left(\frac{243\,a^3}{880\,b^4} - \frac{3\,x^6}{22\,b} + \frac{27\,a\,x^4}{176\,b^2} - \frac{81\,a^2\,x^2}{440\,b^3}\right)$$

input
```
int(x^7/(a + b*x^2)^(1/3),x)
```

output
```
-(a + b*x^2)^(2/3)*((243*a^3)/(880*b^4) - (3*x^6)/(22*b) + (27*a*x^4)/(176
*b^2) - (81*a^2*x^2)/(440*b^3))
```

Reduce [F]

$$\int \frac{x^7}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^7}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x^7/(b*x^2+a)^(1/3),x)
```

output
```
int(x**7/(a + b*x**2)**(1/3),x)
```

3.751 $\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx=\frac{3a^2(a+bx^2)^{2/3}}{4b^3}-\frac{3a(a+bx^2)^{5/3}}{5b^3}+\frac{3(a+bx^2)^{8/3}}{16b^3}$$

output
```
3/4*a^2*(b*x^2+a)^(2/3)/b^3-3/5*a*(b*x^2+a)^(5/3)/b^3+3/16*(b*x^2+a)^(8/3)
/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx=\frac{3(a+bx^2)^{2/3}\left(9a^2-6abx^2+5b^2x^4\right)}{80b^3}$$

input
```
Integrate[x^5/(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(2/3)*(9*a^2 - 6*a*b*x^2 + 5*b^2*x^4))/(80*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt[3]{bx^2+a}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2\sqrt[3]{bx^2+a}}-\frac{2\left(bx^2+a\right)^{2/3}a}{b^2}+\frac{\left(bx^2+a\right)^{5/3}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3a^2\left(a+bx^2\right)^{2/3}}{2b^3}+\frac{3\left(a+bx^2\right)^{8/3}}{8b^3}-\frac{6a\left(a+bx^2\right)^{5/3}}{5b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^(1/3),x]
```

output
```
((3*a^2*(a + b*x^2)^(2/3))/(2*b^3) - (6*a*(a + b*x^2)^(5/3))/(5*b^3) + (3*
(a + b*x^2)^(8/3))/(8*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{2}{3}}(5b^2x^4-6ab\,x^2+9a^2)}{80b^3}$	36
trager	$\frac{3(b\,x^2+a)^{\frac{2}{3}}(5b^2x^4-6ab\,x^2+9a^2)}{80b^3}$	36
risch	$\frac{3(b\,x^2+a)^{\frac{2}{3}}(5b^2x^4-6ab\,x^2+9a^2)}{80b^3}$	36
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{2}{3}}(5b^2x^4-6ab\,x^2+9a^2)}{80b^3}$	36
orering	$\frac{3(b\,x^2+a)^{\frac{2}{3}}(5b^2x^4-6ab\,x^2+9a^2)}{80b^3}$	36

input
```
int(x^5/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/80*(b*x^2+a)^(2/3)*(5*b^2*x^4-6*a*b*x^2+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(5\,b^2x^4 - 6\,abx^2 + 9\,a^2)(bx^2+a)^{\frac{2}{3}}}{80\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/80*(5*b^2*x^4 - 6*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(2/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(54) = 108$.

Time = 1.00 (sec) , antiderivative size = 631, normalized size of antiderivative = 10.69

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \frac{27a^{\frac{32}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} - \frac{27a^{\frac{32}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} + \frac{63a^{\frac{29}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} - \frac{81a^{\frac{29}{3}}bx^2}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} + \frac{42a^{\frac{26}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} - \frac{81a^{\frac{26}{3}}b^2x^4}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} + \frac{18a^{\frac{23}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} - \frac{27a^{\frac{23}{3}}b^3x^6}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} + \frac{27a^{\frac{20}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6} + \frac{15a^{\frac{17}{3}}b^5x^{10}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{80a^8b^3+240a^7b^4x^2+240a^6b^5x^4+80a^5b^6x^6}$$

input

```
integrate(x**5/(b*x**2+a)**(1/3),x)
```

output
```
27*a**(32/3)*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 24
0*a**6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(32/3)/(80*a**8*b**3 + 240*a
**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 63*a**(29/3)*b*x
**2*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b*
*5*x**4 + 80*a**5*b**6*x**6) - 81*a**(29/3)*b*x**2/(80*a**8*b**3 + 240*a**
7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 42*a**(26/3)*b**2*
x**4*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b
**5*x**4 + 80*a**5*b**6*x**6) - 81*a**(26/3)*b**2*x**4/(80*a**8*b**3 + 240
*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 18*a**(23/3)*b
**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a*
*6*b**5*x**4 + 80*a**5*b**6*x**6) - 27*a**(23/3)*b**3*x**6/(80*a**8*b**3 +
 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 27*a**(20/
3)*b**4*x**8*(1 + b*x**2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 24
0*a**6*b**5*x**4 + 80*a**5*b**6*x**6) + 15*a**(17/3)*b**5*x**10*(1 + b*x**
2/a)**(2/3)/(80*a**8*b**3 + 240*a**7*b**4*x**2 + 240*a**6*b**5*x**4 + 80*a
**5*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{8}{3}}}{16\,b^3} - \frac{3\,(bx^2+a)^{\frac{5}{3}}a}{5\,b^3} + \frac{3\,(bx^2+a)^{\frac{2}{3}}a^2}{4\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/16*(b*x^2 + a)^(8/3)/b^3 - 3/5*(b*x^2 + a)^(5/3)*a/b^3 + 3/4*(b*x^2 + a)
^(2/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{2}{3}}a^2}{4\,b^3} + \frac{3\left(5\,(bx^2+a)^{\frac{8}{3}} - 16\,(bx^2+a)^{\frac{5}{3}}a\right)}{80\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/4*(b*x^2 + a)^(2/3)*a^2/b^3 + 3/80*(5*(b*x^2 + a)^(8/3) - 16*(b*x^2 + a)
^(5/3)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \left(b\,x^2+a\right)^{2/3}\left(\frac{27\,a^2}{80\,b^3} + \frac{3\,x^4}{16\,b} - \frac{9\,a\,x^2}{40\,b^2}\right)$$

input
```
int(x^5/(a + b*x^2)^(1/3),x)
```

output
```
(a + b*x^2)^(2/3)*((27*a^2)/(80*b^3) + (3*x^4)/(16*b) - (9*a*x^2)/(40*b^2)
)
```

Reduce [F]

$$\int \frac{x^5}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^5}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x^5/(b*x^2+a)^(1/3),x)
```

output
```
int(x**5/(a + b*x**2)**(1/3),x)
```

3.752 $\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx=-\frac{3a(a+bx^2)^{2/3}}{4b^2}+\frac{3(a+bx^2)^{5/3}}{10b^2}$$

output
```
-3/4*a*(b*x^2+a)^(2/3)/b^2+3/10*(b*x^2+a)^(5/3)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx=\frac{3(a+bx^2)^{2/3}\,(-3a+2bx^2)}{20b^2}$$

input
```
Integrate[x^3/(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(2/3)*(-3*a + 2*b*x^2))/(20*b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt[3]{bx^2+a}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^{2/3}}{b}-\frac{a}{b\sqrt[3]{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3(a+bx^2)^{5/3}}{5b^2}-\frac{3a(a+bx^2)^{2/3}}{2b^2}\right)$$

input
```
Int[x^3/(a + b*x^2)^(1/3),x]
```

output
```
((-3*a*(a + b*x^2)^(2/3))/(2*b^2) + (3*(a + b*x^2)^(5/3))/(5*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
trager	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
risch	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
pseudoelliptic	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25
orering	$-\frac{3(b\,x^2+a)^{\frac{2}{3}}(-2b\,x^2+3a)}{20b^2}$	25

input
```
int(x^3/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-3/20*(b*x^2+a)^(2/3)*(-2*b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(2\,bx^2-3\,a)(bx^2+a)^{\frac{2}{3}}}{20\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/20*(2*b*x^2 - 3*a)*(b*x^2 + a)^(2/3)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(34) = 68.

Time = 0.65 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.68

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = -\frac{9a^{\frac{11}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2} + \frac{9a^{\frac{11}{3}}}{20a^2b^2+20ab^3x^2} - \frac{3a^{\frac{8}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2} + \frac{9a^{\frac{8}{3}}bx^2}{20a^2b^2+20ab^3x^2} + \frac{6a^{\frac{5}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{20a^2b^2+20ab^3x^2}$$

input
```
integrate(x**3/(b*x**2+a)**(1/3),x)
```

output
```
-9*a**(11/3)*(1 + b*x**2/a)**(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**
(11/3)/(20*a**2*b**2 + 20*a*b**3*x**2) - 3*a**(8/3)*b*x**2*(1 + b*x**2/a)*
*(2/3)/(20*a**2*b**2 + 20*a*b**3*x**2) + 9*a**(8/3)*b*x**2/(20*a**2*b**2 +
 20*a*b**3*x**2) + 6*a**(5/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(20*a**2*b**
2 + 20*a*b**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^2} - \frac{3\,(bx^2+a)^{\frac{2}{3}}a}{4\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^2} - \frac{3\,(bx^2+a)^{\frac{2}{3}}a}{4\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b^2 - 3/4*(b*x^2 + a)^(2/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = -\frac{3\,(b\,x^2+a)^{2/3}\,(3\,a-2\,b\,x^2)}{20\,b^2}$$

input
```
int(x^3/(a + b*x^2)^(1/3),x)
```

output
```
-(3*(a + b*x^2)^(2/3)*(3*a - 2*b*x^2))/(20*b^2)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x^3/(b*x^2+a)^(1/3),x)
```

output
```
int(x**3/(a + b*x**2)**(1/3),x)
```

3.753 $\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx=\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

output
```
3/4*(b*x^2+a)^(2/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx=\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

input
```
Integrate[x/(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(2/3))/(4*b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{241\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{3\left(a+bx^2\right)^{2/3}}{4b}$$

input
```
Int[x/(a + b*x^2)^(1/3),x]
```

output
```
(3*(a + b*x^2)^(2/3))/(4*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
derivativedivides	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
default	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
trager	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
risch	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15
orering	$\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b}$	15

input
```
int(x/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/4*(b*x^2+a)^(2/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b}$$

input
```
integrate(x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
3/4*(b*x^2 + a)^(2/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \begin{cases} \frac{3(a+bx^2)^{\frac{2}{3}}}{4b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt[3]{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(1/3),x)
```

output
```
Piecewise((3*(a + b*x**2)**(2/3)/(4*b), Ne(b, 0)), (x**2/(2*a**(1/3)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b}$$

input
```
integrate(x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
3/4*(b*x^2 + a)^(2/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b}$$

input
```
integrate(x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
3/4*(b*x^2 + a)^(2/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\,{\left(b\,x^2+a\right)}^{2/3}}{4\,b}$$

input
```
int(x/(a + b*x^2)^(1/3),x)
```

output
```
(3*(a + b*x^2)^(2/3))/(4*b)
```

Reduce [F]

$$\int \frac{x}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x/(b*x^2+a)^(1/3),x)
```

output
```
int(x/(a + b*x**2)**(1/3),x)
```

3.754 $\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx=\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt[3]{a}}-\frac{\log(x)}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4\sqrt[3]{a}}$$

output
```
1/2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))/a^(1/3
)-1/2*ln(x)/a^(1/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(1/3)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx =\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}\right)}{4\sqrt[3]{a}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(1/3)),x]
```

output
```
(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 2*Log[-a^
(1/3) + (a + b*x^2)^(1/3)] - Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
+ b*x^2)^(2/3)])/(4*a^(1/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {243, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx^2$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(1/3)),x]
```

output
```
((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(1/3)))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+2\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)}{4a^{\frac{1}{3}}}$	83

input
```
int(1/x/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))+2*l
n((b*x^2+a)^(1/3)-a^(1/3))-ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2
/3)))/a^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.73

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \left[\frac{\sqrt{3}a\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^{2}+\sqrt{3}\left(2\,(bx^{2}+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^{2}+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\,(bx^{2}+a)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^{2}}\right)-a^{\frac{2}{3}}\log\left((bx^{2}+a)^{\frac{2}{3}}+(b\right.}{4\,a}\right.$$

input
```
integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*a*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(b*x^2 + a)^(2/
3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 +
a)^(1/3)*a^(2/3) + 3*a)/x^2) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)
^(1/3)*a^(1/3) + a^(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a,
 1/4*(2*sqrt(3)*a^(2/3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))
/a^(1/3)) - a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^
(2/3)) + 2*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3)))/a]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = -\frac{\Gamma\left(\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate(1/x/(b*x**2+a)**(1/3),x)
```

output
```
-gamma(1/3)*hyper((1/3, 1/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(2/3)*gamma(4/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}} + \frac{\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(1/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(1/3)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{1}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{1}{3}}} + \frac{\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{1}{3}}}$$

input
```
integrate(1/x/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(1/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(1/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(1/3)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \frac{\ln\left(\frac{9\,(b\,x^2+a)^{1/3}}{4}-\frac{9\,a^{1/3}}{4}\right)}{2\,a^{1/3}} + \frac{\ln\left(\frac{9\,(b\,x^2+a)^{1/3}}{4}-\frac{9\,a^{1/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{16}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{1/3}} - \frac{\ln\left(\frac{9\,(b\,x^2+a)^{1/3}}{4}-\frac{9\,a^{1/3}\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)^2}{16}\right)\,\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{1/3}}$$

input
```
int(1/(x*(a + b*x^2)^(1/3)),x)
```

output
```
log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3))/4)/(2*a^(1/3)) + (log((9*(a + b*
x^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i - 1)^2)/16)*(3^(1/2)*1i - 1))/(4*a^
(1/3)) - (log((9*(a + b*x^2)^(1/3))/4 - (9*a^(1/3)*(3^(1/2)*1i + 1)^2)/16)
*(3^(1/2)*1i + 1))/(4*a^(1/3))
```

Reduce [F]

$$\int \frac{1}{x\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}x}dx$$

input
```
int(1/x/(b*x^2+a)^(1/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*x),x)
```

3.755 $\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{2/3}}{2ax^2} - \frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2\sqrt{3}a^{4/3}} + \frac{b\log(x)}{6a^{4/3}} - \frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

output
```
-1/2*(b*x^2+a)^(2/3)/a/x^2-1/6*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^
(1/2)/a^(1/3))*3^(1/2)/a^(4/3)+1/6*b*ln(x)/a^(4/3)-1/4*b*ln(a^(1/3)-(b*x^2
+a)^(1/3))/a^(4/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^2}}\,dx = -\frac{6\sqrt[3]{a}(a+bx^2)^{2/3} + 2\sqrt{3}bx^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - bx^2\log\left(a^{2/3}+\right.}{12a^{4/3}x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(1/3)),x]
```

output
```
-1/12*(6*a^(1/3)*(a + b*x^2)^(2/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b
*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)]
 - b*x^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(a^
(4/3)*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \sqrt[3]{a+bx^2}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 \sqrt[3]{bx^2+a}} dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{b \int \frac{1}{x^2 \sqrt[3]{bx^2+a}} dx^2}{3a} - \frac{\left(a+bx^2\right)^{2/3}}{ax^2} \right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2} \left(-\frac{b \left(\frac{3}{2} \int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a} - \frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}} - \frac{\log(x^2)}{2\sqrt[3]{a}} \right)}{3a} - \frac{\left(a+bx^2\right)^{2/3}}{ax^2} \right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(-\frac{b\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^(1/3)),x]
```

output
```
(-((a + b*x^2)^(2/3)/(a*x^2)) - (b*((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1
/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) -
(a + b*x^2)^(1/3)])/(2*a^(1/3))))/(3*a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$\frac{-2b\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,x^{2}-2b\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^{2}+b\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)x^{2}-6\left(b\,x\right.}{12a^{\frac{4}{3}}x^{2}}$

input
```
int(1/x^3/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-2*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)
*x^2-2*b*ln((b*x^2+a)^(1/3)-a^(1/3))*x^2+b*ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1
/3)+(b*x^2+a)^(2/3))*x^2-6*(b*x^2+a)^(2/3)*a^(1/3))/a^(4/3)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.13

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = \left[\frac{3\sqrt{\frac{1}{3}}abx^2\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}-3\,(bx^2+a)^{\frac{1}{3}}(-a)^{\frac{2}{3}}+3\,a}{x^2}\right)+(-a)^{\frac{2}{3}}}{1}\right.$$

$$-\frac{6\sqrt{\frac{1}{3}}abx^2\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-(-a)^{\frac{2}{3}}bx^2\log\left((bx^2+a)^{\frac{2}{3}}-(\right.}{12\,a^2x^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/12*(3*sqrt(1/3)*a*b*x^2*sqrt((-a)^(1/3)/a)*log((2*b*x^2 - 3*sqrt(1/3)*(
2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)^(1/3)*a)*sqrt(
(-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2) + (-a)^(2/3)*b*
x^2*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3)) - 2
*(-a)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) - 6*(b*x^2 + a)^(2/3
)*a)/(a^2*x^2), -1/12*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a)^(1/3)/a)*arctan(sqrt
(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a)) - (-a)^(2/3)
*b*x^2*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/3)*(-a)^(1/3) + (-a)^(2/3))
+ 2*(-a)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3) + (-a)^(1/3)) + 6*(b*x^2 + a)^(
2/3)*a)/(a^2*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx=-\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**2+a)**(1/3),x)
```

output
```
-gamma(4/3)*hyper((1/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(1
/3)*x**(8/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{6\,a^{\frac{4}{3}}} - \frac{(bx^2+a)^{\frac{2}{3}}b}{2\left((bx^2+a)a-a^2\right)} + \frac{b\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{12\,a^{\frac{4}{3}}} - \frac{b\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{6\,a^{\frac{4}{3}}}$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
-1/6*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(4/3) - 1/2*(b*x^2 + a)^(2/3)*b/((b*x^2 + a)*a - a^2) + 1/12*b*log((b*x
^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(4/3) - 1/6*b*log((
b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = -\frac{1}{12}\,b\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{4}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{4}{3}}} + \frac{2\,\log\left(\left|(bx^2+a\right.\right.}{a^{\frac{4}{3}}}\right.$$

input
```
integrate(1/x^3/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
-1/12*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1
/3))/a^(4/3) - log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)
)/a^(4/3) + 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 6*(b*x^2 + a
)^(2/3)/(a*b*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.25

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = -\frac{b\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{6\,a^{4/3}}-\frac{\left(b\,x^2+a\right)^{2/3}}{2\,a\,x^2}+\frac{\ln\left(\frac{\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{4/3}}+\frac{\ln\left(\frac{\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)^2}{16\,a^{5/3}}-\frac{b^2\,\left(b\,x^2+a\right)^{1/3}}{4\,a^2}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{12\,a^{4/3}}$$

input
```
int(1/(x^3*(a + b*x^2)^(1/3)),x)
```

output
```
(log((b - 3^(1/2)*b*1i)^2/(16*a^(5/3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*
(b - 3^(1/2)*b*1i))/(12*a^(4/3)) - (a + b*x^2)^(2/3)/(2*a*x^2) - (b*log((a
 + b*x^2)^(1/3) - a^(1/3)))/(6*a^(4/3)) + (log((b + 3^(1/2)*b*1i)^2/(16*a^
(5/3)) - (b^2*(a + b*x^2)^(1/3))/(4*a^2))*(b + 3^(1/2)*b*1i))/(12*a^(4/3))
```

Reduce [F]

$$\int \frac{1}{x^3\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}x^3}dx$$

input
```
int(1/x^3/(b*x^2+a)^(1/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*x**3),x)
```

3.756 $\int \frac{1}{x^5 \sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{2/3}}{4ax^4} + \frac{b(a+bx^2)^{2/3}}{3a^2x^2} + \frac{b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{7/3}} - \frac{b^2\log(x)}{9a^{7/3}} + \frac{b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{7/3}}$$

output
```
-1/4*(b*x^2+a)^(2/3)/a/x^4+1/3*b*(b*x^2+a)^(2/3)/a^2/x^2+1/9*b^2*arctan(1/
3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)-1/9*b^2*ln(
x)/a^(7/3)+1/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^2}}\,dx = \frac{\frac{3\sqrt[3]{a}(a+bx^2)^{2/3}(-3a+4bx^2)}{x^4} + 4\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 4b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right) - 2b^2\log\left(a^{2/3}\right.}{36a^{7/3}}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^(1/3)),x]
```

output
```
((3*a^(1/3)*(a + b*x^2)^(2/3)*(-3*a + 4*b*x^2))/x^4 + 4*Sqrt[3]*b^2*ArcTan
[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 4*b^2*Log[-a^(1/3) + (a +
b*x^2)^(1/3)] - 2*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2
)^(2/3)])/(36*a^(7/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 52, 52, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{1}{x^6 \sqrt[3]{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(-\frac{2b\int \frac{1}{x^4 \sqrt[3]{bx^2+a}}dx^2}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^4}\right)$$

$$\Big\downarrow \text{ 52}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\int \frac{1}{x^2 \sqrt[3]{bx^2+a}}dx^2}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^4}\right)$$

$$\Big\downarrow \text{ 67}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)}{3a}-\frac{(a\ \cdots$$

$$\downarrow 16$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\left(\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/\cdots}}{2ax^4}\ \cdots$$

$$\downarrow 1082$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\left(-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^4}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{b\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax^2}\right)}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{2ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)^(1/3)),x]
```

output
```
(-1/2*(a + b*x^2)^(2/3)/(a*x^4) - (2*b*(-((a + b*x^2)^(2/3)/(a*x^2)) - (b*
((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - L
og[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(1/3))))/(
3*a)))/(3*a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{4b^2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^2+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^4+4b^2\ln\left(\left(b\,x^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^4-2b^2\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+\left(b\,x^2+a\right)^{\frac{2}{3}}\right)x^4+12}{36a^{\frac{7}{3}}x^4}$

input
```
int(1/x^5/(b*x^2+a)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/36/a^(7/3)*(4*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)
/a^(1/3))*x^4+4*b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4-2*b^2*ln(a^(2/3)+a^(1/
3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))*x^4+12*b*x^2*(b*x^2+a)^(2/3)*a^(1/3)-9
*(b*x^2+a)^(2/3)*a^(4/3))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.36

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx$$
$$= \left[\frac{6\sqrt{\frac{1}{3}}ab^2x^4\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+3\,a}{x^2}\right)-2\,a^{\frac{2}{3}}b^2x^4\log\left(\left(b\right.\right.}{36\,a^3x^4}\right.$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
[1/36*(6*sqrt(1/3)*a*b^2*x^4*sqrt(-1/a^(2/3))*log((2*b*x^2 + 3*sqrt(1/3)*(
2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/
3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 2*a^(2/3)*b^2*x^4*log((b*x
^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*l
og((b*x^2 + a)^(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))
/(a^3*x^4), 1/36*(12*sqrt(1/3)*a^(2/3)*b^2*x^4*arctan(sqrt(1/3)*(2*(b*x^2
+ a)^(1/3) + a^(1/3))/a^(1/3)) - 2*a^(2/3)*b^2*x^4*log((b*x^2 + a)^(2/3) +
 (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 4*a^(2/3)*b^2*x^4*log((b*x^2 + a)^
(1/3) - a^(1/3)) + 3*(4*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(2/3))/(a^3*x^4)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = -\frac{\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\sqrt[3]{b}x^{\frac{14}{3}}\Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**2+a)**(1/3),x)
```

output
```
-gamma(7/3)*hyper((1/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(
1/3)*x**(14/3)*gamma(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{7}{3}}} - \frac{b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{7}{3}}} + \frac{b^2\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{9\,a^{\frac{7}{3}}} + \frac{4\,(bx^2+a)^{\frac{5}{3}}b^2-7\,(bx^2+a)^{\frac{2}{3}}ab^2}{12\left((bx^2+a)^2a^2-2\,(bx^2+a)a^3+a^4\right)}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
1/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(7/3) - 1/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a
^(2/3))/a^(7/3) + 1/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3) + 1/12*
(4*(b*x^2 + a)^(5/3)*b^2 - 7*(b*x^2 + a)^(2/3)*a*b^2)/((b*x^2 + a)^2*a^2 -
 2*(b*x^2 + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{\frac{4\sqrt{3}b^3\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{7}{3}}} - \frac{2\,b^3\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{7}{3}}} + \frac{4\,b^3\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{7}{3}}} + \frac{3\left(4\left(bx^2+a\right)^{\frac{5}{3}}b^3-\right.}{a^2b^2}}{36\,b}$$

input
```
integrate(1/x^5/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
1/36*(4*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(
1/3))/a^(7/3) - 2*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(7/3) + 4*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) + 3
*(4*(b*x^2 + a)^(5/3)*b^3 - 7*(b*x^2 + a)^(2/3)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.46

$$\int \frac{1}{x^5\sqrt[3]{a+bx^2}}\,dx = \frac{b^2\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{9\,a^{7/3}} - \frac{\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{9\,a^4}-\frac{\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)^2}{36\,a^{11/3}}\right)\,\left(b^2+\sqrt{3}\,b^2\,1{}\mathrm{i}\right)}{18\,a^{7/3}} - \frac{\frac{7\,b^2\,\left(b\,x^2+a\right)^{2/3}}{6\,a}-\frac{2\,b^2\,\left(b\,x^2+a\right)^{5/3}}{3\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2} + \frac{b^2\,\ln\left(\frac{b^4\,\left(b\,x^2+a\right)^{1/3}}{9\,a^4}-\frac{b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)^2}{9\,a^{11/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{9\,a^{7/3}}$$

input
```
int(1/(x^5*(a + b*x^2)^(1/3)),x)
```

output
```
(b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(9*a^(7/3)) - (log((b^4*(a + b*x^2)
^(1/3))/(9*a^4) - (3^(1/2)*b^2*1i + b^2)^2/(36*a^(11/3)))*(3^(1/2)*b^2*1i
+ b^2))/(18*a^(7/3)) - ((7*b^2*(a + b*x^2)^(2/3))/(6*a) - (2*b^2*(a + b*x^
2)^(5/3))/(3*a^2))/(2*(a + b*x^2)^2 - 4*a*(a + b*x^2) + 2*a^2) + (b^2*log(
(b^4*(a + b*x^2)^(1/3))/(9*a^4) - (b^4*((3^(1/2)*1i)/2 - 1/2)^2)/(9*a^(11/
3)))*((3^(1/2)*1i)/2 - 1/2))/(9*a^(7/3))
```

Reduce [F]

$$\int \frac{1}{x^5 \sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,x^5}dx$$

input
```
int(1/x^5/(b*x^2+a)^(1/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*x**5),x)
```

3.757 $\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 580

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = -\frac{27ax(a+bx^2)^{2/3}}{91b^2}+\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{81a^2x}{91b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+\frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\ldots\right)}{182b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{27\sqrt{2}3^{3/4}a^{7/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\ldots}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+\ldots}}\right)\ldots\right)}{91b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-27/91*a*x*(b*x^2+a)^(2/3)/b^2+3/13*x^3*(b*x^2+a)^(2/3)/b-81/91*a^2*x/b^2/
((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+81/182*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1
/2))*a^(7/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(
b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((
(1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
,2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-27/91*2^(1/2)*3^(3/4)*a^(7/3)*(a^(1/3)-(b*x
^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2
))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2
+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^
(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(
1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.61 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \frac{3\left(-9a^2x-2abx^3+7b^2x^5+9a^2x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{91b^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(1/3),x]
```

output
```
(3*(-9*a^2*x - 2*a*b*x^3 + 7*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/3)*Hyper
geometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)]))/(91*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {262, 262, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{9a\int \frac{x^2}{\sqrt[3]{bx^2+a}}dx}{13b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{9a\left(\frac{3x(a+bx^2)^{2/3}}{7b}-\frac{3a\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}\right)}{13b}$$

$$\Big\downarrow \text{233}$$

$$\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{9a\left(\frac{3x(a+bx^2)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}\right)}{13b}$$

$$\Big\downarrow \text{833}$$

$$\frac{3x^3(a+bx^2)^{2/3}}{13b}-\frac{9a\left(\frac{3x(a+bx^2)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{14b^2x}\right)}{13b}$$

$$\Big\downarrow \text{760}$$

$$\frac{\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a}\cdots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\cdots}}\right.}{14b^2x}\cdots\right.}{13b}\cdots}{}$$

$\downarrow$ 2418

$$\frac{\frac{3x^3\left(a+bx^2\right)^{2/3}}{13b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\cdots}{\left(1-\sqrt{3}\right)\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots}{}$$

input
```
Int[x^4/(a + b*x^2)^(1/3),x]
```

output
```
(3*x^3*(a + b*x^2)^(2/3))/(13*b) - (9*a*((3*x*(a + b*x^2)^(2/3))/(7*b) - (
9*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
 Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sq
rt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a +
b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/
3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x
^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2)])))/(14*b^2*x)))/(13*b)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/3),x)
```

output
```
int(x^4/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{5}{2} \\ \frac{7}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[3]{a}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/3),x)
```

output
```
x**5*hyper((1/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/3))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(1/3),x)
```

output
```
int(x^4/(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/3),x)
```

output
```
int(x**4/(a + b*x**2)**(1/3),x)
```

3.758 $\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 556

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \frac{3x(a+bx^2)^{2/3}}{7b}+\frac{9ax}{7b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{14b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} + \frac{3\sqrt{2}3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+b}}\right.\right.}{7b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/7*x*(b*x^2+a)^(2/3)/b+9/7*a*x/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-9/
14*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a
^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^
2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(
b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+3/7*2^(1/2)
*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/
3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Ellipti
cF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1
/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx=\frac{3x\left(a+bx^2-a\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{7b\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2,
 -((b*x^2)/a)]))/(7*b*(a + b*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {262, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{3a\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}$$

$$\Big\downarrow \ 233$$

$$\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}$$

$$\Big\downarrow \ 833$$

$$\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{14b^2x}$$

$$\Big\downarrow \ 760$$

$$\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+b\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx\right.}}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}\right.}}}\right.}{14b^2x}$$

$$\Big\downarrow \ 2418$$

$$\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2-}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2-}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}$$

input
```
Int[x^2/(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(a + b*x^2)^(2/3))/(7*b) - (9*a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*
(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
(a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Elliptic
E[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3
) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^
(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)
]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2
)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(14*b^2*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/3),x)
```

output
```
int(x^2/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[3]{a}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/3),x)
```

output
```
x**3*hyper((1/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/3), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^2}{\left(b\,x^2+a\right)^{1/3}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(1/3),x)
```

output
```
int(x^2/(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{3}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/3),x)
```

output
```
int(x**2/(a + b*x**2)**(1/3),x)
```

3.759 $\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 529

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = -\frac{3x}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.\cdots}{2bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{\sqrt{2}3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.\cdots}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*x/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+3/2*3^(1/4)*(1/2*6^(1/2)+1/2*2^
(1/2))*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE
(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-2^(1/2)*3^(3/4)*a^(1/3)*(a^(1/3)-(b*x^2+a)^
(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1
/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.09

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx=\frac{x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/3),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx$$

$\big\downarrow$ 233

$$\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}$$

$\big\downarrow$ 833

$$\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2bx}$$

$\big\downarrow$ 760

$$\frac{3\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}\right.}{2bx}$$

$\big\downarrow$ 2418

$$3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

input
```
Int[(a + b*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt
[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sq
rt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 -
Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqr
t[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b
*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3
))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^
2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))^2)])))/(2*b*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(1/3),x)
```

output
```
int(1/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx=\int \frac{1}{\left(bx^{2}+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[3]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/3),x)
```

output
```
x*hyper((1/3, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/3)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(b\,x^2+a\right)^{1/3}}$$

input
```
int(1/(a + b*x^2)^(1/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/3)*hypergeom([1/3, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(1/3),x)
```

output
```
int(1/(a + b*x**2)**(1/3),x)
```

3.760 $\int \frac{1}{x^2 \sqrt[3]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 546

$$\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{2/3}}{ax} - \frac{bx}{a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-}{2a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{\sqrt{2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7}{\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(2/3)/a/x-b*x/a/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+1/2*3^(1/
4)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(
b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(
1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-
(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-1/3*2^(1/2)*(a^(1/3)-(b
*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/
a^(2/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.47 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{3},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[3]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(1/3)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 1/3, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(1/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}}dx$$

$$\Big\downarrow \ 264$$

$$\frac{b\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

$$\Big\downarrow \ 233$$

$$\frac{\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

$$\Big\downarrow \ 833$$

$$\frac{\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

$$\Big\downarrow \ 760$$

$$\frac{\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\cdots}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right)^2}}}\cdots\right)}{2ax}\cdots$$

$$\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

$$\Big\downarrow \ 2418$$

$$\frac{\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\cdots\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\cdots\right)}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{2/3}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)^(1/3)),x]
```

output
```
-((a + b*x^2)^(2/3)/(a*x)) + (Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3
) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*
x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSi
n[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) -
 (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]) - (2
*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqr
t[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) -
 (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*S
qrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)
))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2])))/(2*a*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{1}{3}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/3),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[3]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b*x^4 + a*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{1}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{\sqrt[3]{a}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/3),x)
```

output
```
-hyper((-1/2, 1/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/3)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}} \, dx = -\frac{3 \left(\frac{a}{b x^2}+1\right)^{1/3} {}_2\mathrm{F}_1\left(\frac{1}{3},\frac{5}{6};\ \frac{11}{6};\ -\frac{a}{b x^2}\right)}{5 x \left(b x^2+a\right)^{1/3}}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/3)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(1/3)*hypergeom([1/3, 5/6], 11/6, -a/(b*x^2)))/(5*x*(a
 + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[3]{a+bx^2}} \, dx = \int \frac{1}{\left(b x^2+a\right)^{\frac{1}{3}} x^2} dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*x**2),x)
```

3.761 $\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 578

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{2/3}}{3ax^3}+\frac{5b(a+bx^2)^{2/3}}{9a^2x}+\frac{5b^2x}{9a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$-\frac{5\sqrt{2+\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7}{6\,3^{3/4}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$+\frac{5\sqrt{2}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-}{9\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(2/3)/a/x^3+5/9*b*(b*x^2+a)^(2/3)/a^2/x+5/9*b^2*x/a^2/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-5/18*(1/2*6^(1/2)+1/2*2^(1/2))*b*(a^(1/3)
-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(1/
4)/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))^2)^(1/2)+5/27*2^(1/2)*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a
^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/
3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*
a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/a^(5/3)/x/(-a^(1/3)*(a^(1/
3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx=-\frac{\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[3]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(1/3)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 1/3, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {264, 264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}dx$$

$\downarrow$ 264

$$-\frac{5b\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx}{9a}-\frac{(a+bx^2)^{2/3}}{3ax^3}$$

$\downarrow$ 264

$$-\frac{5b\left(\frac{b\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{3a}-\frac{(a+bx^2)^{2/3}}{ax}\right)}{9a}-\frac{(a+bx^2)^{2/3}}{3ax^3}$$

$\downarrow$ 233

$$-\frac{5b\left(\frac{\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{(a+bx^2)^{2/3}}{ax}\right)}{9a}-\frac{(a+bx^2)^{2/3}}{3ax^3}$$

$\downarrow$ 833

$$-\frac{5b\left(\frac{\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2ax}-\frac{(a+bx^2)^{2/3}}{ax}\right)}{9a}-\frac{(a+bx^2)^{2/3}}{3ax^3}$$

$\downarrow$ 760

$$-\frac{5b\left(\frac{\sqrt{bx^2}\left(-\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a}\right.}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a}\right.}}}\right)}{2ax}\right.}{9a}$$

$$\frac{(a+bx^2)^{2/3}}{3ax^3}$$

$\downarrow$ 2418

$$-\frac{5b\left(\frac{\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.}{}$$

$$\frac{(a+bx^2)^{2/3}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(1/3)),x]
```

output
```
-1/3*(a + b*x^2)^(2/3)/(a*x^3) - (5*b*(-((a + b*x^2)^(2/3)/(a*x)) + (Sqrt[
b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3
) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/
(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])
*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1
/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*El
lipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*
a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[
-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))^2)])))/(2*a*x)))/(9*a)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^4 (b\,x^2+a)^{\frac{1}{3}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/3),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b*x^6 + a*x^4), x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^2}}\, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/3),x)
```

output
```
-hyper((-3/2, 1/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/3)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^2}}\, dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \sqrt[3]{a+bx^2}}\, dx = \int \frac{1}{(bx^2+a)^{\frac{1}{3}}x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^{1/3}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/3)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[3]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,x^4}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*x**4),x)
```

3.762 $\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = -\frac{3a^3\sqrt[3]{a+bx^2}}{2b^4}+\frac{9a^2(a+bx^2)^{4/3}}{8b^4}-\frac{9a(a+bx^2)^{7/3}}{14b^4}+\frac{3(a+bx^2)^{10/3}}{20b^4}$$

output
```
-3/2*a^3*(b*x^2+a)^(1/3)/b^4+9/8*a^2*(b*x^2+a)^(4/3)/b^4-9/14*a*(b*x^2+a)^
(7/3)/b^4+3/20*(b*x^2+a)^(10/3)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}(-81a^3+27a^2bx^2-18ab^2x^4+14b^3x^6)}{280b^4}$$

input
```
Integrate[x^7/(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(1/3)*(-81*a^3 + 27*a^2*b*x^2 - 18*a*b^2*x^4 + 14*b^3*x^6))
/(280*b^4)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^{2/3}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^{2/3}}+\frac{3\sqrt[3]{bx^2+a}a^2}{b^3}-\frac{3(bx^2+a)^{4/3}a}{b^3}+\frac{(bx^2+a)^{7/3}}{b^3}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{3a^3\sqrt[3]{a+bx^2}}{b^4}+\frac{9a^2(a+bx^2)^{4/3}}{4b^4}+\frac{3(a+bx^2)^{10/3}}{10b^4}-\frac{9a(a+bx^2)^{7/3}}{7b^4}\right)$$

input
```
Int[x^7/(a + b*x^2)^(2/3),x]
```

output
```
((-3*a^3*(a + b*x^2)^(1/3))/b^4 + (9*a^2*(a + b*x^2)^(4/3))/(4*b^4) - (9*a
*(a + b*x^2)^(7/3))/(7*b^4) + (3*(a + b*x^2)^(10/3))/(10*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$	47
trager	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$	47
risch	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$	47
pseudoelliptic	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$	47
orering	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}\left(-14b^{3}x^{6}+18a\,b^{2}x^{4}-27a^{2}b\,x^{2}+81a^{3}\right)}{280b^{4}}$	47

input
```
int(x^7/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/280*(b*x^2+a)^(1/3)*(-14*b^3*x^6+18*a*b^2*x^4-27*a^2*b*x^2+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(14\,b^3x^6 - 18\,ab^2x^4 + 27\,a^2bx^2 - 81\,a^3)(bx^2+a)^{\frac{1}{3}}}{280\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/280*(14*b^3*x^6 - 18*a*b^2*x^4 + 27*a^2*b*x^2 - 81*a^3)*(b*x^2 + a)^(1/3
)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(75) = 150.

Time = 1.57 (sec) , antiderivative size = 1690, normalized size of antiderivative = 21.12

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(b*x**2+a)**(2/3),x)
```

output
```
-243*a**(70/3)*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**
2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1
680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 243*a**(70/3)/(280*a**20*b
**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 +
 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) - 1
377*a**(67/3)*b*x**2*(1 + b*x**2/a)**(1/3)/(280*a**20*b**4 + 1680*a**19*b*
*5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x*
*8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 1458*a**(67/3)*b*x**
2/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**
17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b*
*10*x**12) - 3213*a**(64/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(280*a**20*b**
4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4
200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**14*b**10*x**12) + 364
5*a**(64/3)*b**2*x**4/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*
b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*
x**10 + 280*a**14*b**10*x**12) - 3927*a**(61/3)*b**3*x**6*(1 + b*x**2/a)**
(1/3)/(280*a**20*b**4 + 1680*a**19*b**5*x**2 + 4200*a**18*b**6*x**4 + 5600
*a**17*b**7*x**6 + 4200*a**16*b**8*x**8 + 1680*a**15*b**9*x**10 + 280*a**1
4*b**10*x**12) + 4860*a**(61/3)*b**3*x**6/(280*a**20*b**4 + 1680*a**19*b**
5*x**2 + 4200*a**18*b**6*x**4 + 5600*a**17*b**7*x**6 + 4200*a**16*b**8*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{10}{3}}}{20\,b^4} - \frac{9\,(bx^2+a)^{\frac{7}{3}}a}{14\,b^4} + \frac{9\,(bx^2+a)^{\frac{4}{3}}a^2}{8\,b^4} - \frac{3\,(bx^2+a)^{\frac{1}{3}}a^3}{2\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/20*(b*x^2 + a)^(10/3)/b^4 - 9/14*(b*x^2 + a)^(7/3)*a/b^4 + 9/8*(b*x^2 +
a)^(4/3)*a^2/b^4 - 3/2*(b*x^2 + a)^(1/3)*a^3/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = -\frac{3\,(bx^2+a)^{\frac{1}{3}}a^3}{2\,b^4} + \frac{3\left(14\,(bx^2+a)^{\frac{10}{3}} - 60\,(bx^2+a)^{\frac{7}{3}}a + 105\,(bx^2+a)^{\frac{4}{3}}a^2\right)}{280\,b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
-3/2*(b*x^2 + a)^(1/3)*a^3/b^4 + 3/280*(14*(b*x^2 + a)^(10/3) - 60*(b*x^2
+ a)^(7/3)*a + 105*(b*x^2 + a)^(4/3)*a^2)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.60

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = -\left(b\,x^2+a\right)^{1/3}\left(\frac{243\,a^3}{280\,b^4} - \frac{3\,x^6}{20\,b} + \frac{27\,a\,x^4}{140\,b^2} - \frac{81\,a^2\,x^2}{280\,b^3}\right)$$

input
```
int(x^7/(a + b*x^2)^(2/3),x)
```

output
```
-(a + b*x^2)^(1/3)*((243*a^3)/(280*b^4) - (3*x^6)/(20*b) + (27*a*x^4)/(140
*b^2) - (81*a^2*x^2)/(280*b^3))
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^7}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(x^7/(b*x^2+a)^(2/3),x)
```

output
```
int(x**7/(a + b*x**2)**(2/3),x)
```

3.763 $\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{3a^2\sqrt[3]{a+bx^2}}{2b^3} - \frac{3a(a+bx^2)^{4/3}}{4b^3} + \frac{3(a+bx^2)^{7/3}}{14b^3}$$

output
```
3/2*a^2*(b*x^2+a)^(1/3)/b^3-3/4*a*(b*x^2+a)^(4/3)/b^3+3/14*(b*x^2+a)^(7/3)
/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}(9a^2-3abx^2+2b^2x^4)}{28b^3}$$

input
```
Integrate[x^5/(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(1/3)*(9*a^2 - 3*a*b*x^2 + 2*b^2*x^4))/(28*b^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{2/3}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2(bx^2+a)^{2/3}}-\frac{2\sqrt[3]{bx^2+a}a}{b^2}+\frac{(bx^2+a)^{4/3}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3a^2\sqrt[3]{a+bx^2}}{b^3}+\frac{3(a+bx^2)^{7/3}}{7b^3}-\frac{3a(a+bx^2)^{4/3}}{2b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^(2/3),x]
```

output
```
((3*a^2*(a + b*x^2)^(1/3))/b^3 - (3*a*(a + b*x^2)^(4/3))/(2*b^3) + (3*(a +
 b*x^2)^(7/3))/(7*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

method	result	size
gosper	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
trager	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
risch	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
pseudoelliptic	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36
orering	$\frac{3(b\,x^2+a)^{\frac{1}{3}}(2b^2x^4-3ab\,x^2+9a^2)}{28b^3}$	36

input
```
int(x^5/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/28*(b*x^2+a)^(1/3)*(2*b^2*x^4-3*a*b*x^2+9*a^2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(2\,b^2x^4-3\,abx^2+9\,a^2)(bx^2+a)^{\frac{1}{3}}}{28\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/28*(2*b^2*x^4 - 3*a*b*x^2 + 9*a^2)*(b*x^2 + a)^(1/3)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(54) = 108.

Time = 1.02 (sec) , antiderivative size = 631, normalized size of antiderivative = 10.69

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{27a^{\frac{31}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{27a^{\frac{31}{3}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{72a^{\frac{28}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{81a^{\frac{28}{3}}bx^2}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{60a^{\frac{25}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{81a^{\frac{25}{3}}b^2x^4}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{18a^{\frac{22}{3}}b^3x^6\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} - \frac{27a^{\frac{22}{3}}b^3x^6}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{9a^{\frac{19}{3}}b^4x^8\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6} + \frac{6a^{\frac{16}{3}}b^5x^{10}\sqrt[3]{1+\frac{bx^2}{a}}}{28a^8b^3+84a^7b^4x^2+84a^6b^5x^4+28a^5b^6x^6}$$

input
```
integrate(x**5/(b*x**2+a)**(2/3),x)
```

output
```
27*a**(31/3)*(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*
a**6*b**5*x**4 + 28*a**5*b**6*x**6) - 27*a**(31/3)/(28*a**8*b**3 + 84*a**7
*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 72*a**(28/3)*b*x**2*
(1 + b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**
4 + 28*a**5*b**6*x**6) - 81*a**(28/3)*b*x**2/(28*a**8*b**3 + 84*a**7*b**4*
x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 60*a**(25/3)*b**2*x**4*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
 28*a**5*b**6*x**6) - 81*a**(25/3)*b**2*x**4/(28*a**8*b**3 + 84*a**7*b**4*
x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 18*a**(22/3)*b**3*x**6*(1
+ b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
 28*a**5*b**6*x**6) - 27*a**(22/3)*b**3*x**6/(28*a**8*b**3 + 84*a**7*b**4*
x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6) + 9*a**(19/3)*b**4*x**8*(1 +
 b*x**2/a)**(1/3)/(28*a**8*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 +
28*a**5*b**6*x**6) + 6*a**(16/3)*b**5*x**10*(1 + b*x**2/a)**(1/3)/(28*a**8
*b**3 + 84*a**7*b**4*x**2 + 84*a**6*b**5*x**4 + 28*a**5*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{7}{3}}}{14\,b^3} - \frac{3\,(bx^2+a)^{\frac{4}{3}}a}{4\,b^3} + \frac{3\,(bx^2+a)^{\frac{1}{3}}a^2}{2\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/14*(b*x^2 + a)^(7/3)/b^3 - 3/4*(b*x^2 + a)^(4/3)*a/b^3 + 3/2*(b*x^2 + a)
^(1/3)*a^2/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{1}{3}}a^2}{2\,b^3} + \frac{3\left(2\,(bx^2+a)^{\frac{7}{3}} - 7\,(bx^2+a)^{\frac{4}{3}}a\right)}{28\,b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/2*(b*x^2 + a)^(1/3)*a^2/b^3 + 3/28*(2*(b*x^2 + a)^(7/3) - 7*(b*x^2 + a)^
(4/3)*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.61

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \left(b\,x^2+a\right)^{1/3}\left(\frac{27\,a^2}{28\,b^3}+\frac{3\,x^4}{14\,b}-\frac{9\,a\,x^2}{28\,b^2}\right)$$

input
```
int(x^5/(a + b*x^2)^(2/3),x)
```

output
```
(a + b*x^2)^(1/3)*((27*a^2)/(28*b^3) + (3*x^4)/(14*b) - (9*a*x^2)/(28*b^2)
)
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^5}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(x^5/(b*x^2+a)^(2/3),x)
```

output
```
int(x**5/(a + b*x**2)**(2/3),x)
```

3.764 $\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = -\frac{3a\sqrt[3]{a+bx^2}}{2b^2} + \frac{3(a+bx^2)^{4/3}}{8b^2}$$

output
```
-3/2*a*(b*x^2+a)^(1/3)/b^2+3/8*(b*x^2+a)^(4/3)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = \frac{3(-3a+bx^2)\sqrt[3]{a+bx^2}}{8b^2}$$

input
```
Integrate[x^3/(a + b*x^2)^(2/3),x]
```

output
```
(3*(-3*a + b*x^2)*(a + b*x^2)^(1/3))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{2/3}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{\sqrt[3]{bx^2+a}}{b}-\frac{a}{b\,(bx^2+a)^{2/3}}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{3(a+bx^2)^{4/3}}{4b^2}-\frac{3a\sqrt[3]{a+bx^2}}{b^2}\right)$$

input
```
Int[x^3/(a + b*x^2)^(2/3),x]
```

output
```
((-3*a*(a + b*x^2)^(1/3))/b^2 + (3*(a + b*x^2)^(4/3))/(4*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{3(b x^2+a)^{\frac{1}{3}}(-b x^2+3a)}{8b^2}$	25
trager	$-\frac{3(b x^2+a)^{\frac{1}{3}}(-b x^2+3a)}{8b^2}$	25
risch	$-\frac{3(b x^2+a)^{\frac{1}{3}}(-b x^2+3a)}{8b^2}$	25
pseudoelliptic	$-\frac{3(b x^2+a)^{\frac{1}{3}}(-b x^2+3a)}{8b^2}$	25
orering	$-\frac{3(b x^2+a)^{\frac{1}{3}}(-b x^2+3a)}{8b^2}$	25

input
```
int(x^3/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*(b*x^2+a)^(1/3)*(-b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.61

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{1}{3}}(bx^2-3\,a)}{8\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/8*(b*x^2 + a)^(1/3)*(b*x^2 - 3*a)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(34) = 68.

Time = 0.66 (sec) , antiderivative size = 178, normalized size of antiderivative = 4.68

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = -\frac{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2} + \frac{9a^{\frac{10}{3}}}{8a^2b^2+8ab^3x^2} - \frac{6a^{\frac{7}{3}}bx^2\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2} + \frac{9a^{\frac{7}{3}}bx^2}{8a^2b^2+8ab^3x^2} + \frac{3a^{\frac{4}{3}}b^2x^4\sqrt[3]{1+\frac{bx^2}{a}}}{8a^2b^2+8ab^3x^2}$$

input
```
integrate(x**3/(b*x**2+a)**(2/3),x)
```

output
```
-9*a**(10/3)*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(1
0/3)/(8*a**2*b**2 + 8*a*b**3*x**2) - 6*a**(7/3)*b*x**2*(1 + b*x**2/a)**(1/
3)/(8*a**2*b**2 + 8*a*b**3*x**2) + 9*a**(7/3)*b*x**2/(8*a**2*b**2 + 8*a*b*
*3*x**2) + 3*a**(4/3)*b**2*x**4*(1 + b*x**2/a)**(1/3)/(8*a**2*b**2 + 8*a*b
**3*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b^2} - \frac{3\,(bx^2+a)^{\frac{1}{3}}a}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{4}{3}}}{8\,b^2} - \frac{3\,(bx^2+a)^{\frac{1}{3}}a}{2\,b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/8*(b*x^2 + a)^(4/3)/b^2 - 3/2*(b*x^2 + a)^(1/3)*a/b^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = -\frac{3\,(b\,x^2+a)^{1/3}\,(3\,a-b\,x^2)}{8\,b^2}$$

input
```
int(x^3/(a + b*x^2)^(2/3),x)
```

output
```
-(3*(a + b*x^2)^(1/3)*(3*a - b*x^2))/(8*b^2)
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^3}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(x^3/(b*x^2+a)^(2/3),x)
```

output
```
int(x**3/(a + b*x**2)**(2/3),x)
```

3.765 $\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}}{2b}$$

output
```
3/2*(b*x^2+a)^(1/3)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\sqrt[3]{a+bx^2}}{2b}$$

input
```
Integrate[x/(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(1/3))/(2*b)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{241}$$

$$\frac{3\sqrt[3]{a+bx^2}}{2b}$$

input
```
Int[x/(a + b*x^2)^(2/3),x]
```

output
```
(3*(a + b*x^2)^(1/3))/(2*b)
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
derivativedivides	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
default	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
trager	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
risch	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
pseudoelliptic	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15
orering	$\frac{3(b\,x^{2}+a)^{\frac{1}{3}}}{2b}$	15

input
```
int(x/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/2*(b*x^2+a)^(1/3)/b
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

input
```
integrate(x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/2*(b*x^2 + a)^(1/3)/b
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \begin{cases} \frac{3\sqrt[3]{a+bx^2}}{2b} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{2}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(2/3),x)
```

output
```
Piecewise((3*(a + b*x**2)**(1/3)/(2*b), Ne(b, 0)), (x**2/(2*a**(2/3)), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

input
```
integrate(x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/2*(b*x^2 + a)^(1/3)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{1}{3}}}{2\,b}$$

input
```
integrate(x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
3/2*(b*x^2 + a)^(1/3)/b
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3\,(b\,x^2+a)^{1/3}}{2\,b}$$

input
```
int(x/(a + b*x^2)^(2/3),x)
```

output
```
(3*(a + b*x^2)^(1/3))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{2/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}}{2b}$$

input
```
int(x/(b*x^2+a)^(2/3),x)
```

output
```
(3*(a + b*x**2)**(1/3))/(2*b)
```

3.766 $\int \frac{1}{x(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 86

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{2/3}}-\frac{\log(x)}{2a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{2/3}}$$

output
```
-1/2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))/a^(2/
3)-1/2*ln(x)/a^(2/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(2/3)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = -\frac{2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)+\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}\right)}{4a^{2/3}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(2/3)),x]
```

output
```
-1/4*(2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 2*Lo
g[-a^(1/3) + (a + b*x^2)^(1/3)] + Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3)
+ (a + b*x^2)^(2/3)])/a^(2/3)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {243, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2+a)^{2/3}}dx^2$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2} \left(-\frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3 \int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log (x^2)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2} \left(-\frac{3 \int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log (x^2)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2} \left(\frac{3 \int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3 \log \left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log (x^2)}{2a^{2/3}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(2/3)),x]
```

output
```
(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3))
- Log[x^2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3)))
/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$\frac{-2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)+2\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)}{4a^{\frac{2}{3}}}$	83

input
```
int(1/x/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))+2*
ln((b*x^2+a)^(1/3)-a^(1/3))-ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(
2/3)))/a^(2/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(61) = 122$.

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = -\frac{2\sqrt{3}(a^2)^{\frac{1}{6}}a\arctan\left(\frac{\sqrt{3}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\left(bx^2+a\right)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{3\,a^2}\right)+(a^2)^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+\left(bx^2+a\right)^{\frac{1}{3}}(a^2\right.}{4\,a^2}$$

input
```
integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(a^2)^(1/6)*a*arctan(1/3*sqrt(3)*(a^2)^(1/6)*((a^2)^(1/3)*
a + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + (a^2)^(2/3)*log((b*x^2 + a)^(2
/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3)) - 2*(a^2)^(2/3)*log
((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)))/a^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.48

$$\int \frac{1}{x(a+bx^2)^{2/3}}\,dx = -\frac{\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}x^{\frac{4}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate(1/x/(b*x**2+a)**(2/3),x)
```

output
```
-gamma(2/3)*hyper((2/3, 2/3), (5/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(4/3)*gamma(5/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+bx^2)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}} + \frac{\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(2/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = -\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{2}{3}}} - \frac{\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{2}{3}}} + \frac{\log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{2}{3}}}$$

input
```
integrate(1/x/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
-1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a
^(2/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(2/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(2/3)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.19

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx = \frac{\ln\left(\frac{9\left(b\,x^2+a\right)^{1/3}}{2}-\frac{9\,a^{1/3}}{2}\right)}{2\,a^{2/3}} + \frac{\ln\left(\frac{9\,a^{1/3}\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{4}-\frac{9\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{2/3}} - \frac{\ln\left(\frac{9\,a^{1/3}\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{4}+\frac{9\left(b\,x^2+a\right)^{1/3}}{2}\right)\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}{4\,a^{2/3}}$$

input
```
int(1/(x*(a + b*x^2)^(2/3)),x)
```

output
```
log((9*(a + b*x^2)^(1/3))/2 - (9*a^(1/3))/2)/(2*a^(2/3)) + (log((9*a^(1/3)
*(3^(1/2)*1i - 1))/4 - (9*(a + b*x^2)^(1/3))/2)*(3^(1/2)*1i - 1))/(4*a^(2/
3)) - (log((9*a^(1/3)*(3^(1/2)*1i + 1))/4 + (9*(a + b*x^2)^(1/3))/2)*(3^(1
/2)*1i + 1))/(4*a^(2/3))
```

Reduce [F]

$$\int \frac{1}{x\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}}x}dx$$

input
```
int(1/x/(b*x^2+a)^(2/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*x),x)
```

3.767 $\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 107

$$\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx=-\frac{\sqrt[3]{a+bx^2}}{2ax^2}+\frac{b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}+\frac{b\log(x)}{3a^{5/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{5/3}}$$

output
```
-1/2*(b*x^2+a)^(1/3)/a/x^2+1/3*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^
(1/2)/a^(1/3))*3^(1/2)/a^(5/3)+1/3*b*ln(x)/a^(5/3)-1/2*b*ln(a^(1/3)-(b*x^2
+a)^(1/3))/a^(5/3)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.26

$$\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx=\frac{-3a^{2/3}\sqrt[3]{a+bx^2}+2\sqrt{3}bx^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-2bx^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right.}{6a^{5/3}x^2}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*a^(2/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b*x^2*ArcTan[(1 + (2*(a + b*x^2)
^(1/3))/a^(1/3))/Sqrt[3]] - 2*b*x^2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] + b*
x^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]])/(6*a^(5/
3)*x^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 (bx^2+a)^{2/3}} dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{2b \int \frac{1}{x^2 (bx^2+a)^{2/3}} dx^2}{3a} - \frac{\sqrt[3]{a+bx^2}}{ax^2} \right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2} \left(-\frac{2b \left(-\frac{3 \int \frac{1}{\sqrt[3]{a} - \sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2a^{2/3}} - \frac{3 \int \frac{1}{x^4 + a^{2/3} + \sqrt[3]{a}\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}} - \frac{\log(x^2)}{2a^{2/3}} \right)}{3a} - \frac{\sqrt[3]{a+bx^2}}{ax^2} \right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{3\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(-\frac{2b\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log(x^2)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^(2/3)),x]
```

output
```
(-((a + b*x^2)^(1/3)/(a*x^2)) - (2*b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2
)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)) - Log[x^2]/(2*a^(2/3)) + (3*Log[a^(1/
3) - (a + b*x^2)^(1/3)])/(2*a^(2/3))))/(3*a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.04

method	result
pseudoelliptic	$\frac{2b\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\sqrt{3}\,x^{2}-2b\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^{2}+b\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)x^{2}-3\left(b\,x^{2}-\right.}{6a^{\frac{5}{3}}x^{2}}$

input
```
int(1/x^3/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*b*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)*x
^2-2*b*ln((b*x^2+a)^(1/3)-a^(1/3))*x^2+b*ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3
)+(b*x^2+a)^(2/3))*x^2-3*(b*x^2+a)^(1/3)*a^(2/3))/a^(5/3)/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.67

$$\int \frac{1}{x^3\left(a+bx^2\right)^{2/3}}\,dx = \frac{6\sqrt{\frac{1}{3}}abx^2\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left(\left(-a^2\right)^{\frac{1}{3}}a-2\left(bx^2+a\right)^{\frac{1}{3}}\left(-a^2\right)^{\frac{2}{3}}\right)\sqrt{-\left(-a^2\right)^{\frac{1}{3}}}}{a^2}\right)+\left(-a^2\right)^{\frac{2}{3}}}{}$$

input
```
integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
1/6*(6*sqrt(1/3)*a*b*x^2*sqrt(-(-a^2)^(1/3))*arctan(-sqrt(1/3)*((-a^2)^(1/
3)*a - 2*(b*x^2 + a)^(1/3)*(-a^2)^(2/3))*sqrt(-(-a^2)^(1/3))/a^2) + (-a^2)
^(2/3)*b*x^2*log((b*x^2 + a)^(2/3)*a - (-a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*
(-a^2)^(2/3)) - 2*(-a^2)^(2/3)*b*x^2*log((b*x^2 + a)^(1/3)*a - (-a^2)^(2/3
)) - 3*(b*x^2 + a)^(1/3)*a^2)/(a^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}}\,dx = -\frac{\Gamma\left(\frac{5}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**2+a)**(2/3),x)
```

output
```
-gamma(5/3)*hyper((2/3, 5/3), (8/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*x**(10/3)*gamma(8/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}}\,dx = \frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(2 (bx^2+a)^{\frac{1}{3}} + a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}}\right)}{3 a^{\frac{5}{3}}} - \frac{(bx^2+a)^{\frac{1}{3}} b}{2 \left((bx^2+a)a - a^2\right)} + \frac{b \log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{6 a^{\frac{5}{3}}} - \frac{b \log\left((bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{3 a^{\frac{5}{3}}}$$

input
```
integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/
a^(5/3) - 1/2*(b*x^2 + a)^(1/3)*b/((b*x^2 + a)*a - a^2) + 1/6*b*log((b*x^2
 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a^(5/3) - 1/3*b*log((b*
x^2 + a)^(1/3) - a^(1/3))/a^(5/3)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}}\,dx = \frac{1}{6}b\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{a^{\frac{5}{3}}}+\frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{5}{3}}}-\right.$$

input
```
integrate(1/x^3/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
1/6*b*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3
))/a^(5/3) + log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/
a^(5/3) - 2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(5/3) - 3*(b*x^2 + a)^
(1/3)/(a*b*x^2))
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}}\,dx = \frac{\ln\left(\frac{3\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,(b\,x^2+a)^{1/3}}{a}\right)\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}} + \frac{\ln\left(\frac{3\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{2\,a^{2/3}}+\frac{3\,b\,(b\,x^2+a)^{1/3}}{a}\right)\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{6\,a^{5/3}} - \frac{b\,\ln\left((b\,x^2+a)^{1/3}-a^{1/3}\right)}{3\,a^{5/3}} - \frac{(b\,x^2+a)^{1/3}}{2\,a\,x^2}$$

input
```
int(1/(x^3*(a + b*x^2)^(2/3)),x)
```

output
```
(log((3*(b - 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*b*(a + b*x^2)^(1/3))/a)*(b -
3^(1/2)*b*1i))/(6*a^(5/3)) + (log((3*(b + 3^(1/2)*b*1i))/(2*a^(2/3)) + (3*
b*(a + b*x^2)^(1/3))/a)*(b + 3^(1/2)*b*1i))/(6*a^(5/3)) - (b*log((a + b*x^
2)^(1/3) - a^(1/3)))/(3*a^(5/3)) - (a + b*x^2)^(1/3)/(2*a*x^2)
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,x^3} dx$$

input
```
int(1/x^3/(b*x^2+a)^(2/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*x**3),x)
```

3.768 $\int \frac{1}{x^5 (a+bx^2)^{2/3}} dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}} dx = -\frac{\sqrt[3]{a+bx^2}}{4ax^4} + \frac{5b\sqrt[3]{a+bx^2}}{12a^2x^2} - \frac{5b^2 \arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{6\sqrt{3}a^{8/3}} - \frac{5b^2\log(x)}{18a^{8/3}} + \frac{5b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{12a^{8/3}}$$

output
```
-1/4*(b*x^2+a)^(1/3)/a/x^4+5/12*b*(b*x^2+a)^(1/3)/a^2/x^2-5/18*b^2*arctan(
1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(8/3)-5/18*b^2*
ln(x)/a^(8/3)+5/12*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(8/3)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}} dx = \frac{\frac{3a^{2/3}\sqrt[3]{a+bx^2}(-3a+5bx^2)}{x^4} - 10\sqrt{3}b^2 \arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + 10b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{\cdots}\right.}{36a^{8/3}}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^(2/3)),x]
```

output
```
((3*a^(2/3)*(a + b*x^2)^(1/3)*(-3*a + 5*b*x^2))/x^4 - 10*Sqrt[3]*b^2*ArcTa
n[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 10*b^2*Log[-a^(1/3) + (a
+ b*x^2)^(1/3)] - 5*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x
^2)^(2/3)])/(36*a^(8/3))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 52, 52, 69, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^6 (bx^2+a)^{2/3}} dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{5b \int \frac{1}{x^4 (bx^2+a)^{2/3}} dx^2}{6a} - \frac{\sqrt[3]{a+bx^2}}{2ax^4} \right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{5b \left(-\frac{2b \int \frac{1}{x^2 (bx^2+a)^{2/3}} dx^2}{3a} - \frac{\sqrt[3]{a+bx^2}}{ax^2} \right)}{6a} - \frac{\sqrt[3]{a+bx^2}}{2ax^4} \right)$$

$$\Big\downarrow \text{69}$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int\frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2a^{2/3}}-\frac{3\int\frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)}{6a}-\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{2b\left(-\frac{3\int\frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)}{6a}-\frac{\sqrt[3]{a+bx}}{2ax^4}\right.$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{2b\left(\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)}{6a}-\frac{\sqrt[3]{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(-\frac{5b\left(-\frac{2b\left(-\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{a^{2/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2a^{2/3}}-\frac{\log\left(x^2\right)}{2a^{2/3}}\right)}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax^2}\right)}{6a}-\frac{\sqrt[3]{a+bx^2}}{2ax^4}\right)$$

input
```
Int[1/(x^5*(a + b*x^2)^(2/3)),x]
```

output
```
(-1/2*(a + b*x^2)^(1/3)/(a*x^4) - (5*b*(-((a + b*x^2)^(1/3)/(a*x^2)) - (2*
b*(-((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(2/3)
) - Log[x^2]/(2*a^(2/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(2/3)
)))/(3*a)))/(6*a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 69
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Simp[3/(2*b*q)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1
/3)], x] - Simp[3/(2*b*q^2)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$\frac{-10b^2\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^2+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)x^4+10b^2\ln\left(\left(b\,x^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)x^4-5b^2\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+\left(b\,x^2+a\right)^{\frac{2}{3}}\right)x^4}{36a^{\frac{8}{3}}x^4}$

input
```
int(1/x^5/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/36*(-10*b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/
3))*x^4+10*b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4-5*b^2*ln(a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))*x^4+15*b*x^2*a^(2/3)*(b*x^2+a)^(1/3)-9*(b*x
^2+a)^(1/3)*a^(5/3))/a^(8/3)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{2/3}}\,dx = -\frac{30\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}ab^2x^4\arctan\left(\frac{\sqrt{\frac{1}{3}}(a^2)^{\frac{1}{6}}\left((a^2)^{\frac{1}{3}}a+2\,(bx^2+a)^{\frac{1}{3}}(a^2)^{\frac{2}{3}}\right)}{a^2}\right)+5\,(a^2)^{\frac{2}{3}}b^2x^4\log\left((bx^2+a)^{\frac{2}{3}}a+(a^2)^{\frac{1}{3}}a+(\right.}{36\,a^4x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
-1/36*(30*sqrt(1/3)*(a^2)^(1/6)*a*b^2*x^4*arctan(sqrt(1/3)*(a^2)^(1/6)*((a
^2)^(1/3)*a + 2*(b*x^2 + a)^(1/3)*(a^2)^(2/3))/a^2) + 5*(a^2)^(2/3)*b^2*x^
4*log((b*x^2 + a)^(2/3)*a + (a^2)^(1/3)*a + (b*x^2 + a)^(1/3)*(a^2)^(2/3))
 - 10*(a^2)^(2/3)*b^2*x^4*log((b*x^2 + a)^(1/3)*a - (a^2)^(2/3)) - 3*(5*a^
2*b*x^2 - 3*a^3)*(b*x^2 + a)^(1/3))/(a^4*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}} \, dx = -\frac{\Gamma\left(\frac{8}{3}\right) {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{8}{3} \\ \frac{11}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}} x^{\frac{16}{3}} \Gamma\left(\frac{11}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**2+a)**(2/3),x)
```

output
```
-gamma(8/3)*hyper((2/3, 8/3), (11/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(
2/3)*x**(16/3)*gamma(11/3))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}} \, dx = -\frac{5\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{18a^{\frac{8}{3}}} - \frac{5b^2 \log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{36a^{\frac{8}{3}}} + \frac{5b^2 \log\left((bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{18a^{\frac{8}{3}}} + \frac{5(bx^2+a)^{\frac{4}{3}}b^2 - 8(bx^2+a)^{\frac{1}{3}}ab^2}{12\left((bx^2+a)^2a^2 - 2(bx^2+a)a^3 + a^4\right)}$$

input
```
integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
-5/18*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/
3))/a^(8/3) - 5/36*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(8/3) + 5/18*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(8/3) + 1/
12*(5*(b*x^2 + a)^(4/3)*b^2 - 8*(b*x^2 + a)^(1/3)*a*b^2)/((b*x^2 + a)^2*a^
2 - 2*(b*x^2 + a)*a^3 + a^4)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^5 (a+bx^2)^{2/3}}\, dx =$$

$$-\frac{\frac{10\sqrt{3}b^3 \arctan\left(\frac{\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{a^{\frac{8}{3}}} + \frac{5b^3 \log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{a^{\frac{8}{3}}} - \frac{10b^3 \log\left(\left|\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{a^{\frac{8}{3}}} - \frac{3\left(5\left(bx^2+a\right)^{\frac{4}{3}}b^3 \cdots}{a^2 \cdots}}{36b}$$

input
```
integrate(1/x^5/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
-1/36*(10*sqrt(3)*b^3*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a
^(1/3))/a^(8/3) + 5*b^3*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
+ a^(2/3))/a^(8/3) - 10*b^3*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(8/3)
- 3*(5*(b*x^2 + a)^(4/3)*b^3 - 8*(b*x^2 + a)^(1/3)*a*b^3)/(a^2*b^2*x^4))/b
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{1}{x^5 (a+bx^2)^{2/3}}\, dx &= \frac{5\,b^2\,\ln\left(\left(b\,x^2+a\right)^{1/3}-a^{1/3}\right)}{18\,a^{8/3}} \\ &- \frac{\frac{4\,b^2\,\left(b\,x^2+a\right)^{1/3}}{3\,a}-\frac{5\,b^2\,\left(b\,x^2+a\right)^{4/3}}{6\,a^2}}{2\,\left(b\,x^2+a\right)^2-4\,a\,\left(b\,x^2+a\right)+2\,a^2} \\ &+ \frac{5\,b^2\,\ln\left(\frac{5\,b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a^2}-\frac{5\,b^2\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}} \\ &- \frac{5\,b^2\,\ln\left(\frac{5\,b^2\,\left(b\,x^2+a\right)^{1/3}}{2\,a^2}+\frac{5\,b^2\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{2\,a^{5/3}}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)}{18\,a^{8/3}}\end{aligned}$$

input
```
int(1/(x^5*(a + b*x^2)^(2/3)),x)
```

output
```
(5*b^2*log((a + b*x^2)^(1/3) - a^(1/3)))/(18*a^(8/3)) - ((4*b^2*(a + b*x^2
)^(1/3))/(3*a) - (5*b^2*(a + b*x^2)^(4/3))/(6*a^2))/(2*(a + b*x^2)^2 - 4*a
*(a + b*x^2) + 2*a^2) + (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) - (5*
b^2*((3^(1/2)*1i)/2 - 1/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 - 1/2))/(18*a^(8/
3)) - (5*b^2*log((5*b^2*(a + b*x^2)^(1/3))/(2*a^2) + (5*b^2*((3^(1/2)*1i)/
2 + 1/2))/(2*a^(5/3)))*((3^(1/2)*1i)/2 + 1/2))/(18*a^(8/3))
```

Reduce [F]

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}} x^5}dx$$

input
```
int(1/x^5/(b*x^2+a)^(2/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*x**5),x)
```

3.769 $\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 293

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = -\frac{27ax\sqrt[3]{a+bx^2}}{55b^2}+\frac{3x^3\sqrt[3]{a+bx^2}}{11b} - \frac{27\,3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a}}\right.\right.}{55b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-27/55*a*x*(b*x^2+a)^(1/3)/b^2+3/11*x^3*(b*x^2+a)^(1/3)/b-27/55*3^(3/4)*(1
/2*6^(1/2)-1/2*2^(1/2))*a^2*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1
/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.87 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \frac{3\left(-9a^2x-4abx^3+5b^2x^5+9a^2x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{55b^2(a+bx^2)^{2/3}}$$

input
```
Integrate[x^4/(a + b*x^2)^(2/3),x]
```

output
```
(3*(-9*a^2*x - 4*a*b*x^3 + 5*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(2/3)*Hyper
geometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)]))/(55*b^2*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\int \frac{x^2}{(bx^2+a)^{2/3}}dx}{11b}$$

$$\Big\downarrow \; 262$$

$$\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int \frac{1}{(bx^2+a)^{2/3}}dx}{5b}\right)}{11b}$$

$$\Big\downarrow \; 234$$

$$\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}\right)}{11b}$$

$$\Big\downarrow \text{760}$$

$$\frac{3x^3\sqrt[3]{a+bx^2}}{11b}-\frac{9a\left(\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-7+4\sqrt{3}\right)}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{11b}$$

input
```
Int[x^4/(a + b*x^2)^(2/3),x]
```

output
```
(3*x^3*(a + b*x^2)^(1/3))/(11*b) - (9*a*((3*x*(a + b*x^2)^(1/3))/(5*b) + (
3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3)
+ a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2
)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5
*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))^2)])))/(11*b)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}d x$$

input
```
int(x^4/(b*x^2+a)^(2/3),x)
```

output
```
int(x^4/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx=\int \frac{x^{4}}{{\left(b x^{2}+a\right)}^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix}\frac{2}{3}, \frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{2}{3}}}$$

input
```
integrate(x**4/(b*x**2+a)**(2/3),x)
```

output
```
x**5*hyper((2/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(2/3))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(2/3),x)
```

output
```
int(x^4/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(x^4/(b*x^2+a)^(2/3),x)
```

output
```
int(x**4/(a + b*x**2)**(2/3),x)
```

3.770 $\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 269

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \frac{3x\sqrt[3]{a+bx^2}}{5b} + \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/5*x*(b*x^2+a)^(1/3)/b+3/5*3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*a*(a^(1/3)-(
b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(
-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2
)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{5b(a+bx^2)^{2/3}}$$

input
```
Integrate[x^2/(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2,
 -((b*x^2)/a)]))/(5*b*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{3a\int \frac{1}{(bx^2+a)^{2/3}}dx}{5b}$$

$$\Big\downarrow \; 234$$

$$\frac{3x\sqrt[3]{a+bx^2}}{5b}-\frac{9a\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{10b^2x}$$

$$\Big\downarrow \; 760$$

$$\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right.\right.}{5b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$\frac{3x\sqrt[3]{a+bx^2}}{5b}$$

input
```
Int[x^2/(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3))/(5*b) + (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*(a^(1/3) -
(a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)
^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((
1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*
x^2)^(1/3))], -7 + 4*Sqrt[3]])/(5*b^2*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*
x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(b*x^2+a)^(2/3),x)
```

output
```
int(x^2/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{x^2}{\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{x^2}{\left(bx^2+a\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{2}{3}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}}$$

input
```
integrate(x**2/(b*x**2+a)**(2/3),x)
```

output
```
x**3*hyper((2/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(2/3),x)
```

output
```
int(x^2/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{2/3}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(x^2/(b*x^2+a)^(2/3),x)
```

output
```
int(x**2/(a + b*x**2)**(2/3),x)
```

3.771 $\int \frac{1}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 246

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = -\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right.\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-3^(3/4)*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(
1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)
)^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^
(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.19

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx=\frac{x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{2/3}}$$

input
```
Integrate[(a + b*x^2)^(-2/3),x]
```

output
```
(x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(2/3)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{234}$$

$$\frac{3\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2bx}$$

$$\Big\downarrow \text{760}$$

$$-\frac{3^{3/4}\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),\right.}{bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

input
```
Int[(a + b*x^2)^(-2/3),x]
```

output
```
-((3^(3/4)*Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) +
 a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)
^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(b*
x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)]))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}d x$$

input
```
int(1/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-2/3), x)
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{2}{3} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(2/3),x)
```

output
```
x*hyper((1/2, 2/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(2/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{2/3}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{2/3}}$$

input
```
int(1/(a + b*x^2)^(2/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(2/3)*hypergeom([1/2, 2/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(2/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(a + b*x**2)**(2/3),x)
```

3.772 $\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 265

$$\int \frac{1}{x^2(a+bx^2)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{ax} + \frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(1/3)/a/x+1/3*(1/2*6^(1/2)-1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3
))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*3^(3/4)/a/x/(-a^(1/3
)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.81 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}} \, dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{2}{3},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{2/3}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(2/3)),x]
```

output
```
-(((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/2, 2/3, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(2/3)))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {264, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{1}{(bx^2+a)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{234}$$

$$-\frac{\sqrt{bx^2}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\sqrt[3]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-7+}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$\frac{\sqrt[3]{a+bx^2}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)^(2/3)),x]
```

output
```
-((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/
3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^
(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -
7 + 4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))
)/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(2/3),x)
```

output
```
int(1/x^2/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/(b*x^4 + a*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{2}{3} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{2}{3}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(2/3),x)
```

output
```
-hyper((-1/2, 2/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(2/3)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}} \, dx = -\frac{3 \left(\frac{a}{b\,x^2}+1\right)^{2/3} {}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^2}\right)}{7\,x\,(b\,x^2+a)^{2/3}}$$

input
```
int(1/(x^2*(a + b*x^2)^(2/3)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(2/3)*hypergeom([2/3, 7/6], 13/6, -a/(b*x^2)))/(7*x*(a
 + b*x^2)^(2/3))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(b\,x^2+a)^{\frac{2}{3}}\,x^2} dx$$

input
```
int(1/x^2/(b*x^2+a)^(2/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*x**2),x)
```

3.773 $\int \frac{1}{x^4(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 293

$$\int \frac{1}{x^4(a+bx^2)^{2/3}}\,dx = -\frac{\sqrt[3]{a+bx^2}}{3ax^3}+\frac{7b\sqrt[3]{a+bx^2}}{9a^2x} - \frac{7\sqrt{2-\sqrt{3}}\,b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{9\sqrt[4]{3}a^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(1/3)/a/x^3+7/9*b*(b*x^2+a)^(1/3)/a^2/x-7/27*(1/2*6^(1/2)-1
/2*2^(1/2))*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+
(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(
((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)
),2*I-I*3^(1/2))*3^(3/4)/a^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{2}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{2/3}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(2/3)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-3/2, 2/3, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {264, 264, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 264$$

$$-\frac{7b\int \frac{1}{x^2(bx^2+a)^{2/3}}dx}{9a}-\frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \; 264$$

$$-\frac{7b\left(-\frac{b\int \frac{1}{(bx^2+a)^{2/3}}dx}{3a}-\frac{\sqrt[3]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \; 234$$

$$-\frac{7b\left(-\frac{\sqrt{bx^2}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\sqrt[3]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{760}$$

$$-\frac{7b\left(\frac{\sqrt{2-\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-7+4\sqrt{3}\right)}{\sqrt[4]{3}ax\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}-\right.}{9a}$$

$$\frac{\sqrt[3]{a+bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(2/3)),x]
```

output
```
-1/3*(a + b*x^2)^(1/3)/(a*x^3) - (7*b*(-((a + b*x^2)^(1/3)/(a*x)) + (Sqrt[
2 - Sqrt[3]]*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*Sqrt[-
((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x
^2)^(1/3))^2)])))/(9*a)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b \,x^{2}+a \right)^{\frac{2}{3}}}d x$$

input
```
int(1/x^4/(b*x^2+a)^(2/3),x)
```

output
```
int(1/x^4/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{2/3}}\, dx = \int \frac{1}{{\left(b x^{2} + a\right)}^{\frac{2}{3}} x^{4}}\, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)/(b*x^6 + a*x^4), x)
```

Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{2}{3}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(2/3),x)
```

output
```
-hyper((-3/2, 2/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(2/3)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{2/3}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{2/3}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{2}{3}} x^4}dx$$

input
```
int(1/x^4/(b*x^2+a)^(2/3),x)
```

output
```
int(1/((a + b*x**2)**(2/3)*x**4),x)
```

3.774 $\int \frac{x^7}{(a+bx^2)^{4/3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 80

$$\int \frac{x^7}{(a+bx^2)^{4/3}} \, dx = \frac{3a^3}{2b^4\sqrt[3]{a+bx^2}} + \frac{9a^2(a+bx^2)^{2/3}}{4b^4} - \frac{9a(a+bx^2)^{5/3}}{10b^4} + \frac{3(a+bx^2)^{8/3}}{16b^4}$$

output
```
3/2*a^3/b^4/(b*x^2+a)^(1/3)+9/4*a^2*(b*x^2+a)^(2/3)/b^4-9/10*a*(b*x^2+a)^(
5/3)/b^4+3/16*(b*x^2+a)^(8/3)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x^7}{(a+bx^2)^{4/3}} \, dx = \frac{3(81a^3 + 27a^2bx^2 - 9ab^2x^4 + 5b^3x^6)}{80b^4\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^7/(a + b*x^2)^(4/3),x]
```

output
```
(3*(81*a^3 + 27*a^2*b*x^2 - 9*a*b^2*x^4 + 5*b^3*x^6))/(80*b^4*(a + b*x^2)^
(1/3))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{x^6}{(bx^2+a)^{4/3}}dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(-\frac{a^3}{b^3(bx^2+a)^{4/3}}+\frac{3a^2}{b^3\sqrt[3]{bx^2+a}}-\frac{3(bx^2+a)^{2/3}a}{b^3}+\frac{(bx^2+a)^{5/3}}{b^3}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{3a^3}{b^4\sqrt[3]{a+bx^2}}+\frac{9a^2(a+bx^2)^{2/3}}{2b^4}-\frac{9a(a+bx^2)^{5/3}}{5b^4}+\frac{3(a+bx^2)^{8/3}}{8b^4}\right)$$

input
```
Int[x^7/(a + b*x^2)^(4/3),x]
```

output
```
((3*a^3)/(b^4*(a + b*x^2)^(1/3)) + (9*a^2*(a + b*x^2)^(2/3))/(2*b^4) - (9*
a*(a + b*x^2)^(5/3))/(5*b^4) + (3*(a + b*x^2)^(8/3))/(8*b^4))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.59

method	result	size
gosper	$\frac{\frac{3}{16} b^{3} x^{6}-\frac{27}{80} a \,b^{2} x^{4}+\frac{81}{80} a^{2} b \,x^{2}+\frac{243}{80} a^{3}}{\left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{4}}$	47
trager	$\frac{\frac{3}{16} b^{3} x^{6}-\frac{27}{80} a \,b^{2} x^{4}+\frac{81}{80} a^{2} b \,x^{2}+\frac{243}{80} a^{3}}{\left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{4}}$	47
pseudoelliptic	$\frac{\frac{3}{16} b^{3} x^{6}-\frac{27}{80} a \,b^{2} x^{4}+\frac{81}{80} a^{2} b \,x^{2}+\frac{243}{80} a^{3}}{\left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{4}}$	47
orering	$\frac{\frac{3}{16} b^{3} x^{6}-\frac{27}{80} a \,b^{2} x^{4}+\frac{81}{80} a^{2} b \,x^{2}+\frac{243}{80} a^{3}}{\left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{4}}$	47
risch	$\frac{3\left(5 b^{2} x^{4}-14 a b \,x^{2}+41 a^{2}\right) \left(b \,x^{2}+a \right)^{\frac{2}{3}}}{80 b^{4}}+\frac{3 a^{3}}{2 b^{4} \left(b \,x^{2}+a \right)^{\frac{1}{3}}}$	54

input
```
int(x^7/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/80/(b*x^2+a)^(1/3)*(5*b^3*x^6-9*a*b^2*x^4+27*a^2*b*x^2+81*a^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(5\,b^3x^6-9\,ab^2x^4+27\,a^2bx^2+81\,a^3)(bx^2+a)^{\frac{2}{3}}}{80\,(b^5x^2+ab^4)}$$

input
```
integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/80*(5*b^3*x^6 - 9*a*b^2*x^4 + 27*a^2*b*x^2 + 81*a^3)*(b*x^2 + a)^(2/3)/(
b^5*x^2 + a*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1584 vs. 2(75) = 150.

Time = 1.58 (sec) , antiderivative size = 1584, normalized size of antiderivative = 19.80

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7/(b*x**2+a)**(4/3),x)
```

output
```
243*a**(68/3)*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 +
 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*
a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 243*a**(68/3)/(80*a**20*b**4 +
480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a
**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 1296*a**(6
5/3)*b*x**2*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1
200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a*
*15*b**9*x**10 + 80*a**14*b**10*x**12) - 1458*a**(65/3)*b*x**2/(80*a**20*b
**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 +
1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) + 2808
*a**(62/3)*b**2*x**4*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 480*a**19*b**5
*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8
 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 3645*a**(62/3)*b**2*x**4
/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*
b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x
**12) + 3120*a**(59/3)*b**3*x**6*(1 + b*x**2/a)**(2/3)/(80*a**20*b**4 + 48
0*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 + 1600*a**17*b**7*x**6 + 1200*a**
16*b**8*x**8 + 480*a**15*b**9*x**10 + 80*a**14*b**10*x**12) - 4860*a**(59/
3)*b**3*x**6/(80*a**20*b**4 + 480*a**19*b**5*x**2 + 1200*a**18*b**6*x**4 +
 1600*a**17*b**7*x**6 + 1200*a**16*b**8*x**8 + 480*a**15*b**9*x**10 + 8...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{8}{3}}}{16\,b^4} - \frac{9\,(bx^2+a)^{\frac{5}{3}}a}{10\,b^4} + \frac{9\,(bx^2+a)^{\frac{2}{3}}a^2}{4\,b^4} + \frac{3\,a^3}{2\,(bx^2+a)^{\frac{1}{3}}b^4}$$

input
```
integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/16*(b*x^2 + a)^(8/3)/b^4 - 9/10*(b*x^2 + a)^(5/3)*a/b^4 + 9/4*(b*x^2 + a
)^(2/3)*a^2/b^4 + 3/2*a^3/((b*x^2 + a)^(1/3)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \frac{3\,a^3}{2\,(bx^2+a)^{\frac{1}{3}}b^4} + \frac{3\left(5\,(bx^2+a)^{\frac{8}{3}}b^{28} - 24\,(bx^2+a)^{\frac{5}{3}}ab^{28} + 60\,(bx^2+a)^{\frac{2}{3}}a^2b^{28}\right)}{80\,b^{32}}$$

input
```
integrate(x^7/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/2*a^3/((b*x^2 + a)^(1/3)*b^4) + 3/80*(5*(b*x^2 + a)^(8/3)*b^28 - 24*(b*x
^2 + a)^(5/3)*a*b^28 + 60*(b*x^2 + a)^(2/3)*a^2*b^28)/b^32
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.69

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \frac{180\,a^2\,(b\,x^2+a) - 72\,a\,(b\,x^2+a)^2 + 15\,(b\,x^2+a)^3 + 120\,a^3}{80\,b^4\,(b\,x^2+a)^{1/3}}$$

input
```
int(x^7/(a + b*x^2)^(4/3),x)
```

output
```
(180*a^2*(a + b*x^2) - 72*a*(a + b*x^2)^2 + 15*(a + b*x^2)^3 + 120*a^3)/(8
0*b^4*(a + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{x^7}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^7}{(b\,x^2+a)^{\frac{1}{3}}\,a + (b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x^7/(b*x^2+a)^(4/3),x)
```

output
```
int(x**7/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.775 $\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 59

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = -\frac{3a^2}{2b^3\sqrt[3]{a+bx^2}} - \frac{3a(a+bx^2)^{2/3}}{2b^3} + \frac{3(a+bx^2)^{5/3}}{10b^3}$$

output
```
-3/2*a^2/b^3/(b*x^2+a)^(1/3)-3/2*a*(b*x^2+a)^(2/3)/b^3+3/10*(b*x^2+a)^(5/3
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.64

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = \frac{3(-9a^2-3abx^2+b^2x^4)}{10b^3\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^5/(a + b*x^2)^(4/3),x]
```

output
```
(3*(-9*a^2 - 3*a*b*x^2 + b^2*x^4))/(10*b^3*(a + b*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{x^4}{(bx^2+a)^{4/3}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b^2 (bx^2+a)^{4/3}}-\frac{2a}{b^2 \sqrt[3]{bx^2+a}}+\frac{(bx^2+a)^{2/3}}{b^2}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(-\frac{3a^2}{b^3\sqrt[3]{a+bx^2}}-\frac{3a(a+bx^2)^{2/3}}{b^3}+\frac{3(a+bx^2)^{5/3}}{5b^3}\right)$$

input
```
Int[x^5/(a + b*x^2)^(4/3),x]
```

output
```
((-3*a^2)/(b^3*(a + b*x^2)^(1/3)) - (3*a*(a + b*x^2)^(2/3))/b^3 + (3*(a +
b*x^2)^(5/3))/(5*b^3))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\frac{3}{10} b^{2} x^{4}-\frac{9}{10} a b \,x^{2}-\frac{27}{10} a^{2}}{\left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{3}}$	35
gosper	$-\frac{3\left(-b^{2} x^{4}+3 a b \,x^{2}+9 a^{2}\right)}{10 \left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{3}}$	36
trager	$-\frac{3\left(-b^{2} x^{4}+3 a b \,x^{2}+9 a^{2}\right)}{10 \left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{3}}$	36
orering	$-\frac{3\left(-b^{2} x^{4}+3 a b \,x^{2}+9 a^{2}\right)}{10 \left(b \,x^{2}+a \right)^{\frac{1}{3}} b^{3}}$	36
risch	$-\frac{3 \left(-b \,x^{2}+4 a \right) \left(b \,x^{2}+a \right)^{\frac{2}{3}}}{10 b^{3}}-\frac{3 a^{2}}{2 b^{3} \left(b \,x^{2}+a \right)^{\frac{1}{3}}}$	43

input
```
int(x^5/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/10*(b^2*x^4-3*a*b*x^2-9*a^2)/(b*x^2+a)^(1/3)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(b^2x^4-3\,abx^2-9\,a^2)(bx^2+a)^{\frac{2}{3}}}{10\,(b^4x^2+ab^3)}$$

input
```
integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/10*(b^2*x^4 - 3*a*b*x^2 - 9*a^2)*(b*x^2 + a)^(2/3)/(b^4*x^2 + a*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. 2(54) = 108.

Time = 1.02 (sec) , antiderivative size = 561, normalized size of antiderivative = 9.51

$$\begin{aligned}\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = &-\frac{27a^{\frac{29}{3}}\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &+\frac{27a^{\frac{29}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &-\frac{63a^{\frac{26}{3}}bx^2\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &+\frac{81a^{\frac{26}{3}}bx^2}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &-\frac{42a^{\frac{23}{3}}b^2x^4\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &+\frac{81a^{\frac{23}{3}}b^2x^4}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &-\frac{3a^{\frac{20}{3}}b^3x^6\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &+\frac{27a^{\frac{20}{3}}b^3x^6}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\\ &+\frac{3a^{\frac{17}{3}}b^4x^8\left(1+\frac{bx^2}{a}\right)^{\frac{2}{3}}}{10a^8b^3+30a^7b^4x^2+30a^6b^5x^4+10a^5b^6x^6}\end{aligned}$$

input
```
integrate(x**5/(b*x**2+a)**(4/3),x)
```

output
```
-27*a**(29/3)*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30
*a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 27*a**(29/3)/(10*a**8*b**3 + 30*a**
7*b**4*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 63*a**(26/3)*b*x**2
*(1 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x*
*4 + 10*a**5*b**6*x**6) + 81*a**(26/3)*b*x**2/(10*a**8*b**3 + 30*a**7*b**4
*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 42*a**(23/3)*b**2*x**4*(1
 + b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4
+ 10*a**5*b**6*x**6) + 81*a**(23/3)*b**2*x**4/(10*a**8*b**3 + 30*a**7*b**4
*x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) - 3*a**(20/3)*b**3*x**6*(1
+ b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
 10*a**5*b**6*x**6) + 27*a**(20/3)*b**3*x**6/(10*a**8*b**3 + 30*a**7*b**4*
x**2 + 30*a**6*b**5*x**4 + 10*a**5*b**6*x**6) + 3*a**(17/3)*b**4*x**8*(1 +
 b*x**2/a)**(2/3)/(10*a**8*b**3 + 30*a**7*b**4*x**2 + 30*a**6*b**5*x**4 +
10*a**5*b**6*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.80

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{5}{3}}}{10\,b^3} - \frac{3\,(bx^2+a)^{\frac{2}{3}}a}{2\,b^3} - \frac{3\,a^2}{2\,(bx^2+a)^{\frac{1}{3}}b^3}$$

input
```
integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/10*(b*x^2 + a)^(5/3)/b^3 - 3/2*(b*x^2 + a)^(2/3)*a/b^3 - 3/2*a^2/((b*x^2
 + a)^(1/3)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = -\frac{3\left(\frac{5\,a^2}{(bx^2+a)^{\frac{1}{3}}b} - \frac{(bx^2+a)^{\frac{5}{3}}b^4 - 5\,(bx^2+a)^{\frac{2}{3}}ab^4}{b^5}\right)}{10\,b^2}$$

input
```
integrate(x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
-3/10*(5*a^2/((b*x^2 + a)^(1/3)*b) - ((b*x^2 + a)^(5/3)*b^4 - 5*(b*x^2 + a
)^(2/3)*a*b^4)/b^5)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.69

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = -\frac{15\,a\,(b\,x^2+a) - 3\,(b\,x^2+a)^2 + 15\,a^2}{10\,b^3\,(b\,x^2+a)^{1/3}}$$

input
```
int(x^5/(a + b*x^2)^(4/3),x)
```

output
```
-(15*a*(a + b*x^2) - 3*(a + b*x^2)^2 + 15*a^2)/(10*b^3*(a + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{x^5}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^5}{(b\,x^2+a)^{\frac{1}{3}}\,a + (b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x^5/(b*x^2+a)^(4/3),x)
```

output
```
int(x**5/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.776 $\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 38

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3a}{2b^2\sqrt[3]{a+bx^2}} + \frac{3(a+bx^2)^{2/3}}{4b^2}$$

output
```
3/2*a/b^2/(b*x^2+a)^(1/3)+3/4*(b*x^2+a)^(2/3)/b^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.71

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3(3a+bx^2)}{4b^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^3/(a + b*x^2)^(4/3),x]
```

output
```
(3*(3*a + b*x^2))/(4*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{x^2}{(bx^2+a)^{4/3}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2}\int \left(\frac{1}{b\sqrt[3]{bx^2+a}}-\frac{a}{b\,(bx^2+a)^{4/3}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{3a}{b^2\sqrt[3]{a+bx^2}}+\frac{3(a+bx^2)^{2/3}}{2b^2}\right)$$

input
```
Int[x^3/(a + b*x^2)^(4/3),x]
```

output
```
((3*a)/(b^2*(a + b*x^2)^(1/3)) + (3*(a + b*x^2)^(2/3))/(2*b^2))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

method	result	size
gosper	$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	24
trager	$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	24
orering	$\frac{\frac{3b\,x^2}{4}+\frac{9a}{4}}{(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	24
pseudoelliptic	$\frac{3b\,x^2+9a}{4(b\,x^2+a)^{\frac{1}{3}}\,b^2}$	25
risch	$\frac{3a}{2b^2(b\,x^2+a)^{\frac{1}{3}}}+\frac{3(b\,x^2+a)^{\frac{2}{3}}}{4b^2}$	31

input
```
int(x^3/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
3/4/(b*x^2+a)^(1/3)*(b*x^2+3*a)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(bx^2+3\,a)(bx^2+a)^{\frac{2}{3}}}{4\,(b^3x^2+ab^2)}$$

input
```
integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
3/4*(b*x^2 + 3*a)*(b*x^2 + a)^(2/3)/(b^3*x^2 + a*b^2)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \begin{cases} \frac{9a}{4b^2\sqrt[3]{a+bx^2}} + \frac{3x^2}{4b\sqrt[3]{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3/(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((9*a/(4*b**2*(a + b*x**2)**(1/3)) + 3*x**2/(4*b*(a + b*x**2)**(1
/3)), Ne(b, 0)), (x**4/(4*a**(4/3)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3\,(bx^2+a)^{\frac{2}{3}}}{4\,b^2} + \frac{3\,a}{2\,(bx^2+a)^{\frac{1}{3}}b^2}$$

input
```
integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
3/4*(b*x^2 + a)^(2/3)/b^2 + 3/2*a/((b*x^2 + a)^(1/3)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3\left(\frac{(bx^2+a)^{\frac{2}{3}}}{b}+\frac{2a}{(bx^2+a)^{\frac{1}{3}}b}\right)}{4b}$$

input
```
integrate(x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
3/4*((b*x^2 + a)^(2/3)/b + 2*a/((b*x^2 + a)^(1/3)*b))/b
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \frac{3\,b\,x^2+9\,a}{4\,b^2\,(b\,x^2+a)^{1/3}}$$

input
```
int(x^3/(a + b*x^2)^(4/3),x)
```

output
```
(9*a + 3*b*x^2)/(4*b^2*(a + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{x^3}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^3}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x^3/(b*x^2+a)^(4/3),x)
```

output
```
int(x**3/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.777 $\int \frac{x}{(a+bx^2)^{4/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{x}{(a+bx^2)^{4/3}} \, dx = -\frac{3}{2b\sqrt[3]{a+bx^2}}$$

output
```
-3/2/b/(b*x^2+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a+bx^2)^{4/3}} \, dx = -\frac{3}{2b\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x/(a + b*x^2)^(4/3),x]
```

output
```
-3/(2*b*(a + b*x^2)^(1/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \; 241$$

$$-\frac{3}{2b\sqrt[3]{a+bx^2}}$$

input
```
Int[x/(a + b*x^2)^(4/3),x]
```

output
```
-3/(2*b*(a + b*x^2)^(1/3))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
gosper	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
derivativedivides	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
default	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
trager	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
pseudoelliptic	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15
orering	$-\frac{3}{2b(b\,x^2+a)^{\frac{1}{3}}}$	15

input
```
int(x/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2/b/(b*x^2+a)^(1/3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3\,(bx^2+a)^{\frac{2}{3}}}{2\,(b^2x^2+ab)}$$

input
```
integrate(x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
-3/2*(b*x^2 + a)^(2/3)/(b^2*x^2 + a*b)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = \begin{cases} -\frac{3}{2b\sqrt[3]{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{4}{3}}} & \text{otherwise} \end{cases}$$

input
```
integrate(x/(b*x**2+a)**(4/3),x)
```

output
```
Piecewise((-3/(2*b*(a + b*x**2)**(1/3)), Ne(b, 0)), (x**2/(2*a**(4/3)), Tr
ue))
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3}{2\,(bx^2+a)^{\frac{1}{3}}b}$$

input
```
integrate(x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
-3/2/((b*x^2 + a)^(1/3)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3}{2\,(bx^2+a)^{\frac{1}{3}}b}$$

input
```
integrate(x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
-3/2/((b*x^2 + a)^(1/3)*b)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = -\frac{3}{2\,b\,(b\,x^2+a)^{1/3}}$$

input
```
int(x/(a + b*x^2)^(4/3),x)
```

output
```
-3/(2*b*(a + b*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{x}{(a+bx^2)^{4/3}}\,dx = \int \frac{x}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x/(b*x^2+a)^(4/3),x)
```

output
```
int(x/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.778 $\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx=\frac{3}{2a\sqrt[3]{a+bx^2}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{2a^{4/3}}-\frac{\log(x)}{2a^{4/3}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{4a^{4/3}}$$

output
```
3/2/a/(b*x^2+a)^(1/3)+1/2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3
^(1/2)/a^(1/3))/a^(4/3)-1/2*ln(x)/a^(4/3)+3/4*ln(a^(1/3)-(b*x^2+a)^(1/3))/
a^(4/3)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx=\frac{\frac{6\sqrt[3]{a}}{\sqrt[3]{a+bx^2}}+2\sqrt{3}\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+2\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)-\log\left(a^{2/3}\right.}{4a^{4/3}}$$

input
```
Integrate[1/(x*(a + b*x^2)^(4/3)),x]
```

output
```
((6*a^(1/3))/(a + b*x^2)^(1/3) + 2*Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3
))/a^(1/3))/Sqrt[3]] + 2*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - Log[a^(2/3) +
 a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])/(4*a^(4/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {243, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx^2)^{4/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^2 (bx^2+a)^{4/3}} dx^2$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(\frac{\int \frac{1}{x^2 \sqrt[3]{bx^2+a}} dx^2}{a} + \frac{3}{a \sqrt[3]{a+bx^2}} \right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2} \left(\frac{\frac{3}{2} \int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a} \sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a} - \frac{3 \int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}} d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}} - \frac{\log(x^2)}{2\sqrt[3]{a}}}{a} + \frac{3}{a \sqrt[3]{a+bx^2}} \right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}\left(\frac{-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)$$

input
```
Int[1/(x*(a + b*x^2)^(4/3)),x]
```

output
```
(3/(a*(a + b*x^2)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(
1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x
^2)^(1/3)])/(2*a^(1/3)))/a)/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
pseudoelliptic	$\frac{\sqrt{3}\,\arctan\left(\frac{\left(a^{\frac{1}{3}}+2\left(b\,x^{2}+a\right)^{\frac{1}{3}}\right)\sqrt{3}}{3a^{\frac{1}{3}}}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)\left(b\,x^{2}+a\right)^{\frac{1}{3}}-\frac{\ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)\left(b\right.}{2}}{2\left(b\,x^{2}+a\right)^{\frac{1}{3}}a^{\frac{4}{3}}}$

input
```
int(1/x/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
1/2*(3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*(b*x^
2+a)^(1/3)+ln((b*x^2+a)^(1/3)-a^(1/3))*(b*x^2+a)^(1/3)-1/2*ln(a^(2/3)+a^(1
/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))*(b*x^2+a)^(1/3)+3*a^(1/3))/(b*x^2+a)^
(1/3)/a^(4/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.14

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx = \left[\frac{\sqrt{3}(abx^2+a^2)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+\sqrt{3}\left(2\left(bx^2+a\right)^{\frac{2}{3}}a^{\frac{2}{3}}-\left(bx^2+a\right)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{2}{3}}+}{x^2}\right.}{}\right.$$

$$-\frac{(bx^2+a)a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{2}{3}}+\left(bx^2+a\right)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-2\left(bx^2+a\right)a^{\frac{2}{3}}\log\left(\left(bx^2+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)-\frac{2\sqrt{3}(abx^2+a^2}{}}{4\left(a^2bx^2+a^3\right)}$$

input
```
integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*(a*b*x^2 + a^2)*sqrt(-1/a^(2/3))*log((2*b*x^2 + sqrt(3)*(2*(
b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/3))*sqrt(-1/a^(2/3))
 - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - (b*x^2 + a)*a^(2/3)*log((b*x^
2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) + 2*(b*x^2 + a)*a^(2/3
)*log((b*x^2 + a)^(1/3) - a^(1/3)) + 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a
^3), -1/4*((b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a
^(1/3) + a^(2/3)) - 2*(b*x^2 + a)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1/3))
 - 2*sqrt(3)*(a*b*x^2 + a^2)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(
1/3))/a^(1/3))/a^(1/3) - 6*(b*x^2 + a)^(2/3)*a)/(a^2*b*x^2 + a^3)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx = -\frac{\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}}x^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

input
```
integrate(1/x/(b*x**2+a)**(4/3),x)
```

output
```
-gamma(4/3)*hyper((4/3, 4/3), (7/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(4
/3)*x**(8/3)*gamma(7/3))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{4}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{4}{3}}} + \frac{\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{2\,a^{\frac{4}{3}}} + \frac{3}{2\,(bx^2+a)^{\frac{1}{3}}a}$$

input
```
integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 1/2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(4/3) + 3/2/((b*x^2 + a)^(
1/3)*a)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int \frac{1}{x\left(a+bx^2\right)^{4/3}}\,dx = \frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{2\,a^{\frac{4}{3}}} - \frac{\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{4\,a^{\frac{4}{3}}} + \frac{\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{2\,a^{\frac{4}{3}}} + \frac{3}{2\,(bx^2+a)^{\frac{1}{3}}a}$$

input
```
integrate(1/x/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^
(4/3) - 1/4*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))/a
^(4/3) + 1/2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(4/3) + 3/2/((b*x^2 +
 a)^(1/3)*a)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(a+bx^2)^{4/3}}\,dx = \frac{\ln\left(18\,a\,(b\,x^2+a)^{1/3}-18\,a^{4/3}\right)}{2\,a^{4/3}}+\frac{3}{2\,a\,(b\,x^2+a)^{1/3}}+\frac{\ln\left(18\,a\,(b\,x^2+a)^{1/3}-\frac{9\,a^{4/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)^2}{2}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{4/3}}-\frac{\ln\left(18\,a\,(b\,x^2+a)^{1/3}-\frac{9\,a^{4/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)^2}{2}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4\,a^{4/3}}$$

input
```
int(1/(x*(a + b*x^2)^(4/3)),x)
```

output
```
log(18*a*(a + b*x^2)^(1/3) - 18*a^(4/3))/(2*a^(4/3)) + 3/(2*a*(a + b*x^2)^
(1/3)) + (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3^(1/2)*1i - 1)^2)/2)*(
3^(1/2)*1i - 1))/(4*a^(4/3)) - (log(18*a*(a + b*x^2)^(1/3) - (9*a^(4/3)*(3
^(1/2)*1i + 1)^2)/2)*(3^(1/2)*1i + 1))/(4*a^(4/3))
```

Reduce [F]

$$\int \frac{1}{x(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,ax+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^3}dx$$

input
```
int(1/x/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a*x + (a + b*x**2)**(1/3)*b*x**3),x)
```

3.779 $\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx=-\frac{2b}{a^2\sqrt[3]{a+bx^2}}-\frac{1}{2ax^2\sqrt[3]{a+bx^2}}-\frac{2b\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{7/3}}+\frac{2b\log(x)}{3a^{7/3}}-\frac{b\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{a^{7/3}}$$

output
```
-2*b/a^2/(b*x^2+a)^(1/3)-1/2/a/x^2/(b*x^2+a)^(1/3)-2/3*b*arctan(1/3*(a^(1/
3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*3^(1/2)/a^(7/3)+2/3*b*ln(x)/a^(7/3)
-b*ln(a^(1/3)-(b*x^2+a)^(1/3))/a^(7/3)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx=\frac{-\frac{3\sqrt[3]{a}(a+4bx^2)}{x^2\sqrt[3]{a+bx^2}}-4\sqrt{3}b\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)-4b\log\left(-\sqrt[3]{a}+\sqrt[3]{a+bx^2}\right)+2b}{6a^{7/3}}$$

input
```
Integrate[1/(x^3*(a + b*x^2)^(4/3)),x]
```

output
```
((-3*a^(1/3)*(a + 4*b*x^2))/(x^2*(a + b*x^2)^(1/3)) - 4*Sqrt[3]*b*ArcTan[(
1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] - 4*b*Log[-a^(1/3) + (a + b*x^
2)^(1/3)] + 2*b*Log[a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3
)])/(6*a^(7/3))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {243, 52, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \left(a+bx^2\right)^{4/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int \frac{1}{x^4 \left(bx^2+a\right)^{4/3}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2} \left(-\frac{4b \int \frac{1}{x^2 (bx^2+a)^{4/3}}dx^2}{3a}-\frac{1}{ax^2 \sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2} \left(-\frac{4b \left(\frac{\int \frac{1}{x^2 \sqrt[3]{bx^2+a}}dx^2}{a}+\frac{3}{a \sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2 \sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{67}$$

$$\frac{1}{2}\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a}}\right.$$

$$\Big\downarrow \; 16$$

$$\frac{1}{2}\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{2}\left(-\frac{4b\left(\frac{-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log(x^2)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{2}\left(-\frac{4b\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)$$

input
```
Int[1/(x^3*(a + b*x^2)^(4/3)),x]
```

output
```
(-(1/(a*x^2*(a + b*x^2)^(1/3))) - (4*b*(3/(a*(a + b*x^2)^(1/3)) + ((Sqrt[3
]*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/
(2*a^(1/3)) + (3*Log[a^(1/3) - (a + b*x^2)^(1/3)])/(2*a^(1/3)))/a))/(3*a))
/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{\left(b\,x^{2}+a\right)^{\frac{2}{3}}}{2 a^{2} x^{2}}-\frac{2 b \ln\left(\left(b\,x^{2}+a\right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{3 a^{\frac{7}{3}}}+\frac{b \ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}}\left(b\,x^{2}+a\right)^{\frac{1}{3}}+\left(b\,x^{2}+a\right)^{\frac{2}{3}}\right)}{3 a^{\frac{7}{3}}}-\frac{2 b \sqrt{3}\, \arctan\left(\frac{2\sqrt{3}\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{3 a^{\frac{1}{3}}}+\right.}{3 a^{\frac{7}{3}}}$

input
```
int(1/x^3/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*(b*x^2+a)^(2/3)/x^2-2/3*b/a^(7/3)*ln((b*x^2+a)^(1/3)-a^(1/3))+1/3
*b/a^(7/3)*ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))-2/3*b/a^(7/
3)*3^(1/2)*arctan(2/3*3^(1/2)/a^(1/3)*(b*x^2+a)^(1/3)+1/3*3^(1/2))-3/2*b/a
^2/(b*x^2+a)^(1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(96) = 192$.

Time = 0.08 (sec) , antiderivative size = 453, normalized size of antiderivative = 3.68

$$\int \frac{1}{x^3\left(a+bx^2\right)^{4/3}}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}(ab^2x^4+a^2bx^2)\sqrt{\frac{(-a)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,bx^2-3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}(-a)^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a+(-a)^{\frac{1}{3}}a\right)\sqrt{\frac{(-}{}}}{x^2}\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}(ab^2x^4+a^2bx^2)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\arctan\left(\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{1}{3}}-(-a)^{\frac{1}{3}}\right)\sqrt{-\frac{(-a)^{\frac{1}{3}}}{a}}\right)-2\,(b^2x^4+abx^2)(-a)^{}}{}$$

input
```
integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/6*(6*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt((-a)^(1/3)/a)*log((2*b*x^2
- 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*(-a)^(2/3) - (b*x^2 + a)^(1/3)*a + (-a)
^(1/3)*a)*sqrt((-a)^(1/3)/a) - 3*(b*x^2 + a)^(1/3)*(-a)^(2/3) + 3*a)/x^2)
+ 2*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^(1/
3)*(-a)^(1/3) + (-a)^(2/3)) - 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2
+ a)^(1/3) + (-a)^(1/3)) - 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*b*x
^4 + a^4*x^2), -1/6*(12*sqrt(1/3)*(a*b^2*x^4 + a^2*b*x^2)*sqrt(-(-a)^(1/3)
/a)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) - (-a)^(1/3))*sqrt(-(-a)^(1/3)/a
)) - 2*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x^2 + a)^(2/3) - (b*x^2 + a)^
(1/3)*(-a)^(1/3) + (-a)^(2/3)) + 4*(b^2*x^4 + a*b*x^2)*(-a)^(2/3)*log((b*x
^2 + a)^(1/3) + (-a)^(1/3)) + 3*(4*a*b*x^2 + a^2)*(b*x^2 + a)^(2/3))/(a^3*
b*x^4 + a^4*x^2)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.33

$$\int \frac{1}{x^3 \left(a+bx^2\right)^{4/3}}\,dx = -\frac{\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}} x^{\frac{14}{3}} \Gamma\left(\frac{10}{3}\right)}$$

input
```
integrate(1/x**3/(b*x**2+a)**(4/3),x)
```

output
```
-gamma(7/3)*hyper((4/3, 7/3), (10/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(
4/3)*x**(14/3)*gamma(10/3))
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a+bx^2)^{4/3}} dx = -\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2 (bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}}\right)}{3 a^{\frac{7}{3}}} - \frac{4 (bx^2+a)b - 3 ab}{2 \left((bx^2+a)^{\frac{4}{3}} a^2 - (bx^2+a)^{\frac{1}{3}} a^3\right)} + \frac{b\log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{3 a^{\frac{7}{3}}} - \frac{2 b\log\left((bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{3 a^{\frac{7}{3}}}$$

input
```
integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
-2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(7/3) - 1/2*(4*(b*x^2 + a)*b - 3*a*b)/((b*x^2 + a)^(4/3)*a^2 - (b*x^2 +
 a)^(1/3)*a^3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
 a^(2/3))/a^(7/3) - 2/3*b*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(7/3)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (a+bx^2)^{4/3}} dx = -\frac{2\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2 (bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3 a^{\frac{1}{3}}}\right)}{3 a^{\frac{7}{3}}} + \frac{b\log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}} a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{3 a^{\frac{7}{3}}} - \frac{2 b\log\left(\left|(bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right|\right)}{3 a^{\frac{7}{3}}} - \frac{4 (bx^2+a)b - 3 ab}{2 \left((bx^2+a)^{\frac{4}{3}} - (bx^2+a)^{\frac{1}{3}} a\right) a^2}$$

input
```
integrate(1/x^3/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
-2/3*sqrt(3)*b*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))
/a^(7/3) + 1/3*b*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/
3))/a^(7/3) - 2/3*b*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(7/3) - 1/2*(4
*(b*x^2 + a)*b - 3*a*b)/(((b*x^2 + a)^(4/3) - (b*x^2 + a)^(1/3)*a)*a^2)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (a+bx^2)^{4/3}}\,dx = -\frac{\frac{3\,b}{a}-\frac{4\,b\,\left(b\,x^2+a\right)}{a^2}}{2\,a\,{\left(b\,x^2+a\right)}^{1/3}-2\,{\left(b\,x^2+a\right)}^{4/3}} - \frac{2\,b\,\ln\left(4\,a^{7/3}\,b^2-4\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)}{3\,a^{7/3}} + \frac{\ln\left(a^{7/3}\,{\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)\,\left(b-\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{7/3}} + \frac{\ln\left(a^{7/3}\,{\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}^2-4\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{1/3}\right)\,\left(b+\sqrt{3}\,b\,1{}\mathrm{i}\right)}{3\,a^{7/3}}$$

input
```
int(1/(x^3*(a + b*x^2)^(4/3)),x)
```

output
```
(log(a^(7/3)*(b - 3^(1/2)*b*1i)^2 - 4*a^2*b^2*(a + b*x^2)^(1/3))*(b - 3^(1
/2)*b*1i))/(3*a^(7/3)) - (2*b*log(4*a^(7/3)*b^2 - 4*a^2*b^2*(a + b*x^2)^(1
/3)))/(3*a^(7/3)) - ((3*b)/a - (4*b*(a + b*x^2))/a^2)/(2*a*(a + b*x^2)^(1/
3) - 2*(a + b*x^2)^(4/3)) + (log(a^(7/3)*(b + 3^(1/2)*b*1i)^2 - 4*a^2*b^2*
(a + b*x^2)^(1/3))*(b + 3^(1/2)*b*1i))/(3*a^(7/3))
```

Reduce [F]

$$\int \frac{1}{x^3 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{3}} a\,x^3+\left(b\,x^2+a\right)^{\frac{1}{3}} b\,x^5}dx$$

input
```
int(1/x^3/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a*x**3 + (a + b*x**2)**(1/3)*b*x**5),x)
```

3.780 $\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 159

$$\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx=\frac{7b^2}{3a^3\sqrt[3]{a+bx^2}}-\frac{1}{4ax^4\sqrt[3]{a+bx^2}}+\frac{7b}{12a^2x^2\sqrt[3]{a+bx^2}}+\frac{7b^2\arctan\left(\frac{\sqrt[3]{a}+2\sqrt[3]{a+bx^2}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{10/3}}-\frac{7b^2\log(x)}{9a^{10/3}}+\frac{7b^2\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{6a^{10/3}}$$

output
```
7/3*b^2/a^3/(b*x^2+a)^(1/3)-1/4/a/x^4/(b*x^2+a)^(1/3)+7/12*b/a^2/x^2/(b*x^
2+a)^(1/3)+7/9*b^2*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))
*3^(1/2)/a^(10/3)-7/9*b^2*ln(x)/a^(10/3)+7/6*b^2*ln(a^(1/3)-(b*x^2+a)^(1/3
))/a^(10/3)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5\left(a+bx^2\right)^{4/3}}\,dx=\frac{\frac{3\sqrt[3]{a}\left(-3a^2+7abx^2+28b^2x^4\right)}{x^4\sqrt[3]{a+bx^2}}+28\sqrt{3}b^2\arctan\left(\frac{1+\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}}{\sqrt{3}}\right)+28b^2\log\left(-\sqrt[3]{a}+\sqrt[3]{a}\right.}{36a^{10/3}}$$

input
```
Integrate[1/(x^5*(a + b*x^2)^(4/3)),x]
```

output
```
((3*a^(1/3)*(-3*a^2 + 7*a*b*x^2 + 28*b^2*x^4))/(x^4*(a + b*x^2)^(1/3)) + 2
8*Sqrt[3]*b^2*ArcTan[(1 + (2*(a + b*x^2)^(1/3))/a^(1/3))/Sqrt[3]] + 28*b^2
*Log[-a^(1/3) + (a + b*x^2)^(1/3)] - 14*b^2*Log[a^(2/3) + a^(1/3)*(a + b*x
^2)^(1/3) + (a + b*x^2)^(2/3)])/(36*a^(10/3))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {243, 52, 52, 61, 67, 16, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}} \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{1}{x^6 (bx^2+a)^{4/3}}dx^2$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{7b\int \frac{1}{x^4(bx^2+a)^{4/3}}dx^2}{6a}-\frac{1}{2ax^4\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\int \frac{1}{x^2\left(bx^2+a\right)^{4/3}}dx^2}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)}{6a}-\frac{1}{2ax^4\sqrt[3]{a+bx^2}}\right)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\left(\frac{\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx^2}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)}{6a}-\frac{1}{2ax^4\sqrt[3]{a+bx^2}}\right)$$

$\downarrow$ 67

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}-\frac{3\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{}{ax^2\sqrt[3]{}}\right)}{6a}\right.$$

$\downarrow$ 16

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\left(\frac{\frac{3}{2}\int \frac{1}{x^4+a^{2/3}+\sqrt[3]{a}\sqrt[3]{bx^2+a}}d\sqrt[3]{bx^2+a}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right.}{6a}\right.$$

$\downarrow$ 1082

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\left(\frac{-\frac{3\int \frac{1}{-x^4-3}d\left(\frac{2\sqrt[3]{bx^2+a}}{\sqrt[3]{a}}+1\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)}{6a}-\frac{}{2ax\cdots}\right.$$

$\downarrow$ 217

$$\frac{1}{2}\left(-\frac{7b\left(-\frac{4b\left(\frac{\frac{\sqrt{3}\arctan\left(\frac{\frac{2\sqrt[3]{a+bx^2}}{\sqrt[3]{a}}+1}{\sqrt{3}}\right)}{\sqrt[3]{a}}+\frac{3\log\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{2\sqrt[3]{a}}-\frac{\log\left(x^2\right)}{2\sqrt[3]{a}}}{a}+\frac{3}{a\sqrt[3]{a+bx^2}}\right)}{3a}-\frac{1}{ax^2\sqrt[3]{a+bx^2}}\right)}{6a}-\frac{}{2ax^4\sqrt[3]{}}\right.$$

input

```
Int[1/(x^5*(a + b*x^2)^(4/3)),x]
```

output
```
(-1/2*1/(a*x^4*(a + b*x^2)^(1/3)) - (7*b*(-(1/(a*x^2*(a + b*x^2)^(1/3))) -
 (4*b*(3/(a*(a + b*x^2)^(1/3)) + ((Sqrt[3]*ArcTan[(1 + (2*(a + b*x^2)^(1/3
))/a^(1/3))/Sqrt[3]])/a^(1/3) - Log[x^2]/(2*a^(1/3)) + (3*Log[a^(1/3) - (a
 + b*x^2)^(1/3)])/(2*a^(1/3)))/a))/(3*a)))/(6*a))/2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 67
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, Simp[-Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Simp[3/(2*b)   Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
 x] - Simp[3/(2*b*q)   Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.03

method	result
pseudoelliptic	$\frac{\frac{7 b^{2} \sqrt{3}\, \arctan\left(\frac{\left(a^{\frac{1}{3}}+2 \left(b \,x^{2}+a \right)^{\frac{1}{3}}\right) \sqrt{3}}{3 a^{\frac{1}{3}}}\right) x^{4} \left(b \,x^{2}+a \right)^{\frac{1}{3}}}{9}+\frac{7 b^{2} \ln\left(\left(b \,x^{2}+a \right)^{\frac{1}{3}}-a^{\frac{1}{3}}\right) x^{4} \left(b \,x^{2}+a \right)^{\frac{1}{3}}}{9}-\frac{7 b^{2} \ln\left(a^{\frac{2}{3}}+a^{\frac{1}{3}} \left(b \,x^{2}+a \right)^{\frac{1}{3}}+\left(b x \cdots\right.}{18}}{a^{\frac{10}{3}} x^{4} \left(b \,x^{2}+a \right)^{\frac{1}{3}}}$

input
```
int(1/x^5/(b*x^2+a)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
7/9*(b^2*3^(1/2)*arctan(1/3*(a^(1/3)+2*(b*x^2+a)^(1/3))*3^(1/2)/a^(1/3))*x
^4*(b*x^2+a)^(1/3)+b^2*ln((b*x^2+a)^(1/3)-a^(1/3))*x^4*(b*x^2+a)^(1/3)-1/2
*b^2*ln(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))*x^4*(b*x^2+a)^(1/
3)+3*b^2*x^4*a^(1/3)+3/4*a^(4/3)*b*x^2-9/28*a^(7/3))/(b*x^2+a)^(1/3)/a^(10
/3)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^5 \left(a+bx^2\right)^{4/3}}\,dx = \left[\frac{42\sqrt{\frac{1}{3}}(ab^3x^6+a^2b^2x^4)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}\log\left(\frac{2\,bx^2+3\sqrt{\frac{1}{3}}\left(2\,(bx^2+a)^{\frac{2}{3}}a^{\frac{2}{3}}-(bx^2+a)^{\frac{1}{3}}a-a^{\frac{4}{3}}\right)\sqrt{-\frac{1}{a^{\frac{2}{3}}}}-3}{x^2}\right.}{}\right.$$

$$-\frac{14\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)-28\,(b^3x^6+ab^2x^4)a^{\frac{2}{3}}\log\left((bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right)}{36\,(a^4bx^6+a^5x^4)}$$

input
```
integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
[1/36*(42*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4)*sqrt(-1/a^(2/3))*log((2*b*x^
2 + 3*sqrt(1/3)*(2*(b*x^2 + a)^(2/3)*a^(2/3) - (b*x^2 + a)^(1/3)*a - a^(4/
3))*sqrt(-1/a^(2/3)) - 3*(b*x^2 + a)^(1/3)*a^(2/3) + 3*a)/x^2) - 14*(b^3*x
^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3)
+ a^(2/3)) + 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(1/3) - a^(1
/3)) + 3*(28*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^
6 + a^5*x^4), -1/36*(14*(b^3*x^6 + a*b^2*x^4)*a^(2/3)*log((b*x^2 + a)^(2/3
) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3)) - 28*(b^3*x^6 + a*b^2*x^4)*a^(2/3
)*log((b*x^2 + a)^(1/3) - a^(1/3)) - 84*sqrt(1/3)*(a*b^3*x^6 + a^2*b^2*x^4
)*arctan(sqrt(1/3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3))/a^(1/3) - 3*(2
8*a*b^2*x^4 + 7*a^2*b*x^2 - 3*a^3)*(b*x^2 + a)^(2/3))/(a^4*b*x^6 + a^5*x^4
)]
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = -\frac{\Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} \frac{4}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{4}{3}} x^{\frac{20}{3}} \Gamma\left(\frac{13}{3}\right)}$$

input
```
integrate(1/x**5/(b*x**2+a)**(4/3),x)
```

output
```
-gamma(10/3)*hyper((4/3, 10/3), (13/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b*
*(4/3)*x**(20/3)*gamma(13/3))
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = \frac{7\sqrt{3}b^2 \arctan\left(\frac{\sqrt{3}\left(2(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3a^{\frac{1}{3}}}\right)}{9a^{\frac{10}{3}}} + \frac{28(bx^2+a)^2b^2 - 49(bx^2+a)ab^2 + 18a^2b^2}{12\left((bx^2+a)^{\frac{7}{3}}a^3 - 2(bx^2+a)^{\frac{4}{3}}a^4 + (bx^2+a)^{\frac{1}{3}}a^5\right)} - \frac{7b^2 \log\left((bx^2+a)^{\frac{2}{3}} + (bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}} + a^{\frac{2}{3}}\right)}{18a^{\frac{10}{3}}} + \frac{7b^2 \log\left((bx^2+a)^{\frac{1}{3}} - a^{\frac{1}{3}}\right)}{9a^{\frac{10}{3}}}$$

input
```
integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(10/3) + 1/12*(28*(b*x^2 + a)^2*b^2 - 49*(b*x^2 + a)*a*b^2 + 18*a^2*b^
2)/((b*x^2 + a)^(7/3)*a^3 - 2*(b*x^2 + a)^(4/3)*a^4 + (b*x^2 + a)^(1/3)*a^
5) - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) + a^(2/3))
/a^(10/3) + 7/9*b^2*log((b*x^2 + a)^(1/3) - a^(1/3))/a^(10/3)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = \frac{7\sqrt{3}b^2\arctan\left(\frac{\sqrt{3}\left(2\,(bx^2+a)^{\frac{1}{3}}+a^{\frac{1}{3}}\right)}{3\,a^{\frac{1}{3}}}\right)}{9\,a^{\frac{10}{3}}} - \frac{7\,b^2\log\left((bx^2+a)^{\frac{2}{3}}+(bx^2+a)^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}\right)}{18\,a^{\frac{10}{3}}} + \frac{7\,b^2\log\left(\left|(bx^2+a)^{\frac{1}{3}}-a^{\frac{1}{3}}\right|\right)}{9\,a^{\frac{10}{3}}} + \frac{3\,b^2}{2\,(bx^2+a)^{\frac{1}{3}}a^3} + \frac{10\,(bx^2+a)^{\frac{5}{3}}b^2-13\,(bx^2+a)^{\frac{2}{3}}ab^2}{12\,a^3b^2x^4}$$

input
```
integrate(1/x^5/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
7/9*sqrt(3)*b^2*arctan(1/3*sqrt(3)*(2*(b*x^2 + a)^(1/3) + a^(1/3))/a^(1/3)
)/a^(10/3) - 7/18*b^2*log((b*x^2 + a)^(2/3) + (b*x^2 + a)^(1/3)*a^(1/3) +
a^(2/3))/a^(10/3) + 7/9*b^2*log(abs((b*x^2 + a)^(1/3) - a^(1/3)))/a^(10/3)
 + 3/2*b^2/((b*x^2 + a)^(1/3)*a^3) + 1/12*(10*(b*x^2 + a)^(5/3)*b^2 - 13*(
b*x^2 + a)^(2/3)*a*b^2)/(a^3*b^2*x^4)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.41

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = \frac{\frac{3\,b^2}{a}-\frac{49\,b^2\,(b\,x^2+a)}{6\,a^2}+\frac{14\,b^2\,(b\,x^2+a)^2}{3\,a^3}}{2\,(b\,x^2+a)^{7/3}-4\,a\,(b\,x^2+a)^{4/3}+2\,a^2\,(b\,x^2+a)^{1/3}} + \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\right)}{9\,a^{10/3}} + \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}} - \frac{7\,b^2\,\ln\left(147\,a^3\,b^4\,(b\,x^2+a)^{1/3}-147\,a^{10/3}\,b^4\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)^2\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}\right)}{9\,a^{10/3}}$$

input
```
int(1/(x^5*(a + b*x^2)^(4/3)),x)
```

output
```
((3*b^2)/a - (49*b^2*(a + b*x^2))/(6*a^2) + (14*b^2*(a + b*x^2)^2)/(3*a^3)
)/(2*(a + b*x^2)^(7/3) - 4*a*(a + b*x^2)^(4/3) + 2*a^2*(a + b*x^2)^(1/3))
+ (7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4))/(9*a^(10/3
)) + (7*b^2*log(147*a^3*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)
*1i)/2 - 1/2)^2)*((3^(1/2)*1i)/2 - 1/2))/(9*a^(10/3)) - (7*b^2*log(147*a^3
*b^4*(a + b*x^2)^(1/3) - 147*a^(10/3)*b^4*((3^(1/2)*1i)/2 + 1/2)^2)*((3^(1
/2)*1i)/2 + 1/2))/(9*a^(10/3))
```

Reduce [F]

$$\int \frac{1}{x^5 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a\,x^5+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^7}dx$$

input
```
int(1/x^5/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a*x**5 + (a + b*x**2)**(1/3)*b*x**7),x)
```

3.781 $\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 577

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = -\frac{3x^3}{2b\sqrt[3]{a+bx^2}} + \frac{27x(a+bx^2)^{2/3}}{14b^2} + \frac{81ax}{14b^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$- \frac{81\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{28b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$+ \frac{27\ 3^{3/4}a^{4/3}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right.}{7\sqrt{2}b^3x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3/2*x^3/b/(b*x^2+a)^(1/3)+27/14*x*(b*x^2+a)^(2/3)/b^2+81/14*a*x/b^2/((1-3
^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-81/28*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a
^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+
a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I
*3^(1/2))/b^3/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(
b*x^2+a)^(1/3))^2)^(1/2)+27/14*3^(3/4)*a^(4/3)*(a^(1/3)-(b*x^2+a)^(1/3))*(
(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1
-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)/b^3/x/(-a^(1/3)*
(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \frac{3x\left(9a+2bx^2-9a\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{14b^2\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(9*a + 2*b*x^2 - 9*a*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2
, 3/2, -((b*x^2)/a)]))/(14*b^2*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {252, 262, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{4/3}}dx$$

$$\Big\downarrow \text{ 252}$$

$$\frac{9\int \frac{x^2}{\sqrt[3]{bx^2+a}}dx}{2b}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{9\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{3a\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{7b}\right)}{2b}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{9\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{14b^2x}\right)}{2b}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 833}$$

$$\frac{9\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{14b^2x}\right)}{2b}-\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{ 760}$$

$$\frac{9\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+\cdots}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\cdots\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\cdots}{\left(\left(\cdots\right.\right.}}}\right.}{14b^2x}\right.}{2b}$$

$$\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

$\downarrow$ 2418

$$9\left(\frac{3x\left(a+bx^2\right)^{2/3}}{7b}-\frac{9a\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{\cdots}\right.$$

$$\frac{3x^3}{2b\sqrt[3]{a+bx^2}}$$

input
```
Int[x^4/(a + b*x^2)^(4/3),x]
```

output
```
(-3*x^3)/(2*b*(a + b*x^2)^(1/3)) + (9*((3*x*(a + b*x^2)^(2/3))/(7*b) - (9*
a*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[
(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a
^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqr
t[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - S
qrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt
[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*
x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)
)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2
]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) -
(a + b*x^2)^(1/3))^2)])))/(14*b^2*x)))/(2*b)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{4}{3}}}dx$$

input
```
int(x^4/(b*x^2+a)^(4/3),x)
```

output
```
int(x^4/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{4}{3}}}$$

input
```
integrate(x**4/(b*x**2+a)**(4/3),x)
```

output
```
x**5*hyper((4/3, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(4/3))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{4/3}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(4/3),x)
```

output
```
int(x^4/(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x^4/(b*x^2+a)^(4/3),x)
```

output
```
int(x**4/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.782 $\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 553

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = -\frac{3x}{2b\sqrt[3]{a+bx^2}} - \frac{9x}{2b\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{9\sqrt[4]{3}\sqrt{2+\sqrt{3}}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)\right|\cdots}{4b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{3\ 3^{3/4}\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),\cdots}{\sqrt{2}b^2x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3/2*x/b/(b*x^2+a)^(1/3)-9/2*x/b/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))+9/4
*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(
2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+
a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/b^2/x/(-a^(1/3)*(a^(1/3)-(b*
x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)-3/2*3^(3/4)*a
^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+
a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I
*3^(1/2))*2^(1/2)/b^2/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a
^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = \frac{3x\left(-1+\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{2b\sqrt[3]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(4/3),x]
```

output
```
(3*x*(-1 + (1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2
)/a)]))/(2*b*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {252, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx$$

$\downarrow$ 252

$$\frac{3\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2b}-\frac{3x}{2b\sqrt[3]{a+bx^2}}$$

$\downarrow$ 233

$$\frac{9\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4b^2x}-\frac{3x}{2b\sqrt[3]{a+bx^2}}$$

$\downarrow$ 833

$$\frac{9\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4b^2x}-\frac{3x}{2b\sqrt[3]{a+bx^2}}$$

$\downarrow$ 760

$$\frac{9\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^{2}\ldots}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\ldots}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\ldots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\ldots\right.}}}\right.}{4b^2x}-\frac{3x}{2b\sqrt[3]{a+bx^2}}$$

$\downarrow$ 2418

$$9\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

$$\frac{3x}{2b\sqrt[3]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(4/3),x]
```

output
```
(-3*x)/(2*b*(a + b*x^2)^(1/3)) + (9*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(
a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (
a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE
[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(
1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3
))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sq
rt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(
1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)
^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(4*b^2*x)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{4}{3}}}dx$$

input
```
int(x^2/(b*x^2+a)^(4/3),x)
```

output
```
int(x^2/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx=\int \frac{x^2}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx=\frac{x^3{}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}}$$

input
```
integrate(x**2/(b*x**2+a)**(4/3),x)
```

output
```
x**3*hyper((4/3, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(4/3), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{4/3}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(4/3),x)
```

output
```
int(x^2/(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{4/3}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(x^2/(b*x^2+a)^(4/3),x)
```

output
```
int(x**2/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.783 $\int \frac{1}{(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 552

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{3x}{2a\sqrt[3]{a+bx^2}} + \frac{3x}{2a\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7 \cdots\right.}{4a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+\cdots\right.}{\sqrt{2}a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2*x/a/(b*x^2+a)^(1/3)+3/2*x/a/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-3/4*
3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1
/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))
^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(
1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2
+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*(a^(
1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(
(1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/
3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2
^(1/2)/a^(2/3)/b/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3
)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.11

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{3x - x\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{2a\sqrt[3]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-4/3),x]
```

output
```
(3*x - x*(1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[1/3, 1/2, 3/2, -((b*x^2)/
a)])/(2*a*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 597, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {215, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{4/3}}dx$$

$\big\downarrow$ 215

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{2a}$$

$\big\downarrow$ 233

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{4abx}$$

$\big\downarrow$ 833

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int \frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{4abx}$$

$\big\downarrow$ 760

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-\frac{3\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right.}}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\right.}}}\right.}{4abx}$$

$\big\downarrow$ 2418

$$\frac{3x}{2a\sqrt[3]{a+bx^2}}-$$

$$3\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.$$

input
```
Int[(a + b*x^2)^(-4/3),x]
```

output
```
(3*x)/(2*a*(a + b*x^2)^(1/3)) - (3*Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a
^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a
 + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[
ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3)
- (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1
/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])
 - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)
)*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqr
t[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^
(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(4*a*b*x)
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(b*x^2+a)^(4/3),x)
```

output
```
int(1/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{\left(a+bx^2\right)^{4/3}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{4}{3}}}$$

input
```
integrate(1/(b*x**2+a)**(4/3),x)
```

output
```
x*hyper((1/2, 4/3), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(4/3)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-4/3), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{4}{3};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{4/3}}$$

input
```
int(1/(a + b*x^2)^(4/3),x)
```

output
```
(x*((b*x^2)/a + 1)^(4/3)*hypergeom([1/2, 4/3], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(4/3)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a + (a + b*x**2)**(1/3)*b*x**2),x)
```

3.784 $\int \frac{1}{x^2(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 571

$$\int \frac{1}{x^2(a+bx^2)^{4/3}}\,dx = \frac{3}{2ax\sqrt[3]{a+bx^2}} - \frac{5(a+bx^2)^{2/3}}{2a^2x} - \frac{5bx}{2a^2\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}$$
$$+ \frac{5\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7 \cdots}{4a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$
$$- \frac{5\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+4\cdots}{\sqrt{2}\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2/a/x/(b*x^2+a)^(1/3)-5/2*(b*x^2+a)^(2/3)/a^2/x-5/2*b*x/a^2/((1-3^(1/2))
*a^(1/3)-(b*x^2+a)^(1/3))+5/4*3^(1/4)*(1/2*6^(1/2)+1/2*2^(1/2))*(a^(1/3)-(
b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(
1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE(((1+3^(1/2))*a^(1/3)-(b*
x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))/a^(5/3)
/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3
))^2)^(1/2)-5/6*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3
)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*Elliptic
F(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/
3)),2*I-I*3^(1/2))*2^(1/2)*3^(3/4)/a^(5/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^
(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = -\frac{\sqrt[3]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{4}{3},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax\sqrt[3]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(4/3)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-1/2, 4/3, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(1/3)))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {253, 264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}} \, dx$$

$$\Big\downarrow \; 253$$

$$\frac{5 \int \frac{1}{x^2 \sqrt[3]{bx^2+a}} dx}{2a} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \; 264$$

$$\frac{5 \left(\frac{b \int \frac{1}{\sqrt[3]{bx^2+a}} dx}{3a} - \frac{(a+bx^2)^{2/3}}{ax} \right)}{2a} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \; 233$$

$$\frac{5 \left(\frac{\sqrt{bx^2} \int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}} d\sqrt[3]{bx^2+a}}{2ax} - \frac{(a+bx^2)^{2/3}}{ax} \right)}{2a} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \; 833$$

$$\frac{5 \left(\frac{\sqrt{bx^2} \left(\left(1+\sqrt{3}\right) \sqrt[3]{a} \int \frac{1}{\sqrt{bx^2}} d\sqrt[3]{bx^2+a} - \int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a} - \sqrt[3]{bx^2+a}}{\sqrt{bx^2}} d\sqrt[3]{bx^2+a} \right)}{2ax} - \frac{(a+bx^2)^{2/3}}{ax} \right)}{2a} + \frac{3}{2ax \sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \; 760$$

$$5\left(\frac{\sqrt{bx^2}\left(-\int \frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{El}}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+b}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+}\right.}}}\right)}{2ax}\right)$$

$$\overline{\qquad 2a \qquad}$$

$$\frac{3}{2ax\sqrt[3]{a+bx^2}}$$

$\Big\downarrow$ 2418

$$5\left(\frac{\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}\right),-\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.$$

$$\frac{3}{2ax\sqrt[3]{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(4/3)),x]
```

output
```
3/(2*a*x*(a + b*x^2)^(1/3)) + (5*(-((a + b*x^2)^(2/3)/(a*x)) + (Sqrt[b*x^2
]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)
*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a
^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a
+ b*x^2)^(1/3))^2]*EllipticE[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(
1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt
[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1
/3) - (a + b*x^2)^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)
*(a^(1/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) +
 (a + b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*Ellipti
cF[ArcSin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/
3) - (a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^
(1/3)*(a^(1/3) - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^
(1/3))^2)])))/(2*a*x)))/(2*a)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(4/3),x)
```

output
```
int(1/x^2/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{4}{3}} x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(4/3),x)
```

output
```
-hyper((-1/2, 4/3), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(4/3)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}} x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(4/3)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = -\frac{3\left(\frac{a}{b\,x^2}+1\right)^{4/3}{}_2\mathrm{F}_1\left(\frac{4}{3},\frac{11}{6};\ \frac{17}{6};\ -\frac{a}{b\,x^2}\right)}{11\,x\,(b\,x^2+a)^{4/3}}$$

input
```
int(1/(x^2*(a + b*x^2)^(4/3)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(4/3)*hypergeom([4/3, 11/6], 17/6, -a/(b*x^2)))/(11*x*
(a + b*x^2)^(4/3))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a\,x^2+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^4}dx$$

input
```
int(1/x^2/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a*x**2 + (a + b*x**2)**(1/3)*b*x**4),x)
```

3.785 $\int \frac{1}{x^4(a+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 599

$$\int \frac{1}{x^4(a+bx^2)^{4/3}}\,dx = \frac{3}{2ax^3\sqrt[3]{a+bx^2}} - \frac{11(a+bx^2)^{2/3}}{6a^2x^3} + \frac{55b(a+bx^2)^{2/3}}{18a^3x} + \frac{55b^2x}{18a^3\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)} - \frac{55\sqrt{2+\sqrt{3}}b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right)|-7-}{12\,3^{3/4}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}} + \frac{55b\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}}\right),-7+}{9\sqrt{2}\sqrt[4]{3}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/2/a/x^3/(b*x^2+a)^(1/3)-11/6*(b*x^2+a)^(2/3)/a^2/x^3+55/18*b*(b*x^2+a)^(
2/3)/a^3/x+55/18*b^2*x/a^3/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))-55/36*(1/
2*6^(1/2)+1/2*2^(1/2))*b*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^
2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)
*EllipticE(((1+3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3)),2*I-I*3^(1/2))*3^(1/4)/a^(8/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)
^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)+55/54*b*(a^(1/3)-(b
*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/((1-3^(1
/2))*a^(1/3)-(b*x^2+a)^(1/3))^2)^(1/2)*EllipticF(((1+3^(1/2))*a^(1/3)-(b*x
^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)-(b*x^2+a)^(1/3)),2*I-I*3^(1/2))*2^(1/2)*
3^(3/4)/a^(8/3)/x/(-a^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/((1-3^(1/2))*a^(1/3)
-(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}} \, dx = -\frac{\sqrt[3]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{4}{3},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[3]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(4/3)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/3)*Hypergeometric2F1[-3/2, 4/3, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {253, 264, 264, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{4/3}}\,dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11\int \frac{1}{x^4\sqrt[3]{bx^2+a}}dx}{2a}+\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(-\frac{5b\int \frac{1}{x^2\sqrt[3]{bx^2+a}}dx}{9a}-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}\right)}{2a}+\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(-\frac{5b\left(\frac{b\int \frac{1}{\sqrt[3]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}\right)}{2a}+\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{233}$$

$$\frac{11\left(-\frac{5b\left(\frac{\sqrt{bx^2}\int \frac{\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}\right)}{2a}+\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{11\left(-\frac{5b\left(\frac{\sqrt{bx^2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}\int\frac{1}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}\right)}{2ax}-\frac{\left(a+bx^2\right)^{2/3}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{2/3}}{3ax^3}\right)}{2a}+\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \; 760$$

$$11\left|-\frac{5b\left(\frac{\sqrt{bx^2}\left(-\int\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2+a}}{\sqrt{bx^2}}d\sqrt[3]{bx^2+a}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\cdots\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\cdots}}\cdots}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{\cdots}\right.}}\cdots}\right)}{2ax}\cdots\right)}{9a}\cdots\right.$$
$$\overline{\qquad\qquad 2a \qquad\qquad}$$

$$\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

$\downarrow$ 2418

$$11\left(-\frac{5b\left(\frac{\sqrt{bx^2}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{\left(1+\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2 -}}{\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{bx^2 -}}\right.\right.}{\sqrt[4]{3}\sqrt{bx^2}\sqrt{-\frac{\sqrt[3]{a}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\left(1-\sqrt{3}\right)\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)^2}}}\right.}{}\right.}{}\right.$$

$$\frac{3}{2ax^3\sqrt[3]{a+bx^2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(4/3)),x]
```

output
```
3/(2*a*x^3*(a + b*x^2)^(1/3)) + (11*(-1/3*(a + b*x^2)^(2/3)/(a*x^3) - (5*b
*(-((a + b*x^2)^(2/3)/(a*x)) + (Sqrt[b*x^2]*((-2*Sqrt[b*x^2])/((1 - Sqrt[3
])*a^(1/3) - (a + b*x^2)^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*a^(1/3)*(a^(1
/3) - (a + b*x^2)^(1/3))*Sqrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a +
b*x^2)^(2/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticE[Arc
Sin[((1 + Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (
a + b*x^2)^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3)
 - (a + b*x^2)^(1/3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)]) -
(2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/3))*S
qrt[(a^(2/3) + a^(1/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3))/((1 - Sqrt[3
])*a^(1/3) - (a + b*x^2)^(1/3))^2]*EllipticF[ArcSin[((1 + Sqrt[3])*a^(1/3)
 - (a + b*x^2)^(1/3))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))], -7 + 4
*Sqrt[3]])/(3^(1/4)*Sqrt[b*x^2]*Sqrt[-((a^(1/3)*(a^(1/3) - (a + b*x^2)^(1/
3)))/((1 - Sqrt[3])*a^(1/3) - (a + b*x^2)^(1/3))^2)])))/(2*a*x)))/(9*a)))/
(2*a)
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{4}{3}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(4/3),x)
```

output
```
int(1/x^4/(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(2/3)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{4}{3} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(4/3),x)
```

output
```
-hyper((-3/2, 4/3), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(4/3)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{4}{3}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(4/3)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{x^4 (b\,x^2+a)^{4/3}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(4/3)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(4/3)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{4/3}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{3}}\,a\,x^4+(b\,x^2+a)^{\frac{1}{3}}\,b\,x^6}dx$$

input
```
int(1/x^4/(b*x^2+a)^(4/3),x)
```

output
```
int(1/((a + b*x**2)**(1/3)*a*x**4 + (a + b*x**2)**(1/3)*b*x**6),x)
```

3.786 $\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 195

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = -\frac{5a^2c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{108b^2}+\frac{ac(cx)^{10/3}\sqrt[3]{a+bx^2}}{36b}+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}-\frac{5a^3c^{13/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{54\sqrt{3}b^{8/3}}-\frac{5a^3c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{108b^{8/3}}$$

output
```
-5/108*a^2*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/36*a*c*(c*x)^(10/3)*(b*x^
2+a)^(1/3)/b+1/6*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c-5/162*a^3*c^(13/3)*arctan(
1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(
8/3)-5/108*a^3*c^(13/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^
(8/3)
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.19

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{c^4\sqrt[3]{cx}\left(-15a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+54b^{8/3}x^{16/3}\sqrt[3]{a+bx^2}-\right.}{}$$

input
```
Integrate[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

output
```
(c^4*(c*x)^(1/3)*(-15*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*a*b^(5/3)*
x^(10/3)*(a + b*x^2)^(1/3) + 54*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) - 10*Sq
rt[3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2
)^(1/3))] - 10*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + 5*a^3*Log
[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)])
)/(324*b^(8/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {248, 262, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{9}a\int \frac{(cx)^{13/3}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\int \frac{(cx)^{7/3}}{(bx^2+a)^{2/3}}dx}{6b}\right)+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int\frac{\sqrt[3]{cx}}{\left(bx^2+a\right)^{2/3}}dx}{3b}\right)}{6b}\right)+$$

$$\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int\frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{6b}\right)+$$

$$\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int\frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)}{6b}\right)+$$

$$\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

$$\Big\downarrow \text{853}$$

$$\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\right.}{2b^{2/3}}\right.}{b}\right.}{6b}\right.$$

$$\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}$$

input
```
Int[(c*x)^(13/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*c) + (a*((c*(c*x)^(10/3)*(a + b*x^2)^(
1/3))/(4*b) - (5*a*c^2*((c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) - (a*c*(-(
(c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3))
)/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3
) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/b))/(6*b)))/9
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{13}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x)**(13/3)*(b*x**2+a)**(1/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^{13/3} \sqrt[3]{a+bx^2}\, dx = \int (bx^2+a)^{\frac{1}{3}} (cx)^{\frac{13}{3}}\, dx$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

Giac [F]

$$\int (cx)^{13/3} \sqrt[3]{a+bx^2}\, dx = \int (bx^2+a)^{\frac{1}{3}} (cx)^{\frac{13}{3}}\, dx$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(13/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{13/3} \sqrt[3]{a+bx^2}\, dx = \int (c\,x)^{13/3}\, \left(b\,x^2+a\right)^{1/3}\, dx$$

input
```
int((c*x)^(13/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(13/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int (cx)^{13/3}\sqrt[3]{a+bx^2}\,dx = \frac{c^{\frac{13}{3}}\left(-15x^{\frac{4}{3}}(b\,x^2+a)^{\frac{1}{3}}a^2+9x^{\frac{10}{3}}(b\,x^2+a)^{\frac{1}{3}}ab+54x^{\frac{16}{3}}(b\,x^2+a)^{\frac{1}{3}}b^2+20\left(\int \frac{}{(b\,x}\right.\right.}{324b^2}$$

input
```
int((c*x)^(13/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(1/3)*c**4*( - 15*x**(1/3)*(a + b*x**2)**(1/3)*a**2*x + 9*x**(1/3)*(a
+ b*x**2)**(1/3)*a*b*x**3 + 54*x**(1/3)*(a + b*x**2)**(1/3)*b**2*x**5 + 20
*int((x**(1/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a**3))/(324*b**2)
```

3.787 $\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 164

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{ac(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b}+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c} + \frac{a^2c^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}b^{5/3}}+\frac{a^2c^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{5/3}}$$

output
```
1/12*a*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/4*(c*x)^(10/3)*(b*x^2+a)^(1/3)/c+
1/18*a^2*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/
3))*3^(1/2))*3^(1/2)/b^(5/3)+1/12*a^2*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/
3)*(b*x^2+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.24

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{(cx)^{7/3}\left(3ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)\cdots\right.}{\cdots}$$

input
```
Integrate[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(7/3)*(3*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(10/3)*(
a + b*x^2)^(1/3) + 2*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)
*x^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*a^2*Log[-(b^(1/3)*x^(2/3)) + (a + b*x
^2)^(1/3)] - a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) +
 (a + b*x^2)^(2/3)]))/(36*b^(5/3)*x^(7/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6}a\int \frac{(cx)^{7/3}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx}{3b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int\frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

$\Big\downarrow$ 807

$$\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int\frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

$\Big\downarrow$ 853

$$\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}$$

input
```
Int[(c*x)^(7/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(10/3)*(a + b*x^2)^(1/3))/(4*c) + (a*((c*(c*x)^(4/3)*(a + b*x^2)^(1
/3))/(2*b) - (a*c*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)
*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3
)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/b))/6
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx=\text{Timed out}$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 17.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx=\frac{\sqrt[3]{a}c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((c*x)**(7/3)*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-1/3, 5/3), (8/3,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(8/3))
```

Maxima [F]

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{7}{3}}\,dx$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

Giac [F]

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{7}{3}}\,dx$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx=\int (c\,x)^{7/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int((c*x)^(7/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(7/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int (cx)^{7/3}\sqrt[3]{a+bx^2}\,dx = \frac{c^{\frac{7}{3}}\left(3x^{\frac{4}{3}}(b\,x^2+a)^{\frac{1}{3}}\,a+9x^{\frac{10}{3}}(b\,x^2+a)^{\frac{1}{3}}\,b-4\left(\int\frac{x^{\frac{1}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a^2\right)}{36b}$$

input
```
int((c*x)^(7/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(1/3)*c**2*(3*x**(1/3)*(a + b*x**2)**(1/3)*a*x + 9*x**(1/3)*(a + b*x**
2)**(1/3)*b*x**3 - 4*int((x**(1/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a*
*2))/(36*b)
```

3.788 $\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx=\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}-\frac{a\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2\sqrt{3}b^{2/3}}-\frac{a\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}$$

output

```
1/2*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c-1/6*a*c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c
*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(2/3)-1/4*a*c^(1/3)*
ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.30

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$$
$$= \frac{\sqrt[3]{cx}\left(6b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}-2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-2a\log\left(-\sqrt[3]{b}x^{2/3}+\sqrt[3]{a+bx^2}\right)+a\log}{12b^{2/3}\sqrt[3]{x}}$$

input
```
Integrate[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(1/3)*(6*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) - 2*Sqrt[3]*a*ArcTan[(Sq
rt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 2*a*Log[
-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + a*Log[b^(2/3)*x^(4/3) + b^(1/3)*
x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(12*b^(2/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {248, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx$$
$$\Big\downarrow \; 248$$
$$\frac{1}{3}a\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$
$$\Big\downarrow \; 266$$
$$\frac{a\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$
$$\Big\downarrow \; 807$$

$$\frac{a\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

$$\Big\downarrow \textbf{853}$$

$$\frac{a\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}$$

input
```
Int[(c*x)^(1/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*c) + (a*(-((c^(4/3)*ArcTan[(1 + (2*b^(1
/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3)
)) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2
*b^(2/3))))/(2*c)
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.35

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \frac{\sqrt[3]{a}\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((c*x)**(1/3)*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-1/3, 2/3), (5/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(5/3))
```

Maxima [F]

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int((c*x)^(1/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(1/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int \sqrt[3]{cx}\sqrt[3]{a+bx^2}\,dx = \frac{c^{\frac{1}{3}}\left(3x^{\frac{4}{3}}(b\,x^2+a)^{\frac{1}{3}}+2\left(\int \frac{x^{\frac{1}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)a\right)}{6}$$

input
```
int((c*x)^(1/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(1/3)*(3*x**(1/3)*(a + b*x**2)**(1/3)*x + 2*int((x**(1/3)*(a + b*x**2)
**(1/3))/(a + b*x**2),x)*a))/6
```

3.789 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}} - \frac{\sqrt{3}\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{5/3}} - \frac{3\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{5/3}}$$

output
```
-3/2*(b*x^2+a)^(1/3)/c/(c*x)^(2/3)-1/2*3^(1/2)*b^(1/3)*arctan(1/3*(1+2*b^(
1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))/c^(5/3)-3/4*b^(1/3)*ln(
b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/c^(5/3)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.39

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx =$$

$$-\frac{x\left(6\sqrt[3]{a+bx^2}+2\sqrt{3}\sqrt[3]{b}x^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+2\sqrt[3]{b}x^{2/3}\log\left(-\sqrt[3]{b}x^{2/3}+\sqrt[3]{a+bx^2}\right)-\sqrt[3]{b}x\cdots\right.}{4(cx)^{5/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

output
```
-1/4*(x*(6*(a + b*x^2)^(1/3) + 2*Sqrt[3]*b^(1/3)*x^(2/3)*ArcTan[(Sqrt[3]*b
^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*b^(1/3)*x^(2/
3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - b^(1/3)*x^(2/3)*Log[b^(2/
3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(c*x
)^(5/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {247, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx}{c^2}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{3b\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{3b\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

$$\Big\downarrow \textbf{853}$$

$$\frac{3b\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}$$

input

```
Int[(a + b*x^2)^(1/3)/(c*x)^(5/3),x]
```

output

```
(-3*(a + b*x^2)^(1/3))/(2*c*(c*x)^(2/3)) + (3*b*(-((c^(4/3)*ArcTan[(1 + (2
*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^
(2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)
])/(2*b^(2/3))))/(2*c^3)
```

Defintions of rubi rules used

rule 247

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \frac{\sqrt[3]{a}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(5/3),x)
```

output
```
a**(1/3)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{5/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(5/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{5/3}}\,dx = \frac{\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{5}{3}}}dx}{c^{\frac{5}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(5/3),x)
```

output
```
int((a + b*x**2)**(1/3)/(x**(2/3)*x),x)/(c**(2/3)*c)
```

3.790 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx=-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{8/3}}$$

output
```
-3/8*(b*x^2+a)^(4/3)/a/c/(c*x)^(8/3)
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx=-\frac{3x(a+bx^2)^{4/3}}{8a(cx)^{11/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

output
```
(-3*x*(a + b*x^2)^(4/3))/(8*a*(c*x)^(11/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}} \, dx$$
$$\Big\downarrow \ 242$$
$$-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{8/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(11/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(8/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{3x(b\,x^{2}+a)^{\frac{4}{3}}}{8a(cx)^{\frac{11}{3}}}$	21
orering	$-\frac{3x(b\,x^{2}+a)^{\frac{4}{3}}}{8a(cx)^{\frac{11}{3}}}$	21
risch	$-\frac{3(b\,x^{2}+a)^{\frac{4}{3}}}{8c^{3}(cx)^{\frac{2}{3}}x^{2}a}$	26

input
```
int((b*x^2+a)^(1/3)/(c*x)^(11/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x*(b*x^2+a)^(4/3)/a/(c*x)^(11/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx=-\frac{3\,(bx^2+a)^{\frac{4}{3}}(cx)^{\frac{1}{3}}}{8\,ac^4x^3}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="fricas")
```

output
```
-3/8*(b*x^2 + a)^(4/3)*(c*x)^(1/3)/(a*c^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.

Time = 33.36 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx=\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{4}{3}\right)}{2c^{\frac{11}{3}}x^2\Gamma\left(-\frac{1}{3}\right)}+\frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{4}{3}\right)}{2ac^{\frac{11}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(11/3),x)
```

output
```
b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*c**(11/3)*x**2*gamma(-1/3)
) + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a*c**(11/3)*gamma(-1/3
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = -\frac{3\left(bc^{\frac{1}{3}}x^3+ac^{\frac{1}{3}}x\right)(bx^2+a)^{\frac{1}{3}}}{8\,ac^4x^{\frac{11}{3}}}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="maxima")
```

output
```
-3/8*(b*c^(1/3)*x^3 + a*c^(1/3)*x)*(b*x^2 + a)^(1/3)/(a*c^4*x^(11/3))
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(11/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{11/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(11/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(11/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{11/3}}\,dx = -\frac{3(b\,x^2+a)^{\frac{4}{3}}}{8x^{\frac{8}{3}}c^{\frac{11}{3}}a}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(11/3),x)
```

output
```
( - 3*(a + b*x**2)**(1/3)*(a + b*x**2))/(8*x**(2/3)*c**(2/3)*a*c**3*x**2)
```

3.791 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx=-\frac{3(a+bx^2)^{4/3}}{14ac(cx)^{14/3}}+\frac{9b(a+bx^2)^{4/3}}{56a^2c^3(cx)^{8/3}}$$

output
```
-3/14*(b*x^2+a)^(4/3)/a/c/(c*x)^(14/3)+9/56*b*(b*x^2+a)^(4/3)/a^2/c^3/(c*x
)^(8/3)
```

Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx=-\frac{3x\sqrt[3]{a+bx^2}(4a^2+abx^2-3b^2x^4)}{56a^2(cx)^{17/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*(4*a^2 + a*b*x^2 - 3*b^2*x^4))/(56*a^2*(c*x)^(17/3
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{3\int \frac{(bx^2+a)^{4/3}}{(cx)^{17/3}}dx}{4a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

$$\Big\downarrow \text{242}$$

$$\frac{9\left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(17/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^
2*c*(c*x)^(14/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-3b\,x^{2}+4a\right)}{56a^{2}\left(cx\right)^{\frac{17}{3}}}$	31
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-3b\,x^{2}+4a\right)}{56a^{2}\left(cx\right)^{\frac{17}{3}}}$	31
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b^{2}x^{4}+ab\,x^{2}+4a^{2}\right)}{56c^{5}\left(cx\right)^{\frac{2}{3}}x^{4}a^{2}}$	46

input
```
int((b*x^2+a)^(1/3)/(c*x)^(17/3),x,method=_RETURNVERBOSE)
```

output
```
-3/56*x*(b*x^2+a)^(4/3)*(-3*b*x^2+4*a)/a^2/(c*x)^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx = \frac{3\,(3\,b^2x^4-abx^2-4\,a^2)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{56\,a^2c^6x^5}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="fricas")
```

output
```
3/56*(3*b^2*x^4 - a*b*x^2 - 4*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^6*
x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(17/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{17}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(17/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(17/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx = \int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{17/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(17/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(17/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{17/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,(3b^2x^4-ab\,x^2-4a^2)}{56x^{\frac{14}{3}}c^{\frac{17}{3}}a^2}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(17/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - 4*a**2 - a*b*x**2 + 3*b**2*x**4))/(56*x**(2/3)*
c**(2/3)*a**2*c**5*x**4)
```

3.792 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{20ac(cx)^{20/3}} + \frac{9b(a+bx^2)^{4/3}}{70a^2c^3(cx)^{14/3}} - \frac{27b^2(a+bx^2)^{4/3}}{280a^3c^5(cx)^{8/3}}$$

output
```
-3/20*(b*x^2+a)^(4/3)/a/c/(c*x)^(20/3)+9/70*b*(b*x^2+a)^(4/3)/a^2/c^3/(c*x
)^(14/3)-27/280*b^2*(b*x^2+a)^(4/3)/a^3/c^5/(c*x)^(8/3)
```

Mathematica [A] (verified)

Time = 2.87 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = -\frac{3x(a+bx^2)^{4/3}(14a^2-12abx^2+9b^2x^4)}{280a^3(cx)^{23/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

output
```
(-3*x*(a + b*x^2)^(4/3)*(14*a^2 - 12*a*b*x^2 + 9*b^2*x^4))/(280*a^3*(c*x)^
(23/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{3\int \frac{\left(bx^2+a\right)^{4/3}}{(cx)^{23/3}}dx}{2a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{3\left(-\frac{3\int \frac{\left(bx^2+a\right)^{7/3}}{(cx)^{23/3}}dx}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}\right)}{2a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{3\left(\frac{9\left(a+bx^2\right)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{20/3}}\right)}{2a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{20/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(23/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(20/3)) - (3*((-3*(a + b*x^2)^(7/3))/(
14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140*a^2*c*(c*x)^(20/3))))/(
2*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(9b^{2}x^{4}-12ab\,x^{2}+14a^{2}\right)}{280a^{3}\left(cx\right)^{\frac{23}{3}}}$	42
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(9b^{2}x^{4}-12ab\,x^{2}+14a^{2}\right)}{280a^{3}\left(cx\right)^{\frac{23}{3}}}$	42
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{3}x^{6}-3a\,b^{2}x^{4}+2a^{2}b\,x^{2}+14a^{3}\right)}{280c^{7}\left(cx\right)^{\frac{2}{3}}x^{6}a^{3}}$	58

input
```
int((b*x^2+a)^(1/3)/(c*x)^(23/3),x,method=_RETURNVERBOSE)
```

output
```
-3/280*x*(b*x^2+a)^(4/3)*(9*b^2*x^4-12*a*b*x^2+14*a^2)/a^3/(c*x)^(23/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = -\frac{3\,(9\,b^3x^6-3\,ab^2x^4+2\,a^2bx^2+14\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^3c^8x^7}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="fricas")
```

output
```
-3/280*(9*b^3*x^6 - 3*a*b^2*x^4 + 2*a^2*b*x^2 + 14*a^3)*(b*x^2 + a)^(1/3)*
(c*x)^(1/3)/(a^3*c^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(23/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(23/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(23/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{23/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(23/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(23/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{23/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,(-9b^3x^6+3a\,b^2x^4-2a^2b\,x^2-14a^3)}{280x^{\frac{20}{3}}c^{\frac{23}{3}}a^3}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(23/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - 14*a**3 - 2*a**2*b*x**2 + 3*a*b**2*x**4 - 9*b**
3*x**6))/(280*x**(2/3)*c**(2/3)*a**3*c**7*x**6)
```

3.793 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = -\frac{3(a+bx^2)^{4/3}}{26ac(cx)^{26/3}} + \frac{27b(a+bx^2)^{4/3}}{260a^2c^3(cx)^{20/3}} - \frac{81b^2(a+bx^2)^{4/3}}{910a^3c^5(cx)^{14/3}} + \frac{243b^3(a+bx^2)^{4/3}}{3640a^4c^7(cx)^{8/3}}$$

output
```
-3/26*(b*x^2+a)^(4/3)/a/c/(c*x)^(26/3)+27/260*b*(b*x^2+a)^(4/3)/a^2/c^3/(c
*x)^(20/3)-81/910*b^2*(b*x^2+a)^(4/3)/a^3/c^5/(c*x)^(14/3)+243/3640*b^3*(b
*x^2+a)^(4/3)/a^4/c^7/(c*x)^(8/3)
```

Mathematica [A] (verified)

Time = 4.52 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = -\frac{3x(a+bx^2)^{4/3}(140a^3-126a^2bx^2+108ab^2x^4-81b^3x^6)}{3640a^4(cx)^{29/3}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

output
```
(-3*x*(a + b*x^2)^(4/3)*(140*a^3 - 126*a^2*b*x^2 + 108*a*b^2*x^4 - 81*b^3*
x^6))/(3640*a^4*(c*x)^(29/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{9\int \frac{\left(bx^2+a\right)^{4/3}}{(cx)^{29/3}}dx}{4a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{9\left(-\frac{6\int \frac{\left(bx^2+a\right)^{7/3}}{(cx)^{29/3}}dx}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}\right)}{4a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{9\left(-\frac{6\left(-\frac{3\int \frac{\left(bx^2+a\right)^{10/3}}{(cx)^{29/3}}dx}{10a}-\frac{3\left(a+bx^2\right)^{10/3}}{20ac(cx)^{26/3}}\right)}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}\right)}{4a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{9\left(-\frac{6\left(\frac{9\left(a+bx^2\right)^{13/3}}{260a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{10/3}}{20ac(cx)^{26/3}}\right)}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}\right)}{4a}-\frac{3\left(a+bx^2\right)^{4/3}}{8ac(cx)^{26/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(29/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(8*a*c*(c*x)^(26/3)) - (9*((-3*(a + b*x^2)^(7/3))/(
14*a*c*(c*x)^(26/3)) - (6*((-3*(a + b*x^2)^(10/3))/(20*a*c*(c*x)^(26/3)) +
 (9*(a + b*x^2)^(13/3))/(260*a^2*c*(c*x)^(26/3))))/(7*a)))/(4*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-81b^{3}x^{6}+108a\,b^{2}x^{4}-126a^{2}b\,x^{2}+140a^{3}\right)}{3640a^{4}\left(cx\right)^{\frac{29}{3}}}$	53
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{4}{3}}\left(-81b^{3}x^{6}+108a\,b^{2}x^{4}-126a^{2}b\,x^{2}+140a^{3}\right)}{3640a^{4}\left(cx\right)^{\frac{29}{3}}}$	53
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{4}x^{8}+27a\,b^{3}x^{6}-18a^{2}b^{2}x^{4}+14a^{3}b\,x^{2}+140a^{4}\right)}{3640c^{9}\left(cx\right)^{\frac{2}{3}}x^{8}a^{4}}$	69

input
```
int((b*x^2+a)^(1/3)/(c*x)^(29/3),x,method=_RETURNVERBOSE)
```

output
```
-3/3640*x*(b*x^2+a)^(4/3)*(-81*b^3*x^6+108*a*b^2*x^4-126*a^2*b*x^2+140*a^3
)/a^4/(c*x)^(29/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \frac{3\,(81\,b^4x^8 - 27\,ab^3x^6 + 18\,a^2b^2x^4 - 14\,a^3bx^2 - 140\,a^4)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{3640\,a^4c^{10}x^9}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="fricas")
```

output
```
3/3640*(81*b^4*x^8 - 27*a*b^3*x^6 + 18*a^2*b^2*x^4 - 14*a^3*b*x^2 - 140*a^
4)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^4*c^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(29/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \frac{3\,(81\,b^4x^9 - 27\,ab^3x^7 + 18\,a^2b^2x^5 - 14\,a^3bx^3 - 140\,a^4x)(bx^2+a)^{\frac{1}{3}}}{3640\,a^4c^{\frac{29}{3}}x^{\frac{29}{3}}}$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="maxima")
```

output
```
3/3640*(81*b^4*x^9 - 27*a*b^3*x^7 + 18*a^2*b^2*x^5 - 14*a^3*b*x^3 - 140*a^
4*x)*(b*x^2 + a)^(1/3)/(a^4*c^(29/3)*x^(29/3))
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(29/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(29/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{29/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(29/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(29/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{29/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,(81b^4x^8-27a\,b^3x^6+18a^2b^2x^4-14a^3b\,x^2-140a^4)}{3640x^{\frac{26}{3}}c^{\frac{29}{3}}a^4}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(29/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - 140*a**4 - 14*a**3*b*x**2 + 18*a**2*b**2*x**4 -
 27*a*b**3*x**6 + 81*b**4*x**8))/(3640*x**(2/3)*c**(2/3)*a**4*c**9*x**8)
```

3.794 $\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 451

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx = -\frac{14a^2c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b^2} + \frac{2ac(cx)^{7/3}\sqrt[3]{a+bx^2}}{45b} + \frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$$+\frac{7a^2c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}\right.\right.\cdots}{135\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-14/135*a^2*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+2/45*a*c*(c*x)^(7/3)*(b*x^
2+a)^(1/3)/b+1/5*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+7/405*a^2*c^(7/3)*(c*x)^(1
/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3
)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a
)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2
)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a
)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^
(1/2)+1/4*2^(1/2))*3^(3/4)/b^2/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x
)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x
)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.23

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\frac{c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(\sqrt[3]{1+\frac{bx^2}{a}}(-7a^2+2abx^2+9b^2x^4)+7a^2\,\text{Hypergeometric2F1}\left(\right.\right.}{45b^2\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

output
```
(c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((1 + (b*x^2)/a)^(1/3)*(-7*a^2 + 2*a*b*
x^2 + 9*b^2*x^4) + 7*a^2*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))
/(45*b^2*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {248, 262, 262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{2}{15}a\int \frac{(cx)^{10/3}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\int \frac{(cx)^{4/3}}{(bx^2+a)^{2/3}}dx}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{(cx)^{2/3}\left(bx^2+a\right)^{2/3}}dx}{3b}\right)}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{ 771}$$

$$\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}\right)}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

$\downarrow$ 766

$$\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{Ellipt}}{2\sqrt[4]{3}b\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}\right.}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\right.}}}\right.}{9b}\right.$$

$$\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}$$

input
```
Int[(c*x)^(10/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(13/3)*(a + b*x^2)^(1/3))/(5*c) + (2*a*((c*(c*x)^(7/3)*(a + b*x^2)^
(1/3))/(3*b) - (7*a*c^2*((c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (a*c^(1/3)*
(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3
)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x
)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))
/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(2*3^(1
/4)*b*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*
Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1
 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/(9*b)))/15
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int (cx)^{\frac{10}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}d x$$

input
```
int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c^3*x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 77.68 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.10

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\frac{\sqrt[3]{a}c^{\frac{10}{3}}x^{\frac{13}{3}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{13}{6}\\ \frac{19}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((c*x)**(10/3)*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*c**(10/3)*x**(13/3)*gamma(13/6)*hyper((-1/3, 13/6), (19/6,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(19/6))
```

Maxima [F]

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

Giac [F]

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{10}{3}}\,dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\int (c\,x)^{10/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int((c*x)^(10/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(10/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int (cx)^{10/3}\sqrt[3]{a+bx^2}\,dx=\frac{c^{\frac{10}{3}}\left(-42x^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}a^2+18x^{\frac{7}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}ab+81x^{\frac{13}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}b^2+14\left(\int \frac{(b x}{x^{\frac{2}{3}}}\right.\right.}{405b^2}$$

input
```
int((c*x)^(10/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(1/3)*c**3*( - 42*x**(1/3)*(a + b*x**2)**(1/3)*a**2 + 18*x**(1/3)*(a +
 b*x**2)**(1/3)*a*b*x**2 + 81*x**(1/3)*(a + b*x**2)**(1/3)*b**2*x**4 + 14*
int((a + b*x**2)**(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*a**3))/(405*b**2
)
```

3.795 $\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 418

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\frac{2ac\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b}+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

$$-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(\cdots}{\sqrt[3]{a+b\cdots}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(\cdots}{\sqrt[3]{a+b\cdots}}}\cdots\right.\right.}{9\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
2/9*a*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+1/3*(c*x)^(7/3)*(b*x^2+a)^(1/3)/c-1/
27*a*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x
^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)
*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*
x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)
*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*
x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c
^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^
(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.20

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left((a+bx^2)\sqrt[3]{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{6},\frac{7}{6},-\frac{bx^2}{a}\right)\right)}{3b\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

output
```
(c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/3) - a*Hy
pergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/3)
)
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {248, 262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{9}a\int \frac{(cx)^{4/3}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}a\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{3b}\right)+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{9}a\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{b}\right)+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{771}$$

$$\frac{2}{9}a\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}\right)+\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{766}$$

$$\frac{2}{9}a\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\right.}{c^{2/3}-\left(1+\right.}\right.\right.}{2\sqrt[4]{3}b\sqrt{1-bx^2}\,(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{a}{ac^2+}}}\right.$$

$$\frac{(cx)^{7/3}\sqrt[3]{a+bx^2}}{3c}$$

input
```
Int[(c*x)^(4/3)*(a + b*x^2)^(1/3),x]
```

output
```
((c*x)^(7/3)*(a + b*x^2)^(1/3))/(3*c) + (2*a*((c*(c*x)^(1/3)*(a + b*x^2)^(
1/3))/b - (a*c^(1/3)*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(
4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 +
Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])
*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 +
 Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/
(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^
(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2])))/9
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int (cx)^{\frac{4}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}}dx$$

input
```
int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)*c*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\frac{\sqrt[3]{a}c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(4/3)*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-1/3, 7/6), (13/6,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(13/6))
```

Maxima [F]

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

Giac [F]

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{4}{3}}\,dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\int (c\,x)^{4/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int((c*x)^(4/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(4/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int (cx)^{4/3}\sqrt[3]{a+bx^2}\,dx=\frac{c^{\frac{4}{3}}\left(6x^{\frac{1}{3}}(b\,x^2+a)^{\frac{1}{3}}\,a+9x^{\frac{7}{3}}(b\,x^2+a)^{\frac{1}{3}}\,b-2\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{2}{3}}a+x^{\frac{8}{3}}b}dx\right)a^2\right)}{27b}$$

input
```
int((c*x)^(4/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(1/3)*c*(6*x**(1/3)*(a + b*x**2)**(1/3)*a + 9*x**(1/3)*(a + b*x**2)**(
1/3)*b*x**2 - 2*int((a + b*x**2)**(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*
a**2))/(27*b)
```

3.796 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 381

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c} + \frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}\right.\right.}{\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
(c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(
1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^
(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^
(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3
)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^
(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/c^(5/
3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*
x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^
(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{3x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{6},\frac{7}{6},-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/6, 7/6, -((b*x^2)/a)])/((
c*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {248, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx$$

$\big\downarrow$ 248

$$\frac{2}{3}a\int \frac{1}{(cx)^{2/3}\left(bx^2+a\right)^{2/3}}dx+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

$\big\downarrow$ 266

$$\frac{2a\int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

$\big\downarrow$ 771

$$\frac{2a\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{c\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

$\big\downarrow$ 766

$$\frac{a\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)}{\sqrt[4]{3}c^{5/3}\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(2/3),x]
```

output
```
((c*x)^(1/3)*(a + b*x^2)^(1/3))/c + (a*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x
)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3)
)/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/
3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))], (2 + Sqrt[3])/4])/(3^(1/4)*c^(5/3)*Sqrt[1 - b*x^2]*(a + b*x^
2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^
(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3)
)^2)])
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{2}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \frac{\sqrt[3]{a}\sqrt[3]{x}\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(2/3),x)
```

output
```
a**(1/3)*x**(1/3)*gamma(1/6)*hyper((-1/3, 1/6), (7/6,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx=\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{2/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(2/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{2/3}}\,dx=\frac{3x^{\frac{1}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+2\left(\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^{\frac{2}{3}}a+x^{\frac{8}{3}}b}dx\right)a}{3c^{\frac{2}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(2/3),x)
```

output
```
(3*x**(1/3)*(a + b*x**2)**(1/3) + 2*int((a + b*x**2)**(1/3)/(x**(2/3)*a +
x**(2/3)*b*x**2),x)*a)/(3*c**(2/3))
```

3.797 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 391

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

$$+\frac{3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(c}{\sqrt[3]{a+b}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(c}{\sqrt[3]{a+b}}}\right.\right.}{5ac^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-3/5*(b*x^2+a)^(1/3)/c/(c*x)^(5/3)+1/5*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^(1/
3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(
4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)
-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM
(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)
-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2)
)/a/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^
(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^
(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.14

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = -\frac{3x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{3},\frac{1}{6},-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-5/6, -1/3, 1/6, -((b*x^2)/a)])/
(5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {247, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx$$

$$\Big\downarrow \; 247$$

$$\frac{2b\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{5c^2}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

$$\Big\downarrow \; 266$$

$$\frac{6b\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{5c^3}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

$$\Big\downarrow \; 771$$

$$\frac{6b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5c^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

$$\Big\downarrow \; 766$$

$$\frac{3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}(2+\cdots}{5c^{11/3}\sqrt{1-bx^2}\,(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

$$\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(8/3),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(c^(2/3)
 - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2
/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*Elliptic
F[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqr
t[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(5*c^(11/3)*Sqrt[1 - b*x^2]
*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)
^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(
c*x)^(2/3))^2)])
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{3}}}{\left(cx\right)^{\frac{8}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx=-\frac{\sqrt[3]{b}\,{}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}x}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(8/3),x)
```

output
```
-b**(1/3)*hyper((-1/3, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(8/3)
*x)
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{8/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(8/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{8/3}}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}+2x^{\frac{5}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{2}{3}}a+x^{\frac{8}{3}}b}dx\right)b}{5x^{\frac{5}{3}}c^{\frac{8}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(8/3),x)
```

output
```
( - 3*(a + b*x**2)**(1/3) + 2*x**(2/3)*int((a + b*x**2)**(1/3)/(x**(2/3)*a
 + x**(2/3)*b*x**2),x)*b*x)/(5*x**(2/3)*c**(2/3)*c**2*x)
```

3.798 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 422

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}-\frac{6b\sqrt[3]{a+bx^2}}{55ac^3(cx)^{5/3}}$$

$$-\frac{3\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{}}{\sqrt[3]{a}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{}}{\sqrt[3]{a}}}\right.\right.}{55a^2c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-3/11*(b*x^2+a)^(1/3)/c/(c*x)^(11/3)-6/55*b*(b*x^2+a)^(1/3)/a/c^3/(c*x)^(5
/3)-3/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2
/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)
*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(
2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))
*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(
2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^2/c^(17/3)/(-b^(1/3)*(c*
x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^
(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.13

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = -\frac{3x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{1}{3},-\frac{5}{6},-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -1/3, -5/6, -((b*x^2)/a)]
)/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 264, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{2b\int \frac{1}{(cx)^{8/3}(bx^2+a)^{2/3}}dx}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\left(-\frac{3b\int \frac{1}{(cx)^{2/3}\left(bx^2+a\right)^{2/3}}dx}{5ac^2}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\left(-\frac{9b\int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{5ac^3}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

$$\Big\downarrow \text{771}$$

$$\frac{2b\left(-\frac{9b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5ac^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

$$\Big\downarrow \text{766}$$

$$\frac{2b\left(-\frac{3\ 3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10ac^{11/3}\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\right.}{11c^2}$$

$$\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(14/3),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(11*c*(c*x)^(11/3)) + (2*b*((-3*(a + b*x^2)^(1/3))/
(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2
/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c
^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) -
 (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)
^(2/3))], (2 + Sqrt[3])/4])/(10*a*c^(11/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/
3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3)
- b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])
))/(11*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(14/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(14/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(14/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \int \frac{(b\,x^2+a)^{1/3}}{(c\,x)^{14/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(14/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(14/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{14/3}}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}+2x^{\frac{11}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{8}{3}}a+x^{\frac{14}{3}}b}dx\right)b}{11x^{\frac{11}{3}}c^{\frac{14}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(14/3),x)
```

output
```
( - 3*(a + b*x**2)**(1/3) + 2*x**(2/3)*int((a + b*x**2)**(1/3)/(x**(2/3)*a
*x**2 + x**(2/3)*b*x**4),x)*b*x**3)/(11*x**(2/3)*c**(2/3)*c**4*x**3)
```

3.799 $\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx=\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
3/5*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 5/6],[11/6],-b*x^2/a)/c/(
1+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx=\frac{3x(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

output
```
(3*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[3]{a+bx^2}\int (cx)^{2/3}\sqrt[3]{\frac{bx^2}{a}+1}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{3(cx)^{5/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(c*x)^(2/3)*(a + b*x^2)^(1/3),x]
```

output
```
(3*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 5/6, 11/6, -((b*x
^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{2}{3}} \left(b\,x^{2}+a\right)^{\frac{1}{3}} dx$$

input
```
int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

output
```
int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

Fricas [F]

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx = \frac{\sqrt[3]{a}c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(2/3)*(b*x**2+a)**(1/3),x)
```

output
```
a**(1/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-1/3, 5/6), (11/6,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(11/6))
```

Maxima [F]

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Giac [F]

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)*(c*x)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx=\int (c\,x)^{2/3}\,\left(b\,x^2+a\right)^{1/3}\,dx$$

input
```
int((c*x)^(2/3)*(a + b*x^2)^(1/3),x)
```

output
```
int((c*x)^(2/3)*(a + b*x^2)^(1/3), x)
```

Reduce [F]

$$\int (cx)^{2/3}\sqrt[3]{a+bx^2}\,dx=\frac{c^{\frac{2}{3}}\left(3x^{\frac{5}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+2\left(\int \frac{x^{\frac{2}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right)a\right)}{7}$$

input
```
int((c*x)^(2/3)*(b*x^2+a)^(1/3),x)
```

output
```
(c**(2/3)*(3*x**(2/3)*(a + b*x**2)**(1/3)*x + 2*int((x**(2/3)*(a + b*x**2)
**(1/3))/(a + b*x**2),x)*a))/7
```

3.800 $\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
3/2*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-1/3, 1/3],[4/3],-b*x^2/a)/c/(1
+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \frac{3x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

output
```
(3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^2)/a)])/(2
*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[3]{a+bx^2}\int \frac{\sqrt[3]{\frac{bx^2}{a}+1}}{\sqrt[3]{cx}}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{3(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(1/3),x]
```

output
```
(3*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^
2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \frac{\sqrt[3]{a}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(1/3),x)
```

output
```
a**(1/3)*x**(2/3)*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(1/3)*gamma(4/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx=\int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{1/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(1/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{\sqrt[3]{cx}}\,dx=\frac{3x^{\frac{2}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}+2\left(\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}}{x^{\frac{1}{3}}a+x^{\frac{7}{3}}b}dx\right)a}{4c^{\frac{1}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(1/3),x)
```

output
```
(3*x**(2/3)*(a + b*x**2)**(1/3) + 2*int((a + b*x**2)**(1/3)/(x**(1/3)*a +
x**(1/3)*b*x**2),x)*a)/(4*c**(1/3))
```

3.801 $\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
-3*(b*x^2+a)^(1/3)*hypergeom([-1/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(1
+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = -\frac{3x\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/
((c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[3]{a+bx^2} \int \frac{\sqrt[3]{\frac{bx^2}{a}+1}}{(cx)^{4/3}}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{3\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/3)/(c*x)^(4/3),x]
```

output
```
(-3*(a + b*x^2)^(1/3)*Hypergeometric2F1[-1/3, -1/6, 5/6, -((b*x^2)/a)])/(c
*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}dx$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

output
```
int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

Fricas [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \frac{\sqrt[3]{a}\,\Gamma\left(-\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},-\frac{1}{6}\\ \frac{5}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\,\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate((b*x**2+a)**(1/3)/(c*x)**(4/3),x)
```

output
```
a**(1/3)*gamma(-1/6)*hyper((-1/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Maxima [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/3)/(c*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/3)/(c*x)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \int \frac{\left(b\,x^2+a\right)^{1/3}}{\left(c\,x\right)^{4/3}}\,dx$$

input
```
int((a + b*x^2)^(1/3)/(c*x)^(4/3),x)
```

output
```
int((a + b*x^2)^(1/3)/(c*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{a+bx^2}}{(cx)^{4/3}}\,dx = \frac{-3\left(b\,x^2+a\right)^{\frac{1}{3}}+2x^{\frac{1}{3}}\left(\int \frac{\left(b\,x^2+a\right)^{\frac{1}{3}}x}{x^{\frac{1}{3}}a+x^{\frac{7}{3}}b}dx\right)b}{x^{\frac{1}{3}}c^{\frac{4}{3}}}$$

input
```
int((b*x^2+a)^(1/3)/(c*x)^(4/3),x)
```

output
```
( - 3*(a + b*x**2)**(1/3) + 2*x**(1/3)*int(((a + b*x**2)**(1/3)*x)/(x**(1/
3)*a + x**(1/3)*b*x**2),x)*b)/(x**(1/3)*c**(1/3)*c)
```

3.802 $\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 223

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx = -\frac{5a^3c^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{324b^2} + \frac{a^2c(cx)^{10/3}\sqrt[3]{a+bx^2}}{108b} + \frac{a(cx)^{16/3}\sqrt[3]{a+bx^2}}{18c} + \frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c} - \frac{5a^4c^{13/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{162\sqrt{3}b^{8/3}} - \frac{5a^4c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{324b^{8/3}}$$

output
```
-5/324*a^3*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/108*a^2*c*(c*x)^(10/3)*(b
*x^2+a)^(1/3)/b+1/18*a*(c*x)^(16/3)*(b*x^2+a)^(1/3)/c+1/8*(c*x)^(16/3)*(b*
x^2+a)^(4/3)/c-5/486*a^4*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2
/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)-5/324*a^4*c^(13/3)*ln(b^(1/3
)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 2.57 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.16

$$\int (cx)^{13/3}\left(a+bx^2\right)^{4/3}\,dx=\frac{c^4\sqrt[3]{cx}\Big(-30a^3b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+18a^2b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+351ab^{8/3}x^{16/3}\sqrt[3]{a+bx^2}+243b^{1}\cdots}{\cdots}$$

input
```
Integrate[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

output
```
(c^4*(c*x)^(1/3)*(-30*a^3*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 18*a^2*b^(5/
3)*x^(10/3)*(a + b*x^2)^(1/3) + 351*a*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) +
 243*b^(11/3)*x^(22/3)*(a + b*x^2)^(1/3) - 20*Sqrt[3]*a^4*ArcTan[(Sqrt[3]*
b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 20*a^4*Log[-(b
^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] + 10*a^4*Log[b^(2/3)*x^(4/3) + b^(1/3
)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(1944*b^(8/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {248, 248, 262, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{13/3}\left(a+bx^2\right)^{4/3}\,dx$$

$$\Big\downarrow \; 248$$

$$\frac{1}{3}a\int (cx)^{13/3}\sqrt[3]{bx^2+a}dx+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \; 248$$

$$\frac{1}{3}a\left(\frac{1}{9}a\int \frac{(cx)^{13/3}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}\right)+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\int \frac{(cx)^{7/3}}{(bx^2+a)^{2/3}}dx}{6b}\right)+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}\right)+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int \frac{\sqrt[3]{cx}}{\left(bx^2+a\right)^{2/3}}dx}{3b}\right)}{6b}\right)+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}\right)+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int \frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{6b}\right)+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}\right)+\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int\frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)}{6b}\right)+\frac{(cx)^{16/3}\sqrt[3]{a+bx^2}}{6c}\right)+$$

$$\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

$$\Big\downarrow \text{853}$$

$$\frac{1}{3}a\left(\frac{1}{9}a\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{\cdots}\right.}{2b^{2/3}}\right.}{b}\right.}{6b}\right.\right.$$

$$\frac{(cx)^{16/3}\left(a+bx^2\right)^{4/3}}{8c}$$

input
```
Int[(c*x)^(13/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(16/3)*(a + b*x^2)^(4/3))/(8*c) + (a*(((c*x)^(16/3)*(a + b*x^2)^(1/
3))/(6*c) + (a*((c*(c*x)^(10/3)*(a + b*x^2)^(1/3))/(4*b) - (5*a*c^2*((c*(c
*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) - (a*c*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/
3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))
) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*
b^(2/3))))/b))/(6*b)))/9))/3
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{13}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx = \text{Timed out}$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} \, dx = \text{Timed out}$$

input
```
integrate((c*x)**(13/3)*(b*x**2+a)**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{13}{3}} dx$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

Giac [F]

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{13}{3}} dx$$

input
```
integrate((c*x)^(13/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(13/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{13/3} \left(a+bx^2\right)^{4/3} dx = \int (c\,x)^{13/3} \left(b\,x^2+a\right)^{4/3} dx$$

input
```
int((c*x)^(13/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(13/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int (cx)^{13/3} (a + bx^2)^{4/3} \, dx = \frac{c^{\frac{13}{3}} \left(-30x^{\frac{4}{3}} (b\,x^2 + a)^{\frac{1}{3}} a^3 + 18x^{\frac{10}{3}} (b\,x^2 + a)^{\frac{1}{3}} a^2 b + 351x^{\frac{16}{3}} (b\,x^2 + a)^{\frac{1}{3}} a\,b^2 + 243x^{\frac{22}{3}} (b\,x^2 + a\right.}{1944b^2}$$

input
```
int((c*x)^(13/3)*(b*x^2+a)^(4/3),x)
```

output
```
(c**(1/3)*c**4*( - 30*x**(1/3)*(a + b*x**2)**(1/3)*a**3*x + 18*x**(1/3)*(a
 + b*x**2)**(1/3)*a**2*b*x**3 + 351*x**(1/3)*(a + b*x**2)**(1/3)*a*b**2*x*
*5 + 243*x**(1/3)*(a + b*x**2)**(1/3)*b**3*x**7 + 40*int((x**(1/3)*(a + b*
x**2)**(1/3))/(a + b*x**2),x)*a**4))/(1944*b**2)
```

3.803 $\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 192

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{a^2c(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b}+\frac{a(cx)^{10/3}\sqrt[3]{a+bx^2}}{9c} + \frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}+\frac{2a^3c^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{27\sqrt{3}b^{5/3}} + \frac{a^3c^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{5/3}}$$

output
```
1/27*a^2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/9*a*(c*x)^(10/3)*(b*x^2+a)^(1/3
)/c+1/6*(c*x)^(10/3)*(b*x^2+a)^(4/3)/c+2/81*a^3*c^(7/3)*arctan(1/3*(1+2*b^
(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)+1/27*a
^3*c^(7/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.20

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} dx = \frac{(cx)^{7/3} \left(6a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+45ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+27b^{8/3}x^{16/3}\sqrt[3]{a+bx^2}+4\sqrt{3}a^3\operatorname{arct}\right.}{}$$

input
```
Integrate[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(7/3)*(6*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 45*a*b^(5/3)*x^(10
/3)*(a + b*x^2)^(1/3) + 27*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) + 4*Sqrt[3]*
a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3
))] + 4*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - 2*a^3*Log[b^(2/3
)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(162*
b^(5/3)*x^(7/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {248, 248, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{4}{9}a\int (cx)^{7/3}\sqrt[3]{bx^2+a}dx+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

$$\Big\downarrow \text{248}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\int \frac{(cx)^{7/3}}{\left(bx^2+a\right)^{2/3}}dx+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}\right)+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx}{3b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}\right)+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

$$\Big\downarrow \text{266}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}\right)+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

$$\Big\downarrow \text{807}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)+\frac{(cx)^{10/3}\sqrt[3]{a+bx^2}}{4c}\right)+\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

$$\Big\downarrow \text{853}$$

$$\frac{4}{9}a\left(\frac{1}{6}a\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{b}\right)+\frac{(cx)^{10}}{}\right.$$

$$\frac{(cx)^{10/3}\left(a+bx^2\right)^{4/3}}{6c}$$

input
```
Int[(c*x)^(7/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(10/3)*(a + b*x^2)^(4/3))/(6*c) + (4*a*(((c*x)^(10/3)*(a + b*x^2)^(
1/3))/(4*c) + (a*((c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) - (a*c*(-((c^(4/
3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt
[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a
 + (b*x)/c)^(1/3)])/(2*b^(2/3))))/b))/6))/9
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{7}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \text{Timed out}$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 37.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{a^{\frac{4}{3}} c^{\frac{7}{3}} x^{\frac{10}{3}} \Gamma\left(\frac{5}{3}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, \frac{5}{3} \\ \frac{8}{3} \end{matrix}\middle| \frac{bx^{2} e^{i\pi}}{a} \right)}{2\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((c*x)**(7/3)*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*c**(7/3)*x**(10/3)*gamma(5/3)*hyper((-4/3, 5/3), (8/3,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(8/3))
```

Maxima [F]

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}} \, dx$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

Giac [F]

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{7}{3}} \, dx$$

input
```
integrate((c*x)^(7/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(7/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \int (c\,x)^{7/3} \left(b\,x^2+a\right)^{4/3} \, dx$$

input
```
int((c*x)^(7/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(7/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int (cx)^{7/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{c^{\frac{7}{3}} \left(6x^{\frac{4}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}} a^2+45x^{\frac{10}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}} ab+27x^{\frac{16}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}} b^2-8\left(\int \frac{x^{\frac{1}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right) a^3\right)}{162b}$$

input
```
int((c*x)^(7/3)*(b*x^2+a)^(4/3),x)
```

output
```
(c**(1/3)*c**2*(6*x**(1/3)*(a + b*x**2)**(1/3)*a**2*x + 45*x**(1/3)*(a + b
*x**2)**(1/3)*a*b*x**3 + 27*x**(1/3)*(a + b*x**2)**(1/3)*b**2*x**5 - 8*int
((x**(1/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a**3))/(162*b)
```

3.804 $\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 163

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\frac{a(cx)^{4/3}\sqrt[3]{a+bx^2}}{3c}+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}-\frac{a^2\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{2/3}}-\frac{a^2\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{6b^{2/3}}$$

output

```
1/3*a*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c+1/4*(c*x)^(4/3)*(b*x^2+a)^(4/3)/c-1/9*
a^2*c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3
^(1/2))*3^(1/2)/b^(2/3)-1/6*a^2*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*
x^2+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.25

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\frac{\sqrt[3]{cx}\left(21ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}-4\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-\right.}{36b^{2/3}}$$

input
```
Integrate[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(1/3)*(21*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(10/3)*
(a + b*x^2)^(1/3) - 4*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3
)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 4*a^2*Log[-(b^(1/3)*x^(2/3)) + (a + b*
x^2)^(1/3)] + 2*a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3
) + (a + b*x^2)^(2/3)]))/(36*b^(2/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 248, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}dx$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{3}a\int \sqrt[3]{cx}\sqrt[3]{bx^2+a}dx+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

$$\Big\downarrow \text{248}$$

$$\frac{2}{3}a\left(\frac{1}{3}a\int \frac{\sqrt[3]{cx}}{\left(bx^2+a\right)^{2/3}}dx+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{3}a\left(\frac{a\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

$$\Big\downarrow \text{807}$$

$$\frac{2}{3}a\left(\frac{a\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

$$\Big\downarrow \text{853}$$

$$\frac{2}{3}a\left(\frac{a\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)+\frac{(cx)^{4/3}\left(a+bx^2\right)^{4/3}}{4c}$$

input
```
Int[(c*x)^(1/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(4/3)*(a + b*x^2)^(4/3))/(4*c) + (2*a*(((c*x)^(4/3)*(a + b*x^2)^(1/
3))/(2*c) + (a*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a
 + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3)*(
c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/(2*c)))/3
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int (cx)^{\frac{1}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\text{Timed out}$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.28

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\frac{a^{\frac{4}{3}}\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((c*x)**(1/3)*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*c**(1/3)*x**(4/3)*gamma(2/3)*hyper((-4/3, 2/3), (5/3,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(5/3))
```

Maxima [F]

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{1}{3}}\,dx$$

input
```
integrate((c*x)^(1/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\int (c\,x)^{1/3}\,\left(b\,x^2+a\right)^{4/3}\,dx$$

input
```
int((c*x)^(1/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(1/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int \sqrt[3]{cx}\left(a+bx^2\right)^{4/3}\,dx=\frac{c^{\frac{1}{3}}\left(21x^{\frac{4}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}a+9x^{\frac{10}{3}}\left(b\,x^2+a\right)^{\frac{1}{3}}b+8\left(\int \frac{x^{\frac{1}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right)a^2\right)}{36}$$

input
```
int((c*x)^(1/3)*(b*x^2+a)^(4/3),x)
```

output
```
(c**(1/3)*(21*x**(1/3)*(a + b*x**2)**(1/3)*a*x + 9*x**(1/3)*(a + b*x**2)**
(1/3)*b*x**3 + 8*int((x**(1/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a**2))
/36
```

3.805 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \frac{2b(cx)^{4/3}\sqrt[3]{a+bx^2}}{c^3} - \frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}} - \frac{2a\sqrt[3]{b}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}c^{5/3}} - \frac{a\sqrt[3]{b}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{c^{5/3}}$$

output

```
2*b*(c*x)^(4/3)*(b*x^2+a)^(1/3)/c^3-3/2*(b*x^2+a)^(4/3)/c/(c*x)^(2/3)-2/3*
a*b^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(
1/2))*3^(1/2)/c^(5/3)-a*b^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(
1/3))/c^(5/3)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \frac{x\left(-9a\sqrt[3]{a+bx^2}+3bx^2\sqrt[3]{a+bx^2}-4\sqrt{3}a\sqrt[3]{b}x^{2/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-4a\right.}{}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

output
```
(x*(-9*a*(a + b*x^2)^(1/3) + 3*b*x^2*(a + b*x^2)^(1/3) - 4*Sqrt[3]*a*b^(1/
3)*x^(2/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^
2)^(1/3))] - 4*a*b^(1/3)*x^(2/3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3
)] + 2*a*b^(1/3)*x^(2/3)*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)
^(1/3) + (a + b*x^2)^(2/3)]))/(6*(c*x)^(5/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 248, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{4b\int \sqrt[3]{cx}\sqrt[3]{bx^2+a}dx}{c^2}-\frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

$$\Big\downarrow \text{248}$$

$$\frac{4b\left(\frac{1}{3}a\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)}{c^2}-\frac{3(a+bx^2)^{4/3}}{2c(cx)^{2/3}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\left(\frac{a\int \frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

$\downarrow$ 807

$$\frac{4b\left(\frac{a\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

$\downarrow$ 853

$$\frac{4b\left(\frac{a\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c}+\frac{(cx)^{4/3}\sqrt[3]{a+bx^2}}{2c}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{2c(cx)^{2/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(5/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(2*c*(c*x)^(2/3)) + (4*b*(((c*x)^(4/3)*(a + b*x^2)^
(1/3))/(2*c) + (a*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)
*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3
)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/(2*c)))/c^2
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3},-\frac{1}{3} \\ \frac{2}{3} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{5}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(5/3),x)
```

output
```
a**(4/3)*gamma(-1/3)*hyper((-4/3, -1/3), (2/3,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(5/3)*x**(2/3)*gamma(2/3))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{5}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(5/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{5/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(5/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(5/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{5/3}}\,dx = \frac{-9(b\,x^2+a)^{\frac{1}{3}}\,a+3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+8x^{\frac{2}{3}}\left(\int \frac{x^{\frac{1}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)ab}{6x^{\frac{2}{3}}c^{\frac{5}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(5/3),x)
```

output
```
( - 9*(a + b*x**2)**(1/3)*a + 3*(a + b*x**2)**(1/3)*b*x**2 + 8*x**(2/3)*in
t((x**(1/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a*b)/(6*x**(2/3)*c**(2/3)
*c)
```

3.806 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = -\frac{3b\sqrt[3]{a+bx^2}}{2c^3(cx)^{2/3}} - \frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}} - \frac{\sqrt{3}b^{4/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2c^{11/3}} - \frac{3b^{4/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4c^{11/3}}$$

output
```
-3/2*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(2/3)-3/8*(b*x^2+a)^(4/3)/c/(c*x)^(8/3)-1
/2*3^(1/2)*b^(4/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(
1/3))*3^(1/2))/c^(11/3)-3/4*b^(4/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+
a)^(1/3))/c^(11/3)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx =$$
$$-\frac{x\left(3a\sqrt[3]{a+bx^2}+15bx^2\sqrt[3]{a+bx^2}+4\sqrt{3}b^{4/3}x^{8/3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+4b^{4/3}x^{8/3}\log\left(-\sqrt[3]{b}x^{2/3}\right.\right.}{8(cx)^{11/3}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

output
```
-1/8*(x*(3*a*(a + b*x^2)^(1/3) + 15*b*x^2*(a + b*x^2)^(1/3) + 4*Sqrt[3]*b^
(4/3)*x^(8/3)*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b
*x^2)^(1/3))] + 4*b^(4/3)*x^(8/3)*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/
3)] - 2*b^(4/3)*x^(8/3)*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^
(1/3) + (a + b*x^2)^(2/3)]))/(c*x)^(11/3)
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 247, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx$$
$$\Big\downarrow \; 247$$
$$\frac{b\int \frac{\sqrt[3]{bx^2+a}}{(cx)^{5/3}}dx}{c^2}-\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}}$$
$$\Big\downarrow \; 247$$

$$\frac{b\left(\frac{b\int \frac{\sqrt[3]{cx}}{\left(bx^2+a\right)^{2/3}}dx}{c^2}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{b\left(\frac{3b\int \frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{b\left(\frac{3b\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}\right)}{c^2}-\frac{3\left(a+bx^2\right)^{4/3}}{8c(cx)^{8/3}}$$

$$\Big\downarrow \text{ 853}$$

$$\frac{b\left(\frac{3b\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c^3}-\frac{3\sqrt[3]{a+bx^2}}{2c(cx)^{2/3}}\right)}{c^2}-\frac{3(a+bx^2)^{4/3}}{8c(cx)^{8/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(11/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(8*c*(c*x)^(8/3)) + (b*((-3*(a + b*x^2)^(1/3))/(2*c
*(c*x)^(2/3)) + (3*b*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2
/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(
1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/(2*c^3)))/
c^2
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 33.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.34

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \frac{a^{\frac{4}{3}}\Gamma\left(-\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{4}{3} \\ -\frac{1}{3} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{11}{3}}x^{\frac{8}{3}}\Gamma\left(-\frac{1}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(11/3),x)
```

output
```
a**(4/3)*gamma(-4/3)*hyper((-4/3, -4/3), (-1/3,), b*x**2*exp_polar(I*pi)/a
)/(2*c**(11/3)*x**(8/3)*gamma(-1/3))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{11}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(11/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(11/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{11/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(11/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(11/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{11/3}}\,dx = \frac{-3(b\,x^2+a)^{\frac{1}{3}}\,a-3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+8x^{\frac{8}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{5}{3}}}dx\right)b}{8x^{\frac{8}{3}}c^{\frac{11}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(11/3),x)
```

output
```
( - 3*(a + b*x**2)**(1/3)*a - 3*(a + b*x**2)**(1/3)*b*x**2 + 8*x**(2/3)*in
t((a + b*x**2)**(1/3)/(x**(2/3)*x),x)*b*x**2)/(8*x**(2/3)*c**(2/3)*c**3*x*
*2)
```

3.807 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx = -\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

output
```
-3/14*(b*x^2+a)^(7/3)/a/c/(c*x)^(14/3)
```

Mathematica [A] (verified)

Time = 2.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx = -\frac{3x(a+bx^2)^{7/3}}{14a(cx)^{17/3}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

output
```
(-3*x*(a + b*x^2)^(7/3))/(14*a*(c*x)^(17/3))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx$$
$$\Big\downarrow \; 242$$
$$-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{14/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(17/3),x]
```

output
```
(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(14/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}}{14a\left(cx\right)^{\frac{17}{3}}}$	21
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}}{14a\left(cx\right)^{\frac{17}{3}}}$	21
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(b^{2}x^{4}+2ab\,x^{2}+a^{2}\right)}{14c^{5}\left(cx\right)^{\frac{2}{3}}x^{4}a}$	44

input
```
int((b*x^2+a)^(4/3)/(c*x)^(17/3),x,method=_RETURNVERBOSE)
```

output
```
-3/14*x*(b*x^2+a)^(7/3)/a/(c*x)^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx = -\frac{3\left(b^2x^4+2\,abx^2+a^2\right)\left(bx^2+a\right)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{14\,ac^6x^5}$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="fricas")
```

output
```
-3/14*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^6*x^5
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{17/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(17/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a + bx^2)^{4/3}}{(cx)^{17/3}} \, dx = \int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}} \, dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

Giac [F]

$$\int \frac{(a + bx^2)^{4/3}}{(cx)^{17/3}} \, dx = \int \frac{(bx^2 + a)^{\frac{4}{3}}}{(cx)^{\frac{17}{3}}} \, dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(17/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(17/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)^{4/3}}{(cx)^{17/3}} \, dx = \int \frac{(b\,x^2 + a)^{4/3}}{(c\,x)^{17/3}} \, dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(17/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(17/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{17/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,(-b^2x^4-2ab\,x^2-a^2)}{14x^{\frac{14}{3}}c^{\frac{17}{3}}a}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(17/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - a**2 - 2*a*b*x**2 - b**2*x**4))/(14*x**(2/3)*c*
*(2/3)*a*c**5*x**4)
```

3.808 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}} dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}} dx = -\frac{3(a+bx^2)^{7/3}}{20ac(cx)^{20/3}} + \frac{9b(a+bx^2)^{7/3}}{140a^2c^3(cx)^{14/3}}$$

output
```
-3/20*(b*x^2+a)^(7/3)/a/c/(c*x)^(20/3)+9/140*b*(b*x^2+a)^(7/3)/a^2/c^3/(c*
x)^(14/3)
```

Mathematica [A] (verified)

Time = 3.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}} dx = -\frac{3x(7a-3bx^2)(a+bx^2)^{7/3}}{140a^2(cx)^{23/3}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

output
```
(-3*x*(7*a - 3*b*x^2)*(a + b*x^2)^(7/3))/(140*a^2*(c*x)^(23/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{3\int \frac{(bx^2+a)^{7/3}}{(cx)^{23/3}}dx}{7a}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(23/3),x]
```

output
```
(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (9*(a + b*x^2)^(10/3))/(140
*a^2*c*(c*x)^(20/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{3x(b\,x^{2}+a)^{\frac{7}{3}}(-3b\,x^{2}+7a)}{140a^{2}(cx)^{\frac{23}{3}}}$	31
orering	$-\frac{3x(b\,x^{2}+a)^{\frac{7}{3}}(-3b\,x^{2}+7a)}{140a^{2}(cx)^{\frac{23}{3}}}$	31
risch	$-\frac{3(b\,x^{2}+a)^{\frac{1}{3}}(-3b^{3}x^{6}+a\,b^{2}x^{4}+11a^{2}b\,x^{2}+7a^{3})}{140c^{7}(cx)^{\frac{2}{3}}x^{6}a^{2}}$	57

input
```
int((b*x^2+a)^(4/3)/(c*x)^(23/3),x,method=_RETURNVERBOSE)
```

output
```
-3/140*x*(b*x^2+a)^(7/3)*(-3*b*x^2+7*a)/a^2/(c*x)^(23/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx = \frac{3\,(3\,b^3x^6-ab^2x^4-11\,a^2bx^2-7\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{140\,a^2c^8x^7}$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="fricas")
```

output
```
3/140*(3*b^3*x^6 - a*b^2*x^4 - 11*a^2*b*x^2 - 7*a^3)*(b*x^2 + a)^(1/3)*(c*
x)^(1/3)/(a^2*c^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(23/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{23}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(23/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(23/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx=\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{23/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(23/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(23/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{23/3}}\,dx=\frac{3(b\,x^2+a)^{\frac{1}{3}}\,(3b^3x^6-a\,b^2x^4-11a^2b\,x^2-7a^3)}{140x^{\frac{20}{3}}\,c^{\frac{23}{3}}\,a^2}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(23/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - 7*a**3 - 11*a**2*b*x**2 - a*b**2*x**4 + 3*b**3*
x**6))/(140*x**(2/3)*c**(2/3)*a**2*c**7*x**6)
```

3.809 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx=-\frac{3(a+bx^2)^{7/3}}{26ac(cx)^{26/3}}+\frac{9b(a+bx^2)^{7/3}}{130a^2c^3(cx)^{20/3}}-\frac{27b^2(a+bx^2)^{7/3}}{910a^3c^5(cx)^{14/3}}$$

output
```
-3/26*(b*x^2+a)^(7/3)/a/c/(c*x)^(26/3)+9/130*b*(b*x^2+a)^(7/3)/a^2/c^3/(c*
x)^(20/3)-27/910*b^2*(b*x^2+a)^(7/3)/a^3/c^5/(c*x)^(14/3)
```

Mathematica [A] (verified)

Time = 4.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx=-\frac{3x(a+bx^2)^{7/3}(35a^2-21abx^2+9b^2x^4)}{910a^3(cx)^{29/3}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

output
```
(-3*x*(a + b*x^2)^(7/3)*(35*a^2 - 21*a*b*x^2 + 9*b^2*x^4))/(910*a^3*(c*x)^
(29/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{4/3}}{(cx)^{29/3}}\,dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{6\int \frac{\left(bx^2+a\right)^{7/3}}{(cx)^{29/3}}dx}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{6\left(-\frac{3\int \frac{\left(bx^2+a\right)^{10/3}}{(cx)^{29/3}}dx}{10a}-\frac{3\left(a+bx^2\right)^{10/3}}{20ac(cx)^{26/3}}\right)}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{6\left(\frac{9\left(a+bx^2\right)^{13/3}}{260a^2c(cx)^{26/3}}-\frac{3\left(a+bx^2\right)^{10/3}}{20ac(cx)^{26/3}}\right)}{7a}-\frac{3\left(a+bx^2\right)^{7/3}}{14ac(cx)^{26/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(29/3),x]
```

output
```
(-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(26/3)) - (6*((-3*(a + b*x^2)^(10/3))
/(20*a*c*(c*x)^(26/3)) + (9*(a + b*x^2)^(13/3))/(260*a^2*c*(c*x)^(26/3))))
/(7*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(9b^{2}x^{4}-21ab\,x^{2}+35a^{2}\right)}{910a^{3}\left(cx\right)^{\frac{29}{3}}}$	42
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{7}{3}}\left(9b^{2}x^{4}-21ab\,x^{2}+35a^{2}\right)}{910a^{3}\left(cx\right)^{\frac{29}{3}}}$	42
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{4}x^{8}-3a\,b^{3}x^{6}+2a^{2}b^{2}x^{4}+49a^{3}b\,x^{2}+35a^{4}\right)}{910c^{9}\left(cx\right)^{\frac{2}{3}}x^{8}a^{3}}$	69

input
```
int((b*x^2+a)^(4/3)/(c*x)^(29/3),x,method=_RETURNVERBOSE)
```

output
```
-3/910*x*(b*x^2+a)^(7/3)*(9*b^2*x^4-21*a*b*x^2+35*a^2)/a^3/(c*x)^(29/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx=-\frac{3\,(9\,b^4x^8-3\,ab^3x^6+2\,a^2b^2x^4+49\,a^3bx^2+35\,a^4)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{910\,a^3c^{10}x^9}$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="fricas")
```

output
```
-3/910*(9*b^4*x^8 - 3*a*b^3*x^6 + 2*a^2*b^2*x^4 + 49*a^3*b*x^2 + 35*a^4)*(
b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(29/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{29}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(29/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(29/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{29/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(29/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(29/3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{29/3}}\,dx = \frac{3(b\,x^2+a)^{\frac{1}{3}}\,(-9b^4x^8+3a\,b^3x^6-2a^2b^2x^4-49a^3b\,x^2-35a^4)}{910x^{\frac{26}{3}}\,c^{\frac{29}{3}}\,a^3}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(29/3),x)
```

output
```
(3*(a + b*x**2)**(1/3)*( - 35*a**4 - 49*a**3*b*x**2 - 2*a**2*b**2*x**4 + 3
*a*b**3*x**6 - 9*b**4*x**8))/(910*x**(2/3)*c**(2/3)*a**3*c**9*x**8)
```

3.810 $\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 479

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx = -\frac{16a^3c^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{405b^2} + \frac{16a^2c(cx)^{7/3}\sqrt[3]{a+bx^2}}{945b} + \frac{8a(cx)^{13/3}\sqrt[3]{a+bx^2}}{105c} + \frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$
$$+ \frac{8a^3c^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}\cdots\right.\right.}{405\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-16/405*a^3*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+16/945*a^2*c*(c*x)^(7/3)*(
b*x^2+a)^(1/3)/b+8/105*a*(c*x)^(13/3)*(b*x^2+a)^(1/3)/c+1/7*(c*x)^(13/3)*(
b*x^2+a)^(4/3)/c+8/1215*a^3*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b
^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a
)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*
b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2
/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*
b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^2
/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^
2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.21

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} dx = \frac{c^3 \sqrt[3]{cx} \sqrt[3]{a+bx^2} \left(-\left(\left(7a-15bx^2\right)\left(a+bx^2\right)^2 \sqrt[3]{1+\frac{bx^2}{a}}\right) + 7a^3 \text{Hypergeometric2F1}\left(-\frac{4}{3},\right.\right.}{105b^2 \sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

output
```
(c^3*(c*x)^(1/3)*(a + b*x^2)^(1/3)*(-((7*a - 15*b*x^2)*(a + b*x^2)^2*(1 +
(b*x^2)/a)^(1/3)) + 7*a^3*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)])
)/(105*b^2*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {248, 248, 262, 262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{8}{21} a \int (cx)^{10/3} \sqrt[3]{bx^2+a}dx+\frac{(cx)^{13/3} \left(a+bx^2\right)^{4/3}}{7 c}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{8}{21} a \left(\frac{2}{15} a \int \frac{(cx)^{10/3}}{\left(bx^2+a\right)^{2/3}}dx+\frac{(cx)^{13/3} \sqrt[3]{a+bx^2}}{5 c}\right)+\frac{(cx)^{13/3} \left(a+bx^2\right)^{4/3}}{7 c}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{8}{21} a \left(\frac{2}{15} a \left(\frac{c (cx)^{7/3} \sqrt[3]{a+bx^2}}{3 b}-\frac{7 a c^2 \int \frac{(cx)^{4/3}}{\left(bx^2+a\right)^{2/3}}dx}{9 b}\right)+\frac{(cx)^{13/3} \sqrt[3]{a+bx^2}}{5 c}\right)+\frac{(cx)^{13/3} \left(a+bx^2\right)^{4/3}}{7 c}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{8}{21} a \left(\frac{2}{15} a \left(\frac{c (cx)^{7/3} \sqrt[3]{a+bx^2}}{3 b}-\frac{7 a c^2 \left(\frac{c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{b}-\frac{a c^2 \int \frac{1}{(cx)^{2/3} \left(bx^2+a\right)^{2/3}}dx}{3 b}\right)}{9 b}\right)+\frac{(cx)^{13/3} \sqrt[3]{a+bx^2}}{5 c}\right)+\frac{(cx)^{13/3} \left(a+bx^2\right)^{4/3}}{7 c}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{8}{21}a\left(\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int\frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}\right)+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

$\big\downarrow$ 771

$$\frac{8}{21}a\left(\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int\frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}\right)}{9b}\right)+\frac{(cx)^{13/3}\sqrt[3]{a+bx^2}}{5c}\right)+\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

$\big\downarrow$ 766

$$\frac{8}{21}a\left(\frac{2}{15}a\left(\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}}{2\sqrt[4]{3}b\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2}}{\left(c^{2/3}-(\right.}}}\right.}{9b}\right.\right.$$

$$\frac{(cx)^{13/3}\left(a+bx^2\right)^{4/3}}{7c}$$

input
```
Int[(c*x)^(10/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(13/3)*(a + b*x^2)^(4/3))/(7*c) + (8*a*(((c*x)^(13/3)*(a + b*x^2)^(
1/3))/(5*c) + (2*a*((c*(c*x)^(7/3)*(a + b*x^2)^(1/3))/(3*b) - (7*a*c^2*((c
*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (a*c^(1/3)*(c*x)^(1/3)*(c^(2/3) - b^(1
/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*
x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCo
s[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[1 - b*x^2]*(a +
b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)
*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(
2/3))^2])))/(9*b)))/15))/21
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int (cx)^{\frac{10}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^{2}+a\right)^{\frac{4}{3}} (cx)^{\frac{10}{3}} \, dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*c^3*x^5 + a*c^3*x^3)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Sympy [F(-1)]

Timed out.

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} \, dx = \text{Timed out}$$

input
```
integrate((c*x)**(10/3)*(b*x**2+a)**(4/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{10}{3}}\,dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

Giac [F]

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{10}{3}}\,dx$$

input
```
integrate((c*x)^(10/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(10/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{10/3}\left(a+bx^2\right)^{4/3}\,dx=\int (c\,x)^{10/3}\left(b\,x^2+a\right)^{4/3}\,dx$$

input
```
int((c*x)^(10/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(10/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int (cx)^{10/3} \left(a+bx^2\right)^{4/3} dx = \frac{c^{\frac{10}{3}} \left(-336x^{\frac{1}{3}} (b\,x^2+a)^{\frac{1}{3}} a^3 + 144x^{\frac{7}{3}} (b\,x^2+a)^{\frac{1}{3}} a^2 b + 1863x^{\frac{13}{3}} (b\,x^2+a)^{\frac{1}{3}} a\,b^2 + 1215x^{\frac{19}{3}} (b\,x^2 \right.}{8505b^2}$$

input

```
int((c*x)^(10/3)*(b*x^2+a)^(4/3),x)
```

output

```
(c**(1/3)*c**3*( - 336*x**(1/3)*(a + b*x**2)**(1/3)*a**3 + 144*x**(1/3)*(a
 + b*x**2)**(1/3)*a**2*b*x**2 + 1863*x**(1/3)*(a + b*x**2)**(1/3)*a*b**2*x
**4 + 1215*x**(1/3)*(a + b*x**2)**(1/3)*b**3*x**6 + 112*int((a + b*x**2)**
(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*a**4))/(8505*b**2)
```

3.811 $\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 448

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{16a^2c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{135b} + \frac{8a(cx)^{7/3}\sqrt[3]{a+bx^2}}{45c} + \frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c} - \frac{8a^2\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}\cdots\right.\right.}{135\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
16/135*a^2*c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b+8/45*a*(c*x)^(7/3)*(b*x^2+a)^(1
/3)/c+1/5*(c*x)^(7/3)*(b*x^2+a)^(4/3)/c-8/405*a^2*c^(1/3)*(c*x)^(1/3)*(b*x
^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3
)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*Invers
eJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/
4*2^(1/2))*3^(3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b
*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b
*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.20

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(\left(a+bx^2\right)^2\sqrt[3]{1+\frac{bx^2}{a}} - a^2 \,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{6},\frac{7}{6},-\frac{bx^2}{a}\right)\right)}{5b\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

output
```
(c*(c*x)^(1/3)*(a + b*x^2)^(1/3)*((a + b*x^2)^2*(1 + (b*x^2)/a)^(1/3) - a^
2*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)]))/(5*b*(1 + (b*x^2)/a)^(
1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {248, 248, 262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx$$

$$\Big\downarrow \ 248$$

$$\frac{8}{15} a \int (cx)^{4/3} \sqrt[3]{bx^2+a}dx+\frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c}$$

$$\Big\downarrow \ 248$$

$$\frac{8}{15} a \left(\frac{2}{9} a \int \frac{(cx)^{4/3}}{\left(bx^2+a\right)^{2/3}}dx+\frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c}\right)+\frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c}$$

$$\Big\downarrow \ 262$$

$$\frac{8}{15} a \left(\frac{2}{9} a \left(\frac{c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{b}-\frac{a c^2 \int \frac{1}{(cx)^{2/3} \left(bx^2+a\right)^{2/3}}dx}{3b}\right)+\frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c}\right)+\frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c}$$

$$\Big\downarrow \ 266$$

$$\frac{8}{15} a \left(\frac{2}{9} a \left(\frac{c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{b}-\frac{a c \int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)+\frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c}\right)+\frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c}$$

$$\Big\downarrow \ 771$$

$$\frac{8}{15} a \left(\frac{2}{9} a \left(\frac{c \sqrt[3]{cx} \sqrt[3]{a+bx^2}}{b}-\frac{a c \int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b \sqrt{a+bx^2} \sqrt{\frac{a c^2}{a c^2+b c^2 x^2}}}\right)+\frac{(cx)^{7/3} \sqrt[3]{a+bx^2}}{3c}\right)+\frac{(cx)^{7/3} \left(a+bx^2\right)^{4/3}}{5c}$$

$$\Big\downarrow \text{766}$$

$$\frac{8}{15}a\left(\frac{2}{9}a\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/}}{c^{2/}}\right.\right.}{2\sqrt[4]{3}b\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}}\right.\right.$$

$$\frac{(cx)^{7/3}\left(a+bx^2\right)^{4/3}}{5c}$$

input
```
Int[(c*x)^(4/3)*(a + b*x^2)^(4/3),x]
```

output
```
((c*x)^(7/3)*(a + b*x^2)^(4/3))/(5*c) + (8*a*(((c*x)^(7/3)*(a + b*x^2)^(1/
3))/(3*c) + (2*a*((c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (a*c^(1/3)*(c*x)^(
1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)
^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3)
)^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/
3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*S
qrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-(
(b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt
[3])*b^(1/3)*(c*x)^(2/3))^2]))))/9))/15
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int (cx)^{\frac{4}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}d x$$

input
```
int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}} \, dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*c*x^3 + a*c*x)*(b*x^2 + a)^(1/3)*(c*x)^(1/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.10

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{a^{\frac{4}{3}} c^{\frac{4}{3}} x^{\frac{7}{3}} \Gamma\left(\frac{7}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{7}{6} \\ \frac{13}{6} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(4/3)*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*c**(4/3)*x**(7/3)*gamma(7/6)*hyper((-4/3, 7/6), (13/6,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(13/6))
```

Maxima [F]

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}} \, dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

Giac [F]

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{4}{3}} \, dx$$

input
```
integrate((c*x)^(4/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{4/3} \left(a+bx^2\right)^{4/3} \, dx = \int {\left(c\,x\right)}^{4/3}\,{\left(b\,x^2+a\right)}^{4/3} \,d x$$

input
```
int((c*x)^(4/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(4/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int (cx)^{4/3} (a + bx^2)^{4/3} \, dx = \frac{c^{\frac{4}{3}} \left(48x^{\frac{1}{3}} (b\,x^2+a)^{\frac{1}{3}} a^2 + 153x^{\frac{7}{3}} (b\,x^2+a)^{\frac{1}{3}} ab + 81x^{\frac{13}{3}} (b\,x^2+a)^{\frac{1}{3}} b^2 - 16\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{2}{3}} a + x^{\frac{8}{3}} b} dx\right) a^3\right)}{405b}$$

input
```
int((c*x)^(4/3)*(b*x^2+a)^(4/3),x)
```

output
```
(c**(1/3)*c*(48*x**(1/3)*(a + b*x**2)**(1/3)*a**2 + 153*x**(1/3)*(a + b*x*
*2)**(1/3)*a*b*x**2 + 81*x**(1/3)*(a + b*x**2)**(1/3)*b**2*x**4 - 16*int((
a + b*x**2)**(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*a**3))/(405*b)
```

3.812 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 414

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9c} + \frac{\sqrt[3]{cx}(a+bx^2)^{4/3}}{3c}$$

$$+\frac{8a\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2}}{\sqrt[3]{a+bx^2}}}\right.\right.}{9\sqrt[4]{3}c^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
8/9*a*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c+1/3*(c*x)^(1/3)*(b*x^2+a)^(4/3)/c+8/27
*a*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3
))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/
3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/
3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/
3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/
3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/c^(5/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/
3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2
))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{6},\frac{7}{6},-\frac{bx^2}{a}\right)}{(cx)^{2/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

output
```
(3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/6, 7/6, -((b*x^2)/a)])/
((c*x)^(2/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.85, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {248, 248, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx$$

$\big\downarrow$ 248

$$\frac{8}{9}a\int \frac{\sqrt[3]{bx^2+a}}{(cx)^{2/3}}dx+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}$$

$\big\downarrow$ 248

$$\frac{8}{9}a\left(\frac{2}{3}a\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c}\right)+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}$$

$\big\downarrow$ 266

$$\frac{8}{9}a\left(\frac{2a\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c}\right)+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}$$

$\big\downarrow$ 771

$$\frac{8}{9}a\left(\frac{2a\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{c\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}+\frac{\sqrt[3]{cx}\,\sqrt[3]{a+bx^2}}{c}\right)+\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}$$

$\big\downarrow$ 766

$$\frac{8}{9}a\left(\frac{a\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}(2+\right.}{\sqrt[4]{3}c^{5/3}\sqrt{1-bx^2}\,(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

$$\frac{\sqrt[3]{cx}\,(a+bx^2)^{4/3}}{3c}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(2/3),x]
```

output
```
((c*x)^(1/3)*(a + b*x^2)^(4/3))/(3*c) + (8*a*(((c*x)^(1/3)*(a + b*x^2)^(1/
3))/c + (a*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(
1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b
^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(
c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/
4])/(3^(1/4)*c^(5/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2
 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))
)/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/9
```

Defintions of rubi rules used

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.11

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \frac{a^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{1}{6}\right){}_2F_1\left(\begin{matrix}-\frac{4}{3},\frac{1}{6}\\ \frac{7}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{2}{3}}\Gamma\left(\frac{7}{6}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(2/3),x)
```

output
```
a**(4/3)*x**(1/3)*gamma(1/6)*hyper((-4/3, 1/6), (7/6,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(2/3)*gamma(7/6))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{2/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(2/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(2/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{2/3}}\,dx = \frac{33x^{\frac{1}{3}}(b\,x^2+a)^{\frac{1}{3}}\,a+9x^{\frac{7}{3}}(b\,x^2+a)^{\frac{1}{3}}\,b+16\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{2}{3}}a+x^{\frac{8}{3}}b}dx\right)a^2}{27c^{\frac{2}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(2/3),x)
```

output
```
(33*x**(1/3)*(a + b*x**2)**(1/3)*a + 9*x**(1/3)*(a + b*x**2)**(1/3)*b*x**2
 + 16*int((a + b*x**2)**(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*a**2)/(27*
c**(2/3))
```

3.813 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 414

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{5c^3} - \frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

$$+\frac{8b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx)^{2}}{\sqrt[3]{a+bx^2}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2}}{\sqrt[3]{a+bx^2}}}\right.\right.}{5\sqrt[4]{3}c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
8/5*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)/c^3-3/5*(b*x^2+a)^(4/3)/c/(c*x)^(5/3)+8/
15*b*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1
/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(
2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(
1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(
2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(
1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^
(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(
1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = -\frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{5}{6},\frac{1}{6},-\frac{bx^2}{a}\right)}{5(cx)^{8/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

output
```
(-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -5/6, 1/6, -((b*x^2)/a)]
)/(5*(c*x)^(8/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 248, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx$$

$\big\downarrow$ 247

$$\frac{8b\int \frac{\sqrt[3]{bx^2+a}}{(cx)^{2/3}}dx}{5c^2}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

$\big\downarrow$ 248

$$\frac{8b\left(\frac{2}{3}a\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}\right)}{5c^2}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

$\big\downarrow$ 266

$$\frac{8b\left(\frac{2a\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}\right)}{5c^2}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

$\big\downarrow$ 771

$$\frac{8b\left(\frac{2a\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{c\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{c}\right)}{5c^2}-\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

$\big\downarrow$ 766

$$\frac{8b\left(\frac{a\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}c^{5/3}\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}+\frac{\sqrt[3]{cx}\sqrt[3]{\cdots}}{\cdots}\right)}{5c^2}$$

$$\frac{3(a+bx^2)^{4/3}}{5c(cx)^{5/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(8/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(5*c*(c*x)^(5/3)) + (8*b*(((c*x)^(1/3)*(a + b*x^2)^
(1/3))/c + (a*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) +
b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3]
)*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3
)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3
])/4])/(3^(1/4)*c^(5/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*
c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/
3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/(5*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{4}{3}}}{\left(cx\right)^{\frac{8}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \frac{b^{\frac{4}{3}} x \, {}_{2}F_{1}\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{8}{3}}}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(8/3),x)
```

output
```
b**(4/3)*x*hyper((-4/3, -1/2), (1/2,), a*exp_polar(I*pi)/(b*x**2))/c**(8/3
)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(8/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(8/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \int \frac{{\left(b\,x^2+a\right)}^{4/3}}{{\left(c\,x\right)}^{8/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(8/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(8/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{8/3}}\,dx = \frac{-9(b\,x^2+a)^{\frac{1}{3}}\,a+15(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+16x^{\frac{5}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{2}{3}}\,a+x^{\frac{8}{3}}\,b}dx\right)ab}{15x^{\frac{5}{3}}c^{\frac{8}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(8/3),x)
```

output
```
( - 9*(a + b*x**2)**(1/3)*a + 15*(a + b*x**2)**(1/3)*b*x**2 + 16*x**(2/3)*
int((a + b*x**2)**(1/3)/(x**(2/3)*a + x**(2/3)*b*x**2),x)*a*b*x)/(15*x**(2
/3)*c**(2/3)*c**2*x)
```

3.814 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 419

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = -\frac{24b\sqrt[3]{a+bx^2}}{55c^3(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

$$+\frac{8\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\mathrm{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a\cdots}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a\cdots}}}\cdots\right.\right.}{55ac^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-24/55*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(5/3)-3/11*(b*x^2+a)^(4/3)/c/(c*x)^(11/
3)+8/55*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/
3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*
c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2
/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*
b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2
/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a/c^(17/3)/(-b^(1/3)*(c*x)^
(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/
3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = -\frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{6},-\frac{4}{3},-\frac{5}{6},-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

output
```
(-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-11/6, -4/3, -5/6, -((b*x^2)/a
)])/(11*(c*x)^(14/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.86, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {247, 247, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx$$

$\big\downarrow$ 247

$$\frac{8b\int \frac{\sqrt[3]{bx^2+a}}{(cx)^{8/3}}dx}{11c^2}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

$\big\downarrow$ 247

$$\frac{8b\left(\frac{2b\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{5c^2}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}\right)}{11c^2}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

$\big\downarrow$ 266

$$\frac{8b\left(\frac{6b\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{5c^3}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}\right)}{11c^2}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

$\big\downarrow$ 771

$$\frac{8b\left(\frac{6b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5c^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5c(cx)^{5/3}}\right)}{11c^2}-\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

$\big\downarrow$ 766

$$\frac{8b\left(\frac{3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5c^{11/3}\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{}}{5c}\right)}{11c^2}$$

$$\frac{3(a+bx^2)^{4/3}}{11c(cx)^{11/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(14/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(11*c*(c*x)^(11/3)) + (8*b*((-3*(a + b*x^2)^(1/3))/
(5*c*(c*x)^(5/3)) + (3^(3/4)*b*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))
*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/
3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1
- Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/
3))], (2 + Sqrt[3])/4])/(5*c^(11/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt
[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/
3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/(11*
c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^5*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 177.40 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = -\frac{b^{\frac{4}{3}}\,{}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{14}{3}}x}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(14/3),x)
```

output
```
-b**(4/3)*hyper((-4/3, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(14/3
)*x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(14/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(14/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{14/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(14/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(14/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{14/3}}\,dx = \frac{-9(b\,x^2+a)^{\frac{1}{3}}\,a-33(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2-16x^{\frac{11}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{8}{3}}a+x^{\frac{14}{3}}b}dx\right)ab}{33x^{\frac{11}{3}}c^{\frac{14}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(14/3),x)
```

output
```
( - 9*(a + b*x**2)**(1/3)*a - 33*(a + b*x**2)**(1/3)*b*x**2 - 16*x**(2/3)*
int((a + b*x**2)**(1/3)/(x**(2/3)*a*x**2 + x**(2/3)*b*x**4),x)*a*b*x**3)/(
33*x**(2/3)*c**(2/3)*c**4*x**3)
```

3.815 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 450

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = -\frac{24b\sqrt[3]{a+bx^2}}{187c^3(cx)^{11/3}} - \frac{48b^2\sqrt[3]{a+bx^2}}{935ac^5(cx)^{5/3}} - \frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$$-\frac{24\,3^{3/4}b^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{a}\cdots}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt[3]{a}\cdots}}\right.\right.\cdots}{935a^2c^{23/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-24/187*b*(b*x^2+a)^(1/3)/c^3/(c*x)^(11/3)-48/935*b^2*(b*x^2+a)^(1/3)/a/c^
5/(c*x)^(5/3)-3/17*(b*x^2+a)^(4/3)/c/(c*x)^(17/3)-24/935*3^(3/4)*b^3*(c*x)
^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(
4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^
2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(
1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^
2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4
*6^(1/2)+1/4*2^(1/2))/a^2/c^(23/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = -\frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{17}{6},-\frac{4}{3},-\frac{11}{6},-\frac{bx^2}{a}\right)}{17(cx)^{20/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

output
```
(-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-17/6, -4/3, -11/6, -((b*x^2)/
a)])/(17*(c*x)^(20/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {247, 247, 264, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx$$

$\Big\downarrow$ 247

$$\frac{8b\int \frac{\sqrt[3]{bx^2+a}}{(cx)^{14/3}}dx}{17c^2}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$\Big\downarrow$ 247

$$\frac{8b\left(\frac{2b\int \frac{1}{(cx)^{8/3}(bx^2+a)^{2/3}}dx}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}\right)}{17c^2}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$\Big\downarrow$ 264

$$\frac{8b\left(\frac{2b\left(-\frac{3b\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{5ac^2}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}\right)}{17c^2}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$\Big\downarrow$ 266

$$\frac{8b\left(\frac{2b\left(-\frac{9b\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{5ac^3}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}\right)}{17c^2}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$\Big\downarrow$ 771

$$\frac{8b\left(\frac{2b\left(-\frac{9b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5ac^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11c^2}-\frac{3\sqrt[3]{a+bx^2}}{11c(cx)^{11/3}}\right)}{17c^2}-\frac{3(a+bx^2)^{4/3}}{17c(cx)^{17/3}}$$

$\Big\downarrow$ 766

$$8b\left(\frac{2b\left(-\frac{3\,3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10ac^{11/3}\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{\cdots}}{5a\cdots}\right)}{11c^2}\right.$$

$$\overline{\qquad 17c^2 \qquad}$$

$$\frac{3\left(a+bx^2\right)^{4/3}}{17c(cx)^{17/3}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(20/3),x]
```

output
```
(-3*(a + b*x^2)^(4/3))/(17*c*(c*x)^(17/3)) + (8*b*((-3*(a + b*x^2)^(1/3))/
(11*c*(c*x)^(11/3)) + (2*b*((-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (
3*3^(3/4)*b*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^
(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*
b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*
(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])
/4])/(10*a*c^(11/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2
+ b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))
/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/(11*c^2)))/(17*c^2)
```

Defintions of rubi rules used

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(1/3)/(c^7*x^7), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(20/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx=\int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{20}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(20/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(20/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx=\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{20/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(20/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(20/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{20/3}}\,dx=\frac{-27(b\,x^2+a)^{\frac{1}{3}}\,a-51(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2-16x^{\frac{17}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{14}{3}}a+x^{\frac{20}{3}}b}dx\right)ab}{153x^{\frac{17}{3}}c^{\frac{20}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(20/3),x)
```

output
```
( - 27*(a + b*x**2)**(1/3)*a - 51*(a + b*x**2)**(1/3)*b*x**2 - 16*x**(2/3)
*int((a + b*x**2)**(1/3)/(x**(2/3)*a*x**4 + x**(2/3)*b*x**6),x)*a*b*x**5)/
(153*x**(2/3)*c**(2/3)*c**6*x**5)
```

3.816 $\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
3/5*a*(c*x)^(5/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 5/6],[11/6],-b*x^2/a)/c
/(1+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{3ax(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

output
```
(3*a*x*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -(
(b*x^2)/a)])/(5*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx$$

$$\Big\downarrow \ 279$$

$$\frac{a\sqrt[3]{a+bx^2}\int (cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{4/3}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \ 278$$

$$\frac{3a(cx)^{5/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(c*x)^(2/3)*(a + b*x^2)^(4/3),x]
```

output
```
(3*a*(c*x)^(5/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 5/6, 11/6, -((b
*x^2)/a)])/(5*c*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^{\frac{2}{3}} \left(b\,x^{2}+a\right)^{\frac{4}{3}}dx$$

input
```
int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

output
```
int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

Fricas [F]

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3}\,dx=\int \left(bx^2+a\right)^{\frac{4}{3}}(cx)^{\frac{2}{3}}\,dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{a^{\frac{4}{3}} c^{\frac{2}{3}} x^{\frac{5}{3}} \Gamma\left(\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, \frac{5}{6} \\ \frac{11}{6} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(2/3)*(b*x**2+a)**(4/3),x)
```

output
```
a**(4/3)*c**(2/3)*x**(5/3)*gamma(5/6)*hyper((-4/3, 5/6), (11/6,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(11/6))
```

Maxima [F]

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}} \, dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Giac [F]

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \int \left(bx^2+a\right)^{\frac{4}{3}} (cx)^{\frac{2}{3}} \, dx$$

input
```
integrate((c*x)^(2/3)*(b*x^2+a)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)*(c*x)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \int (c\,x)^{2/3} \left(b\,x^2+a\right)^{4/3} dx$$

input
```
int((c*x)^(2/3)*(a + b*x^2)^(4/3),x)
```

output
```
int((c*x)^(2/3)*(a + b*x^2)^(4/3), x)
```

Reduce [F]

$$\int (cx)^{2/3} \left(a+bx^2\right)^{4/3} \, dx = \frac{c^{\frac{2}{3}} \left(45x^{\frac{5}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}} a+21x^{\frac{11}{3}} \left(b\,x^2+a\right)^{\frac{1}{3}} b+16 \left(\int \frac{x^{\frac{2}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right) a^2\right)}{91}$$

input
```
int((c*x)^(2/3)*(b*x^2+a)^(4/3),x)
```

output
```
(c**(2/3)*(45*x**(2/3)*(a + b*x**2)**(1/3)*a*x + 21*x**(2/3)*(a + b*x**2)*
*(1/3)*b*x**3 + 16*int((x**(2/3)*(a + b*x**2)**(1/3))/(a + b*x**2),x)*a**2
))/91
```

3.817 $\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx=\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
3/2*a*(c*x)^(2/3)*(b*x^2+a)^(1/3)*hypergeom([-4/3, 1/3],[4/3],-b*x^2/a)/c/
(1+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx=\frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

output
```
(3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^2)/a)])/
(2*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx$$

$$\Big\downarrow \; 279$$

$$\frac{a\sqrt[3]{a+bx^2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{4/3}}{\sqrt[3]{cx}}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \; 278$$

$$\frac{3a(cx)^{2/3}\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(1/3),x]
```

output
```
(3*a*(c*x)^(2/3)*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*
x^2)/a)])/(2*c*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx = \frac{a^{\frac{4}{3}}x^{\frac{2}{3}}\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{4}{3},\frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[3]{c}\Gamma\left(\frac{4}{3}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(1/3),x)
```

output
```
a**(4/3)*x**(2/3)*gamma(1/3)*hyper((-4/3, 1/3), (4/3,), b*x**2*exp_polar(I
*pi)/a)/(2*c**(1/3)*gamma(4/3))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(1/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx=\int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{1/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(1/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(1/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{\sqrt[3]{cx}}\,dx=\frac{9x^{\frac{2}{3}}(b\,x^2+a)^{\frac{1}{3}}\,a+3x^{\frac{8}{3}}(b\,x^2+a)^{\frac{1}{3}}\,b+4\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}}{x^{\frac{1}{3}}a+x^{\frac{7}{3}}b}dx\right)a^2}{10c^{\frac{1}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(1/3),x)
```

output
```
(9*x**(2/3)*(a + b*x**2)**(1/3)*a + 3*x**(2/3)*(a + b*x**2)**(1/3)*b*x**2
+ 4*int((a + b*x**2)**(1/3)/(x**(1/3)*a + x**(1/3)*b*x**2),x)*a**2)/(10*c*
*(1/3))
```

3.818 $\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = -\frac{3a\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{1+\frac{bx^2}{a}}}$$

output
```
-3*a*(b*x^2+a)^(1/3)*hypergeom([-4/3, -1/6],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/
(1+b*x^2/a)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = -\frac{3ax\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{(cx)^{4/3}\sqrt[3]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

output
```
(-3*a*x*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)]
)/((c*x)^(4/3)*(1 + (b*x^2)/a)^(1/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt[3]{a+bx^2}\int \frac{\left(\frac{bx^2}{a}+1\right)^{4/3}}{(cx)^{4/3}}dx}{\sqrt[3]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{3a\sqrt[3]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{4}{3},-\frac{1}{6},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\sqrt[3]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(4/3)/(c*x)^(4/3),x]
```

output
```
(-3*a*(a + b*x^2)^(1/3)*Hypergeometric2F1[-4/3, -1/6, 5/6, -((b*x^2)/a)])/
(c*(c*x)^(1/3)*(1 + (b*x^2)/a)^(1/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}dx$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

output
```
int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(4/3)*(c*x)^(2/3)/(c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.32 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \frac{a^{\frac{4}{3}}\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{4}{3}, -\frac{1}{6} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{4}{3}}\sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate((b*x**2+a)**(4/3)/(c*x)**(4/3),x)
```

output
```
a**(4/3)*gamma(-1/6)*hyper((-4/3, -1/6), (5/6,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(4/3)*x**(1/3)*gamma(5/6))
```

Maxima [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \int \frac{(bx^2+a)^{\frac{4}{3}}}{(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate((b*x^2+a)^(4/3)/(c*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(4/3)/(c*x)^(4/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \int \frac{(b\,x^2+a)^{4/3}}{(c\,x)^{4/3}}\,dx$$

input
```
int((a + b*x^2)^(4/3)/(c*x)^(4/3),x)
```

output
```
int((a + b*x^2)^(4/3)/(c*x)^(4/3), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{4/3}}{(cx)^{4/3}}\,dx = \frac{-21(b\,x^2+a)^{\frac{1}{3}}\,a+3(b\,x^2+a)^{\frac{1}{3}}\,b\,x^2+16x^{\frac{1}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{1}{3}}\,x}{x^{\frac{1}{3}}\,a+x^{\frac{7}{3}}\,b}dx\right)ab}{7x^{\frac{1}{3}}c^{\frac{4}{3}}}$$

input
```
int((b*x^2+a)^(4/3)/(c*x)^(4/3),x)
```

output
```
( - 21*(a + b*x**2)**(1/3)*a + 3*(a + b*x**2)**(1/3)*b*x**2 + 16*x**(1/3)*
int(((a + b*x**2)**(1/3)*x)/(x**(1/3)*a + x**(1/3)*b*x**2),x)*a*b)/(7*x**(
1/3)*c**(1/3)*c)
```

3.819 $\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 198

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \frac{10a^2c^5(cx)^{4/3}\sqrt[3]{a+bx^2}}{27b^3} - \frac{2ac^3(cx)^{10/3}\sqrt[3]{a+bx^2}}{9b^2} + \frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} + \frac{20a^3c^{19/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{27\sqrt{3}b^{11/3}} + \frac{10a^3c^{19/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{27b^{11/3}}$$

output
```
10/27*a^2*c^5*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^3-2/9*a*c^3*(c*x)^(10/3)*(b*x^
2+a)^(1/3)/b^2+1/6*c*(c*x)^(16/3)*(b*x^2+a)^(1/3)/b+20/81*a^3*c^(19/3)*arc
tan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)
/b^(11/3)+10/27*a^3*c^(19/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3
))/b^(11/3)
```

Mathematica [A] (verified)

Time = 5.81 (sec), antiderivative size = 233, normalized size of antiderivative = 1.18

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^6\sqrt[3]{cx}\left(60a^2b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}-36ab^{5/3}x^{10/3}\sqrt[3]{a+bx^2}+27b^{8/3}x^{16/3}\sqrt[3]{a+bx^2}+40\sqrt{}\right.}{}$$

input
```
Integrate[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c^6*(c*x)^(1/3)*(60*a^2*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) - 36*a*b^(5/3)*
x^(10/3)*(a + b*x^2)^(1/3) + 27*b^(8/3)*x^(16/3)*(a + b*x^2)^(1/3) + 40*Sq
rt[3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2
)^(1/3))] + 40*a^3*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - 20*a^3*Lo
g[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]
))/(162*b^(11/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec), antiderivative size = 216, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {262, 262, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}-\frac{8ac^2\int \frac{(cx)^{13/3}}{(bx^2+a)^{2/3}}dx}{9b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}-\frac{8ac^2\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\int\frac{(cx)^{7/3}}{\left(bx^2+a\right)^{2/3}}dx}{6b}\right)}{9b}$$

$$\Big\downarrow \ 262$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}-\frac{8ac^2\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int\frac{\sqrt[3]{cx}}{\left(bx^2+a\right)^{2/3}}dx}{3b}\right)}{6b}\right)}{9b}$$

$$\Big\downarrow \ 266$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}-\frac{8ac^2\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int\frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{6b}\right)}{9b}$$

$$\Big\downarrow \ 807$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b}-\frac{8ac^2\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int\frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)}{6b}\right)}{9b}$$

$$\Big\downarrow \ 853$$

$$\frac{c(cx)^{16/3}\sqrt[3]{a+bx^2}}{6b} - \frac{8ac^2\left(\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} - \frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}} - \frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}} - \sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right.}{b}\right.}{6b}\right.}{9b}$$

input
```
Int[(c*x)^(19/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(16/3)*(a + b*x^2)^(1/3))/(6*b) - (8*a*c^2*((c*(c*x)^(10/3)*(a +
b*x^2)^(1/3))/(4*b) - (5*a*c^2*((c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) -
(a*c*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c
)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3)
)/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/b))/(6*b)))/(9*b)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{19}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(19/3)/(b*x**2+a)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{19}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{19}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(19/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(19/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{19/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(19/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(19/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{19/3}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{19}{3}}\left(\int \frac{x^{\frac{19}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(19/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int((x**(1/3)*x**6)/(a + b*x**2)**(2/3),x)*c**6
```

3.820 $\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = -\frac{5ac^3(cx)^{4/3}\sqrt[3]{a+bx^2}}{12b^2}+\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b} - \frac{5a^2c^{13/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{6\sqrt{3}b^{8/3}} - \frac{5a^2c^{13/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{12b^{8/3}}$$

output
```
-5/12*a*c^3*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b^2+1/4*c*(c*x)^(10/3)*(b*x^2+a)^(
1/3)/b-5/18*a^2*c^(13/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^
2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(8/3)-5/12*a^2*c^(13/3)*ln(b^(1/3)*(c*x)^(2
/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(8/3)
```

Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.24

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^4\sqrt[3]{cx}\left(-15ab^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+9b^{5/3}x^{10/3}\sqrt[3]{a+bx^2}-10\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a}}\right.\right.}{}$$

input
```
Integrate[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c^4*(c*x)^(1/3)*(-15*a*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 9*b^(5/3)*x^(1
0/3)*(a + b*x^2)^(1/3) - 10*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(
b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] - 10*a^2*Log[-(b^(1/3)*x^(2/3)) +
(a + b*x^2)^(1/3)] + 5*a^2*Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^
2)^(1/3) + (a + b*x^2)^(2/3)]))/(36*b^(8/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {262, 262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\int \frac{(cx)^{7/3}}{(bx^2+a)^{2/3}}dx}{6b}$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx}{3b}\right)}{6b}$$

$\Big\downarrow$ 266

$$\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int \frac{cx}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{6b}$$

$\Big\downarrow$ 807

$$\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}\right)}{6b}$$

$\Big\downarrow$ 853

$$\frac{c(cx)^{10/3}\sqrt[3]{a+bx^2}}{4b}-\frac{5ac^2\left(\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{b}\right)}{6b}$$

input
```
Int[(c*x)^(13/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(10/3)*(a + b*x^2)^(1/3))/(4*b) - (5*a*c^2*((c*(c*x)^(4/3)*(a + b
*x^2)^(1/3))/(2*b) - (a*c*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/
(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log
[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/b))/(
6*b)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{13}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(13/3)/(b*x**2+a)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{13}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(13/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(13/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{13/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(13/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(13/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{13/3}}{\left(a+bx^2\right)^{2/3}}\,dx = c^{\frac{13}{3}}\left(\int \frac{x^{\frac{13}{3}}}{\left(b\,x^2+a\right)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(13/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int((x**(1/3)*x**4)/(a + b*x**2)**(2/3),x)*c**4
```

3.821 $\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b} + \frac{ac^{7/3}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{\sqrt{3}b^{5/3}} + \frac{ac^{7/3}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{2b^{5/3}}$$

output
```
1/2*c*(c*x)^(4/3)*(b*x^2+a)^(1/3)/b+1/3*a*c^(7/3)*arctan(1/3*(1+2*b^(1/3)*
(c*x)^(2/3)/c^(2/3)/(b*x^2+a)^(1/3))*3^(1/2))*3^(1/2)/b^(5/3)+1/2*a*c^(7/3
)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2+a)^(1/3))/b^(5/3)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.33

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \frac{(cx)^{7/3}\left(3b^{2/3}x^{4/3}\sqrt[3]{a+bx^2}+2\sqrt{3}a\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)+2a\log\left(-\sqrt[3]{b}x^{2/3}\right.\right.}{6b^{5/3}x^{7/3}}$$

input
```
Integrate[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

output
```
((c*x)^(7/3)*(3*b^(2/3)*x^(4/3)*(a + b*x^2)^(1/3) + 2*Sqrt[3]*a*ArcTan[(Sq
rt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3) + 2*(a + b*x^2)^(1/3))] + 2*a*Log[
-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] - a*Log[b^(2/3)*x^(4/3) + b^(1/3)*
x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/3)]))/(6*b^(5/3)*x^(7/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {262, 266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac^2\int \frac{\sqrt[3]{cx}}{(bx^2+a)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{2ac\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\int\frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{b}$$

$\Big\downarrow$ 853

$$\frac{c(cx)^{4/3}\sqrt[3]{a+bx^2}}{2b}-\frac{ac\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{b}$$

input
```
Int[(c*x)^(7/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(4/3)*(a + b*x^2)^(1/3))/(2*b) - (a*c*(-((c^(4/3)*ArcTan[(1 + (2*
b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(
2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/c^(2/3) - (a + (b*x)/c)^(1/3)]
)/(2*b^(2/3))))/b
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{7}{3}}}{(b\,x^{2}+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 15.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.34

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^{\frac{7}{3}}x^{\frac{10}{3}}\Gamma\left(\frac{5}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{3}\\ \frac{8}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{8}{3}\right)}$$

input
```
integrate((c*x)**(7/3)/(b*x**2+a)**(2/3),x)
```

output
```
c**(7/3)*x**(10/3)*gamma(5/3)*hyper((2/3, 5/3), (8/3,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(2/3)*gamma(8/3))
```

Maxima [F]

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{7}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(7/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{7/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(7/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(7/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/3}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{7}{3}}\left(\int \frac{x^{\frac{7}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(7/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int((x**(1/3)*x**2)/(a + b*x**2)**(2/3),x)*c**2
```

3.822 $\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 106

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = -\frac{\sqrt{3}\sqrt[3]{c}\arctan\left(\frac{1+\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+bx^2}}}{\sqrt{3}}\right)}{2b^{2/3}} - \frac{3\sqrt[3]{c}\log\left(\sqrt[3]{b}(cx)^{2/3}-c^{2/3}\sqrt[3]{a+bx^2}\right)}{4b^{2/3}}$$

output
```
-1/2*3^(1/2)*c^(1/3)*arctan(1/3*(1+2*b^(1/3)*(c*x)^(2/3)/c^(2/3)/(b*x^2+a)
^(1/3))*3^(1/2))/b^(2/3)-3/4*c^(1/3)*ln(b^(1/3)*(c*x)^(2/3)-c^(2/3)*(b*x^2
+a)^(1/3))/b^(2/3)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = \frac{\sqrt[3]{cx}\left(-2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}x^{2/3}}{\sqrt[3]{b}x^{2/3}+2\sqrt[3]{a+bx^2}}\right)-2\log\left(-\sqrt[3]{b}x^{2/3}+\sqrt[3]{a+bx^2}\right)+\log\left(b^{2/}\right.\right.}{4b^{2/3}\sqrt[3]{x}}$$

input
```
Integrate[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

output
```
((c*x)^(1/3)*(-2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x^(2/3))/(b^(1/3)*x^(2/3)
 + 2*(a + b*x^2)^(1/3))] - 2*Log[-(b^(1/3)*x^(2/3)) + (a + b*x^2)^(1/3)] +
 Log[b^(2/3)*x^(4/3) + b^(1/3)*x^(2/3)*(a + b*x^2)^(1/3) + (a + b*x^2)^(2/
3)]))/(4*b^(2/3)*x^(1/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.21 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {266, 807, 853}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{3\int \frac{cx}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}$$

$$\Big\downarrow \text{807}$$

$$\frac{3\int \frac{(cx)^{2/3}}{\left(a+\frac{bx}{c}\right)^{2/3}}d(cx)^{2/3}}{2c}$$

$$\Big\downarrow \text{853}$$

$$\frac{3\left(-\frac{c^{4/3}\arctan\left(\frac{\frac{2\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}\sqrt[3]{a+\frac{bx}{c}}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{2/3}}-\frac{c^{4/3}\log\left(\frac{\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}}-\sqrt[3]{a+\frac{bx}{c}}\right)}{2b^{2/3}}\right)}{2c}$$

input
```
Int[(c*x)^(1/3)/(a + b*x^2)^(2/3),x]
```

output
```
(3*(-((c^(4/3)*ArcTan[(1 + (2*b^(1/3)*(c*x)^(2/3))/(c^(2/3)*(a + (b*x)/c)^
(1/3)))/Sqrt[3]])/(Sqrt[3]*b^(2/3))) - (c^(4/3)*Log[(b^(1/3)*(c*x)^(2/3))/
c^(2/3) - (a + (b*x)/c)^(1/3)])/(2*b^(2/3))))/(2*c)
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 853
```
Int[(x_)/((a_) + (b_.)*(x_)^3)^(2/3), x_Symbol] :> With[{q = Rt[b, 3]}, Sim
p[-ArcTan[(1 + 2*q*(x/(a + b*x^3)^(1/3)))/Sqrt[3]]/(Sqrt[3]*q^2), x] - Simp
[Log[q*x - (a + b*x^3)^(1/3)]/(2*q^2), x]] /; FreeQ[{a, b}, x]
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = \frac{\sqrt[3]{c}x^{\frac{4}{3}}\Gamma\left(\frac{2}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\ \frac{5}{3}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{5}{3}\right)}$$

input
```
integrate((c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

output
```
c**(1/3)*x**(4/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(2/3)*gamma(5/3))
```

Maxima [F]

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(cx)^{\frac{1}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(cx)^{\frac{1}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(c\,x)^{1/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(1/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(1/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{\sqrt[3]{cx}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{1}{3}}\left(\int \frac{x^{\frac{1}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int(x**(1/3)/(a + b*x**2)**(2/3),x)
```

3.823 $\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

output
```
-3/2*(b*x^2+a)^(1/3)/a/c/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3x\sqrt[3]{a+bx^2}}{2a(cx)^{5/3}}$$

input
```
Integrate[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3))/(2*a*(c*x)^(5/3))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx$$
$$\Big\downarrow \text{ 242}$$
$$-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{2/3}}$$

input
```
Int[1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(2/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{3x(b\,x^2+a)^{\frac{1}{3}}}{2a(cx)^{\frac{5}{3}}}$	21
orering	$-\frac{3x(b\,x^2+a)^{\frac{1}{3}}}{2a(cx)^{\frac{5}{3}}}$	21
risch	$-\frac{3(b\,x^2+a)^{\frac{1}{3}}}{2ac(cx)^{\frac{2}{3}}}$	23

input
```
int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/2*x*(b*x^2+a)^(1/3)/a/(c*x)^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)^{2/3}}\,dx = -\frac{3\,(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{2\,ac^2x}$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
-3/2*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{1}{(cx)^{5/3}(a+bx^2)^{2/3}}\,dx = \frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{1}{3}\right)}{2ac^{\frac{5}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(c*x)**(5/3)/(b*x**2+a)**(2/3),x)
```

output
```
b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-1/3)/(2*a*c**(5/3)*gamma(2/3))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\left(bc^{\frac{1}{3}}x^3+ac^{\frac{1}{3}}x\right)}{2\left(bx^2+a\right)^{\frac{2}{3}}ac^2x^{\frac{5}{3}}}$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
-3/2*(b*c^(1/3)*x^3 + a*c^(1/3)*x)/((b*x^2 + a)^(2/3)*a*c^2*x^(5/3))
```

Giac [F]

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(5/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = \int \frac{1}{(c\,x)^{5/3}\,\left(b\,x^2+a\right)^{2/3}}\,dx$$

input
```
int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(5/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{\int \frac{1}{x^{\frac{5}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}dx}{c^{\frac{5}{3}}}$$

input
```
int(1/(c*x)^(5/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x),x)/(c**(2/3)*c)
```

3.824 $\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{8ac(cx)^{8/3}}+\frac{9b\sqrt[3]{a+bx^2}}{8a^2c^3(cx)^{2/3}}$$

output
```
-3/8*(b*x^2+a)^(1/3)/a/c/(c*x)^(8/3)+9/8*b*(b*x^2+a)^(1/3)/a^2/c^3/(c*x)^(
2/3)
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3x(a-3bx^2)\sqrt[3]{a+bx^2}}{8a^2(cx)^{11/3}}$$

input
```
Integrate[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(a - 3*b*x^2)*(a + b*x^2)^(1/3))/(8*a^2*(c*x)^(11/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/3} (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \ 246$$

$$-\frac{3 \int \frac{\sqrt[3]{bx^2+a}}{(cx)^{11/3}}dx}{a} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

$$\Big\downarrow \ 242$$

$$\frac{9\left(a+bx^2\right)^{4/3}}{8a^2c(cx)^{8/3}} - \frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{8/3}}$$

input
```
Int[1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(8/3)) + (9*(a + b*x^2)^(4/3))/(8*a^2*
c*(c*x)^(8/3))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)}{8a^{2}\left(cx\right)^{\frac{11}{3}}}$	29
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)}{8a^{2}\left(cx\right)^{\frac{11}{3}}}$	29
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-3b\,x^{2}+a\right)}{8c^{3}\left(cx\right)^{\frac{2}{3}}a^{2}x^{2}}$	34

input
```
int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x*(b*x^2+a)^(1/3)*(-3*b*x^2+a)/a^2/(c*x)^(11/3)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx=\frac{3\,(3\,bx^2-a)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{8\,a^2c^4x^3}$$

input
```
integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/8*(3*b*x^2 - a)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^2*c^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 68.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{1}{(cx)^{11/3} (a+bx^2)^{2/3}}\,dx = -\frac{\sqrt[3]{b}\sqrt[3]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{4}{3}\right)}{6ac^{\frac{11}{3}}x^2\Gamma\left(\frac{2}{3}\right)} + \frac{b^{\frac{4}{3}}\sqrt[3]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{4}{3}\right)}{2a^2c^{\frac{11}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(c*x)**(11/3)/(b*x**2+a)**(2/3),x)
```

output
```
-b**(1/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(6*a*c**(11/3)*x**2*gamma(2/
3)) + b**(4/3)*(a/(b*x**2) + 1)**(1/3)*gamma(-4/3)/(2*a**2*c**(11/3)*gamma
(2/3))
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/3} (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/3} (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}(cx)^{\frac{11}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(11/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{11/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

input
```
int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(11/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{11/3}\left(a+bx^2\right)^{2/3}}\,dx=\frac{\int \frac{1}{x^{\frac{11}{3}} \left(b \,x^{2}+a \right)^{\frac{2}{3}}}d x}{c^{\frac{11}{3}}}$$

input
```
int(1/(c*x)^(11/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x**3),x)/(c**(2/3)*c**3)
```

3.825 $\int \frac{1}{(cx)^{17/3}(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{14ac(cx)^{14/3}}+\frac{9b\sqrt[3]{a+bx^2}}{28a^2c^3(cx)^{8/3}}-\frac{27b^2\sqrt[3]{a+bx^2}}{28a^3c^5(cx)^{2/3}}$$

output
```
-3/14*(b*x^2+a)^(1/3)/a/c/(c*x)^(14/3)+9/28*b*(b*x^2+a)^(1/3)/a^2/c^3/(c*x
)^(8/3)-27/28*b^2*(b*x^2+a)^(1/3)/a^3/c^5/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 3.55 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3x\sqrt[3]{a+bx^2}(2a^2-3abx^2+9b^2x^4)}{28a^3(cx)^{17/3}}$$

input
```
Integrate[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*(2*a^2 - 3*a*b*x^2 + 9*b^2*x^4))/(28*a^3*(c*x)^(17
/3))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{17/3} (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \; 246$$

$$-\frac{6 \int \frac{\sqrt[3]{bx^2+a}}{(cx)^{17/3}}dx}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

$$\Big\downarrow \; 246$$

$$-\frac{6 \left(-\frac{3 \int \frac{\left(bx^2+a\right)^{4/3}}{(cx)^{17/3}}dx}{4a}-\frac{3 \left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}\right)}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

$$\Big\downarrow \; 242$$

$$-\frac{6 \left(\frac{9 \left(a+bx^2\right)^{7/3}}{56a^2c(cx)^{14/3}}-\frac{3 \left(a+bx^2\right)^{4/3}}{8ac(cx)^{14/3}}\right)}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2ac(cx)^{14/3}}$$

input
```
Int[1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(14/3)) - (6*((-3*(a + b*x^2)^(4/3))/(
8*a*c*(c*x)^(14/3)) + (9*(a + b*x^2)^(7/3))/(56*a^2*c*(c*x)^(14/3))))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)}{28a^{3}\left(cx\right)^{\frac{17}{3}}}$	42
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)}{28a^{3}\left(cx\right)^{\frac{17}{3}}}$	42
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(9b^{2}x^{4}-3ab\,x^{2}+2a^{2}\right)}{28c^{5}\left(cx\right)^{\frac{2}{3}}a^{3}x^{4}}$	47

input
```
int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/28*x*(b*x^2+a)^(1/3)*(9*b^2*x^4-3*a*b*x^2+2*a^2)/a^3/(c*x)^(17/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{17/3}\,(a+bx^2)^{2/3}}\,dx = -\frac{3\,(9\,b^2x^4-3\,abx^2+2\,a^2)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{28\,a^3c^6x^5}$$

input
```
integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
-3/28*(9*b^2*x^4 - 3*a*b*x^2 + 2*a^2)*(b*x^2 + a)^(1/3)*(c*x)^(1/3)/(a^3*c
^6*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{17/3}\,(a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(17/3)/(b*x**2+a)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{17/3}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}\,(cx)^{\frac{17}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{17/3}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}\,(cx)^{\frac{17}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(17/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{{\left(c\,x\right)}^{17/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

input
```
int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(17/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{17/3}\left(a+bx^2\right)^{2/3}}\,dx=\frac{\int \frac{1}{x^{\frac{17}{3}} \left(b \,x^{2}+a \right)^{\frac{2}{3}}}d x}{c^{\frac{17}{3}}}$$

input
```
int(1/(c*x)^(17/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x**5),x)/(c**(2/3)*c**5)
```

3.826 $\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{20ac(cx)^{20/3}} + \frac{27b\sqrt[3]{a+bx^2}}{140a^2c^3(cx)^{14/3}} - \frac{81b^2\sqrt[3]{a+bx^2}}{280a^3c^5(cx)^{8/3}} + \frac{243b^3\sqrt[3]{a+bx^2}}{280a^4c^7(cx)^{2/3}}$$

output
```
-3/20*(b*x^2+a)^(1/3)/a/c/(c*x)^(20/3)+27/140*b*(b*x^2+a)^(1/3)/a^2/c^3/(c
*x)^(14/3)-81/280*b^2*(b*x^2+a)^(1/3)/a^3/c^5/(c*x)^(8/3)+243/280*b^3*(b*x
^2+a)^(1/3)/a^4/c^7/(c*x)^(2/3)
```

Mathematica [A] (verified)

Time = 5.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = -\frac{3x\sqrt[3]{a+bx^2}(14a^3-18a^2bx^2+27ab^2x^4-81b^3x^6)}{280a^4(cx)^{23/3}}$$

input
```
Integrate[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(a + b*x^2)^(1/3)*(14*a^3 - 18*a^2*b*x^2 + 27*a*b^2*x^4 - 81*b^3*x^6
))/(280*a^4*(c*x)^(23/3))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{23/3} \left(a+bx^2\right)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{9 \int \frac{\sqrt[3]{bx^2+a}}{(cx)^{23/3}}dx}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2 a c (cx)^{20/3}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{9 \left(-\frac{3 \int \frac{\left(bx^2+a\right)^{4/3}}{(cx)^{23/3}}dx}{2 a}-\frac{3 \left(a+bx^2\right)^{4/3}}{8 a c (cx)^{20/3}}\right)}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2 a c (cx)^{20/3}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{9 \left(-\frac{3 \left(-\frac{3 \int \frac{\left(bx^2+a\right)^{7/3}}{(cx)^{23/3}}dx}{7 a}-\frac{3 \left(a+bx^2\right)^{7/3}}{14 a c (cx)^{20/3}}\right)}{2 a}-\frac{3 \left(a+bx^2\right)^{4/3}}{8 a c (cx)^{20/3}}\right)}{a}-\frac{3 \sqrt[3]{a+bx^2}}{2 a c (cx)^{20/3}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{9\left(-\frac{3\left(\frac{9(a+bx^2)^{10/3}}{140a^2c(cx)^{20/3}}-\frac{3(a+bx^2)^{7/3}}{14ac(cx)^{20/3}}\right)}{2a}-\frac{3(a+bx^2)^{4/3}}{8ac(cx)^{20/3}}\right)}{a}-\frac{3\sqrt[3]{a+bx^2}}{2ac(cx)^{20/3}}$$

input
```
Int[1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(2*a*c*(c*x)^(20/3)) - (9*((-3*(a + b*x^2)^(4/3))/(
8*a*c*(c*x)^(20/3)) - (3*((-3*(a + b*x^2)^(7/3))/(14*a*c*(c*x)^(20/3)) + (
9*(a + b*x^2)^(10/3))/(140*a^2*c*(c*x)^(20/3))))/(2*a)))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{3}x^{6}+27a\,b^{2}x^{4}-18a^{2}b\,x^{2}+14a^{3}\right)}{280a^{4}\left(cx\right)^{\frac{23}{3}}}$	53
orering	$-\frac{3x\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{3}x^{6}+27a\,b^{2}x^{4}-18a^{2}b\,x^{2}+14a^{3}\right)}{280a^{4}\left(cx\right)^{\frac{23}{3}}}$	53
risch	$-\frac{3\left(b\,x^{2}+a\right)^{\frac{1}{3}}\left(-81b^{3}x^{6}+27a\,b^{2}x^{4}-18a^{2}b\,x^{2}+14a^{3}\right)}{280c^{7}\left(cx\right)^{\frac{2}{3}}a^{4}x^{6}}$	58

input
```
int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3/280*x*(b*x^2+a)^(1/3)*(-81*b^3*x^6+27*a*b^2*x^4-18*a^2*b*x^2+14*a^3)/a^
4/(c*x)^(23/3)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.48

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{3\,(81\,b^3x^6-27\,ab^2x^4+18\,a^2bx^2-14\,a^3)(bx^2+a)^{\frac{1}{3}}(cx)^{\frac{1}{3}}}{280\,a^4c^8x^7}$$

input
```
integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
3/280*(81*b^3*x^6 - 27*a*b^2*x^4 + 18*a^2*b*x^2 - 14*a^3)*(b*x^2 + a)^(1/3
)*(c*x)^(1/3)/(a^4*c^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{23/3}\left(a+bx^2\right)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(23/3)/(b*x**2+a)**(2/3),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{23/3} (a+bx^2)^{2/3}} \, dx = \frac{3 \left(81 \, b^4 x^9 + 54 \, a b^3 x^7 - 9 \, a^2 b^2 x^5 + 4 \, a^3 b x^3 - 14 \, a^4 x\right)}{280 \, {\left(b x^2 + a\right)}^{\frac{2}{3}} a^4 c^{\frac{23}{3}} x^{\frac{23}{3}}}$$

input
```
integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
3/280*(81*b^4*x^9 + 54*a*b^3*x^7 - 9*a^2*b^2*x^5 + 4*a^3*b*x^3 - 14*a^4*x)
/((b*x^2 + a)^(2/3)*a^4*c^(23/3)*x^(23/3))
```

Giac [F]

$$\int \frac{1}{(cx)^{23/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{{\left(b x^2 + a\right)}^{\frac{2}{3}} \left(c x\right)^{\frac{23}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(23/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{23/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{{\left(c\,x\right)}^{23/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

input
```
int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(23/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{23/3} (a+bx^2)^{2/3}} \, dx = \frac{\int \frac{1}{x^{\frac{23}{3}} (b\,x^2+a)^{\frac{2}{3}}}dx}{c^{\frac{23}{3}}}$$

input
```
int(1/(c*x)^(23/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x**7),x)/(c**(2/3)*c**7)
```

3.827 $\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 421

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = -\frac{7ac^3\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{9b^2}+\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b} + \frac{7ac^{7/3}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}\cdots}{\sqrt[3]{a+\cdots}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}\cdots}{\sqrt[3]{a+\cdots}}}\right.\cdots\right.}{18\sqrt[4]{3}b^2\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-7/9*a*c^3*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b^2+1/3*c*(c*x)^(7/3)*(b*x^2+a)^(1/
3)/b+7/54*a*c^(7/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/
3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*
c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2
/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*
b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2
/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/b^2/(-b^(1/3)*(c*x)
^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2
/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.21

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^3\sqrt[3]{cx}\left(-7a^2-4abx^2+3b^2x^4+7a^2\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},-\frac{bx^2}{a}\right)\right.}{9b^2\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c^3*(c*x)^(1/3)*(-7*a^2 - 4*a*b*x^2 + 3*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(
2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)]))/(9*b^2*(a + b*x^2)^(
2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {262, 262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\int \frac{(cx)^{4/3}}{(bx^2+a)^{2/3}}dx}{9b}$$

$$\Big\downarrow \; 262$$

$$\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{(cx)^{2/3}\left(bx^2+a\right)^{2/3}}dx}{3b}\right)}{9b}$$

$$\Big\downarrow \; 266$$

$$\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{b}\right)}{9b}$$

$$\Big\downarrow \; 771$$

$$\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}\right)}{9b}$$

$$\Big\downarrow \; 766$$

$$\frac{c(cx)^{7/3}\sqrt[3]{a+bx^2}}{3b}-$$

$$\frac{7ac^2\left(\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right.\right.}{2\sqrt[4]{3}b\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}\right)}{9b}$$

input
```
Int[(c*x)^(10/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(7/3)*(a + b*x^2)^(1/3))/(3*b) - (7*a*c^2*((c*(c*x)^(1/3)*(a + b*
x^2)^(1/3))/b - (a*c^(1/3)*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqr
t[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) -
 (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sq
rt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))]
, (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a
*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*
(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])))/(9*b)
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(cx)^{\frac{10}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{10}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/3)*c^3*x^3/(b*x^2 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 74.79 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.10

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^{\frac{10}{3}}x^{\frac{13}{3}}\Gamma\left(\frac{13}{6}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{13}{6}\\\frac{19}{6}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{19}{6}\right)}$$

input
```
integrate((c*x)**(10/3)/(b*x**2+a)**(2/3),x)
```

output
```
c**(10/3)*x**(13/3)*gamma(13/6)*hyper((2/3, 13/6), (19/6,), b*x**2*exp_pol
ar(I*pi)/a)/(2*a**(2/3)*gamma(19/6))
```

Maxima [F]

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(cx)^{\frac{10}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(cx)^{\frac{10}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(10/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(10/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx=\int \frac{(c\,x)^{10/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(10/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(10/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{10/3}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{10}{3}}\left(\int \frac{x^{\frac{10}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(10/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int((x**(1/3)*x**3)/(a + b*x**2)**(2/3),x)*c**3
```

3.828 $\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 388

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b} - \frac{\sqrt[3]{c}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{b}(cx}{\sqrt[3]{a+bx}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx}{\sqrt[3]{a+bx}}}\right.\right.}{2\sqrt[4]{3}b\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
c*(c*x)^(1/3)*(b*x^2+a)^(1/3)/b-1/6*c^(1/3)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c
^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/
(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3
^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arcc
os((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3
^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(
3/4)/b/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.17

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c\sqrt[3]{cx}\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},-\frac{bx^2}{a}\right)\right)}{b\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(1/3)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6,
 2/3, 7/6, -((b*x^2)/a)]))/(b*(a + b*x^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {262, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{3b}$$

$$\Big\downarrow \text{266}$$

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{b}$$

$$\Big\downarrow \text{771}$$

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{ac\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{b\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

$$\Big\downarrow \text{766}$$

$$\frac{c\sqrt[3]{cx}\sqrt[3]{a+bx^2}}{b}-\frac{a\sqrt[3]{c}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}(2+\sqrt{3})\right)}{2\sqrt[4]{3}b\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

input
```
Int[(c*x)^(4/3)/(a + b*x^2)^(2/3),x]
```

output
```
(c*(c*x)^(1/3)*(a + b*x^2)^(1/3))/b - (a*c^(1/3)*(c*x)^(1/3)*(c^(2/3) - b^
(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(
c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[Arc
Cos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])
*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(2*3^(1/4)*b*Sqrt[1 - b*x^2]*(a
+ b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/
3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)
^(2/3))^2)])
```

Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{(cx)^{\frac{4}{3}}}{(b\,x^{2}+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/3)*c*x/(b*x^2 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.11

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^{\frac{4}{3}}x^{\frac{7}{3}}\Gamma\left(\frac{7}{6}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{6}\\ \frac{13}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{13}{6}\right)}$$

input
```
integrate((c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

output
```
c**(4/3)*x**(7/3)*gamma(7/6)*hyper((2/3, 7/6), (13/6,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(2/3)*gamma(13/6))
```

Maxima [F]

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{4}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(4/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{4/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(4/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(4/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{4/3}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{4}{3}}\left(\int \frac{x^{\frac{4}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(1/3)*int((x**(1/3)*x)/(a + b*x**2)**(2/3),x)*c
```

3.829 $\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 364

$$\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{3^{3/4}\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \text{EllipticF}}{2ac^{5/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(c}{\sqrt[3]{a+bx}}\right)}}}$$

output
```
1/2*3^(3/4)*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^
2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x
^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x
^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a/c^(5/3)/(-b^(1/3)*(c*x)^(2/3)*(c^
(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(
1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.15

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx=\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{2}{3},\frac{7}{6},-\frac{bx^2}{a}\right)}{(cx)^{2/3}(a+bx^2)^{2/3}}$$

input
```
Integrate[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/6, 2/3, 7/6, -((b*x^2)/a)])
/((c*x)^(2/3)*(a + b*x^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{2/3}(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 266$$

$$\frac{3\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{c}$$

$$\Big\downarrow \ 771$$

$$\frac{3\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{c\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

$$\Big\downarrow \ 766$$

$$\frac{3^{3/4}\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{}\right.\right.}{2c^{5/3}\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

input
```
Int[1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(3^(3/4)*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/
3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(
1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*
x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4]
)/(2*c^(5/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2
*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/
3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)])
```

Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{2}{3}} (b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{2/3} (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.09

$$\int \frac{1}{(cx)^{2/3} (a+bx^2)^{2/3}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{2}{3}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{2}{3}}c^{\frac{2}{3}}x}$$

input
```
integrate(1/(c*x)**(2/3)/(b*x**2+a)**(2/3),x)
```

output
```
-hyper((1/2, 2/3), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(2/3)*c**(2/3)*
x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{2/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(2/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{2/3} \left(a+bx^2\right)^{2/3}} \, dx=\int \frac{1}{{\left(c\,x\right)}^{2/3}\,{\left(b\,x^2+a\right)}^{2/3}} \,d x$$

input
```
int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(2/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{2/3} \left(a+bx^2\right)^{2/3}} \, dx=\frac{\int \frac{1}{x^{\frac{2}{3}} \left(b \,x^{2}+a \right)^{\frac{2}{3}}}d x}{c^{\frac{2}{3}}}$$

input
```
int(1/(c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)),x)/c**(2/3)
```

3.830 $\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 394

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx = -\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

$$-\frac{3\,3^{3/4}b\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{(a+bx^2)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\ \mathrm{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{(1-\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}{c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{\cdots}}{\sqrt[3]{a+\cdots}}}\right.\right.}{10a^2c^{11/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{(1+\sqrt{3})\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-3/5*(b*x^2+a)^(1/3)/a/c/(c*x)^(5/3)-3/10*3^(3/4)*b*(c*x)^(1/3)*(b*x^2+a)^
(1/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x
)^(4/3)/(b*x^2+a)^(2/3)+b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2
/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((c^(2/3)-(1-3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2
/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1
/2))/a^2/c^(11/3)/(-b^(1/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^
2+a)^(1/3))/(b*x^2+a)^(1/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^
2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.14

$$\int \frac{1}{(cx)^{8/3} (a+bx^2)^{2/3}} \, dx = -\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{2}{3},\frac{1}{6},-\frac{bx^2}{a}\right)}{5(cx)^{8/3} (a+bx^2)^{2/3}}$$

input
```
Integrate[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-5/6, 2/3, 1/6, -((b*x^2)/a)
])/(5*(c*x)^(8/3)*(a + b*x^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {264, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{8/3} (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \ 264$$

$$-\frac{3b\int \frac{1}{(cx)^{2/3}(bx^2+a)^{2/3}}dx}{5ac^2}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \ 266$$

$$-\frac{9b\int \frac{1}{(bx^2+a)^{2/3}}d\sqrt[3]{cx}}{5ac^3}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \ 771$$

$$-\frac{9b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5ac^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

$$\Big\downarrow \ 766$$

$$-\frac{3\,3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}(2}{10ac^{11/3}\sqrt{1-bx^2}\left(a+bx^2\right)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}$$

$$\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}$$

input
```
Int[1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(c^(
2/3) - b^(1/3)*(c*x)^(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) +
b^(2/3)*(c*x)^(4/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*Elli
pticF[ArcCos[(c^(2/3) - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 +
 Sqrt[3])*b^(1/3)*(c*x)^(2/3))], (2 + Sqrt[3])/4])/(10*a*c^(11/3)*Sqrt[1 -
 b*x^2]*(a + b*x^2)^(2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3
)*(c*x)^(2/3)*(c^(2/3) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^
(1/3)*(c*x)^(2/3))^2)])
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{8}{3}} \left(b\,x^{2}+a\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{8/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{8}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.84 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.12

$$\int \frac{1}{(cx)^{8/3} (a+bx^2)^{2/3}} \, dx = \frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{2}{3} \\ \frac{1}{6} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} c^{\frac{8}{3}} x^{\frac{5}{3}} \Gamma\left(\frac{1}{6}\right)}$$

input
```
integrate(1/(c*x)**(8/3)/(b*x**2+a)**(2/3),x)
```

output
```
gamma(-5/6)*hyper((-5/6, 2/3), (1/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/
3)*c**(8/3)*x**(5/3)*gamma(1/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{8/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{8}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{2}{3}}(cx)^{\frac{8}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(8/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx=\int \frac{1}{(c\,x)^{8/3}\,(b\,x^2+a)^{2/3}}\,dx$$

input
```
int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(8/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{8/3}(a+bx^2)^{2/3}}\,dx=\frac{\int \frac{1}{x^{\frac{8}{3}}(b\,x^2+a)^{\frac{2}{3}}}dx}{c^{\frac{8}{3}}}$$

input
```
int(1/(c*x)^(8/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x**2),x)/(c**(2/3)*c**2)
```

3.831 $\int \frac{1}{(cx)^{14/3}(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 425

$$\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}+\frac{27b\sqrt[3]{a+bx^2}}{55a^2c^3(cx)^{5/3}}$$
$$+\frac{27\,3^{3/4}b^2\sqrt[3]{cx}\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)\sqrt{\frac{c^{4/3}+\frac{b^{2/3}(cx)^{4/3}}{\left(a+bx^2\right)^{2/3}}+\frac{\sqrt[3]{b}c^{2/3}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}}{\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\frac{\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{a\cdots}}}{c^{2/3}-\frac{\left(1+\sqrt{3}\right)\cdots}{\sqrt[3]{a\cdots}}}\cdots\right.\right.}{110a^3c^{17/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\frac{\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)}{\sqrt[3]{a+bx^2}\left(c^{2/3}-\frac{\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{\sqrt[3]{a+bx^2}}\right)^2}}}$$

output
```
-3/11*(b*x^2+a)^(1/3)/a/c/(c*x)^(11/3)+27/55*b*(b*x^2+a)^(1/3)/a^2/c^3/(c*
x)^(5/3)+27/110*3^(3/4)*b^2*(c*x)^(1/3)*(b*x^2+a)^(1/3)*(c^(2/3)-b^(1/3)*(
c*x)^(2/3)/(b*x^2+a)^(1/3))*((c^(4/3)+b^(2/3)*(c*x)^(4/3)/(b*x^2+a)^(2/3)+
b^(1/3)*c^(2/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((c^(2/3)-(1-3
^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(c^(2/3)-(1+3^(1/2))*b^(1/3)*
(c*x)^(2/3)/(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^3/c^(17/3)/(-b^(1
/3)*(c*x)^(2/3)*(c^(2/3)-b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))/(b*x^2+a)^(1
/3)/(c^(2/3)-(1+3^(1/2))*b^(1/3)*(c*x)^(2/3)/(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.13

$$\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{11}{6},\frac{2}{3},-\frac{5}{6},-\frac{bx^2}{a}\right)}{11(cx)^{14/3}\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-11/6, 2/3, -5/6, -((b*x^2)/
a)])/(11*(c*x)^(14/3)*(a + b*x^2)^(2/3))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.263$, Rules used = {264, 264, 266, 771, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{9b\int \frac{1}{(cx)^{8/3}(bx^2+a)^{2/3}}dx}{11ac^2}-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{9b\left(-\frac{3b\int \frac{1}{(cx)^{2/3}\left(bx^2+a\right)^{2/3}}dx}{5ac^2}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11ac^2}-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{9b\left(-\frac{9b\int \frac{1}{\left(bx^2+a\right)^{2/3}}d\sqrt[3]{cx}}{5ac^3}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11ac^2}-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

$$\Big\downarrow \text{771}$$

$$-\frac{9b\left(-\frac{9b\int \frac{1}{\sqrt{1-bx^2}}d\frac{\sqrt[3]{cx}}{\sqrt[6]{bx^2+a}}}{5ac^3\sqrt{a+bx^2}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}}-\frac{3\sqrt[3]{a+bx^2}}{5ac(cx)^{5/3}}\right)}{11ac^2}-\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

$$\Big\downarrow \text{766}$$

$$-\frac{9b\left(-\frac{3\ 3^{3/4}b\sqrt[3]{cx}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)\sqrt{\frac{b^{2/3}(cx)^{4/3}+\sqrt[3]{b}c^{2/3}(cx)^{2/3}+c^{4/3}}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{c^{2/3}-\left(1-\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}{c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{10ac^{11/3}\sqrt{1-bx^2}(a+bx^2)^{2/3}\sqrt{-\frac{\sqrt[3]{b}(cx)^{2/3}\left(c^{2/3}-\sqrt[3]{b}(cx)^{2/3}\right)}{\left(c^{2/3}-\left(1+\sqrt{3}\right)\sqrt[3]{b}(cx)^{2/3}\right)^2}}\sqrt{\frac{ac^2}{ac^2+bc^2x^2}}} - \right.}{11ac^2}$$

$$\frac{3\sqrt[3]{a+bx^2}}{11ac(cx)^{11/3}}$$

input
```
Int[1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(a + b*x^2)^(1/3))/(11*a*c*(c*x)^(11/3)) - (9*b*((-3*(a + b*x^2)^(1/3)
)/(5*a*c*(c*x)^(5/3)) - (3*3^(3/4)*b*(c*x)^(1/3)*(c^(2/3) - b^(1/3)*(c*x)^
(2/3))*Sqrt[(c^(4/3) + b^(1/3)*c^(2/3)*(c*x)^(2/3) + b^(2/3)*(c*x)^(4/3))/
(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2]*EllipticF[ArcCos[(c^(2/3)
 - (1 - Sqrt[3])*b^(1/3)*(c*x)^(2/3))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*
x)^(2/3))], (2 + Sqrt[3])/4])/(10*a*c^(11/3)*Sqrt[1 - b*x^2]*(a + b*x^2)^(
2/3)*Sqrt[(a*c^2)/(a*c^2 + b*c^2*x^2)]*Sqrt[-((b^(1/3)*(c*x)^(2/3)*(c^(2/3
) - b^(1/3)*(c*x)^(2/3)))/(c^(2/3) - (1 + Sqrt[3])*b^(1/3)*(c*x)^(2/3))^2)
])))/(11*a*c^2)
```

Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 771
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a/(a + b*x^n))^(p + 1
/n)*(a + b*x^n)^(p + 1/n)   Subst[Int[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x
/(a + b*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0]
&& NeQ[p, -2^(-1)] && LtQ[Denominator[p + 1/n], Denominator[p]]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{14}{3}} (b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{14/3} (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{14}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(1/3)/(b*c^5*x^7 + a*c^5*x^5), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{14/3} (a+bx^2)^{2/3}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(14/3)/(b*x**2+a)**(2/3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{14/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{14}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{14/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{14}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(14/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{14/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(c\,x)^{14/3}\,(b\,x^2+a)^{2/3}} \, dx$$

input
```
int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(14/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{14/3}\left(a+bx^2\right)^{2/3}}\,dx = \frac{\int \frac{1}{x^{\frac{14}{3}}\left(b\,x^2+a\right)^{\frac{2}{3}}}dx}{c^{\frac{14}{3}}}$$

input
```
int(1/(c*x)^(14/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x**2)**(2/3)*x**4),x)/(c**(2/3)*c**4)
```

3.832 $\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}} \, dx = \frac{3(cx)^{5/3}\left(1+\frac{bx^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\,(a+bx^2)^{2/3}}$$

output
```
3/5*(c*x)^(5/3)*(1+b*x^2/a)^(2/3)*hypergeom([2/3, 5/6],[11/6],-b*x^2/a)/c/
(b*x^2+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}} \, dx = \frac{3x(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{2/3}}$$

input
```
Integrate[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

output
```
(3*x*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -
((b*x^2)/a)])/(5*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3} \int \frac{(cx)^{2/3}}{\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{278}$$

$$\frac{3(cx)^{5/3} \left(\frac{bx^2}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(\frac{2}{3},\frac{5}{6},\frac{11}{6},-\frac{bx^2}{a}\right)}{5c\,(a+bx^2)^{2/3}}$$

input
```
Int[(c*x)^(2/3)/(a + b*x^2)^(2/3),x]
```

output
```
(3*(c*x)^(5/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[2/3, 5/6, 11/6, -((
b*x^2)/a)])/(5*c*(a + b*x^2)^(2/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{2}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

output
```
int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \frac{c^{\frac{2}{3}}x^{\frac{5}{3}}\Gamma\left(\frac{5}{6}\right)\,{}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{5}{6}\\ \frac{11}{6}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}}\Gamma\left(\frac{11}{6}\right)}$$

input
```
integrate((c*x)**(2/3)/(b*x**2+a)**(2/3),x)
```

output
```
c**(2/3)*x**(5/3)*gamma(5/6)*hyper((2/3, 5/6), (11/6,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(2/3)*gamma(11/6))
```

Maxima [F]

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Giac [F]

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(cx)^{\frac{2}{3}}}{(bx^2+a)^{\frac{2}{3}}}\,dx$$

input
```
integrate((c*x)^(2/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate((c*x)^(2/3)/(b*x^2 + a)^(2/3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = \int \frac{(c\,x)^{2/3}}{(b\,x^2+a)^{2/3}}\,dx$$

input
```
int((c*x)^(2/3)/(a + b*x^2)^(2/3),x)
```

output
```
int((c*x)^(2/3)/(a + b*x^2)^(2/3), x)
```

Reduce [F]

$$\int \frac{(cx)^{2/3}}{(a+bx^2)^{2/3}}\,dx = c^{\frac{2}{3}}\left(\int \frac{x^{\frac{2}{3}}}{(b\,x^2+a)^{\frac{2}{3}}}dx\right)$$

input
```
int((c*x)^(2/3)/(b*x^2+a)^(2/3),x)
```

output
```
c**(2/3)*int(x**(2/3)/(a + b*x**2)**(2/3),x)
```

3.833 $\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \frac{3(cx)^{2/3}\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\,(a+bx^2)^{2/3}}$$

output
```
3/2*(c*x)^(2/3)*(1+b*x^2/a)^(2/3)*hypergeom([1/3, 2/3],[4/3],-b*x^2/a)/c/(
b*x^2+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2\sqrt[3]{cx}\,(a+bx^2)^{2/3}}$$

input
```
Integrate[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^2)/a)])
/(2*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3}\int \frac{1}{\sqrt[3]{cx}\left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{(a+bx^2)^{2/3}}$$

$$\Big\downarrow \ 278$$

$$\frac{3(cx)^{2/3}\left(\frac{bx^2}{a}+1\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx^2}{a}\right)}{2c\,(a+bx^2)^{2/3}}$$

input

```
Int[1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x]
```

output

```
(3*(c*x)^(2/3)*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b
*x^2)/a)])/(2*c*(a + b*x^2)^(2/3))
```

Defintions of rubi rules used

rule 278

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{3}} (b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}\,(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \frac{\Gamma\left(-\frac{1}{3}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2b^{\frac{2}{3}}\sqrt[3]{c}x^{\frac{2}{3}}\Gamma\left(\frac{2}{3}\right)}$$

input
```
integrate(1/(c*x)**(1/3)/(b*x**2+a)**(2/3),x)
```

output
```
gamma(-1/3)*hyper((1/3, 2/3), (4/3,), a*exp_polar(I*pi)/(b*x**2))/(2*b**(2
/3)*c**(1/3)*x**(2/3)*gamma(2/3))
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}\,(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}}\,(cx)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \int \frac{1}{(c\,x)^{1/3}\,(b\,x^2+a)^{2/3}}\,dx$$

input
```
int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(1/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{cx}\,(a+bx^2)^{2/3}}\,dx = \frac{\int \frac{1}{x^{\frac{1}{3}}(b\,x^2+a)^{\frac{2}{3}}}dx}{c^{\frac{1}{3}}}$$

input
```
int(1/(c*x)^(1/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(1/3)*(a + b*x**2)**(2/3)),x)/c**(1/3)
```

3.834 $\int \frac{1}{(cx)^{4/3}(a+bx^2)^{2/3}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\left(a+bx^2\right)^{2/3}}$$

output
```
-3*(1+b*x^2/a)^(2/3)*hypergeom([-1/6, 2/3],[5/6],-b*x^2/a)/c/(c*x)^(1/3)/(
b*x^2+a)^(2/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx=-\frac{3x\left(1+\frac{bx^2}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-\frac{bx^2}{a}\right)}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}$$

input
```
Integrate[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*x*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)
])/((c*x)^(4/3)*(a + b*x^2)^(2/3))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{4/3} (a+bx^2)^{2/3}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{2/3} \int \frac{1}{(cx)^{4/3} \left(\frac{bx^2}{a}+1\right)^{2/3}}dx}{(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{3\left(\frac{bx^2}{a}+1\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{2}{3},\frac{5}{6},-\frac{bx^2}{a}\right)}{c\sqrt[3]{cx}\,(a+bx^2)^{2/3}}$$

input
```
Int[1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x]
```

output
```
(-3*(1 + (b*x^2)/a)^(2/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -((b*x^2)/a)])
/(c*(c*x)^(1/3)*(a + b*x^2)^(2/3))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{4}{3}} (b\,x^2+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{4/3} (a+bx^2)^{2/3}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{2}{3}} (cx)^{\frac{4}{3}}} \, dx$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/3)*(c*x)^(2/3)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{(cx)^{4/3} (a+bx^2)^{2/3}} \, dx = \frac{\Gamma\left(-\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{2}{3} \\ \frac{5}{6} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{2}{3}} c^{\frac{4}{3}} \sqrt[3]{x}\Gamma\left(\frac{5}{6}\right)}$$

input
```
integrate(1/(c*x)**(4/3)/(b*x**2+a)**(2/3),x)
```

output
```
gamma(-1/6)*hyper((-1/6, 2/3), (5/6,), b*x**2*exp_polar(I*pi)/a)/(2*a**(2/
3)*c**(4/3)*x**(1/3)*gamma(5/6))
```

Maxima [F]

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{4/3}\left(a+bx^2\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{2}{3}}(cx)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(2/3)*(c*x)^(4/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{4/3} (a+bx^2)^{2/3}}\,dx = \int \frac{1}{(c\,x)^{4/3}\,(b\,x^2+a)^{2/3}}\,dx$$

input
```
int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)),x)
```

output
```
int(1/((c*x)^(4/3)*(a + b*x^2)^(2/3)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{4/3} (a+bx^2)^{2/3}}\,dx = \frac{\int \frac{1}{x^{\frac{4}{3}}(b\,x^2+a)^{\frac{2}{3}}}dx}{c^{\frac{4}{3}}}$$

input
```
int(1/(c*x)^(4/3)/(b*x^2+a)^(2/3),x)
```

output
```
int(1/(x**(1/3)*(a + b*x**2)**(2/3)*x),x)/(c**(1/3)*c)
```

3.835 $\int x^4 \sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = -\frac{4a^2x\sqrt[4]{a+bx^2}}{77b^2}+\frac{2ax^3\sqrt[4]{a+bx^2}}{77b}+\frac{2}{11}x^5\sqrt[4]{a+bx^2}+\frac{8a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{5/2}(a+bx^2)^{3/4}}$$

output

```
-4/77*a^2*x*(b*x^2+a)^(1/4)/b^2+2/77*a*x^3*(b*x^2+a)^(1/4)/b+2/11*x^5*(b*x
^2+a)^(1/4)+8/77*a^(7/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1
/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.77

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = \frac{2x\sqrt[4]{a+bx^2}\left(\sqrt[4]{1+\frac{bx^2}{a}}(-6a^2+abx^2+7b^2x^4)+6a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-6*a^2 + a*b*x^2 + 7*b^2*x^
4) + 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)]))/(77*b^2*(1 +
(b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[4]{a+bx^2} \, dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{11} a \int \frac{x^4}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{11} x^5 \sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{11} a \left(\frac{2 x^3 \sqrt[4]{a+bx^2}}{7 b}-\frac{6 a \int \frac{x^2}{\left(bx^2+a\right)^{3/4}}dx}{7 b}\right)+\frac{2}{11} x^5 \sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{11} a \left(\frac{2 x^3 \sqrt[4]{a+bx^2}}{7 b}-\frac{6 a \left(\frac{2 x \sqrt[4]{a+bx^2}}{3 b}-\frac{2 a \int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{3 b}\right)}{7 b}\right)+\frac{2}{11} x^5 \sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b(a+bx^2)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}$$

$\downarrow$ 229

$$\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}$$

input
```
Int[x^4*(a + b*x^2)^(1/4),x]
```

output
```
(2*x^5*(a + b*x^2)^(1/4))/11 + (a*((2*x^3*(a + b*x^2)^(1/4))/(7*b) - (6*a*
((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*Elliptic
F[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))))/(7*b)
))/11
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{4} \left(b\,x^{2}+a \right)^{\frac{1}{4}}d x$$

input
```
int(x^4*(b*x^2+a)^(1/4),x)
```

output
```
int(x^4*(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = \int \left(bx^{2}+a\right)^{\frac{1}{4}}x^{4}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int x^4\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt[4]{a}x^5{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4\sqrt[4]{a+bx^2}\,dx=\int (bx^2+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*x^4, x)
```

Giac [F]

$$\int x^4\sqrt[4]{a+bx^2}\,dx=\int (bx^2+a)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = \int x^4 \left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/4),x)
```

output
```
int(x^4*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^4 \sqrt[4]{a+bx^2}\,dx = \frac{-\frac{4(b\,x^2+a)^{\frac{1}{4}}a^2x}{77}+\frac{2(b\,x^2+a)^{\frac{1}{4}}ab\,x^3}{77}+\frac{2(b\,x^2+a)^{\frac{1}{4}}b^2x^5}{11}+\frac{4\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^3}{77}}{b^2}$$

input
```
int(x^4*(b*x^2+a)^(1/4),x)
```

output
```
(2*( - 2*(a + b*x**2)**(1/4)*a**2*x + (a + b*x**2)**(1/4)*a*b*x**3 + 7*(a
+ b*x**2)**(1/4)*b**2*x**5 + 2*int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a**
3))/(77*b**2)
```

3.836 $\int x^2 \sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 97

$$\int x^2 \sqrt[4]{a+bx^2}\,dx=\frac{2ax\sqrt[4]{a+bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a+bx^2}-\frac{4a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a+bx^2\right)^{3/4}}$$

output
```
2/21*a*x*(b*x^2+a)^(1/4)/b+2/7*x^3*(b*x^2+a)^(1/4)-4/21*a^(5/2)*(1+b*x^2/a
)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(b*
x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.92 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.64

$$\int x^2 \sqrt[4]{a+bx^2}\,dx=\frac{2x\sqrt[4]{a+bx^2}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}\right)}{7b}$$

input
```
Integrate[x^2*(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4)*(a + b*x^2 - (a*Hypergeometric2F1[-1/4, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(7*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {248, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[4]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{7}a\int \frac{x^2}{(bx^2+a)^{3/4}}dx+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\left(a+bx^2\right)^{3/4}}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}$$

input
```
Int[x^2*(a + b*x^2)^(1/4),x]
```

output
```
(2*x^3*(a + b*x^2)^(1/4))/7 + (a*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/
2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b
^(3/2)*(a + b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a \right)^{\frac{1}{4}}d x$$

input
```
int(x^2*(b*x^2+a)^(1/4),x)
```

output
```
int(x^2*(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int x^2 \sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.30

$$\int x^2 \sqrt[4]{a+bx^2}\,dx = \frac{\sqrt[4]{a}x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \sqrt[4]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[4]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[4]{a+bx^2}\,dx = \int x^2 \left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/4),x)
```

output
```
int(x^2*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^2\sqrt[4]{a+bx^2}\,dx = \frac{\frac{2(b\,x^2+a)^{\frac{1}{4}}a x}{21}+\frac{2(b\,x^2+a)^{\frac{1}{4}}b\,x^3}{7}-\frac{2\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2}{21}}{b}$$

input
```
int(x^2*(b*x^2+a)^(1/4),x)
```

output
```
(2*((a + b*x**2)**(1/4)*a*x + 3*(a + b*x**2)**(1/4)*b*x**3 - int((a + b*x*
*2)**(1/4)/(a + b*x**2),x)*a**2))/(21*b)
```

3.837 $\int \sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 75

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{2}{3}x\sqrt[4]{a+bx^2}+\frac{2a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

output
```
2/3*x*(b*x^2+a)^(1/4)+2/3*a^(3/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*ar
ctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.61

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4),x]
```

output
```
(x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a+bx^2}\,dx$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{3}a\int \frac{1}{(bx^2+a)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{231}$$

$$\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

$$\Big\downarrow \text{229}$$

$$\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}$$

input
```
Int[(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int((b*x^2+a)^(1/4),x)
```

output
```
int((b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.35

$$\int \sqrt[4]{a+bx^2}\,dx = \sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{1/4}}$$

input
```
int((a + b*x^2)^(1/4),x)
```

output
```
(x*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a+bx^2}\,dx=\frac{2(b\,x^2+a)^{\frac{1}{4}}\,x}{3}+\frac{\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int((b*x^2+a)^(1/4),x)
```

output
```
(2*(a + b*x**2)**(1/4)*x + int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a)/3
```

3.838 $\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 72

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx=-\frac{\sqrt[4]{a+bx^2}}{x}+\frac{\sqrt{a}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{(a+bx^2)^{3/4}}$$

output
```
-(b*x^2+a)^(1/4)/x+a^(1/2)*b^(1/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*a
rctan(b^(1/2)*x/a^(1/2)),2^(1/2))/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.94 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx=-\frac{\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/x^2,x]
```

output
```
-(((a + b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(1/4)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {247, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}b\int \frac{1}{(bx^2+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^2}}{x}$$

$$\Big\downarrow \text{231}$$

$$\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}$$

input
```
Int[(a + b*x^2)^(1/4)/x^2,x]
```

output
```
-((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a + b*x^2)^(3/4)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{x^2}dx$$

input
```
int((b*x^2+a)^(1/4)/x^2,x)
```

output
```
int((b*x^2+a)^(1/4)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.40

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(1/4)/x**2,x)
```

output
```
-a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = -\frac{2\,(b\,x^2+a)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ -\frac{a}{b\,x^2}\right)}{x\,\left(\frac{a}{b\,x^2}+1\right)^{1/4}}$$

input
```
int((a + b*x^2)^(1/4)/x^2,x)
```

output
```
-(2*(a + b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(1/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^2}\,dx = \frac{-2(b\,x^2+a)^{\frac{1}{4}}-\left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((b*x^2+a)^(1/4)/x^2,x)
```

output
```
( - 2*(a + b*x**2)**(1/4) - int((a + b*x**2)**(1/4)/(a*x**2 + b*x**4),x)*a
*x)/x
```

3.839 $\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a+bx^2}}{3x^3}-\frac{b\sqrt[4]{a+bx^2}}{6ax}-\frac{b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\,(a+bx^2)^{3/4}}$$

output
```
-1/3*(b*x^2+a)^(1/4)/x^3-1/6*b*(b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1+b*x^2/a)
^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/(b*x
^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {247, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{6} b \int \frac{1}{x^2 \left(bx^2+a\right)^{3/4}} dx - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{6} b \left(-\frac{b \int \frac{1}{\left(bx^2+a\right)^{3/4}} dx}{2a} - \frac{\sqrt[4]{a+bx^2}}{ax}\right) - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{1}{6} b \left(-\frac{b \left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}} dx}{2a \left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a+bx^2}}{ax}\right) - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{1}{6} b \left(-\frac{\sqrt{b} \left(\frac{bx^2}{a}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2} \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 2\right)}{\sqrt{a} \left(a+bx^2\right)^{3/4}} - \frac{\sqrt[4]{a+bx^2}}{ax}\right) - \frac{\sqrt[4]{a+bx^2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(1/4)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(1/4)/x^3 + (b*(-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1
+ (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(
a + b*x^2)^(3/4))))/6
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{x^4}dx$$

input
```
int((b*x^2+a)^(1/4)/x^4,x)
```

output
```
int((b*x^2+a)^(1/4)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(1/4)/x**4,x)
```

output
```
-a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx=\int \frac{(b\,x^2+a)^{1/4}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(1/4)/x^4,x)
```

output
```
int((a + b*x^2)^(1/4)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^4}\,dx = \frac{-2(b\,x^2+a)^{\frac{1}{4}} - \left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^6+a\,x^4}dx\right)a\,x^3}{5x^3}$$

input
```
int((b*x^2+a)^(1/4)/x^4,x)
```

output
```
( - 2*(a + b*x**2)**(1/4) - int((a + b*x**2)**(1/4)/(a*x**4 + b*x**6),x)*a
*x**3)/(5*x**3)
```

3.840 $\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a+bx^2}}{5x^5}-\frac{b\sqrt[4]{a+bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a+bx^2}}{12a^2x}+\frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a+bx^2\right)^{3/4}}$$

output
```
-1/5*(b*x^2+a)^(1/4)/x^5-1/30*b*(b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(b*x^2+a)^(
1/4)/a^2/x+1/12*b^(5/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/
2)*x/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx$$

$$\Big\downarrow \; 247$$

$$\frac{1}{10}b\int \frac{1}{x^4\left(bx^2+a\right)^{3/4}}dx-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

$$\Big\downarrow \; 264$$

$$\frac{1}{10}b\left(-\frac{5b\int \frac{1}{x^2\left(bx^2+a\right)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

$$\Big\downarrow \; 264$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

$$\Big\downarrow \; 231$$

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

$\downarrow$ 229

$$\frac{1}{10}b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a+bx^2}}{5x^5}$$

input
```
Int[(a + b*x^2)^(1/4)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(1/4)/x^5 + (b*(-1/3*(a + b*x^2)^(1/4)/(a*x^3) - (5*b*(-(
(a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))))/(6*a)))/10
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{x^6}dx$$

input
```
int((b*x^2+a)^(1/4)/x^6,x)
```

output
```
int((b*x^2+a)^(1/4)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(1/4)/x**6,x)
```

output
```
-a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = \int \frac{{\left(b\,x^2+a\right)}^{1/4}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(1/4)/x^6,x)
```

output
```
int((a + b*x^2)^(1/4)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{x^6}\,dx = \frac{-2(b\,x^2+a)^{\frac{1}{4}}-\left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^8+a\,x^6}dx\right)a\,x^5}{9x^5}$$

input
```
int((b*x^2+a)^(1/4)/x^6,x)
```

output
```
( - 2*(a + b*x**2)**(1/4) - int((a + b*x**2)**(1/4)/(a*x**6 + b*x**8),x)*a
*x**5)/(9*x**5)
```

3.841 $\int x^4 \sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x^4 \sqrt[4]{a-bx^2}\,dx = -\frac{4a^2x\sqrt[4]{a-bx^2}}{77b^2} - \frac{2ax^3\sqrt[4]{a-bx^2}}{77b} + \frac{2}{11}x^5\sqrt[4]{a-bx^2} + \frac{8a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{5/2}\left(a-bx^2\right)^{3/4}}$$

output
```
-4/77*a^2*x*(-b*x^2+a)^(1/4)/b^2-2/77*a*x^3*(-b*x^2+a)^(1/4)/b+2/11*x^5*(-
b*x^2+a)^(1/4)+8/77*a^(7/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b
^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int x^4 \sqrt[4]{a-bx^2}\,dx = -\frac{2x\sqrt[4]{a-bx^2}\left(\sqrt[4]{1-\frac{bx^2}{a}}\left(6a^2+abx^2-7b^2x^4\right)-6a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{77b^2\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(a - b*x^2)^(1/4),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*((1 - (b*x^2)/a)^(1/4)*(6*a^2 + a*b*x^2 - 7*b^2*x^
4) - 6*a^2*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a]))/(77*b^2*(1 - (b*
x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {248, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[4]{a-bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{11}a\int \frac{x^4}{\left(a-bx^2\right)^{3/4}}dx+\frac{2}{11}x^5\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{11}a\left(\frac{6a\int \frac{x^2}{\left(a-bx^2\right)^{3/4}}dx}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{11}a\left(\frac{6a\left(\frac{2a\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{11}a\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}$$

$\downarrow$ 230

$$\frac{1}{11}a\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}$$

input
```
Int[x^4*(a - b*x^2)^(1/4),x]
```

output
```
(2*x^5*(a - b*x^2)^(1/4))/11 + (a*((-2*x^3*(a - b*x^2)^(1/4))/(7*b) + (6*a
*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*Ellipt
icF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))))/(7*
b)))/11
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{4}\left(-b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int(x^4*(-b*x^2+a)^(1/4),x)
```

output
```
int(x^4*(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int x^4 \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int x^4 \sqrt[4]{a - bx^2}\, dx = \frac{\sqrt[4]{a} x^5 {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x**5*hyper((-1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Maxima [F]

$$\int x^4 \sqrt[4]{a - bx^2}\, dx = \int \left(-bx^2 + a\right)^{\frac{1}{4}} x^4\, dx$$

input
```
integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[4]{a - bx^2}\, dx = \int \left(-bx^2 + a\right)^{\frac{1}{4}} x^4\, dx$$

input
```
integrate(x^4*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[4]{a-bx^2}\,dx = \int x^4\,\left(a-b\,x^2\right)^{1/4}\,dx$$

input
```
int(x^4*(a - b*x^2)^(1/4),x)
```

output
```
int(x^4*(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^4 \sqrt[4]{a-bx^2}\,dx$$
$$= \frac{-\frac{4(-b\,x^2+a)^{\frac{1}{4}}a^2x}{77}-\frac{2(-b\,x^2+a)^{\frac{1}{4}}ab\,x^3}{77}+\frac{2(-b\,x^2+a)^{\frac{1}{4}}b^2x^5}{11}+\frac{4\left(\int\frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a^3}{77}}{b^2}$$

input
```
int(x^4*(-b*x^2+a)^(1/4),x)
```

output
```
(2*( - 2*(a - b*x**2)**(1/4)*a**2*x - (a - b*x**2)**(1/4)*a*b*x**3 + 7*(a
- b*x**2)**(1/4)*b**2*x**5 + 2*int((a - b*x**2)**(1/4)/(a - b*x**2),x)*a**
3))/(77*b**2)
```

3.842 $\int x^2\sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int x^2\sqrt[4]{a-bx^2}\,dx=-\frac{2ax\sqrt[4]{a-bx^2}}{21b}+\frac{2}{7}x^3\sqrt[4]{a-bx^2}+\frac{4a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(a-bx^2\right)^{3/4}}$$

output
```
-2/21*a*x*(-b*x^2+a)^(1/4)/b+2/7*x^3*(-b*x^2+a)^(1/4)+4/21*a^(5/2)*(1-b*x^
2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/
(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^2\sqrt[4]{a-bx^2}\,dx=\frac{2x\sqrt[4]{a-bx^2}\left(-a+bx^2+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}\right)}{7b}$$

input
```
Integrate[x^2*(a - b*x^2)^(1/4),x]
```

output
```
(2*x*(a - b*x^2)^(1/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-1/4, 1/2, 3/2,
(b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(7*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {248, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[4]{a-bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{7}a\int \frac{x^2}{(a-bx^2)^{3/4}}dx+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{7}a\left(\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{7}a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{7}a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a-bx^2}$$

input
```
Int[x^2*(a - b*x^2)^(1/4),x]
```

output
```
(2*x^3*(a - b*x^2)^(1/4))/7 + (a*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3
/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*
b^(3/2)*(a - b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^2 \left(-b\,x^2+a\right)^{\frac{1}{4}}dx$$

input
```
int(x^2*(-b*x^2+a)^(1/4),x)
```

output
```
int(x^2*(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \frac{\sqrt[4]{a}x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x**3*hyper((-1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}} x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \int x^2 \left(a-b\,x^2\right)^{1/4}\,dx$$

input
```
int(x^2*(a - b*x^2)^(1/4),x)
```

output
```
int(x^2*(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int x^2 \sqrt[4]{a-bx^2}\,dx = \frac{-\frac{2(-b\,x^2+a)^{\frac{1}{4}}a x}{21}+\frac{2(-b\,x^2+a)^{\frac{1}{4}}b\,x^3}{7}+\frac{2\left(\int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2}{21}}{b}$$

input
```
int(x^2*(-b*x^2+a)^(1/4),x)
```

output
```
(2*( - (a - b*x**2)**(1/4)*a*x + 3*(a - b*x**2)**(1/4)*b*x**3 + int((a - b
*x**2)**(1/4)/(a - b*x**2),x)*a**2))/(21*b)
```

3.843 $\int \sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{2}{3}x\sqrt[4]{a-bx^2}+\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
2/3*x*(-b*x^2+a)^(1/4)+2/3*a^(3/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*a
rcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int \sqrt[4]{a-bx^2}\,dx=\frac{x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4),x]
```

output
```
(x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b
*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{a-bx^2}\,dx$$

$$\Big\downarrow \; 211$$

$$\frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \; 231$$

$$\frac{a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

$$\Big\downarrow \; 230$$

$$\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}$$

input
```
Int[(a - b*x^2)^(1/4),x]
```

output
```
(2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int((-b*x^2+a)^(1/4),x)
```

output
```
int((-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{a-bx^2}\,dx=\int \left(-bx^2+a\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \sqrt[4]{a-bx^2}\,dx=\sqrt[4]{a}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*x*hyper((-1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \sqrt[4]{a-bx^2}\,dx=\int (-bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{a-bx^2}\,dx=\int (-bx^2+a)^{\frac{1}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \sqrt[4]{a-bx^2}\,dx = \frac{x\,(a-b\,x^2)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{1/4}}$$

input
```
int((a - b*x^2)^(1/4),x)
```

output
```
(x*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(1/4)
```

Reduce [F]

$$\int \sqrt[4]{a-bx^2}\,dx = \frac{2(-b\,x^2+a)^{\frac{1}{4}}\,x}{3} + \frac{\left(\int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a}{3}$$

input
```
int((-b*x^2+a)^(1/4),x)
```

output
```
(2*(a - b*x**2)**(1/4)*x + int((a - b*x**2)**(1/4)/(a - b*x**2),x)*a)/3
```

3.844 $\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx=-\frac{\sqrt[4]{a-bx^2}}{x}-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a-bx^2\right)^{3/4}}$$

output
```
-(-b*x^2+a)^(1/4)/x-a^(1/2)*b^(1/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*
arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx=-\frac{\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{4},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/x^2,x]
```

output
```
-(((a - b*x^2)^(1/4)*Hypergeometric2F1[-1/2, -1/4, 1/2, (b*x^2)/a])/(x*(1
- (b*x^2)/a)^(1/4)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {247, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2} \, dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{2}b\int \frac{1}{(a-bx^2)^{3/4}}dx-\frac{\sqrt[4]{a-bx^2}}{x}$$

$$\Big\downarrow \text{231}$$

$$-\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}$$

$$\Big\downarrow \text{230}$$

$$-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}$$

input
```
Int[(a - b*x^2)^(1/4)/x^2,x]
```

output
```
-((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(3/4)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^{2}+a)^{\frac{1}{4}}}{x^{2}}dx$$

input
```
int((-b*x^2+a)^(1/4)/x^2,x)
```

output
```
int((-b*x^2+a)^(1/4)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx=\int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2}, -\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

input
```
integrate((-b*x**2+a)**(1/4)/x**2,x)
```

output
```
-a**(1/4)*hyper((-1/2, -1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = -\frac{2\,(a-b\,x^2)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{4};\ \frac{5}{4};\ \frac{a}{b\,x^2}\right)}{x\,\left(1-\frac{a}{b\,x^2}\right)^{1/4}}$$

input
```
int((a - b*x^2)^(1/4)/x^2,x)
```

output
```
-(2*(a - b*x^2)^(1/4)*hypergeom([-1/4, 1/4], 5/4, a/(b*x^2)))/(x*(1 - a/(b
*x^2))^(1/4))
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^2}\,dx = \frac{-2(-b\,x^2+a)^{\frac{1}{4}}-\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((-b*x^2+a)^(1/4)/x^2,x)
```

output
```
( - 2*(a - b*x**2)**(1/4) - int((a - b*x**2)**(1/4)/(a*x**2 - b*x**4),x)*a
*x)/x
```

3.845 $\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a-bx^2}}{3x^3}+\frac{b\sqrt[4]{a-bx^2}}{6ax} - \frac{b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{a}\,(a-bx^2)^{3/4}}$$

output
```
-1/3*(-b*x^2+a)^(1/4)/x^3+1/6*b*(-b*x^2+a)^(1/4)/a/x-1/6*b^(3/2)*(1-b*x^2/
a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/(-
b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/x^4,x]
```

output
```
-1/3*((a - b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -1/4, -1/2, (b*x^2)/a])/(x
^3*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {247, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4} dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{6}b\int \frac{1}{x^2 (a-bx^2)^{3/4}}dx-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{6}b\left(\frac{b\int \frac{1}{(a-bx^2)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

$$\Big\downarrow \text{231}$$

$$-\frac{1}{6}b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a (a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

$$\Big\downarrow \text{230}$$

$$-\frac{1}{6}b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a} (a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}$$

input
```
Int[(a - b*x^2)^(1/4)/x^4,x]
```

output
```
-1/3*(a - b*x^2)^(1/4)/x^3 - (b*(-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1
- (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(
a - b*x^2)^(3/4))))/6
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{x^4}dx$$

input
```
int((-b*x^2+a)^(1/4)/x^4,x)
```

output
```
int((-b*x^2+a)^(1/4)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((-b*x**2+a)**(1/4)/x**4,x)
```

output
```
-a**(1/4)*hyper((-3/2, -1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3
)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^4,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = \int \frac{(a-b\,x^2)^{1/4}}{x^4}\,dx$$

input
```
int((a - b*x^2)^(1/4)/x^4,x)
```

output
```
int((a - b*x^2)^(1/4)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^4}\,dx = \frac{-2(-b\,x^2+a)^{\frac{1}{4}} - \left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^6+a\,x^4}dx\right)a\,x^3}{5x^3}$$

input
```
int((-b*x^2+a)^(1/4)/x^4,x)
```

output
```
( - 2*(a - b*x**2)**(1/4) - int((a - b*x**2)**(1/4)/(a*x**4 - b*x**6),x)*a
*x**3)/(5*x**3)
```

3.846 $\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a-bx^2}}{5x^5}+\frac{b\sqrt[4]{a-bx^2}}{30ax^3}+\frac{b^2\sqrt[4]{a-bx^2}}{12a^2x} - \frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12a^{3/2}\left(a-bx^2\right)^{3/4}}$$

output
```
-1/5*(-b*x^2+a)^(1/4)/x^5+1/30*b*(-b*x^2+a)^(1/4)/a/x^3+1/12*b^2*(-b*x^2+a
)^(1/4)/a^2/x-1/12*b^(5/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^
(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/x^6,x]
```

output
```
-1/5*((a - b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -1/4, -3/2, (b*x^2)/a])/(x
^5*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {247, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{10}b\int \frac{1}{x^4\left(a-bx^2\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{10}b\left(\frac{5b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{10}b\left(\frac{5b\left(\frac{b\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

$$\Big\downarrow \text{231}$$

$$-\frac{1}{10}b\left(\frac{5b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

$\big\downarrow$ 230

$$-\frac{1}{10}b\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)-\frac{\sqrt[4]{a-bx^2}}{5x^5}$$

input
```
Int[(a - b*x^2)^(1/4)/x^6,x]
```

output
```
-1/5*(a - b*x^2)^(1/4)/x^5 - (b*(-1/3*(a - b*x^2)^(1/4)/(a*x^3) + (5*b*(-(
(a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))))/(6*a)))/10
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^{2}+a)^{\frac{1}{4}}}{x^{6}}dx$$

input
```
int((-b*x^2+a)^(1/4)/x^6,x)
```

output
```
int((-b*x^2+a)^(1/4)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = -\frac{\sqrt[4]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2},-\frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((-b*x**2+a)**(1/4)/x**6,x)
```

output
```
-a**(1/4)*hyper((-5/2, -1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5
)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/x^6,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = \int \frac{\left(a-b\,x^2\right)^{1/4}}{x^6}\,dx$$

input
```
int((a - b*x^2)^(1/4)/x^6,x)
```

output
```
int((a - b*x^2)^(1/4)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{x^6}\,dx = \frac{-2(-b\,x^2+a)^{\frac{1}{4}}-\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^8+a\,x^6}dx\right)a\,x^5}{9x^5}$$

input
```
int((-b*x^2+a)^(1/4)/x^6,x)
```

output
```
( - 2*(a - b*x**2)**(1/4) - int((a - b*x**2)**(1/4)/(a*x**6 - b*x**8),x)*a
*x**5)/(9*x**5)
```

3.847 $\int x^4(a+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 143

$$\int x^4\left(a+bx^2\right)^{3/4}\,dx=\frac{8a^3x}{65b^2\sqrt[4]{a+bx^2}}-\frac{4a^2x(a+bx^2)^{3/4}}{65b^2}+\frac{2ax^3(a+bx^2)^{3/4}}{39b}+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}-\frac{8a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
8/65*a^3*x/b^2/(b*x^2+a)^(1/4)-4/65*a^2*x*(b*x^2+a)^(3/4)/b^2+2/39*a*x^3*(
b*x^2+a)^(3/4)/b+2/13*x^5*(b*x^2+a)^(3/4)-8/65*a^(7/2)*(1+b*x^2/a)^(1/4)*E
llipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(1/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.65

$$\int x^4\left(a+bx^2\right)^{3/4}\,dx=\frac{2x(a+bx^2)^{3/4}\left(\left(1+\frac{bx^2}{a}\right)^{3/4}(-2a^2+abx^2+3b^2x^4)+2a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\right.\right.}{39b^2\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[x^4*(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2)^(3/4)*((1 + (b*x^2)/a)^(3/4)*(-2*a^2 + a*b*x^2 + 3*b^2*x^
4) + 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)]))/(39*b^2*(1 +
(b*x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {248, 262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2\right)^{3/4}dx$$

$$\Big\downarrow \text{248}$$

$$\frac{3}{13}a\int \frac{x^4}{\sqrt[4]{bx^2+a}}dx+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{13}a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\int \frac{x^2}{\sqrt[4]{bx^2+a}}dx}{3b}\right)+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{13}a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}\right)}{3b}\right)+\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

$\big\downarrow$ 227

$$\frac{3}{13}a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)+$$

$$\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

$\big\downarrow$ 225

$$\frac{3}{13}a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)+$$

$$\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

$\big\downarrow$ 212

$$\frac{3}{13}a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)+$$

$$\frac{2}{13}x^5\left(a+bx^2\right)^{3/4}$$

input
```
Int[x^4*(a + b*x^2)^(3/4),x]
```

output
```
(2*x^5*(a + b*x^2)^(3/4))/13 + (3*a*((2*x^3*(a + b*x^2)^(3/4))/(9*b) - (2*
a*((2*x*(a + b*x^2)^(3/4))/(5*b) - (2*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 +
(b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4))))/(3*b)))/13
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int(x^4*(b*x^2+a)^(3/4),x)
```

output
```
int(x^4*(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{3/4} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{3}{4}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int x^4 (a + bx^2)^{3/4} \, dx = \frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a + bx^2)^{3/4} \, dx = \int (bx^2 + a)^{\frac{3}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2)^{3/4} \, dx = \int (bx^2 + a)^{\frac{3}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{3/4} \, dx = \int x^4 \left(b x^2 + a\right)^{3/4} dx$$

input
```
int(x^4*(a + b*x^2)^(3/4),x)
```

output
```
int(x^4*(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^4 (a+bx^2)^{3/4} \, dx = \frac{-\frac{4(b x^2+a)^{\frac{3}{4}} a^2 x}{65} + \frac{2(b x^2+a)^{\frac{3}{4}} a b x^3}{39} + \frac{2(b x^2+a)^{\frac{3}{4}} b^2 x^5}{13} + \frac{4\left(\int \frac{1}{(b x^2+a)^{\frac{1}{4}}} dx\right) a^3}{65}}{b^2}$$

input
```
int(x^4*(b*x^2+a)^(3/4),x)
```

output
```
(2*( - 6*(a + b*x**2)**(3/4)*a**2*x + 5*(a + b*x**2)**(3/4)*a*b*x**3 + 15*
(a + b*x**2)**(3/4)*b**2*x**5 + 6*int((a + b*x**2)**(3/4)/(a + b*x**2),x)*
a**3))/(195*b**2)
```

3.848 $\int x^2(a+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 119

$$\int x^2\left(a+bx^2\right)^{3/4}\,dx=-\frac{4a^2x}{15b\sqrt[4]{a+bx^2}}+\frac{2ax(a+bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
-4/15*a^2*x/b/(b*x^2+a)^(1/4)+2/15*a*x*(b*x^2+a)^(3/4)/b+2/9*x^3*(b*x^2+a)
^(3/4)+4/15*a^(5/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a
^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.87 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.52

$$\int x^2\left(a+bx^2\right)^{3/4}\,dx=\frac{2x(a+bx^2)^{3/4}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2)^(3/4)*(a + b*x^2 - (a*Hypergeometric2F1[-3/4, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(3/4)))/(9*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/4} dx$$

$$\Big\downarrow \ 248$$

$$\frac{1}{3}a\int \frac{x^2}{\sqrt[4]{bx^2+a}}dx+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{3}a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}\right)+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \ 227$$

$$\frac{1}{3}a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}\right)+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \ 225$$

$$\frac{1}{3}a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}\right)+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{3}a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}\right)+\frac{2}{9}x^3\left(a+bx^2\right)^{3/4}$$

input
```
Int[x^2*(a + b*x^2)^(3/4),x]
```

output
```
(2*x^3*(a + b*x^2)^(3/4))/9 + (a*((2*x*(a + b*x^2)^(3/4))/(5*b) - (2*a*(1
+ (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4))))/3
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int(x^2*(b*x^2+a)^(3/4),x)
```

output
```
int(x^2*(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{3/4} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{3}{4}} x^{2} \,d x}$$

input
```
integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.24

$$\int x^2(a+bx^2)^{3/4}\,dx = \frac{a^{\frac{3}{4}}x^3\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2(a+bx^2)^{3/4}\,dx = \int (bx^2+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^2)^{3/4}\,dx = \int (bx^2+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a+bx^2)^{3/4} \, dx = \int x^2 \left(b x^2+a\right)^{3/4} dx$$

input
```
int(x^2*(a + b*x^2)^(3/4),x)
```

output
```
int(x^2*(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^2 (a+bx^2)^{3/4} \, dx = \frac{\frac{2(b x^2+a)^{\frac{3}{4}} a x}{15}+\frac{2(b x^2+a)^{\frac{3}{4}} b x^3}{9}-\frac{2\left(\int \frac{1}{(b x^2+a)^{\frac{1}{4}}}dx\right) a^2}{15}}{b}$$

input
```
int(x^2*(b*x^2+a)^(3/4),x)
```

output
```
(2*(3*(a + b*x**2)**(3/4)*a*x + 5*(a + b*x**2)**(3/4)*b*x**3 - 3*int((a +
b*x**2)**(3/4)/(a + b*x**2),x)*a**2))/(45*b)
```

3.849 $\int (a+bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \left(a+bx^2\right)^{3/4}\,dx=\frac{6ax}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}-\frac{6a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
6/5*a*x/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)-6/5*a^(3/2)*(1+b*x^2/a)^(1/4
)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^2+a)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^{3/4}\,dx=\frac{x\left(a+bx^2\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(3/4),x]
```

output
```
(x*(a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt[4]{bx^2+a}}dx+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{227}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{225}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

$$\Big\downarrow \text{212}$$

$$\frac{3a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5\sqrt[4]{a+bx^2}}+\frac{2}{5}x\left(a+bx^2\right)^{3/4}$$

input
```
Int[(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2)^(3/4))/5 + (3*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)
/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b
]))/(5*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int((b*x^2+a)^(3/4),x)
```

output
```
int((b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{3/4}\,dx=\int \left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int \left(a+bx^2\right)^{3/4}\,dx=a^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/4}\,dx=\int \left(bx^2+a\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/4} dx = \int \left(bx^2+a\right)^{\frac{3}{4}} dx$$

input
```
integrate((b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \left(a+bx^2\right)^{3/4} dx = \frac{x\left(b\,x^2+a\right)^{3/4}\,{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/4}}$$

input
```
int((a + b*x^2)^(3/4),x)
```

output
```
(x*(a + b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/4)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/4} dx = \frac{2(b\,x^2+a)^{\frac{3}{4}}\,x}{5} + \frac{3\left(\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)a}{5}$$

input
```
int((b*x^2+a)^(3/4),x)
```

output
```
(2*(a + b*x**2)**(3/4)*x + 3*int((a + b*x**2)**(3/4)/(a + b*x**2),x)*a)/5
```

3.850 $\int \frac{(a+bx^2)^{3/4}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{(a+bx^2)^{3/4}}{x^2} dx = \frac{3bx}{\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a+bx^2}}$$

output
```
3*b*x/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/x-3*a^(1/2)*b^(1/2)*(1+b*x^2/a)^(1/4
)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.98 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int \frac{(a+bx^2)^{3/4}}{x^2} dx = -\frac{(a+bx^2)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(3/4)/x^2,x]
```

output
```
-(((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(3/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/4}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{3}{2}b\int \frac{1}{\sqrt[4]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{3/4}}{x}$$

$$\Big\downarrow \text{227}$$

$$\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{x}$$

$$\Big\downarrow \text{225}$$

$$\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{x}$$

$$\Big\downarrow \text{212}$$

$$\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{x}$$

input
```
Int[(a + b*x^2)^(3/4)/x^2,x]
```

output
```
-((a + b*x^2)^(3/4)/x) + (3*b*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)
^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))
/(2*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^2}dx$$

input
```
int((b*x^2+a)^(3/4)/x^2,x)
```

output
```
int((b*x^2+a)^(3/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.33

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = -\frac{a^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(3/4)/x**2,x)
```

output
```
-a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.45

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = \frac{2\,(b\,x^2+a)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\ \frac{3}{4};\ -\frac{a}{b\,x^2}\right)}{x\,\left(\frac{a}{b\,x^2}+1\right)^{3/4}}$$

input
```
int((a + b*x^2)^(3/4)/x^2,x)
```

output
```
(2*(a + b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(3/4))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^2}\,dx = \frac{2(b\,x^2+a)^{\frac{3}{4}}+3\left(\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((b*x^2+a)^(3/4)/x^2,x)
```

output
```
(2*(a + b*x**2)**(3/4) + 3*int((a + b*x**2)**(3/4)/(a*x**2 + b*x**4),x)*a*
x)/x
```

3.851 $\int \frac{(a+bx^2)^{3/4}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{(a+bx^2)^{3/4}}{x^4} dx = \frac{b^2x}{2a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{3x^3} - \frac{b(a+bx^2)^{3/4}}{2ax} - \frac{b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a+bx^2}}$$

output
```
1/2*b^2*x/a/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/x^3-1/2*b*(b*x^2+a)^(3/4)/
a/x-1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/
2))),2^(1/2))/a^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.42

$$\int \frac{(a+bx^2)^{3/4}}{x^4} dx = -\frac{(a+bx^2)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(3/4)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{bx^2+a}}dx-\frac{(a+bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2}b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{2a}-\frac{(a+bx^2)^{3/4}}{ax}\right)-\frac{(a+bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)-\frac{(a+bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{212}$$

$$\frac{1}{2}b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)-\frac{\left(a+bx^2\right)^{3/4}}{3x^3}$$

input
```
Int[(a + b*x^2)^(3/4)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(3/4)/x^3 + (b*(-((a + b*x^2)^(3/4)/(a*x)) + (b*(1 + (b*x
^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a*(a + b*x^2)^(1/4))))/2
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{3}{4}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(3/4)/x^4,x)
```

output
```
int((b*x^2+a)^(3/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = -\frac{a^{\frac{3}{4}}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(3/4)/x**4,x)
```

output
```
-a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{3/4}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(3/4)/x^4,x)
```

output
```
int((a + b*x^2)^(3/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^4}\,dx = \frac{-2(b\,x^2+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{b\,x^6+a\,x^4}dx\right)a\,x^3}{3x^3}$$

input
```
int((b*x^2+a)^(3/4)/x^4,x)
```

output
```
( - 2*(a + b*x**2)**(3/4) - 3*int((a + b*x**2)**(3/4)/(a*x**4 + b*x**6),x)
*a*x**3)/(3*x**3)
```

3.852 $\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 145

$$\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx=-\frac{3b^3x}{20a^2\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{5x^5}-\frac{b\left(a+bx^2\right)^{3/4}}{10ax^3}+\frac{3b^2\left(a+bx^2\right)^{3/4}}{20a^2x}+\frac{3b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a+bx^2}}$$

output

```
-3/20*b^3*x/a^2/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/x^5-1/10*b*(b*x^2+a)^(
3/4)/a/x^3+3/20*b^2*(b*x^2+a)^(3/4)/a^2/x+3/20*b^(5/2)*(1+b*x^2/a)^(1/4)*E
llipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{\left(a+bx^2\right)^{3/4}}{x^6}\,dx=-\frac{\left(a+bx^2\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(3/4)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {247, 264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{3}{10}b\int \frac{1}{x^4\sqrt[4]{bx^2+a}}dx-\frac{(a+bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{10}b\left(-\frac{b\int \frac{1}{x^2\sqrt[4]{bx^2+a}}dx}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}\right)-\frac{(a+bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{2a}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}\right)-\frac{(a+bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{227}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}\right)-\frac{(a+bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}\right)-\frac{(a+bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{3}{10}b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}\right)-\frac{(a+bx^2)^{3/4}}{5x^5}$$

input
```
Int[(a + b*x^2)^(3/4)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(3/4)/x^5 + (3*b*(-1/3*(a + b*x^2)^(3/4)/(a*x^3) - (b*(-(
(a + b*x^2)^(3/4)/(a*x)) + (b*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)
^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))
/(2*a*(a + b*x^2)^(1/4))))/(2*a)))/10
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{x^6}dx$$

input
```
int((b*x^2+a)^(3/4)/x^6,x)
```

output
```
int((b*x^2+a)^(3/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx=\int \frac{(bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.23

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(3/4)/x**6,x)
```

output
```
-a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx=\int \frac{(b\,x^2+a)^{3/4}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(3/4)/x^6,x)
```

output
```
int((a + b*x^2)^(3/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/4}}{x^6}\,dx=\frac{-2(b\,x^2+a)^{\frac{3}{4}}-3\left(\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{b\,x^8+a\,x^6}dx\right)a\,x^5}{7x^5}$$

input
```
int((b*x^2+a)^(3/4)/x^6,x)
```

output
```
( - 2*(a + b*x**2)**(3/4) - 3*int((a + b*x**2)**(3/4)/(a*x**6 + b*x**8),x)
*a*x**5)/(7*x**5)
```

3.853 $\int x^4(a-bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int x^4(a-bx^2)^{3/4}\,dx=-\frac{4a^2x(a-bx^2)^{3/4}}{65b^2}-\frac{2ax^3(a-bx^2)^{3/4}}{39b}+\frac{2}{13}x^5(a-bx^2)^{3/4}+\frac{8a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{a-bx^2}}$$

output
```
-4/65*a^2*x*(-b*x^2+a)^(3/4)/b^2-2/39*a*x^3*(-b*x^2+a)^(3/4)/b+2/13*x^5*(-
b*x^2+a)^(3/4)+8/65*a^(7/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(
1/2)*x/a^(1/2))),2^(1/2))/b^(5/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int x^4(a-bx^2)^{3/4}\,dx=-\frac{2x(a-bx^2)^{3/4}\left(\left(1-\frac{bx^2}{a}\right)^{3/4}(2a^2+abx^2-3b^2x^4)-2a^2\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{39b^2\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[x^4*(a - b*x^2)^(3/4),x]
```

output
```
(-2*x*(a - b*x^2)^(3/4)*((1 - (b*x^2)/a)^(3/4)*(2*a^2 + a*b*x^2 - 3*b^2*x^
4) - 2*a^2*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a]))/(39*b^2*(1 - (b*
x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {248, 262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a-bx^2\right)^{3/4} \, dx$$

$$\Big\downarrow \ 248$$

$$\frac{3}{13} a \int \frac{x^4}{\sqrt[4]{a-bx^2}}dx+\frac{2}{13} x^5 \left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 262$$

$$\frac{3}{13} a \left(\frac{2 a \int \frac{x^2}{\sqrt[4]{a-bx^2}}dx}{3 b}-\frac{2 x^3 \left(a-bx^2\right)^{3/4}}{9 b}\right)+\frac{2}{13} x^5 \left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 262$$

$$\frac{3}{13} a \left(\frac{2 a \left(\frac{2 a \int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5 b}-\frac{2 x \left(a-bx^2\right)^{3/4}}{5 b}\right)}{3 b}-\frac{2 x^3 \left(a-bx^2\right)^{3/4}}{9 b}\right)+\frac{2}{13} x^5 \left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 227$$

$$\frac{3}{13}a\left(\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{3b}-\frac{2x^3(a-bx^2)^{3/4}}{9b}\right)+$$
$$\frac{2}{13}x^5(a-bx^2)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{3}{13}a\left(\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{3b}-\frac{2x^3(a-bx^2)^{3/4}}{9b}\right)+$$
$$\frac{2}{13}x^5(a-bx^2)^{3/4}$$

input
```
Int[x^4*(a - b*x^2)^(3/4),x]
```

output
```
(2*x^5*(a - b*x^2)^(3/4))/13 + (3*a*((-2*x^3*(a - b*x^2)^(3/4))/(9*b) + (2
*a*((-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*Elli
pticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))))/(
3*b)))/13
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^4 \left(-b\,x^2+a\right)^{\frac{3}{4}}dx$$

input
```
int(x^4*(-b*x^2+a)^(3/4),x)
```

output
```
int(x^4*(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int x^4 (a - bx^2)^{3/4} \, dx = \int (-bx^2 + a)^{\frac{3}{4}} x^4 \, dx$$

input
```
integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int x^4 (a - bx^2)^{3/4} \, dx = \frac{a^{\frac{3}{4}} x^5 {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(-b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x**5*hyper((-3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a - bx^2)^{3/4} \, dx = \int (-bx^2 + a)^{\frac{3}{4}} x^4 \, dx$$

input
```
integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

Giac [F]

$$\int x^4 \left(a-bx^2\right)^{3/4}\,dx=\int \left(-bx^2+a\right)^{\frac{3}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a-bx^2\right)^{3/4}\,dx=\int x^4\,\left(a-b\,x^2\right)^{3/4}\,dx$$

input
```
int(x^4*(a - b*x^2)^(3/4),x)
```

output
```
int(x^4*(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^4 \left(a-bx^2\right)^{3/4}\,dx=\frac{-\frac{4\left(-b\,x^2+a\right)^{\frac{3}{4}}a^2x}{65}-\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}}a b\,x^3}{39}+\frac{2\left(-b\,x^2+a\right)^{\frac{3}{4}}b^2x^5}{13}+\frac{4\left(\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx\right)a^3}{65}}{b^2}$$

input
```
int(x^4*(-b*x^2+a)^(3/4),x)
```

output
```
(2*( - 6*(a - b*x**2)**(3/4)*a**2*x - 5*(a - b*x**2)**(3/4)*a*b*x**3 + 15*
(a - b*x**2)**(3/4)*b**2*x**5 + 6*int((a - b*x**2)**(3/4)/(a - b*x**2),x)*
a**3))/(195*b**2)
```

3.854 $\int x^2(a-bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int x^2\left(a-bx^2\right)^{3/4}\,dx=-\frac{2ax(a-bx^2)^{3/4}}{15b}+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}+\frac{4a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
-2/15*a*x*(-b*x^2+a)^(3/4)/b+2/9*x^3*(-b*x^2+a)^(3/4)+4/15*a^(5/2)*(1-b*x^
2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(
-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.89 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.63

$$\int x^2\left(a-bx^2\right)^{3/4}\,dx=\frac{2x(a-bx^2)^{3/4}\left(-a+bx^2+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}\right)}{9b}$$

input
```
Integrate[x^2*(a - b*x^2)^(3/4),x]
```

output
```
(2*x*(a - b*x^2)^(3/4)*(-a + b*x^2 + (a*Hypergeometric2F1[-3/4, 1/2, 3/2,
(b*x^2)/a])/(1 - (b*x^2)/a)^(3/4)))/(9*b)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {248, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a-bx^2\right)^{3/4} \, dx$$

$$\Big\downarrow \ 248$$

$$\frac{1}{3}a\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{3}a\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 227$$

$$\frac{1}{3}a\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \ 226$$

$$\frac{1}{3}a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)+\frac{2}{9}x^3\left(a-bx^2\right)^{3/4}$$

input
```
Int[x^2*(a - b*x^2)^(3/4),x]
```

output
```
(2*x^3*(a - b*x^2)^(3/4))/9 + (a*((-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3
/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*
b^(3/2)*(a - b*x^2)^(1/4))))/3
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{2} \left(-b\,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int(x^2*(-b*x^2+a)^(3/4),x)
```

output
```
int(x^2*(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int x^2 \left(a-bx^2\right)^{3/4} \, dx = \int { {\left(-b x^{2} + a\right)}^{\frac{3}{4}} x^{2} \,d x}$$

input
```
integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.31

$$\int x^2 \left(a-bx^2\right)^{3/4} \, dx = \frac{a^{\frac{3}{4}} x^{3} {{}_{2}F_{1}\left(\begin{matrix} - \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{2 i \pi}}{a}} \right)}}{3}$$

input
```
integrate(x**2*(-b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x**3*hyper((-3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Maxima [F]

$$\int x^2(a-bx^2)^{3/4}\,dx=\int(-bx^2+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

Giac [F]

$$\int x^2(a-bx^2)^{3/4}\,dx=\int(-bx^2+a)^{\frac{3}{4}}x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a-bx^2)^{3/4}\,dx=\int x^2\,\left(a-b\,x^2\right)^{3/4}\,dx$$

input
```
int(x^2*(a - b*x^2)^(3/4),x)
```

output
```
int(x^2*(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int x^2\left(a-bx^2\right)^{3/4}\,dx=\frac{-\frac{2(-b\,x^2+a)^{\frac{3}{4}}a x}{15}+\frac{2(-b\,x^2+a)^{\frac{3}{4}}b\,x^3}{9}+\frac{2\left(\int\frac{1}{(-b\,x^2+a)^{\frac{1}{4}}}dx\right)a^2}{15}}{b}$$

input
```
int(x^2*(-b*x^2+a)^(3/4),x)
```

output
```
(2*( - 3*(a - b*x**2)**(3/4)*a*x + 5*(a - b*x**2)**(3/4)*b*x**3 + 3*int((a
 - b*x**2)**(3/4)/(a - b*x**2),x)*a**2))/(45*b)
```

3.855 $\int (a-bx^2)^{3/4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 78

$$\int (a-bx^2)^{3/4}\,dx=\frac{2}{5}x(a-bx^2)^{3/4}+\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2/5*x*(-b*x^2+a)^(3/4)+6/5*a^(3/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arc
sin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.60

$$\int (a-bx^2)^{3/4}\,dx=\frac{x(a-bx^2)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(3/4),x]
```

output
```
(x*(a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b
*x^2)/a)^(3/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {211, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{3/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{5}a\int \frac{1}{\sqrt[4]{a-bx^2}}dx+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \text{227}$$

$$\frac{3a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{6a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5\sqrt{b}\sqrt[4]{a-bx^2}}+\frac{2}{5}x\left(a-bx^2\right)^{3/4}$$

input
```
Int[(a - b*x^2)^(3/4),x]
```

output
```
(2*x*(a - b*x^2)^(3/4))/5 + (6*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a \right)^{\frac{3}{4}}d x$$

input
```
int((-b*x^2+a)^(3/4),x)
```

output
```
int((-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{3/4}\,dx=\int \left(-bx^{2}+a\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int (a-bx^2)^{3/4}\,dx = a^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix} -\frac{3}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(3/4),x)
```

output
```
a**(3/4)*x*hyper((-3/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int (a-bx^2)^{3/4}\,dx = \int (-bx^2+a)^{\frac{3}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int (a-bx^2)^{3/4}\,dx = \int (-bx^2+a)^{\frac{3}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int (a-bx^2)^{3/4} dx = \frac{x\,(a-b\,x^2)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{3/4}}$$

input
```
int((a - b*x^2)^(3/4),x)
```

output
```
(x*(a - b*x^2)^(3/4)*hypergeom([-3/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(3/4)
```

Reduce [F]

$$\int (a-bx^2)^{3/4} dx = \frac{2(-b\,x^2+a)^{\frac{3}{4}}\,x}{5} + \frac{3\left(\int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}}dx\right)a}{5}$$

input
```
int((-b*x^2+a)^(3/4),x)
```

output
```
(2*(a - b*x**2)**(3/4)*x + 3*int((a - b*x**2)**(3/4)/(a - b*x**2),x)*a)/5
```

3.856 $\int \frac{(a-bx^2)^{3/4}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{(a-bx^2)^{3/4}}{x^2} dx = -\frac{(a-bx^2)^{3/4}}{x} - \frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}$$

output
```
-(-b*x^2+a)^(3/4)/x-3*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*
arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.66

$$\int \frac{(a-bx^2)^{3/4}}{x^2} dx = -\frac{(a-bx^2)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(3/4)/x^2,x]
```

output
```
-(((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/4, -1/2, 1/2, (b*x^2)/a])/(x*(1
- (b*x^2)/a)^(3/4)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {247, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{3/4}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{3}{2}b\int \frac{1}{\sqrt[4]{a-bx^2}}dx-\frac{\left(a-bx^2\right)^{3/4}}{x}$$

$$\Big\downarrow \text{227}$$

$$-\frac{3b\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{x}$$

$$\Big\downarrow \text{226}$$

$$-\frac{3\sqrt{a}\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{x}$$

input
```
Int[(a - b*x^2)^(3/4)/x^2,x]
```

output
```
-((a - b*x^2)^(3/4)/x) - (3*Sqrt[a]*Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a - b*x^2)^(1/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{x^2}dx$$

input
```
int((-b*x^2+a)^(3/4)/x^2,x)
```

output
```
int((-b*x^2+a)^(3/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.41

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

input
```
integrate((-b*x**2+a)**(3/4)/x**2,x)
```

output
```
-a**(3/4)*hyper((-3/4, -1/2), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx=\int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx=\frac{2\,(a-b\,x^2)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4},-\frac{1}{4};\ \frac{3}{4};\ \frac{a}{b\,x^2}\right)}{x\,\left(1-\frac{a}{b\,x^2}\right)^{3/4}}$$

input
```
int((a - b*x^2)^(3/4)/x^2,x)
```

output
```
(2*(a - b*x^2)^(3/4)*hypergeom([-3/4, -1/4], 3/4, a/(b*x^2)))/(x*(1 - a/(b
*x^2))^(3/4))
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^2}\,dx=\frac{2(-b\,x^2+a)^{\frac{3}{4}}+3\left(\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{-b\,x^4+a\,x^2}dx\right)ax}{x}$$

input
```
int((-b*x^2+a)^(3/4)/x^2,x)
```

output
```
(2*(a - b*x**2)**(3/4) + 3*int((a - b*x**2)**(3/4)/(a*x**2 - b*x**4),x)*a*
x)/x
```

3.857 $\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = -\frac{(a-bx^2)^{3/4}}{3x^3}+\frac{b(a-bx^2)^{3/4}}{2ax}+\frac{b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2\sqrt{a}\sqrt[4]{a-bx^2}}$$

output
```
-1/3*(-b*x^2+a)^(3/4)/x^3+1/2*b*(-b*x^2+a)^(3/4)/a/x+1/2*b^(3/2)*(1-b*x^2/
a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/(-b
*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.50

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = -\frac{(a-bx^2)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(3/4)/x^4,x]
```

output
```
-1/3*((a - b*x^2)^(3/4)*Hypergeometric2F1[-3/2, -3/4, -1/2, (b*x^2)/a])/(x
^3*(1 - (b*x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {247, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/4}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{2}b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}dx-\frac{(a-bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{2}b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{(a-bx^2)^{3/4}}{ax}\right)-\frac{(a-bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{227}$$

$$-\frac{1}{2}b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}\right)-\frac{(a-bx^2)^{3/4}}{3x^3}$$

$$\Big\downarrow \text{226}$$

$$-\frac{1}{2}b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}\right)-\frac{(a-bx^2)^{3/4}}{3x^3}$$

input
```
Int[(a - b*x^2)^(3/4)/x^4,x]
```

output
```
-1/3*(a - b*x^2)^(3/4)/x^3 - (b*(-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1
- (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(
a - b*x^2)^(1/4))))/2
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{x^4}dx$$

input
```
int((-b*x^2+a)^(3/4)/x^4,x)
```

output
```
int((-b*x^2+a)^(3/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((-b*x**2+a)**(3/4)/x**4,x)
```

output
```
-a**(3/4)*hyper((-3/2, -3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3
)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^4,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = \int \frac{(a-b\,x^2)^{3/4}}{x^4}\,dx$$

input
```
int((a - b*x^2)^(3/4)/x^4,x)
```

output
```
int((a - b*x^2)^(3/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^4}\,dx = \frac{-2(-b\,x^2+a)^{\frac{3}{4}}-3\left(\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{-b\,x^6+a\,x^4}dx\right)a\,x^3}{3x^3}$$

input
```
int((-b*x^2+a)^(3/4)/x^4,x)
```

output
```
( - 2*(a - b*x**2)**(3/4) - 3*int((a - b*x**2)**(3/4)/(a*x**4 - b*x**6),x)
*a*x**3)/(3*x**3)
```

3.858 $\int \frac{(a-bx^2)^{3/4}}{x^6} dx$

Optimal result

Integrand size = 16, antiderivative size = 128

$$\int \frac{(a-bx^2)^{3/4}}{x^6} dx = -\frac{(a-bx^2)^{3/4}}{5x^5} + \frac{b(a-bx^2)^{3/4}}{10ax^3} + \frac{3b^2(a-bx^2)^{3/4}}{20a^2x} + \frac{3b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
-1/5*(-b*x^2+a)^(3/4)/x^5+1/10*b*(-b*x^2+a)^(3/4)/a/x^3+3/20*b^2*(-b*x^2+a
)^(3/4)/a^2/x+3/20*b^(5/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1
/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.41

$$\int \frac{(a-bx^2)^{3/4}}{x^6} dx = -\frac{(a-bx^2)^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(3/4)/x^6,x]
```

output
```
-1/5*((a - b*x^2)^(3/4)*Hypergeometric2F1[-5/2, -3/4, -3/2, (b*x^2)/a])/(x
^5*(1 - (b*x^2)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {247, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx$$

$$\Big\downarrow \; 247$$

$$-\frac{3}{10}b\int \frac{1}{x^4\sqrt[4]{a-bx^2}}dx-\frac{(a-bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \; 264$$

$$-\frac{3}{10}b\left(\frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}dx}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)-\frac{(a-bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \; 264$$

$$-\frac{3}{10}b\left(\frac{b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{(a-bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)-\frac{(a-bx^2)^{3/4}}{5x^5}$$

$$\Big\downarrow \; 227$$

$$-\frac{3}{10}b\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)-\frac{(a-bx^2)^{3/4}}{5x^5}$$

$\Big\downarrow$ 226

$$-\frac{3}{10}b\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)-\frac{(a-bx^2)^{3/4}}{5x^5}$$

input
```
Int[(a - b*x^2)^(3/4)/x^6,x]
```

output
```
-1/5*(a - b*x^2)^(3/4)/x^5 - (3*b*(-1/3*(a - b*x^2)^(3/4)/(a*x^3) + (b*(-(
(a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin
[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))))/(2*a)))/10
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{x^6}dx$$

input
```
int((-b*x^2+a)^(3/4)/x^6,x)
```

output
```
int((-b*x^2+a)^(3/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx=\int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx = -\frac{a^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((-b*x**2+a)**(3/4)/x**6,x)
```

output
```
-a**(3/4)*hyper((-5/2, -3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5
)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{3}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(3/4)/x^6,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(3/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx=\int \frac{(a-b\,x^2)^{3/4}}{x^6}\,dx$$

input
```
int((a - b*x^2)^(3/4)/x^6,x)
```

output
```
int((a - b*x^2)^(3/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{3/4}}{x^6}\,dx=\frac{-2(-b\,x^2+a)^{\frac{3}{4}}-3\left(\int \frac{(-b\,x^2+a)^{\frac{3}{4}}}{-b\,x^8+a\,x^6}dx\right)a\,x^5}{7x^5}$$

input
```
int((-b*x^2+a)^(3/4)/x^6,x)
```

output
```
( - 2*(a - b*x**2)**(3/4) - 3*int((a - b*x**2)**(3/4)/(a*x**6 - b*x**8),x)
*a*x**5)/(7*x**5)
```

3.859 $\int x^4(a+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 142

$$\int x^4\left(a+bx^2\right)^{5/4}\,dx=-\frac{4a^3x\sqrt[4]{a+bx^2}}{231b^2}+\frac{2a^2x^3\sqrt[4]{a+bx^2}}{231b}+\frac{2}{33}ax^5\sqrt[4]{a+bx^2}+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}+\frac{8a^{9/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{5/2}\left(a+bx^2\right)^{3/4}}$$

output
```
-4/231*a^3*x*(b*x^2+a)^(1/4)/b^2+2/231*a^2*x^3*(b*x^2+a)^(1/4)/b+2/33*a*x^
5*(b*x^2+a)^(1/4)+2/15*x^5*(b*x^2+a)^(5/4)+8/231*a^(9/2)*(1+b*x^2/a)^(3/4)
*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(b*x^2+a)^
(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56

$$\int x^4(a+bx^2)^{5/4}\,dx = \frac{2x\sqrt[4]{a+bx^2}\left(-\left(\left(6a-11bx^2\right)\left(a+bx^2\right)^2\right)+\frac{6a^3\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}\right)}{165b^2}$$

input
```
Integrate[x^4*(a + b*x^2)^(5/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4)*(-((6*a - 11*b*x^2)*(a + b*x^2)^2) + (6*a^3*Hyperge
ometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(165*b^2
)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {248, 248, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2\right)^{5/4}dx$$

$$\Big\downarrow \ 248$$

$$\frac{1}{3}a\int x^4\sqrt[4]{bx^2+a}dx+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \ 248$$

$$\frac{1}{3}a\left(\frac{1}{11}a\int\frac{x^4}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{11}x^5\sqrt[4]{a+bx^2}\right)+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \ 262$$

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\int \frac{x^2}{(bx^2+a)^{3/4}}dx}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}\right)+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

$\downarrow$ 262

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}\right)+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

$\downarrow$ 231

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b(a+bx^2)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}\right)+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

$\downarrow$ 229

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}\right)}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a+bx^2}\right)+\frac{2}{15}x^5\left(a+bx^2\right)^{5/4}$$

input
```
Int[x^4*(a + b*x^2)^(5/4),x]
```

output
```
(2*x^5*(a + b*x^2)^(5/4))/15 + (a*((2*x^5*(a + b*x^2)^(1/4))/11 + (a*((2*x
^3*(a + b*x^2)^(1/4))/(7*b) - (6*a*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(
3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3
*b^(3/2)*(a + b*x^2)^(3/4))))/(7*b)))/11))/3
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{\frac{5}{4}}d x$$

input
```
int(x^4*(b*x^2+a)^(5/4),x)
```

output
```
int(x^4*(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int x^4 (a+bx^2)^{5/4} \, dx = \int (bx^2+a)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.20

$$\int x^4 (a+bx^2)^{5/4} \, dx = \frac{a^{\frac{5}{4}} x^5 {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x**5*hyper((-5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a + bx^2)^{5/4} \, dx = \int (bx^2 + a)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2)^{5/4} \, dx = \int (bx^2 + a)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{5/4} \, dx = \int x^4 \left(b\,x^2 + a\right)^{5/4} \, dx$$

input
```
int(x^4*(a + b*x^2)^(5/4),x)
```

output
```
int(x^4*(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x^4(a+bx^2)^{5/4}\,dx=\frac{-\frac{4(bx^2+a)^{\frac{1}{4}}a^3x}{231}+\frac{2(bx^2+a)^{\frac{1}{4}}a^2bx^3}{231}+\frac{32(bx^2+a)^{\frac{1}{4}}ab^2x^5}{165}+\frac{2(bx^2+a)^{\frac{1}{4}}b^3x^7}{15}+\frac{4\left(\int\frac{1}{(bx^2+a)^{\frac{3}{4}}}dx\right)a^4}{231}}{b^2}$$

input
```
int(x^4*(b*x^2+a)^(5/4),x)
```

output
```
(2*( - 10*(a + b*x**2)**(1/4)*a**3*x + 5*(a + b*x**2)**(1/4)*a**2*b*x**3 +
 112*(a + b*x**2)**(1/4)*a*b**2*x**5 + 77*(a + b*x**2)**(1/4)*b**3*x**7 +
10*int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a**4))/(1155*b**2)
```

3.860 $\int x^2(a+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int x^2\left(a+bx^2\right)^{5/4}\,dx=\frac{10a^2x\sqrt[4]{a+bx^2}}{231b}+\frac{10}{77}ax^3\sqrt[4]{a+bx^2}+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}-\frac{20a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{3/2}\left(a+bx^2\right)^{3/4}}$$

output
```
10/231*a^2*x*(b*x^2+a)^(1/4)/b+10/77*a*x^3*(b*x^2+a)^(1/4)+2/11*x^3*(b*x^2
+a)^(5/4)-20/231*a^(7/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1
/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

$$\int x^2\left(a+bx^2\right)^{5/4}\,dx=\frac{2x\sqrt[4]{a+bx^2}\left(\left(a+bx^2\right)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}\right)}{11b}$$

input
```
Integrate[x^2*(a + b*x^2)^(5/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-5/4, 1/2,
3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/4)))/(11*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {248, 248, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{5/4} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{11}a\int x^2\sqrt[4]{bx^2+a}dx+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{11}a\left(\frac{1}{7}a\int \frac{x^2}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{7}x^3\sqrt[4]{a+bx^2}\right)+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}\right)+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\left(a+bx^2\right)^{3/4}}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}\right)+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}\right)+\frac{2}{7}x^3\sqrt[4]{a+bx^2}\right)+\frac{2}{11}x^3\left(a+bx^2\right)^{5/4}$$

input
```
Int[x^2*(a + b*x^2)^(5/4),x]
```

output
```
(2*x^3*(a + b*x^2)^(5/4))/11 + (5*a*((2*x^3*(a + b*x^2)^(1/4))/7 + (a*((2*
x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[Ar
cTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))))/7))/11
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a \right)^{\frac{5}{4}}d x$$

input
```
int(x^2*(b*x^2+a)^(5/4),x)
```

output
```
int(x^2*(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{5/4} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{5}{4}} x^{2} \,d x }$$

input
```
integrate(x^2*(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.25

$$\int x^2 \left(a+bx^2\right)^{5/4} \, dx = \frac{a^{\frac{5}{4}} x^{3} {{}_{2}F_{1}\left(\begin{matrix} - \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x**3*hyper((-5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a + bx^2)^{5/4} \, dx = \int (bx^2 + a)^{\frac{5}{4}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^{5/4} \, dx = \int (bx^2 + a)^{\frac{5}{4}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^{5/4} \, dx = \int x^2 \left(b x^2 + a\right)^{5/4} \, dx$$

input
```
int(x^2*(a + b*x^2)^(5/4),x)
```

output
```
int(x^2*(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x^2(a+bx^2)^{5/4}\,dx = \frac{\frac{10(b\,x^2+a)^{\frac{1}{4}}a^2x}{231}+\frac{24(b\,x^2+a)^{\frac{1}{4}}ab\,x^3}{77}+\frac{2(b\,x^2+a)^{\frac{1}{4}}b^2x^5}{11}-\frac{10\left(\int\frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^3}{231}}{b}$$

input
```
int(x^2*(b*x^2+a)^(5/4),x)
```

output
```
(2*(5*(a + b*x**2)**(1/4)*a**2*x + 36*(a + b*x**2)**(1/4)*a*b*x**3 + 21*(a
 + b*x**2)**(1/4)*b**2*x**5 - 5*int((a + b*x**2)**(1/4)/(a + b*x**2),x)*a*
*3))/(231*b)
```

3.861 $\int (a+bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 92

$$\int \left(a+bx^2\right)^{5/4}\,dx=\frac{10}{21}ax\sqrt[4]{a+bx^2}+\frac{2}{7}x\left(a+bx^2\right)^{5/4}+\frac{10a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}\left(a+bx^2\right)^{3/4}}$$

output
```
10/21*a*x*(b*x^2+a)^(1/4)+2/7*x*(b*x^2+a)^(5/4)+10/21*a^(5/2)*(1+b*x^2/a)^
(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^
2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \left(a+bx^2\right)^{5/4}\,dx=\frac{ax\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/4),x]
```

output
```
(a*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/4}dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\int \sqrt[4]{bx^2+a}dx+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\left(\frac{1}{3}a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{7}a\left(\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{7}a\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)+\frac{2}{7}x\left(a+bx^2\right)^{5/4}$$

input
```
Int[(a + b*x^2)^(5/4),x]
```

output
```
(2*x*(a + b*x^2)^(5/4))/7 + (5*a*((2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(
1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[
b]*(a + b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{4}}d x$$

input
```
int((b*x^2+a)^(5/4),x)
```

output
```
int((b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \left(a + bx^2\right)^{5/4} dx = \int \left(bx^2 + a\right)^{\frac{5}{4}} dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.28

$$\int \left(a + bx^2\right)^{5/4} dx = a^{\frac{5}{4}} x {}_2F_1\left(\left.\begin{matrix} -\frac{5}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a + bx^2\right)^{5/4} dx = \int \left(bx^2 + a\right)^{\frac{5}{4}} dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/4} dx = \int \left(bx^2+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.40

$$\int \left(a+bx^2\right)^{5/4} dx = \frac{x\left(b\,x^2+a\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/4}}$$

input
```
int((a + b*x^2)^(5/4),x)
```

output
```
(x*(a + b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/4)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/4} dx = \frac{16(b\,x^2+a)^{\frac{1}{4}}\,ax}{21} + \frac{2(b\,x^2+a)^{\frac{1}{4}}\,b\,x^3}{7} + \frac{5\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2}{21}$$

input
```
int((b*x^2+a)^(5/4),x)
```

output
```
(16*(a + b*x**2)**(1/4)*a*x + 6*(a + b*x**2)**(1/4)*b*x**3 + 5*int((a + b*
x**2)**(1/4)/(a + b*x**2),x)*a**2)/21
```

3.862 $\int \frac{(a+bx^2)^{5/4}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int \frac{(a+bx^2)^{5/4}}{x^2} dx = \frac{5}{3}bx\sqrt[4]{a+bx^2} - \frac{(a+bx^2)^{5/4}}{x} + \frac{5a^{3/2}\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3(a+bx^2)^{3/4}}$$

output
```
5/3*b*x*(b*x^2+a)^(1/4)-(b*x^2+a)^(5/4)/x+5/3*a^(3/2)*b^(1/2)*(1+b*x^2/a)^
(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/(b*x^2+a)^(3/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{5/4}}{x^2} dx = -\frac{a\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/4)/x^2,x]
```

output
```
-((a*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, -((b*x^2)/a)])/(
x*(1 + (b*x^2)/a)^(1/4)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 211, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/4}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{5}{2}b\int \sqrt[4]{bx^2+a}dx-\frac{\left(a+bx^2\right)^{5/4}}{x}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{2}b\left(\frac{1}{3}a\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{5/4}}{x}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{2}b\left(\frac{a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{5/4}}{x}$$

$$\Big\downarrow \text{229}$$

$$\frac{5}{2}b\left(\frac{2a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{5/4}}{x}$$

input
```
Int[(a + b*x^2)^(5/4)/x^2,x]
```

output
```
-((a + b*x^2)^(5/4)/x) + (5*b*((2*x*(a + b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 +
 (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*
(a + b*x^2)^(3/4))))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{x^2}dx$$

input
```
int((b*x^2+a)^(5/4)/x^2,x)
```

output
```
int((b*x^2+a)^(5/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.32

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = -\frac{a^{\frac{5}{4}}{}_2F_1\left(\begin{matrix} -\frac{5}{4}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(5/4)/x**2,x)
```

output
```
-a**(5/4)*hyper((-5/4, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = \frac{2\,(b\,x^2+a)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},-\frac{3}{4};\ \frac{1}{4};\ -\frac{a}{b\,x^2}\right)}{3\,x\,\left(\frac{a}{b\,x^2}+1\right)^{5/4}}$$

input
```
int((a + b*x^2)^(5/4)/x^2,x)
```

output
```
(2*(a + b*x^2)^(5/4)*hypergeom([-5/4, -3/4], 1/4, -a/(b*x^2)))/(3*x*(a/(b*
x^2) + 1)^(5/4))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^2}\,dx = \frac{-8(b\,x^2+a)^{\frac{1}{4}}\,a+2(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2-5\left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^4+a\,x^2}dx\right)a^2x}{3x}$$

input
```
int((b*x^2+a)^(5/4)/x^2,x)
```

output
```
( - 8*(a + b*x**2)**(1/4)*a + 2*(a + b*x**2)**(1/4)*b*x**2 - 5*int((a + b*
x**2)**(1/4)/(a*x**2 + b*x**4),x)*a**2*x)/(3*x)
```

3.863 $\int \frac{(a+bx^2)^{5/4}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{(a+bx^2)^{5/4}}{x^4} dx = -\frac{5b\sqrt[4]{a+bx^2}}{6x} - \frac{(a+bx^2)^{5/4}}{3x^3} + \frac{5\sqrt{a}b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6(a+bx^2)^{3/4}}$$

output
```
-5/6*b*(b*x^2+a)^(1/4)/x-1/3*(b*x^2+a)^(5/4)/x^3+5/6*a^(1/2)*b^(3/2)*(1+b*
x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/(b*x^2
+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)^{5/4}}{x^4} dx = -\frac{a\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/4)/x^4,x]
```

output
```
-1/3*(a*(a + b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, -((b*x^2)/a)
])/(x^3*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {247, 247, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/4}}{x^4}dx$$

$$\Big\downarrow \ 247$$

$$\frac{5}{6}b\int \frac{\sqrt[4]{bx^2+a}}{x^2}dx-\frac{\left(a+bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \ 247$$

$$\frac{5}{6}b\left(\frac{1}{2}b\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx-\frac{\sqrt[4]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \ 231$$

$$\frac{5}{6}b\left(\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \ 229$$

$$\frac{5}{6}b\left(\frac{\sqrt{a}\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{5/4}}{3x^3}$$

input
```
Int[(a + b*x^2)^(5/4)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(5/4)/x^3 + (5*b*(-((a + b*x^2)^(1/4)/x) + (Sqrt[a]*Sqrt[
b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a +
 b*x^2)^(3/4)))/6
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{x^4}dx$$

input
```
int((b*x^2+a)^(5/4)/x^4,x)
```

output
```
int((b*x^2+a)^(5/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.35

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx=-\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(5/4)/x**4,x)
```

output
```
-a**(5/4)*hyper((-3/2, -5/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(5/4)/x^4,x)
```

output
```
int((a + b*x^2)^(5/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^4}\,dx = \frac{-2(b\,x^2+a)^{\frac{1}{4}}\,a-12(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2-5\left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^4+a\,x^2}dx\right)ab\,x^3}{6x^3}$$

input
```
int((b*x^2+a)^(5/4)/x^4,x)
```

output
```
( - 2*(a + b*x**2)**(1/4)*a - 12*(a + b*x**2)**(1/4)*b*x**2 - 5*int((a + b
*x**2)**(1/4)/(a*x**2 + b*x**4),x)*a*b*x**3)/(6*x**3)
```

3.864 $\int \frac{(a+bx^2)^{5/4}}{x^6} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{(a+bx^2)^{5/4}}{x^6} \, dx = -\frac{b\sqrt[4]{a+bx^2}}{6x^3} - \frac{b^2\sqrt[4]{a+bx^2}}{12ax} - \frac{(a+bx^2)^{5/4}}{5x^5} - \frac{b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12\sqrt{a}\,(a+bx^2)^{3/4}}$$

output
```
-1/6*b*(b*x^2+a)^(1/4)/x^3-1/12*b^2*(b*x^2+a)^(1/4)/a/x-1/5*(b*x^2+a)^(5/4
)/x^5-1/12*b^(5/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/
a^(1/2)),2^(1/2))/a^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{(a+bx^2)^{5/4}}{x^6} \, dx = -\frac{a\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(5/4)/x^6,x]
```

output
```
-1/5*(a*(a + b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, -((b*x^2)/a)
])/(x^5*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 247, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}b\int \frac{\sqrt[4]{bx^2+a}}{x^4}dx-\frac{(a+bx^2)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{2}b\left(\frac{1}{6}b\int \frac{1}{x^2\,(bx^2+a)^{3/4}}dx-\frac{\sqrt[4]{a+bx^2}}{3x^3}\right)-\frac{(a+bx^2)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)-\frac{\sqrt[4]{a+bx^2}}{3x^3}\right)-\frac{(a+bx^2)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{231}$$

$$\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)-\frac{\sqrt[4]{a+bx^2}}{3x^3}\right)-\frac{(a+bx^2)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{2}b\left(\frac{1}{6}b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)-\frac{\sqrt[4]{a+bx^2}}{3x^3}\right)-\frac{(a+bx^2)^{5/4}}{5x^5}$$

input
```
Int[(a + b*x^2)^(5/4)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/4)/x^5 + (b*(-1/3*(a + b*x^2)^(1/4)/x^3 + (b*(-((a + b
*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))))/6))/2
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{4}}}{x^6}dx$$

input
```
int((b*x^2+a)^(5/4)/x^6,x)
```

output
```
int((b*x^2+a)^(5/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.28

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = -\frac{a^{\frac{5}{4}}{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(5/4)/x**6,x)
```

output
```
-a**(5/4)*hyper((-5/2, -5/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/4)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{5/4}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(5/4)/x^6,x)
```

output
```
int((a + b*x^2)^(5/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/4}}{x^6}\,dx = \frac{-8(b\,x^2+a)^{\frac{1}{4}}\,a-18(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+5\left(\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{b\,x^8+a\,x^6}dx\right)a^2x^5}{45x^5}$$

input
```
int((b*x^2+a)^(5/4)/x^6,x)
```

output
```
( - 8*(a + b*x**2)**(1/4)*a - 18*(a + b*x**2)**(1/4)*b*x**2 + 5*int((a + b
*x**2)**(1/4)/(a*x**6 + b*x**8),x)*a**2*x**5)/(45*x**5)
```

3.865 $\int x^4(a-bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 148

$$\int x^4\left(a-bx^2\right)^{5/4}\,dx=-\frac{4a^3x\sqrt[4]{a-bx^2}}{231b^2}-\frac{2a^2x^3\sqrt[4]{a-bx^2}}{231b}+\frac{2}{33}ax^5\sqrt[4]{a-bx^2}+\frac{2}{15}x^5\left(a-bx^2\right)^{5/4}+\frac{8a^{9/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{5/2}\left(a-bx^2\right)^{3/4}}$$

output
```
-4/231*a^3*x*(-b*x^2+a)^(1/4)/b^2-2/231*a^2*x^3*(-b*x^2+a)^(1/4)/b+2/33*a*
x^5*(-b*x^2+a)^(1/4)+2/15*x^5*(-b*x^2+a)^(5/4)+8/231*a^(9/2)*(1-b*x^2/a)^(
3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(-b*x^
2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.80 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.55

$$\int x^4(a-bx^2)^{5/4}\,dx=\frac{2x\sqrt[4]{a-bx^2}\left(-(a-bx^2)^2(6a+11bx^2)+\frac{6a^3\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}\right)}{165b^2}$$

input
```
Integrate[x^4*(a - b*x^2)^(5/4),x]
```

output
```
(2*x*(a - b*x^2)^(1/4)*(-((a - b*x^2)^2*(6*a + 11*b*x^2)) + (6*a^3*Hyperge
ometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(165*b^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {248, 248, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a-bx^2)^{5/4}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{3}a\int x^4\sqrt[4]{a-bx^2}dx+\frac{2}{15}x^5(a-bx^2)^{5/4}$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{3}a\left(\frac{1}{11}a\int\frac{x^4}{(a-bx^2)^{3/4}}dx+\frac{2}{11}x^5\sqrt[4]{a-bx^2}\right)+\frac{2}{15}x^5(a-bx^2)^{5/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{6a\int\frac{x^2}{(a-bx^2)^{3/4}}dx}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}\right)+\frac{2}{15}x^5\left(a-bx^2\right)^{5/4}$$

$\Big\downarrow$ 262

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{6a\left(\frac{2a\int\frac{1}{\left(a-bx^2\right)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}\right)+\frac{2}{15}x^5\left(a-bx^2\right)^{5/4}$$

$\Big\downarrow$ 231

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}\right)+\frac{2}{15}x^5\left(a-bx^2\right)^{5/4}$$

$\Big\downarrow$ 230

$$\frac{1}{3}a\left(\frac{1}{11}a\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)+\frac{2}{11}x^5\sqrt[4]{a-bx^2}\right)+\frac{2}{15}x^5\left(a-bx^2\right)^{5/4}$$

input

```
Int[x^4*(a - b*x^2)^(5/4),x]
```

output
```
(2*x^5*(a - b*x^2)^(5/4))/15 + (a*((2*x^5*(a - b*x^2)^(1/4))/11 + (a*((-2*
x^3*(a - b*x^2)^(1/4))/(7*b) + (6*a*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a
^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(3*b^(3/2)*(a - b*x^2)^(3/4))))/(7*b)))/11))/3
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^{4} \left(-b \,x^{2}+a \right)^{\frac{5}{4}}d x$$

input
```
int(x^4*(-b*x^2+a)^(5/4),x)
```

output
```
int(x^4*(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int x^4 \left(a - bx^2\right)^{5/4} \, dx = \int \left(-bx^2 + a\right)^{\frac{5}{4}} x^4 \, dx$$

input
```
integrate(x^4*(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral(-(b*x^6 - a*x^4)*(-b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.21

$$\int x^4 \left(a - bx^2\right)^{5/4} \, dx = \frac{a^{\frac{5}{4}} x^5 {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(-b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x**5*hyper((-5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/5
```

Maxima [F]

$$\int x^4{(a-bx^2)}^{5/4}\,dx=\int {(-bx^2+a)}^{\frac{5}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4)*x^4, x)
```

Giac [F]

$$\int x^4{(a-bx^2)}^{5/4}\,dx=\int {(-bx^2+a)}^{\frac{5}{4}}x^4\,dx$$

input
```
integrate(x^4*(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4{(a-bx^2)}^{5/4}\,dx=\int x^4\,{\left(a-b\,x^2\right)}^{5/4}\,dx$$

input
```
int(x^4*(a - b*x^2)^(5/4),x)
```

output
```
int(x^4*(a - b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x^4 (a - bx^2)^{5/4} \, dx = \frac{-\frac{4(-b x^2+a)^{\frac{1}{4}} a^3 x}{231} - \frac{2(-b x^2+a)^{\frac{1}{4}} a^2 b x^3}{231} + \frac{32(-b x^2+a)^{\frac{1}{4}} a b^2 x^5}{165} - \frac{2(-b x^2+a)^{\frac{1}{4}} b^3 x^7}{15} + \frac{4\left(\int \frac{1}{(-b x^2+a)^{\frac{3}{4}}} dx\right) a^4}{231}}{b^2}$$

input
```
int(x^4*(-b*x^2+a)^(5/4),x)
```

output
```
(2*( - 10*(a - b*x**2)**(1/4)*a**3*x - 5*(a - b*x**2)**(1/4)*a**2*b*x**3 +
 112*(a - b*x**2)**(1/4)*a*b**2*x**5 - 77*(a - b*x**2)**(1/4)*b**3*x**7 +
10*int((a - b*x**2)**(1/4)/(a - b*x**2),x)*a**4))/(1155*b**2)
```

3.866 $\int x^2(a-bx^2)^{5/4}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int x^2\left(a-bx^2\right)^{5/4}\,dx=-\frac{10a^2x\sqrt[4]{a-bx^2}}{231b}+\frac{10}{77}ax^3\sqrt[4]{a-bx^2}+\frac{2}{11}x^3\left(a-bx^2\right)^{5/4}+\frac{20a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{231b^{3/2}\left(a-bx^2\right)^{3/4}}$$

output
```
-10/231*a^2*x*(-b*x^2+a)^(1/4)/b+10/77*a*x^3*(-b*x^2+a)^(1/4)+2/11*x^3*(-b
*x^2+a)^(5/4)+20/231*a^(7/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(
b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.57

$$\int x^2\left(a-bx^2\right)^{5/4}\,dx=\frac{2x\sqrt[4]{a-bx^2}\left(-\left(a-bx^2\right)^2+\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}\right)}{11b}$$

input
```
Integrate[x^2*(a - b*x^2)^(5/4),x]
```

output
```
(2*x*(a - b*x^2)^(1/4)*(-(a - b*x^2)^2 + (a^2*Hypergeometric2F1[-5/4, 1/2,
 3/2, (b*x^2)/a])/(1 - (b*x^2)/a)^(1/4)))/(11*b)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {248, 248, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a-b x^2\right)^{5/4} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{11} a \int x^2 \sqrt[4]{a-b x^2}dx+\frac{2}{11} x^3 \left(a-b x^2\right)^{5/4}$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{11} a \left(\frac{1}{7} a \int \frac{x^2}{\left(a-b x^2\right)^{3/4}}dx+\frac{2}{7} x^3 \sqrt[4]{a-b x^2}\right)+\frac{2}{11} x^3 \left(a-b x^2\right)^{5/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{11} a \left(\frac{1}{7} a \left(\frac{2 a \int \frac{1}{\left(a-b x^2\right)^{3/4}}dx}{3 b}-\frac{2 x \sqrt[4]{a-b x^2}}{3 b}\right)+\frac{2}{7} x^3 \sqrt[4]{a-b x^2}\right)+\frac{2}{11} x^3 \left(a-b x^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{11} a \left(\frac{1}{7} a \left(\frac{2 a \left(1-\frac{b x^2}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{b x^2}{a}\right)^{3/4}}dx}{3 b \left(a-b x^2\right)^{3/4}}-\frac{2 x \sqrt[4]{a-b x^2}}{3 b}\right)+\frac{2}{7} x^3 \sqrt[4]{a-b x^2}\right)+\frac{2}{11} x^3 \left(a-b x^2\right)^{5/4}$$

$$\Big\downarrow \text{230}$$

$$\frac{5}{11}a\left(\frac{1}{7}a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)+\frac{2}{7}x^3\sqrt[4]{a-bx^2}\right)+\frac{2}{11}x^3\left(a-bx^2\right)^{5/4}$$

input
```
Int[x^2*(a - b*x^2)^(5/4),x]
```

output
```
(2*x^3*(a - b*x^2)^(5/4))/11 + (5*a*((2*x^3*(a - b*x^2)^(1/4))/7 + (a*((-2
*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))))/7))/11
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int x^2 (-b\,x^2+a)^{\frac{5}{4}}\,dx$$

input
```
int(x^2*(-b*x^2+a)^(5/4),x)
```

output
```
int(x^2*(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int x^2 (a-bx^2)^{5/4}\,dx = \int (-bx^2+a)^{\frac{5}{4}} x^2\,dx$$

input
```
integrate(x^2*(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral(-(b*x^4 - a*x^2)*(-b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.25

$$\int x^2 (a-bx^2)^{5/4}\,dx = \frac{a^{\frac{5}{4}} x^3 {}_2F_1\left(\begin{matrix} -\frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(-b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x**3*hyper((-5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a - bx^2)^{5/4} \, dx = \int (-bx^2 + a)^{\frac{5}{4}} x^2 \, dx$$

input
```
integrate(x^2*(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4)*x^2, x)
```

Giac [F]

$$\int x^2 (a - bx^2)^{5/4} \, dx = \int (-bx^2 + a)^{\frac{5}{4}} x^2 \, dx$$

input
```
integrate(x^2*(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a - bx^2)^{5/4} \, dx = \int x^2 \left(a - b x^2\right)^{5/4} \, dx$$

input
```
int(x^2*(a - b*x^2)^(5/4),x)
```

output
```
int(x^2*(a - b*x^2)^(5/4), x)
```

Reduce [F]

$$\int x^2 \left(a-bx^2\right)^{5/4} dx = \frac{-\frac{10(-b\,x^2+a)^{\frac{1}{4}}a^2x}{231} + \frac{24(-b\,x^2+a)^{\frac{1}{4}}ab\,x^3}{77} - \frac{2(-b\,x^2+a)^{\frac{1}{4}}b^2x^5}{11} + \frac{10\left(\int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a^3}{231}}{b}$$

input
```
int(x^2*(-b*x^2+a)^(5/4),x)
```

output
```
(2*( - 5*(a - b*x**2)**(1/4)*a**2*x + 36*(a - b*x**2)**(1/4)*a*b*x**3 - 21
*(a - b*x**2)**(1/4)*b**2*x**5 + 5*int((a - b*x**2)**(1/4)/(a - b*x**2),x)
*a**3))/(231*b)
```

3.867 $\int (a-bx^2)^{5/4} dx$

Optimal result

Integrand size = 12, antiderivative size = 96

$$\int (a-bx^2)^{5/4} \, dx = \frac{10}{21}ax\sqrt[4]{a-bx^2} + \frac{2}{7}x(a-bx^2)^{5/4} + \frac{10a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21\sqrt{b}(a-bx^2)^{3/4}}$$

output
```
10/21*a*x*(-b*x^2+a)^(1/4)+2/7*x*(-b*x^2+a)^(5/4)+10/21*a^(5/2)*(1-b*x^2/a
)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.50

$$\int (a-bx^2)^{5/4} \, dx = \frac{ax\sqrt[4]{a-bx^2} \operatorname{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(5/4),x]
```

output
```
(a*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, 1/2, 3/2, (b*x^2)/a])/(1 -
(b*x^2)/a)^(1/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {211, 211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a-bx^2\right)^{5/4} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\int \sqrt[4]{a-bx^2}dx+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{7}a\left(\frac{1}{3}a\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{231}$$

$$\frac{5}{7}a\left(\frac{a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

$$\Big\downarrow \text{230}$$

$$\frac{5}{7}a\left(\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}\left(a-bx^2\right)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)+\frac{2}{7}x\left(a-bx^2\right)^{5/4}$$

input
```
Int[(a - b*x^2)^(5/4),x]
```

output
```
(2*x*(a - b*x^2)^(5/4))/7 + (5*a*((2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(
1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[
b]*(a - b*x^2)^(3/4))))/7
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \left(-b\,x^{2}+a \right)^{\frac{5}{4}}d x$$

input
```
int((-b*x^2+a)^(5/4),x)
```

output
```
int((-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \left(a-bx^2\right)^{5/4}\,dx=\int \left(-bx^2+a\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(5/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.28

$$\int \left(a-bx^2\right)^{5/4}\,dx=a^{\frac{5}{4}}x\,{}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)$$

input
```
integrate((-b*x**2+a)**(5/4),x)
```

output
```
a**(5/4)*x*hyper((-5/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)
```

Maxima [F]

$$\int \left(a-bx^2\right)^{5/4}\,dx=\int \left(-bx^2+a\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \left(a-bx^2\right)^{5/4} dx = \int \left(-bx^2+a\right)^{\frac{5}{4}} dx$$

input
```
integrate((-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.40

$$\int \left(a-bx^2\right)^{5/4} dx = \frac{x\left(a-b\,x^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{5/4}}$$

input
```
int((a - b*x^2)^(5/4),x)
```

output
```
(x*(a - b*x^2)^(5/4)*hypergeom([-5/4, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(5/4)
```

Reduce [F]

$$\int \left(a-bx^2\right)^{5/4} dx = \frac{16\left(-b\,x^2+a\right)^{\frac{1}{4}} a x}{21} - \frac{2\left(-b\,x^2+a\right)^{\frac{1}{4}} b\,x^3}{7} + \frac{5\left(\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{3}{4}}}dx\right) a^2}{21}$$

input
```
int((-b*x^2+a)^(5/4),x)
```

output
```
(16*(a - b*x**2)**(1/4)*a*x - 6*(a - b*x**2)**(1/4)*b*x**3 + 5*int((a - b*
x**2)**(1/4)/(a - b*x**2),x)*a**2)/21
```

3.868 $\int \frac{(a-bx^2)^{5/4}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 96

$$\int \frac{(a-bx^2)^{5/4}}{x^2} dx = -\frac{5}{3}bx\sqrt[4]{a-bx^2} - \frac{(a-bx^2)^{5/4}}{x} - \frac{5a^{3/2}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3(a-bx^2)^{3/4}}$$

output
```
-5/3*b*x*(-b*x^2+a)^(1/4)-(-b*x^2+a)^(5/4)/x-5/3*a^(3/2)*b^(1/2)*(1-b*x^2/
a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/(-b*x^2+a)
^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.53

$$\int \frac{(a-bx^2)^{5/4}}{x^2} dx = -\frac{a\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(5/4)/x^2,x]
```

output
```
-((a*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/4, -1/2, 1/2, (b*x^2)/a])/(x*(
1 - (b*x^2)/a)^(1/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {247, 211, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a-bx^2)^{5/4}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{5}{2}b\int \sqrt[4]{a-bx^2}dx-\frac{(a-bx^2)^{5/4}}{x}$$

$$\Big\downarrow \text{211}$$

$$-\frac{5}{2}b\left(\frac{1}{3}a\int \frac{1}{(a-bx^2)^{3/4}}dx+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)-\frac{(a-bx^2)^{5/4}}{x}$$

$$\Big\downarrow \text{231}$$

$$-\frac{5}{2}b\left(\frac{a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3(a-bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)-\frac{(a-bx^2)^{5/4}}{x}$$

$$\Big\downarrow \text{230}$$

$$-\frac{5}{2}b\left(\frac{2a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{b}(a-bx^2)^{3/4}}+\frac{2}{3}x\sqrt[4]{a-bx^2}\right)-\frac{(a-bx^2)^{5/4}}{x}$$

input
```
Int[(a - b*x^2)^(5/4)/x^2,x]
```

output
```
-((a - b*x^2)^(5/4)/x) - (5*b*((2*x*(a - b*x^2)^(1/4))/3 + (2*a^(3/2)*(1 -
 (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[b]*
(a - b*x^2)^(3/4))))/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{5}{4}}}{x^2}dx$$

input
```
int((-b*x^2+a)^(5/4)/x^2,x)
```

output
```
int((-b*x^2+a)^(5/4)/x^2,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^2}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^2}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^2,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(5/4)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.32

$$\int \frac{(a-bx^2)^{5/4}}{x^2}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{4},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{x}$$

input
```
integrate((-b*x**2+a)**(5/4)/x**2,x)
```

output
```
-a**(5/4)*hyper((-5/4, -1/2), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a - bx^2)^{5/4}}{x^2} \, dx = \int \frac{(-bx^2 + a)^{\frac{5}{4}}}{x^2} \, dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^2,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^2, x)
```

Giac [F]

$$\int \frac{(a - bx^2)^{5/4}}{x^2} \, dx = \int \frac{(-bx^2 + a)^{\frac{5}{4}}}{x^2} \, dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^2,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.43

$$\int \frac{(a - bx^2)^{5/4}}{x^2} \, dx = \frac{2\,(a - b\,x^2)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4}, -\frac{3}{4};\ \frac{1}{4};\ \frac{a}{b\,x^2}\right)}{3\,x\,\left(1 - \frac{a}{b\,x^2}\right)^{5/4}}$$

input
```
int((a - b*x^2)^(5/4)/x^2,x)
```

output
```
(2*(a - b*x^2)^(5/4)*hypergeom([-5/4, -3/4], 1/4, a/(b*x^2)))/(3*x*(1 - a/
(b*x^2))^(5/4))
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^2}\,dx = \frac{-8(-b\,x^2+a)^{\frac{1}{4}}\,a-2(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2-5\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^4+a\,x^2}dx\right)a^2x}{3x}$$

input
```
int((-b*x^2+a)^(5/4)/x^2,x)
```

output
```
( - 8*(a - b*x**2)**(1/4)*a - 2*(a - b*x**2)**(1/4)*b*x**2 - 5*int((a - b*
x**2)**(1/4)/(a*x**2 - b*x**4),x)*a**2*x)/(3*x)
```

3.869 $\int \frac{\left(a-bx^2\right)^{5/4}}{x^4} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 100

$$\int \frac{\left(a-bx^2\right)^{5/4}}{x^4} \, dx = \frac{5b\sqrt[4]{a-bx^2}}{6x} - \frac{\left(a-bx^2\right)^{5/4}}{3x^3} + \frac{5\sqrt{a}b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\left(a-bx^2\right)^{3/4}}$$

output
```
5/6*b*(-b*x^2+a)^(1/4)/x-1/3*(-b*x^2+a)^(5/4)/x^3+5/6*a^(1/2)*b^(3/2)*(1-b
*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/(-b*x
^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.53

$$\int \frac{\left(a-bx^2\right)^{5/4}}{x^4} \, dx = -\frac{a\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(5/4)/x^4,x]
```

output
```
-1/3*(a*(a - b*x^2)^(1/4)*Hypergeometric2F1[-3/2, -5/4, -1/2, (b*x^2)/a])/
(x^3*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {247, 247, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/4}}{x^4}dx$$

$$\Big\downarrow \text{ 247}$$

$$-\frac{5}{6}b\int \frac{\sqrt[4]{a-bx^2}}{x^2}dx-\frac{\left(a-bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{ 247}$$

$$-\frac{5}{6}b\left(-\frac{1}{2}b\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^2}}{x}\right)-\frac{\left(a-bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{ 231}$$

$$-\frac{5}{6}b\left(-\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}\right)-\frac{\left(a-bx^2\right)^{5/4}}{3x^3}$$

$$\Big\downarrow \text{ 230}$$

$$-\frac{5}{6}b\left(-\frac{\sqrt{a}\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{x}\right)-\frac{\left(a-bx^2\right)^{5/4}}{3x^3}$$

input
```
Int[(a - b*x^2)^(5/4)/x^4,x]
```

output
```
-1/3*(a - b*x^2)^(5/4)/x^3 - (5*b*(-((a - b*x^2)^(1/4)/x) - (Sqrt[a]*Sqrt[
b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a -
 b*x^2)^(3/4)))/6
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^{2}+a)^{\frac{5}{4}}}{x^{4}}dx$$

input
```
int((-b*x^2+a)^(5/4)/x^4,x)
```

output
```
int((-b*x^2+a)^(5/4)/x^4,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^4,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(5/4)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.36

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle|\, \frac{bx^2e^{2i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((-b*x**2+a)**(5/4)/x**4,x)
```

output
```
-a**(5/4)*hyper((-3/2, -5/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*x**3
)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^4,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^4, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^4}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^4,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = \int \frac{(a-b\,x^2)^{5/4}}{x^4}\,dx$$

input
```
int((a - b*x^2)^(5/4)/x^4,x)
```

output
```
int((a - b*x^2)^(5/4)/x^4, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^4}\,dx = \frac{-2(-b\,x^2+a)^{\frac{1}{4}}\,a+12(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+5\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^4+a\,x^2}dx\right)ab\,x^3}{6x^3}$$

input
```
int((-b*x^2+a)^(5/4)/x^4,x)
```

output
```
( - 2*(a - b*x**2)**(1/4)*a + 12*(a - b*x**2)**(1/4)*b*x**2 + 5*int((a - b
*x**2)**(1/4)/(a*x**2 - b*x**4),x)*a*b*x**3)/(6*x**3)
```

3.870 $\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \frac{b\sqrt[4]{a-bx^2}}{6x^3}-\frac{b^2\sqrt[4]{a-bx^2}}{12ax}-\frac{(a-bx^2)^{5/4}}{5x^5}+\frac{b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12\sqrt{a}\,(a-bx^2)^{3/4}}$$

output
```
1/6*b*(-b*x^2+a)^(1/4)/x^3-1/12*b^2*(-b*x^2+a)^(1/4)/a/x-1/5*(-b*x^2+a)^(5
/4)/x^5+1/12*b^(5/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*
x/a^(1/2)),2^(1/2))/a^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.42

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = -\frac{a\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(5/4)/x^6,x]
```

output
```
-1/5*(a*(a - b*x^2)^(1/4)*Hypergeometric2F1[-5/2, -5/4, -3/2, (b*x^2)/a])/
(x^5*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {247, 247, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a-bx^2\right)^{5/4}}{x^6}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{2}b\int \frac{\sqrt[4]{a-bx^2}}{x^4}dx-\frac{\left(a-bx^2\right)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$-\frac{1}{2}b\left(-\frac{1}{6}b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}dx-\frac{\sqrt[4]{a-bx^2}}{3x^3}\right)-\frac{\left(a-bx^2\right)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{2}b\left(-\frac{1}{6}b\left(\frac{b\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}\right)-\frac{\left(a-bx^2\right)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{231}$$

$$-\frac{1}{2}b\left(-\frac{1}{6}b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}\right)-\frac{\left(a-bx^2\right)^{5/4}}{5x^5}$$

$$\Big\downarrow \text{230}$$

$$-\frac{1}{2}b\left(-\frac{1}{6}b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)-\frac{\sqrt[4]{a-bx^2}}{3x^3}\right)-\frac{(a-bx^2)^{5/4}}{5x^5}$$

input
```
Int[(a - b*x^2)^(5/4)/x^6,x]
```

output
```
-1/5*(a - b*x^2)^(5/4)/x^5 - (b*(-1/3*(a - b*x^2)^(1/4)/x^3 - (b*(-((a - b
*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt
[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))))/6))/2
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{5}{4}}}{x^6}dx$$

input
```
int((-b*x^2+a)^(5/4)/x^6,x)
```

output
```
int((-b*x^2+a)^(5/4)/x^6,x)
```

Fricas [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^6,x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(5/4)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.29

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = -\frac{a^{\frac{5}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((-b*x**2+a)**(5/4)/x**6,x)
```

output
```
-a**(5/4)*hyper((-5/2, -5/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*x**5
)
```

Maxima [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^6,x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^6, x)
```

Giac [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \int \frac{(-bx^2+a)^{\frac{5}{4}}}{x^6}\,dx$$

input
```
integrate((-b*x^2+a)^(5/4)/x^6,x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(5/4)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \int \frac{(a-b\,x^2)^{5/4}}{x^6}\,dx$$

input
```
int((a - b*x^2)^(5/4)/x^6,x)
```

output
```
int((a - b*x^2)^(5/4)/x^6, x)
```

Reduce [F]

$$\int \frac{(a-bx^2)^{5/4}}{x^6}\,dx = \frac{-8(-b\,x^2+a)^{\frac{1}{4}}\,a+18(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+5\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^8+a\,x^6}dx\right)a^2x^5}{45x^5}$$

input
```
int((-b*x^2+a)^(5/4)/x^6,x)
```

output
```
( - 8*(a - b*x**2)**(1/4)*a + 18*(a - b*x**2)**(1/4)*b*x**2 + 5*int((a - b
*x**2)**(1/4)/(a*x**6 - b*x**8),x)*a**2*x**5)/(45*x**5)
```

3.871 $\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 146

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx=-\frac{16a^3x}{39b^3\sqrt[4]{a+bx^2}}+\frac{8a^2x(a+bx^2)^{3/4}}{39b^3}-\frac{20ax^3(a+bx^2)^{3/4}}{117b^2}+\frac{2x^5(a+bx^2)^{3/4}}{13b}+\frac{16a^{7/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a+bx^2}}$$

output

```
-16/39*a^3*x/b^3/(b*x^2+a)^(1/4)+8/39*a^2*x*(b*x^2+a)^(3/4)/b^3-20/117*a*x
^3*(b*x^2+a)^(3/4)/b^2+2/13*x^5*(b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1+b*x^2/a
)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(7/2)/(b*x
^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.94 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.62

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \frac{2\left(12a^3x+2a^2bx^3-ab^2x^5+9b^3x^7-12a^3x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{117b^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(1/4),x]
```

output
```
(2*(12*a^3*x + 2*a^2*b*x^3 - a*b^2*x^5 + 9*b^3*x^7 - 12*a^3*x*(1 + (b*x^2)
/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(117*b^3*(a + b
*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {262, 262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\int \frac{x^4}{\sqrt[4]{bx^2+a}}dx}{13b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\int \frac{x^2}{\sqrt[4]{bx^2+a}}dx}{3b}\right)}{13b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\int\frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}\right)}{3b}\right)}{13b}$$

$\Big\downarrow$ 227

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)}{13b}$$

$\Big\downarrow$ 225

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)}{13b}$$

$\Big\downarrow$ 212

$$\frac{2x^5\left(a+bx^2\right)^{3/4}}{13b}-\frac{10a\left(\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}\right)}{13b}$$

input
```
Int[x^6/(a + b*x^2)^(1/4),x]
```

output
```
(2*x^5*(a + b*x^2)^(3/4))/(13*b) - (10*a*((2*x^3*(a + b*x^2)^(3/4))/(9*b)
- (2*a*((2*x*(a + b*x^2)^(3/4))/(5*b) - (2*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/
(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2
, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4))))/(3*b)))/(13*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/4),x)
```

output
```
int(x^6/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.18

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

input
```
integrate(x**6/(b*x**2+a)**(1/4),x)
```

output
```
x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(1/4),x)
```

output
```
int(x^6/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/4),x)
```

output
```
int(x**6/(a + b*x**2)**(1/4),x)
```

3.872 $\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx=\frac{8a^2x}{15b^2\sqrt[4]{a+bx^2}}-\frac{4ax(a+bx^2)^{3/4}}{15b^2}+\frac{2x^3(a+bx^2)^{3/4}}{9b}-\frac{8a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
8/15*a^2*x/b^2/(b*x^2+a)^(1/4)-4/15*a*x*(b*x^2+a)^(3/4)/b^2+2/9*x^3*(b*x^2
+a)^(3/4)/b-8/15*a^(5/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2
)*x/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.76 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx=\frac{2\left(-6a^2x-abx^3+5b^2x^5+6a^2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(1/4),x]
```

output
```
(2*(-6*a^2*x - a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(45*b^2*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {262, 262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\int \frac{x^2}{\sqrt[4]{bx^2+a}}dx}{3b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}\right)}{3b}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x^3\left(a+bx^2\right)^{3/4}}{9b}-\frac{2a\left(\frac{2x\left(a+bx^2\right)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}\right)}{3b}$$

$$\Big\downarrow \text{225}$$

$$\frac{2x^3(a+bx^2)^{3/4}}{9b}-\frac{2a\left(\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}$$

$$\Big\downarrow \text{212}$$

$$\frac{2x^3(a+bx^2)^{3/4}}{9b}-\frac{2a\left(\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}\right)}{3b}$$

input
```
Int[x^4/(a + b*x^2)^(1/4),x]
```

output
```
(2*x^3*(a + b*x^2)^(3/4))/(9*b) - (2*a*((2*x*(a + b*x^2)^(3/4))/(5*b) - (2
*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*Ellipti
cE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(5*b*(a + b*x^2)^(1/4))))/
(3*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/4),x)
```

output
```
int(x^4/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.22

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/4),x)
```

output
```
x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^4}{{\left(b\,x^2+a\right)}^{1/4}} \,dx$$

input
```
int(x^4/(a + b*x^2)^(1/4),x)
```

output
```
int(x^4/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/4),x)
```

output
```
int(x**4/(a + b*x**2)**(1/4),x)
```

3.873 $\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 98

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = -\frac{4ax}{5b\sqrt[4]{a+bx^2}}+\frac{2x(a+bx^2)^{3/4}}{5b}+\frac{4a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
-4/5*a*x/b/(b*x^2+a)^(1/4)+2/5*x*(b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1+b*x^2/a)
^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^
2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.64 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.63

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \frac{2x\left(a+bx^2-a\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{5b\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2,
 -((b*x^2)/a)]))/(5*b*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{5b}$$

$$\Big\downarrow \ 227$$

$$\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{5b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 225$$

$$\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{5b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 212$$

$$\frac{2x(a+bx^2)^{3/4}}{5b}-\frac{2a\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{5b\sqrt[4]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(1/4),x]
```

output
```
(2*x*(a + b*x^2)^(3/4))/(5*b) - (2*a*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*
x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sq
rt[b]))/(5*b*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/4),x)
```

output
```
int(x^2/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^2}{\left(bx^2+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.28

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/4),x)
```

output
```
x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(1/4),x)
```

output
```
int(x^2/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/4),x)
```

output
```
int(x**2/(a + b*x**2)**(1/4),x)
```

3.874 $\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 71

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx=\frac{2x}{\sqrt[4]{a+bx^2}}-\frac{2\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
2*x/(b*x^2+a)^(1/4)-2*a^(1/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b
^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx=\frac{x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/4),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{227}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{225}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{\sqrt[4]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/4),x]
```

output
```
((1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(a + b*x^2)^(1/4)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.34

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\, \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/4}{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{1/4}}$$

input
```
int(1/(a + b*x^2)^(1/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(a + b*x**2)**(1/4),x)
```

3.875 $\int \frac{1}{x^2 \sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 93

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^2}}\,dx = \frac{bx}{a\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a+bx^2}}$$

output
```
b*x/a/(b*x^2+a)^(1/4)-(b*x^2+a)^(3/4)/a/x-b^(1/2)*(1+b*x^2/a)^(1/4)*Ellipt
icE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.69 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 \sqrt[4]{a+bx^2}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(1/4)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \ 264$$

$$\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{ax}$$

$$\Big\downarrow \ 227$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}$$

$$\Big\downarrow \ 225$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}$$

$$\Big\downarrow \ 212$$

$$\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)^(1/4)),x]
```

output
```
-((a + b*x^2)^(3/4)/(a*x)) + (b*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/
a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]
))/(2*a*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(1/4),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx=-\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[4]{a}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx = -\frac{2\left(\frac{a}{b\,x^2}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{a}{b\,x^2}\right)}{3\,x\,(b\,x^2+a)^{1/4}}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/4)),x)
```

output
```
-(2*(a/(b*x^2) + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -a/(b*x^2)))/(3*x*(a
+ b*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}}x^2}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*x**2),x)
```

3.876 $\int \frac{1}{x^4 \sqrt[4]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^2}} \, dx = -\frac{b^2 x}{2a^2 \sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{3ax^3} + \frac{b(a+bx^2)^{3/4}}{2a^2 x} + \frac{b^{3/2} \sqrt[4]{1+\frac{bx^2}{a}} E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{3/2} \sqrt[4]{a+bx^2}}$$

output

```
-1/2*b^2*x/a^2/(b*x^2+a)^(1/4)-1/3*(b*x^2+a)^(3/4)/a/x^3+1/2*b*(b*x^2+a)^(
3/4)/a^2/x+1/2*b^(3/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*
x/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^2}} \, dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 \sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(1/4)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{1}{x^2\sqrt[4]{bx^2+a}}dx}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{227}$$

$$-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{225}$$

$$-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{212}$$

$$-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^2}}-\frac{(a+bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a+bx^2)^{3/4}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(1/4)),x]
```

output
```
-1/3*(a + b*x^2)^(3/4)/(a*x^3) - (b*(-((a + b*x^2)^(3/4)/(a*x)) + (b*(1 +
(b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTa
n[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[b]))/(2*a*(a + b*x^2)^(1/4))))/(2*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^4/(b*x^2+a)^(1/4),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[4]{a}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*x**4),x)
```

3.877 $\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 148

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx = \frac{7b^3x}{20a^3\sqrt[4]{a+bx^2}} - \frac{(a+bx^2)^{3/4}}{5ax^5} + \frac{7b(a+bx^2)^{3/4}}{30a^2x^3} - \frac{7b^2(a+bx^2)^{3/4}}{20a^3x} - \frac{7b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
7/20*b^3*x/a^3/(b*x^2+a)^(1/4)-1/5*(b*x^2+a)^(3/4)/a/x^5+7/30*b*(b*x^2+a)^
(3/4)/a^2/x^3-7/20*b^2*(b*x^2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1+b*x^2/a)^(1/4
)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/(b*x^2+a)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(1/4)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {264, 264, 264, 227, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{7b\int \frac{1}{x^4\sqrt[4]{bx^2+a}}dx}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{7b\left(-\frac{b\int \frac{1}{x^2\sqrt[4]{bx^2+a}}dx}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{7b\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^2+a}}dx}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{227}$$

$$-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

$\downarrow$ 225

$$-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

$\downarrow$ 212

$$-\frac{7b\left(-\frac{b\left(\frac{b\sqrt[4]{\frac{bx^2}{a}+1}\left(\frac{2x}{\sqrt[4]{\frac{bx^2}{a}+1}}-\frac{2\sqrt{a}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}}\right)}{2a\sqrt[4]{a+bx^2}}-\frac{\left(a+bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a+bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a+bx^2\right)^{3/4}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^2)^(1/4)),x]
```

output
```
-1/5*(a + b*x^2)^(3/4)/(a*x^5) - (7*b*(-1/3*(a + b*x^2)^(3/4)/(a*x^3) - (b
*(-((a + b*x^2)^(3/4)/(a*x)) + (b*(1 + (b*x^2)/a)^(1/4)*((2*x)/(1 + (b*x^2
)/a)^(1/4) - (2*Sqrt[a]*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/Sqrt[
b]))/(2*a*(a + b*x^2)^(1/4))))/(2*a)))/(10*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(1/4),x)
```

output
```
int(1/x^6/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[4]{a}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}} \, dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{1/4}} \, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[4]{a+bx^2}} \, dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}} x^6} dx$$

input
```
int(1/x^6/(b*x^2+a)^(1/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*x**6),x)
```

3.878 $\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = -\frac{8a^2x(a-bx^2)^{3/4}}{39b^3} - \frac{20ax^3(a-bx^2)^{3/4}}{117b^2} - \frac{2x^5(a-bx^2)^{3/4}}{13b} + \frac{16a^{7/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{39b^{7/2}\sqrt[4]{a-bx^2}}$$

output

```
-8/39*a^2*x*(-b*x^2+a)^(3/4)/b^3-20/117*a*x^3*(-b*x^2+a)^(3/4)/b^2-2/13*x^
5*(-b*x^2+a)^(3/4)/b+16/39*a^(7/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arc
sin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.94 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \frac{2x\left(-12a^3+2a^2bx^2+ab^2x^4+9b^3x^6+12a^3\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{117b^3\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^6/(a - b*x^2)^(1/4),x]
```

output
```
(2*x*(-12*a^3 + 2*a^2*b*x^2 + a*b^2*x^4 + 9*b^3*x^6 + 12*a^3*(1 - (b*x^2)/
a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(117*b^3*(a - b*x^2
)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {262, 262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{10a\int \frac{x^4}{\sqrt[4]{a-bx^2}}dx}{13b}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{10a\left(\frac{2a\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{10a\left(\frac{2a\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{10a\left(\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

$$\Big\downarrow \; 226$$

$$\frac{10a\left(\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{13b}-\frac{2x^5\left(a-bx^2\right)^{3/4}}{13b}$$

input
```
Int[x^6/(a - b*x^2)^(1/4),x]
```

output
```
(-2*x^5*(a - b*x^2)^(3/4))/(13*b) + (10*a*((-2*x^3*(a - b*x^2)^(3/4))/(9*b
) + (2*a*((-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4
)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4
))))/(3*b)))/(13*b)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^{6}}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-b*x^2+a)^(1/4),x)
```

output
```
int(x^6/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^{6}}{(-bx^{2}+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{7\sqrt[4]{a}}$$

input
```
integrate(x**6/(-b*x**2+a)**(1/4),x)
```

output
```
x**7*hyper((1/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(1/4))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^6}{(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(x^6/(a - b*x^2)^(1/4),x)
```

output
```
int(x^6/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^6}{(-b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-b*x^2+a)^(1/4),x)
```

output
```
int(x**6/(a - b*x**2)**(1/4),x)
```

3.879 $\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx=-\frac{4ax(a-bx^2)^{3/4}}{15b^2}-\frac{2x^3(a-bx^2)^{3/4}}{9b}+\frac{8a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15b^{5/2}\sqrt[4]{a-bx^2}}$$

output
```
-4/15*a*x*(-b*x^2+a)^(3/4)/b^2-2/9*x^3*(-b*x^2+a)^(3/4)/b+8/15*a^(5/2)*(1-
b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(5/
2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.79 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx=\frac{2\left(-6a^2x+abx^3+5b^2x^5+6a^2x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{45b^2\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^4/(a - b*x^2)^(1/4),x]
```

output
```
(2*(-6*a^2*x + a*b*x^3 + 5*b^2*x^5 + 6*a^2*x*(1 - (b*x^2)/a)^(1/4)*Hyperge
ometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(45*b^2*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2a\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2a\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

$$\Big\downarrow \text{227}$$

$$\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

$$\Big\downarrow \text{226}$$

$$\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}$$

input
```
Int[x^4/(a - b*x^2)^(1/4),x]
```

output
```
(-2*x^3*(a - b*x^2)^(3/4))/(9*b) + (2*a*((-2*x*(a - b*x^2)^(3/4))/(5*b) +
(4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(5*b^(3/2)*(a - b*x^2)^(1/4))))/(3*b)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^4}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(-b*x^2+a)^(1/4),x)
```

output
```
int(x^4/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^4}{\left(-bx^2+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*x^4/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = \frac{x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}}$$

input
```
integrate(x**4/(-b*x**2+a)**(1/4),x)
```

output
```
x**5*hyper((1/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^4}{(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(x^4/(a - b*x^2)^(1/4),x)
```

output
```
int(x^4/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{a-bx^2}}\,dx=\int \frac{x^4}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(-b*x^2+a)^(1/4),x)
```

output
```
int(x**4/(a - b*x**2)**(1/4),x)
```

3.880 $\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx=-\frac{2x(a-bx^2)^{3/4}}{5b}+\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
-2/5*x*(-b*x^2+a)^(3/4)/b+4/5*a^(3/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*
arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.62 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx=\frac{2x\left(-a+bx^2+a\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{5b\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^2/(a - b*x^2)^(1/4),x]
```

output
```
(2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2
, (b*x^2)/a]))/(5*b*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

$$\Big\downarrow \text{ 227}$$

$$\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}$$

input
```
Int[x^2/(a - b*x^2)^(1/4),x]
```

output
```
(-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*Elliptic
E[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/4),x)
```

output
```
int(x^2/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \frac{x^3{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}}$$

input
```
integrate(x**2/(-b*x**2+a)**(1/4),x)
```

output
```
x**3*hyper((1/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^2}{(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(x^2/(a - b*x^2)^(1/4),x)
```

output
```
int(x^2/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{x^2}{(-b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(1/4),x)
```

output
```
int(x**2/(a - b*x**2)**(1/4),x)
```

3.881 $\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2*a^(1/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2
^(1/2))/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[4]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(-1/4),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a])/(a -
b*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{227}$$

$$\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

input
```
Int[(a - b*x^2)^(-1/4),x]
```

output
```
(2*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{\left(-bx^{2}+a\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}}$$

input
```
integrate(1/(-b*x**2+a)**(1/4),x)
```

output
```
x*hyper((1/4, 1/2), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(1/4)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\frac{x\,\left(1-\frac{b\,x^2}{a}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(a-b\,x^2\right)^{1/4}}$$

input
```
int(1/(a - b*x^2)^(1/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(1/4)*hypergeom([1/4, 1/2], 3/2, (b*x^2)/a))/(a - b*x^2
)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(a - b*x**2)**(1/4),x)
```

3.882 $\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 79

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = -\frac{(a-bx^2)^{3/4}}{ax} - \frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}$$

output
```
-(-b*x^2+a)^(3/4)/a/x-b^(1/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b
^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(1/4)),x]
```

output
```
-(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (b*x^2)/a])/(x*
(a - b*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{ax}$$

$$\Big\downarrow \text{227}$$

$$-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}$$

input
```
Int[1/(x^2*(a - b*x^2)^(1/4)),x]
```

output
```
-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-b\,x^{2}+a \right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/x^2/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^2}} \, dx = \int \frac{1}{{\left(-b x^{2} + a\right)}^{\frac{1}{4}} x^{2}} \,d x$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{\sqrt[4]{a}x}$$

input
```
integrate(1/x**2/(-b*x**2+a)**(1/4),x)
```

output
```
-hyper((-1/2, 1/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(1/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^2}} \, dx = -\frac{2 \left(1-\frac{a}{b x^2}\right)^{1/4} {}_2F_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{a}{b x^2}\right)}{3 x \left(a-b x^2\right)^{1/4}}$$

input
```
int(1/(x^2*(a - b*x^2)^(1/4)),x)
```

output
```
-(2*(1 - a/(b*x^2))^(1/4)*hypergeom([1/4, 3/4], 7/4, a/(b*x^2)))/(3*x*(a -
 b*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{a-bx^2}} \, dx = \int \frac{1}{\left(-b x^2+a\right)^{\frac{1}{4}} x^2} dx$$

input
```
int(1/x^2/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*x**2),x)
```

3.883 $\int \frac{1}{x^4 \sqrt[4]{a-bx^2}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^2}} \, dx = -\frac{(a-bx^2)^{3/4}}{3ax^3} - \frac{b(a-bx^2)^{3/4}}{2a^2x} - \frac{b^{3/2} \sqrt[4]{1-\frac{bx^2}{a}} E\left(\frac{1}{2} \arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) \middle| 2\right)}{2a^{3/2} \sqrt[4]{a-bx^2}}$$

output
```
-1/3*(-b*x^2+a)^(3/4)/a/x^3-1/2*b*(-b*x^2+a)^(3/4)/a^2/x-1/2*b^(3/2)*(1-b*
x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)
/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4 \sqrt[4]{a-bx^2}} \, dx = -\frac{\sqrt[4]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{1}{4}, -\frac{1}{2}, \frac{bx^2}{a}\right)}{3x^3 \sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(1/4)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (b*x^2)/a])
/(x^3*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{227}$$

$$\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

$$\Big\downarrow \text{226}$$

$$\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}$$

input
```
Int[1/(x^4*(a - b*x^2)^(1/4)),x]
```

output
```
-1/3*(a - b*x^2)^(3/4)/(a*x^3) + (b*(-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]
*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[
a]*(a - b*x^2)^(1/4))))/(2*a)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^4 \left(-b\,x^2+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/x^4/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx = -\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{3\sqrt[4]{a}x^3}$$

input
```
integrate(1/x**4/(-b*x**2+a)**(1/4),x)
```

output
```
-hyper((-3/2, 1/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(1/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{x^4\,(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*x***4),x)
```

3.884 $\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = -\frac{(a-bx^2)^{3/4}}{5ax^5}-\frac{7b(a-bx^2)^{3/4}}{30a^2x^3}-\frac{7b^2(a-bx^2)^{3/4}}{20a^3x} -\frac{7b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{5/2}\sqrt[4]{a-bx^2}}$$

output
```
-1/5*(-b*x^2+a)^(3/4)/a/x^5-7/30*b*(-b*x^2+a)^(3/4)/a^2/x^3-7/20*b^2*(-b*x
^2+a)^(3/4)/a^3/x-7/20*b^(5/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(
b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^6*(a - b*x^2)^(1/4)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (b*x^2)/a])
/(x^5*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {264, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{7b\int \frac{1}{x^4 \sqrt[4]{a-bx^2}}dx}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{7b\left(\frac{b\int \frac{1}{x^2 \sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{7b\left(\frac{b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

$$\Big\downarrow \text{227}$$

$$\frac{7b\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

$\Big\downarrow$ 226

$$\frac{7b\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}$$

input
```
Int[1/(x^6*(a - b*x^2)^(1/4)),x]
```

output
```
-1/5*(a - b*x^2)^(3/4)/(a*x^5) + (7*b*(-1/3*(a - b*x^2)^(3/4)/(a*x^3) + (b
*(-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[Ar
cSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))))/(2*a)))/(10
*a)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(-b\,x^{2}+a \right)^{\frac{1}{4}}}d x$$

input
```
int(1/x^6/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/x^6/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5\sqrt[4]{a}x^5}$$

input
```
integrate(1/x**6/(-b*x**2+a)**(1/4),x)
```

output
```
-hyper((-5/2, 1/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(1/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{x^6\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

input
```
int(1/(x^6*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}}x^6}dx$$

input
```
int(1/x^6/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*x**6),x)
```

3.885 $\int \frac{x^6}{\left(a+bx^2\right)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/4}} \, dx = \frac{40a^2x\sqrt[4]{a+bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a+bx^2}}{77b^2} + \frac{2x^5\sqrt[4]{a+bx^2}}{11b} - \frac{80a^{7/2}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{7/2}\left(a+bx^2\right)^{3/4}}$$

output
```
40/77*a^2*x*(b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(b*x^2+a)^(1/4)/b^2+2/11*x^5*(
b*x^2+a)^(1/4)/b-80/77*a^(7/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcta
n(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/4}} \, dx = \frac{2\left(20a^3x + 10a^2bx^3 - 3ab^2x^5 + 7b^3x^7 - 20a^3x\left(1+\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4}\right.\right.}{77b^3\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[x^6/(a + b*x^2)^(3/4),x]
```

output
```
(2*(20*a^3*x + 10*a^2*b*x^3 - 3*a*b^2*x^5 + 7*b^3*x^7 - 20*a^3*x*(1 + (b*x
^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(77*b^3*(a +
 b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {262, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{10a\int \frac{x^4}{(bx^2+a)^{3/4}}dx}{11b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{10a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\int \frac{x^2}{(bx^2+a)^{3/4}}dx}{7b}\right)}{11b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{10a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)}{7b}\right)}{11b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{10a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\left(a+bx^2\right)^{3/4}}\right)}{7b}\right)}{11b}$$

$\Big\downarrow$ 229

$$\frac{2x^5\sqrt[4]{a+bx^2}}{11b}-\frac{10a\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}\right)}{7b}\right)}{11b}$$

input
```
Int[x^6/(a + b*x^2)^(3/4),x]
```

output
```
(2*x^5*(a + b*x^2)^(1/4))/(11*b) - (10*a*((2*x^3*(a + b*x^2)^(1/4))/(7*b)
- (6*a*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*E
llipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4)))
)/(7*b)))/(11*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(3/4),x)
```

output
```
int(x^6/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

input
```
integrate(x**6/(b*x**2+a)**(3/4),x)
```

output
```
x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/4))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(3/4),x)
```

output
```
int(x^6/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(3/4),x)
```

output
```
int(x**6/(a + b*x**2)**(3/4),x)
```

3.886 $\int \frac{x^4}{(a+bx^2)^{3/4}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{x^4}{(a+bx^2)^{3/4}} \, dx = -\frac{4ax\sqrt[4]{a+bx^2}}{7b^2}+\frac{2x^3\sqrt[4]{a+bx^2}}{7b} + \frac{8a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7b^{5/2}(a+bx^2)^{3/4}}$$

output
```
-4/7*a*x*(b*x^2+a)^(1/4)/b^2+2/7*x^3*(b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1+b*x^
2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/
(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.99 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.78

$$\int \frac{x^4}{(a+bx^2)^{3/4}} \, dx = \frac{2\left(-2a^2x-abx^3+b^2x^5+2a^2x\left(1+\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{7b^2(a+bx^2)^{3/4}}$$

input
```
Integrate[x^4/(a + b*x^2)^(3/4),x]
```

output
```
(2*(-2*a^2*x - a*b*x^3 + b^2*x^5 + 2*a^2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeom
etric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)]))/(7*b^2*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\int \frac{x^2}{(bx^2+a)^{3/4}}dx}{7b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)}{7b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b(a+bx^2)^{3/4}}\right)}{7b}$$

$$\Big\downarrow \text{229}$$

$$\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}\right)}{7b}$$

input
```
Int[x^4/(a + b*x^2)^(3/4),x]
```

output
```
(2*x^3*(a + b*x^2)^(1/4))/(7*b) - (6*a*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4
*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(3*b^(3/2)*(a + b*x^2)^(3/4))))/(7*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(3/4),x)
```

output
```
int(x^4/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

input
```
integrate(x**4/(b*x**2+a)**(3/4),x)
```

output
```
x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(3/4),x)
```

output
```
int(x^4/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(3/4),x)
```

output
```
int(x**4/(a + b*x**2)**(3/4),x)
```

3.887 $\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx=\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}$$

output
```
2/3*x*(b*x^2+a)^(1/4)/b-4/3*a^(3/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*
arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.58 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx=\frac{2x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{3b(a+bx^2)^{3/4}}$$

input
```
Integrate[x^2/(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2,
 -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\,(a+bx^2)^{3/4}}$$

input
```
Int[x^2/(a + b*x^2)^(3/4),x]
```

output
```
(2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF
[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(3/4),x)
```

output
```
int(x^2/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

input
```
integrate(x**2/(b*x**2+a)**(3/4),x)
```

output
```
x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(3/4),x)
```

output
```
int(x^2/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{3/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(3/4),x)
```

output
```
int(x**2/(a + b*x**2)**(3/4),x)
```

3.888 $\int \frac{1}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{2\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
2*a^(1/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),
2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(-3/4),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

input
```
Int[(a + b*x^2)^(-3/4),x]
```

output
```
(2*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{3/4}}$$

input
```
int(1/(a + b*x^2)^(3/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(a + b*x**2)**(3/4),x)
```

3.889 $\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{ax} - \frac{\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}$$

output
```
-(b*x^2+a)^(1/4)/a/x-b^(1/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2(a+bx^2)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\,(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/4)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 231}$$

$$-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 229}$$

$$-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/4)),x]
```

output
```
-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcT
an[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(3/4),x)
```

output
```
int(1/x^2/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{3/4}}\, dx=\int \frac{1}{{\left(bx^{2}+a\right)}^{\frac{3}{4}} x^{2}}\, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{4}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/4),x)
```

output
```
-hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = -\frac{2\left(\frac{a}{b\,x^2}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{a}{b\,x^2}\right)}{5\,x\,(b\,x^2+a)^{3/4}}$$

input
```
int(1/(x^2*(a + b*x^2)^(3/4)),x)
```

output
```
-(2*(a/(b*x^2) + 1)^(3/4)*hypergeom([3/4, 5/4], 9/4, -a/(b*x^2)))/(5*x*(a
+ b*x^2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/x^2/(b*x^2+a)^(3/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*x**2),x)
```

3.890 $\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{3ax^3}+\frac{5b\sqrt[4]{a+bx^2}}{6a^2x}+\frac{5b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6a^{3/2}\left(a+bx^2\right)^{3/4}}$$

output

```
-1/3*(b*x^2+a)^(1/4)/a/x^3+5/6*b*(b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1+b*x^
2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/
(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^4\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(3/4)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{5b\int \frac{1}{x^2(bx^2+a)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{5b\left(-\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{231}$$

$$-\frac{5b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{229}$$

$$-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(3/4)),x]
```

output
```
-1/3*(a + b*x^2)^(1/4)/(a*x^3) - (5*b*(-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[
b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqr
t[a]*(a + b*x^2)^(3/4))))/(6*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(3/4),x)
```

output
```
int(1/x^4/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^4 (a + bx^2)^{3/4}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{3}{4}} x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(3/4),x)
```

output
```
-hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{x^4 (b\,x^2+a)^{3/4}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(b*x^2+a)^(3/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*x***4),x)
```

3.891 $\int \frac{1}{x^6(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{x^6(a+bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a+bx^2}}{5ax^5}+\frac{3b\sqrt[4]{a+bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a+bx^2}}{4a^3x} - \frac{3b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{5/2}(a+bx^2)^{3/4}}$$

output
```
-1/5*(b*x^2+a)^(1/4)/a/x^5+3/10*b*(b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(b*x^2+a
)^(1/4)/a^3/x-3/4*b^(5/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(
1/2)*x/a^(1/2)),2^(1/2))/a^(5/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6(a+bx^2)^{3/4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(3/4)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{3/4}} \, dx$$
$$\Big\downarrow \text{264}$$
$$-\frac{9b\int \frac{1}{x^4(bx^2+a)^{3/4}}dx}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$
$$\Big\downarrow \text{264}$$
$$-\frac{9b\left(-\frac{5b\int \frac{1}{x^2\left(bx^2+a\right)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$
$$\Big\downarrow \text{264}$$
$$-\frac{9b\left(-\frac{5b\left(-\frac{b\int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$
$$\Big\downarrow \text{231}$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{229}$$

$$-\frac{9b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}$$

input

```
Int[1/(x^6*(a + b*x^2)^(3/4)),x]
```

output

```
-1/5*(a + b*x^2)^(1/4)/(a*x^5) - (9*b*(-1/3*(a + b*x^2)^(1/4)/(a*x^3) - (5
*b*(-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))))/(6*a)))/(
10*a)
```

Defintions of rubi rules used

rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)    Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(3/4),x)
```

output
```
int(1/x^6/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/4}}\, dx = \int \frac{1}{{\left(b x^{2} + a\right)}^{\frac{3}{4}} x^{6}}\, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^6 (a+bx^2)^{3/4}}\, dx = -\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{5 a^{\frac{3}{4}} x^{5}}$$

input
```
integrate(1/x**6/(b*x**2+a)**(3/4),x)
```

output
```
-hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{3/4}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/4}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}} x^6}dx$$

input
```
int(1/x^6/(b*x^2+a)^(3/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*x**6),x)
```

3.892 $\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = -\frac{40a^2x\sqrt[4]{a-bx^2}}{77b^3} - \frac{20ax^3\sqrt[4]{a-bx^2}}{77b^2} - \frac{2x^5\sqrt[4]{a-bx^2}}{11b} + \frac{80a^{7/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77b^{7/2}(a-bx^2)^{3/4}}$$

output
```
-40/77*a^2*x*(-b*x^2+a)^(1/4)/b^3-20/77*a*x^3*(-b*x^2+a)^(1/4)/b^2-2/11*x^
5*(-b*x^2+a)^(1/4)/b+80/77*a^(7/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*a
rcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = \frac{2\left(-20a^3x + 10a^2bx^3 + 3ab^2x^5 + 7b^3x^7 + 20a^3x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2}\right.\right.}{77b^3(a-bx^2)^{3/4}}$$

input
```
Integrate[x^6/(a - b*x^2)^(3/4),x]
```

output
```
(2*(-20*a^3*x + 10*a^2*b*x^3 + 3*a*b^2*x^5 + 7*b^3*x^7 + 20*a^3*x*(1 - (b*
x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(77*b^3*(a - b
*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {262, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{10a\int \frac{x^4}{(a-bx^2)^{3/4}}dx}{11b}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

$$\Big\downarrow \text{262}$$

$$\frac{10a\left(\frac{6a\int \frac{x^2}{(a-bx^2)^{3/4}}dx}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{11b}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

$$\Big\downarrow \text{262}$$

$$\frac{10a\left(\frac{6a\left(\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{11b}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

$$\Big\downarrow \text{231}$$

$$\frac{10a\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{11b}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

$$\Big\downarrow \; 230$$

$$\frac{10a\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{11b}-\frac{2x^5\sqrt[4]{a-bx^2}}{11b}$$

input
```
Int[x^6/(a - b*x^2)^(3/4),x]
```

output
```
(-2*x^5*(a - b*x^2)^(1/4))/(11*b) + (10*a*((-2*x^3*(a - b*x^2)^(1/4))/(7*b
) + (6*a*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4
)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4
))))/(7*b)))/(11*b)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^6}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(-b*x^2+a)^(3/4),x)
```

output
```
int(x^6/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx=\int \frac{x^6}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*x^6/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx=\frac{x^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{3}{4}}}$$

input
```
integrate(x**6/(-b*x**2+a)**(3/4),x)
```

output
```
x**7*hyper((3/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(3/4))
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^6}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(x^6/(a - b*x^2)^(3/4),x)
```

output
```
int(x^6/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^6}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(-b*x^2+a)^(3/4),x)
```

output
```
int(x**6/(a - b*x**2)**(3/4),x)
```

3.893 $\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = -\frac{4ax\sqrt[4]{a-bx^2}}{7b^2}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}+\frac{8a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7b^{5/2}(a-bx^2)^{3/4}}$$

output

```
-4/7*a*x*(-b*x^2+a)^(1/4)/b^2-2/7*x^3*(-b*x^2+a)^(1/4)/b+8/7*a^(5/2)*(1-b*
x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2
)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \frac{2x\left(-2a^2+abx^2+b^2x^4+2a^2\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{7b^2(a-bx^2)^{3/4}}$$

input
```
Integrate[x^4/(a - b*x^2)^(3/4),x]
```

output
```
(2*x*(-2*a^2 + a*b*x^2 + b^2*x^4 + 2*a^2*(1 - (b*x^2)/a)^(3/4)*Hypergeomet
ric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(7*b^2*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{6a\int \frac{x^2}{(a-bx^2)^{3/4}}dx}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

$$\Big\downarrow \text{262}$$

$$\frac{6a\left(\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

$$\Big\downarrow \text{231}$$

$$\frac{6a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

$$\Big\downarrow \text{230}$$

$$\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}$$

input
```
Int[x^4/(a - b*x^2)^(3/4),x]
```

output
```
(-2*x^3*(a - b*x^2)^(1/4))/(7*b) + (6*a*((-2*x*(a - b*x^2)^(1/4))/(3*b) +
(4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(3*b^(3/2)*(a - b*x^2)^(3/4))))/(7*b)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^{4}}{\left(-b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(x^4/(-b*x^2+a)^(3/4),x)
```

output
```
int(x^4/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*x^4/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}}$$

input
```
integrate(x**4/(-b*x**2+a)**(3/4),x)
```

output
```
x**5*hyper((3/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4))
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^4}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(x^4/(a - b*x^2)^(3/4),x)
```

output
```
int(x^4/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^4}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(-b*x^2+a)^(3/4),x)
```

output
```
int(x**4/(a - b*x**2)**(3/4),x)
```

3.894 $\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}}{3b} + \frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}$$

output
```
-2/3*x*(-b*x^2+a)^(1/4)/b+4/3*a^(3/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/
2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.68 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \frac{2x\left(-a+bx^2+a\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{3b(a-bx^2)^{3/4}}$$

input
```
Integrate[x^2/(a - b*x^2)^(3/4),x]
```

output
```
(2*x*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2
, (b*x^2)/a]))/(3*b*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

$$\Big\downarrow \; 231$$

$$\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

$$\Big\downarrow \; 230$$

$$\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}$$

input
```
Int[x^2/(a - b*x^2)^(3/4),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*Elliptic
F[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(3/4),x)
```

output
```
int(x^2/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*x^2/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}}}$$

input
```
integrate(x**2/(-b*x**2+a)**(3/4),x)
```

output
```
x**3*hyper((3/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^2}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(x^2/(a - b*x^2)^(3/4),x)
```

output
```
int(x^2/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{3/4}}\,dx = \int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(3/4),x)
```

output
```
int(x**2/(a - b*x**2)**(3/4),x)
```

3.895 $\int \frac{1}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
2*a^(1/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),
2^(1/2))/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)}{(a-bx^2)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(-3/4),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])/(a -
 b*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{231}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{2\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

input
```
Int[(a - b*x^2)^(-3/4),x]
```

output
```
(2*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2,
2])/(Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.45

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(3/4),x)
```

output
```
x*hyper((1/2, 3/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(3/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{3/4}}$$

input
```
int(1/(a - b*x^2)^(3/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(3/4)*hypergeom([1/2, 3/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(a - b*x**2)**(3/4),x)
```

3.896 $\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{ax}+\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a-bx^2)^{3/4}}$$

output
```
-(-b*x^2+a)^(1/4)/a/x+b^(1/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin
(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.88 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^2(a-bx^2)^{3/4}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},\frac{bx^2}{a}\right)}{x\,(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(3/4)),x]
```

output
```
-(((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (b*x^2)/a])/(x*
(a - b*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \; 264$$

$$\frac{b\int \frac{1}{(a-bx^2)^{3/4}}dx}{2a} - \frac{\sqrt[4]{a-bx^2}}{ax}$$

$$\Big\downarrow \; 231$$

$$\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a\,(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}$$

$$\Big\downarrow \; 230$$

$$\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}$$

input
```
Int[1/(x^2*(a - b*x^2)^(3/4)),x]
```

output
```
-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcS
in[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/x^2/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{a^{\frac{3}{4}} x}$$

input
```
integrate(1/x**2/(-b*x**2+a)**(3/4),x)
```

output
```
-hyper((-1/2, 3/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(3/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a-bx^2)^{3/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 (a - bx^2)^{3/4}} \, dx = -\frac{2 \left(1 - \frac{a}{b x^2}\right)^{3/4} {}_2\mathrm{F}_1\left(\frac{3}{4}, \frac{5}{4}; \frac{9}{4}; \frac{a}{b x^2}\right)}{5 x (a - b x^2)^{3/4}}$$

input
```
int(1/(x^2*(a - b*x^2)^(3/4)),x)
```

output
```
-(2*(1 - a/(b*x^2))^(3/4)*hypergeom([3/4, 5/4], 9/4, a/(b*x^2)))/(5*x*(a -
 b*x^2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a - bx^2)^{3/4}} \, dx = \int \frac{1}{(-b x^2 + a)^{\frac{3}{4}} x^2} dx$$

input
```
int(1/x^2/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*x**2),x)
```

3.897 $\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{3ax^3} - \frac{5b\sqrt[4]{a-bx^2}}{6a^2x} + \frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6a^{3/2}(a-bx^2)^{3/4}}$$

output
```
-1/3*(-b*x^2+a)^(1/4)/a/x^3-5/6*b*(-b*x^2+a)^(1/4)/a^2/x+5/6*b^(3/2)*(1-b*
x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2
)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.49

$$\int \frac{1}{x^4(a-bx^2)^{3/4}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(3/4)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (b*x^2)/a])
/(x^3*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{5b \int \frac{1}{x^2(a-bx^2)^{3/4}}dx}{6a} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{5b \left(\frac{b\int \frac{1}{(a-bx^2)^{3/4}}dx}{2a} - \frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

$$\Big\downarrow \text{231}$$

$$\frac{5b \left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

$$\Big\downarrow \text{230}$$

$$\frac{5b \left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a} - \frac{\sqrt[4]{a-bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*(a - b*x^2)^(3/4)),x]
```

output
```
-1/3*(a - b*x^2)^(1/4)/(a*x^3) + (5*b*(-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[
b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqr
t[a]*(a - b*x^2)^(3/4))))/(6*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^4 (-b x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/x^4/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{3a^{\frac{3}{4}} x^3}$$

input
```
integrate(1/x**4/(-b*x**2+a)**(3/4),x)
```

output
```
-hyper((-3/2, 3/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(3/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{x^4 (a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/(x^4*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*x**4),x)
```

3.898 $\int \frac{1}{x^6(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 131

$$\int \frac{1}{x^6(a-bx^2)^{3/4}}\,dx = -\frac{\sqrt[4]{a-bx^2}}{5ax^5}-\frac{3b\sqrt[4]{a-bx^2}}{10a^2x^3}-\frac{3b^2\sqrt[4]{a-bx^2}}{4a^3x}+\frac{3b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{5/2}(a-bx^2)^{3/4}}$$

output

```
-1/5*(-b*x^2+a)^(1/4)/a/x^5-3/10*b*(-b*x^2+a)^(1/4)/a^2/x^3-3/4*b^2*(-b*x^
2+a)^(1/4)/a^3/x+3/4*b^(5/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(5/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6(a-bx^2)^{3/4}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^6*(a - b*x^2)^(3/4)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (b*x^2)/a])
/(x^5*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {264, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{9b\int \frac{1}{x^4(a-bx^2)^{3/4}}dx}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{9b\left(\frac{5b\int \frac{1}{x^2\left(a-bx^2\right)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{9b\left(\frac{5b\left(\frac{b\int \frac{1}{\left(a-bx^2\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

$$\Big\downarrow \text{231}$$

$$\frac{9b\left(\frac{5b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

$$\Big\downarrow \textbf{230}$$

$$\frac{9b\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}$$

input
```
Int[1/(x^6*(a - b*x^2)^(3/4)),x]
```

output
```
-1/5*(a - b*x^2)^(1/4)/(a*x^5) + (9*b*(-1/3*(a - b*x^2)^(1/4)/(a*x^3) + (5
*b*(-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[
ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))))/(6*a)))/(
10*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(-b\,x^{2}+a \right)^{\frac{3}{4}}}d x$$

input
```
int(1/x^6/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/x^6/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/4}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6 (a-bx^2)^{3/4}}\,dx=-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2},\frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{3}{4}}x^5}$$

input
```
integrate(1/x**6/(-b*x**2+a)**(3/4),x)
```

output
```
-hyper((-5/2, 3/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(3/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(a-b x^2\right)^{3/4}} \, dx = \int \frac{1}{\left(-b x^2+a\right)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(a-b x^2\right)^{3/4}} \, dx = \int \frac{1}{\left(-b x^2+a\right)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{x^6\,(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/(x^6*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,x^6}dx$$

input
```
int(1/x^6/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*x**6),x)
```

3.899 $\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx=\frac{8a^2x}{3b^3\sqrt[4]{a+bx^2}}-\frac{4ax^3}{9b^2\sqrt[4]{a+bx^2}}+\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{16a^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
8/3*a^2*x/b^3/(b*x^2+a)^(1/4)-4/9*a*x^3/b^2/(b*x^2+a)^(1/4)+2/9*x^5/b/(b*x
^2+a)^(1/4)-16/3*a^(5/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2
)*x/a^(1/2))),2^(1/2))/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.41 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx=\frac{2\left(-12a^2x-2abx^3+b^2x^5+12a^2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{9b^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(5/4),x]
```

output
```
(2*(-12*a^2*x - 2*a*b*x^3 + b^2*x^5 + 12*a^2*x*(1 + (b*x^2)/a)^(1/4)*Hyper
geometric2F1[1/4, 1/2, 3/2, -((b*x^2)/a)]))/(9*b^3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {250, 250, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{ 250}$$

$$\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{10a\int \frac{x^4}{(bx^2+a)^{5/4}}dx}{9b}$$

$$\Big\downarrow \text{ 250}$$

$$\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{10a\left(\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\int \frac{x^2}{(bx^2+a)^{5/4}}dx}{5b}\right)}{9b}$$

$$\Big\downarrow \text{ 250}$$

$$\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{10a\left(\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2a\int \frac{1}{(bx^2+a)^{5/4}}dx}{b}\right)}{5b}\right)}{9b}$$

$$\Big\downarrow \text{ 213}$$

$$\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{10a\left(\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{b\sqrt[4]{a+bx^2}}\right)}{5b}\right)}{9b}$$

$$\Big\downarrow \text{212}$$

$$\frac{2x^5}{9b\sqrt[4]{a+bx^2}}-\frac{10a\left(\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}\right)}{5b}\right)}{9b}$$

input

```
Int[x^6/(a + b*x^2)^(5/4),x]
```

output

```
(2*x^5)/(9*b*(a + b*x^2)^(1/4)) - (10*a*((2*x^3)/(5*b*(a + b*x^2)^(1/4)) -
 (6*a*((2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*Elli
pticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(5*
b)))/(9*b)
```

Defintions of rubi rules used

rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(5/4),x)
```

output
```
int(x^6/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix} \middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

input
```
integrate(x**6/(b*x**2+a)**(5/4),x)
```

output
```
x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/4))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(5/4),x)
```

output
```
int(x^6/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^6/(b*x^2+a)^(5/4),x)
```

output
```
int(x**6/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)
```

3.900 $\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = -\frac{12ax}{5b^2\sqrt[4]{a+bx^2}}+\frac{2x^3}{5b\sqrt[4]{a+bx^2}}+\frac{24a^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
-12/5*a*x/b^2/(b*x^2+a)^(1/4)+2/5*x^3/b/(b*x^2+a)^(1/4)+24/5*a^(3/2)*(1+b*
x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(5/2)
/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \frac{2\left(6ax+bx^3-6ax\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{5b^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(5/4),x]
```

output
```
(2*(6*a*x + b*x^3 - 6*a*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2
, 3/2, -((b*x^2)/a)]))/(5*b^2*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {250, 250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{250}$$

$$\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\int \frac{x^2}{(bx^2+a)^{5/4}}dx}{5b}$$

$$\Big\downarrow \text{250}$$

$$\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2a\int \frac{1}{(bx^2+a)^{5/4}}dx}{b}\right)}{5b}$$

$$\Big\downarrow \text{213}$$

$$\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{b\sqrt[4]{a+bx^2}}\right)}{5b}$$

$$\Big\downarrow \text{212}$$

$$\frac{2x^3}{5b\sqrt[4]{a+bx^2}}-\frac{6a\left(\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}\right)}{5b}$$

input
```
Int[x^4/(a + b*x^2)^(5/4),x]
```

output
```
(2*x^3)/(5*b*(a + b*x^2)^(1/4)) - (6*a*((2*x)/(b*(a + b*x^2)^(1/4)) - (4*S
qrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/
(b^(3/2)*(a + b*x^2)^(1/4))))/(5*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(5/4),x)
```

output
```
int(x^4/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

input
```
integrate(x**4/(b*x**2+a)**(5/4),x)
```

output
```
x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(5/4),x)
```

output
```
int(x^4/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^4/(b*x^2+a)^(5/4),x)
```

output
```
int(x**4/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)
```

3.901 $\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
2*x/b/(b*x^2+a)^(1/4)-4*a^(1/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan
(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.85 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{\left(a+bx^2\right)^{5/4}}\,dx=\frac{2x\left(-1+\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(5/4),x]
```

output
```
(2*x*(-1 + (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2
)/a)]))/(b*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {250, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \ 250$$

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2a\int \frac{1}{(bx^2+a)^{5/4}}dx}{b}$$

$$\Big\downarrow \ 213$$

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{2\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 212$$

$$\frac{2x}{b\sqrt[4]{a+bx^2}}-\frac{4\sqrt{a}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(5/4),x]
```

output
```
(2*x)/(b*(a + b*x^2)^(1/4)) - (4*Sqrt[a]*(1 + (b*x^2)/a)^(1/4)*EllipticE[A
rcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(5/4),x)
```

output
```
int(x^2/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx=\int \frac{x^2}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

input
```
integrate(x**2/(b*x**2+a)**(5/4),x)
```

output
```
x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(5/4),x)
```

output
```
int(x^2/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{5/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^2/(b*x^2+a)^(5/4),x)
```

output
```
int(x**2/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)
```

3.902 $\int \frac{1}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx=\frac{2\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
2*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/
a^(1/2)/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx=\frac{2x-x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-5/4),x]
```

output
```
(2*x - x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x^2)/
a)])/(a*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx$$
$$\Big\downarrow \; 213$$
$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{a\sqrt[4]{a+bx^2}}$$
$$\Big\downarrow \; 212$$
$$\frac{2\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-5/4),x]
```

output
```
(2*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqr
t[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(b*x^2+a)^(5/4),x)
```

output
```
int(1/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.43

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{5}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(5/4),x)
```

output
```
x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/4}}$$

input
```
int(1/(a + b*x^2)^(5/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/4)*hypergeom([1/2, 5/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(5/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x)
```

3.903 $\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 76

$$\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx = -\frac{1}{ax\sqrt[4]{a+bx^2}} - \frac{3\sqrt{b}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
-1/a/x/(b*x^2+a)^(1/4)-3*b^(1/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcta
n(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.68

$$\int \frac{1}{x^2(a+bx^2)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(5/4)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{ 251}$$

$$-\frac{3b\int \frac{1}{(bx^2+a)^{5/4}}dx}{2a}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{ 213}$$

$$-\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{2a^2\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{ 212}$$

$$-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(5/4)),x]
```

output
```
-(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[
ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{5}{4}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(5/4),x)
```

output
```
int(1/x^2/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^2 (a+bx^2)^{5/4}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{5}{4}} x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(5/4),x)
```

output
```
-hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/4}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/4}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{5/4}}\,dx = -\frac{2\left(\frac{a}{b\,x^2}+1\right)^{5/4}{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ -\frac{a}{b\,x^2}\right)}{7\,x\,\left(b\,x^2+a\right)^{5/4}}$$

input
```
int(1/(x^2*(a + b*x^2)^(5/4)),x)
```

output
```
-(2*(a/(b*x^2) + 1)^(5/4)*hypergeom([5/4, 7/4], 11/4, -a/(b*x^2)))/(7*x*(a
 + b*x^2)^(5/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{5/4}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}} a\,x^2+\left(b\,x^2+a\right)^{\frac{1}{4}} b\,x^4}dx$$

input
```
int(1/x^2/(b*x^2+a)^(5/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a*x**2 + (a + b*x**2)**(1/4)*b*x**4),x)
```

3.904 $\int \frac{1}{x^4(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{1}{x^4(a+bx^2)^{5/4}}\,dx = -\frac{1}{3ax^3\sqrt[4]{a+bx^2}}+\frac{7b}{6a^2x\sqrt[4]{a+bx^2}} + \frac{7b^{3/2}\sqrt[4]{1+\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
-1/3/a/x^3/(b*x^2+a)^(1/4)+7/6*b/a^2/x/(b*x^2+a)^(1/4)+7/2*b^(3/2)*(1+b*x^
2/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/(
b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.53

$$\int \frac{1}{x^4(a+bx^2)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(5/4)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {251, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{251}$$

$$-\frac{7b\int \frac{1}{x^2(bx^2+a)^{5/4}}dx}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{7b\left(-\frac{3b\int \frac{1}{(bx^2+a)^{5/4}}dx}{2a}-\frac{1}{ax\sqrt[4]{a+bx^2}}\right)}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{213}$$

$$-\frac{7b\left(-\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{2a^2\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}\right)}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{7b\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}\right)}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(5/4)),x]
```

output
```
-1/3*1/(a*x^3*(a + b*x^2)^(1/4)) - (7*b*(-(1/(a*x*(a + b*x^2)^(1/4))) - (3
*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*EllipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2]
)/(a^(3/2)*(a + b*x^2)^(1/4))))/(6*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(5/4),x)
```

output
```
int(1/x^4/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.31

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{4}} x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(5/4),x)
```

output
```
-hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/4}} dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} x^4} dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/4}} dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} x^4} dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{5/4}} dx = \int \frac{1}{x^4 (b x^2+a)^{5/4}} dx$$

input
```
int(1/(x^4*(a + b*x^2)^(5/4)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/4}} \, dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{4}}\,a\,x^4+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^6}dx$$

input
```
int(1/x^4/(b*x^2+a)^(5/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a*x**4 + (a + b*x**2)**(1/4)*b*x**6),x)
```

3.905 $\int \frac{1}{x^6(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 126

$$\int \frac{1}{x^6(a+bx^2)^{5/4}}\,dx = -\frac{1}{5ax^5\sqrt[4]{a+bx^2}}+\frac{11b}{30a^2x^3\sqrt[4]{a+bx^2}} -\frac{77b^2}{60a^3x\sqrt[4]{a+bx^2}}-\frac{77b^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a+bx^2}}$$

output

```
-1/5/a/x^5/(b*x^2+a)^(1/4)+11/30*b/a^2/x^3/(b*x^2+a)^(1/4)-77/60*b^2/a^3/x
/(b*x^2+a)^(1/4)-77/20*b^(5/2)*(1+b*x^2/a)^(1/4)*EllipticE(sin(1/2*arctan(
b^(1/2)*x/a^(1/2))),2^(1/2))/a^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^6(a+bx^2)^{5/4}}\,dx = -\frac{\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(5/4)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, -((b*x^2)/a
)])/(a*x^5*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {251, 251, 251, 213, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{251}$$

$$-\frac{11 b \int \frac{1}{x^4 \left(bx^2+a\right)^{5/4}}dx}{10 a}-\frac{1}{5 a x^5 \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{11 b \left(-\frac{7 b \int \frac{1}{x^2 \left(bx^2+a\right)^{5/4}}dx}{6 a}-\frac{1}{3 a x^3 \sqrt[4]{a+bx^2}}\right)}{10 a}-\frac{1}{5 a x^5 \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{11 b \left(-\frac{7 b \left(-\frac{3 b \int \frac{1}{\left(bx^2+a\right)^{5/4}}dx}{2 a}-\frac{1}{a x \sqrt[4]{a+bx^2}}\right)}{6 a}-\frac{1}{3 a x^3 \sqrt[4]{a+bx^2}}\right)}{10 a}-\frac{1}{5 a x^5 \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{213}$$

$$-\frac{11b\left(-\frac{7b\left(-\frac{3b\sqrt[4]{\frac{bx^2}{a}+1}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/4}}dx}{2a^2\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}\right)}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}\right)}{10a}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

$\Big\downarrow$ 212

$$-\frac{11b\left(-\frac{7b\left(-\frac{3\sqrt{b}\sqrt[4]{\frac{bx^2}{a}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a+bx^2}}-\frac{1}{ax\sqrt[4]{a+bx^2}}\right)}{6a}-\frac{1}{3ax^3\sqrt[4]{a+bx^2}}\right)}{10a}-\frac{1}{5ax^5\sqrt[4]{a+bx^2}}$$

input

```
Int[1/(x^6*(a + b*x^2)^(5/4)),x]
```

output

```
-1/5*1/(a*x^5*(a + b*x^2)^(1/4)) - (11*b*(-1/3*1/(a*x^3*(a + b*x^2)^(1/4))
 - (7*b*(-(1/(a*x*(a + b*x^2)^(1/4))) - (3*Sqrt[b]*(1 + (b*x^2)/a)^(1/4)*E
llipticE[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(a^(3/2)*(a + b*x^2)^(1/4))))/
(6*a)))/(10*a)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 213
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a*(a + b*x^2)^(1/4))   Int[1/(1 + b*(x^2/a))^(5/4), x], x] /; FreeQ[{a, b},
 x] && PosQ[a] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(5/4),x)
```

output
```
int(1/x^6/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/4}}\,dx = \int \frac{1}{\left(bx^{2}+a\right)^{\frac{5}{4}}x^{6}}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.25

$$\int \frac{1}{x^6 (a+bx^2)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{4}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(5/4),x)
```

output
```
-hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{5/4}}\, dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{5/4}}\, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(5/4)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/4}}\, dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{4}} a\,x^6+\left(b\,x^2+a\right)^{\frac{1}{4}} b\,x^8} dx$$

input
```
int(1/x^6/(b*x^2+a)^(5/4),x)
```

output
```
int(1/((a + b*x**2)**(1/4)*a*x**6 + (a + b*x**2)**(1/4)*b*x**8),x)
```

3.906 $\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int \frac{x^6}{\left(a-bx^2\right)^{5/4}}\,dx=\frac{2x^5}{b\sqrt[4]{a-bx^2}}+\frac{8ax\left(a-bx^2\right)^{3/4}}{3b^3}+\frac{20x^3\left(a-bx^2\right)^{3/4}}{9b^2}-\frac{16a^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3b^{7/2}\sqrt[4]{a-bx^2}}$$

output
```
2*x^5/b/(-b*x^2+a)^(1/4)+8/3*a*x*(-b*x^2+a)^(3/4)/b^3+20/9*x^3*(-b*x^2+a)^
(3/4)/b^2-16/3*a^(5/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*
x/a^(1/2))),2^(1/2))/b^(7/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx = -\frac{2x\left(-12a^2+2abx^2+b^2x^4+12a^2\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{9b^3\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^6/(a - b*x^2)^(5/4),x]
```

output
```
(-2*x*(-12*a^2 + 2*a*b*x^2 + b^2*x^4 + 12*a^2*(1 - (b*x^2)/a)^(1/4)*Hyperg
eometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]))/(9*b^3*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {252, 262, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\int \frac{x^4}{\sqrt[4]{a-bx^2}}dx}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\left(\frac{2a\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx}{3b}-\frac{2x^3(a-bx^2)^{3/4}}{9b}\right)}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\left(\frac{2a\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{b}$$

$\Big\downarrow$ 227

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\left(\frac{2a\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{b}$$

$\Big\downarrow$ 226

$$\frac{2x^5}{b\sqrt[4]{a-bx^2}}-\frac{10\left(\frac{2a\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x\left(a-bx^2\right)^{3/4}}{5b}\right)}{3b}-\frac{2x^3\left(a-bx^2\right)^{3/4}}{9b}\right)}{b}$$

input
```
Int[x^6/(a - b*x^2)^(5/4),x]
```

output
```
(2*x^5)/(b*(a - b*x^2)^(1/4)) - (10*((-2*x^3*(a - b*x^2)^(3/4))/(9*b) + (2
*a*((-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(1/4)*Elli
pticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(5*b^(3/2)*(a - b*x^2)^(1/4))))/(
3*b)))/b
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^6}{(-b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(x^6/(-b*x^2+a)^(5/4),x)
```

output
```
int(x^6/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\int \frac{x^6}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\frac{x^7{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{5}{4}}}$$

input
```
integrate(x**6/(-b*x**2+a)**(5/4),x)
```

output
```
x**7*hyper((5/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(5/4))
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\int \frac{x^6}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\int \frac{x^6}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\int \frac{x^6}{(a-b\,x^2)^{5/4}}\,dx$$

input
```
int(x^6/(a - b*x^2)^(5/4),x)
```

output
```
int(x^6/(a - b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{5/4}}\,dx=\int \frac{x^6}{(-b\,x^2+a)^{\frac{1}{4}}\,a-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^6/(-b*x^2+a)^(5/4),x)
```

output
```
int(x**6/((a - b*x**2)**(1/4)*a - (a - b*x**2)**(1/4)*b*x**2),x)
```

3.907 $\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 101

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx=\frac{2x^3}{b\sqrt[4]{a-bx^2}}+\frac{12x(a-bx^2)^{3/4}}{5b^2}-\frac{24a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{5/2}\sqrt[4]{a-bx^2}}$$

output
```
2*x^3/b/(-b*x^2+a)^(1/4)+12/5*x*(-b*x^2+a)^(3/4)/b^2-24/5*a^(3/2)*(1-b*x^2
/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(5/2)/(-
b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.65

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx=-\frac{2\left(-6ax+bx^3+6ax\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{5b^2\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^4/(a - b*x^2)^(5/4),x]
```

output
```
(-2*(-6*a*x + b*x^3 + 6*a*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1
/2, 3/2, (b*x^2)/a]))/(5*b^2*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {252, 262, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x^3}{b\sqrt[4]{a-bx^2}}-\frac{6\int \frac{x^2}{\sqrt[4]{a-bx^2}}dx}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3}{b\sqrt[4]{a-bx^2}}-\frac{6\left(\frac{2a\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{5b}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{b}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x^3}{b\sqrt[4]{a-bx^2}}-\frac{6\left(\frac{2a\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{5b\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{b}$$

$$\Big\downarrow \text{226}$$

$$\frac{2x^3}{b\sqrt[4]{a-bx^2}}-\frac{6\left(\frac{4a^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5b^{3/2}\sqrt[4]{a-bx^2}}-\frac{2x(a-bx^2)^{3/4}}{5b}\right)}{b}$$

input
```
Int[x^4/(a - b*x^2)^(5/4),x]
```

output
```
(2*x^3)/(b*(a - b*x^2)^(1/4)) - (6*((-2*x*(a - b*x^2)^(3/4))/(5*b) + (4*a^
(3/2)*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(
5*b^(3/2)*(a - b*x^2)^(1/4))))/b
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^4}{(-b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(x^4/(-b*x^2+a)^(5/4),x)
```

output
```
int(x^4/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \frac{x^5{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}}}$$

input
```
integrate(x**4/(-b*x**2+a)**(5/4),x)
```

output
```
x**5*hyper((5/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4))
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^4}{(a-b\,x^2)^{5/4}}\,dx$$

input
```
int(x^4/(a - b*x^2)^(5/4),x)
```

output
```
int(x^4/(a - b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^4}{(-b\,x^2+a)^{\frac{1}{4}}\,a-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^4/(-b*x^2+a)^(5/4),x)
```

output
```
int(x**4/((a - b*x**2)**(1/4)*a - (a - b*x**2)**(1/4)*b*x**2),x)
```

3.908 $\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 77

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \frac{2x}{b\sqrt[4]{a-bx^2}} - \frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
2*x/b/(-b*x^2+a)^(1/4)-4*a^(1/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsi
n(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.92 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \frac{2x-2x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{b\sqrt[4]{a-bx^2}}$$

input
```
Integrate[x^2/(a - b*x^2)^(5/4),x]
```

output
```
(2*x - 2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/
a])/(b*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {252, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{2\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{b}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{b\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2x}{b\sqrt[4]{a-bx^2}}-\frac{4\sqrt{a}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a-bx^2}}$$

input
```
Int[x^2/(a - b*x^2)^(5/4),x]
```

output
```
(2*x)/(b*(a - b*x^2)^(1/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(1/4)*EllipticE[A
rcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(b^(3/2)*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{\left(-b\,x^2+a\right)^{\frac{5}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(5/4),x)
```

output
```
int(x^2/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^2}{\left(-bx^2+a\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.38

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}}$$

input
```
integrate(x**2/(-b*x**2+a)**(5/4),x)
```

output
```
x**3*hyper((5/4, 3/2), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^2}{(a-b\,x^2)^{5/4}}\,dx$$

input
```
int(x^2/(a - b*x^2)^(5/4),x)
```

output
```
int(x^2/(a - b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{5/4}}\,dx = \int \frac{x^2}{(-b\,x^2+a)^{\frac{1}{4}}\,a-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(x^2/(-b*x^2+a)^(5/4),x)
```

output
```
int(x**2/((a - b*x**2)**(1/4)*a - (a - b*x**2)**(1/4)*b*x**2),x)
```

3.909 $\int \frac{1}{(a-bx^2)^{5/4}} dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{1}{(a-bx^2)^{5/4}} dx = \frac{2x}{a\sqrt[4]{a-bx^2}} - \frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
2*x/a/(-b*x^2+a)^(1/4)-2*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2
)*x/a^(1/2))),2^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a-bx^2)^{5/4}} dx = \frac{2x - x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[4]{a-bx^2}}$$

input
```
Integrate[(a - b*x^2)^(-5/4),x]
```

output
```
(2*x - x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (b*x^2)/a]
)/(a*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {215, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{215}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{a}$$

$$\Big\downarrow \text{227}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{a\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2x}{a\sqrt[4]{a-bx^2}}-\frac{2\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a-bx^2}}$$

input
```
Int[(a - b*x^2)^(-5/4),x]
```

output
```
(2*x)/(a*(a - b*x^2)^(1/4)) - (2*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*Sqrt[b]*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.34

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(5/4),x)
```

output
```
x*hyper((1/2, 5/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(5/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{5/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{5/4}}$$

input
```
int(1/(a - b*x^2)^(5/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(5/4)*hypergeom([1/2, 5/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(5/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,a-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx$$

input
```
int(1/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a - (a - b*x**2)**(1/4)*b*x**2),x)
```

3.910 $\int \frac{1}{x^2(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx=\frac{2}{ax\sqrt[4]{a-bx^2}}-\frac{3(a-bx^2)^{3/4}}{a^2x}-\frac{3\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
2/a/x/(-b*x^2+a)^(1/4)-3*(-b*x^2+a)^(3/4)/a^2/x-3*b^(1/2)*(1-b*x^2/a)^(1/4
)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/(-b*x^2+a)
^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.54

$$\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx=-\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{4},\frac{1}{2},\frac{bx^2}{a}\right)}{ax\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(5/4)),x]
```

output
```
-(((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/2, 5/4, 1/2, (b*x^2)/a])/(a*
x*(a - b*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {253, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \ 253$$

$$\frac{3 \int \frac{1}{x^2 \sqrt[4]{a-bx^2}}dx}{a} + \frac{2}{ax\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \ 264$$

$$\frac{3 \left(-\frac{b \int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a} - \frac{(a-bx^2)^{3/4}}{ax}\right)}{a} + \frac{2}{ax\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \ 227$$

$$\frac{3 \left(-\frac{b \sqrt[4]{1-\frac{bx^2}{a}} \int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}} - \frac{(a-bx^2)^{3/4}}{ax}\right)}{a} + \frac{2}{ax\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \ 226$$

$$\frac{3\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{(a-bx^2)^{3/4}}{ax}\right)}{a}+\frac{2}{ax\sqrt[4]{a-bx^2}}$$

input
```
Int[1/(x^2*(a - b*x^2)^(5/4)),x]
```

output
```
2/(a*x*(a - b*x^2)^(1/4)) + (3*(-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 -
 (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a
 - b*x^2)^(1/4))))/a
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^2\left(-b\,x^2+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/x^2/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^2\left(a-bx^2\right)^{5/4}}\,dx=-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{5}{4}}x}$$

input
```
integrate(1/x**2/(-b*x**2+a)**(5/4),x)
```

output
```
-hyper((-1/2, 5/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(5/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.41

$$\int \frac{1}{x^2 (a-bx^2)^{5/4}}\,dx = -\frac{2\left(1-\frac{a}{b\,x^2}\right)^{5/4}{}_2\mathrm{F}_1\left(\frac{5}{4},\frac{7}{4};\ \frac{11}{4};\ \frac{a}{b\,x^2}\right)}{7\,x\,(a-b\,x^2)^{5/4}}$$

input
```
int(1/(x^2*(a - b*x^2)^(5/4)),x)
```

output
```
-(2*(1 - a/(b*x^2))^(5/4)*hypergeom([5/4, 7/4], 11/4, a/(b*x^2)))/(7*x*(a
- b*x^2)^(5/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^2)^{5/4}}\,dx = \int \frac{1}{\left(-b\,x^2+a\right)^{\frac{1}{4}} a\,x^2-\left(-b\,x^2+a\right)^{\frac{1}{4}} b\,x^4}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a*x**2 - (a - b*x**2)**(1/4)*b*x**4),x)
```

3.911 $\int \frac{1}{x^4(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^4(a-bx^2)^{5/4}}\,dx = \frac{2}{ax^3\sqrt[4]{a-bx^2}} - \frac{7(a-bx^2)^{3/4}}{3a^2x^3} - \frac{7b(a-bx^2)^{3/4}}{2a^3x} - \frac{7b^{3/2}\sqrt[4]{1-\frac{bx^2}{a}}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2a^{5/2}\sqrt[4]{a-bx^2}}$$

output
```
2/a/x^3/(-b*x^2+a)^(1/4)-7/3*(-b*x^2+a)^(3/4)/a^2/x^3-7/2*b*(-b*x^2+a)^(3/
4)/a^3/x-7/2*b^(3/2)*(1-b*x^2/a)^(1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/
a^(1/2))),2^(1/2))/a^(5/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4(a-bx^2)^{5/4}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(5/4)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, (b*x^2)/a])
/(a*x^3*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {253, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{7\int \frac{1}{x^4\sqrt[4]{a-bx^2}}dx}{a}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{b\int \frac{1}{x^2\sqrt[4]{a-bx^2}}dx}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)}{a}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{7\left(\frac{b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{(a-bx^2)^{3/4}}{ax}\right)}{2a}-\frac{(a-bx^2)^{3/4}}{3ax^3}\right)}{a}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{227}$$

$$\frac{7\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int\frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{a}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \textbf{226}$$

$$\frac{7\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{a}+\frac{2}{ax^3\sqrt[4]{a-bx^2}}$$

input
```
Int[1/(x^4*(a - b*x^2)^(5/4)),x]
```

output
```
2/(a*x^3*(a - b*x^2)^(1/4)) + (7*(-1/3*(a - b*x^2)^(3/4)/(a*x^3) + (b*(-((
a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(1/4))))/(2*a)))/a
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^4 \left(-b\,x^2+a\right)^{\frac{5}{4}}}dx$$

input
```
int(1/x^4/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/x^4/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{5}{4}}x^3}$$

input
```
integrate(1/x**4/(-b*x**2+a)**(5/4),x)
```

output
```
-hyper((-3/2, 5/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(5/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{5}{4}} x^4} \, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}} \, dx = \int \frac{1}{x^4 (a-b\,x^2)^{5/4}} \, dx$$

input
```
int(1/(x^4*(a - b*x^2)^(5/4)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{5/4}} \, dx = \int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,a\,x^4 - (-b\,x^2+a)^{\frac{1}{4}}\,b\,x^6} dx$$

input
```
int(1/x^4/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a*x**4 - (a - b*x**2)**(1/4)*b*x**6),x)
```

3.912 $\int \frac{1}{x^6(a-bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 151

$$\int \frac{1}{x^6(a-bx^2)^{5/4}}\,dx = \frac{2}{ax^5\sqrt[4]{a-bx^2}}-\frac{11(a-bx^2)^{3/4}}{5a^2x^5}-\frac{77b(a-bx^2)^{3/4}}{30a^3x^3}-\frac{77b^2(a-bx^2)^{3/4}}{20a^4x}-\frac{77b^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20a^{7/2}\sqrt[4]{a-bx^2}}$$

output
```
2/a/x^5/(-b*x^2+a)^(1/4)-11/5*(-b*x^2+a)^(3/4)/a^2/x^5-77/30*b*(-b*x^2+a)^
(3/4)/a^3/x^3-77/20*b^2*(-b*x^2+a)^(3/4)/a^4/x-77/20*b^(5/2)*(1-b*x^2/a)^(
1/4)*EllipticE(sin(1/2*arcsin(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(7/2)/(-b*x^2
+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^6(a-bx^2)^{5/4}}\,dx = -\frac{\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/(x^6*(a - b*x^2)^(5/4)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/2, 5/4, -3/2, (b*x^2)/a])
/(a*x^5*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {253, 264, 264, 264, 227, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a-bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{253}$$

$$\frac{11 \int \frac{1}{x^6 \sqrt[4]{a-bx^2}}dx}{a}+\frac{2}{ax^5 \sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11 \left(\frac{7b \int \frac{1}{x^4 \sqrt[4]{a-bx^2}}dx}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}\right)}{a}+\frac{2}{ax^5 \sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11 \left(\frac{7b \left(\frac{b \int \frac{1}{x^2 \sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}\right)}{a}+\frac{2}{ax^5 \sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{11\left(\frac{7b\left(\frac{b\left(-\frac{b\int \frac{1}{\sqrt[4]{a-bx^2}}dx}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}\right)}{a}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

$\big\downarrow$ 227

$$\frac{11\left(\frac{7b\left(\frac{b\left(-\frac{b\sqrt[4]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[4]{1-\frac{bx^2}{a}}}dx}{2a\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}\right)}{a}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

$\big\downarrow$ 226

$$\frac{11\left(\frac{7b\left(\frac{b\left(-\frac{\sqrt{b}\sqrt[4]{1-\frac{bx^2}{a}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt[4]{a-bx^2}}-\frac{\left(a-bx^2\right)^{3/4}}{ax}\right)}{2a}-\frac{\left(a-bx^2\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{\left(a-bx^2\right)^{3/4}}{5ax^5}\right)}{a}+\frac{2}{ax^5\sqrt[4]{a-bx^2}}$$

input
```
Int[1/(x^6*(a - b*x^2)^(5/4)),x]
```

output
```
2/(a*x^5*(a - b*x^2)^(1/4)) + (11*(-1/5*(a - b*x^2)^(3/4)/(a*x^5) + (7*b*(
-1/3*(a - b*x^2)^(3/4)/(a*x^3) + (b*(-((a - b*x^2)^(3/4)/(a*x)) - (Sqrt[b]
*(1 - (b*x^2)/a)^(1/4)*EllipticE[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[
a]*(a - b*x^2)^(1/4))))/(2*a)))/(10*a)))/a
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 227
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(1/4)/(
a + b*x^2)^(1/4)   Int[1/(1 + b*(x^2/a))^(1/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(-b\,x^{2}+a \right)^{\frac{5}{4}}}d x$$

input
```
int(1/x^6/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/x^6/(-b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a-bx^2\right)^{5/4}}\, dx = \int \frac{1}{{\left(-b x^{2} + a\right)}^{\frac{5}{4}} x^{6}}\, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(3/4)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^6 (a - bx^2)^{5/4}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{4} \\ -\frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{2i\pi}}{a}\right)}{5a^{\frac{5}{4}} x^5}$$

input
```
integrate(1/x**6/(-b*x**2+a)**(5/4),x)
```

output
```
-hyper((-5/2, 5/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(5/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a - bx^2)^{5/4}} \, dx = \int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a - bx^2)^{5/4}} \, dx = \int \frac{1}{(-bx^2 + a)^{\frac{5}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(5/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^2)^{5/4}}\, dx = \int \frac{1}{x^6\,(a-b\,x^2)^{5/4}}\, dx$$

input
```
int(1/(x^6*(a - b*x^2)^(5/4)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(5/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^2)^{5/4}}\, dx = \int \frac{1}{(-b\,x^2+a)^{\frac{1}{4}}\,a\,x^6-(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^8} dx$$

input
```
int(1/x^6/(-b*x^2+a)^(5/4),x)
```

output
```
int(1/((a - b*x**2)**(1/4)*a*x**6 - (a - b*x**2)**(1/4)*b*x**8),x)
```

3.913 $\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx=-\frac{2x^5}{3b(a+bx^2)^{3/4}}-\frac{40ax\sqrt[4]{a+bx^2}}{21b^3}+\frac{20x^3\sqrt[4]{a+bx^2}}{21b^2}+\frac{80a^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{7/2}(a+bx^2)^{3/4}}$$

output
```
-2/3*x^5/b/(b*x^2+a)^(3/4)-40/21*a*x*(b*x^2+a)^(1/4)/b^3+20/21*x^3*(b*x^2+
a)^(1/4)/b^2+80/21*a^(5/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^
(1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx=\frac{-40a^2x-20abx^3+6b^2x^5+40a^2x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)}{21b^3(a+bx^2)^{3/4}}$$

input
```
Integrate[x^6/(a + b*x^2)^(7/4),x]
```

output
```
(-40*a^2*x - 20*a*b*x^3 + 6*b^2*x^5 + 40*a^2*x*(1 + (b*x^2)/a)^(3/4)*Hyper
geometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a)])/(21*b^3*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {252, 262, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{10\int \frac{x^4}{(bx^2+a)^{3/4}}dx}{3b}-\frac{2x^5}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{10\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\int \frac{x^2}{(bx^2+a)^{3/4}}dx}{7b}\right)}{3b}-\frac{2x^5}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{10\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)}{7b}\right)}{3b}-\frac{2x^5}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{10\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\left(a+bx^2\right)^{3/4}}\right)}{7b}\right)}{3b}-\frac{2x^5}{3b\left(a+bx^2\right)^{3/4}}$$

$\downarrow$ 229

$$\frac{10\left(\frac{2x^3\sqrt[4]{a+bx^2}}{7b}-\frac{6a\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a+bx^2\right)^{3/4}}\right)}{7b}\right)}{3b}-\frac{2x^5}{3b\left(a+bx^2\right)^{3/4}}$$

input
```
Int[x^6/(a + b*x^2)^(7/4),x]
```

output
```
(-2*x^5)/(3*b*(a + b*x^2)^(3/4)) + (10*((2*x^3*(a + b*x^2)^(1/4))/(7*b) -
(6*a*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*a^(3/2)*(1 + (b*x^2)/a)^(3/4)*Ell
ipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))))/
(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int(x^6/(b*x^2+a)^(7/4),x)
```

output
```
int(x^6/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{4}}}$$

input
```
integrate(x**6/(b*x**2+a)**(7/4),x)
```

output
```
x**7*hyper((7/4, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/4))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(7/4),x)
```

output
```
int(x^6/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^6/(b*x^2+a)^(7/4),x)
```

output
```
int(x**6/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)
```

3.914 $\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx=-\frac{2x^3}{3b(a+bx^2)^{3/4}}+\frac{4x\sqrt[4]{a+bx^2}}{3b^2}-\frac{8a^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{5/2}(a+bx^2)^{3/4}}$$

output
```
-2/3*x^3/b/(b*x^2+a)^(3/4)+4/3*x*(b*x^2+a)^(1/4)/b^2-8/3*a^(3/2)*(1+b*x^2/
a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.66

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx=\frac{2\left(2ax+bx^3-2ax\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{3b^2(a+bx^2)^{3/4}}$$

input
```
Integrate[x^4/(a + b*x^2)^(7/4),x]
```

output
```
(2*(2*a*x + b*x^3 - 2*a*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4
, 3/2, -((b*x^2)/a)]))/(3*b^2*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {252, 262, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2\int \frac{x^2}{(bx^2+a)^{3/4}}dx}{b}-\frac{2x^3}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{262}$$

$$\frac{2\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}\right)}{b}-\frac{2x^3}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{2\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{2a\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b(a+bx^2)^{3/4}}\right)}{b}-\frac{2x^3}{3b(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{2\left(\frac{2x\sqrt[4]{a+bx^2}}{3b}-\frac{4a^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}\right)}{b}-\frac{2x^3}{3b\left(a+bx^2\right)^{3/4}}$$

input
```
Int[x^4/(a + b*x^2)^(7/4),x]
```

output
```
(-2*x^3)/(3*b*(a + b*x^2)^(3/4)) + (2*((2*x*(a + b*x^2)^(1/4))/(3*b) - (4*
a^(3/2)*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(3*b^(3/2)*(a + b*x^2)^(3/4))))/b
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int(x^4/(b*x^2+a)^(7/4),x)
```

output
```
int(x^4/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \frac{x^5{}_2F_1\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{4}}}$$

input
```
integrate(x**4/(b*x**2+a)**(7/4),x)
```

output
```
x**5*hyper((7/4, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/4))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(7/4),x)
```

output
```
int(x^4/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^4/(b*x^2+a)^(7/4),x)
```

output
```
int(x**4/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)
```

3.915 $\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx=-\frac{2x}{3b(a+bx^2)^{3/4}}+\frac{4\sqrt{a}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a+bx^2)^{3/4}}$$

output
```
-2/3*x/b/(b*x^2+a)^(3/4)+4/3*a^(1/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2
*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx=\frac{2x\left(-1+\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{3b(a+bx^2)^{3/4}}$$

input
```
Integrate[x^2/(a + b*x^2)^(7/4),x]
```

output
```
(2*x*(-1 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2
)/a)]))/(3*b*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {252, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2\int \frac{1}{(bx^2+a)^{3/4}}dx}{3b}-\frac{2x}{3b\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3b\,(a+bx^2)^{3/4}}-\frac{2x}{3b\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{4\sqrt{a}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\,(a+bx^2)^{3/4}}-\frac{2x}{3b\,(a+bx^2)^{3/4}}$$

input
```
Int[x^2/(a + b*x^2)^(7/4),x]
```

output
```
(-2*x)/(3*b*(a + b*x^2)^(3/4)) + (4*Sqrt[a]*(1 + (b*x^2)/a)^(3/4)*Elliptic
F[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int(x^2/(b*x^2+a)^(7/4),x)
```

output
```
int(x^2/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{3}{2}, \frac{7}{4} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{4}}}$$

input
```
integrate(x**2/(b*x**2+a)**(7/4),x)
```

output
```
x**3*hyper((3/2, 7/4), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(7/4),x)
```

output
```
int(x^2/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{7/4}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^2/(b*x^2+a)^(7/4),x)
```

output
```
int(x**2/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)
```

3.916 $\int \frac{1}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 78

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx=\frac{2x}{3a(a+bx^2)^{3/4}}+\frac{2\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a+bx^2)^{3/4}}$$

output
```
2/3*x/a/(b*x^2+a)^(3/4)+2/3*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b
^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx=\frac{x\left(2+\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{3a(a+bx^2)^{3/4}}$$

input
```
Integrate[(a + b*x^2)^(-7/4),x]
```

output
```
(x*(2 + (1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, -((b*x^2)/a
)]))/(3*a*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {215, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{1}{(bx^2+a)^{3/4}}dx}{3a}+\frac{2x}{3a\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \ 231$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{3a\,(a+bx^2)^{3/4}}+\frac{2x}{3a\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \ 229$$

$$\frac{2\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}\,(a+bx^2)^{3/4}}+\frac{2x}{3a\,(a+bx^2)^{3/4}}$$

input
```
Int[(a + b*x^2)^(-7/4),x]
```

output
```
(2*x)/(3*a*(a + b*x^2)^(3/4)) + (2*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(1/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

input
```
integrate(1/(b*x**2+a)**(7/4),x)
```

output
```
x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/4)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/4}}$$

input
```
int(1/(a + b*x^2)^(7/4),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/4)*hypergeom([1/2, 7/4], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/4)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(7/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2),x)
```

3.917 $\int \frac{1}{x^2(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{x^2(a+bx^2)^{7/4}}\,dx = \frac{2}{3ax(a+bx^2)^{3/4}} - \frac{5\sqrt[4]{a+bx^2}}{3a^2x} - \frac{5\sqrt{b}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a+bx^2)^{3/4}}$$

output
```
2/3/a/x/(b*x^2+a)^(3/4)-5/3*(b*x^2+a)^(1/4)/a^2/x-5/3*b^(1/2)*(1+b*x^2/a)^
(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/(b*x^
2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^2(a+bx^2)^{7/4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{4},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(7/4)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 7/4, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {253, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{5 \int \frac{1}{x^2 (bx^2+a)^{3/4}}dx}{3a}+\frac{2}{3ax \left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5 \left(-\frac{b \int \frac{1}{\left(bx^2+a\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{3a}+\frac{2}{3ax \left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{5 \left(-\frac{b \left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{3a}+\frac{2}{3ax \left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{5\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{3a}+\frac{2}{3ax\left(a+bx^2\right)^{3/4}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x*(a + b*x^2)^(3/4)) + (5*(-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1
 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*
(a + b*x^2)^(3/4))))/(3*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(7/4),x)
```

output
```
int(1/x^2/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^2 (a+bx^2)^{7/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{4}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(7/4),x)
```

output
```
-hyper((-1/2, 7/4), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2)^{7/4}} dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{4}} x^2} dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^2)^{7/4}} dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{4}} x^2} dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^2 (a + bx^2)^{7/4}} dx = -\frac{2 \left(\frac{a}{b x^2} + 1\right)^{7/4} {}_2F_1\left(\frac{7}{4}, \frac{9}{4}; \frac{13}{4}; -\frac{a}{b x^2}\right)}{9 x (b x^2 + a)^{7/4}}$$

input
```
int(1/(x^2*(a + b*x^2)^(7/4)),x)
```

output
```
-(2*(a/(b*x^2) + 1)^(7/4)*hypergeom([7/4, 9/4], 13/4, -a/(b*x^2)))/(9*x*(a
 + b*x^2)^(7/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{7/4}} \, dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{4}}\,a\,x^2+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^4}dx$$

input
```
int(1/x^2/(b*x^2+a)^(7/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*a*x**2 + (a + b*x**2)**(3/4)*b*x**4),x)
```

3.918 $\int \frac{1}{x^4(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{1}{x^4(a+bx^2)^{7/4}}\,dx = \frac{2}{3ax^3(a+bx^2)^{3/4}} - \frac{\sqrt[4]{a+bx^2}}{a^2x^3} + \frac{5b\sqrt[4]{a+bx^2}}{2a^3x} + \frac{5b^{3/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2a^{5/2}(a+bx^2)^{3/4}}$$

output
```
2/3/a/x^3/(b*x^2+a)^(3/4)-(b*x^2+a)^(1/4)/a^2/x^3+5/2*b*(b*x^2+a)^(1/4)/a^
3/x+5/2*b^(3/2)*(1+b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(
1/2)),2^(1/2))/a^(5/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.45

$$\int \frac{1}{x^4(a+bx^2)^{7/4}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{4},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(7/4)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 7/4, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {253, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{3\int \frac{1}{x^4(bx^2+a)^{3/4}}dx}{a}+\frac{2}{3ax^3(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(-\frac{5b\int \frac{1}{x^2(bx^2+a)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{3\left(-\frac{5b\left(-\frac{b\int \frac{1}{(bx^2+a)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{3\left(-\frac{5b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \; 229$$

$$\frac{3\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3\left(a+bx^2\right)^{3/4}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x^3*(a + b*x^2)^(3/4)) + (3*(-1/3*(a + b*x^2)^(1/4)/(a*x^3) - (5*b*
(-((a + b*x^2)^(1/4)/(a*x)) - (Sqrt[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[Arc
Tan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a + b*x^2)^(3/4))))/(6*a)))/a
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(1/x^4/(b*x^2+a)^(7/4),x)
```

output
```
int(1/x^4/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{7/4}}\, dx = \int \frac{1}{{\left(bx^{2} + a\right)}^{\frac{7}{4}} x^{4}}\, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{7}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{7}{4}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(7/4),x)
```

output
```
-hyper((-3/2, 7/4), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{7/4}} \,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(7/4)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/4}} \, dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}} a\,x^4+\left(b\,x^2+a\right)^{\frac{3}{4}} b\,x^6}dx$$

input
```
int(1/x^4/(b*x^2+a)^(7/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*a*x**4 + (a + b*x**2)**(3/4)*b*x**6),x)
```

3.919 $\int \frac{1}{x^6(a+bx^2)^{7/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 147

$$\int \frac{1}{x^6(a+bx^2)^{7/4}} dx = \frac{2}{3ax^5(a+bx^2)^{3/4}} - \frac{13\sqrt[4]{a+bx^2}}{15a^2x^5} + \frac{13b\sqrt[4]{a+bx^2}}{10a^3x^3} - \frac{13b^2\sqrt[4]{a+bx^2}}{4a^4x} - \frac{13b^{5/2}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{7/2}(a+bx^2)^{3/4}}$$

output
```
2/3/a/x^5/(b*x^2+a)^(3/4)-13/15*(b*x^2+a)^(1/4)/a^2/x^5+13/10*b*(b*x^2+a)^
(1/4)/a^3/x^3-13/4*b^2*(b*x^2+a)^(1/4)/a^4/x-13/4*b^(5/2)*(1+b*x^2/a)^(3/4
)*InverseJacobiAM(1/2*arctan(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(7/2)/(b*x^2+a)
^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int \frac{1}{x^6(a+bx^2)^{7/4}} dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{4},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(7/4)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 7/4, -3/2, -((b*x^2)/a
)])/(a*x^5*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {253, 264, 264, 264, 231, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}} dx$$

$$\Big\downarrow \text{253}$$

$$\frac{13 \int \frac{1}{x^6 (bx^2+a)^{3/4}}dx}{3a} + \frac{2}{3ax^5 (a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{13 \left(-\frac{9b \int \frac{1}{x^4 \left(bx^2+a\right)^{3/4}}dx}{10a} - \frac{\sqrt[4]{a+bx^2}}{5ax^5}\right)}{3a} + \frac{2}{3ax^5 (a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{13 \left(-\frac{9b \left(-\frac{5b \int \frac{1}{x^2 \left(bx^2+a\right)^{3/4}}dx}{6a} - \frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a} - \frac{\sqrt[4]{a+bx^2}}{5ax^5}\right)}{3a} + \frac{2}{3ax^5 (a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{13\left(-\frac{9b\left(-\frac{5b\left(-\frac{b\int\frac{1}{(bx^2+a)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5(a+bx^2)^{3/4}}$$

$\Big\downarrow$ 231

$$\frac{13\left(-\frac{9b\left(-\frac{5b\left(-\frac{b\left(\frac{bx^2}{a}+1\right)^{3/4}\int\frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/4}}dx}{2a(a+bx^2)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5(a+bx^2)^{3/4}}$$

$\Big\downarrow$ 229

$$\frac{13\left(-\frac{9b\left(-\frac{5b\left(-\frac{\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^2\right)^{3/4}}-\frac{\sqrt[4]{a+bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a+bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a+bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\left(a+bx^2\right)^{3/4}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x^5*(a + b*x^2)^(3/4)) + (13*(-1/5*(a + b*x^2)^(1/4)/(a*x^5) - (9*b
*(-1/3*(a + b*x^2)^(1/4)/(a*x^3) - (5*b*(-((a + b*x^2)^(1/4)/(a*x)) - (Sqr
t[b]*(1 + (b*x^2)/a)^(3/4)*EllipticF[ArcTan[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(S
qrt[a]*(a + b*x^2)^(3/4))))/(6*a)))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(7/4),x)
```

output
```
int(1/x^6/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\, dx = \int \frac{1}{{\left(bx^{2} + a\right)}^{\frac{7}{4}} x^{6}}\, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\, dx = -\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{7}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{5a^{\frac{7}{4}}x^{5}}$$

input
```
integrate(1/x**6/(b*x**2+a)**(7/4),x)
```

output
```
-hyper((-5/2, 7/4), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{7/4}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(7/4)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/4}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{4}} a\,x^6+\left(b\,x^2+a\right)^{\frac{3}{4}} b\,x^8}dx$$

input
```
int(1/x^6/(b*x^2+a)^(7/4),x)
```

output
```
int(1/((a + b*x**2)**(3/4)*a*x**6 + (a + b*x**2)**(3/4)*b*x**8),x)
```

3.920 $\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \frac{2x^5}{3b(a-bx^2)^{3/4}} + \frac{40ax\sqrt[4]{a-bx^2}}{21b^3} + \frac{20x^3\sqrt[4]{a-bx^2}}{21b^2} - \frac{80a^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21b^{7/2}(a-bx^2)^{3/4}}$$

output
```
2/3*x^5/b/(-b*x^2+a)^(3/4)+40/21*a*x*(-b*x^2+a)^(1/4)/b^3+20/21*x^3*(-b*x^
2+a)^(1/4)/b^2-80/21*a^(5/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(
b^(1/2)*x/a^(1/2)),2^(1/2))/b^(7/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.48 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.63

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = -\frac{2\left(-20a^2x+10abx^3+3b^2x^5+20a^2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{21b^3(a-bx^2)^{3/4}}$$

input
```
Integrate[x^6/(a - b*x^2)^(7/4),x]
```

output
```
(-2*(-20*a^2*x + 10*a*b*x^3 + 3*b^2*x^5 + 20*a^2*x*(1 - (b*x^2)/a)^(3/4)*H
ypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a]))/(21*b^3*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {252, 262, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a-bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x^5}{3b(a-bx^2)^{3/4}}-\frac{10\int \frac{x^4}{(a-bx^2)^{3/4}}dx}{3b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5}{3b(a-bx^2)^{3/4}}-\frac{10\left(\frac{6a\int \frac{x^2}{(a-bx^2)^{3/4}}dx}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{3b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^5}{3b(a-bx^2)^{3/4}}-\frac{10\left(\frac{6a\left(\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{3b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x^5}{3b\left(a-bx^2\right)^{3/4}}-\frac{10\left(\frac{6a\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{3b}$$

$\Big\downarrow$ 230

$$\frac{2x^5}{3b\left(a-bx^2\right)^{3/4}}-\frac{10\left(\frac{6a\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}\left(a-bx^2\right)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{7b}-\frac{2x^3\sqrt[4]{a-bx^2}}{7b}\right)}{3b}$$

input
```
Int[x^6/(a - b*x^2)^(7/4),x]
```

output
```
(2*x^5)/(3*b*(a - b*x^2)^(3/4)) - (10*((-2*x^3*(a - b*x^2)^(1/4))/(7*b) +
(6*a*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*a^(3/2)*(1 - (b*x^2)/a)^(3/4)*El
lipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))))
/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^{6}}{\left(-b\,x^{2}+a\right)^{\frac{7}{4}}}d x$$

input
```
int(x^6/(-b*x^2+a)^(7/4),x)
```

output
```
int(x^6/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^{6}}{{\left(-b x^{2} + a\right)}^{\frac{7}{4}}}\,{d x}$$

input
```
integrate(x^6/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.23

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{7}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{7a^{\frac{7}{4}}}$$

input
```
integrate(x**6/(-b*x**2+a)**(7/4),x)
```

output
```
x**7*hyper((7/4, 7/2), (9/2,), b*x**2*exp_polar(2*I*pi)/a)/(7*a**(7/4))
```

Maxima [F]

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^6}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^6/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^6}{(a-b\,x^2)^{7/4}}\,dx$$

input
```
int(x^6/(a - b*x^2)^(7/4),x)
```

output
```
int(x^6/(a - b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^6}{(-b\,x^2+a)^{\frac{3}{4}}\,a-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^6/(-b*x^2+a)^(7/4),x)
```

output
```
int(x**6/((a - b*x**2)**(3/4)*a - (a - b*x**2)**(3/4)*b*x**2),x)
```

3.921 $\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx=\frac{2x^3}{3b(a-bx^2)^{3/4}}+\frac{4x\sqrt[4]{a-bx^2}}{3b^2}-\frac{8a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{5/2}(a-bx^2)^{3/4}}$$

output
```
2/3*x^3/b/(-b*x^2+a)^(3/4)+4/3*x*(-b*x^2+a)^(1/4)/b^2-8/3*a^(3/2)*(1-b*x^2
/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(5/2)/(
-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.64

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx=-\frac{2\left(-2ax+bx^3+2ax\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)\right)}{3b^2(a-bx^2)^{3/4}}$$

input
```
Integrate[x^4/(a - b*x^2)^(7/4),x]
```

output
```
(-2*(-2*a*x + b*x^3 + 2*a*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3
/4, 3/2, (b*x^2)/a]))/(3*b^2*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {252, 262, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a-bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x^3}{3b(a-bx^2)^{3/4}}-\frac{2\int \frac{x^2}{(a-bx^2)^{3/4}}dx}{b}$$

$$\Big\downarrow \text{262}$$

$$\frac{2x^3}{3b(a-bx^2)^{3/4}}-\frac{2\left(\frac{2a\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x^3}{3b(a-bx^2)^{3/4}}-\frac{2\left(\frac{2a\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{b}$$

$$\Big\downarrow \text{230}$$

$$\frac{2x^3}{3b\left(a-bx^2\right)^{3/4}}-\frac{2\left(\frac{4a^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}-\frac{2x\sqrt[4]{a-bx^2}}{3b}\right)}{b}$$

input
```
Int[x^4/(a - b*x^2)^(7/4),x]
```

output
```
(2*x^3)/(3*b*(a - b*x^2)^(3/4)) - (2*((-2*x*(a - b*x^2)^(1/4))/(3*b) + (4*
a^(3/2)*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])
/(3*b^(3/2)*(a - b*x^2)^(3/4))))/b
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^{4}}{\left(-b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int(x^4/(-b*x^2+a)^(7/4),x)
```

output
```
int(x^4/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^{4}}{\left(-bx^{2}+a\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \frac{x^{5}{}_{2}F_{1}\left(\begin{matrix}\frac{7}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|{\frac{bx^{2}e^{2i\pi}}{a}}\right)}{5a^{\frac{7}{4}}}$$

input
```
integrate(x**4/(-b*x**2+a)**(7/4),x)
```

output
```
x**5*hyper((7/4, 5/2), (7/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(7/4))
```

Maxima [F]

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^4}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^4/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^4}{(a-b\,x^2)^{7/4}}\,dx$$

input
```
int(x^4/(a - b*x^2)^(7/4),x)
```

output
```
int(x^4/(a - b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^4}{(-b\,x^2+a)^{\frac{3}{4}}\,a-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^4/(-b*x^2+a)^(7/4),x)
```

output
```
int(x**4/((a - b*x**2)**(3/4)*a - (a - b*x**2)**(3/4)*b*x**2),x)
```

3.922 $\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx=\frac{2x}{3b(a-bx^2)^{3/4}}-\frac{4\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}$$

output
```
2/3*x/b/(-b*x^2+a)^(3/4)-4/3*a^(1/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2
*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(3/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx=\frac{2x-2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{bx^2}{a}\right)}{3b(a-bx^2)^{3/4}}$$

input
```
Integrate[x^2/(a - b*x^2)^(7/4),x]
```

output
```
(2*x - 2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/
a])/(3*b*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {252, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a-bx^2)^{7/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{2x}{3b(a-bx^2)^{3/4}}-\frac{2\int \frac{1}{(a-bx^2)^{3/4}}dx}{3b}$$

$$\Big\downarrow \text{231}$$

$$\frac{2x}{3b(a-bx^2)^{3/4}}-\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3b(a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{2x}{3b(a-bx^2)^{3/4}}-\frac{4\sqrt{a}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3b^{3/2}(a-bx^2)^{3/4}}$$

input
```
Int[x^2/(a - b*x^2)^(7/4),x]
```

output
```
(2*x)/(3*b*(a - b*x^2)^(3/4)) - (4*Sqrt[a]*(1 - (b*x^2)/a)^(3/4)*EllipticF
[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*b^(3/2)*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{x^2}{(-b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int(x^2/(-b*x^2+a)^(7/4),x)
```

output
```
int(x^2/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx=\int \frac{x^2}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.36

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{4}}}$$

input
```
integrate(x**2/(-b*x**2+a)**(7/4),x)
```

output
```
x**3*hyper((3/2, 7/4), (5/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/4))
```

Maxima [F]

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^2}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(x^2/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^2}{(a-b\,x^2)^{7/4}}\,dx$$

input
```
int(x^2/(a - b*x^2)^(7/4),x)
```

output
```
int(x^2/(a - b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(a-bx^2)^{7/4}}\,dx = \int \frac{x^2}{(-b\,x^2+a)^{\frac{3}{4}}\,a-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(x^2/(-b*x^2+a)^(7/4),x)
```

output
```
int(x**2/((a - b*x**2)**(3/4)*a - (a - b*x**2)**(3/4)*b*x**2),x)
```

3.923 $\int \frac{1}{(a-bx^2)^{7/4}} dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(a-bx^2)^{7/4}} dx = \frac{2x}{3a(a-bx^2)^{3/4}} + \frac{2\left(1-\frac{bx^2}{a}\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right), 2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}}$$

output
```
2/3*x/a/(-b*x^2+a)^(3/4)+2/3*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

$$\int \frac{1}{(a-bx^2)^{7/4}} dx = \frac{x\left(2+\left(1-\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{bx^2}{a}\right)\right)}{3a(a-bx^2)^{3/4}}$$

input
```
Integrate[(a - b*x^2)^(-7/4),x]
```

output
```
(x*(2 + (1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (b*x^2)/a])
)/(3*a*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {215, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \ 215$$

$$\frac{\int \frac{1}{(a-bx^2)^{3/4}}dx}{3a}+\frac{2x}{3a(a-bx^2)^{3/4}}$$

$$\Big\downarrow \ 231$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{3a(a-bx^2)^{3/4}}+\frac{2x}{3a(a-bx^2)^{3/4}}$$

$$\Big\downarrow \ 230$$

$$\frac{2\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}\sqrt{b}(a-bx^2)^{3/4}}+\frac{2x}{3a(a-bx^2)^{3/4}}$$

input
```
Int[(a - b*x^2)^(-7/4),x]
```

output
```
(2*x)/(3*a*(a - b*x^2)^(3/4)) + (2*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[
(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(3*Sqrt[a]*Sqrt[b]*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

Maple [F]

$$\int \frac{1}{\left(-b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx=\int \frac{1}{\left(-bx^{2}+a\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.32

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{7}{4} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}}$$

input
```
integrate(1/(-b*x**2+a)**(7/4),x)
```

output
```
x*hyper((1/2, 7/4), (3/2,), b*x**2*exp_polar(2*I*pi)/a)/a**(7/4)
```

Maxima [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.47

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{7/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(a-b\,x^2)^{7/4}}$$

input
```
int(1/(a - b*x^2)^(7/4),x)
```

output
```
(x*(1 - (b*x^2)/a)^(7/4)*hypergeom([1/2, 7/4], 3/2, (b*x^2)/a))/(a - b*x^2
)^(7/4)
```

Reduce [F]

$$\int \frac{1}{(a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,a-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx$$

input
```
int(1/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*a - (a - b*x**2)**(3/4)*b*x**2),x)
```

3.924 $\int \frac{1}{x^2(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 105

$$\int \frac{1}{x^2(a-bx^2)^{7/4}}\,dx = \frac{2}{3ax(a-bx^2)^{3/4}} - \frac{5\sqrt[4]{a-bx^2}}{3a^2x} + \frac{5\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}(a-bx^2)^{3/4}}$$

output
```
2/3/a/x/(-b*x^2+a)^(3/4)-5/3*(-b*x^2+a)^(1/4)/a^2/x+5/3*b^(1/2)*(1-b*x^2/a
)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/(-b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.55 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{1}{x^2(a-bx^2)^{7/4}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{4},\frac{1}{2},\frac{bx^2}{a}\right)}{ax(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(a - b*x^2)^(7/4)),x]
```

output
```
-(((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/2, 7/4, 1/2, (b*x^2)/a])/(a*
x*(a - b*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {253, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{5 \int \frac{1}{x^2 (a-bx^2)^{3/4}}dx}{3a}+\frac{2}{3ax (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{264}$$

$$\frac{5 \left(\frac{b \int \frac{1}{\left(a-bx^2\right)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{3a}+\frac{2}{3ax (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{231}$$

$$\frac{5 \left(\frac{b \left(1-\frac{bx^2}{a}\right)^{3/4} \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{3a}+\frac{2}{3ax (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{5\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{3a}+\frac{2}{3ax\left(a-bx^2\right)^{3/4}}$$

input
```
Int[1/(x^2*(a - b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x*(a - b*x^2)^(3/4)) + (5*(-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1
 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*
(a - b*x^2)^(3/4))))/(3*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(-b\,x^{2}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(1/x^2/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/x^2/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}}\, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}x^2}\, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}}\, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{a^{\frac{7}{4}}x}$$

input
```
integrate(1/x**2/(-b*x**2+a)**(7/4),x)
```

output
```
-hyper((-1/2, 7/4), (1/2,), b*x**2*exp_polar(2*I*pi)/a)/(a**(7/4)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}} \, dx = -\frac{2 \left(1-\frac{a}{b x^2}\right)^{7/4} {}_2F_1\left(\frac{7}{4}, \frac{9}{4};\ \frac{13}{4};\ \frac{a}{b x^2}\right)}{9 x (a-b x^2)^{7/4}}$$

input
```
int(1/(x^2*(a - b*x^2)^(7/4)),x)
```

output
```
-(2*(1 - a/(b*x^2))^(7/4)*hypergeom([7/4, 9/4], 13/4, a/(b*x^2)))/(9*x*(a
- b*x^2)^(7/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,a\,x^2-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^4}dx$$

input
```
int(1/x^2/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*a*x**2 - (a - b*x**2)**(3/4)*b*x**4),x)
```

3.925 $\int \frac{1}{x^4(a-bx^2)^{7/4}} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}} \, dx = \frac{2}{3ax^3 (a-bx^2)^{3/4}} - \frac{\sqrt[4]{a-bx^2}}{a^2x^3} - \frac{5b\sqrt[4]{a-bx^2}}{2a^3x} + \frac{5b^{3/2}\left(1-\frac{bx^2}{a}\right)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{2a^{5/2}(a-bx^2)^{3/4}}$$

output
```
2/3/a/x^3/(-b*x^2+a)^(3/4)-(-b*x^2+a)^(1/4)/a^2/x^3-5/2*b*(-b*x^2+a)^(1/4)
/a^3/x+5/2*b^(3/2)*(1-b*x^2/a)^(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/
a^(1/2)),2^(1/2))/a^(5/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}} \, dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{4},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3 (a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(a - b*x^2)^(7/4)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/2, 7/4, -1/2, (b*x^2)/a])
/(a*x^3*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {253, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(a-b x^2\right)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{3 \int \frac{1}{x^4 \left(a-b x^2\right)^{3/4}}dx}{a}+\frac{2}{3 a x^3 \left(a-b x^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{3 \left(\frac{5 b \int \frac{1}{x^2 \left(a-b x^2\right)^{3/4}}dx}{6 a}-\frac{\sqrt[4]{a-b x^2}}{3 a x^3}\right)}{a}+\frac{2}{3 a x^3 \left(a-b x^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{3 \left(\frac{5 b \left(\frac{b \int \frac{1}{\left(a-b x^2\right)^{3/4}}dx}{2 a}-\frac{\sqrt[4]{a-b x^2}}{a x}\right)}{6 a}-\frac{\sqrt[4]{a-b x^2}}{3 a x^3}\right)}{a}+\frac{2}{3 a x^3 \left(a-b x^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 231}$$

$$\frac{3\left(\frac{5b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3(a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{3\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}(a-bx^2)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{a}+\frac{2}{3ax^3(a-bx^2)^{3/4}}$$

input
```
Int[1/(x^4*(a - b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x^3*(a - b*x^2)^(3/4)) + (3*(-1/3*(a - b*x^2)^(1/4)/(a*x^3) + (5*b*
(-((a - b*x^2)^(1/4)/(a*x)) + (Sqrt[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[Arc
Sin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(Sqrt[a]*(a - b*x^2)^(3/4))))/(6*a)))/a
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{4} \left(-b\,x^{2}+a \right)^{\frac{7}{4}}}d x$$

input
```
int(1/x^4/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/x^4/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}}\, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}x^4}\, dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{7}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{3a^{\frac{7}{4}}x^3}$$

input
```
integrate(1/x**4/(-b*x**2+a)**(7/4),x)
```

output
```
-hyper((-3/2, 7/4), (-1/2,), b*x**2*exp_polar(2*I*pi)/a)/(3*a**(7/4)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{x^4 (a-b x^2)^{7/4}} \, dx$$

input
```
int(1/(x^4*(a - b*x^2)^(7/4)),x)
```

output
```
int(1/(x^4*(a - b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{(-b x^2+a)^{\frac{3}{4}} a x^4 - (-b x^2+a)^{\frac{3}{4}} b x^6} dx$$

input
```
int(1/x^4/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*a*x**4 - (a - b*x**2)**(3/4)*b*x**6),x)
```

3.926 $\int \frac{1}{x^6(a-bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 153

$$\int \frac{1}{x^6(a-bx^2)^{7/4}}\,dx = \frac{2}{3ax^5(a-bx^2)^{3/4}} - \frac{13\sqrt[4]{a-bx^2}}{15a^2x^5} - \frac{13b\sqrt[4]{a-bx^2}}{10a^3x^3} - \frac{13b^2\sqrt[4]{a-bx^2}}{4a^4x} + \frac{13b^{5/2}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{4a^{7/2}(a-bx^2)^{3/4}}$$

output
```
2/3/a/x^5/(-b*x^2+a)^(3/4)-13/15*(-b*x^2+a)^(1/4)/a^2/x^5-13/10*b*(-b*x^2+
a)^(1/4)/a^3/x^3-13/4*b^2*(-b*x^2+a)^(1/4)/a^4/x+13/4*b^(5/2)*(1-b*x^2/a)^
(3/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(7/2)/(-b*x
^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.36

$$\int \frac{1}{x^6(a-bx^2)^{7/4}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{4},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(x^6*(a - b*x^2)^(7/4)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-5/2, 7/4, -3/2, (b*x^2)/a])
/(a*x^5*(a - b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {253, 264, 264, 264, 231, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 253}$$

$$\frac{13 \int \frac{1}{x^6 (a-bx^2)^{3/4}}dx}{3a}+\frac{2}{3ax^5 (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{13 \left(\frac{9b \int \frac{1}{x^4 \left(a-bx^2\right)^{3/4}}dx}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5 (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{13 \left(\frac{9b \left(\frac{5b \int \frac{1}{x^2 \left(a-bx^2\right)^{3/4}}dx}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5 (a-bx^2)^{3/4}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{13\left(\frac{9b\left(\frac{5b\left(\frac{b\int\frac{1}{(a-bx^2)^{3/4}}dx}{2a}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\left(a-bx^2\right)^{3/4}}$$

$\big\downarrow$ 231

$$\frac{13\left(\frac{9b\left(\frac{5b\left(\frac{b\left(1-\frac{bx^2}{a}\right)^{3/4}\int\frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/4}}dx}{2a\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\left(a-bx^2\right)^{3/4}}$$

$\big\downarrow$ 230

$$\frac{13\left(\frac{9b\left(\frac{5b\left(\frac{\sqrt{b}\left(1-\frac{bx^2}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a-bx^2\right)^{3/4}}-\frac{\sqrt[4]{a-bx^2}}{ax}\right)}{6a}-\frac{\sqrt[4]{a-bx^2}}{3ax^3}\right)}{10a}-\frac{\sqrt[4]{a-bx^2}}{5ax^5}\right)}{3a}+\frac{2}{3ax^5\left(a-bx^2\right)^{3/4}}$$

input
```
Int[1/(x^6*(a - b*x^2)^(7/4)),x]
```

output
```
2/(3*a*x^5*(a - b*x^2)^(3/4)) + (13*(-1/5*(a - b*x^2)^(1/4)/(a*x^5) + (9*b
*(-1/3*(a - b*x^2)^(1/4)/(a*x^3) + (5*b*(-((a - b*x^2)^(1/4)/(a*x)) + (Sqr
t[b]*(1 - (b*x^2)/a)^(3/4)*EllipticF[ArcSin[(Sqrt[b]*x)/Sqrt[a]]/2, 2])/(S
qrt[a]*(a - b*x^2)^(3/4))))/(6*a)))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 231
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(1 + b*(x^2/a))^(3/4)/(
a + b*x^2)^(3/4)   Int[1/(1 + b*(x^2/a))^(3/4), x], x] /; FreeQ[{a, b}, x]
&& PosQ[a]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(-b\,x^{2}+a \right)^{\frac{7}{4}}}dx$$

input
```
int(1/x^6/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/x^6/(-b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{7}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{5a^{\frac{7}{4}}x^5}$$

input
```
integrate(1/x**6/(-b*x**2+a)**(7/4),x)
```

output
```
-hyper((-5/2, 7/4), (-3/2,), b*x**2*exp_polar(2*I*pi)/a)/(5*a**(7/4)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}} \, dx = \int \frac{1}{(-bx^2+a)^{\frac{7}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(7/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{x^6 \left(a-b\,x^2\right)^{7/4}}\,dx$$

input
```
int(1/(x^6*(a - b*x^2)^(7/4)),x)
```

output
```
int(1/(x^6*(a - b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a-bx^2)^{7/4}}\,dx = \int \frac{1}{(-b\,x^2+a)^{\frac{3}{4}}\,a\,x^6-(-b\,x^2+a)^{\frac{3}{4}}\,b\,x^8}dx$$

input
```
int(1/x^6/(-b*x^2+a)^(7/4),x)
```

output
```
int(1/((a - b*x**2)**(3/4)*a*x**6 - (a - b*x**2)**(3/4)*b*x**8),x)
```

3.927 $\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = -\frac{128x}{1053\sqrt[4]{2+3x^2}} + \frac{32x(2+3x^2)^{3/4}}{1053} - \frac{40x^3(2+3x^2)^{3/4}}{1053} + \frac{2}{39}x^5(2+3x^2)^{3/4} + \frac{128\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

output
```
-128/1053*x/(3*x^2+2)^(1/4)+32/1053*x*(3*x^2+2)^(3/4)-40/1053*x^3*(3*x^2+2
)^(3/4)+2/39*x^5*(3*x^2+2)^(3/4)+128/3159*2^(1/4)*EllipticE(sin(1/2*arctan
(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.55

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \frac{2x\left((2+3x^2)^{3/4}(16-20x^2+27x^4) - 16\ 2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{1053}$$

input
```
Integrate[x^6/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*((2 + 3*x^2)^(3/4)*(16 - 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometric
2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/1053
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {262, 262, 262, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{39}x^5\left(3x^2+2\right)^{3/4}-\frac{20}{39}\int \frac{x^4}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{39}x^5\left(3x^2+2\right)^{3/4}-\frac{20}{39}\left(\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{3x^2+2}}dx\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{39}x^5\left(3x^2+2\right)^{3/4}-\frac{20}{39}\left(\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2+2}}dx\right)\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{2}{39}x^5\left(3x^2+2\right)^{3/4}-\frac{20}{39}\left(\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{\left(3x^2+2\right)^{5/4}}dx\right)\right)\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{2}{39}x^5\left(3x^2+2\right)^{3/4}-\frac{20}{39}\left(\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\right)\right)\right)$$

input
```
Int[x^6/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x^5*(2 + 3*x^2)^(3/4))/39 - (20*((2*x^3*(2 + 3*x^2)^(3/4))/27 - (4*((2*
x*(2 + 3*x^2)^(3/4))/15 - (4*((2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*Ellipti
cE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]))/15))/9))/39
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.20

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], -\frac{3 x^{2}}{2}\right)}{14}$	20
risch	$\frac{2 x \left(27 x^{4}-20 x^{2}+16\right) \left(3 x^{2}+2\right)^{\frac{3}{4}}}{1053}-\frac{32\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{1053}$	43

input
```
int(x^6/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^{6}}{\left(3\,x^{2}+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^6/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^{7} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{14}$$

input
```
integrate(x**6/(3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^6}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^6}{(3\,x^2+2)^{1/4}}\,dx$$

input
```
int(x^6/(3*x^2 + 2)^(1/4),x)
```

output
```
int(x^6/(3*x^2 + 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^6}{(3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(3*x^2+2)^(1/4),x)
```

output
```
int(x**6/(3*x**2 + 2)**(1/4),x)
```

3.928 $\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx=\frac{32x}{135\sqrt[4]{2+3x^2}}-\frac{8}{135}x\left(2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(2+3x^2\right)^{3/4}-\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

output
```
32/135*x/(3*x^2+2)^(1/4)-8/135*x*(3*x^2+2)^(3/4)+2/27*x^3*(3*x^2+2)^(3/4)-
32/405*2^(1/4)*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx=\frac{2}{135}x\left(\left(2+3x^2\right)^{3/4}\left(-4+5x^2\right)+4\,2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)$$

input
```
Integrate[x^4/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*((2 + 3*x^2)^(3/4)*(-4 + 5*x^2) + 4*2^(3/4)*Hypergeometric2F1[1/4, 1/
2, 3/2, (-3*x^2)/2]))/135
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2+2}}dx\right)$$

$$\Big\downarrow \text{225}$$

$$\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{\left(3x^2+2\right)^{5/4}}dx\right)\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{2}{27}x^3\left(3x^2+2\right)^{3/4}-\frac{4}{9}\left(\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\right)\right)$$

input
```
Int[x^4/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x^3*(2 + 3*x^2)^(3/4))/27 - (4*((2*x*(2 + 3*x^2)^(3/4))/15 - (4*((2*x)/
(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3
]))/15))/9
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.25

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right],\left[\frac{7}{2}\right],-\frac{3 x^{2}}{2}\right)}{10}$	20
risch	$\frac{2 x\left(5 x^{2}-4\right)\left(3 x^{2}+2\right)^{\frac{3}{4}}}{135}+\frac{8\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{135}$	38

input
```
int(x^4/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^4/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.33

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{1/4}}\,dx$$

input
```
int(x^4/(3*x^2 + 2)^(1/4),x)
```

output
```
int(x^4/(3*x^2 + 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^4}{(3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(3*x^2+2)^(1/4),x)
```

output
```
int(x**4/(3*x**2 + 2)**(1/4),x)
```

3.929 $\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx=-\frac{8x}{15\sqrt[4]{2+3x^2}}+\frac{2}{15}x\left(2+3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

output
```
-8/15*x/(3*x^2+2)^(1/4)+2/15*x*(3*x^2+2)^(3/4)+8/45*2^(1/4)*EllipticE(sin(
1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.98 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx=\frac{2}{15}x\left(\left(2+3x^2\right)^{3/4}-2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)$$

input
```
Integrate[x^2/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*((2 + 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2
)/2]))/15
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{225}$$

$$\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{\left(3x^2+2\right)^{5/4}}dx\right)$$

$$\Big\downarrow \text{212}$$

$$\frac{2}{15}x\left(3x^2+2\right)^{3/4}-\frac{4}{15}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\right)$$

input
```
Int[x^2/(2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*(2 + 3*x^2)^(3/4))/15 - (4*((2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*Elli
pticE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]))/15
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], -\frac{3 x^{2}}{2}\right)}{6}$	20
risch	$\frac{2 x \left(3 x^{2}+2\right)^{\frac{3}{4}}}{15}-\frac{2\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{15}$	31

input
```
int(x^2/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^2/(3*x^2 + 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}x^3\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{1/4}}\,dx$$

input
```
int(x^2/(3*x^2 + 2)^(1/4),x)
```

output
```
int(x^2/(3*x^2 + 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{x^2}{(3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(3*x^2+2)^(1/4),x)
```

output
```
int(x**2/(3*x**2 + 2)**(1/4),x)
```

3.930 $\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx=\frac{2x}{\sqrt[4]{2+3x^2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

output
```
2*x/(3*x^2+2)^(1/4)-2/3*2^(1/4)*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2
^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

input
```
Integrate[(2 + 3*x^2)^(-1/4),x]
```

output
```
(x*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/2^(1/4)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3x^2+2}}dx$$
$$\Big\downarrow \text{ 225}$$
$$\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{(3x^2+2)^{5/4}}dx$$
$$\Big\downarrow \text{ 212}$$
$$\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

input
```
Int[(2 + 3*x^2)^(-1/4),x]
```

output
```
(2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/
Sqrt[3]
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{2}$	18

input
```
int(1/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \frac{2^{\frac{3}{4}} x\, {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \frac{8^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

input
```
int(1/(3*x^2 + 2)^(1/4),x)
```

output
```
(8^(1/4)*x*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(1/(3*x^2+2)^(1/4),x)
```

output
```
int(1/(3*x**2 + 2)**(1/4),x)
```

3.931 $\int \frac{1}{x^2 \sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^2}}\,dx = \frac{3x}{2\sqrt[4]{2+3x^2}} - \frac{(2+3x^2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

output
```
3/2*x/(3*x^2+2)^(1/4)-1/2*(3*x^2+2)^(3/4)/x-1/2*2^(1/4)*EllipticE(sin(1/2*
arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^2 \sqrt[4]{2+3x^2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{3x^2}{2}\right)}{\sqrt[4]{2}x}$$

input
```
Integrate[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2]/(2^(1/4)*x))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {264, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{3}{4}\int \frac{1}{\sqrt[4]{3x^2+2}}dx-\frac{\left(3x^2+2\right)^{3/4}}{2x}$$

$$\Big\downarrow \text{ 225}$$

$$\frac{3}{4}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{\left(3x^2+2\right)^{5/4}}dx\right)-\frac{\left(3x^2+2\right)^{3/4}}{2x}$$

$$\Big\downarrow \text{ 212}$$

$$\frac{3}{4}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\right)-\frac{\left(3x^2+2\right)^{3/4}}{2x}$$

input
```
Int[1/(x^2*(2 + 3*x^2)^(1/4)),x]
```

output
```
-1/2*(2 + 3*x^2)^(3/4)/x + (3*((2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*Ellipt
icE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]))/4
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.32

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	20
risch	$-\frac{\left(3x^2+2\right)^{\frac{3}{4}}}{2x}+\frac{3\,2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}$	33

input
```
int(1/x^2/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx=\int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(3/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.46

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx=-\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx=\int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx = -\frac{2\,3^{3/4}\,\left(\frac{2}{x^2}+3\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^2}\right)}{9\,x\,(3\,x^2+2)^{1/4}}$$

input
```
int(1/(x^2*(3*x^2 + 2)^(1/4)),x)
```

output
```
-(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x
*(3*x^2 + 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3x^2+2)^{\frac{1}{4}}\,x^2}dx$$

input
```
int(1/x^2/(3*x^2+2)^(1/4),x)
```

output
```
int(1/((3*x**2 + 2)**(1/4)*x**2),x)
```

3.932 $\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\,dx = -\frac{9x}{8\sqrt[4]{2+3x^2}} - \frac{(2+3x^2)^{3/4}}{6x^3} + \frac{3(2+3x^2)^{3/4}}{8x} + \frac{3\sqrt{3}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

output
```
-9/8*x/(3*x^2+2)^(1/4)-1/6*(3*x^2+2)^(3/4)/x^3+3/8*(3*x^2+2)^(3/4)/x+3/8*2
^(1/4)*EllipticE(sin(1/2*arctan(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{3x^2}{2}\right)}{3\sqrt[4]{2}x^3}$$

input
```
Integrate[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2]/(2^(1/4)*x^3)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 264, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[4]{3x^2+2}} \, dx$$

$$\Big\downarrow \ 264$$

$$-\frac{3}{4} \int \frac{1}{x^2 \sqrt[4]{3x^2+2}} dx - \frac{\left(3x^2+2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \ 264$$

$$-\frac{3}{4} \left(\frac{3}{4} \int \frac{1}{\sqrt[4]{3x^2+2}} dx - \frac{\left(3x^2+2\right)^{3/4}}{2x}\right) - \frac{\left(3x^2+2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \ 225$$

$$-\frac{3}{4} \left(\frac{3}{4} \left(\frac{2x}{\sqrt[4]{3x^2+2}} - 2 \int \frac{1}{\left(3x^2+2\right)^{5/4}} dx\right) - \frac{\left(3x^2+2\right)^{3/4}}{2x}\right) - \frac{\left(3x^2+2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \ 212$$

$$-\frac{3}{4} \left(\frac{3}{4} \left(\frac{2x}{\sqrt[4]{3x^2+2}} - \frac{2\sqrt[4]{2} E\left(\frac{1}{2} \arctan\left(\sqrt{\frac{3}{2}} x\right) \middle| 2\right)}{\sqrt{3}}\right) - \frac{\left(3x^2+2\right)^{3/4}}{2x}\right) - \frac{\left(3x^2+2\right)^{3/4}}{6x^3}$$

input
```
Int[1/(x^4*(2 + 3*x^2)^(1/4)),x]
```

output
```
-1/6*(2 + 3*x^2)^(3/4)/x^3 - (3*(-1/2*(2 + 3*x^2)^(3/4)/x + (3*((2*x)/(2 +
 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]))/
4))/4
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],-\frac{3 x^{2}}{2}\right)}{6 x^{3}}$	20
risch	$\frac{27 x^{4}+6 x^{2}-8}{24 x^{3} \left(3 x^{2}+2\right)^{\frac{1}{4}}}-\frac{9\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{32}$	45

input
```
int(1/x^4/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(3/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx = -\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\, dx = \int \frac{1}{\left(3\,x^2+2\right)^{\frac{1}{4}} x^4}\, dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\, dx = \int \frac{1}{x^4 \left(3\,x^2+2\right)^{1/4}}\, dx$$

input
```
int(1/(x^4*(3*x^2 + 2)^(1/4)),x)
```

output
```
int(1/(x^4*(3*x^2 + 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \sqrt[4]{2+3x^2}}\, dx = \int \frac{1}{\left(3x^2+2\right)^{\frac{1}{4}} x^4} dx$$

input
```
int(1/x^4/(3*x^2+2)^(1/4),x)
```

output
```
int(1/((3*x**2 + 2)**(1/4)*x**4),x)
```

3.933 $\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx=\frac{189x}{160\sqrt[4]{2+3x^2}}-\frac{(2+3x^2)^{3/4}}{10x^5}+\frac{7(2+3x^2)^{3/4}}{40x^3}-\frac{63(2+3x^2)^{3/4}}{160x}-\frac{63\sqrt{3}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

output
```
189/160*x/(3*x^2+2)^(1/4)-1/10*(3*x^2+2)^(3/4)/x^5+7/40*(3*x^2+2)^(3/4)/x^
3-63/160*(3*x^2+2)^(3/4)/x-63/160*2^(1/4)*EllipticE(sin(1/2*arctan(1/2*x*6
^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.29

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{3x^2}{2}\right)}{5\sqrt[4]{2}x^5}$$

input
```
Integrate[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2]/(2^(1/4)*x^5)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 264, 264, 225, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{3x^2+2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{21}{20}\int \frac{1}{x^4 \sqrt[4]{3x^2+2}}dx-\frac{\left(3x^2+2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{21}{20}\left(-\frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{3x^2+2}}dx-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}\right)-\frac{\left(3x^2+2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{21}{20}\left(-\frac{3}{4}\left(\frac{3}{4}\int \frac{1}{\sqrt[4]{3x^2+2}}dx-\frac{\left(3x^2+2\right)^{3/4}}{2x}\right)-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}\right)-\frac{\left(3x^2+2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{225}$$

$$-\frac{21}{20}\left(-\frac{3}{4}\left(\frac{3}{4}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-2\int \frac{1}{\left(3x^2+2\right)^{5/4}}dx\right)-\frac{\left(3x^2+2\right)^{3/4}}{2x}\right)-\frac{\left(3x^2+2\right)^{3/4}}{6x^3}\right)-\frac{\left(3x^2+2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{212}$$

$$-\frac{21}{20}\left(-\frac{3}{4}\left(\frac{3}{4}\left(\frac{2x}{\sqrt[4]{3x^2+2}}-\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}\right)-\frac{(3x^2+2)^{3/4}}{2x}\right)-\frac{(3x^2+2)^{3/4}}{6x^3}\right)-\frac{(3x^2+2)^{3/4}}{10x^5}$$

input
```
Int[1/(x^6*(2 + 3*x^2)^(1/4)),x]
```

output
```
-1/10*(2 + 3*x^2)^(3/4)/x^5 - (21*(-1/6*(2 + 3*x^2)^(3/4)/x^3 - (3*(-1/2*(
2 + 3*x^2)^(3/4)/x + (3*((2*x)/(2 + 3*x^2)^(1/4) - (2*2^(1/4)*EllipticE[Ar
cTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]))/4))/4))/20
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 225
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(x/(a + b*x^2)^(1/4))
, x] - Simp[a   Int[1/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b}, x] && GtQ[
a, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.20

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \text{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{10x^{5}}$	20
risch	$-\frac{189x^{6}+42x^{4}-8x^{2}+32}{160x^{5}\left(3x^{2}+2\right)^{\frac{1}{4}}}+\frac{189\,2^{\frac{3}{4}} x\, \text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{640}$	50

input
```
int(1/x^6/(3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx=\int \frac{1}{\left(3\,x^{2}+2\right)^{\frac{1}{4}}x^{6}}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(3/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx=-\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{x^6\,(3\,x^2+2)^{1/4}}\,dx$$

input
```
int(1/(x^6*(3*x^2 + 2)^(1/4)),x)
```

output
```
int(1/(x^6*(3*x^2 + 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[4]{2+3x^2}}\,dx = \int \frac{1}{(3x^2+2)^{\frac{1}{4}}\,x^6}dx$$

input
```
int(1/x^6/(3*x^2+2)^(1/4),x)
```

output
```
int(1/((3*x**2 + 2)**(1/4)*x**6),x)
```

3.934 $\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx=-\frac{32x(2-3x^2)^{3/4}}{1053}-\frac{40x^3(2-3x^2)^{3/4}}{1053}-\frac{2}{39}x^5(2-3x^2)^{3/4}+\frac{128\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{1053\sqrt{3}}$$

output
```
-32/1053*x*(-3*x^2+2)^(3/4)-40/1053*x^3*(-3*x^2+2)^(3/4)-2/39*x^5*(-3*x^2+
2)^(3/4)+128/3159*2^(1/4)*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2)
)*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.98 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx=-\frac{2x\left((2-3x^2)^{3/4}(16+20x^2+27x^4)-16\ 2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{1053}$$

input
```
Integrate[x^6/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x*((2 - 3*x^2)^(3/4)*(16 + 20*x^2 + 27*x^4) - 16*2^(3/4)*Hypergeometri
c2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/1053
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 262, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{39}\int \frac{x^4}{\sqrt[4]{2-3x^2}}dx-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{39}\left(\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2-3x^2}}dx-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}\right)-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}dx-\frac{2}{15}x\left(2-3x^2\right)^{3/4}\right)-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}\right)-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}\right)-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}\right)-\frac{2}{39}x^5\left(2-3x^2\right)^{3/4}$$

input
```
Int[x^6/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x^5*(2 - 3*x^2)^(3/4))/39 + (20*((-2*x^3*(2 - 3*x^2)^(3/4))/27 + (4*((
-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2
])/(15*Sqrt[3])))/9))/39
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], \frac{3 x^{2}}{2}\right)}{14}$	20
risch	$\frac{2 x \left(27 x^{4}+20 x^{2}+16\right) \left(3 x^{2}-2\right)}{1053 \left(-3 x^{2}+2\right)^{\frac{1}{4}}}+\frac{32\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{1053}$	50

input
```
int(x^6/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^7 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{14}$$

input
```
integrate(x**6/(-3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**7*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^6}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^6}{\left(2-3\,x^2\right)^{1/4}}\,dx$$

input
```
int(x^6/(2 - 3*x^2)^(1/4),x)
```

output
```
int(x^6/(2 - 3*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^6}{\left(-3x^2+2\right)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-3*x^2+2)^(1/4),x)
```

output
```
int(x**6/( - 3*x**2 + 2)**(1/4),x)
```

3.935 $\int \frac{x^4}{\sqrt[4]{2-3x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}} \, dx = -\frac{8}{135}x\left(2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}+\frac{32\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{135\sqrt{3}}$$

output
```
-8/135*x*(-3*x^2+2)^(3/4)-2/27*x^3*(-3*x^2+2)^(3/4)+32/405*2^(1/4)*Ellipti
cE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.78 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}} \, dx = -\frac{2}{135}x\left(\left(2-3x^2\right)^{3/4}\left(4+5x^2\right) - 4\, 2^{3/4}\, \text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)$$

input
```
Integrate[x^4/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x*((2 - 3*x^2)^(3/4)*(4 + 5*x^2) - 4*2^(3/4)*Hypergeometric2F1[1/4, 1/
2, 3/2, (3*x^2)/2]))/135
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 262, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\int \frac{x^2}{\sqrt[4]{2-3x^2}}dx-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\left(\frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}dx-\frac{2}{15}x\left(2-3x^2\right)^{3/4}\right)-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{4}{9}\left(\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}\right)-\frac{2}{27}x^3\left(2-3x^2\right)^{3/4}$$

input
```
Int[x^4/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x^3*(2 - 3*x^2)^(3/4))/27 + (4*((-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/
4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/(15*Sqrt[3])))/9
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right], \left[\frac{7}{2}\right], \frac{3 x^{2}}{2}\right)}{10}$	20
risch	$\frac{2 x \left(5 x^{2}+4\right) \left(3 x^{2}-2\right)}{135 \left(-3 x^{2}+2\right)^{\frac{1}{4}}}+\frac{8\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{135}$	45

input
```
int(x^4/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)*x^4/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \frac{2^{\frac{3}{4}}x^5{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(-3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**5*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^4}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^4}{(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^4}{(2-3\,x^2)^{1/4}}\,dx$$

input
```
int(x^4/(2 - 3*x^2)^(1/4),x)
```

output
```
int(x^4/(2 - 3*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^4}{(-3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(-3*x^2+2)^(1/4),x)
```

output
```
int(x**4/( - 3*x**2 + 2)**(1/4),x)
```

3.936 $\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx=-\frac{2}{15}x\left(2-3x^2\right)^{3/4}+\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}$$

output
```
-2/15*x*(-3*x^2+2)^(3/4)+8/45*2^(1/4)*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/
2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.69 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx=-\frac{2}{15}x\left(\left(2-3x^2\right)^{3/4}-2^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)$$

input
```
Integrate[x^2/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x*((2 - 3*x^2)^(3/4) - 2^(3/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2
)/2]))/15
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {262, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{15}\int \frac{1}{\sqrt[4]{2-3x^2}}dx-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

$$\Big\downarrow \text{226}$$

$$\frac{8\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{15\sqrt{3}}-\frac{2}{15}x\left(2-3x^2\right)^{3/4}$$

input
```
Int[x^2/(2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x*(2 - 3*x^2)^(3/4))/15 + (8*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2,
2])/(15*Sqrt[3])
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], \frac{3 x^{2}}{2}\right)}{6}$	20
risch	$\frac{2 x \left(3 x^{2}-2\right)}{15 \left(-3 x^{2}+2\right)^{\frac{1}{4}}}+\frac{2\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{15}$	38

input
```
int(x^2/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^3 {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(-3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x**3*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^2 + 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^2}{(2-3\,x^2)^{1/4}}\,dx$$

input
```
int(x^2/(2 - 3*x^2)^(1/4),x)
```

output
```
int(x^2/(2 - 3*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{x^2}{(-3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-3*x^2+2)^(1/4),x)
```

output
```
int(x**2/( - 3*x**2 + 2)**(1/4),x)
```

3.937 $\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx=\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

output
```
2/3*2^(1/4)*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx=\frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)}{\sqrt[4]{2}}$$

input
```
Integrate[(2 - 3*x^2)^(-1/4),x]
```

output
```
(x*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/2^(1/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx$$

$$\Big\downarrow \text{226}$$

$$\frac{2\sqrt[4]{2}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{\sqrt{3}}$$

input
```
Int[(2 - 3*x^2)^(-1/4),x]
```

output
```
(2*2^(1/4)*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2}$	18

input
```
int(1/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \frac{2^{\frac{3}{4}}x\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(-3*x**2+2)**(1/4),x)
```

output
```
2**(3/4)*x*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \frac{2^{3/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

input
```
int(1/(2 - 3*x^2)^(1/4),x)
```

output
```
(2^(3/4)*x*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3x^2+2)^{\frac{1}{4}}}dx$$

input
```
int(1/(-3*x^2+2)^(1/4),x)
```

output
```
int(1/( - 3*x**2 + 2)**(1/4),x)
```

3.938 $\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}\,dx = -\frac{(2-3x^2)^{3/4}}{2x} - \frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}$$

output
```
-1/2*(-3*x^2+2)^(3/4)/x-1/2*2^(1/4)*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2)
)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.74 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{3x^2}{2}\right)}{\sqrt[4]{2}x}$$

input
```
Integrate[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2]/(2^(1/4)*x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {264, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[4]{2-3x^2}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{3}{4} \int \frac{1}{\sqrt[4]{2-3x^2}}dx - \frac{(2-3x^2)^{3/4}}{2x}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}} - \frac{(2-3x^2)^{3/4}}{2x}$$

input
```
Int[1/(x^2*(2 - 3*x^2)^(1/4)),x]
```

output
```
-1/2*(2 - 3*x^2)^(3/4)/x - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2
^(3/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \text{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2x}$	20
risch	$\frac{3x^2-2}{2x(-3x^2+2)^{\frac{1}{4}}}-\frac{3\,2^{\frac{3}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8}$	40

input
```
int(1/x^2/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx=\int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx=-\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(-3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx = -\frac{2\,3^{3/4}\,\left(3-\frac{2}{x^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,\left(2-3\,x^2\right)^{1/4}}$$

input
```
int(1/(x^2*(2 - 3*x^2)^(1/4)),x)
```

output
```
-(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*
(2 - 3*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}} x^2}dx$$

input
```
int(1/x^2/(-3*x^2+2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(1/4)*x**2),x)
```

3.939 $\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}\,dx = -\frac{(2-3x^2)^{3/4}}{6x^3} - \frac{3(2-3x^2)^{3/4}}{8x} - \frac{3\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{4\ 2^{3/4}}$$

output
```
-1/6*(-3*x^2+2)^(3/4)/x^3-3/8*(-3*x^2+2)^(3/4)/x-3/8*2^(1/4)*EllipticE(sin
(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},\frac{3x^2}{2}\right)}{3\sqrt[4]{2}x^3}$$

input
```
Integrate[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2]/(2^(1/4)*x^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {264, 264, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}dx-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}\left(-\frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}}dx-\frac{\left(2-3x^2\right)^{3/4}}{2x}\right)-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \text{226}$$

$$\frac{3}{4}\left(-\frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}-\frac{\left(2-3x^2\right)^{3/4}}{2x}\right)-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}$$

input
```
Int[1/(x^4*(2 - 3*x^2)^(1/4)),x]
```

output
```
-1/6*(2 - 3*x^2)^(3/4)/x^3 + (3*(-1/2*(2 - 3*x^2)^(3/4)/x - (Sqrt[3]*Ellip
ticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^(3/4)))/4
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6x^3}$	20
risch	$\frac{27x^4-6x^2-8}{24x^3\left(-3x^2+2\right)^{\frac{1}{4}}}-\frac{9\,2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32}$	45

input
```
int(1/x^4/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = -\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle|\, \frac{3x^2e^{2i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(-3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{x^4\,\left(2-3\,x^2\right)^{1/4}}\,dx$$

input
```
int(1/(x^4*(2 - 3*x^2)^(1/4)),x)
```

output
```
int(1/(x^4*(2 - 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{\left(-3x^2+2\right)^{\frac{1}{4}}x^4}dx$$

input
```
int(1/x^4/(-3*x^2+2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(1/4)*x**4),x)
```

3.940 $\int \frac{1}{x^6 \sqrt[4]{2-3x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}} \, dx = -\frac{(2-3x^2)^{3/4}}{10x^5} - \frac{7(2-3x^2)^{3/4}}{40x^3} - \frac{63(2-3x^2)^{3/4}}{160x} - \frac{63\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{80\ 2^{3/4}}$$

output
```
-1/10*(-3*x^2+2)^(3/4)/x^5-7/40*(-3*x^2+2)^(3/4)/x^3-63/160*(-3*x^2+2)^(3/
4)/x-63/160*2^(1/4)*EllipticE(sin(1/2*arcsin(1/2*x*6^(1/2))),2^(1/2))*3^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}} \, dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2}, \frac{1}{4}, -\frac{3}{2}, \frac{3x^2}{2}\right)}{5\sqrt[4]{2}x^5}$$

input
```
Integrate[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2]/(2^(1/4)*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 264, 264, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\int \frac{1}{x^4 \sqrt[4]{2-3x^2}}dx-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\left(\frac{3}{4}\int \frac{1}{x^2 \sqrt[4]{2-3x^2}}dx-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}\right)-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\left(\frac{3}{4}\left(-\frac{3}{4}\int \frac{1}{\sqrt[4]{2-3x^2}}dx-\frac{\left(2-3x^2\right)^{3/4}}{2x}\right)-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}\right)-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{226}$$

$$\frac{21}{20}\left(\frac{3}{4}\left(-\frac{\sqrt{3}E\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right)\middle|2\right)}{2^{3/4}}-\frac{\left(2-3x^2\right)^{3/4}}{2x}\right)-\frac{\left(2-3x^2\right)^{3/4}}{6x^3}\right)-\frac{\left(2-3x^2\right)^{3/4}}{10x^5}$$

input
```
Int[1/(x^6*(2 - 3*x^2)^(1/4)),x]
```

output
```
-1/10*(2 - 3*x^2)^(3/4)/x^5 + (21*(-1/6*(2 - 3*x^2)^(3/4)/x^3 + (3*(-1/2*(
2 - 3*x^2)^(3/4)/x - (Sqrt[3]*EllipticE[ArcSin[Sqrt[3/2]*x]/2, 2])/2^(3/4)
))/4))/20
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],\frac{3x^2}{2}\right)}{10x^5}$	20
risch	$\frac{189x^6-42x^4-8x^2-32}{160x^5\left(-3x^2+2\right)^{\frac{1}{4}}}-\frac{189\, 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{640}$	50

input
```
int(1/x^6/(-3*x^2+2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^2+2\right)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx = -\frac{2^{\frac{3}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(-3*x**2+2)**(1/4),x)
```

output
```
-2**(3/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5
)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{x^6\,(2-3\,x^2)^{1/4}}\,dx$$

input
```
int(1/(x^6*(2 - 3*x^2)^(1/4)),x)
```

output
```
int(1/(x^6*(2 - 3*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[4]{2-3x^2}}\,dx = \int \frac{1}{(-3x^2+2)^{\frac{1}{4}}\,x^6}dx$$

input
```
int(1/x^6/(-3*x^2+2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(1/4)*x**6),x)
```

3.941 $\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \frac{160x\sqrt[4]{2+3x^2}}{2079} - \frac{40}{693}x^3\sqrt[4]{2+3x^2} + \frac{2}{33}x^5\sqrt[4]{2+3x^2} - \frac{320\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2079\sqrt{3}}$$

output

```
160/2079*x*(3*x^2+2)^(1/4)-40/693*x^3*(3*x^2+2)^(1/4)+2/33*x^5*(3*x^2+2)^(
1/4)-320/6237*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*x*6^(1/2)),2^(1/2))*3
^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \frac{2x\left(\sqrt[4]{2+3x^2}(80-60x^2+63x^4) - 80\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{2079}$$

input
```
Integrate[x^6/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*((2 + 3*x^2)^(1/4)*(80 - 60*x^2 + 63*x^4) - 80*2^(1/4)*Hypergeometric
2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/2079
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 262, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{33}x^5\sqrt[4]{3x^2+2}-\frac{20}{33}\int \frac{x^4}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{33}x^5\sqrt[4]{3x^2+2}-\frac{20}{33}\left(\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\int \frac{x^2}{(3x^2+2)^{3/4}}dx\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{33}x^5\sqrt[4]{3x^2+2}-\frac{20}{33}\left(\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\left(\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4}{9}\int \frac{1}{(3x^2+2)^{3/4}}dx\right)\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{2}{33}x^5\sqrt[4]{3x^2+2}-\frac{20}{33}\left(\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\left(\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}\right)\right)$$

input
```
Int[x^6/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x^5*(2 + 3*x^2)^(1/4))/33 - (20*((2*x^3*(2 + 3*x^2)^(1/4))/21 - (4*((2*
x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(
9*Sqrt[3])))/7))/33
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], -\frac{3 x^{2}}{2}\right)}{14}$	20
risch	$\frac{2 x \left(63 x^{4}-60 x^{2}+80\right) \left(3 x^{2}+2\right)^{\frac{1}{4}}}{2079}-\frac{160\, 2^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{2079}$	43

input
```
int(x^6/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^6/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.33

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{14}$$

input
```
integrate(x**6/(3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2+2)^{3/4}}\,dx$$

input
```
int(x^6/(3*x^2 + 2)^(3/4),x)
```

output
```
int(x^6/(3*x^2 + 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(3*x^2+2)^(3/4),x)
```

output
```
int(x**6/(3*x**2 + 2)**(3/4),x)
```

3.942 $\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = -\frac{8}{63}x\sqrt[4]{2+3x^2}+\frac{2}{21}x^3\sqrt[4]{2+3x^2}+\frac{16\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{63\sqrt{3}}$$

output
```
-8/63*x*(3*x^2+2)^(1/4)+2/21*x^3*(3*x^2+2)^(1/4)+16/189*2^(3/4)*InverseJac
obiAM(1/2*arctan(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \frac{2}{63}x\left((-4+3x^2)\sqrt[4]{2+3x^2}+4\sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)\right)$$

input
```
Integrate[x^4/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*((-4 + 3*x^2)*(2 + 3*x^2)^(1/4) + 4*2^(1/4)*Hypergeometric2F1[1/2, 3/
4, 3/2, (-3*x^2)/2]))/63
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 262, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\int \frac{x^2}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\left(\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4}{9}\int \frac{1}{(3x^2+2)^{3/4}}dx\right)$$

$$\Big\downarrow \text{229}$$

$$\frac{2}{21}x^3\sqrt[4]{3x^2+2}-\frac{4}{7}\left(\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}\right)$$

input
```
Int[x^4/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x^3*(2 + 3*x^2)^(1/4))/21 - (4*((2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*
EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(9*Sqrt[3])))/7
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{7}{2}\right], -\frac{3 x^{2}}{2}\right)}{10}$	20
risch	$\frac{2 x \left(3 x^{2}-4\right) \left(3 x^{2}+2\right)^{\frac{1}{4}}}{63}+\frac{8\, 2^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{63}$	38

input
```
int(x^4/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx=\int \frac{x^{4}}{\left(3\,x^{2}+2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^4/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.42

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^5{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2+2)^{3/4}}\,dx$$

input
```
int(x^4/(3*x^2 + 2)^(3/4),x)
```

output
```
int(x^4/(3*x^2 + 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(3*x^2+2)^(3/4),x)
```

output
```
int(x**4/(3*x**2 + 2)**(3/4),x)
```

3.943 $\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \frac{2}{9}x\sqrt[4]{2+3x^2} - \frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

output

```
2/9*x*(3*x^2+2)^(1/4)-4/27*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*x*6^(1/2
)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.76 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \frac{2}{9}x\left(\sqrt[4]{2+3x^2} - \sqrt[4]{2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)\right)$$

input

```
Integrate[x^2/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*((2 + 3*x^2)^(1/4) - 2^(1/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2
)/2]))/9
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {262, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4}{9}\int \frac{1}{(3x^2+2)^{3/4}}dx$$

$$\Big\downarrow \text{ 229}$$

$$\frac{2}{9}x\sqrt[4]{3x^2+2}-\frac{4\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

input
```
Int[x^2/(2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*(2 + 3*x^2)^(1/4))/9 - (4*2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right],\left[\frac{5}{2}\right],-\frac{3 x^{2}}{2}\right)}{6}$	20
risch	$\frac{2 x\left(3 x^{2}+2\right)^{\frac{1}{4}}}{9}-\frac{2\, 2^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{9}$	31

input
```
int(x^2/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx=\int \frac{x^{2}}{\left(3\,x^{2}+2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^2/(3*x^2 + 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^3{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{5}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2+2)^{3/4}}\,dx$$

input
```
int(x^2/(3*x^2 + 2)^(3/4),x)
```

output
```
int(x^2/(3*x^2 + 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(3*x^2+2)^(3/4),x)
```

output
```
int(x**2/(3*x**2 + 2)**(3/4),x)
```

3.944 $\int \frac{1}{(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

output
```
1/3*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \frac{x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)}{2^{3/4}}$$

input
```
Integrate[(2 + 3*x^2)^(-3/4),x]
```

output
```
(x*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/2^(3/4)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+2)^{3/4}} \, dx$$

$$\Big\downarrow \text{229}$$

$$\frac{2^{3/4} \text{EllipticF}\left(\frac{1}{2} \arctan\left(\sqrt{\frac{3}{2}} x\right), 2\right)}{\sqrt{3}}$$

input
```
Int[(2 + 3*x^2)^(-3/4),x]
```

output
```
(2^(3/4)*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{2}$	18

input
```
int(1/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)/2)/2
```

Maxima [F]

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \frac{2^{1/4}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2}$$

input
```
int(1/(3*x^2 + 2)^(3/4),x)
```

output
```
(2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/2
```

Reduce [F]

$$\int \frac{1}{(2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(1/(3*x^2+2)^(3/4),x)
```

output
```
int(1/(3*x**2 + 2)**(3/4),x)
```

3.945 $\int \frac{1}{x^2(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx=-\frac{\sqrt[4]{2+3x^2}}{2x}-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}$$

output
```
-1/2*(3*x^2+2)^(1/4)/x-1/4*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*x*6^(1/2
)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{3x^2}{2}\right)}{2^{3/4}x}$$

input
```
Integrate[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2]/(2^(3/4)*x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {264, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (3x^2+2)^{3/4}} \, dx$$

$$\Big\downarrow \; 264$$

$$-\frac{3}{4} \int \frac{1}{(3x^2+2)^{3/4}} dx - \frac{\sqrt[4]{3x^2+2}}{2x}$$

$$\Big\downarrow \; 229$$

$$-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}} - \frac{\sqrt[4]{3x^2+2}}{2x}$$

input
```
Int[1/(x^2*(2 + 3*x^2)^(3/4)),x]
```

output
```
-1/2*(2 + 3*x^2)^(1/4)/x - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(
2*2^(1/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.41

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{2x}$	20
risch	$-\frac{\left(3x^2+2\right)^{\frac{1}{4}}}{2x}-\frac{3\, 2^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{8}$	33

input
```
int(1/x^2/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx=\int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(1/4)/(3*x^4 + 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx=-\frac{\sqrt[4]{2}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\right|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}}\,dx = -\frac{2\,3^{1/4}\,\left(\frac{2}{x^2}+3\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\,(3\,x^2+2)^{3/4}}$$

input
```
int(1/(x^2*(3*x^2 + 2)^(3/4)),x)
```

output
```
-(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*
x*(3*x^2 + 2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (2+3x^2)^{3/4}} \, dx = \int \frac{1}{(3x^2+2)^{\frac{3}{4}} x^2} dx$$

input
```
int(1/x^2/(3*x^2+2)^(3/4),x)
```

output
```
int(1/((3*x**2 + 2)**(3/4)*x**2),x)
```

3.946 $\int \frac{1}{x^4(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{1}{x^4(2+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2+3x^2}}{6x^3}+\frac{5\sqrt[4]{2+3x^2}}{8x}+\frac{5\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{8\sqrt[4]{2}}$$

output
```
-1/6*(3*x^2+2)^(1/4)/x^3+5/8*(3*x^2+2)^(1/4)/x+5/16*2^(3/4)*InverseJacobiA
M(1/2*arctan(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4(2+3x^2)^{3/4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

input
```
Integrate[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2]/(2^(3/4)*x^3)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {264, 264, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (3x^2+2)^{3/4}}\,dx$$

$$\Big\downarrow \ 264$$

$$-\frac{5}{4}\int \frac{1}{x^2 (3x^2+2)^{3/4}}dx-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

$$\Big\downarrow \ 264$$

$$-\frac{5}{4}\left(-\frac{3}{4}\int \frac{1}{(3x^2+2)^{3/4}}dx-\frac{\sqrt[4]{3x^2+2}}{2x}\right)-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

$$\Big\downarrow \ 229$$

$$-\frac{5}{4}\left(-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{3x^2+2}}{2x}\right)-\frac{\sqrt[4]{3x^2+2}}{6x^3}$$

input
```
Int[1/(x^4*(2 + 3*x^2)^(3/4)),x]
```

output
```
-1/6*(2 + 3*x^2)^(1/4)/x^3 - (5*(-1/2*(2 + 3*x^2)^(1/4)/x - (Sqrt[3]*Ellip
ticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2*2^(1/4))))/4
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \text{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{6x^3}$	20
risch	$\frac{45x^4+18x^2-8}{24x^3\left(3x^2+2\right)^{\frac{3}{4}}}+\frac{15\,2^{\frac{1}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}$	45

input
```
int(1/x^4/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(2+3x^2\right)^{3/4}}\,dx=\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(1/4)/(3*x^6 + 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.48

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}} \, dx = \int \frac{1}{x^4 (3\,x^2+2)^{3/4}} \, dx$$

input
```
int(1/(x^4*(3*x^2 + 2)^(3/4)),x)
```

output
```
int(1/(x^4*(3*x^2 + 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (2+3x^2)^{3/4}} \, dx = \int \frac{1}{(3x^2+2)^{\frac{3}{4}} x^4} dx$$

input
```
int(1/x^4/(3*x^2+2)^(3/4),x)
```

output
```
int(1/((3*x**2 + 2)**(3/4)*x**4),x)
```

3.947 $\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2+3x^2}}{10x^5}+\frac{9\sqrt[4]{2+3x^2}}{40x^3} -\frac{27\sqrt[4]{2+3x^2}}{32x}-\frac{27\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}$$

output
```
-1/10*(3*x^2+2)^(1/4)/x^5+9/40*(3*x^2+2)^(1/4)/x^3-27/32*(3*x^2+2)^(1/4)/x
-27/64*2^(3/4)*InverseJacobiAM(1/2*arctan(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6(2+3x^2)^{3/4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{3x^2}{2}\right)}{5\ 2^{3/4}x^5}$$

input
```
Integrate[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2]/(2^(3/4)*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 264, 264, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (3x^2+2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{27}{20} \int \frac{1}{x^4 (3x^2+2)^{3/4}}dx - \frac{\sqrt[4]{3x^2+2}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{27}{20} \left(-\frac{5}{4} \int \frac{1}{x^2 (3x^2+2)^{3/4}}dx - \frac{\sqrt[4]{3x^2+2}}{6x^3}\right) - \frac{\sqrt[4]{3x^2+2}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{27}{20} \left(-\frac{5}{4} \left(-\frac{3}{4} \int \frac{1}{(3x^2+2)^{3/4}}dx - \frac{\sqrt[4]{3x^2+2}}{2x}\right) - \frac{\sqrt[4]{3x^2+2}}{6x^3}\right) - \frac{\sqrt[4]{3x^2+2}}{10x^5}$$

$$\Big\downarrow \text{229}$$

$$-\frac{27}{20} \left(-\frac{5}{4} \left(-\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}} - \frac{\sqrt[4]{3x^2+2}}{2x}\right) - \frac{\sqrt[4]{3x^2+2}}{6x^3}\right) - \frac{\sqrt[4]{3x^2+2}}{10x^5}$$

input
```
Int[1/(x^6*(2 + 3*x^2)^(3/4)),x]
```

output
```
-1/10*(2 + 3*x^2)^(1/4)/x^5 - (27*(-1/6*(2 + 3*x^2)^(1/4)/x^3 - (5*(-1/2*(
2 + 3*x^2)^(1/4)/x - (Sqrt[3]*EllipticF[ArcTan[Sqrt[3/2]*x]/2, 2])/(2*2^(1
/4))))/4))/20
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2},\frac{3}{4}\right],\left[-\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{10x^{5}}$	20
risch	$-\frac{405x^{6}+162x^{4}-24x^{2}+32}{160x^{5}\left(3x^{2}+2\right)^{\frac{3}{4}}}-\frac{81\,2^{\frac{1}{4}}x \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],-\frac{3x^{2}}{2}\right)}{128}$	50

input
```
int(1/x^6/(3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6\left(2+3x^2\right)^{3/4}}\,dx=\int \frac{1}{\left(3\,x^2+2\right)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^(1/4)/(3*x^8 + 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.38

$$\int \frac{1}{x^6 (2+3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 + 2)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\,(2+3x^2)^{3/4}}\,dx=\int \frac{1}{x^6\,(3\,x^2+2)^{3/4}}\,dx$$

input
```
int(1/(x^6*(3*x^2 + 2)^(3/4)),x)
```

output
```
int(1/(x^6*(3*x^2 + 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\,(2+3x^2)^{3/4}}\,dx=\int \frac{1}{(3x^2+2)^{\frac{3}{4}}\,x^6}dx$$

input
```
int(1/x^6/(3*x^2+2)^(3/4),x)
```

output
```
int(1/((3*x**2 + 2)**(3/4)*x**6),x)
```

3.948 $\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 83

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = -\frac{160x\sqrt[4]{2-3x^2}}{2079}-\frac{40}{693}x^3\sqrt[4]{2-3x^2}-\frac{2}{33}x^5\sqrt[4]{2-3x^2}+\frac{320\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2079\sqrt{3}}$$

output
```
-160/2079*x*(-3*x^2+2)^(1/4)-40/693*x^3*(-3*x^2+2)^(1/4)-2/33*x^5*(-3*x^2+
2)^(1/4)+320/6237*2^(3/4)*InverseJacobiAM(1/2*arcsin(1/2*x*6^(1/2)),2^(1/2
))*3^(1/2)
```

Mathematica [A] (verified)

Time = 5.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.71

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \frac{-6x\sqrt[4]{2-3x^2}(80+60x^2+63x^4)+320\ 2^{3/4}\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{6237}$$

input
```
Integrate[x^6/(2 - 3*x^2)^(3/4),x]
```

output
```
(-6*x*(2 - 3*x^2)^(1/4)*(80 + 60*x^2 + 63*x^4) + 320*2^(3/4)*Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/6237
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 262, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(2-3x^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\int \frac{x^4}{(2-3x^2)^{3/4}}dx-\frac{2}{33}x^5\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\left(\frac{4}{7}\int \frac{x^2}{(2-3x^2)^{3/4}}dx-\frac{2}{21}x^3\sqrt[4]{2-3x^2}\right)-\frac{2}{33}x^5\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\left(\frac{4}{7}\left(\frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}}dx-\frac{2}{9}x\sqrt[4]{2-3x^2}\right)-\frac{2}{21}x^3\sqrt[4]{2-3x^2}\right)-\frac{2}{33}x^5\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \text{230}$$

$$\frac{20}{33}\left(\frac{4}{7}\left(\frac{4\ 2^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}\right)-\frac{2}{21}x^3\sqrt[4]{2-3x^2}\right)-\frac{2}{33}x^5\sqrt[4]{2-3x^2}$$

input
```
Int[x^6/(2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x^5*(2 - 3*x^2)^(1/4))/33 + (20*((-2*x^3*(2 - 3*x^2)^(1/4))/21 + (4*((
-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2]
)/(9*Sqrt[3])))/7))/33
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], \frac{3 x^{2}}{2}\right)}{14}$	20

input
```
int(x^6/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)*x^6/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^7\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{3x^2e^{2i\pi}}{2}\right)}{14}$$

input
```
integrate(x**6/(-3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**7*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(2*I*pi)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(2-3\,x^2)^{3/4}}\,dx$$

input
```
int(x^6/(2 - 3*x^2)^(3/4),x)
```

output
```
int(x^6/(2 - 3*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(-3*x^2+2)^(3/4),x)
```

output
```
int(x**6/( - 3*x**2 + 2)**(3/4),x)
```

3.949 $\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = -\frac{8}{63}x\sqrt[4]{2-3x^2}-\frac{2}{21}x^3\sqrt[4]{2-3x^2}+\frac{16\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{63\sqrt{3}}$$

output
```
-8/63*x*(-3*x^2+2)^(1/4)-2/21*x^3*(-3*x^2+2)^(1/4)+16/189*2^(3/4)*InverseJ
acobiAM(1/2*arcsin(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 5.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{189}\left(3x\sqrt[4]{2-3x^2}(4+3x^2)-8\ 2^{3/4}\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)\right)$$

input
```
Integrate[x^4/(2 - 3*x^2)^(3/4),x]
```

output
```
(-2*(3*x*(2 - 3*x^2)^(1/4)*(4 + 3*x^2) - 8*2^(3/4)*Sqrt[3]*EllipticF[ArcSi
n[Sqrt[3/2]*x]/2, 2]))/189
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {262, 262, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{4}{7}\int \frac{x^2}{(2-3x^2)^{3/4}}dx-\frac{2}{21}x^3\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \; 262$$

$$\frac{4}{7}\left(\frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}}dx-\frac{2}{9}x\sqrt[4]{2-3x^2}\right)-\frac{2}{21}x^3\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \; 230$$

$$\frac{4}{7}\left(\frac{4\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}\right)-\frac{2}{21}x^3\sqrt[4]{2-3x^2}$$

input
```
Int[x^4/(2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x^3*(2 - 3*x^2)^(1/4))/21 + (4*((-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4
)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(9*Sqrt[3])))/7
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.31

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{7}{2}\right], \frac{3 x^{2}}{2}\right)}{10}$	20

input
```
int(x^4/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^{4}}{\left(-3\,x^{2}+2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)*x^4/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.45

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^5 {}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(-3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**5*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(2*I*pi)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(2-3\,x^2)^{3/4}}\,dx$$

input
```
int(x^4/(2 - 3*x^2)^(3/4),x)
```

output
```
int(x^4/(2 - 3*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(-3*x^2+2)^(3/4),x)
```

output
```
int(x**4/( - 3*x**2 + 2)**(3/4),x)
```

3.950 $\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

output
```
-2/9*x*(-3*x^2+2)^(1/4)+4/27*2^(3/4)*InverseJacobiAM(1/2*arcsin(1/2*x*6^(1
/2)),2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 4.77 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{2-3x^2}+\frac{4\,2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}$$

input
```
Integrate[x^2/(2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2
])/(9*Sqrt[3])
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {262, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\int \frac{1}{(2-3x^2)^{3/4}}dx-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

$$\Big\downarrow \text{230}$$

$$\frac{4\ 2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{9\sqrt{3}}-\frac{2}{9}x\sqrt[4]{2-3x^2}$$

input
```
Int[x^2/(2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x*(2 - 3*x^2)^(1/4))/9 + (4*2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2
])/(9*Sqrt[3])
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.43

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], \frac{3 x^{2}}{2}\right)}{6}$	20

input
```
int(x^2/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)*x^2/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.62

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^3\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(-3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x**3*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(2*I*pi)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^2 + 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(2-3\,x^2)^{3/4}}\,dx$$

input
```
int(x^2/(2 - 3*x^2)^(3/4),x)
```

output
```
int(x^2/(2 - 3*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-3*x^2+2)^(3/4),x)
```

output
```
int(x**2/( - 3*x**2 + 2)**(3/4),x)
```

3.951 $\int \frac{1}{(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 27

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

output
```
1/3*2^(3/4)*InverseJacobiAM(1/2*arcsin(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

input
```
Integrate[(2 - 3*x^2)^(-3/4),x]
```

output
```
(2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \; 230$$

$$\frac{2^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{\sqrt{3}}$$

input
```
Int[(2 - 3*x^2)^(-3/4),x]
```

output
```
(2^(3/4)*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/Sqrt[3]
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^{2}}{2}\right)}{2}$	18

input
```
int(1/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)/(3*x^2 - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\right|\frac{3x^2e^{2i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(-3*x**2+2)**(3/4),x)
```

output
```
2**(1/4)*x*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(2*I*pi)/2)/2
```

Maxima [F]

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \frac{2^{1/4}\,x\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2}$$

input
```
int(1/(2 - 3*x^2)^(3/4),x)
```

output
```
(2^(1/4)*x*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/2
```

Reduce [F]

$$\int \frac{1}{(2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2+2)^{\frac{3}{4}}}dx$$

input
```
int(1/(-3*x^2+2)^(3/4),x)
```

output
```
int(1/( - 3*x**2 + 2)**(3/4),x)
```

3.952 $\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2-3x^2}}{2x} + \frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}$$

output
```
-1/2*(-3*x^2+2)^(1/4)/x+1/4*2^(3/4)*InverseJacobiAM(1/2*arcsin(1/2*x*6^(1/
2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2(2-3x^2)^{3/4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},\frac{3x^2}{2}\right)}{2^{3/4}x}$$

input
```
Integrate[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

output
```
-(Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2]/(2^(3/4)*x))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {264, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}} \, dx$$
$$\Big\downarrow \text{ 264}$$
$$\frac{3}{4}\int \frac{1}{(2-3x^2)^{3/4}}dx - \frac{\sqrt[4]{2-3x^2}}{2x}$$
$$\Big\downarrow \text{ 230}$$
$$\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}} - \frac{\sqrt[4]{2-3x^2}}{2x}$$

input
```
Int[1/(x^2*(2 - 3*x^2)^(3/4)),x]
```

output
```
-1/2*(2 - 3*x^2)^(1/4)/x + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(
2*2^(1/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.41

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2x}$	20

input
```
int(1/x^2/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2},\frac{3}{4} \\ \frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{2i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(-3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.73

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = -\frac{2\,3^{1/4}\left(3-\frac{2}{x^2}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x\,(2-3\,x^2)^{3/4}}$$

input
```
int(1/(x^2*(2 - 3*x^2)^(3/4)),x)
```

output
```
-(2*3^(1/4)*(3 - 2/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x
*(2 - 3*x^2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2+2)^{\frac{3}{4}} x^2}dx$$

input
```
int(1/x^2/(-3*x^2+2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(3/4)*x**2),x)
```

3.953 $\int \frac{1}{x^4(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 67

$$\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx=-\frac{\sqrt[4]{2-3x^2}}{6x^3}-\frac{5\sqrt[4]{2-3x^2}}{8x}+\frac{5\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{8\sqrt[4]{2}}$$

output
```
-1/6*(-3*x^2+2)^(1/4)/x^3-5/8*(-3*x^2+2)^(1/4)/x+5/16*2^(3/4)*InverseJacob
iAM(1/2*arcsin(1/2*x*6^(1/2)),2^(1/2))*3^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.43

$$\int \frac{1}{x^4\left(2-3x^2\right)^{3/4}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},\frac{3x^2}{2}\right)}{3\ 2^{3/4}x^3}$$

input
```
Integrate[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2]/(2^(3/4)*x^3)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {264, 264, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{4}\int \frac{1}{x^2 (2-3x^2)^{3/4}}dx-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{4}\left(\frac{3}{4}\int \frac{1}{(2-3x^2)^{3/4}}dx-\frac{\sqrt[4]{2-3x^2}}{2x}\right)-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

$$\Big\downarrow \text{230}$$

$$\frac{5}{4}\left(\frac{\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{2\sqrt[4]{2}}-\frac{\sqrt[4]{2-3x^2}}{2x}\right)-\frac{\sqrt[4]{2-3x^2}}{6x^3}$$

input
```
Int[1/(x^4*(2 - 3*x^2)^(3/4)),x]
```

output
```
-1/6*(2 - 3*x^2)^(1/4)/x^3 + (5*(-1/2*(2 - 3*x^2)^(1/4)/x + (Sqrt[3]*Ellip
ticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2*2^(1/4))))/4
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6x^3}$	20

input
```
int(1/x^4/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{\left(-3\,x^2+2\right)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.51

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx = -\frac{\sqrt[4]{2} {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix} \middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(-3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(2*I*pi)/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}} x^4} \, dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}} x^4} \, dx$$

input
```
integrate(1/x^4/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx = \int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx$$

input
```
int(1/(x^4*(2 - 3*x^2)^(3/4)),x)
```

output
```
int(1/(x^4*(2 - 3*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (2-3x^2)^{3/4}} \, dx = \int \frac{1}{(-3x^2+2)^{\frac{3}{4}} x^4} dx$$

input
```
int(1/x^4/(-3*x^2+2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(3/4)*x***4),x)
```

3.954 $\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 85

$$\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2-3x^2}}{10x^5}-\frac{9\sqrt[4]{2-3x^2}}{40x^3}-\frac{27\sqrt[4]{2-3x^2}}{32x}+\frac{27\sqrt{3}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\sqrt{\frac{3}{2}}x\right),2\right)}{32\sqrt[4]{2}}$$

output
```
-1/10*(-3*x^2+2)^(1/4)/x^5-9/40*(-3*x^2+2)^(1/4)/x^3-27/32*(-3*x^2+2)^(1/4
)/x+27/64*2^(3/4)*InverseJacobiAM(1/2*arcsin(1/2*x*6^(1/2)),2^(1/2))*3^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6(2-3x^2)^{3/4}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},\frac{3x^2}{2}\right)}{5\ 2^{3/4}x^5}$$

input
```
Integrate[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

output
```
-1/5*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2]/(2^(3/4)*x^5)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 264, 264, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(2-3 x^2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \int \frac{1}{x^4 \left(2-3 x^2\right)^{3/4}}dx-\frac{\sqrt[4]{2-3 x^2}}{10 x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \left(\frac{5}{4} \int \frac{1}{x^2 \left(2-3 x^2\right)^{3/4}}dx-\frac{\sqrt[4]{2-3 x^2}}{6 x^3}\right)-\frac{\sqrt[4]{2-3 x^2}}{10 x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \left(\frac{5}{4} \left(\frac{3}{4} \int \frac{1}{\left(2-3 x^2\right)^{3/4}}dx-\frac{\sqrt[4]{2-3 x^2}}{2 x}\right)-\frac{\sqrt[4]{2-3 x^2}}{6 x^3}\right)-\frac{\sqrt[4]{2-3 x^2}}{10 x^5}$$

$$\Big\downarrow \text{230}$$

$$\frac{27}{20} \left(\frac{5}{4} \left(\frac{\sqrt{3} \text{EllipticF}\left(\frac{1}{2} \arcsin \left(\sqrt{\frac{3}{2}} x\right),2\right)}{2 \sqrt[4]{2}}-\frac{\sqrt[4]{2-3 x^2}}{2 x}\right)-\frac{\sqrt[4]{2-3 x^2}}{6 x^3}\right)-\frac{\sqrt[4]{2-3 x^2}}{10 x^5}$$

input
```
Int[1/(x^6*(2 - 3*x^2)^(3/4)),x]
```

output
```
-1/10*(2 - 3*x^2)^(1/4)/x^5 + (27*(-1/6*(2 - 3*x^2)^(1/4)/x^3 + (5*(-1/2*(
2 - 3*x^2)^(1/4)/x + (Sqrt[3]*EllipticF[ArcSin[Sqrt[3/2]*x]/2, 2])/(2*2^(1
/4))))/4))/20
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.24

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right],\left[-\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{10 x^{5}}$	20

input
```
int(1/x^6/(-3*x^2+2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}} x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = -\frac{\sqrt[4]{2}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^2 e^{2i\pi}}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(-3*x**2+2)**(3/4),x)
```

output
```
-2**(1/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(2*I*pi)/2)/(10*x**5
)
```

Maxima [F]

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}} x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2+2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 + 2)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{x^6\,(2-3\,x^2)^{3/4}}\,dx$$

input
```
int(1/(x^6*(2 - 3*x^2)^(3/4)),x)
```

output
```
int(1/(x^6*(2 - 3*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2+2)^{\frac{3}{4}}\,x^6}dx$$

input
```
int(1/x^6/(-3*x^2+2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 + 2)**(3/4)*x**6),x)
```

3.955 $\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 258

$$\begin{aligned}\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{32x(-2+3x^2)^{3/4}}{1053}+\frac{40x^3(-2+3x^2)^{3/4}}{1053} \\ &\quad+\frac{2}{39}x^5(-2+3x^2)^{3/4}+\frac{128x\sqrt[4]{-2+3x^2}}{1053\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} \\ &\quad-\frac{128\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x} \\ &\quad+\frac{64\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}x}\end{aligned}$$

output
```
32/1053*x*(3*x^2-2)^(3/4)+40/1053*x^3*(3*x^2-2)^(3/4)+2/39*x^5*(3*x^2-2)^(
3/4)+128*x*(3*x^2-2)^(1/4)/(1053*2^(1/2)+1053*(3*x^2-2)^(1/2))-128/3159*2^
(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*El
lipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x+
64/3159*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)
^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))
*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.26

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2x\left(-32+8x^2+6x^4+81x^6+16\,2^{3/4}\sqrt[4]{2-3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{1053\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[x^6/(-2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*(-32 + 8*x^2 + 6*x^4 + 81*x^6 + 16*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeo
metric2F1[1/4, 1/2, 3/2, (3*x^2)/2]))/(1053*(-2 + 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {262, 262, 262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{3x^2-2}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{20}{39}\int \frac{x^4}{\sqrt[4]{3x^2-2}}dx+\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 262$$

$$\frac{20}{39}\left(\frac{4}{9}\int \frac{x^2}{\sqrt[4]{3x^2-2}}dx+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3\right)+\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 262$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2-2}}dx+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3\right)+\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 228$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3\right)+$$
$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt{x^2}\int \frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3\right)+$$
$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 834$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}\right.$$
$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \; 27$$

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3\right.$$
$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$\downarrow$ 761

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\right.\right.$$

$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

$\downarrow$ 1510

$$\frac{20}{39}\left(\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E}{\sqrt{x^2}}\right.}{15\sqrt{3}x}\right.\right.$$

$$\frac{2}{39}\left(3x^2-2\right)^{3/4}x^5$$

input
```
Int[x^6/(-2 + 3*x^2)^(1/4),x]
```

output
```
(2*x^5*(-2 + 3*x^2)^(3/4))/39 + (20*((2*x^3*(-2 + 3*x^2)^(3/4))/27 + (4*((
2*x*(-2 + 3*x^2)^(3/4))/15 + (8*Sqrt[x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2)
^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-
2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2
)^(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])
^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(
1/4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(15*Sqrt[3]*x)))/9))/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.16

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], \frac{3 x^{2}}{2}\right)}{14 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	42
risch	$\frac{2 x \left(27 x^{4}+20 x^{2}+16\right) \left(3 x^{2}-2\right)^{\frac{3}{4}}}{1053}+\frac{32\, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{1053 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	65

input
```
int(x^6/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^7*hype
rgeom([1/4,7/2],[9/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^{6}}{\left(3\,x^{2}-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^6/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^{7} e^{-\frac{i\pi}{4}} {{}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{3x^{2}}{2}} \right)}}{14}$$

input
```
integrate(x**6/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2/2)/14
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{1/4}}\,dx$$

input
```
int(x^6/(3*x^2 - 2)^(1/4),x)
```

output
```
int(x^6/(3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^6}{(3x^2-2)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(3*x^2-2)^(1/4),x)
```

output
```
int(x**6/(3*x**2 - 2)**(1/4),x)
```

3.956 $\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 240

$$\begin{aligned}\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{8}{135}x\left(-2+3x^2\right)^{3/4}+\frac{2}{27}x^3\left(-2+3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2+3x^2}}{135\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}\\ &\quad-\frac{32\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}\\ &\quad+\frac{16\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}x}\end{aligned}$$

output
```
8/135*x*(3*x^2-2)^(3/4)+2/27*x^3*(3*x^2-2)^(3/4)+32*x*(3*x^2-2)^(1/4)/(135
*2^(1/2)+135*(3*x^2-2)^(1/2))-32/405*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2)
)^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^
(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x+16/405*2^(1/4)*(x^2/(2^(1/2)+(3*x^2
-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2
*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.77 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2x\left(-8+2x^2+15x^4+4\ 2^{3/4}\sqrt[4]{2-3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{135\sqrt[4]{-2+3x^2}}$$

input

```
Integrate[x^4/(-2 + 3*x^2)^(1/4),x]
```

output

```
(2*x*(-8 + 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (3*x^2)/2]))/(135*(-2 + 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {262, 262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\int \frac{x^2}{\sqrt[4]{3x^2-2}}dx+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\left(\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2-2}}dx+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{228}$$

$$\frac{4}{9}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{9}\left(\frac{8\sqrt{x^2}\int \frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{ 834}$$

$$\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x\right)+\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{ 761}$$

$$\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2\right.\right.$$

$$\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{ 1510}$$

$$\frac{4}{9}\left(\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\operatorname{arc}\right.}{\sqrt{x^2}}\right.}{15\sqrt{3}x}\right.$$

$$\frac{2}{27}\left(3x^2-2\right)^{3/4}x^3$$

input
```
Int[x^4/(-2 + 3*x^2)^(1/4),x]
```

output
```
(2*x^3*(-2 + 3*x^2)^(3/4))/27 + (4*((2*x*(-2 + 3*x^2)^(3/4))/15 + (8*Sqrt[
x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2])
- (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3
*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (
Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(1
5*Sqrt[3]*x)))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.18

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right],\left[\frac{7}{2}\right], \frac{3 x^{2}}{2}\right)}{10 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	42
risch	$\frac{2 x \left(5 x^{2}+4\right) \left(3 x^{2}-2\right)^{\frac{3}{4}}}{135}+\frac{8 \, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{135 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	60

input
```
int(x^4/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^5*hype
rgeom([1/4,5/2],[7/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^4/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.12

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10}$$

input
```
integrate(x**4/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2/2)/10
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{1/4}}\,dx$$

input
```
int(x^4/(3*x^2 - 2)^(1/4),x)
```

output
```
int(x^4/(3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^4}{(3x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
int(x^4/(3*x^2-2)^(1/4),x)
```

output
```
int(x**4/(3*x**2 - 2)**(1/4),x)
```

3.957 $\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 222

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2}{15}x\left(-2+3x^2\right)^{3/4}+\frac{8x\sqrt[4]{-2+3x^2}}{15\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} - \frac{8\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}x} + \frac{4\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}x}$$

output
```
2/15*x*(3*x^2-2)^(3/4)+8*x*(3*x^2-2)^(1/4)/(15*2^(1/2)+15*(3*x^2-2)^(1/2))
-8/45*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(
1/2))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^
(1/2)/x+4/45*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x
^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(
1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2x\left(-2+3x^2+2^{3/4}\sqrt[4]{2-3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{15\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[x^2/(-2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*(-2 + 3*x^2 + 2^(3/4)*(2 - 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3
/2, (3*x^2)/2]))/(15*(-2 + 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{15}\int \frac{1}{\sqrt[4]{3x^2-2}}dx+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \text{228}$$

$$\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \text{27}$$

$$\frac{8\sqrt{x^2}\int \frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \ 834$$

$$\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \ 27$$

$$\frac{8\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \ 761$$

$$\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{15\sqrt{3}x}+\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \ 1510$$

$$\frac{8\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\arctan\left(\right.\right.}{\sqrt{x^2}}\right.}{15\sqrt{3}x}$$

$$\frac{2}{15}\left(3x^2-2\right)^{3/4}x$$

input
```
Int[x^2/(-2 + 3*x^2)^(1/4),x]
```

output
```
(2*x*(-2 + 3*x^2)^(3/4))/15 + (8*Sqrt[x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2
)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[
-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^
2)^(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2]
)^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^
(1/4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(15*Sqrt[3]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], \frac{3 x^{2}}{2}\right)}{6 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	42
risch	$\frac{2 x \left(3 x^{2}-2\right)^{\frac{3}{4}}}{15}+\frac{2\, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{15 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	53

input
```
int(x^2/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x^3*hyper
geom([1/4,3/2],[5/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^2}{\left(3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral(x^2/(3*x^2 - 2)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.13

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^3 e^{-\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2}{2}\right)}{6}$$

input
```
integrate(x**2/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2/2)/6
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx=\int \frac{x^2}{{\left(3\,x^2-2\right)}^{1/4}} \,dx$$

input
```
int(x^2/(3*x^2 - 2)^(1/4),x)
```

output
```
int(x^2/(3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{-2+3x^2}}\,dx=\int \frac{x^2}{(3x^2-2)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(3*x^2-2)^(1/4),x)
```

output
```
int(x**2/(3*x**2 - 2)**(1/4),x)
```

3.958 $\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 199

$$\begin{aligned}\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx &= \frac{2x\sqrt[4]{-2+3x^2}}{\sqrt{2}+\sqrt{-2+3x^2}}\\ &\quad - \frac{2\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x}\\ &\quad + \frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}x}\end{aligned}$$

output
```
2*x*(3*x^2-2)^(1/4)/(2^(1/2)+(3*x^2-2)^(1/2))-2/3*2^(1/4)*(x^2/(2^(1/2)+(3
*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*EllipticE(sin(2*arctan(1
/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x+1/3*2^(1/4)*(x^2/(2^(1
/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*
arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.66 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.22

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx=\frac{x\sqrt[4]{1-\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},\frac{3x^2}{2}\right)}{\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[(-2 + 3*x^2)^(-1/4),x]
```

output
```
(x*(1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (3*x^2)/2])/(-2
+ 3*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \; 228$$

$$\frac{\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{x^2}\int \frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{\sqrt{3}x}$$

$$\Big\downarrow \; 834$$

$$\frac{2\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{\sqrt{3}x}$$

$\Big\downarrow$ 27

$$\frac{2\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{\sqrt{3}x}$$

$\Big\downarrow$ 761

$$\frac{2\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{\sqrt{3}x}$$

$\Big\downarrow$ 1510

$$\frac{2\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\arctan\left(\right.\right.}{\sqrt{x^2}}\right.}{\sqrt{3}x}$$

input
```
Int[(-2 + 3*x^2)^(-1/4),x]
```

output
```
(2*Sqrt[x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 +
3*x^2]) - (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqr
t[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[
x^2] + (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^
2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[x^
2])))/(Sqrt[3]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.20

method	result	size
meijerg	$\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2 \operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{1}{4}}}$	40

input
```
int(1/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)*x*hyperge
om([1/4,1/2],[3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(-1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.14

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{3x^2}{2}\right)}{2}$$

input
```
integrate(1/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2/2)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{3/4}\,x\,(2-3\,x^2)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,(3\,x^2-2)^{1/4}}$$

input
```
int(1/(3*x^2 - 2)^(1/4),x)
```

output
```
(2^(3/4)*x*(2 - 3*x^2)^(1/4)*hypergeom([1/4, 1/2], 3/2, (3*x^2)/2))/(2*(3*
x^2 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{1}{4}}}dx$$

input
```
int(1/(3*x^2-2)^(1/4),x)
```

output
```
int(1/(3*x**2 - 2)**(1/4),x)
```

3.959 $\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx &= \frac{(-2+3x^2)^{3/4}}{2x}-\frac{3x\sqrt[4]{-2+3x^2}}{2\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}\\ &+\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2^{3/4}x}\\ &-\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}\end{aligned}$$

output
```
1/2*(3*x^2-2)^(3/4)/x-3*x*(3*x^2-2)^(1/4)/(2*2^(1/2)+2*(3*x^2-2)^(1/2))+1/
2*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2)
)*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2
)/x-1/4*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)
^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))
*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.73 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx=-\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},\frac{3x^2}{2}\right)}{x\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

output
```
-(((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (3*x^2)/2])/(x*
(-2 + 3*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{228}$$

$$\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}$$

$$\Big\downarrow \text{27}$$

$$\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\int \frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}$$

$\Big\downarrow$ 834

$$\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}$$

$\Big\downarrow$ 27

$$\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}$$

$\Big\downarrow$ 761

$$\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int \frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}$$

$\Big\downarrow$ 1510

$$\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)E\left(2\operatorname{arcta}\right.}{\sqrt{x^2}}\right.}{2x}$$

input
```
Int[1/(x^2*(-2 + 3*x^2)^(1/4)),x]
```

output
```
(-2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*
x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sq
rt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3
*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x
^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)
/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(2*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.19

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{1}{2}\right], \frac{3 x^{2}}{2}\right)}{2 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}} x}$	42
risch	$\frac{\left(3 x^{2}-2\right)^{\frac{3}{4}}}{2 x}-\frac{3\, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{8 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	55

input
```
int(1/x^2/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x*hyperg
eom([-1/2,1/4],[1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\, dx = \int \frac{1}{\left(3\, x^{2} - 2\right)^{\frac{1}{4}} x^{2}}\, dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(3/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\, dx = \frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{1}{4} \\ \frac{1}{2} \end{matrix}\middle| {\frac{3x^{2}}{2}} \right)}{2x}$$

input
```
integrate(1/x**2/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx=\int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{-2+3x^2}}\,dx=\int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\,dx = -\frac{2\,3^{3/4}\,\left(3-\frac{2}{x^2}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{2}{3\,x^2}\right)}{9\,x\,\left(3\,x^2-2\right)^{1/4}}$$

input
```
int(1/(x^2*(3*x^2 - 2)^(1/4)),x)
```

output
```
-(2*3^(3/4)*(3 - 2/x^2)^(1/4)*hypergeom([1/4, 3/4], 7/4, 2/(3*x^2)))/(9*x*
(3*x^2 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2 \sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{1}{4}}\,x^2}dx$$

input
```
int(1/x^2/(3*x^2-2)^(1/4),x)
```

output
```
int(1/((3*x**2 - 2)**(1/4)*x**2),x)
```

3.960 $\int \frac{1}{x^4 \sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 242

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \frac{(-2+3x^2)^{3/4}}{6x^3}+\frac{3(-2+3x^2)^{3/4}}{8x}-\frac{9x\sqrt[4]{-2+3x^2}}{8\left(\sqrt{2}+\sqrt{-2+3x^2}\right)} + \frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\ 2^{3/4}x} - \frac{3\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\ 2^{3/4}x}$$

output
```
1/6*(3*x^2-2)^(3/4)/x^3+3/8*(3*x^2-2)^(3/4)/x-9*x*(3*x^2-2)^(1/4)/(8*2^(1/
2)+8*(3*x^2-2)^(1/2))+3/8*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*
(2^(1/2)+(3*x^2-2)^(1/2))*EllipticE(sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/
4))),1/2*2^(1/2))*3^(1/2)/x-3/16*2^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)
^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1
/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = -\frac{\sqrt[4]{1-\frac{3x^2}{2}}\ \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

output
```
-1/3*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (3*x^2)/2])
/(x^3*(-2 + 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {264, 264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{3x^2-2}}dx+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{3}{4}\int \frac{1}{\sqrt[4]{3x^2-2}}dx\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$$\Big\downarrow \text{228}$$

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{x^2}\int\frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$\Big\downarrow$ 27

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\int\frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$\Big\downarrow$ 834

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$\Big\downarrow$ 27

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$\Big\downarrow$ 761

$$\frac{3}{4}\left(\frac{\left(3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}\right)+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}$$

$\Big\downarrow$ 1510

$$\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\cdots\right.}{\cdots}\right.}{2x}\right.$$

$$\frac{(3x^2-2)^{3/4}}{6x^3}$$

input
```
Int[1/(x^4*(-2 + 3*x^2)^(1/4)),x]
```

output
```
(-2 + 3*x^2)^(3/4)/(6*x^3) + (3*((-2 + 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[
x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2])
- (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3
*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (
Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Elli
pticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(2
*x)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.17

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[-\frac{1}{2}\right], \frac{3 x^{2}}{2}\right)}{6 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}} x^{3}}$	42
risch	$\frac{27 x^{4}-6 x^{2}-8}{24 x^{3} \left(3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{9\, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{32 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	67

input
```
int(1/x^4/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x^3*hype
rgeom([-3/2,1/4],[-1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(3/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{4}\\-\frac{1}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{x^4\,(3\,x^2-2)^{1/4}}\,dx$$

input
```
int(1/(x^4*(3*x^2 - 2)^(1/4)),x)
```

output
```
int(1/(x^4*(3*x^2 - 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(3*x^2-2)^(1/4),x)
```

output
```
int(1/((3*x**2 - 2)**(1/4)*x**4),x)
```

3.961 $\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 260

$$\begin{aligned}\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx &= \frac{\left(-2+3x^2\right)^{3/4}}{10x^5}+\frac{7\left(-2+3x^2\right)^{3/4}}{40x^3}+\frac{63\left(-2+3x^2\right)^{3/4}}{160x}-\frac{189x\sqrt[4]{-2+3x^2}}{160\left(\sqrt{2}+\sqrt{-2+3x^2}\right)}\\&\quad+\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{80\ 2^{3/4}x}\\&\quad-\frac{63\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\ 2^{3/4}x}\end{aligned}$$

output
```
1/10*(3*x^2-2)^(3/4)/x^5+7/40*(3*x^2-2)^(3/4)/x^3+63/160*(3*x^2-2)^(3/4)/x
-189*x*(3*x^2-2)^(1/4)/(160*2^(1/2)+160*(3*x^2-2)^(1/2))+63/160*2^(1/4)*(x
^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*EllipticE(
sin(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x-63/320*2
^(1/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*I
nverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/
x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx = -\frac{\sqrt[4]{1-\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-2+3x^2}}$$

input
```
Integrate[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

output
```
-1/5*((1 - (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (3*x^2)/2])
/(x^5*(-2 + 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {264, 264, 264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt[4]{3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{3x^2-2}}dx+\frac{\left(3x^2-2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\left(\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{3x^2-2}}dx+\frac{\left(3x^2-2\right)^{3/4}}{6x^3}\right)+\frac{\left(3x^2-2\right)^{3/4}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{3}{4}\int\frac{1}{\sqrt[4]{3x^2-2}}dx\right)+\frac{(3x^2-2)^{3/4}}{6x^3}\right)+\frac{(3x^2-2)^{3/4}}{10x^5}$$

$\downarrow$ 228

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{x^2}\int\frac{\sqrt{\frac{2}{3}}\sqrt{3x^2-2}}{\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}\right)+\frac{(3x^2-2)^{3/4}}{6x^3}\right)+\frac{(3x^2-2)^{3/4}}{10x^5}$$

$\downarrow$ 27

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\int\frac{\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}}{2x}\right)+\frac{(3x^2-2)^{3/4}}{6x^3}\right)+\frac{(3x^2-2)^{3/4}}{10x^5}$$

$\downarrow$ 834

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\sqrt{2}\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{6}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}\right)+\frac{(3x^2-2)^{3/4}}{6x^3}\right)+\frac{(3x^2-2)^{3/4}}{10x^5}$$

$\downarrow$ 27

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}-\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\sqrt[4]{3x^2-2}\right)}{2x}\right)+\frac{(3x^2-2)^{3/4}}{6x^3}\right)+\frac{(3x^2-2)^{3/4}}{10x^5}$$

$\downarrow$ 761

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\int\frac{\sqrt{2}-\sqrt{3x^2-2}}{\sqrt{3}\sqrt{x^2}}d\right.}{2x}\right.\right.$$

$$\frac{(3x^2-2)^{3/4}}{10x^5}$$

$$\downarrow \text{1510}$$

$$\frac{21}{20}\left(\frac{3}{4}\left(\frac{(3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{x^2}\left(\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{x^2}}-\frac{\sqrt[4]{2}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{\right.}}}}{}\right.}{2x}\right.\right.$$

$$\frac{(3x^2-2)^{3/4}}{10x^5}$$

input
```
Int[1/(x^6*(-2 + 3*x^2)^(1/4)),x]
```

output
```
(-2 + 3*x^2)^(3/4)/(10*x^5) + (21*((-2 + 3*x^2)^(3/4)/(6*x^3) + (3*((-2 +
3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[x^2]*((Sqrt[3]*Sqrt[x^2]*(-2 + 3*x^2)^(
1/4))/(Sqrt[2] + Sqrt[-2 + 3*x^2]) - (2^(1/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2
+ 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticE[2*ArcTan[(-2 + 3*x^2)^
(1/4)/2^(1/4)], 1/2])/Sqrt[x^2] + (Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2
]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/
4)], 1/2])/(2^(3/4)*Sqrt[x^2])))/(2*x)))/4))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.16

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2},\frac{1}{4}\right],\left[-\frac{3}{2}\right],\frac{3 x^{2}}{2}\right)}{10 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}} x^{5}}$	42
risch	$\frac{189 x^{6}-42 x^{4}-8 x^{2}-32}{160 x^{5} \left(3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{189\, 2^{\frac{3}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{1}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],\frac{3 x^{2}}{2}\right)}{640 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{1}{4}}}$	72

input
```
int(1/x^6/(3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(3/4)/signum(-1+3/2*x^2)^(1/4)*(-signum(-1+3/2*x^2))^(1/4)/x^5*hyp
ergeom([-5/2,1/4],[-3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(3/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$\int \frac{1}{x^6\sqrt[4]{-2+3x^2}}\,dx = \frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{4}\\-\frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{x^6\,(3\,x^2-2)^{1/4}}\,dx$$

input
```
int(1/(x^6*(3*x^2 - 2)^(1/4)),x)
```

output
```
int(1/(x^6*(3*x^2 - 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2+3x^2}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{1}{4}}\,x^6}dx$$

input

```
int(1/x^6/(3*x^2-2)^(1/4),x)
```

output

```
int(1/((3*x**2 - 2)**(1/4)*x**6),x)
```

3.962 $\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 260

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = -\frac{32x(-2-3x^2)^{3/4}}{1053}+\frac{40x^3(-2-3x^2)^{3/4}}{1053} - \frac{2}{39}x^5(-2-3x^2)^{3/4}-\frac{128x\sqrt[4]{-2-3x^2}}{1053\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{128\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{1053\sqrt{3}x} + \frac{64\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{1053\sqrt{3}x}$$

output
```
-32/1053*x*(-3*x^2-2)^(3/4)+40/1053*x^3*(-3*x^2-2)^(3/4)-2/39*x^5*(-3*x^2-
2)^(3/4)-128*x*(-3*x^2-2)^(1/4)/(1053*2^(1/2)+1053*(-3*x^2-2)^(1/2))-128/3
159*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^
(1/2))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*
3^(1/2)/x+64/3159*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/
2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)
),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.26

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2x\left(32+8x^2-6x^4+81x^6-16\,2^{3/4}\sqrt[4]{2+3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{1053\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[x^6/(-2 - 3*x^2)^(1/4),x]
```

output
```
(2*x*(32 + 8*x^2 - 6*x^4 + 81*x^6 - 16*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeom
etric2F1[1/4, 1/2, 3/2, (-3*x^2)/2]))/(1053*(-2 - 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {262, 262, 262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{20}{39}\int \frac{x^4}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{20}{39}\left(-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3\right)-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \text{ 262}$$

$$-\frac{20}{39}\left(-\frac{4}{9}\left(-\frac{4}{15}\int\frac{1}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3\right)-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$\Big\downarrow$ 228

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int\frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3\right)-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$\Big\downarrow$ 27

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\int\frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3\right)-\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$\Big\downarrow$ 834

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int\frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\right.$$
$$\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$\Big\downarrow$ 27

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int\frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-\right.\right.$$
$$\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$\Big\downarrow$ 761

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int\frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3\right)}{15\sqrt{3}x}\right)\right)$$

$$\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

$$\Big\downarrow \ 1510$$

$$-\frac{20}{39}\left(-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\right.}{}\right)}{15\sqrt{3}x}\right)\right)$$

$$\frac{2}{39}\left(-3x^2-2\right)^{3/4}x^5$$

input
```
Int[x^6/(-2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x^5*(-2 - 3*x^2)^(3/4))/39 - (20*((-2*x^3*(-2 - 3*x^2)^(3/4))/27 - (4*
((-2*x*(-2 - 3*x^2)^(3/4))/15 + (8*Sqrt[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3
*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] +
 Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2
 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-
2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^
2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[-x^2])))/(15*Sqrt[3]*x)))/9))/39
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{7}{2}\right],\left[\frac{9}{2}\right],-\frac{3 x^{2}}{2}\right)}{14}$	23
risch	$\frac{2 x\left(27 x^{4}-20 x^{2}+16\right)\left(3 x^{2}+2\right)}{1053\left(-3 x^{2}-2\right)^{\frac{1}{4}}}+\frac{32(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{1053}$	53

input
```
int(x^6/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/14*(-1)^(3/4)*2^(3/4)*x^7*hypergeom([1/4,7/2],[9/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx=\int \frac{x^{6}}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/3159*(3159*x*integral(256/3159*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) -
2*(81*x^6 - 60*x^4 + 48*x^2 - 64)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.13

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx=\frac{2^{\frac{3}{4}} x^{7} e^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{1}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}}\right)}{14}$$

input
```
integrate(x**6/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**7*exp(-I*pi/4)*hyper((1/4, 7/2), (9/2,), 3*x**2*exp_polar(I*pi
)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^6}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^6}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^6}{\left(-3\,x^2-2\right)^{1/4}}\,dx$$

input
```
int(x^6/(- 3*x^2 - 2)^(1/4),x)
```

output
```
int(x^6/(- 3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^6}{\left(-3x^2-2\right)^{\frac{1}{4}}}dx$$

input
```
int(x^6/(-3*x^2-2)^(1/4),x)
```

output
```
int(x**6/( - 3*x**2 - 2)**(1/4),x)
```

3.963 $\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 242

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \frac{8}{135}x\left(-2-3x^2\right)^{3/4}-\frac{2}{27}x^3\left(-2-3x^2\right)^{3/4}+\frac{32x\sqrt[4]{-2-3x^2}}{135\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}$$
$$+\frac{32\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{135\sqrt{3}x}$$
$$-\frac{16\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{135\sqrt{3}x}$$

output
```
8/135*x*(-3*x^2-2)^(3/4)-2/27*x^3*(-3*x^2-2)^(3/4)+32*x*(-3*x^2-2)^(1/4)/(
135*2^(1/2)+135*(-3*x^2-2)^(1/2))+32/405*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)
^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*EllipticE(sin(2*arctan(1/2*(-3
*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x-16/405*2^(1/4)*(-x^2/(2^(1/
2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2
*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2x\left(-8-2x^2+15x^4+4\,2^{3/4}\sqrt[4]{2+3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{135\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[x^4/(-2 - 3*x^2)^(1/4),x]
```

output
```
(2*x*(-8 - 2*x^2 + 15*x^4 + 4*2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[
1/4, 1/2, 3/2, (-3*x^2)/2]))/(135*(-2 - 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {262, 262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{4}{9}\int \frac{x^2}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{262}$$

$$-\frac{4}{9}\left(-\frac{4}{15}\int \frac{1}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{228}$$

$$-\frac{4}{9}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\int \frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{834}$$

$$-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}\right)-\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{761}$$

$$-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right.}{15\sqrt{3}x}\right.$$

$$\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

$$\Big\downarrow \text{1510}$$

$$-\frac{4}{9}\left(\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2\cdots\right.}{}\right)}{15\sqrt{3}x}\right.$$

$$\frac{2}{27}\left(-3x^2-2\right)^{3/4}x^3$$

input
```
Int[x^4/(-2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x^3*(-2 - 3*x^2)^(3/4))/27 - (4*((-2*x*(-2 - 3*x^2)^(3/4))/15 + (8*Sqr
t[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^
2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt
[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-
x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3
*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt
[-x^2])))/(15*Sqrt[3]*x)))/9
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{5}{2}\right], \left[\frac{7}{2}\right], -\frac{3 x^{2}}{2}\right)}{10}$	23
risch	$\frac{2 x \left(5 x^{2}-4\right) \left(3 x^{2}+2\right)}{135 \left(-3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{8 (-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{135}$	48

input
```
int(x^4/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-1)^(3/4)*2^(3/4)*x^5*hypergeom([1/4,5/2],[7/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^4}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/405*(405*x*integral(-64/405*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(
15*x^4 - 12*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.14

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2^{\frac{3}{4}}x^5e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**5*exp(-I*pi/4)*hyper((1/4, 5/2), (7/2,), 3*x**2*exp_polar(I*pi
)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^4}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^4}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^4}{\left(-3\,x^2-2\right)^{1/4}}\,dx$$

input
```
int(x^4/(- 3*x^2 - 2)^(1/4),x)
```

output
```
int(x^4/(- 3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^4}{\left(-3x^2-2\right)^{\frac{1}{4}}}dx$$

input
```
int(x^4/(-3*x^2-2)^(1/4),x)
```

output
```
int(x**4/( - 3*x**2 - 2)**(1/4),x)
```

3.964 $\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 224

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx=-\frac{2}{15}x\left(-2-3x^2\right)^{3/4}-\frac{8x\sqrt[4]{-2-3x^2}}{15\left(\sqrt{2}+\sqrt{-2-3x^2}\right)}-\frac{8\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{15\sqrt{3}x}+\frac{4\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{15\sqrt{3}x}$$

output
```
-2/15*x*(-3*x^2-2)^(3/4)-8*x*(-3*x^2-2)^(1/4)/(15*2^(1/2)+15*(-3*x^2-2)^(1
/2))-8/45*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x
^2-2)^(1/2))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(
1/2))*3^(1/2)/x+4/45*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^
(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3
/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.26

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx=\frac{2x\left(2+3x^2-2^{3/4}\sqrt[4]{2+3x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{15\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[x^2/(-2 - 3*x^2)^(1/4),x]
```

output
```
(2*x*(2 + 3*x^2 - 2^(3/4)*(2 + 3*x^2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/
2, (-3*x^2)/2]))/(15*(-2 - 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {262, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{4}{15}\int \frac{1}{\sqrt[4]{-3x^2-2}}dx-\frac{2}{15}\left(-3x^2-2\right)^{3/4}x$$

$$\Big\downarrow \text{228}$$

$$\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\sqrt{-x^2}\int \frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

$$\Big\downarrow \text{834}$$

$$\frac{8\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

$$\Big\downarrow \text{761}$$

$$\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{15\sqrt{3}x}-\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

$$\Big\downarrow \text{1510}$$

$$\frac{8\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right.}{\sqrt{-x}}\right.}{15\sqrt{3}x}$$

$$\frac{2}{15}x\left(-3x^2-2\right)^{3/4}$$

input
```
Int[x^2/(-2 - 3*x^2)^(1/4),x]
```

output
```
(-2*x*(-2 - 3*x^2)^(3/4))/15 + (8*Sqrt[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3*
x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] +
Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2
- 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2
 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2
)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[-x^2])))/(15*Sqrt[3]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], -\frac{3 x^{2}}{2}\right)}{6}$	23
risch	$\frac{2 x \left(3 x^{2}+2\right)}{15 \left(-3 x^{2}-2\right)^{\frac{1}{4}}}+\frac{2 (-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{15}$	41

input
```
int(x^2/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-1)^(3/4)*2^(3/4)*x^3*hypergeom([1/4,3/2],[5/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^2}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/45*(45*x*integral(16/45*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(3*x^
2 - 4)*(-3*x^2 - 2)^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.15

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2^{\frac{3}{4}} x^3 e^{-\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x**3*exp(-I*pi/4)*hyper((1/4, 3/2), (5/2,), 3*x**2*exp_polar(I*pi
)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^2 - 2)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^2}{\left(-3\,x^2-2\right)^{1/4}}\,dx$$

input
```
int(x^2/(- 3*x^2 - 2)^(1/4),x)
```

output
```
int(x^2/(- 3*x^2 - 2)^(1/4), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{x^2}{\left(-3x^2-2\right)^{\frac{1}{4}}}dx$$

input
```
int(x^2/(-3*x^2-2)^(1/4),x)
```

output
```
int(x**2/( - 3*x**2 - 2)**(1/4),x)
```

3.965 $\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 202

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2x\sqrt[4]{-2-3x^2}}{\sqrt{2}+\sqrt{-2-3x^2}} + \frac{2\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{\sqrt{3}x} - \frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt{3}x}$$

output
```
2*x*(-3*x^2-2)^(1/4)/(2^(1/2)+(-3*x^2-2)^(1/2))+2/3*2^(1/4)*(-x^2/(2^(1/2)
+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*EllipticE(sin(2*arc
tan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x-1/3*2^(1/4)*(-x^
2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJa
cobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.73 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx=\frac{x\sqrt[4]{1+\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{3x^2}{2}\right)}{\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[(-2 - 3*x^2)^(-1/4),x]
```

output
```
(x*(1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, (-3*x^2)/2])/(-2
 - 3*x^2)^(1/4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{228}$$

$$-\frac{\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\sqrt{-x^2}\int \frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{\sqrt{3}x}$$

$$\Big\downarrow \text{834}$$

$$-\frac{2\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{\sqrt{3}x}$$

$\big\downarrow$ 27

$$-\frac{2\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{\sqrt{3}x}$$

$\big\downarrow$ 761

$$-\frac{2\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{\sqrt{3}x}$$

$\big\downarrow$ 1510

$$-\frac{2\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\right.}{\sqrt{-}}\right.}{\sqrt{3}x}$$

input
```
Int[(-2 - 3*x^2)^(-1/4),x]
```

output
```
(-2*Sqrt[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2
 - 3*x^2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2]
 + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])
/Sqrt[-x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt
[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/
4)*Sqrt[-x^2])))/(Sqrt[3]*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.10

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{2}$	21

input
```
int(1/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1)^(3/4)*2^(3/4)*x*hypergeom([1/4,1/2],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/3*(3*x*integral(-4/3*(-3*x^2 - 2)^(3/4)/(3*x^4 + 2*x^2), x) - 2*(-3*x^2
- 2)^(3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.16

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2^{\frac{3}{4}}xe^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\, \frac{3x^2e^{i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*x*exp(-I*pi/4)*hyper((1/4, 1/2), (3/2,), 3*x**2*exp_polar(I*pi)/2
)/2
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.17

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \frac{2^{3/4}\,x\,(3\,x^2+2)^{1/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2\,(-3\,x^2-2)^{1/4}}$$

input
```
int(1/(- 3*x^2 - 2)^(1/4),x)
```

output
```
(2^(3/4)*x*(3*x^2 + 2)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(3*x^2)/2))/(2*(-
 3*x^2 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3x^2-2)^{\frac{1}{4}}}dx$$

input
```
int(1/(-3*x^2-2)^(1/4),x)
```

output
```
int(1/( - 3*x**2 - 2)**(1/4),x)
```

3.966 $\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 223

$$\begin{aligned}\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx &= \frac{\left(-2-3x^2\right)^{3/4}}{2x} + \frac{3x\sqrt[4]{-2-3x^2}}{2\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} \\ &+ \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right) E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{2^{3/4}x} \\ &- \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right) \operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2\,2^{3/4}x}\end{aligned}$$

output
```
1/2*(-3*x^2-2)^(3/4)/x+3*x*(-3*x^2-2)^(1/4)/(2*2^(1/2)+2*(-3*x^2-2)^(1/2))
+1/2*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)
^(1/2))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))
*3^(1/2)/x-1/4*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+
(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1
/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.77 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.21

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = -\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{4},\frac{1}{2},-\frac{3x^2}{2}\right)}{x\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

output
```
-(((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-1/2, 1/4, 1/2, (-3*x^2)/2])/(x
*(-2 - 3*x^2)^(1/4)))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{4}\int \frac{1}{\sqrt[4]{-3x^2-2}}dx+\frac{\left(-3x^2-2\right)^{3/4}}{2x}$$

$$\Big\downarrow \text{228}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\int \frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}$$

$$\Big\downarrow \text{834}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{2x}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{2x}$$

$$\Big\downarrow \text{761}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{2x}$$

$$\Big\downarrow \text{1510}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\frac{\sqrt[4]{2}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\right.}{\sqrt{-}}\right.}{2x}$$

input
```
Int[1/(x^2*(-2 - 3*x^2)^(1/4)),x]
```

output
```
(-2 - 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 -
3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2]
+ Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-
2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[
-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x
^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[-x^2])))/(2*x)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.10

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right],\left[\frac{1}{2}\right],-\frac{3 x^{2}}{2}\right)}{2 x}$	23
risch	$-\frac{3 x^{2}+2}{2 x\left(-3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{3(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{8}$	43

input
```
int(1/x^2/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1)^(3/4)*2^(3/4)/x*hypergeom([-1/2,1/4],[1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx=\int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}}x^{2}}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/2*(2*x*integral(-3/4*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(
3/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2 \sqrt[4]{-2-3x^2}}\,dx=\frac{2^{\frac{3}{4}}e^{\frac{3i\pi}{4}}\,{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^{2}e^{i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-1/2, 1/4), (1/2,), 3*x**2*exp_polar(I*pi)/2
)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = -\frac{2\,3^{3/4}\,\left(\frac{2}{x^2}+3\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{2}{3\,x^2}\right)}{9\,x\,\left(-3\,x^2-2\right)^{1/4}}$$

input
```
int(1/(x^2*(- 3*x^2 - 2)^(1/4)),x)
```

output
```
-(2*3^(3/4)*(2/x^2 + 3)^(1/4)*hypergeom([1/4, 3/4], 7/4, -2/(3*x^2)))/(9*x
*(- 3*x^2 - 2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3x^2-2\right)^{\frac{1}{4}}x^2}dx$$

input
```
int(1/x^2/(-3*x^2-2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(1/4)*x**2),x)
```

3.967 $\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 244

$$\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx = \frac{(-2-3x^2)^{3/4}}{6x^3} - \frac{3(-2-3x^2)^{3/4}}{8x} - \frac{9x\sqrt[4]{-2-3x^2}}{8\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} - \frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{4\ 2^{3/4}x} + \frac{3\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{8\ 2^{3/4}x}$$

output

```
1/6*(-3*x^2-2)^(3/4)/x^3-3/8*(-3*x^2-2)^(3/4)/x-9*x*(-3*x^2-2)^(1/4)/(8*2^
(1/2)+8*(-3*x^2-2)^(1/2))-3/8*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^
(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*EllipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/
4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x+3/16*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)
^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(
-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.20

$$\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx = -\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{4},-\frac{1}{2},-\frac{3x^2}{2}\right)}{3x^3\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

output
```
-1/3*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-3/2, 1/4, -1/2, (-3*x^2)/2]
)/(x^3*(-2 - 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {264, 264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{3}{4}\int \frac{1}{\sqrt[4]{-3x^2-2}}dx+\frac{\left(-3x^2-2\right)^{3/4}}{2x}\right)$$

$$\Big\downarrow \text{228}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int \frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\int \frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}\right)$$

$$\Big\downarrow \ 834$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{2x}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int \frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int \frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right)}{2x}\right)$$

$$\Big\downarrow \ 761$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}}-\int \frac{\sqrt{2}-\sqrt{-3}}{\sqrt{3}\sqrt{-}}\right.}{2x}\right.$$

$$\Big\downarrow \ 1510$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{6x^3} - \frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x} - \frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2^{3/4}\sqrt{-x^2}} - \frac{\sqrt[4]{2}\sqrt{-\frac{}{(\sqrt{}}}\cdots}{}\right.}{2x}\cdots\right.$$

input
```
Int[1/(x^4*(-2 - 3*x^2)^(1/4)),x]
```

output
```
(-2 - 3*x^2)^(3/4)/(6*x^3) - (3*((-2 - 3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[
-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3*x^2)^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]
) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-
2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-x^
2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x
^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[-
x^2])))/(2*x)))/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[-\frac{1}{2}\right],-\frac{3x^2}{2}\right)}{6x^3}$	23
risch	$\frac{27x^4+6x^2-8}{24x^3\left(-3x^2-2\right)^{\frac{1}{4}}}+\frac{9(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3x^2}{2}\right)}{32}$	48

input
```
int(1/x^4/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-1)^(3/4)*2^(3/4)/x^3*hypergeom([-3/2,1/4],[-1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/24*(24*x^3*integral(9/16*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) - (9*x^2 - 4
)*(-3*x^2 - 2)^(3/4))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx = \frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2 e^{i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-3/2, 1/4), (-1/2,), 3*x**2*exp_polar(I*pi)/
2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 \sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{1}{4}} x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{x^4\,(-3\,x^2-2)^{1/4}}\,dx$$

input
```
int(1/(x^4*(- 3*x^2 - 2)^(1/4)),x)
```

output
```
int(1/(x^4*(- 3*x^2 - 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3x^2-2)^{\frac{1}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-3*x^2-2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(1/4)*x**4),x)
```

3.968 $\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 262

$$\begin{aligned}\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}} \, dx &= \frac{(-2-3x^2)^{3/4}}{10x^5} - \frac{7(-2-3x^2)^{3/4}}{40x^3} + \frac{63(-2-3x^2)^{3/4}}{160x} + \frac{189x\sqrt[4]{-2-3x^2}}{160\left(\sqrt{2}+\sqrt{-2-3x^2}\right)} \\ &+ \frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)E\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right)|\frac{1}{2}\right)}{80\ 2^{3/4}x} \\ &- \frac{63\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{160\ 2^{3/4}x}\end{aligned}$$

output

```
1/10*(-3*x^2-2)^(3/4)/x^5-7/40*(-3*x^2-2)^(3/4)/x^3+63/160*(-3*x^2-2)^(3/4
)/x+189*x*(-3*x^2-2)^(1/4)/(160*2^(1/2)+160*(-3*x^2-2)^(1/2))+63/160*2^(1/
4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*El
lipticE(sin(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4))),1/2*2^(1/2))*3^(1/2)/x
-63/320*2^(1/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2
-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1
/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx = -\frac{\sqrt[4]{1+\frac{3x^2}{2}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{4},-\frac{3}{2},-\frac{3x^2}{2}\right)}{5x^5\sqrt[4]{-2-3x^2}}$$

input
```
Integrate[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

output
```
-1/5*((1 + (3*x^2)/2)^(1/4)*Hypergeometric2F1[-5/2, 1/4, -3/2, (-3*x^2)/2]
)/(x^5*(-2 - 3*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {264, 264, 264, 228, 27, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-\frac{21}{20}\int \frac{1}{x^4\sqrt[4]{-3x^2-2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\int \frac{1}{x^2\sqrt[4]{-3x^2-2}}dx\right)$$

$$\Big\downarrow \text{ 264}$$

$$\frac{(-3x^2-2)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{(-3x^2-2)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{3}{4}\int\frac{1}{\sqrt[4]{-3x^2-2}}dx+\frac{(-3x^2-2)^{3/4}}{2x}\right)\right)$$

$$\Big\downarrow \; 228$$

$$\frac{(-3x^2-2)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{(-3x^2-2)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{(-3x^2-2)^{3/4}}{2x}-\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int\frac{\sqrt{\frac{2}{3}}\sqrt{-3x^2-2}}{\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}\right)\right)$$

$$\Big\downarrow \; 27$$

$$\frac{(-3x^2-2)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{(-3x^2-2)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{(-3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\int\frac{\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}}{2x}\right)\right)$$

$$\Big\downarrow \; 834$$

$$\frac{(-3x^2-2)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{(-3x^2-2)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{(-3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\sqrt{2}\int\frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{6}\sqrt{-x^2}}d\sqrt[4]{-3x^2}\right.}{2x}\right.\right.$$

$$\Big\downarrow \; 27$$

$$\frac{(-3x^2-2)^{3/4}}{10x^5}-\frac{21}{20}\left(\frac{(-3x^2-2)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{(-3x^2-2)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\sqrt{2}\int\frac{1}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}-\int\frac{\sqrt{2}-\sqrt{-3x^2-2}}{\sqrt{3}\sqrt{-x^2}}d\sqrt[4]{-3x^2-2}\right.}{2x}\right.\right.$$

$$\Big\downarrow \; 761$$

$$\frac{21}{20}\left(\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2}}{\sqrt[4]{2}}\right.\right.}{2^{3/4}\sqrt{-x^2}}\right.}{2x}\right.\right.$$

$$\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-$$

$\downarrow$ 1510

$$\frac{\left(-3x^2-2\right)^{3/4}}{10x^5}-$$

$$\frac{21}{20}\left(\frac{\left(-3x^2-2\right)^{3/4}}{6x^3}-\frac{3}{4}\left(\frac{\left(-3x^2-2\right)^{3/4}}{2x}-\frac{\sqrt{3}\sqrt{-x^2}\left(\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2}}{\sqrt[4]{2}}\right.\right.}{2^{3/4}\sqrt{-x^2}}\right.}{}\right.\right.$$

input

```
Int[1/(x^6*(-2 - 3*x^2)^(1/4)),x]
```

output

```
(-2 - 3*x^2)^(3/4)/(10*x^5) - (21*((-2 - 3*x^2)^(3/4)/(6*x^3) - (3*((-2 -
3*x^2)^(3/4)/(2*x) - (Sqrt[3]*Sqrt[-x^2]*((Sqrt[3]*Sqrt[-x^2]*(-2 - 3*x^2)
^(1/4))/(Sqrt[2] + Sqrt[-2 - 3*x^2]) - (2^(1/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt
[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticE[2*ArcTan[(-2 - 3*
x^2)^(1/4)/2^(1/4)], 1/2])/Sqrt[-x^2] + (Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3
*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1
/4)/2^(1/4)], 1/2])/(2^(3/4)*Sqrt[-x^2])))/(2*x)))/4))/20
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 228
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[x^2/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; Fre
eQ[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.09

method	result	size
meijerg	$\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{1}{4}\right],\left[-\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{10 x^{5}}$	23
risch	$-\frac{189 x^{6}+42 x^{4}-8 x^{2}+32}{160 x^{5}\left(-3 x^{2}-2\right)^{\frac{1}{4}}}-\frac{189(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right],\left[\frac{3}{2}\right],-\frac{3 x^{2}}{2}\right)}{640}$	53

input
```
int(1/x^6/(-3*x^2-2)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*(-1)^(3/4)*2^(3/4)/x^5*hypergeom([-5/2,1/4],[-3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^{2}-2\right)^{\frac{1}{4}} x^{6}}\,dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="fricas")
```

output
```
1/160*(160*x^5*integral(-189/320*(-3*x^2 - 2)^(3/4)/(3*x^2 + 2), x) + (63*
x^4 - 28*x^2 + 16)*(-3*x^2 - 2)^(3/4))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}}\,dx = \frac{2^{\frac{3}{4}} e^{\frac{3i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{1}{4} \\ -\frac{3}{2} \end{matrix}\middle| {\frac{3x^{2}e^{i\pi}}{2}} \right)}{10x^{5}}$$

input
```
integrate(1/x**6/(-3*x**2-2)**(1/4),x)
```

output
```
2**(3/4)*exp(3*I*pi/4)*hyper((-5/2, 1/4), (-3/2,), 3*x**2*exp_polar(I*pi)/
2)/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{1}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(1/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{x^6\,{\left(-3\,x^2-2\right)}^{1/4}}\,dx$$

input
```
int(1/(x^6*(- 3*x^2 - 2)^(1/4)),x)
```

output
```
int(1/(x^6*(- 3*x^2 - 2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[4]{-2-3x^2}}\,dx = \int \frac{1}{\left(-3x^2-2\right)^{\frac{1}{4}} x^6}dx$$

input
```
int(1/x^6/(-3*x^2-2)^(1/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(1/4)*x**6),x)
```

3.969 $\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 138

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \frac{160x\sqrt[4]{-2+3x^2}}{2079} + \frac{40}{693}x^3\sqrt[4]{-2+3x^2} + \frac{2}{33}x^5\sqrt[4]{-2+3x^2} + \frac{160\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}x}$$

output
```
160/2079*x*(3*x^2-2)^(1/4)+40/693*x^3*(3*x^2-2)^(1/4)+2/33*x^5*(3*x^2-2)^(
1/4)+160/6237*2^(3/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*
x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^
(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \frac{2x\left(-160+120x^2+54x^4+189x^6+80\sqrt[4]{2}(2-3x^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2}\right.\right.}{2079\,(-2+3x^2)^{3/4}}$$

input
```
Integrate[x^6/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*(-160 + 120*x^2 + 54*x^4 + 189*x^6 + 80*2^(1/4)*(2 - 3*x^2)^(3/4)*Hyp
ergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2]))/(2079*(-2 + 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {262, 262, 262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(3x^2-2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\int \frac{x^4}{(3x^2-2)^{3/4}}dx+\frac{2}{33}\sqrt[4]{3x^2-2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\left(\frac{4}{7}\int \frac{x^2}{(3x^2-2)^{3/4}}dx+\frac{2}{21}\sqrt[4]{3x^2-2}x^3\right)+\frac{2}{33}\sqrt[4]{3x^2-2}x^5$$

$$\Big\downarrow \text{262}$$

$$\frac{20}{33}\left(\frac{4}{7}\left(\frac{4}{9}\int \frac{1}{(3x^2-2)^{3/4}}dx+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\sqrt[4]{3x^2-2}x^3\right)+\frac{2}{33}\sqrt[4]{3x^2-2}x^5$$

$$\Big\downarrow \text{232}$$

$$\frac{20}{33}\left(\frac{4}{7}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{9x}+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\sqrt[4]{3x^2-2}x^3\right)+\frac{2}{33}\sqrt[4]{3x^2-2}x^5$$

$$\Big\downarrow \text{761}$$

$$\frac{20}{33}\left(\frac{4}{7}\left(\frac{2\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\right.$$

$$\frac{2}{33}\sqrt[4]{3x^2-2}x^5$$

input
```
Int[x^6/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x^5*(-2 + 3*x^2)^(1/4))/33 + (20*((2*x^3*(-2 + 3*x^2)^(1/4))/21 + (4*((
2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2
])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2
^(1/4)], 1/2])/(9*Sqrt[3]*x)))/7))/33
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{7}{2}\right], \left[\frac{9}{2}\right], \frac{3 x^{2}}{2}\right)}{14 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}}}$	42
risch	$\frac{2 x \left(63 x^{4}+60 x^{2}+80\right) \left(3 x^{2}-2\right)^{\frac{1}{4}}}{2079}+\frac{160\, 2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{2079 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}}}$	65

input
```
int(x^6/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/14*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^7*hype
rgeom([3/4,7/2],[9/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\, dx = \int \frac{x^{6}}{\left(3\, x^{2} - 2\right)^{\frac{3}{4}}}\, dx$$

input
```
integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^6/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.22

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^7e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{14}$$

input
```
integrate(x**6/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2/2)/14
```

Maxima [F]

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^6/(3*x^2 - 2)^(3/4),x)
```

output
```
int(x^6/(3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^6}{(3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(3*x^2-2)^(3/4),x)
```

output
```
int(x**6/(3*x**2 - 2)**(3/4),x)
```

3.970 $\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 120

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \frac{8}{63}x\sqrt[4]{-2+3x^2}+\frac{2}{21}x^3\sqrt[4]{-2+3x^2} + \frac{8\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}x}$$

output
```
8/63*x*(3*x^2-2)^(1/4)+2/21*x^3*(3*x^2-2)^(1/4)+8/189*2^(3/4)*(x^2/(2^(1/2
)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*ar
ctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \frac{2x\left(-8+6x^2+9x^4+4\sqrt[4]{2}(2-3x^2)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{63\left(-2+3x^2\right)^{3/4}}$$

input
```
Integrate[x^4/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*(-8 + 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1
/2, 3/4, 3/2, (3*x^2)/2]))/(63*(-2 + 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(3x^2-2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{7}\int \frac{x^2}{(3x^2-2)^{3/4}}dx+\frac{2}{21}\sqrt[4]{3x^2-2}x^3$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{7}\left(\frac{4}{9}\int \frac{1}{(3x^2-2)^{3/4}}dx+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\sqrt[4]{3x^2-2}x^3$$

$$\Big\downarrow \text{232}$$

$$\frac{4}{7}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{9x}+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\sqrt[4]{3x^2-2}x^3$$

$$\Big\downarrow \text{761}$$

$$\frac{4}{7}\left(\frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}+\frac{2}{9}\sqrt[4]{3x^2-2}x\right)+\frac{2}{21}\sqrt[4]{3x^2-2}x^3$$

input
```
Int[x^4/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x^3*(-2 + 3*x^2)^(1/4))/21 + (4*((2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4
)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*El
lipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)))/7
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.35

method	result	size
meijerg	$\frac{2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right], \left[\frac{7}{2}\right], \frac{3 x^{2}}{2}\right)}{10 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}}}$	42
risch	$\frac{2 x \left(3 x^{2}+4\right) \left(3 x^{2}-2\right)^{\frac{1}{4}}}{63}+\frac{8 2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{63 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}}}$	60

input
```
int(x^4/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^5*hype
rgeom([3/4,5/2],[7/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^{4}}{\left(3\,x^{2}-2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^4/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.26

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^{5}e^{-\frac{3i\pi}{4}}{}_{2}F_{1}\left(\left.\begin{matrix}\frac{3}{4}, \frac{5}{2}\\\frac{7}{2}\end{matrix}\right|\frac{3x^{2}}{2}\right)}{10}$$

input
```
integrate(x**4/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2/2)/10
```

Maxima [F]

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^4/(3*x^2 - 2)^(3/4),x)
```

output
```
int(x^4/(3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^4}{(3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(3*x^2-2)^(3/4),x)
```

output
```
int(x**4/(3*x**2 - 2)**(3/4),x)
```

3.971 $\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 102

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \frac{2}{9}x\sqrt[4]{-2+3x^2} + \frac{2\,2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}$$

output
```
2/9*x*(3*x^2-2)^(1/4)+2/27*2^(3/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)
*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^
(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \frac{2x\left(-2+3x^2+\sqrt[4]{2}(2-3x^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{3x^2}{2}\right)\right)}{9\,(-2+3x^2)^{3/4}}$$

input
```
Integrate[x^2/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*(-2 + 3*x^2 + 2^(1/4)*(2 - 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3
/2, (3*x^2)/2]))/(9*(-2 + 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(3x^2-2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{4}{9}\int \frac{1}{(3x^2-2)^{3/4}}dx+\frac{2}{9}\sqrt[4]{3x^2-2}x$$

$$\Big\downarrow \text{232}$$

$$\frac{4\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{9x}+\frac{2}{9}\sqrt[4]{3x^2-2}x$$

$$\Big\downarrow \text{761}$$

$$\frac{2\ 2^{3/4}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}+\frac{2}{9}\sqrt[4]{3x^2-2}x$$

input
```
Int[x^2/(-2 + 3*x^2)^(3/4),x]
```

output
```
(2*x*(-2 + 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^
2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/
2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.41

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{5}{2}\right],\frac{3x^2}{2}\right)}{6\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	42
risch	$\frac{2x\left(3x^2-2\right)^{\frac{1}{4}}}{9}+\frac{2\,2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{9\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	53

input
```
int(x^2/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x^3*hyper
geom([3/4,3/2],[5/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral(x^2/(3*x^2 - 2)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.30

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^3e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{6}$$

input
```
integrate(x**2/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2/2)/6
```

Maxima [F]

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^2/(3*x^2 - 2)^(3/4),x)
```

output
```
int(x^2/(3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(-2+3x^2)^{3/4}}\,dx = \int \frac{x^2}{(3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(3*x^2-2)^(3/4),x)
```

output
```
int(x**2/(3*x**2 - 2)**(3/4),x)
```

3.972 $\int \frac{1}{(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 82

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx=\frac{\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

output
```
1/6*2^(3/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/
2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(
1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.52

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx=\frac{x\left(1-\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\frac{3x^2}{2}\right)}{(-2+3x^2)^{3/4}}$$

input
```
Integrate[(-2 + 3*x^2)^(-3/4),x]
```

output
```
(x*(1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (3*x^2)/2])/(-2
+ 3*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2)^{3/4}}dx$$
$$\Big\downarrow \text{232}$$
$$\frac{\sqrt{\frac{2}{3}}\sqrt{x^2}\int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{x}$$
$$\Big\downarrow \text{761}$$
$$\frac{\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

input
```
Int[(-2 + 3*x^2)^(-3/4),x]
```

output
```
(Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*Ell
ipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x)
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

method	result	size
meijerg	$\frac{2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{2\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	40

input
```
int(1/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)*x*hyperge
om([1/2,3/4],[3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(-3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.35

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}xe^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{3x^2}{2}\right)}{2}$$

input
```
integrate(1/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2/2)/2
```

Maxima [F]

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.41

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \frac{2^{1/4}\,x\,(2-3\,x^2)^{3/4}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ \frac{3\,x^2}{2}\right)}{2\,(3\,x^2-2)^{3/4}}$$

input
```
int(1/(3*x^2 - 2)^(3/4),x)
```

output
```
(2^(1/4)*x*(2 - 3*x^2)^(3/4)*hypergeom([1/2, 3/4], 3/2, (3*x^2)/2))/(2*(3*
x^2 - 2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{(-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(1/(3*x^2-2)^(3/4),x)
```

output
```
int(1/(3*x**2 - 2)**(3/4),x)
```

3.973 $\int \frac{1}{x^2(-2+3x^2)^{3/4}} dx$

Optimal result

Integrand size = 15, antiderivative size = 104

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}} dx = \frac{\sqrt[4]{-2+3x^2}}{2x} + \frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right) \text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right), \frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

output
```
1/2*(3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*
(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(
3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}} dx = -\frac{\left(1-\frac{3x^2}{2}\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{3x^2}{2}\right)}{x(-2+3x^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

output
```
-(((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (3*x^2)/2])/(x*
(-2 + 3*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(3x^2-2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{3}{4} \int \frac{1}{\left(3x^2-2\right)^{3/4}} dx + \frac{\sqrt[4]{3x^2-2}}{2x}$$

$$\Big\downarrow \text{ 232}$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{x^2} \int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}} d\sqrt[4]{3x^2-2}}{2x} + \frac{\sqrt[4]{3x^2-2}}{2x}$$

$$\Big\downarrow \text{ 761}$$

$$\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}} \left(\sqrt{3x^2-2}+\sqrt{2}\right) \text{EllipticF}\left(2 \arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right), \frac{1}{2}\right)}{4\sqrt[4]{2}x} + \frac{\sqrt[4]{3x^2-2}}{2x}$$

input
```
Int[1/(x^2*(-2 + 3*x^2)^(3/4)),x]
```

output
```
(-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^
2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1
/4)], 1/2])/(4*2^(1/4)*x)
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.40

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}\operatorname{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{1}{2}\right],\frac{3x^2}{2}\right)}{2\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}x}$	42
risch	$\frac{\left(3x^2-2\right)^{\frac{1}{4}}}{2x}+\frac{3\,2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}}x\operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{8\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	55

input
```
int(1/x^2/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x*hyperg
eom([-1/2,3/4],[1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2 (-2 + 3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2 - 2)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(1/4)/(3*x^4 - 2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.28

$$\int \frac{1}{x^2 (-2 + 3x^2)^{3/4}} \, dx = \frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{3}{4} \\ \frac{1}{2} \end{matrix} \middle| \frac{3x^2}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2/2)/(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (-2 + 3x^2)^{3/4}} \, dx = \int \frac{1}{(3\,x^2 - 2)^{\frac{3}{4}} x^2} \, dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.22

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}}\,dx = -\frac{2\,3^{1/4}\left(\frac{1}{x^2}\right)^{3/4}{}_2\mathrm{F}_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ \frac{2}{3\,x^2}\right)}{15\,x}$$

input
```
int(1/(x^2*(3*x^2 - 2)^(3/4)),x)
```

output
```
-(2*3^(1/4)*(1/x^2)^(3/4)*hypergeom([3/4, 5/4], 9/4, 2/(3*x^2)))/(15*x)
```

Reduce [F]

$$\int \frac{1}{x^2 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/x^2/(3*x^2-2)^(3/4),x)
```

output
```
int(1/((3*x**2 - 2)**(3/4)*x**2),x)
```

3.974 $\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 122

$$\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-2+3x^2}}{6x^3}+\frac{5\sqrt[4]{-2+3x^2}}{8x} + \frac{5\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}x}$$

output
```
1/6*(3*x^2-2)^(1/4)/x^3+5/8*(3*x^2-2)^(1/4)/x+5/32*2^(3/4)*(x^2/(2^(1/2)+(
3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^(1/2))*InverseJacobiAM(2*arcta
n(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4(-2+3x^2)^{3/4}}\,dx = -\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},\frac{3x^2}{2}\right)}{3x^3(-2+3x^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

output
```
-1/3*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (3*x^2)/2])
/(x^3*(-2 + 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {264, 264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (3x^2-2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{4} \int \frac{1}{x^2 (3x^2-2)^{3/4}}dx + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{4} \left(\frac{3}{4} \int \frac{1}{(3x^2-2)^{3/4}}dx + \frac{\sqrt[4]{3x^2-2}}{2x} \right) + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

$$\Big\downarrow \text{232}$$

$$\frac{5}{4} \left(\frac{\sqrt{\frac{3}{2}} \sqrt{x^2} \int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{2x} + \frac{\sqrt[4]{3x^2-2}}{2x} \right) + \frac{\sqrt[4]{3x^2-2}}{6x^3}$$

$$\Big\downarrow \text{761}$$

$$\frac{5}{4}\left(\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{3x^2-2}}{2x}\right)+\frac{\sqrt[4]{3x^2-2}}{6x^3}$$

input
```
Int[1/(x^4*(-2 + 3*x^2)^(3/4)),x]
```

output
```
(-2 + 3*x^2)^(1/4)/(6*x^3) + (5*((-2 + 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[
x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sqrt[2] + Sqrt[-2 + 3*x^2])*EllipticF
[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(1/4)*x)))/4
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.34

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[-\frac{1}{2}\right],\frac{3x^2}{2}\right)}{6 \operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}} x^3}$	42
risch	$\frac{45x^4-18x^2-8}{24x^3(3x^2-2)^{\frac{3}{4}}}+\frac{15\, 2^{\frac{1}{4}}\left(-\operatorname{signum}\left(-1+\frac{3x^2}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{3}{2}\right],\frac{3x^2}{2}\right)}{32 \operatorname{signum}\left(-1+\frac{3x^2}{2}\right)^{\frac{3}{4}}}$	67

input
```
int(1/x^4/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x^3*hype
rgeom([-3/2,3/4],[-1/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\, dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}} x^4}\, dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(1/4)/(3*x^6 - 2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\, dx = \frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_{2}F_{1}\left(\left.\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\right| \frac{3x^2}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2/2)/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{x^4\,(3\,x^2-2)^{3/4}}\,dx$$

input
```
int(1/(x^4*(3*x^2 - 2)^(3/4)),x)
```

output
```
int(1/(x^4*(3*x^2 - 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (-2+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2-2)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(3*x^2-2)^(3/4),x)
```

output
```
int(1/((3*x**2 - 2)**(3/4)*x**4),x)
```

3.975 $\int \frac{1}{x^6(-2+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 140

$$\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx=\frac{\sqrt[4]{-2+3x^2}}{10x^5}+\frac{9\sqrt[4]{-2+3x^2}}{40x^3}+\frac{27\sqrt[4]{-2+3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{2}+\sqrt{-2+3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2+3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2+3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}x}$$

output
```
1/10*(3*x^2-2)^(1/4)/x^5+9/40*(3*x^2-2)^(1/4)/x^3+27/32*(3*x^2-2)^(1/4)/x+
27/128*2^(3/4)*(x^2/(2^(1/2)+(3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(3*x^2-2)^
(1/2))*InverseJacobiAM(2*arctan(1/2*(3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*
3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6\left(-2+3x^2\right)^{3/4}}\,dx=-\frac{\left(1-\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},\frac{3x^2}{2}\right)}{5x^5\left(-2+3x^2\right)^{3/4}}$$

input
```
Integrate[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

output
```
-1/5*((1 - (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (3*x^2)/2])
/(x^5*(-2 + 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 264, 264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (3x^2-2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \int \frac{1}{x^4 (3x^2-2)^{3/4}}dx + \frac{\sqrt[4]{3x^2-2}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \left(\frac{5}{4} \int \frac{1}{x^2 (3x^2-2)^{3/4}}dx + \frac{\sqrt[4]{3x^2-2}}{6x^3}\right) + \frac{\sqrt[4]{3x^2-2}}{10x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{27}{20} \left(\frac{5}{4} \left(\frac{3}{4} \int \frac{1}{(3x^2-2)^{3/4}}dx + \frac{\sqrt[4]{3x^2-2}}{2x}\right) + \frac{\sqrt[4]{3x^2-2}}{6x^3}\right) + \frac{\sqrt[4]{3x^2-2}}{10x^5}$$

$$\Big\downarrow \text{232}$$

$$\frac{27}{20} \left(\frac{5}{4} \left(\frac{\sqrt{\frac{3}{2}}\sqrt{x^2} \int \frac{1}{\sqrt{\frac{1}{2}(3x^2-2)+1}}d\sqrt[4]{3x^2-2}}{2x} + \frac{\sqrt[4]{3x^2-2}}{2x}\right) + \frac{\sqrt[4]{3x^2-2}}{6x^3}\right) + \frac{\sqrt[4]{3x^2-2}}{10x^5}$$

$$\Big\downarrow \text{761}$$

$$\frac{27}{20}\left(\frac{5}{4}\left(\frac{\sqrt{3}\sqrt{\frac{x^2}{\left(\sqrt{3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{3x^2-2}}{2x}\right)+\frac{\sqrt[4]{3x^2-}}{6x^3}\right.$$

$$\frac{\sqrt[4]{3x^2-2}}{10x^5}$$

input
```
Int[1/(x^6*(-2 + 3*x^2)^(3/4)),x]
```

output
```
(-2 + 3*x^2)^(1/4)/(10*x^5) + (27*((-2 + 3*x^2)^(1/4)/(6*x^3) + (5*((-2 +
3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[x^2/(Sqrt[2] + Sqrt[-2 + 3*x^2])^2]*(Sq
rt[2] + Sqrt[-2 + 3*x^2])*EllipticF[2*ArcTan[(-2 + 3*x^2)^(1/4)/2^(1/4)],
1/2])/(4*2^(1/4)*x)))/4))/20
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.30

method	result	size
meijerg	$-\frac{2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right], \left[-\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{10 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}} x^{5}}$	42
risch	$\frac{405 x^{6}-162 x^{4}-24 x^{2}-32}{160 x^{5} \left(3 x^{2}-2\right)^{\frac{3}{4}}}+\frac{81\, 2^{\frac{1}{4}} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{\frac{3}{4}} x \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], \frac{3 x^{2}}{2}\right)}{128 \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{\frac{3}{4}}}$	72

input
```
int(1/x^6/(3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*2^(1/4)/signum(-1+3/2*x^2)^(3/4)*(-signum(-1+3/2*x^2))^(3/4)/x^5*hyp
ergeom([-5/2,3/4],[-3/2],3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(-2+3 x^2\right)^{3/4}}\, dx = \int \frac{1}{\left(3\, x^{2} - 2\right)^{\frac{3}{4}} x^{6}}\, dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^(1/4)/(3*x^8 - 2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.23

$$\int \frac{1}{x^6 \left(-2+3 x^2\right)^{3/4}}\, dx = \frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix}\middle| \frac{3x^{2}}{2} \right)}{10 x^{5}}$$

input
```
integrate(1/x**6/(3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2/2)/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(-2+3x^2\right)^{3/4}} \, dx = \int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(-2+3x^2\right)^{3/4}} \, dx = \int \frac{1}{\left(3\,x^2-2\right)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((3*x^2 - 2)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(-2+3x^2\right)^{3/4}}\,dx=\int \frac{1}{x^6\,{\left(3\,x^2-2\right)}^{3/4}} \,dx$$

input
```
int(1/(x^6*(3*x^2 - 2)^(3/4)),x)
```

output
```
int(1/(x^6*(3*x^2 - 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-2+3x^2\right)^{3/4}}\,dx=\int \frac{1}{\left(3x^{2}-2\right)^{\frac{3}{4}} x^{6}}dx$$

input
```
int(1/x^6/(3*x^2-2)^(3/4),x)
```

output
```
int(1/((3*x**2 - 2)**(3/4)*x**6),x)
```

3.976 $\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 139

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx=-\frac{160x\sqrt[4]{-2-3x^2}}{2079}+\frac{40}{693}x^3\sqrt[4]{-2-3x^2}-\frac{2}{33}x^5\sqrt[4]{-2-3x^2}+\frac{160\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{2079\sqrt{3}x}$$

output

```
-160/2079*x*(-3*x^2-2)^(1/4)+40/693*x^3*(-3*x^2-2)^(1/4)-2/33*x^5*(-3*x^2-
2)^(1/4)+160/6237*2^(3/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/
2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)
),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.49

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \frac{2x\left(160+120x^2-54x^4+189x^6-80\sqrt[4]{2}(2+3x^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},\right.\right.}{2079\,(-2-3x^2)^{3/4}}$$

input
```
Integrate[x^6/(-2 - 3*x^2)^(3/4),x]
```

output
```
(2*x*(160 + 120*x^2 - 54*x^4 + 189*x^6 - 80*2^(1/4)*(2 + 3*x^2)^(3/4)*Hype
rgeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2]))/(2079*(-2 - 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {262, 262, 262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(-3x^2-2)^{3/4}}dx$$

$$\Big\downarrow \ 262$$

$$-\frac{20}{33}\int \frac{x^4}{(-3x^2-2)^{3/4}}dx-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5$$

$$\Big\downarrow \ 262$$

$$-\frac{20}{33}\left(-\frac{4}{7}\int \frac{x^2}{(-3x^2-2)^{3/4}}dx-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3\right)-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5$$

$$\Big\downarrow \ 262$$

$$-\frac{20}{33}\left(-\frac{4}{7}\left(-\frac{4}{9}\int \frac{1}{(-3x^2-2)^{3/4}}dx-\frac{2}{9}\sqrt[4]{-3x^2-2}x\right)-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3\right)-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5$$

$\big\downarrow$ 232

$$-\frac{20}{33}\left(-\frac{4}{7}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}(-3x^2-2)+1}}d\sqrt[4]{-3x^2-2}}{9x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}\right)-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3\right)-\frac{2}{33}\sqrt[4]{-3x^2-2}x^5$$

$\big\downarrow$ 761

$$-\frac{20}{33}\left(-\frac{4}{7}\left(\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}-\frac{2}{9}x\sqrt[4]{-3x\right.\right.$$

$$\frac{2}{33}\sqrt[4]{-3x^2-2}x^5$$

input
```
Int[x^6/(-2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x^5*(-2 - 3*x^2)^(1/4))/33 - (20*((-2*x^3*(-2 - 3*x^2)^(1/4))/21 - (4*
((-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 -
3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(
1/4)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)))/7))/33
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.17

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x^{7} \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{7}{2}\right],\left[\frac{9}{2}\right],-\frac{3 x^{2}}{2}\right)}{14}$	23

input
```
int(x^6/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/14*(-1)^(1/4)*2^(1/4)*x^7*hypergeom([3/4,7/2],[9/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^{6}}{(-3\,x^{2}-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
-2/2079*(63*x^5 - 60*x^3 + 80*x)*(-3*x^2 - 2)^(1/4) + integral(320/2079*(-
3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.26

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^7e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{14}$$

input
```
integrate(x**6/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**7*exp(-3*I*pi/4)*hyper((3/4, 7/2), (9/2,), 3*x**2*exp_polar(I*
pi)/2)/14
```

Maxima [F]

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^6/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^6/(- 3*x^2 - 2)^(3/4),x)
```

output
```
int(x^6/(- 3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^6}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^6}{(-3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^6/(-3*x^2-2)^(3/4),x)
```

output
```
int(x**6/( - 3*x**2 - 2)**(3/4),x)
```

3.977 $\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 121

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \frac{8}{63}x\sqrt[4]{-2-3x^2}-\frac{2}{21}x^3\sqrt[4]{-2-3x^2} - \frac{8\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{63\sqrt{3}x}$$

output
```
8/63*x*(-3*x^2-2)^(1/4)-2/21*x^3*(-3*x^2-2)^(1/4)-8/189*2^(3/4)*(-x^2/(2^(
1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM
(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.52

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \frac{2x\left(-8-6x^2+9x^4+4\sqrt[4]{2}(2+3x^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{63\,(-2-3x^2)^{3/4}}$$

input
```
Integrate[x^4/(-2 - 3*x^2)^(3/4),x]
```

output
```
(2*x*(-8 - 6*x^2 + 9*x^4 + 4*2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1
/2, 3/4, 3/2, (-3*x^2)/2]))/(63*(-2 - 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(-3x^2-2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$-\frac{4}{7}\int \frac{x^2}{(-3x^2-2)^{3/4}}dx-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3$$

$$\Big\downarrow \text{262}$$

$$-\frac{4}{7}\left(-\frac{4}{9}\int \frac{1}{(-3x^2-2)^{3/4}}dx-\frac{2}{9}\sqrt[4]{-3x^2-2}x\right)-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3$$

$$\Big\downarrow \text{232}$$

$$-\frac{4}{7}\left(\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}(-3x^2-2)+1}}d\sqrt[4]{-3x^2-2}}{9x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}\right)-\frac{2}{21}\sqrt[4]{-3x^2-2}x^3$$

$$\Big\downarrow \text{761}$$

$$-\frac{4}{7}\left(\frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}-\frac{2}{9}x\sqrt[4]{-3x^2-2}\right)$$

$$\frac{2}{21}\sqrt[4]{-3x^2-2}x^3$$

input
```
Int[x^4/(-2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x^3*(-2 - 3*x^2)^(1/4))/21 - (4*((-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3
/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2
])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)))/7
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.19

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x^{5} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{5}{2}\right],\left[\frac{7}{2}\right],-\frac{3 x^{2}}{2}\right)}{10}$	23

input
```
int(x^4/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/10*(-1)^(1/4)*2^(1/4)*x^5*hypergeom([3/4,5/2],[7/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^{4}}{\left(-3\,x^{2}-2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
-2/63*(3*x^3 - 4*x)*(-3*x^2 - 2)^(1/4) + integral(-16/63*(-3*x^2 - 2)^(1/4
)/(3*x^2 + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.30

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^5e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{10}$$

input
```
integrate(x**4/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**5*exp(-3*I*pi/4)*hyper((3/4, 5/2), (7/2,), 3*x**2*exp_polar(I*
pi)/2)/10
```

Maxima [F]

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^4/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^4/(- 3*x^2 - 2)^(3/4),x)
```

output
```
int(x^4/(- 3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^4}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^4}{(-3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^4/(-3*x^2-2)^(3/4),x)
```

output
```
int(x**4/( - 3*x**2 - 2)**(3/4),x)
```

3.978 $\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 103

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = -\frac{2}{9}x\sqrt[4]{-2-3x^2} + \frac{2\,2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x}$$

output
```
-2/9*x*(-3*x^2-2)^(1/4)+2/27*2^(3/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(
1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1
/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.56

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \frac{2x\left(2+3x^2-\sqrt[4]{2}(2+3x^2)^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)\right)}{9\,(-2-3x^2)^{3/4}}$$

input
```
Integrate[x^2/(-2 - 3*x^2)^(3/4),x]
```

output
```
(2*x*(2 + 3*x^2 - 2^(1/4)*(2 + 3*x^2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/
2, (-3*x^2)/2]))/(9*(-2 - 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {262, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(-3x^2-2)^{3/4}} \, dx$$

$$\Big\downarrow \ 262$$

$$-\frac{4}{9}\int \frac{1}{(-3x^2-2)^{3/4}}dx - \frac{2}{9}\sqrt[4]{-3x^2-2}x$$

$$\Big\downarrow \ 232$$

$$\frac{4\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}(-3x^2-2)+1}}d\sqrt[4]{-3x^2-2}}{9x} - \frac{2}{9}x\sqrt[4]{-3x^2-2}$$

$$\Big\downarrow \ 761$$

$$\frac{2\ 2^{3/4}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{9\sqrt{3}x} - \frac{2}{9}x\sqrt[4]{-3x^2-2}$$

input
```
Int[x^2/(-2 - 3*x^2)^(3/4),x]
```

output
```
(-2*x*(-2 - 3*x^2)^(1/4))/9 + (2*2^(3/4)*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3
*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1
/4)/2^(1/4)], 1/2])/(9*Sqrt[3]*x)
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.22

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x^{3} \operatorname{hypergeom}\left(\left[\frac{3}{4}, \frac{3}{2}\right], \left[\frac{5}{2}\right], -\frac{3 x^{2}}{2}\right)}{6}$	23

input
```
int(x^2/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-1)^(1/4)*2^(1/4)*x^3*hypergeom([3/4,3/2],[5/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
-2/9*(-3*x^2 - 2)^(1/4)*x + integral(4/9*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x
)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.35

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}x^3e^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6}$$

input
```
integrate(x**2/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x**3*exp(-3*I*pi/4)*hyper((3/4, 3/2), (5/2,), 3*x**2*exp_polar(I*
pi)/2)/6
```

Maxima [F]

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

Giac [F]

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(x^2/(-3*x^2 - 2)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3\,x^2-2)^{3/4}}\,dx$$

input
```
int(x^2/(- 3*x^2 - 2)^(3/4),x)
```

output
```
int(x^2/(- 3*x^2 - 2)^(3/4), x)
```

Reduce [F]

$$\int \frac{x^2}{(-2-3x^2)^{3/4}}\,dx = \int \frac{x^2}{(-3x^2-2)^{\frac{3}{4}}}dx$$

input
```
int(x^2/(-3*x^2-2)^(3/4),x)
```

output
```
int(x**2/( - 3*x**2 - 2)**(3/4),x)
```

3.979 $\int \frac{1}{(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 84

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = -\frac{\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

output
```
-1/6*2^(3/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)
^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2)
)*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 4.69 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.51

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = \frac{x\left(1+\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{3}{2},-\frac{3x^2}{2}\right)}{(-2-3x^2)^{3/4}}$$

input
```
Integrate[(-2 - 3*x^2)^(-3/4),x]
```

output
```
(x*(1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[1/2, 3/4, 3/2, (-3*x^2)/2])/(-2
 - 3*x^2)^(3/4)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-2)^{3/4}}dx$$
$$\Big\downarrow \ 232$$
$$-\frac{\sqrt{\frac{2}{3}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}(-3x^2-2)+1}}d\sqrt[4]{-3x^2-2}}{x}$$
$$\Big\downarrow \ 761$$
$$-\frac{\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{\sqrt[4]{2}\sqrt{3}x}$$

input
```
Int[(-2 - 3*x^2)^(-3/4),x]
```

output
```
-((Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2]
)*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(2^(1/4)*Sqrt[3]*x
))
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.25

method	result	size
meijerg	$-\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x \,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{2}\right], -\frac{3 x^{2}}{2}\right)}{2}$	21

input
```
int(1/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1)^(1/4)*2^(1/4)*x*hypergeom([1/2,3/4],[3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}xe^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{3}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2}$$

input
```
integrate(1/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*x*exp(-3*I*pi/4)*hyper((1/2, 3/4), (3/2,), 3*x**2*exp_polar(I*pi)
/2)/2
```

Maxima [F]

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx=\frac{2^{1/4}\,x\,{\left(3\,x^2+2\right)}^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{3}{2};\ -\frac{3\,x^2}{2}\right)}{2\,{\left(-3\,x^2-2\right)}^{3/4}}$$

input
```
int(1/(- 3*x^2 - 2)^(3/4),x)
```

output
```
(2^(1/4)*x*(3*x^2 + 2)^(3/4)*hypergeom([1/2, 3/4], 3/2, -(3*x^2)/2))/(2*(-
 3*x^2 - 2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{(-2-3x^2)^{3/4}}\,dx=\int \frac{1}{\left(-3x^{2}-2\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(-3*x^2-2)^(3/4),x)
```

output
```
int(1/( - 3*x**2 - 2)**(3/4),x)
```

3.980 $\int \frac{1}{x^2(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{1}{x^2 (-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{2x} + \frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}$$

output
```
1/2*(-3*x^2-2)^(1/4)/x+1/8*2^(3/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/
2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4
)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 5.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.44

$$\int \frac{1}{x^2 (-2-3x^2)^{3/4}}\,dx = -\frac{\left(1+\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{3x^2}{2}\right)}{x(-2-3x^2)^{3/4}}$$

input
```
Integrate[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

output
```
-(((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, (-3*x^2)/2])/(x
*(-2 - 3*x^2)^(3/4)))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(-3 x^2-2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{2x}-\frac{3}{4}\int \frac{1}{\left(-3x^2-2\right)^{3/4}}dx$$

$$\Big\downarrow \text{232}$$

$$\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}\left(-3x^2-2\right)+1}}d\sqrt[4]{-3x^2-2}}{2x}+\frac{\sqrt[4]{-3x^2-2}}{2x}$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{-3x^2-2}}{2x}$$

input
```
Int[1/(x^2*(-2 - 3*x^2)^(3/4)),x]
```

output
```
(-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2]
)^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2
^(1/4)], 1/2])/(4*2^(1/4)*x)
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.22

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{1}{2}\right], -\frac{3x^2}{2}\right)}{2x}$	23

input
```
int(1/x^2/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-1)^(1/4)*2^(1/4)/x*hypergeom([-1/2,3/4],[1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
1/2*(2*x*integral(3/4*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (-3*x^2 - 2)^(1
/4))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.32

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2x}$$

input
```
integrate(1/x**2/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-1/2, 3/4), (1/2,), 3*x**2*exp_polar(I*pi)/2)/
(2*x)
```

Maxima [F]

$$\int \frac{1}{x^2\left(-2-3x^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-3\,x^2-2\right)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^2}\,dx$$

input
```
integrate(1/x^2/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^2 (-2-3x^2)^{3/4}}\,dx = -\frac{2\,3^{1/4}\left(\frac{2}{x^2}+3\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{2}{3\,x^2}\right)}{15\,x\,(-3\,x^2-2)^{3/4}}$$

input
```
int(1/(x^2*(- 3*x^2 - 2)^(3/4)),x)
```

output
```
-(2*3^(1/4)*(2/x^2 + 3)^(3/4)*hypergeom([3/4, 5/4], 9/4, -2/(3*x^2)))/(15*
x*(- 3*x^2 - 2)^(3/4))
```

Reduce [F]

$$\int \frac{1}{x^2 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2-2)^{\frac{3}{4}}\,x^2}dx$$

input
```
int(1/x^2/(-3*x^2-2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(3/4)*x**2),x)
```

3.981 $\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 123

$$\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{-2-3x^2}}{6x^3}-\frac{5\sqrt[4]{-2-3x^2}}{8x} - \frac{5\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{16\sqrt[4]{2}x}$$

output
```
1/6*(-3*x^2-2)^(1/4)/x^3-5/8*(-3*x^2-2)^(1/4)/x-5/32*2^(3/4)*(-x^2/(2^(1/2
)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x^2-2)^(1/2))*InverseJacobiAM(2*
arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.39

$$\int \frac{1}{x^4(-2-3x^2)^{3/4}}\,dx = -\frac{\left(1+\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{4},-\frac{1}{2},-\frac{3x^2}{2}\right)}{3x^3(-2-3x^2)^{3/4}}$$

input
```
Integrate[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

output
```
-1/3*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-3/2, 3/4, -1/2, (-3*x^2)/2]
)/(x^3*(-2 - 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {264, 264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \left(-3 x^2-2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2 \left(-3x^2-2\right)^{3/4}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\left(\frac{\sqrt[4]{-3x^2-2}}{2x}-\frac{3}{4}\int \frac{1}{\left(-3x^2-2\right)^{3/4}}dx\right)$$

$$\Big\downarrow \text{232}$$

$$\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\left(\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}\left(-3x^2-2\right)+1}}d\sqrt[4]{-3x^2-2}}{2x}+\frac{\sqrt[4]{-3x^2-2}}{2x}\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\left(\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x}+\frac{\sqrt[4]{-3x^2-2}}{2x}\right)$$

input
```
Int[1/(x^4*(-2 - 3*x^2)^(3/4)),x]
```

output
```
(-2 - 3*x^2)^(1/4)/(6*x^3) - (5*((-2 - 3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[
-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*(Sqrt[2] + Sqrt[-2 - 3*x^2])*Ellipt
icF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)], 1/2])/(4*2^(1/4)*x)))/4
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.19

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right],\left[-\frac{1}{2}\right],-\frac{3 x^{2}}{2}\right)}{6 x^{3}}$	23

input
```
int(1/x^4/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-1)^(1/4)*2^(1/4)/x^3*hypergeom([-3/2,3/4],[-1/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
1/24*(24*x^3*integral(-15/16*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) - (15*x^2
- 4)*(-3*x^2 - 2)^(1/4))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.30

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \frac{\sqrt[4]{2}e^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{4} \\ -\frac{1}{2} \end{matrix}\middle| \frac{3x^2e^{i\pi}}{2}\right)}{6x^3}$$

input
```
integrate(1/x**4/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-3/2, 3/4), (-1/2,), 3*x**2*exp_polar(I*pi)/2)
/(6*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^4}\,dx$$

input
```
integrate(1/x^4/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{x^4\,(-3\,x^2-2)^{3/4}}\,dx$$

input
```
int(1/(x^4*(- 3*x^2 - 2)^(3/4)),x)
```

output
```
int(1/(x^4*(- 3*x^2 - 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2-2)^{\frac{3}{4}}\,x^4}dx$$

input
```
int(1/x^4/(-3*x^2-2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(3/4)*x**4),x)
```

3.982 $\int \frac{1}{x^6(-2-3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 141

$$\int \frac{1}{x^6\left(-2-3x^2\right)^{3/4}}\,dx=\frac{\sqrt[4]{-2-3x^2}}{10x^5}-\frac{9\sqrt[4]{-2-3x^2}}{40x^3}+\frac{27\sqrt[4]{-2-3x^2}}{32x}+\frac{27\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{2}+\sqrt{-2-3x^2}\right)^2}}\left(\sqrt{2}+\sqrt{-2-3x^2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-2-3x^2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{64\sqrt[4]{2}x}$$

output
```
1/10*(-3*x^2-2)^(1/4)/x^5-9/40*(-3*x^2-2)^(1/4)/x^3+27/32*(-3*x^2-2)^(1/4)
/x+27/128*2^(3/4)*(-x^2/(2^(1/2)+(-3*x^2-2)^(1/2))^2)^(1/2)*(2^(1/2)+(-3*x
^2-2)^(1/2))*InverseJacobiAM(2*arctan(1/2*(-3*x^2-2)^(1/4)*2^(3/4)),1/2*2^
(1/2))*3^(1/2)/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.34

$$\int \frac{1}{x^6\left(-2-3x^2\right)^{3/4}}\,dx=-\frac{\left(1+\frac{3x^2}{2}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{4},-\frac{3}{2},-\frac{3x^2}{2}\right)}{5x^5\left(-2-3x^2\right)^{3/4}}$$

input
```
Integrate[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

output
```
-1/5*((1 + (3*x^2)/2)^(3/4)*Hypergeometric2F1[-5/2, 3/4, -3/2, (-3*x^2)/2]
)/(x^5*(-2 - 3*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 264, 264, 232, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6\left(-3x^2-2\right)^{3/4}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{10x^5}-\frac{27}{20}\int \frac{1}{x^4\left(-3x^2-2\right)^{3/4}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{10x^5}-\frac{27}{20}\left(\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\int \frac{1}{x^2\left(-3x^2-2\right)^{3/4}}dx\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{\sqrt[4]{-3x^2-2}}{10x^5}-\frac{27}{20}\left(\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\left(\frac{\sqrt[4]{-3x^2-2}}{2x}-\frac{3}{4}\int \frac{1}{\left(-3x^2-2\right)^{3/4}}dx\right)\right)$$

$$\Big\downarrow \text{232}$$

$$\frac{\sqrt[4]{-3x^2-2}}{10x^5}-\frac{27}{20}\left(\frac{\sqrt[4]{-3x^2-2}}{6x^3}-\frac{5}{4}\left(\frac{\sqrt{\frac{3}{2}}\sqrt{-x^2}\int \frac{1}{\sqrt{\frac{1}{2}\left(-3x^2-2\right)+1}}d\sqrt[4]{-3x^2-2}}{2x}+\frac{\sqrt[4]{-3x^2-2}}{2x}\right)\right)$$

$$\Big\downarrow \text{761}$$

$$\frac{\sqrt[4]{-3x^2-2}}{10x^5} - \frac{27}{20}\left(\frac{\sqrt[4]{-3x^2-2}}{6x^3} - \frac{5}{4}\left(\frac{\sqrt{3}\sqrt{-\frac{x^2}{\left(\sqrt{-3x^2-2}+\sqrt{2}\right)^2}}\left(\sqrt{-3x^2-2}+\sqrt{2}\right)\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{-3x^2-2}}{\sqrt[4]{2}}\right),\frac{1}{2}\right)}{4\sqrt[4]{2}x} + \right.\right.$$

input
```
Int[1/(x^6*(-2 - 3*x^2)^(3/4)),x]
```

output
```
(-2 - 3*x^2)^(1/4)/(10*x^5) - (27*((-2 - 3*x^2)^(1/4)/(6*x^3) - (5*((-2 -
3*x^2)^(1/4)/(2*x) + (Sqrt[3]*Sqrt[-(x^2/(Sqrt[2] + Sqrt[-2 - 3*x^2])^2)]*
(Sqrt[2] + Sqrt[-2 - 3*x^2])*EllipticF[2*ArcTan[(-2 - 3*x^2)^(1/4)/2^(1/4)
], 1/2])/(4*2^(1/4)*x)))/4))/20
```

Defintions of rubi rules used

rule 232
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[2*(Sqrt[(-b)*(x^2/a)]/(
b*x))   Subst[Int[1/Sqrt[1 - x^4/a], x], x, (a + b*x^2)^(1/4)], x] /; FreeQ
[{a, b}, x] && NegQ[a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.16

method	result	size
meijerg	$\frac{(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[-\frac{5}{2}, \frac{3}{4}\right], \left[-\frac{3}{2}\right], -\frac{3x^2}{2}\right)}{10x^5}$	23

input
```
int(1/x^6/(-3*x^2-2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/10*(-1)^(1/4)*2^(1/4)/x^5*hypergeom([-5/2,3/4],[-3/2],-3/2*x^2)
```

Fricas [F]

$$\int \frac{1}{x^6 (-2-3x^2)^{3/4}} \, dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}} x^6} \, dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="fricas")
```

output
```
1/160*(160*x^5*integral(81/64*(-3*x^2 - 2)^(1/4)/(3*x^2 + 2), x) + (135*x^
4 - 36*x^2 + 16)*(-3*x^2 - 2)^(1/4))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.26

$$\int \frac{1}{x^6 (-2-3x^2)^{3/4}} \, dx = \frac{\sqrt[4]{2} e^{\frac{i\pi}{4}} {}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{4} \\ -\frac{3}{2} \end{matrix} \middle| \frac{3x^2 e^{i\pi}}{2}\right)}{10x^5}$$

input
```
integrate(1/x**6/(-3*x**2-2)**(3/4),x)
```

output
```
2**(1/4)*exp(I*pi/4)*hyper((-5/2, 3/4), (-3/2,), 3*x**2*exp_polar(I*pi)/2)
/(10*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2)^{\frac{3}{4}}x^6}\,dx$$

input
```
integrate(1/x^6/(-3*x^2-2)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-3*x^2 - 2)^(3/4)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (-2-3x^2)^{3/4}}\,dx = \int \frac{1}{x^6\,(-3\,x^2-2)^{3/4}}\,dx$$

input
```
int(1/(x^6*(- 3*x^2 - 2)^(3/4)),x)
```

output
```
int(1/(x^6*(- 3*x^2 - 2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-2-3x^2\right)^{3/4}} \, dx = \int \frac{1}{\left(-3x^2-2\right)^{\frac{3}{4}} x^6} dx$$

input
```
int(1/x^6/(-3*x^2-2)^(3/4),x)
```

output
```
int(1/(( - 3*x**2 - 2)**(3/4)*x**6),x)
```

3.983 $\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 261

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx=-\frac{ac(cx)^{3/2}\sqrt[4]{a-bx^2}}{16b}+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}-\frac{3a^2c^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}}+\frac{3a^2c^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{32\sqrt{2}b^{7/4}}-\frac{3a^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{32\sqrt{2}b^{7/4}}$$

output
```
-1/16*a*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(-b*x^2+a)^(1/4)/
c+3/64*a^2*c^(5/2)*arctan(-1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a
)^(1/4))*2^(1/2)/b^(7/4)+3/64*a^2*c^(5/2)*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(
1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(7/4)-3/64*a^2*c^(5/2)*arctanh(2^
(1/2)*b^(1/4)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(1/2)+b^(1/2)*c^(1/2)*x/(-b*
x^2+a)^(1/2)))*2^(1/2)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.68

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{(cx)^{5/2}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}(-a+4bx^2)+3\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}x+\sqrt{a-bx^2}}\right)-\right.}{64b^{7/4}x^{5/2}}$$

input
```
Integrate[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

output
```
((c*x)^(5/2)*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4)*(-a + 4*b*x^2) + 3*Sqrt[
2]*a^2*ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) +
Sqrt[a - b*x^2])] - 3*Sqrt[2]*a^2*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(S
qrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(64*b^(7/4)*x^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {248, 262, 266, 854, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{8}a\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}dx+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{8}a\left(\frac{3ac^2\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}dx}{4b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{8}a\left(\frac{3ac\int \frac{cx}{(a-bx^2)^{3/4}}d\sqrt{cx}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{8}a\left(\frac{3ac\int \frac{c^3x}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}a\left(\frac{3ac^3\int \frac{cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{826}$$

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$\downarrow$ 217

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}\right)+$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$\downarrow$ 1479

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\cdots}}{\cdots}}{2\sqrt{b}}\cdots\right.}{2b}\cdots\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$\downarrow$ 25

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}(\ldots}{\sqrt[4]{b}\left(xc+\ldots\right.}}{\ldots}}{2\sqrt{b}}\right.}{2b}\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int\frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}}{\sqrt[4]{a-}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}}{\sqrt[4]{b}\sqrt[4]{}}}}{}}{2\sqrt{b}}\right)}{2b}\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{8}a\left(\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}\right)}{2b}\right.$$

$$\frac{(cx)^{7/2}\sqrt[4]{a-bx^2}}{4c}$$

input
```
Int[(c*x)^(5/2)*(a - b*x^2)^(1/4),x]
```

output
```
((c*x)^(7/2)*(a - b*x^2)^(1/4))/(4*c) + (a*(-1/2*(c*(c*x)^(3/2)*(a - b*x^2
)^(1/4))/b + (3*a*c^3*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*
(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/
4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*S
qrt[b]) - (-1/2*Log[c + Sqrt[b]*c*x - (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/
(a - b*x^2)^(1/4)]/(Sqrt[2]*b^(1/4)*Sqrt[c]) + Log[c + Sqrt[b]*c*x + (Sqrt
[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[
c]))/(2*Sqrt[b])))/(2*b)))/8
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int (cx)^{\frac{5}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}d x$$

input
```
int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{5/2} \sqrt[4]{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.18

$$\int (cx)^{5/2} \sqrt[4]{a-bx^2}\,dx = \frac{\sqrt[4]{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right) {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{4}, \frac{7}{4} \\ \frac{11}{4} \end{matrix}\middle| \frac{bx^{2}e^{2i\pi}}{a} \right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)*(-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*e
xp_polar(2*I*pi)/a)/(2*gamma(11/4))
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \int (-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \int (-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \int (c\,x)^{5/2}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

input
```
int((c*x)^(5/2)*(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(5/2)*(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int (cx)^{5/2}\sqrt[4]{a-bx^2}\,dx = \frac{\sqrt{c}\,c^2\left(-2\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}\,ax+8\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^3+3\left(\int \frac{\sqrt{x}}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2\right)}{32b}$$

input
```
int((c*x)^(5/2)*(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*c**2*( - 2*sqrt(x)*(a - b*x**2)**(1/4)*a*x + 8*sqrt(x)*(a - b*x**
2)**(1/4)*b*x**3 + 3*int((sqrt(x)*(a - b*x**2)**(1/4))/(a - b*x**2),x)*a**
2))/(32*b)
```

3.984 $\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx=\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}-\frac{a\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}}+\frac{a\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{3/4}}-\frac{a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{4\sqrt{2}b^{3/4}}$$

output
```
1/2*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/c+1/8*a*c^(1/2)*arctan(-1+2^(1/2)*b^(1/4)
*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(3/4)+1/8*a*c^(1/2)*arcta
n(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(3/4)-
1/8*a*c^(1/2)*arctanh(2^(1/2)*b^(1/4)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(1/2
)+b^(1/2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^(1/2)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.71

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{\sqrt{cx}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}+\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}x+\sqrt{a-bx^2}}\right)-\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{b}x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{8b^{3/4}\sqrt{x}}$$

input

```
Integrate[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

output

```
(Sqrt[c*x]*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4) + Sqrt[2]*a*ArcTan[(Sqrt[2
]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) + Sqrt[a - b*x^2])] - S
qrt[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]*(a
 - b*x^2)^(1/4))]))/(8*b^(3/4)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.30, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {248, 266, 854, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{4}a\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}dx+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$$\Big\downarrow \text{266}$$

$$\frac{a\int \frac{cx}{(a-bx^2)^{3/4}}d\sqrt{cx}}{2c}+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$$\Big\downarrow \text{854}$$

$$\frac{a\int \frac{c^3x}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2c}+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\Big\downarrow$ 27

$$\frac{1}{2}ac\int \frac{cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\Big\downarrow$ 826

$$\frac{1}{2}ac\left(\frac{\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\Big\downarrow$ 1476

$$\frac{1}{2}ac\left(\frac{\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\Big\downarrow$ 1082

$$\frac{1}{2}ac\left(\frac{\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\Big\downarrow$ 217

$$\frac{1}{2}ac\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\big\downarrow$ 1479

$$\frac{1}{2}ac\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\big\downarrow$ 25

$$\frac{1}{2}ac\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\cdots}{\sqrt[4]{b}\left(xc\cdots\right.}}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\big\downarrow$ 27

$$\frac{1}{2}ac\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{\cdots}}{\sqrt[4]{a-\cdots}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\cdots}{\sqrt[4]{b}\sqrt[4]{\cdots}}}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$$\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

$\big\downarrow$ 1103

$$\frac{1}{2}ac\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}\right.$$

$$\frac{(cx)^{3/2}\sqrt[4]{a-bx^2}}{2c}$$

input
```
Int[Sqrt[c*x]*(a - b*x^2)^(1/4),x]
```

output
```
((c*x)^(3/2)*(a - b*x^2)^(1/4))/(2*c) + (a*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/
4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + Ar
cTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]
*b^(1/4)*Sqrt[c]))/(2*Sqrt[b]) - (-1/2*Log[c + Sqrt[b]*c*x - (Sqrt[2]*b^(1
/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(Sqrt[2]*b^(1/4)*Sqrt[c]) + Log[
c + Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(
2*Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b])))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \sqrt{cx}\,\left(-b\,x^{2}+a\right)^{\frac{1}{4}}dx$$

input
```
int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.21

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{\sqrt[4]{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^{2}e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)*(-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp
_polar(2*I*pi)/a)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \int \left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \int \sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

input
```
int((c*x)^(1/2)*(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(1/2)*(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \sqrt{cx}\sqrt[4]{a-bx^2}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}\,x+\left(\int \frac{\sqrt{x}}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)a\right)}{4}$$

input
```
int((c*x)^(1/2)*(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*(a - b*x**2)**(1/4)*x + int((sqrt(x)*(a - b*x**2)**(1/
4))/(a - b*x**2),x)*a))/4
```

3.985 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 214

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}+\frac{\sqrt[4]{b}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}} - \frac{\sqrt[4]{b}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}c^{3/2}}+\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{\sqrt{2}c^{3/2}}$$

output
```
-2*(-b*x^2+a)^(1/4)/c/(c*x)^(1/2)-1/2*b^(1/4)*arctan(-1+2^(1/2)*b^(1/4)*(c
*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/c^(3/2)-1/2*b^(1/4)*arctan(1+2
^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/c^(3/2)+1/2*b
^(1/4)*arctanh(2^(1/2)*b^(1/4)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(1/2)+b^(1/
2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \frac{x\left(-4\sqrt[4]{a-bx^2}+\sqrt{2}\sqrt[4]{b}\sqrt{x}\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{\sqrt{b}x-\sqrt{a-bx^2}}\right)+\sqrt{2}\sqrt[4]{b}\sqrt{x}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x+}{\sqrt{2}\sqrt[4]{b}\sqrt{x}}\right.\right.}{2(cx)^{3/2}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

output
```
(x*(-4*(a - b*x^2)^(1/4) + Sqrt[2]*b^(1/4)*Sqrt[x]*ArcTan[(Sqrt[2]*b^(1/4)
*Sqrt[x]*(a - b*x^2)^(1/4))/(Sqrt[b]*x - Sqrt[a - b*x^2])] + Sqrt[2]*b^(1/
4)*Sqrt[x]*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]*
(a - b*x^2)^(1/4))]))/(2*(c*x)^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.37, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {247, 266, 854, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \; 247$$

$$-\frac{b\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}dx}{c^2}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 266$$

$$-\frac{2b\int \frac{cx}{(a-bx^2)^{3/4}}d\sqrt{cx}}{c^3}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 854$$

$$-\frac{2b\int \frac{c^3 x}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{c^3}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 27

$$-\frac{2b\int \frac{cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{c}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 826

$$-\frac{2b\left(\frac{\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{c}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 1476

$$-\frac{2b\left(\frac{\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{c}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 1082

$$-\frac{2b\left(\frac{\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{c}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\Big\downarrow$ 217

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{c}-\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\downarrow$ 1479

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{c}\cdots\right.}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{\cdots}}\cdots\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{c}\cdots$$

$$\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\downarrow$ 25

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{c}+\frac{\sqrt{\cdots}}{\sqrt[4]{\cdots}}\right.}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\frac{\cdots}{\sqrt[4]{\cdots}}\right.}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right)}{c}\cdots$$

$$\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\downarrow$ 27

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int\frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}}{2\sqrt{b}\sqrt{c}}}{2\sqrt{b}}\right)}{c}$$

$$\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

$\downarrow$ 1103

$$-\frac{2b\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}\right)}{c}-$$

$$\frac{2\sqrt[4]{a-bx^2}}{c\sqrt{cx}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(3/2),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(c*Sqrt[c*x]) - (2*b*((-(ArcTan[1 - (Sqrt[2]*b^(1/4
)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + Arc
Tan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*
b^(1/4)*Sqrt[c]))/(2*Sqrt[b]) - (-1/2*Log[c + Sqrt[b]*c*x - (Sqrt[2]*b^(1/
4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(Sqrt[2]*b^(1/4)*Sqrt[c]) + Log[c
 + Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2
*Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b])))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{4}, -\frac{1}{4} \\ \frac{3}{4} \end{matrix}\middle| \frac{bx^2e^{2i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

output
```
a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(2*I*pi)/
a)/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(a-b\,x^2)^{1/4}}{(c\,x)^{3/2}}\,dx$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(3/2),x)
```

output
```
int((a - b*x^2)^(1/4)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

3.986 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

output
```
-2/5*(-b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx=-\frac{2x(a-bx^2)^{5/4}}{5a(cx)^{7/2}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

output
```
(-2*x*(a - b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}} \, dx$$

$$\Big\downarrow \; 242$$

$$-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(7/2),x]
```

output
```
(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}{5a\left(cx\right)^{\frac{7}{2}}}$	22
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}}{5a\left(cx\right)^{\frac{7}{2}}}$	22
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}}{5\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{3}x^{2}a}$	54

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(-b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = \frac{2\,(bx^2-a)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(b*x^2 - a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.42 (sec) , antiderivative size = 178, normalized size of antiderivative = 6.14

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = \begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right| > 1 \\ \frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

output
```
Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*g
amma(-1/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*g
amma(-1/4)), Abs(a/(b*x**2)) > 1), (b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(
I*pi/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4)) - b**(5/4)*(-a/(b*x**2)
+ 1)**(1/4)*exp(I*pi/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4)), True))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = -\frac{(a-b\,x^2)^{1/4}\left(\frac{2}{5\,c^3}-\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(7/2),x)
```

output
```
-((a - b*x^2)^(1/4)*(2/(5*c^3) - (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}\,dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(b\,x^2-a)}{5\sqrt{x}\,a\,c^4x^2}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(7/2),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - a + b*x**2))/(5*sqrt(x)*a*c**4*x**2)
```

3.987 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}} \, dx = -\frac{2(a-bx^2)^{5/4}}{9ac(cx)^{9/2}} - \frac{8b(a-bx^2)^{5/4}}{45a^2c^3(cx)^{5/2}}$$

output
```
-2/9*(-b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)-8/45*b*(-b*x^2+a)^(5/4)/a^2/c^3/(c*x
)^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}} \, dx = -\frac{2x\sqrt[4]{a-bx^2}(5a^2-abx^2-4b^2x^4)}{45a^2(cx)^{11/2}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*(5*a^2 - a*b*x^2 - 4*b^2*x^4))/(45*a^2*(c*x)^(11/2
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}} \, dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{4\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}dx}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(11/2),x]
```

output
```
(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2
*c*(c*x)^(9/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	32
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	32
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(-4b^{2}x^{4}-ab\,x^{2}+5a^{2}\right)}{45\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{5}x^{4}a^{2}}$	75

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(-b*x^2+a)^(5/4)*(4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \frac{2\,(4\,b^2x^4+abx^2-5\,a^2)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
2/45*(4*b^2*x^4 + a*b*x^2 - 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x
^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 41.65 (sec) , antiderivative size = 462, normalized size of antiderivative = 7.70

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \begin{cases} -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)} \\ \frac{5a^3b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{x^2\left(-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)\right)} - \frac{6a^2b^{\frac{9}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}e^{\frac{i\pi}{4}}\Gamma\left(-\frac{9}{4}\right)}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+8a^2b^2c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)} - \frac{3ab^{\frac{13}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}\cdots}}{-8a^3bc^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)+\cdots} \end{cases}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

output
```
Piecewise((-5*b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x*
*4*gamma(-1/4)) + b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(8*a*c**(11
/2)*x**2*gamma(-1/4)) + b**(9/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-9/4)/(2*a*
*2*c**(11/2)*gamma(-1/4)), Abs(a/(b*x**2)) > 1), (5*a**3*b**(5/4)*(-a/(b*x
**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(x**2*(-8*a**3*b*c**(11/2)*x**2*g
amma(-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4))) - 6*a**2*b**(9/4)*(-
a/(b*x**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2*g
amma(-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) - 3*a*b**(13/4)*x**2*
(-a/(b*x**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2
*gamma(-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)) + 4*b**(17/4)*x**4*
(-a/(b*x**2) + 1)**(1/4)*exp(I*pi/4)*gamma(-9/4)/(-8*a**3*b*c**(11/2)*x**2
*gamma(-1/4) + 8*a**2*b**2*c**(11/2)*x**4*gamma(-1/4)), True))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \frac{(a-b\,x^2)^{1/4}\left(\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{2}{9\,c^5}+\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(11/2),x)
```

output
```
((a - b*x^2)^(1/4)*((2*b*x^2)/(45*a*c^5) - 2/(9*c^5) + (8*b^2*x^4)/(45*a^2
*c^5)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}\,dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(4b^2x^4+ab\,x^2-5a^2)}{45\sqrt{x}\,a^2c^6x^4}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - 5*a**2 + a*b*x**2 + 4*b**2*x**4))/(45*s
qrt(x)*a**2*c**6*x**4)
```

3.988 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx=-\frac{2(a-bx^2)^{5/4}}{13ac(cx)^{13/2}}-\frac{16b(a-bx^2)^{5/4}}{117a^2c^3(cx)^{9/2}}-\frac{64b^2(a-bx^2)^{5/4}}{585a^3c^5(cx)^{5/2}}$$

output
```
-2/13*(-b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)-16/117*b*(-b*x^2+a)^(5/4)/a^2/c^3/
(c*x)^(9/2)-64/585*b^2*(-b*x^2+a)^(5/4)/a^3/c^5/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx=-\frac{2x(a-bx^2)^{5/4}(45a^2+40abx^2+32b^2x^4)}{585a^3(cx)^{15/2}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

output
```
(-2*x*(a - b*x^2)^(5/4)*(45*a^2 + 40*a*b*x^2 + 32*b^2*x^4))/(585*a^3*(c*x)
^(15/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx$$

$$\Big\downarrow \ 246$$

$$-\frac{8\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{15/2}}dx}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

$$\Big\downarrow \ 246$$

$$-\frac{8\left(-\frac{4\int \frac{\left(a-bx^2\right)^{9/4}}{(cx)^{15/2}}dx}{9a}-\frac{2\left(a-bx^2\right)^{9/4}}{9ac(cx)^{13/2}}\right)}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

$$\Big\downarrow \ 242$$

$$-\frac{8\left(\frac{8\left(a-bx^2\right)^{13/4}}{117a^2c(cx)^{13/2}}-\frac{2\left(a-bx^2\right)^{9/4}}{9ac(cx)^{13/2}}\right)}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(15/2),x]
```

output
```
(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) - (8*((-2*(a - b*x^2)^(9/4))/(
9*a*c*(c*x)^(13/2)) + (8*(a - b*x^2)^(13/4))/(117*a^2*c*(c*x)^(13/2))))/(5
*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}+40ab\,x^{2}+45a^{2}\right)}{585a^{3}\left(cx\right)^{\frac{15}{2}}}$	43
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}+40ab\,x^{2}+45a^{2}\right)}{585a^{3}\left(cx\right)^{\frac{15}{2}}}$	43
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(-32b^{3}x^{6}-8a\,b^{2}x^{4}-5a^{2}b\,x^{2}+45a^{3}\right)}{585\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{7}x^{6}a^{3}}$	86

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*x*(-b*x^2+a)^(5/4)*(32*b^2*x^4+40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \frac{2\,(32\,b^3x^6+8\,ab^2x^4+5\,a^2bx^2-45\,a^3)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

output
```
2/585*(32*b^3*x^6 + 8*a*b^2*x^4 + 5*a^2*b*x^2 - 45*a^3)*(-b*x^2 + a)^(1/4)
*sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \frac{(a-b\,x^2)^{1/4}\left(\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{2}{13\,c^7}+\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(15/2),x)
```

output
```
((a - b*x^2)^(1/4)*((2*b*x^2)/(117*a*c^7) - 2/(13*c^7) + (16*b^2*x^4)/(585
*a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{15/2}}\,dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(32b^3x^6+8a\,b^2x^4+5a^2b\,x^2-45a^3)}{585\sqrt{x}\,a^3c^8x^6}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - 45*a**3 + 5*a**2*b*x**2 + 8*a*b**2*x**4
 + 32*b**3*x**6))/(585*sqrt(x)*a**3*c**8*x**6)
```

3.989 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 124

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = -\frac{2(a-bx^2)^{5/4}}{17ac(cx)^{17/2}} - \frac{24b(a-bx^2)^{5/4}}{221a^2c^3(cx)^{13/2}} - \frac{64b^2(a-bx^2)^{5/4}}{663a^3c^5(cx)^{9/2}} - \frac{256b^3(a-bx^2)^{5/4}}{3315a^4c^7(cx)^{5/2}}$$

output
```
-2/17*(-b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)-24/221*b*(-b*x^2+a)^(5/4)/a^2/c^3/
(c*x)^(13/2)-64/663*b^2*(-b*x^2+a)^(5/4)/a^3/c^5/(c*x)^(9/2)-256/3315*b^3*
(-b*x^2+a)^(5/4)/a^4/c^7/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 1.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = -\frac{2x(a-bx^2)^{5/4}(195a^3+180a^2bx^2+160ab^2x^4+128b^3x^6)}{3315a^4(cx)^{19/2}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

output
```
(-2*x*(a - b*x^2)^(5/4)*(195*a^3 + 180*a^2*b*x^2 + 160*a*b^2*x^4 + 128*b^3
*x^6))/(3315*a^4*(c*x)^(19/2))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\int \frac{(a-bx^2)^{5/4}}{(cx)^{19/2}}dx}{5a}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\left(-\frac{8\int \frac{(a-bx^2)^{9/4}}{(cx)^{19/2}}dx}{9a}-\frac{2(a-bx^2)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\left(-\frac{8\left(-\frac{4\int \frac{(a-bx^2)^{13/4}}{(cx)^{19/2}}dx}{13a}-\frac{2(a-bx^2)^{13/4}}{13ac(cx)^{17/2}}\right)}{9a}-\frac{2(a-bx^2)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2(a-bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{12\left(-\frac{8\left(\frac{8\left(a-bx^2\right)^{17/4}}{221a^2c(cx)^{17/2}}-\frac{2\left(a-bx^2\right)^{13/4}}{13ac(cx)^{17/2}}\right)}{9a}-\frac{2\left(a-bx^2\right)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(19/2),x]
```

output
```
(-2*(a - b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) - (12*((-2*(a - b*x^2)^(9/4))/
(9*a*c*(c*x)^(17/2)) - (8*((-2*(a - b*x^2)^(13/4))/(13*a*c*(c*x)^(17/2)) +
 (8*(a - b*x^2)^(17/4))/(221*a^2*c*(c*x)^(17/2))))/(9*a)))/(5*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(128b^{3}x^{6}+160a\,b^{2}x^{4}+180a^{2}b\,x^{2}+195a^{3}\right)}{3315a^{4}\left(cx\right)^{\frac{19}{2}}}$	54
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{5}{4}}\left(128b^{3}x^{6}+160a\,b^{2}x^{4}+180a^{2}b\,x^{2}+195a^{3}\right)}{3315a^{4}\left(cx\right)^{\frac{19}{2}}}$	54
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(-128b^{4}x^{8}-32a\,b^{3}x^{6}-20a^{2}b^{2}x^{4}-15a^{3}b\,x^{2}+195a^{4}\right)}{3315\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{9}x^{8}a^{4}}$	97

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3315*x*(-b*x^2+a)^(5/4)*(128*b^3*x^6+160*a*b^2*x^4+180*a^2*b*x^2+195*a^
3)/a^4/(c*x)^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \frac{2\left(128\,b^4x^8 + 32\,ab^3x^6 + 20\,a^2b^2x^4 + 15\,a^3bx^2 - 195\,a^4\right)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

output
```
2/3315*(128*b^4*x^8 + 32*a*b^3*x^6 + 20*a^2*b^2*x^4 + 15*a^3*b*x^2 - 195*a
^4)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \frac{(a-b\,x^2)^{1/4}\left(\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{2}{17\,c^9}+\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}+\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(19/2),x)
```

output
```
((a - b*x^2)^(1/4)*((2*b*x^2)/(221*a*c^9) - 2/(17*c^9) + (8*b^2*x^4)/(663*
a^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) + (256*b^4*x^8)/(3315*a^4*c^9)))/(x
^8*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{19/2}}\,dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(128b^4x^8+32a\,b^3x^6+20a^2b^2x^4+15a^3b\,x^2-195a^4)}{3315\sqrt{x}\,a^4c^{10}x^8}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - 195*a**4 + 15*a**3*b*x**2 + 20*a**2*b**
2*x**4 + 32*a*b**3*x**6 + 128*b**4*x**8))/(3315*sqrt(x)*a**4*c**10*x**8)
```

3.990 $\int (cx)^{3/2} \sqrt[4]{a-bx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 122

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=-\frac{ac\sqrt{cx}\sqrt[4]{a-bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}-\frac{a^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\left(a-bx^2\right)^{3/4}}$$

output
```
-1/6*a*c*(c*x)^(1/2)*(-b*x^2+a)^(1/4)/b+1/3*(c*x)^(5/2)*(-b*x^2+a)^(1/4)/c
-1/6*a^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccsc(b^(1
/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.72

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}\left(\left(-a+bx^2\right)\sqrt[4]{1-\frac{bx^2}{a}}+a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

output
```
(c*Sqrt[c*x]*(a - b*x^2)^(1/4)*((-a + b*x^2)*(1 - (b*x^2)/a)^(1/4) + a*Hyp
ergeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a]))/(3*b*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {248, 262, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2} \sqrt[4]{a-bx^2} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6} a \int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}} dx + \frac{(cx)^{5/2} \sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6} a \left(\frac{ac^2 \int \frac{1}{\sqrt{cx}(a-bx^2)^{3/4}} dx}{2b} - \frac{c\sqrt{cx} \sqrt[4]{a-bx^2}}{b} \right) + \frac{(cx)^{5/2} \sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{6} a \left(\frac{ac \int \frac{1}{(a-bx^2)^{3/4}} d\sqrt{cx}}{b} - \frac{c\sqrt{cx} \sqrt[4]{a-bx^2}}{b} \right) + \frac{(cx)^{5/2} \sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{6} a \left(\frac{ac(cx)^{3/2} \left(1-\frac{a}{bx^2}\right)^{3/4} \int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4} (cx)^{3/2}} d\sqrt{cx}}{b\left(a-bx^2\right)^{3/4}} - \frac{c\sqrt{cx} \sqrt[4]{a-bx^2}}{b} \right) + \frac{(cx)^{5/2} \sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{6}a\left(-\frac{ac(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int\frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{b\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}a\left(-\frac{ac(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int\frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{2b\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}$$

$$\Big\downarrow \text{230}$$

$$\frac{1}{6}a\left(-\frac{\sqrt{a}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a-bx^2}}{3c}$$

input
```
Int[(c*x)^(3/2)*(a - b*x^2)^(1/4),x]
```

output
```
((c*x)^(5/2)*(a - b*x^2)^(1/4))/(3*c) + (a*(-((c*Sqrt[c*x]*(a - b*x^2)^(1/
4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt
[a]*c^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a - b*x^2)^(3/4))))/6
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int (cx)^{\frac{3}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}dx$$

input
```
int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.39

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\frac{\sqrt[4]{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)*(-b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*ex
p_polar(2*I*pi)/a)/(2*gamma(9/4))
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\int \left(-bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\int (c\,x)^{3/2}\,\left(a-b\,x^2\right)^{1/4}\,dx$$

input
```
int((c*x)^(3/2)*(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(3/2)*(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int (cx)^{3/2}\sqrt[4]{a-bx^2}\,dx=\frac{\sqrt{c}\,c\left(-2\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}\,a+4\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}\,b\,x^2+\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^3+ax}dx\right)a^2\right)}{12b}$$

input
```
int((c*x)^(3/2)*(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(x)*(a - b*x**2)**(1/4)*a + 4*sqrt(x)*(a - b*x**2)**(
1/4)*b*x**2 + int((sqrt(x)*(a - b*x**2)**(1/4))/(a*x - b*x**3),x)*a**2))/(
12*b)
```

3.991 $\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

output
```
(c*x)^(1/2)*(-b*x^2+a)^(1/4)/c-a^(1/2)*b^(1/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/
2)*InverseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),2^(1/2))/c^2/(-b*x^2+a)^(
3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \frac{2x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

output
```
(2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, (b*x^2)/a])/(Sqrt
[c*x]*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {248, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx+\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}$$

$$\Big\downarrow \text{266}$$

$$\frac{a\int \frac{1}{\left(a-bx^2\right)^{3/4}}d\sqrt{cx}}{c}+\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}$$

$$\Big\downarrow \text{768}$$

$$\frac{a(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{c\left(a-bx^2\right)^{3/4}}+\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{a(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{c\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{a(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{2c\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$\frac{\sqrt{cx}\sqrt[4]{a-bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{c^2\left(a-bx^2\right)^{3/4}}$$

input
```
Int[(a - b*x^2)^(1/4)/Sqrt[c*x],x]
```

output
```
(Sqrt[c*x]*(a - b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(
c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(c^2*(a - b*x^
2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{\sqrt{cx}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.42

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = -\frac{i\sqrt[4]{b}xe^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{1}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt{c}}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(1/2),x)
```

output
```
-I*b**(1/4)*x*exp(3*I*pi/4)*hyper((-1/2, -1/4), (1/2,), a/(b*x**2))/sqrt(c
)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx=\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{\sqrt{c\,x}}\,dx$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(1/2),x)
```

output
```
int((a - b*x^2)^(1/4)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{\sqrt{cx}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}}{x}dx\right)}{c}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a - b*x**2)**(1/4))/x,x))/c
```

3.992 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}} + \frac{2b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}c^4\left(a-bx^2\right)^{3/4}}$$

output

```
-2/3*(-b*x^2+a)^(1/4)/c/(c*x)^(3/2)+2/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3
/2)*InverseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/c^4/(-b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

input

```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, (b*x^2)/a])/(3*
(c*x)^(5/2)*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {247, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{b\int \frac{1}{\sqrt{cx}(a-bx^2)^{3/4}}dx}{3c^2}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2b\int \frac{1}{(a-bx^2)^{3/4}}d\sqrt{cx}}{3c^3}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3c^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3c^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3c^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{230}$$

$$\frac{2b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3\sqrt{a}c^4(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3c(cx)^{3/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(5/2),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) + (2*b^(3/2)*(1 - a/(b*x^2))^(3/4
)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*Sqrt[a]*
c^4*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^{2}+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = -\frac{i\sqrt[4]{b}e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{a}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

output
```
-I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 1/2), (3/2,), a/(b*x**2))/(c**(5/2)*
x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx=\int \frac{{\left(a-b\,x^2\right)}^{1/4}}{{\left(c\,x\right)}^{5/2}} \,d x$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(5/2),x)
```

output
```
int((a - b*x^2)^(1/4)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{5/2}}\,dx=\frac{\sqrt{c}\,\left(-2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}-\sqrt{x}\,\left(\int \frac{\sqrt{x}\,\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{-b\,x^{5}+a\,x^{3}}d x\right) a x\right)}{2\sqrt{x}\,c^{3} x}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*(a - b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a - b*x**2)**(1
/4))/(a*x**3 - b*x**5),x)*a*x))/(2*sqrt(x)*c**3*x)
```

3.993 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 127

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}} + \frac{2b\sqrt[4]{a-bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a-bx^2\right)^{3/4}}$$

output
```
-2/7*(-b*x^2+a)^(1/4)/c/(c*x)^(7/2)+2/21*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^(3
/2)+4/21*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccsc(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/c^6/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, (b*x^2)/a])/(7
*(c*x)^(9/2)*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {247, 264, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{b\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}dx}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(\frac{2b\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{4b\int \frac{1}{\left(a-bx^2\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{b\left(\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$\Big\downarrow$ 858

$$-\frac{b\left(-\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$\Big\downarrow$ 807

$$-\frac{b\left(-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

$\Big\downarrow$ 230

$$-\frac{b\left(-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a-bx^2}}{7c(cx)^{7/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(9/2),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) - (b*((-2*(a - b*x^2)^(1/4))/(3*a
*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[A
rcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a - b*x^2)^(3/4))))/
(7*c^2)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{9}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \int \frac{\left(-bx^{2}+a\right)^{\frac{1}{4}}}{\left(cx\right)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.31

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \frac{i\sqrt[4]{b}e^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3c^{\frac{9}{2}}x^3}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

output
```
I*b**(1/4)*exp(3*I*pi/4)*hyper((-1/4, 3/2), (5/2,), a/(b*x**2))/(3*c**(9/2
)*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(a-b\,x^2)^{1/4}}{(c\,x)^{9/2}}\,dx$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(9/2),x)
```

output
```
int((a - b*x^2)^(1/4)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{c}\left(-2(-b\,x^2+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^7+a\,x^5}dx\right)a\,x^3\right)}{6\sqrt{x}\,c^5x^3}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

output
```
(sqrt(c)*( - 2*(a - b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a - b*x**2)**(1
/4))/(a*x**5 - b*x**7),x)*a*x**3))/(6*sqrt(x)*c**5*x**3)
```

3.994 $\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 159

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx=-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}+\frac{2b\sqrt[4]{a-bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a-bx^2}}{77a^2c^5(cx)^{3/2}}+\frac{8b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a-bx^2\right)^{3/4}}$$

output
```
-2/11*(-b*x^2+a)^(1/4)/c/(c*x)^(11/2)+2/77*b*(-b*x^2+a)^(1/4)/a/c^3/(c*x)^
(7/2)+4/77*b^2*(-b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)+8/77*b^(7/2)*(1-a/b/x^
2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),2^(1/2)
)/a^(5/2)/c^8/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx=-\frac{2x\sqrt[4]{a-bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, (b*x^2)/a])/(
11*(c*x)^(13/2)*(1 - (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {247, 264, 264, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}dx$$

$$\Big\downarrow \text{247}$$

$$-\frac{b\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}dx}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(\frac{6b\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}dx}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{b\left(\frac{6b\left(\frac{2b\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{b\left(\frac{6b\left(\frac{4b\int \frac{1}{\left(a-bx^2\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$\Big\downarrow$ 768

$$-\frac{b\left(\frac{6b\left(\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$\Big\downarrow$ 858

$$-\frac{b\left(\frac{6b\left(-\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$\Big\downarrow$ 807

$$-\frac{b\left(\frac{6b\left(-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

$\Big\downarrow$ 230

$$-\frac{b\left(\frac{6b\left(-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a-bx^2}}{11c(cx)^{11/2}}$$

input
```
Int[(a - b*x^2)^(1/4)/(c*x)^(13/2),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) - (b*((-2*(a - b*x^2)^(1/4))/(7
*a*c*(c*x)^(7/2)) + (6*b*((-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*
b^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^2*x)
/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a - b*x^2)^(3/4))))/(7*a*c^2)))/(11*c^2)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}dx$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

output
```
int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx=\int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

output
```
integral((-b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 139.76 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.25

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx=-\frac{i\sqrt[4]{b}e^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{5c^{\frac{13}{2}}x^5}$$

input
```
integrate((-b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

output
```
-I*b**(1/4)*exp(-I*pi/4)*hyper((-1/4, 5/2), (7/2,), a/(b*x**2))/(5*c**(13/
2)*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx=\int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(-bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((-b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

output
```
integrate((-b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(a-b\,x^2)^{1/4}}{(c\,x)^{13/2}}\,dx$$

input
```
int((a - b*x^2)^(1/4)/(c*x)^(13/2),x)
```

output
```
int((a - b*x^2)^(1/4)/(c*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{13/2}}\,dx = \frac{\sqrt{c}\left(-2(-b\,x^2+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}}{-b\,x^9+a\,x^7}dx\right)a\,x^5\right)}{10\sqrt{x}\,c^7x^5}$$

input
```
int((-b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

output
```
(sqrt(c)*( - 2*(a - b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a - b*x**2)**(1
/4))/(a*x**7 - b*x**9),x)*a*x**5))/(10*sqrt(x)*c**7*x**5)
```

3.995 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 228

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{c\sqrt{cx}(a-bx^2)^{3/4}}{2b} - \frac{ac^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} + \frac{ac^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{5/4}} + \frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{4\sqrt{2}b^{5/4}}$$

output
```
-1/2*c*(c*x)^(1/2)*(-b*x^2+a)^(3/4)/b+1/8*a*c^(3/2)*arctan(-1+2^(1/2)*b^(1
/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(5/4)+1/8*a*c^(3/2)*ar
ctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(5/
4)+1/8*a*c^(3/2)*arctanh(2^(1/2)*b^(1/4)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(
1/2)+b^(1/2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^(1/2)/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.71

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \frac{(cx)^{3/2}\left(-4\sqrt[4]{b}\sqrt{x}(a-bx^2)^{3/4}+\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}x+\sqrt{a-bx^2}}\right)+\sqrt{2}a\text{arctanh}\left(\right.}{8b^{5/4}x^{3/2}}$$

input
```
Integrate[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

output
```
((c*x)^(3/2)*(-4*b^(1/4)*Sqrt[x]*(a - b*x^2)^(3/4) + Sqrt[2]*a*ArcTan[(Sqr
t[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x) + Sqrt[a - b*x^2])]
+ Sqrt[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sqrt[x]
*(a - b*x^2)^(1/4))]))/(8*b^(5/4)*x^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.28, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {262, 266, 770, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}dx}{4b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$$\Big\downarrow \text{266}$$

$$\frac{ac\int \frac{1}{\sqrt[4]{a-bx^2}}d\sqrt{cx}}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$$\Big\downarrow \text{770}$$

$$\frac{ac\int \frac{1}{bx^2+1}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 755

$$\frac{ac\left(\frac{\int \frac{c^2\left(c-\sqrt{b}cx\right)}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2c}+\frac{\int \frac{c^2\left(\sqrt{b}xc+c\right)}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2c}\right)}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 27

$$\frac{ac\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 1476

$$\frac{ac\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)\right)}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 1082

$$\frac{ac\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)\right)}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 217

$$\frac{ac\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)\right)}{2b}-\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 1479

$$\frac{ac\left(\frac{1}{2}c\left(-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan\cdots}{\cdots}\right.\cdots\right)}{2b}$$

$$\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 25

$$\frac{ac\left(\frac{1}{2}c\left(\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan\left(\cdots\right)}{\sqrt{\cdots}}\cdots\right)\right)}{2b}$$

$$\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 27

$$\frac{ac\left(\frac{1}{2}c\left(\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+\cdots\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\cdots\right)\right)}{2b}$$

$$\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

$\Big\downarrow$ 1103

$$\frac{ac\left(\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{c}}{\sqrt[4]{a-bx^2}}\right.}{2\sqrt{2}\sqrt[4]{b}}\right.\right.}{2b}$$

$$\frac{c\sqrt{cx}\left(a-bx^2\right)^{3/4}}{2b}$$

input
```
Int[(c*x)^(3/2)/(a - b*x^2)^(1/4),x]
```

output
```
-1/2*(c*Sqrt[c*x]*(a - b*x^2)^(3/4))/b + (a*c*((c*(-(ArcTan[1 - (Sqrt[2]*b
^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]))
+ ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqr
t[2]*b^(1/4)*Sqrt[c])))/2 + (c*(-1/2*Log[c + Sqrt[b]*c*x - (Sqrt[2]*b^(1/4
)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(Sqrt[2]*b^(1/4)*Sqrt[c]) + Log[c
+ Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*
Sqrt[2]*b^(1/4)*Sqrt[c])))/2))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{2}}}{(-b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.42

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = -\frac{4\left(-bx^{2}+a\right)^{\frac{3}{4}}\sqrt{cx}c+\left(-\frac{a^{4}c^{6}}{b^{5}}\right)^{\frac{1}{4}}b\log\left(\frac{(-bx^{2}+a)^{\frac{3}{4}}\sqrt{cx}ac+\left(-\frac{a^{4}c^{6}}{b^{5}}\right)^{\frac{1}{4}}(b^{2}x^{2}-ab)}{bx^{2}-a}\right)-\left(-\frac{a^{4}c^{6}}{b^{5}}\right)^{\frac{1}{4}}b\log\left(\frac{(-bx^{2}+a)^{\frac{3}{4}}\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/8*(4*(-b*x^2 + a)^(3/4)*sqrt(c*x)*c + (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x
^2 + a)^(3/4)*sqrt(c*x)*a*c + (-a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2
 - a)) - (-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c - (-
a^4*c^6/b^5)^(1/4)*(b^2*x^2 - a*b))/(b*x^2 - a)) - I*(-a^4*c^6/b^5)^(1/4)*
b*log(((-b*x^2 + a)^(3/4)*sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(I*b^2*x^2
- I*a*b))/(b*x^2 - a)) + I*(-a^4*c^6/b^5)^(1/4)*b*log(((-b*x^2 + a)^(3/4)*
sqrt(c*x)*a*c - (-a^4*c^6/b^5)^(1/4)*(-I*b^2*x^2 + I*a*b))/(b*x^2 - a)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.20

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(1/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(c\,x)^{3/2}}{(a-b\,x^2)^{1/4}}\,dx$$

input
```
int((c*x)^(3/2)/(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(3/2)/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a-bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x}{(-b\,x^2+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int((c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x)/(a - b*x**2)**(1/4),x)*c
```

3.996 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 190

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = -\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}$$

output
```
1/2*arctan(-1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2
)/b^(1/4)/c^(1/2)+1/2*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2
+a)^(1/4))*2^(1/2)/b^(1/4)/c^(1/2)+1/2*arctanh(2^(1/2)*b^(1/4)*(c*x)^(1/2)
/(-b*x^2+a)^(1/4)/(c^(1/2)+b^(1/2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^(1/2)/b^
(1/4)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.66

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{x}\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}x+\sqrt{a-bx^2}}\right)+\operatorname{arctanh}\left(\frac{\sqrt{b}x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

output
```
(Sqrt[x]*(ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*x)
 + Sqrt[a - b*x^2])] + ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1
/4)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(Sqrt[2]*b^(1/4)*Sqrt[c*x])
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {266, 770, 755, 27, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{\sqrt[4]{a-bx^2}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{770}$$

$$\frac{2\int \frac{1}{bx^2+1}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{c}$$

$$\Big\downarrow \text{755}$$

$$\frac{2\left(\frac{\int \frac{c^2\left(c-\sqrt{b}cx\right)}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2c}+\frac{\int \frac{c^2\left(\sqrt{b}xc+c\right)}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2c}\right)}{c}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{c}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)\right)}{c}$$

$$\Big\downarrow \text{1082}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)\right)}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)\right)}{c}$$

$$\Big\downarrow \text{1479}$$

$$\frac{2\left(\frac{1}{2}c\left(-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan \cdots}{\cdots}\right.}{c}$$

$\downarrow$ 25

$$\frac{2\left(\frac{1}{2}c\left(\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\right)}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{\cdots}}{\sqrt{c}\sqrt[4]{\cdots}}\right.}{\sqrt{2}\cdots}\right.}{c}$$

$\downarrow$ 27

$$\frac{2\left(\frac{1}{2}c\left(\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right.}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right.}{c}$$

$\downarrow$ 1103

$$\frac{2\left(\frac{1}{2}c\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}\right)+\frac{1}{2}c\left(\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}\cdots\right.}{2\sqrt{2}\sqrt[4]{b}\sqrt{\cdots}}\right.}{c}$$

input
```
Int[1/(Sqrt[c*x]*(a - b*x^2)^(1/4)),x]
```

output
```
(2*((c*(-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4
))]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(S
qrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c])))/2 + (c*(-1/2*Log[c
+ Sqrt[b]*c*x - (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(Sq
rt[2]*b^(1/4)*Sqrt[c]) + Log[c + Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c]*Sq
rt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c])))/2))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx$$
$$= -\frac{1}{2}\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}+(bcx^2-ac)\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2-a}\right)$$
$$+\frac{1}{2}\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(bcx^2-ac)\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2-a}\right)$$
$$+\frac{1}{2}i\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(i\,bcx^2-i\,ac)\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2-a}\right)$$
$$-\frac{1}{2}i\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(-i\,bcx^2+i\,ac)\left(-\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2-a}\right)$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 - a*c
)*(-1/(b*c^2))^(1/4))/(b*x^2 - a)) + 1/2*(-1/(b*c^2))^(1/4)*log(((-b*x^2 +
 a)^(3/4)*sqrt(c*x) - (b*c*x^2 - a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a)) + 1
/2*I*(-1/(b*c^2))^(1/4)*log(((-b*x^2 + a)^(3/4)*sqrt(c*x) - (I*b*c*x^2 - I
*a*c)*(-1/(b*c^2))^(1/4))/(b*x^2 - a)) - 1/2*I*(-1/(b*c^2))^(1/4)*log(((-b
*x^2 + a)^(3/4)*sqrt(c*x) - (-I*b*c*x^2 + I*a*c)*(-1/(b*c^2))^(1/4))/(b*x^
2 - a))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(2*I*pi)/a)/(
2*a**(1/4)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(1/2)*(a - b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{x}\,\sqrt{c}\,(-2b\,x^2+2a)}{(-b\,x^2+a)^{\frac{3}{4}}\,\sqrt{-b\,x^2+a}\,c}$$

input
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(x)*sqrt(c)*(a - b*x**2)**(1/4)*(a - b*x**2 + a - b*x**2))/(sqrt(a -
b*x**2)*c*(a - b*x**2))
```

3.997 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 29

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

output
```
-2/3*(-b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2x(a-bx^2)^{3/4}}{3a(cx)^{5/2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2x(-b\,x^2+a)^{\frac{3}{4}}}{3a(cx)^{\frac{5}{2}}}$	22
orering	$-\frac{2x(-b\,x^2+a)^{\frac{3}{4}}}{3a(cx)^{\frac{5}{2}}}$	22
risch	$-\frac{2(-b\,x^2+a)^{\frac{3}{4}}}{3c^2\sqrt{cx}\,ax}$	27

input
```
int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(-b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\,(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-2/3*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.78 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.03

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = \begin{cases} \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{for } \left|\frac{a}{bx^2}\right|>1 \\ -\frac{b^{\frac{3}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}e^{-\frac{i\pi}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

output
```
Piecewise((b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamm
a(1/4)), Abs(a/(b*x**2)) > 1), (-b**(3/4)*(-a/(b*x**2) + 1)**(3/4)*exp(-I*
pi/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4)), True))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\,{\left(a-b\,x^2\right)}^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(5/2)*(a - b*x^2)^(1/4)),x)
```

output
```
-(2*(a - b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.59

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{c}\,((-b\,x^2+a)\,a-4(-b\,x^2+a)\,b\,x^2-3a^2+3ab\,x^2)}{3\,(-b\,x^2+a)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{-b\,x^2+a}\,a\,c^3x}$$

input
```
int(1/(c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a - b*x**2)**(1/4)*((a - b*x**2)*a - 4*(a - b*x**2)*b*x**2 - 3*a
**2 + 3*a*b*x**2))/(3*sqrt(x)*sqrt(a - b*x**2)*a*c**3*x*(a - b*x**2))
```

3.998 $\int \frac{1}{(cx)^{9/2} \sqrt[4]{a-bx^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{1}{(cx)^{9/2} \sqrt[4]{a-bx^2}} \, dx = -\frac{2(a-bx^2)^{3/4}}{7ac(cx)^{7/2}} - \frac{8b(a-bx^2)^{3/4}}{21a^2c^3(cx)^{3/2}}$$

output
```
-2/7*(-b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)-8/21*b*(-b*x^2+a)^(3/4)/a^2/c^3/(c*x
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{9/2} \sqrt[4]{a-bx^2}} \, dx = -\frac{2x(a-bx^2)^{3/4} (3a+4bx^2)}{21a^2(cx)^{9/2}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(3/4)*(3*a + 4*b*x^2))/(21*a^2*(c*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{4\int \frac{(a-bx^2)^{3/4}}{(cx)^{9/2}}dx}{3a}-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{8(a-bx^2)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2(a-bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

input
```
Int[1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a - b*x^2)^(7/4))/(21*a^2
*c*(c*x)^(7/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(4b\,x^{2}+3a\right)}{21a^{2}\left(cx\right)^{\frac{9}{2}}}$	32
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(4b\,x^{2}+3a\right)}{21a^{2}\left(cx\right)^{\frac{9}{2}}}$	32
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(4b\,x^{2}+3a\right)}{21c^{4}\sqrt{cx}\,a^{2}x^{3}}$	37

input
```
int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*x*(-b*x^2+a)^(3/4)*(4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\,(4\,bx^2+3\,a)(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-2/21*(4*b*x^2 + 3*a)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 20.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 5.72

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = \begin{cases} -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)} - \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}-1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)} \\ -\frac{3a^2b^{\frac{7}{4}}\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} - \frac{ab^{\frac{11}{4}}x^2\left(-\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{-8a^3bc^{\frac{9}{2}}x^2e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)+8a^2b^2c^{\frac{9}{2}}x^4e^{\frac{i\pi}{4}}\Gamma\left(\frac{1}{4}\right)} + \frac{4b}{-8a^3bc^{\frac{9}{2}}x} \end{cases}$$

input
```
integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(1/4),x)
```

output
```
Piecewise((-3*b**(3/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x
**2*gamma(1/4)) - b**(7/4)*(a/(b*x**2) - 1)**(3/4)*gamma(-7/4)/(2*a**2*c**
(9/2)*gamma(1/4)), Abs(a/(b*x**2)) > 1), (-3*a**2*b**(7/4)*(-a/(b*x**2) +
1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1/4) + 8*
a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) - a*b**(11/4)*x**2*(-a/(b*
x**2) + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)*gamma(1
/4) + 8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)) + 4*b**(15/4)*x**4
*(-a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(-8*a**3*b*c**(9/2)*x**2*exp(I*pi/4)
*gamma(1/4) + 8*a**2*b**2*c**(9/2)*x**4*exp(I*pi/4)*gamma(1/4)), True))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{\left(a-b\,x^2\right)^{3/4}\left(\frac{2}{7\,a\,c^4}+\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(9/2)*(a - b*x^2)^(1/4)),x)
```

output
```
-((a - b*x^2)^(3/4)*(2/(7*a*c^4) + (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{c}\,(3(-b\,x^2+a)\,a^2+8(-b\,x^2+a)\,ab\,x^2-32(-b\,x^2+a)\,b^2x^4-21a^3+21a^2b\,x^2}{63\,(-b\,x^2+a)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{-b\,x^2+a}\,a^2c^5x^3}$$

input
```
int(1/(c*x)^(9/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a - b*x**2)**(1/4)*(3*(a - b*x**2)*a**2 + 8*(a - b*x**2)*a*b*x**
2 - 32*(a - b*x**2)*b**2*x**4 - 21*a**3 + 21*a**2*b*x**2))/(63*sqrt(x)*sqr
t(a - b*x**2)*a**2*c**5*x**3*(a - b*x**2))
```

3.999 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2(a-bx^2)^{3/4}}{11ac(cx)^{11/2}}-\frac{16b(a-bx^2)^{3/4}}{77a^2c^3(cx)^{7/2}}-\frac{64b^2(a-bx^2)^{3/4}}{231a^3c^5(cx)^{3/2}}$$

output
```
-2/11*(-b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)-16/77*b*(-b*x^2+a)^(3/4)/a^2/c^3/(
c*x)^(7/2)-64/231*b^2*(-b*x^2+a)^(3/4)/a^3/c^5/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2x(a-bx^2)^{3/4}(21a^2+24abx^2+32b^2x^4)}{231a^3(cx)^{13/2}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(3/4)*(21*a^2 + 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*(c*x)
^(13/2))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{8\int \frac{\left(a-bx^2\right)^{3/4}}{(cx)^{13/2}}dx}{3a}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{8\left(-\frac{4\int \frac{\left(a-bx^2\right)^{7/4}}{(cx)^{13/2}}dx}{7a}-\frac{2\left(a-bx^2\right)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{8\left(\frac{8\left(a-bx^2\right)^{11/4}}{77a^2c(cx)^{11/2}}-\frac{2\left(a-bx^2\right)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2\left(a-bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

input
```
Int[1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*(a - b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) - (8*((-2*(a - b*x^2)^(7/4))/(
7*a*c*(c*x)^(11/2)) + (8*(a - b*x^2)^(11/4))/(77*a^2*c*(c*x)^(11/2))))/(3*
a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}+24ab\,x^{2}+21a^{2}\right)}{231a^{3}\left(cx\right)^{\frac{13}{2}}}$	43
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}+24ab\,x^{2}+21a^{2}\right)}{231a^{3}\left(cx\right)^{\frac{13}{2}}}$	43
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}+24ab\,x^{2}+21a^{2}\right)}{231c^{6}\sqrt{cx}\,a^{3}x^{5}}$	48

input
```
int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/231*x*(-b*x^2+a)^(3/4)*(32*b^2*x^4+24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\,(32\,b^2x^4+24\,abx^2+21\,a^2)(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-2/231*(32*b^2*x^4 + 24*a*b*x^2 + 21*a^2)*(-b*x^2 + a)^(3/4)*sqrt(c*x)/(a^
3*c^7*x^6)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 171.19 (sec) , antiderivative size = 1221, normalized size of antiderivative = 13.27

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(1/4),x)
```

output
```
Piecewise((-21*a**4*b**(19/4)*(a/(b*x**2) - 1)**(3/4)*exp(-3*I*pi/4)*gamma
(-11/4)/(32*a**5*b**4*c**(13/2)*x**4*exp(I*pi/4)*gamma(1/4) - 64*a**4*b**5
*c**(13/2)*x**6*exp(I*pi/4)*gamma(1/4) + 32*a**3*b**6*c**(13/2)*x**8*exp(I
*pi/4)*gamma(1/4)) + 18*a**3*b**(23/4)*x**2*(a/(b*x**2) - 1)**(3/4)*exp(-3
*I*pi/4)*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4*exp(I*pi/4)*gamma(1/4)
- 64*a**4*b**5*c**(13/2)*x**6*exp(I*pi/4)*gamma(1/4) + 32*a**3*b**6*c**(13
/2)*x**8*exp(I*pi/4)*gamma(1/4)) - 5*a**2*b**(27/4)*x**4*(a/(b*x**2) - 1)*
*(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4*exp(I*pi/4
)*gamma(1/4) - 64*a**4*b**5*c**(13/2)*x**6*exp(I*pi/4)*gamma(1/4) + 32*a**
3*b**6*c**(13/2)*x**8*exp(I*pi/4)*gamma(1/4)) + 40*a*b**(31/4)*x**6*(a/(b*
x**2) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4
*exp(I*pi/4)*gamma(1/4) - 64*a**4*b**5*c**(13/2)*x**6*exp(I*pi/4)*gamma(1/
4) + 32*a**3*b**6*c**(13/2)*x**8*exp(I*pi/4)*gamma(1/4)) - 32*b**(35/4)*x*
*8*(a/(b*x**2) - 1)**(3/4)*exp(-3*I*pi/4)*gamma(-11/4)/(32*a**5*b**4*c**(1
3/2)*x**4*exp(I*pi/4)*gamma(1/4) - 64*a**4*b**5*c**(13/2)*x**6*exp(I*pi/4)
*gamma(1/4) + 32*a**3*b**6*c**(13/2)*x**8*exp(I*pi/4)*gamma(1/4)), Abs(a/(
b*x**2)) > 1), (-21*a**4*b**(19/4)*(-a/(b*x**2) + 1)**(3/4)*gamma(-11/4)/(
32*a**5*b**4*c**(13/2)*x**4*exp(I*pi/4)*gamma(1/4) - 64*a**4*b**5*c**(13/2
)*x**6*exp(I*pi/4)*gamma(1/4) + 32*a**3*b**6*c**(13/2)*x**8*exp(I*pi/4)*ga
mma(1/4)) + 18*a**3*b**(23/4)*x**2*(-a/(b*x**2) + 1)**(3/4)*gamma(-11/4...
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{(a-b\,x^2)^{3/4}\left(\frac{2}{11\,a\,c^6}+\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(13/2)*(a - b*x^2)^(1/4)),x)
```

output
```
-((a - b*x^2)^(3/4)*(2/(11*a*c^6) + (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)
/(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.26

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{c}\,(7(-b\,x^2+a)\,a^3+12(-b\,x^2+a)\,a^2b\,x^2+32(-b\,x^2+a)\,a\,b^2x^4-128(-b\,x^2}{385\,(-b\,x^2+a)^{\frac{3}{4}}\sqrt{x}\,\sqrt{-b\,x^2+a}\,a^3c^7x^5}$$

input
```
int(1/(c*x)^(13/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a - b*x**2)**(1/4)*(7*(a - b*x**2)*a**3 + 12*(a - b*x**2)*a**2*b
*x**2 + 32*(a - b*x**2)*a*b**2*x**4 - 128*(a - b*x**2)*b**3*x**6 - 77*a**4
 + 77*a**3*b*x**2))/(385*sqrt(x)*sqrt(a - b*x**2)*a**3*c**7*x**5*(a - b*x*
*2))
```

3.1000 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx=-\frac{ac^3(a-bx^2)^{3/4}}{2b^2\sqrt{cx}}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}+\frac{a^{3/2}c^2\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a-bx^2}}$$

output
```
-1/2*a*c^3*(-b*x^2+a)^(3/4)/b^2/(c*x)^(1/2)-1/3*c*(c*x)^(3/2)*(-b*x^2+a)^(
3/4)/b+1/2*a^(3/2)*c^2*(1-a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arc
csc(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx=\frac{c(cx)^{3/2}\left(-a+bx^2+a\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{a-bx^2}}$$

input
```
Integrate[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

output
```
(c*(c*x)^(3/2)*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4
, 3/4, 7/4, (b*x^2)/a]))/(3*b*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {262, 256, 258, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}dx$$
$$\Big\downarrow \text{262}$$
$$\frac{ac^2\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}dx}{2b}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$
$$\Big\downarrow \text{256}$$
$$\frac{ac^2\left(-\frac{ac^2\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}dx}{2b}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}\right)}{2b}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$
$$\Big\downarrow \text{258}$$
$$\frac{ac^2\left(-\frac{a\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}dx}{2b\sqrt[4]{a-bx^2}}-\frac{c\left(a-bx^2\right)^{3/4}}{b\sqrt{cx}}\right)}{2b}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$
$$\Big\downarrow \text{858}$$

$$\frac{ac^2\left(\frac{a\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x}}{2b\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}\right)}{2b}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

$$\Big\downarrow \text{226}$$

$$\frac{ac^2\left(\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}\right)}{2b}-\frac{c(cx)^{3/2}\left(a-bx^2\right)^{3/4}}{3b}$$

input
```
Int[(c*x)^(5/2)/(a - b*x^2)^(1/4),x]
```

output
```
-1/3*(c*(c*x)^(3/2)*(a - b*x^2)^(3/4))/b + (a*c^2*(-((c*(a - b*x^2)^(3/4))
/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcSi
n[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[b]*(a - b*x^2)^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 256
```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[c*((a +
b*x^2)^(3/4)/(b*Sqrt[c*x])), x] + Simp[a*(c^2/(2*b))   Int[1/((c*x)^(3/2)*(
a + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 258
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 +
a/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{2}}}{(-b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.36

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right) {}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(-b*x**2+a)**(1/4),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(2
*I*pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx=\int \frac{{\left(c\,x\right)}^{5/2}}{{\left(a-b\,x^2\right)}^{1/4}} \,d x$$

input
```
int((c*x)^(5/2)/(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(5/2)/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a-bx^2}}\,dx=\sqrt{c}\,\left(\int \frac{\sqrt{x}\,x^{2}}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}d x \right) c^{2}$$

input
```
int((c*x)^(5/2)/(-b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**2)/(a - b*x**2)**(1/4),x)*c**2
```

3.1001 $\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx=-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}+\frac{\sqrt{a}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}$$

output
```
-c*(-b*x^2+a)^(3/4)/b/(c*x)^(1/2)+a^(1/2)*(1-a/b/x^2)^(1/4)*(c*x)^(1/2)*El
lipticE(sin(1/2*arccsc(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(-b*x^2+a)^(1/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx=\frac{2x\sqrt{cx}\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{bx^2}{a}\right)}{3\sqrt[4]{a-bx^2}}$$

input
```
Integrate[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

output
```
(2*x*Sqrt[c*x]*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (b*x
^2)/a])/(3*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {256, 258, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{256}$$

$$-\frac{ac^2\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}dx}{2b}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

$$\Big\downarrow \text{258}$$

$$-\frac{a\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}dx}{2b\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

$$\Big\downarrow \text{858}$$

$$\frac{a\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x}}{2b\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

$$\Big\downarrow \text{226}$$

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a-bx^2}}-\frac{c(a-bx^2)^{3/4}}{b\sqrt{cx}}$$

input
```
Int[Sqrt[c*x]/(a - b*x^2)^(1/4),x]
```

output
```
-((c*(a - b*x^2)^(3/4))/(b*Sqrt[c*x])) + (Sqrt[a]*(1 - a/(b*x^2))^(1/4)*Sq
rt[c*x]*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[b]*(a - b*x^2)^
(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 256
```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[c*((a +
b*x^2)^(3/4)/(b*Sqrt[c*x])), x] + Simp[a*(c^2/(2*b))   Int[1/((c*x)^(3/2)*(
a + b*x^2)^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 258
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 +
a/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(-b*x**2+a)**(1/4),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(-b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \int \frac{\sqrt{c\,x}}{(a-b\,x^2)^{1/4}}\,dx$$

input
```
int((c*x)^(1/2)/(a - b*x^2)^(1/4),x)
```

output
```
int((c*x)^(1/2)/(a - b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}}{(-b\,x^2+a)^{\frac{1}{4}}}dx\right)$$

input
```
int((c*x)^(1/2)/(-b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int(sqrt(x)/(a - b*x**2)**(1/4),x)
```

3.1002 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2\sqrt{b}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a-bx^2}}$$

output
```
-2*b^(1/2)*(1-a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccsc(b^(1/2)*
x/a^(1/2))),2^(1/2))/a^(1/2)/c^2/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, (b*x^2)/a])/
((c*x)^(3/2)*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {258, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{258}$$

$$\frac{\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}dx}{c^2\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x}}{c^2\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a-bx^2}}$$

input
```
Int[1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*Sqrt[b]*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcSin[Sqrt[a]/(Sqrt
[b]*x)]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 258
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 +
a/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{2}}\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 - a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.47

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{\sqrt[4]{b}c^{\frac{3}{2}}x}$$

input
```
integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(1/4),x)
```

output
```
I*exp(I*pi/4)*hyper((1/4, 1/2), (3/2,), a/(b*x**2))/(b**(1/4)*c**(3/2)*x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(3/2)*(a - b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{3}{4}}}{-b\,x^4+a\,x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a - b*x**2)**(3/4))/(a*x**2 - b*x**4),x))/c**2
```

3.1003 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}$$

output
```
-2/5*(-b*x^2+a)^(3/4)/a/c/(c*x)^(5/2)-4/5*b^(3/2)*(1-a/b/x^2)^(1/4)*(c*x)^
(1/2)*EllipticE(sin(1/2*arccsc(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/c^4/(-
b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, (b*x^2)/a])
/(5*(c*x)^(7/2)*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {264, 258, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}dx}{5ac^2}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{ 258}$$

$$\frac{2b\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}dx}{5ac^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{2b\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x}}{5ac^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{ 226}$$

$$-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*(a - b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1
/4)*Sqrt[c*x]*EllipticE[ArcSin[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(5*a^(3/2)*c^4*
(a - b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 258
```
Int[1/((((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 +
a/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{7}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 - a*c^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = -\frac{ie^{-\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3\sqrt[4]{b}c^{\frac{7}{2}}x^3}$$

input
```
integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(1/4),x)
```

output
```
-I*exp(-3*I*pi/4)*hyper((1/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(1/4)*c**(7/
2)*x**3)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx=\int \frac{1}{(c\,x)^{7/2}\,(a-b\,x^2)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(7/2)*(a - b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}\,dx=\frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(b\,x^2-a)}{5\sqrt{x}\,\sqrt{-b\,x^2+a}\,a\,c^4x^2}$$

input
```
int(1/(c*x)^(7/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - a + b*x**2))/(5*sqrt(x)*sqrt(a - b*x**2
)*a*c**4*x**2)
```

3.1004 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 130

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}-\frac{4b(a-bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}}-\frac{8b^{5/2}\sqrt[4]{1-\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a-bx^2}}$$

output
```
-2/9*(-b*x^2+a)^(3/4)/a/c/(c*x)^(9/2)-4/15*b*(-b*x^2+a)^(3/4)/a^2/c^3/(c*x
)^(5/2)-8/15*b^(5/2)*(1-a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccs
c(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/c^6/(-b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx=-\frac{2x\sqrt[4]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a-bx^2}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, (b*x^2)/a])
/(9*(c*x)^(11/2)*(a - b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {264, 264, 258, 858, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}dx$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2b\int \frac{1}{(cx)^{7/2}\sqrt[4]{a-bx^2}}dx}{3ac^2}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2b\left(\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt[4]{a-bx^2}}dx}{5ac^2}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{ 258}$$

$$\frac{2b\left(\frac{2b\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}x^2}dx}{5ac^4\sqrt[4]{a-bx^2}}-\frac{2\left(a-bx^2\right)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2\left(a-bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{2b\left(-\frac{2b\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}\int \frac{1}{\sqrt[4]{1-\frac{a}{bx^2}}}d\frac{1}{x}}{5ac^4\sqrt[4]{a-bx^2}}-\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \ 226$$

$$\frac{2b\left(-\frac{4b^{3/2}\sqrt{cx}\sqrt[4]{1-\frac{a}{bx^2}}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a-bx^2}}-\frac{2(a-bx^2)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2(a-bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

input
```
Int[1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x]
```

output
```
(-2*(a - b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) + (2*b*((-2*(a - b*x^2)^(3/4))/
(5*a*c*(c*x)^(5/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE
[ArcSin[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(5*a^(3/2)*c^4*(a - b*x^2)^(1/4))))/(3
*a*c^2)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 258
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[S
qrt[c*x]*((1 + a/(b*x^2))^(1/4)/(c^2*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 +
a/(b*x^2))^(1/4)), x], x] /; FreeQ[{a, b, c}, x] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}} \left(-b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{\left(-bx^{2}+a\right)^{\frac{1}{4}} (cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 - a*c^6*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 54.66 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \frac{ie^{\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{5\sqrt[4]{b}c^{\frac{11}{2}}x^5}$$

input
```
integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(1/4),x)
```

output
```
I*exp(I*pi/4)*hyper((1/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(1/4)*c**(11/2)*
x**5)
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \int \frac{1}{(c\,x)^{11/2}\,\left(a-b\,x^2\right)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(11/2)*(a - b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.45

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a-bx^2}}\,dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}\,(4b^2x^4+ab\,x^2-5a^2)}{45\sqrt{x}\,\sqrt{-b\,x^2+a}\,a^2c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(-b*x^2+a)^(1/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - 5*a**2 + a*b*x**2 + 4*b**2*x**4))/(45*s
qrt(x)*sqrt(a - b*x**2)*a**2*c**6*x**4)
```

3.1005 $\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 228

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = -\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b} - \frac{3ac^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} + \frac{3ac^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{4\sqrt{2}b^{7/4}} - \frac{3ac^{5/2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{4\sqrt{2}b^{7/4}}$$

output

```
-1/2*c*(c*x)^(3/2)*(-b*x^2+a)^(1/4)/b+3/8*a*c^(5/2)*arctan(-1+2^(1/2)*b^(1
/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(7/4)+3/8*a*c^(5/2)*ar
ctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4))*2^(1/2)/b^(7/
4)-3/8*a*c^(5/2)*arctanh(2^(1/2)*b^(1/4)*(c*x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(
1/2)+b^(1/2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^(1/2)/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx =$$
$$-\frac{(cx)^{5/2}\left(4b^{3/4}x^{3/2}\sqrt[4]{a-bx^2}+3\sqrt{2}a\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{\sqrt{b}x-\sqrt{a-bx^2}}\right)+3\sqrt{2}a\text{arctanh}\left(\frac{\sqrt{b}x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{8b^{7/4}x^{5/2}}$$

input
```
Integrate[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

output
```
-1/8*((c*x)^(5/2)*(4*b^(3/4)*x^(3/2)*(a - b*x^2)^(1/4) + 3*Sqrt[2]*a*ArcTa
n[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(Sqrt[b]*x - Sqrt[a - b*x^2]
)] + 3*Sqrt[2]*a*ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^(1/4)*Sq
rt[x]*(a - b*x^2)^(1/4))]))/(b^(7/4)*x^(5/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.32, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {262, 266, 854, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx$$
$$\Big\downarrow \text{262}$$
$$\frac{3ac^2\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}dx}{4b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$
$$\Big\downarrow \text{266}$$
$$\frac{3ac\int \frac{cx}{(a-bx^2)^{3/4}}d\sqrt{cx}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{854}$$

$$\frac{3ac\int \frac{c^3x}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3ac^3\int \frac{cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{826}$$

$$\frac{3ac^3\left(\frac{\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{1476}$$

$$\frac{3ac^3\left(\frac{\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3ac^3\left(\frac{\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \text{217}$$

$$\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int\frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)}{2b}-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \; 1479$$

$$\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{\ldots}\right.}{\sqrt[4]{b}\left(xc+\ldots\right.}\ldots}{\ldots}}{2\sqrt{b}}\ldots\right.}{2b}\ldots$$

$$-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{c}+\ldots\right.}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}+\ldots\right.}\ldots}{\ldots}}{2\sqrt{b}}\ldots\right.}{2b}\ldots$$

$$-\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$$\Big\downarrow \; 27$$

$$\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int\frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-b}}}}{2\sqrt{b}\sqrt{c}}}{2\sqrt{b}}\right.}{2b}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

$\Big\downarrow$ 1103

$$\frac{3ac^3\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}\right)}{2b}-$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a-bx^2}}{2b}$$

input
```
Int[(c*x)^(5/2)/(a - b*x^2)^(3/4),x]
```

output
```
-1/2*(c*(c*x)^(3/2)*(a - b*x^2)^(1/4))/b + (3*a*c^3*((-(ArcTan[1 - (Sqrt[2
]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]
)) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))]/(
Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b]) - (-1/2*Log[c + Sqrt[b]*c*x - (Sqrt[
2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(Sqrt[2]*b^(1/4)*Sqrt[c])
 + Log[c + Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(
1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{2}}}{(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.20

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(2
*I*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(c\,x)^{5/2}}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int((c*x)^(5/2)/(a - b*x^2)^(3/4),x)
```

output
```
int((c*x)^(5/2)/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a-bx^2)^{3/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^2}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)c^2$$

input
```
int((c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**2)/(a - b*x**2)**(3/4),x)*c**2
```

3.1006 $\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 191

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = -\frac{\sqrt{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} + \frac{\sqrt{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}b^{3/4}} - \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}\left(\sqrt{c}+\frac{\sqrt{b}\sqrt{cx}}{\sqrt{a-bx^2}}\right)}\right)}{\sqrt{2}b^{3/4}}$$

output
```
1/2*c^(1/2)*arctan(-1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)/(-b*x^2+a)^(1/4)
)*2^(1/2)/b^(3/4)+1/2*c^(1/2)*arctan(1+2^(1/2)*b^(1/4)*(c*x)^(1/2)/c^(1/2)
/(-b*x^2+a)^(1/4))*2^(1/2)/b^(3/4)-1/2*c^(1/2)*arctanh(2^(1/2)*b^(1/4)*(c*
x)^(1/2)/(-b*x^2+a)^(1/4)/(c^(1/2)+b^(1/2)*c^(1/2)*x/(-b*x^2+a)^(1/2)))*2^
(1/2)/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \frac{\sqrt{cx}\left(\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}{-\sqrt{b}x+\sqrt{a-bx^2}}\right)-\operatorname{arctanh}\left(\frac{\sqrt{b}x+\sqrt{a-bx^2}}{\sqrt{2}\sqrt[4]{b}\sqrt{x}\sqrt[4]{a-bx^2}}\right)\right)}{\sqrt{2}b^{3/4}\sqrt{x}}$$

input
```
Integrate[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

output
```
(Sqrt[c*x]*(ArcTan[(Sqrt[2]*b^(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))/(-(Sqrt[b]*
x) + Sqrt[a - b*x^2])] - ArcTanh[(Sqrt[b]*x + Sqrt[a - b*x^2])/(Sqrt[2]*b^
(1/4)*Sqrt[x]*(a - b*x^2)^(1/4))]))/(Sqrt[2]*b^(3/4)*Sqrt[x])
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.39, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {266, 854, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\int \frac{cx}{(a-bx^2)^{3/4}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{2\int \frac{c^3x}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$2c\int \frac{cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}$$

$$\Big\downarrow \text{826}$$

$$2c\left(\frac{\int \frac{\sqrt{b}xc+c}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1476}$$

$$2c\left(\frac{\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}+\frac{\int \frac{1}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1082}$$

$$2c\left(\frac{\frac{\int \frac{1}{-cx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int \frac{1}{-cx-1}d\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{217}$$

$$2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\int \frac{c-\sqrt{b}cx}{bx^2c^2+c^2}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{1479}$$

$$2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\int -\frac{\cdots}{\sqrt[4]{b}\cdots}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$\downarrow$ 25

$$2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{\sqrt[4]{b}\left(xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}\right)}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}+\frac{\int \frac{\sqrt{2}\left(\cdots\right)}{\sqrt[4]{b}\left(xc+\cdots\right)}\cdots}{\cdots}}{2\sqrt{b}}\cdots\right.$$

$\downarrow$ 27

$$2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{c}-\frac{2\sqrt[4]{b}\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{xc+\frac{c}{\sqrt{b}}-\frac{\sqrt{2}\sqrt{cx}\sqrt{c}}{\sqrt[4]{b}\sqrt[4]{a-bx^2}}}d\frac{\sqrt{cx}}{\sqrt[4]{a-bx^2}}}{2\sqrt{2}\sqrt{b}\sqrt{c}}+\frac{\int \frac{\sqrt{c}+\frac{\sqrt{2}\sqrt[4]{b}\cdots}{\sqrt[4]{a-\cdots}}}{xc+\frac{c}{\sqrt{b}}+\frac{\sqrt{2}\sqrt{c}\cdots}{\sqrt[4]{b}\sqrt[4]{a\cdots}}}\cdots}{2\cdots}}{2\sqrt{b}}\cdots\right.$$

$\downarrow$ 1103

$$2c\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}+1\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a-bx^2}}\right)}{\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}-\frac{\frac{\log\left(\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}-\frac{\log\left(-\frac{\sqrt{2}\sqrt[4]{b}\sqrt{c}\sqrt{cx}}{\sqrt[4]{a-bx^2}}+\sqrt{b}cx+c\right)}{2\sqrt{2}\sqrt[4]{b}\sqrt{c}}}{2\sqrt{b}}\right.$$

input
```
Int[Sqrt[c*x]/(a - b*x^2)^(3/4),x]
```

output
```
2*c*((-(ArcTan[1 - (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a - b*x^2)^(1/4))
]/(Sqrt[2]*b^(1/4)*Sqrt[c])) + ArcTan[1 + (Sqrt[2]*b^(1/4)*Sqrt[c*x])/(Sqr
t[c]*(a - b*x^2)^(1/4))]/(Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b]) - (-1/2*Lo
g[c + Sqrt[b]*c*x - (Sqrt[2]*b^(1/4)*Sqrt[c]*Sqrt[c*x])/(a - b*x^2)^(1/4)]
/(Sqrt[2]*b^(1/4)*Sqrt[c]) + Log[c + Sqrt[b]*c*x + (Sqrt[2]*b^(1/4)*Sqrt[c
]*Sqrt[c*x])/(a - b*x^2)^(1/4)]/(2*Sqrt[2]*b^(1/4)*Sqrt[c]))/(2*Sqrt[b]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.24

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(2*I
*pi)/a)/(2*a**(3/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \int \frac{\sqrt{cx}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx = \int \frac{\sqrt{c\,x}}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int((c*x)^(1/2)/(a - b*x^2)^(3/4),x)
```

output
```
int((c*x)^(1/2)/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(a-bx^2)^{3/4}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{x}}{(-b\,x^2+a)^{\frac{3}{4}}}dx\right)$$

input
```
int((c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

output
```
sqrt(c)*int(sqrt(x)/(a - b*x**2)**(3/4),x)
```

3.1007 $\int \frac{1}{(cx)^{3/2}(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 27

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

output
```
-2*(-b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a-bx^2}}{a(cx)^{3/2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2} (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \; 242$$

$$-\frac{2 \sqrt[4]{a-bx^2}}{ac\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{a\left(c x\right)^{\frac{3}{2}}}$	22
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}}{a\left(c x\right)^{\frac{3}{2}}}$	22
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}}{\sqrt{c x}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}} c a}$	51

input
```
int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2*x*(-b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{2\left(-bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.33

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx = \begin{cases} \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{for}\ \left|\frac{a}{bx^2}\right|>1 \\ -\frac{\sqrt[4]{b}\sqrt[4]{-\frac{a}{bx^2}+1}\,e^{-\frac{3i\pi}{4}}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

output
```
Piecewise((b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamm
a(3/4)), Abs(a/(b*x**2)) > 1), (-b**(1/4)*(-a/(b*x**2) + 1)**(1/4)*exp(-3*
I*pi/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4)), True))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\left(a-bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}\left(cx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{1}{(cx)^{3/2}\,(a-bx^2)^{3/4}}\,dx = -\frac{2\,(a-b\,x^2)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(3/2)*(a - b*x^2)^(3/4)),x)
```

output
```
-(2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{3/2}\,(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{1}{4}}}{\sqrt{x}\,a\,c^2}$$

input
```
int(1/(c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

output
```
( - 2*sqrt(c)*(a - b*x**2)**(1/4))/(sqrt(x)*a*c**2)
```

3.1008 $\int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}} dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}} dx = -\frac{2\sqrt[4]{a-bx^2}}{5ac(cx)^{5/2}} - \frac{8b\sqrt[4]{a-bx^2}}{5a^2c^3\sqrt{cx}}$$

output
```
-2/5*(-b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)-8/5*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

$$\int \frac{1}{(cx)^{7/2}(a-bx^2)^{3/4}} dx = -\frac{2x\sqrt[4]{a-bx^2}(a+4bx^2)}{5a^2(cx)^{7/2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*(a + 4*b*x^2))/(5*a^2*(c*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2} (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{4 \int \frac{\sqrt[4]{a-bx^2}}{(cx)^{7/2}}dx}{a}-\frac{2 \sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{8 \left(a-bx^2\right)^{5/4}}{5a^2c(cx)^{5/2}}-\frac{2 \sqrt[4]{a-bx^2}}{ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a - b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(5/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(4b\,x^{2}+a\right)}{5a^{2}\left(cx\right)^{\frac{7}{2}}}$	30
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(4b\,x^{2}+a\right)}{5a^{2}\left(cx\right)^{\frac{7}{2}}}$	30
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(4b\,x^{2}+a\right)}{5\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{3}a^{2}x^{2}}$	62

input
```
int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(-b*x^2+a)^(1/4)*(4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2\,(4\,bx^2+a)(-bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2/5*(4*b*x^2 + a)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.88 (sec) , antiderivative size = 352, normalized size of antiderivative = 5.87

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx = \begin{cases} -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}-1}\,\Gamma\left(-\frac{5}{4}\right)}{2a^2c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)} \\ -\frac{a^2b^{\frac{5}{4}}\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} - \frac{3ab^{\frac{9}{4}}x^2\sqrt[4]{-\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{5}{4}\right)}{-8a^3bc^{\frac{7}{2}}x^2e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)+8a^2b^2c^{\frac{7}{2}}x^4e^{\frac{3i\pi}{4}}\Gamma\left(\frac{3}{4}\right)} + - \end{cases}$$

input
```
integrate(1/(c*x)**(7/2)/(-b*x**2+a)**(3/4),x)
```

output
```
Piecewise((-b**(1/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**
2*gamma(3/4)) - b**(5/4)*(a/(b*x**2) - 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7
/2)*gamma(3/4)), Abs(a/(b*x**2)) > 1), (-a**2*b**(5/4)*(-a/(b*x**2) + 1)**
(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamma(3/4) + 8*a*
*2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) - 3*a*b**(9/4)*x**2*(-a/(b
*x**2) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*I*pi/4)*gamm
a(3/4) + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)) + 4*b**(13/4)
*x**4*(-a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(-8*a**3*b*c**(7/2)*x**2*exp(3*
I*pi/4)*gamma(3/4) + 8*a**2*b**2*c**(7/2)*x**4*exp(3*I*pi/4)*gamma(3/4)),
True))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{\left(a-b\,x^2\right)^{1/4}\left(\frac{2}{5\,a\,c^3}+\frac{8\,b\,x^2}{5\,a^2\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(7/2)*(a - b*x^2)^(3/4)),x)
```

output
```
-((a - b*x^2)^(1/4)*(2/(5*a*c^3) + (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{7/2}\left(a-bx^2\right)^{3/4}}\,dx=\frac{2\sqrt{c}\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(-4b\,x^2-a\right)}{5\sqrt{x}\,a^2c^4x^2}$$

input
```
int(1/(c*x)^(7/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - a - 4*b*x**2))/(5*sqrt(x)*a**2*c**4*x**
2)
```

3.1009 $\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{9ac(cx)^{9/2}} - \frac{16b\sqrt[4]{a-bx^2}}{45a^2c^3(cx)^{5/2}} - \frac{64b^2\sqrt[4]{a-bx^2}}{45a^3c^5\sqrt{cx}}$$

output
```
-2/9*(-b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)-16/45*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c*
x)^(5/2)-64/45*b^2*(-b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{11/2}(a-bx^2)^{3/4}}\,dx = -\frac{2x\sqrt[4]{a-bx^2}(5a^2+8abx^2+32b^2x^4)}{45a^3(cx)^{11/2}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a - b*x^2)^(1/4)*(5*a^2 + 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*(c*x)^(1
1/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2} (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{8\int \frac{\sqrt[4]{a-bx^2}}{(cx)^{11/2}}dx}{a}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{8\left(-\frac{4\int \frac{\left(a-bx^2\right)^{5/4}}{(cx)^{11/2}}dx}{5a}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}\right)}{a}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{8\left(\frac{8\left(a-bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2\left(a-bx^2\right)^{5/4}}{5ac(cx)^{9/2}}\right)}{a}-\frac{2\sqrt[4]{a-bx^2}}{ac(cx)^{9/2}}$$

input
```
Int[1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) - (8*((-2*(a - b*x^2)^(5/4))/(5*a
*c*(c*x)^(9/2)) + (8*(a - b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}+8ab\,x^{2}+5a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{11}{2}}}$	43
orering	$-\frac{2x\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}+8ab\,x^{2}+5a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{11}{2}}}$	43
risch	$-\frac{2\left(-b\,x^{2}+a\right)^{\frac{1}{4}}\left(\left(-b\,x^{2}+a\right)^{3}\right)^{\frac{1}{4}}\left(32b^{2}x^{4}+8ab\,x^{2}+5a^{2}\right)}{45\sqrt{cx}\,\left(-\left(b\,x^{2}-a\right)^{3}\right)^{\frac{1}{4}}c^{5}a^{3}x^{4}}$	75

input
```
int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(-b*x^2+a)^(1/4)*(32*b^2*x^4+8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{(cx)^{11/2} (a-bx^2)^{3/4}} \, dx = -\frac{2 \left(32 \, b^2 x^4 + 8 \, a b x^2 + 5 \, a^2\right) (-b x^2 + a)^{\frac{1}{4}} \sqrt{c x}}{45 \, a^3 c^6 x^5}$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2/45*(32*b^2*x^4 + 8*a*b*x^2 + 5*a^2)*(-b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c
^6*x^5)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 108.39 (sec) , antiderivative size = 1263, normalized size of antiderivative = 13.73

$$\int \frac{1}{(cx)^{11/2} (a-bx^2)^{3/4}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(c*x)**(11/2)/(-b*x**2+a)**(3/4),x)
```

output
```
Piecewise((-5*a**4*b**(17/4)*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9
/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c
**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3
*I*pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) - 1)**(1/4)*exp(-
I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4)
- 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(
11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 21*a**2*b**(25/4)*x**4*(a/(b*x**2)
- 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*
pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) +
32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) + 56*a*b**(29/4)*x**
6*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)
*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*exp(3*I*pi/4)
*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(3/4)) - 32*b
**(33/4)*x**8*(a/(b*x**2) - 1)**(1/4)*exp(-I*pi/4)*gamma(-9/4)/(32*a**5*b*
*4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4*b**5*c**(11/2)*x**6*e
xp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*exp(3*I*pi/4)*gamma(
3/4)), Abs(a/(b*x**2)) > 1), (-5*a**4*b**(17/4)*(-a/(b*x**2) + 1)**(1/4)*g
amma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*exp(3*I*pi/4)*gamma(3/4) - 64*a**4
*b**5*c**(11/2)*x**6*exp(3*I*pi/4)*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**
8*exp(3*I*pi/4)*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(-a/(b*x**2) + 1)**...
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{\left(a-b\,x^2\right)^{1/4}\left(\frac{2}{9\,a\,c^5}+\frac{16\,b\,x^2}{45\,a^2\,c^5}+\frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(11/2)*(a - b*x^2)^(3/4)),x)
```

output
```
-((a - b*x^2)^(1/4)*(2/(9*a*c^5) + (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/
(45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{11/2}\left(a-bx^2\right)^{3/4}}\,dx=\frac{2\sqrt{c}\left(-b\,x^2+a\right)^{\frac{1}{4}}\left(-32b^2x^4-8ab\,x^2-5a^2\right)}{45\sqrt{x}\,a^3c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(1/4)*( - 5*a**2 - 8*a*b*x**2 - 32*b**2*x**4))/(4
5*sqrt(x)*a**3*c**6*x**4)
```

3.1010 $\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = -\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b} - \frac{\sqrt{a}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a-bx^2)^{3/4}}$$

output
```
-c*(c*x)^(1/2)*(-b*x^2+a)^(1/4)/b-a^(1/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*In
verseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(-b*x^2+a)^(3
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.75

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \frac{c\sqrt{cx}\left(-a+bx^2+a\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx^2}{a}\right)\right)}{b\,(a-bx^2)^{3/4}}$$

input
```
Integrate[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

output
```
(c*Sqrt[c*x]*(-a + b*x^2 + a*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4,
3/4, 5/4, (b*x^2)/a]))/(b*(a - b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {262, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{ac^2\int \frac{1}{\sqrt{cx}(a-bx^2)^{3/4}}dx}{2b}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{ac\int \frac{1}{(a-bx^2)^{3/4}}d\sqrt{cx}}{b}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{ac(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{b\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{ac(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{b\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{ac(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{2b\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

$$\Big\downarrow \text{230}$$

$$-\frac{\sqrt{a}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a-bx^2\right)^{3/4}}-\frac{c\sqrt{cx}\sqrt[4]{a-bx^2}}{b}$$

input
```
Int[(c*x)^(3/2)/(a - b*x^2)^(3/4),x]
```

output
```
-((c*Sqrt[c*x]*(a - b*x^2)^(1/4))/b) - (Sqrt[a]*(1 - a/(b*x^2))^(3/4)*(c*x
)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a - b*x
^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{2}}}{\left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)*c*x/(b*x^2 - a), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{bx^2e^{2i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(-b*x**2+a)**(3/4),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(2*
I*pi)/a)/(2*a**(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(-bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(-b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \int \frac{(c\,x)^{3/2}}{(a-b\,x^2)^{3/4}}\,dx$$

input
```
int((c*x)^(3/2)/(a - b*x^2)^(3/4),x)
```

output
```
int((c*x)^(3/2)/(a - b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a-bx^2)^{3/4}}\,dx = \frac{\sqrt{c}\,c\left(-2\sqrt{x}\,(-b\,x^2+a)^{\frac{1}{4}}+\left(\int \frac{(-b\,x^2+a)^{\frac{1}{4}}}{\sqrt{x}\,a-\sqrt{x}\,b\,x^2}dx\right)a\right)}{2b}$$

input
```
int((c*x)^(3/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(sqrt(c)*c*( - 2*sqrt(x)*(a - b*x**2)**(1/4) + int((a - b*x**2)**(1/4)/(sq
rt(x)*a - sqrt(x)*b*x**2),x)*a))/(2*b)
```

3.1011 $\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 68

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt{b}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}c^2\,(a-bx^2)^{3/4}}$$

output
```
-2*b^(1/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccsc(b^(1/2
)*x/a^(1/2)),2^(1/2))/a^(1/2)/c^2/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{bx^2}{a}\right)}{\sqrt{cx}\,(a-bx^2)^{3/4}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

output
```
(2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (b*x^2)/a])/(S
qrt[c*x]*(a - b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{(a-bx^2)^{3/4}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$\frac{2(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{c\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{c\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{c\left(a-bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^2\left(a-bx^2\right)^{3/4}}$$

input
```
Int[1/(Sqrt[c*x]*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*Sqrt[b]*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^
2*x)/Sqrt[b]]/2, 2])/(Sqrt[a]*c^2*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,(-b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 - a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \frac{ie^{-\frac{i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{cx}}$$

input
```
integrate(1/(c*x)**(1/2)/(-b*x**2+a)**(3/4),x)
```

output
```
I*exp(-I*pi/4)*hyper((1/2, 3/4), (3/2,), a/(b*x**2))/(b**(3/4)*sqrt(c)*x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(1/2)*(a - b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(a-bx^2)^{3/4}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(-b\,x^2+a)^{\frac{5}{4}}}{b^2x^5-2ab\,x^3+a^2x}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a - b*x**2)**(5/4))/(a**2*x - 2*a*b*x**3 + b**2*x**
5),x))/c
```

3.1012 $\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}} - \frac{4b^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\mathrm{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}}$$

output
```
-2/3*(-b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)-4/3*b^(3/2)*(1-a/b/x^2)^(3/4)*(c*x)^
(3/2)*InverseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/c^4/(
-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.57

$$\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx = -\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},\frac{bx^2}{a}\right)}{3(cx)^{5/2}(a-bx^2)^{3/4}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, (b*x^2)/a])/
(3*(c*x)^(5/2)*(a - b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {264, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\int \frac{1}{\sqrt{cx}(a-bx^2)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{4b\int \frac{1}{(a-bx^2)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{768}$$

$$\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3
/4)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2
)*c^4*(a - b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} \left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/2} \left(a-bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}} (cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 - a*c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.39

$$\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx = -\frac{ie^{\frac{3i\pi}{4}}\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{a}{bx^2}\right)}{3b^{\frac{3}{4}}c^{\frac{5}{2}}x^3}$$

input
```
integrate(1/(c*x)**(5/2)/(-b*x**2+a)**(3/4),x)
```

output
```
-I*exp(3*I*pi/4)*hyper((3/4, 3/2), (5/2,), a/(b*x**2))/(3*b**(3/4)*c**(5/2
)*x**3)
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(5/2)*(a - b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.38

$$\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}\,dx = -\frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{3}{4}}}{3\sqrt{x}\,\sqrt{-b\,x^2+a}\,a\,c^3x}$$

input
```
int(1/(c*x)^(5/2)/(-b*x^2+a)^(3/4),x)
```

output
```
( - 2*sqrt(c)*(a - b*x**2)**(3/4))/(3*sqrt(x)*sqrt(a - b*x**2)*a*c**3*x)
```

3.1013 $\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}} dx$

Optimal result

Integrand size = 20, antiderivative size = 130

$$\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}} dx = -\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}} - \frac{4b\sqrt[4]{a-bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6(a-bx^2)^{3/4}}$$

output
```
-2/7*(-b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)-4/7*b*(-b*x^2+a)^(1/4)/a^2/c^3/(c*x)
^(3/2)-8/7*b^(5/2)*(1-a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccs
c(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(5/2)/c^6/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}} dx = -\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},\frac{bx^2}{a}\right)}{7(cx)^{9/2}(a-bx^2)^{3/4}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, (b*x^2)/a])
/(7*(c*x)^(9/2)*(a - b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {264, 264, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}dx$$

$$\Big\downarrow \ 264$$

$$\frac{6b\int \frac{1}{(cx)^{5/2}(a-bx^2)^{3/4}}dx}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 264$$

$$\frac{6b\left(\frac{2b\int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 266$$

$$\frac{6b\left(\frac{4b\int \frac{1}{\left(a-bx^2\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 768$$

$$\frac{6b\left(\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{6b\left(-\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{6b\left(-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3ac^3(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{230}$$

$$\frac{6b\left(-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4(a-bx^2)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}$$

input
```
Int[1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) + (6*b*((-2*(a - b*x^2)^(1/4))/
(3*a*c*(c*x)^(3/2)) - (4*b^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*Ellipti
cF[ArcSin[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a - b*x^2)^(3/4)
)))/(7*a*c^2)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{9}{2}} \left(-b\,x^{2}+a\right)^{\frac{3}{4}}}d x$$

input
```
int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}} (cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 - a*c^5*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.28

$$\int \frac{1}{(cx)^{9/2}\left(a-bx^2\right)^{3/4}}\,dx = \frac{ie^{-\frac{i\pi}{4}} {}_{2}F_{1}\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{a}{bx^{2}}} \right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^{5}}$$

input
```
integrate(1/(c*x)**(9/2)/(-b*x**2+a)**(3/4),x)
```

output
```
I*exp(-I*pi/4)*hyper((3/4, 5/2), (7/2,), a/(b*x**2))/(5*b**(3/4)*c**(9/2)*
x**5)
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(-bx^2+a)^{\frac{3}{4}}\,(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/2}\,(a-bx^2)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{9/2}\,(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(9/2)*(a - b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.37

$$\int \frac{1}{(cx)^{9/2} (a-bx^2)^{3/4}} \, dx = \frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{3}{4}}\,(-4b\,x^2-3a)}{21\sqrt{x}\,\sqrt{-b\,x^2+a}\,a^2c^5x^3}$$

input
```
int(1/(c*x)^(9/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(3/4)*( - 3*a - 4*b*x**2))/(21*sqrt(x)*sqrt(a - b
*x**2)*a**2*c**5*x**3)
```

3.1014 $\int \frac{1}{(cx)^{13/2}(a-bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 162

$$\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}-\frac{20b\sqrt[4]{a-bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a-bx^2}}{77a^3c^5(cx)^{3/2}}-\frac{80b^{7/2}\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\csc^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a-bx^2\right)^{3/4}}$$

output
```
-2/11*(-b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)-20/77*b*(-b*x^2+a)^(1/4)/a^2/c^3/(
c*x)^(7/2)-40/77*b^2*(-b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)-80/77*b^(7/2)*(1
-a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccsc(b^(1/2)*x/a^(1/2)),
2^(1/2))/a^(7/2)/c^8/(-b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.35

$$\int \frac{1}{(cx)^{13/2}\left(a-bx^2\right)^{3/4}}\,dx=-\frac{2x\left(1-\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a-bx^2\right)^{3/4}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 - (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, (b*x^2)/a]
)/(11*(c*x)^(13/2)*(a - b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {264, 264, 264, 266, 768, 858, 807, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2} (a-bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$\frac{10b \int \frac{1}{(cx)^{9/2}(a-bx^2)^{3/4}}dx}{11ac^2} - \frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{10b \left(\frac{6b \int \frac{1}{(cx)^{5/2}\left(a-bx^2\right)^{3/4}}dx}{7ac^2} - \frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2} - \frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{10b \left(\frac{6b \left(\frac{2b \int \frac{1}{\sqrt{cx}\left(a-bx^2\right)^{3/4}}dx}{3ac^2} - \frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2} - \frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2} - \frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{10b\left(\frac{6b\left(\frac{4b\int\frac{1}{\left(a-bx^2\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$\Big\downarrow$ 768

$$\frac{10b\left(\frac{6b\left(\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int\frac{1}{\left(1-\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$\Big\downarrow$ 858

$$\frac{10b\left(\frac{6b\left(-\frac{4b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int\frac{1}{\sqrt{cx}\left(1-\frac{ac^4x^2}{b}\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$\Big\downarrow$ 807

$$\frac{10b\left(\frac{6b\left(-\frac{2b(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\int\frac{1}{\left(1-\frac{ac^3x}{b}\right)^{3/4}}d(cx)}{3ac^3\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

$\Big\downarrow$ 230

$$\frac{10b\left(\frac{6b\left(-\frac{4b^{3/2}(cx)^{3/2}\left(1-\frac{a}{bx^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4\left(a-bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a-bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a-bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a-bx^2}}{11ac(cx)^{11/2}}$$

input
```
Int[1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x]
```

output
```
(-2*(a - b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) + (10*b*((-2*(a - b*x^2)^(1/4
))/(7*a*c*(c*x)^(7/2)) + (6*b*((-2*(a - b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2))
- (4*b^(3/2)*(1 - a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcSin[(Sqrt[a]*c
^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a - b*x^2)^(3/4))))/(7*a*c^2)))/(11*
a*c^2)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{13}{2}} \left(-b\,x^{2}+a\right)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{13/2} \left(a-bx^2\right)^{3/4}}\, dx = \int \frac{1}{\left(-bx^{2}+a\right)^{\frac{3}{4}} (cx)^{\frac{13}{2}}}\, dx$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 - a*c^7*x^7), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2} (a - bx^2)^{3/4}} \, dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(13/2)/(-b*x**2+a)**(3/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2} (a - bx^2)^{3/4}} \, dx = \int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{13}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2} (a - bx^2)^{3/4}} \, dx = \int \frac{1}{(-bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{13}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((-b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2}\,(a-bx^2)^{3/4}}\,dx=\int \frac{1}{(c\,x)^{13/2}\,(a-b\,x^2)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(13/2)*(a - b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{13/2}\,(a-bx^2)^{3/4}}\,dx=\frac{2\sqrt{c}\,(-b\,x^2+a)^{\frac{3}{4}}\,(-32b^2x^4-24ab\,x^2-21a^2)}{231\sqrt{x}\,\sqrt{-b\,x^2+a}\,a^3c^7x^5}$$

input
```
int(1/(c*x)^(13/2)/(-b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a - b*x**2)**(3/4)*( - 21*a**2 - 24*a*b*x**2 - 32*b**2*x**4))/
(231*sqrt(x)*sqrt(a - b*x**2)*a**3*c**7*x**5)
```

3.1015 $\int (cx)^{5/2} \sqrt[4]{a+bx^2}\, dx$

Optimal result

Integrand size = 19, antiderivative size = 147

$$\int (cx)^{5/2} \sqrt[4]{a+bx^2}\, dx = \frac{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}{16b} + \frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c} + \frac{3a^2c^{5/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}} - \frac{3a^2c^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{32b^{7/4}}$$

output
```
1/16*a*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+1/4*(c*x)^(7/2)*(b*x^2+a)^(1/4)/c+3
/32*a^2*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(7/4
)-3/32*a^2*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^
(7/4)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.74

$$\int (cx)^{5/2} \sqrt[4]{a+bx^2}\, dx = \frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}(a+4bx^2) + 3a^2\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) - 3a^2\text{arctanh}\left(\right.}{32b^{7/4}x^{5/2}}$$

input
```
Integrate[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

output
```
((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4)*(a + 4*b*x^2) + 3*a^2*Ar
cTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] - 3*a^2*ArcTanh[(b^(1/4)*Sqrt[x]
)/(a + b*x^2)^(1/4)]))/(32*b^(7/4)*x^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {248, 262, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{5/2} \sqrt[4]{a+bx^2} \, dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{8} a \int \frac{(cx)^{5/2}}{(bx^2+a)^{3/4}} dx + \frac{(cx)^{7/2} \sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{8} a \left(\frac{c(cx)^{3/2} \sqrt[4]{a+bx^2}}{2b} - \frac{3ac^2 \int \frac{\sqrt{cx}}{(bx^2+a)^{3/4}} dx}{4b} \right) + \frac{(cx)^{7/2} \sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{8} a \left(\frac{c(cx)^{3/2} \sqrt[4]{a+bx^2}}{2b} - \frac{3ac \int \frac{cx}{(bx^2+a)^{3/4}} d\sqrt{cx}}{2b} \right) + \frac{(cx)^{7/2} \sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{854}$$

$$\frac{1}{8} a \left(\frac{c(cx)^{3/2} \sqrt[4]{a+bx^2}}{2b} - \frac{3ac \int \frac{c^3 x}{c^2-bc^2x^2} d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2b} \right) + \frac{(cx)^{7/2} \sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8} a \left(\frac{c(cx)^{3/2} \sqrt[4]{a+bx^2}}{2b} - \frac{3ac^3 \int \frac{cx}{c^2-bc^2x^2} d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2b} \right) + \frac{(cx)^{7/2} \sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{8}a\left(\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{218}$$

$$\frac{1}{8}a\left(\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{8}a\left(\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{2b}\right)+\frac{(cx)^{7/2}\sqrt[4]{a+bx^2}}{4c}$$

input
```
Int[(c*x)^(5/2)*(a + b*x^2)^(1/4),x]
```

output
```
((c*x)^(7/2)*(a + b*x^2)^(1/4))/(4*c) + (a*((c*(c*x)^(3/2)*(a + b*x^2)^(1/
4))/(2*b) - (3*a*c^3*(-1/2*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)
^(1/4))]/(b^(3/4)*Sqrt[c]) + ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x
^2)^(1/4))]/(2*b^(3/4)*Sqrt[c])))/(2*b)))/8
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int (cx)^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}}dx$$

input
```
int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.31

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \frac{\sqrt[4]{a}c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*c**(5/2)*x**(7/2)*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(11/4))
```

Maxima [F]

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Giac [F]

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}\,dx$$

input
```
integrate((c*x)^(5/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx=\int (c\,x)^{5/2}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int((c*x)^(5/2)*(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(5/2)*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int (cx)^{5/2}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt{c}\,c^2\left(2\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,ax+8\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,b\,x^3-3\left(\int \frac{\sqrt{x}}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a^2\right)}{32b}$$

input
```
int((c*x)^(5/2)*(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(x)*(a + b*x**2)**(1/4)*a*x + 8*sqrt(x)*(a + b*x**2)*
*(1/4)*b*x**3 - 3*int((sqrt(x)*(a + b*x**2)**(1/4))/(a + b*x**2),x)*a**2))
/(32*b)
```

3.1016 $\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx = \frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c} - \frac{a\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}} + \frac{a\sqrt{c}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{3/4}}$$

output
```
1/2*(c*x)^(3/2)*(b*x^2+a)^(1/4)/c-1/4*a*c^(1/2)*arctan(b^(1/4)*(c*x)^(1/2)
/c^(1/2)/(b*x^2+a)^(1/4))/b^(3/4)+1/4*a*c^(1/2)*arctanh(b^(1/4)*(c*x)^(1/2
)/c^(1/2)/(b*x^2+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx = \frac{\sqrt{cx}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2} - a\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{3/4}\sqrt{x}}$$

input
```
Integrate[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

output
```
(Sqrt[c*x]*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) - a*ArcTan[(b^(1/4)*Sqrt[x
])/(a + b*x^2)^(1/4)] + a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(
4*b^(3/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {248, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{4}a\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{3/4}}dx+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \text{266}$$

$$\frac{a\int \frac{cx}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{2c}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \text{854}$$

$$\frac{a\int \frac{c^3x}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2c}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}ac\int \frac{cx}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{2}ac\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \; 218$$

$$\frac{1}{2}ac\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}ac\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)+\frac{(cx)^{3/2}\sqrt[4]{a+bx^2}}{2c}$$

input
```
Int[Sqrt[c*x]*(a + b*x^2)^(1/4),x]
```

output
```
((c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*c) + (a*c*(-1/2*ArcTan[(b^(1/4)*Sqrt[c*
x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[c]) + ArcTanh[(b^(1/4)*Sqrt
[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[c])))/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}dx$$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.40

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx = \frac{\sqrt[4]{a}\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*sqrt(c)*x**(3/2)*gamma(3/4)*hyper((-1/4, 3/4), (7/4,), b*x**2*exp
_polar(I*pi)/a)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{4}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Giac [F]

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx=\int (bx^2+a)^{\frac{1}{4}}\sqrt{cx}\,dx$$

input
```
integrate((c*x)^(1/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx=\int \sqrt{c\,x}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int((c*x)^(1/2)*(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(1/2)*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \sqrt{cx}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt{c}\left(2\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,x+\left(\int \frac{\sqrt{x}}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)a\right)}{4}$$

input
```
int((c*x)^(1/2)*(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*(a + b*x**2)**(1/4)*x + int((sqrt(x)*(a + b*x**2)**(1/
4))/(a + b*x**2),x)*a))/4
```

3.1017 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}-\frac{\sqrt[4]{b}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}+\frac{\sqrt[4]{b}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{c^{3/2}}$$

output
```
-2*(b*x^2+a)^(1/4)/c/(c*x)^(1/2)-b^(1/4)*arctan(b^(1/4)*(c*x)^(1/2)/c^(1/2
)/(b*x^2+a)^(1/4))/c^(3/2)+b^(1/4)*arctanh(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*
x^2+a)^(1/4))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx=\frac{x\left(-2\sqrt[4]{a+bx^2}-\sqrt[4]{b}\sqrt{x}\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)+\sqrt[4]{b}\sqrt{x}\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{(cx)^{3/2}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

output
```
(x*(-2*(a + b*x^2)^(1/4) - b^(1/4)*Sqrt[x]*ArcTan[(b^(1/4)*Sqrt[x])/(a + b
*x^2)^(1/4)] + b^(1/4)*Sqrt[x]*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)
]))/(c*x)^(3/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {247, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{\sqrt{cx}}{(bx^2+a)^{3/4}}dx}{c^2}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2b\int \frac{cx}{(bx^2+a)^{3/4}}d\sqrt{cx}}{c^3}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{854}$$

$$\frac{2b\int \frac{c^3x}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{c^3}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{cx}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{c}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \text{827}$$

$$\frac{2b\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{c}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 218$$

$$\frac{2b\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{c}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

$$\Big\downarrow \; 221$$

$$\frac{2b\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{c}-\frac{2\sqrt[4]{a+bx^2}}{c\sqrt{cx}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(3/2),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(c*Sqrt[c*x]) + (2*b*(-1/2*ArcTan[(b^(1/4)*Sqrt[c*x
])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[c]) + ArcTanh[(b^(1/4)*Sqrt[
c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[c])))/c
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{(b\,x^2 + a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

output
```
int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt[4]{a}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2c^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(3/2),x)
```

output
```
a**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), b*x**2*exp_polar(I*pi)/a)
/(2*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{(c\,x)^{3/2}}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(3/2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{x^2}dx\right)}{c^2}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a + b*x**2)**(1/4))/x**2,x))/c**2
```

3.1018 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{5/2}}$$

output
```
-2/5*(b*x^2+a)^(5/4)/a/c/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=-\frac{2x(a+bx^2)^{5/4}}{5a(cx)^{7/2}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

output
```
(-2*x*(a + b*x^2)^(5/4))/(5*a*(c*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}} \, dx$$

$$\Big\downarrow \; 242$$

$$-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{5/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(7/2),x]
```

output
```
(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(5/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2x(b\,x^2+a)^{\frac{5}{4}}}{5a(cx)^{\frac{7}{2}}}$	21
orering	$-\frac{2x(b\,x^2+a)^{\frac{5}{4}}}{5a(cx)^{\frac{7}{2}}}$	21
risch	$-\frac{2(b\,x^2+a)^{\frac{5}{4}}}{5c^3\sqrt{cx}\,x^2a}$	26

input
```
int((b*x^2+a)^(1/4)/(c*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(b*x^2+a)^(5/4)/a/(c*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\,(bx^2+a)^{\frac{5}{4}}\sqrt{cx}}{5\,ac^4x^3}$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/5*(b*x^2 + a)^(5/4)*sqrt(c*x)/(a*c^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(24) = 48.

Time = 4.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.79

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx = \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2c^{\frac{7}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2ac^{\frac{7}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(7/2),x)
```

output
```
b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*c**(7/2)*x**2*gamma(-1/4))
 + b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a*c**(7/2)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=\int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx=-\frac{(b\,x^2+a)^{1/4}\left(\frac{2}{5\,c^3}+\frac{2\,b\,x^2}{5\,a\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(7/2),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(5*c^3) + (2*b*x^2)/(5*a*c^3)))/(x^2*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{7/2}}\,dx = -\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{5}{4}}}{5\sqrt{x}\,a\,c^4x^2}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(7/2),x)
```

output
```
( - 2*sqrt(c)*(a + b*x**2)**(1/4)*(a + b*x**2))/(5*sqrt(x)*a*c**4*x**2)
```

3.1019 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{9ac(cx)^{9/2}} + \frac{8b(a+bx^2)^{5/4}}{45a^2c^3(cx)^{5/2}}$$

output
```
-2/9*(b*x^2+a)^(5/4)/a/c/(c*x)^(9/2)+8/45*b*(b*x^2+a)^(5/4)/a^2/c^3/(c*x)^
(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{2x\sqrt[4]{a+bx^2}(5a^2+abx^2-4b^2x^4)}{45a^2(cx)^{11/2}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*(5*a^2 + a*b*x^2 - 4*b^2*x^4))/(45*a^2*(c*x)^(11/2
))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}} \, dx$$

$$\Big\downarrow \ 246$$

$$-\frac{4\int \frac{(bx^2+a)^{5/4}}{(cx)^{11/2}}dx}{5a}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

$$\Big\downarrow \ 242$$

$$\frac{8(a+bx^2)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{9/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(11/2),x]
```

output
```
(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2
*c*(c*x)^(9/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(-4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	31
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(-4b\,x^{2}+5a\right)}{45a^{2}\left(cx\right)^{\frac{11}{2}}}$	31
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b^{2}x^{4}+ab\,x^{2}+5a^{2}\right)}{45c^{5}\sqrt{cx}\,x^{4}a^{2}}$	46

input
```
int((b*x^2+a)^(1/4)/(c*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(b*x^2+a)^(5/4)*(-4*b*x^2+5*a)/a^2/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = \frac{2\,(4\,b^2x^4-abx^2-5\,a^2)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^2c^6x^5}$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="fricas")
```

output
```
2/45*(4*b^2*x^4 - a*b*x^2 - 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^6*x^
5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(51) = 102$.

Time = 40.58 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{5\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{9}{4}\right)}{8c^{\frac{11}{2}}x^4\Gamma\left(-\frac{1}{4}\right)} - \frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{9}{4}\right)}{8ac^{\frac{11}{2}}x^2\Gamma\left(-\frac{1}{4}\right)} + \frac{b^{\frac{9}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{9}{4}\right)}{2a^2c^{\frac{11}{2}}\Gamma\left(-\frac{1}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(11/2),x)
```

output
```
-5*b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*c**(11/2)*x**4*gamma(-1
/4)) - b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(8*a*c**(11/2)*x**2*ga
mma(-1/4)) + b**(9/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(2*a**2*c**(11/2
)*gamma(-1/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(11/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(11/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2}{9\,c^5}+\frac{2\,b\,x^2}{45\,a\,c^5}-\frac{8\,b^2\,x^4}{45\,a^2\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(11/2),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(9*c^5) + (2*b*x^2)/(45*a*c^5) - (8*b^2*x^4)/(45*a^
2*c^5)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{11/2}}\,dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(4b^2x^4-ab\,x^2-5a^2)}{45\sqrt{x}\,a^2c^6x^4}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(11/2),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 5*a**2 - a*b*x**2 + 4*b**2*x**4))/(45*s
qrt(x)*a**2*c**6*x**4)
```

3.1020 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{13ac(cx)^{13/2}}+\frac{16b(a+bx^2)^{5/4}}{117a^2c^3(cx)^{9/2}}-\frac{64b^2(a+bx^2)^{5/4}}{585a^3c^5(cx)^{5/2}}$$

output
```
-2/13*(b*x^2+a)^(5/4)/a/c/(c*x)^(13/2)+16/117*b*(b*x^2+a)^(5/4)/a^2/c^3/(c
*x)^(9/2)-64/585*b^2*(b*x^2+a)^(5/4)/a^3/c^5/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{2x(a+bx^2)^{5/4}(45a^2-40abx^2+32b^2x^4)}{585a^3(cx)^{15/2}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

output
```
(-2*x*(a + b*x^2)^(5/4)*(45*a^2 - 40*a*b*x^2 + 32*b^2*x^4))/(585*a^3*(c*x)
^(15/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}} \, dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{8\int \frac{(bx^2+a)^{5/4}}{(cx)^{15/2}}dx}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{8\left(-\frac{4\int \frac{(bx^2+a)^{9/4}}{(cx)^{15/2}}dx}{9a}-\frac{2\left(a+bx^2\right)^{9/4}}{9ac(cx)^{13/2}}\right)}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

$$\Big\downarrow \text{ 242}$$

$$-\frac{8\left(\frac{8\left(a+bx^2\right)^{13/4}}{117a^2c(cx)^{13/2}}-\frac{2\left(a+bx^2\right)^{9/4}}{9ac(cx)^{13/2}}\right)}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{13/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(15/2),x]
```

output
```
(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(13/2)) - (8*((-2*(a + b*x^2)^(9/4))/(
9*a*c*(c*x)^(13/2)) + (8*(a + b*x^2)^(13/4))/(117*a^2*c*(c*x)^(13/2))))/(5
*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}-40ab\,x^{2}+45a^{2}\right)}{585a^{3}\left(cx\right)^{\frac{15}{2}}}$	42
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{5}{4}}\left(32b^{2}x^{4}-40ab\,x^{2}+45a^{2}\right)}{585a^{3}\left(cx\right)^{\frac{15}{2}}}$	42
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{3}x^{6}-8a\,b^{2}x^{4}+5a^{2}b\,x^{2}+45a^{3}\right)}{585c^{7}\sqrt{cx}\,x^{6}a^{3}}$	58

input
```
int((b*x^2+a)^(1/4)/(c*x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/585*x*(b*x^2+a)^(5/4)*(32*b^2*x^4-40*a*b*x^2+45*a^2)/a^3/(c*x)^(15/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{2\,(32\,b^3x^6-8\,ab^2x^4+5\,a^2bx^2+45\,a^3)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{585\,a^3c^8x^7}$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="fricas")
```

output
```
-2/585*(32*b^3*x^6 - 8*a*b^2*x^4 + 5*a^2*b*x^2 + 45*a^3)*(b*x^2 + a)^(1/4)
*sqrt(c*x)/(a^3*c^8*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{15}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(15/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(15/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = -\frac{\left(b\,x^2+a\right)^{1/4}\left(\frac{2}{13\,c^7}+\frac{2\,b\,x^2}{117\,a\,c^7}-\frac{16\,b^2\,x^4}{585\,a^2\,c^7}+\frac{64\,b^3\,x^6}{585\,a^3\,c^7}\right)}{x^6\,\sqrt{c\,x}}$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(15/2),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(13*c^7) + (2*b*x^2)/(117*a*c^7) - (16*b^2*x^4)/(58
5*a^2*c^7) + (64*b^3*x^6)/(585*a^3*c^7)))/(x^6*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{15/2}}\,dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(-32b^3x^6+8a\,b^2x^4-5a^2b\,x^2-45a^3)}{585\sqrt{x}\,a^3c^8x^6}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(15/2),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 45*a**3 - 5*a**2*b*x**2 + 8*a*b**2*x**4
 - 32*b**3*x**6))/(585*sqrt(x)*a**3*c**8*x**6)
```

3.1021 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = -\frac{2(a+bx^2)^{5/4}}{17ac(cx)^{17/2}} + \frac{24b(a+bx^2)^{5/4}}{221a^2c^3(cx)^{13/2}} - \frac{64b^2(a+bx^2)^{5/4}}{663a^3c^5(cx)^{9/2}} + \frac{256b^3(a+bx^2)^{5/4}}{3315a^4c^7(cx)^{5/2}}$$

output
```
-2/17*(b*x^2+a)^(5/4)/a/c/(c*x)^(17/2)+24/221*b*(b*x^2+a)^(5/4)/a^2/c^3/(c
*x)^(13/2)-64/663*b^2*(b*x^2+a)^(5/4)/a^3/c^5/(c*x)^(9/2)+256/3315*b^3*(b*
x^2+a)^(5/4)/a^4/c^7/(c*x)^(5/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = -\frac{2x(a+bx^2)^{5/4}(195a^3-180a^2bx^2+160ab^2x^4-128b^3x^6)}{3315a^4(cx)^{19/2}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

output
```
(-2*x*(a + b*x^2)^(5/4)*(195*a^3 - 180*a^2*b*x^2 + 160*a*b^2*x^4 - 128*b^3
*x^6))/(3315*a^4*(c*x)^(19/2))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\int \frac{\left(bx^2+a\right)^{5/4}}{(cx)^{19/2}}dx}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\left(-\frac{8\int \frac{\left(bx^2+a\right)^{9/4}}{(cx)^{19/2}}dx}{9a}-\frac{2\left(a+bx^2\right)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{12\left(-\frac{8\left(-\frac{4\int \frac{\left(bx^2+a\right)^{13/4}}{(cx)^{19/2}}dx}{13a}-\frac{2\left(a+bx^2\right)^{13/4}}{13ac(cx)^{17/2}}\right)}{9a}-\frac{2\left(a+bx^2\right)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2\left(a+bx^2\right)^{5/4}}{5ac(cx)^{17/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{12\left(-\frac{8\left(\frac{8(a+bx^2)^{17/4}}{221a^2c(cx)^{17/2}}-\frac{2(a+bx^2)^{13/4}}{13ac(cx)^{17/2}}\right)}{9a}-\frac{2(a+bx^2)^{9/4}}{9ac(cx)^{17/2}}\right)}{5a}-\frac{2(a+bx^2)^{5/4}}{5ac(cx)^{17/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(19/2),x]
```

output
```
(-2*(a + b*x^2)^(5/4))/(5*a*c*(c*x)^(17/2)) - (12*((-2*(a + b*x^2)^(9/4))/
(9*a*c*(c*x)^(17/2)) - (8*((-2*(a + b*x^2)^(13/4))/(13*a*c*(c*x)^(17/2)) +
 (8*(a + b*x^2)^(17/4))/(221*a^2*c*(c*x)^(17/2))))/(9*a)))/(5*a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.44

method	result	size
gosper	$-\frac{2x(b\,x^2+a)^{\frac{5}{4}}\left(-128b^3x^6+160a\,b^2x^4-180a^2b\,x^2+195a^3\right)}{3315a^4(cx)^{\frac{19}{2}}}$	53
orering	$-\frac{2x(b\,x^2+a)^{\frac{5}{4}}\left(-128b^3x^6+160a\,b^2x^4-180a^2b\,x^2+195a^3\right)}{3315a^4(cx)^{\frac{19}{2}}}$	53
risch	$-\frac{2(b\,x^2+a)^{\frac{1}{4}}\left(-128b^4x^8+32a\,b^3x^6-20a^2b^2x^4+15a^3b\,x^2+195a^4\right)}{3315c^9\sqrt{cx}\,x^8a^4}$	69

input
```
int((b*x^2+a)^(1/4)/(c*x)^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3315*x*(b*x^2+a)^(5/4)*(-128*b^3*x^6+160*a*b^2*x^4-180*a^2*b*x^2+195*a^
3)/a^4/(c*x)^(19/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = \frac{2\,(128\,b^4x^8-32\,ab^3x^6+20\,a^2b^2x^4-15\,a^3bx^2-195\,a^4)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{3315\,a^4c^{10}x^9}$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="fricas")
```

output
```
2/3315*(128*b^4*x^8 - 32*a*b^3*x^6 + 20*a^2*b^2*x^4 - 15*a^3*b*x^2 - 195*a
^4)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^4*c^10*x^9)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(19/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{19}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(19/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(19/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = -\frac{(b\,x^2+a)^{1/4}\left(\frac{2}{17\,c^9}+\frac{2\,b\,x^2}{221\,a\,c^9}-\frac{8\,b^2\,x^4}{663\,a^2\,c^9}+\frac{64\,b^3\,x^6}{3315\,a^3\,c^9}-\frac{256\,b^4\,x^8}{3315\,a^4\,c^9}\right)}{x^8\,\sqrt{c\,x}}$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(19/2),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(17*c^9) + (2*b*x^2)/(221*a*c^9) - (8*b^2*x^4)/(663
*a^2*c^9) + (64*b^3*x^6)/(3315*a^3*c^9) - (256*b^4*x^8)/(3315*a^4*c^9)))/(
x^8*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{19/2}}\,dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(128b^4x^8-32a\,b^3x^6+20a^2b^2x^4-15a^3b\,x^2-195a^4)}{3315\sqrt{x}\,a^4c^{10}x^8}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(19/2),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 195*a**4 - 15*a**3*b*x**2 + 20*a**2*b**
2*x**4 - 32*a*b**3*x**6 + 128*b**4*x**8))/(3315*sqrt(x)*a**4*c**10*x**8)
```

3.1022 $\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 152

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=-\frac{a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}}{12b^2}+\frac{ac(cx)^{5/2}\sqrt[4]{a+bx^2}}{30b}+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}-\frac{a^{5/2}c^2\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{12b^{3/2}\left(a+bx^2\right)^{3/4}}$$

output
```
-1/12*a^2*c^3*(c*x)^(1/2)*(b*x^2+a)^(1/4)/b^2+1/30*a*c*(c*x)^(5/2)*(b*x^2+
a)^(1/4)/b+1/5*(c*x)^(9/2)*(b*x^2+a)^(1/4)/c-1/12*a^(5/2)*c^2*(1+a/b/x^2)^
(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/b
^(3/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.67

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\frac{c^3\sqrt{cx}\sqrt[4]{a+bx^2}\left(\sqrt[4]{1+\frac{bx^2}{a}}\left(-5a^2+abx^2+6b^2x^4\right)+5a^2\,\text{Hypergeometric2F1}\left(-\right.\right.}{30b^2\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

output
```
(c^3*Sqrt[c*x]*(a + b*x^2)^(1/4)*((1 + (b*x^2)/a)^(1/4)*(-5*a^2 + a*b*x^2
+ 6*b^2*x^4) + 5*a^2*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(30
*b^2*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {248, 262, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{7/2} \sqrt[4]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{10}a\int \frac{(cx)^{7/2}}{(bx^2+a)^{3/4}}dx+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\int \frac{(cx)^{3/2}}{(bx^2+a)^{3/4}}dx}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\left(bx^2+a\right)^{3/4}}dx}{2b}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac\int\frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{b}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$\downarrow$ 768

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{b(a+bx^2)^{3/4}}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$\downarrow$ 858

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{b(a+bx^2)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$\downarrow$ 807

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{2b(a+bx^2)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

$$\downarrow \text{229}$$

$$\frac{1}{10}a\left(\frac{c(cx)^{5/2}\sqrt[4]{a+bx^2}}{3b}-\frac{5ac^2\left(\frac{\sqrt{a}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}(a+bx^2)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)}{6b}\right)+\frac{(cx)^{9/2}\sqrt[4]{a+bx^2}}{5c}$$

input

```
Int[(c*x)^(7/2)*(a + b*x^2)^(1/4),x]
```

output

```
((c*x)^(9/2)*(a + b*x^2)^(1/4))/(5*c) + (a*((c*(c*x)^(5/2)*(a + b*x^2)^(1/
4))/(3*b) - (5*a*c^2*((c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/
(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2]
)/(Sqrt[b]*(a + b*x^2)^(3/4))))/(6*b)))/10
```

Defintions of rubi rules used

rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 248

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int (cx)^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}} dx$$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c^3*x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.84 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt[4]{a}c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*c**(7/2)*x**(9/2)*gamma(9/4)*hyper((-1/4, 9/4), (13/4,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(13/4))
```

Maxima [F]

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

Giac [F]

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{7}{2}}\,dx$$

input
```
integrate((c*x)^(7/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\int (c\,x)^{7/2}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int((c*x)^(7/2)*(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(7/2)*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int (cx)^{7/2}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt{c}\,c^3\left(-10\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,a^2+4\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,ab\,x^2+24\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,b^2x^4+5\right.}{120b^2}$$

input
```
int((c*x)^(7/2)*(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*c**3*( - 10*sqrt(x)*(a + b*x**2)**(1/4)*a**2 + 4*sqrt(x)*(a + b*x
**2)**(1/4)*a*b*x**2 + 24*sqrt(x)*(a + b*x**2)**(1/4)*b**2*x**4 + 5*int((s
qrt(x)*(a + b*x**2)**(1/4))/(a*x + b*x**3),x)*a**3))/(120*b**2)
```

3.1023 $\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 118

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\frac{ac\sqrt{cx}\sqrt[4]{a+bx^2}}{6b}+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}+\frac{a^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{6\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
1/6*a*c*(c*x)^(1/2)*(b*x^2+a)^(1/4)/b+1/3*(c*x)^(5/2)*(b*x^2+a)^(1/4)/c+1/
6*a^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^(1/2)
*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.72

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}\left((a+bx^2)\sqrt[4]{1+\frac{bx^2}{a}}-a\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

output
```
(c*Sqrt[c*x]*(a + b*x^2)^(1/4)*((a + b*x^2)*(1 + (b*x^2)/a)^(1/4) - a*Hype
rgeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)]))/(3*b*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {248, 262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{6}a\int \frac{(cx)^{3/2}}{(bx^2+a)^{3/4}}dx+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}a\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{\sqrt{cx}(bx^2+a)^{3/4}}dx}{2b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{266}$$

$$\frac{1}{6}a\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{768}$$

$$\frac{1}{6}a\left(\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{b\left(a+bx^2\right)^{3/4}}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{858}$$

$$\frac{1}{6}a\left(\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{b\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{6}a\left(\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int\frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{2b\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

$$\Big\downarrow \text{229}$$

$$\frac{1}{6}a\left(\frac{\sqrt{a}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}\right)+\frac{(cx)^{5/2}\sqrt[4]{a+bx^2}}{3c}$$

input
```
Int[(c*x)^(3/2)*(a + b*x^2)^(1/4),x]
```

output
```
((c*x)^(5/2)*(a + b*x^2)^(1/4))/(3*c) + (a*((c*Sqrt[c*x]*(a + b*x^2)^(1/4)
)/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]
*c^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^2)^(3/4))))/6
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int (cx)^{\frac{3}{2}} \left(b\,x^{2}+a\right)^{\frac{1}{4}}dx$$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)*c*x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.39

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt[4]{a}c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)*(b*x**2+a)**(1/4),x)
```

output
```
a**(1/4)*c**(3/2)*x**(5/2)*gamma(5/4)*hyper((-1/4, 5/4), (9/4,), b*x**2*ex
p_polar(I*pi)/a)/(2*gamma(9/4))
```

Maxima [F]

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Giac [F]

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{4}}(cx)^{\frac{3}{2}}\,dx$$

input
```
integrate((c*x)^(3/2)*(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)*(c*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\int (c\,x)^{3/2}\,\left(b\,x^2+a\right)^{1/4}\,dx$$

input
```
int((c*x)^(3/2)*(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(3/2)*(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int (cx)^{3/2}\sqrt[4]{a+bx^2}\,dx=\frac{\sqrt{c}\,c\left(2\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,a+4\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2-\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{b\,x^3+ax}dx\right)a^2\right)}{12b}$$

input
```
int((c*x)^(3/2)*(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*c*(2*sqrt(x)*(a + b*x**2)**(1/4)*a + 4*sqrt(x)*(a + b*x**2)**(1/4
)*b*x**2 - int((sqrt(x)*(a + b*x**2)**(1/4))/(a*x + b*x**3),x)*a**2))/(12*
b)
```

3.1024 $\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c} - \frac{\sqrt{a}\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

output

```
(c*x)^(1/2)*(b*x^2+a)^(1/4)/c-a^(1/2)*b^(1/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2
)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/c^2/(b*x^2+a)^(3/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \frac{2x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{cx}\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

output
```
(2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 5/4, -((b*x^2)/a)])/(S
qrt[c*x]*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {248, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}dx$$

$$\Big\downarrow \ 248$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{cx}\,(bx^2+a)^{3/4}}dx+\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}$$

$$\Big\downarrow \ 266$$

$$\frac{a\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{c}+\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}$$

$$\Big\downarrow \ 768$$

$$\frac{a(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{c\,(a+bx^2)^{3/4}}+\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}$$

$$\Big\downarrow \ 858$$

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{a(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{c\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \ 807$$

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{a(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{2c\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$\frac{\sqrt{cx}\sqrt[4]{a+bx^2}}{c}-\frac{\sqrt{a}\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{c^2\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(a + b*x^2)^(1/4)/Sqrt[c*x],x]
```

output
```
(Sqrt[c*x]*(a + b*x^2)^(1/4))/c - (Sqrt[a]*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(
c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(c^2*(a + b*x^
2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{4}}}{\sqrt{c x}}dx$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

output
```
int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt[4]{a}\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(1/2),x)
```

output
```
a**(1/4)*sqrt(x)*gamma(1/4)*hyper((-1/4, 1/4), (5/4,), b*x**2*exp_polar(I*
pi)/a)/(2*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{\sqrt{cx}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/sqrt(c*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{\sqrt{c\,x}}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(1/2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{\sqrt{cx}}\,dx = \frac{\sqrt{c}\left(2\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}+\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{b\,x^3+ax}dx\right)a\right)}{2c}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(x)*(a + b*x**2)**(1/4) + int((sqrt(x)*(a + b*x**2)**(1/4)
)/(a*x + b*x**3),x)*a))/(2*c)
```

3.1025 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}-\frac{2b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3\sqrt{a}c^4\left(a+bx^2\right)^{3/4}}$$

output
```
-2/3*(b*x^2+a)^(1/4)/c/(c*x)^(3/2)-2/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/
2)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(1/2)/c^4/(b*x
^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},-\frac{1}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-3/4, -1/4, 1/4, -((b*x^2)/a)])/
(3*(c*x)^(5/2)*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {247, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{b\int \frac{1}{\sqrt{cx}(bx^2+a)^{3/4}}dx}{3c^2}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2b\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{3c^3}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3c^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 858}$$

$$-\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3c^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 807}$$

$$-\frac{b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3c^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

$$\Big\downarrow \text{ 229}$$

$$-\frac{2b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3\sqrt{a}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3c(cx)^{3/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(5/2),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(3*c*(c*x)^(3/2)) - (2*b^(3/2)*(1 + a/(b*x^2))^(3/4
)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*Sqrt[a]*
c^4*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

output
```
int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.34

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = -\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{c^{\frac{5}{2}}x}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(5/2),x)
```

output
```
-b**(1/4)*hyper((-1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(c**(5/2)
*x)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{(c\,x)^{5/2}}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(5/2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2(b\,x^2+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{b\,x^5+a\,x^3}dx\right)ax\right)}{2\sqrt{x}\,c^3x}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*(a + b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x**2)**(1
/4))/(a*x**3 + b*x**5),x)*a*x))/(2*sqrt(x)*c**3*x)
```

3.1026 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 123

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}-\frac{2b\sqrt[4]{a+bx^2}}{21ac^3(cx)^{3/2}} + \frac{4b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{21a^{3/2}c^6\left(a+bx^2\right)^{3/4}}$$

output
```
-2/7*(b*x^2+a)^(1/4)/c/(c*x)^(7/2)-2/21*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(3/2
)+4/21*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^
(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/c^6/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{2x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{4},-\frac{1}{4},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7(cx)^{9/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-7/4, -1/4, -3/4, -((b*x^2)/a)])
/(7*(c*x)^(9/2)*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {247, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{(cx)^{5/2}(bx^2+a)^{3/4}}dx}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\left(bx^2+a\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{768}$$

$$\frac{b\left(-\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{b\left(\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{807}$$

$$\frac{b\left(\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

$$\Big\downarrow \text{229}$$

$$\frac{b\left(\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7c^2}-\frac{2\sqrt[4]{a+bx^2}}{7c(cx)^{7/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(9/2),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(7*c*(c*x)^(7/2)) + (b*((-2*(a + b*x^2)^(1/4))/(3*a
*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[A
rcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a + b*x^2)^(3/4))))/
(7*c^2)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}dx$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

output
```
int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^5*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 13.78 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.29

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = -\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{3c^{\frac{9}{2}}x^3}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(9/2),x)
```

output
```
-b**(1/4)*hyper((-1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*c**(9/
2)*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(9/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{(c\,x)^{9/2}}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(9/2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{9/2}}\,dx = \frac{\sqrt{c}\left(-2(b\,x^2+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{b\,x^7+a\,x^5}dx\right)a\,x^3\right)}{6\sqrt{x}\,c^5x^3}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(9/2),x)
```

output
```
(sqrt(c)*( - 2*(a + b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x**2)**(1
/4))/(a*x**5 + b*x**7),x)*a*x**3))/(6*sqrt(x)*c**5*x**3)
```

3.1027 $\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 154

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}-\frac{2b\sqrt[4]{a+bx^2}}{77ac^3(cx)^{7/2}}+\frac{4b^2\sqrt[4]{a+bx^2}}{77a^2c^5(cx)^{3/2}} - \frac{8b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{5/2}c^8\left(a+bx^2\right)^{3/4}}$$

output
```
-2/11*(b*x^2+a)^(1/4)/c/(c*x)^(11/2)-2/77*b*(b*x^2+a)^(1/4)/a/c^3/(c*x)^(7
/2)+4/77*b^2*(b*x^2+a)^(1/4)/a^2/c^5/(c*x)^(3/2)-8/77*b^(7/2)*(1+a/b/x^2)^
(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/a
^(5/2)/c^8/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{2x\sqrt[4]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{11}{4},-\frac{1}{4},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\sqrt[4]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*Hypergeometric2F1[-11/4, -1/4, -7/4, -((b*x^2)/a)]
)/(11*(c*x)^(13/2)*(1 + (b*x^2)/a)^(1/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {247, 264, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{b\int \frac{1}{(cx)^{9/2}(bx^2+a)^{3/4}}dx}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{b\left(-\frac{6b\int \frac{1}{(cx)^{5/2}\left(bx^2+a\right)^{3/4}}dx}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\left(bx^2+a\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{b\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \textbf{768}$$

$$\frac{b\left(-\frac{6b\left(-\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \textbf{858}$$

$$\frac{b\left(-\frac{6b\left(\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \textbf{807}$$

$$\frac{b\left(-\frac{6b\left(\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

$$\Big\downarrow \textbf{229}$$

$$\frac{b\left(-\frac{6b\left(\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11c^2}-\frac{2\sqrt[4]{a+bx^2}}{11c(cx)^{11/2}}$$

input
```
Int[(a + b*x^2)^(1/4)/(c*x)^(13/2),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(11*c*(c*x)^(11/2)) + (b*((-2*(a + b*x^2)^(1/4))/(7
*a*c*(c*x)^(7/2)) - (6*b*((-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*
b^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)
/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a + b*x^2)^(3/4))))/(7*a*c^2)))/(11*c^2)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}dx$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

output
```
int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(c^7*x^7), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 137.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = -\frac{\sqrt[4]{b}\,{}_2F_1\left(\begin{matrix} -\frac{1}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5c^{\frac{13}{2}}x^5}$$

input
```
integrate((b*x**2+a)**(1/4)/(c*x)**(13/2),x)
```

output
```
-b**(1/4)*hyper((-1/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*c**(13
/2)*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(bx^2+a)^{\frac{1}{4}}}{(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^(1/4)/(c*x)^(13/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/4)/(c*x)^(13/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = \int \frac{(b\,x^2+a)^{1/4}}{(c\,x)^{13/2}}\,dx$$

input
```
int((a + b*x^2)^(1/4)/(c*x)^(13/2),x)
```

output
```
int((a + b*x^2)^(1/4)/(c*x)^(13/2), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{a+bx^2}}{(cx)^{13/2}}\,dx = \frac{\sqrt{c}\left(-2(b\,x^2+a)^{\frac{1}{4}}-\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{1}{4}}}{b\,x^9+a\,x^7}dx\right)a\,x^5\right)}{10\sqrt{x}\,c^7x^5}$$

input
```
int((b*x^2+a)^(1/4)/(c*x)^(13/2),x)
```

output
```
(sqrt(c)*( - 2*(a + b*x**2)**(1/4) - sqrt(x)*int((sqrt(x)*(a + b*x**2)**(1
/4))/(a*x**7 + b*x**9),x)*a*x**5))/(10*sqrt(x)*c**7*x**5)
```

3.1028 $\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx=\frac{c\sqrt{cx}(a+bx^2)^{3/4}}{2b}-\frac{ac^{3/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}-\frac{ac^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{5/4}}$$

output
```
1/2*c*(c*x)^(1/2)*(b*x^2+a)^(3/4)/b-1/4*a*c^(3/2)*arctan(b^(1/4)*(c*x)^(1/
2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(5/4)-1/4*a*c^(3/2)*arctanh(b^(1/4)*(c*x)^(1
/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx=\frac{(cx)^{3/2}\left(2\sqrt[4]{b}\sqrt{x}(a+bx^2)^{3/4}-a\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{5/4}x^{3/2}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

output
```
((c*x)^(3/2)*(2*b^(1/4)*Sqrt[x]*(a + b*x^2)^(3/4) - a*ArcTan[(b^(1/4)*Sqrt
[x])/(a + b*x^2)^(1/4)] - a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))
/(4*b^(5/4)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {262, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac^2\int \frac{1}{\sqrt{cx}\sqrt[4]{bx^2+a}}dx}{4b}$$

$$\Big\downarrow \ 266$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{cx}}{2b}$$

$$\Big\downarrow \ 770$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac\int \frac{1}{1-bx^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2b}$$

$$\Big\downarrow \ 756$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}c\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}\right)}{2b}$$

$$\Big\downarrow \ 218$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac\left(\frac{1}{2}c\int\frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\sqrt{cx}\left(a+bx^2\right)^{3/4}}{2b}-\frac{ac\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{2b}$$

input
```
Int[(c*x)^(3/2)/(a + b*x^2)^(1/4),x]
```

output
```
(c*Sqrt[c*x]*(a + b*x^2)^(3/4))/(2*b) - (a*c*((Sqrt[c]*ArcTan[(b^(1/4)*Sqr
t[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]))/(2*b^(1/4)) + (Sqrt[c]*ArcTanh[(b^(1
/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.57

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{4\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}c - \left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}} b\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}ac+\left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}}(b^2x^2+ab)}{bx^2+a}\right) + \left(\frac{a^4c^6}{b^5}\right)^{\frac{1}{4}} b\log\left(\frac{(bx}{}\right.}{}$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
1/8*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c - (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 +
 a)^(3/4)*sqrt(c*x)*a*c + (a^4*c^6/b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)
) + (a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c - (a^4*c^6/
b^5)^(1/4)*(b^2*x^2 + a*b))/(b*x^2 + a)) + I*(a^4*c^6/b^5)^(1/4)*b*log(((b
*x^2 + a)^(3/4)*sqrt(c*x)*a*c - (a^4*c^6/b^5)^(1/4)*(I*b^2*x^2 + I*a*b))/(
b*x^2 + a)) - I*(a^4*c^6/b^5)^(1/4)*b*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c
 - (a^4*c^6/b^5)^(1/4)*(-I*b^2*x^2 - I*a*b))/(b*x^2 + a)))/b
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.38

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((1/4, 5/4), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(1/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{\sqrt[4]{a+bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)c$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x)/(a + b*x**2)**(1/4),x)*c
```

3.1029 $\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{\sqrt[4]{b}\sqrt{c}}$$

output
```
arctan(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(1/4)/c^(1/2)+arctan
h(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(1/4)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{x}\left(\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + \operatorname{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{\sqrt[4]{b}\sqrt{cx}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

output
```
(Sqrt[x]*(ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4)*S
qrt[x])/(a + b*x^2)^(1/4)]))/(b^(1/4)*Sqrt[c*x])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{770}$$

$$\frac{2\int \frac{1}{1-bx^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{c}$$

$$\Big\downarrow \text{756}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}c\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}\right)}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{c}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{c}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^2)^(1/4)),x]
```

output
```
(2*((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(2*b
^(1/4)) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))
])/(2*b^(1/4))))/c
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.89

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx =& \frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}+(bcx^2+ac)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)\\&-\frac{1}{2}\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(bcx^2+ac)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)\\&-\frac{1}{2}i\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(i\,bcx^2+i\,ac)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)\\&+\frac{1}{2}i\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}-(-i\,bcx^2-i\,ac)\left(\frac{1}{bc^2}\right)^{\frac{1}{4}}}{bx^2+a}\right)\end{aligned}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) + (b*c*x^2 + a*c)*(
1/(b*c^2))^(1/4))/(b*x^2 + a)) - 1/2*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3
/4)*sqrt(c*x) - (b*c*x^2 + a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a)) - 1/2*I*(1
/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x) - (I*b*c*x^2 + I*a*c)*(1/
(b*c^2))^(1/4))/(b*x^2 + a)) + 1/2*I*(1/(b*c^2))^(1/4)*log(((b*x^2 + a)^(3
/4)*sqrt(c*x) - (-I*b*c*x^2 - I*a*c)*(1/(b*c^2))^(1/4))/(b*x^2 + a))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt{c}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

output
```
sqrt(x)*gamma(1/4)*hyper((1/4, 1/4), (5/4,), b*x**2*exp_polar(I*pi)/a)/(2*
a**(1/4)*sqrt(c)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.45

$$\int \frac{1}{\sqrt{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{x}\,\sqrt{c}\,(2b\,x^2+2a)}{(b\,x^2+a)^{\frac{3}{4}}\,\sqrt{b\,x^2+a}\,c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(x)*sqrt(c)*(a + b*x**2)**(1/4)*(a + b*x**2 + a + b*x**2))/(sqrt(a +
b*x**2)*c*(a + b*x**2))
```

3.1030 $\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

output
```
-2/3*(b*x^2+a)^(3/4)/a/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2x(a+bx^2)^{3/4}}{3a(cx)^{5/2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(3/4))/(3*a*(c*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx$$
$$\Big\downarrow \text{242}$$
$$-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(3/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a\left(cx\right)^{\frac{5}{2}}}$	21
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a\left(cx\right)^{\frac{5}{2}}}$	21
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3c^{2}\sqrt{cx}\,ax}$	26

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x^2+a)^(3/4)/a/(c*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{3\,ac^3x^2}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-2/3*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*c^3*x^2)
```

Sympy [A] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = \frac{b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2ac^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

output
```
b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-3/4)/(2*a*c**(5/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\,{\left(b\,x^2+a\right)}^{3/4}}{3\,a\,c^2\,x\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(1/4)),x)
```

output
```
-(2*(a + b*x^2)^(3/4))/(3*a*c^2*x*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{1}{(cx)^{5/2}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{c}\,((b\,x^2+a)\,a+4(b\,x^2+a)\,b\,x^2-3a^2-3ab\,x^2)}{3\,(b\,x^2+a)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,c^3x}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a + b*x**2)**(1/4)*((a + b*x**2)*a + 4*(a + b*x**2)*b*x**2 - 3*a
**2 - 3*a*b*x**2))/(3*sqrt(x)*sqrt(a + b*x**2)*a*c**3*x*(a + b*x**2))
```

3.1031 $\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2(a+bx^2)^{3/4}}{7ac(cx)^{7/2}}+\frac{8b(a+bx^2)^{3/4}}{21a^2c^3(cx)^{3/2}}$$

output
```
-2/7*(b*x^2+a)^(3/4)/a/c/(c*x)^(7/2)+8/21*b*(b*x^2+a)^(3/4)/a^2/c^3/(c*x)^
(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2x(3a-4bx^2)\left(a+bx^2\right)^{3/4}}{21a^2(cx)^{9/2}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(3*a - 4*b*x^2)*(a + b*x^2)^(3/4))/(21*a^2*(c*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{4\int \frac{(bx^2+a)^{3/4}}{(cx)^{9/2}}dx}{3a} - \frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{8(a+bx^2)^{7/4}}{21a^2c(cx)^{7/2}} - \frac{2(a+bx^2)^{3/4}}{3ac(cx)^{7/2}}$$

input
```
Int[1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2
*c*(c*x)^(7/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)}{21a^{2}\left(cx\right)^{\frac{9}{2}}}$	31
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)}{21a^{2}\left(cx\right)^{\frac{9}{2}}}$	31
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-4b\,x^{2}+3a\right)}{21c^{4}\sqrt{cx}\,a^{2}x^{3}}$	36

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*x*(b*x^2+a)^(3/4)*(-4*b*x^2+3*a)/a^2/(c*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = \frac{2\,(4\,bx^2-3\,a)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{21\,a^2c^5x^4}$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
2/21*(4*b*x^2 - 3*a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*c^5*x^4)
```

Sympy [A] (verification not implemented)

Time = 19.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.38

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{3b^{\frac{3}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{8ac^{\frac{9}{2}}x^2\Gamma\left(\frac{1}{4}\right)} + \frac{b^{\frac{7}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{2a^2c^{\frac{9}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

output
```
-3*b**(3/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(8*a*c**(9/2)*x**2*gamma(1
/4)) + b**(7/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(2*a**2*c**(9/2)*gamma
(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(9/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{(b\,x^2+a)^{3/4}\left(\frac{2}{7\,a\,c^4}-\frac{8\,b\,x^2}{21\,a^2\,c^4}\right)}{x^3\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(1/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*(2/(7*a*c^4) - (8*b*x^2)/(21*a^2*c^4)))/(x^3*(c*x)^(1/
2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.59

$$\int \frac{1}{(cx)^{9/2}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{c}\,(3(b\,x^2+a)\,a^2-8(b\,x^2+a)\,ab\,x^2-32(b\,x^2+a)\,b^2x^4-21a^3-21a^2b\,x^2)}{63\,(b\,x^2+a)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2c^5x^3}$$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a + b*x**2)**(1/4)*(3*(a + b*x**2)*a**2 - 8*(a + b*x**2)*a*b*x**
2 - 32*(a + b*x**2)*b**2*x**4 - 21*a**3 - 21*a**2*b*x**2))/(63*sqrt(x)*sqr
t(a + b*x**2)*a**2*c**5*x**3*(a + b*x**2))
```

3.1032 $\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2(a+bx^2)^{3/4}}{11ac(cx)^{11/2}} + \frac{16b(a+bx^2)^{3/4}}{77a^2c^3(cx)^{7/2}} - \frac{64b^2(a+bx^2)^{3/4}}{231a^3c^5(cx)^{3/2}}$$

output
```
-2/11*(b*x^2+a)^(3/4)/a/c/(c*x)^(11/2)+16/77*b*(b*x^2+a)^(3/4)/a^2/c^3/(c*
x)^(7/2)-64/231*b^2*(b*x^2+a)^(3/4)/a^3/c^5/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2x(a+bx^2)^{3/4}(21a^2-24abx^2+32b^2x^4)}{231a^3(cx)^{13/2}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(3/4)*(21*a^2 - 24*a*b*x^2 + 32*b^2*x^4))/(231*a^3*(c*x)
^(13/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{8\int \frac{(bx^2+a)^{3/4}}{(cx)^{13/2}}dx}{3a}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{8\left(-\frac{4\int \frac{(bx^2+a)^{7/4}}{(cx)^{13/2}}dx}{7a}-\frac{2(a+bx^2)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{8\left(\frac{8(a+bx^2)^{11/4}}{77a^2c(cx)^{11/2}}-\frac{2(a+bx^2)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2\left(a+bx^2\right)^{3/4}}{3ac(cx)^{11/2}}$$

input
```
Int[1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*(a + b*x^2)^(3/4))/(3*a*c*(c*x)^(11/2)) - (8*((-2*(a + b*x^2)^(7/4))/(
7*a*c*(c*x)^(11/2)) + (8*(a + b*x^2)^(11/4))/(77*a^2*c*(c*x)^(11/2))))/(3*
a)
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}-24ab\,x^{2}+21a^{2}\right)}{231a^{3}\left(cx\right)^{\frac{13}{2}}}$	42
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}-24ab\,x^{2}+21a^{2}\right)}{231a^{3}\left(cx\right)^{\frac{13}{2}}}$	42
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(32b^{2}x^{4}-24ab\,x^{2}+21a^{2}\right)}{231c^{6}\sqrt{cx}\,a^{3}x^{5}}$	47

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/231*x*(b*x^2+a)^(3/4)*(32*b^2*x^4-24*a*b*x^2+21*a^2)/a^3/(c*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\,(32\,b^2x^4-24\,abx^2+21\,a^2)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{231\,a^3c^7x^6}$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
-2/231*(32*b^2*x^4 - 24*a*b*x^2 + 21*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3
*c^7*x^6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(82) = 164$.

Time = 168.84 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.43

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = \frac{21a^4b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{32a^5b^4c^{\frac{13}{2}}x^4\Gamma\left(\frac{1}{4}\right)+64a^4b^5c^{\frac{13}{2}}x^6\Gamma\left(\frac{1}{4}\right)+32a^3b^6c^{\frac{13}{2}}x^8\Gamma\left(\frac{1}{4}\right)} + \frac{18a^3b^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{32a^5b^4c^{\frac{13}{2}}x^4\Gamma\left(\frac{1}{4}\right)+64a^4b^5c^{\frac{13}{2}}x^6\Gamma\left(\frac{1}{4}\right)+32a^3b^6c^{\frac{13}{2}}x^8\Gamma\left(\frac{1}{4}\right)} + \frac{5a^2b^{\frac{27}{4}}x^4\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{32a^5b^4c^{\frac{13}{2}}x^4\Gamma\left(\frac{1}{4}\right)+64a^4b^5c^{\frac{13}{2}}x^6\Gamma\left(\frac{1}{4}\right)+32a^3b^6c^{\frac{13}{2}}x^8\Gamma\left(\frac{1}{4}\right)} + \frac{40ab^{\frac{31}{4}}x^6\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{32a^5b^4c^{\frac{13}{2}}x^4\Gamma\left(\frac{1}{4}\right)+64a^4b^5c^{\frac{13}{2}}x^6\Gamma\left(\frac{1}{4}\right)+32a^3b^6c^{\frac{13}{2}}x^8\Gamma\left(\frac{1}{4}\right)} + \frac{32b^{\frac{35}{4}}x^8\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{11}{4}\right)}{32a^5b^4c^{\frac{13}{2}}x^4\Gamma\left(\frac{1}{4}\right)+64a^4b^5c^{\frac{13}{2}}x^6\Gamma\left(\frac{1}{4}\right)+32a^3b^6c^{\frac{13}{2}}x^8\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(13/2)/(b*x**2+a)**(1/4),x)
```

output
```
21*a**4*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-11/4)/(32*a**5*b**4*c**(1
3/2)*x**4*gamma(1/4) + 64*a**4*b**5*c**(13/2)*x**6*gamma(1/4) + 32*a**3*b*
*6*c**(13/2)*x**8*gamma(1/4)) + 18*a**3*b**(23/4)*x**2*(a/(b*x**2) + 1)**(
3/4)*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4*gamma(1/4) + 64*a**4*b**5*c
**(13/2)*x**6*gamma(1/4) + 32*a**3*b**6*c**(13/2)*x**8*gamma(1/4)) + 5*a**
2*b**(27/4)*x**4*(a/(b*x**2) + 1)**(3/4)*gamma(-11/4)/(32*a**5*b**4*c**(13
/2)*x**4*gamma(1/4) + 64*a**4*b**5*c**(13/2)*x**6*gamma(1/4) + 32*a**3*b**
6*c**(13/2)*x**8*gamma(1/4)) + 40*a*b**(31/4)*x**6*(a/(b*x**2) + 1)**(3/4)
*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4*gamma(1/4) + 64*a**4*b**5*c**(1
3/2)*x**6*gamma(1/4) + 32*a**3*b**6*c**(13/2)*x**8*gamma(1/4)) + 32*b**(35
/4)*x**8*(a/(b*x**2) + 1)**(3/4)*gamma(-11/4)/(32*a**5*b**4*c**(13/2)*x**4
*gamma(1/4) + 64*a**4*b**5*c**(13/2)*x**6*gamma(1/4) + 32*a**3*b**6*c**(13
/2)*x**8*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(13/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{(b\,x^2+a)^{3/4}\left(\frac{2}{11\,a\,c^6}-\frac{16\,b\,x^2}{77\,a^2\,c^6}+\frac{64\,b^2\,x^4}{231\,a^3\,c^6}\right)}{x^5\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(13/2)*(a + b*x^2)^(1/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*(2/(11*a*c^6) - (16*b*x^2)/(77*a^2*c^6) + (64*b^2*x^4)
/(231*a^3*c^6)))/(x^5*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{1}{(cx)^{13/2}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{c}\,(7(b\,x^2+a)\,a^3-12(b\,x^2+a)\,a^2b\,x^2+32(b\,x^2+a)\,a\,b^2x^4+128(b\,x^2+a)\,b^3x}{385\,(b\,x^2+a)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{b\,x^2+a}\,a^3c^7x^5}$$

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*(a + b*x**2)**(1/4)*(7*(a + b*x**2)*a**3 - 12*(a + b*x**2)*a**2*b
*x**2 + 32*(a + b*x**2)*a*b**2*x**4 + 128*(a + b*x**2)*b**3*x**6 - 77*a**4
 - 77*a**3*b*x**2))/(385*sqrt(x)*sqrt(a + b*x**2)*a**3*c**7*x**5*(a + b*x*
*2))
```

3.1033 $\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{7a^2c^3(cx)^{3/2}}{20b^2\sqrt[4]{a+bx^2}} - \frac{7ac^3(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{30b^2} + \frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b} + \frac{7a^{5/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{5/2}\sqrt[4]{a+bx^2}}$$

output
```
7/20*a^2*c^3*(c*x)^(3/2)/b^2/(b*x^2+a)^(1/4)-7/30*a*c^3*(c*x)^(3/2)*(b*x^2
+a)^(3/4)/b^2+1/5*c*(c*x)^(7/2)*(b*x^2+a)^(3/4)/b+7/20*a^(5/2)*c^4*(1+a/b/
x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2
))/b^(5/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^3(cx)^{3/2}\left(-7a^2-abx^2+6b^2x^4+7a^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{30b^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

output
```
(c^3*(c*x)^(3/2)*(-7*a^2 - a*b*x^2 + 6*b^2*x^4 + 7*a^2*(1 + (b*x^2)/a)^(1/
4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b*x^2)/a)]))/(30*b^2*(a + b*x^2)^(1
/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {262, 262, 255, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\int \frac{(cx)^{5/2}}{\sqrt[4]{bx^2+a}}dx}{10b}$$

$$\Big\downarrow \ 262$$

$$\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\left(\frac{c(cx)^{3/2}(a+bx^2)^{3/4}}{3b}-\frac{ac^2\int \frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}dx}{2b}\right)}{10b}$$

$$\Big\downarrow \ 255$$

$$\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\left(\frac{c(cx)^{3/2}(a+bx^2)^{3/4}}{3b}-\frac{ac^2\left(\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{1}{2}a\int \frac{\sqrt{cx}}{(bx^2+a)^{5/4}}dx\right)}{2b}\right)}{10b}$$

$$\Big\downarrow \ 249$$

$$\frac{\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\left(\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b\sqrt[4]{a+bx^2}}\right)}{2b}\right)}{10b}}{}$$

$\Big\downarrow$ 858

$$\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\left(\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}\right)}{10b}$$

$\Big\downarrow$ 212

$$\frac{c(cx)^{7/2}\left(a+bx^2\right)^{3/4}}{5b}-\frac{7ac^2\left(\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}\right)}{10b}$$

input
```
Int[(c*x)^(9/2)/(a + b*x^2)^(1/4),x]
```

output
```
(c*(c*x)^(7/2)*(a + b*x^2)^(3/4))/(5*b) - (7*a*c^2*((c*(c*x)^(3/2)*(a + b*
x^2)^(3/4))/(3*b) - (a*c^2*((x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1
+ a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/
(Sqrt[b]*(a + b*x^2)^(1/4))))/(2*b)))/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 255
```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Simp[a/2   Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x
], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{9}{2}}}{(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^{2}+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^4*x^4/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 27.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.28

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\right|\frac{bx^{2}e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**2+a)**(1/4),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/4)*hyper((1/4, 11/4), (15/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(1/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(c\,x)^{9/2}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{\sqrt[4]{a+bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^4}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)c^4$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**4)/(a + b*x**2)**(1/4),x)*c**4
```

3.1034 $\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = -\frac{ac(cx)^{3/2}}{2b\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b} - \frac{a^{3/2}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
-1/2*a*c*(c*x)^(3/2)/b/(b*x^2+a)^(1/4)+1/3*c*(c*x)^(3/2)*(b*x^2+a)^(3/4)/b
-1/2*a^(3/2)*c^2*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^
(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.57

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c(cx)^{3/2}\left(a+bx^2-a\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{3b\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

output
```
(c*(c*x)^(3/2)*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4,
 3/4, 7/4, -((b*x^2)/a)]))/(3*b*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {262, 255, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\int \frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}dx}{2b}$$

$$\Big\downarrow \text{255}$$

$$\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{1}{2}a\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx\right)}{2b}$$

$$\Big\downarrow \text{249}$$

$$\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$\Big\downarrow \text{858}$$

$$\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

$$\Big\downarrow \text{212}$$

$$\frac{c(cx)^{3/2}\left(a+bx^2\right)^{3/4}}{3b}-\frac{ac^2\left(\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}\right)}{2b}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^2)^(1/4),x]
```

output
```
(c*(c*x)^(3/2)*(a + b*x^2)^(3/4))/(3*b) - (a*c^2*((x*Sqrt[c*x])/(a + b*x^2
)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a
]/(Sqrt[b]*x)]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))))/(2*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 255
```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Simp[a/2   Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x
], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c^2*x^2/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.36

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(1/4),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((1/4, 7/4), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(1/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{\sqrt[4]{a+bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^2}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)c^2$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**2)/(a + b*x**2)**(1/4),x)*c**2
```

3.1035 $\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 85

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx=\frac{(cx)^{3/2}}{c\sqrt[4]{a+bx^2}}+\frac{\sqrt{a}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
(c*x)^(3/2)/c/(b*x^2+a)^(1/4)+a^(1/2)*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*Ellipt
icE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx=\frac{2x\sqrt{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

output
```
(2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((b
*x^2)/a)])/(3*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {255, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \ 255$$

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{1}{2}a\int \frac{\sqrt{cx}}{(bx^2+a)^{5/4}}dx$$

$$\Big\downarrow \ 249$$

$$\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}-\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 858$$

$$\frac{a\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \ 212$$

$$\frac{\sqrt{a}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{a+bx^2}}+\frac{x\sqrt{cx}}{\sqrt[4]{a+bx^2}}$$

input
```
Int[Sqrt[c*x]/(a + b*x^2)^(1/4),x]
```

output
```
(x*Sqrt[c*x])/(a + b*x^2)^(1/4) + (Sqrt[a]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]
*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[b]*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 255
```
Int[Sqrt[(c_)*(x_)]/((a_) + (b_.)*(x_)^2)^(1/4), x_Symbol] :> Simp[x*(Sqrt[
c*x]/(a + b*x^2)^(1/4)), x] - Simp[a/2   Int[Sqrt[c*x]/(a + b*x^2)^(5/4), x
], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(1/4),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((1/4, 3/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(1/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{\sqrt[4]{a+bx^2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(1/4),x)
```

output
```
sqrt(c)*int(sqrt(x)/(a + b*x**2)**(1/4),x)
```

3.1036 $\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{2\sqrt{b}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}$$

output
```
-2/c/(c*x)^(1/2)/(b*x^2+a)^(1/4)+2*b^(1/2)*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*E
llipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(1/2)/c^2/(b*x^2+a)
^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx=-\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 1/4, 3/4, -((b*x^2)/a)
])/((c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {257, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{257}$$

$$-\frac{b\int \frac{\sqrt{cx}}{(bx^2+a)^{5/4}}dx}{c^2}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x]
```

output
```
-2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt
[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2
)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 257
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Simp[b/c^2   Int[Sqrt[c*x]/(a + b*x
^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{2}}\,(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{\sqrt[4]{b}c^{\frac{3}{2}}x}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((1/4, 1/2), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(1/4)*c**(3/2)*
x)
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/2}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{3}{4}}}{b\,x^4+a\,x^2}dx\right)}{c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(1/4),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a + b*x**2)**(3/4))/(a*x**2 + b*x**4),x))/c**2
```

3.1037 $\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = \frac{4b}{5ac^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}} - \frac{4b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{3/2}c^4\sqrt[4]{a+bx^2}}$$

output
```
4/5*b/a/c^3/(c*x)^(1/2)/(b*x^2+a)^(1/4)-2/5*(b*x^2+a)^(3/4)/a/c/(c*x)^(5/2
)-4/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/
2)*x/a^(1/2))),2^(1/2))/a^(3/2)/c^4/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{1}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 1/4, -1/4, -((b*x^2)/a
)])/(5*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {264, 257, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt[4]{bx^2+a}}dx}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{257}$$

$$-\frac{2b\left(-\frac{b\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{c^2}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{2b\left(-\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2b\left(\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{2b\left(\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*(a + b*x^2)^(3/4))/(5*a*c*(c*x)^(5/2)) - (2*b*(-2/(c*Sqrt[c*x]*(a + b*
x^2)^(1/4)) + (2*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[
Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(1/4))))/(5*a*c^2)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 257
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Simp[b/c^2   Int[Sqrt[c*x]/(a + b*x
^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{7}{2}}\,(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^4*x^6 + a*c^4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{3\sqrt[4]{b}c^{\frac{7}{2}}x^3}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((1/4, 3/2), (5/2,), a*exp_polar(I*pi)/(b*x**2))/(3*b**(1/4)*c**(7/2
)*x**3)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{\left(c\,x\right)^{7/2}\,\left(b\,x^2+a\right)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.29

$$\int \frac{1}{(cx)^{7/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{5}{4}}}{5\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,c^4x^2}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(1/4),x)
```

output
```
( - 2*sqrt(c)*(a + b*x**2)**(1/4)*(a + b*x**2))/(5*sqrt(x)*sqrt(a + b*x**2
)*a*c**4*x**2)
```

3.1038 $\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{8b^2}{15a^2c^5\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}} + \frac{4b(a+bx^2)^{3/4}}{15a^2c^3(cx)^{5/2}} + \frac{8b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{15a^{5/2}c^6\sqrt[4]{a+bx^2}}$$

output
```
-8/15*b^2/a^2/c^5/(c*x)^(1/2)/(b*x^2+a)^(1/4)-2/9*(b*x^2+a)^(3/4)/a/c/(c*x
)^(9/2)+4/15*b*(b*x^2+a)^(3/4)/a^2/c^3/(c*x)^(5/2)+8/15*b^(5/2)*(1+a/b/x^2
)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/
a^(5/2)/c^6/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{1}{4},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 1/4, -5/4, -((b*x^2)/a
)])/(9*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {264, 264, 257, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\int \frac{1}{(cx)^{7/2}\sqrt[4]{bx^2+a}}dx}{3ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\left(-\frac{2b\int \frac{1}{(cx)^{3/2}\sqrt[4]{bx^2+a}}dx}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{257}$$

$$-\frac{2b\left(-\frac{2b\left(-\frac{b\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{c^2}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2\left(a+bx^2\right)^{3/4}}{9ac(cx)^{9/2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{2b\left(-\frac{2b\left(-\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

$\Big\downarrow$ 858

$$-\frac{2b\left(-\frac{2b\left(\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

$\Big\downarrow$ 212

$$-\frac{2b\left(-\frac{2b\left(\frac{2\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}c^2\sqrt[4]{a+bx^2}}-\frac{2}{c\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2(a+bx^2)^{3/4}}{5ac(cx)^{5/2}}\right)}{3ac^2}-\frac{2(a+bx^2)^{3/4}}{9ac(cx)^{9/2}}$$

input
```
Int[1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x]
```

output
```
(-2*(a + b*x^2)^(3/4))/(9*a*c*(c*x)^(9/2)) - (2*b*((-2*(a + b*x^2)^(3/4))/
(5*a*c*(c*x)^(5/2)) - (2*b*(-2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (2*Sqrt[b
]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2,
 2])/(Sqrt[a]*c^2*(a + b*x^2)^(1/4))))/(5*a*c^2)))/(3*a*c^2)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 257
```
Int[1/(((c_.)*(x_))^(3/2)*((a_) + (b_.)*(x_)^2)^(1/4)), x_Symbol] :> Simp[-
2/(c*Sqrt[c*x]*(a + b*x^2)^(1/4)), x] - Simp[b/c^2   Int[Sqrt[c*x]/(a + b*x
^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b*c^6*x^8 + a*c^6*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 53.86 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.22

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{5\sqrt[4]{b}c^{\frac{11}{2}}x^5}$$

input
```
integrate(1/(c*x)**(11/2)/(b*x**2+a)**(1/4),x)
```

output
```
-hyper((1/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(1/4)*c**(11/
2)*x**5)
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{11/2}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(11/2)*(a + b*x^2)^(1/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{11/2}\sqrt[4]{a+bx^2}}\,dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(4b^2x^4-ab\,x^2-5a^2)}{45\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(1/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 5*a**2 - a*b*x**2 + 4*b**2*x**4))/(45*s
qrt(x)*sqrt(a + b*x**2)*a**2*c**6*x**4)
```

3.1039 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 117

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b} + \frac{3ac^{5/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}} - \frac{3ac^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{7/4}}$$

output
```
1/2*c*(c*x)^(3/2)*(b*x^2+a)^(1/4)/b+3/4*a*c^(5/2)*arctan(b^(1/4)*(c*x)^(1/
2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(7/4)-3/4*a*c^(5/2)*arctanh(b^(1/4)*(c*x)^(1
/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(7/4)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.83

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \frac{(cx)^{5/2}\left(2b^{3/4}x^{3/2}\sqrt[4]{a+bx^2}+3a\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)-3a\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{4b^{7/4}x^{5/2}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

output
```
((c*x)^(5/2)*(2*b^(3/4)*x^(3/2)*(a + b*x^2)^(1/4) + 3*a*ArcTan[(b^(1/4)*Sq
rt[x])/(a + b*x^2)^(1/4)] - 3*a*ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4
)]))/(4*b^(7/4)*x^(5/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {262, 266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^2\int \frac{\sqrt{cx}}{(bx^2+a)^{3/4}}dx}{4b}$$

$$\Big\downarrow \text{266}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac\int \frac{cx}{(bx^2+a)^{3/4}}d\sqrt{cx}}{2b}$$

$$\Big\downarrow \text{854}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac\int \frac{c^3x}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\int \frac{cx}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2b}$$

$$\Big\downarrow \text{827}$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)}{2b}$$

$$\Big\downarrow \ 218$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{2b}$$

$$\Big\downarrow \ 221$$

$$\frac{c(cx)^{3/2}\sqrt[4]{a+bx^2}}{2b}-\frac{3ac^3\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)}{2b}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^2)^(3/4),x]
```

output
```
(c*(c*x)^(3/2)*(a + b*x^2)^(1/4))/(2*b) - (3*a*c^3*(-1/2*ArcTan[(b^(1/4)*S
qrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(3/4)*Sqrt[c]) + ArcTanh[(b^(1/4
)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(2*b^(3/4)*Sqrt[c])))/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^{2}+a)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.38

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((3/4, 7/4), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(3/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(3/4),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{3/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^2}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)c^2$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**2)/(a + b*x**2)**(3/4),x)*c**2
```

3.1040 $\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = -\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}} + \frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{3/4}}$$

output
```
-c^(1/2)*arctan(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(3/4)+c^(1/
2)*arctanh(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(3/4)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \frac{\sqrt{cx}\left(-\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) + \mathrm{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)\right)}{b^{3/4}\sqrt{x}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

output
```
(Sqrt[c*x]*(-ArcTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] + ArcTanh[(b^(1/4
)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(b^(3/4)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {266, 854, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2\int \frac{cx}{(bx^2+a)^{3/4}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{ 854}$$

$$\frac{2\int \frac{c^3x}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$2c\int \frac{cx}{c^2-bc^2x^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}$$

$$\Big\downarrow \text{ 827}$$

$$2c\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}\right)$$

$$\Big\downarrow \text{ 218}$$

$$2c\left(\frac{\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{2\sqrt{b}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)$$

$$\downarrow \text{221}$$

$$2c\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2b^{3/4}\sqrt{c}}\right)$$

input
```
Int[Sqrt[c*x]/(a + b*x^2)^(3/4),x]
```

output
```
2*c*(-1/2*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(b^(3/4)
*Sqrt[c]) + ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))]/(2*b^
(3/4)*Sqrt[c]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 854
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + (m +
 1)/n)   Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n
)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -
2^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 3/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(3/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(3/4),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{3/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

output
```
sqrt(c)*int(sqrt(x)/(a + b*x**2)**(3/4),x)
```

3.1041 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

output
```
-2*(b*x^2+a)^(1/4)/a/c/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}}{a(cx)^{3/2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4))/(a*(c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2} (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{242}$$

$$-\frac{2\sqrt[4]{a+bx^2}}{ac\sqrt{cx}}$$

input
```
Int[1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(a*c*Sqrt[c*x])
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
gosper	$-\frac{2x(b\,x^2+a)^{\frac{1}{4}}}{a(cx)^{\frac{3}{2}}}$	21
orering	$-\frac{2x(b\,x^2+a)^{\frac{1}{4}}}{a(cx)^{\frac{3}{2}}}$	21
risch	$-\frac{2(b\,x^2+a)^{\frac{1}{4}}}{ac\sqrt{cx}}$	23

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2*x*(b*x^2+a)^(1/4)/a/(c*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\,(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{ac^2x}$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a*c^2*x)
```

Sympy [A] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx = \frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{1}{4}\right)}{2ac^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

output
```
b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-1/4)/(2*a*c**(3/2)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(3/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\left(b\,x^2+a\right)^{1/4}}{a\,c\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-(2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

$$\int \frac{1}{(cx)^{3/2} (a+bx^2)^{3/4}} \, dx = -\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}}{\sqrt{x}\,a\,c^2}$$

input

```
int(1/(c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

output

```
( - 2*sqrt(c)*(a + b*x**2)**(1/4))/(sqrt(x)*a*c**2)
```

3.1042 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{5ac(cx)^{5/2}}+\frac{8b\sqrt[4]{a+bx^2}}{5a^2c^3\sqrt{cx}}$$

output
```
-2/5*(b*x^2+a)^(1/4)/a/c/(c*x)^(5/2)+8/5*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x(a-4bx^2)\sqrt[4]{a+bx^2}}{5a^2(cx)^{7/2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a - 4*b*x^2)*(a + b*x^2)^(1/4))/(5*a^2*(c*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 246}$$

$$-\frac{4 \int \frac{\sqrt[4]{bx^2+a}}{(cx)^{7/2}}dx}{a} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

$$\Big\downarrow \text{ 242}$$

$$\frac{8(a+bx^2)^{5/4}}{5a^2c(cx)^{5/2}} - \frac{2\sqrt[4]{a+bx^2}}{ac(cx)^{5/2}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(5/2)) + (8*(a + b*x^2)^(5/4))/(5*a^2*c*
(c*x)^(5/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)}{5a^{2}\left(cx\right)^{\frac{7}{2}}}$	29
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)}{5a^{2}\left(cx\right)^{\frac{7}{2}}}$	29
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(-4b\,x^{2}+a\right)}{5c^{3}\sqrt{cx}\,a^{2}x^{2}}$	34

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/5*x*(b*x^2+a)^(1/4)*(-4*b*x^2+a)/a^2/(c*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = \frac{2\,(4\,bx^2-a)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{5\,a^2c^4x^3}$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
2/5*(4*b*x^2 - a)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^2*c^4*x^3)
```

Sympy [A] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.34

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{\sqrt[4]{b}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{8ac^{\frac{7}{2}}x^2\Gamma\left(\frac{3}{4}\right)}+\frac{b^{\frac{5}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(-\frac{5}{4}\right)}{2a^2c^{\frac{7}{2}}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(3/4),x)
```

output
```
-b**(1/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(8*a*c**(7/2)*x**2*gamma(3/4
)) + b**(5/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-5/4)/(2*a**2*c**(7/2)*gamma(3
/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(7/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{3/4}} \, dx = -\frac{(b\,x^2+a)^{1/4} \left(\frac{2}{5\,a\,c^3}-\frac{8\,b\,x^2}{5\,a^2\,c^3}\right)}{x^2\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(5*a*c^3) - (8*b*x^2)/(5*a^2*c^3)))/(x^2*(c*x)^(1/2
))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{3/4}} \, dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(4b\,x^2-a)}{5\sqrt{x}\,a^2c^4x^2}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - a + 4*b*x**2))/(5*sqrt(x)*a**2*c**4*x**
2)
```

3.1043 $\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 89

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{9ac(cx)^{9/2}}+\frac{16b\sqrt[4]{a+bx^2}}{45a^2c^3(cx)^{5/2}}-\frac{64b^2\sqrt[4]{a+bx^2}}{45a^3c^5\sqrt{cx}}$$

output
```
-2/9*(b*x^2+a)^(1/4)/a/c/(c*x)^(9/2)+16/45*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x)
^(5/2)-64/45*b^2*(b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx=-\frac{2x\sqrt[4]{a+bx^2}(5a^2-8abx^2+32b^2x^4)}{45a^3(cx)^{11/2}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(a + b*x^2)^(1/4)*(5*a^2 - 8*a*b*x^2 + 32*b^2*x^4))/(45*a^3*(c*x)^(1
1/2))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2} (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{246}$$

$$-\frac{8 \int \frac{\sqrt[4]{bx^2+a}}{(cx)^{11/2}}dx}{a}-\frac{2 \sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

$$\Big\downarrow \text{246}$$

$$-\frac{8 \left(-\frac{4 \int \frac{\left(bx^2+a\right)^{5/4}}{(cx)^{11/2}}dx}{5a}-\frac{2 \left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}\right)}{a}-\frac{2 \sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

$$\Big\downarrow \text{242}$$

$$-\frac{8 \left(\frac{8 \left(a+bx^2\right)^{9/4}}{45a^2c(cx)^{9/2}}-\frac{2 \left(a+bx^2\right)^{5/4}}{5ac(cx)^{9/2}}\right)}{a}-\frac{2 \sqrt[4]{a+bx^2}}{ac(cx)^{9/2}}$$

input
```
Int[1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(a*c*(c*x)^(9/2)) - (8*((-2*(a + b*x^2)^(5/4))/(5*a
*c*(c*x)^(9/2)) + (8*(a + b*x^2)^(9/4))/(45*a^2*c*(c*x)^(9/2))))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.47

method	result	size
gosper	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}-8ab\,x^{2}+5a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{11}{2}}}$	42
orering	$-\frac{2x\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}-8ab\,x^{2}+5a^{2}\right)}{45a^{3}\left(cx\right)^{\frac{11}{2}}}$	42
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{1}{4}}\left(32b^{2}x^{4}-8ab\,x^{2}+5a^{2}\right)}{45c^{5}\sqrt{cx}\,a^{3}x^{4}}$	47

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2/45*x*(b*x^2+a)^(1/4)*(32*b^2*x^4-8*a*b*x^2+5*a^2)/a^3/(c*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\,(32\,b^2x^4-8\,abx^2+5\,a^2)(bx^2+a)^{\frac{1}{4}}\sqrt{cx}}{45\,a^3c^6x^5}$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
-2/45*(32*b^2*x^4 - 8*a*b*x^2 + 5*a^2)*(b*x^2 + a)^(1/4)*sqrt(c*x)/(a^3*c^
6*x^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(82) = 164$.

Time = 106.18 (sec) , antiderivative size = 483, normalized size of antiderivative = 5.43

$$\int \frac{1}{(cx)^{11/2}\,(a+bx^2)^{3/4}}\,dx = \frac{5a^4b^{\frac{17}{4}}\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)} + \frac{2a^3b^{\frac{21}{4}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)} + \frac{21a^2b^{\frac{25}{4}}x^4\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)} + \frac{56ab^{\frac{29}{4}}x^6\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)} + \frac{32b^{\frac{33}{4}}x^8\sqrt[4]{\frac{a}{bx^2}+1}\,\Gamma\left(-\frac{9}{4}\right)}{32a^5b^4c^{\frac{11}{2}}x^4\Gamma\left(\frac{3}{4}\right)+64a^4b^5c^{\frac{11}{2}}x^6\Gamma\left(\frac{3}{4}\right)+32a^3b^6c^{\frac{11}{2}}x^8\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(11/2)/(b*x**2+a)**(3/4),x)
```

output
```
5*a**4*b**(17/4)*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/
2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6
*c**(11/2)*x**8*gamma(3/4)) + 2*a**3*b**(21/4)*x**2*(a/(b*x**2) + 1)**(1/4
)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(1
1/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 21*a**2*b
**(25/4)*x**4*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*
x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c*
*(11/2)*x**8*gamma(3/4)) + 56*a*b**(29/4)*x**6*(a/(b*x**2) + 1)**(1/4)*gam
ma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma(3/4) + 64*a**4*b**5*c**(11/2)*
x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x**8*gamma(3/4)) + 32*b**(33/4)*x
**8*(a/(b*x**2) + 1)**(1/4)*gamma(-9/4)/(32*a**5*b**4*c**(11/2)*x**4*gamma
(3/4) + 64*a**4*b**5*c**(11/2)*x**6*gamma(3/4) + 32*a**3*b**6*c**(11/2)*x*
*8*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(11/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{11/2} (a+bx^2)^{3/4}} \, dx = -\frac{(b\,x^2+a)^{1/4} \left(\frac{2}{9\,a\,c^5} - \frac{16\,b\,x^2}{45\,a^2\,c^5} + \frac{64\,b^2\,x^4}{45\,a^3\,c^5}\right)}{x^4\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(11/2)*(a + b*x^2)^(3/4)),x)
```

output
```
-((a + b*x^2)^(1/4)*(2/(9*a*c^5) - (16*b*x^2)/(45*a^2*c^5) + (64*b^2*x^4)/
(45*a^3*c^5)))/(x^4*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

$$\int \frac{1}{(cx)^{11/2} (a+bx^2)^{3/4}} \, dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}} \left(-32b^2x^4+8ab\,x^2-5a^2\right)}{45\sqrt{x}\,a^3c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 5*a**2 + 8*a*b*x**2 - 32*b**2*x**4))/(4
5*sqrt(x)*a**3*c**6*x**4)
```

3.1044 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 86

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx=\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}+\frac{\sqrt{a}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{b}\,(a+bx^2)^{3/4}}$$

output
```
c*(c*x)^(1/2)*(b*x^2+a)^(1/4)/b+a^(1/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*Inve
rseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/b^(1/2)/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.77

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx=\frac{c\sqrt{cx}\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^2}{a}\right)\right)}{b\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

output
```
(c*Sqrt[c*x]*(a + b*x^2 - a*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3
/4, 5/4, -((b*x^2)/a)]))/(b*(a + b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {262, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac^2\int \frac{1}{\sqrt{cx}(bx^2+a)^{3/4}}dx}{2b}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{b}$$

$$\Big\downarrow \text{ 768}$$

$$\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}-\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{b\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{b\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{ac(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{2b\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}$$

$$\Big\downarrow \text{ 229}$$

$$\frac{\sqrt{a}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{b}\left(a+bx^2\right)^{3/4}}+\frac{c\sqrt{cx}\sqrt[4]{a+bx^2}}{b}$$

input
```
Int[(c*x)^(3/2)/(a + b*x^2)^(3/4),x]
```

output
```
(c*Sqrt[c*x]*(a + b*x^2)^(1/4))/b + (Sqrt[a]*(1 + a/(b*x^2))^(3/4)*(c*x)^(
3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(Sqrt[b]*(a + b*x^2)
^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^{2}+a)^{\frac{3}{4}}}dx$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

output
```
int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x)*c*x/(b*x^2 + a)^(3/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.51

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(3/4),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((3/4, 5/4), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(3/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{3}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(3/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{3/4}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(3/4),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(3/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{3/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x}{(b\,x^2+a)^{\frac{3}{4}}}dx\right)c$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(3/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x)/(a + b*x**2)**(3/4),x)*c
```

3.1045 $\int \frac{1}{\sqrt{cx}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx=-\frac{2\sqrt{b}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{\sqrt{a}c^2\,(a+bx^2)^{3/4}}$$

output
```
-2*b^(1/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^(1/2
)*x/a^(1/2)),2^(1/2))/a^(1/2)/c^2/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx=\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{bx^2}{a}\right)}{\sqrt{cx}\,(a+bx^2)^{3/4}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

output
```
(2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((b*x^2)/a)])
/(Sqrt[c*x]*(a + b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}dx$$

$$\Big\downarrow \text{266}$$

$$\frac{2\int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{c}$$

$$\Big\downarrow \text{768}$$

$$\frac{2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{c\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{2(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{c\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{c\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{229}$$

$$-\frac{2\sqrt{b}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{\sqrt{a}c^2\,(a+bx^2)^{3/4}}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*Sqrt[b]*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^
2*x)/Sqrt[b]]/2, 2])/(Sqrt[a]*c^2*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{\sqrt{cx}\,(b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.47

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{3}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{b^{\frac{3}{4}}\sqrt{cx}}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(3/4),x)
```

output
```
-hyper((1/2, 3/4), (3/2,), a*exp_polar(I*pi)/(b*x**2))/(b**(3/4)*sqrt(c)*x
)
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*sqrt(c*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \int \frac{1}{\sqrt{c\,x}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(3/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{3/4}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^{\frac{5}{4}}}{b^2x^5+2ab\,x^3+a^2x}dx\right)}{c}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(3/4),x)
```

output
```
(sqrt(c)*int((sqrt(x)*(a + b*x**2)**(5/4))/(a**2*x + 2*a*b*x**3 + b**2*x**
5),x))/c
```

3.1046 $\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 97

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}} + \frac{4b^{3/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\,\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}$$

output
```
-2/3*(b*x^2+a)^(1/4)/a/c/(c*x)^(3/2)+4/3*b^(3/2)*(1+a/b/x^2)^(3/4)*(c*x)^(
3/2)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(1/2))/a^(3/2)/c^4/(b
*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.58

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = -\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{3}{4},\frac{1}{4},-\frac{bx^2}{a}\right)}{3(cx)^{5/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-3/4, 3/4, 1/4, -((b*x^2)/a)
])/(3*(c*x)^(5/2)*(a + b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{2b \int \frac{1}{\sqrt{cx}(bx^2+a)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{4b \int \frac{1}{(bx^2+a)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 768}$$

$$-\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 858}$$

$$\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

$$\Big\downarrow \text{229}$$

$$\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3
/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2
)*c^4*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{2}} (b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} (cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^3*x^5 + a*c^3*x^3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.49

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} -\frac{3}{4}, \frac{3}{4} \\ \frac{1}{4} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2a^{\frac{3}{4}}c^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(3/4),x)
```

output
```
gamma(-3/4)*hyper((-3/4, 3/4), (1/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(3/
4)*c**(5/2)*x**(3/2)*gamma(1/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{3/4}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{3}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^2)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{5/2}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.37

$$\int \frac{1}{(cx)^{5/2}\,(a+bx^2)^{3/4}}\,dx = -\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{3}{4}}}{3\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,c^3x}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(3/4),x)
```

output
```
( - 2*sqrt(c)*(a + b*x**2)**(3/4))/(3*sqrt(x)*sqrt(a + b*x**2)*a*c**3*x)
```

3.1047 $\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx = -\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}+\frac{4b\sqrt[4]{a+bx^2}}{7a^2c^3(cx)^{3/2}} - \frac{8b^{5/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{7a^{5/2}c^6(a+bx^2)^{3/4}}$$

output
```
-2/7*(b*x^2+a)^(1/4)/a/c/(c*x)^(7/2)+4/7*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*x)^(
3/2)-8/7*b^(5/2)*(1+a/b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(
b^(1/2)*x/a^(1/2)),2^(1/2))/a^(5/2)/c^6/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.44

$$\int \frac{1}{(cx)^{9/2}(a+bx^2)^{3/4}}\,dx = -\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{3}{4},-\frac{3}{4},-\frac{bx^2}{a}\right)}{7(cx)^{9/2}(a+bx^2)^{3/4}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-7/4, 3/4, -3/4, -((b*x^2)/a
)])/(7*(c*x)^(9/2)*(a + b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {264, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{6b\int \frac{1}{(cx)^{5/2}(bx^2+a)^{3/4}}dx}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$-\frac{6b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\left(bx^2+a\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{6b\left(-\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 858$$

$$-\frac{6b\left(\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 807$$

$$-\frac{6b\left(\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3ac^3(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

$$\Big\downarrow \ 229$$

$$-\frac{6b\left(\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4(a+bx^2)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}$$

input
```
Int[1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*(a + b*x^2)^(1/4))/(7*a*c*(c*x)^(7/2)) - (6*b*((-2*(a + b*x^2)^(1/4))/
(3*a*c*(c*x)^(3/2)) + (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*Ellipti
cF[ArcTan[(Sqrt[a]*c^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a + b*x^2)^(3/4)
)))/(7*a*c^2)
```

Defintions of rubi rules used

rule 229
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{9}{2}} (b\,x^2+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^5*x^7 + a*c^5*x^5), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.54 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} \frac{3}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{3}{4}}c^{\frac{9}{2}}x^5}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**2+a)**(3/4),x)
```

output
```
-hyper((3/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(3/4)*c**(9/2
)*x**5)
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} (cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(9/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}}\,dx = \int \frac{1}{(c\,x)^{9/2}\,(b\,x^2+a)^{3/4}}\,dx$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{3/4}} \, dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{3}{4}}\,(4b\,x^2-3a)}{21\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2c^5x^3}$$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(3/4)*( - 3*a + 4*b*x**2))/(21*sqrt(x)*sqrt(a + b
*x**2)*a**2*c**5*x**3)
```

3.1048 $\int \frac{1}{(cx)^{13/2}(a+bx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}+\frac{20b\sqrt[4]{a+bx^2}}{77a^2c^3(cx)^{7/2}}-\frac{40b^2\sqrt[4]{a+bx^2}}{77a^3c^5(cx)^{3/2}}+\frac{80b^{7/2}\left(1+\frac{a}{bx^2}\right)^{3/4}(cx)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),2\right)}{77a^{7/2}c^8\left(a+bx^2\right)^{3/4}}$$

output
```
-2/11*(b*x^2+a)^(1/4)/a/c/(c*x)^(11/2)+20/77*b*(b*x^2+a)^(1/4)/a^2/c^3/(c*
x)^(7/2)-40/77*b^2*(b*x^2+a)^(1/4)/a^3/c^5/(c*x)^(3/2)+80/77*b^(7/2)*(1+a/
b/x^2)^(3/4)*(c*x)^(3/2)*InverseJacobiAM(1/2*arccot(b^(1/2)*x/a^(1/2)),2^(
1/2))/a^(7/2)/c^8/(b*x^2+a)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{3/4}}\,dx=-\frac{2x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{11}{4},\frac{3}{4},-\frac{7}{4},-\frac{bx^2}{a}\right)}{11(cx)^{13/2}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-11/4, 3/4, -7/4, -((b*x^2)/
a)])/(11*(c*x)^(13/2)*(a + b*x^2)^(3/4))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {264, 264, 264, 266, 768, 858, 807, 229}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2} \left(a+bx^2\right)^{3/4}} \, dx$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{10b \int \frac{1}{(cx)^{9/2}(bx^2+a)^{3/4}}dx}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{10b\left(-\frac{6b\int \frac{1}{(cx)^{5/2}\left(bx^2+a\right)^{3/4}}dx}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{1}{\sqrt{cx}\left(bx^2+a\right)^{3/4}}dx}{3ac^2}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{ 266}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{4b\int \frac{1}{\left(bx^2+a\right)^{3/4}}d\sqrt{cx}}{3ac^3}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{768}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{3/4}(cx)^{3/2}}d\sqrt{cx}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{10b\left(-\frac{6b\left(\frac{4b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\sqrt{cx}\left(\frac{ax^2c^4}{b}+1\right)^{3/4}}d\frac{1}{\sqrt{cx}}}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow \text{807}$$

$$-\frac{10b\left(-\frac{6b\left(\frac{2b(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\int \frac{1}{\left(\frac{axc^3}{b}+1\right)^{3/4}}d(cx)}{3ac^3\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

$$\Big\downarrow 229$$

$$-\frac{10b\left(-\frac{6b\left(\frac{4b^{3/2}(cx)^{3/2}\left(\frac{a}{bx^2}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}c^2x}{\sqrt{b}}\right),2\right)}{3a^{3/2}c^4\left(a+bx^2\right)^{3/4}}-\frac{2\sqrt[4]{a+bx^2}}{3ac(cx)^{3/2}}\right)}{7ac^2}-\frac{2\sqrt[4]{a+bx^2}}{7ac(cx)^{7/2}}\right)}{11ac^2}-\frac{2\sqrt[4]{a+bx^2}}{11ac(cx)^{11/2}}$$

input

```
Int[1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x]
```

output

```
(-2*(a + b*x^2)^(1/4))/(11*a*c*(c*x)^(11/2)) - (10*b*((-2*(a + b*x^2)^(1/4
))/(7*a*c*(c*x)^(7/2)) - (6*b*((-2*(a + b*x^2)^(1/4))/(3*a*c*(c*x)^(3/2))
+ (4*b^(3/2)*(1 + a/(b*x^2))^(3/4)*(c*x)^(3/2)*EllipticF[ArcTan[(Sqrt[a]*c
^2*x)/Sqrt[b]]/2, 2])/(3*a^(3/2)*c^4*(a + b*x^2)^(3/4))))/(7*a*c^2)))/(11*
a*c^2)
```

Defintions of rubi rules used

rule 229

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[b/a, 2])
)*EllipticF[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 264

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 768
```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Simp[x^3*((1 + a/(b*x^4))^(3
/4)/(a + b*x^4)^(3/4))   Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ
[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{13}{2}} (b\,x^{2}+a)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

output
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{3/4}}\, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{4}} (cx)^{\frac{13}{2}}}\, dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*sqrt(c*x)/(b*c^7*x^9 + a*c^7*x^7), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2} (a + bx^2)^{3/4}} \, dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(13/2)/(b*x**2+a)**(3/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2} (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{13}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2} (a + bx^2)^{3/4}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{3}{4}} (cx)^{\frac{13}{2}}} \, dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/4)*(c*x)^(13/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{3/4}} \, dx = \int \frac{1}{(c\,x)^{13/2} \left(b\,x^2+a\right)^{3/4}} \, dx$$

input
```
int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)),x)
```

output
```
int(1/((c*x)^(13/2)*(a + b*x^2)^(3/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{3/4}} \, dx = \frac{2\sqrt{c}\left(b\,x^2+a\right)^{\frac{3}{4}} \left(-32b^2x^4+24ab\,x^2-21a^2\right)}{231\sqrt{x}\,\sqrt{b\,x^2+a}\,a^3c^7x^5}$$

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(3/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(3/4)*( - 21*a**2 + 24*a*b*x**2 - 32*b**2*x**4))/
(231*sqrt(x)*sqrt(a + b*x**2)*a**3*c**7*x**5)
```

3.1049 $\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 146

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}} \, dx = \frac{5ac^3\sqrt{cx}}{2b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}} - \frac{5ac^{7/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}} - \frac{5ac^{7/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{4b^{9/4}}$$

output
```
5/2*a*c^3*(c*x)^(1/2)/b^2/(b*x^2+a)^(1/4)+1/2*c*(c*x)^(5/2)/b/(b*x^2+a)^(1
/4)-5/4*a*c^(7/2)*arctan(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(9
/4)-5/4*a*c^(7/2)*arctanh(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(b*x^2+a)^(1/4))/b^(
9/4)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}} \, dx = \frac{c^3\sqrt{cx}\left(2\sqrt[4]{b}\sqrt{x}(5a+bx^2) - 5a\sqrt[4]{a+bx^2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right) - 5a\sqrt[4]{a+bx^2}\text{arctanh}\right.}{4b^{9/4}\sqrt{x}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c^3*Sqrt[c*x]*(2*b^(1/4)*Sqrt[x]*(5*a + b*x^2) - 5*a*(a + b*x^2)^(1/4)*Ar
cTan[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)] - 5*a*(a + b*x^2)^(1/4)*ArcTanh[
(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]))/(4*b^(9/4)*Sqrt[x]*(a + b*x^2)^(1/4
))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {250, 252, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\int \frac{(cx)^{3/2}}{(bx^2+a)^{5/4}}dx}{4b}$$

$$\Big\downarrow \text{252}$$

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt[4]{bx^2+a}}dx}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

$$\Big\downarrow \text{266}$$

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{2c\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{cx}}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

$$\Big\downarrow \text{770}$$

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{2c\int \frac{1}{1-bx^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

$\Big\downarrow$ 756

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{2c\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}c\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

$\Big\downarrow$ 218

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{2c\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

$\Big\downarrow$ 221

$$\frac{c(cx)^{5/2}}{2b\sqrt[4]{a+bx^2}}-\frac{5ac^2\left(\frac{2c\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}\right)}{4b}$$

input
```
Int[(c*x)^(7/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*(c*x)^(5/2))/(2*b*(a + b*x^2)^(1/4)) - (5*a*c^2*((-2*c*Sqrt[c*x])/(b*(a
 + b*x^2)^(1/4)) + (2*c*((Sqrt[c]*ArcTan[(b^(1/4)*Sqrt[c*x])/(Sqrt[c]*(a +
 b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[c]*ArcTanh[(b^(1/4)*Sqrt[c*x])/(Sqrt[
c]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/b))/(4*b)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(b\,x^{2}+a\right)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.74

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \frac{4\,(bc^3x^2+5\,ac^3)(bx^2+a)^{\frac{3}{4}}\sqrt{cx}-5\left(\frac{a^4c^{14}}{b^9}\right)^{\frac{1}{4}}(b^3x^2+ab^2)\log\left(\frac{5\left((bx^2+a)^{\frac{3}{4}}\sqrt{cx}ac^3+\left(\frac{a^4c^{1}}{b^9}\right.\right.}{bx^2+a}\right.}{}$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
1/8*(4*(b*c^3*x^2 + 5*a*c^3)*(b*x^2 + a)^(3/4)*sqrt(c*x) - 5*(a^4*c^14/b^9
)^(1/4)*(b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 + (a^4*
c^14/b^9)^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a)) + 5*(a^4*c^14/b^9)^(1/4)*(
b^3*x^2 + a*b^2)*log(5*((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)
^(1/4)*(b^3*x^2 + a*b^2))/(b*x^2 + a)) - 5*(a^4*c^14/b^9)^(1/4)*(-I*b^3*x^
2 - I*a*b^2)*log(5*((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/
4)*(I*b^3*x^2 + I*a*b^2))/(b*x^2 + a)) - 5*(a^4*c^14/b^9)^(1/4)*(I*b^3*x^2
 + I*a*b^2)*log(5*((b*x^2 + a)^(3/4)*sqrt(c*x)*a*c^3 - (a^4*c^14/b^9)^(1/4
)*(-I*b^3*x^2 - I*a*b^2))/(b*x^2 + a)))/(b^3*x^2 + a*b^2)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.97 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.30

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^{\frac{7}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{5}{4},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate((c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

output
```
c**(7/2)*x**(9/2)*gamma(9/4)*hyper((5/4, 9/4), (13/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(13/4))
```

Maxima [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{7}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(7/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(c\,x)^{7/2}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(7/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(7/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{7/2}}{(a+bx^2)^{5/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^3}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^3$$

input
```
int((c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**3)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*
x**2),x)*c**3
```

3.1050 $\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx = -\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}} + \frac{c^{3/2}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}} + \frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{b^{5/4}}$$

output
```
-2*c*(c*x)^(1/2)/b/(b*x^2+a)^(1/4)+c^(3/2)*arctan(b^(1/4)*(c*x)^(1/2)/c^(1
/2)/(b*x^2+a)^(1/4))/b^(5/4)+c^(3/2)*arctanh(b^(1/4)*(c*x)^(1/2)/c^(1/2)/(
b*x^2+a)^(1/4))/b^(5/4)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c\sqrt{cx}\left(-\frac{2\sqrt[4]{b}}{\sqrt[4]{a+bx^2}} + \frac{\arctan\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}} + \frac{\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a+bx^2}}\right)}{\sqrt{x}}\right)}{b^{5/4}}$$

input
```
Integrate[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*Sqrt[c*x]*((-2*b^(1/4))/(a + b*x^2)^(1/4) + ArcTan[(b^(1/4)*Sqrt[x])/(a
 + b*x^2)^(1/4)]/Sqrt[x] + ArcTanh[(b^(1/4)*Sqrt[x])/(a + b*x^2)^(1/4)]/Sq
rt[x]))/b^(5/4)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {252, 266, 770, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{c^2\int \frac{1}{\sqrt{cx}\sqrt[4]{bx^2+a}}dx}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2c\int \frac{1}{\sqrt[4]{bx^2+a}}d\sqrt{cx}}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{770}$$

$$\frac{2c\int \frac{1}{1-bx^2}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{756}$$

$$\frac{2c\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{1}{2}c\int \frac{1}{\sqrt{b}xc+c}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{2c\left(\frac{1}{2}c\int \frac{1}{c-\sqrt{b}cx}d\frac{\sqrt{cx}}{\sqrt[4]{bx^2+a}}+\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2c\left(\frac{\sqrt{c}\arctan\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}+\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt[4]{b}\sqrt{cx}}{\sqrt{c}\sqrt[4]{a+bx^2}}\right)}{2\sqrt[4]{b}}\right)}{b}-\frac{2c\sqrt{cx}}{b\sqrt[4]{a+bx^2}}$$

input
```
Int[(c*x)^(3/2)/(a + b*x^2)^(5/4),x]
```

output
```
(-2*c*Sqrt[c*x])/(b*(a + b*x^2)^(1/4)) + (2*c*((Sqrt[c]*ArcTan[(b^(1/4)*Sq
rt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(2*b^(1/4)) + (Sqrt[c]*ArcTanh[(b^(
1/4)*Sqrt[c*x])/(Sqrt[c]*(a + b*x^2)^(1/4))])/(2*b^(1/4))))/b
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 770
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^(p + 1/n)   Subst[In
t[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
 b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1
/n]
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.06

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx =$$

$$-\frac{4\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}c-(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a)^{\frac{3}{4}}\sqrt{cx}c+(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}}{bx^2+a}\right)+(b^2x^2+ab)\left(\frac{c^6}{b^5}\right)^{\frac{1}{4}}\log\left(\frac{(bx^2+a}{}\right.}{}$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
-1/2*(4*(b*x^2 + a)^(3/4)*sqrt(c*x)*c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*lo
g(((b*x^2 + a)^(3/4)*sqrt(c*x)*c + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2
 + a)) + (b^2*x^2 + a*b)*(c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*
c - (b^2*x^2 + a*b)*(c^6/b^5)^(1/4))/(b*x^2 + a)) - (-I*b^2*x^2 - I*a*b)*(
c^6/b^5)^(1/4)*log(((b*x^2 + a)^(3/4)*sqrt(c*x)*c - (I*b^2*x^2 + I*a*b)*(c
^6/b^5)^(1/4))/(b*x^2 + a)) - (I*b^2*x^2 + I*a*b)*(c^6/b^5)^(1/4)*log(((b*
x^2 + a)^(3/4)*sqrt(c*x)*c - (-I*b^2*x^2 - I*a*b)*(c^6/b^5)^(1/4))/(b*x^2
+ a)))/(b^2*x^2 + a*b)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.87 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.41

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^{\frac{3}{2}}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right) {}_{2}F_{1}\left(\begin{matrix} \frac{5}{4}, \frac{5}{4} \\ \frac{9}{4} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

output
```
c**(3/2)*x**(5/2)*gamma(5/4)*hyper((5/4, 5/4), (9/4,), b*x**2*exp_polar(I*
pi)/a)/(2*a**(5/4)*gamma(9/4))
```

Maxima [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx=\int \frac{(cx)^{\frac{3}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx=\int \frac{(c\,x)^{3/2}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(3/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(3/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/2}}{(a+bx^2)^{5/4}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{x}\,x}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c$$

input
```
int((c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**
2),x)*c
```

3.1051 $\int \frac{1}{\sqrt{cx}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

output
```
2*(c*x)^(1/2)/a/c/(b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2x}{a\sqrt{cx}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

output
```
(2*x)/(a*Sqrt[c*x]*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \; 242$$

$$\frac{2\sqrt{cx}}{ac\sqrt[4]{a+bx^2}}$$

input
```
Int[1/(Sqrt[c*x]*(a + b*x^2)^(5/4)),x]
```

output
```
(2*Sqrt[c*x])/(a*c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
gosper	$\frac{2x}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} a\sqrt{cx}}$	21
orering	$\frac{2x}{\left(b\,x^{2}+a\right)^{\frac{1}{4}} a\sqrt{cx}}$	21

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
2*x/(b*x^2+a)^(1/4)/a/(c*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2\,(bx^2+a)^{\frac{3}{4}}\sqrt{cx}}{abcx^2+a^2c}$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
2*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a*b*c*x^2 + a^2*c)
```

Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{\Gamma\left(\frac{1}{4}\right)}{2a\sqrt[4]{b}\sqrt{c}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

output
```
gamma(1/4)/(2*a*b**(1/4)*sqrt(c)*(a/(b*x**2) + 1)**(1/4)*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}\sqrt{cx}}\,dx$$

input
```
integrate(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*sqrt(c*x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2\,x\,(b\,x^2+a)^{3/4}}{(a^2+b\,a\,x^2)\,\sqrt{c\,x}}$$

input
```
int(1/((c*x)^(1/2)*(a + b*x^2)^(5/4)),x)
```

output
```
(2*x*(a + b*x^2)^(3/4))/((a^2 + a*b*x^2)*(c*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{cx}\,(a+bx^2)^{5/4}}\,dx = \frac{2\sqrt{x}\,\sqrt{c}}{(b\,x^2+a)^{\frac{1}{4}}\,ac}$$

input
```
int(1/(c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(x)*sqrt(c)*(a + b*x**2)**(3/4))/(a*c*(a + b*x**2))
```

3.1052 $\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2}{ac(cx)^{3/2}\sqrt[4]{a+bx^2}}-\frac{8\left(a+bx^2\right)^{3/4}}{3a^2c(cx)^{3/2}}$$

output
```
2/a/c/(c*x)^(3/2)/(b*x^2+a)^(1/4)-8/3*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.62

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2x(a+4bx^2)}{3a^2(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(a + 4*b*x^2))/(3*a^2*(c*x)^(5/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/2} (a+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{246}$$

$$\frac{4 \int \frac{1}{(cx)^{5/2} \sqrt[4]{bx^2+a}}dx}{a} + \frac{2}{ac(cx)^{3/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{ac(cx)^{3/2} \sqrt[4]{a+bx^2}} - \frac{8(a+bx^2)^{3/4}}{3a^2c(cx)^{3/2}}$$

input
```
Int[1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x]
```

output
```
2/(a*c*(c*x)^(3/2)*(a + b*x^2)^(1/4)) - (8*(a + b*x^2)^(3/4))/(3*a^2*c*(c*
x)^(3/2))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.53

method	result	size
gosper	$-\frac{2x\left(4b\,x^{2}+a\right)}{3\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{2}\left(cx\right)^{\frac{5}{2}}}$	29
orering	$-\frac{2x\left(4b\,x^{2}+a\right)}{3\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{2}\left(cx\right)^{\frac{5}{2}}}$	29
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}}{3a^{2}x\,c^{2}\sqrt{cx}}-\frac{2bx}{a^{2}c^{2}\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	51

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(4*b*x^2+a)/(b*x^2+a)^(1/4)/a^2/(c*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2\left(4\,bx^2+a\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{3\left(a^2bc^3x^4+a^3c^3x^2\right)}$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
-2/3*(4*b*x^2 + a)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^2*b*c^3*x^4 + a^3*c^3*x^
2)
```

Sympy [A] (verification not implemented)

Time = 7.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}}\,dx = \frac{\Gamma\left(-\frac{3}{4}\right)}{8a\sqrt[4]{b}c^{\frac{5}{2}}x^2\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)} + \frac{b^{\frac{3}{4}}\Gamma\left(-\frac{3}{4}\right)}{2a^2c^{\frac{5}{2}}\sqrt[4]{\frac{a}{bx^2}+1}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

output
```
gamma(-3/4)/(8*a*b**(1/4)*c**(5/2)*x**2*(a/(b*x**2) + 1)**(1/4)*gamma(5/4)
) + b**(3/4)*gamma(-3/4)/(2*a**2*c**(5/2)*(a/(b*x**2) + 1)**(1/4)*gamma(5/
4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/2}(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(cx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(5/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{2}{3\,a\,b\,c^2}+\frac{8\,x^2}{3\,a^2\,c^2}\right)}{x^3\,\sqrt{c\,x}+\frac{a\,x\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(5/2)*(a + b*x^2)^(5/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*(2/(3*a*b*c^2) + (8*x^2)/(3*a^2*c^2)))/(x^3*(c*x)^(1/2
) + (a*x*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{1}{(cx)^{5/2}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2\sqrt{c}\left(-4b\,x^2-a\right)}{3\left(b\,x^2+a\right)^{\frac{1}{4}}\sqrt{x}\,a^2c^3x}$$

input
```
int(1/(c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(3/4)*( - a - 4*b*x**2))/(3*sqrt(x)*a**2*c**3*x*(
a + b*x**2))
```

3.1053 $\int \frac{1}{(cx)^{9/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 84

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2}{ac(cx)^{7/2}\sqrt[4]{a+bx^2}}-\frac{16\left(a+bx^2\right)^{3/4}}{7a^2c(cx)^{7/2}}+\frac{64b\left(a+bx^2\right)^{3/4}}{21a^3c^3(cx)^{3/2}}$$

output
```
2/a/c/(c*x)^(7/2)/(b*x^2+a)^(1/4)-16/7*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(7/2)+6
4/21*b*(b*x^2+a)^(3/4)/a^3/c^3/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{2x(3a^2-8abx^2-32b^2x^4)}{21a^3(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(3*a^2 - 8*a*b*x^2 - 32*b^2*x^4))/(21*a^3*(c*x)^(9/2)*(a + b*x^2)^(1
/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{9/2} \left(a+bx^2\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{246}$$

$$\frac{8 \int \frac{1}{(cx)^{9/2} \sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ac(cx)^{7/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{246}$$

$$\frac{8 \left(-\frac{4 \int \frac{\left(bx^2+a\right)^{3/4}}{(cx)^{9/2}}dx}{3a}-\frac{2 \left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}\right)}{a}+\frac{2}{ac(cx)^{7/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{8 \left(\frac{8 \left(a+bx^2\right)^{7/4}}{21a^2c(cx)^{7/2}}-\frac{2 \left(a+bx^2\right)^{3/4}}{3ac(cx)^{7/2}}\right)}{a}+\frac{2}{ac(cx)^{7/2} \sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x]
```

output
```
2/(a*c*(c*x)^(7/2)*(a + b*x^2)^(1/4)) + (8*((-2*(a + b*x^2)^(3/4))/(3*a*c*
(c*x)^(7/2)) + (8*(a + b*x^2)^(7/4))/(21*a^2*c*(c*x)^(7/2))))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.50

method	result	size
gosper	$-\frac{2x\left(-32b^{2}x^{4}-8ab\,x^{2}+3a^{2}\right)}{21\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{3}\left(cx\right)^{\frac{9}{2}}}$	42
orering	$-\frac{2x\left(-32b^{2}x^{4}-8ab\,x^{2}+3a^{2}\right)}{21\left(b\,x^{2}+a\right)^{\frac{1}{4}}a^{3}\left(cx\right)^{\frac{9}{2}}}$	42
risch	$-\frac{2\left(b\,x^{2}+a\right)^{\frac{3}{4}}\left(-11b\,x^{2}+3a\right)}{21a^{3}x^{3}c^{4}\sqrt{cx}}+\frac{2b^{2}x}{a^{3}c^{4}\sqrt{cx}\,\left(b\,x^{2}+a\right)^{\frac{1}{4}}}$	63

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/21*x*(-32*b^2*x^4-8*a*b*x^2+3*a^2)/(b*x^2+a)^(1/4)/a^3/(c*x)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.73

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2\left(32\,b^2x^4+8\,abx^2-3\,a^2\right)\left(bx^2+a\right)^{\frac{3}{4}}\sqrt{cx}}{21\left(a^3bc^5x^6+a^4c^5x^4\right)}$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
2/21*(32*b^2*x^4 + 8*a*b*x^2 - 3*a^2)*(b*x^2 + a)^(3/4)*sqrt(c*x)/(a^3*b*c
^5*x^6 + a^4*c^5*x^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(75) = 150$.

Time = 65.83 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.57

$$\int \frac{1}{(cx)^{9/2}(a+bx^2)^{5/4}}\,dx = -\frac{3a^3b^{\frac{19}{4}}\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)} + \frac{5a^2b^{\frac{23}{4}}x^2\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)} + \frac{40ab^{\frac{27}{4}}x^4\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)} + \frac{32b^{\frac{31}{4}}x^6\left(\frac{a}{bx^2}+1\right)^{\frac{3}{4}}\Gamma\left(-\frac{7}{4}\right)}{32a^5b^4c^{\frac{9}{2}}x^2\Gamma\left(\frac{5}{4}\right)+64a^4b^5c^{\frac{9}{2}}x^4\Gamma\left(\frac{5}{4}\right)+32a^3b^6c^{\frac{9}{2}}x^6\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(1/(c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

output
```
-3*a**3*b**(19/4)*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/
2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*
c**(9/2)*x**6*gamma(5/4)) + 5*a**2*b**(23/4)*x**2*(a/(b*x**2) + 1)**(3/4)*
gamma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)
*x**4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**6*gamma(5/4)) + 40*a*b**(27/4)
*x**4*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*a**5*b**4*c**(9/2)*x**2*gamm
a(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4) + 32*a**3*b**6*c**(9/2)*x**
6*gamma(5/4)) + 32*b**(31/4)*x**6*(a/(b*x**2) + 1)**(3/4)*gamma(-7/4)/(32*
a**5*b**4*c**(9/2)*x**2*gamma(5/4) + 64*a**4*b**5*c**(9/2)*x**4*gamma(5/4)
 + 32*a**3*b**6*c**(9/2)*x**6*gamma(5/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(9/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{1}{(cx)^{9/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{16\,x^2}{21\,a^2\,c^4}-\frac{2}{7\,a\,b\,c^4}+\frac{64\,b\,x^4}{21\,a^3\,c^4}\right)}{x^5\,\sqrt{c\,x}+\frac{a\,x^3\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(9/2)*(a + b*x^2)^(5/4)),x)
```

output
```
((a + b*x^2)^(3/4)*((16*x^2)/(21*a^2*c^4) - 2/(7*a*b*c^4) + (64*b*x^4)/(21
*a^3*c^4)))/(x^5*(c*x)^(1/2) + (a*x^3*(c*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.56

$$\int \frac{1}{(cx)^{9/2} (a+bx^2)^{5/4}} \, dx = \frac{2\sqrt{c}\,(32b^2x^4+8ab\,x^2-3a^2)}{21\,(b\,x^2+a)^{\frac{1}{4}}\sqrt{x}\,a^3c^5x^3}$$

input
```
int(1/(c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(3/4)*( - 3*a**2 + 8*a*b*x**2 + 32*b**2*x**4))/(2
1*sqrt(x)*a**3*c**5*x**3*(a + b*x**2))
```

3.1054 $\int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}}\,dx = \frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}} - \frac{24(a+bx^2)^{3/4}}{11a^2c(cx)^{11/2}} + \frac{192b(a+bx^2)^{3/4}}{77a^3c^3(cx)^{7/2}} - \frac{256b^2(a+bx^2)^{3/4}}{77a^4c^5(cx)^{3/2}}$$

output
```
2/a/c/(c*x)^(11/2)/(b*x^2+a)^(1/4)-24/11*(b*x^2+a)^(3/4)/a^2/c/(c*x)^(11/2
)+192/77*b*(b*x^2+a)^(3/4)/a^3/c^3/(c*x)^(7/2)-256/77*b^2*(b*x^2+a)^(3/4)/
a^4/c^5/(c*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \frac{1}{(cx)^{13/2}(a+bx^2)^{5/4}}\,dx = -\frac{2x(7a^3-12a^2bx^2+32ab^2x^4+128b^3x^6)}{77a^4(cx)^{13/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(7*a^3 - 12*a^2*b*x^2 + 32*a*b^2*x^4 + 128*b^3*x^6))/(77*a^4*(c*x)^(
13/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {246, 246, 246, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{246}$$

$$\frac{12 \int \frac{1}{(cx)^{13/2} \sqrt[4]{bx^2+a}}dx}{a}+\frac{2}{ac(cx)^{11/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{246}$$

$$\frac{12 \left(-\frac{8 \int \frac{\left(bx^2+a\right)^{3/4}}{(cx)^{13/2}}dx}{3a}-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}\right)}{a}+\frac{2}{ac(cx)^{11/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{246}$$

$$\frac{12 \left(-\frac{8 \left(-\frac{4 \int \frac{\left(bx^2+a\right)^{7/4}}{(cx)^{13/2}}dx}{7a}-\frac{2\left(a+bx^2\right)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}\right)}{a}+\frac{2}{ac(cx)^{11/2} \sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{242}$$

$$\frac{12\left(-\frac{8\left(\frac{8(a+bx^2)^{11/4}}{77a^2c(cx)^{11/2}}-\frac{2(a+bx^2)^{7/4}}{7ac(cx)^{11/2}}\right)}{3a}-\frac{2(a+bx^2)^{3/4}}{3ac(cx)^{11/2}}\right)}{a}+\frac{2}{ac(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x]
```

output
```
2/(a*c*(c*x)^(11/2)*(a + b*x^2)^(1/4)) + (12*((-2*(a + b*x^2)^(3/4))/(3*a*
c*(c*x)^(11/2)) - (8*((-2*(a + b*x^2)^(7/4))/(7*a*c*(c*x)^(11/2)) + (8*(a
+ b*x^2)^(11/4))/(77*a^2*c*(c*x)^(11/2))))/(3*a)))/a
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 246
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(a*c*2*(p + 1))), x] + Simp[(m + 2*p + 3)/(
a*2*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
, p}, x] && ILtQ[Simplify[(m + 1)/2 + p + 1], 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.46

method	result	size
gosper	$-\frac{2x\left(128b^3x^6+32a\,b^2x^4-12a^2b\,x^2+7a^3\right)}{77\left(b\,x^2+a\right)^{\frac{1}{4}}a^4\left(cx\right)^{\frac{13}{2}}}$	53
orering	$-\frac{2x\left(128b^3x^6+32a\,b^2x^4-12a^2b\,x^2+7a^3\right)}{77\left(b\,x^2+a\right)^{\frac{1}{4}}a^4\left(cx\right)^{\frac{13}{2}}}$	53
risch	$-\frac{2\left(b\,x^2+a\right)^{\frac{3}{4}}\left(51b^2x^4-19ab\,x^2+7a^2\right)}{77a^4x^5c^6\sqrt{cx}}-\frac{2b^3x}{a^4c^6\sqrt{cx}\,\left(b\,x^2+a\right)^{\frac{1}{4}}}$	74

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2/77*x*(128*b^3*x^6+32*a*b^2*x^4-12*a^2*b*x^2+7*a^3)/(b*x^2+a)^(1/4)/a^4/
(c*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.63

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{5/4}} dx = -\frac{2 (128 b^3x^6 + 32 ab^2x^4 - 12 a^2bx^2 + 7 a^3)(bx^2 + a)^{\frac{3}{4}} \sqrt{cx}}{77 (a^4bc^7x^8 + a^5c^7x^6)}$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
-2/77*(128*b^3*x^6 + 32*a*b^2*x^4 - 12*a^2*b*x^2 + 7*a^3)*(b*x^2 + a)^(3/4
)*sqrt(c*x)/(a^4*b*c^7*x^8 + a^5*c^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{5/4}} dx = \text{Timed out}$$

input
```
integrate(1/(c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{13}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(13/2)), x)
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74

$$\int \frac{1}{(cx)^{13/2}\left(a+bx^2\right)^{5/4}}\,dx=-\frac{\left(b\,x^2+a\right)^{3/4}\left(\frac{2}{11\,a\,b\,c^6}-\frac{24\,x^2}{77\,a^2\,c^6}+\frac{64\,b\,x^4}{77\,a^3\,c^6}+\frac{256\,b^2\,x^6}{77\,a^4\,c^6}\right)}{x^7\,\sqrt{c\,x}+\frac{a\,x^5\,\sqrt{c\,x}}{b}}$$

input
```
int(1/((c*x)^(13/2)*(a + b*x^2)^(5/4)),x)
```

output
```
-((a + b*x^2)^(3/4)*(2/(11*a*b*c^6) - (24*x^2)/(77*a^2*c^6) + (64*b*x^4)/(
77*a^3*c^6) + (256*b^2*x^6)/(77*a^4*c^6)))/(x^7*(c*x)^(1/2) + (a*x^5*(c*x)
^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.50

$$\int \frac{1}{(cx)^{13/2} (a+bx^2)^{5/4}} dx = \frac{2\sqrt{c}\,(-128b^3x^6 - 32a\,b^2x^4 + 12a^2b\,x^2 - 7a^3)}{77\,(b\,x^2+a)^{\frac{1}{4}}\,\sqrt{x}\,a^4c^7x^5}$$

input
```
int(1/(c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(3/4)*( - 7*a**3 + 12*a**2*b*x**2 - 32*a*b**2*x**
4 - 128*b**3*x**6))/(77*sqrt(x)*a**4*c**7*x**5*(a + b*x**2))
```

3.1055 $\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 155

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \frac{77a^2c^5(cx)^{3/2}}{60b^3\sqrt[4]{a+bx^2}} - \frac{11ac^3(cx)^{7/2}}{30b^2\sqrt[4]{a+bx^2}} + \frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}} + \frac{77a^{5/2}c^6\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{20b^{7/2}\sqrt[4]{a+bx^2}}$$

output
```
77/60*a^2*c^5*(c*x)^(3/2)/b^3/(b*x^2+a)^(1/4)-11/30*a*c^3*(c*x)^(7/2)/b^2/
(b*x^2+a)^(1/4)+1/5*c*(c*x)^(11/2)/b/(b*x^2+a)^(1/4)+77/20*a^(5/2)*c^6*(1+
a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^
(1/2))/b^(7/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.56

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^5(cx)^{3/2}\left(77a^2-22abx^2+12b^2x^4-77a^2\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\right.\right.}{60b^3\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c^5*(c*x)^(3/2)*(77*a^2 - 22*a*b*x^2 + 12*b^2*x^4 - 77*a^2*(1 + (b*x^2)/a
)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(60*b^3*(a + b*x^
2)^(1/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {250, 250, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\int \frac{(cx)^{9/2}}{(bx^2+a)^{5/4}}dx}{10b}$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\left(\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\int \frac{(cx)^{5/2}}{\left(bx^2+a\right)^{5/4}}dx}{6b}\right)}{10b}$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\left(\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{2b}\right)}{6b}\right)}{10b}$$

$$\Big\downarrow \text{249}$$

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\left(\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}$$

$\downarrow$ 858

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\left(\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}$$

$\downarrow$ 212

$$\frac{c(cx)^{11/2}}{5b\sqrt[4]{a+bx^2}}-\frac{11ac^2\left(\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{3\sqrt{a}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}\right)}{10b}$$

input

```
Int[(c*x)^(13/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*(c*x)^(11/2))/(5*b*(a + b*x^2)^(1/4)) - (11*a*c^2*((c*(c*x)^(7/2))/(3*b
*(a + b*x^2)^(1/4)) - (7*a*c^2*((c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3
*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqr
t[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(6*b)))/(10*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{13}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^6*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \text{Timed out}$$

input
```
integrate((c*x)**(13/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{13}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(13/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(13/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(c\,x)^{13/2}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(13/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(13/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{13/2}}{(a+bx^2)^{5/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^6}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^6$$

input
```
int((c*x)^(13/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**6)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*
x**2),x)*c**6
```

3.1056 $\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = -\frac{7ac^3(cx)^{3/2}}{6b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}} - \frac{7a^{3/2}c^4\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{2b^{5/2}\sqrt[4]{a+bx^2}}$$

output

```
-7/6*a*c^3*(c*x)^(3/2)/b^2/(b*x^2+a)^(1/4)+1/3*c*(c*x)^(7/2)/b/(b*x^2+a)^(
1/4)-7/2*a^(3/2)*c^4*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arcco
t(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(5/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.60

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^3(cx)^{3/2}\left(-7a+2bx^2+7a\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{6b^2\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c^3*(c*x)^(3/2)*(-7*a + 2*b*x^2 + 7*a*(1 + (b*x^2)/a)^(1/4)*Hypergeometri
c2F1[3/4, 5/4, 7/4, -((b*x^2)/a)]))/(6*b^2*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {250, 250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\int \frac{(cx)^{5/2}}{(bx^2+a)^{5/4}}dx}{6b}$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\int \frac{\sqrt{cx}}{(bx^2+a)^{5/4}}dx}{2b}\right)}{6b}$$

$$\Big\downarrow \text{249}$$

$$\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}\right)}{6b}$$

$$\Big\downarrow \text{858}$$

$$\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int\frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}$$

$$\Big\downarrow \text{212}$$

$$\frac{c(cx)^{7/2}}{3b\sqrt[4]{a+bx^2}}-\frac{7ac^2\left(\frac{3\sqrt{a}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}\right)}{6b}$$

input
```
Int[(c*x)^(9/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*(c*x)^(7/2))/(3*b*(a + b*x^2)^(1/4)) - (7*a*c^2*((c*(c*x)^(3/2))/(b*(a
+ b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE
[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*x^2)^(1/4))))/(6*b)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{9}{2}}}{(b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^4*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 36.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.35

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^{\frac{9}{2}}x^{\frac{11}{2}}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{15}{4}\right)}$$

input
```
integrate((c*x)**(9/2)/(b*x**2+a)**(5/4),x)
```

output
```
c**(9/2)*x**(11/2)*gamma(11/4)*hyper((5/4, 11/4), (15/4,), b*x**2*exp_pola
r(I*pi)/a)/(2*a**(5/4)*gamma(15/4))
```

Maxima [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{9}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(9/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(9/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(c\,x)^{9/2}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(9/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(9/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{9/2}}{(a+bx^2)^{5/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^4}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^4$$

input
```
int((c*x)^(9/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**4)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*
x**2),x)*c**4
```

3.1057 $\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 90

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx=\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}+\frac{3\sqrt{a}c^2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}$$

output
```
c*(c*x)^(3/2)/b/(b*x^2+a)^(1/4)+3*a^(1/2)*c^2*(1+a/b/x^2)^(1/4)*(c*x)^(1/2
)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/b^(3/2)/(b*x^2+a)^
(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.67

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx=\frac{c(cx)^{3/2}\left(1-\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^2}{a}\right)\right)}{b\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*(c*x)^(3/2)*(1 - (1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4,
 -((b*x^2)/a)]))/(b*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {250, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}dx$$

$$\Big\downarrow \text{250}$$

$$\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\int \frac{\sqrt{cx}}{(bx^2+a)^{5/4}}dx}{2b}$$

$$\Big\downarrow \text{249}$$

$$\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}-\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{2b^2\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{3ac^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{2b^2\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{3\sqrt{a}c^2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{a+bx^2}}+\frac{c(cx)^{3/2}}{b\sqrt[4]{a+bx^2}}$$

input
```
Int[(c*x)^(5/2)/(a + b*x^2)^(5/4),x]
```

output
```
(c*(c*x)^(3/2))/(b*(a + b*x^2)^(1/4)) + (3*Sqrt[a]*c^2*(1 + a/(b*x^2))^(1/
4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(b^(3/2)*(a + b*
x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 250
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[2*c*((
c*x)^(m - 1)/(b*(2*m - 3)*(a + b*x^2)^(1/4))), x] - Simp[2*a*c^2*((m - 1)/(
b*(2*m - 3)))   Int[(c*x)^(m - 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b,
 c}, x] && PosQ[b/a] && IntegerQ[2*m] && GtQ[m, 3/2]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{2}}}{(b\,x^{2}+a)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)*c^2*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2),
x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.49

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \frac{c^{\frac{5}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate((c*x)**(5/2)/(b*x**2+a)**(5/4),x)
```

output
```
c**(5/2)*x**(7/2)*gamma(7/4)*hyper((5/4, 7/4), (11/4,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(5/4)*gamma(11/4))
```

Maxima [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(cx)^{\frac{5}{2}}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(5/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/2)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \int \frac{(c\,x)^{5/2}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(5/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(5/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/2}}{(a+bx^2)^{5/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}\,x^2}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)c^2$$

input
```
int((c*x)^(5/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int((sqrt(x)*x**2)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*
x**2),x)*c**2
```

3.1058 $\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

output
```
-2*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2
))),2^(1/2))/a^(1/2)/b^(1/2)/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \frac{2x\sqrt{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{5}{4},\frac{7}{4},-\frac{bx^2}{a}\right)}{3a\sqrt[4]{a+bx^2}}$$

input
```
Integrate[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

output
```
(2*x*Sqrt[c*x]*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[3/4, 5/4, 7/4, -((b
*x^2)/a)])/(3*a*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{249}$$

$$\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{b\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{\sqrt{a}\sqrt{b}\sqrt[4]{a+bx^2}}$$

input
```
Int[Sqrt[c*x]/(a + b*x^2)^(5/4),x]
```

output
```
(-2*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/
2, 2])/(Sqrt[a]*Sqrt[b]*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{\sqrt{cx}}{(b\,x^{2}+a)^{\frac{5}{4}}}dx$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

output
```
int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \frac{\sqrt{c}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((c*x)**(1/2)/(b*x**2+a)**(5/4),x)
```

output
```
sqrt(c)*x**(3/2)*gamma(3/4)*hyper((3/4, 5/4), (7/4,), b*x**2*exp_polar(I*p
i)/a)/(2*a**(5/4)*gamma(7/4))
```

Maxima [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

Giac [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \int \frac{\sqrt{cx}}{(bx^2+a)^{\frac{5}{4}}}\,dx$$

input
```
integrate((c*x)^(1/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x)/(b*x^2 + a)^(5/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \int \frac{\sqrt{c\,x}}{(b\,x^2+a)^{5/4}}\,dx$$

input
```
int((c*x)^(1/2)/(a + b*x^2)^(5/4),x)
```

output
```
int((c*x)^(1/2)/(a + b*x^2)^(5/4), x)
```

Reduce [F]

$$\int \frac{\sqrt{cx}}{(a+bx^2)^{5/4}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{x}}{(b\,x^2+a)^{\frac{1}{4}}\,a+(b\,x^2+a)^{\frac{1}{4}}\,b\,x^2}dx\right)$$

input
```
int((c*x)^(1/2)/(b*x^2+a)^(5/4),x)
```

output
```
sqrt(c)*int(sqrt(x)/((a + b*x**2)**(1/4)*a + (a + b*x**2)**(1/4)*b*x**2),x
)
```

3.1059 $\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{4\sqrt{b}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}$$

output
```
-2/a/c/(c*x)^(1/2)/(b*x^2+a)^(1/4)+4*b^(1/2)*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)
*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/a^(3/2)/c^2/(b*x^2+
a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{bx^2}{a}\right)}{a(cx)^{3/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((b*x^2)/a)
])/(a*(c*x)^(3/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{251}$$

$$-\frac{2b\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{ac^2}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{212}$$

$$\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x]
```

output
```
-2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sq
rt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(a^(3/2)*c^2*(a + b*x
^2)^(1/4))
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{2}} (b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

output
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*c^
2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{5}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{5}{4}}c^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(1/(c*x)**(3/2)/(b*x**2+a)**(5/4),x)
```

output
```
gamma(-1/4)*hyper((-1/4, 5/4), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*a**(5/
4)*c**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}}(cx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx = \int \frac{1}{(c\,x)^{3/2}\,(b\,x^2+a)^{5/4}}\,dx$$

input
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int(1/((c*x)^(3/2)*(a + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.35

$$\int \frac{1}{(cx)^{3/2}(a+bx^2)^{5/4}}\,dx = -\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{b\,x^2+a}\,a\,c^2}$$

input
```
int(1/(c*x)^(3/2)/(b*x^2+a)^(5/4),x)
```

output
```
( - 2*sqrt(c)*(a + b*x**2)**(1/4))/(sqrt(x)*sqrt(a + b*x**2)*a*c**2)
```

3.1060 $\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 126

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}}\,dx = -\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}} + \frac{12b}{5a^2c^3\sqrt{cx}\sqrt[4]{a+bx^2}} - \frac{24b^{3/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}\,E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{5a^{5/2}c^4\sqrt[4]{a+bx^2}}$$

output

```
-2/5/a/c/(c*x)^(5/2)/(b*x^2+a)^(1/4)+12/5*b/a^2/c^3/(c*x)^(1/2)/(b*x^2+a)^
(1/4)-24/5*b^(3/2)*(1+a/b/x^2)^(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(
b^(1/2)*x/a^(1/2))),2^(1/2))/a^(5/2)/c^4/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.47

$$\int \frac{1}{(cx)^{7/2}(a+bx^2)^{5/4}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{5}{4},-\frac{1}{4},-\frac{bx^2}{a}\right)}{5a(cx)^{7/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-5/4, 5/4, -1/4, -((b*x^2)/a
)])/(5*a*(c*x)^(7/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {251, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}dx$$

$$\Big\downarrow \text{251}$$

$$-\frac{6b\int \frac{1}{(cx)^{3/2}(bx^2+a)^{5/4}}dx}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{6b\left(-\frac{2b\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{ac^2}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{6b\left(-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{858}$$

$$-\frac{6b\left(\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \; 212$$

$$-\frac{6b\left(\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x]
```

output
```
-2/(5*a*c*(c*x)^(5/2)*(a + b*x^2)^(1/4)) - (6*b*(-2/(a*c*Sqrt[c*x]*(a + b*
x^2)^(1/4)) + (4*Sqrt[b]*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[
Sqrt[a]/(Sqrt[b]*x)]/2, 2])/(a^(3/2)*c^2*(a + b*x^2)^(1/4))))/(5*a*c^2)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{7}{2}} (b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

output
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{5/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{4}} (cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^4*x^8 + 2*a*b*c^4*x^6 + a^2*c^
4*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 22.97 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.27

$$\int \frac{1}{(cx)^{7/2} (a+bx^2)^{5/4}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{5b^{\frac{5}{4}}c^{\frac{7}{2}}x^5}$$

input
```
integrate(1/(c*x)**(7/2)/(b*x**2+a)**(5/4),x)
```

output
```
-hyper((5/4, 5/2), (7/2,), a*exp_polar(I*pi)/(b*x**2))/(5*b**(5/4)*c**(7/2
)*x**5)
```

Maxima [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{\left(c\,x\right)^{7/2}\,\left(b\,x^2+a\right)^{5/4}}\,dx$$

input
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int(1/((c*x)^(7/2)*(a + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.37

$$\int \frac{1}{(cx)^{7/2}\left(a+bx^2\right)^{5/4}}\,dx=\frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\,(4b\,x^2-a)}{5\sqrt{x}\,\sqrt{b\,x^2+a}\,a^2c^4x^2}$$

input
```
int(1/(c*x)^(7/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - a + 4*b*x**2))/(5*sqrt(x)*sqrt(a + b*x*
*2)*a**2*c**4*x**2)
```

3.1061 $\int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 157

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}}\,dx = -\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}+\frac{4b}{9a^2c^3(cx)^{5/2}\sqrt[4]{a+bx^2}} - \frac{8b^2}{3a^3c^5\sqrt{cx}\sqrt[4]{a+bx^2}}+\frac{16b^{5/2}\sqrt[4]{1+\frac{a}{bx^2}}\sqrt{cx}E\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\middle|2\right)}{3a^{7/2}c^6\sqrt[4]{a+bx^2}}$$

output
```
-2/9/a/c/(c*x)^(9/2)/(b*x^2+a)^(1/4)+4/9*b/a^2/c^3/(c*x)^(5/2)/(b*x^2+a)^(
1/4)-8/3*b^2/a^3/c^5/(c*x)^(1/2)/(b*x^2+a)^(1/4)+16/3*b^(5/2)*(1+a/b/x^2)^
(1/4)*(c*x)^(1/2)*EllipticE(sin(1/2*arccot(b^(1/2)*x/a^(1/2))),2^(1/2))/a^
(7/2)/c^6/(b*x^2+a)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int \frac{1}{(cx)^{11/2}(a+bx^2)^{5/4}}\,dx = -\frac{2x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{5}{4},-\frac{5}{4},-\frac{bx^2}{a}\right)}{9a(cx)^{11/2}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

output
```
(-2*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-9/4, 5/4, -5/4, -((b*x^2)/a
)])/(9*a*(c*x)^(11/2)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {251, 251, 251, 249, 858, 212}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{11/2} \left(a+bx^2\right)^{5/4}} \, dx$$

$$\Big\downarrow \text{251}$$

$$-\frac{10b \int \frac{1}{(cx)^{7/2}(bx^2+a)^{5/4}}dx}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{10b\left(-\frac{6b\int \frac{1}{(cx)^{3/2}\left(bx^2+a\right)^{5/4}}dx}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{251}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2b\int \frac{\sqrt{cx}}{\left(bx^2+a\right)^{5/4}}dx}{ac^2}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{249}$$

$$-\frac{10b\left(-\frac{6b\left(-\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}x^2}dx}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

$\downarrow$ 858

$$-\frac{10b\left(-\frac{6b\left(\frac{2\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}\int \frac{1}{\left(\frac{a}{bx^2}+1\right)^{5/4}}d\frac{1}{x}}{ac^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

$\downarrow$ 212

$$-\frac{10b\left(-\frac{6b\left(\frac{4\sqrt{b}\sqrt{cx}\sqrt[4]{\frac{a}{bx^2}+1}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)\middle|2\right)}{a^{3/2}c^2\sqrt[4]{a+bx^2}}-\frac{2}{ac\sqrt{cx}\sqrt[4]{a+bx^2}}\right)}{5ac^2}-\frac{2}{5ac(cx)^{5/2}\sqrt[4]{a+bx^2}}\right)}{9ac^2}-\frac{2}{9ac(cx)^{9/2}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x]
```

output
```
-2/(9*a*c*(c*x)^(9/2)*(a + b*x^2)^(1/4)) - (10*b*(-2/(5*a*c*(c*x)^(5/2)*(a
 + b*x^2)^(1/4)) - (6*b*(-2/(a*c*Sqrt[c*x]*(a + b*x^2)^(1/4)) + (4*Sqrt[b]
*(1 + a/(b*x^2))^(1/4)*Sqrt[c*x]*EllipticE[ArcTan[Sqrt[a]/(Sqrt[b]*x)]/2,
2])/(a^(3/2)*c^2*(a + b*x^2)^(1/4))))/(5*a*c^2)))/(9*a*c^2)
```

Defintions of rubi rules used

rule 212
```
Int[((a_) + (b_.)*(x_)^2)^(-5/4), x_Symbol] :> Simp[(2/(a^(5/4)*Rt[b/a, 2])
)*EllipticE[(1/2)*ArcTan[Rt[b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

rule 249
```
Int[Sqrt[(c_.)*(x_)]/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[Sqrt[c*
x]*((1 + a/(b*x^2))^(1/4)/(b*(a + b*x^2)^(1/4)))   Int[1/(x^2*(1 + a/(b*x^2
))^(5/4)), x], x] /; FreeQ[{a, b, c}, x] && PosQ[b/a]
```

rule 251
```
Int[((c_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2)^(5/4), x_Symbol] :> Simp[(c*x)^
(m + 1)/(a*c*(m + 1)*(a + b*x^2)^(1/4)), x] - Simp[b*((2*m + 1)/(2*a*c^2*(m
 + 1)))   Int[(c*x)^(m + 2)/(a + b*x^2)^(5/4), x], x] /; FreeQ[{a, b, c}, x
] && PosQ[b/a] && IntegerQ[2*m] && LtQ[m, -1]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{11}{2}} (b\,x^2+a)^{\frac{5}{4}}}dx$$

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

output
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{4}}\,(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*sqrt(c*x)/(b^2*c^6*x^10 + 2*a*b*c^6*x^8 + a^2*c
^6*x^6), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx=\text{Timed out}$$

input
```
integrate(1/(c*x)**(11/2)/(b*x**2+a)**(5/4),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{4}}\,(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{5}{4}}(cx)^{\frac{11}{2}}}\,dx$$

input
```
integrate(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/4)*(c*x)^(11/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = \int \frac{1}{(c\,x)^{11/2}\,\left(b\,x^2+a\right)^{5/4}}\,dx$$

input
```
int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)),x)
```

output
```
int(1/((c*x)^(11/2)*(a + b*x^2)^(5/4)), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.36

$$\int \frac{1}{(cx)^{11/2}\left(a+bx^2\right)^{5/4}}\,dx = \frac{2\sqrt{c}\,(b\,x^2+a)^{\frac{1}{4}}\left(-32b^2x^4+8ab\,x^2-5a^2\right)}{45\sqrt{x}\,\sqrt{b\,x^2+a}\,a^3c^6x^4}$$

input
```
int(1/(c*x)^(11/2)/(b*x^2+a)^(5/4),x)
```

output
```
(2*sqrt(c)*(a + b*x**2)**(1/4)*( - 5*a**2 + 8*a*b*x**2 - 32*b**2*x**4))/(4
5*sqrt(x)*sqrt(a + b*x**2)*a**3*c**6*x**4)
```

3.1062 $\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4(cx)^{9/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{8},\frac{17}{8},-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

output
```
4/9*(c*x)^(9/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 9/8],[17/8],-b*x^2/a)/c/
(b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4x(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{8},\frac{17}{8},-\frac{bx^2}{a}\right)}{9\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -
((b*x^2)/a)])/(9*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}dx$$

$$\big\downarrow \text{279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{(cx)^{5/4}}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\big\downarrow \text{278}$$

$$\frac{4(cx)^{9/4}\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{9}{8},\frac{17}{8},-\frac{bx^2}{a}\right)}{9c\sqrt[4]{a+bx^2}}$$

input
```
Int[(c*x)^(5/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 9/8, 17/8, -((
b*x^2)/a)])/(9*c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/4)*c*x/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{9}{8}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

output
```
c**(5/4)*x**(9/4)*gamma(9/8)*hyper((1/4, 9/8), (17/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(1/4)*gamma(17/8))
```

Maxima [F]

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(c\,x)^{5/4}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(5/4)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(5/4)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/4}}{\sqrt[4]{a+bx^2}}\,dx = c^{\frac{5}{4}}\left(\int \frac{x^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

output
```
c**(1/4)*int((x**(1/4)*x)/(a + b*x**2)**(1/4),x)*c
```

3.1063 $\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4(cx)^{7/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

output
```
4/7*(c*x)^(7/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 7/8],[15/8],-b*x^2/a)/c/
(b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4x(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -
((b*x^2)/a)])/(7*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{(cx)^{3/4}}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{7/4}\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{7}{8},\frac{15}{8},-\frac{bx^2}{a}\right)}{7c\sqrt[4]{a+bx^2}}$$

input
```
Int[(c*x)^(3/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 7/8, 15/8, -((
b*x^2)/a)])/(7*c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.56 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right) {}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{7}{8} \\ \frac{15}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

output
```
c**(3/4)*x**(7/4)*gamma(7/8)*hyper((1/4, 7/8), (15/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(1/4)*gamma(15/8))
```

Maxima [F]

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(c\,x)^{3/4}}{(b\,x^2+a)^{1/4}}\,dx$$

input
```
int((c*x)^(3/4)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(3/4)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/4}}{\sqrt[4]{a+bx^2}}\,dx = c^{\frac{3}{4}}\left(\int \frac{x^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{1}{4}}}dx\right)$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

output
```
c**(3/4)*int(x**(3/4)/(a + b*x**2)**(1/4),x)
```

3.1064 $\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4(cx)^{5/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

output
```
4/5*(c*x)^(5/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 5/8],[13/8],-b*x^2/a)/c/
(b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx=\frac{4x\sqrt[4]{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5\sqrt[4]{a+bx^2}}$$

input
```
Integrate[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -
((b*x^2)/a)])/(5*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{\sqrt[4]{cx}}{\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{5}{8},\frac{13}{8},-\frac{bx^2}{a}\right)}{5c\sqrt[4]{a+bx^2}}$$

input
```
Int[(c*x)^(1/4)/(a + b*x^2)^(1/4),x]
```

output
```
(4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 5/8, 13/8, -((
b*x^2)/a)])/(5*c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^{2}+a)^{\frac{1}{4}}}dx$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

output
```
int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{5}{8}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

output
```
c**(1/4)*x**(5/4)*gamma(5/8)*hyper((1/4, 5/8), (13/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(1/4)*gamma(13/8))
```

Maxima [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{1}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(1/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx=\int \frac{{\left(c\,x\right)}^{1/4}}{{\left(b\,x^2+a\right)}^{1/4}}\,dx$$

input
```
int((c*x)^(1/4)/(a + b*x^2)^(1/4),x)
```

output
```
int((c*x)^(1/4)/(a + b*x^2)^(1/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{cx}}{\sqrt[4]{a+bx^2}}\,dx=c^{\frac{1}{4}}\left(\int \frac{x^{\frac{1}{4}}}{\left(b\,x^{2}+a\right)^{\frac{1}{4}}}d x\right)$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

output
```
c**(1/4)*int(x**(1/4)/(a + b*x**2)**(1/4),x)
```

3.1065 $\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx=\frac{4(cx)^{3/4}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{8},\frac{11}{8},-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

output
```
4/3*(c*x)^(3/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/4, 3/8],[11/8],-b*x^2/a)/c/
(b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{8},\frac{11}{8},-\frac{bx^2}{a}\right)}{3\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((b*x^2)/a)]
)/(3*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{8},\frac{11}{8},-\frac{bx^2}{a}\right)}{3c\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/4, 3/8, 11/8, -((
b*x^2)/a)])/(3*c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{4}} (b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.76

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{3}{8}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}\sqrt[4]{c}\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/(c*x)**(1/4)/(b*x**2+a)**(1/4),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((1/4, 3/8), (11/8,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(1/4)*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{1/4}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{cx}\sqrt[4]{a+bx^2}}\,dx = \frac{\int \frac{1}{x^{\frac{1}{4}}(b\,x^2+a)^{\frac{1}{4}}}dx}{c^{\frac{1}{4}}}$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x**2)**(1/4)),x)/c**(1/4)
```

3.1066 $\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx=\frac{4\sqrt[4]{cx}\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

output
```
4*(c*x)^(1/4)*(1+b*x^2/a)^(1/4)*hypergeom([1/8, 1/4],[9/8],-b*x^2/a)/c/(b*
x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{(cx)^{3/4}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b*x^2)/a)])
/((c*x)^(3/4)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{(cx)^{3/4}\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4\sqrt[4]{cx}\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{c\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[1/8, 1/4, 9/8, -((b
*x^2)/a)])/(c*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (b\,x^2+a)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(c*x)^(1/4)/(b*c*x^3 + a*c*x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\sqrt[4]{x}\,\Gamma\left(\frac{1}{8}\right)\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{4}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x**2+a)**(1/4),x)
```

output
```
x**(1/4)*gamma(1/8)*hyper((1/8, 1/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(1/4)*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\int \frac{1}{x^{\frac{3}{4}}(b\,x^2+a)^{\frac{1}{4}}}dx}{c^{\frac{3}{4}}}$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x**2)**(1/4)),x)/c**(3/4)
```

3.1067 $\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx=-\frac{4\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

output
```
-4*(1+b*x^2/a)^(1/4)*hypergeom([-1/8, 1/4],[7/8],-b*x^2/a)/c/(c*x)^(1/4)/(
b*x^2+a)^(1/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx=-\frac{4x\sqrt[4]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{(cx)^{5/4}\sqrt[4]{a+bx^2}}$$

input
```
Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(-4*x*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)
])/((c*x)^(5/4)*(a + b*x^2)^(1/4))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\sqrt[4]{\frac{bx^2}{a}+1}\int \frac{1}{(cx)^{5/4}\sqrt[4]{\frac{bx^2}{a}+1}}dx}{\sqrt[4]{a+bx^2}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{4\sqrt[4]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{c\sqrt[4]{cx}\sqrt[4]{a+bx^2}}$$

input
```
Int[1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x]
```

output
```
(-4*(1 + (b*x^2)/a)^(1/4)*Hypergeometric2F1[-1/8, 1/4, 7/8, -((b*x^2)/a)])
/(c*(c*x)^(1/4)*(a + b*x^2)^(1/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{4}}\left(b\,x^{2}+a\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}\,(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/4)*(c*x)^(3/4)/(b*c^2*x^4 + a*c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\Gamma\left(-\frac{1}{8}\right){}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{1}{4}\\ \frac{7}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt[4]{a}c^{\frac{5}{4}}\sqrt[4]{x}\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate(1/(c*x)**(5/4)/(b*x**2+a)**(1/4),x)
```

output
```
gamma(-1/8)*hyper((-1/8, 1/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(1/
4)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{4}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/4)*(c*x)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \int \frac{1}{(c\,x)^{5/4}\,(b\,x^2+a)^{1/4}}\,dx$$

input
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)),x)
```

output
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(1/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/4}\sqrt[4]{a+bx^2}}\,dx = \frac{\int \frac{1}{x^{\frac{5}{4}}(b\,x^2+a)^{\frac{1}{4}}}dx}{c^{\frac{5}{4}}}$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x**2)**(1/4)*x),x)/(c**(1/4)*c)
```

3.1068 $\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx=\frac{4(cx)^{9/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{4},\frac{17}{8},-\frac{bx^2}{a}\right)}{9ac\,(a+bx^2)^{3/4}}$$

output
```
4/9*(c*x)^(9/4)*(1+b*x^2/a)^(3/4)*hypergeom([9/8, 7/4],[17/8],-b*x^2/a)/a/
c/(b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx=\frac{4x(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{4},\frac{17}{8},-\frac{bx^2}{a}\right)}{9a\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*x*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -
((b*x^2)/a)])/(9*a*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}} dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{(cx)^{5/4}}{\left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a\left(a+bx^2\right)^{3/4}}$$
$$\Big\downarrow \text{278}$$
$$\frac{4(cx)^{9/4}\left(\frac{bx^2}{a}+1\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{4},\frac{17}{8},-\frac{bx^2}{a}\right)}{9ac\left(a+bx^2\right)^{3/4}}$$

input
```
Int[(c*x)^(5/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*(c*x)^(9/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[9/8, 7/4, 17/8, -((
b*x^2)/a)])/(9*a*c*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{5}{4}}}{(b\,x^{2}+a)^{\frac{7}{4}}}dx$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

output
```
int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)*c*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 11.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \frac{c^{\frac{5}{4}}x^{\frac{9}{4}}\Gamma\left(\frac{9}{8}\right){}_{2}F_{1}\left(\begin{matrix}\frac{9}{8},\frac{7}{4}\\ \frac{17}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{17}{8}\right)}$$

input
```
integrate((c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

output
```
c**(5/4)*x**(9/4)*gamma(9/8)*hyper((9/8, 7/4), (17/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(7/4)*gamma(17/8))
```

Maxima [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{5}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(5/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(c\,x)^{5/4}}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int((c*x)^(5/4)/(a + b*x^2)^(7/4),x)
```

output
```
int((c*x)^(5/4)/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{5/4}}{(a+bx^2)^{7/4}}\,dx = c^{\frac{5}{4}}\left(\int \frac{x^{\frac{5}{4}}}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)$$

input
```
int((c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

output
```
c**(1/4)*int((x**(1/4)*x)/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x
**2),x)*c
```

3.1069 $\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \frac{4(cx)^{7/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{4},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac\,(a+bx^2)^{3/4}}$$

output
```
4/7*(c*x)^(7/4)*(1+b*x^2/a)^(3/4)*hypergeom([7/8, 7/4],[15/8],-b*x^2/a)/a/
c/(b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \frac{4x(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{4},\frac{15}{8},-\frac{bx^2}{a}\right)}{7a\,(a+bx^2)^{3/4}}$$

input
```
Integrate[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*x*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -
((b*x^2)/a)])/(7*a*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{(cx)^{3/4}}{\left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{7/4}\left(\frac{bx^2}{a}+1\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{4},\frac{15}{8},-\frac{bx^2}{a}\right)}{7ac\,(a+bx^2)^{3/4}}$$

input
```
Int[(c*x)^(3/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*(c*x)^(7/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[7/8, 7/4, 15/8, -((
b*x^2)/a)])/(7*a*c*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

output
```
int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \frac{c^{\frac{3}{4}}x^{\frac{7}{4}}\Gamma\left(\frac{7}{8}\right)\,{}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{7}{4}\\ \frac{15}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{15}{8}\right)}$$

input
```
integrate((c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

output
```
c**(3/4)*x**(7/4)*gamma(7/8)*hyper((7/8, 7/4), (15/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(7/4)*gamma(15/8))
```

Maxima [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{3}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(3/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(c\,x)^{3/4}}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int((c*x)^(3/4)/(a + b*x^2)^(7/4),x)
```

output
```
int((c*x)^(3/4)/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{(cx)^{3/4}}{(a+bx^2)^{7/4}}\,dx = c^{\frac{3}{4}}\left(\int \frac{x^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)$$

input
```
int((c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

output
```
c**(3/4)*int(x**(3/4)/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2)
,x)
```

3.1070 $\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}} dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}} \, dx = \frac{4(cx)^{5/4}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{4},\frac{13}{8},-\frac{bx^2}{a}\right)}{5ac\left(a+bx^2\right)^{3/4}}$$

output
```
4/5*(c*x)^(5/4)*(1+b*x^2/a)^(3/4)*hypergeom([5/8, 7/4],[13/8],-b*x^2/a)/a/
c/(b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}} \, dx = \frac{4x\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{4},\frac{13}{8},-\frac{bx^2}{a}\right)}{5a\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*x*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -
((b*x^2)/a)])/(5*a*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{\sqrt[4]{cx}}{\left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{5/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{4},\frac{13}{8},-\frac{bx^2}{a}\right)}{5ac\,(a+bx^2)^{3/4}}$$

input
```
Int[(c*x)^(1/4)/(a + b*x^2)^(7/4),x]
```

output
```
(4*(c*x)^(5/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[5/8, 7/4, 13/8, -((
b*x^2)/a)])/(5*a*c*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(cx)^{\frac{1}{4}}}{(b\,x^{2}+a)^{\frac{7}{4}}}dx$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

output
```
int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx=\int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = \frac{\sqrt[4]{c}x^{\frac{5}{4}}\Gamma\left(\frac{5}{8}\right){}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{7}{4}\\ \frac{13}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\Gamma\left(\frac{13}{8}\right)}$$

input
```
integrate((c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

output
```
c**(1/4)*x**(5/4)*gamma(5/8)*hyper((5/8, 7/4), (13/8,), b*x**2*exp_polar(I
*pi)/a)/(2*a**(7/4)*gamma(13/8))
```

Maxima [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

Giac [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(cx)^{\frac{1}{4}}}{(bx^2+a)^{\frac{7}{4}}}\,dx$$

input
```
integrate((c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate((c*x)^(1/4)/(b*x^2 + a)^(7/4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = \int \frac{(c\,x)^{1/4}}{(b\,x^2+a)^{7/4}}\,dx$$

input
```
int((c*x)^(1/4)/(a + b*x^2)^(7/4),x)
```

output
```
int((c*x)^(1/4)/(a + b*x^2)^(7/4), x)
```

Reduce [F]

$$\int \frac{\sqrt[4]{cx}}{(a+bx^2)^{7/4}}\,dx = c^{\frac{1}{4}}\left(\int \frac{x^{\frac{1}{4}}}{(b\,x^2+a)^{\frac{3}{4}}\,a+(b\,x^2+a)^{\frac{3}{4}}\,b\,x^2}dx\right)$$

input
```
int((c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

output
```
c**(1/4)*int(x**(1/4)/((a + b*x**2)**(3/4)*a + (a + b*x**2)**(3/4)*b*x**2)
,x)
```

3.1071 $\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 61

$$\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx=\frac{4(cx)^{3/4}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{4},\frac{11}{8},-\frac{bx^2}{a}\right)}{3ac\left(a+bx^2\right)^{3/4}}$$

output
```
4/3*(c*x)^(3/4)*(1+b*x^2/a)^(3/4)*hypergeom([3/8, 7/4],[11/8],-b*x^2/a)/a/
c/(b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt[4]{cx}\left(a+bx^2\right)^{7/4}}\,dx=\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{4},\frac{11}{8},-\frac{bx^2}{a}\right)}{3a\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((b*x^2)/a)]
)/(3*a*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4}\int \frac{1}{\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a\,(a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{278}$$

$$\frac{4(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{4},\frac{11}{8},-\frac{bx^2}{a}\right)}{3ac\,(a+bx^2)^{3/4}}$$

input
```
Int[1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(4*(c*x)^(3/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[3/8, 7/4, 11/8, -((
b*x^2)/a)])/(3*a*c*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{1}{4}} \left(b\,x^{2}+a\right)^{\frac{7}{4}}}dx$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x)
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \frac{x^{\frac{3}{4}}\Gamma\left(\frac{3}{8}\right)\,{}_{2}F_{1}\left(\begin{matrix}\frac{3}{8},\frac{7}{4}\\ \frac{11}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}\sqrt[4]{c}\,\Gamma\left(\frac{11}{8}\right)}$$

input
```
integrate(1/(c*x)**(1/4)/(b*x**2+a)**(7/4),x)
```

output
```
x**(3/4)*gamma(3/8)*hyper((3/8, 7/4), (11/8,), b*x**2*exp_polar(I*pi)/a)/(
2*a**(7/4)*c**(1/4)*gamma(11/8))
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}\,(cx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(1/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(c\,x)^{1/4}\,(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)),x)
```

output
```
int(1/((c*x)^(1/4)*(a + b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{cx}\,(a+bx^2)^{7/4}}\,dx = \frac{\int \frac{1}{x^{\frac{1}{4}}(b\,x^2+a)^{\frac{3}{4}}a+x^{\frac{9}{4}}(b\,x^2+a)^{\frac{3}{4}}b}dx}{c^{\frac{1}{4}}}$$

input
```
int(1/(c*x)^(1/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x**2)**(3/4)*a + x**(1/4)*(a + b*x**2)**(3/4)*b*x**
2),x)/c**(1/4)
```

3.1072 $\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{(cx)^{3/4}\,(a+bx^2)^{7/4}}\,dx=\frac{4\sqrt[4]{cx}\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{ac\,(a+bx^2)^{3/4}}$$

output
```
4*(c*x)^(1/4)*(1+b*x^2/a)^(3/4)*hypergeom([1/8, 7/4],[9/8],-b*x^2/a)/a/c/(
b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{1}{(cx)^{3/4}\,(a+bx^2)^{7/4}}\,dx=\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{a(cx)^{3/4}\,(a+bx^2)^{3/4}}$$

input
```
Integrate[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b*x^2)/a)])
/(a*(c*x)^(3/4)*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{3/4} (a+bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{1}{(cx)^{3/4}\left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a\left(a+bx^2\right)^{3/4}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{4\sqrt[4]{cx}\left(\frac{bx^2}{a}+1\right)^{3/4} \text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{4},\frac{9}{8},-\frac{bx^2}{a}\right)}{ac\left(a+bx^2\right)^{3/4}}$$

input
```
Int[1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(4*(c*x)^(1/4)*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[1/8, 7/4, 9/8, -((b
*x^2)/a)])/(a*c*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{3}{4}} (b\,x^{2}+a)^{\frac{7}{4}}}dx$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}\,(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(1/4)/(b^2*c*x^5 + 2*a*b*c*x^3 + a^2*c*x)
, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx = \frac{\sqrt[4]{x}\Gamma\left(\frac{1}{8}\right){}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{7}{4}\\ \frac{9}{8}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{3}{4}}\Gamma\left(\frac{9}{8}\right)}$$

input
```
integrate(1/(c*x)**(3/4)/(b*x**2+a)**(7/4),x)
```

output
```
x**(1/4)*gamma(1/8)*hyper((1/8, 7/4), (9/8,), b*x**2*exp_polar(I*pi)/a)/(2
*a**(7/4)*c**(3/4)*gamma(9/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{3/4}\left(a+bx^2\right)^{7/4}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{4}}(cx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(3/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(c\,x)^{3/4}\,(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)),x)
```

output
```
int(1/((c*x)^(3/4)*(a + b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{3/4}(a+bx^2)^{7/4}}\,dx = \frac{\int \frac{1}{x^{\frac{3}{4}}(b\,x^2+a)^{\frac{3}{4}}a+x^{\frac{11}{4}}(b\,x^2+a)^{\frac{3}{4}}b}dx}{c^{\frac{3}{4}}}$$

input
```
int(1/(c*x)^(3/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(x**(3/4)*(a + b*x**2)**(3/4)*a + x**(3/4)*(a + b*x**2)**(3/4)*b*x**
2),x)/c**(3/4)
```

3.1073 $\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx=-\frac{4\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx}\left(a+bx^2\right)^{3/4}}$$

output
```
-4*(1+b*x^2/a)^(3/4)*hypergeom([-1/8, 7/4],[7/8],-b*x^2/a)/a/c/(c*x)^(1/4)
/(b*x^2+a)^(3/4)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

$$\int \frac{1}{(cx)^{5/4}\left(a+bx^2\right)^{7/4}}\,dx=-\frac{4x\left(1+\frac{bx^2}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{a(cx)^{5/4}\left(a+bx^2\right)^{3/4}}$$

input
```
Integrate[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(-4*x*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)
])/(a*(c*x)^(5/4)*(a + b*x^2)^(3/4))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(cx)^{5/4} (a+bx^2)^{7/4}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/4} \int \frac{1}{(cx)^{5/4} \left(\frac{bx^2}{a}+1\right)^{7/4}}dx}{a (a+bx^2)^{3/4}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{4\left(\frac{bx^2}{a}+1\right)^{3/4} \text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{4},\frac{7}{8},-\frac{bx^2}{a}\right)}{ac\sqrt[4]{cx} (a+bx^2)^{3/4}}$$

input
```
Int[1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x]
```

output
```
(-4*(1 + (b*x^2)/a)^(3/4)*Hypergeometric2F1[-1/8, 7/4, 7/8, -((b*x^2)/a)])
/(a*c*(c*x)^(1/4)*(a + b*x^2)^(3/4))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{(cx)^{\frac{5}{4}} (b\,x^2+a)^{\frac{7}{4}}}dx$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/4)*(c*x)^(3/4)/(b^2*c^2*x^6 + 2*a*b*c^2*x^4 + a^2*
c^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.98 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.81

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \frac{\Gamma\left(-\frac{1}{8}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{7}{4} \\ \frac{7}{8} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2a^{\frac{7}{4}}c^{\frac{5}{4}}\sqrt[4]{x}\,\Gamma\left(\frac{7}{8}\right)}$$

input
```
integrate(1/(c*x)**(5/4)/(b*x**2+a)**(7/4),x)
```

output
```
gamma(-1/8)*hyper((-1/8, 7/4), (7/8,), b*x**2*exp_polar(I*pi)/a)/(2*a**(7/
4)*c**(5/4)*x**(1/4)*gamma(7/8))
```

Maxima [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

Giac [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{4}}(cx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/4)*(c*x)^(5/4)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \int \frac{1}{(c\,x)^{5/4}\,(b\,x^2+a)^{7/4}}\,dx$$

input
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)),x)
```

output
```
int(1/((c*x)^(5/4)*(a + b*x^2)^(7/4)), x)
```

Reduce [F]

$$\int \frac{1}{(cx)^{5/4}(a+bx^2)^{7/4}}\,dx = \frac{\int \frac{1}{x^{\frac{5}{4}}(b\,x^2+a)^{\frac{3}{4}}a+x^{\frac{13}{4}}(b\,x^2+a)^{\frac{3}{4}}b}dx}{c^{\frac{5}{4}}}$$

input
```
int(1/(c*x)^(5/4)/(b*x^2+a)^(7/4),x)
```

output
```
int(1/(x**(1/4)*(a + b*x**2)**(3/4)*a*x + x**(1/4)*(a + b*x**2)**(3/4)*b*x
**3),x)/(c**(1/4)*c)
```

3.1074 $\int x^6 \sqrt[6]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 338

$$\int x^6\sqrt[6]{a+bx^2}\,dx = \frac{81a^3x\sqrt[6]{a+bx^2}}{2816b^3} - \frac{9a^2x^3\sqrt[6]{a+bx^2}}{704b^2} + \frac{3ax^5\sqrt[6]{a+bx^2}}{352b} + \frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$- \frac{81\,3^{3/4}a^{11/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right.\cdots}{\sqrt[3]{a}-\left(1+\sqrt{3}\right.\cdots}\right.\right.\cdots}{5632b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
81/2816*a^3*x*(b*x^2+a)^(1/6)/b^3-9/704*a^2*x^3*(b*x^2+a)^(1/6)/b^2+3/352*
a*x^5*(b*x^2+a)^(1/6)/b+3/22*x^7*(b*x^2+a)^(1/6)-81/5632*3^(3/4)*a^(11/3)*
(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3
)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJ
acobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3
)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.31

$$\int x^6 \sqrt[6]{a+bx^2}\,dx$$
$$= \frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{1+\frac{bx^2}{a}}(27a^3-3a^2bx^2+2ab^2x^4+32b^3x^6)-27a^3\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{704b^3\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^6*(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(27*a^3 - 3*a^2*b*x^2 + 2*a*
b^2*x^4 + 32*b^3*x^6) - 27*a^3*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)
/a)]))/(704*b^3*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {248, 262, 262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[6]{a+bx^2}\,dx$$
$$\Big\downarrow \text{ 248}$$
$$\frac{1}{22}a\int \frac{x^6}{(bx^2+a)^{5/6}}dx+\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$
$$\Big\downarrow \text{ 262}$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\int \frac{x^4}{(bx^2+a)^{5/6}}dx}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int \frac{x^2}{\left(bx^2+a\right)^{5/6}}dx}{10b}\right)}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx}{4b}\right)}{10b}\right)}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{ 236}$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}\right)}{16b}\right)+$$

$$\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \; 234$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx}}{4b}\right.}{10b}\right.}{16b}\right.$$

$$\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

$$\downarrow 760$$

$$\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{(}}\right)}{}\right)}{16b}\right)$$

$$\frac{3}{22}x^7\sqrt[6]{a+bx^2}$$

input
```
Int[x^6*(a + b*x^2)^(1/6),x]
```

output
```
(3*x^7*(a + b*x^2)^(1/6))/22 + (a*((3*x^5*(a + b*x^2)^(1/6))/(16*b) - (15*
a*((3*x^3*(a + b*x^2)^(1/6))/(10*b) - (9*a*((3*x*(a + b*x^2)^(1/6))/(4*b)
- (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^
(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1
 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x
*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (
b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
^2)])))/(10*b)))/(16*b)))/22
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{6} \left(b\,x^{2}+a\right)^{\frac{1}{6}}d x$$

input
```
int(x^6*(b*x^2+a)^(1/6),x)
```

output
```
int(x^6*(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int x^{6}\sqrt[6]{a+bx^{2}}\,dx=\int \left(bx^{2}+a\right)^{\frac{1}{6}}x^{6}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^{6}\sqrt[6]{a+bx^{2}}\,dx=\frac{\sqrt[6]{a}x^{7}{}_{2}F_{1}\left(\begin{matrix}-\frac{1}{6},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(1/6),x)
```

output
```
a**(1/6)*x**7*hyper((-1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 \sqrt[6]{a + bx^2}\, dx = \int (bx^2 + a)^{\frac{1}{6}} x^6\, dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^6, x)
```

Giac [F]

$$\int x^6 \sqrt[6]{a + bx^2}\, dx = \int (bx^2 + a)^{\frac{1}{6}} x^6\, dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[6]{a + bx^2}\, dx = \int x^6 \left(b\, x^2 + a\right)^{1/6} dx$$

input
```
int(x^6*(a + b*x^2)^(1/6),x)
```

output
```
int(x^6*(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int x^6 \sqrt[6]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{6}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(1/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**6,x)
```

3.1075 $\int x^4 \sqrt[6]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 314

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = -\frac{27a^2x\sqrt[6]{a+bx^2}}{640b^2}+\frac{3ax^3\sqrt[6]{a+bx^2}}{160b}+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$
$$+\frac{27\,3^{3/4}a^{8/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right.}{\sqrt[3]{a}-\left(1+\sqrt{3}\right.}\right.\right.}{1280b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-27/640*a^2*x*(b*x^2+a)^(1/6)/b^2+3/160*a*x^3*(b*x^2+a)^(1/6)/b+3/16*x^5*(
b*x^2+a)^(1/6)+27/1280*3^(3/4)*a^(8/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^
(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1
/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))
*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2
^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1
/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.30

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \frac{3x\sqrt[6]{a+bx^2}\left(\sqrt[6]{1+\frac{bx^2}{a}}(-9a^2+abx^2+10b^2x^4)+9a^2\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{160b^2\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*((1 + (b*x^2)/a)^(1/6)*(-9*a^2 + a*b*x^2 + 10*b^2*x
^4) + 9*a^2*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)]))/(160*b^2*(1
+ (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {248, 262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[6]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{16}a\int \frac{x^4}{(bx^2+a)^{5/6}}dx+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int\frac{x^2}{(bx^2+a)^{5/6}}dx}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(bx^2+a\right)^{5/6}}dx}{4b}\right)}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{236}$$

$$\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{234}$$

$$\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{}{a}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+b}}\right.}}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{}{\left(-\right.}}}\right.}{10b}\right.$$

$$\frac{3}{16}x^5\sqrt[6]{a+bx^2}$$

input
```
Int[x^4*(a + b*x^2)^(1/6),x]
```

output
```
(3*x^5*(a + b*x^2)^(1/6))/16 + (a*((3*x^3*(a + b*x^2)^(1/6))/(10*b) - (9*a
*((3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b
*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1
/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a +
b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(10*b)))/16
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^4 \left(b\,x^2+a\right)^{\frac{1}{6}}dx$$

input
```
int(x^4*(b*x^2+a)^(1/6),x)
```

output
```
int(x^4*(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \frac{\sqrt[6]{a}x^5 {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(1/6),x)
```

output
```
a**(1/6)*x**5*hyper((-1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}} x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[6]{a+bx^2}\,dx = \int x^4\,\left(b\,x^2+a\right)^{1/6}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/6),x)
```

output
```
int(x^4*(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int x^4\sqrt[6]{a+bx^2}\,dx=\int \left(b\,x^2+a\right)^{\frac{1}{6}}x^4dx$$

input
```
int(x^4*(b*x^2+a)^(1/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**4,x)
```

3.1076 $\int x^2 \sqrt[6]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\int x^2\sqrt[6]{a+bx^2}\,dx=\frac{3ax\sqrt[6]{a+bx^2}}{40b}+\frac{3}{10}x^3\sqrt[6]{a+bx^2}-\frac{3\ 3^{3/4}a^{5/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)}\right.\right.}{80b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/40*a*x*(b*x^2+a)^(1/6)/b+3/10*x^3*(b*x^2+a)^(1/6)-3/80*3^(3/4)*a^(5/3)*(
b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJa
cobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(
b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.21

$$\int x^2\sqrt[6]{a+bx^2}\,dx=\frac{3x\sqrt[6]{a+bx^2}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}\right)}{10b}$$

input
```
Integrate[x^2*(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*(a + b*x^2 - (a*Hypergeometric2F1[-1/6, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(10*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {248, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt[6]{a+bx^2}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{10}a\int\frac{x^2}{\left(bx^2+a\right)^{5/6}}dx+\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(bx^2+a\right)^{5/6}}dx}{4b}\right)+\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \text{236}$$

$$\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)+\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

$\Big\downarrow$ 234

$$\frac{1}{10}a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)+$$

$$\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

$\Big\downarrow$ 760

$$\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{E}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1}}{\left(-\sqrt[3]{1-\frac{}{a}}\right.}}}\right.$$

$$\frac{3}{10}x^3\sqrt[6]{a+bx^2}$$

input
```
Int[x^2*(a + b*x^2)^(1/6),x]
```

output
```
(3*x^3*(a + b*x^2)^(1/6))/10 + (a*((3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3
/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1
- (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2
)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*E
llipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a +
b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2])]))/1
0
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^2 (b x^2 + a)^{\frac{1}{6}} dx$$

input
```
int(x^2*(b*x^2+a)^(1/6),x)
```

output
```
int(x^2*(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int x^2 \sqrt[6]{a + bx^2} \, dx = \int (bx^2 + a)^{\frac{1}{6}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int x^2 \sqrt[6]{a + bx^2} \, dx = \frac{\sqrt[6]{a} x^3 {}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(1/6),x)
```

output
```
a**(1/6)*x**3*hyper((-1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^2, x)
```

Giac [F]

$$\int x^2 \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}} x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[6]{a+bx^2}\,dx = \int x^2\,\left(b\,x^2+a\right)^{1/6}\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/6),x)
```

output
```
int(x^2*(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int x^2 \sqrt[6]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{6}} x^2 dx$$

input
```
int(x^2*(b*x^2+a)^(1/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**2,x)
```

3.1077 $\int \sqrt[6]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 268

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{3}{4}x\sqrt[6]{a+bx^2} + \frac{3^{3/4}a^{2/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{8bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*x*(b*x^2+a)^(1/6)+1/8*3^(3/4)*a^(2/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+
a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3
^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/
2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/
4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.17

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{x\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6),x]
```

output
```
(x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-1/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {211, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[6]{a+bx^2}\,dx$$

$$\Big\downarrow \ 211$$

$$\frac{1}{4}a\int \frac{1}{(bx^2+a)^{5/6}}dx+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \ 236$$

$$\frac{a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3}{4}x\sqrt[6]{a+bx^2}$$

$$\Big\downarrow \ 234$$

$$\frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$\Big\downarrow \; 760$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\cdots}}{-\sqrt[3]{\cdots}}\right.\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$
$$\frac{3}{4}x\sqrt[6]{a+bx^2}$$

input
```
Int[(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a
 + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(
1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -
7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/
2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^
2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (b\,x^2+a)^{\frac{1}{6}}\,dx$$

input
```
int((b*x^2+a)^(1/6),x)
```

output
```
int((b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

$$\int \sqrt[6]{a+bx^2}\,dx = \sqrt[6]{a}x\,{}_2F_1\left(\begin{matrix} -\frac{1}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/6),x)
```

output
```
a**(1/6)*x*hyper((-1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6), x)
```

Giac [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.14

$$\int \sqrt[6]{a+bx^2}\,dx = \frac{x\,(b\,x^2+a)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/6}}$$

input
```
int((a + b*x^2)^(1/6),x)
```

output
```
(x*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/6)
```

Reduce [F]

$$\int \sqrt[6]{a+bx^2}\,dx = \int (b\,x^2+a)^{\frac{1}{6}}\,dx$$

input
```
int((b*x^2+a)^(1/6),x)
```

output
```
int((a + b*x**2)**(1/6),x)
```

3.1078 $\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 265

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[6]{a+bx^2}}{x} + \frac{\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{2\sqrt[4]{3}\sqrt[3]{a}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(1/6)/x+1/6*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)
+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(
1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3)
)/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/
a^(1/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))
*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{6},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6)/x^2,x]
```

output
```
-(((a + b*x^2)^(1/6)*Hypergeometric2F1[-1/2, -1/6, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(1/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {247, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{3}b\int \frac{1}{(bx^2+a)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{x}$$

$$\Big\downarrow \text{236}$$

$$\frac{b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

$$\Big\downarrow \text{234}$$

$$-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2x\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{b}{}}}{-\sqrt[3]{1-\frac{}{b}}}\right.\right.}{\sqrt[4]{3}x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

$$\frac{\sqrt[6]{a+bx^2}}{x}$$

input
```
Int[(a + b*x^2)^(1/6)/x^2,x]
```

output
```
-((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(
a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a +
b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[
-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b
*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{6}}}{x^{2}}dx$$

input
```
int((b*x^2+a)^(1/6)/x^2,x)
```

output
```
int((b*x^2+a)^(1/6)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(1/6)/x**2,x)
```

output
```
-a**(1/6)*hyper((-1/2, -1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{3\,(b\,x^2+a)^{1/6}\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{3};\ \frac{4}{3};\ -\frac{a}{b\,x^2}\right)}{2\,x\,\left(\frac{a}{b\,x^2}+1\right)^{1/6}}$$

input
```
int((a + b*x^2)^(1/6)/x^2,x)
```

output
```
-(3*(a + b*x^2)^(1/6)*hypergeom([-1/6, 1/3], 4/3, -a/(b*x^2)))/(2*x*(a/(b*
x^2) + 1)^(1/6))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.06

$$\int \frac{\sqrt[6]{a+bx^2}}{x^2}\,dx = -\frac{(b\,x^2+a)^{\frac{7}{6}}}{ax}$$

input
```
int((b*x^2+a)^(1/6)/x^2,x)
```

output
```
( - (a + b*x**2)**(5/6)*(a + b*x**2))/((a + b*x**2)**(2/3)*a*x)
```

3.1079 $\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 290

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3x^3} - \frac{b\sqrt[6]{a+bx^2}}{9ax} - \frac{b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{9\sqrt[4]{3}a^{4/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(1/6)/x^3-1/9*b*(b*x^2+a)^(1/6)/a/x-1/27*b*(b*x^2+a)^(1/6)*
(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((
a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))
),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b
*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.18

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx=-\frac{\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(1/6)*Hypergeometric2F1[-3/2, -1/6, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {247, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{9}b\int \frac{1}{x^2\left(bx^2+a\right)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{9}b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{236}$$

$$\frac{1}{9}b\left(-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{234}$$

$$\frac{1}{9}b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

$$\Big\downarrow \text{760}$$

$$\frac{1}{9}b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\cdots}{-\cdots}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.$$

$$\frac{\sqrt[6]{a+bx^2}}{3x^3}$$

input
```
Int[(a + b*x^2)^(1/6)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(1/6)/x^3 + (b*(-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 -
Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/
(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin
[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^
2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/
3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/9
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{6}}}{x^4}dx$$

input
```
int((b*x^2+a)^(1/6)/x^4,x)
```

output
```
int((b*x^2+a)^(1/6)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(1/6)/x**4,x)
```

output
```
-a**(1/6)*hyper((-3/2, -1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{1/6}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(1/6)/x^4,x)
```

output
```
int((a + b*x^2)^(1/6)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^4}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}} + (b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^8+2ab\,x^6+a^2x^4}dx\right)a\,x^3}{4\,(b\,x^2+a)^{\frac{2}{3}}\,x^3}$$

input
```
int((b*x^2+a)^(1/6)/x^4,x)
```

output
```
( - (a + b*x**2)**(5/6) + (a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a**
2*x**4 + 2*a*b*x**6 + b**2*x**8),x)*a*x**3)/(4*(a + b*x**2)**(2/3)*x**3)
```

3.1080 $\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 316

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[6]{a+bx^2}}{5x^5} - \frac{b\sqrt[6]{a+bx^2}}{45ax^3} + \frac{8b^2\sqrt[6]{a+bx^2}}{135a^2x} + \frac{8b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+b}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+b}}\right.\right.}{135\sqrt[4]{3}a^{7/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/5*(b*x^2+a)^(1/6)/x^5-1/45*b*(b*x^2+a)^(1/6)/a/x^3+8/135*b^2*(b*x^2+a)^
(1/6)/a^2/x+8/405*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+
a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1
/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a
^(7/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.16

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(1/6)*Hypergeometric2F1[-5/2, -1/6, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {247, 264, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{15}b\int \frac{1}{x^4\left(bx^2+a\right)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{15}b\left(-\frac{8b\int \frac{1}{x^2(bx^2+a)^{5/6}}dx}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{15}b\left(-\frac{8b\left(-\frac{2b\int\frac{1}{\left(bx^2+a\right)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

$\big\downarrow$ 236

$$\frac{1}{15}b\left(-\frac{8b\left(-\frac{2b\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

$\big\downarrow$ 234

$$\frac{1}{15}b\left(-\frac{8b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

$\big\downarrow$ 760

$$\frac{1}{15}b\left(-\frac{8b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{b}{}}}{-\sqrt[3]{1-\frac{b}{}}}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{9a}\right.$$

$$\frac{\sqrt[6]{a+bx^2}}{5x^5}$$

input
```
Int[(a + b*x^2)^(1/6)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(1/6)/x^5 + (b*(-1/3*(a + b*x^2)^(1/6)/(a*x^3) - (8*b*(-(
(a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2)
)]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(
a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/
(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b
*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqr
t[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]
*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/
(a + b*x^2))^(1/3))^2)])))/(9*a)))/15
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{6}}}{x^6}dx$$

input
```
int((b*x^2+a)^(1/6)/x^6,x)
```

output
```
int((b*x^2+a)^(1/6)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(1/6)/x**6,x)
```

output
```
-a**(1/6)*hyper((-5/2, -1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{1/6}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(1/6)/x^6,x)
```

output
```
int((a + b*x^2)^(1/6)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^6}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}}+(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^{10}+2ab\,x^8+a^2x^6}dx\right)a\,x^5}{6\,(b\,x^2+a)^{\frac{2}{3}}\,x^5}$$

input
```
int((b*x^2+a)^(1/6)/x^6,x)
```

output
```
( - (a + b*x**2)**(5/6) + (a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a**
2*x**6 + 2*a*b*x**8 + b**2*x**10),x)*a*x**5)/(6*(a + b*x**2)**(2/3)*x**5)
```

3.1081 $\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 340

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[6]{a+bx^2}}{7x^7} - \frac{b\sqrt[6]{a+bx^2}}{105ax^5} + \frac{2b^2\sqrt[6]{a+bx^2}}{135a^2x^3} - \frac{16b^3\sqrt[6]{a+bx^2}}{405a^3x} - \frac{16b^3\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+}}\right.\right.}{405\sqrt[4]{3}a^{10/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/7*(b*x^2+a)^(1/6)/x^7-1/105*b*(b*x^2+a)^(1/6)/a/x^5+2/135*b^2*(b*x^2+a)
^(1/6)/a^2/x^3-16/405*b^3*(b*x^2+a)^(1/6)/a^3/x-16/1215*b^3*(b*x^2+a)^(1/6
)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2
/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos
((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3
))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(10/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.15

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{6},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/6)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(1/6)*Hypergeometric2F1[-7/2, -1/6, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 264, 264, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{21}b\int \frac{1}{x^6\left(bx^2+a\right)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{21}b\left(-\frac{14b\int \frac{1}{x^4\left(bx^2+a\right)^{5/6}}dx}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}\right)-\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{21}b\left(-\frac{14b\left(-\frac{8b\int\frac{1}{x^2\left(bx^2+a\right)^{5/6}}dx}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}\right)-\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$\big\downarrow$ 264

$$\frac{1}{21}b\left(-\frac{14b\left(-\frac{8b\left(-\frac{2b\int\frac{1}{\left(bx^2+a\right)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}\right)-\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$\big\downarrow$ 236

$$\frac{1}{21}b\left(-\frac{14b\left(-\frac{8b\left(-\frac{2b\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}\right)-\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$\big\downarrow$ 234

$$\frac{1}{21}b\left(-\frac{14b\left(-\frac{8b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}\right)$$

$$\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

$\downarrow$ 760

$$\frac{1}{21}b\left(-\frac{14b\left(-\frac{8b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-}}{-\sqrt[3]{1-}}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{9a}\right.}{15a}\right.$$

$$\frac{\sqrt[6]{a+bx^2}}{7x^7}$$

input
```
Int[(a + b*x^2)^(1/6)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(1/6)/x^7 + (b*(-1/5*(a + b*x^2)^(1/6)/(a*x^5) - (14*b*(-
1/3*(a + b*x^2)^(1/6)/(a*x^3) - (8*b*(-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt
[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*
x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
(b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2)
)^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(9*a)))
/(15*a)))/21
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{1}{6}}}{x^{8}}dx$$

input
```
int((b*x^2+a)^(1/6)/x^8,x)
```

output
```
int((b*x^2+a)^(1/6)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[6]{a}\,{}_2F_1\left(\begin{matrix}-\frac{7}{2}, -\frac{1}{6}\\ -\frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(1/6)/x**8,x)
```

output
```
-a**(1/6)*hyper((-7/2, -1/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{1}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/6)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/6)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx=\int \frac{{\left(b\,x^2+a\right)}^{1/6}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(1/6)/x^8,x)
```

output
```
int((a + b*x^2)^(1/6)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt[6]{a+bx^2}}{x^8}\,dx=\frac{-\left(b\,x^2+a\right)^{\frac{5}{6}}+\left(b\,x^2+a\right)^{\frac{2}{3}}\left(\int \frac{\left(b\,x^2+a\right)^{\frac{7}{6}}}{b^2x^{12}+2ab\,x^{10}+a^2x^8}dx\right)a\,x^7}{8\left(b\,x^2+a\right)^{\frac{2}{3}}x^7}$$

input
```
int((b*x^2+a)^(1/6)/x^8,x)
```

output
```
( - (a + b*x**2)**(5/6) + (a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a**
2*x**8 + 2*a*b*x**10 + b**2*x**12),x)*a*x**7)/(8*(a + b*x**2)**(2/3)*x**7)
```

3.1082 $\int x^6(a+bx^2)^{5/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 655

$$\int x^6\left(a+bx^2\right)^{5/6}\,dx=\frac{405a^3x(a+bx^2)^{5/6}}{11648b^3}-\frac{45a^2x^3(a+bx^2)^{5/6}}{1456b^2}+\frac{3ax^5(a+bx^2)^{5/6}}{104b}$$

$$+\frac{3}{26}x^7\left(a+bx^2\right)^{5/6}+\frac{1215\left(1+\sqrt{3}\right)a^4x\sqrt[6]{a+bx^2}}{23296b^3\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}+\frac{1215\sqrt[4]{3}a^{13/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/}}{(}}}{23296}$$

output
```
405/11648*a^3*x*(b*x^2+a)^(5/6)/b^3-45/1456*a^2*x^3*(b*x^2+a)^(5/6)/b^2+3/
104*a*x^5*(b*x^2+a)^(5/6)/b+3/26*x^7*(b*x^2+a)^(5/6)+1215/23296*(1+3^(1/2)
)*a^4*x*(b*x^2+a)^(1/6)/b^3/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))+1215/232
96*3^(1/4)*a^(13/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^
(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3
))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-
(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*
x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3
))^2)^(1/2)+405/46592*3^(3/4)*(1-3^(1/2))*a^(13/3)*(b*x^2+a)^(1/6)*(a^(1/3
)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(
1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)
-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6
^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(
1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.16

$$\int x^6(a+bx^2)^{5/6}\,dx=\frac{3x(a+bx^2)^{5/6}\left(\left(1+\frac{bx^2}{a}\right)^{5/6}(27a^3-15a^2bx^2+14ab^2x^4+56b^3x^6)-27a^3\,\text{Hypergeometric}\ldots\right.}{1456b^3\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[x^6*(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2)^(5/6)*((1 + (b*x^2)/a)^(5/6)*(27*a^3 - 15*a^2*b*x^2 + 14*
a*b^2*x^4 + 56*b^3*x^6) - 27*a^3*Hypergeometric2F1[-5/6, 1/2, 3/2, -((b*x^
2)/a)]))/(1456*b^3*(1 + (b*x^2)/a)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {248, 262, 262, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^2\right)^{5/6} dx$$

$$\Big\downarrow \ 248$$

$$\frac{5}{26} a \int \frac{x^6}{\sqrt[6]{bx^2+a}}dx+\frac{3}{26} x^7 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \ 262$$

$$\frac{5}{26} a \left(\frac{3 x^5 \left(a+bx^2\right)^{5/6}}{20 b}-\frac{3 a \int \frac{x^4}{\sqrt[6]{bx^2+a}}dx}{4 b}\right)+\frac{3}{26} x^7 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \ 262$$

$$\frac{5}{26} a \left(\frac{3 x^5 \left(a+bx^2\right)^{5/6}}{20 b}-\frac{3 a \left(\frac{3 x^3 \left(a+bx^2\right)^{5/6}}{14 b}-\frac{9 a \int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{14 b}\right)}{4 b}\right)+\frac{3}{26} x^7 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \ 262$$

$$\frac{5}{26}a\left(\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}\right)}{14b}\right)}{4b}\right)+$$

$$\frac{3}{26}x^7\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \ 235$$

$$\frac{5}{26}a\left(\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{8b}\right)}{14b}\right)}{4b}\right)+$$

$$\frac{3}{26}x^7\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \ 214$$

$$\frac{5}{26}a\left(\frac{3x^5(a+bx^2)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}\right)}{8b}\right)}{14b}\right)}{4b}\right)-$$

$$\frac{3}{26}x^7(a+bx^2)^{5/6}$$

$$\Big\downarrow \text{233}$$

$$\frac{5}{26}a\left(\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{}{2\sqrt[6]{}}\right.}{8b}\right.}{14b}\right.}{4b}\right.$$

↓ 833

$$\frac{5}{26}a\left(\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{\ldots}}\right.}{\ldots}\right.}{14b}\right.}{4b}\right.$$

↓ 760

$$3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{b\ldots}{bx^2\ldots}}\right.$$

$$3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\ldots\right)}{\ldots}\right)}{\ldots}\right.$$

↓ 2418

$$\frac{3}{26}\left(bx^2+a\right)^{5/6}x^7+$$

$$\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-\frac{bx^2}{bx^2\ldots}}\right.$$

$$3a\sqrt{-\frac{bx^2}{bx^2+a}}$$

$$3a\left(\frac{3x}{2\sqrt[6]{bx^2+a}}+\right.$$

$$9a\left(\frac{3x\left(bx^2+a\right)^{5/6}}{8b}-\right.$$

$$3a\left(\frac{3x^3\left(bx^2+a\right)^{5/6}}{14b}-\right.$$

input
```
Int[x^6*(a + b*x^2)^(5/6),x]
```

output
```
(3*x^7*(a + b*x^2)^(5/6))/26 + (5*a*((3*x^5*(a + b*x^2)^(5/6))/(20*b) - (3
*a*((3*x^3*(a + b*x^2)^(5/6))/(14*b) - (9*a*((3*x*(a + b*x^2)^(5/6))/(8*b)
 - (3*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(
(-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1
/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))
^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x
^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a +
b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)
]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)
/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6)
)))/(8*b)))/(14*b)))/(4*b)))/26
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^{6} \left(b\,x^{2}+a \right)^{\frac{5}{6}}d x$$

input
```
int(x^6*(b*x^2+a)^(5/6),x)
```

output
```
int(x^6*(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int x^6 \left(a+bx^2\right)^{5/6} \, dx = \int { \left (b x^{2} + a\right)}^{\frac {5}{6}} x^{6}\, dx$$

input
```
integrate(x^6*(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.04

$$\int x^6 \left(a+bx^2\right)^{5/6} \, dx = \frac {a^{\frac {5}{6}} x^{7} {{}_{2}F_{1}\left (\begin {matrix} - \frac {5}{6}, \frac {7}{2} \\ \frac {9}{2} \end {matrix}\middle | {\frac {b x^{2} e^{i \pi }}{a}} \right)}}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(5/6),x)
```

output
```
a**(5/6)*x**7*hyper((-5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a+bx^2)^{5/6} \, dx = \int (bx^2+a)^{\frac{5}{6}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^6, x)
```

Giac [F]

$$\int x^6 (a+bx^2)^{5/6} \, dx = \int (bx^2+a)^{\frac{5}{6}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a+bx^2)^{5/6} \, dx = \int x^6 \left(b x^2+a\right)^{5/6} \, dx$$

input
```
int(x^6*(a + b*x^2)^(5/6),x)
```

output
```
int(x^6*(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int x^6 \left(a+bx^2\right)^{5/6} \, dx = \int \left(b\,x^2+a\right)^{\frac{5}{6}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(5/6)*x**6,x)
```

3.1083 $\int x^4(a+bx^2)^{5/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 631

$$\int x^4\left(a+bx^2\right)^{5/6}\,dx=-\frac{27a^2x(a+bx^2)^{5/6}}{448b^2}+\frac{3ax^3(a+bx^2)^{5/6}}{56b}$$

$$+\frac{3}{20}x^5\left(a+bx^2\right)^{5/6}-\frac{81\left(1+\sqrt{3}\right)a^3x\sqrt[6]{a+bx^2}}{896b^2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}-\frac{81\sqrt[4]{3}a^{10/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{\cdots}}{\left(\sqrt[3]{a}\cdots\right.}}\cdots}{896b^3x\sqrt{\cdots}}$$

output
```
-27/448*a^2*x*(b*x^2+a)^(5/6)/b^2+3/56*a*x^3*(b*x^2+a)^(5/6)/b+3/20*x^5*(b
*x^2+a)^(5/6)-81/896*(1+3^(1/2))*a^3*x*(b*x^2+a)^(1/6)/b^2/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))-81/896*3^(1/4)*a^(10/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b
*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(
b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/
2)+1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-27/1792*3^(3/4)*(1-3^(1/2))*a^(10/3
)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1
/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*Invers
eJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(a^(1
/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.90 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.15

$$\int x^4(a+bx^2)^{5/6}\,dx=\frac{3x(a+bx^2)^{5/6}\left(\left(1+\frac{bx^2}{a}\right)^{5/6}(-9a^2+5abx^2+14b^2x^4)+9a^2\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2}\right.\right.}{280b^2\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[x^4*(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2)^(5/6)*((1 + (b*x^2)/a)^(5/6)*(-9*a^2 + 5*a*b*x^2 + 14*b^2
*x^4) + 9*a^2*Hypergeometric2F1[-5/6, 1/2, 3/2, -((b*x^2)/a)]))/(280*b^2*(
1 + (b*x^2)/a)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {248, 262, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{5/6} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{1}{4} a \int \frac{x^4}{\sqrt[6]{bx^2+a}}dx+\frac{3}{20} x^5 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4} a \left(\frac{3 x^3 \left(a+bx^2\right)^{5/6}}{14 b}-\frac{9 a \int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{14 b}\right)+\frac{3}{20} x^5 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4} a \left(\frac{3 x^3 \left(a+bx^2\right)^{5/6}}{14 b}-\frac{9 a \left(\frac{3 x \left(a+bx^2\right)^{5/6}}{8 b}-\frac{3 a \int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8 b}\right)}{14 b}\right)+\frac{3}{20} x^5 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{235}$$

$$\frac{1}{4} a \left(\frac{3 x^3 \left(a+bx^2\right)^{5/6}}{14 b}-\frac{9 a \left(\frac{3 x \left(a+bx^2\right)^{5/6}}{8 b}-\frac{3 a \left(\frac{3 x}{2 \sqrt[6]{a+bx^2}}-\frac{1}{2} a \int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{8 b}\right)}{14 b}\right)+$$

$$\frac{3}{20} x^5 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{214}$$

$$\frac{1}{4}a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{8b}\right)}{14b}\right)+$$

$$\frac{3}{20}x^5\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{233}$$

$$\frac{1}{4}a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)}{14b}\right)+$$

$$\frac{3}{20}x^5\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \; 833$$

$$\frac{1}{4}a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+\cdots}}\cdots}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}\cdots}}\cdots\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\cdots\right)}{8b}\cdots\right)}{14b}\cdots\right)$$

$$\frac{3}{20}x^5\left(a+bx^2\right)^{5/6}$$

$\downarrow$ 760

$$\frac{1}{4}a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.$$

↓ 2418

$$\frac{1}{4}a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+b}}\cdots}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}\cdots\right.\right.}{\cdots}\right.}{\cdots}\right.$$

input
```
Int[x^4*(a + b*x^2)^(5/6),x]
```

output
```
(3*x^5*(a + b*x^2)^(5/6))/20 + (a*((3*x^3*(a + b*x^2)^(5/6))/(14*b) - (9*a
*((3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a
*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 -
Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*
EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(
a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt
[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1
- (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/
3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*
Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/
3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a +
b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b)))/(14*b)))/4
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^4 (b x^2 + a)^{\frac{5}{6}} dx$$

input
```
int(x^4*(b*x^2+a)^(5/6),x)
```

output
```
int(x^4*(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int x^4 (a + bx^2)^{5/6} dx = \int (bx^2 + a)^{\frac{5}{6}} x^4 dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int x^4 (a + bx^2)^{5/6} dx = \frac{a^{\frac{5}{6}} x^5 {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(5/6),x)
```

output
```
a**(5/6)*x**5*hyper((-5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4\left(a+bx^2\right)^{5/6}\,dx=\int \left(bx^2+a\right)^{\frac{5}{6}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^4, x)
```

Giac [F]

$$\int x^4\left(a+bx^2\right)^{5/6}\,dx=\int \left(bx^2+a\right)^{\frac{5}{6}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(a+bx^2\right)^{5/6}\,dx=\int x^4\,\left(b\,x^2+a\right)^{5/6}\,dx$$

input
```
int(x^4*(a + b*x^2)^(5/6),x)
```

output
```
int(x^4*(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int x^4 (a + bx^2)^{5/6} \, dx = \int (b\,x^2 + a)^{\frac{5}{6}} x^4 dx$$

input
```
int(x^4*(b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(5/6)*x**4,x)
```

3.1084 $\int x^2(a+bx^2)^{5/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 607

$$\int x^2\left(a+bx^2\right)^{5/6}\,dx=\frac{15ax(a+bx^2)^{5/6}}{112b}$$

$$+\frac{3}{14}x^3\left(a+bx^2\right)^{5/6}+\frac{45\left(1+\sqrt{3}\right)a^2x\sqrt[6]{a+bx^2}}{224b\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{45\sqrt[4]{3}a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{224b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{15\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}}{\sqrt[3]{a}}\right.\right.}{448b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
15/112*a*x*(b*x^2+a)^(5/6)/b+3/14*x^3*(b*x^2+a)^(5/6)+45/224*(1+3^(1/2))*a
^2*x*(b*x^2+a)^(1/6)/b/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))+45/224*3^(1/4
)*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x
^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2
)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/
3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2
)+15/448*3^(3/4)*(1-3^(1/2))*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1
/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(
b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(
1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.10

$$\int x^2\left(a+bx^2\right)^{5/6}\,dx=\frac{3x(a+bx^2)^{5/6}\left(a+bx^2-\frac{a\,\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/6}}\right)}{14b}$$

input
```
Integrate[x^2*(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2)^(5/6)*(a + b*x^2 - (a*Hypergeometric2F1[-5/6, 1/2, 3/2, -
((b*x^2)/a)])/(1 + (b*x^2)/a)^(5/6)))/(14*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {248, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{5/6} dx$$

$$\Big\downarrow \text{248}$$

$$\frac{5}{14} a \int \frac{x^2}{\sqrt[6]{bx^2+a}} dx + \frac{3}{14} x^3 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{262}$$

$$\frac{5}{14} a \left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{3a \int \frac{1}{\sqrt[6]{bx^2+a}} dx}{8b}\right) + \frac{3}{14} x^3 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{235}$$

$$\frac{5}{14} a \left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{3a \left(\frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{1}{2} a \int \frac{1}{\left(bx^2+a\right)^{7/6}} dx\right)}{8b}\right) + \frac{3}{14} x^3 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{214}$$

$$\frac{5}{14} a \left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{3a \left(\frac{3x}{2\sqrt[6]{a+bx^2}} - \frac{a \int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}} d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3} \left(a+bx^2\right)^{2/3}}\right)}{8b}\right) + \frac{3}{14} x^3 \left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{233}$$

$$\frac{5}{14}a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)+$$

$$\frac{3}{14}x^3\left(a+bx^2\right)^{5/6}$$

$\Big\downarrow$ 833

$$\frac{5}{14}a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{8b}\right.$$

$$\frac{3}{14}x^3\left(a+bx^2\right)^{5/6}$$

$\Big\downarrow$ 760

$$\frac{5}{14}a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right.}{\ldots}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{\ldots}}\right.}{\ldots}\right.$$

$$\frac{3}{14}x^3(a+bx^2)^{5/6}$$

$\downarrow$ 2418

$$\frac{5}{14}a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}\right.}}}\right.}{}\right.}{}\right.$$

$$\frac{3}{14}x^3\left(a+bx^2\right)^{5/6}$$

input
```
Int[x^2*(a + b*x^2)^(5/6),x]
```

output
```
(3*x^3*(a + b*x^2)^(5/6))/14 + (5*a*((3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*
((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt
[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/
3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqr
t[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x
^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)
]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/
3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-
((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*
x^2))^(1/3))^2]))))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b
)))/14
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int x^2 \left(b\,x^2+a\right)^{\frac{5}{6}}dx$$

input
```
int(x^2*(b*x^2+a)^(5/6),x)
```

output
```
int(x^2*(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int x^2 \left(a+bx^2\right)^{5/6} dx = \frac{a^{\frac{5}{6}} x^3 {}_2F_1\left(\begin{matrix} -\frac{5}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(5/6),x)
```

output
```
a**(5/6)*x**3*hyper((-5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^{5/6} \, dx = \int (bx^2 + a)^{\frac{5}{6}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^{5/6} \, dx = \int x^2 \left(b x^2 + a\right)^{5/6} \, dx$$

input
```
int(x^2*(a + b*x^2)^(5/6),x)
```

output
```
int(x^2*(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int x^2 (a + bx^2)^{5/6} \, dx = \int (b x^2 + a)^{\frac{5}{6}} x^2 dx$$

input
```
int(x^2*(b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(5/6)*x**2,x)
```

3.1085 $\int (a+bx^2)^{5/6}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 580

$$\int (a+bx^2)^{5/6}\,dx = \frac{3}{8}x\left(a+bx^2\right)^{5/6} - \frac{15\left(1+\sqrt{3}\right)ax\sqrt[6]{a+bx^2}}{16\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{15\sqrt[4]{3}a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{16bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{5\;3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-}{\sqrt[3]{a}-}\right.\right.}{32bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/8*x*(b*x^2+a)^(5/6)-15*(1+3^(1/2))*a*x*(b*x^2+a)^(1/6)/(16*a^(1/3)-16*(1
+3^(1/2))*(b*x^2+a)^(1/3))-15/16*3^(1/4)*a^(4/3)*(b*x^2+a)^(1/6)*(a^(1/3)-
(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/
3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))
*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-5/32*3^(3/4)*(1-3^(1/2))*a^(4/3)*(b
*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+
(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJac
obiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b
*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \left(a+bx^2\right)^{5/6} dx = \frac{x\left(a+bx^2\right)^{5/6} \text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6),x]
```

output
```
(x*(a + b*x^2)^(5/6)*Hypergeometric2F1[-5/6, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {211, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/6}dx$$

$$\Big\downarrow \text{211}$$

$$\frac{5}{8}a\int \frac{1}{\sqrt[6]{bx^2+a}}dx+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{235}$$

$$\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{214}$$

$$\frac{5}{8}a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{233}$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)+\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{833}$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{}{2\sqrt{}}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$$\Big\downarrow \text{760}$$

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{\cdots}}{\left(-\sqrt[3]{1-\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

$\downarrow$ 2418

$$\frac{5}{8}a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+\cdots}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+\cdots}}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}\right.$$

$$\frac{3}{8}x\left(a+bx^2\right)^{5/6}$$

input
```
Int[(a + b*x^2)^(5/6),x]
```

output

```
(3*x*(a + b*x^2)^(5/6))/8 + (5*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[
-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
- (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a
+ b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellipt
icE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*
x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(
1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x
^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]
*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-
1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2)
)^(2/3)*(a + b*x^2)^(1/6))))/8
```

Defintions of rubi rules used

rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{6}}d x$$

input
```
int((b*x^2+a)^(5/6),x)
```

output
```
int((b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{5/6}\,dx=\int \left(bx^2+a\right)^{\frac{5}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.04

$$\int \left(a+bx^2\right)^{5/6}\,dx=a^{\frac{5}{6}}x\,{}_2F_1\left(\left.\begin{matrix}-\frac{5}{6},\frac{1}{2}\\\frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/6),x)
```

output
```
a**(5/6)*x*hyper((-5/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/6}\,dx=\int \left(bx^2+a\right)^{\frac{5}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(bx^2+a\right)^{\frac{5}{6}} dx$$

input
```
integrate((b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \left(a+bx^2\right)^{5/6} dx = \frac{x\left(b\,x^2+a\right)^{5/6}\,{}_2\mathrm{F}_1\left(-\frac{5}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/6}}$$

input
```
int((a + b*x^2)^(5/6),x)
```

output
```
(x*(a + b*x^2)^(5/6)*hypergeom([-5/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/6)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/6} dx = \int \left(b\,x^2+a\right)^{\frac{5}{6}} dx$$

input
```
int((b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(5/6),x)
```

3.1086 $\int \frac{(a+bx^2)^{5/6}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 574

$$\int \frac{(a+bx^2)^{5/6}}{x^2} dx = -\frac{(a+bx^2)^{5/6}}{x} - \frac{5(1+\sqrt{3})\, bx\sqrt[6]{a+bx^2}}{2\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{5\sqrt[4]{3}\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right)\right|}{2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{5(1-\sqrt{3})\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{}}{\sqrt[3]{a}-(1+\sqrt{}}\right.\right.}{4\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(5/6)/x-5*(1+3^(1/2))*b*x*(b*x^2+a)^(1/6)/(2*a^(1/3)-2*(1+3^(1/
2))*(b*x^2+a)^(1/3))-5/2*3^(1/4)*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a
)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^
(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+
a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4
*2^(1/2))/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))^2)^(1/2)-5/12*(1-3^(1/2))*a^(1/3)*(b*x^2+a)^(1/6)*(a^(
1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(
a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1
/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/
4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.55 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.09

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx = -\frac{(a+bx^2)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{6},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6)/x^2,x]
```

output
```
-(((a + b*x^2)^(5/6)*Hypergeometric2F1[-5/6, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(5/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 721, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/6}}{x^2}\,dx$$

$\Big\downarrow$ 247

$$\frac{5}{3}b\int \frac{1}{\sqrt[6]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\Big\downarrow$ 235

$$\frac{5}{3}b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)-\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\Big\downarrow$ 214

$$\frac{5}{3}b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)-\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\Big\downarrow$ 233

$$\frac{5}{3}b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\Big\downarrow$ 833

$$\frac{5}{3}b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{}{2\sqrt[6]{}}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\Big\downarrow$ 760

$$\frac{5}{3}b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{\cdots}}{\left(-\sqrt[3]{1-\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{x}$$

$\downarrow$ 2418

$$\frac{5}{3}b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{x}$$

input
```
Int[(a + b*x^2)^(5/6)/x^2,x]
```

output
```
-((a + b*x^2)^(5/6)/x) + (5*b*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((
b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (
b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b
*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE
[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
- (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2
)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
(b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*El
lipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 +
 x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]))))/(4*b*x*(a/(a + b*x^2))^(
2/3)*(a + b*x^2)^(1/6))))/3
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{x^2}dx$$

input
```
int((b*x^2+a)^(5/6)/x^2,x)
```

output
```
int((b*x^2+a)^(5/6)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx = -\frac{a^{\frac{5}{6}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(5/6)/x**2,x)
```

output
```
-a**(5/6)*hyper((-5/6, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx=\int \frac{(bx^2+a)^{\frac{5}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx=\frac{3\,(b\,x^2+a)^{5/6}\,{}_2\mathrm{F}_1\left(-\frac{5}{6},-\frac{1}{3};\ \frac{2}{3};\ -\frac{a}{b\,x^2}\right)}{2\,x\,\left(\frac{a}{b\,x^2}+1\right)^{5/6}}$$

input
```
int((a + b*x^2)^(5/6)/x^2,x)
```

output
```
(3*(a + b*x^2)^(5/6)*hypergeom([-5/6, -1/3], 2/3, -a/(b*x^2)))/(2*x*(a/(b*
x^2) + 1)^(5/6))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{5/6}}{x^2}\,dx=\frac{\sqrt{b\,x^2+a}\,(-b^2x^4-2ab\,x^2-a^2)}{(b\,x^2+a)^{\frac{2}{3}}\,ax}$$

input
```
int((b*x^2+a)^(5/6)/x^2,x)
```

output
```
(sqrt(a + b*x**2)*( - a**2 - 2*a*b*x**2 - b**2*x**4))/((a + b*x**2)**(2/3)
*a*x)
```

3.1087 $\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 605

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = -\frac{(a+bx^2)^{5/6}}{3x^3}-\frac{5b(a+bx^2)^{5/6}}{9ax}-\frac{5\left(1+\sqrt{3}\right)b^2x\sqrt[6]{a+bx^2}}{9a\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{5b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2+\cdots}{3\,3^{3/4}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{5\left(1-\sqrt{3}\right)b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\cdots}\right.\right.}{18\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(5/6)/x^3-5/9*b*(b*x^2+a)^(5/6)/a/x-5/9*(1+3^(1/2))*b^2*x*(
b*x^2+a)^(1/6)/a/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))-5/9*b*(b*x^2+a)^(1/
6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(
2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)
-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(
1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(2/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-5/54*(1-3
^(1/2))*b*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x
^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2
)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(2/3)/x/(-(
b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1
/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = -\frac{(a+bx^2)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(5/6)*Hypergeometric2F1[-3/2, -5/6, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 751, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {247, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/6}}{x^4}dx$$

$$\Big\downarrow \text{ 247}$$

$$\frac{5}{9}b\int \frac{1}{x^2\sqrt[6]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{5}{9}b\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)-\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$$\Big\downarrow \text{ 235}$$

$$\frac{5}{9}b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)-\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$$\Big\downarrow \text{ 214}$$

$$\frac{5}{9}b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)-\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$$\Big\downarrow \text{ 233}$$

$$\frac{5}{9}b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)-$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$\big\downarrow$ 833

$$\frac{5}{9}b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+\right.}}}{3a}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$\big\downarrow$ 760

$$\frac{5}{9}b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+b\cdots}}\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{3a}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

$\downarrow$ 2418

$$\frac{5}{9}b\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3x^3}$$

input
```
Int[(a + b*x^2)^(5/6)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(5/6)/x^3 + (5*b*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x
)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 +
 x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) +
 (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1
+ x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (
b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7
+ 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(
a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]) -
(2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*S
qrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 -
(b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))
^(1/3))^2]))))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)))/9
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{6}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(5/6)/x^4,x)
```

output
```
int((b*x^2+a)^(5/6)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{5}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.06

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx=-\frac{a^{\frac{5}{6}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{6}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(5/6)/x**4,x)
```

output
```
-a**(5/6)*hyper((-3/2, -5/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{5/6}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(5/6)/x^4,x)
```

output
```
int((a + b*x^2)^(5/6)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^4}\,dx = \frac{(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{b\,x^6+a\,x^4}dx\right)a\,x^3-\sqrt{b\,x^2+a}\,a-\sqrt{b\,x^2+a}\,b\,x^2}{4\,(b\,x^2+a)^{\frac{2}{3}}\,x^3}$$

input
```
int((b*x^2+a)^(5/6)/x^4,x)
```

output
```
((a + b*x**2)**(2/3)*int((a + b*x**2)**(5/6)/(a*x**4 + b*x**6),x)*a*x**3 -
 sqrt(a + b*x**2)*a - sqrt(a + b*x**2)*b*x**2)/(4*(a + b*x**2)**(2/3)*x**3
)
```

3.1088 $\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 633

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = -\frac{(a+bx^2)^{5/6}}{5x^5}-\frac{b(a+bx^2)^{5/6}}{9ax^3}+\frac{4b^2(a+bx^2)^{5/6}}{27a^2x}+\frac{4(1+\sqrt{3})\,b^3x\sqrt[6]{a+bx^2}}{27a^2\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}+\frac{4b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2+}{9\,3^{3/4}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}+\frac{2(1-\sqrt{3})\,b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})}{\sqrt[3]{a}-(1+\sqrt{3})}\right.\right.}{27\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/5*(b*x^2+a)^(5/6)/x^5-1/9*b*(b*x^2+a)^(5/6)/a/x^3+4/27*b^2*(b*x^2+a)^(5
/6)/a^2/x+4/27*(1+3^(1/2))*b^3*x*(b*x^2+a)^(1/6)/a^2/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))+4/27*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2
/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a
)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/
4)/a^(5/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/
2))*(b*x^2+a)^(1/3))^2)^(1/2)+2/81*(1-3^(1/2))*b^2*(b*x^2+a)^(1/6)*(a^(1/3
)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(
1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)
-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6
^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)
^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx=-\frac{(a+bx^2)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(5/6)*Hypergeometric2F1[-5/2, -5/6, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 781, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {247, 264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/6}}{x^6}dx$$

$$\Big\downarrow \ 247$$

$$\frac{1}{3}b\int \frac{1}{x^4\sqrt[6]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{3}b\left(-\frac{4b\int \frac{1}{x^2\sqrt[6]{bx^2+a}}dx}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$$\Big\downarrow \ 235$$

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$$\Big\downarrow \ 214$$

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$\downarrow$ 233

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)-\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$\Big\downarrow$ 833

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+}}\right.}{3a}}{9a}\right.$$

$$\frac{\left(a+bx^2\right)^{5/6}}{5x^5}$$

$\downarrow$ 760

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1\cdots}}{\left(-\sqrt[3]{1-\frac{\cdots}{a\cdots}}\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}\right.}{3a\cdots}\right.}{9a\cdots}\right.$$

↓ 2418

$$\frac{1}{3}b\left(-\frac{4b\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.\right.}{}\right.}{}\right.}{}\right.$$

input
```
Int[(a + b*x^2)^(5/6)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(5/6)/x^5 + (b*(-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4*b*(-(
(a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-
((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3]
 - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellipti
cE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x
^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
- (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*
EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1
 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))
^(2/3)*(a + b*x^2)^(1/6))))/(3*a)))/(9*a)))/3
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{\frac{5}{6}}}{x^{6}}d x$$

input
```
int((b*x^2+a)^(5/6)/x^6,x)
```

output
```
int((b*x^2+a)^(5/6)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = -\frac{a^{\frac{5}{6}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{5}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(5/6)/x**6,x)
```

output
```
-a**(5/6)*hyper((-5/2, -5/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{5/6}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(5/6)/x^6,x)
```

output
```
int((a + b*x^2)^(5/6)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^6}\,dx = \frac{(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{b\,x^8+a\,x^6}dx\right)a\,x^5-\sqrt{b\,x^2+a}\,a-\sqrt{b\,x^2+a}\,b\,x^2}{6\,(b\,x^2+a)^{\frac{2}{3}}\,x^5}$$

input
```
int((b*x^2+a)^(5/6)/x^6,x)
```

output
```
((a + b*x**2)**(2/3)*int((a + b*x**2)**(5/6)/(a*x**6 + b*x**8),x)*a*x**5 -
 sqrt(a + b*x**2)*a - sqrt(a + b*x**2)*b*x**2)/(6*(a + b*x**2)**(2/3)*x**5
)
```

3.1089 $\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 657

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = -\frac{(a+bx^2)^{5/6}}{7x^7}-\frac{b(a+bx^2)^{5/6}}{21ax^5}+\frac{10b^2(a+bx^2)^{5/6}}{189a^2x^3}$$
$$-\frac{40b^3(a+bx^2)^{5/6}}{567a^3x}-\frac{40\left(1+\sqrt{3}\right)b^4x\sqrt[6]{a+bx^2}}{567a^3\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$
$$-\frac{40b^3\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2}{189\ 3^{3/4}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{20\left(1-\sqrt{3}\right)b^3\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right.}{\sqrt[3]{a}-\left(1+\sqrt{3}\right.}\right.\right.}{567\sqrt[4]{3}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/7*(b*x^2+a)^(5/6)/x^7-1/21*b*(b*x^2+a)^(5/6)/a/x^5+10/189*b^2*(b*x^2+a)
^(5/6)/a^2/x^3-40/567*b^3*(b*x^2+a)^(5/6)/a^3/x-40/567*(1+3^(1/2))*b^4*x*(
b*x^2+a)^(1/6)/a^3/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))-40/567*b^3*(b*x^2
+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x
^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(
a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(8/3)/x/(-(b*x^2+a)^(1/3)*(
a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-20
/1701*(1-3^(1/2))*b^3*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+
a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1
/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a
^(8/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.08

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = -\frac{(a+bx^2)^{5/6}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{6},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(5/6)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(5/6)*Hypergeometric2F1[-7/2, -5/6, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 811, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {247, 264, 264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/6}}{x^8}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{5}{21}b\int \frac{1}{x^6\sqrt[6]{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{21}b\left(-\frac{2b\int \frac{1}{x^4\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\int \frac{1}{x^2\sqrt[6]{bx^2+a}}dx}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$$\Big\downarrow \text{264}$$

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$$\Big\downarrow \ 235$$

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int\frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$$\Big\downarrow \ 214$$

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)-\frac{\left(a+bx^2\right)^{5/6}}{7x^7}$$

$\big\downarrow$ 233

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{(a+bx^2)^{5/6}}{ax}\right)}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}\right)}{3a}-\frac{(a+$$

↓ 833

$$\frac{5}{21}b\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{\cdots}{2\cdots}\cdots\right)}{3a}\cdots\right)}{9a}\cdots\right)}{3a}\cdots\right.$$

↓ 760

$$2b\left(-\frac{4b\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^{\cdots}}{(a+bx\cdots}}}\right.\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}}{3a\cdots}}{9a\cdots}\right.\right.$$

↓ 2418

$$\frac{3a\sqrt{-\frac{bx^2}{bx^2+a}}\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)\sqrt{\frac{\frac{x^2}{bx^2+a}+\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1\right)^2}}\,E\left(\arcsin\left(\frac{-\sqrt[3]{\cdots}}{-\sqrt[3]{\cdots}}\right.\right.}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\left(-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1\right)^2}}}\right.}{}$$

$$2b\left(\frac{3x}{2\sqrt[6]{bx^2+a}}+\cdots\right.$$

$$4b$$

$$2b\quad-$$

input
```
Int[(a + b*x^2)^(5/6)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(5/6)/x^7 + (5*b*(-1/5*(a + b*x^2)^(5/6)/(a*x^5) - (2*b*(
-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4*b*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*(
(3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)
))/(9*a)))/(3*a)))/21
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{x^8}dx$$

input
```
int((b*x^2+a)^(5/6)/x^8,x)
```

output
```
int((b*x^2+a)^(5/6)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.05

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = -\frac{a^{\frac{5}{6}}{}_2F_1\left(\begin{matrix} -\frac{7}{2}, -\frac{5}{6} \\ -\frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(5/6)/x**8,x)
```

output
```
-a**(5/6)*hyper((-7/2, -5/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{5}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(5/6)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/6)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = \int \frac{(b\,x^2+a)^{5/6}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(5/6)/x^8,x)
```

output
```
int((a + b*x^2)^(5/6)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/6}}{x^8}\,dx = \frac{(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{b\,x^{10}+a\,x^8}dx\right)a\,x^7-\sqrt{b\,x^2+a}\,a-\sqrt{b\,x^2+a}\,b\,x^2}{8\,(b\,x^2+a)^{\frac{2}{3}}\,x^7}$$

input
```
int((b*x^2+a)^(5/6)/x^8,x)
```

output
```
((a + b*x**2)**(2/3)*int((a + b*x**2)**(5/6)/(a*x**8 + b*x**10),x)*a*x**7
- sqrt(a + b*x**2)*a - sqrt(a + b*x**2)*b*x**2)/(8*(a + b*x**2)**(2/3)*x**
7)
```

3.1090 $\int x^6(a+bx^2)^{7/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 359

$$\int x^6\left(a+bx^2\right)^{7/6}\,dx=\frac{81a^4x\sqrt[6]{a+bx^2}}{11264b^3}-\frac{9a^3x^3\sqrt[6]{a+bx^2}}{2816b^2}+\frac{3a^2x^5\sqrt[6]{a+bx^2}}{1408b}+\frac{3}{88}ax^7\sqrt[6]{a+bx^2}+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}-\frac{81\,3^{3/4}a^{14/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{}}\right.\right.}{22528b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
81/11264*a^4*x*(b*x^2+a)^(1/6)/b^3-9/2816*a^3*x^3*(b*x^2+a)^(1/6)/b^2+3/14
08*a^2*x^5*(b*x^2+a)^(1/6)/b+3/88*a*x^7*(b*x^2+a)^(1/6)+3/28*x^7*(b*x^2+a)
^(7/6)-81/22528*3^(3/4)*a^(14/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))
*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(
b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^
2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2)
)/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(
b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.99 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.25

$$\int x^6\left(a+bx^2\right)^{7/6}\,dx=\frac{3x\sqrt[6]{a+bx^2}\left(\left(a+bx^2\right)^2\left(135a^2-240abx^2+352b^2x^4\right)-\frac{135a^4\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}\right)}{9856b^3}$$

input
```
Integrate[x^6*(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*((a + b*x^2)^2*(135*a^2 - 240*a*b*x^2 + 352*b^2*x^4
) - (135*a^4*Hypergeometric2F1[-7/6, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)
/a)^(1/6)))/(9856*b^3)
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {248, 248, 262, 262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6\left(a+bx^2\right)^{7/6}dx$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{4}a\int x^6\sqrt[6]{bx^2+a}dx+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{ 248}$$

$$\frac{1}{4}a\left(\frac{1}{22}a\int\frac{x^6}{\left(bx^2+a\right)^{5/6}}dx+\frac{3}{22}x^7\sqrt[6]{a+bx^2}\right)+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\int \frac{x^4}{(bx^2+a)^{5/6}}dx}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}\right)+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int \frac{x^2}{\left(bx^2+a\right)^{5/6}}dx}{10b}\right)}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}\right)+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx}{4b}\right)}{10b}\right)}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+bx^2}\right)+\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\downarrow \text{236}$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}\right)}{16b}\right)+\frac{3}{22}x^7\sqrt[6]{a+\right.$$

$$\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \; 234$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a}}{4}\right.}{10b}\right.}{16b}\right.\right.$$

$$\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

$$\downarrow 760$$

$$3\,3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx}{a+}}\right.$$

$$9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{\cdots}{4b^2x\sqrt[3]{\frac{a}{a+b}}}\right)$$

$$15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\right.$$

$$\frac{1}{4}a\left(\frac{1}{22}a\left(\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{\cdots}{16b}\right)\right)$$

$$\frac{3}{28}x^7\left(a+bx^2\right)^{7/6}$$

input
```
Int[x^6*(a + b*x^2)^(7/6),x]
```

output
```
(3*x^7*(a + b*x^2)^(7/6))/28 + (a*((3*x^7*(a + b*x^2)^(1/6))/22 + (a*((3*x
^5*(a + b*x^2)^(1/6))/(16*b) - (15*a*((3*x^3*(a + b*x^2)^(1/6))/(10*b) - (
9*a*((3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-
((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1
/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))
^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a
 + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(10*b)))/(16*b)))/22))/4
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{6} \left(b \,x^{2}+a \right)^{\frac{7}{6}}d x$$

input
```
int(x^6*(b*x^2+a)^(7/6),x)
```

output
```
int(x^6*(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int x^6 \left(a+bx^2\right)^{7/6} \, dx=\int { {\left(b x^{2} + a\right)}^{\frac{7}{6}} x^{6} \,d x }$$

input
```
integrate(x^6*(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^8 + a*x^6)*(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.08

$$\int x^6 \left(a+bx^2\right)^{7/6} \, dx=\frac{a^{\frac{7}{6}} x^{7} {{}_{2}F_{1}\left(\begin{matrix} - \frac{7}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(7/6),x)
```

output
```
a**(7/6)*x**7*hyper((-7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a + bx^2)^{7/6} \, dx = \int (bx^2 + a)^{\frac{7}{6}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^2)^{7/6} \, dx = \int (bx^2 + a)^{\frac{7}{6}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^2)^{7/6} \, dx = \int x^6 \left(b x^2 + a\right)^{7/6} \, dx$$

input
```
int(x^6*(a + b*x^2)^(7/6),x)
```

output
```
int(x^6*(a + b*x^2)^(7/6), x)
```

Reduce [F]

$$\int x^6 (a + bx^2)^{7/6} \, dx = \left(\int (b\,x^2 + a)^{\frac{1}{6}} \, x^8 dx\right) b + \left(\int (b\,x^2 + a)^{\frac{1}{6}} \, x^6 dx\right) a$$

input
```
int(x^6*(b*x^2+a)^(7/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**8,x)*b + int((a + b*x**2)**(1/6)*x**6,x)*a
```

3.1091 $\int x^4(a+bx^2)^{7/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 335

$$\int x^4\left(a+bx^2\right)^{7/6}\,dx=-\frac{189a^3x\sqrt[6]{a+bx^2}}{14080b^2}+\frac{21a^2x^3\sqrt[6]{a+bx^2}}{3520b}+\frac{21}{352}ax^5\sqrt[6]{a+bx^2}+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}+\frac{189\,3^{3/4}a^{11/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\ldots}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\ldots}\right.\right.}{28160b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-189/14080*a^3*x*(b*x^2+a)^(1/6)/b^2+21/3520*a^2*x^3*(b*x^2+a)^(1/6)/b+21/
352*a*x^5*(b*x^2+a)^(1/6)+3/22*x^5*(b*x^2+a)^(7/6)+189/28160*3^(3/4)*a^(11
/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^
(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*Inve
rseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/
2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(a^
(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.24

$$\int x^4\left(a+bx^2\right)^{7/6}\,dx=\frac{3x\sqrt[6]{a+bx^2}\left(-\left(\left(9a-16bx^2\right)\left(a+bx^2\right)^2\right)+\frac{9a^3\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}\right)}{352b^2}$$

input
```
Integrate[x^4*(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*(-((9*a - 16*b*x^2)*(a + b*x^2)^2) + (9*a^3*Hyperge
ometric2F1[-7/6, 1/2, 3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(352*b^2
)
```

Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.30, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {248, 248, 262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+bx^2\right)^{7/6}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{7}{22}a\int x^4\sqrt[6]{bx^2+a}dx+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{248}$$

$$\frac{7}{22}a\left(\frac{1}{16}a\int\frac{x^4}{\left(bx^2+a\right)^{5/6}}dx+\frac{3}{16}x^5\sqrt[6]{a+bx^2}\right)+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$$\big\downarrow \text{262}$$

$$\frac{7}{22}a\left(\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int\frac{x^2}{(bx^2+a)^{5/6}}dx}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}\right)+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$$\big\downarrow \text{262}$$

$$\frac{7}{22}a\left(\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{(bx^2+a)^{5/6}}dx}{4b}\right)}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}\right)+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$$\big\downarrow \text{236}$$

$$\frac{7}{22}a\left(\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}\right)+\frac{3}{16}x^5\sqrt[6]{a+bx^2}\right)+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$$\big\downarrow \text{234}$$

$$\frac{7}{22}a\left(\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)}{10b}\right)+\frac{3}{16}x\ldots\right)+\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

$\big\downarrow$ 760

$$\frac{7}{22}a\left(\frac{1}{16}a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{\ldots}}{\left(-\sqrt[3]{1-\ldots}\right.}}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1\ldots}}\right.}{10b}\right.\right.$$

$$\frac{3}{22}x^5\left(a+bx^2\right)^{7/6}$$

input
```
Int[x^4*(a + b*x^2)^(7/6),x]
```

output
```
(3*x^5*(a + b*x^2)^(7/6))/22 + (7*a*((3*x^5*(a + b*x^2)^(1/6))/16 + (a*((3
*x^3*(a + b*x^2)^(1/6))/(10*b) - (9*a*((3*x*(a + b*x^2)^(1/6))/(4*b) - (3*
3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)
*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^
2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - S
qrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(
a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2
)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])
))/(10*b)))/16))/22
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}+a\right)^{\frac{7}{6}}d x$$

input
```
int(x^4*(b*x^2+a)^(7/6),x)
```

output
```
int(x^4*(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int x^4\left(a+bx^2\right)^{7/6}\,dx=\int\left(bx^2+a\right)^{\frac{7}{6}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^6 + a*x^4)*(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^4\left(a+bx^2\right)^{7/6}\,dx=\frac{a^{\frac{7}{6}}x^5{}_2F_1\left(\begin{matrix}-\frac{7}{6},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(7/6),x)
```

output
```
a**(7/6)*x**5*hyper((-7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4(a+bx^2)^{7/6}\,dx = \int (bx^2+a)^{\frac{7}{6}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^2)^{7/6}\,dx = \int (bx^2+a)^{\frac{7}{6}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{7/6}\,dx = \int x^4\,\left(b\,x^2+a\right)^{7/6}\,dx$$

input
```
int(x^4*(a + b*x^2)^(7/6),x)
```

output
```
int(x^4*(a + b*x^2)^(7/6), x)
```

Reduce [F]

$$\int x^4 (a + bx^2)^{7/6} \, dx = \left(\int (b\,x^2 + a)^{\frac{1}{6}} x^6 dx \right) b + \left(\int (b\,x^2 + a)^{\frac{1}{6}} x^4 dx \right) a$$

input
```
int(x^4*(b*x^2+a)^(7/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**6,x)*b + int((a + b*x**2)**(1/6)*x**4,x)*a
```

3.1092 $\int x^2(a+bx^2)^{7/6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 311

$$\int x^2(a+bx^2)^{7/6}\,dx = \frac{21a^2x\sqrt[6]{a+bx^2}}{640b}+\frac{21}{160}ax^3\sqrt[6]{a+bx^2}+\frac{3}{16}x^3(a+bx^2)^{7/6} - \frac{21\,3^{3/4}a^{8/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right), \dots\right)}{1280b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
21/640*a^2*x*(b*x^2+a)^(1/6)/b+21/160*a*x^3*(b*x^2+a)^(1/6)+3/16*x^3*(b*x^
2+a)^(7/6)-21/1280*3^(3/4)*a^(8/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3
))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))
*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*
x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/
2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))
*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.22

$$\int x^2(a+bx^2)^{7/6}\,dx=\frac{3x\sqrt[6]{a+bx^2}\left((a+bx^2)^2-\frac{a^2\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}\right)}{16b}$$

input
```
Integrate[x^2*(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6)*((a + b*x^2)^2 - (a^2*Hypergeometric2F1[-7/6, 1/2,
3/2, -((b*x^2)/a)])/(1 + (b*x^2)/a)^(1/6)))/(16*b)
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.30, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {248, 248, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)^{7/6}\,dx$$

$$\Big\downarrow \text{248}$$

$$\frac{7}{16}a\int x^2\sqrt[6]{bx^2+a}dx+\frac{3}{16}x^3(a+bx^2)^{7/6}$$

$$\Big\downarrow \text{248}$$

$$\frac{7}{16}a\left(\frac{1}{10}a\int\frac{x^2}{(bx^2+a)^{5/6}}dx+\frac{3}{10}x^3\sqrt[6]{a+bx^2}\right)+\frac{3}{16}x^3(a+bx^2)^{7/6}$$

$$\Big\downarrow \text{262}$$

$$\frac{7}{16}a\left(\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{(bx^2+a)^{5/6}}dx}{4b}\right)+\frac{3}{10}x^3\sqrt[6]{a+bx^2}\right)+\frac{3}{16}x^3\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \ 236$$

$$\frac{7}{16}a\left(\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)+\frac{3}{10}x^3\sqrt[6]{a+bx^2}\right)+\frac{3}{16}x^3\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \ 234$$

$$\frac{7}{16}a\left(\frac{1}{10}a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)+\frac{3}{10}x^3\sqrt[6]{a+bx^2}\right)+\frac{3}{16}x^3\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \ 760$$

$$\frac{7}{16}a\left(\frac{1}{10}a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\cdots}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}\cdots\right.}}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}\right.\right.$$

$$\frac{3}{16}x^3\left(a+bx^2\right)^{7/6}$$

input
```
Int[x^2*(a + b*x^2)^(7/6),x]
```

output
```
(3*x^3*(a + b*x^2)^(7/6))/16 + (7*a*((3*x^3*(a + b*x^2)^(1/6))/10 + (a*((3
*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2
)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sq
rt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (
b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^
2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))^2)])))/10))/16
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 248
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 2*p + 1))), x] + Simp[2*a*(p/(m + 2*p + 1))
  Int[(c*x)^m*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && GtQ[
p, 0] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a \right)^{\frac{7}{6}}d x$$

input
```
int(x^2*(b*x^2+a)^(7/6),x)
```

output
```
int(x^2*(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{7/6} \, dx = \int (bx^2+a)^{\frac{7}{6}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^4 + a*x^2)*(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.09

$$\int x^2 \left(a+bx^2\right)^{7/6} \, dx = \frac{a^{\frac{7}{6}} x^3 {}_2F_1\left(\begin{matrix} -\frac{7}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(7/6),x)
```

output
```
a**(7/6)*x**3*hyper((-7/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2(a+bx^2)^{7/6}\,dx = \int (bx^2+a)^{\frac{7}{6}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^2, x)
```

Giac [F]

$$\int x^2(a+bx^2)^{7/6}\,dx = \int (bx^2+a)^{\frac{7}{6}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+bx^2)^{7/6}\,dx = \int x^2\,(b\,x^2+a)^{7/6}\,dx$$

input
```
int(x^2*(a + b*x^2)^(7/6),x)
```

output
```
int(x^2*(a + b*x^2)^(7/6), x)
```

Reduce [F]

$$\int x^2(a+bx^2)^{7/6}\,dx=\left(\int (b\,x^2+a)^{\frac{1}{6}}\,x^4dx\right)b+\left(\int (b\,x^2+a)^{\frac{1}{6}}\,x^2dx\right)a$$

input
```
int(x^2*(b*x^2+a)^(7/6),x)
```

output
```
int((a + b*x**2)**(1/6)*x**4,x)*b + int((a + b*x**2)**(1/6)*x**2,x)*a
```

3.1093 $\int \left(a+bx^2\right)^{7/6} dx$

Optimal result

Integrand size = 11, antiderivative size = 285

$$\int \left(a+bx^2\right)^{7/6} dx = \frac{21}{40}ax\sqrt[6]{a+bx^2}+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$
$$+\frac{7\,3^{3/4}a^{5/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right.\right.}{80bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
21/40*a*x*(b*x^2+a)^(1/6)+3/10*x*(b*x^2+a)^(7/6)+7/80*3^(3/4)*a^(5/3)*(b*x
^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b
*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x
^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.16

$$\int \left(a+bx^2\right)^{7/6} dx = \frac{ax\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6),x]
```

output
```
(a*x*(a + b*x^2)^(1/6)*Hypergeometric2F1[-7/6, 1/2, 3/2, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {211, 211, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{7/6} dx$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{10}a\int \sqrt[6]{bx^2+a}dx+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{10}a\left(\frac{1}{4}a\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$$\Big\downarrow \text{236}$$

$$\frac{7}{10}a\left(\frac{a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$\Big\downarrow$ 234

$$\frac{7}{10}a\left(\frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}\right)+\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

$\Big\downarrow$ 760

$$\frac{7}{10}a\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.$$

$$\frac{3}{10}x\left(a+bx^2\right)^{7/6}$$

input
```
Int[(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(a + b*x^2)^(7/6))/10 + (7*a*((3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sq
rt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Siniti
cF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^
(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/10
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int (b\,x^2 + a)^{\frac{7}{6}}\,dx$$

input
```
int((b*x^2+a)^(7/6),x)
```

output
```
int((b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{7/6} dx = \int \left(bx^2+a\right)^{\frac{7}{6}} dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.09

$$\int \left(a+bx^2\right)^{7/6} dx = a^{\frac{7}{6}} x {}_2F_1\left(\begin{matrix} -\frac{7}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(7/6),x)
```

output
```
a**(7/6)*x*hyper((-7/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{7/6} dx = \int \left(bx^2+a\right)^{\frac{7}{6}} dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{7/6}\,dx=\int \left(bx^2+a\right)^{\frac{7}{6}}\,dx$$

input
```
integrate((b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.13

$$\int \left(a+bx^2\right)^{7/6}\,dx=\frac{x\,{\left(b\,x^2+a\right)}^{7/6}\,{}_2\mathrm{F}_1\left(-\frac{7}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{7/6}}$$

input
```
int((a + b*x^2)^(7/6),x)
```

output
```
(x*(a + b*x^2)^(7/6)*hypergeom([-7/6, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(7/6)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{7/6}\,dx=\left(\int \left(b\,x^2+a\right)^{\frac{1}{6}}\,dx\right)a+\left(\int \left(b\,x^2+a\right)^{\frac{1}{6}}\,x^2dx\right)b$$

input
```
int((b*x^2+a)^(7/6),x)
```

output
```
int((a + b*x**2)**(1/6),x)*a + int((a + b*x**2)**(1/6)*x**2,x)*b
```

3.1094 $\int \frac{(a+bx^2)^{7/6}}{x^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 282

$$\int \frac{(a+bx^2)^{7/6}}{x^2} \, dx = \frac{7}{4} bx \sqrt[6]{a+bx^2} - \frac{(a+bx^2)^{7/6}}{x} + \frac{7a^{2/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx}}\right.\right.}{8\sqrt[4]{3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
7/4*b*x*(b*x^2+a)^(1/6)-(b*x^2+a)^(7/6)/x+7/24*a^(2/3)*(b*x^2+a)^(1/6)*(a^
(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/
(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(
1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1
/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/
3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.62 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^2)^{7/6}}{x^2} dx = -\frac{a\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{6},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6)/x^2,x]
```

output
```
-((a*(a + b*x^2)^(1/6)*Hypergeometric2F1[-7/6, -1/2, 1/2, -((b*x^2)/a)])/(
x*(1 + (b*x^2)/a)^(1/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.33, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 211, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/6}}{x^2} dx$$

$$\Big\downarrow \text{247}$$

$$\frac{7}{3}b\int \sqrt[6]{bx^2+a}dx-\frac{(a+bx^2)^{7/6}}{x}$$

$$\Big\downarrow \text{211}$$

$$\frac{7}{3}b\left(\frac{1}{4}a\int \frac{1}{(bx^2+a)^{5/6}}dx+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)-\frac{(a+bx^2)^{7/6}}{x}$$

$$\Big\downarrow \text{236}$$

$$\frac{7}{3}b\left(\frac{a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}+\frac{3}{4}x\sqrt[6]{a+bx^2}\right)-\frac{\left(a+bx^2\right)^{7/6}}{x}$$

$\Big\downarrow$ 234

$$\frac{7}{3}b\left(\frac{3}{4}x\sqrt[6]{a+bx^2}-\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8bx\sqrt[3]{\frac{a}{a+bx^2}}}\right)-\frac{\left(a+bx^2\right)^{7/6}}{x}$$

$\Big\downarrow$ 760

$$\frac{7}{3}b\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{4bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.$$

$$\frac{\left(a+bx^2\right)^{7/6}}{x}$$

input
```
Int[(a + b*x^2)^(7/6)/x^2,x]
```

output
```
-((a + b*x^2)^(7/6)/x) + (7*b*((3*x*(a + b*x^2)^(1/6))/4 + (3^(3/4)*Sqrt[2
 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*
x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x
^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[A
rcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
(b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b*x*(a/(a + b*x^2))^(1/3
)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(
1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/3
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{x^2}dx$$

input
```
int((b*x^2+a)^(7/6)/x^2,x)
```

output
```
int((b*x^2+a)^(7/6)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = -\frac{a^{\frac{7}{6}}{}_2F_1\left(\begin{matrix} -\frac{7}{6}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(7/6)/x**2,x)
```

output
```
-a**(7/6)*hyper((-7/6, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.14

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = \frac{3\,(b\,x^2+a)^{7/6}\,{}_2\mathrm{F}_1\left(-\frac{7}{6},-\frac{2}{3};\ \frac{1}{3};\ -\frac{a}{b\,x^2}\right)}{4\,x\,\left(\frac{a}{b\,x^2}+1\right)^{7/6}}$$

input
```
int((a + b*x^2)^(7/6)/x^2,x)
```

output
```
(3*(a + b*x^2)^(7/6)*hypergeom([-7/6, -2/3], 1/3, -a/(b*x^2)))/(4*x*(a/(b*
x^2) + 1)^(7/6))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^2}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}}\,a-(b\,x^2+a)^{\frac{5}{6}}\,b\,x^2+(b\,x^2+a)^{\frac{2}{3}}\left(\int (b\,x^2+a)^{\frac{1}{6}}\,dx\right)bx}{(b\,x^2+a)^{\frac{2}{3}}\,x}$$

input
```
int((b*x^2+a)^(7/6)/x^2,x)
```

output
```
( - (a + b*x**2)**(5/6)*a - (a + b*x**2)**(5/6)*b*x**2 + (a + b*x**2)**(2/
3)*int((a + b*x**2)**(1/6),x)*b*x)/((a + b*x**2)**(2/3)*x)
```

3.1095 $\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 287

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = -\frac{7b\sqrt[6]{a+bx^2}}{9x} - \frac{(a+bx^2)^{7/6}}{3x^3} + \frac{7b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\Bigg(\arccos\Bigg(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}}{18\sqrt[4]{3}\sqrt[3]{a}\,x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-7/9*b*(b*x^2+a)^(1/6)/x-1/3*(b*x^2+a)^(7/6)/x^3+7/54*b*(b*x^2+a)^(1/6)*(a
^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^
(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),
1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(1/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x
^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.18

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = -\frac{a\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6)/x^4,x]
```

output
```
-1/3*(a*(a + b*x^2)^(1/6)*Hypergeometric2F1[-3/2, -7/6, -1/2, -((b*x^2)/a)
])/(x^3*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.28 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {247, 247, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/6}}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{7}{9}b\int \frac{\sqrt[6]{bx^2+a}}{x^2}dx-\frac{(a+bx^2)^{7/6}}{3x^3}$$

$$\Big\downarrow \text{247}$$

$$\frac{7}{9}b\left(\frac{1}{3}b\int \frac{1}{(bx^2+a)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{x}\right)-\frac{(a+bx^2)^{7/6}}{3x^3}$$

$$\Big\downarrow \text{236}$$

$$\frac{7}{9}b\left(\frac{b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{7/6}}{3x^3}$$

$\downarrow$ 234

$$\frac{7}{9}b\left(-\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2x\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{x}\right)-\frac{\left(a+bx^2\right)^{7/6}}{3x^3}$$

$\downarrow$ 760

$$\frac{7}{9}b\left(\frac{\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1}}{-\sqrt[3]{1}}\right.\right.}{\sqrt[4]{3}x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.$$

$$\frac{\left(a+bx^2\right)^{7/6}}{3x^3}$$

input
```
Int[(a + b*x^2)^(7/6)/x^4,x]
```

output
```
-1/3*(a + b*x^2)^(7/6)/x^3 + (7*b*(-((a + b*x^2)^(1/6)/x) + (Sqrt[2 - Sqrt
[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3
))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1
+ Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(
a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*x*(a/(a + b*x^2))^(1/3)*Sqr
t[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2])))/9
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{7}{6}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(7/6)/x^4,x)
```

output
```
int((b*x^2+a)^(7/6)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = -\frac{a^{\frac{7}{6}}{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{7}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(7/6)/x**4,x)
```

output
```
-a**(7/6)*hyper((-3/2, -7/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{7/6}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(7/6)/x^4,x)
```

output
```
int((a + b*x^2)^(7/6)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.13

$$\int \frac{(a+bx^2)^{7/6}}{x^4}\,dx = \frac{(b\,x^2+a)^{\frac{1}{6}}\,(-b^2x^4-2ab\,x^2-a^2)}{3a\,x^3}$$

input
```
int((b*x^2+a)^(7/6)/x^4,x)
```

output
```
((a + b*x**2)**(5/6)*( - a**2 - 2*a*b*x**2 - b**2*x**4))/(3*(a + b*x**2)**
(2/3)*a*x**3)
```

3.1096 $\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 313

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = -\frac{7b\sqrt[6]{a+bx^2}}{45x^3} - \frac{7b^2\sqrt[6]{a+bx^2}}{135ax} - \frac{(a+bx^2)^{7/6}}{5x^5} - \frac{7b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{135\sqrt[4]{3}a^{4/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-7/45*b*(b*x^2+a)^(1/6)/x^3-7/135*b^2*(b*x^2+a)^(1/6)/a/x-1/5*(b*x^2+a)^(7
/6)/x^5-7/405*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1
/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))
^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(4/
3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.17

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = -\frac{a\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6)/x^6,x]
```

output
```
-1/5*(a*(a + b*x^2)^(1/6)*Hypergeometric2F1[-5/2, -7/6, -3/2, -((b*x^2)/a)
])/(x^5*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {247, 247, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/6}}{x^6}dx$$

$$\Big\downarrow \text{247}$$

$$\frac{7}{15}b\int \frac{\sqrt[6]{bx^2+a}}{x^4}dx-\frac{(a+bx^2)^{7/6}}{5x^5}$$

$$\Big\downarrow \text{247}$$

$$\frac{7}{15}b\left(\frac{1}{9}b\int \frac{1}{x^2(bx^2+a)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{3x^3}\right)-\frac{(a+bx^2)^{7/6}}{5x^5}$$

$$\Big\downarrow \text{264}$$

$$\frac{7}{15}b\left(\frac{1}{9}b\left(-\frac{2b\int \frac{1}{(bx^2+a)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7/6}}{5x^5}$$

$$\Big\downarrow \text{236}$$

$$\frac{7}{15}b\left(\frac{1}{9}b\left(-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7/6}}{5x^5}$$

$$\Big\downarrow \text{234}$$

$$\frac{7}{15}b\left(\frac{1}{9}b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)-\frac{\sqrt[6]{a+bx^2}}{3x^3}\right)-\frac{\left(a+bx^2\right)^{7/6}}{5x^5}$$

$$\Big\downarrow \text{760}$$

$$\frac{7}{15}b\left(\frac{1}{9}b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\text{arcsi}\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right.}}}\right.\right.$$

$$\frac{\left(a+bx^2\right)^{7/6}}{5x^5}$$

input
```
Int[(a + b*x^2)^(7/6)/x^6,x]
```

output
```
-1/5*(a + b*x^2)^(7/6)/x^5 + (7*b*(-1/3*(a + b*x^2)^(1/6)/x^3 + (b*(-((a +
 b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(
a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a +
b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]
])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqr
t[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))^2)])))/9))/15
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{7}{6}}}{x^{6}}dx$$

input
```
int((b*x^2+a)^(7/6)/x^6,x)
```

output
```
int((b*x^2+a)^(7/6)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx=\int \frac{(bx^2+a)^{\frac{7}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.11

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = -\frac{a^{\frac{7}{6}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{7}{6}\\ -\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(7/6)/x**6,x)
```

output
```
-a**(7/6)*hyper((-5/2, -7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{7/6}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(7/6)/x^6,x)
```

output
```
int((a + b*x^2)^(7/6)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^6}\,dx = \frac{-5(b\,x^2+a)^{\frac{5}{6}}\,a-6(b\,x^2+a)^{\frac{5}{6}}\,b\,x^2-(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^{10}+2ab\,x^8+a^2x^6}dx\right)a^2x^5}{24\,(b\,x^2+a)^{\frac{2}{3}}\,x^5}$$

input
```
int((b*x^2+a)^(7/6)/x^6,x)
```

output
```
( - 5*(a + b*x**2)**(5/6)*a - 6*(a + b*x**2)**(5/6)*b*x**2 - (a + b*x**2)*
*(2/3)*int((a + b*x**2)**(7/6)/(a**2*x**6 + 2*a*b*x**8 + b**2*x**10),x)*a*
*2*x**5)/(24*(a + b*x**2)**(2/3)*x**5)
```

3.1097 $\int \frac{\left(a+bx^2\right)^{7/6}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 337

$$\int \frac{\left(a+bx^2\right)^{7/6}}{x^8}\,dx = -\frac{b\sqrt[6]{a+bx^2}}{15x^5}-\frac{b^2\sqrt[6]{a+bx^2}}{135ax^3}+\frac{8b^3\sqrt[6]{a+bx^2}}{405a^2x}-\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$
$$+\frac{8b^3\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{405\sqrt[4]{3}a^{7/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/15*b*(b*x^2+a)^(1/6)/x^5-1/135*b^2*(b*x^2+a)^(1/6)/a/x^3+8/405*b^3*(b*x
^2+a)^(1/6)/a^2/x-1/7*(b*x^2+a)^(7/6)/x^7+8/1215*b^3*(b*x^2+a)^(1/6)*(a^(1
/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a
^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/
3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4
*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+
a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.15

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = -\frac{a\sqrt[6]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{6},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\sqrt[6]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(7/6)/x^8,x]
```

output
```
-1/7*(a*(a + b*x^2)^(1/6)*Hypergeometric2F1[-7/2, -7/6, -5/2, -((b*x^2)/a)
])/(x^7*(1 + (b*x^2)/a)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {247, 247, 264, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx$$

$$\Big\downarrow \; 247$$

$$\frac{1}{3}b\int \frac{\sqrt[6]{bx^2+a}}{x^6}dx-\frac{(a+bx^2)^{7/6}}{7x^7}$$

$$\Big\downarrow \; 247$$

$$\frac{1}{3}b\left(\frac{1}{15}b\int \frac{1}{x^4\,(bx^2+a)^{5/6}}dx-\frac{\sqrt[6]{a+bx^2}}{5x^5}\right)-\frac{(a+bx^2)^{7/6}}{7x^7}$$

$$\Big\downarrow \; 264$$

$$\frac{1}{3}b\left(\frac{1}{15}b\left(-\frac{8b\int \frac{1}{x^2(bx^2+a)^{5/6}}dx}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$

$\downarrow$ 264

$$\frac{1}{3}b\left(\frac{1}{15}b\left(-\frac{8b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$

$\downarrow$ 236

$$\frac{1}{3}b\left(\frac{1}{15}b\left(-\frac{8b\left(-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$

$\downarrow$ 234

$$\frac{1}{3}b\left(\frac{1}{15}b\left(-\frac{8b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)-\frac{\sqrt[6]{a+bx^2}}{5x^5}\right)-\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$

$\downarrow$ 760

$$\frac{1}{3}b\left(\frac{1}{15}b\left(-\frac{8b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\cdots}}{-\sqrt[3]{\cdots}}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right)}{9a}\right.\right.$$

$$\frac{\left(a+bx^2\right)^{7/6}}{7x^7}$$

input
```
Int[(a + b*x^2)^(7/6)/x^8,x]
```

output
```
-1/7*(a + b*x^2)^(7/6)/x^7 + (b*(-1/5*(a + b*x^2)^(1/6)/x^5 + (b*(-1/3*(a
+ b*x^2)^(1/6)/(a*x^3) - (8*b*(-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sq
rt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a
 + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1
/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(
1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)
/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)
*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1
/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(9*a)))/15))/3
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{7}{6}}}{x^{8}}dx$$

input
```
int((b*x^2+a)^(7/6)/x^8,x)
```

output
```
int((b*x^2+a)^(7/6)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/6)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.10

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = -\frac{a^{\frac{7}{6}}\,{}_2F_1\left(\begin{matrix} -\frac{7}{2}, -\frac{7}{6} \\ -\frac{5}{2} \end{matrix}\middle|\, \frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(7/6)/x**8,x)
```

output
```
-a**(7/6)*hyper((-7/2, -7/6), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{7}{6}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/6)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/6)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = \int \frac{(b\,x^2+a)^{7/6}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(7/6)/x^8,x)
```

output
```
int((a + b*x^2)^(7/6)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/6}}{x^8}\,dx = \frac{-7(b\,x^2+a)^{\frac{5}{6}}\,a-8(b\,x^2+a)^{\frac{5}{6}}\,b\,x^2-(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^{12}+2ab\,x^{10}+a^2x^8}dx\right)a^2x^7}{48\,(b\,x^2+a)^{\frac{2}{3}}\,x^7}$$

input

```
int((b*x^2+a)^(7/6)/x^8,x)
```

output

```
( - 7*(a + b*x**2)**(5/6)*a - 8*(a + b*x**2)**(5/6)*b*x**2 - (a + b*x**2)*
*(2/3)*int((a + b*x**2)**(7/6)/(a**2*x**8 + 2*a*b*x**10 + b**2*x**12),x)*a
**2*x**7)/(48*(a + b*x**2)**(2/3)*x**7)
```

3.1098 $\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 634

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{81a^2x(a+bx^2)^{5/6}}{448b^3} - \frac{9ax^3(a+bx^2)^{5/6}}{56b^2} + \frac{3x^5(a+bx^2)^{5/6}}{20b} + \frac{243\left(1+\sqrt{3}\right)a^3x\sqrt[6]{a+bx^2}}{896b^3\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$
$$+ \frac{243\sqrt[4]{3}a^{10/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+}}\right.\right.}{896b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$
$$+ \frac{81\ 3^{3/4}\left(1-\sqrt{3}\right)a^{10/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\right.\right.}{1792b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
81/448*a^2*x*(b*x^2+a)^(5/6)/b^3-9/56*a*x^3*(b*x^2+a)^(5/6)/b^2+3/20*x^5*(
b*x^2+a)^(5/6)/b+243/896*(1+3^(1/2))*a^3*x*(b*x^2+a)^(1/6)/b^3/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))+243/896*3^(1/4)*a^(10/3)*(b*x^2+a)^(1/6)*(a^(1/
3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/
2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)+81/1792*3^(3/4)*(1-3^(1/2))*a^
(10/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+
a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*I
nverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^
(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*
(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.14

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{3\left(135a^3x+15a^2bx^3-8ab^2x^5+112b^3x^7-135a^3x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{2240b^3\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(1/6),x]
```

output
```
(3*(135*a^3*x + 15*a^2*b*x^3 - 8*a*b^2*x^5 + 112*b^3*x^7 - 135*a^3*x*(1 +
(b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(2240*b^
3*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {262, 262, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\int \frac{x^4}{\sqrt[6]{bx^2+a}}dx}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{14b}\right)}{4b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}\right)}{14b}\right)}{4b}$$

$$\Big\downarrow \text{235}$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{8b}\right)}{14b}\right)}{4b}$$

$$\Big\downarrow \; 214$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{8b}\right)}{14b}\right)}{4b}$$

$$\Big\downarrow \; 233$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)}{14b}\right)}{4b}$$

$$\Big\downarrow \ 833$$

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b}-$$

$$\frac{3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\cdots\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\cdots\right.}{8b}\cdots\right)}{14b}\cdots\right)}{4b}$$

$\downarrow$ 760

$$\frac{3x^5\left(a+bx^2\right)^{5/6}}{20b} -$$

$$3a\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b} - \frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b} - \frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}} - \frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1\cdots}\right.}{\cdots}\right.}{4bx\left(\frac{\cdots}{a+\cdots}\right.}\right.}{\cdots}\right.$$

↓ 2418

$$3a\left(\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}(1+\sqrt{3})\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{E}\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-\frac{\cdots}{a+\cdots}}\cdots\right)\cdots}}}\cdots\right)\cdots\right)}{\cdots}\right)}{\cdots}\cdots\right)$$

$$\frac{3x^5(a+bx^2)^{5/6}}{20b}-\cdots$$

input
```
Int[x^6/(a + b*x^2)^(1/6),x]
```

output
```
(3*x^5*(a + b*x^2)^(5/6))/(20*b) - (3*a*((3*x^3*(a + b*x^2)^(5/6))/(14*b)
- (9*a*((3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*((3*x)/(2*(a + b*x^2)^(1/6))
+ (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])
/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[
3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 -
 (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 +
 x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1
+ Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2)
 + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2
))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/
3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^
(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2
))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a
/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b)))/(14*b)))/(4*b)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/6),x)
```

output
```
int(x^6/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.04

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \frac{x^7{}_2F_1\left(\begin{matrix}\frac{1}{6},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7\sqrt[6]{a}}$$

input
```
integrate(x**6/(b*x**2+a)**(1/6),x)
```

output
```
x**7*hyper((1/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/6))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/6), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{1/6}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(1/6),x)
```

output
```
int(x^6/(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/6),x)
```

output
```
int(x**6/(a + b*x**2)**(1/6),x)
```

3.1099 $\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 610

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = -\frac{27ax(a+bx^2)^{5/6}}{112b^2}+\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{81\left(1+\sqrt{3}\right)a^2x\sqrt[6]{a+bx^2}}{224b^2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$
$$-\frac{81\sqrt[4]{3}a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{224b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$
$$-\frac{27\,3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\right.\right.}{448b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-27/112*a*x*(b*x^2+a)^(5/6)/b^2+3/14*x^3*(b*x^2+a)^(5/6)/b-81/224*(1+3^(1/
2))*a^2*x*(b*x^2+a)^(1/6)/b^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))-81/224
*3^(1/4)*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/
3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^
2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+
3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^3/x/(-(b*x^2
+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^
2)^(1/2)-27/448*3^(3/4)*(1-3^(1/2))*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(
1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+
1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.13

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \frac{3\left(-9a^2x-abx^3+8b^2x^5+9a^2x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{112b^2\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(1/6),x]
```

output
```
(3*(-9*a^2*x - a*b*x^3 + 8*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hyperge
ometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)]))/(112*b^2*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {262, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{14b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}\right)}{14b}$$

$$\Big\downarrow \text{235}$$

$$\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx\right)}{8b}\right)}{14b}$$

$$\Big\downarrow \text{214}$$

$$\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}\right)}{8b}\right)}{14b}$$

$$\Big\downarrow \text{233}$$

$$\frac{\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)}{14b}}{}$$

$\downarrow$ 833

$$\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{8b}\right.}{14b}$$

$\downarrow$ 760

$$\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-$$

$$9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\cdots}{a+\cdots}}\ \cdots}{\sqrt[4]{3}\sqrt{\frac{\cdots}{\left(a\cdots\right.}}\ \cdots}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\ \cdots}}{8b\ \cdots}\right.$$

$$\overline{\qquad 14b \qquad}$$

↓ 2418

$$\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1\cdots}}{-\sqrt[3]{1\cdots}}\right)\cdots\right)}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\cdots}\cdots\right)\cdots\right)}{3a\cdots}\cdots\right)}{\cdots}$$

input
```
Int[x^4/(a + b*x^2)^(1/6),x]
```

output
```
(3*x^3*(a + b*x^2)^(5/6))/(14*b) - (9*a*((3*x*(a + b*x^2)^(5/6))/(8*b) - (
3*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*
Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))
^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))
*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1
- (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/
3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 -
(b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))
^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^
2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sq
rt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a
+ b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/
(8*b)))/(14*b)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/6),x)
```

output
```
int(x^4/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.04

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5\sqrt[6]{a}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/6),x)
```

output
```
x**5*hyper((1/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/6), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{1/6}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(1/6),x)
```

output
```
int(x^4/(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/6),x)
```

output
```
int(x**4/(a + b*x**2)**(1/6),x)
```

3.1100 $\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 586

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \frac{3x\left(a+bx^2\right)^{5/6}}{8b} + \frac{9\left(1+\sqrt{3}\right)ax\sqrt[6]{a+bx^2}}{16b\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{9\sqrt[4]{3}a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right.\right.}{16b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\right.\right.}{32b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/8*x*(b*x^2+a)^(5/6)/b+9/16*(1+3^(1/2))*a*x*(b*x^2+a)^(1/6)/b/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))+9/16*3^(1/4)*a^(4/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(
b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3
)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*
(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1
/2)+1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3
)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)+3/32*3^(3/4)*(1-3^(1/2))*a^(4/3)*(
b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)
+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJa
cobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(
b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)
-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.85 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx=\frac{3x\left(a+bx^2-a\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{8b\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2,
 -((b*x^2)/a)]))/(8*b*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx$$

$\Big\downarrow$ 262

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}$$

$\Big\downarrow$ 235

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx\right)}{8b}$$

$\Big\downarrow$ 214

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}\right)}{8b}$$

$\Big\downarrow$ 233

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}$$

$\Big\downarrow$ 833

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}$$

$$\Big\downarrow \text{760}$$

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-$$

$$\frac{3a\left(\dfrac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\dfrac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+b\ldots}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{\ldots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1\ldots}{\left(-\sqrt[3]{\ldots}\right.}}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{8b}$$

$$\Big\downarrow \text{2418}$$

$$\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-$$

$$3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1}\right)\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}\right.$$

input
```
Int[x^2/(a + b*x^2)^(1/6),x]
```

output
```
(3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*S
qrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*El
lipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3
] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a
+ b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3
])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 -
(b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sq
rt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*
x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2 + a)^{\frac{1}{6}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/6),x)
```

output
```
int(x^2/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3\sqrt[6]{a}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/6),x)
```

output
```
x**3*hyper((1/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/6), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{1/6}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(1/6),x)
```

output
```
int(x^2/(a + b*x^2)^(1/6), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/6),x)
```

output
```
int(x**2/(a + b*x**2)**(1/6),x)
```

3.1101 $\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 563

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = -\frac{3(1+\sqrt{3})\,x\sqrt[6]{a+bx^2}}{2\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{3}\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\Big(\arccos\Big(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}}{2bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{3^{3/4}(1-\sqrt{3})\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\operatorname{EllipticF}\Big(\arccos\Big(\frac{\sqrt[3]{a}-}{\sqrt[3]{a}-}}{4bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*(1+3^(1/2))*x*(b*x^2+a)^(1/6)/(2*a^(1/3)-2*(1+3^(1/2))*(b*x^2+a)^(1/3))
-3/2*3^(1/4)*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a
^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/
3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)
-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b/x/(-(b*x
^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3)
)^2)^(1/2)-1/4*3^(3/4)*(1-3^(1/2))*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+
3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1
/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1
/4*2^(1/2))/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^
(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.08

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[6]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/6, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 699, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{235}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx$$

$$\Big\downarrow \text{214}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\cdots}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\cdots\right)}}\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{\cdots}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}$$

$\downarrow$ 2418

$$\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{\cdots}$$

$$\frac{3x}{2\sqrt[6]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/6),x]
```

output
```
(3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(1/6),x)
```

output
```
int(1/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{6}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/6),x)
```

output
```
x*hyper((1/6, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/6)
```

Maxima [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/6), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.07

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{1/6}}$$

input
```
int(1/(a + b*x^2)^(1/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/6)*hypergeom([1/6, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/6)
```

Reduce [F]

$$\int \frac{1}{\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(1/6),x)
```

output
```
int(1/(a + b*x**2)**(1/6),x)
```

3.1102 $\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 576

$$\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{5/6}}{ax} - \frac{\left(1+\sqrt{3}\right) bx\sqrt[6]{a+bx^2}}{a\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{\sqrt[4]{3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(\cdots}{a^{2/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\cdots}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\cdots}\right.\right.\cdots}{2\sqrt[4]{3}a^{2/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(5/6)/a/x-(1+3^(1/2))*b*x*(b*x^2+a)^(1/6)/a/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))-3^(1/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(
2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+
a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a
^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a^(2
/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*
x^2+a)^(1/3))^2)^(1/2)-1/6*(1-3^(1/2))*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^
(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1
/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))
*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2
^(1/2))*3^(3/4)/a^(2/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(
1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.87 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2 \sqrt[6]{a+bx^2}} \, dx = -\frac{\sqrt[6]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{6},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(1/6)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 1/6, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(1/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \sqrt[6]{a+bx^2}}\,dx$$

$$\Big\downarrow \text{264}$$

$$\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \text{235}$$

$$\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \text{214}$$

$$\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \text{233}$$

$$\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \text{833}$$

$$\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}$$

$$\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \; 760$$

$$\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\cdots}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{\cdots}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{3a}$$

$$\frac{\left(a+bx^2\right)^{5/6}}{ax}$$

$$\Big\downarrow \; 2418$$

$$\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1}\right)\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{}\right.}{\frac{\left(a+bx^2\right)^{5/6}}{ax}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(1/6)),x]
```

output
```
-((a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt
[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellip
ticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b
*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2
))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{1}{6}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/6),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[6]{a}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/6),x)
```

output
```
-hyper((-1/2, 1/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/6)*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = -\frac{3\left(\frac{a}{b\,x^2}+1\right)^{1/6}\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{2}{3};\ \frac{5}{3};\ -\frac{a}{b\,x^2}\right)}{4\,x\,\left(b\,x^2+a\right)^{1/6}}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/6)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(1/6)*hypergeom([1/6, 2/3], 5/3, -a/(b*x^2)))/(4*x*(a
+ b*x^2)^(1/6))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^2\sqrt[6]{a+bx^2}}\,dx = -\frac{\sqrt{b\,x^2+a}\,\left(b\,x^2+a\right)^{\frac{1}{3}}}{ax}$$

input
```
int(1/x^2/(b*x^2+a)^(1/6),x)
```

output
```
( - sqrt(a + b*x**2)*(a + b*x**2))/((a + b*x**2)**(2/3)*a*x)
```

3.1103 $\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 608

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = -\frac{(a+bx^2)^{5/6}}{3ax^3}+\frac{4b(a+bx^2)^{5/6}}{9a^2x}+\frac{4\left(1+\sqrt{3}\right)b^2x\sqrt[6]{a+bx^2}}{9a^2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{4b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2}{3\,3^{3/4}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{2\left(1-\sqrt{3}\right)b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{}}{\sqrt[3]{a}-(1+\sqrt{}}\right.\right.}{9\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(5/6)/a/x^3+4/9*b*(b*x^2+a)^(5/6)/a^2/x+4/9*(1+3^(1/2))*b^2
*x*(b*x^2+a)^(1/6)/a^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))+4/9*b*(b*x^2+
a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a
^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3)
)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(5/3)/x/(-(b*x^2+a)^(1/3)*(a
^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)+2/2
7*(1-3^(1/2))*b*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3
)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2
)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1
/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)
/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2
+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = -\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(1/6)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 1/6, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 757, normalized size of antiderivative = 1.25, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{4b\int \frac{1}{x^2 \sqrt[6]{bx^2+a}}dx}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{4b\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

$$\Big\downarrow \text{235}$$

$$-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

$$\Big\downarrow \text{214}$$

$$-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

$$\Big\downarrow \text{233}$$

$$-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{(a+bx^2)^{5/6}}{ax}\right)}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}$$

$\Big\downarrow$ 833

$$-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left((1+\sqrt{3})\int \frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}\right.}{9a}$$

$$\frac{(a+bx^2)^{5/6}}{3ax^3}$$

$\Big\downarrow$ 760

$$-\frac{4b\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{\ldots}{a+\ldots}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\ldots\right.}}\ldots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\ldots}}\right.\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\ldots\right)}{3a}\ldots\right)}{9a}\ldots$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

$\downarrow$ 2418

$$-\frac{4b\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{\cdots}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{\cdots}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(1/6)),x]
```

output
```
-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4*b*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*(
(3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)
))/(9*a)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^4 (b x^2 + a)^{\frac{1}{6}}} dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/6),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{1}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[6]{a}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/6),x)
```

output
```
-hyper((-3/2, 1/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/6)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx=\int \frac{1}{x^4\,(b\,x^2+a)^{1/6}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/6)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/6)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[6]{a+bx^2}}\,dx=\frac{(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{b^2x^8+2ab\,x^6+a^2x^4}dx\right)a\,x^3-\sqrt{b\,x^2+a}}{4\,(b\,x^2+a)^{\frac{2}{3}}\,x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(1/6),x)
```

output
```
((a + b*x**2)**(2/3)*int((a + b*x**2)**(5/6)/(a**2*x**4 + 2*a*b*x**6 + b**
2*x**8),x)*a*x**3 - sqrt(a + b*x**2))/(4*(a + b*x**2)**(2/3)*x**3)
```

3.1104 $\int \frac{1}{x^6 \sqrt[6]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 636

$$\int \frac{1}{x^6 \sqrt[6]{a+bx^2}} \, dx$$

$$= -\frac{(a+bx^2)^{5/6}}{5ax^5} + \frac{2b(a+bx^2)^{5/6}}{9a^2x^3} - \frac{8b^2(a+bx^2)^{5/6}}{27a^3x} - \frac{8(1+\sqrt{3})\, b^3 x \sqrt[6]{a+bx^2}}{27a^3 \left(\sqrt[3]{a} - (1+\sqrt{3})\, \sqrt[3]{a+bx^2}\right)}$$

$$- \frac{8b^2 \sqrt[6]{a+bx^2} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}} E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right) | \frac{1}{4} (}{9\ 3^{3/4} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{4(1-\sqrt{3})\, b^2 \sqrt[6]{a+bx^2} \left(\sqrt[3]{a} - \sqrt[3]{a+bx^2}\right) \sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}} \operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-}{\sqrt[3]{a}-(1+}\right.\right.}{27\sqrt[4]{3} a^{8/3} x \sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/5*(b*x^2+a)^(5/6)/a/x^5+2/9*b*(b*x^2+a)^(5/6)/a^2/x^3-8/27*b^2*(b*x^2+a
)^(5/6)/a^3/x-8/27*(1+3^(1/2))*b^3*x*(b*x^2+a)^(1/6)/a^3/(a^(1/3)-(1+3^(1/
2))*(b*x^2+a)^(1/3))-8/27*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((
a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3
^(1/4)/a^(8/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3
^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-4/81*(1-3^(1/2))*b^2*(b*x^2+a)^(1/6)*(a^
(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/
(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(
1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1
/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(8/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^
2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^6\sqrt[6]{a+bx^2}}\,dx=-\frac{\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(1/6)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 1/6, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {264, 264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\int \frac{1}{x^4\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\left(-\frac{4b\int \frac{1}{x^2\sqrt[6]{bx^2+a}}dx}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$$\Big\downarrow \text{235}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$$\Big\downarrow \text{214}$$

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$$\Big\downarrow \ 233$$

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$\downarrow$ 833

$$-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+\cdots}}\cdots\right)}{3a}\cdots\right)}{9a}\cdots\right)}{3a}$$

$$\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}$$

$\downarrow$ 760

$$2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1\cdots}}{\left(-\sqrt[3]{1-\frac{\cdots}{a\cdots}}\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-\cdots}}\cdots\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}\cdots\right.}{3a\cdots}\cdots\right.}{9a\cdots}\cdots\right.$$

↓ 2418

$$2b\left(-\frac{\left(\cdots\right.}{4b\left(\frac{\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+\cdots}}\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+\cdots}}\cdots}\right.\right.\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\cdots}\right.\cdots}{2b\cdots}\right.\cdots}\right.\cdots$$

input
```
Int[1/(x^6*(a + b*x^2)^(1/6)),x]
```

output
```
-1/5*(a + b*x^2)^(5/6)/(a*x^5) - (2*b*(-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4
*b*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*
Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - S
qrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1
- (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2
)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*E
llipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a
 + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
 - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[
3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 -
 (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1
- Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*S
qrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b
*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)))/(9*a)))/(3*a)
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a \right)^{\frac{1}{6}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(1/6),x)
```

output
```
int(1/x^6/(b*x^2+a)^(1/6),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[6]{a + bx^2}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^6 \sqrt[6]{a + bx^2}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{6} \\ -\frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5 \sqrt[6]{a} x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(1/6),x)
```

output
```
-hyper((-5/2, 1/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/6)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[6]{a + bx^2}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{1}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{6}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/6)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx = \int \frac{1}{x^6\,(b\,x^2+a)^{1/6}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(1/6)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(1/6)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[6]{a+bx^2}}\,dx = \frac{(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{5}{6}}}{b^2x^{10}+2ab\,x^8+a^2x^6}dx\right)a\,x^5-\sqrt{b\,x^2+a}}{6\,(b\,x^2+a)^{\frac{2}{3}}\,x^5}$$

input
```
int(1/x^6/(b*x^2+a)^(1/6),x)
```

output
```
((a + b*x**2)**(2/3)*int((a + b*x**2)**(5/6)/(a**2*x**6 + 2*a*b*x**8 + b**
2*x**10),x)*a*x**5 - sqrt(a + b*x**2))/(6*(a + b*x**2)**(2/3)*x**5)
```

3.1105 $\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 317

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx=\frac{81a^2x\sqrt[6]{a+bx^2}}{128b^3}-\frac{9ax^3\sqrt[6]{a+bx^2}}{32b^2}+\frac{3x^5\sqrt[6]{a+bx^2}}{16b}$$
$$-\frac{81\,3^{3/4}a^{8/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{\cdots}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\cdots}}\right.\right.}{256b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
81/128*a^2*x*(b*x^2+a)^(1/6)/b^3-9/32*a*x^3*(b*x^2+a)^(1/6)/b^2+3/16*x^5*(
b*x^2+a)^(1/6)/b-81/256*3^(3/4)*a^(8/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)
^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2)
)*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*
2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.85 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.28

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \frac{3x\left(27a^3+15a^2bx^2-4ab^2x^4+8b^3x^6-27a^3\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\right.\right.}{128b^3\left(a+bx^2\right)^{5/6}}$$

input
```
Integrate[x^6/(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(27*a^3 + 15*a^2*b*x^2 - 4*a*b^2*x^4 + 8*b^3*x^6 - 27*a^3*(1 + (b*x^2
)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(128*b^3*(a +
b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.31, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {262, 262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \; 262$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\int \frac{x^4}{(bx^2+a)^{5/6}}dx}{16b}$$

$$\Big\downarrow \; 262$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int \frac{x^2}{(bx^2+a)^{5/6}}dx}{10b}\right)}{16b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(bx^2+a\right)^{5/6}}dx}{4b}\right)}{10b}\right)}{16b}$$

$$\Big\downarrow \text{236}$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}\right)}{16b}$$

$$\Big\downarrow \text{234}$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b}-\frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)}{10b}\right)}{16b}$$

$$\Big\downarrow \text{760}$$

$$\frac{3x^5\sqrt[6]{a+bx^2}}{16b} - \frac{15a\left(\frac{3x^3\sqrt[6]{a+bx^2}}{10b} - \frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}\,a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\cdots}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+\cdots\right.}}}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1\cdots}}{\left(-\sqrt[3]{1-\cdots}\right.}}}\cdots\right)}{10b}\cdots\right)}{16b}\cdots$$

input

```
Int[x^6/(a + b*x^2)^(5/6),x]
```

output

```
(3*x^5*(a + b*x^2)^(1/6))/(16*b) - (15*a*((3*x^3*(a + b*x^2)^(1/6))/(10*b)
 - (9*a*((3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sq
rt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2)
)^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^
3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(10*b)))/(16*b)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{5}{6}}}dx$$

input
```
int(x^6/(b*x^2+a)^(5/6),x)
```

output
```
int(x^6/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{5}{6}}}$$

input
```
integrate(x**6/(b*x**2+a)**(5/6),x)
```

output
```
x**7*hyper((5/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/6))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/6), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{5/6}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(5/6),x)
```

output
```
int(x^6/(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{5}{6}}}dx$$

input
```
int(x^6/(b*x^2+a)^(5/6),x)
```

output
```
int(x**6/(a + b*x**2)**(5/6),x)
```

3.1106 $\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 293

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = -\frac{27ax\sqrt[6]{a+bx^2}}{40b^2}+\frac{3x^3\sqrt[6]{a+bx^2}}{10b} + \frac{27\,3^{3/4}a^{5/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{80b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output

```
-27/40*a*x*(b*x^2+a)^(1/6)/b^2+3/10*x^3*(b*x^2+a)^(1/6)/b+27/80*3^(3/4)*a^
(5/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a
)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*In
verseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(
a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.73 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \frac{3\left(-9a^2x-5abx^3+4b^2x^5+9a^2x\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{40b^2(a+bx^2)^{5/6}}$$

input
```
Integrate[x^4/(a + b*x^2)^(5/6),x]
```

output
```
(3*(-9*a^2*x - 5*a*b*x^3 + 4*b^2*x^5 + 9*a^2*x*(1 + (b*x^2)/a)^(5/6)*Hyper
geometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)]))/(40*b^2*(a + b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.30 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {262, 262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\int \frac{x^2}{(bx^2+a)^{5/6}}dx}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{(bx^2+a)^{5/6}}dx}{4b}\right)}{10b}$$

$$\Big\downarrow \text{236}$$

$$\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int\frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}\right)}{10b}$$

$\downarrow$ 234

$$\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int\frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}\right)}{10b}$$

$\downarrow$ 760

$$\frac{3x^3\sqrt[6]{a+bx^2}}{10b}-\frac{9a\left(\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3\,3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\dots\right.\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right)}{10b}$$

input
```
Int[x^4/(a + b*x^2)^(5/6),x]
```

output
```
(3*x^3*(a + b*x^2)^(1/6))/(10*b) - (9*a*((3*x*(a + b*x^2)^(1/6))/(4*b) - (
3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/
6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 -
(b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)
)^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 -
 Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a
/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x
^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)
])))/(10*b)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{5}{6}}}dx$$

input
```
int(x^4/(b*x^2+a)^(5/6),x)
```

output
```
int(x^4/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}}}$$

input
```
integrate(x**4/(b*x**2+a)**(5/6),x)
```

output
```
x**5*hyper((5/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/6), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{5/6}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(5/6),x)
```

output
```
int(x^4/(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{5}{6}}}\,dx$$

input
```
int(x^4/(b*x^2+a)^(5/6),x)
```

output
```
int(x**4/(a + b*x**2)**(5/6),x)
```

3.1107 $\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 271

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \frac{3x\sqrt[6]{a+bx^2}}{4b} - \frac{3\,3^{3/4}a^{2/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a}\cdots}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a}\cdots}\right.\right.}{8b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/4*x*(b*x^2+a)^(1/6)/b-3/8*3^(3/4)*a^(2/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^
2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(
1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+
1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.95 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.23

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \frac{3x\left(a+bx^2-a\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{4b(a+bx^2)^{5/6}}$$

input
```
Integrate[x^2/(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2 - a*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2,
 -((b*x^2)/a)]))/(4*b*(a + b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {262, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \ 262$$

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{(bx^2+a)^{5/6}}dx}{4b}$$

$$\Big\downarrow \ 236$$

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-\frac{3a\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{4b\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \ 234$$

$$\frac{9a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{8b^2x\sqrt[3]{\frac{a}{a+bx^2}}}+\frac{3x\sqrt[6]{a+bx^2}}{4b}$$

$\Big\downarrow$ 760

$$\frac{3x\sqrt[6]{a+bx^2}}{4b}-$$

$$\frac{3\ 3^{3/4}\sqrt{2-\sqrt{3}}a\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{\ldots}}{-\sqrt[3]{\ldots}}\right.\right.}{4b^2x\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

input
```
Int[x^2/(a + b*x^2)^(5/6),x]
```

output
```
(3*x*(a + b*x^2)^(1/6))/(4*b) - (3*3^(3/4)*Sqrt[2 - Sqrt[3]]*a*Sqrt[-((b*x
^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*
Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3]
- (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
))], -7 + 4*Sqrt[3]])/(4*b^2*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*
x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{6}}}dx$$

input
```
int(x^2/(b*x^2+a)^(5/6),x)
```

output
```
int(x^2/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{5}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}}$$

input
```
integrate(x**2/(b*x**2+a)**(5/6),x)
```

output
```
x**3*hyper((5/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/6), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{5/6}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(5/6),x)
```

output
```
int(x^2/(a + b*x^2)^(5/6), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{5/6}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{5}{6}}}dx$$

input
```
int(x^2/(b*x^2+a)^(5/6),x)
```

output
```
int(x**2/(a + b*x**2)**(5/6),x)
```

3.1108 $\int \frac{1}{(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 251

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{3^{3/4}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\cdots\right.\right.}{2\sqrt[3]{a}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
1/2*3^(3/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b
*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1
/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-
(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/a^(1/3)/b/x/(-(b*x^
2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))
^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.18

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/6}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/6}}$$

input
```
Integrate[(a + b*x^2)^(-5/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[1/2, 5/6, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/6)
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \text{236}$$

$$\frac{\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}$$

$$\Big\downarrow \text{234}$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2bx\sqrt[3]{\frac{a}{a+bx^2}}}$$

$$\Big\downarrow 760$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1\cdots}}{-\sqrt[3]{1\cdots}}\right.\right.}{bx\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

input
```
Int[(a + b*x^2)^(-5/6),x]
```

output
```
(3^(3/4)*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(b*x*(a/(a + b
*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a
+ b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{6}}}d x$$

input
```
int(1/(b*x^2+a)^(5/6),x)
```

output
```
int(1/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-5/6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.10

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{5}{6} \\ \frac{3}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(5/6),x)
```

output
```
x*hyper((1/2, 5/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.15

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/6}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/6}}$$

input
```
int(1/(a + b*x^2)^(5/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/6)*hypergeom([1/2, 5/6], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/6)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/6}}\,dx = \int \frac{(b\,x^2+a)^{\frac{2}{3}}}{\sqrt{b\,x^2+a}\,a+\sqrt{b\,x^2+a}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(5/6),x)
```

output
```
int((a + b*x**2)**(2/3)/(sqrt(a + b*x**2)*a + sqrt(a + b*x**2)*b*x**2),x)
```

3.1109 $\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 266

$$\int \frac{1}{x^2\left(a+bx^2\right)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{ax} - \frac{\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right),\right.}{\sqrt[4]{3}a^{4/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-(b*x^2+a)^(1/6)/a/x-1/3*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/
3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)
^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/
3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4
)/a^(4/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.18

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/6} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{6},\frac{1}{2},-\frac{bx^2}{a}\right)}{x(a+bx^2)^{5/6}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(5/6)),x]
```

output
```
-(((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-1/2, 5/6, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(5/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{2b\int \frac{1}{(bx^2+a)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{236}$$

$$-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{234}$$

$$\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}$$

$$\Big\downarrow \text{760}$$

$$-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\dots}}{-\sqrt[3]{1-\dots}}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{(a+bx^2)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}$$

$$\frac{\sqrt[6]{a+bx^2}}{ax}$$

input
```
Int[1/(x^2*(a + b*x^2)^(5/6)),x]
```

output
```
-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^
2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2
/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2
)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*S
qrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2
)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2
)/(a + b*x^2))^(1/3))^2)])
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{5}{6}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(5/6),x)
```

output
```
int(1/x^2/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{6} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{6}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(5/6),x)
```

output
```
-hyper((-1/2, 5/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/6)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.15

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = -\frac{3\left(\frac{a}{b\,x^2}+1\right)^{5/6}\,{}_2\mathrm{F}_1\left(\frac{5}{6},\frac{4}{3};\ \frac{7}{3};\ -\frac{a}{b\,x^2}\right)}{8\,x\,(b\,x^2+a)^{5/6}}$$

input
```
int(1/(x^2*(a + b*x^2)^(5/6)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(5/6)*hypergeom([5/6, 4/3], 7/3, -a/(b*x^2)))/(8*x*(a
+ b*x^2)^(5/6))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/6}}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}}-2(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^4+2ab\,x^2+a^2}dx\right)bx}{(b\,x^2+a)^{\frac{2}{3}}\,ax}$$

input
```
int(1/x^2/(b*x^2+a)^(5/6),x)
```

output
```
( - (a + b*x**2)**(5/6) - 2*(a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a
**2 + 2*a*b*x**2 + b**2*x**4),x)*b*x)/((a + b*x**2)**(2/3)*a*x)
```

3.1110 $\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 293

$$\int \frac{1}{x^4(a+bx^2)^{5/6}}\,dx = -\frac{\sqrt[6]{a+bx^2}}{3ax^3}+\frac{8b\sqrt[6]{a+bx^2}}{9a^2x} + \frac{8b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{9\sqrt[4]{3}a^{7/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/3*(b*x^2+a)^(1/6)/a/x^3+8/9*b*(b*x^2+a)^(1/6)/a^2/x+8/27*b*(b*x^2+a)^(1
/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^
(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arcc
os((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1
/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(7/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3
)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.17

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}} \, dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/6} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{5/6}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(5/6)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-3/2, 5/6, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.29 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.31, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}} \, dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{8b\int \frac{1}{x^2 (bx^2+a)^{5/6}}dx}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{264}$$

$$-\frac{8b\left(-\frac{2b\int \frac{1}{(bx^2+a)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{236}$$

$$-\frac{8b\left(-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 234

$$-\frac{8b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

$\Big\downarrow$ 760

$$-\frac{8b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\cdots}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\cdots\right.}{9a}\cdots$$

$$\frac{\sqrt[6]{a+bx^2}}{3ax^3}$$

input
```
Int[1/(x^4*(a + b*x^2)^(5/6)),x]
```

output
```
-1/3*(a + b*x^2)^(1/6)/(a*x^3) - (8*b*(-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqr
t[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a + b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b
*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*
x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[
ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 -
 (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2
))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*
x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(9*a)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{5}{6}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(5/6),x)
```

output
```
int(1/x^4/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{6}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(5/6),x)
```

output
```
-hyper((-3/2, 5/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/6)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}} x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{x^4 (b\,x^2+a)^{5/6}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(5/6)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(5/6)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/6}}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}} - 4(b\,x^2+a)^{\frac{2}{3}} \left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^6+2ab\,x^4+a^2x^2}dx\right) b\,x^3}{3\,(b\,x^2+a)^{\frac{2}{3}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(5/6),x)
```

output
```
( - (a + b*x**2)**(5/6) - 4*(a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a
**2*x**2 + 2*a*b*x**4 + b**2*x**6),x)*b*x**3)/(3*(a + b*x**2)**(2/3)*a*x**
3)
```

3.1111 $\int \frac{1}{x^6 \left(a+bx^2\right)^{5/6}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 319

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/6}} \, dx = -\frac{\sqrt[6]{a+bx^2}}{5ax^5} + \frac{14b\sqrt[6]{a+bx^2}}{45a^2x^3} - \frac{112b^2\sqrt[6]{a+bx^2}}{135a^3x} - \frac{112b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+\left(a+bx^2\right)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\ \text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+b}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+b}}\right.\right.}{135\sqrt[4]{3}a^{10/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-1/5*(b*x^2+a)^(1/6)/a/x^5+14/45*b*(b*x^2+a)^(1/6)/a^2/x^3-112/135*b^2*(b*
x^2+a)^(1/6)/a^3/x-112/405*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*(
(a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*
x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+
a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*
3^(3/4)/a^(10/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1
+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.16

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/6} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 (a+bx^2)^{5/6}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(5/6)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(5/6)*Hypergeometric2F1[-5/2, 5/6, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(5/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {264, 264, 264, 236, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}}\,dx$$

$$\Big\downarrow \text{264}$$

$$-\frac{14b\int \frac{1}{x^4(bx^2+a)^{5/6}}dx}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{14b\left(-\frac{8b\int \frac{1}{x^2\left(bx^2+a\right)^{5/6}}dx}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{264}$$

$$-\frac{14b\left(-\frac{8b\left(-\frac{2b\int \frac{1}{\left(bx^2+a\right)^{5/6}}dx}{3a}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

$\downarrow$ 236

$$-\frac{14b\left(-\frac{8b\left(-\frac{2b\int \frac{1}{\left(1-\frac{bx^2}{bx^2+a}\right)^{2/3}}d\frac{x}{\sqrt{bx^2+a}}}{3a\sqrt[3]{\frac{a}{a+bx^2}}\sqrt[3]{a+bx^2}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

$\downarrow$ 234

$$-\frac{14b\left(-\frac{8b\left(\frac{\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{ax\sqrt[3]{\frac{a}{a+bx^2}}}-\frac{\sqrt[6]{a+bx^2}}{ax}\right)}{9a}-\frac{\sqrt[6]{a+bx^2}}{3ax^3}\right)}{15a}-\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

$\downarrow$ 760

$$-\frac{14b\left(-\frac{8b\left(-\frac{2\sqrt{2-\sqrt{3}}\sqrt{-\frac{bx^2}{a+bx^2}}\sqrt[6]{a+bx^2}\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}{-\sqrt[3]{1-\frac{bx^2}{bx^2+}}}\right.\right.}{\sqrt[4]{3}ax\sqrt[3]{\frac{a}{a+bx^2}}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{9a}\right.}{15a}$$

$$\frac{\sqrt[6]{a+bx^2}}{5ax^5}$$

input
```
Int[1/(x^6*(a + b*x^2)^(5/6)),x]
```

output
```
-1/5*(a + b*x^2)^(1/6)/(a*x^5) - (14*b*(-1/3*(a + b*x^2)^(1/6)/(a*x^3) - (
8*b*(-((a + b*x^2)^(1/6)/(a*x)) - (2*Sqrt[2 - Sqrt[3]]*Sqrt[-((b*x^2)/(a +
 b*x^2))]*(a + b*x^2)^(1/6)*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1
+ x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (
b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)
/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7
+ 4*Sqrt[3]])/(3^(1/4)*a*x*(a/(a + b*x^2))^(1/3)*Sqrt[-1 + x^3/(a + b*x^2)
^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (
b*x^2)/(a + b*x^2))^(1/3))^2)])))/(9*a)))/(15*a)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 236
```
Int[((a_) + (b_.)*(x_)^2)^(-5/6), x_Symbol] :> Simp[1/((a/(a + b*x^2))^(1/3
)*(a + b*x^2)^(1/3))   Subst[Int[1/(1 - b*x^2)^(2/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{5}{6}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(5/6),x)
```

output
```
int(1/x^6/(b*x^2+a)^(5/6),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/6)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}} \, dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{6} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{5}{6}} x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(5/6),x)
```

output
```
-hyper((-5/2, 5/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/6)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{6}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/6)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}}\,dx = \int \frac{1}{x^6 (b\,x^2+a)^{5/6}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(5/6)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(5/6)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/6}}\,dx = \frac{-(b\,x^2+a)^{\frac{5}{6}} - 6(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{7}{6}}}{b^2x^8+2ab\,x^6+a^2x^4}dx\right)b\,x^5}{5\,(b\,x^2+a)^{\frac{2}{3}}\,a\,x^5}$$

input
```
int(1/x^6/(b*x^2+a)^(5/6),x)
```

output
```
( - (a + b*x**2)**(5/6) - 6*(a + b*x**2)**(2/3)*int((a + b*x**2)**(7/6)/(a
**2*x**4 + 2*a*b*x**6 + b**2*x**8),x)*b*x**5)/(5*(a + b*x**2)**(2/3)*a*x**
5)
```

3.1112 $\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 629

$$\begin{aligned}\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx &= -\frac{3x^5}{b\sqrt[6]{a+bx^2}} - \frac{405ax(a+bx^2)^{5/6}}{112b^3} \\ &+ \frac{45x^3(a+bx^2)^{5/6}}{14b^2} - \frac{1215\left(1+\sqrt{3}\right)a^2x\sqrt[6]{a+bx^2}}{224b^3\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)} \\ &- \frac{1215\sqrt[4]{3}a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx}}\right.\right.}{224b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}} \\ &- \frac{405\ 3^{3/4}\left(1-\sqrt{3}\right)a^{7/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\right.\right.}{448b^4x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}\end{aligned}$$

output
```
-3*x^5/b/(b*x^2+a)^(1/6)-405/112*a*x*(b*x^2+a)^(5/6)/b^3+45/14*x^3*(b*x^2+
a)^(5/6)/b^2-1215/224*(1+3^(1/2))*a^2*x*(b*x^2+a)^(1/6)/b^3/(a^(1/3)-(1+3^
(1/2))*(b*x^2+a)^(1/3))-1215/224*3^(1/4)*a^(7/3)*(b*x^2+a)^(1/6)*(a^(1/3)-
(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/
3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))
*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(
1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/
3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-405/448*3^(3/4)*(1-3^(1/2))*a^(7/
3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(
1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*Inver
seJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^4/x/(-(b*x^2+a)^(1/3)*(a^(
1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.72 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.13

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \frac{405a^2x-90abx^3+48b^2x^5-405a^2x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{224b^3\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(7/6),x]
```

output
```
(405*a^2*x - 90*a*b*x^3 + 48*b^2*x^5 - 405*a^2*x*(1 + (b*x^2)/a)^(1/6)*Hyp
ergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(224*b^3*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {252, 262, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{7/6}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{15\int \frac{x^4}{\sqrt[6]{bx^2+a}}dx}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{15\left(\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{14b}\right)}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{15\left(\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}\right)}{14b}\right)}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{235}$$

$$\frac{15\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int\frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{8b}\right)}{14b}\right)}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$\downarrow$ 214

$$\frac{15\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{8b}\right)}{14b}\right)}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$\downarrow$ 233

$$\frac{15\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)}{14b}\right)}{b}-\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$\downarrow$ 833

$$\frac{15\left(\frac{3x^3(a+bx^2)^{5/6}}{14b}-\frac{9a\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left((1+\sqrt{3})\int\frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{8b}\right.}{14b}\right.}{b}$$

$$\frac{3x^5}{b\sqrt[6]{a+bx^2}}$$

$\downarrow$ 760

$$2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1\right.$$

$$3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\right.$$

$$3a\,\overline{\qquad}$$

$$4bx\left(\frac{}{a+}\right.$$

$$9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\right.$$

$$15\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\right.$$

↓ 2418

$$15\left(\frac{3x^3\left(a+bx^2\right)^{5/6}}{14b}-\frac{9a\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{El}\ldots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\ldots}}{\left(-\sqrt[3]{1-\frac{\ldots}{a+\ldots}}\ldots\right.}}}\ldots\right.\right.}{\ldots}\right.}{\ldots}\right.$$

input
```
Int[x^6/(a + b*x^2)^(7/6),x]
```

output
```
(-3*x^5)/(b*(a + b*x^2)^(1/6)) + (15*((3*x^3*(a + b*x^2)^(5/6))/(14*b) - (
9*a*((3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*((3*x)/(2*(a + b*x^2)^(1/6)) + (
3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^
2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - S
qrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^
3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + S
qrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) +
(1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/
4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a
 + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b)))/(14*b)))/b
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{7}{6}}}dx$$

input
```
int(x^6/(b*x^2+a)^(7/6),x)
```

output
```
int(x^6/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.04

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \frac{x^7{}_2F_1\left(\left.\begin{matrix}\frac{7}{6},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{6}}}$$

input
```
integrate(x**6/(b*x**2+a)**(7/6),x)
```

output
```
x**7*hyper((7/6, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/6))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/6), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{7/6}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(7/6),x)
```

output
```
int(x^6/(a + b*x^2)^(7/6), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{7/6}}\,dx = \frac{-87(b\,x^2+a)^{\frac{2}{3}}\left(\int \frac{(b\,x^2+a)^{\frac{2}{3}}}{(b\,x^2+a)^{\frac{5}{6}}a+(b\,x^2+a)^{\frac{5}{6}}b\,x^2}dx\right)a^3+87\sqrt{b\,x^2+a}\,a^2x-62\sqrt{b\,x^2+a}\,ab}{32\,(b\,x^2+a)^{\frac{2}{3}}\,b^3}$$

input
```
int(x^6/(b*x^2+a)^(7/6),x)
```

output
```
( - 87*(a + b*x**2)**(2/3)*int((a + b*x**2)**(2/3)/((a + b*x**2)**(5/6)*a
+ (a + b*x**2)**(5/6)*b*x**2),x)*a**3 + 87*sqrt(a + b*x**2)*a**2*x - 62*sq
rt(a + b*x**2)*a*b*x**3 + 16*sqrt(a + b*x**2)*b**2*x**5)/(32*(a + b*x**2)*
*(2/3)*b**3)
```

3.1113 $\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 605

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = -\frac{3x^3}{b\sqrt[6]{a+bx^2}} + \frac{27x(a+bx^2)^{5/6}}{8b^2} + \frac{81\left(1+\sqrt{3}\right)ax\sqrt[6]{a+bx^2}}{16b^2\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{81\sqrt[4]{3}a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right.}{16b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{27\ 3^{3/4}\left(1-\sqrt{3}\right)a^{4/3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}}{\sqrt[3]{a}}\right.\right.}{32b^3x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*x^3/b/(b*x^2+a)^(1/6)+27/8*x*(b*x^2+a)^(5/6)/b^2+81/16*(1+3^(1/2))*a*x*
(b*x^2+a)^(1/6)/b^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))+81/16*3^(1/4)*a^
(4/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a
)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*El
lipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/b^3/x/(-(b*x^2+a)^(1/3)*(
a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)+27
/32*3^(3/4)*(1-3^(1/2))*a^(4/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*
((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2
+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))
/b^3/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.63 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \frac{3x\left(-9a+2bx^2+9a\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)\right)}{16b^2\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(7/6),x]
```

output
```
(3*x*(-9*a + 2*b*x^2 + 9*a*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/
6, 3/2, -((b*x^2)/a)]))/(16*b^2*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {252, 262, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{7/6}}dx$$

$$\Big\downarrow \text{252}$$

$$\frac{9\int \frac{x^2}{\sqrt[6]{bx^2+a}}dx}{b}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{262}$$

$$\frac{9\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{8b}\right)}{b}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{235}$$

$$\frac{9\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx\right)}{8b}\right)}{b}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{214}$$

$$\frac{9\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}\right)}{8b}\right)}{b}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{233}$$

$$\frac{9\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{8b}\right)}{b}-\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{9\left(\frac{3x(a+bx^2)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left((1+\sqrt{3})\int\frac{1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{(bx^2+a)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\right.}{8b}\right)}{b}$$

$$\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{760}$$

$$9\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}\cdots}{\left(-\sqrt[3]{\cdots}\right)\cdots}}\cdots}{\sqrt[4]{3}\sqrt{\frac{\cdots}{\left(a+b\cdots\right)\cdots}}\cdots}\cdots\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}\cdots\right)}{8b}\cdots}{b}\cdots$$

$$\frac{3x^3}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 2418$$

$$\frac{9\left(\frac{3x\left(a+bx^2\right)^{5/6}}{8b}-\frac{3a\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\operatorname{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-}}{-\sqrt[3]{1-}}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.\right.}{}\right.}{\frac{3x^3}{b\sqrt[6]{a+bx^2}}}$$

input
```
Int[x^4/(a + b*x^2)^(7/6),x]
```

output
```
(-3*x^3)/(b*(a + b*x^2)^(1/6)) + (9*((3*x*(a + b*x^2)^(5/6))/(8*b) - (3*a*
((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt
[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/
3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqr
t[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))]
, -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x
^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)
]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/
3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt
[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
(1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^
(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-
((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*
x^2))^(1/3))^2]))))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(8*b
)))/b
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{7}{6}}}dx$$

input
```
int(x^4/(b*x^2+a)^(7/6),x)
```

output
```
int(x^4/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.04

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{7}{6}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}}$$

input
```
integrate(x**4/(b*x**2+a)**(7/6),x)
```

output
```
x**5*hyper((7/6, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/6), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{7/6}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(7/6),x)
```

output
```
int(x^4/(a + b*x^2)^(7/6), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{1}{6}}\,a+(b\,x^2+a)^{\frac{1}{6}}\,b\,x^2}dx$$

input
```
int(x^4/(b*x^2+a)^(7/6),x)
```

output
```
int(x**4/((a + b*x**2)**(1/6)*a + (a + b*x**2)**(1/6)*b*x**2),x)
```

3.1114 $\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 583

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = -\frac{3x}{b\sqrt[6]{a+bx^2}} - \frac{9\left(1+\sqrt{3}\right)x\sqrt[6]{a+bx^2}}{2b\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{9\sqrt[4]{3}\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)\right|}{2b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{3\,3^{3/4}\left(1-\sqrt{3}\right)\sqrt[3]{a}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-}{\sqrt[3]{a}-}\right.\right.}{4b^2x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
-3*x/b/(b*x^2+a)^(1/6)-9/2*(1+3^(1/2))*x*(b*x^2+a)^(1/6)/b/(a^(1/3)-(1+3^(
1/2))*(b*x^2+a)^(1/3))-9/2*3^(1/4)*a^(1/3)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2
+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+
3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^
2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+
3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-3/4*3^(3/4)*(1-3^(1/2))*a^(1/3)*(b*x^2+
a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^
2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM
(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+
a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))/b^2/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^
2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.10

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx=\frac{3x-3x\sqrt[6]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{2b\sqrt[6]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(7/6),x]
```

output
```
(3*x - 3*x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2
)/a)])/(2*b*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {252, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx$$

$$\Big\downarrow \; 252$$

$$\frac{3\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{b}-\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 235$$

$$\frac{3\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{(bx^2+a)^{7/6}}dx\right)}{b}-\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 214$$

$$\frac{3\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}\right)}{b}-\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 233$$

$$\frac{3\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{b}-\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 833$$

$$\frac{3\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{b}$$

$$\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$\big\downarrow$ 760

$$\frac{3\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\cdots}{\left(-\sqrt[3]{1\cdots}\right.}}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{b}$$

$$\frac{3x}{b\sqrt[6]{a+bx^2}}$$

$\big\downarrow$ 2418

$$3\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1}\right),\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{4}\right.$$

$$\frac{3x}{b\sqrt[6]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(7/6),x]
```

output
```
(-3*x)/(b*(a + b*x^2)^(1/6)) + (3*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt
[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellip
ticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b
*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2
))^(2/3)*(a + b*x^2)^(1/6))))/b
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{7}{6}}}dx$$

input
```
int(x^2/(b*x^2+a)^(7/6),x)
```

output
```
int(x^2/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{7}{6}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}}$$

input
```
integrate(x**2/(b*x**2+a)**(7/6),x)
```

output
```
x**3*hyper((7/6, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/6), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{7/6}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(7/6),x)
```

output
```
int(x^2/(a + b*x^2)^(7/6), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{(a+bx^2)^{7/6}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)x^2}{(b\,x^2+a)^{\frac{1}{6}}\,ab}$$

input
```
int(x^2/(b*x^2+a)^(7/6),x)
```

output
```
(sqrt(b)*sqrt(a)*(a + b*x**2)**(5/6)*atan((b*x)/(sqrt(b)*sqrt(a)))*x**2)/(
a*b*(a + b*x**2))
```

3.1115 $\int \frac{1}{(a+bx^2)^{7/6}} dx$

Optimal result

Integrand size = 11, antiderivative size = 579

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{3x}{a\sqrt[6]{a+bx^2}} + \frac{3\left(1+\sqrt{3}\right)x\sqrt[6]{a+bx^2}}{a\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{3\sqrt[4]{3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2\cdots}{a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{3^{3/4}\left(1-\sqrt{3}\right)\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\cdots}{\sqrt[3]{a}-\left(1+\sqrt{3}\cdots}\right.\right.}{2a^{2/3}bx\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3*x/a/(b*x^2+a)^(1/6)+3*(1+3^(1/2))*x*(b*x^2+a)^(1/6)/a/(a^(1/3)-(1+3^(1/2
))*(b*x^2+a)^(1/3))+3*3^(1/4)*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((
a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2
/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/a
^(2/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2)
)*(b*x^2+a)^(1/3))^2)^(1/2)+1/2*3^(3/4)*(1-3^(1/2))*(b*x^2+a)^(1/6)*(a^(1/
3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^
(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3
)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*
6^(1/2)+1/4*2^(1/2))/a^(2/3)/b/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.08

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx=\frac{x\sqrt[6]{1+\frac{bx^2}{a}}\ \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[6]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-7/6),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[1/2, 7/6, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx$$

$$\Big\downarrow \; 214$$

$$\frac{\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{\left(\frac{a}{a+bx^2}\right)^{2/3}(a+bx^2)^{2/3}}$$

$$\Big\downarrow \; 233$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 833$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 760$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+b\cdots}}\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{\cdots}}\right.}{2bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 2418$$

$$-\frac{3\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{\cdots}$$

input

```
Int[(a + b*x^2)^(-7/6),x]
```

output

```
(-3*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1
 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]
*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b
*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^
2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - S
qrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^
3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqr
t[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + S
qrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) +
(1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^
(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))
/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/
4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^
(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(2*b*x*(a/(a
 + b*x^2))^(2/3)*(a + b*x^2)^(1/6))
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{7}{6}}}dx$$

input
```
int(1/(b*x^2+a)^(7/6),x)
```

output
```
int(1/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.04

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{6}\\\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}}$$

input
```
integrate(1/(b*x**2+a)**(7/6),x)
```

output
```
x*hyper((1/2, 7/6), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/6)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/6), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/6), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/6}{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/6}}$$

input
```
int(1/(a + b*x^2)^(7/6),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/6)*hypergeom([1/2, 7/6], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/6)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int \frac{1}{(a+bx^2)^{7/6}}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right)}{(b\,x^2+a)^{\frac{1}{6}}\,ab}$$

input
```
int(1/(b*x^2+a)^(7/6),x)
```

output
```
(sqrt(b)*sqrt(a)*(a + b*x**2)**(5/6)*atan((b*x)/(sqrt(b)*sqrt(a))))/(a*b*(
a + b*x**2))
```

3.1116 $\int \frac{1}{x^2(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 593

$$\int \frac{1}{x^2(a+bx^2)^{7/6}}\,dx = \frac{3}{ax\sqrt[6]{a+bx^2}} - \frac{4(a+bx^2)^{5/6}}{a^2x} - \frac{4(1+\sqrt{3})\,bx\sqrt[6]{a+bx^2}}{a^2\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)}$$

$$-\frac{4\sqrt[4]{3}\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2\ldots}{a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$-\frac{2(1-\sqrt{3})\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{\ldots}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{\ldots}}\right.\right.}{\sqrt[4]{3}a^{5/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/a/x/(b*x^2+a)^(1/6)-4*(b*x^2+a)^(5/6)/a^2/x-4*(1+3^(1/2))*b*x*(b*x^2+a)^
(1/6)/a^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))-4*3^(1/4)*(b*x^2+a)^(1/6)*
(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1
-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2
),1/4*6^(1/2)+1/4*2^(1/2))/a^(5/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^
(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-2/3*(1-3^(1/2))*(b*x
^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b
*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacob
iAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x
^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(5/3)/x/(-(b*x^2+a)^(1/3)
*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 7.46 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2 (a+bx^2)^{7/6}} \, dx = -\frac{\sqrt[6]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{6},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(7/6)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-1/2, 7/6, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(1/6)))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 752, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {253, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{7/6}}\,dx$$

$$\Big\downarrow \; 253$$

$$\frac{4\int \frac{1}{x^2 \sqrt[6]{bx^2+a}}dx}{a}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 264$$

$$\frac{4\left(\frac{2b\int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{a}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 235$$

$$\frac{4\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int \frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{a}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 214$$

$$\frac{4\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{a}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 233$$

$$\frac{4\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int \frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{a}+\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$\big\downarrow$ 833

$$\frac{4\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int \frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\right)}{a}$$

$$\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$\big\downarrow$ 760

$$4\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+b\cdots}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\cdots\right.}}}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{\cdots}{\left(-\sqrt[3]{\cdots}\right.}}}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}\right.}{3a}\right.}{a}$$

$$\frac{3}{ax\sqrt[6]{a+bx^2}}$$

$\downarrow$ 2418

$$\frac{4\left(\frac{2b\left(3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.\right.}{\cdots}\right)}{\cdots}$$

$$\frac{3}{ax\sqrt[6]{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(7/6)),x]
```

output
```
3/(a*x*(a + b*x^2)^(1/6)) + (4*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x)/(
2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^
3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3
^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x
^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x
^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4
*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a +
 b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*
Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1
 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1
/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)))/a
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^2\left(b\,x^2+a\right)^{\frac{7}{6}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(7/6),x)
```

output
```
int(1/x^2/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{7}{6}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\left(a+bx^2\right)^{7/6}}\,dx=-\frac{{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{6}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{6}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(7/6),x)
```

output
```
-hyper((-1/2, 7/6), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/6)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + bx^2)^{7/6}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a + bx^2)^{7/6}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{6}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2 (a + bx^2)^{7/6}} \, dx = -\frac{3 \left(\frac{a}{b\,x^2} + 1\right)^{7/6} {}_2F_1\left(\frac{7}{6}, \frac{5}{3};\ \frac{8}{3};\ -\frac{a}{b\,x^2}\right)}{10\,x\,(b\,x^2 + a)^{7/6}}$$

input
```
int(1/(x^2*(a + b*x^2)^(7/6)),x)
```

output
```
-(3*(a/(b*x^2) + 1)^(7/6)*hypergeom([7/6, 5/3], 8/3, -a/(b*x^2)))/(10*x*(a
 + b*x^2)^(7/6))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^2 (a+bx^2)^{7/6}} \, dx = \frac{-2\sqrt{b}\,\sqrt{a}\, atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) x - a}{(b\,x^2+a)^{\frac{1}{6}}\, a^2 x}$$

input
```
int(1/x^2/(b*x^2+a)^(7/6),x)
```

output
```
((a + b*x**2)**(5/6)*( - 2*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*x
 - a))/(a**2*x*(a + b*x**2))
```

3.1117 $\int \frac{1}{x^4(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 627

$$\int \frac{1}{x^4\left(a+bx^2\right)^{7/6}}\,dx = \frac{3}{ax^3\sqrt[6]{a+bx^2}} - \frac{10\left(a+bx^2\right)^{5/6}}{3a^2x^3}$$

$$+\frac{40b\left(a+bx^2\right)^{5/6}}{9a^3x} + \frac{40\left(1+\sqrt{3}\right)b^2x\sqrt[6]{a+bx^2}}{9a^3\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$+\frac{40b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(2-}{3\,3^{3/4}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

$$+\frac{20\left(1-\sqrt{3}\right)b\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-(1-\sqrt{3})}{\sqrt[3]{a}-(1+\sqrt{3})}\right.\right.}{9\sqrt[4]{3}a^{8/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/a/x^3/(b*x^2+a)^(1/6)-10/3*(b*x^2+a)^(5/6)/a^2/x^3+40/9*b*(b*x^2+a)^(5/6
)/a^3/x+40/9*(1+3^(1/2))*b^2*x*(b*x^2+a)^(1/6)/a^3/(a^(1/3)-(1+3^(1/2))*(b
*x^2+a)^(1/3))+40/9*b*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+
a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1
/3))^2)^(1/2)*EllipticE((1-(a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3
)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a
^(8/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*
(b*x^2+a)^(1/3))^2)^(1/2)+20/27*(1-3^(1/2))*b*(b*x^2+a)^(1/6)*(a^(1/3)-(b*
x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-
(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3
^(1/2))*(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2
)+1/4*2^(1/2))*3^(3/4)/a^(8/3)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3
))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}} \, dx = -\frac{\sqrt[6]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{6},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(7/6)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-3/2, 7/6, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 782, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {253, 264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{10 \int \frac{1}{x^4 \sqrt[6]{bx^2+a}}dx}{a}+\frac{3}{ax^3 \sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{10 \left(-\frac{4b \int \frac{1}{x^2 \sqrt[6]{bx^2+a}}dx}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}\right)}{a}+\frac{3}{ax^3 \sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{10 \left(-\frac{4b \left(\frac{2b \int \frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{(a+bx^2)^{5/6}}{ax}\right)}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}\right)}{a}+\frac{3}{ax^3 \sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{235}$$

$$\frac{10 \left(-\frac{4b \left(\frac{2b \left(\frac{3x}{2 \sqrt[6]{a+bx^2}}-\frac{1}{2} a \int \frac{1}{(bx^2+a)^{7/6}}dx\right)}{3a}-\frac{(a+bx^2)^{5/6}}{ax}\right)}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}\right)}{a}+\frac{3}{ax^3 \sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 214$$

$$\frac{10\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int \frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{a}+\frac{3}{ax^3\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \; 233$$

$$\frac{10\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3}+\frac{a}{ax^3\sqrt[6]{a+bx^2}}$$

$\downarrow$ 833

$$10\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+b}}\right.}{3a}\right.}{9a}\right.}{a}$$

$$\frac{3}{ax^3\sqrt[6]{a+bx^2}}$$

$\downarrow$ 760

10 $$-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int \frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\cdots}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+\cdots}}\cdots\right.}}\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\cdots}\right.}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}\cdots}\cdots\right.}{3a\cdots}\cdots\right.}{9a\cdots}$$

↓ 2418

$$10 - \frac{\left(\frac{\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{1-\frac{bx^2}{a+bx^2}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\cdots}{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\cdots}\right.\right.}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}}{\left(-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}-\sqrt{3}+1\right)^2}}}\right.}{2b}\cdots\right)}{4b}\cdots\right)}{\cdots}$$

input
```
Int[1/(x^4*(a + b*x^2)^(7/6)),x]
```

output
```
3/(a*x^3*(a + b*x^2)^(1/6)) + (10*(-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4*b*(
-((a + b*x^2)^(5/6)/(a*x)) + (2*b*((3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt
[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1
 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a
 + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*Ellip
ticE[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] -
 (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b
*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2
]*EllipticF[ArcSin[(1 + Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sq
rt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(
1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2
))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)))/(9*a)))/a
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{7}{6}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(7/6),x)
```

output
```
int(1/x^4/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{7}{6} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{6}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(7/6),x)
```

output
```
-hyper((-3/2, 7/6), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/6)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^{7/6}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(7/6)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(7/6)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^4 (a+bx^2)^{7/6}} dx = \frac{8\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b\,x^3 - a^2 + 4ab\,x^2}{3\,(b\,x^2+a)^{\frac{1}{6}}\,a^3x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(7/6),x)
```

output
```
((a + b*x**2)**(5/6)*(8*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*b*x*
*3 - a**2 + 4*a*b*x**2))/(3*a**3*x**3*(a + b*x**2))
```

3.1118 $\int \frac{1}{x^6(a+bx^2)^{7/6}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 655

$$\int \frac{1}{x^6(a+bx^2)^{7/6}}\,dx = \frac{3}{ax^5\sqrt[6]{a+bx^2}} - \frac{16(a+bx^2)^{5/6}}{5a^2x^5} + \frac{32b(a+bx^2)^{5/6}}{9a^3x^3}$$

$$- \frac{128b^2(a+bx^2)^{5/6}}{27a^4x} - \frac{128\left(1+\sqrt{3}\right)b^3x\sqrt[6]{a+bx^2}}{27a^4\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)}$$

$$- \frac{128b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,E\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+bx^2}}{\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}}\right)|\frac{1}{4}(\cdots}{9\,3^{3/4}a^{11/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

$$- \frac{64\left(1-\sqrt{3}\right)b^2\sqrt[6]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)\sqrt{\frac{a^{2/3}+\sqrt[3]{a}\sqrt[3]{a+bx^2}+(a+bx^2)^{2/3}}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{a}-\left(1-\sqrt{\cdots}\right.}{\sqrt[3]{a}-\left(1+\sqrt{\cdots}\right.}\right.\right.}{27\sqrt[4]{3}a^{11/3}x\sqrt{-\frac{\sqrt[3]{a+bx^2}\left(\sqrt[3]{a}-\sqrt[3]{a+bx^2}\right)}{\left(\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+bx^2}\right)^2}}}$$

output
```
3/a/x^5/(b*x^2+a)^(1/6)-16/5*(b*x^2+a)^(5/6)/a^2/x^5+32/9*b*(b*x^2+a)^(5/6
)/a^3/x^3-128/27*b^2*(b*x^2+a)^(5/6)/a^4/x-128/27*(1+3^(1/2))*b^3*x*(b*x^2
+a)^(1/6)/a^4/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))-128/27*b^2*(b*x^2+a)^(
1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3)*(b*x^2+a)^(1/3)+(b*x^2+a)
^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)*EllipticE((1-(a^(1/
3)-(1-3^(1/2))*(b*x^2+a)^(1/3))^2/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)
^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*3^(1/4)/a^(11/3)/x/(-(b*x^2+a)^(1/3)*(a^(1
/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2)^(1/2)-64/81*
(1-3^(1/2))*b^2*(b*x^2+a)^(1/6)*(a^(1/3)-(b*x^2+a)^(1/3))*((a^(2/3)+a^(1/3
)*(b*x^2+a)^(1/3)+(b*x^2+a)^(2/3))/(a^(1/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))^2
)^(1/2)*InverseJacobiAM(arccos((a^(1/3)-(1-3^(1/2))*(b*x^2+a)^(1/3))/(a^(1
/3)-(1+3^(1/2))*(b*x^2+a)^(1/3))),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/a^(11/3
)/x/(-(b*x^2+a)^(1/3)*(a^(1/3)-(b*x^2+a)^(1/3))/(a^(1/3)-(1+3^(1/2))*(b*x^
2+a)^(1/3))^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.08

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx = -\frac{\sqrt[6]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{6},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5\sqrt[6]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(7/6)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/6)*Hypergeometric2F1[-5/2, 7/6, -3/2, -((b*x^2)/a
)])/(a*x^5*(a + b*x^2)^(1/6))
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {253, 264, 264, 264, 235, 214, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx$$

$$\Big\downarrow \text{253}$$

$$\frac{16\int \frac{1}{x^6 \sqrt[6]{bx^2+a}}dx}{a}+\frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{16\left(-\frac{2b\int \frac{1}{x^4 \sqrt[6]{bx^2+a}}dx}{3a}-\frac{(a+bx^2)^{5/6}}{5ax^5}\right)}{a}+\frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{16\left(-\frac{2b\left(-\frac{4b\int \frac{1}{x^2 \sqrt[6]{bx^2+a}}dx}{9a}-\frac{(a+bx^2)^{5/6}}{3ax^3}\right)}{3a}-\frac{(a+bx^2)^{5/6}}{5ax^5}\right)}{a}+\frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{16\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\int\frac{1}{\sqrt[6]{bx^2+a}}dx}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)}{a}+\frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

$\Big\downarrow$ 235

$$\frac{16\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{1}{2}a\int\frac{1}{\left(bx^2+a\right)^{7/6}}dx\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)}{a}+\frac{3}{ax^5\sqrt[6]{a+bx^2}}$$

$\Big\downarrow$ 214

$$\frac{16\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3x}{2\sqrt[6]{a+bx^2}}-\frac{a\int\frac{1}{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}d\frac{x}{\sqrt{bx^2+a}}}{2\left(\frac{a}{a+bx^2}\right)^{2/3}\left(a+bx^2\right)^{2/3}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right)}{\frac{3^{a}}{ax^5\sqrt[6]{a+bx^2}}}+$$

↓ 233

$$16\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\int\frac{\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{3x}{2\sqrt[6]{a+bx^2}}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{ax}\right)}{9a}-\frac{\left(a+bx^2\right)^{5/6}}{3ax^3}\right)}{3a}-\frac{\left(a+bx^2\right)^{5/6}}{5ax^5}\right.$$

↓ 833

$$16\left(-\frac{2b\left(-\frac{4b\left(\frac{2b\left(\frac{3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)}{4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}}+\frac{\ldots}{2\sqrt[6]{a\ldots}}\right.}{3a}\right.}{9a}\right.}{3a}\right.$$

↓ 760

$$2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(1-\sqrt[3]{1-\frac{bx^2}{a+bx^2}}\right)\sqrt{\frac{\frac{x^2}{a+bx^2}+\sqrt[3]{\cdots}}{\left(-\sqrt[3]{1-\cdots}\right.}}$$

$$3a\sqrt{-\frac{bx^2}{a+bx^2}}\left(-\int\frac{-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+\sqrt{3}+1}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}}\,d\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\frac{\cdots}{\sqrt[4]{3}\sqrt{\frac{x^3}{\left(a+bx^2\right)^{3/\cdots}}}}\right.$$

$$2b \qquad 4bx\left(\frac{a}{a+bx^2}\right)^{2/3}\sqrt[6]{a+bx^2}$$

$$4b \qquad 3a$$

$$2b \quad - \qquad 9a$$

↓ 2418

$$\left(2b\left(-\frac{\left(4b\left(2b\left(\frac{3x}{2\sqrt[6]{bx^2+a}}+\frac{3a\sqrt{-\frac{bx^2}{bx^2+a}}\left(\frac{\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}\right)\sqrt{\frac{\frac{x^2}{bx^2+a}+\sqrt[3]{1-\frac{bx^2}{bx^2+a}}+1}{\left(-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1\right)^2}}E\left(\arcsin\left(\frac{-\sqrt[3]{1-\cdots}}{-\sqrt[3]{1-\cdots}}\right.\right.}{\sqrt{\frac{x^3}{\left(bx^2+a\right)^{3/2}}-1}\sqrt{-\frac{1-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}}{\left(-\sqrt[3]{1-\frac{bx^2}{bx^2+a}}-\sqrt{3}+1\right)^2}}}\right.\right.\right.}{\cdots}\right.\right.$$

input
```
Int[1/(x^6*(a + b*x^2)^(7/6)),x]
```

output
```
3/(a*x^5*(a + b*x^2)^(1/6)) + (16*(-1/5*(a + b*x^2)^(5/6)/(a*x^5) - (2*b*(
-1/3*(a + b*x^2)^(5/6)/(a*x^3) - (4*b*(-((a + b*x^2)^(5/6)/(a*x)) + (2*b*(
(3*x)/(2*(a + b*x^2)^(1/6)) + (3*a*Sqrt[-((b*x^2)/(a + b*x^2))]*((-2*Sqrt[
-1 + x^3/(a + b*x^2)^(3/2)])/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3
)) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))*Sqrt
[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1
 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - (1 - (b*
x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))],
 -7 + 4*Sqrt[3]])/(Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-((1 - (1 - (b*x^
2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2)]
) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3
))*Sqrt[(1 + x^2/(a + b*x^2) + (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[
3] - (1 - (b*x^2)/(a + b*x^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - (
1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x^2))^(
1/3))], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-1 + x^3/(a + b*x^2)^(3/2)]*Sqrt[-(
(1 - (1 - (b*x^2)/(a + b*x^2))^(1/3))/(1 - Sqrt[3] - (1 - (b*x^2)/(a + b*x
^2))^(1/3))^2)])))/(4*b*x*(a/(a + b*x^2))^(2/3)*(a + b*x^2)^(1/6))))/(3*a)
))/(9*a)))/(3*a)))/a
```

Defintions of rubi rules used

rule 214
```
Int[((a_) + (b_.)*(x_)^2)^(-7/6), x_Symbol] :> Simp[1/((a + b*x^2)^(2/3)*(a
/(a + b*x^2))^(2/3))   Subst[Int[1/(1 - b*x^2)^(1/3), x], x, x/Sqrt[a + b*x
^2]], x] /; FreeQ[{a, b}, x]
```

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 235
```
Int[((a_) + (b_.)*(x_)^2)^(-1/6), x_Symbol] :> Simp[3*(x/(2*(a + b*x^2)^(1/
6))), x] - Simp[a/2   Int[1/(a + b*x^2)^(7/6), x], x] /; FreeQ[{a, b}, x]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{x^6 \left(b\,x^2+a\right)^{\frac{7}{6}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(7/6),x)
```

output
```
int(1/x^6/(b*x^2+a)^(7/6),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{7/6}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/6)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{7/6}}\,dx = -\frac{{}_2F_1\left(\left.\begin{matrix} -\frac{5}{2}, \frac{7}{6} \\ -\frac{3}{2} \end{matrix}\right| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{6}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(7/6),x)
```

output
```
-hyper((-5/2, 7/6), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/6)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{7}{6}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/6),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/6)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx = \int \frac{1}{x^6 (b x^2+a)^{7/6}} \, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(7/6)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(7/6)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^6 (a+bx^2)^{7/6}} \, dx = \frac{-16\sqrt{b}\,\sqrt{a}\,atan\left(\frac{bx}{\sqrt{b}\,\sqrt{a}}\right) b^2x^5 - a^3 + 2a^2b\,x^2 - 8a\,b^2x^4}{5\,(b\,x^2+a)^{\frac{1}{6}}\,a^4x^5}$$

input
```
int(1/x^6/(b*x^2+a)^(7/6),x)
```

output
```
((a + b*x**2)**(5/6)*( - 16*sqrt(b)*sqrt(a)*atan((b*x)/(sqrt(b)*sqrt(a)))*
b**2*x**5 - a**3 + 2*a**2*b*x**2 - 8*a*b**2*x**4))/(5*a**4*x**5*(a + b*x**
2))
```

3.1119 $\int x^6 \sqrt[8]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^6 \sqrt[8]{a+bx^2}\,dx=\frac{x^7 \sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
1/7*x^7*(b*x^2+a)^(1/8)*hypergeom([-1/8, 7/2],[9/2],-b*x^2/a)/(1+b*x^2/a)^
(1/8)
```

Mathematica [A] (verified)

Time = 8.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^6 \sqrt[8]{a+bx^2}\,dx=\frac{x^7 \sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^6*(a + b*x^2)^(1/8),x]
```

output
```
(x^7*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[8]{a+bx^2} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2} \int x^6 \sqrt[8]{\frac{bx^2}{a}+1}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7 \sqrt[8]{a+bx^2} \text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[x^6*(a + b*x^2)^(1/8),x]
```

output
```
(x^7*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6} \left(b \,x^{2}+a \right)^{\frac{1}{8}}d x$$

input
```
int(x^6*(b*x^2+a)^(1/8),x)
```

output
```
int(x^6*(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \int \left(bx^{2} + a\right)^{\frac{1}{8}} x^{6}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \frac{\sqrt[8]{a}x^7{}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(1/8),x)
```

output
```
a**(1/8)*x**7*hyper((-1/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^6, x)
```

Giac [F]

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \int \left(bx^2+a\right)^{\frac{1}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \int x^6 \left(b\,x^2+a\right)^{1/8}\,dx$$

input
```
int(x^6*(a + b*x^2)^(1/8),x)
```

output
```
int(x^6*(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int x^6 \sqrt[8]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(1/8),x)
```

output
```
int((a + b*x**2)**(1/8)*x**6,x)
```

3.1120 $\int x^4 \sqrt[8]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^4 \sqrt[8]{a+bx^2}\,dx=\frac{x^5\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
1/5*x^5*(b*x^2+a)^(1/8)*hypergeom([-1/8, 5/2],[7/2],-b*x^2/a)/(1+b*x^2/a)^
(1/8)
```

Mathematica [A] (verified)

Time = 8.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^4 \sqrt[8]{a+bx^2}\,dx=\frac{x^5\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(a + b*x^2)^(1/8),x]
```

output
```
(x^5*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[8]{a+bx^2}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2}\int x^4 \sqrt[8]{\frac{bx^2}{a}+1}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5 \sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[x^4*(a + b*x^2)^(1/8),x]
```

output

```
(x^5*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4} \left(b\,x^{2}+a \right)^{\frac{1}{8}}d x$$

input
```
int(x^4*(b*x^2+a)^(1/8),x)
```

output
```
int(x^4*(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \int \left(bx^{2} + a\right)^{\frac{1}{8}} x^{4}\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \frac{\sqrt[8]{a}x^5{}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(1/8),x)
```

output
```
a**(1/8)*x**5*hyper((-1/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \int x^4 \left(b\,x^2+a\right)^{1/8}\,dx$$

input
```
int(x^4*(a + b*x^2)^(1/8),x)
```

output
```
int(x^4*(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int x^4 \sqrt[8]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{8}} x^4 dx$$

input
```
int(x^4*(b*x^2+a)^(1/8),x)
```

output
```
int((a + b*x**2)**(1/8)*x**4,x)
```

3.1121 $\int x^2 \sqrt[8]{a+bx^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^2 \sqrt[8]{a+bx^2}\,dx = \frac{x^3 \sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
1/3*x^3*(b*x^2+a)^(1/8)*hypergeom([-1/8, 3/2],[5/2],-b*x^2/a)/(1+b*x^2/a)^
(1/8)
```

Mathematica [A] (verified)

Time = 8.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^2 \sqrt[8]{a+bx^2}\,dx = \frac{x^3 \sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*(a + b*x^2)^(1/8),x]
```

output
```
(x^3*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[8]{a+bx^2} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2} \int x^2 \sqrt[8]{\frac{bx^2}{a}+1}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3 \sqrt[8]{a+bx^2} \text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[x^2*(a + b*x^2)^(1/8),x]
```

output
```
(x^3*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a \right)^{\frac{1}{8}}d x$$

input
```
int(x^2*(b*x^2+a)^(1/8),x)
```

output
```
int(x^2*(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int x^2 \sqrt[8]{a+bx^2}\,dx = \int \left(bx^{2}+a\right)^{\frac{1}{8}}x^{2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^2\sqrt[8]{a+bx^2}\,dx=\frac{\sqrt[8]{a}x^3{}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(1/8),x)
```

output
```
a**(1/8)*x**3*hyper((-1/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2\sqrt[8]{a+bx^2}\,dx=\int\left(bx^2+a\right)^{\frac{1}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt[8]{a+bx^2}\,dx=\int\left(bx^2+a\right)^{\frac{1}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \sqrt[8]{a+bx^2}\,dx = \int x^2\,{\left(b\,x^2+a\right)}^{1/8} \,dx$$

input
```
int(x^2*(a + b*x^2)^(1/8),x)
```

output
```
int(x^2*(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int x^2 \sqrt[8]{a+bx^2}\,dx = \int {\left(b\,x^2+a\right)}^{\frac{1}{8}} x^2 dx$$

input
```
int(x^2*(b*x^2+a)^(1/8),x)
```

output
```
int((a + b*x**2)**(1/8)*x**2,x)
```

3.1122 $\int \sqrt[8]{a+bx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \sqrt[8]{a+bx^2}\,dx=\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
x*(b*x^2+a)^(1/8)*hypergeom([-1/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(1/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \sqrt[8]{a+bx^2}\,dx=\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8),x]
```

output
```
(x*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[8]{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\sqrt[8]{a+bx^2}\int \sqrt[8]{\frac{bx^2}{a}+1}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8),x]
```

output
```
(x*(a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2+a)^(1/8),x)
```

output
```
int((b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \sqrt[8]{a+bx^2}\,dx=\int \left(bx^2+a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \sqrt[8]{a+bx^2}\,dx=\sqrt[8]{a}x{}_2F_1\left(\begin{matrix} -\frac{1}{8},\frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(1/8),x)
```

output
```
a**(1/8)*x*hyper((-1/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \sqrt[8]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8), x)
```

Giac [F]

$$\int \sqrt[8]{a+bx^2}\,dx = \int (bx^2+a)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \sqrt[8]{a+bx^2}\,dx = \frac{x\,(b\,x^2+a)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{1/8}}$$

input
```
int((a + b*x^2)^(1/8),x)
```

output
```
(x*(a + b*x^2)^(1/8)*hypergeom([-1/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(1/8)
```

Reduce [F]

$$\int \sqrt[8]{a+bx^2}\,dx = \int \left(b\,x^2+a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2+a)^(1/8),x)
```

output
```
int((a + b*x**2)**(1/8),x)
```

3.1123 $\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx=-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
-(b*x^2+a)^(1/8)*hypergeom([-1/2, -1/8],[1/2],-b*x^2/a)/x/(1+b*x^2/a)^(1/8
)
```

Mathematica [A] (verified)

Time = 8.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx=-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(1/8)*Hypergeometric2F1[-1/2, -1/8, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(1/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2}\int \frac{\sqrt[8]{\frac{bx^2}{a}+1}}{x^2}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(1/8)*Hypergeometric2F1[-1/2, -1/8, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(1/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{8}}}{x^2}dx$$

input
```
int((b*x^2+a)^(1/8)/x^2,x)
```

output
```
int((b*x^2+a)^(1/8)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx = -\frac{\sqrt[8]{a}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(1/8)/x**2,x)
```

output
```
-a**(1/8)*hyper((-1/2, -1/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx = -\frac{4\,{\left(b\,x^2+a\right)}^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{3}{8};\ \frac{11}{8};\ -\frac{a}{b\,x^2}\right)}{3\,x\,{\left(\frac{a}{b\,x^2}+1\right)}^{1/8}}$$

input
```
int((a + b*x^2)^(1/8)/x^2,x)
```

output
```
-(4*(a + b*x^2)^(1/8)*hypergeom([-1/8, 3/8], 11/8, -a/(b*x^2)))/(3*x*(a/(b
*x^2) + 1)^(1/8))
```

Reduce [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^2}\,dx$$
$$= \frac{-36(b\,x^2+a)^{\frac{7}{8}}\,a-20(b\,x^2+a)^{\frac{7}{8}}\,b\,x^2+5(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{5}{8}}a+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2}dx\right)abx+25(b\,x^2+a)^{\frac{3}{4}}\left(\right.}{36\,(b\,x^2+a)^{\frac{3}{4}}\,ax}$$

input
```
int((b*x^2+a)^(1/8)/x^2,x)
```

output
```
( - 36*(a + b*x**2)**(7/8)*a - 20*(a + b*x**2)**(7/8)*b*x**2 + 5*(a + b*x*
*2)**(3/4)*int((a + b*x**2)**(3/4)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**
(5/8)*b*x**2),x)*a*b*x + 25*(a + b*x**2)**(3/4)*int(x**2/(a + b*x**2)**(7/
8),x)*b**2*x)/(36*(a + b*x**2)**(3/4)*a*x)
```

3.1124 $\int \frac{\sqrt[8]{a+bx^2}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4} \, dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
-1/3*(b*x^2+a)^(1/8)*hypergeom([-3/2, -1/8],[-1/2],-b*x^2/a)/x^3/(1+b*x^2/
a)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4} \, dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(1/8)*Hypergeometric2F1[-3/2, -1/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt[8]{a+bx^2}\int \frac{\sqrt[8]{\frac{bx^2}{a}+1}}{x^4}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \ 278$$

$$-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(1/8)*Hypergeometric2F1[-3/2, -1/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{8}}}{x^4}dx$$

input
```
int((b*x^2+a)^(1/8)/x^4,x)
```

output
```
int((b*x^2+a)^(1/8)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx = -\frac{\sqrt[8]{a}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{1}{8}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(1/8)/x**4,x)
```

output
```
-a**(1/8)*hyper((-3/2, -1/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{1/8}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(1/8)/x^4,x)
```

output
```
int((a + b*x^2)^(1/8)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^4}\,dx = \frac{-272(b\,x^2+a)^{\frac{7}{8}}\,a+220(b\,x^2+a)^{\frac{7}{8}}\,b\,x^2+615(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2+a)^{\frac{3}{4}}}{(b\,x^2+a)^{\frac{5}{8}}\,a+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2}dx\right)b^2x^3}{816\,(b\,x^2+a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int((b*x^2+a)^(1/8)/x^4,x)
```

output
```
( - 272*(a + b*x**2)**(7/8)*a + 220*(a + b*x**2)**(7/8)*b*x**2 + 615*(a +
b*x**2)**(3/4)*int((a + b*x**2)**(3/4)/((a + b*x**2)**(5/8)*a + (a + b*x**
2)**(5/8)*b*x**2),x)*b**2*x**3)/(816*(a + b*x**2)**(3/4)*a*x**3)
```

3.1125 $\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
-1/5*(b*x^2+a)^(1/8)*hypergeom([-5/2, -1/8],[-3/2],-b*x^2/a)/x^5/(1+b*x^2/
a)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(1/8)*Hypergeometric2F1[-5/2, -1/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2}\int \frac{\sqrt[8]{\frac{bx^2}{a}+1}}{x^6}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(1/8)*Hypergeometric2F1[-5/2, -1/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{1}{8}}}{x^6}dx$$

input
```
int((b*x^2+a)^(1/8)/x^6,x)
```

output
```
int((b*x^2+a)^(1/8)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx=\int \frac{(bx^2+a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = -\frac{\sqrt[8]{a}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(1/8)/x**6,x)
```

output
```
-a**(1/8)*hyper((-5/2, -1/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{1/8}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(1/8)/x^6,x)
```

output
```
int((a + b*x^2)^(1/8)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^6}\,dx$$
$$= \frac{-960(b\,x^2+a)^{\frac{7}{8}}\,a^3 + 304(b\,x^2+a)^{\frac{7}{8}}\,a^2b\,x^2 - 1580(b\,x^2+a)^{\frac{7}{8}}\,a\,b^2x^4 - 2108(b\,x^2+a)^{\frac{7}{8}}\,b^3x^6 + 1188(b\,x^2}{4}$$

input
```
int((b*x^2+a)^(1/8)/x^6,x)
```

output
```
( - 960*(a + b*x**2)**(7/8)*a**3 + 304*(a + b*x**2)**(7/8)*a**2*b*x**2 - 1
580*(a + b*x**2)**(7/8)*a*b**2*x**4 - 2108*(a + b*x**2)**(7/8)*b**3*x**6 +
 1188*(a + b*x**2)**(3/4)*int((a + b*x**2)**(3/4)/((a + b*x**2)**(5/8)*a +
 (a + b*x**2)**(5/8)*b*x**2),x)*a*b**3*x**5 + 2635*(a + b*x**2)**(3/4)*int
(x**2/(a + b*x**2)**(7/8),x)*b**4*x**5 + 527*(a + b*x**2)**(3/4)*int(1/(a
+ b*x**2)**(7/8),x)*a*b**3*x**5)/(4800*(a + b*x**2)**(3/4)*a**3*x**5)
```

3.1126 $\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\sqrt[8]{1+\frac{bx^2}{a}}}$$

output
```
-1/7*(b*x^2+a)^(1/8)*hypergeom([-7/2, -1/8],[-5/2],-b*x^2/a)/x^7/(1+b*x^2/
a)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\sqrt[8]{1+\frac{bx^2}{a}}}$$

input
```
Integrate[(a + b*x^2)^(1/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(1/8)*Hypergeometric2F1[-7/2, -1/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{a+bx^2}\int \frac{\sqrt[8]{\frac{bx^2}{a}+1}}{x^8}dx}{\sqrt[8]{\frac{bx^2}{a}+1}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\sqrt[8]{\frac{bx^2}{a}+1}}$$

input
```
Int[(a + b*x^2)^(1/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(1/8)*Hypergeometric2F1[-7/2, -1/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{1}{8}}}{x^{8}}dx$$

input
```
int((b*x^2+a)^(1/8)/x^8,x)
```

output
```
int((b*x^2+a)^(1/8)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx=\int \frac{(bx^2+a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = -\frac{\sqrt[8]{a}\,{}_2F_1\left(\begin{matrix} -\frac{7}{2},-\frac{1}{8} \\ -\frac{5}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(1/8)/x**8,x)
```

output
```
-a**(1/8)*hyper((-7/2, -1/8), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(1/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(1/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = \int \frac{(b\,x^2+a)^{1/8}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(1/8)/x^8,x)
```

output
```
int((a + b*x^2)^(1/8)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{a+bx^2}}{x^8}\,dx = \frac{-6336(b\,x^2+a)^{\frac{7}{8}}\,a^4+1296(b\,x^2+a)^{\frac{7}{8}}\,a^3b\,x^2-3180(b\,x^2+a)^{\frac{7}{8}}\,a^2b^2x^4-11160(b\,x^2+a)^{\frac{7}{8}}\,a\,b^3x^6-646}{}$$

input
```
int((b*x^2+a)^(1/8)/x^8,x)
```

output
```
( - 6336*(a + b*x**2)**(7/8)*a**4 + 1296*(a + b*x**2)**(7/8)*a**3*b*x**2 -
 3180*(a + b*x**2)**(7/8)*a**2*b**2*x**4 - 11160*(a + b*x**2)**(7/8)*a*b**
3*x**6 - 6460*(a + b*x**2)**(7/8)*b**4*x**8 + 8075*(a + b*x**2)**(3/4)*int
(x**2/(a + b*x**2)**(7/8),x)*b**5*x**7 - 1910*(a + b*x**2)**(3/4)*int(1/(a
 + b*x**2)**(7/8),x)*a*b**4*x**7 - 17625*(a + b*x**2)**(3/4)*int(1/((a + b
*x**2)**(7/8)*x**2),x)*a**2*b**3*x**7)/(44352*(a + b*x**2)**(3/4)*a**4*x**
7)
```

3.1127 $\int x^6(a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^6\left(a+bx^2\right)^{3/8}\,dx=\frac{x^7(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
1/7*x^7*(b*x^2+a)^(3/8)*hypergeom([-3/8, 7/2],[9/2],-b*x^2/a)/(1+b*x^2/a)^
(3/8)
```

Mathematica [A] (verified)

Time = 9.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^6\left(a+bx^2\right)^{3/8}\,dx=\frac{x^7(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^6*(a + b*x^2)^(3/8),x]
```

output
```
(x^7*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^2\right)^{3/8} dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(a+bx^2\right)^{3/8} \int x^6 \left(\frac{bx^2}{a}+1\right)^{3/8} dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^7 \left(a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7 \left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[x^6*(a + b*x^2)^(3/8),x]
```

output
```
(x^7*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6}\left(b\,x^{2}+a\right)^{\frac{3}{8}}d x$$

input
```
int(x^6*(b*x^2+a)^(3/8),x)
```

output
```
int(x^6*(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int x^{6}\left(a+bx^{2}\right)^{3/8}\,dx=\int\left(bx^{2}+a\right)^{\frac{3}{8}}x^{6}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^{6}\left(a+bx^{2}\right)^{3/8}\,dx=\frac{a^{\frac{3}{8}}x^{7}{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(3/8),x)
```

output
```
a**(3/8)*x**7*hyper((-3/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a + bx^2)^{3/8} \, dx = \int (bx^2 + a)^{\frac{3}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^2)^{3/8} \, dx = \int (bx^2 + a)^{\frac{3}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^2)^{3/8} \, dx = \int x^6 \left(b\,x^2 + a\right)^{3/8} \, dx$$

input
```
int(x^6*(a + b*x^2)^(3/8),x)
```

output
```
int(x^6*(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int x^6 \left(a+bx^2\right)^{3/8} dx = \int \left(b\,x^2+a\right)^{\frac{3}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(3/8),x)
```

output
```
int((a + b*x**2)**(3/8)*x**6,x)
```

3.1128 $\int x^4(a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^4\left(a+bx^2\right)^{3/8}\,dx=\frac{x^5(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
1/5*x^5*(b*x^2+a)^(3/8)*hypergeom([-3/8, 5/2],[7/2],-b*x^2/a)/(1+b*x^2/a)^
(3/8)
```

Mathematica [A] (verified)

Time = 9.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^{3/8}\,dx=\frac{x^5(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^4*(a + b*x^2)^(3/8),x]
```

output
```
(x^5*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{3/8} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(a+bx^2\right)^{3/8} \int x^4 \left(\frac{bx^2}{a}+1\right)^{3/8}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^5 \left(a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5 \left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[x^4*(a + b*x^2)^(3/8),x]
```

output
```
(x^5*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}+a\right)^{\frac{3}{8}}dx$$

input
```
int(x^4*(b*x^2+a)^(3/8),x)
```

output
```
int(x^4*(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{3/8} \, dx = \int \left(bx^2+a\right)^{\frac{3}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^4 \left(a+bx^2\right)^{3/8} \, dx = \frac{a^{\frac{3}{8}} x^5 {}_2F_1\left(\begin{matrix} -\frac{3}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(3/8),x)
```

output
```
a**(3/8)*x**5*hyper((-3/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4(a+bx^2)^{3/8}\,dx=\int (bx^2+a)^{\frac{3}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^2)^{3/8}\,dx=\int (bx^2+a)^{\frac{3}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{3/8}\,dx=\int x^4\,\left(b\,x^2+a\right)^{3/8}\,dx$$

input
```
int(x^4*(a + b*x^2)^(3/8),x)
```

output
```
int(x^4*(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int x^4(a+bx^2)^{3/8}\,dx=\int (b\,x^2+a)^{\frac{3}{8}}\,x^4dx$$

input
```
int(x^4*(b*x^2+a)^(3/8),x)
```

output
```
int((a + b*x**2)**(3/8)*x**4,x)
```

3.1129 $\int x^2(a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^2\left(a+bx^2\right)^{3/8}\,dx=\frac{x^3(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
1/3*x^3*(b*x^2+a)^(3/8)*hypergeom([-3/8, 3/2],[5/2],-b*x^2/a)/(1+b*x^2/a)^
(3/8)
```

Mathematica [A] (verified)

Time = 8.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^{3/8}\,dx=\frac{x^3(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^2*(a + b*x^2)^(3/8),x]
```

output
```
(x^3*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{3/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{3/8} \int x^2 \left(\frac{bx^2}{a}+1\right)^{3/8}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3 \left(a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3 \left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[x^2*(a + b*x^2)^(3/8),x]
```

output
```
(x^3*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a \right)^{\frac{3}{8}}d x$$

input
```
int(x^2*(b*x^2+a)^(3/8),x)
```

output
```
int(x^2*(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{3/8} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{3}{8}} x^{2} \,d x }$$

input
```
integrate(x^2*(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^2 \left(a+bx^2\right)^{3/8} \, dx = \frac{a^{\frac{3}{8}} x^{3} {{}_{2}F_{1}\left(\begin{matrix} - \frac{3}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(3/8),x)
```

output
```
a**(3/8)*x**3*hyper((-3/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a + bx^2)^{3/8} \, dx = \int (bx^2 + a)^{\frac{3}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^{3/8} \, dx = \int (bx^2 + a)^{\frac{3}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^{3/8} \, dx = \int x^2 \left(b\,x^2 + a\right)^{3/8} \, dx$$

input
```
int(x^2*(a + b*x^2)^(3/8),x)
```

output
```
int(x^2*(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int x^2 (a + bx^2)^{3/8} \, dx = \int (b\,x^2 + a)^{\frac{3}{8}} \, x^2 dx$$

input
```
int(x^2*(b*x^2+a)^(3/8),x)
```

output
```
int((a + b*x**2)**(3/8)*x**2,x)
```

3.1130 $\int (a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{3/8}\,dx=\frac{x(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
x*(b*x^2+a)^(3/8)*hypergeom([-3/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(3/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{3/8}\,dx=\frac{x(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8),x]
```

output
```
(x*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/8} dx$$

$$\Big\downarrow \; 238$$

$$\frac{\left(a+bx^2\right)^{3/8} \int \left(\frac{bx^2}{a}+1\right)^{3/8} dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \; 237$$

$$\frac{x\left(a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8),x]
```

output
```
(x*(a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{3}{8}}d x$$

input
```
int((b*x^2+a)^(3/8),x)
```

output
```
int((b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{3/8}\,dx=\int \left(bx^{2}+a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{3/8}\,dx=a^{\frac{3}{8}}x\,{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(3/8),x)
```

output
```
a**(3/8)*x*hyper((-3/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{3/8}\,dx=\int \left(bx^2+a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{3/8}\,dx=\int \left(bx^2+a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{3/8}\,dx=\frac{x\,\left(b\,x^2+a\right)^{3/8}\,{}_2\mathrm{F}_1\left(-\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/8}}$$

input
```
int((a + b*x^2)^(3/8),x)
```

output
```
(x*(a + b*x^2)^(3/8)*hypergeom([-3/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(3/8)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{3/8} dx = \int \left(b\,x^2+a\right)^{\frac{3}{8}} dx$$

input
```
int((b*x^2+a)^(3/8),x)
```

output
```
int((a + b*x**2)**(3/8),x)
```

3.1131 $\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = -\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
-(b*x^2+a)^(3/8)*hypergeom([-1/2, -3/8],[1/2],-b*x^2/a)/x/(1+b*x^2/a)^(3/8
)
```

Mathematica [A] (verified)

Time = 8.75 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = -\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(3/8)*Hypergeometric2F1[-1/2, -3/8, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(3/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/8}}{x^2} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{3/8} \int \frac{\left(\frac{bx^2}{a}+1\right)^{3/8}}{x^2}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(3/8)*Hypergeometric2F1[-1/2, -3/8, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(3/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{8}}}{x^2}dx$$

input
```
int((b*x^2+a)^(3/8)/x^2,x)
```

output
```
int((b*x^2+a)^(3/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = -\frac{a^{\frac{3}{8}}{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{8}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(3/8)/x**2,x)
```

output
```
-a**(3/8)*hyper((-1/2, -3/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = -\frac{4\,(b\,x^2+a)^{3/8}\,{}_2F_1\left(-\frac{3}{8},\frac{1}{8};\ \frac{9}{8};\ -\frac{a}{b\,x^2}\right)}{x\,\left(\frac{a}{b\,x^2}+1\right)^{3/8}}$$

input
```
int((a + b*x^2)^(3/8)/x^2,x)
```

output
```
-(4*(a + b*x^2)^(3/8)*hypergeom([-3/8, 1/8], 9/8, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(3/8))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^2}\,dx = \frac{-4(b\,x^2+a)^{\frac{1}{8}}\,a-4(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2-5(b\,x^2+a)^{\frac{3}{4}}\left(\int\frac{1}{(b\,x^2+a)^{\frac{5}{8}}}dx\right)bx}{4\,(b\,x^2+a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2+a)^(3/8)/x^2,x)
```

output
```
( - 4*(a + b*x**2)**(1/8)*a - 4*(a + b*x**2)**(1/8)*b*x**2 - 5*(a + b*x**2
)**(3/4)*int(1/(a + b*x**2)**(5/8),x)*b*x)/(4*(a + b*x**2)**(3/4)*x)
```

3.1132 $\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = -\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
-1/3*(b*x^2+a)^(3/8)*hypergeom([-3/2, -3/8],[-1/2],-b*x^2/a)/x^3/(1+b*x^2/
a)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = -\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(3/8)*Hypergeometric2F1[-3/2, -3/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{3/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/8}}{x^4}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(3/8)*Hypergeometric2F1[-3/2, -3/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{8}}}{x^4}dx$$

input
```
int((b*x^2+a)^(3/8)/x^4,x)
```

output
```
int((b*x^2+a)^(3/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx=-\frac{a^{\frac{3}{8}}{}_2F_1\left(\begin{matrix} -\frac{3}{2},-\frac{3}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(3/8)/x**4,x)
```

output
```
-a**(3/8)*hyper((-3/2, -3/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{3/8}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(3/8)/x^4,x)
```

output
```
int((a + b*x^2)^(3/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^4}\,dx = \frac{-144(b\,x^2+a)^{\frac{1}{8}}\,a^2+36(b\,x^2+a)^{\frac{1}{8}}\,ab\,x^2+100(b\,x^2+a)^{\frac{1}{8}}\,b^2x^4+225(b\,x^2+a)^{\frac{3}{4}}\left(\int\right.}{432\,(b\,x^2+}$$

input
```
int((b*x^2+a)^(3/8)/x^4,x)
```

output
```
( - 144*(a + b*x**2)**(1/8)*a**2 + 36*(a + b*x**2)**(1/8)*a*b*x**2 + 100*(
a + b*x**2)**(1/8)*b**2*x**4 + 225*(a + b*x**2)**(3/4)*int(x**2/((a + b*x*
*2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**3 + 325*(a + b*x**2)
**(3/4)*int(1/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*a*b*
*2*x**3)/(432*(a + b*x**2)**(3/4)*a*x**3)
```

3.1133 $\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx=-\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
-1/5*(b*x^2+a)^(3/8)*hypergeom([-5/2, -3/8],[-3/2],-b*x^2/a)/x^5/(1+b*x^2/
a)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx=-\frac{(a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(3/8)*Hypergeometric2F1[-5/2, -3/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/8}}{x^6}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{3/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/8}}{x^6}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{3/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(3/8)*Hypergeometric2F1[-5/2, -3/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{8}}}{x^6}dx$$

input
```
int((b*x^2+a)^(3/8)/x^6,x)
```

output
```
int((b*x^2+a)^(3/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = -\frac{a^{\frac{3}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{3}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(3/8)/x**6,x)
```

output
```
-a**(3/8)*hyper((-5/2, -3/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{3/8}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(3/8)/x^6,x)
```

output
```
int((a + b*x^2)^(3/8)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^6}\,dx = \frac{-816(b\,x^2+a)^{\frac{1}{8}}\,a^2-476(b\,x^2+a)^{\frac{1}{8}}\,ab\,x^2+100(b\,x^2+a)^{\frac{1}{8}}\,b^2x^4+725(b\,x^2+a)^{\frac{3}{4}}\left(\int\right.}{4080\,(b\,x^2}$$

input
```
int((b*x^2+a)^(3/8)/x^6,x)
```

output
```
( - 816*(a + b*x**2)**(1/8)*a**2 - 476*(a + b*x**2)**(1/8)*a*b*x**2 + 100*
(a + b*x**2)**(1/8)*b**2*x**4 + 725*(a + b*x**2)**(3/4)*int(1/((a + b*x**2
)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*a*b**2*x**5 + 425*(a + b*
x**2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)
*b**3*x**5)/(4080*(a + b*x**2)**(3/4)*a*x**5)
```

3.1134 $\int \frac{(a+bx^2)^{3/8}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{3/8}}{x^8} dx = -\frac{(a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

output
```
-1/7*(b*x^2+a)^(3/8)*hypergeom([-7/2, -3/8],[-5/2],-b*x^2/a)/x^7/(1+b*x^2/
a)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{3/8}}{x^8} dx = -\frac{(a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(3/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(3/8)*Hypergeometric2F1[-7/2, -3/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/8}}{x^8}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{3/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{3/8}}{x^8}dx}{\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(a+bx^2\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(\frac{bx^2}{a}+1\right)^{3/8}}$$

input
```
Int[(a + b*x^2)^(3/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(3/8)*Hypergeometric2F1[-7/2, -3/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{3}{8}}}{x^8}dx$$

input
```
int((b*x^2+a)^(3/8)/x^8,x)
```

output
```
int((b*x^2+a)^(3/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = -\frac{a^{\frac{3}{8}}{}_2F_1\left(\begin{matrix} -\frac{7}{2},-\frac{3}{8} \\ -\frac{5}{2} \end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(3/8)/x**8,x)
```

output
```
-a**(3/8)*hyper((-7/2, -3/8), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(3/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(3/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(b\,x^2+a)^{3/8}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(3/8)/x^8,x)
```

output
```
int((a + b*x^2)^(3/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{3/8}}{x^8}\,dx = \frac{-80(b\,x^2+a)^{\frac{1}{8}}\,a^2-60(b\,x^2+a)^{\frac{1}{8}}\,ab\,x^2+4(b\,x^2+a)^{\frac{1}{8}}\,b^2x^4+45(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2+}\right.}{560\,(b\,x^2+}$$

input
```
int((b*x^2+a)^(3/8)/x^8,x)
```

output
```
( - 80*(a + b*x**2)**(1/8)*a**2 - 60*(a + b*x**2)**(1/8)*a*b*x**2 + 4*(a +
 b*x**2)**(1/8)*b**2*x**4 + 45*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(5
/8)*a*x**4 + (a + b*x**2)**(5/8)*b*x**6),x)*a*b**2*x**7 + 25*(a + b*x**2)*
*(3/4)*int(1/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*
b**3*x**7)/(560*(a + b*x**2)**(3/4)*a*x**7)
```

3.1135 $\int x^6(a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^6\left(a+bx^2\right)^{5/8}\,dx=\frac{x^7(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
1/7*x^7*(b*x^2+a)^(5/8)*hypergeom([-5/8, 7/2],[9/2],-b*x^2/a)/(1+b*x^2/a)^
(5/8)
```

Mathematica [A] (verified)

Time = 9.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^6\left(a+bx^2\right)^{5/8}\,dx=\frac{x^7(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^6*(a + b*x^2)^(5/8),x]
```

output
```
(x^7*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (a+bx^2)^{5/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{5/8} \int x^6 \left(\frac{bx^2}{a}+1\right)^{5/8} dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7 (a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[x^6*(a + b*x^2)^(5/8),x]
```

output
```
(x^7*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6} \left(b\,x^{2}+a \right)^{\frac{5}{8}}d x$$

input
```
int(x^6*(b*x^2+a)^(5/8),x)
```

output
```
int(x^6*(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int x^6 \left(a+bx^2\right)^{5/8} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{5}{8}} x^{6} \,d x }$$

input
```
integrate(x^6*(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^6 \left(a+bx^2\right)^{5/8} \, dx = \frac{a^{\frac{5}{8}} x^{7} {{}_{2}F_{1}\left(\begin{matrix} - \frac{5}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi }}{a}} \right)}}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(5/8),x)
```

output
```
a**(5/8)*x**7*hyper((-5/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a + bx^2)^{5/8} dx = \int (bx^2 + a)^{\frac{5}{8}} x^6 dx$$

input
```
integrate(x^6*(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^2)^{5/8} dx = \int (bx^2 + a)^{\frac{5}{8}} x^6 dx$$

input
```
integrate(x^6*(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^2)^{5/8} dx = \int x^6 (b x^2 + a)^{5/8} dx$$

input
```
int(x^6*(a + b*x^2)^(5/8),x)
```

output
```
int(x^6*(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int x^6 \left(a+bx^2\right)^{5/8} dx = \int \left(b\,x^2+a\right)^{\frac{5}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(5/8),x)
```

output
```
int((a + b*x**2)**(5/8)*x**6,x)
```

3.1136 $\int x^4(a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^4\left(a+bx^2\right)^{5/8}\,dx=\frac{x^5(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
1/5*x^5*(b*x^2+a)^(5/8)*hypergeom([-5/8, 5/2],[7/2],-b*x^2/a)/(1+b*x^2/a)^
(5/8)
```

Mathematica [A] (verified)

Time = 9.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^{5/8}\,dx=\frac{x^5(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^4*(a + b*x^2)^(5/8),x]
```

output
```
(x^5*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{5/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{5/8} \int x^4 \left(\frac{bx^2}{a}+1\right)^{5/8}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5 \left(a+bx^2\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5 \left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[x^4*(a + b*x^2)^(5/8),x]
```

output
```
(x^5*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{\frac{5}{8}}d x$$

input
```
int(x^4*(b*x^2+a)^(5/8),x)
```

output
```
int(x^4*(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{5/8} \, dx=\int { {\left(b x^{2} + a\right)}^{\frac{5}{8}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^4 \left(a+bx^2\right)^{5/8} \, dx=\frac{a^{\frac{5}{8}} x^{5} {{}_{2}F_{1}\left(\begin{matrix} - \frac{5}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi }}{a}} \right)}}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(5/8),x)
```

output
```
a**(5/8)*x**5*hyper((-5/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a + bx^2)^{5/8} \, dx = \int (bx^2 + a)^{\frac{5}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2)^{5/8} \, dx = \int (bx^2 + a)^{\frac{5}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^{5/8} \, dx = \int x^4 \left(b\,x^2 + a\right)^{5/8} \, dx$$

input
```
int(x^4*(a + b*x^2)^(5/8),x)
```

output
```
int(x^4*(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int x^4 (a + bx^2)^{5/8} \, dx = \int (b\, x^2 + a)^{\frac{5}{8}} x^4 dx$$

input
```
int(x^4*(b*x^2+a)^(5/8),x)
```

output
```
int((a + b*x**2)**(5/8)*x**4,x)
```

3.1137 $\int x^2(a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^2\left(a+bx^2\right)^{5/8}\,dx=\frac{x^3(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
1/3*x^3*(b*x^2+a)^(5/8)*hypergeom([-5/8, 3/2],[5/2],-b*x^2/a)/(1+b*x^2/a)^
(5/8)
```

Mathematica [A] (verified)

Time = 9.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^{5/8}\,dx=\frac{x^3(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^2*(a + b*x^2)^(5/8),x]
```

output
```
(x^3*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{5/8} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(a+bx^2\right)^{5/8} \int x^2 \left(\frac{bx^2}{a}+1\right)^{5/8}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^3 \left(a+bx^2\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3 \left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[x^2*(a + b*x^2)^(5/8),x]
```

output
```
(x^3*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}+a \right)^{\frac{5}{8}}d x$$

input
```
int(x^2*(b*x^2+a)^(5/8),x)
```

output
```
int(x^2*(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int x^2 \left(a+bx^2\right)^{5/8} \, dx=\int \left(bx^{2}+a\right)^{\frac{5}{8}}x^{2}\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^2 \left(a+bx^2\right)^{5/8} \, dx=\frac{a^{\frac{5}{8}}x^{3}{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{8},\frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(5/8),x)
```

output
```
a**(5/8)*x**3*hyper((-5/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a + bx^2)^{5/8} \, dx = \int (bx^2 + a)^{\frac{5}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^{5/8} \, dx = \int (bx^2 + a)^{\frac{5}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^{5/8} \, dx = \int x^2 \left(b\,x^2 + a\right)^{5/8} \, dx$$

input
```
int(x^2*(a + b*x^2)^(5/8),x)
```

output
```
int(x^2*(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int x^2\left(a+bx^2\right)^{5/8}\,dx=\int \left(b\,x^2+a\right)^{\frac{5}{8}}x^2dx$$

input
```
int(x^2*(b*x^2+a)^(5/8),x)
```

output
```
int((a + b*x**2)**(5/8)*x**2,x)
```

3.1138 $\int (a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{5/8}\,dx=\frac{x(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
x*(b*x^2+a)^(5/8)*hypergeom([-5/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(5/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{5/8}\,dx=\frac{x(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8),x]
```

output
```
(x*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^{5/8} \, dx$$
$$\Big\downarrow \text{238}$$
$$\frac{(a+bx^2)^{5/8} \int \left(\frac{bx^2}{a}+1\right)^{5/8} dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$
$$\Big\downarrow \text{237}$$
$$\frac{x(a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8),x]
```

output
```
(x*(a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{5}{8}}d x$$

input
```
int((b*x^2+a)^(5/8),x)
```

output
```
int((b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{5/8}\,dx=\int \left(bx^{2}+a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{5/8}\,dx=a^{\frac{5}{8}}x\,{}_2F_1\left(\begin{matrix} -\frac{5}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^{2}e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(5/8),x)
```

output
```
a**(5/8)*x*hyper((-5/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{5/8} dx = \int \left(bx^2+a\right)^{\frac{5}{8}} dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{5/8} dx = \int \left(bx^2+a\right)^{\frac{5}{8}} dx$$

input
```
integrate((b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{5/8} dx = \frac{x\left(b\,x^2+a\right)^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/8}}$$

input
```
int((a + b*x^2)^(5/8),x)
```

output
```
(x*(a + b*x^2)^(5/8)*hypergeom([-5/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(5/8)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{5/8}\, dx=\int \left(b\,x^{2}+a\right)^{\frac{5}{8}}d x$$

input
```
int((b*x^2+a)^(5/8),x)
```

output
```
int((a + b*x**2)**(5/8),x)
```

3.1139 $\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = -\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
-(b*x^2+a)^(5/8)*hypergeom([-5/8, -1/2],[1/2],-b*x^2/a)/x/(1+b*x^2/a)^(5/8
)
```

Mathematica [A] (verified)

Time = 9.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = -\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(5/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/8}}{x^2}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{5/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{5/8}}{x^2}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(5/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{8}}}{x^{2}}dx$$

input
```
int((b*x^2+a)^(5/8)/x^2,x)
```

output
```
int((b*x^2+a)^(5/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx=\int \frac{(bx^2+a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx=-\frac{a^{\frac{5}{8}}{}_2F_1\left(\begin{matrix}-\frac{5}{8},-\frac{1}{2}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(5/8)/x**2,x)
```

output
```
-a**(5/8)*hyper((-5/8, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = \frac{4\,(b\,x^2+a)^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},-\frac{1}{8};\ \frac{7}{8};\ -\frac{a}{b\,x^2}\right)}{x\,\left(\frac{a}{b\,x^2}+1\right)^{5/8}}$$

input
```
int((a + b*x^2)^(5/8)/x^2,x)
```

output
```
(4*(a + b*x^2)^(5/8)*hypergeom([-5/8, -1/8], 7/8, -a/(b*x^2)))/(x*(a/(b*x^
2) + 1)^(5/8))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^2}\,dx = \frac{-32(b\,x^2+a)^{\frac{3}{8}}\,a-32(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2+5(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}a+(b\,x^2+a)^{\frac{3}{8}}b\,x^2}dx\right)b^2x}{27\,(b\,x^2+a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2+a)^(5/8)/x^2,x)
```

output
```
( - 32*(a + b*x**2)**(3/8)*a - 32*(a + b*x**2)**(3/8)*b*x**2 + 5*(a + b*x*
*2)**(3/4)*int(x**2/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x
)*b**2*x - 5*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (a +
b*x**2)**(3/8)*b*x**4),x)*a**2*x)/(27*(a + b*x**2)**(3/4)*x)
```

3.1140 $\int \frac{\left(a+bx^2\right)^{5/8}}{x^4} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\left(a+bx^2\right)^{5/8}}{x^4} \, dx = -\frac{\left(a+bx^2\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 \left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
-1/3*(b*x^2+a)^(5/8)*hypergeom([-3/2, -5/8],[-1/2],-b*x^2/a)/x^3/(1+b*x^2/
a)^(5/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2\right)^{5/8}}{x^4} \, dx = -\frac{\left(a+bx^2\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 \left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(5/8)*Hypergeometric2F1[-3/2, -5/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{5/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{5/8}}{x^4}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(5/8)*Hypergeometric2F1[-3/2, -5/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{5}{8}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(5/8)/x^4,x)
```

output
```
int((b*x^2+a)^(5/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx=-\frac{a^{\frac{5}{8}}{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{8}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(5/8)/x**4,x)
```

output
```
-a**(5/8)*hyper((-3/2, -5/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{5/8}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(5/8)/x^4,x)
```

output
```
int((a + b*x^2)^(5/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^4}\,dx = \frac{-16(b\,x^2+a)^{\frac{3}{8}}\,a^2-56(b\,x^2+a)^{\frac{3}{8}}\,ab\,x^2-20(b\,x^2+a)^{\frac{3}{8}}\,b^2x^4+5(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2+}\right.}{}$$

input
```
int((b*x^2+a)^(5/8)/x^4,x)
```

output
```
( - 16*(a + b*x**2)**(3/8)*a**2 - 56*(a + b*x**2)**(3/8)*a*b*x**2 - 20*(a
+ b*x**2)**(3/8)*b**2*x**4 + 5*(a + b*x**2)**(3/4)*int(x**2/((a + b*x**2)*
*(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*b**3*x**3 + 20*(a + b*x**2)**(3/
4)*int(1/((a + b*x**2)**(3/8)*a*x**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a**3
*x**3 - 20*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*
x**2)**(3/8)*b*x**4),x)*a**2*b*x**3 - 60*(a + b*x**2)**(3/4)*int(1/((a + b
*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*a*b**2*x**3)/(68*(a + b*x
**2)**(3/4)*a*x**3)
```

3.1141 $\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = -\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
-1/5*(b*x^2+a)^(5/8)*hypergeom([-5/2, -5/8],[-3/2],-b*x^2/a)/x^5/(1+b*x^2/
a)^(5/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = -\frac{(a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(5/8)*Hypergeometric2F1[-5/2, -5/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/8}}{x^6} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{(a+bx^2)^{5/8} \int \frac{\left(\frac{bx^2}{a}+1\right)^{5/8}}{x^6}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{(a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(5/8)*Hypergeometric2F1[-5/2, -5/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{8}}}{x^6}dx$$

input
```
int((b*x^2+a)^(5/8)/x^6,x)
```

output
```
int((b*x^2+a)^(5/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = -\frac{a^{\frac{5}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{5}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(5/8)/x**6,x)
```

output
```
-a**(5/8)*hyper((-5/2, -5/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(b\,x^2+a)^{5/8}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(5/8)/x^6,x)
```

output
```
int((a + b*x^2)^(5/8)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^6}\,dx = \frac{-48(b\,x^2+a)^{\frac{3}{8}}\,a^2-88(b\,x^2+a)^{\frac{3}{8}}\,ab\,x^2-20(b\,x^2+a)^{\frac{3}{8}}\,b^2x^4+60(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2}\right.}{}$$

input
```
int((b*x^2+a)^(5/8)/x^6,x)
```

output
```
( - 48*(a + b*x**2)**(3/8)*a**2 - 88*(a + b*x**2)**(3/8)*a*b*x**2 - 20*(a
+ b*x**2)**(3/8)*b**2*x**4 + 60*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(
3/8)*a*x**6 + (a + b*x**2)**(3/8)*b*x**8),x)*a**3*x**5 - 60*(a + b*x**2)**
(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a
**2*b*x**5 - 180*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (
a + b*x**2)**(3/8)*b*x**4),x)*a*b**2*x**5 - 35*(a + b*x**2)**(3/4)*int(1/(
(a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*b**3*x**5)/(300*(a
+ b*x**2)**(3/4)*a*x**5)
```

3.1142 $\int \frac{(a+bx^2)^{5/8}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{5/8}}{x^8} dx = -\frac{(a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

output
```
-1/7*(b*x^2+a)^(5/8)*hypergeom([-7/2, -5/8],[-5/2],-b*x^2/a)/x^7/(1+b*x^2/
a)^(5/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{5/8}}{x^8} dx = -\frac{(a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(5/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(5/8)*Hypergeometric2F1[-7/2, -5/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/8}}{x^8} \, dx$$

$$\Big\downarrow \ \textbf{279}$$

$$\frac{(a+bx^2)^{5/8} \int \frac{\left(\frac{bx^2}{a}+1\right)^{5/8}}{x^8}dx}{\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

$$\Big\downarrow \ \textbf{278}$$

$$-\frac{(a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(\frac{bx^2}{a}+1\right)^{5/8}}$$

input
```
Int[(a + b*x^2)^(5/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(5/8)*Hypergeometric2F1[-7/2, -5/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{5}{8}}}{x^8}dx$$

input
```
int((b*x^2+a)^(5/8)/x^8,x)
```

output
```
int((b*x^2+a)^(5/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(bx^2+a)^{\frac{5}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{5/8}}{x^8}\,dx=-\frac{a^{\frac{5}{8}}{}_2F_1\left(\begin{matrix} -\frac{7}{2},-\frac{5}{8} \\ -\frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(5/8)/x**8,x)
```

output
```
-a**(5/8)*hyper((-7/2, -5/8), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^8} \, dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^8} \, dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^8} \, dx = \int \frac{(bx^2+a)^{\frac{5}{8}}}{x^8} \, dx$$

input
```
integrate((b*x^2+a)^(5/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(5/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(b\,x^2+a)^{5/8}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(5/8)/x^8,x)
```

output
```
int((a + b*x^2)^(5/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{5/8}}{x^8}\,dx=\frac{-64(b\,x^2+a)^{\frac{3}{8}}\,a^3-96(b\,x^2+a)^{\frac{3}{8}}\,a^2b\,x^2-16(b\,x^2+a)^{\frac{3}{8}}\,a\,b^2x^4+140(b\,x^2+a)^{\frac{3}{8}}\,b^3x^6}{}$$

input
```
int((b*x^2+a)^(5/8)/x^8,x)
```

output
```
( - 64*(a + b*x**2)**(3/8)*a**3 - 96*(a + b*x**2)**(3/8)*a**2*b*x**2 - 16*
(a + b*x**2)**(3/8)*a*b**2*x**4 + 140*(a + b*x**2)**(3/8)*b**3*x**6 + 80*(
a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**8 + (a + b*x**2)**(3/8)
*b*x**10),x)*a**4*x**7 - 80*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)
*a*x**6 + (a + b*x**2)**(3/8)*b*x**8),x)*a**3*b*x**7 - 240*(a + b*x**2)**(
3/4)*int(1/((a + b*x**2)**(3/8)*a*x**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a*
*2*b**2*x**7 + 80*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 +
(a + b*x**2)**(3/8)*b*x**4),x)*a*b**3*x**7 + 315*(a + b*x**2)**(3/4)*int(1
/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*b**4*x**7)/(528*(
a + b*x**2)**(3/4)*a**2*x**7)
```

3.1143 $\int x^6(a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^6(a+bx^2)^{7/8}\,dx=\frac{x^7(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
1/7*x^7*(b*x^2+a)^(7/8)*hypergeom([-7/8, 7/2],[9/2],-b*x^2/a)/(1+b*x^2/a)^
(7/8)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^6(a+bx^2)^{7/8}\,dx=\frac{x^7(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^6*(a + b*x^2)^(7/8),x]
```

output
```
(x^7*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(a+bx^2\right)^{7/8} dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(a+bx^2\right)^{7/8} \int x^6 \left(\frac{bx^2}{a}+1\right)^{7/8} dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^7 \left(a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7 \left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[x^6*(a + b*x^2)^(7/8),x]
```

output
```
(x^7*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 7/2, 9/2, -((b*x^2)/a)])/(7
*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6} \left(b\,x^{2}+a \right)^{\frac{7}{8}}d x$$

input
```
int(x^6*(b*x^2+a)^(7/8),x)
```

output
```
int(x^6*(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int x^6 \left(a+bx^2\right)^{7/8}\,dx=\int {\left(bx^{2}+a\right)}^{\frac{7}{8}}x^{6}\,dx$$

input
```
integrate(x^6*(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^6 \left(a+bx^2\right)^{7/8}\,dx=\frac{a^{\frac{7}{8}}x^{7}{}_{2}F_{1}\left(\begin{matrix}-\frac{7}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2+a)**(7/8),x)
```

output
```
a**(7/8)*x**7*hyper((-7/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a + bx^2)^{7/8} \, dx = \int (bx^2 + a)^{\frac{7}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^2)^{7/8} \, dx = \int (bx^2 + a)^{\frac{7}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^2)^{7/8} \, dx = \int x^6 \, (b\,x^2 + a)^{7/8} \, dx$$

input
```
int(x^6*(a + b*x^2)^(7/8),x)
```

output
```
int(x^6*(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int x^6 \left(a+bx^2\right)^{7/8} \, dx = \int \left(b\,x^2+a\right)^{\frac{7}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(7/8)*x**6,x)
```

3.1144 $\int x^4(a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^4\left(a+bx^2\right)^{7/8}\,dx=\frac{x^5(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
1/5*x^5*(b*x^2+a)^(7/8)*hypergeom([-7/8, 5/2],[7/2],-b*x^2/a)/(1+b*x^2/a)^
(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^4\left(a+bx^2\right)^{7/8}\,dx=\frac{x^5(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^4*(a + b*x^2)^(7/8),x]
```

output
```
(x^5*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+bx^2\right)^{7/8} dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{7/8} \int x^4 \left(\frac{bx^2}{a}+1\right)^{7/8} dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5 \left(a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5 \left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[x^4*(a + b*x^2)^(7/8),x]
```

output
```
(x^5*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 5/2, 7/2, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4} \left(b \,x^{2}+a \right)^{\frac{7}{8}}d x$$

input
```
int(x^4*(b*x^2+a)^(7/8),x)
```

output
```
int(x^4*(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^{7/8} \, dx = \int { {\left(b x^{2} + a\right)}^{\frac{7}{8}} x^{4} \,d x }$$

input
```
integrate(x^4*(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^4 \left(a+bx^2\right)^{7/8} \, dx = \frac{a^{\frac{7}{8}} x^{5} {{}_{2}F_{1}\left(\begin{matrix} - \frac{7}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}}{5}$$

input
```
integrate(x**4*(b*x**2+a)**(7/8),x)
```

output
```
a**(7/8)*x**5*hyper((-7/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4(a+bx^2)^{7/8}\,dx=\int (bx^2+a)^{\frac{7}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^4, x)
```

Giac [F]

$$\int x^4(a+bx^2)^{7/8}\,dx=\int (bx^2+a)^{\frac{7}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4(a+bx^2)^{7/8}\,dx=\int x^4\,\left(b\,x^2+a\right)^{7/8}\,dx$$

input
```
int(x^4*(a + b*x^2)^(7/8),x)
```

output
```
int(x^4*(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int x^4 (a + bx^2)^{7/8} \, dx = \int (b\,x^2 + a)^{\frac{7}{8}} \, x^4 dx$$

input
```
int(x^4*(b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(7/8)*x**4,x)
```

3.1145 $\int x^2(a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^2\left(a+bx^2\right)^{7/8}\,dx=\frac{x^3(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
1/3*x^3*(b*x^2+a)^(7/8)*hypergeom([-7/8, 3/2],[5/2],-b*x^2/a)/(1+b*x^2/a)^
(7/8)
```

Mathematica [A] (verified)

Time = 9.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)^{7/8}\,dx=\frac{x^3(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^2*(a + b*x^2)^(7/8),x]
```

output
```
(x^3*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+bx^2\right)^{7/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{7/8} \int x^2 \left(\frac{bx^2}{a}+1\right)^{7/8}dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3 \left(a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3 \left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[x^2*(a + b*x^2)^(7/8),x]
```

output
```
(x^3*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 3/2, 5/2, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^2\left(b\,x^2+a\right)^{\frac{7}{8}}dx$$

input
```
int(x^2*(b*x^2+a)^(7/8),x)
```

output
```
int(x^2*(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int x^2(a+bx^2)^{7/8}\,dx=\int\left(bx^2+a\right)^{\frac{7}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.57

$$\int x^2(a+bx^2)^{7/8}\,dx=\frac{a^{\frac{7}{8}}x^3{}_2F_1\left(\begin{matrix}-\frac{7}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**(7/8),x)
```

output
```
a**(7/8)*x**3*hyper((-7/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 \left(a+bx^2\right)^{7/8} dx = \int \left(bx^2+a\right)^{\frac{7}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^2, x)
```

Giac [F]

$$\int x^2 \left(a+bx^2\right)^{7/8} dx = \int \left(bx^2+a\right)^{\frac{7}{8}} x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a+bx^2\right)^{7/8} dx = \int x^2 \left(b\,x^2+a\right)^{7/8} dx$$

input
```
int(x^2*(a + b*x^2)^(7/8),x)
```

output
```
int(x^2*(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int x^2 (a+bx^2)^{7/8} \, dx = \int (b\,x^2+a)^{\frac{7}{8}} x^2 dx$$

input
```
int(x^2*(b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(7/8)*x**2,x)
```

3.1146 $\int (a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int (a+bx^2)^{7/8}\,dx=\frac{x(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
x*(b*x^2+a)^(7/8)*hypergeom([-7/8, 1/2],[3/2],-b*x^2/a)/(1+b*x^2/a)^(7/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^{7/8}\,dx=\frac{x(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8),x]
```

output
```
(x*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(7/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{7/8} dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(a+bx^2\right)^{7/8} \int \left(\frac{bx^2}{a}+1\right)^{7/8} dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8),x]
```

output
```
(x*(a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, -((b*x^2)/a)])/(1 +
 (b*x^2)/a)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{\frac{7}{8}}dx$$

input
```
int((b*x^2+a)^(7/8),x)
```

output
```
int((b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^{7/8}\,dx=\int \left(bx^{2}+a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57

$$\int \left(a+bx^2\right)^{7/8}\,dx=a^{\frac{7}{8}}x\,{}_{2}F_{1}\left(\begin{matrix} -\frac{7}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**(7/8),x)
```

output
```
a**(7/8)*x*hyper((-7/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^{7/8} \, dx = \int \left(bx^2+a\right)^{\frac{7}{8}} \, dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8), x)
```

Giac [F]

$$\int \left(a+bx^2\right)^{7/8} \, dx = \int \left(bx^2+a\right)^{\frac{7}{8}} \, dx$$

input
```
integrate((b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \left(a+bx^2\right)^{7/8} \, dx = \frac{x\left(b\,x^2+a\right)^{7/8}\,{}_2\mathrm{F}_1\left(-\frac{7}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{7/8}}$$

input
```
int((a + b*x^2)^(7/8),x)
```

output
```
(x*(a + b*x^2)^(7/8)*hypergeom([-7/8, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a +
 1)^(7/8)
```

Reduce [F]

$$\int \left(a+bx^2\right)^{7/8} \, dx = \int \left(b\,x^2+a\right)^{\frac{7}{8}}dx$$

input
```
int((b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(7/8),x)
```

3.1147 $\int \frac{(a+bx^2)^{7/8}}{x^2} dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^{7/8}}{x^2} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
-(b*x^2+a)^(7/8)*hypergeom([-7/8, -1/2],[1/2],-b*x^2/a)/x/(1+b*x^2/a)^(7/8
)
```

Mathematica [A] (verified)

Time = 9.68 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{7/8}}{x^2} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(7/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{(a+bx^2)^{7/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{7/8}}{x^2}dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},-\frac{1}{2},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8)/x^2,x]
```

output
```
-(((a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, -1/2, 1/2, -((b*x^2)/a)])/(x*
(1 + (b*x^2)/a)^(7/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{8}}}{x^2}dx$$

input
```
int((b*x^2+a)^(7/8)/x^2,x)
```

output
```
int((b*x^2+a)^(7/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = -\frac{a^{\frac{7}{8}}{}_2F_1\left(\begin{matrix} -\frac{7}{8}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**(7/8)/x**2,x)
```

output
```
-a**(7/8)*hyper((-7/8, -1/2), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = \frac{4\,(b\,x^2+a)^{7/8}\,{}_2F_1\left(-\frac{7}{8},-\frac{3}{8};\ \frac{5}{8};\ -\frac{a}{b\,x^2}\right)}{3\,x\,\left(\frac{a}{b\,x^2}+1\right)^{7/8}}$$

input
```
int((a + b*x^2)^(7/8)/x^2,x)
```

output
```
(4*(a + b*x^2)^(7/8)*hypergeom([-7/8, -3/8], 5/8, -a/(b*x^2)))/(3*x*(a/(b*
x^2) + 1)^(7/8))
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^2}\,dx = \frac{-56(b\,x^2+a)^{\frac{5}{8}}\,a-56(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2+25(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2+a}}{(b\,x^2+a)^{\frac{5}{8}}\,a\,x^2+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^4}dx\right)}{81\,(b\,x^2+a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2+a)^(7/8)/x^2,x)
```

output
```
( - 56*(a + b*x**2)**(5/8)*a - 56*(a + b*x**2)**(5/8)*b*x**2 + 25*(a + b*x
**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2
)**(5/8)*b*x**4),x)*a**2*x - 25*(a + b*x**2)**(3/4)*int((sqrt(a + b*x**2)*
x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**2*x)/(81*
(a + b*x**2)**(3/4)*x)
```

3.1148 $\int \frac{(a+bx^2)^{7/8}}{x^4} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{7/8}}{x^4} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
-1/3*(b*x^2+a)^(7/8)*hypergeom([-3/2, -7/8],[-1/2],-b*x^2/a)/x^3/(1+b*x^2/
a)^(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{7/8}}{x^4} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(7/8)*Hypergeometric2F1[-3/2, -7/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{(a+bx^2)^{7/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{7/8}}{x^4}dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{(a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8)/x^4,x]
```

output
```
-1/3*((a + b*x^2)^(7/8)*Hypergeometric2F1[-3/2, -7/8, -1/2, -((b*x^2)/a)])
/(x^3*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}+a)^{\frac{7}{8}}}{x^{4}}dx$$

input
```
int((b*x^2+a)^(7/8)/x^4,x)
```

output
```
int((b*x^2+a)^(7/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx=\int \frac{(bx^2+a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx=-\frac{a^{\frac{7}{8}}{}_{2}F_{1}\left(\begin{matrix}-\frac{3}{2},-\frac{7}{8}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2+a)**(7/8)/x**4,x)
```

output
```
-a**(7/8)*hyper((-3/2, -7/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^4, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx = \int \frac{(b\,x^2+a)^{7/8}}{x^4}\,dx$$

input
```
int((a + b*x^2)^(7/8)/x^4,x)
```

output
```
int((a + b*x^2)^(7/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^4}\,dx = \frac{-612(b\,x^2+a)^{\frac{5}{8}}\,a^2-892(b\,x^2+a)^{\frac{5}{8}}\,ab\,x^2-280(b\,x^2+a)^{\frac{5}{8}}\,b^2x^4-865(b\,x^2+a)^{\frac{3}{4}}\left(\int\right.}{}$$

input
```
int((b*x^2+a)^(7/8)/x^4,x)
```

output
```
( - 612*(a + b*x**2)**(5/8)*a**2 - 892*(a + b*x**2)**(5/8)*a*b*x**2 - 280*
(a + b*x**2)**(5/8)*b**2*x**4 - 865*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**
2)/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*a**2*b*x**
3 - 1215*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a +
 (a + b*x**2)**(5/8)*b*x**2),x)*a*b**2*x**3 - 350*(a + b*x**2)**(3/4)*int(
(sqrt(a + b*x**2)*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**
2),x)*b**3*x**3)/(1836*(a + b*x**2)**(3/4)*a*x**3)
```

3.1149 $\int \frac{\left(a+bx^2\right)^{7/8}}{x^6}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\left(a+bx^2\right)^{7/8}}{x^6}\,dx = -\frac{\left(a+bx^2\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
-1/5*(b*x^2+a)^(7/8)*hypergeom([-5/2, -7/8],[-3/2],-b*x^2/a)/x^5/(1+b*x^2/
a)^(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+bx^2\right)^{7/8}}{x^6}\,dx = -\frac{\left(a+bx^2\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(7/8)*Hypergeometric2F1[-5/2, -7/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{7/8}}{x^6}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(a+bx^2\right)^{7/8}\int \frac{\left(\frac{bx^2}{a}+1\right)^{7/8}}{x^6}dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(a+bx^2\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8)/x^6,x]
```

output
```
-1/5*((a + b*x^2)^(7/8)*Hypergeometric2F1[-5/2, -7/8, -3/2, -((b*x^2)/a)])
/(x^5*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{8}}}{x^6}dx$$

input
```
int((b*x^2+a)^(7/8)/x^6,x)
```

output
```
int((b*x^2+a)^(7/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx = -\frac{a^{\frac{7}{8}}{}_2F_1\left(\begin{matrix} -\frac{5}{2},-\frac{7}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2+a)**(7/8)/x**6,x)
```

output
```
-a**(7/8)*hyper((-5/2, -7/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^6, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx=\int \frac{(b\,x^2+a)^{7/8}}{x^6}\,dx$$

input
```
int((a + b*x^2)^(7/8)/x^6,x)
```

output
```
int((a + b*x^2)^(7/8)/x^6, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^6}\,dx=\frac{-1500(b\,x^2+a)^{\frac{5}{8}}\,a^2-1748(b\,x^2+a)^{\frac{5}{8}}\,ab\,x^2-248(b\,x^2+a)^{\frac{5}{8}}\,b^2x^4-2319(b\,x^2+a)^{\frac{3}{4}}}{}$$

input
```
int((b*x^2+a)^(7/8)/x^6,x)
```

output
```
( - 1500*(a + b*x**2)**(5/8)*a**2 - 1748*(a + b*x**2)**(5/8)*a*b*x**2 - 24
8*(a + b*x**2)**(5/8)*b**2*x**4 - 2319*(a + b*x**2)**(3/4)*int(sqrt(a + b*
x**2)/((a + b*x**2)**(5/8)*a*x**4 + (a + b*x**2)**(5/8)*b*x**6),x)*a**2*b*
x**5 - 3125*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*
a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*a*b**2*x**5 - 806*(a + b*x**2)**(3
/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x*
*2),x)*b**3*x**5)/(7500*(a + b*x**2)**(3/4)*a*x**5)
```

3.1150 $\int \frac{(a+bx^2)^{7/8}}{x^8} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^{7/8}}{x^8} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

output
```
-1/7*(b*x^2+a)^(7/8)*hypergeom([-7/2, -7/8],[-5/2],-b*x^2/a)/x^7/(1+b*x^2/
a)^(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^{7/8}}{x^8} dx = -\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(1+\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(7/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(7/8)*Hypergeometric2F1[-7/2, -7/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{7/8}}{x^8} dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(a+bx^2)^{7/8} \int \frac{\left(\frac{bx^2}{a}+1\right)^{7/8}}{x^8} dx}{\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{8},-\frac{5}{2},-\frac{bx^2}{a}\right)}{7x^7\left(\frac{bx^2}{a}+1\right)^{7/8}}$$

input
```
Int[(a + b*x^2)^(7/8)/x^8,x]
```

output
```
-1/7*((a + b*x^2)^(7/8)*Hypergeometric2F1[-7/2, -7/8, -5/2, -((b*x^2)/a)])
/(x^7*(1 + (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^{\frac{7}{8}}}{x^8}dx$$

input
```
int((b*x^2+a)^(7/8)/x^8,x)
```

output
```
int((b*x^2+a)^(7/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx=\int \frac{(bx^2+a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx=-\frac{a^{\frac{7}{8}}{}_2F_1\left(\begin{matrix} -\frac{7}{2},-\frac{7}{8} \\ -\frac{5}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2+a)**(7/8)/x**8,x)
```

output
```
-a**(7/8)*hyper((-7/2, -7/8), (-5/2,), b*x**2*exp_polar(I*pi)/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^8, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(bx^2+a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2+a)^(7/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(7/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(b\,x^2+a)^{7/8}}{x^8}\,dx$$

input
```
int((a + b*x^2)^(7/8)/x^8,x)
```

output
```
int((a + b*x^2)^(7/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^{7/8}}{x^8}\,dx = \frac{-584496(b\,x^2+a)^{\frac{5}{8}}\,a^4 - 645840(b\,x^2+a)^{\frac{5}{8}}\,a^3b\,x^2 + 362268(b\,x^2+a)^{\frac{5}{8}}\,a^2b^2x^4 - 1882}{}$$

input
```
int((b*x^2+a)^(7/8)/x^8,x)
```

output
```
( - 584496*(a + b*x**2)**(5/8)*a**4 - 645840*(a + b*x**2)**(5/8)*a**3*b*x*
*2 + 362268*(a + b*x**2)**(5/8)*a**2*b**2*x**4 - 188272*(a + b*x**2)**(5/8
)*a*b**3*x**6 - 611884*(a + b*x**2)**(5/8)*b**4*x**8 - 948780*(a + b*x**2)
**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**6 + (a + b*x**2)**(
5/8)*b*x**8),x)*a**4*b*x**7 + 1713635*(a + b*x**2)**(3/4)*int(sqrt(a + b*x
**2)/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*a**2*b**
3*x**7 - 764855*(a + b*x**2)**(3/4)*int((sqrt(a + b*x**2)*x**2)/((a + b*x*
*2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**5*x**7)/(4091472*(a + b*x
**2)**(3/4)*a**3*x**7)
```

3.1151 $\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^7\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{a+bx^2}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(1/8)*hypergeom([1/8, 7/2],[9/2],-b*x^2/a)/(b*x^2+a)^(
1/8)
```

Mathematica [A] (verified)

Time = 8.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^7\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(1/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^6}{\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\sqrt[8]{a+bx^2}}$$

input
```
Int[x^6/(a + b*x^2)^(1/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{1}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/8),x)
```

output
```
int(x^6/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx=\int \frac{x^6}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{1}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7\sqrt[8]{a}}$$

input
```
integrate(x**6/(b*x**2+a)**(1/8),x)
```

output
```
x**7*hyper((1/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(1/8))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^6}{{\left(b\,x^2+a\right)}^{1/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(1/8),x)
```

output
```
int(x^6/(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^6}{\left(b\,x^2+a\right)^{\frac{1}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(1/8),x)
```

output
```
int(x**6/(a + b*x**2)**(1/8),x)
```

3.1152 $\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^5\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{a+bx^2}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(1/8)*hypergeom([1/8, 5/2],[7/2],-b*x^2/a)/(b*x^2+a)^(
1/8)
```

Mathematica [A] (verified)

Time = 8.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^5\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(1/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^4}{\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\sqrt[8]{a+bx^2}}$$

input
```
Int[x^4/(a + b*x^2)^(1/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{1}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/8),x)
```

output
```
int(x^4/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{1}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[8]{a}}$$

input
```
integrate(x**4/(b*x**2+a)**(1/8),x)
```

output
```
x**5*hyper((1/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/8))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2+a\right)^{1/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(1/8),x)
```

output
```
int(x^4/(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2+a\right)^{\frac{1}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(1/8),x)
```

output
```
int(x**4/(a + b*x**2)**(1/8),x)
```

3.1153 $\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^3\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{a+bx^2}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(1/8)*hypergeom([1/8, 3/2],[5/2],-b*x^2/a)/(b*x^2+a)^(
1/8)
```

Mathematica [A] (verified)

Time = 7.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx=\frac{x^3\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(1/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^2}{\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\sqrt[8]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(1/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{1}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/8),x)
```

output
```
int(x^2/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \frac{x^3 {}_2F_1\left(\begin{matrix} \frac{1}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[8]{a}}$$

input
```
integrate(x**2/(b*x**2+a)**(1/8),x)
```

output
```
x**3*hyper((1/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/8))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^2}{{\left(b\,x^2+a\right)}^{1/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(1/8),x)
```

output
```
int(x^2/(a + b*x^2)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{x^2}{\left(b\,x^2+a\right)^{\frac{1}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(1/8),x)
```

output
```
int(x**2/(a + b*x**2)**(1/8),x)
```

3.1154 $\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx=\frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/8)*hypergeom([1/8, 1/2],[3/2],-b*x^2/a)/(b*x^2+a)^(1/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx=\frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[8]{a+bx^2}}dx$$
$$\Big\downarrow \text{238}$$
$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{1}{\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$
$$\Big\downarrow \text{237}$$
$$\frac{x\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{\sqrt[8]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(1/8),x)
```

output
```
int(1/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[8]{a}}$$

input
```
integrate(1/(b*x**2+a)**(1/8),x)
```

output
```
x*hyper((1/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(1/8)
```

Maxima [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-1/8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx=\frac{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{1/8}\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^2+a\right)}^{1/8}}$$

input
```
int(1/(a + b*x^2)^(1/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(1/8)*hypergeom([1/8, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(1/8)
```

Reduce [F]

$$\int \frac{1}{\sqrt[8]{a+bx^2}}\,dx=\int \frac{1}{\left(b\,x^2+a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(1/8),x)
```

output
```
int(1/(a + b*x**2)**(1/8),x)
```

3.1155 $\int \frac{1}{x^2 \sqrt[8]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{a+bx^2}}$$

output
```
-(1+b*x^2/a)^(1/8)*hypergeom([-1/2, 1/8],[1/2],-b*x^2/a)/x/(b*x^2+a)^(1/8)
```

Mathematica [A] (verified)

Time = 8.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(1/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 1/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(1/8)))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[8]{a+bx^2}}dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{1}{x^2\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \ 278$$

$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x\sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(1/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 1/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(1/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(1/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[8]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2\sqrt[8]{a+bx^2}}\,dx=-\frac{{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{8}\\ \frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{\sqrt[8]{a}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(1/8),x)
```

output
```
-hyper((-1/2, 1/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(1/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2\sqrt[8]{a+bx^2}}\,dx = -\frac{4\left(\frac{a}{b\,x^2}+1\right)^{1/8}{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{5}{8};\ \frac{13}{8};\ -\frac{a}{b\,x^2}\right)}{5\,x\,\left(b\,x^2+a\right)^{1/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(1/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(1/8)*hypergeom([1/8, 5/8], 13/8, -a/(b*x^2)))/(5*x*(a
 + b*x^2)^(1/8))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[8]{a+bx^2}}\,dx$$
$$= \frac{-36(b\,x^2+a)^{\frac{5}{8}}\,a-20(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2-45(b\,x^2+a)^{\frac{3}{4}}\left(\int\frac{\sqrt{b\,x^2+a}}{(b\,x^2+a)^{\frac{5}{8}}a+(b\,x^2+a)^{\frac{5}{8}}b\,x^2}dx\right)abx-25(b\,x^2+a)^{\frac{3}{4}}}{36\,(b\,x^2+a)^{\frac{3}{4}}\,ax}$$

input
```
int(1/x^2/(b*x^2+a)^(1/8),x)
```

output
```
( - 36*(a + b*x**2)**(5/8)*a - 20*(a + b*x**2)**(5/8)*b*x**2 - 45*(a + b*x
**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5
/8)*b*x**2),x)*a*b*x - 25*(a + b*x**2)**(3/4)*int((sqrt(a + b*x**2)*x**2)/
((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**2*x)/(36*(a + b
*x**2)**(3/4)*a*x)
```

3.1156 $\int \frac{1}{x^4 \sqrt[8]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^4 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 \sqrt[8]{a+bx^2}}$$

output
```
-1/3*(1+b*x^2/a)^(1/8)*hypergeom([-3/2, 1/8],[-1/2],-b*x^2/a)/x^3/(b*x^2+a
)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 \sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(1/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 1/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}dx$$
$$\Big\downarrow \ 279$$
$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{1}{x^4\sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$
$$\Big\downarrow \ 278$$
$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(1/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 1/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(1/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx=-\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2},\frac{1}{8} \\ -\frac{1}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3\sqrt[8]{a}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(1/8),x)
```

output
```
-hyper((-3/2, 1/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(1/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^{1/8}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(1/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(1/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt[8]{a+bx^2}}\,dx = \frac{-68(b\,x^2+a)^{\frac{5}{8}}\,a-20(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2-125(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2+a}}{(b\,x^2+a)^{\frac{5}{8}}\,a\,x^2+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^4}dx\right)a b\,x^3-65(b\,x^2+}{204\,(b\,x^2+a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(1/8),x)
```

output
```
( - 68*(a + b*x**2)**(5/8)*a - 20*(a + b*x**2)**(5/8)*b*x**2 - 125*(a + b*
x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**
2)**(5/8)*b*x**4),x)*a*b*x**3 - 65*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2
)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**2*x**3)/(204*
(a + b*x**2)**(3/4)*a*x**3)
```

3.1157 $\int \frac{1}{x^6 \sqrt[8]{a+bx^2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 \sqrt[8]{a+bx^2}}$$

output
```
-1/5*(1+b*x^2/a)^(1/8)*hypergeom([-5/2, 1/8],[-3/2],-b*x^2/a)/x^5/(b*x^2+a
)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}} \, dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 \sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(1/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 1/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}} dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{x^6 \sqrt[8]{\frac{bx^2}{a}+1}}dx}{\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 \sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(1/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 1/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a \right)^{\frac{1}{8}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(1/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{1}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5\sqrt[8]{a}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(1/8),x)
```

output
```
-hyper((-5/2, 1/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(1/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}} x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{1}{8}} x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(1/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = \int \frac{1}{x^6\,(b\,x^2+a)^{1/8}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(1/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(1/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \sqrt[8]{a+bx^2}}\,dx = \frac{-2160(b\,x^2+a)^{\frac{5}{8}}\,a^3 + 1044(b\,x^2+a)^{\frac{5}{8}}\,a^2 b\,x^2 - 656(b\,x^2+a)^{\frac{5}{8}}\,a\,b^2x^4 - 2132(b\,x^2+a)^{\frac{5}{8}}\,b^3x^6 + 4825(b\,x}{10800\,(b\,x^2+}$$

input
```
int(1/x^6/(b*x^2+a)^(1/8),x)
```

output
```
( - 2160*(a + b*x**2)**(5/8)*a**3 + 1044*(a + b*x**2)**(5/8)*a**2*b*x**2 -
 656*(a + b*x**2)**(5/8)*a*b**2*x**4 - 2132*(a + b*x**2)**(5/8)*b**3*x**6
+ 4825*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**
2 + (a + b*x**2)**(5/8)*b*x**4),x)*a**2*b**2*x**5 - 2665*(a + b*x**2)**(3/
4)*int((sqrt(a + b*x**2)*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8
)*b*x**2),x)*b**4*x**5)/(10800*(a + b*x**2)**(3/4)*a**3*x**5)
```

3.1158 $\int \frac{x^6}{\left(a+bx^2\right)^{3/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/8}}\,dx=\frac{x^7\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(a+bx^2\right)^{3/8}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(3/8)*hypergeom([3/8, 7/2],[9/2],-b*x^2/a)/(b*x^2+a)^(
3/8)
```

Mathematica [A] (verified)

Time = 8.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{\left(a+bx^2\right)^{3/8}}\,dx=\frac{x^7\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(a+bx^2\right)^{3/8}}$$

input
```
Integrate[x^6/(a + b*x^2)^(3/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{3/8}} \, dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{x^6}{\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^7 \left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\,(a+bx^2)^{3/8}}$$

input
```
Int[x^6/(a + b*x^2)^(3/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{3}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(3/8),x)
```

output
```
int(x^6/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^{6}}{{\left(bx^{2}+a\right)}^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \frac{x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{3}{8},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7a^{\frac{3}{8}}}$$

input
```
integrate(x**6/(b*x**2+a)**(3/8),x)
```

output
```
x**7*hyper((3/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(3/8))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{3/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(3/8),x)
```

output
```
int(x^6/(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(3/8),x)
```

output
```
int(x**6/(a + b*x**2)**(3/8),x)
```

3.1159 $\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx=\frac{x^5\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{3/8}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(3/8)*hypergeom([3/8, 5/2],[7/2],-b*x^2/a)/(b*x^2+a)^(
3/8)
```

Mathematica [A] (verified)

Time = 8.82 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx=\frac{x^5\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{3/8}}$$

input
```
Integrate[x^4/(a + b*x^2)^(3/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{x^4}{\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{3/8}}$$

input
```
Int[x^4/(a + b*x^2)^(3/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(3/8),x)
```

output
```
int(x^4/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{8}}}$$

input
```
integrate(x**4/(b*x**2+a)**(3/8),x)
```

output
```
x**5*hyper((3/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/8))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{3/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(3/8),x)
```

output
```
int(x^4/(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(3/8),x)
```

output
```
int(x**4/(a + b*x**2)**(3/8),x)
```

3.1160 $\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx=\frac{x^3\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3(a+bx^2)^{3/8}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(3/8)*hypergeom([3/8, 3/2],[5/2],-b*x^2/a)/(b*x^2+a)^(
3/8)
```

Mathematica [A] (verified)

Time = 8.49 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx=\frac{x^3\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3(a+bx^2)^{3/8}}$$

input
```
Integrate[x^2/(a + b*x^2)^(3/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx$$
$$\Big\downarrow \ 279$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$
$$\Big\downarrow \ 278$$
$$\frac{x^3\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3(a+bx^2)^{3/8}}$$

input
```
Int[x^2/(a + b*x^2)^(3/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(3/8),x)
```

output
```
int(x^2/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{8}}}$$

input
```
integrate(x**2/(b*x**2+a)**(3/8),x)
```

output
```
x**3*hyper((3/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/8))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{3/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(3/8),x)
```

output
```
int(x^2/(a + b*x^2)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(3/8),x)
```

output
```
int(x**2/(a + b*x**2)**(3/8),x)
```

3.1161 $\int \frac{1}{(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

output
```
x*(1+b*x^2/a)^(3/8)*hypergeom([3/8, 1/2],[3/2],-b*x^2/a)/(b*x^2+a)^(3/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/8)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx$$
$$\Big\downarrow \ 238$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$
$$\Big\downarrow \ 237$$
$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{3/8}}$$

input
```
Int[(a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(3/8),x)
```

output
```
int(1/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{3}{8}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{3}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(3/8),x)
```

output
```
x*hyper((3/8, 1/2), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(3/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-3/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{3/8}\,{}_2\mathrm{F}_1\left(\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{3/8}}$$

input
```
int(1/(a + b*x^2)^(3/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(3/8)*hypergeom([3/8, 1/2], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(3/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{3/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(3/8),x)
```

output
```
int(1/(a + b*x**2)**(3/8),x)
```

3.1162 $\int \frac{1}{x^2(a+bx^2)^{3/8}} dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2(a+bx^2)^{3/8}} dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x(a+bx^2)^{3/8}}$$

output
```
-(1+b*x^2/a)^(3/8)*hypergeom([-1/2, 3/8],[1/2],-b*x^2/a)/x/(b*x^2+a)^(3/8)
```

Mathematica [A] (verified)

Time = 8.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^2)^{3/8}} dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x(a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(3/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 3/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(3/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}} \, dx$$
$$\Big\downarrow \ 279$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^2\left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$
$$\Big\downarrow \ 278$$
$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(3/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 3/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(3/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{3}{8}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(3/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}}\,dx=-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{3}{8} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{a^{\frac{3}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(3/8),x)
```

output
```
-hyper((-1/2, 3/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(3/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}}\,dx = -\frac{4 \left(\frac{a}{b\,x^2}+1\right)^{3/8} {}_2\mathrm{F}_1\left(\frac{3}{8},\frac{7}{8};\ \frac{15}{8};\ -\frac{a}{b\,x^2}\right)}{7\,x\,\left(b\,x^2+a\right)^{3/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(3/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(3/8)*hypergeom([3/8, 7/8], 15/8, -a/(b*x^2)))/(7*x*(a
 + b*x^2)^(3/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{3/8}}\,dx = \frac{-36(b\,x^2+a)^{\frac{3}{8}}\,a-20(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2+5(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}a+(b\,x^2+a)^{\frac{3}{8}}b\,x^2}dx\right)}{36\left(b\,x^2+a\right)^{\frac{3}{4}}ax}$$

input
```
int(1/x^2/(b*x^2+a)^(3/8),x)
```

output
```
( - 36*(a + b*x**2)**(3/8)*a - 20*(a + b*x**2)**(3/8)*b*x**2 + 5*(a + b*x*
*2)**(3/4)*int(x**2/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x
)*b**2*x - 15*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a + b*x*
*2)**(3/8)*b*x**2),x)*a*b*x)/(36*(a + b*x**2)**(3/4)*a*x)
```

3.1163 $\int \frac{1}{x^4(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{3/8}}$$

output
```
-1/3*(1+b*x^2/a)^(3/8)*hypergeom([-3/2, 3/8],[-1/2],-b*x^2/a)/x^3/(b*x^2+a
)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(3/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 3/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^4 \left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(3/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 3/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{3}{8}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(3/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{3}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{3}{8}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(3/8),x)
```

output
```
-hyper((-3/2, 3/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(3/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}} \, dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{3/8}} \,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(3/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(3/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{3/8}} \, dx = \frac{-68(b\,x^2+a)^{\frac{3}{8}}\,a-20(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2-95(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}\,a\,x^2+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^4}d\right.}{204\,(b\,x^2+a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(3/8),x)
```

output
```
( - 68*(a + b*x**2)**(3/8)*a - 20*(a + b*x**2)**(3/8)*b*x**2 - 95*(a + b*x
**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*x**2)**(3/8)*b*x**4
),x)*a*b*x**3 - 35*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a +
 b*x**2)**(3/8)*b*x**2),x)*b**2*x**3)/(204*(a + b*x**2)**(3/4)*a*x**3)
```

3.1164 $\int \frac{1}{x^6(a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 (a+bx^2)^{3/8}}$$

output
```
-1/5*(1+b*x^2/a)^(3/8)*hypergeom([-5/2, 3/8],[-3/2],-b*x^2/a)/x^5/(b*x^2+a
)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 (a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(3/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 3/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}} \, dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^6 \left(\frac{bx^2}{a}+1\right)^{3/8}}dx}{(a+bx^2)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(3/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 3/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a \right)^{\frac{3}{8}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(3/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{3}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{3/8}}\,dx=-\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{3}{8} \\ -\frac{3}{2} \end{matrix}\middle|\, \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{3}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(3/8),x)
```

output
```
-hyper((-5/2, 3/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(3/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{3}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(3/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{3/8}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(3/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(3/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{3/8}}\,dx = \frac{-80(b\,x^2+a)^{\frac{3}{8}}\,a^2-16(b\,x^2+a)^{\frac{3}{8}}\,ab\,x^2+140(b\,x^2+a)^{\frac{3}{8}}\,b^2x^4-140(b\,x^2+a)^{\frac{3}{4}}\left(\int\right.}{}$$

input
```
int(1/x^6/(b*x^2+a)^(3/8),x)
```

output
```
( - 80*(a + b*x**2)**(3/8)*a**2 - 16*(a + b*x**2)**(3/8)*a*b*x**2 + 140*(a
 + b*x**2)**(3/8)*b**2*x**4 - 140*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)*
*(3/8)*a*x**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a**2*b*x**5 + 80*(a + b*x**
2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*x**2)**(3/8)*b*x**4),
x)*a*b**2*x**5 + 315*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a
 + b*x**2)**(3/8)*b*x**2),x)*b**3*x**5)/(400*(a + b*x**2)**(3/4)*a**2*x**5
)
```

3.1165 $\int \frac{x^6}{(a+bx^2)^{5/8}} dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^6}{(a+bx^2)^{5/8}} dx = \frac{x^7\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7(a+bx^2)^{5/8}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(5/8)*hypergeom([5/8, 7/2],[9/2],-b*x^2/a)/(b*x^2+a)^(
5/8)
```

Mathematica [A] (verified)

Time = 8.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^2)^{5/8}} dx = \frac{x^7\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7(a+bx^2)^{5/8}}$$

input
```
Integrate[x^6/(a + b*x^2)^(5/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{x^6}{\left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\,(a+bx^2)^{5/8}}$$

input
```
Int[x^6/(a + b*x^2)^(5/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{6}}{\left(b\,x^{2}+a\right)^{\frac{5}{8}}}d x$$

input
```
int(x^6/(b*x^2+a)^(5/8),x)
```

output
```
int(x^6/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx=\int \frac{x^{6}}{(bx^{2}+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx=\frac{x^{7}{}_{2}F_{1}\left(\begin{matrix}\frac{5}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^{2}e^{i\pi}}{a}\right)}{7a^{\frac{5}{8}}}$$

input
```
integrate(x**6/(b*x**2+a)**(5/8),x)
```

output
```
x**7*hyper((5/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(5/8))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{5/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(5/8),x)
```

output
```
int(x^6/(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(5/8),x)
```

output
```
int(x**6/(a + b*x**2)**(5/8),x)
```

3.1166 $\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \frac{x^5\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5(a+bx^2)^{5/8}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(5/8)*hypergeom([5/8, 5/2],[7/2],-b*x^2/a)/(b*x^2+a)^(
5/8)
```

Mathematica [A] (verified)

Time = 8.84 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \frac{x^5\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5(a+bx^2)^{5/8}}$$

input
```
Integrate[x^4/(a + b*x^2)^(5/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{5/8}}dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{x^4}{\left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$
$$\Big\downarrow \text{278}$$
$$\frac{x^5\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5(a+bx^2)^{5/8}}$$

input
```
Int[x^4/(a + b*x^2)^(5/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(5/8),x)
```

output
```
int(x^4/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \frac{x^5\,{}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{8}}}$$

input
```
integrate(x**4/(b*x**2+a)**(5/8),x)
```

output
```
x**5*hyper((5/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/8))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{5/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(5/8),x)
```

output
```
int(x^4/(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(5/8),x)
```

output
```
int(x**4/(a + b*x**2)**(5/8),x)
```

3.1167 $\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \frac{x^3\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\,(a+bx^2)^{5/8}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(5/8)*hypergeom([5/8, 3/2],[5/2],-b*x^2/a)/(b*x^2+a)^(
5/8)
```

Mathematica [A] (verified)

Time = 8.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \frac{x^3\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\,(a+bx^2)^{5/8}}$$

input
```
Integrate[x^2/(a + b*x^2)^(5/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3(a+bx^2)^{5/8}}$$

input
```
Int[x^2/(a + b*x^2)^(5/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(5/8),x)
```

output
```
int(x^2/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \frac{x^3{}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{8}}}$$

input
```
integrate(x**2/(b*x**2+a)**(5/8),x)
```

output
```
x**3*hyper((5/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/8))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{5/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(5/8),x)
```

output
```
int(x^2/(a + b*x^2)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(5/8),x)
```

output
```
int(x**2/(a + b*x**2)**(5/8),x)
```

3.1168 $\int \frac{1}{(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

output
```
x*(1+b*x^2/a)^(5/8)*hypergeom([1/2, 5/8],[3/2],-b*x^2/a)/(b*x^2+a)^(5/8)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

input
```
Integrate[(a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx$$
$$\Big\downarrow \ 238$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$
$$\Big\downarrow \ 237$$
$$\frac{x\left(\frac{bx^2}{a}+1\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{5/8}}$$

input
```
Int[(a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{5}{8}}}d x$$

input
```
int(1/(b*x^2+a)^(5/8),x)
```

output
```
int(1/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx=\frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(5/8),x)
```

output
```
x*hyper((1/2, 5/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(5/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-5/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{5/8}\,{}_2F_1\left(\frac{1}{2},\frac{5}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{5/8}}$$

input
```
int(1/(a + b*x^2)^(5/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(5/8)*hypergeom([1/2, 5/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(5/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{5/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(5/8),x)
```

output
```
int(1/(a + b*x**2)**(5/8),x)
```

3.1169 $\int \frac{1}{x^2(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{5/8}}$$

output
```
-(1+b*x^2/a)^(5/8)*hypergeom([-1/2, 5/8],[1/2],-b*x^2/a)/x/(b*x^2+a)^(5/8)
```

Mathematica [A] (verified)

Time = 8.57 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{5/8}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(5/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-1/2, 5/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(5/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}} \, dx$$
$$\Big\downarrow \ 279$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \int \frac{1}{x^2 \left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$
$$\Big\downarrow \ 278$$
$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{5/8}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(5/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-1/2, 5/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(5/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\,x^2+a\right)^{\frac{5}{8}}}dx$$

input
```
int(1/x^2/(b*x^2+a)^(5/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{5}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{5}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(5/8),x)
```

output
```
-hyper((-1/2, 5/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(5/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = -\frac{4\left(\frac{a}{b\,x^2}+1\right)^{5/8}{}_2\mathrm{F}_1\left(\frac{5}{8},\frac{9}{8};\ \frac{17}{8};\ -\frac{a}{b\,x^2}\right)}{9\,x\,(b\,x^2+a)^{5/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(5/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(5/8)*hypergeom([5/8, 9/8], 17/8, -a/(b*x^2)))/(9*x*(a
 + b*x^2)^(5/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{5/8}}\,dx = \frac{-36(b\,x^2+a)^{\frac{1}{8}}\,a-20(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2-45(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{5}{8}}a+(b\,x^2+a)^{\frac{5}{8}}b\,x^2}dx\right)}{36\,(b\,x^2+a)^{\frac{3}{4}}\,ax}$$

input
```
int(1/x^2/(b*x^2+a)^(5/8),x)
```

output
```
( - 36*(a + b*x**2)**(1/8)*a - 20*(a + b*x**2)**(1/8)*b*x**2 - 45*(a + b*x
**2)**(3/4)*int(x**2/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),
x)*b**2*x - 65*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a + (a + b*x
**2)**(5/8)*b*x**2),x)*a*b*x)/(36*(a + b*x**2)**(3/4)*a*x)
```

3.1170 $\int \frac{1}{x^4(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{5/8}}$$

output
```
-1/3*(1+b*x^2/a)^(5/8)*hypergeom([-3/2, 5/8],[-1/2],-b*x^2/a)/x^3/(b*x^2+a
)^(5/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^{5/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{5/8}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(5/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-3/2, 5/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \int \frac{1}{x^4 \left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{(a+bx^2)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{5/8}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(5/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-3/2, 5/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4 (b\,x^2+a)^{\frac{5}{8}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(5/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{5}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{5}{8}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(5/8),x)
```

output
```
-hyper((-3/2, 5/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(5/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{x^4\,(b\,x^2+a)^{5/8}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(5/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(5/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{5/8}}\,dx = \frac{-68(b\,x^2+a)^{\frac{1}{8}}\,a-20(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2-145(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2+a)^{\frac{5}{8}}\,a\,x^2+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^4}\right.}{204\,(b\,x^2+a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2+a)^(5/8),x)
```

output
```
( - 68*(a + b*x**2)**(1/8)*a - 20*(a + b*x**2)**(1/8)*b*x**2 - 145*(a + b*
x**2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**
4),x)*a*b*x**3 - 85*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a + (a
+ b*x**2)**(5/8)*b*x**2),x)*b**2*x**3)/(204*(a + b*x**2)**(3/4)*a*x**3)
```

3.1171 $\int \frac{1}{x^6(a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^6(a+bx^2)^{5/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5(a+bx^2)^{5/8}}$$

output
```
-1/5*(1+b*x^2/a)^(5/8)*hypergeom([-5/2, 5/8],[-3/2],-b*x^2/a)/x^5/(b*x^2+a
)^(5/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(a+bx^2)^{5/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5(a+bx^2)^{5/8}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(5/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-5/2, 5/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(5/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/8}}dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \int \frac{1}{x^6 \left(\frac{bx^2}{a}+1\right)^{5/8}}dx}{\left(a+bx^2\right)^{5/8}}$$
$$\Big\downarrow \text{278}$$
$$-\frac{\left(\frac{bx^2}{a}+1\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 \left(a+bx^2\right)^{5/8}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(5/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(5/8)*Hypergeometric2F1[-5/2, 5/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}+a\right)^{\frac{5}{8}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(5/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(3/8)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6 (a+bx^2)^{5/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{5}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(5/8),x)
```

output
```
-hyper((-5/2, 5/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(5/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(5/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{5/8}} \, dx = \int \frac{1}{x^6 \left(b x^2+a\right)^{5/8}} \, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(5/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(5/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{5/8}} \, dx = \frac{-80(b x^2+a)^{\frac{1}{8}} a^2-16(b x^2+a)^{\frac{1}{8}} ab x^2+180(b x^2+a)^{\frac{1}{8}} b^2 x^4-180(b x^2+a)^{\frac{3}{4}} \left(\int \right.}{}$$

input
```
int(1/x^6/(b*x^2+a)^(5/8),x)
```

output
```
( - 80*(a + b*x**2)**(1/8)*a**2 - 16*(a + b*x**2)**(1/8)*a*b*x**2 + 180*(a
 + b*x**2)**(1/8)*b**2*x**4 - 180*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)*
*(5/8)*a*x**4 + (a + b*x**2)**(5/8)*b*x**6),x)*a**2*b*x**5 + 80*(a + b*x**
2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),
x)*a*b**2*x**5 + 405*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(5/8)*a + (a
 + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5)/(400*(a + b*x**2)**(3/4)*a**2*x**5
)
```

3.1172 $\int \frac{x^6}{\left(a+bx^2\right)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^6}{\left(a+bx^2\right)^{7/8}}\,dx = \frac{x^7\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(a+bx^2\right)^{7/8}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(7/8)*hypergeom([7/8, 7/2],[9/2],-b*x^2/a)/(b*x^2+a)^(
7/8)
```

Mathematica [A] (verified)

Time = 8.97 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{\left(a+bx^2\right)^{7/8}}\,dx = \frac{x^7\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7\left(a+bx^2\right)^{7/8}}$$

input
```
Integrate[x^6/(a + b*x^2)^(7/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{7/8}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8}\int \frac{x^6}{\left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7\left(\frac{bx^2}{a}+1\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7(a+bx^2)^{7/8}}$$

input
```
Int[x^6/(a + b*x^2)^(7/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(7/8),x)
```

output
```
int(x^6/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 + a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \frac{x^7\,{}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{7}{2}\\\frac{9}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{7}{8}}}$$

input
```
integrate(x**6/(b*x**2+a)**(7/8),x)
```

output
```
x**7*hyper((7/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(7/8))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{7/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(7/8),x)
```

output
```
int(x^6/(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(7/8),x)
```

output
```
int(x**6/(a + b*x**2)**(7/8),x)
```

3.1173 $\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx=\frac{x^5\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{7/8}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(7/8)*hypergeom([7/8, 5/2],[7/2],-b*x^2/a)/(b*x^2+a)^(
7/8)
```

Mathematica [A] (verified)

Time = 8.94 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx=\frac{x^5\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{7/8}}$$

input
```
Integrate[x^4/(a + b*x^2)^(7/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8}\int \frac{x^4}{\left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^5\left(\frac{bx^2}{a}+1\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5\,(a+bx^2)^{7/8}}$$

input
```
Int[x^4/(a + b*x^2)^(7/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(7/8),x)
```

output
```
int(x^4/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 + a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \frac{x^5\,{}_2F_1\left(\left.\begin{matrix}\frac{7}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{7}{8}}}$$

input
```
integrate(x**4/(b*x**2+a)**(7/8),x)
```

output
```
x**5*hyper((7/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/8))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{7/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(7/8),x)
```

output
```
int(x^4/(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(7/8),x)
```

output
```
int(x**4/(a + b*x**2)**(7/8),x)
```

3.1174 $\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \frac{x^3\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\,(a+bx^2)^{7/8}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(7/8)*hypergeom([7/8, 3/2],[5/2],-b*x^2/a)/(b*x^2+a)^(
7/8)
```

Mathematica [A] (verified)

Time = 8.43 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \frac{x^3\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\,(a+bx^2)^{7/8}}$$

input
```
Integrate[x^2/(a + b*x^2)^(7/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^3\left(\frac{bx^2}{a}+1\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3\,(a+bx^2)^{7/8}}$$

input
```
Int[x^2/(a + b*x^2)^(7/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(7/8),x)
```

output
```
int(x^2/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 + a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \frac{x^3\,{}_2F_1\left(\begin{matrix} \frac{7}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{7}{8}}}$$

input
```
integrate(x**2/(b*x**2+a)**(7/8),x)
```

output
```
x**3*hyper((7/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/8))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{7/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(7/8),x)
```

output
```
int(x^2/(a + b*x^2)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{7}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(7/8),x)
```

output
```
int(x**2/(a + b*x**2)**(7/8),x)
```

3.1175 $\int \frac{1}{(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 46

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

output
```
x*(1+b*x^2/a)^(7/8)*hypergeom([1/2, 7/8],[3/2],-b*x^2/a)/(b*x^2+a)^(7/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

input
```
Integrate[(a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(7/8)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{7/8}} \, dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{7/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{(a+bx^2)^{7/8}}$$

input
```
Int[(a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, -((b*x^2)/a)])/(
a + b*x^2)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{7}{8}}}d x$$

input
```
int(1/(b*x^2+a)^(7/8),x)
```

output
```
int(1/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(-7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{7}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(7/8),x)
```

output
```
x*hyper((1/2, 7/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(7/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-7/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+bx^2)^{7/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{7/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{7/8}}$$

input
```
int(1/(a + b*x^2)^(7/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(7/8)*hypergeom([1/2, 7/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(7/8)
```

Reduce [F]

$$\int \frac{1}{\left(a+bx^2\right)^{7/8}}\,dx = \int \frac{\left(b\,x^2+a\right)^{\frac{3}{4}}}{\left(b\,x^2+a\right)^{\frac{5}{8}} a+\left(b\,x^2+a\right)^{\frac{5}{8}} b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(7/8),x)
```

output
```
int((a + b*x**2)**(3/4)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**
2),x)
```

3.1176 $\int \frac{1}{x^2(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{1}{x^2(a+bx^2)^{7/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x(a+bx^2)^{7/8}}$$

output
```
-(1+b*x^2/a)^(7/8)*hypergeom([-1/2, 7/8],[1/2],-b*x^2/a)/x/(b*x^2+a)^(7/8)
```

Mathematica [A] (verified)

Time = 8.71 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^2)^{7/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x(a+bx^2)^{7/8}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(7/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-1/2, 7/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(7/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{7/8}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \int \frac{1}{x^2 \left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{x (a+bx^2)^{7/8}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(7/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-1/2, 7/8, 1/2, -((b*x^2)/a)])/
(x*(a + b*x^2)^(7/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{7}{8}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(7/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/(b*x^4 + a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{x^2 (a+bx^2)^{7/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{7}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{a^{\frac{7}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(7/8),x)
```

output
```
-hyper((-1/2, 7/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(7/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^2 (a + bx^2)^{7/8}} \, dx = -\frac{4 \left(\frac{a}{b x^2} + 1\right)^{7/8} {}_2\mathrm{F}_1\left(\frac{7}{8}, \frac{11}{8}; \frac{19}{8}; -\frac{a}{b x^2}\right)}{11 x (b x^2 + a)^{7/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(7/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(7/8)*hypergeom([7/8, 11/8], 19/8, -a/(b*x^2)))/(11*x*
(a + b*x^2)^(7/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (a + bx^2)^{7/8}} \, dx = \frac{-4(b x^2 + a)^{\frac{7}{8}} - 9(b x^2 + a)^{\frac{3}{4}} \left(\int \frac{(b x^2+a)^{\frac{3}{4}}}{(b x^2+a)^{\frac{5}{8}} a + (b x^2+a)^{\frac{5}{8}} b x^2} dx\right) b x}{4 (b x^2 + a)^{\frac{3}{4}} a x}$$

input
```
int(1/x^2/(b*x^2+a)^(7/8),x)
```

output
```
( - 4*(a + b*x**2)**(7/8) - 9*(a + b*x**2)**(3/4)*int((a + b*x**2)**(3/4)/
((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b*x)/(4*(a + b*x**
2)**(3/4)*a*x)
```

3.1177 $\int \frac{1}{x^4(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^4\left(a+bx^2\right)^{7/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{7/8}}$$

output
```
-1/3*(1+b*x^2/a)^(7/8)*hypergeom([-3/2, 7/8],[-1/2],-b*x^2/a)/x^3/(b*x^2+a
)^(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^{7/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3\left(a+bx^2\right)^{7/8}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(7/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-3/2, 7/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \int \frac{1}{x^4 \left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3x^3 (a+bx^2)^{7/8}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(7/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-3/2, 7/8, -1/2, -((b*x^2)/a
)])/(x^3*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}+a \right)^{\frac{7}{8}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(7/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/(b*x^6 + a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}}\,dx = -\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{7}{8} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3a^{\frac{7}{8}}x^{3}}$$

input
```
integrate(1/x**4/(b*x**2+a)**(7/8),x)
```

output
```
-hyper((-3/2, 7/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(7/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{x^4 \left(b\,x^2+a\right)^{7/8}}\,dx$$

input
```
int(1/(x^4*(a + b*x^2)^(7/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(7/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (a+bx^2)^{7/8}}\,dx = \frac{-16(b\,x^2+a)^{\frac{7}{8}}\,a^2+68(b\,x^2+a)^{\frac{7}{8}}\,ab\,x^2+68(b\,x^2+a)^{\frac{7}{8}}\,b^2x^4-17(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b}\right.}{48\,(b\,x^2+a)^{\frac{3}{4}}\,a^3}$$

input
```
int(1/x^4/(b*x^2+a)^(7/8),x)
```

output
```
( - 16*(a + b*x**2)**(7/8)*a**2 + 68*(a + b*x**2)**(7/8)*a*b*x**2 + 68*(a
+ b*x**2)**(7/8)*b**2*x**4 - 17*(a + b*x**2)**(3/4)*int((a + b*x**2)**(3/4
)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*a*b**2*x**3 - 85
*(a + b*x**2)**(3/4)*int(x**2/(a + b*x**2)**(7/8),x)*b**3*x**3)/(48*(a + b
*x**2)**(3/4)*a**3*x**3)
```

3.1178 $\int \frac{1}{x^6(a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{1}{x^6\left(a+bx^2\right)^{7/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{7/8}}$$

output
```
-1/5*(1+b*x^2/a)^(7/8)*hypergeom([-5/2, 7/8],[-3/2],-b*x^2/a)/x^5/(b*x^2+a
)^(7/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6\left(a+bx^2\right)^{7/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5\left(a+bx^2\right)^{7/8}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(7/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-5/2, 7/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(7/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{7/8}} \, dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \int \frac{1}{x^6 \left(\frac{bx^2}{a}+1\right)^{7/8}}dx}{(a+bx^2)^{7/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5x^5 (a+bx^2)^{7/8}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(7/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(7/8)*Hypergeometric2F1[-5/2, 7/8, -3/2, -((b*x^2)/a
)])/(x^5*(a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^6 \left(b\,x^2+a\right)^{\frac{7}{8}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(7/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{7}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(1/8)/(b*x^8 + a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.63

$$\int \frac{1}{x^6 (a+bx^2)^{7/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{7}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{7}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(7/8),x)
```

output
```
-hyper((-5/2, 7/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(7/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a + bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{8}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a + bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2 + a)^{\frac{7}{8}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(7/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{7/8}}\,dx=\int \frac{1}{x^6\,(b\,x^2+a)^{7/8}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(7/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(7/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{7/8}}\,dx=\frac{-192(b\,x^2+a)^{\frac{7}{8}}\,a^2+400(b\,x^2+a)^{\frac{7}{8}}\,ab\,x^2-1100(b\,x^2+a)^{\frac{7}{8}}\,b^2x^4-3075(b\,x^2+a)}{960\,(b\,x^2+a)^{\frac{3}{4}}\,a^3x^5}$$

input
```
int(1/x^6/(b*x^2+a)^(7/8),x)
```

output
```
( - 192*(a + b*x**2)**(7/8)*a**2 + 400*(a + b*x**2)**(7/8)*a*b*x**2 - 1100
*(a + b*x**2)**(7/8)*b**2*x**4 - 3075*(a + b*x**2)**(3/4)*int((a + b*x**2)
**(3/4)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5)
/(960*(a + b*x**2)**(3/4)*a**3*x**5)
```

3.1179 $\int \frac{x^6}{(a+bx^2)^{9/8}} dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^6}{(a+bx^2)^{9/8}} dx = \frac{x^7 \sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a\sqrt[8]{a+bx^2}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(1/8)*hypergeom([9/8, 7/2],[9/2],-b*x^2/a)/a/(b*x^2+a)
^(1/8)
```

Mathematica [A] (verified)

Time = 8.73 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^2)^{9/8}} dx = \frac{x^7 \sqrt[8]{1+\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^6/(a + b*x^2)^(9/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*a*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{9/8}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^6}{\left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a\sqrt[8]{a+bx^2}}$$

input
```
Int[x^6/(a + b*x^2)^(9/8),x]
```

output

```
(x^7*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 7/2, 9/2, -((b*x^2)/a)])
/(7*a*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{9}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(9/8),x)
```

output
```
int(x^6/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx=\int \frac{x^6}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{9}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{7a^{\frac{9}{8}}}$$

input
```
integrate(x**6/(b*x**2+a)**(9/8),x)
```

output
```
x**7*hyper((9/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(9/8))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{9/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(9/8),x)
```

output
```
int(x^6/(a + b*x^2)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{9/8}}\,dx = \frac{76(b\,x^2+a)^{\frac{5}{8}}\,a^2x+80(b\,x^2+a)^{\frac{5}{8}}\,ab\,x^3+4(b\,x^2+a)^{\frac{5}{8}}\,b^2x^5-76(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2+a}\right.}{7\,(b\,x^2+a)^{\frac{3}{4}}\,b}$$

input
```
int(x^6/(b*x^2+a)^(9/8),x)
```

output
```
(76*(a + b*x**2)**(5/8)*a**2*x + 80*(a + b*x**2)**(5/8)*a*b*x**3 + 4*(a +
b*x**2)**(5/8)*b**2*x**5 - 76*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a
 + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*a**3 - 69*(a + b*x**2
)**(3/4)*int((sqrt(a + b*x**2)*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)
**(5/8)*b*x**2),x)*a**2*b)/(7*(a + b*x**2)**(3/4)*b**3)
```

3.1180 $\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx=\frac{x^5\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\sqrt[8]{a+bx^2}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(1/8)*hypergeom([9/8, 5/2],[7/2],-b*x^2/a)/a/(b*x^2+a)
^(1/8)
```

Mathematica [A] (verified)

Time = 8.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx=\frac{x^5\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^4/(a + b*x^2)^(9/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*a*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{9/8}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^4}{\left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\sqrt[8]{\frac{bx^2}{a}+1}\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\sqrt[8]{a+bx^2}}$$

input
```
Int[x^4/(a + b*x^2)^(9/8),x]
```

output

```
(x^5*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 5/2, 7/2, -((b*x^2)/a)])
/(5*a*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{2}+a \right)^{\frac{9}{8}}}d x$$

input
```
int(x^4/(b*x^2+a)^(9/8),x)
```

output
```
int(x^4/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^{4}}{(bx^{2}+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \frac{x^5 {}_2F_1\left(\begin{matrix} \frac{9}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{9}{8}}}$$

input
```
integrate(x**4/(b*x**2+a)**(9/8),x)
```

output
```
x**5*hyper((9/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(9/8))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{9/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(9/8),x)
```

output
```
int(x^4/(a + b*x^2)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{9/8}}\,dx = \frac{-4(b\,x^2+a)^{\frac{5}{8}}\,ax - 4(b\,x^2+a)^{\frac{5}{8}}\,b\,x^3 + 4(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2+a}}{(b\,x^2+a)^{\frac{5}{8}}\,a+(b\,x^2+a)^{\frac{5}{8}}\,b\,x^2}dx\right)a^2 +}{(b\,x^2+a)^{\frac{3}{4}}\,b^2}$$

input
```
int(x^4/(b*x^2+a)^(9/8),x)
```

output
```
( - 4*(a + b*x**2)**(5/8)*a*x - 4*(a + b*x**2)**(5/8)*b*x**3 + 4*(a + b*x*
*2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/
8)*b*x**2),x)*a**2 + 3*(a + b*x**2)**(3/4)*int((sqrt(a + b*x**2)*x**2)/((a
 + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*a*b)/((a + b*x**2)**(
3/4)*b**2)
```

3.1181 $\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx=\frac{x^3\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\sqrt[8]{a+bx^2}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(1/8)*hypergeom([9/8, 3/2],[5/2],-b*x^2/a)/a/(b*x^2+a)
^(1/8)
```

Mathematica [A] (verified)

Time = 8.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx=\frac{x^3\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\sqrt[8]{a+bx^2}}$$

input
```
Integrate[x^2/(a + b*x^2)^(9/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*a*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{9/8}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1}\int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\sqrt[8]{a+bx^2}}$$

input
```
Int[x^2/(a + b*x^2)^(9/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 3/2, 5/2, -((b*x^2)/a)])
/(3*a*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{9}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(9/8),x)
```

output
```
int(x^2/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{9}{8}}}$$

input
```
integrate(x**2/(b*x**2+a)**(9/8),x)
```

output
```
x**3*hyper((9/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(9/8))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{9/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(9/8),x)
```

output
```
int(x^2/(a + b*x^2)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{1}{8}}\,a+(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2}dx$$

input
```
int(x^2/(b*x^2+a)^(9/8),x)
```

output
```
int(x**2/((a + b*x**2)**(1/8)*a + (a + b*x**2)**(1/8)*b*x**2),x)
```

3.1182 $\int \frac{1}{(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx=\frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

output
```
x*(1+b*x^2/a)^(1/8)*hypergeom([1/2, 9/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(1/8)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx=\frac{x\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

input
```
Integrate[(a + b*x^2)^(-9/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{9/8}} dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[8]{\frac{bx^2}{a}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\sqrt[8]{a+bx^2}}$$

input
```
Int[(a + b*x^2)^(-9/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, -((b*x^2)/a)])/(
a*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+a\right)^{\frac{9}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(9/8),x)
```

output
```
int(1/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \frac{x\,{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{9}{8} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(9/8),x)
```

output
```
x*hyper((1/2, 9/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(9/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-9/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-9/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{9/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{9/8}}$$

input
```
int(1/(a + b*x^2)^(9/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(9/8)*hypergeom([1/2, 9/8], 3/2, -(b*x^2)/a))/(a + b*x^
2)^(9/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{9/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{1}{8}}\,a+(b\,x^2+a)^{\frac{1}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(9/8),x)
```

output
```
int(1/((a + b*x**2)**(1/8)*a + (a + b*x**2)**(1/8)*b*x**2),x)
```

3.1183 $\int \frac{1}{x^2(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx=-\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax\sqrt[8]{a+bx^2}}$$

output
```
-(1+b*x^2/a)^(1/8)*hypergeom([-1/2, 9/8],[1/2],-b*x^2/a)/a/x/(b*x^2+a)^(1/
8)
```

Mathematica [A] (verified)

Time = 8.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx=-\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax\sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(9/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 9/8, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(1/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{x^2 \left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a \sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax \sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(9/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 9/8, 1/2, -((b*x^2)/a)])/
(a*x*(a + b*x^2)^(1/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{9}{8}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(9/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{9}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(9/8),x)
```

output
```
-hyper((-1/2, 9/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(9/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}} \, dx = -\frac{4 \left(\frac{a}{b x^2}+1\right)^{9/8} {}_2F_1\left(\frac{9}{8}, \frac{13}{8}; \frac{21}{8}; -\frac{a}{b x^2}\right)}{13 x (b x^2+a)^{9/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(9/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(9/8)*hypergeom([9/8, 13/8], 21/8, -a/(b*x^2)))/(13*x*
(a + b*x^2)^(9/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (a+bx^2)^{9/8}} \, dx = \int \frac{1}{(b x^2+a)^{\frac{1}{8}} a x^2 + (b x^2+a)^{\frac{1}{8}} b x^4} dx$$

input
```
int(1/x^2/(b*x^2+a)^(9/8),x)
```

output
```
int(1/((a + b*x**2)**(1/8)*a*x**2 + (a + b*x**2)**(1/8)*b*x**4),x)
```

3.1184 $\int \frac{1}{x^4(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}}\,dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{9}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[8]{a+bx^2}}$$

output
```
-1/3*(1+b*x^2/a)^(1/8)*hypergeom([-3/2, 9/8],[-1/2],-b*x^2/a)/a/x^3/(b*x^2
+a)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}}\,dx = -\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{9}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(9/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 9/8, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{x^4 \left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a\sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1} \, \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{9}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(9/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 9/8, -1/2, -((b*x^2)/a
)])/(a*x^3*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2+a\right)^{\frac{9}{8}}}dx$$

input
```
int(1/x^4/(b*x^2+a)^(9/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{9/8}}\,dx = \int \frac{1}{\left(bx^2+a\right)^{\frac{9}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{9}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{9}{8}}x^3}$$

input
```
integrate(1/x**4/(b*x**2+a)**(9/8),x)
```

output
```
-hyper((-3/2, 9/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(9/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{9/8}}\,dx=\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{9/8}} \,d x$$

input
```
int(1/(x^4*(a + b*x^2)^(9/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(9/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{9/8}}\,dx=\frac{-36(b\,x^{2}+a)^{\frac{5}{8}} a^{2}-92(b\,x^{2}+a)^{\frac{5}{8}} ab\,x^{2}+52(b\,x^{2}+a)^{\frac{5}{8}} b^{2}x^{4}-281(b\,x^{2}+a)^{\frac{3}{4}} \left(\int$$

input
```
int(1/x^4/(b*x^2+a)^(9/8),x)
```

output
```
( - 36*(a + b*x**2)**(5/8)*a**2 - 92*(a + b*x**2)**(5/8)*a*b*x**2 + 52*(a
+ b*x**2)**(5/8)*b**2*x**4 - 281*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/
((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x**4),x)*a**2*b*x**3 -
 243*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a + (a
+ b*x**2)**(5/8)*b*x**2),x)*a*b**2*x**3 + 65*(a + b*x**2)**(3/4)*int((sqrt
(a + b*x**2)*x**2)/((a + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)
*b**3*x**3)/(108*(a + b*x**2)**(3/4)*a**3*x**3)
```

3.1185 $\int \frac{1}{x^6(a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{x^6(a+bx^2)^{9/8}}\,dx=-\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{9}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5\sqrt[8]{a+bx^2}}$$

output
```
-1/5*(1+b*x^2/a)^(1/8)*hypergeom([-5/2, 9/8],[-3/2],-b*x^2/a)/a/x^5/(b*x^2
+a)^(1/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(a+bx^2)^{9/8}}\,dx=-\frac{\sqrt[8]{1+\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{9}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5\sqrt[8]{a+bx^2}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(9/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 9/8, -3/2, -((b*x^2)/a
)])/(a*x^5*(a + b*x^2)^(1/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{\frac{bx^2}{a}+1} \int \frac{1}{x^6 \left(\frac{bx^2}{a}+1\right)^{9/8}}dx}{a \sqrt[8]{a+bx^2}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{\frac{bx^2}{a}+1} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{9}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5 \sqrt[8]{a+bx^2}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(9/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 9/8, -3/2, -((b*x^2)/a
)])/(a*x^5*(a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b \,x^{2}+a \right)^{\frac{9}{8}}}d x$$

input
```
int(1/x^6/(b*x^2+a)^(9/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{9/8}}\, dx = \int { \frac{1}{{\left(b x^{2} + a\right)}^{\frac{9}{8}} x^{6}} \, dx }$$

input
```
integrate(1/x^6/(b*x^2+a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(7/8)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}}\,dx = -\frac{{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{9}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{5a^{\frac{9}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(9/8),x)
```

output
```
-hyper((-5/2, 9/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(9/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{9}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(9/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}} \, dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{9/8}} \, dx$$

input
```
int(1/(x^6*(a + b*x^2)^(9/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(9/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (a+bx^2)^{9/8}} \, dx = \frac{-2700(b\,x^2+a)^{\frac{5}{8}}\,a^3-432(b\,x^2+a)^{\frac{5}{8}}\,a^2 b\,x^2+268(b\,x^2+a)^{\frac{5}{8}}\,a\,b^2 x^4-6500(b\,x^2+$$

input
```
int(1/x^6/(b*x^2+a)^(9/8),x)
```

output
```
( - 2700*(a + b*x**2)**(5/8)*a**3 - 432*(a + b*x**2)**(5/8)*a**2*b*x**2 +
268*(a + b*x**2)**(5/8)*a*b**2*x**4 - 6500*(a + b*x**2)**(5/8)*b**3*x**6 -
 20871*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**
4 + (a + b*x**2)**(5/8)*b*x**6),x)*a**3*b*x**5 + 2500*(a + b*x**2)**(3/4)*
int(sqrt(a + b*x**2)/((a + b*x**2)**(5/8)*a*x**2 + (a + b*x**2)**(5/8)*b*x
**4),x)*a**2*b**2*x**5 + 7371*(a + b*x**2)**(3/4)*int(sqrt(a + b*x**2)/((a
 + b*x**2)**(5/8)*a + (a + b*x**2)**(5/8)*b*x**2),x)*a*b**3*x**5 - 8125*(a
 + b*x**2)**(3/4)*int((sqrt(a + b*x**2)*x**2)/((a + b*x**2)**(5/8)*a + (a
+ b*x**2)**(5/8)*b*x**2),x)*b**4*x**5)/(13500*(a + b*x**2)**(3/4)*a**4*x**
5)
```

3.1186 $\int \frac{x^6}{(a+bx^2)^{11/8}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^6}{(a+bx^2)^{11/8}} \, dx = \frac{x^7 \left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{11}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a (a+bx^2)^{3/8}}$$

output
```
1/7*x^7*(1+b*x^2/a)^(3/8)*hypergeom([11/8, 7/2],[9/2],-b*x^2/a)/a/(b*x^2+a
)^(3/8)
```

Mathematica [A] (verified)

Time = 9.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a+bx^2)^{11/8}} \, dx = \frac{x^7 \left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{11}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a (a+bx^2)^{3/8}}$$

input
```
Integrate[x^6/(a + b*x^2)^(11/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 7/2, 9/2, -((b*x^2)/a)]
)/(7*a*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(a+bx^2)^{11/8}} \, dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{x^6}{\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a\,(a+bx^2)^{3/8}}$$
$$\Big\downarrow \text{278}$$
$$\frac{x^7\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{7}{2},\frac{9}{2},-\frac{bx^2}{a}\right)}{7a\,(a+bx^2)^{3/8}}$$

input
```
Int[x^6/(a + b*x^2)^(11/8),x]
```

output
```
(x^7*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 7/2, 9/2, -((b*x^2)/a)]
)/(7*a*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2+a)^{\frac{11}{8}}}dx$$

input
```
int(x^6/(b*x^2+a)^(11/8),x)
```

output
```
int(x^6/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^6/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \frac{x^7 {}_2F_1\left(\begin{matrix} \frac{11}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7a^{\frac{11}{8}}}$$

input
```
integrate(x**6/(b*x**2+a)**(11/8),x)
```

output
```
x**7*hyper((11/8, 7/2), (9/2,), b*x**2*exp_polar(I*pi)/a)/(7*a**(11/8))
```

Maxima [F]

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 + a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 + a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(b\,x^2+a)^{11/8}}\,dx$$

input
```
int(x^6/(a + b*x^2)^(11/8),x)
```

output
```
int(x^6/(a + b*x^2)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(a+bx^2)^{11/8}}\,dx = \frac{76(b\,x^2+a)^{\frac{3}{8}}\,a^2x+80(b\,x^2+a)^{\frac{3}{8}}\,ab\,x^3+4(b\,x^2+a)^{\frac{3}{8}}\,b^2x^5-69(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2+}\right.}{7\,(b\,x^2+a)^{\frac{3}{4}}\,b}$$

input
```
int(x^6/(b*x^2+a)^(11/8),x)
```

output
```
(76*(a + b*x**2)**(3/8)*a**2*x + 80*(a + b*x**2)**(3/8)*a*b*x**3 + 4*(a +
b*x**2)**(3/8)*b**2*x**5 - 69*(a + b*x**2)**(3/4)*int(x**2/((a + b*x**2)**
(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*a**2*b - 76*(a + b*x**2)**(3/4)*i
nt(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)*a**3)/(7*(a +
 b*x**2)**(3/4)*b**3)
```

3.1187 $\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \frac{x^5\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\,(a+bx^2)^{3/8}}$$

output
```
1/5*x^5*(1+b*x^2/a)^(3/8)*hypergeom([11/8, 5/2],[7/2],-b*x^2/a)/a/(b*x^2+a
)^(3/8)
```

Mathematica [A] (verified)

Time = 9.44 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \frac{x^5\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\,(a+bx^2)^{3/8}}$$

input
```
Integrate[x^4/(a + b*x^2)^(11/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 5/2, 7/2, -((b*x^2)/a)]
)/(5*a*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(a+bx^2)^{11/8}}dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{x^4}{\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a\,(a+bx^2)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^5\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{5}{2},\frac{7}{2},-\frac{bx^2}{a}\right)}{5a\,(a+bx^2)^{3/8}}$$

input
```
Int[x^4/(a + b*x^2)^(11/8),x]
```

output
```
(x^5*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 5/2, 7/2, -((b*x^2)/a)]
)/(5*a*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2+a)^{\frac{11}{8}}}dx$$

input
```
int(x^4/(b*x^2+a)^(11/8),x)
```

output
```
int(x^4/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx=\int \frac{x^4}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx=\frac{x^5\,{}_2F_1\left(\begin{matrix} \frac{11}{8},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{11}{8}}}$$

input
```
integrate(x**4/(b*x**2+a)**(11/8),x)
```

output
```
x**5*hyper((11/8, 5/2), (7/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(11/8))
```

Maxima [F]

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 + a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 + a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(b\,x^2+a)^{11/8}}\,dx$$

input
```
int(x^4/(a + b*x^2)^(11/8),x)
```

output
```
int(x^4/(a + b*x^2)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(a+bx^2)^{11/8}}\,dx = \frac{-4(b\,x^2+a)^{\frac{3}{8}}\,ax-4(b\,x^2+a)^{\frac{3}{8}}\,b\,x^3+3(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}a+(b\,x^2+a)^{\frac{3}{8}}b\,x^2}dx\right)ab}{(b\,x^2+a)^{\frac{3}{4}}\,b^2}$$

input
```
int(x^4/(b*x^2+a)^(11/8),x)
```

output
```
( - 4*(a + b*x**2)**(3/8)*a*x - 4*(a + b*x**2)**(3/8)*b*x**3 + 3*(a + b*x*
*2)**(3/4)*int(x**2/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x
)*a*b + 4*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)*
*(3/8)*b*x**2),x)*a**2)/((a + b*x**2)**(3/4)*b**2)
```

3.1188 $\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx=\frac{x^3\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\left(a+bx^2\right)^{3/8}}$$

output
```
1/3*x^3*(1+b*x^2/a)^(3/8)*hypergeom([11/8, 3/2],[5/2],-b*x^2/a)/a/(b*x^2+a
)^(3/8)
```

Mathematica [A] (verified)

Time = 8.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx=\frac{x^3\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\left(a+bx^2\right)^{3/8}}$$

input
```
Integrate[x^2/(a + b*x^2)^(11/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 3/2, 5/2, -((b*x^2)/a)]
)/(3*a*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx^2)^{11/8}} \, dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{x^2}{\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a\,(a+bx^2)^{3/8}}$$
$$\Big\downarrow \text{278}$$
$$\frac{x^3\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{11}{8},\frac{3}{2},\frac{5}{2},-\frac{bx^2}{a}\right)}{3a\,(a+bx^2)^{3/8}}$$

input
```
Int[x^2/(a + b*x^2)^(11/8),x]
```

output
```
(x^3*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 3/2, 5/2, -((b*x^2)/a)]
)/(3*a*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2+a)^{\frac{11}{8}}}dx$$

input
```
int(x^2/(b*x^2+a)^(11/8),x)
```

output
```
int(x^2/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \frac{x^3\,{}_2F_1\left(\begin{matrix}\frac{11}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{3a^{\frac{11}{8}}}$$

input
```
integrate(x**2/(b*x**2+a)**(11/8),x)
```

output
```
x**3*hyper((11/8, 3/2), (5/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(11/8))
```

Maxima [F]

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 + a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 + a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{11/8}}\,dx$$

input
```
int(x^2/(a + b*x^2)^(11/8),x)
```

output
```
int(x^2/(a + b*x^2)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(b\,x^2+a)^{\frac{3}{8}}\,a+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2}dx$$

input
```
int(x^2/(b*x^2+a)^(11/8),x)
```

output
```
int(x**2/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)
```

3.1189 $\int \frac{1}{(a+bx^2)^{11/8}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)^{11/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

output
```
x*(1+b*x^2/a)^(3/8)*hypergeom([1/2, 11/8],[3/2],-b*x^2/a)/a/(b*x^2+a)^(3/8
)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)^{11/8}} \, dx = \frac{x\left(1+\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

input
```
Integrate[(a + b*x^2)^(-11/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8}\int \frac{1}{\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a\,(a+bx^2)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(\frac{bx^2}{a}+1\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},-\frac{bx^2}{a}\right)}{a\,(a+bx^2)^{3/8}}$$

input
```
Int[(a + b*x^2)^(-11/8),x]
```

output
```
(x*(1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, -((b*x^2)/a)])/
(a*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+a)^{\frac{11}{8}}}dx$$

input
```
int(1/(b*x^2+a)^(11/8),x)
```

output
```
int(1/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.49

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \frac{x_2F_1\left(\begin{matrix}\frac{1}{2}, \frac{11}{8}\\ \frac{3}{2}\end{matrix}\middle|\,\frac{bx^2e^{i\pi}}{a}\right)}{a^{\frac{11}{8}}}$$

input
```
integrate(1/(b*x**2+a)**(11/8),x)
```

output
```
x*hyper((1/2, 11/8), (3/2,), b*x**2*exp_polar(I*pi)/a)/a**(11/8)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^(-11/8), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^(-11/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \frac{x\left(\frac{b\,x^2}{a}+1\right)^{11/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{8};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{(b\,x^2+a)^{11/8}}$$

input
```
int(1/(a + b*x^2)^(11/8),x)
```

output
```
(x*((b*x^2)/a + 1)^(11/8)*hypergeom([1/2, 11/8], 3/2, -(b*x^2)/a))/(a + b*
x^2)^(11/8)
```

Reduce [F]

$$\int \frac{1}{(a+bx^2)^{11/8}}\,dx = \int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}\,a+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^2}dx$$

input
```
int(1/(b*x^2+a)^(11/8),x)
```

output
```
int(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8)*b*x**2),x)
```

3.1190 $\int \frac{1}{x^2(a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{1}{x^2(a+bx^2)^{11/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{11}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax(a+bx^2)^{3/8}}$$

output
```
-(1+b*x^2/a)^(3/8)*hypergeom([-1/2, 11/8],[1/2],-b*x^2/a)/a/x/(b*x^2+a)^(3
/8)
```

Mathematica [A] (verified)

Time = 9.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+bx^2)^{11/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{11}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax(a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^2*(a + b*x^2)^(11/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 11/8, 1/2, -((b*x^2)/a)])
/(a*x*(a + b*x^2)^(3/8)))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+bx^2)^{11/8}} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^2 \left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a (a+bx^2)^{3/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{11}{8},\frac{1}{2},-\frac{bx^2}{a}\right)}{ax (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^2*(a + b*x^2)^(11/8)),x]
```

output
```
-(((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 11/8, 1/2, -((b*x^2)/a)])
/(a*x*(a + b*x^2)^(3/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}+a \right)^{\frac{11}{8}}}d x$$

input
```
int(1/x^2/(b*x^2+a)^(11/8),x)
```

output
```
int(1/x^2/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{11/8}}\, dx = \int \frac{1}{\left(bx^{2} + a\right)^{\frac{11}{8}} x^{2}}\, dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.52

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{11/8}}\, dx = -\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{11}{8} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{a^{\frac{11}{8}} x}$$

input
```
integrate(1/x**2/(b*x**2+a)**(11/8),x)
```

output
```
-hyper((-1/2, 11/8), (1/2,), b*x**2*exp_polar(I*pi)/a)/(a**(11/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{11/8}}\,dx = -\frac{4\left(\frac{a}{b\,x^2}+1\right)^{11/8}{}_2\mathrm{F}_1\left(\frac{11}{8},\frac{15}{8};\ \frac{23}{8};\ -\frac{a}{b\,x^2}\right)}{15\,x\left(b\,x^2+a\right)^{11/8}}$$

input
```
int(1/(x^2*(a + b*x^2)^(11/8)),x)
```

output
```
-(4*(a/(b*x^2) + 1)^(11/8)*hypergeom([11/8, 15/8], 23/8, -a/(b*x^2)))/(15*
x*(a + b*x^2)^(11/8))
```

Reduce [F]

$$\int \frac{1}{x^2 \left(a+bx^2\right)^{11/8}}\,dx = \int \frac{1}{\left(b\,x^2+a\right)^{\frac{3}{8}} a\,x^2+\left(b\,x^2+a\right)^{\frac{3}{8}} b\,x^4}dx$$

input
```
int(1/x^2/(b*x^2+a)^(11/8),x)
```

output
```
int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*x**2)**(3/8)*b*x**4),x)
```

3.1191 $\int \frac{1}{x^4(a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{x^4\left(a+bx^2\right)^{11/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{11}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\left(a+bx^2\right)^{3/8}}$$

output
```
-1/3*(1+b*x^2/a)^(3/8)*hypergeom([-3/2, 11/8],[-1/2],-b*x^2/a)/a/x^3/(b*x^
2+a)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a+bx^2\right)^{11/8}}\,dx=-\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{11}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3\left(a+bx^2\right)^{3/8}}$$

input
```
Integrate[1/(x^4*(a + b*x^2)^(11/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 11/8, -1/2, -((b*x^2)/
a)])/(a*x^3*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (a+bx^2)^{11/8}} dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^4 \left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a (a+bx^2)^{3/8}}$$
$$\Big\downarrow \text{278}$$
$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{11}{8},-\frac{1}{2},-\frac{bx^2}{a}\right)}{3ax^3 (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^4*(a + b*x^2)^(11/8)),x]
```

output
```
-1/3*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 11/8, -1/2, -((b*x^2)/
a)])/(a*x^3*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b \,x^{2}+a \right)^{\frac{11}{8}}}d x$$

input
```
int(1/x^4/(b*x^2+a)^(11/8),x)
```

output
```
int(1/x^4/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{{\left(bx^{2}+a\right)}^{\frac{11}{8}}x^{4}}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{11/8}}\,dx=-\frac{{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{11}{8} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}e^{i\pi}}{a}} \right)}{3a^{\frac{11}{8}}x^{3}}$$

input
```
integrate(1/x**4/(b*x**2+a)**(11/8),x)
```

output
```
-hyper((-3/2, 11/8), (-1/2,), b*x**2*exp_polar(I*pi)/a)/(3*a**(11/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{x^4\,{\left(b\,x^2+a\right)}^{11/8}} \,d x$$

input
```
int(1/(x^4*(a + b*x^2)^(11/8)),x)
```

output
```
int(1/(x^4*(a + b*x^2)^(11/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(a+bx^2\right)^{11/8}}\,dx=\frac{-4(b\,x^2+a)^{\frac{3}{8}}\,b+2(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}\,a\,x^4+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^6}d x\right)a^2 x-4(b\,x^2+a)^{\frac{3}{4}}}{2\,(b\,x^2}$$

input
```
int(1/x^4/(b*x^2+a)^(11/8),x)
```

output
```
( - 4*(a + b*x**2)**(3/8)*b + 2*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(
3/8)*a*x**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a**2*x - 4*(a + b*x**2)**(3/4
)*int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*x**2)**(3/8)*b*x**4),x)*a*b*x
 - 9*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)**(3/8
)*b*x**2),x)*b**2*x)/(2*(a + b*x**2)**(3/4)*a**2*x)
```

3.1192 $\int \frac{1}{x^6(a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 54

$$\int \frac{1}{x^6(a+bx^2)^{11/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{11}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5(a+bx^2)^{3/8}}$$

output
```
-1/5*(1+b*x^2/a)^(3/8)*hypergeom([-5/2, 11/8],[-3/2],-b*x^2/a)/a/x^5/(b*x^
2+a)^(3/8)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^6(a+bx^2)^{11/8}}\,dx = -\frac{\left(1+\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{11}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5(a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^6*(a + b*x^2)^(11/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 11/8, -3/2, -((b*x^2)/
a)])/(a*x^5*(a + b*x^2)^(3/8))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (a+bx^2)^{11/8}} dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \int \frac{1}{x^6\left(\frac{bx^2}{a}+1\right)^{11/8}}dx}{a (a+bx^2)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{\left(\frac{bx^2}{a}+1\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{11}{8},-\frac{3}{2},-\frac{bx^2}{a}\right)}{5ax^5 (a+bx^2)^{3/8}}$$

input
```
Int[1/(x^6*(a + b*x^2)^(11/8)),x]
```

output
```
-1/5*((1 + (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 11/8, -3/2, -((b*x^2)/
a)])/(a*x^5*(a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^6\left(b\,x^2+a\right)^{\frac{11}{8}}}dx$$

input
```
int(1/x^6/(b*x^2+a)^(11/8),x)
```

output
```
int(1/x^6/(b*x^2+a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6\left(a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{\left(bx^2+a\right)^{\frac{11}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^(5/8)/(b^2*x^10 + 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^6\left(a+bx^2\right)^{11/8}}\,dx=-\frac{{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{11}{8}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{5a^{\frac{11}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2+a)**(11/8),x)
```

output
```
-hyper((-5/2, 11/8), (-3/2,), b*x**2*exp_polar(I*pi)/a)/(5*a**(11/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (a+bx^2)^{11/8}} dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}} x^6} dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (a+bx^2)^{11/8}} dx = \int \frac{1}{(bx^2+a)^{\frac{11}{8}} x^6} dx$$

input
```
integrate(1/x^6/(b*x^2+a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 + a)^(11/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{11/8}}\,dx = \int \frac{1}{x^6 \left(b\,x^2+a\right)^{11/8}}\,dx$$

input
```
int(1/(x^6*(a + b*x^2)^(11/8)),x)
```

output
```
int(1/(x^6*(a + b*x^2)^(11/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(a+bx^2\right)^{11/8}}\,dx = \frac{-8(b\,x^2+a)^{\frac{3}{8}}\,ab + 12(b\,x^2+a)^{\frac{3}{8}}\,b^2x^2 + 12(b\,x^2+a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2+a)^{\frac{3}{8}}\,a\,x^6+(b\,x^2+a)^{\frac{3}{8}}\,b\,x^8}\cdots\right.}{\cdots}$$

input
```
int(1/x^6/(b*x^2+a)^(11/8),x)
```

output
```
( - 8*(a + b*x**2)**(3/8)*a*b + 12*(a + b*x**2)**(3/8)*b**2*x**2 + 12*(a +
 b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x**6 + (a + b*x**2)**(3/8)*b*
x**8),x)*a**3*x**3 - 24*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a*x
**4 + (a + b*x**2)**(3/8)*b*x**6),x)*a**2*b*x**3 - 22*(a + b*x**2)**(3/4)*
int(1/((a + b*x**2)**(3/8)*a*x**2 + (a + b*x**2)**(3/8)*b*x**4),x)*a*b**2*
x**3 + 27*(a + b*x**2)**(3/4)*int(1/((a + b*x**2)**(3/8)*a + (a + b*x**2)*
*(3/8)*b*x**2),x)*b**3*x**3)/(12*(a + b*x**2)**(3/4)*a**3*x**3)
```

3.1193 $\int x^6 \sqrt[8]{-a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 541

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx$$

$$= -\frac{64a^3x\sqrt[8]{-a+bx^2}}{2639b^3} - \frac{80a^2x^3\sqrt[8]{-a+bx^2}}{7917b^2} - \frac{4ax^5\sqrt[8]{-a+bx^2}}{609b} + \frac{4}{29}x^7\sqrt[8]{-a+bx^2}$$

$$- \frac{128a^4\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdots}}}\right.\right.}{2639\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+ \frac{128a^4\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdots}}}\right.\right.}{2639\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-64/2639*a^3*x*(b*x^2-a)^(1/8)/b^3-80/7917*a^2*x^3*(b*x^2-a)^(1/8)/b^2-4/6
09*a*x^5*(b*x^2-a)^(1/8)/b+4/29*x^7*(b*x^2-a)^(1/8)-128/2639*a^4*(-b*x^2/a
^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2
/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-
a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(
-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))+128
/2639*a^4*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4
)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)
*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^
2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(
b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \frac{x^7 \sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[x^6*(-a + b*x^2)^(1/8),x]
```

output
```
(x^7*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = $\{279, 278\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \sqrt[8]{bx^2-a}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\sqrt[8]{bx^2-a}\int x^6 \sqrt[8]{1-\frac{bx^2}{a}}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^7 \sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[x^6*(-a + b*x^2)^(1/8),x]
```

output
```
(x^7*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^6 \left(b\,x^2-a\right)^{\frac{1}{8}}dx$$

input
```
int(x^6*(b*x^2-a)^(1/8),x)
```

output
```
int(x^6*(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \int (bx^2-a)^{\frac{1}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \frac{\sqrt[8]{a}x^7 e^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2-a)**(1/8),x)
```

output
```
a**(1/8)*x**7*exp(I*pi/8)*hyper((-1/8, 7/2), (9/2,), b*x**2/a)/7
```

Maxima [F]

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^6, x)
```

Giac [F]

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \int x^6 \left(b\,x^2-a\right)^{1/8}\,dx$$

input
```
int(x^6*(b*x^2 - a)^(1/8),x)
```

output
```
int(x^6*(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int x^6 \sqrt[8]{-a+bx^2}\,dx = \int \left(b\,x^2-a\right)^{\frac{1}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2-a)^(1/8),x)
```

output
```
int(( - a + b*x**2)**(1/8)*x**6,x)
```

3.1194 $\int x^4 \sqrt[8]{-a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 515

$$\int x^4 \sqrt[8]{-a+bx^2}\,dx = -\frac{16a^2x\sqrt[8]{-a+bx^2}}{455b^2} - \frac{4ax^3\sqrt[8]{-a+bx^2}}{273b} + \frac{4}{21}x^5\sqrt[8]{-a+bx^2}$$

$$-\frac{32a^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+b\cdots}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a+\cdots}}}\cdots\right.\right.}{455\sqrt{2+\sqrt{2}}b^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{32a^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+b\cdots}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a+\cdots}}}\cdots\right.\right.}{455\sqrt{2+\sqrt{2}}b^3x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-16/455*a^2*x*(b*x^2-a)^(1/8)/b^2-4/273*a*x^3*(b*x^2-a)^(1/8)/b+4/21*x^5*(
b*x^2-a)^(1/8)-32/455*a^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)
^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipti
cF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2
)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/
b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))+32/455*a^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2)
)^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1
/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/
2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(
2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int x^4 \sqrt[8]{-a+bx^2}\,dx = \frac{x^5 \sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[x^4*(-a + b*x^2)^(1/8),x]
```

output
```
(x^5*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \sqrt[8]{bx^2-a}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{bx^2-a}\int x^4 \sqrt[8]{1-\frac{bx^2}{a}}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5 \sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[x^4*(-a + b*x^2)^(1/8),x]
```

output
```
(x^5*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4} \left(b\,x^{2}-a\right)^{\frac{1}{8}}d x$$

input
```
int(x^4*(b*x^2-a)^(1/8),x)
```

output
```
int(x^4*(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int x^4 \sqrt[8]{-a+bx^2}\,dx=\int (bx^2-a)^{\frac{1}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int x^4 \sqrt[8]{-a+bx^2}\,dx=\frac{\sqrt[8]{a}x^5e^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{8},\frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2-a)**(1/8),x)
```

output
```
a**(1/8)*x**5*exp(I*pi/8)*hyper((-1/8, 5/2), (7/2,), b*x**2/a)/5
```

Maxima [F]

$$\int x^4 \sqrt[8]{-a+bx^2}\, dx = \int (bx^2-a)^{\frac{1}{8}} x^4\, dx$$

input
```
integrate(x^4*(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^4, x)
```

Giac [F]

$$\int x^4 \sqrt[8]{-a+bx^2}\, dx = \int (bx^2-a)^{\frac{1}{8}} x^4\, dx$$

input
```
integrate(x^4*(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt[8]{-a+bx^2}\, dx = \int x^4 \left(b\,x^2-a\right)^{1/8} dx$$

input
```
int(x^4*(b*x^2 - a)^(1/8),x)
```

output
```
int(x^4*(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int x^4 \sqrt[8]{-a+bx^2}\,dx = \int \left(b\,x^2-a\right)^{\frac{1}{8}} x^4 dx$$

input
```
int(x^4*(b*x^2-a)^(1/8),x)
```

output
```
int(( - a + b*x**2)**(1/8)*x**4,x)
```

3.1195 $\int x^2 \sqrt[8]{-a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 489

$$\int x^2 \sqrt[8]{-a+bx^2}\,dx = -\frac{4ax\sqrt[8]{-a+bx^2}}{65b}+\frac{4}{13}x^3\sqrt[8]{-a+bx^2}$$
$$-\frac{8a^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-2\frac{\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{65\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$
$$+\frac{8a^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+2\frac{\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{65\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-4/65*a*x*(b*x^2-a)^(1/8)/b+4/13*x^3*(b*x^2-a)^(1/8)-8/65*a^2*(-b*x^2/a^(1
/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^
(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^
(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+
2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/65*a
^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/
2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-(b*x^2-a
)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int x^2 \sqrt[8]{-a+bx^2}\,dx = \frac{x^3\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[x^2*(-a + b*x^2)^(1/8),x]
```

output
```
(x^3*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt[8]{bx^2 - a}\, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\sqrt[8]{bx^2 - a} \int x^2 \sqrt[8]{1 - \frac{bx^2}{a}} dx}{\sqrt[8]{1 - \frac{bx^2}{a}}}$$

$$\Big\downarrow \; 278$$

$$\frac{x^3 \sqrt[8]{bx^2 - a}\, \text{Hypergeometric2F1}\left(-\frac{1}{8}, \frac{3}{2}, \frac{5}{2}, \frac{bx^2}{a}\right)}{3 \sqrt[8]{1 - \frac{bx^2}{a}}}$$

input
```
Int[x^2*(-a + b*x^2)^(1/8),x]
```

output
```
(x^3*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}-a \right)^{\frac{1}{8}}d x$$

input
```
int(x^2*(b*x^2-a)^(1/8),x)
```

output
```
int(x^2*(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int x^2 \sqrt[8]{-a+bx^2}\, dx = \int \left(bx^{2} - a\right)^{\frac{1}{8}} x^{2}\, dx$$

input
```
integrate(x^2*(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int x^2 \sqrt[8]{-a+bx^2}\, dx = \frac{\sqrt[8]{a} x^{3} e^{\frac{i\pi}{8}} {{}_{2}F_{1}\left(\begin{matrix} -\frac{1}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{bx^{2}}{a}} \right)}}{3}$$

input
```
integrate(x**2*(b*x**2-a)**(1/8),x)
```

output
```
a**(1/8)*x**3*exp(I*pi/8)*hyper((-1/8, 3/2), (5/2,), b*x**2/a)/3
```

Maxima [F]

$$\int x^2\sqrt[8]{-a+bx^2}\,dx=\int \left(bx^2-a\right)^{\frac{1}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^2, x)
```

Giac [F]

$$\int x^2\sqrt[8]{-a+bx^2}\,dx=\int \left(bx^2-a\right)^{\frac{1}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt[8]{-a+bx^2}\,dx=\int x^2\,\left(b\,x^2-a\right)^{1/8}\,dx$$

input
```
int(x^2*(b*x^2 - a)^(1/8),x)
```

output
```
int(x^2*(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int x^2 \sqrt[8]{-a+bx^2}\,dx = \int \left(b\,x^2-a\right)^{\frac{1}{8}} x^2 dx$$

input
```
int(x^2*(b*x^2-a)^(1/8),x)
```

output
```
int(( - a + b*x**2)**(1/8)*x**2,x)
```

3.1196 $\int \sqrt[8]{-a+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 461

$$\int \sqrt[8]{-a+bx^2}\,dx = \frac{4}{5}x\sqrt[8]{-a+bx^2} - \frac{2a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-2\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{5\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)} + \frac{2a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+2\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{5\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/5*x*(b*x^2-a)^(1/8)-2/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-
a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellip
ticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1
/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^
(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \sqrt[8]{-a+bx^2}\,dx=\frac{x\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8),x]
```

output
```
(x*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[8]{bx^2-a}\,dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[8]{bx^2-a}\int \sqrt[8]{1-\frac{bx^2}{a}}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8),x]
```

output
```
(x*(-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2-a)^(1/8),x)
```

output
```
int((b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \sqrt[8]{-a+bx^2}\,dx=\int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \sqrt[8]{-a+bx^2}\,dx=\sqrt[8]{a}xe^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)$$

input
```
integrate((b*x**2-a)**(1/8),x)
```

output
```
a**(1/8)*x*exp(I*pi/8)*hyper((-1/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8), x)
```

Giac [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(bx^2-a\right)^{\frac{1}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \sqrt[8]{-a+bx^2}\,dx = \frac{x\,\left(b\,x^2-a\right)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{1/8}}$$

input
```
int((b*x^2 - a)^(1/8),x)
```

output
```
(x*(b*x^2 - a)^(1/8)*hypergeom([-1/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(1/8)
```

Reduce [F]

$$\int \sqrt[8]{-a+bx^2}\,dx = \int \left(b\,x^2-a\right)^{\frac{1}{8}}dx$$

input
```
int((b*x^2-a)^(1/8),x)
```

output
```
int(( - a + b*x**2)**(1/8),x)
```

3.1197 $\int \frac{\sqrt[8]{-a+bx^2}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 453

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2} dx = -\frac{\sqrt[8]{-a+bx^2}}{x}$$

$$+\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-(b*x^2-a)^(1/8)/x+1/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3
/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(
1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a
^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/x/(
a^(1/4)+(b*x^2-a)^(1/4))-1/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx=-\frac{\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/2, -1/8, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(1/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{bx^2-a}}{x^2}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{bx^2-a}\int \frac{\sqrt[8]{1-\frac{bx^2}{a}}}{x^2}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{1}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(1/8)*Hypergeometric2F1[-1/2, -1/8, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(1/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{1}{8}}}{x^2}dx$$

input
```
int((b*x^2-a)^(1/8)/x^2,x)
```

output
```
int((b*x^2-a)^(1/8)/x^2,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx = \frac{\sqrt[8]{a}e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, -\frac{1}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{x}$$

input
```
integrate((b*x**2-a)**(1/8)/x**2,x)
```

output
```
a**(1/8)*exp(-7*I*pi/8)*hyper((-1/2, -1/8), (1/2,), b*x**2/a)/x
```

Maxima [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx=\int \frac{(bx^2-a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx=\int \frac{(bx^2-a)^{\frac{1}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx=-\frac{4\,(b\,x^2-a)^{1/8}\,{}_2\mathrm{F}_1\left(-\frac{1}{8},\frac{3}{8};\ \frac{11}{8};\ \frac{a}{b\,x^2}\right)}{3\,x\,\left(1-\frac{a}{b\,x^2}\right)^{1/8}}$$

input
```
int((b*x^2 - a)^(1/8)/x^2,x)
```

output
```
-(4*(b*x^2 - a)^(1/8)*hypergeom([-1/8, 3/8], 11/8, a/(b*x^2)))/(3*x*(1 - a
/(b*x^2))^(1/8))
```

Reduce [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^2}\,dx$$

$$= \frac{-36(b\,x^2-a)^{\frac{7}{8}}\,a+20(b\,x^2-a)^{\frac{7}{8}}\,b\,x^2-5(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}a-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2}dx\right)abx+25(b\,x^2-a)^{\frac{3}{4}}\left(\right.}{36\,(b\,x^2-a)^{\frac{3}{4}}\,ax}$$

input

```
int((b*x^2-a)^(1/8)/x^2,x)
```

output

```
( - 36*( - a + b*x**2)**(7/8)*a + 20*( - a + b*x**2)**(7/8)*b*x**2 - 5*( -
 a + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a -
 ( - a + b*x**2)**(5/8)*b*x**2),x)*a*b*x + 25*( - a + b*x**2)**(3/4)*int((
( - a + b*x**2)**(3/4)*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**
(5/8)*b*x**2),x)*b**2*x)/(36*( - a + b*x**2)**(3/4)*a*x)
```

3.1198 $\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 487

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = -\frac{\sqrt[8]{-a+bx^2}}{3x^3} + \frac{b\sqrt[8]{-a+bx^2}}{12ax}$$

$$+\frac{b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\,(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right)\cdots\right)}{8\sqrt{2+\sqrt{2}}\,ax\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\,(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right)\cdots\right)}{8\sqrt{2+\sqrt{2}}\,ax\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/3*(b*x^2-a)^(1/8)/x^3+1/12*b*(b*x^2-a)^(1/8)/a/x+1/8*b*(-b*x^2/a^(1/2)/
(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4
)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4
)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a/x/(a^(1/4)+(b*x^2-a)^(1/4))-1/8*b*(-b*x^
2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x
^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2
),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = -\frac{\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-3/2, -1/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{bx^2-a}}{x^4}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{bx^2-a}\int \frac{\sqrt[8]{1-\frac{bx^2}{a}}}{x^4}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-3/2, -1/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{1}{8}}}{x^4}dx$$

input
```
int((b*x^2-a)^(1/8)/x^4,x)
```

output
```
int((b*x^2-a)^(1/8)/x^4,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = \frac{\sqrt[8]{a}e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},-\frac{1}{8}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2-a)**(1/8)/x**4,x)
```

output
```
a**(1/8)*exp(-7*I*pi/8)*hyper((-3/2, -1/8), (-1/2,), b*x**2/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx = \int \frac{(b\,x^2-a)^{1/8}}{x^4}\,dx$$

input
```
int((b*x^2 - a)^(1/8)/x^4,x)
```

output
```
int((b*x^2 - a)^(1/8)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^4}\,dx$$
$$= \frac{-68(b\,x^2-a)^{\frac{7}{8}}\,a+20(b\,x^2-a)^{\frac{7}{8}}\,b\,x^2+75(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^4}dx\right)ab\,x^3-15(b\,x^2-\cdots}{204\,(b\,x^2-a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int((b*x^2-a)^(1/8)/x^4,x)
```

output
```
( - 68*( - a + b*x**2)**(7/8)*a + 20*( - a + b*x**2)**(7/8)*b*x**2 + 75*(
- a + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a*
x**2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b*x**3 - 15*( - a + b*x**2)**(3
/4)*int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)
**(5/8)*b*x**2),x)*b**2*x**3)/(204*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1199 $\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 517

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = -\frac{\sqrt[8]{-a+bx^2}}{5x^5} + \frac{b\sqrt[8]{-a+bx^2}}{60ax^3} + \frac{11b^2\sqrt[8]{-a+bx^2}}{240a^2x}$$

$$+ \frac{11b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+b}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$- \frac{11b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+b}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/5*(b*x^2-a)^(1/8)/x^5+1/60*b*(b*x^2-a)^(1/8)/a/x^3+11/240*b^2*(b*x^2-a)
^(1/8)/a^2/x+11/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(
3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF
(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/
a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^
2/x/(a^(1/4)+(b*x^2-a)^(1/4))-11/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^
(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4
))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)
*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+
2^(1/2))^(1/2)/a^2/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = -\frac{\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-5/2, -1/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{bx^2-a}}{x^6}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{bx^2-a}\int \frac{\sqrt[8]{1-\frac{bx^2}{a}}}{x^6}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{1}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-5/2, -1/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{1}{8}}}{x^6}dx$$

input
```
int((b*x^2-a)^(1/8)/x^6,x)
```

output
```
int((b*x^2-a)^(1/8)/x^6,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = \frac{\sqrt[8]{a}e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, -\frac{1}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2-a)**(1/8)/x**6,x)
```

output
```
a**(1/8)*exp(-7*I*pi/8)*hyper((-5/2, -1/8), (-3/2,), b*x**2/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx = \int \frac{(b\,x^2-a)^{1/8}}{x^6}\,dx$$

input
```
int((b*x^2 - a)^(1/8)/x^6,x)
```

output
```
int((b*x^2 - a)^(1/8)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^6}\,dx$$
$$= \frac{-240(b\,x^2-a)^{\frac{7}{8}}\,a^2-76(b\,x^2-a)^{\frac{7}{8}}\,ab\,x^2+124(b\,x^2-a)^{\frac{7}{8}}\,b^2x^4+333(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}}\right.}{1200\,(b\,x^2-a)^{\frac{3}{4}}\,a^2x^5}$$

input
```
int((b*x^2-a)^(1/8)/x^6,x)
```

output
```
( - 240*( - a + b*x**2)**(7/8)*a**2 - 76*( - a + b*x**2)**(7/8)*a*b*x**2 +
 124*( - a + b*x**2)**(7/8)*b**2*x**4 + 333*( - a + b*x**2)**(3/4)*int(( -
 a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8
)*b*x**4),x)*a*b**2*x**5 - 93*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**
(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x
**5)/(1200*( - a + b*x**2)**(3/4)*a**2*x**5)
```

3.1200 $\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 543

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = -\frac{\sqrt[8]{-a+bx^2}}{7x^7}+\frac{b\sqrt[8]{-a+bx^2}}{140ax^5}+\frac{19b^2\sqrt[8]{-a+bx^2}}{1680a^2x^3}+\frac{209b^3\sqrt[8]{-a+bx^2}}{6720a^3x}$$

$$+\frac{209b^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a\cdots}}}\right.\right.}{4480\sqrt{2+\sqrt{2}}\,a^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{209b^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a\cdots}}}\right.\right.}{4480\sqrt{2+\sqrt{2}}\,a^3x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/7*(b*x^2-a)^(1/8)/x^7+1/140*b*(b*x^2-a)^(1/8)/a/x^5+19/1680*b^2*(b*x^2-
a)^(1/8)/a^2/x^3+209/6720*b^3*(b*x^2-a)^(1/8)/a^3/x+209/4480*b^3*(-b*x^2/a
^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2
/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-
a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(
-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-209
/4480*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4
)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)
*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^
2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^3/x/(a^(1/4)-(
b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx=-\frac{\sqrt[8]{-a+bx^2}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Integrate[(-a + b*x^2)^(1/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-7/2, -1/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[8]{bx^2-a}}{x^8}\,dx$$

$$\Big\downarrow \; 279$$

$$\frac{\sqrt[8]{bx^2-a}\int \frac{\sqrt[8]{1-\frac{bx^2}{a}}}{x^8}dx}{\sqrt[8]{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\sqrt[8]{bx^2-a}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{1}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\sqrt[8]{1-\frac{bx^2}{a}}}$$

input
```
Int[(-a + b*x^2)^(1/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(1/8)*Hypergeometric2F1[-7/2, -1/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{1}{8}}}{x^8}dx$$

input
```
int((b*x^2-a)^(1/8)/x^8,x)
```

output
```
int((b*x^2-a)^(1/8)/x^8,x)
```

Fricas [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = \frac{\sqrt[8]{a}e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{7}{2},-\frac{1}{8} \\ -\frac{5}{2} \end{matrix}\middle|\,\frac{bx^2}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2-a)**(1/8)/x**8,x)
```

output
```
a**(1/8)*exp(-7*I*pi/8)*hyper((-7/2, -1/8), (-5/2,), b*x**2/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^8, x)
```

Giac [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{1}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(1/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(1/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx = \int \frac{(b\,x^2-a)^{1/8}}{x^8}\,dx$$

input
```
int((b*x^2 - a)^(1/8)/x^8,x)
```

output
```
int((b*x^2 - a)^(1/8)/x^8, x)
```

Reduce [F]

$$\int \frac{\sqrt[8]{-a+bx^2}}{x^8}\,dx$$

$$= \frac{-1584(b\,x^2-a)^{\frac{7}{8}}\,a^3-324(b\,x^2-a)^{\frac{7}{8}}\,a^2b\,x^2+944(b\,x^2-a)^{\frac{7}{8}}\,a\,b^2x^4-380(b\,x^2-a)^{\frac{7}{8}}\,b^3x^6+4371(b\,x^2-$$

input

```
int((b*x^2-a)^(1/8)/x^8,x)
```

output

```
( - 1584*( - a + b*x**2)**(7/8)*a**3 - 324*( - a + b*x**2)**(7/8)*a**2*b*x
**2 + 944*( - a + b*x**2)**(7/8)*a*b**2*x**4 - 380*( - a + b*x**2)**(7/8)*
b**3*x**6 + 4371*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/4)/(( - a
+ b*x**2)**(5/8)*a*x**4 - ( - a + b*x**2)**(5/8)*b*x**6),x)*a**2*b**2*x**7
 - 2976*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)
**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b**3*x**7 + 285*( - a
 + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a - (
 - a + b*x**2)**(5/8)*b*x**2),x)*b**4*x**7)/(11088*( - a + b*x**2)**(3/4)*
a**3*x**7)
```

3.1201 $\int x^6(-a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 545

$$\int x^6\left(-a+bx^2\right)^{3/8}\,dx=-\frac{192a^3x(-a+bx^2)^{3/8}}{4991b^3}-\frac{16a^2x^3(-a+bx^2)^{3/8}}{713b^2}-\frac{12ax^5(-a+bx^2)^{3/8}}{713b}+\frac{4}{31}x^7\left(-a+bx^2\right)^{3/8}+\frac{384a^{17/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\cdots}\right.\right.}{4991\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}+\sqrt[4]{-a+\cdots}\right.}$$

output
```
-192/4991*a^3*x*(b*x^2-a)^(3/8)/b^3-16/713*a^2*x^3*(b*x^2-a)^(3/8)/b^2-12/
713*a*x^5*(b*x^2-a)^(3/8)/b+4/31*x^7*(b*x^2-a)^(3/8)+384/4991*a^(17/4)*(-b
*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1
/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(
b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(
1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4
))+384/4991*a^(17/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8
)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1
/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(
1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x
/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int x^6 \left(-a+bx^2\right)^{3/8} dx = \frac{x^7(-a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^6*(-a + b*x^2)^(3/8),x]
```

output
```
(x^7*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(bx^2-a\right)^{3/8} dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{3/8} \int x^6 \left(1-\frac{bx^2}{a}\right)^{3/8} dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7 \left(bx^2-a\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7 \left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[x^6*(-a + b*x^2)^(3/8),x]
```

output
```
(x^7*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6} \left(b\,x^{2}-a\right)^{\frac{3}{8}}d x$$

input
```
int(x^6*(b*x^2-a)^(3/8),x)
```

output
```
int(x^6*(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int x^6 \left(-a+bx^2\right)^{3/8}\,dx = \int \left(bx^2-a\right)^{\frac{3}{8}} x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int x^6 \left(-a+bx^2\right)^{3/8}\,dx = \frac{a^{\frac{3}{8}} x^7 e^{\frac{3i\pi}{8}} {}_2F_1\left(\begin{matrix} -\frac{3}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2-a)**(3/8),x)
```

output
```
a**(3/8)*x**7*exp(3*I*pi/8)*hyper((-3/8, 7/2), (9/2,), b*x**2/a)/7
```

Maxima [F]

$$\int x^6(-a+bx^2)^{3/8}\,dx=\int (bx^2-a)^{\frac{3}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^6, x)
```

Giac [F]

$$\int x^6(-a+bx^2)^{3/8}\,dx=\int (bx^2-a)^{\frac{3}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6(-a+bx^2)^{3/8}\,dx=\int x^6\,(b\,x^2-a)^{3/8}\,dx$$

input
```
int(x^6*(b*x^2 - a)^(3/8),x)
```

output
```
int(x^6*(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int x^6 \left(-a+bx^2\right)^{3/8} \, dx = \int \left(b\,x^2-a\right)^{\frac{3}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2-a)^(3/8),x)
```

output
```
int(( - a + b*x**2)**(3/8)*x**6,x)
```

3.1202 $\int x^4(-a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 519

$$\int x^4(-a+bx^2)^{3/8}\,dx = -\frac{48a^2x(-a+bx^2)^{3/8}}{805b^2} - \frac{4ax^3(-a+bx^2)^{3/8}}{115b} + \frac{4}{23}x^5(-a+bx^2)^{3/8} + \frac{96a^{13/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\cdots}\right.\right.}{805\sqrt{2+\sqrt{2}}b^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+b\cdots}\right.}$$

output
```
-48/805*a^2*x*(b*x^2-a)^(3/8)/b^2-4/115*a*x^3*(b*x^2-a)^(3/8)/b+4/23*x^5*(
b*x^2-a)^(3/8)+96/805*a^(13/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x
^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)
^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))+96/805*a^(13/4)*(-b*x^2/a^(1/2)/(b*x^
2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(
1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2)
)^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int x^4 \left(-a+bx^2\right)^{3/8} \, dx = \frac{x^5 (-a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5 \left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^4*(-a + b*x^2)^(3/8),x]
```

output
```
(x^5*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(bx^2-a\right)^{3/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{3/8} \int x^4 \left(1-\frac{bx^2}{a}\right)^{3/8}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5 \left(bx^2-a\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5 \left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[x^4*(-a + b*x^2)^(3/8),x]
```

output
```
(x^5*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4} \left(b\,x^{2}-a \right)^{\frac{3}{8}}d x$$

input
```
int(x^4*(b*x^2-a)^(3/8),x)
```

output
```
int(x^4*(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int x^4 \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} x^4 dx$$

input
```
integrate(x^4*(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int x^4 \left(-a+bx^2\right)^{3/8} dx = \frac{a^{\frac{3}{8}} x^5 e^{\frac{3i\pi}{8}} {}_2F_1\left(\begin{matrix} -\frac{3}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2-a)**(3/8),x)
```

output
```
a**(3/8)*x**5*exp(3*I*pi/8)*hyper((-3/8, 5/2), (7/2,), b*x**2/a)/5
```

Maxima [F]

$$\int x^4 \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} x^4 dx$$

input
```
integrate(x^4*(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^4, x)
```

Giac [F]

$$\int x^4 \left(-a+bx^2\right)^{3/8} \, dx = \int \left(bx^2 - a\right)^{\frac{3}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \left(-a+bx^2\right)^{3/8} \, dx = \int x^4 \left(b\,x^2 - a\right)^{3/8} \, dx$$

input
```
int(x^4*(b*x^2 - a)^(3/8),x)
```

output
```
int(x^4*(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int x^4 \left(-a+bx^2\right)^{3/8} \, dx = \int \left(b\,x^2 - a\right)^{\frac{3}{8}} x^4 dx$$

input
```
int(x^4*(b*x^2-a)^(3/8),x)
```

output
```
int(( - a + b*x**2)**(3/8)*x**4,x)
```

3.1203 $\int x^2(-a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 493

$$\int x^2\left(-a+bx^2\right)^{3/8}\,dx=-\frac{4ax\left(-a+bx^2\right)^{3/8}}{35b}+\frac{4}{15}x^3\left(-a+bx^2\right)^{3/8}$$

$$+\frac{8a^{9/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{35\sqrt{2+\sqrt{2}}\,b^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{8a^{9/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{35\sqrt{2+\sqrt{2}}\,b^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-4/35*a*x*(b*x^2-a)^(3/8)/b+4/15*x^3*(b*x^2-a)^(3/8)+8/35*a^(9/4)*(-b*x^2/
a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^
2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2
-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),
(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/
35*a^(9/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/
4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4
)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x
^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-
(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int x^2 \left(-a+bx^2\right)^{3/8} dx = \frac{x^3(-a+bx^2)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[x^2*(-a + b*x^2)^(3/8),x]
```

output
```
(x^3*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(bx^2-a\right)^{3/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{3/8} \int x^2 \left(1-\frac{bx^2}{a}\right)^{3/8} dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3 \left(bx^2-a\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3 \left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[x^2*(-a + b*x^2)^(3/8),x]
```

output
```
(x^3*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}-a \right)^{\frac{3}{8}}d x$$

input
```
int(x^2*(b*x^2-a)^(3/8),x)
```

output
```
int(x^2*(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int x^2 \left(-a+b x^2\right)^{3/8} \, dx = \int { \left(b x^{2} - a\right)}^{\frac{3}{8}} x^{2} \,d x$$

input
```
integrate(x^2*(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int x^2 \left(-a+b x^2\right)^{3/8} \, dx = \frac{a^{\frac{3}{8}} x^{3} e^{\frac{3 i \pi}{8}} {{}_{2}F_{1}\left(\begin{matrix} - \frac{3}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2}}{a}} \right)}}{3}$$

input
```
integrate(x**2*(b*x**2-a)**(3/8),x)
```

output
```
a**(3/8)*x**3*exp(3*I*pi/8)*hyper((-3/8, 3/2), (5/2,), b*x**2/a)/3
```

Maxima [F]

$$\int x^2 (-a+bx^2)^{3/8}\,dx = \int (bx^2-a)^{\frac{3}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^2, x)
```

Giac [F]

$$\int x^2 (-a+bx^2)^{3/8}\,dx = \int (bx^2-a)^{\frac{3}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (-a+bx^2)^{3/8}\,dx = \int x^2\,\left(b\,x^2-a\right)^{3/8}\,dx$$

input
```
int(x^2*(b*x^2 - a)^(3/8),x)
```

output
```
int(x^2*(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int x^2 \left(-a+bx^2\right)^{3/8} \, dx = \int \left(b\,x^2-a\right)^{\frac{3}{8}} x^2 dx$$

input
```
int(x^2*(b*x^2-a)^(3/8),x)
```

output
```
int(( - a + b*x**2)**(3/8)*x**2,x)
```

3.1204 $\int (-a+bx^2)^{3/8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 469

$$\int \left(-a+bx^2\right)^{3/8}\,dx = \frac{4}{7}x\left(-a+bx^2\right)^{3/8}$$

$$+\frac{6a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{7\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{6a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{7\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/7*x*(b*x^2-a)^(3/8)+6/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(
b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)
*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2
-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2)
)^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+6/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x^2-a
)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^
2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4
)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10

$$\int \left(-a+bx^2\right)^{3/8} dx = \frac{x\left(-a+bx^2\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8),x]
```

output
```
(x*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2-a\right)^{3/8} dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(bx^2-a\right)^{3/8} \int \left(1-\frac{bx^2}{a}\right)^{3/8} dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(bx^2-a\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(3/8),x]
```

output
```
(x*(-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{3}{8}}dx$$

input
```
int((b*x^2-a)^(3/8),x)
```

output
```
int((b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \left(-a+bx^2\right)^{3/8}\,dx=\int \left(bx^{2}-a\right)^{\frac{3}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.06

$$\int \left(-a+bx^2\right)^{3/8}\,dx=a^{\frac{3}{8}}xe^{\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^{2}}{a}\right)$$

input
```
integrate((b*x**2-a)**(3/8),x)
```

output
```
a**(3/8)*x*exp(3*I*pi/8)*hyper((-3/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8), x)
```

Giac [F]

$$\int \left(-a+bx^2\right)^{3/8} dx = \int \left(bx^2-a\right)^{\frac{3}{8}} dx$$

input
```
integrate((b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.08

$$\int \left(-a+bx^2\right)^{3/8} dx = \frac{x\left(b\,x^2-a\right)^{3/8}\,{}_2\mathrm{F}_1\left(-\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{3/8}}$$

input
```
int((b*x^2 - a)^(3/8),x)
```

output
```
(x*(b*x^2 - a)^(3/8)*hypergeom([-3/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(3/8)
```

Reduce [F]

$$\int \left(-a+bx^2\right)^{3/8} dx = \int \left(b\,x^2-a\right)^{\frac{3}{8}} dx$$

input
```
int((b*x^2-a)^(3/8),x)
```

output
```
int(( - a + b*x**2)**(3/8),x)
```

3.1205 $\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 463

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx = -\frac{(-a+bx^2)^{3/8}}{x}$$

$$-\frac{3\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{3\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-(b*x^2-a)^(3/8)/x-3/2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x
^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)
^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-3/2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/
2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^
(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(
1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.84 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx = -\frac{(-a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(3/8)*Hypergeometric2F1[-1/2, -3/8, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(3/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{3/8}}{x^2}dx$$

$\Big\downarrow$ 279

$$\frac{(bx^2-a)^{3/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{3/8}}{x^2}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$\Big\downarrow$ 278

$$-\frac{(bx^2-a)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-\frac{3}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(3/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(3/8)*Hypergeometric2F1[-1/2, -3/8, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(3/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}-a)^{\frac{3}{8}}}{x^{2}}dx$$

input
```
int((b*x^2-a)^(3/8)/x^2,x)
```

output
```
int((b*x^2-a)^(3/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx = \frac{a^{\frac{3}{8}}e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{3}{8}\\ \frac{1}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{x}$$

input
```
integrate((b*x**2-a)**(3/8)/x**2,x)
```

output
```
a**(3/8)*exp(-5*I*pi/8)*hyper((-1/2, -3/8), (1/2,), b*x**2/a)/x
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^2, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx=-\frac{4\,(b\,x^2-a)^{3/8}\,{}_2F_1\left(-\frac{3}{8},\frac{1}{8};\ \frac{9}{8};\ \frac{a}{b\,x^2}\right)}{x\,\left(1-\frac{a}{b\,x^2}\right)^{3/8}}$$

input
```
int((b*x^2 - a)^(3/8)/x^2,x)
```

output
```
-(4*(b*x^2 - a)^(3/8)*hypergeom([-3/8, 1/8], 9/8, a/(b*x^2)))/(x*(1 - a/(b
*x^2))^(3/8))
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^2}\,dx = \frac{4(b\,x^2-a)^{\frac{1}{8}}\,a-4(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2-5(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}}dx\right)bx}{4\,(b\,x^2-a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2-a)^(3/8)/x^2,x)
```

output
```
(4*( - a + b*x**2)**(1/8)*a - 4*( - a + b*x**2)**(1/8)*b*x**2 - 5*( - a +
b*x**2)**(3/4)*int(1/( - a + b*x**2)**(5/8),x)*b*x)/(4*( - a + b*x**2)**(3
/4)*x)
```

3.1206 $\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 491

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx = -\frac{(-a+bx^2)^{3/8}}{3x^3} + \frac{b(-a+bx^2)^{3/8}}{4ax}$$

$$-\frac{b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{8\sqrt{2+\sqrt{2}}a^{3/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{8\sqrt{2+\sqrt{2}}a^{3/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/3*(b*x^2-a)^(3/8)/x^3+1/4*b*(b*x^2-a)^(3/8)/a/x-1/8*b*(-b*x^2/a^(1/2)/(
b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)
/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)
/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(
1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-1/8*b*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/4)/x/(a^(1/4)-(b*x^2-a
)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx = -\frac{(-a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/2, -3/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{3/8}}{x^4}\,dx$$

$$\Big\downarrow \; 279$$

$$\frac{(bx^2-a)^{3/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{3/8}}{x^4}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{(bx^2-a)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{3}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input

```
Int[(-a + b*x^2)^(3/8)/x^4,x]
```

output

```
-1/3*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-3/2, -3/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{3}{8}}}{x^4}dx$$

input
```
int((b*x^2-a)^(3/8)/x^4,x)
```

output
```
int((b*x^2-a)^(3/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx = \frac{a^{\frac{3}{8}}e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{3}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2-a)**(3/8)/x**4,x)
```

output
```
a**(3/8)*exp(-5*I*pi/8)*hyper((-3/2, -3/8), (-1/2,), b*x**2/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^4, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx=\int \frac{(b\,x^2-a)^{3/8}}{x^4}\,dx$$

input
```
int((b*x^2 - a)^(3/8)/x^4,x)
```

output
```
int((b*x^2 - a)^(3/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^4}\,dx = \frac{144(b\,x^2-a)^{\frac{1}{8}}\,a^2+36(b\,x^2-a)^{\frac{1}{8}}\,ab\,x^2-100(b\,x^2-a)^{\frac{1}{8}}\,b^2x^4+225(b\,x^2-a)^{\frac{3}{4}}\left(\int\right.}{432\,(b\,x^2-}$$

input
```
int((b*x^2-a)^(3/8)/x^4,x)
```

output
```
(144*( - a + b*x**2)**(1/8)*a**2 + 36*( - a + b*x**2)**(1/8)*a*b*x**2 - 10
0*( - a + b*x**2)**(1/8)*b**2*x**4 + 225*( - a + b*x**2)**(3/4)*int(x**2/(
( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**3 - 3
25*( - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2
)**(5/8)*b*x**2),x)*a*b**2*x**3)/(432*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1207 $\int \frac{(-a+bx^2)^{3/8}}{x^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 521

$$\int \frac{(-a+bx^2)^{3/8}}{x^6} dx = -\frac{(-a+bx^2)^{3/8}}{5x^5} + \frac{b(-a+bx^2)^{3/8}}{20ax^3} + \frac{9b^2(-a+bx^2)^{3/8}}{80a^2x}$$

$$-\frac{9b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^{7/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{9b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^{7/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/5*(b*x^2-a)^(3/8)/x^5+1/20*b*(b*x^2-a)^(3/8)/a/x^3+9/80*b^2*(b*x^2-a)^(
3/8)/a^2/x-9/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8
)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/
2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(
1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(7/
4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-9/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^
(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4
))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)
*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+
2^(1/2))^(1/2)/a^(7/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx = -\frac{(-a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-5/2, -3/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{3/8}}{x^6}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{3/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{3/8}}{x^6}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{3}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(3/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-5/2, -3/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{3}{8}}}{x^6}dx$$

input
```
int((b*x^2-a)^(3/8)/x^6,x)
```

output
```
int((b*x^2-a)^(3/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx = \frac{a^{\frac{3}{8}}e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{3}{8}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2-a)**(3/8)/x**6,x)
```

output
```
a**(3/8)*exp(-5*I*pi/8)*hyper((-5/2, -3/8), (-3/2,), b*x**2/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^6, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx=\int \frac{(bx^2-a)^{\frac{3}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx=\int \frac{(b\,x^2-a)^{3/8}}{x^6}\,dx$$

input
```
int((b*x^2 - a)^(3/8)/x^6,x)
```

output
```
int((b*x^2 - a)^(3/8)/x^6, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^6}\,dx = \frac{816(b\,x^2-a)^{\frac{1}{8}}\,a^2-476(b\,x^2-a)^{\frac{1}{8}}\,ab\,x^2-100(b\,x^2-a)^{\frac{1}{8}}\,b^2x^4-725(b\,x^2-a)^{\frac{3}{4}}\left(\int\right.}{4080\,(b\,x}$$

input
```
int((b*x^2-a)^(3/8)/x^6,x)
```

output
```
(816*( - a + b*x**2)**(1/8)*a**2 - 476*( - a + b*x**2)**(1/8)*a*b*x**2 - 1
00*( - a + b*x**2)**(1/8)*b**2*x**4 - 725*( - a + b*x**2)**(3/4)*int(1/((
- a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b**2*x**
5 + 425*( - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2)**(5/8)*a - ( - a + b
*x**2)**(5/8)*b*x**2),x)*b**3*x**5)/(4080*( - a + b*x**2)**(3/4)*a*x**5)
```

3.1208 $\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 547

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = -\frac{(-a+bx^2)^{3/8}}{7x^7} + \frac{3b(-a+bx^2)^{3/8}}{140ax^5}$$

$$+ \frac{17b^2(-a+bx^2)^{3/8}}{560a^2x^3} + \frac{153b^3(-a+bx^2)^{3/8}}{2240a^3x}$$

$$- \frac{153b^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\Big(\sqrt{2}-\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}}{\sqrt[4]{-a+b}}}}{4480\sqrt{2+\sqrt{2}}a^{11/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$- \frac{153b^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\Bigg(\arcsin\Bigg(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\Big(\sqrt{2}+\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}}{\sqrt[4]{-a+b}}}}{4480\sqrt{2+\sqrt{2}}a^{11/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-1/7*(b*x^2-a)^(3/8)/x^7+3/140*b*(b*x^2-a)^(3/8)/a/x^5+17/560*b^2*(b*x^2-a
)^(3/8)/a^2/x^3+153/2240*b^3*(b*x^2-a)^(3/8)/a^3/x-153/4480*b^3*(-b*x^2/a^
(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/
a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a
)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-
2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(11/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))
-153/4480*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^
(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(
1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(
b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(11/4)/x/(
a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = -\frac{(-a+bx^2)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(3/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-7/2, -3/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{3/8}}{x^8}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{3/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{3/8}}{x^8}dx}{\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{3/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{3}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(3/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(3/8)*Hypergeometric2F1[-7/2, -3/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{3}{8}}}{x^8}dx$$

input
```
int((b*x^2-a)^(3/8)/x^8,x)
```

output
```
int((b*x^2-a)^(3/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.07

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \frac{a^{\frac{3}{8}}e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{7}{2},-\frac{3}{8}\\-\frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2-a)**(3/8)/x**8,x)
```

output
```
a**(3/8)*exp(-5*I*pi/8)*hyper((-7/2, -3/8), (-5/2,), b*x**2/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^8, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{3}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(3/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(3/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \int \frac{(b\,x^2-a)^{3/8}}{x^8}\,dx$$

input
```
int((b*x^2 - a)^(3/8)/x^8,x)
```

output
```
int((b*x^2 - a)^(3/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{3/8}}{x^8}\,dx = \frac{80(b\,x^2-a)^{\frac{1}{8}}\,a^2-60(b\,x^2-a)^{\frac{1}{8}}\,ab\,x^2-4(b\,x^2-a)^{\frac{1}{8}}\,b^2x^4-45(b\,x^2-a)^{\frac{3}{4}}\left(\int\frac{}{(b\,x^2-}\right.}{560\,(b\,x^2-}$$

input
```
int((b*x^2-a)^(3/8)/x^8,x)
```

output
```
(80*( - a + b*x**2)**(1/8)*a**2 - 60*( - a + b*x**2)**(1/8)*a*b*x**2 - 4*(
 - a + b*x**2)**(1/8)*b**2*x**4 - 45*( - a + b*x**2)**(3/4)*int(1/(( - a +
 b*x**2)**(5/8)*a*x**4 - ( - a + b*x**2)**(5/8)*b*x**6),x)*a*b**2*x**7 + 2
5*( - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*
x**2)**(5/8)*b*x**4),x)*b**3*x**7)/(560*( - a + b*x**2)**(3/4)*a*x**7)
```

3.1209 $\int x^6(-a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 995

$$\int x^6\left(-a+bx^2\right)^{5/8}\,dx=\text{Too large to display}$$

output
```
-64/1683*a^3*x*(b*x^2-a)^(5/8)/b^3-16/561*a^2*x^3*(b*x^2-a)^(5/8)/b^2-4/16
5*a*x^5*(b*x^2-a)^(5/8)/b+4/33*x^7*(b*x^2-a)^(5/8)+128/1683*(2+2^(1/2))^(1
/2)*a^(9/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/
4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/
4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-
128/1683*(2+2^(1/2))^(1/2)*a^(9/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*
(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^
2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^
(1/4)-(b*x^2-a)^(1/4))-128/1683*a^(9/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(
1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*
(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2
^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))+128/1683*a^(9/2)*(-b*x^2/a^(
1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/
a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)
^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2
+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.66 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int x^6\left(-a+bx^2\right)^{5/8}\,dx=\frac{x^7\left(-a+bx^2\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^6*(-a + b*x^2)^(5/8),x]
```

output
```
(x^7*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(bx^2-a\right)^{5/8} dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{5/8} \int x^6 \left(1-\frac{bx^2}{a}\right)^{5/8} dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7 \left(bx^2-a\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7 \left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[x^6*(-a + b*x^2)^(5/8),x]
```

output
```
(x^7*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6}\left(b\,x^{2}-a\right)^{\frac{5}{8}}dx$$

input
```
int(x^6*(b*x^2-a)^(5/8),x)
```

output
```
int(x^6*(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int x^6\left(-a+bx^2\right)^{5/8}\,dx=\int \left(bx^2-a\right)^{\frac{5}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^6\left(-a+bx^2\right)^{5/8}\,dx=\frac{a^{\frac{5}{8}}x^7e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2-a)**(5/8),x)
```

output
```
a**(5/8)*x**7*exp(5*I*pi/8)*hyper((-5/8, 7/2), (9/2,), b*x**2/a)/7
```

Maxima [F]

$$\int x^6 \left(-a+bx^2\right)^{5/8} dx = \int \left(bx^2-a\right)^{\frac{5}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^6, x)
```

Giac [F]

$$\int x^6 \left(-a+bx^2\right)^{5/8} dx = \int \left(bx^2-a\right)^{\frac{5}{8}} x^6 \, dx$$

input
```
integrate(x^6*(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 \left(-a+bx^2\right)^{5/8} dx = \int x^6 \left(b\,x^2-a\right)^{5/8} dx$$

input
```
int(x^6*(b*x^2 - a)^(5/8),x)
```

output
```
int(x^6*(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int x^6 \left(-a + bx^2\right)^{5/8} \, dx = \int \left(b\,x^2 - a\right)^{\frac{5}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2-a)^(5/8),x)
```

output
```
int(( - a + b*x**2)**(5/8)*x**6,x)
```

3.1210 $\int x^4(-a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 969

$$\int x^4(-a+bx^2)^{5/8}\,dx = \text{Too large to display}$$

output
```
-16/255*a^2*x*(b*x^2-a)^(5/8)/b^2-4/85*a*x^3*(b*x^2-a)^(5/8)/b+4/25*x^5*(b
*x^2-a)^(5/8)+32/255*(2+2^(1/2))^(1/2)*a^(7/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(
1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)
^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2
^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2
))/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-32/255*(2+2^(1/2))^(1/2)*a^(7/2)*(-b*x^
2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x
^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2
),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))-32/255*a^(7/2)*(-b
*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1
/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(
b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(
1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4
))+32/255*a^(7/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(
-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*
(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2
))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a
^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.48 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int x^4(-a+bx^2)^{5/8}\,dx=\frac{x^5(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^4*(-a + b*x^2)^(5/8),x]
```

output
```
(x^5*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(bx^2-a\right)^{5/8} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(bx^2-a\right)^{5/8} \int x^4 \left(1-\frac{bx^2}{a}\right)^{5/8}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^5 \left(bx^2-a\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5 \left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[x^4*(-a + b*x^2)^(5/8),x]
```

output
```
(x^5*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}-a\right)^{\frac{5}{8}}dx$$

input
```
int(x^4*(b*x^2-a)^(5/8),x)
```

output
```
int(x^4*(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int x^4\left(-a+bx^2\right)^{5/8}\,dx=\int \left(bx^2-a\right)^{\frac{5}{8}}x^4\,dx$$

input
```
integrate(x^4*(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^4\left(-a+bx^2\right)^{5/8}\,dx=\frac{a^{\frac{5}{8}}x^5e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2-a)**(5/8),x)
```

output
```
a**(5/8)*x**5*exp(5*I*pi/8)*hyper((-5/8, 5/2), (7/2,), b*x**2/a)/5
```

Maxima [F]

$$\int x^4 (-a+bx^2)^{5/8} \, dx = \int (bx^2-a)^{\frac{5}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^4, x)
```

Giac [F]

$$\int x^4 (-a+bx^2)^{5/8} \, dx = \int (bx^2-a)^{\frac{5}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (-a+bx^2)^{5/8} \, dx = \int x^4 \left(b\,x^2-a\right)^{5/8} \, dx$$

input
```
int(x^4*(b*x^2 - a)^(5/8),x)
```

output
```
int(x^4*(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int x^4\left(-a+bx^2\right)^{5/8}\,dx=\int \left(b\,x^2-a\right)^{\frac{5}{8}}x^4dx$$

input
```
int(x^4*(b*x^2-a)^(5/8),x)
```

output
```
int(( - a + b*x**2)**(5/8)*x**4,x)
```

3.1211 $\int x^2(-a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 943

$$\int x^2(-a+bx^2)^{5/8}\,dx = \text{Too large to display}$$

output
```
-20/153*a*x*(b*x^2-a)^(5/8)/b+4/17*x^3*(b*x^2-a)^(5/8)+40/153*(2+2^(1/2))^
(1/2)*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(
1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(
1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(
b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4)
)-40/153*(2+2^(1/2))^(1/2)*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*
(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^
2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^
(1/4)-(b*x^2-a)^(1/4))-40/153*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(
1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+40/153*a^(5/2)*(-b*x^2/a^(1/2)
/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/
4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2
^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int x^2(-a+bx^2)^{5/8}\,dx=\frac{x^3(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[x^2*(-a + b*x^2)^(5/8),x]
```

output
```
(x^3*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(bx^2-a)^{5/8}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{5/8}\int x^2\left(1-\frac{bx^2}{a}\right)^{5/8}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3(bx^2-a)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[x^2*(-a + b*x^2)^(5/8),x]
```

output
```
(x^3*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b\,x^{2}-a \right)^{\frac{5}{8}}d x$$

input
```
int(x^2*(b*x^2-a)^(5/8),x)
```

output
```
int(x^2*(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int x^2(-a+bx^2)^{5/8}\,dx=\int (bx^2-a)^{\frac{5}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^2(-a+bx^2)^{5/8}\,dx=\frac{a^{\frac{5}{8}}x^3e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2-a)**(5/8),x)
```

output
```
a**(5/8)*x**3*exp(5*I*pi/8)*hyper((-5/8, 3/2), (5/2,), b*x**2/a)/3
```

Maxima [F]

$$\int x^2(-a+bx^2)^{5/8}\,dx=\int (bx^2-a)^{\frac{5}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^2, x)
```

Giac [F]

$$\int x^2\left(-a+bx^2\right)^{5/8}\,dx=\int \left(bx^2-a\right)^{\frac{5}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(-a+bx^2\right)^{5/8}\,dx=\int x^2\,\left(b\,x^2-a\right)^{5/8}\,dx$$

input
```
int(x^2*(b*x^2 - a)^(5/8),x)
```

output
```
int(x^2*(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int x^2\left(-a+bx^2\right)^{5/8}\,dx=\int \left(b\,x^2-a\right)^{\frac{5}{8}}x^2dx$$

input
```
int(x^2*(b*x^2-a)^(5/8),x)
```

output
```
int(( - a + b*x**2)**(5/8)*x**2,x)
```

3.1212 $\int (-a+bx^2)^{5/8}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 919

$$\int (-a+bx^2)^{5/8}\,dx = \text{Too large to display}$$

output
```
4/9*x*(b*x^2-a)^(5/8)+10/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^
2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*
x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(
1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2)
)^(1/2))/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-10/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*
x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1
/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b
*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1
/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)-(b*x^2-a)^(1/4))-10/9*a^(3/2)*(-b*x
^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*
x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/
2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+1
0/9*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1
/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/
4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(
b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \left(-a+bx^2\right)^{5/8} dx = \frac{x(-a+bx^2)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8),x]
```

output
```
(x*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(5/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2-a\right)^{5/8} dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(bx^2-a\right)^{5/8} \int \left(1-\frac{bx^2}{a}\right)^{5/8} dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(bx^2-a\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8),x]
```

output
```
(x*(-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{5}{8}}d x$$

input
```
int((b*x^2-a)^(5/8),x)
```

output
```
int((b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \left(-a+bx^2\right)^{5/8}\,dx=\int \left(bx^2-a\right)^{\frac{5}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \left(-a+bx^2\right)^{5/8} \, dx = a^{\frac{5}{8}} x e^{\frac{5i\pi}{8}} {}_2F_1\left(\begin{matrix} -\frac{5}{8}, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)$$

input
```
integrate((b*x**2-a)**(5/8),x)
```

output
```
a**(5/8)*x*exp(5*I*pi/8)*hyper((-5/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \left(-a+bx^2\right)^{5/8} \, dx = \int \left(bx^2-a\right)^{\frac{5}{8}} \, dx$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8), x)
```

Giac [F]

$$\int \left(-a+bx^2\right)^{5/8} \, dx = \int \left(bx^2-a\right)^{\frac{5}{8}} \, dx$$

input
```
integrate((b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \left(-a+bx^2\right)^{5/8}\,dx=\frac{x\,{\left(b\,x^2-a\right)}^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(1-\frac{b\,x^2}{a}\right)}^{5/8}}$$

input
```
int((b*x^2 - a)^(5/8),x)
```

output
```
(x*(b*x^2 - a)^(5/8)*hypergeom([-5/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(5/8)
```

Reduce [F]

$$\int \left(-a+bx^2\right)^{5/8}\,dx=\int {\left(b\,x^2-a\right)}^{\frac{5}{8}}\,dx$$

input
```
int((b*x^2-a)^(5/8),x)
```

output
```
int(( - a + b*x**2)**(5/8),x)
```

3.1213 $\int \frac{(-a+bx^2)^{5/8}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 907

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = -\frac{(-a+bx^2)^{5/8}}{x}$$

$$-\frac{5\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+}}}\right.\right.}{2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{5\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{5\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{5\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{2\sqrt{2+\sqrt{2}}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-(b*x^2-a)^(5/8)/x-5/2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-
a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/x/(a^(1/4)+(b*x^2-a)^(1/4))+5/2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(
1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/
a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)
^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2
+2*2^(1/2))^(1/2))/x/(a^(1/4)-(b*x^2-a)^(1/4))+5/2*a^(1/2)*(-b*x^2/a^(1/2)
/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/
4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/
4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2
^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-5/2*a^(1/2)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = -\frac{(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{8},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, -1/2, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(5/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{5/8}}{x^2} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{5/8} \int \frac{\left(1-\frac{bx^2}{a}\right)^{5/8}}{x^2}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{8},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/8, -1/2, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(5/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}-a)^{\frac{5}{8}}}{x^{2}}dx$$

input
```
int((b*x^2-a)^(5/8)/x^2,x)
```

output
```
int((b*x^2-a)^(5/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \frac{a^{\frac{5}{8}}e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{8}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{x}$$

input
```
integrate((b*x**2-a)**(5/8)/x**2,x)
```

output
```
a**(5/8)*exp(-3*I*pi/8)*hyper((-5/8, -1/2), (1/2,), b*x**2/a)/x
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^2, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \frac{4\,(b\,x^2-a)^{5/8}\,{}_2\mathrm{F}_1\left(-\frac{5}{8},-\frac{1}{8};\ \frac{7}{8};\ \frac{a}{b\,x^2}\right)}{x\,\left(1-\frac{a}{b\,x^2}\right)^{5/8}}$$

input
```
int((b*x^2 - a)^(5/8)/x^2,x)
```

output
```
(4*(b*x^2 - a)^(5/8)*hypergeom([-5/8, -1/8], 7/8, a/(b*x^2)))/(x*(1 - a/(b
*x^2))^(5/8))
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^2}\,dx = \frac{4(b\,x^2-a)^{\frac{3}{8}}\,a-4(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2-5(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}x^2}dx\right)ax}{9\,(b\,x^2-a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2-a)^(5/8)/x^2,x)
```

output
```
(4*( - a + b*x**2)**(3/8)*a - 4*( - a + b*x**2)**(3/8)*b*x**2 - 5*( - a +
b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**2),x)
*a*x)/(9*( - a + b*x**2)**(3/4)*x)
```

3.1214 $\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 937

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx = \text{Too large to display}$$

output

```
-1/3*(b*x^2-a)^(5/8)/x^3+5/12*b*(b*x^2-a)^(5/8)/a/x+5/24*(2+2^(1/2))^(1/2)
*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/
2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-5/24*
(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)
*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/
2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1
/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/2)/x/(a^(1/4)-(b*x^
2-a)^(1/4))-5/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*
((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*
(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/
2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)
/x/(a^(1/4)+(b*x^2-a)^(1/4))+5/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)
*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1
/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x
^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2)/a^(1/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx = -\frac{(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-3/2, -5/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{5/8}}{x^4}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{5/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{5/8}}{x^4}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{5}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-3/2, -5/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2 - a)^{\frac{5}{8}}}{x^4}dx$$

input
```
int((b*x^2-a)^(5/8)/x^4,x)
```

output
```
int((b*x^2-a)^(5/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx=\int \frac{(bx^2-a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.04

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx=\frac{a^{\frac{5}{8}}e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},-\frac{5}{8}\\ -\frac{1}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2-a)**(5/8)/x**4,x)
```

output
```
a**(5/8)*exp(-3*I*pi/8)*hyper((-3/2, -5/8), (-1/2,), b*x**2/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx=\int \frac{(bx^2-a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^4, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx = \int \frac{(b\,x^2-a)^{5/8}}{x^4}\,dx$$

input
```
int((b*x^2 - a)^(5/8)/x^4,x)
```

output
```
int((b*x^2 - a)^(5/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^4}\,dx = \frac{16(b\,x^2-a)^{\frac{3}{8}}\,a^2-36(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^2+20(b\,x^2-a)^{\frac{3}{8}}\,b^2x^4-5(b\,x^2-a)^{\frac{3}{4}}\left(\int\frac{1}{(b\,x^2-}\right.}{68\,(b}$$

input
```
int((b*x^2-a)^(5/8)/x^4,x)
```

output
```
(16*( - a + b*x**2)**(3/8)*a**2 - 36*( - a + b*x**2)**(3/8)*a*b*x**2 + 20*
( - a + b*x**2)**(3/8)*b**2*x**4 - 5*( - a + b*x**2)**(3/4)*int(( - a + b*
x**2)**(1/4)/( - a + b*x**2)**(5/8),x)*b**2*x**3 - 20*( - a + b*x**2)**(3/
4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**4),x)*a**2*x**3 -
 20*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5
/8)*x**2),x)*a*b*x**3)/(68*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1215 $\int \frac{(-a+bx^2)^{5/8}}{x^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 971

$$\int \frac{(-a+bx^2)^{5/8}}{x^6} dx = \text{Too large to display}$$

output
```
-1/5*(b*x^2-a)^(5/8)/x^5+1/12*b*(b*x^2-a)^(5/8)/a/x^3+7/48*b^2*(b*x^2-a)^(
5/8)/a^2/x+7/96*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(
1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2
)/x/(a^(1/4)+(b*x^2-a)^(1/4))-7/96*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(
b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4
)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)
/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(
1/2))^(1/2))/a^(3/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))-7/96*b^2*(-b*x^2/a^(1/2)/
(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4
)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4
)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/96*b
^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/
2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/2)/x/(a^(1/4)-(b*x
^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{5/8}}{x^6}\,dx = -\frac{(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/2, -5/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2-a\right)^{5/8}}{x^6}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{5/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{5/8}}{x^6}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(bx^2-a\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{5}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-5/2, -5/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}-a)^{\frac{5}{8}}}{x^{6}}dx$$

input
```
int((b*x^2-a)^(5/8)/x^6,x)
```

output
```
int((b*x^2-a)^(5/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.04

$$\int \frac{(-a+bx^2)^{5/8}}{x^6}\,dx = \frac{a^{\frac{5}{8}}e^{-\frac{3i\pi}{8}}{}_2F_1\left(\left.\begin{matrix}-\frac{5}{2},-\frac{5}{8}\\-\frac{3}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2-a)**(5/8)/x**6,x)
```

output
```
a**(5/8)*exp(-3*I*pi/8)*hyper((-5/2, -5/8), (-3/2,), b*x**2/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^6, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{5}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(-a+bx^2\right)^{5/8}}{x^6}\,dx=\int \frac{\left(b\,x^2-a\right)^{5/8}}{x^6}\,dx$$

input
```
int((b*x^2 - a)^(5/8)/x^6,x)
```

output
```
int((b*x^2 - a)^(5/8)/x^6, x)
```

Reduce [F]

$$\int \frac{\left(-a+bx^2\right)^{5/8}}{x^6}\,dx=\frac{48(b\,x^2-a)^{\frac{3}{8}}\,a^2-68(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^2+20(b\,x^2-a)^{\frac{3}{8}}\,b^2x^4-60(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2}\right.}{300}$$

input
```
int((b*x^2-a)^(5/8)/x^6,x)
```

output
```
(48*( - a + b*x**2)**(3/8)*a**2 - 68*( - a + b*x**2)**(3/8)*a*b*x**2 + 20*
( - a + b*x**2)**(3/8)*b**2*x**4 - 60*( - a + b*x**2)**(3/4)*int(( - a + b
*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**6),x)*a**2*x**5 - 60*( - a + b*x*
*2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**4),x)*a*b
*x**5 + 35*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x*
*2)**(5/8)*x**2),x)*b**2*x**5)/(300*( - a + b*x**2)**(3/4)*a*x**5)
```

3.1216 $\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 997

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx = \text{Too large to display}$$

output
```
-1/7*(b*x^2-a)^(5/8)/x^7+1/28*b*(b*x^2-a)^(5/8)/a/x^5+5/112*b^2*(b*x^2-a)^
(5/8)/a^2/x^3+5/64*b^3*(b*x^2-a)^(5/8)/a^3/x+5/128*(2+2^(1/2))^(1/2)*b^3*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-5/128*(2+
2^(1/2))^(1/2)*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*
(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2
*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/
2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/2)/x/(a^(1/4)-(b*x^2
-a)^(1/4))-5/128*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8
)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/
2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(
1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/
2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+5/128*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^
(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4
))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)
*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+
2^(1/2))^(1/2)/a^(5/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx = -\frac{(-a+bx^2)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(5/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-7/2, -5/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{5/8}}{x^8}\,dx$$

$$\Big\downarrow \; 279$$

$$\frac{(bx^2-a)^{5/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{5/8}}{x^8}dx}{\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{(bx^2-a)^{5/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{5}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(5/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(5/8)*Hypergeometric2F1[-7/2, -5/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^{2}-a)^{\frac{5}{8}}}{x^{8}}dx$$

input
```
int((b*x^2-a)^(5/8)/x^8,x)
```

output
```
int((b*x^2-a)^(5/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(bx^2-a)^{\frac{5}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.04

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\frac{a^{\frac{5}{8}}e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{7}{2},-\frac{5}{8}\\-\frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2-a)**(5/8)/x**8,x)
```

output
```
a**(5/8)*exp(-3*I*pi/8)*hyper((-7/2, -5/8), (-5/2,), b*x**2/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(bx^2-a)^{\frac{5}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^8, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(bx^2-a)^{\frac{5}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(5/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(5/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\int \frac{(b\,x^2-a)^{5/8}}{x^8}\,dx$$

input
```
int((b*x^2 - a)^(5/8)/x^8,x)
```

output
```
int((b*x^2 - a)^(5/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{5/8}}{x^8}\,dx=\frac{16(b\,x^2-a)^{\frac{3}{8}}\,a^2-20(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^2+4(b\,x^2-a)^{\frac{3}{8}}\,b^2x^4-20(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2-}\right.}{132\,(}$$

input
```
int((b*x^2-a)^(5/8)/x^8,x)
```

output
```
(16*( - a + b*x**2)**(3/8)*a**2 - 20*( - a + b*x**2)**(3/8)*a*b*x**2 + 4*(
 - a + b*x**2)**(3/8)*b**2*x**4 - 20*( - a + b*x**2)**(3/4)*int(( - a + b*
x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**8),x)*a**2*x**7 - 20*( - a + b*x**
2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*x**6),x)*a*b*
x**7 + 15*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**
2)**(5/8)*x**4),x)*b**2*x**7)/(132*( - a + b*x**2)**(3/4)*a*x**7)
```

3.1217 $\int x^6(-a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1019

$$\int x^6\left(-a+bx^2\right)^{7/8}\,dx=\text{Too large to display}$$

output
```
-256/5643*a^4*x/b^3/(b*x^2-a)^(1/8)-64/1881*a^3*x*(b*x^2-a)^(7/8)/b^3-16/5
13*a^2*x^3*(b*x^2-a)^(7/8)/b^2-4/135*a*x^5*(b*x^2-a)^(7/8)/b+4/35*x^7*(b*x
^2-a)^(7/8)-128/5643*(2+2^(1/2))^(1/2)*a^(19/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^
(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a
)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+
2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/
2))/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-128/5643*(2+2^(1/2))^(1/2)*a^(19/4)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))+128/5643*a^(19
/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1
/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(
1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-
a)^(1/4))+128/5643*a^(19/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-
a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1
/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2
)/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int x^6 \left(-a+bx^2\right)^{7/8} dx = \frac{x^7(-a+bx^2)^{7/8} \operatorname{Hypergeometric2F1}\left(-\frac{7}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^6*(-a + b*x^2)^(7/8),x]
```

output
```
(x^7*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \left(bx^2-a\right)^{7/8} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{7/8} \int x^6 \left(1-\frac{bx^2}{a}\right)^{7/8} dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7 \left(bx^2-a\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7 \left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[x^6*(-a + b*x^2)^(7/8),x]
```

output
```
(x^7*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 7/2, 9/2, (b*x^2)/a])/(7*(
1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{6}\left(b\,x^{2}-a\right)^{\frac{7}{8}}dx$$

input
```
int(x^6*(b*x^2-a)^(7/8),x)
```

output
```
int(x^6*(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int x^6\left(-a+bx^2\right)^{7/8}\,dx=\int\left(bx^2-a\right)^{\frac{7}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^6\left(-a+bx^2\right)^{7/8}\,dx=\frac{a^{\frac{7}{8}}x^7e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{7}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2-a)**(7/8),x)
```

output
```
a**(7/8)*x**7*exp(7*I*pi/8)*hyper((-7/8, 7/2), (9/2,), b*x**2/a)/7
```

Maxima [F]

$$\int x^6(-a+bx^2)^{7/8}\,dx = \int (bx^2-a)^{\frac{7}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^6, x)
```

Giac [F]

$$\int x^6(-a+bx^2)^{7/8}\,dx = \int (bx^2-a)^{\frac{7}{8}}x^6\,dx$$

input
```
integrate(x^6*(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6(-a+bx^2)^{7/8}\,dx = \int x^6\,\left(b\,x^2-a\right)^{7/8}\,dx$$

input
```
int(x^6*(b*x^2 - a)^(7/8),x)
```

output
```
int(x^6*(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int x^6 \left(-a+bx^2\right)^{7/8} dx = \int \left(b\,x^2-a\right)^{\frac{7}{8}} x^6 dx$$

input
```
int(x^6*(b*x^2-a)^(7/8),x)
```

output
```
int(( - a + b*x**2)**(7/8)*x**6,x)
```

3.1218 $\int x^4(-a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 993

$$\int x^4\left(-a+bx^2\right)^{7/8}\,dx = \text{Too large to display}$$

output
```
-448/5643*a^3*x/b^2/(b*x^2-a)^(1/8)-112/1881*a^2*x*(b*x^2-a)^(7/8)/b^2-28/
513*a*x^3*(b*x^2-a)^(7/8)/b+4/27*x^5*(b*x^2-a)^(7/8)-224/5643*(2+2^(1/2))^
(1/2)*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^
(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^
(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/
(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4
))-224/5643*(2+2^(1/2))^(1/2)*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1
/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(
b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x
/(a^(1/4)-(b*x^2-a)^(1/4))+224/5643*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/
2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(
1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(
1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))+224/5643*a^(15/4)*(-b*x
^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/
4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*
x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/
2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int x^4\left(-a+bx^2\right)^{7/8}\,dx=\frac{x^5(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^4*(-a + b*x^2)^(7/8),x]
```

output
```
(x^5*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(bx^2-a\right)^{7/8}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{7/8}\int x^4\left(1-\frac{bx^2}{a}\right)^{7/8}dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\left(bx^2-a\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[x^4*(-a + b*x^2)^(7/8),x]
```

output
```
(x^5*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 5/2, 7/2, (b*x^2)/a])/(5*(
1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}-a\right)^{\frac{7}{8}}dx$$

input
```
int(x^4*(b*x^2-a)^(7/8),x)
```

output
```
int(x^4*(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int x^4 \left(-a+bx^2\right)^{7/8} \, dx=\int \left(bx^{2}-a\right)^{\frac{7}{8}} x^{4} \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^4 \left(-a+bx^2\right)^{7/8} \, dx=\frac{a^{\frac{7}{8}} x^{5} e^{\frac{7 i \pi}{8}} {{}_{2}F_{1}\left(\begin{matrix} - \frac{7}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| {\frac{b x^{2}}{a}} \right)}}{5}$$

input
```
integrate(x**4*(b*x**2-a)**(7/8),x)
```

output
```
a**(7/8)*x**5*exp(7*I*pi/8)*hyper((-7/8, 5/2), (7/2,), b*x**2/a)/5
```

Maxima [F]

$$\int x^4 (-a + bx^2)^{7/8} \, dx = \int (bx^2 - a)^{\frac{7}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^4, x)
```

Giac [F]

$$\int x^4 (-a + bx^2)^{7/8} \, dx = \int (bx^2 - a)^{\frac{7}{8}} x^4 \, dx$$

input
```
integrate(x^4*(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (-a + bx^2)^{7/8} \, dx = \int x^4 \left(b x^2 - a\right)^{7/8} \, dx$$

input
```
int(x^4*(b*x^2 - a)^(7/8),x)
```

output
```
int(x^4*(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int x^4 \left(-a+bx^2\right)^{7/8} \, dx = \int \left(b\,x^2-a\right)^{\frac{7}{8}} x^4 dx$$

input
```
int(x^4*(b*x^2-a)^(7/8),x)
```

output
```
int(( - a + b*x**2)**(7/8)*x**4,x)
```

3.1219 $\int x^2(-a+bx^2)^{7/8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 967

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\text{Too large to display}$$

output
```
-112/627*a^2*x/b/(b*x^2-a)^(1/8)-28/209*a*x*(b*x^2-a)^(7/8)/b+4/19*x^3*(b*
x^2-a)^(7/8)-56/627*(2+2^(1/2))^(1/2)*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(
1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)
^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2
^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2
))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))-56/627*(2+2^(1/2))^(1/2)*a^(11/4)*(-b*x
^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/
4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*
x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/
2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))+56/627*a^(11/4)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1
/4))+56/627*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8
)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1
/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(
1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x
/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.63 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int x^2(-a+bx^2)^{7/8}\,dx=\frac{x^3(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[x^2*(-a + b*x^2)^(7/8),x]
```

output
```
(x^3*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(bx^2-a\right)^{7/8} dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(bx^2-a\right)^{7/8} \int x^2 \left(1-\frac{bx^2}{a}\right)^{7/8} dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^3 \left(bx^2-a\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3 \left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[x^2*(-a + b*x^2)^(7/8),x]
```

output
```
(x^3*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 3/2, 5/2, (b*x^2)/a])/(3*(
1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2}\left(b\,x^{2}-a\right)^{\frac{7}{8}}dx$$

input
```
int(x^2*(b*x^2-a)^(7/8),x)
```

output
```
int(x^2*(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\int\left(bx^2-a\right)^{\frac{7}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\frac{a^{\frac{7}{8}}x^3e^{\frac{7i\pi}{8}}{}_2F_1\left(\begin{matrix}-\frac{7}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3}$$

input
```
integrate(x**2*(b*x**2-a)**(7/8),x)
```

output
```
a**(7/8)*x**3*exp(7*I*pi/8)*hyper((-7/8, 3/2), (5/2,), b*x**2/a)/3
```

Maxima [F]

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\int \left(bx^2-a\right)^{\frac{7}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^2, x)
```

Giac [F]

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\int \left(bx^2-a\right)^{\frac{7}{8}}x^2\,dx$$

input
```
integrate(x^2*(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(-a+bx^2\right)^{7/8}\,dx=\int x^2\left(b\,x^2-a\right)^{7/8}\,dx$$

input
```
int(x^2*(b*x^2 - a)^(7/8),x)
```

output
```
int(x^2*(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int x^2 \left(-a+bx^2\right)^{7/8} \, dx = \int \left(b\,x^2-a\right)^{\frac{7}{8}} x^2 dx$$

input
```
int(x^2*(b*x^2-a)^(7/8),x)
```

output
```
int(( - a + b*x**2)**(7/8)*x**2,x)
```

3.1220 $\int (-a+bx^2)^{7/8} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 938

$$\int \left(-a+bx^2\right)^{7/8} \, dx = \text{Too large to display}$$

output

```
-28/33*a*x/(b*x^2-a)^(1/8)+4/11*x*(b*x^2-a)^(7/8)-14/33*(2+2^(1/2))^(1/2)*
a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(
b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(
2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-14/33*
(2+2^(1/2))^(1/2)*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)
^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipt
icE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2
)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)-(b*x^
2-a)^(1/4))+14/33*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)
^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipti
cF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2
)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/
b/x/(a^(1/4)+(b*x^2-a)^(1/4))+14/33*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2
))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(
1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1
/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \left(-a+bx^2\right)^{7/8} dx = \frac{x\left(-a+bx^2\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(7/8),x]
```

output
```
(x*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(7/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx^2-a\right)^{7/8} dx$$

$$\Big\downarrow \text{ 238}$$

$$\frac{\left(bx^2-a\right)^{7/8} \int \left(1-\frac{bx^2}{a}\right)^{7/8} dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{ 237}$$

$$\frac{x\left(bx^2-a\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{7}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(7/8),x]
```

output
```
(x*(-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, 1/2, 3/2, (b*x^2)/a])/(1 - (
b*x^2)/a)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}-a\right)^{\frac{7}{8}}d x$$

input
```
int((b*x^2-a)^(7/8),x)
```

output
```
int((b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \left(-a+bx^2\right)^{7/8} \, dx=\int \left(b x^{2}-a\right)^{\frac{7}{8}}\,{d x}$$

input
```
integrate((b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.89 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \left(-a+bx^2\right)^{7/8}\,dx = a^{\frac{7}{8}}xe^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{7}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)$$

input
```
integrate((b*x**2-a)**(7/8),x)
```

output
```
a**(7/8)*x*exp(7*I*pi/8)*hyper((-7/8, 1/2), (3/2,), b*x**2/a)
```

Maxima [F]

$$\int \left(-a+bx^2\right)^{7/8}\,dx = \int \left(bx^2-a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8), x)
```

Giac [F]

$$\int \left(-a+bx^2\right)^{7/8}\,dx = \int \left(bx^2-a\right)^{\frac{7}{8}}\,dx$$

input
```
integrate((b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \left(-a+bx^2\right)^{7/8} dx = \frac{x\,(b\,x^2-a)^{7/8}\,{}_2\mathrm{F}_1\left(-\frac{7}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{\left(1-\frac{b\,x^2}{a}\right)^{7/8}}$$

input
```
int((b*x^2 - a)^(7/8),x)
```

output
```
(x*(b*x^2 - a)^(7/8)*hypergeom([-7/8, 1/2], 3/2, (b*x^2)/a))/(1 - (b*x^2)/
a)^(7/8)
```

Reduce [F]

$$\int \left(-a+bx^2\right)^{7/8} dx = \int \left(b\,x^2-a\right)^{\frac{7}{8}} dx$$

input
```
int((b*x^2-a)^(7/8),x)
```

output
```
int(( - a + b*x**2)**(7/8),x)
```

3.1221 $\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 926

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \text{Too large to display}$$

output
```
7/3*b*x/(b*x^2-a)^(1/8)-(b*x^2-a)^(7/8)/x+7/6*(2+2^(1/2))^(1/2)*a^(3/4)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(
1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/6*(2+2^(1/2))^(1
/2)*a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1
/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/
4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/x/(a^(1/4)-(b*x^2-a)^(1/4))-7/6*
a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(
b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(
2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/x/(a^(1/4)+(b*x^2-
a)^(1/4))-7/6*a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/
8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(
1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^
(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/x/(a
^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = -\frac{(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(7/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, -1/2, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(7/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{7/8}}{x^2}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{7/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{7/8}}{x^2}dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{8},-\frac{1}{2},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(7/8)/x^2,x]
```

output
```
-(((-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/8, -1/2, 1/2, (b*x^2)/a])/(x*(1
 - (b*x^2)/a)^(7/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{7}{8}}}{x^2}dx$$

input
```
int((b*x^2-a)^(7/8)/x^2,x)
```

output
```
int((b*x^2-a)^(7/8)/x^2,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \frac{a^{\frac{7}{8}}e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{7}{8}, -\frac{1}{2} \\ \frac{1}{2} \end{matrix}\middle|\, \frac{bx^2}{a}\right)}{x}$$

input
```
integrate((b*x**2-a)**(7/8)/x**2,x)
```

output
```
a**(7/8)*exp(-I*pi/8)*hyper((-7/8, -1/2), (1/2,), b*x**2/a)/x
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^2, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^2}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \frac{4\,(b\,x^2-a)^{7/8}\,{}_2\mathrm{F}_1\left(-\frac{7}{8},-\frac{3}{8};\ \frac{5}{8};\ \frac{a}{b\,x^2}\right)}{3\,x\,\left(1-\frac{a}{b\,x^2}\right)^{7/8}}$$

input
```
int((b*x^2 - a)^(7/8)/x^2,x)
```

output
```
(4*(b*x^2 - a)^(7/8)*hypergeom([-7/8, -3/8], 5/8, a/(b*x^2)))/(3*x*(1 - a/
(b*x^2))^(7/8))
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^2}\,dx = \frac{4(b\,x^2-a)^{\frac{5}{8}}\,a-4(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2-5(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2-a}}{(b\,x^2-a)^{\frac{5}{8}}\,x^2}dx\right)a x}{9\,(b\,x^2-a)^{\frac{3}{4}}\,x}$$

input
```
int((b*x^2-a)^(7/8)/x^2,x)
```

output
```
(4*( - a + b*x**2)**(5/8)*a - 4*( - a + b*x**2)**(5/8)*b*x**2 - 5*( - a +
b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*x**2),x)*a*
x)/(9*( - a + b*x**2)**(3/4)*x)
```

3.1222 $\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 961

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx = \text{Too large to display}$$

output
```
-7/12*b^2*x/a/(b*x^2-a)^(1/8)-1/3*(b*x^2-a)^(7/8)/x^3+7/12*b*(b*x^2-a)^(7/
8)/a/x-7/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*
x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*E
llipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a
)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/x/(a
^(1/4)+(b*x^2-a)^(1/4))-7/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2
-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/a^(1/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))+7/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^
(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a
)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+
2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2)/a^(1/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/24*b*(-b*x^2/a
^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^
2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-
a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(
-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx = -\frac{(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(7/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-3/2, -7/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{7/8}}{x^4}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{(bx^2-a)^{7/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{7/8}}{x^4}dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{(bx^2-a)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{7}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(7/8)/x^4,x]
```

output
```
-1/3*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-3/2, -7/8, -1/2, (b*x^2)/a])/(
x^3*(1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{7}{8}}}{x^4}dx$$

input
```
int((b*x^2-a)^(7/8)/x^4,x)
```

output
```
int((b*x^2-a)^(7/8)/x^4,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^4,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.04

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx = \frac{a^{\frac{7}{8}}e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{3}{2}, -\frac{7}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{3x^3}$$

input
```
integrate((b*x**2-a)**(7/8)/x**4,x)
```

output
```
a**(7/8)*exp(-I*pi/8)*hyper((-3/2, -7/8), (-1/2,), b*x**2/a)/(3*x**3)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^4, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx=\int \frac{(bx^2-a)^{\frac{7}{8}}}{x^4}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^4,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx=\int \frac{(b\,x^2-a)^{7/8}}{x^4}\,dx$$

input
```
int((b*x^2 - a)^(7/8)/x^4,x)
```

output
```
int((b*x^2 - a)^(7/8)/x^4, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^4}\,dx=\frac{16(b\,x^2-a)^{\frac{5}{8}}\,a^2-36(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^2+20(b\,x^2-a)^{\frac{5}{8}}\,b^2x^4+25(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x}}{(b\,x^2}\right.}{68\,(}$$

input
```
int((b*x^2-a)^(7/8)/x^4,x)
```

output
```
(16*( - a + b*x**2)**(5/8)*a**2 - 36*( - a + b*x**2)**(5/8)*a*b*x**2 + 20*
( - a + b*x**2)**(5/8)*b**2*x**4 + 25*( - a + b*x**2)**(3/4)*int(sqrt( - a
 + b*x**2)/( - a + b*x**2)**(5/8),x)*b**2*x**3 - 20*( - a + b*x**2)**(3/4)
*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*x**4),x)*a**2*x**3 - 20*(
 - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*x**2
),x)*a*b*x**3)/(68*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1223 $\int \frac{(-a+bx^2)^{7/8}}{x^6} dx$

Optimal result

Integrand size = 17, antiderivative size = 995

$$\int \frac{(-a+bx^2)^{7/8}}{x^6} dx = \text{Too large to display}$$

output
```
-7/48*b^3*x/a^2/(b*x^2-a)^(1/8)-1/5*(b*x^2-a)^(7/8)/x^5+7/60*b*(b*x^2-a)^(
7/8)/a/x^3+7/48*b^2*(b*x^2-a)^(7/8)/a^2/x-7/96*(2+2^(1/2))^(1/2)*b^2*(-b*x
^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*
x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/
2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-7/96*(2+2^(1/
2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^
(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(
1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(
b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/x/(a^(1/4)-(b*x^2-a)^(
1/4))+7/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^
(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^
(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/
(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/4)/x/(
a^(1/4)+(b*x^2-a)^(1/4))+7/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(
b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2
)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2
-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2)
)^(1/2)/a^(5/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = -\frac{(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(7/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-5/2, -7/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx^2-a\right)^{7/8}}{x^6} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(bx^2-a\right)^{7/8} \int \frac{\left(1-\frac{bx^2}{a}\right)^{7/8}}{x^6}dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(bx^2-a\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},-\frac{7}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 \left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(7/8)/x^6,x]
```

output
```
-1/5*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-5/2, -7/8, -3/2, (b*x^2)/a])/(
x^5*(1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{7}{8}}}{x^6}dx$$

input
```
int((b*x^2-a)^(7/8)/x^6,x)
```

output
```
int((b*x^2-a)^(7/8)/x^6,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^6,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.03

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \frac{a^{\frac{7}{8}}e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},-\frac{7}{8}\\ -\frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{5x^5}$$

input
```
integrate((b*x**2-a)**(7/8)/x**6,x)
```

output
```
a**(7/8)*exp(-I*pi/8)*hyper((-5/2, -7/8), (-3/2,), b*x**2/a)/(5*x**5)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^6, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^6}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^6,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \int \frac{(b\,x^2-a)^{7/8}}{x^6}\,dx$$

input
```
int((b*x^2 - a)^(7/8)/x^6,x)
```

output
```
int((b*x^2 - a)^(7/8)/x^6, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^6}\,dx = \frac{48(b\,x^2-a)^{\frac{5}{8}}\,a^2-68(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^2+20(b\,x^2-a)^{\frac{5}{8}}\,b^2x^4-60(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b}}{(b\,x^2}\right.}{300}$$

input
```
int((b*x^2-a)^(7/8)/x^6,x)
```

output
```
(48*( - a + b*x**2)**(5/8)*a**2 - 68*( - a + b*x**2)**(5/8)*a*b*x**2 + 20*
( - a + b*x**2)**(5/8)*b**2*x**4 - 60*( - a + b*x**2)**(3/4)*int(sqrt( - a
 + b*x**2)/(( - a + b*x**2)**(5/8)*x**6),x)*a**2*x**5 - 60*( - a + b*x**2)
**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*x**4),x)*a*b*x**5
+ 65*( - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8
)*x**2),x)*b**2*x**5)/(300*( - a + b*x**2)**(3/4)*a*x**5)
```

3.1224 $\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1021

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \text{Too large to display}$$

output
```
-13/192*b^4*x/a^3/(b*x^2-a)^(1/8)-1/7*(b*x^2-a)^(7/8)/x^7+1/20*b*(b*x^2-a)
^(7/8)/a/x^5+13/240*b^2*(b*x^2-a)^(7/8)/a^2/x^3+13/192*b^3*(b*x^2-a)^(7/8)
/a^3/x-13/384*(2+2^(1/2))^(1/2)*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)
*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x
^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/
x/(a^(1/4)+(b*x^2-a)^(1/4))-13/384*(2+2^(1/2))^(1/2)*b^3*(-b*x^2/a^(1/2)/(
b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4
)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)
/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(
1/2))^(1/2))/a^(9/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))+13/384*b^3*(-b*x^2/a^(1/2
)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1
/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(9/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))+13/3
84*b^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(
b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2
^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a
)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(9/4)/x/(a^(1/4)-
(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = -\frac{(-a+bx^2)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(7/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/2, -7/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx^2-a)^{7/8}}{x^8}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{(bx^2-a)^{7/8}\int \frac{\left(1-\frac{bx^2}{a}\right)^{7/8}}{x^8}dx}{\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{(bx^2-a)^{7/8}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{8},-\frac{5}{2},\frac{bx^2}{a}\right)}{7x^7\left(1-\frac{bx^2}{a}\right)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(7/8)/x^8,x]
```

output
```
-1/7*((-a + b*x^2)^(7/8)*Hypergeometric2F1[-7/2, -7/8, -5/2, (b*x^2)/a])/(
x^7*(1 - (b*x^2)/a)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2-a)^{\frac{7}{8}}}{x^8}dx$$

input
```
int((b*x^2-a)^(7/8)/x^8,x)
```

output
```
int((b*x^2-a)^(7/8)/x^8,x)
```

Fricas [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^8,x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/x^8, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.03

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \frac{a^{\frac{7}{8}}e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{7}{2}, -\frac{7}{8} \\ -\frac{5}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{7x^7}$$

input
```
integrate((b*x**2-a)**(7/8)/x**8,x)
```

output
```
a**(7/8)*exp(-I*pi/8)*hyper((-7/2, -7/8), (-5/2,), b*x**2/a)/(7*x**7)
```

Maxima [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^8, x)
```

Giac [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(bx^2-a)^{\frac{7}{8}}}{x^8}\,dx$$

input
```
integrate((b*x^2-a)^(7/8)/x^8,x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(7/8)/x^8, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \int \frac{(b\,x^2-a)^{7/8}}{x^8}\,dx$$

input
```
int((b*x^2 - a)^(7/8)/x^8,x)
```

output
```
int((b*x^2 - a)^(7/8)/x^8, x)
```

Reduce [F]

$$\int \frac{(-a+bx^2)^{7/8}}{x^8}\,dx = \frac{16(b\,x^2-a)^{\frac{5}{8}}\,a^2-20(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^2+4(b\,x^2-a)^{\frac{5}{8}}\,b^2x^4-20(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x}}{(b\,x^2-}\right.}{132\,(}$$

input
```
int((b*x^2-a)^(7/8)/x^8,x)
```

output
```
(16*( - a + b*x**2)**(5/8)*a**2 - 20*( - a + b*x**2)**(5/8)*a*b*x**2 + 4*(
 - a + b*x**2)**(5/8)*b**2*x**4 - 20*( - a + b*x**2)**(3/4)*int(sqrt( - a
+ b*x**2)/(( - a + b*x**2)**(5/8)*x**8),x)*a**2*x**7 - 20*( - a + b*x**2)*
*(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*x**6),x)*a*b*x**7 +
 21*( - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)
*x**4),x)*b**2*x**7)/(132*( - a + b*x**2)**(3/4)*a*x**7)
```

3.1225 $\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 996

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx = \text{Too large to display}$$

output
```
1280/5643*a^3*x/b^3/(b*x^2-a)^(1/8)+320/1881*a^2*x*(b*x^2-a)^(7/8)/b^3+80/
513*a*x^3*(b*x^2-a)^(7/8)/b^2+4/27*x^5*(b*x^2-a)^(7/8)/b+640/5643*(2+2^(1/
2))^(1/2)*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*
((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*
(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/
2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)+(b*x^2-a)^
(1/4))+640/5643*(2+2^(1/2))^(1/2)*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2)
)^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1
/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/
2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b
^4/x/(a^(1/4)-(b*x^2-a)^(1/4))-640/5643*a^(15/4)*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-
a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-640/5643*a^(15/4)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(b*x^2-a)^(1
/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^7\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(1/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\sqrt[8]{bx^2-a}}dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^6}{\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$
$$\Big\downarrow \text{278}$$
$$\frac{x^7\sqrt[8]{1-\frac{bx^2}{a}}\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\sqrt[8]{bx^2-a}}$$

input
```
Int[x^6/(-a + b*x^2)^(1/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{\left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(1/8),x)
```

output
```
int(x^6/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{x^6}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 - a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^7 e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7\sqrt[8]{a}}$$

input
```
integrate(x**6/(b*x**2-a)**(1/8),x)
```

output
```
x**7*exp(-I*pi/8)*hyper((1/8, 7/2), (9/2,), b*x**2/a)/(7*a**(1/8))
```

Maxima [F]

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{x^6}{{\left(b\,x^2-a\right)}^{1/8}} \,d x$$

input
```
int(x^6/(b*x^2 - a)^(1/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^6}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{x^6}{\left(b\,x^2-a\right)^{\frac{1}{8}}}d x$$

input
```
int(x^6/(b*x^2-a)^(1/8),x)
```

output
```
int(x**6/( - a + b*x**2)**(1/8),x)
```

3.1226 $\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 970

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \text{Too large to display}$$

output
```
64/209*a^2*x/b^2/(b*x^2-a)^(1/8)+48/209*a*x*(b*x^2-a)^(7/8)/b^2+4/19*x^3*(
b*x^2-a)^(7/8)/b+32/209*(2+2^(1/2))^(1/2)*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-
a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^
2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/
4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^
(1/2))/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))+32/209*(2+2^(1/2))^(1/2)*a^(11/4)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))-32/209*a^(11/
4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/
2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a
)^(1/4))-32/209*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^
(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipti
cF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)
/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b
^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^5\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(1/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt[8]{bx^2-a}}dx$$

$$\Big\downarrow \; 279$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^4}{\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \; 278$$

$$\frac{x^5\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\sqrt[8]{bx^2-a}}$$

input
```
Int[x^4/(-a + b*x^2)^(1/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2-a)^{\frac{1}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(1/8),x)
```

output
```
int(x^4/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{x^4}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 - a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^5 e^{-\frac{i\pi}{8}} {}_2F_1\left(\begin{matrix} \frac{1}{8}, \frac{5}{2} \\ \frac{7}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5\sqrt[8]{a}}$$

input
```
integrate(x**4/(b*x**2-a)**(1/8),x)
```

output
```
x**5*exp(-I*pi/8)*hyper((1/8, 5/2), (7/2,), b*x**2/a)/(5*a**(1/8))
```

Maxima [F]

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^4}{{\left(b\,x^2-a\right)}^{1/8}}\,dx$$

input
```
int(x^4/(b*x^2 - a)^(1/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^4}{\left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(1/8),x)
```

output
```
int(x**4/( - a + b*x**2)**(1/8),x)
```

3.1227 $\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 944

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \text{Too large to display}$$

output
```
16/33*a*x/b/(b*x^2-a)^(1/8)+4/11*x*(b*x^2-a)^(7/8)/b+8/33*(2+2^(1/2))^(1/2
)*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)
+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)
*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^
2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/
33*(2+2^(1/2))^(1/2)*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)-
(b*x^2-a)^(1/4))-8/33*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))-8/33*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2
-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.00 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^3\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(1/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt[8]{bx^2-a}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^2}{\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^3 \sqrt[8]{1 - \frac{bx^2}{a}} \text{Hypergeometric2F1}\left(\frac{1}{8}, \frac{3}{2}, \frac{5}{2}, \frac{bx^2}{a}\right)}{3 \sqrt[8]{bx^2 - a}}$$

input
```
Int[x^2/(-a + b*x^2)^(1/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2 - a)^{\frac{1}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(1/8),x)
```

output
```
int(x^2/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 - a)^(1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \frac{x^3 e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{3\sqrt[8]{a}}$$

input
```
integrate(x**2/(b*x**2-a)**(1/8),x)
```

output
```
x**3*exp(-I*pi/8)*hyper((1/8, 3/2), (5/2,), b*x**2/a)/(3*a**(1/8))
```

Maxima [F]

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(1/8), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(1/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{1/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(1/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(1/8), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{\frac{1}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(1/8),x)
```

output
```
int(x**2/( - a + b*x**2)**(1/8),x)
```

3.1228 $\int \frac{1}{\sqrt[8]{-a+bx^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 919

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{4x}{3\sqrt[8]{-a+bx^2}}$$

$$+\frac{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdots}}}\right.\right.\cdots}{3bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{2}}a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdots}}}\right.\right.\cdots}{3bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+\cdots}}}\right.\right.\cdots}{3\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2a^{3/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+\cdots}}}\right.\right.\cdots}{3\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/3*x/(b*x^2-a)^(1/8)+2/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2
-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x
^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1
/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))
^(1/2))/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-b*x^
2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4
))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x
^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2
),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)-(b*x^2-a)^(1/4))-2/3*a^(3/4)*(-b*x^2/
a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^
2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2
-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),
(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2/3*
a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(
2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx=\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[(-a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(1/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[8]{bx^2-a}}dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{1}{\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{\sqrt[8]{bx^2-a}}$$

input
```
Int[(-a + b*x^2)^(-1/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(1/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{1}{8}}}d x$$

input
```
int(1/(b*x^2-a)^(1/8),x)
```

output
```
int(1/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-1/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.03

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \frac{xe^{-\frac{i\pi}{8}}{}_2F_1\left(\begin{matrix}\frac{1}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{\sqrt[8]{a}}$$

input
```
integrate(1/(b*x**2-a)**(1/8),x)
```

output
```
x*exp(-I*pi/8)*hyper((1/8, 1/2), (3/2,), b*x**2/a)/a**(1/8)
```

Maxima [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-1/8), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-1/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx=\frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{1/8}\,{}_2\mathrm{F}_1\left(\frac{1}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(b\,x^2-a\right)}^{1/8}}$$

input
```
int(1/(b*x^2 - a)^(1/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(1/8)*hypergeom([1/8, 1/2], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(1/8)
```

Reduce [F]

$$\int \frac{1}{\sqrt[8]{-a+bx^2}}\,dx=\int \frac{1}{\left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(1/8),x)
```

output
```
int(1/( - a + b*x**2)**(1/8),x)
```

3.1229 $\int \frac{1}{x^2 \sqrt[8]{-a+bx^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 929

$$\int \frac{1}{x^2 \sqrt[8]{-a+bx^2}} \, dx = \text{Too large to display}$$

output

```
-b*x/a/(b*x^2-a)^(1/8)+(b*x^2-a)^(7/8)/a/x-1/2*(2+2^(1/2))^(1/2)*(-b*x^2/a
^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2
/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-
a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(
-2+2*2^(1/2))^(1/2))/a^(1/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-1/2*(2+2^(1/2))^(
1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*
x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(
1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^
(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))+1/2
*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a
)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)
-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4
))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/4)/x/(a^(1/4)+(b*x^2
-a)^(1/4))+1/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a
^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^
(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/
(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/4)/x/(
a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = -\frac{\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(1/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 1/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(1/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt[8]{bx^2-a}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{1}{x^2\sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{ 278}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(1/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 1/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(1/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2\left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/x^2/(b*x^2-a)^(1/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = \frac{e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{1}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{\sqrt[8]{a}x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(1/8),x)
```

output
```
exp(7*I*pi/8)*hyper((-1/2, 1/8), (1/2,), b*x**2/a)/(a**(1/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = -\frac{4\left(1-\frac{a}{b\,x^2}\right)^{1/8}{}_2F_1\left(\frac{1}{8},\frac{5}{8};\ \frac{13}{8};\ \frac{a}{b\,x^2}\right)}{5\,x\,(b\,x^2-a)^{1/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(1/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(1/8)*hypergeom([1/8, 5/8], 13/8, a/(b*x^2)))/(5*x*(b*
x^2 - a)^(1/8))
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt[8]{-a+bx^2}}\,dx = \frac{-36(b\,x^2-a)^{\frac{5}{8}}\,a+20(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2+45(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2-a}}{(b\,x^2-a)^{\frac{5}{8}}\,a-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2}dx\right)abx-25(b\,x^2-a)^{\frac{3}{4}}}{36\,(b\,x^2-a)^{\frac{3}{4}}\,ax}$$

input
```
int(1/x^2/(b*x^2-a)^(1/8),x)
```

output
```
( - 36*( - a + b*x**2)**(5/8)*a + 20*( - a + b*x**2)**(5/8)*b*x**2 + 45*(
- a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a - (
 - a + b*x**2)**(5/8)*b*x**2),x)*a*b*x - 25*( - a + b*x**2)**(3/4)*int((sq
rt( - a + b*x**2)*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)
*b*x**2),x)*b**2*x)/(36*( - a + b*x**2)**(3/4)*a*x)
```

3.1230 $\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 964

$$\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}}\,dx = \text{Too large to display}$$

output
```
-5/12*b^2*x/a^2/(b*x^2-a)^(1/8)+1/3*(b*x^2-a)^(7/8)/a/x^3+5/12*b*(b*x^2-a)
^(7/8)/a^2/x-5/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(
1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4
)/x/(a^(1/4)+(b*x^2-a)^(1/4))-5/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*
x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/
(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a
^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/
2))^(1/2))/a^(5/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))+5/24*b*(-b*x^2/a^(1/2)/(b*x
^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^
(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))+5/24*b*(-b
*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(
1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(
b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(
1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/4)/x/(a^(1/4)-(b*x^2-a)^
(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}} \, dx = -\frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 \sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(1/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 1/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \sqrt[8]{bx^2-a}} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}} \int \frac{1}{x^4 \sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 \sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(1/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 1/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\,x^2-a\right)^{\frac{1}{8}}}dx$$

input
```
int(1/x^4/(b*x^2-a)^(1/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^4\sqrt[8]{-a+bx^2}}\,dx = \frac{e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{1}{8}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3\sqrt[8]{a}x^3}$$

input
```
integrate(1/x**4/(b*x**2-a)**(1/8),x)
```

output
```
exp(7*I*pi/8)*hyper((-3/2, 1/8), (-1/2,), b*x**2/a)/(3*a**(1/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{x^4\,(b\,x^2-a)^{1/8}}\,dx$$

input
```
int(1/(x^4*(b*x^2 - a)^(1/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(1/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \sqrt[8]{-a+bx^2}}\,dx = \frac{-68(b\,x^2-a)^{\frac{5}{8}}\,a+20(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2+125(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2-a}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^4}dx\right)a b\,x^3-65(b\,x^2-}{204\,(b\,x^2-a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2-a)^(1/8),x)
```

output
```
( - 68*( - a + b*x**2)**(5/8)*a + 20*( - a + b*x**2)**(5/8)*b*x**2 + 125*(
 - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a*x*
*2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b*x**3 - 65*( - a + b*x**2)**(3/4
)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/
8)*b*x**2),x)*b**2*x**3)/(204*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1231 $\int \frac{1}{x^6 \sqrt[8]{-a+bx^2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 998

$$\int \frac{1}{x^6 \sqrt[8]{-a+bx^2}}\,dx = \text{Too large to display}$$

output
```
-13/48*b^3*x/a^3/(b*x^2-a)^(1/8)+1/5*(b*x^2-a)^(7/8)/a/x^5+13/60*b*(b*x^2-
a)^(7/8)/a^2/x^3+13/48*b^2*(b*x^2-a)^(7/8)/a^3/x-13/96*(2+2^(1/2))^(1/2)*b
^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/
2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-13/96
*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3
/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE
(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a
^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/x/(a^(1/4)-(b
*x^2-a)^(1/4))+13/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^
(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elliptic
F(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)
/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a
^(9/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))+13/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/
2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^
(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(
1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/a^(9/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^6 \sqrt[8]{-a+bx^2}}\,dx = -\frac{\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(1/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 1/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 \sqrt[8]{bx^2-a}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}} \int \frac{1}{x^6 \sqrt[8]{1-\frac{bx^2}{a}}}dx}{\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{1}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 \sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(1/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 1/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a \right)^{\frac{1}{8}}}d x$$

input
```
int(1/x^6/(b*x^2-a)^(1/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(1/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \sqrt[8]{-a+bx^2}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(1/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^6\sqrt[8]{-a+bx^2}}\,dx = \frac{e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{1}{8}\\-\frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{5\sqrt[8]{a}x^5}$$

input
```
integrate(1/x**6/(b*x**2-a)**(1/8),x)
```

output
```
exp(7*I*pi/8)*hyper((-5/2, 1/8), (-3/2,), b*x**2/a)/(5*a**(1/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(1/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\sqrt[8]{-a+bx^2}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{1}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(1/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(1/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\sqrt[8]{-a+bx^2}}\,dx=\int \frac{1}{x^6\,{\left(b\,x^2-a\right)}^{1/8}}\,dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(1/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(1/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6\sqrt[8]{-a+bx^2}}\,dx = \frac{-240(b\,x^2-a)^{\frac{5}{8}}\,a^2-116(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^2+164(b\,x^2-a)^{\frac{5}{8}}\,b^2x^4+773(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2-a}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a}\right.}{1200\,(b\,x^2-a)^{\frac{3}{4}}\,a^2x^5}$$

input
```
int(1/x^6/(b*x^2-a)^(1/8),x)
```

output
```
( - 240*( - a + b*x**2)**(5/8)*a**2 - 116*( - a + b*x**2)**(5/8)*a*b*x**2
+ 164*( - a + b*x**2)**(5/8)*b**2*x**4 + 773*( - a + b*x**2)**(3/4)*int(sq
rt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*
b*x**4),x)*a*b**2*x**5 - 533*( - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2
)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5)
/(1200*( - a + b*x**2)**(3/4)*a**2*x**5)
```

3.1232 $\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 972

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \text{Too large to display}$$

output
```
64/255*a^2*x*(b*x^2-a)^(5/8)/b^3+16/85*a*x^3*(b*x^2-a)^(5/8)/b^2+4/25*x^5*
(b*x^2-a)^(5/8)/b-128/255*(2+2^(1/2))^(1/2)*a^(7/2)*(-b*x^2/a^(1/2)/(b*x^2
-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x
^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1
/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))
^(1/2))/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))+128/255*(2+2^(1/2))^(1/2)*a^(7/2)*
(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a
)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+
2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4)
)^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))+128/255*a^(7
/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1
/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(
1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-
a)^(1/4))-128/255*a^(7/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)
^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipt
icF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2
)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/
b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.91 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \frac{x^7\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(3/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2-a)^{3/8}} \, dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{x^6}{\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^7 \left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{3}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\,(bx^2-a)^{3/8}}$$

input
```
Int[x^6/(-a + b*x^2)^(3/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(3/8),x)
```

output
```
int(x^6/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 - a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \frac{x^7e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix} \frac{3}{8}, \frac{7}{2} \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{7a^{\frac{3}{8}}}$$

input
```
integrate(x**6/(b*x**2-a)**(3/8),x)
```

output
```
x**7*exp(-3*I*pi/8)*hyper((3/8, 7/2), (9/2,), b*x**2/a)/(7*a**(3/8))
```

Maxima [F]

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^6}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{x^6}{\left(b\,x^2-a\right)^{3/8}}\,dx$$

input
```
int(x^6/(b*x^2 - a)^(3/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^6}{\left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{x^6}{\left(b\,x^2-a\right)^{\frac{3}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(3/8),x)
```

output
```
int(x**6/( - a + b*x**2)**(3/8),x)
```

3.1233 $\int \frac{x^4}{(-a+bx^2)^{3/8}} dx$

Optimal result

Integrand size = 17, antiderivative size = 946

$$\int \frac{x^4}{(-a+bx^2)^{3/8}} dx = \text{Too large to display}$$

output

```
16/51*a*x*(b*x^2-a)^(5/8)/b^2+4/17*x^3*(b*x^2-a)^(5/8)/b-32/51*(2+2^(1/2))
^(1/2)*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^
(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^
(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/
(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4
))+32/51*(2+2^(1/2))^(1/2)*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*
(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^
2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^
(1/4)-(b*x^2-a)^(1/4))+32/51*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2
)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1
/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*
x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1
/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-32/51*a^(5/2)*(-b*x^2/a^(1/2)/(
b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4
)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)
/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(
1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.83 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx = \frac{x^5\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5(-a+bx^2)^{3/8}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(3/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2-a)^{3/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{x^4}{\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^5\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5(bx^2-a)^{3/8}}$$

input
```
Int[x^4/(-a + b*x^2)^(3/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(3/8),x)
```

output
```
int(x^4/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 - a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx = \frac{x^5e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{5a^{\frac{3}{8}}}$$

input
```
integrate(x**4/(b*x**2-a)**(3/8),x)
```

output
```
x**5*exp(-3*I*pi/8)*hyper((3/8, 5/2), (7/2,), b*x**2/a)/(5*a**(3/8))
```

Maxima [F]

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx=\int \frac{x^4}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx=\int \frac{x^4}{(b\,x^2-a)^{3/8}}\,dx$$

input
```
int(x^4/(b*x^2 - a)^(3/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(-a+bx^2)^{3/8}}\,dx=\int \frac{x^4}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(3/8),x)
```

output
```
int(x**4/( - a + b*x**2)**(3/8),x)
```

3.1234 $\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 922

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx = \text{Too large to display}$$

output
```
4/9*x*(b*x^2-a)^(5/8)/b-8/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*x^2/a^(1/2)/(b*x
^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^
(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2
))^(1/2))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))+8/9*a^(3/2)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(
1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/
4))-8/9*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(
a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a
^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))
/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(
1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.50 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx = \frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(3/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-a)^{3/8}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{x^2}{\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(bx^2-a)^{3/8}}$$

input
```
Int[x^2/(-a + b*x^2)^(3/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{2}}{\left(b\,x^{2}-a\right)^{\frac{3}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(3/8),x)
```

output
```
int(x^2/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx=\int \frac{x^2}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 - a)^(3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx=\frac{x^3e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{3a^{\frac{3}{8}}}$$

input
```
integrate(x**2/(b*x**2-a)**(3/8),x)
```

output
```
x**3*exp(-3*I*pi/8)*hyper((3/8, 3/2), (5/2,), b*x**2/a)/(3*a**(3/8))
```

Maxima [F]

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx=\int \frac{x^2}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(3/8), x)
```

Giac [F]

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(3/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{3/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(3/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(3/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(-a+bx^2)^{3/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(3/8),x)
```

output
```
int(x**2/( - a + b*x**2)**(3/8),x)
```

3.1235 $\int \frac{1}{\left(-a+bx^2\right)^{3/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 893

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx =$$

$$-\frac{2\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{2+\sqrt{2}}\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{2\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b/x/(a^(1/4)+(b
*x^2-a)^(1/4))+2*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2)
)^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1
/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/
2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b
/x/(a^(1/4)-(b*x^2-a)^(1/4))+2*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1
/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^
(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(
b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^
2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(
1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2)
)^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(3/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{3/8}} \, dx$$

$$\Big\downarrow \text{238}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(\frac{3}{8},\frac{1}{2},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(-3/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[3/8, 1/2, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(3/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] && !IntegerQ[2*p] && !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(3/8),x)
```

output
```
int(1/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-3/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx=\frac{xe^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{3}{8},\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{3}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(3/8),x)
```

output
```
x*exp(-3*I*pi/8)*hyper((3/8, 1/2), (3/2,), b*x**2/a)/a**(3/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-3/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{3}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-3/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{3/8}\,{}_2\mathrm{F}_1\left(\frac{3}{8},\frac{1}{2};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{3/8}}$$

input
```
int(1/(b*x^2 - a)^(3/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(3/8)*hypergeom([3/8, 1/2], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(3/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(3/8),x)
```

output
```
int(1/( - a + b*x**2)**(3/8),x)
```

3.1236 $\int \frac{1}{x^2\left(-a+bx^2\right)^{3/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 909

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{3/8}}\,dx = \frac{\left(-a+bx^2\right)^{5/8}}{ax}$$

$$+\frac{\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{a}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{a}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}\sqrt{a}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$+\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}\sqrt{a}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
(b*x^2-a)^(5/8)/a/x+1/2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))
^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4
))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2
)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^
(1/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-1/2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b
*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)
/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/
a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1
/2))^(1/2))/a^(1/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))-1/2*(-b*x^2/a^(1/2)/(b*x^2
-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x
^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1
/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))
^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+1/2*(-b*x^2/
a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))
^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2
-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),
(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/x/(a^(1/4)-(b*x^2-a)^(1/4)
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.64 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 (-a+bx^2)^{3/8}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x (-a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(3/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 3/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(3/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2-a)^{3/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{x^2 \left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x (bx^2-a)^{3/8}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(3/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 3/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(3/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}-a\right)^{\frac{3}{8}}}dx$$

input
```
int(1/x^2/(b*x^2-a)^(3/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{3}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{3/8}}\,dx=\frac{e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{8}\\ \frac{1}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{a^{\frac{3}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(3/8),x)
```

output
```
exp(5*I*pi/8)*hyper((-1/2, 3/8), (1/2,), b*x**2/a)/(a**(3/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{3/8}}\, dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{3}{8}} x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{3/8}}\, dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{3}{8}} x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{3/8}}\,dx = -\frac{4\left(1-\frac{a}{b\,x^2}\right)^{3/8}\,{}_2\mathrm{F}_1\left(\frac{3}{8},\frac{7}{8};\ \frac{15}{8};\ \frac{a}{b\,x^2}\right)}{7\,x\left(b\,x^2-a\right)^{3/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(3/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(3/8)*hypergeom([3/8, 7/8], 15/8, a/(b*x^2)))/(7*x*(b*
x^2 - a)^(3/8))
```

Reduce [F]

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{3/8}}\,dx = \frac{-36(b\,x^2-a)^{\frac{3}{8}}\,a+20(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2+15(b\,x^2-a)^{\frac{3}{4}}\left(\int\frac{(b\,x^2-a)^{\frac{1}{4}}}{(b\,x^2-a)^{\frac{5}{8}}a-(b\,x^2-a)^{\frac{5}{8}}b\,x^2}dx\right.}{36\,(b\,x^2-a)^{\frac{3}{4}}\,ax}$$

input
```
int(1/x^2/(b*x^2-a)^(3/8),x)
```

output
```
( - 36*( - a + b*x**2)**(3/8)*a + 20*( - a + b*x**2)**(3/8)*b*x**2 + 15*(
- a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a
- ( - a + b*x**2)**(5/8)*b*x**2),x)*a*b*x + 5*( - a + b*x**2)**(3/4)*int((
( - a + b*x**2)**(1/4)*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**
(5/8)*b*x**2),x)*b**2*x)/(36*( - a + b*x**2)**(3/4)*a*x)
```

3.1237 $\int \frac{1}{x^4(-a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 940

$$\int \frac{1}{x^4\,(-a+bx^2)^{3/8}}\,dx = \text{Too large to display}$$

output

```
1/3*(b*x^2-a)^(5/8)/a/x^3+7/12*b*(b*x^2-a)^(5/8)/a^2/x+7/24*(2+2^(1/2))^(1
/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*
x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^
(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)
^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-7/
24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3
/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE
(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a
^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2)/x/(a^(1/4)-(b
*x^2-a)^(1/4))-7/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/
8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1
/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^
(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3
/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1
/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(
b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^
(1/2))^(1/2)/a^(3/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^4 (-a+bx^2)^{3/8}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 (-a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(3/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 3/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2-a)^{3/8}}\,dx$$

$$\Big\downarrow \; 279$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{x^4\left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{3}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 (bx^2-a)^{3/8}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(3/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 3/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}-a \right)^{\frac{3}{8}}}d x$$

input
```
int(1/x^4/(b*x^2-a)^(3/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(-a+b x^2\right)^{3/8}} \, dx=\int \frac{1}{\left(b x^2-a\right)^{\frac{3}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^4 \left(-a+b x^2\right)^{3/8}} \, dx=\frac{e^{\frac{5 i \pi}{8}} {}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2}, \frac{3}{8} \\ -\frac{1}{2} \end{matrix}\middle| {\frac{b x^{2}}{a}} \right)}{3 a^{\frac{3}{8}} x^{3}}$$

input
```
integrate(1/x**4/(b*x**2-a)**(3/8),x)
```

output
```
exp(5*I*pi/8)*hyper((-3/2, 3/8), (-1/2,), b*x**2/a)/(3*a**(3/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 \left(-a+b x^2\right)^{3/8}} \, dx=\int \frac{1}{\left(b x^2-a\right)^{\frac{3}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{3/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{3}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-a+bx^2)^{3/8}}\,dx = \int \frac{1}{x^4\,(b\,x^2-a)^{3/8}}\,dx$$

input
```
int(1/(x^4*(b*x^2 - a)^(3/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(3/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{3/8}}\,dx = \frac{-68(b\,x^2-a)^{\frac{3}{8}}\,a+20(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2+95(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2-a)^{\frac{1}{4}}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^4}\right.}{204\,(b\,x^2-a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2-a)^(3/8),x)
```

output
```
( - 68*( - a + b*x**2)**(3/8)*a + 20*( - a + b*x**2)**(3/8)*b*x**2 + 95*(
- a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a*
x**2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b*x**3 - 35*( - a + b*x**2)**(3
/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)
**(5/8)*b*x**2),x)*b**2*x**3)/(204*( - a + b*x**2)**(3/4)*a*x**3)
```

3.1238 $\int \frac{1}{x^6(-a+bx^2)^{3/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 974

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{3/8}}\,dx = \text{Too large to display}$$

output
```
1/5*(b*x^2-a)^(5/8)/a/x^5+1/4*b*(b*x^2-a)^(5/8)/a^2/x^3+7/16*b^2*(b*x^2-a)
^(5/8)/a^3/x+7/32*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(
1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*
(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5
/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-7/32*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)
/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/
4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2
^(1/2))^(1/2))/a^(5/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))-7/32*b^2*(-b*x^2/a^(1/2
)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1
/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/32
*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*
x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(
1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^
(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/2)/x/(a^(1/4)-(b
*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^6 (-a+bx^2)^{3/8}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5(-a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(3/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 3/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (bx^2-a)^{3/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{x^6 \left(1-\frac{bx^2}{a}\right)^{3/8}}dx}{(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{3}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 (bx^2-a)^{3/8}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(3/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 3/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a \right)^{\frac{3}{8}}}d x$$

input
```
int(1/x^6/(b*x^2-a)^(3/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(3/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\, dx = \int \frac{1}{\left(bx^{2}-a\right)^{\frac{3}{8}}x^{6}}\, dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(3/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\, dx = \frac{e^{\frac{5i\pi}{8}}\,{}_{2}F_{1}\left(\begin{matrix} -\frac{5}{2}, \frac{3}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^{2}}{a} \right)}{5a^{\frac{3}{8}}x^{5}}$$

input
```
integrate(1/x**6/(b*x**2-a)**(3/8),x)
```

output
```
exp(5*I*pi/8)*hyper((-5/2, 3/8), (-3/2,), b*x**2/a)/(5*a**(3/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{3}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(3/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{3}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(3/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(3/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\,dx=\int \frac{1}{x^6\,{\left(b\,x^2-a\right)}^{3/8}} \,dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(3/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(3/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{3/8}}\,dx=\frac{-240(b\,x^2-a)^{\frac{3}{8}}\,a^2-92(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^2+140(b\,x^2-a)^{\frac{3}{8}}\,b^2x^4+485(b\,x^2-a)^{\frac{3}{4}}}{1200\,(}$$

input
```
int(1/x^6/(b*x^2-a)^(3/8),x)
```

output
```
( - 240*( - a + b*x**2)**(3/8)*a**2 - 92*( - a + b*x**2)**(3/8)*a*b*x**2 +
 140*( - a + b*x**2)**(3/8)*b**2*x**4 + 485*( - a + b*x**2)**(3/4)*int(( -
 a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8
)*b*x**4),x)*a*b**2*x**5 - 245*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)*
*(1/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*
x**5)/(1200*( - a + b*x**2)**(3/4)*a**2*x**5)
```

3.1239 $\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 522

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \frac{64a^2x(-a+bx^2)^{3/8}}{161b^3}+\frac{16ax^3(-a+bx^2)^{3/8}}{69b^2}+\frac{4x^5(-a+bx^2)^{3/8}}{23b}$$

$$-\frac{128a^{13/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdot}}}\right.\right.}{161\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{128a^{13/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a\cdot}}}\right.\right.}{161\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
64/161*a^2*x*(b*x^2-a)^(3/8)/b^3+16/69*a*x^3*(b*x^2-a)^(3/8)/b^2+4/23*x^5*
(b*x^2-a)^(3/8)/b-128/161*a^(13/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*
(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2
)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^
2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2
))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-128/161*a^(13/4)*(-b*x^2/a^(1/2)/
(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/
4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4
)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^
(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.94 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \frac{x^7\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\,(-a+bx^2)^{5/8}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(5/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2-a)^{5/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\int \frac{x^6}{\left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^7\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\,(bx^2-a)^{5/8}}$$

input
```
Int[x^6/(-a + b*x^2)^(5/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(5/8),x)
```

output
```
int(x^6/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 - a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \frac{x^7e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7a^{\frac{5}{8}}}$$

input
```
integrate(x**6/(b*x**2-a)**(5/8),x)
```

output
```
x**7*exp(-5*I*pi/8)*hyper((5/8, 7/2), (9/2,), b*x**2/a)/(7*a**(5/8))
```

Maxima [F]

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{5/8}}\,dx$$

input
```
int(x^6/(b*x^2 - a)^(5/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(5/8),x)
```

output
```
int(x**6/( - a + b*x**2)**(5/8),x)
```

3.1240 $\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 496

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \frac{16ax(-a+bx^2)^{3/8}}{35b^2} + \frac{4x^3(-a+bx^2)^{3/8}}{15b}$$

$$-\frac{32a^{9/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+b\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+\cdots}}}\cdots\right.\right.}{35\sqrt{2+\sqrt{2}}b^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{32a^{9/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+b\cdots}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+\cdots}}}\cdots\right.\right.}{35\sqrt{2+\sqrt{2}}b^3x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
16/35*a*x*(b*x^2-a)^(3/8)/b^2+4/15*x^3*(b*x^2-a)^(3/8)/b-32/35*a^(9/4)*(-b
*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1
/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(
b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(
1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4
))-32/35*a^(9/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-
(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(
a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2)
)/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^
(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.90 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx=\frac{x^5\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\,(-a+bx^2)^{5/8}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(5/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2-a)^{5/8}} \, dx$$
$$\Big\downarrow \text{ 279}$$
$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \int \frac{x^4}{\left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$
$$\Big\downarrow \text{ 278}$$
$$\frac{x^5 \left(1-\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(\frac{5}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5 (bx^2-a)^{5/8}}$$

input
```
Int[x^4/(-a + b*x^2)^(5/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(5/8),x)
```

output
```
int(x^4/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 - a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \frac{x^5e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{5a^{\frac{5}{8}}}$$

input
```
integrate(x**4/(b*x**2-a)**(5/8),x)
```

output
```
x**5*exp(-5*I*pi/8)*hyper((5/8, 5/2), (7/2,), b*x**2/a)/(5*a**(5/8))
```

Maxima [F]

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(b\,x^2-a)^{5/8}}\,dx$$

input
```
int(x^4/(b*x^2 - a)^(5/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^4}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(5/8),x)
```

output
```
int(x**4/( - a + b*x**2)**(5/8),x)
```

3.1241 $\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 472

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \frac{4x(-a+bx^2)^{3/8}}{7b}$$

$$-\frac{8a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{7\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{8a^{5/4}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{7\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/7*x*(b*x^2-a)^(3/8)/b-8/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)
*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x
^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))-8/7*a^(5/4)*(-b*x^2/a^(1/2)/(b*x
^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(
b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^
(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \frac{x^3\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(-a+bx^2)^{5/8}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(5/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-a)^{5/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\int \frac{x^2}{\left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{x^3\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{5}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(bx^2-a)^{5/8}}$$

input
```
Int[x^2/(-a + b*x^2)^(5/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[5/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(5/8),x)
```

output
```
int(x^2/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 - a)^(5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \frac{x^3e^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{5}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3a^{\frac{5}{8}}}$$

input
```
integrate(x**2/(b*x**2-a)**(5/8),x)
```

output
```
x**3*exp(-5*I*pi/8)*hyper((5/8, 3/2), (5/2,), b*x**2/a)/(3*a**(5/8))
```

Maxima [F]

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(5/8), x)
```

Giac [F]

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(5/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{5/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(5/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(5/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(-a+bx^2)^{5/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(5/8),x)
```

output
```
int(x**2/( - a + b*x**2)**(5/8),x)
```

3.1242 $\int \frac{1}{(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 447

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx =$$

$$-\frac{2\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{2\sqrt[4]{a}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{\sqrt{2+\sqrt{2}}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
-2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4
)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4
)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x
^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b
*x^2-a)^(1/4))-2*a^(1/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^
(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipti
cF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)
/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b
/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{5/8}}$$

input
```
Integrate[(-a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(5/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{5/8}}\,dx$$

$$\Big\downarrow \; 238$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$

$$\Big\downarrow \; 237$$

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{5/8}}$$

input
```
Int[(-a + b*x^2)^(-5/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[1/2, 5/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(5/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(5/8),x)
```

output
```
int(1/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-5/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{xe^{-\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{5}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(5/8),x)
```

output
```
x*exp(-5*I*pi/8)*hyper((1/2, 5/8), (3/2,), b*x**2/a)/a**(5/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-5/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-5/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{5/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{5/8}}$$

input
```
int(1/(b*x^2 - a)^(5/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(5/8)*hypergeom([1/2, 5/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(5/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(5/8),x)
```

output
```
int(1/( - a + b*x**2)**(5/8),x)
```

3.1243 $\int \frac{1}{x^2(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 465

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{5/8}}\,dx=\frac{\left(-a+bx^2\right)^{3/8}}{ax}$$

$$-\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-2\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}a^{3/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+2\frac{\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}a^{3/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
(b*x^2-a)^(3/8)/a/x-1/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(
3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF
(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/
a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^
(3/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-1/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/
2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^
(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b
*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(
1/2))^(1/2)/a^(3/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\,dx=-\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x\left(-a+bx^2\right)^{5/8}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(5/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-1/2, 5/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(5/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2-a)^{5/8}} \, dx$$

$$\Big\downarrow \textbf{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \int \frac{1}{x^2 \left(1-\frac{bx^2}{a}\right)^{5/8}}dx}{(bx^2-a)^{5/8}}$$

$$\Big\downarrow \textbf{278}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x (bx^2-a)^{5/8}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(5/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-1/2, 5/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(5/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^2 \left(b\, x^2 - a\right)^{\frac{5}{8}}}dx$$

input
```
int(1/x^2/(b*x^2-a)^(5/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\, dx = \int \frac{1}{\left(bx^2 - a\right)^{\frac{5}{8}} x^2}\, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\, dx = \frac{e^{\frac{3i\pi}{8}}\, {}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{5}{8} \\ \frac{1}{2} \end{matrix}\right| \frac{bx^2}{a}\right)}{a^{\frac{5}{8}} x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(5/8),x)
```

output
```
exp(3*I*pi/8)*hyper((-1/2, 5/8), (1/2,), b*x**2/a)/(a**(5/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{5}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{5}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\,dx = -\frac{4\left(1-\frac{a}{b\,x^2}\right)^{5/8}{}_2\mathrm{F}_1\left(\frac{5}{8},\frac{9}{8};\ \frac{17}{8};\ \frac{a}{b\,x^2}\right)}{9\,x\left(b\,x^2-a\right)^{5/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(5/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(5/8)*hypergeom([5/8, 9/8], 17/8, a/(b*x^2)))/(9*x*(b*
x^2 - a)^(5/8))
```

Reduce [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{5/8}}\,dx = \frac{-36(b\,x^2-a)^{\frac{1}{8}}\,a+20(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2-45(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{x^2}{(b\,x^2-a)^{\frac{5}{8}}a-(b\,x^2-a)^{\frac{5}{8}}b\,x^2}d}{36\,(b\,x^2-a)^{\frac{3}{4}}\,ax}$$

input

```
int(1/x^2/(b*x^2-a)^(5/8),x)
```

output

```
( - 36*( - a + b*x**2)**(1/8)*a + 20*( - a + b*x**2)**(1/8)*b*x**2 - 45*(
- a + b*x**2)**(3/4)*int(x**2/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)*
*(5/8)*b*x**2),x)*b**2*x + 65*( - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2
)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*a*b*x)/(36*( - a + b*x**2)*
*(3/4)*a*x)
```

3.1244 $\int \frac{1}{x^4(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 494

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{5/8}}\,dx = \frac{\left(-a+bx^2\right)^{3/8}}{3ax^3}+\frac{3b\left(-a+bx^2\right)^{3/8}}{4a^2x}$$

$$-\frac{3b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{8\sqrt{2+\sqrt{2}}a^{7/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{3b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{8\sqrt{2+\sqrt{2}}a^{7/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
1/3*(b*x^2-a)^(3/8)/a/x^3+3/4*b*(b*x^2-a)^(3/8)/a^2/x-3/8*b*(-b*x^2/a^(1/2
)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1
/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*
2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(7/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-3/8*
b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2
)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/
4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(7/4)/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^4 (-a+bx^2)^{5/8}} \, dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 (-a+bx^2)^{5/8}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(5/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-3/2, 5/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2 - a)^{5/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1 - \frac{bx^2}{a}\right)^{5/8} \int \frac{1}{x^4 \left(1 - \frac{bx^2}{a}\right)^{5/8}} dx}{(bx^2 - a)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(1 - \frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{3}{2}, \frac{5}{8}, -\frac{1}{2}, \frac{bx^2}{a}\right)}{3x^3 (bx^2 - a)^{5/8}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(5/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-3/2, 5/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}-a \right)^{\frac{5}{8}}}d x$$

input
```
int(1/x^4/(b*x^2-a)^(5/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{5/8}}\,dx=\int \frac{1}{\left(bx^{2}-a\right)^{\frac{5}{8}}x^{4}}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{5/8}}\,dx=\frac{e^{\frac{3i\pi}{8}}{}_{2}F_{1}\left(\begin{matrix} -\frac{3}{2},\frac{5}{8} \\ -\frac{1}{2} \end{matrix}\middle|{\frac{bx^{2}}{a}}\right)}{3a^{\frac{5}{8}}x^{3}}$$

input
```
integrate(1/x**4/(b*x**2-a)**(5/8),x)
```

output
```
exp(3*I*pi/8)*hyper((-3/2, 5/8), (-1/2,), b*x**2/a)/(3*a**(5/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-a+bx^2)^{5/8}}\,dx = \int \frac{1}{x^4\,(b\,x^2-a)^{5/8}}\,dx$$

input
```
int(1/(x^4*(b*x^2 - a)^(5/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(5/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{5/8}}\,dx = \frac{-68(b\,x^2-a)^{\frac{1}{8}}\,a + 20(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2 + 145(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{1}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}\,b\,x}\right.}{204\,(b\,x^2-a)^{\frac{3}{4}}\,a\,x^3}$$

input
```
int(1/x^4/(b*x^2-a)^(5/8),x)
```

output
```
( - 68*( - a + b*x**2)**(1/8)*a + 20*( - a + b*x**2)**(1/8)*b*x**2 + 145*(
 - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**
2)**(5/8)*b*x**4),x)*a*b*x**3 - 85*( - a + b*x**2)**(3/4)*int(1/(( - a + b
*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**2*x**3)/(204*( - a
+ b*x**2)**(3/4)*a*x**3)
```

3.1245 $\int \frac{1}{x^6(-a+bx^2)^{5/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 524

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{5/8}}\,dx = \frac{\left(-a+bx^2\right)^{3/8}}{5ax^5} + \frac{17b\left(-a+bx^2\right)^{3/8}}{60a^2x^3} + \frac{51b^2\left(-a+bx^2\right)^{3/8}}{80a^3x}$$

$$-\frac{51b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^{11/4}x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{51b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\ \text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^{11/4}x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
1/5*(b*x^2-a)^(3/8)/a/x^5+17/60*b*(b*x^2-a)^(3/8)/a^2/x^3+51/80*b^2*(b*x^2
-a)^(3/8)/a^3/x-51/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a
)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipt
icF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/
2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
/a^(11/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-51/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2
-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/(2+2^(1/2))^(1/2)/a^(11/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^6 (-a+bx^2)^{5/8}}\,dx=-\frac{\left(1-\frac{bx^2}{a}\right)^{5/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5(-a+bx^2)^{5/8}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(5/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-5/2, 5/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(5/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (bx^2-a)^{5/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \int \frac{1}{x^6 \left(1-\frac{bx^2}{a}\right)^{5/8}} dx}{(bx^2-a)^{5/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{5/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{5}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 (bx^2-a)^{5/8}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(5/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(5/8)*Hypergeometric2F1[-5/2, 5/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(5/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a\right)^{\frac{5}{8}}}dx$$

input
```
int(1/x^6/(b*x^2-a)^(5/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(5/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(5/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(3/8)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^6 (-a+bx^2)^{5/8}}\,dx = \frac{e^{\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{5}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5a^{\frac{5}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2-a)**(5/8),x)
```

output
```
exp(3*I*pi/8)*hyper((-5/2, 5/8), (-3/2,), b*x**2/a)/(5*a**(5/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(5/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{5}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(5/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(5/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{5/8}}\,dx = \int \frac{1}{x^6\left(b\,x^2-a\right)^{5/8}}\,dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(5/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(5/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{5/8}}\,dx = \frac{-1360(b\,x^2-a)^{\frac{1}{8}}\,a^2+1892(b\,x^2-a)^{\frac{1}{8}}\,ab\,x^2+1440(b\,x^2-a)^{\frac{1}{8}}\,b^2x^4+7920(b\,x^2}{}$$

input
```
int(1/x^6/(b*x^2-a)^(5/8),x)
```

output
```
( - 1360*( - a + b*x**2)**(1/8)*a**2 + 1892*( - a + b*x**2)**(1/8)*a*b*x**
2 + 1440*( - a + b*x**2)**(1/8)*b**2*x**4 + 7920*( - a + b*x**2)**(3/4)*in
t(1/(( - a + b*x**2)**(5/8)*a*x**4 - ( - a + b*x**2)**(5/8)*b*x**6),x)*a**
2*b*x**5 - 10385*( - a + b*x**2)**(3/4)*int(1/(( - a + b*x**2)**(5/8)*a*x*
*2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b**2*x**5)/(6800*( - a + b*x**2)*
*(3/4)*a**2*x**5)
```

3.1246 $\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 518

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \frac{64a^2x\sqrt[8]{-a+bx^2}}{91b^3} + \frac{80ax^3\sqrt[8]{-a+bx^2}}{273b^2} + \frac{4x^5\sqrt[8]{-a+bx^2}}{21b}$$
$$+ \frac{128a^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a+b\cdots}}}\cdots\right.\right.}{91\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$
$$- \frac{128a^3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{-a+b\cdots}}}\cdots\right.\right.}{91\sqrt{2+\sqrt{2}}b^4x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
64/91*a^2*x*(b*x^2-a)^(1/8)/b^3+80/273*a*x^3*(b*x^2-a)^(1/8)/b^2+4/21*x^5*
(b*x^2-a)^(1/8)/b+128/91*a^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2
-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elli
pticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(
1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/
2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-128/91*a^3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1
/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)
^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^
(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2)
)/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \frac{x^7\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\left(-a+bx^2\right)^{7/8}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(7/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2-a)^{7/8}}\,dx$$
$$\Big\downarrow \text{279}$$
$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\int \frac{x^6}{\left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$
$$\Big\downarrow \text{278}$$
$$\frac{x^7\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7\,(bx^2-a)^{7/8}}$$

input
```
Int[x^6/(-a + b*x^2)^(7/8),x]
```

output
```
(x^7*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 7/2, 9/2, (b*x^2)/a])/(7
*(-a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(7/8),x)
```

output
```
int(x^6/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^6/(b*x^2 - a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \frac{x^7e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7a^{\frac{7}{8}}}$$

input
```
integrate(x**6/(b*x**2-a)**(7/8),x)
```

output
```
x**7*exp(-7*I*pi/8)*hyper((7/8, 7/2), (9/2,), b*x**2/a)/(7*a**(7/8))
```

Maxima [F]

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{7/8}}\,dx$$

input
```
int(x^6/(b*x^2 - a)^(7/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(7/8),x)
```

output
```
int(x**6/( - a + b*x**2)**(7/8),x)
```

3.1247 $\int \frac{x^4}{\left(-a+bx^2\right)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 492

$$\int \frac{x^4}{\left(-a+bx^2\right)^{7/8}}\,dx = \frac{48ax\sqrt[8]{-a+bx^2}}{65b^2} + \frac{4x^3\sqrt[8]{-a+bx^2}}{13b}$$

$$+ \frac{96a^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{65\sqrt{2+\sqrt{2}}\,b^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$- \frac{96a^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{65\sqrt{2+\sqrt{2}}\,b^3x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
48/65*a*x*(b*x^2-a)^(1/8)/b^2+4/13*x^3*(b*x^2-a)^(1/8)/b+96/65*a^2*(-b*x^2
/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))
^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^
2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2)
,(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-9
6/65*a^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)
-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*
(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2
-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b
*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.97 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \frac{x^5\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\,(-a+bx^2)^{7/8}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(7/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2-a)^{7/8}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \int \frac{x^4}{\left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^5 \left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(\frac{7}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5\,(bx^2-a)^{7/8}}$$

input
```
Int[x^4/(-a + b*x^2)^(7/8),x]
```

output
```
(x^5*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 5/2, 7/2, (b*x^2)/a])/(5
*(-a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^{4}}{\left(b\,x^{2}-a\right)^{\frac{7}{8}}}d x$$

input
```
int(x^4/(b*x^2-a)^(7/8),x)
```

output
```
int(x^4/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^{4}}{(bx^{2}-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^4/(b*x^2 - a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \frac{x^{5}e^{-\frac{7i\pi}{8}}\,{}_{2}F_{1}\left(\begin{matrix}\frac{7}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\middle|\,\frac{bx^{2}}{a}\right)}{5a^{\frac{7}{8}}}$$

input
```
integrate(x**4/(b*x**2-a)**(7/8),x)
```

output
```
x**5*exp(-7*I*pi/8)*hyper((7/8, 5/2), (7/2,), b*x**2/a)/(5*a**(7/8))
```

Maxima [F]

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(b\,x^2-a)^{7/8}}\,dx$$

input
```
int(x^4/(b*x^2 - a)^(7/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^4}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(7/8),x)
```

output
```
int(x**4/( - a + b*x**2)**(7/8),x)
```

3.1248 $\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 464

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx = \frac{4x\sqrt[8]{-a+bx^2}}{5b}$$

$$+\frac{8a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{5\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{8a\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right)}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{5\sqrt{2+\sqrt{2}}b^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
4/5*x*(b*x^2-a)^(1/8)/b+8/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))-8/5*a*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))
^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/
4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2
)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2
+2^(1/2))^(1/2)/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.47 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx = \frac{x^3\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(-a+bx^2)^{7/8}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(7/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-a)^{7/8}}\,dx$$

$$\Big\downarrow \ 279$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\int \frac{x^2}{\left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \ 278$$

$$\frac{x^3\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{7}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3\,(bx^2-a)^{7/8}}$$

input
```
Int[x^2/(-a + b*x^2)^(7/8),x]
```

output
```
(x^3*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[7/8, 3/2, 5/2, (b*x^2)/a])/(3
*(-a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(7/8),x)
```

output
```
int(x^2/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral(x^2/(b*x^2 - a)^(7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.07

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx = \frac{x^3e^{-\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{7}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3a^{\frac{7}{8}}}$$

input
```
integrate(x**2/(b*x**2-a)**(7/8),x)
```

output
```
x**3*exp(-7*I*pi/8)*hyper((7/8, 3/2), (5/2,), b*x**2/a)/(3*a**(7/8))
```

Maxima [F]

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx=\int \frac{x^2}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(7/8), x)
```

Giac [F]

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx=\int \frac{x^2}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(7/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx=\int \frac{x^2}{(b\,x^2-a)^{7/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(7/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(7/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(-a+bx^2)^{7/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(7/8),x)
```

output
```
int(x**2/( - a + b*x**2)**(7/8),x)
```

3.1249 $\int \frac{1}{(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 437

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}}{\cdots}}\right.\right.}{\sqrt{2+\sqrt{2}}\,bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)} - \frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\operatorname{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{\sqrt{2+\sqrt{2}}\,bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-
a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2
)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/
4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(
1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(
2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx=\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(-a+bx^2)^{7/8}}$$

input
```
Integrate[(-a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(7/8)
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{7/8}}\,dx$$

$$\Big\downarrow \ 238$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \ 237$$

$$\frac{x\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{(bx^2-a)^{7/8}}$$

input
```
Int[(-a + b*x^2)^(-7/8),x]
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[1/2, 7/8, 3/2, (b*x^2)/a])/(-a
+ b*x^2)^(7/8)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2-a)^{\frac{7}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(7/8),x)
```

output
```
int(1/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(-7/8), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = \frac{xe^{-\frac{7i\pi}{8}}{}_2F_1\left(\begin{matrix} \frac{1}{2},\frac{7}{8} \\ \frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{a^{\frac{7}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(7/8),x)
```

output
```
x*exp(-7*I*pi/8)*hyper((1/2, 7/8), (3/2,), b*x**2/a)/a**(7/8)
```

Maxima [F]

$$\int \frac{1}{\left(-a+bx^2\right)^{7/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-7/8), x)
```

Giac [F]

$$\int \frac{1}{\left(-a+bx^2\right)^{7/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{7}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-7/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.09

$$\int \frac{1}{\left(-a+bx^2\right)^{7/8}}\,dx = \frac{x\,{\left(1-\frac{b\,x^2}{a}\right)}^{7/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{{\left(b\,x^2-a\right)}^{7/8}}$$

input
```
int(1/(b*x^2 - a)^(7/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(7/8)*hypergeom([1/2, 7/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(7/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{7/8}}\,dx = -\left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}\,a-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(7/8),x)
```

output
```
 - int(( - a + b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)*
*(5/8)*b*x**2),x)
```

3.1250 $\int \frac{1}{x^2(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 461

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{7/8}}\,dx = \frac{\sqrt[8]{-a+bx^2}}{ax}$$

$$+ \frac{3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}ax\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$- \frac{3\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{2}\right.}{\sqrt[4]{-a+bx^2}}}\right.\right.}{2\sqrt{2+\sqrt{2}}ax\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
(b*x^2-a)^(1/8)/a/x+3/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(
3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF
(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/
a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a/
x/(a^(1/4)+(b*x^2-a)^(1/4))-3/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*
x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*
EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a
)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^
(1/2)/a/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.74 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^2 (-a+bx^2)^{7/8}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{7/8}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x(-a+bx^2)^{7/8}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(7/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-1/2, 7/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(7/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2-a)^{7/8}} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \int \frac{1}{x^2 \left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{7}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{x (bx^2-a)^{7/8}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(7/8)),x]
```

output
```
-(((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-1/2, 7/8, 1/2, (b*x^2)/a])/(x*
(-a + b*x^2)^(7/8)))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b\,x^{2}-a\right)^{\frac{7}{8}}}dx$$

input
```
int(1/x^2/(b*x^2-a)^(7/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{7/8}}\, dx = \int \frac{1}{\left(bx^{2}-a\right)^{\frac{7}{8}} x^{2}}\, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/(b*x^4 - a*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 \left(-a+bx^2\right)^{7/8}}\, dx = \frac{e^{\frac{i\pi}{8}}\, {}_{2}F_{1}\left(\begin{matrix} -\frac{1}{2}, \frac{7}{8} \\ \frac{1}{2} \end{matrix}\middle| {\frac{bx^{2}}{a}} \right)}{a^{\frac{7}{8}} x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(7/8),x)
```

output
```
exp(I*pi/8)*hyper((-1/2, 7/8), (1/2,), b*x**2/a)/(a**(7/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2(-a+bx^2)^{7/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.09

$$\int \frac{1}{x^2(-a+bx^2)^{7/8}}\,dx = -\frac{4\left(1-\frac{a}{b\,x^2}\right)^{7/8}\,{}_2\mathrm{F}_1\left(\frac{7}{8},\frac{11}{8};\ \frac{19}{8};\ \frac{a}{b\,x^2}\right)}{11\,x\,(b\,x^2-a)^{7/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(7/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(7/8)*hypergeom([7/8, 11/8], 19/8, a/(b*x^2)))/(11*x*(
b*x^2 - a)^(7/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{7/8}}\,dx = \frac{4(b\,x^2-a)^{\frac{7}{8}} - 9(b\,x^2-a)^{\frac{3}{4}} \left(\int \frac{(b\,x^2-a)^{\frac{3}{4}}}{(b\,x^2-a)^{\frac{5}{8}}a-(b\,x^2-a)^{\frac{5}{8}}b\,x^2}dx\right) bx}{4\,(b\,x^2-a)^{\frac{3}{4}}\,ax}$$

input
```
int(1/x^2/(b*x^2-a)^(7/8),x)
```

output
```
(4*( - a + b*x**2)**(7/8) - 9*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**
(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b*x)/(
4*( - a + b*x**2)**(3/4)*a*x)
```

3.1251 $\int \frac{1}{x^4(-a+bx^2)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 490

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{7/8}}\,dx = \frac{\sqrt[8]{-a+bx^2}}{3ax^3} + \frac{11b\sqrt[8]{-a+bx^2}}{12a^2x}$$

$$+\frac{11b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{8\sqrt{2+\sqrt{2}}\,a^2x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{11b\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{8\sqrt{2+\sqrt{2}}\,a^2x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
1/3*(b*x^2-a)^(1/8)/a/x^3+11/12*b*(b*x^2-a)^(1/8)/a^2/x+11/8*b*(-b*x^2/a^(
1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a
^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)
^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2
+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^2/x/(a^(1/4)+(b*x^2-a)^(1/4))-11/8*
b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2
)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/
4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^2/x/(a^(1/4)-(b*x^2-a)
^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11

$$\int \frac{1}{x^4 (-a+bx^2)^{7/8}} dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 (-a+bx^2)^{7/8}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(7/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-3/2, 7/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2-a)^{7/8}} \, dx$$
$$\Big\downarrow \text{ 279}$$
$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \int \frac{1}{x^4 \left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$
$$\Big\downarrow \text{ 278}$$
$$-\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{7}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3x^3 (bx^2-a)^{7/8}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(7/8)),x]
```

output
```
-1/3*((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-3/2, 7/8, -1/2, (b*x^2)/a])
/(x^3*(-a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4\left(b\,x^2-a\right)^{\frac{7}{8}}}dx$$

input
```
int(1/x^4/(b*x^2-a)^(7/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{7/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{7}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/(b*x^6 - a*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.07

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{7/8}}\,dx=\frac{e^{\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{3}{2},\frac{7}{8}\\-\frac{1}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3a^{\frac{7}{8}}x^3}$$

input
```
integrate(1/x**4/(b*x**2-a)**(7/8),x)
```

output
```
exp(I*pi/8)*hyper((-3/2, 7/8), (-1/2,), b*x**2/a)/(3*a**(7/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{7/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}} x^4} \, dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-a+bx^2)^{7/8}} \, dx = \int \frac{1}{x^4 (b\,x^2-a)^{7/8}} \, dx$$

input
```
int(1/(x^4*(b*x^2 - a)^(7/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(7/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{7/8}}\,dx = \frac{16(b\,x^2-a)^{\frac{7}{8}}\,a^2+68(b\,x^2-a)^{\frac{7}{8}}\,ab\,x^2-68(b\,x^2-a)^{\frac{7}{8}}\,b^2x^4+17(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{}{(b}\right.}{48\,(b\,x^2-}$$

input
```
int(1/x^4/(b*x^2-a)^(7/8),x)
```

output
```
(16*( - a + b*x**2)**(7/8)*a**2 + 68*( - a + b*x**2)**(7/8)*a*b*x**2 - 68*
( - a + b*x**2)**(7/8)*b**2*x**4 + 17*( - a + b*x**2)**(3/4)*int(( - a + b
*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x
)*a*b**2*x**3 - 85*( - a + b*x**2)**(3/4)*int((( - a + b*x**2)**(3/4)*x**2
)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**3)
/(48*( - a + b*x**2)**(3/4)*a**3*x**3)
```

3.1252 $\int \frac{1}{x^6\left(-a+bx^2\right)^{7/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 520

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{7/8}}\,dx = \frac{\sqrt[8]{-a+bx^2}}{5ax^5}+\frac{19b\sqrt[8]{-a+bx^2}}{60a^2x^3}+\frac{209b^2\sqrt[8]{-a+bx^2}}{240a^3x}$$

$$+\frac{209b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+bx}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^3x\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)}$$

$$-\frac{209b^2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}\left(-a+bx^2\right)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\text{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx}}{\sqrt[4]{a}}\right.}{\sqrt[4]{-a+b}}}\right.\right.}{160\sqrt{2+\sqrt{2}}a^3x\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)}$$

output
```
1/5*(b*x^2-a)^(1/8)/a/x^5+19/60*b*(b*x^2-a)^(1/8)/a^2/x^3+209/240*b^2*(b*x
^2-a)^(1/8)/a^3/x+209/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/a^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-209/160*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^
(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-
a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+
2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2)/a^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10

$$\int \frac{1}{x^6 (-a+bx^2)^{7/8}}\,dx = -\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 (-a+bx^2)^{7/8}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(7/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-5/2, 7/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(7/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (bx^2-a)^{7/8}} \, dx$$

$$\Big\downarrow \; 279$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \int \frac{1}{x^6 \left(1-\frac{bx^2}{a}\right)^{7/8}}dx}{(bx^2-a)^{7/8}}$$

$$\Big\downarrow \; 278$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{7/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5x^5 (bx^2-a)^{7/8}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(7/8)),x]
```

output
```
-1/5*((1 - (b*x^2)/a)^(7/8)*Hypergeometric2F1[-5/2, 7/8, -3/2, (b*x^2)/a])
/(x^5*(-a + b*x^2)^(7/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a \right)^{\frac{7}{8}}}d x$$

input
```
int(1/x^6/(b*x^2-a)^(7/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(7/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{7/8}}\, dx = \int \frac{1}{(bx^2-a)^{\frac{7}{8}}x^6}\, dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(7/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(1/8)/(b*x^8 - a*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{7/8}}\, dx = \frac{e^{\frac{i\pi}{8}}{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{7}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5a^{\frac{7}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2-a)**(7/8),x)
```

output
```
exp(I*pi/8)*hyper((-5/2, 7/8), (-3/2,), b*x**2/a)/(5*a**(7/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{7/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{7}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(7/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{7/8}}\,dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{7}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(7/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(7/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{7/8}}\,dx = \int \frac{1}{x^6\left(b\,x^2-a\right)^{7/8}}\,dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(7/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(7/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{7/8}}\,dx = \frac{48(b\,x^2-a)^{\frac{7}{8}}\,a^2+100(b\,x^2-a)^{\frac{7}{8}}\,ab\,x^2-100(b\,x^2-a)^{\frac{7}{8}}\,b^2x^4-375(b\,x^2-a)^{\frac{3}{4}}\,\Big(}{240\,(b\,x}$$

input
```
int(1/x^6/(b*x^2-a)^(7/8),x)
```

output
```
(48*( - a + b*x**2)**(7/8)*a**2 + 100*( - a + b*x**2)**(7/8)*a*b*x**2 - 10
0*( - a + b*x**2)**(7/8)*b**2*x**4 - 375*( - a + b*x**2)**(3/4)*int(( - a
+ b*x**2)**(3/4)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*b
*x**4),x)*a*b**2*x**5 + 75*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(3/
4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5
)/(240*( - a + b*x**2)**(3/4)*a**3*x**5)
```

3.1253 $\int \frac{x^6}{(-a+bx^2)^{9/8}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 991

$$\int \frac{x^6}{(-a+bx^2)^{9/8}} \, dx = \text{Too large to display}$$

output
```
1280/209*a^2*x/b^3/(b*x^2-a)^(1/8)-4*x^5/b/(b*x^2-a)^(1/8)+960/209*a*x*(b*
x^2-a)^(7/8)/b^3+80/19*x^3*(b*x^2-a)^(7/8)/b^2+640/209*(2+2^(1/2))^(1/2)*a
^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(
b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(
2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))+640/
209*(2+2^(1/2))^(1/2)*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x
^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*E
llipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)
^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4
)-(b*x^2-a)^(1/4))-640/209*a^(11/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)
*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x
^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/
2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))-640/209*a^(11/4)*(-b*x^2/a^(1/2)
/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1
/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/
4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2
^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.81 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = -\frac{x^7\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7a\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(9/8),x]
```

output
```
-1/7*(x^7*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 7/2, 9/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2-a)^{9/8}}\,dx$$

$$\Big\downarrow \ 279$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^6}{\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \ 278$$

$$-\frac{x^7\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7a\sqrt[8]{bx^2-a}}$$

input
```
Int[x^6/(-a + b*x^2)^(9/8),x]
```

output
```
-1/7*(x^7*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 7/2, 9/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2-a)^{\frac{9}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(9/8),x)
```

output
```
int(x^6/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \frac{x^7 e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7a^{\frac{9}{8}}}$$

input
```
integrate(x**6/(b*x**2-a)**(9/8),x)
```

output
```
x**7*exp(7*I*pi/8)*hyper((9/8, 7/2), (9/2,), b*x**2/a)/(7*a**(9/8))
```

Maxima [F]

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{9/8}}\,dx$$

input
```
int(x^6/(b*x^2 - a)^(9/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(-a+bx^2)^{9/8}}\,dx = \frac{76(b\,x^2-a)^{\frac{5}{8}}\,a^2x-80(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^3+4(b\,x^2-a)^{\frac{5}{8}}\,b^2x^5-76(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x^2}\right.}{7\,(b\,x^2-a)^{\frac{3}{4}}}$$

input
```
int(x^6/(b*x^2-a)^(9/8),x)
```

output
```
(76*( - a + b*x**2)**(5/8)*a**2*x - 80*( - a + b*x**2)**(5/8)*a*b*x**3 + 4
*( - a + b*x**2)**(5/8)*b**2*x**5 - 76*( - a + b*x**2)**(3/4)*int(sqrt( -
a + b*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*
a**3 + 69*( - a + b*x**2)**(3/4)*int((sqrt( - a + b*x**2)*x**2)/(( - a + b
*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*a**2*b)/(7*( - a + b*x
**2)**(3/4)*b**3)
```

3.1254 $\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 965

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx = \text{Too large to display}$$

output
```
64/11*a*x/b^2/(b*x^2-a)^(1/8)-4*x^3/b/(b*x^2-a)^(1/8)+48/11*x*(b*x^2-a)^(7
/8)/b^2+32/11*(2+2^(1/2))^(1/2)*a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(
1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))
^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*
(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/
x/(a^(1/4)+(b*x^2-a)^(1/4))+32/11*(2+2^(1/2))^(1/2)*a^(7/4)*(-b*x^2/a^(1/2
)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(
1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1
/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*
2^(1/2))^(1/2))/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))-32/11*a^(7/4)*(-b*x^2/a^(1
/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^
(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^
(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+
2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-32/11*
a^(7/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(
2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-
a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*
x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.72 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx = -\frac{x^5\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5a\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(9/8),x]
```

output
```
-1/5*(x^5*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 5/2, 7/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2-a)^{9/8}}\,dx$$

$$\Big\downarrow \ 279$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^4}{\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \ 278$$

$$-\frac{x^5\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5a\sqrt[8]{bx^2-a}}$$

input
```
Int[x^4/(-a + b*x^2)^(9/8),x]
```

output
```
-1/5*(x^5*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 5/2, 7/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2-a)^{\frac{9}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(9/8),x)
```

output
```
int(x^4/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx=\int \frac{x^4}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx=\frac{x^5e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{5}{2}\\\frac{7}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{5a^{\frac{9}{8}}}$$

input
```
integrate(x**4/(b*x**2-a)**(9/8),x)
```

output
```
x**5*exp(7*I*pi/8)*hyper((9/8, 5/2), (7/2,), b*x**2/a)/(5*a**(9/8))
```

Maxima [F]

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^4}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(-a+bx^2\right)^{9/8}}\,dx=\int \frac{x^4}{{\left(b\,x^2-a\right)}^{9/8}} \,d x$$

input
```
int(x^4/(b*x^2 - a)^(9/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(-a+bx^2\right)^{9/8}}\,dx=\frac{4(b\,x^{2}-a)^{\frac{5}{8}} a x -4(b\,x^{2}-a)^{\frac{5}{8}} b\,x^{3}-4(b\,x^{2}-a)^{\frac{3}{4}} \left(\int \frac{\sqrt{b\,x^{2}-a}}{(b\,x^{2}-a)^{\frac{5}{8}} a -(b\,x^{2}-a)^{\frac{5}{8}} b\,x^{2}}d x \right) a^{2} -}{(b\,x^{2}-a)^{\frac{3}{4}} b^{2}}$$

input
```
int(x^4/(b*x^2-a)^(9/8),x)
```

output
```
(4*( - a + b*x**2)**(5/8)*a*x - 4*( - a + b*x**2)**(5/8)*b*x**3 - 4*( - a
+ b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a
 + b*x**2)**(5/8)*b*x**2),x)*a**2 + 3*( - a + b*x**2)**(3/4)*int((sqrt( -
a + b*x**2)*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**
2),x)*a*b)/(( - a + b*x**2)**(3/4)*b**2)
```

3.1255 $\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 922

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \text{Too large to display}$$

output

```
4/3*x/b/(b*x^2-a)^(1/8)+8/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-b*x^2/a^(1/2)/(b*x
^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^
(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2
))^(1/2))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/3*(2+2^(1/2))^(1/2)*a^(3/4)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))-8/3*a^(3/4)*(-
b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(
1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*
(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^
(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1/
4))-8/3*a^(3/4)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(
a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a
^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))
/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(
1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = -\frac{x^3\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3a\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(9/8),x]
```

output
```
-1/3*(x^3*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 3/2, 5/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-a)^{9/8}}\,dx$$

$$\Big\downarrow \ 279$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{x^2}{\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \ 278$$

$$-\frac{x^3\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{9}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3a\sqrt[8]{bx^2-a}}$$

input
```
Int[x^2/(-a + b*x^2)^(9/8),x]
```

output
```
-1/3*(x^3*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[9/8, 3/2, 5/2, (b*x^2)/a
])/(a*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2-a)^{\frac{9}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(9/8),x)
```

output
```
int(x^2/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \frac{x^3 e^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{9}{8},\frac{3}{2}\\ \frac{5}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{3a^{\frac{9}{8}}}$$

input
```
integrate(x**2/(b*x**2-a)**(9/8),x)
```

output
```
x**3*exp(7*I*pi/8)*hyper((9/8, 3/2), (5/2,), b*x**2/a)/(3*a**(9/8))
```

Maxima [F]

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(9/8), x)
```

Giac [F]

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(9/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{9/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(9/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(9/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(-a+bx^2)^{9/8}}\,dx = -\left(\int \frac{x^2}{(b\,x^2-a)^{\frac{1}{8}}\,a-(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2}dx\right)$$

input
```
int(x^2/(b*x^2-a)^(9/8),x)
```

output
```
- int(x**2/(( - a + b*x**2)**(1/8)*a - ( - a + b*x**2)**(1/8)*b*x**2),x)
```

3.1256 $\int \frac{1}{(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 893

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \frac{2\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{\cdots}}{\cdots}}\cdots\right.\right.}{\sqrt[4]{a}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)\cdots}$$

$$+\frac{2\sqrt{2+\sqrt{2}}\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}E\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\cdots\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{\sqrt[4]{a}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)\cdots}$$

$$-\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{\frac{\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{-\frac{\sqrt[4]{a}\left(\sqrt{2}-\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{\sqrt{2+\sqrt{2}}\sqrt[4]{a}bx\left(\sqrt[4]{a}+\sqrt[4]{-a+bx^2}\right)\cdots}$$

$$-\frac{2\sqrt{-\frac{bx^2}{\sqrt{a}\sqrt{-a+bx^2}}}(-a+bx^2)^{3/8}\sqrt{-\frac{\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)^2}{\sqrt[4]{a}\sqrt[4]{-a+bx^2}}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{1}{2}\sqrt{\frac{\sqrt[4]{a}\left(\sqrt{2}+\frac{2\sqrt[4]{-a+bx^2}}{\sqrt[4]{a}}+\sqrt{\cdots}\right.}{\sqrt[4]{-a+bx^2}}}\cdots\right.\right.}{\sqrt{2+\sqrt{2}}\sqrt[4]{a}bx\left(\sqrt[4]{a}-\sqrt[4]{-a+bx^2}\right)\cdots}$$

output
```
2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)
*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2
*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1
/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/b/x/(a^(1/4)+(b*
x^2-a)^(1/4))+2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(
b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2
)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2
-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(1/4)/b/
x/(a^(1/4)-(b*x^2-a)^(1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/a^(1/4)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2
))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(
1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1
/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/
(2+2^(1/2))^(1/2)/a^(1/4)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = -\frac{x\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[(-a + b*x^2)^(-9/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, (b*x^2)/a])/(a
*(-a + b*x^2)^(1/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{9/8}} \, dx$$

$$\Big\downarrow \text{238}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}} \int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{237}$$

$$-\frac{x\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\sqrt[8]{bx^2-a}}$$

input
```
Int[(-a + b*x^2)^(-9/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[1/2, 9/8, 3/2, (b*x^2)/a])/(a
*(-a + b*x^2)^(1/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2-a)^{\frac{9}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(9/8),x)
```

output
```
int(1/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx=\int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx=\frac{xe^{\frac{7i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{9}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(9/8),x)
```

output
```
x*exp(7*I*pi/8)*hyper((1/2, 9/8), (3/2,), b*x**2/a)/a**(9/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-9/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-9/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{9/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{9/8}}$$

input
```
int(1/(b*x^2 - a)^(9/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(9/8)*hypergeom([1/2, 9/8], 3/2, (b*x^2)/a))/(b*x^2 - a
)^(9/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{9/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{1}{8}}\,a-(b\,x^2-a)^{\frac{1}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(9/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(1/8)*a - ( - a + b*x**2)**(1/8)*b*x**2),x)
```

3.1257 $\int \frac{1}{x^2(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 951

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{9/8}}\,dx = \text{Too large to display}$$

output
```
-4/a/x/(b*x^2-a)^(1/8)+5*b*x/a^2/(b*x^2-a)^(1/8)-5*(b*x^2-a)^(7/8)/a^2/x+5
/2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8
)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/
2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(
1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/x/(a^(1/4)+(b*x
^2-a)^(1/4))+5/2*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*
(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/
2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^
2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/4)/x
/(a^(1/4)-(b*x^2-a)^(1/4))-5/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x
^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)
^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)/a^(5/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-5/2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/
2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^
(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(
1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))
/(2+2^(1/2))^(1/2)/a^(5/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.60 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 (-a+bx^2)^{9/8}}\,dx = \frac{\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{ax\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 9/8, 1/2, (b*x^2)/a])/(a*x*
(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2-a)^{9/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}}\int \frac{1}{x^2\left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a\sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{9}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{ax\sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-1/2, 9/8, 1/2, (b*x^2)/a])/(a*x*
(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{2}-a \right)^{\frac{9}{8}}}d x$$

input
```
int(1/x^2/(b*x^2-a)^(9/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^2 (-a+bx^2)^{9/8}}\,dx = \frac{e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{9}{8} \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{a^{\frac{9}{8}}x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(9/8),x)
```

output
```
exp(-I*pi/8)*hyper((-1/2, 9/8), (1/2,), b*x**2/a)/(a**(9/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (-a + bx^2)^{9/8}} \, dx = \int \frac{1}{(bx^2 - a)^{\frac{9}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^2 (-a + bx^2)^{9/8}} \, dx = -\frac{4 \left(1 - \frac{a}{b x^2}\right)^{9/8} {}_2F_1\left(\frac{9}{8}, \frac{13}{8}; \frac{21}{8}; \frac{a}{b x^2}\right)}{13 x (b x^2 - a)^{9/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(9/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(9/8)*hypergeom([9/8, 13/8], 21/8, a/(b*x^2)))/(13*x*(
b*x^2 - a)^(9/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (-a + bx^2)^{9/8}} \, dx = -\left(\int \frac{1}{(b x^2 - a)^{\frac{1}{8}} a x^2 - (b x^2 - a)^{\frac{1}{8}} b x^4} dx \right)$$

input
```
int(1/x^2/(b*x^2-a)^(9/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(1/8)*a*x**2 - ( - a + b*x**2)**(1/8)*b*x**4),x
)
```

3.1258 $\int \frac{1}{x^4(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 985

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{9/8}}\,dx = \text{Too large to display}$$

output
```
-4/a/x^3/(b*x^2-a)^(1/8)+65/12*b^2*x/a^3/(b*x^2-a)^(1/8)-13/3*(b*x^2-a)^(7
/8)/a^2/x^3-65/12*b*(b*x^2-a)^(7/8)/a^3/x+65/24*(2+2^(1/2))^(1/2)*b*(-b*x^
2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4)
)^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x
^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2
),(-2+2*2^(1/2))^(1/2))/a^(9/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))+65/24*(2+2^(1/
2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1
/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/
4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(9/4)/x/(a^(1/4)-(b*x^2-a)^(1/
4))-65/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/
4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/
4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*
x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(9/4)/x/(a^(
1/4)+(b*x^2-a)^(1/4))-65/24*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1
/2)/a^(9/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^4 (-a+bx^2)^{9/8}} \, dx = \frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{ Hypergeometric2F1}\left(-\frac{3}{2},\frac{9}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 9/8, -1/2, (b*x^2)/a])/(3*a
*x^3*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2-a)^{9/8}} \, dx$$

$$\Big\downarrow \text{ 279}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}} \int \frac{1}{x^4 \left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a \sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{ 278}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{9}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3 \sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-3/2, 9/8, -1/2, (b*x^2)/a])/(3*a
*x^3*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^4 \left(b\, x^2 - a\right)^{\frac{9}{8}}}dx$$

input
```
int(1/x^4/(b*x^2-a)^(9/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{9/8}}\, dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{9}{8}} x^4}\, dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{9/8}}\, dx = \frac{e^{-\frac{i\pi}{8}}\, {}_2F_1\left(\begin{matrix} -\frac{3}{2}, \frac{9}{8} \\ -\frac{1}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{3a^{\frac{9}{8}} x^3}$$

input
```
integrate(1/x**4/(b*x**2-a)**(9/8),x)
```

output
```
exp(-I*pi/8)*hyper((-3/2, 9/8), (-1/2,), b*x**2/a)/(3*a**(9/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{9/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (-a+bx^2)^{9/8}}\,dx = \int \frac{1}{x^4\,(b\,x^2-a)^{9/8}}\,dx$$

input
```
int(1/(x^4*(b*x^2 - a)^(9/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(9/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{9/8}}\,dx = \frac{4(b\,x^2-a)^{\frac{5}{8}}\,a-16(b\,x^2-a)^{\frac{5}{8}}\,b\,x^2-37(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{\sqrt{b\,x^2-a}}{(b\,x^2-a)^{\frac{5}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{5}{8}}\,b\,x^4}dx}{12\,(b\,x^2-a)^{\frac{3}{4}}\,a^2x^3}$$

input
```
int(1/x^4/(b*x^2-a)^(9/8),x)
```

output
```
(4*( - a + b*x**2)**(5/8)*a - 16*( - a + b*x**2)**(5/8)*b*x**2 - 37*( - a
+ b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a*x**2 -
( - a + b*x**2)**(5/8)*b*x**4),x)*a*b*x**3 + 40*( - a + b*x**2)**(3/4)*int
(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*
x**2),x)*b**2*x**3)/(12*( - a + b*x**2)**(3/4)*a**2*x**3)
```

3.1259 $\int \frac{1}{x^6(-a+bx^2)^{9/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1019

$$\int \frac{1}{x^6\left(-a+bx^2\right)^{9/8}}\,dx = \text{Too large to display}$$

output
```
-4/a/x^5/(b*x^2-a)^(1/8)+91/16*b^3*x/a^4/(b*x^2-a)^(1/8)-21/5*(b*x^2-a)^(7
/8)/a^2/x^5-91/20*b*(b*x^2-a)^(7/8)/a^3/x^3-91/16*b^2*(b*x^2-a)^(7/8)/a^4/
x+91/32*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ell
ipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(13/4)/x/(a^
(1/4)+(b*x^2-a)^(1/4))+91/32*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-
a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x
^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/
4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^
(1/2))/a^(13/4)/x/(a^(1/4)-(b*x^2-a)^(1/4))-91/32*b^2*(-b*x^2/a^(1/2)/(b*x
^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b
*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^
(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2
))^(1/2))/(2+2^(1/2))^(1/2)/a^(13/4)/x/(a^(1/4)+(b*x^2-a)^(1/4))-91/32*b^2
*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-
a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)
+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4
))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(13/4)/x/(a^(1/4)-(b*x^
2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.05

$$\int \frac{1}{x^6 (-a+bx^2)^{9/8}} \, dx = \frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{9}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5\sqrt[8]{-a+bx^2}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 9/8, -3/2, (b*x^2)/a])/(5*a
*x^5*(-a + b*x^2)^(1/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (bx^2-a)^{9/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\sqrt[8]{1-\frac{bx^2}{a}} \int \frac{1}{x^6 \left(1-\frac{bx^2}{a}\right)^{9/8}}dx}{a \sqrt[8]{bx^2-a}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\sqrt[8]{1-\frac{bx^2}{a}} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{9}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5 \sqrt[8]{bx^2-a}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(9/8)),x]
```

output
```
((1 - (b*x^2)/a)^(1/8)*Hypergeometric2F1[-5/2, 9/8, -3/2, (b*x^2)/a])/(5*a
*x^5*(-a + b*x^2)^(1/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a\right)^{\frac{9}{8}}}dx$$

input
```
int(1/x^6/(b*x^2-a)^(9/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(9/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{9/8}}\, dx = \int \frac{1}{\left(bx^2-a\right)^{\frac{9}{8}} x^6}\, dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(9/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(7/8)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{9/8}}\, dx = \frac{e^{-\frac{i\pi}{8}}\,{}_2F_1\left(\begin{matrix} -\frac{5}{2}, \frac{9}{8} \\ -\frac{3}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{5a^{\frac{9}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2-a)**(9/8),x)
```

output
```
exp(-I*pi/8)*hyper((-5/2, 9/8), (-3/2,), b*x**2/a)/(5*a**(9/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{9/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(9/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{9/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{9}{8}} x^6} \, dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(9/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(9/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{9/8}}\, dx = \int \frac{1}{x^6 \left(b\,x^2-a\right)^{9/8}}\, dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(9/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(9/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{9/8}}\, dx = \frac{3024(b\,x^2-a)^{\frac{5}{8}}\,a^2-16964(b\,x^2-a)^{\frac{5}{8}}\,ab\,x^2+8900(b\,x^2-a)^{\frac{5}{8}}\,b^2x^4-72816(b\,x^2}{}$$

input
```
int(1/x^6/(b*x^2-a)^(9/8),x)
```

output
```
(3024*( - a + b*x**2)**(5/8)*a**2 - 16964*( - a + b*x**2)**(5/8)*a*b*x**2
+ 8900*( - a + b*x**2)**(5/8)*b**2*x**4 - 72816*( - a + b*x**2)**(3/4)*int
(sqrt( - a + b*x**2)/(( - a + b*x**2)**(5/8)*a*x**4 - ( - a + b*x**2)**(5/
8)*b*x**6),x)*a**2*b*x**5 + 92921*( - a + b*x**2)**(3/4)*int(sqrt( - a + b
*x**2)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*b*x**4),x)*
a*b**2*x**5 - 28925*( - a + b*x**2)**(3/4)*int(sqrt( - a + b*x**2)/(( - a
+ b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5)/(15120*(
 - a + b*x**2)**(3/4)*a**3*x**5)
```

3.1260 $\int \frac{x^6}{\left(-a+bx^2\right)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 969

$$\int \frac{x^6}{\left(-a+bx^2\right)^{11/8}}\,dx = \text{Too large to display}$$

output
```
-4/3*x^5/b/(b*x^2-a)^(3/8)+320/153*a*x*(b*x^2-a)^(5/8)/b^3+80/51*x^3*(b*x^
2-a)^(5/8)/b^2-640/153*(2+2^(1/2))^(1/2)*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)
^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-
a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)
+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1
/2))/b^4/x/(a^(1/4)+(b*x^2-a)^(1/4))+640/153*(2+2^(1/2))^(1/2)*a^(5/2)*(-b
*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(
1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(
b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(
1/2),(-2+2*2^(1/2))^(1/2))/b^4/x/(a^(1/4)-(b*x^2-a)^(1/4))+640/153*a^(5/2)
*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a
)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)
-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4
))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4/x/(a^(1/4)+(b*x^2-a)^
(1/4))-640/153*a^(5/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3
/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF
(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a
^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^4
/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.67 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = -\frac{x^7\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7a\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[x^6/(-a + b*x^2)^(11/8),x]
```

output
```
-1/7*(x^7*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 7/2, 9/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{(bx^2-a)^{11/8}}\,dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{x^6}{\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a\,(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^7\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{7}{2},\frac{9}{2},\frac{bx^2}{a}\right)}{7a\,(bx^2-a)^{3/8}}$$

input
```
Int[x^6/(-a + b*x^2)^(11/8),x]
```

output
```
-1/7*(x^7*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 7/2, 9/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^6}{(b\,x^2-a)^{\frac{11}{8}}}dx$$

input
```
int(x^6/(b*x^2-a)^(11/8),x)
```

output
```
int(x^6/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^6/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \frac{x^7e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{11}{8},\frac{7}{2}\\ \frac{9}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{7a^{\frac{11}{8}}}$$

input
```
integrate(x**6/(b*x**2-a)**(11/8),x)
```

output
```
x**7*exp(5*I*pi/8)*hyper((11/8, 7/2), (9/2,), b*x**2/a)/(7*a**(11/8))
```

Maxima [F]

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^6/(b*x^2 - a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^6/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^6/(b*x^2 - a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^6}{(b\,x^2-a)^{11/8}}\,dx$$

input
```
int(x^6/(b*x^2 - a)^(11/8),x)
```

output
```
int(x^6/(b*x^2 - a)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^6}{(-a+bx^2)^{11/8}}\,dx = \frac{76(b\,x^2-a)^{\frac{3}{8}}\,a^2x-80(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^3+4(b\,x^2-a)^{\frac{3}{8}}\,b^2x^5-76(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{}{(b\,x}\right.}{7\,(b\,x^2-a)}$$

input
```
int(x^6/(b*x^2-a)^(11/8),x)
```

output
```
(76*( - a + b*x**2)**(3/8)*a**2*x - 80*( - a + b*x**2)**(3/8)*a*b*x**3 + 4
*( - a + b*x**2)**(3/8)*b**2*x**5 - 76*( - a + b*x**2)**(3/4)*int(( - a +
b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),
x)*a**3 + 69*( - a + b*x**2)**(3/4)*int((( - a + b*x**2)**(1/4)*x**2)/(( -
 a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*a**2*b)/(7*( - a
 + b*x**2)**(3/4)*b**3)
```

3.1261 $\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 945

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \text{Too large to display}$$

output
```
-4/3*x^3/b/(b*x^2-a)^(3/8)+16/9*x*(b*x^2-a)^(5/8)/b^2-32/9*(2+2^(1/2))^(1/
2)*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4
)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4
)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x
^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))+3
2/9*(2+2^(1/2))^(1/2)*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^
2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^
(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/b^3/x/(a^(1/4)
-(b*x^2-a)^(1/4))+32/9*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x
^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*El
lipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)
^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(
1/2)/b^3/x/(a^(1/4)+(b*x^2-a)^(1/4))-32/9*a^(3/2)*(-b*x^2/a^(1/2)/(b*x^2-a
)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^
2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4
)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(
1/2))/(2+2^(1/2))^(1/2)/b^3/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.49 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = -\frac{x^5\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5a\left(-a+bx^2\right)^{3/8}}$$

input
```
Integrate[x^4/(-a + b*x^2)^(11/8),x]
```

output
```
-1/5*(x^5*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 5/2, 7/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(bx^2-a)^{11/8}}dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{x^4}{\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a\left(bx^2-a\right)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^5\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{5}{2},\frac{7}{2},\frac{bx^2}{a}\right)}{5a\left(bx^2-a\right)^{3/8}}$$

input
```
Int[x^4/(-a + b*x^2)^(11/8),x]
```

output
```
-1/5*(x^5*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 5/2, 7/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^4}{(b\,x^2-a)^{\frac{11}{8}}}dx$$

input
```
int(x^4/(b*x^2-a)^(11/8),x)
```

output
```
int(x^4/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^4/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \frac{x^5 e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\left.\begin{matrix}\frac{11}{8},\frac{5}{2}\\ \frac{7}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{5a^{\frac{11}{8}}}$$

input
```
integrate(x**4/(b*x**2-a)**(11/8),x)
```

output
```
x**5*exp(5*I*pi/8)*hyper((11/8, 5/2), (7/2,), b*x**2/a)/(5*a**(11/8))
```

Maxima [F]

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^4/(b*x^2 - a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^4/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^4/(b*x^2 - a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^4}{(b\,x^2-a)^{11/8}}\,dx$$

input
```
int(x^4/(b*x^2 - a)^(11/8),x)
```

output
```
int(x^4/(b*x^2 - a)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^4}{(-a+bx^2)^{11/8}}\,dx = \frac{4(b\,x^2-a)^{\frac{3}{8}}\,ax-4(b\,x^2-a)^{\frac{3}{8}}\,b\,x^3-4(b\,x^2-a)^{\frac{3}{4}}\left(\int \frac{(b\,x^2-a)^{\frac{1}{4}}}{(b\,x^2-a)^{\frac{5}{8}}a-(b\,x^2-a)^{\frac{5}{8}}b\,x^2}dx\right)a^2}{(b\,x^2-a)^{\frac{3}{4}}\,b^2}$$

input
```
int(x^4/(b*x^2-a)^(11/8),x)
```

output
```
(4*( - a + b*x**2)**(3/8)*a*x - 4*( - a + b*x**2)**(3/8)*b*x**3 - 4*( - a
+ b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a - (
- a + b*x**2)**(5/8)*b*x**2),x)*a**2 + 3*( - a + b*x**2)**(3/4)*int((( - a
 + b*x**2)**(1/4)*x**2)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)
*b*x**2),x)*a*b)/(( - a + b*x**2)**(3/4)*b**2)
```

3.1262 $\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 922

$$\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx = \text{Too large to display}$$

output

```
-4/3*x/b/(b*x^2-a)^(3/8)-8/3*(2+2^(1/2))^(1/2)*a^(1/2)*(-b*x^2/a^(1/2)/(b*
x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(
b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a
^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/
2))^(1/2))/b^2/x/(a^(1/4)+(b*x^2-a)^(1/4))+8/3*(2+2^(1/2))^(1/2)*a^(1/2)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/b^2/x/(a^(1/4)-(b*x^2-a)^(1/4))+8/3*a^(1/2)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^(1/4)+(b*x^2-a)^(1
/4))-8/3*a^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-
(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(
a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2)
)/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/b^2/x/(a^
(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 8.94 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx = -\frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3a\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[x^2/(-a + b*x^2)^(11/8),x]
```

output
```
-1/3*(x^3*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 3/2, 5/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(bx^2-a)^{11/8}}dx$$

$$\Big\downarrow \ 279$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{x^2}{\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a\,(bx^2-a)^{3/8}}$$

$$\Big\downarrow \ 278$$

$$-\frac{x^3\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{11}{8},\frac{3}{2},\frac{5}{2},\frac{bx^2}{a}\right)}{3a\,(bx^2-a)^{3/8}}$$

input
```
Int[x^2/(-a + b*x^2)^(11/8),x]
```

output
```
-1/3*(x^3*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[11/8, 3/2, 5/2, (b*x^2)/
a])/(a*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{x^2}{(b\,x^2-a)^{\frac{11}{8}}}dx$$

input
```
int(x^2/(b*x^2-a)^(11/8),x)
```

output
```
int(x^2/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{x^2}{\left(-a+bx^2\right)^{11/8}}\,dx = \int \frac{x^2}{\left(bx^2-a\right)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)*x^2/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.03

$$\int \frac{x^2}{\left(-a+bx^2\right)^{11/8}}\,dx = \frac{x^3 e^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix} \frac{11}{8}, \frac{3}{2} \\ \frac{5}{2} \end{matrix}\middle| \frac{bx^2}{a}\right)}{3a^{\frac{11}{8}}}$$

input
```
integrate(x**2/(b*x**2-a)**(11/8),x)
```

output
```
x**3*exp(5*I*pi/8)*hyper((11/8, 3/2), (5/2,), b*x**2/a)/(3*a**(11/8))
```

Maxima [F]

$$\int \frac{x^2}{\left(-a+bx^2\right)^{11/8}}\,dx = \int \frac{x^2}{\left(bx^2-a\right)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*x^2 - a)^(11/8), x)
```

Giac [F]

$$\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(x^2/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(x^2/(b*x^2 - a)^(11/8), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx = \int \frac{x^2}{(b\,x^2-a)^{11/8}}\,dx$$

input
```
int(x^2/(b*x^2 - a)^(11/8),x)
```

output
```
int(x^2/(b*x^2 - a)^(11/8), x)
```

Reduce [F]

$$\int \frac{x^2}{(-a+bx^2)^{11/8}}\,dx = -\left(\int \frac{x^2}{(b\,x^2-a)^{\frac{3}{8}}\,a-(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2}dx\right)$$

input
```
int(x^2/(b*x^2-a)^(11/8),x)
```

output
```
- int(x**2/(( - a + b*x**2)**(3/8)*a - ( - a + b*x**2)**(3/8)*b*x**2),x)
```

3.1263 $\int \frac{1}{(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 922

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \text{Too large to display}$$

output
```
-4/3*x/a/(b*x^2-a)^(3/8)-2/3*(2+2^(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(
1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)
^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2
^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2
))/a^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))+2/3*(2+2^(1/2))^(1/2)*(-b*x^2/a^(
1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/
a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)
^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2
+2*2^(1/2))^(1/2))/a^(1/2)/b/x/(a^(1/4)-(b*x^2-a)^(1/4))+2/3*(-b*x^2/a^(1/
2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(
1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(
1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2
*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/b/x/(a^(1/4)+(b*x^2-a)^(1/4))-2
/3*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^
2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/
2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(1/2)/b/x/(a^(1/4)-(b
*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = -\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(-a+bx^2)^{3/8}}$$

input
```
Integrate[(-a + b*x^2)^(-11/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(3/8)))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx^2-a)^{11/8}}\,dx$$

$$\Big\downarrow \text{238}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{1}{\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a\,(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{237}$$

$$-\frac{x\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{8},\frac{3}{2},\frac{bx^2}{a}\right)}{a\,(bx^2-a)^{3/8}}$$

input
```
Int[(-a + b*x^2)^(-11/8),x]
```

output
```
-((x*(1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[1/2, 11/8, 3/2, (b*x^2)/a])/(
a*(-a + b*x^2)^(3/8)))
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}-a\right)^{\frac{11}{8}}}dx$$

input
```
int(1/(b*x^2-a)^(11/8),x)
```

output
```
int(1/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{\left(-a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b^2*x^4 - 2*a*b*x^2 + a^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.03

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \frac{xe^{\frac{5i\pi}{8}}\,{}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{11}{8}\\ \frac{3}{2}\end{matrix}\middle|\frac{bx^2}{a}\right)}{a^{\frac{11}{8}}}$$

input
```
integrate(1/(b*x**2-a)**(11/8),x)
```

output
```
x*exp(5*I*pi/8)*hyper((1/2, 11/8), (3/2,), b*x**2/a)/a**(11/8)
```

Maxima [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate((b*x^2 - a)^(-11/8), x)
```

Giac [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}}\,dx$$

input
```
integrate(1/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate((b*x^2 - a)^(-11/8), x)
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.04

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = \frac{x\left(1-\frac{b\,x^2}{a}\right)^{11/8}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{8};\ \frac{3}{2};\ \frac{b\,x^2}{a}\right)}{(b\,x^2-a)^{11/8}}$$

input
```
int(1/(b*x^2 - a)^(11/8),x)
```

output
```
(x*(1 - (b*x^2)/a)^(11/8)*hypergeom([1/2, 11/8], 3/2, (b*x^2)/a))/(b*x^2 -
 a)^(11/8)
```

Reduce [F]

$$\int \frac{1}{(-a+bx^2)^{11/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}\,a-(b\,x^2-a)^{\frac{3}{8}}\,b\,x^2}dx\right)$$

input
```
int(1/(b*x^2-a)^(11/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(3/8)*a - ( - a + b*x**2)**(3/8)*b*x**2),x)
```

3.1264 $\int \frac{1}{x^2(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 935

$$\int \frac{1}{x^2\left(-a+bx^2\right)^{11/8}}\,dx=\text{Too large to display}$$

output
```
-4/3/a/x/(b*x^2-a)^(3/8)-7/3*(b*x^2-a)^(5/8)/a^2/x-7/6*(2+2^(1/2))^(1/2)*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^
(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+7/6*(2+2^
(1/2))^(1/2)*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(
1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1
/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b
*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(3/2)/x/(a^(1/4)-(b*x^2-a)^(1
/4))+7/6*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+
(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*
(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2
-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(3/2)/x/(a^(1/4
)+(b*x^2-a)^(1/4))-7/6*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3
/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF
(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a
^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(
3/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 9.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}}\,dx = \frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{11}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{ax(-a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^2*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 11/8, 1/2, (b*x^2)/a])/(a*x
*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (bx^2-a)^{11/8}}dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\int \frac{1}{x^2\left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a(bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{11}{8},\frac{1}{2},\frac{bx^2}{a}\right)}{ax(bx^2-a)^{3/8}}$$

input
```
Int[1/(x^2*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-1/2, 11/8, 1/2, (b*x^2)/a])/(a*x
*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{2} \left(b \,x^{2}-a \right)^{\frac{11}{8}}}d x$$

input
```
int(1/x^2/(b*x^2-a)^(11/8),x)
```

output
```
int(1/x^2/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b^2*x^6 - 2*a*b*x^4 + a^2*x^2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}} \, dx = \frac{e^{-\frac{3i\pi}{8}} {}_2F_1\left(\begin{matrix} -\frac{1}{2}, \frac{11}{8} \\ \frac{1}{2} \end{matrix} \middle| \frac{bx^2}{a}\right)}{a^{\frac{11}{8}} x}$$

input
```
integrate(1/x**2/(b*x**2-a)**(11/8),x)
```

output
```
exp(-3*I*pi/8)*hyper((-1/2, 11/8), (1/2,), b*x**2/a)/(a**(11/8)*x)
```

Maxima [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}} \, dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}} x^2} \, dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}x^2}\,dx$$

input
```
integrate(1/x^2/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}}\,dx = -\frac{4\left(1-\frac{a}{b\,x^2}\right)^{11/8}{}_2F_1\left(\frac{11}{8},\frac{15}{8};\ \frac{23}{8};\ \frac{a}{b\,x^2}\right)}{15\,x\,(b\,x^2-a)^{11/8}}$$

input
```
int(1/(x^2*(b*x^2 - a)^(11/8)),x)
```

output
```
-(4*(1 - a/(b*x^2))^(11/8)*hypergeom([11/8, 15/8], 23/8, a/(b*x^2)))/(15*x
*(b*x^2 - a)^(11/8))
```

Reduce [F]

$$\int \frac{1}{x^2 (-a+bx^2)^{11/8}}\,dx = -\left(\int \frac{1}{(b\,x^2-a)^{\frac{3}{8}}\,a\,x^2-(b\,x^2-a)^{\frac{3}{8}}\,b\,x^4}dx\right)$$

input
```
int(1/x^2/(b*x^2-a)^(11/8),x)
```

output
```
- int(1/(( - a + b*x**2)**(3/8)*a*x**2 - ( - a + b*x**2)**(3/8)*b*x**4),x
)
```

3.1265 $\int \frac{1}{x^4(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 963

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{11/8}}\,dx = \text{Too large to display}$$

output
```
-4/3/a/x^3/(b*x^2-a)^(3/8)-5/3*(b*x^2-a)^(5/8)/a^2/x^3-35/12*b*(b*x^2-a)^(
5/8)/a^3/x-35/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2
)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1
/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*
x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(5/2)
/x/(a^(1/4)+(b*x^2-a)^(1/4))+35/24*(2+2^(1/2))^(1/2)*b*(-b*x^2/a^(1/2)/(b*
x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/
(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a
^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/
2))^(1/2))/a^(5/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))+35/24*b*(-b*x^2/a^(1/2)/(b*
x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(
b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a
^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/
2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-35/24*b*(
-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)
^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2
*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))
^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)/a^(5/2)/x/(a^(1/4)-(b*x^2-a
)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^4\left(-a+bx^2\right)^{11/8}}\,dx=\frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{11}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3\left(-a+bx^2\right)^{3/8}}$$

input
```
Integrate[1/(x^4*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 11/8, -1/2, (b*x^2)/a])/(3*
a*x^3*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 (bx^2-a)^{11/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{x^4 \left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a (bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{11}{8},-\frac{1}{2},\frac{bx^2}{a}\right)}{3ax^3 (bx^2-a)^{3/8}}$$

input
```
Int[1/(x^4*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-3/2, 11/8, -1/2, (b*x^2)/a])/(3*
a*x^3*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{4} \left(b\,x^{2}-a\right)^{\frac{11}{8}}}dx$$

input
```
int(1/x^4/(b*x^2-a)^(11/8),x)
```

output
```
int(1/x^4/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{11}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b^2*x^8 - 2*a*b*x^6 + a^2*x^4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.04

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{11/8}}\,dx=\frac{e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\left.\begin{matrix}-\frac{3}{2},\frac{11}{8}\\-\frac{1}{2}\end{matrix}\right|\frac{bx^2}{a}\right)}{3a^{\frac{11}{8}}x^3}$$

input
```
integrate(1/x**4/(b*x**2-a)**(11/8),x)
```

output
```
exp(-3*I*pi/8)*hyper((-3/2, 11/8), (-1/2,), b*x**2/a)/(3*a**(11/8)*x**3)
```

Maxima [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4 (-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}x^4}\,dx$$

input
```
integrate(1/x^4/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{x^4\,{\left(b\,x^2-a\right)}^{11/8}} \,d x$$

input
```
int(1/(x^4*(b*x^2 - a)^(11/8)),x)
```

output
```
int(1/(x^4*(b*x^2 - a)^(11/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^4 \left(-a+bx^2\right)^{11/8}}\,dx=\frac{4(b\,x^{2}-a)^{\frac{3}{8}} a-16(b\,x^{2}-a)^{\frac{3}{8}} b\,x^{2}-31(b\,x^{2}-a)^{\frac{3}{4}} \left(\int \frac{(b\,x^{2}-a)^{\frac{1}{4}}}{(b\,x^{2}-a)^{\frac{5}{8}} a\,x^{2}-(b\,x^{2}-a)^{\frac{5}{8}} b\,x^{4}}d x}{12\,(b\,x^{2}-a)^{\frac{3}{4}} a^{2} x^{3}}$$

input
```
int(1/x^4/(b*x^2-a)^(11/8),x)
```

output
```
(4*( - a + b*x**2)**(3/8)*a - 16*( - a + b*x**2)**(3/8)*b*x**2 - 31*( - a
+ b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a*x**2
 - ( - a + b*x**2)**(5/8)*b*x**4),x)*a*b*x**3 + 34*( - a + b*x**2)**(3/4)*
int(( - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5
/8)*b*x**2),x)*b**2*x**3)/(12*( - a + b*x**2)**(3/4)*a**2*x**3)
```

3.1266 $\int \frac{1}{x^6(-a+bx^2)^{11/8}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 997

$$\int \frac{1}{x^6\,(-a+bx^2)^{11/8}}\,dx = \text{Too large to display}$$

output
```
-4/3/a/x^5/(b*x^2-a)^(3/8)-23/15*(b*x^2-a)^(5/8)/a^2/x^5-23/12*b*(b*x^2-a)
^(5/8)/a^3/x^3-161/48*b^2*(b*x^2-a)^(5/8)/a^4/x-161/96*(2+2^(1/2))^(1/2)*b
^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(3/8)*((a^(1/4)+(b*x^2
-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*EllipticE(1/2*(-a^(1/4)*(2^(1/
2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1
/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(7/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))+161/9
6*(2+2^(1/2))^(1/2)*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a)^(
3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Elliptic
E(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/2)/
a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/a^(7/2)/x/(a^(1/4)-(
b*x^2-a)^(1/4))+161/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^(1/2))^(1/2)*(b*x^2-a
)^(3/8)*((a^(1/4)+(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-a)^(1/4))^(1/2)*Ellipt
icF(1/2*(-a^(1/4)*(2^(1/2)-2*(b*x^2-a)^(1/4)/a^(1/4)+2^(1/2)*(b*x^2-a)^(1/
2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/2))/(2+2^(1/2))^(1/2)
/a^(7/2)/x/(a^(1/4)+(b*x^2-a)^(1/4))-161/96*b^2*(-b*x^2/a^(1/2)/(b*x^2-a)^
(1/2))^(1/2)*(b*x^2-a)^(3/8)*(-(a^(1/4)-(b*x^2-a)^(1/4))^2/a^(1/4)/(b*x^2-
a)^(1/4))^(1/2)*EllipticF(1/2*(a^(1/4)*(2^(1/2)+2*(b*x^2-a)^(1/4)/a^(1/4)+
2^(1/2)*(b*x^2-a)^(1/2)/a^(1/2))/(b*x^2-a)^(1/4))^(1/2),(-2+2*2^(1/2))^(1/
2))/(2+2^(1/2))^(1/2)/a^(7/2)/x/(a^(1/4)-(b*x^2-a)^(1/4))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06

$$\int \frac{1}{x^6 (-a+bx^2)^{11/8}}\,dx = \frac{\left(1-\frac{bx^2}{a}\right)^{3/8}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{11}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5(-a+bx^2)^{3/8}}$$

input
```
Integrate[1/(x^6*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 11/8, -3/2, (b*x^2)/a])/(5*
a*x^5*(-a + b*x^2)^(3/8))
```

Rubi [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^6 (bx^2-a)^{11/8}} \, dx$$

$$\Big\downarrow \text{279}$$

$$-\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \int \frac{1}{x^6 \left(1-\frac{bx^2}{a}\right)^{11/8}}dx}{a (bx^2-a)^{3/8}}$$

$$\Big\downarrow \text{278}$$

$$\frac{\left(1-\frac{bx^2}{a}\right)^{3/8} \text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{11}{8},-\frac{3}{2},\frac{bx^2}{a}\right)}{5ax^5 (bx^2-a)^{3/8}}$$

input
```
Int[1/(x^6*(-a + b*x^2)^(11/8)),x]
```

output
```
((1 - (b*x^2)/a)^(3/8)*Hypergeometric2F1[-5/2, 11/8, -3/2, (b*x^2)/a])/(5*
a*x^5*(-a + b*x^2)^(3/8))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{1}{x^{6} \left(b\,x^{2}-a \right)^{\frac{11}{8}}}dx$$

input
```
int(1/x^6/(b*x^2-a)^(11/8),x)
```

output
```
int(1/x^6/(b*x^2-a)^(11/8),x)
```

Fricas [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{\left(bx^2-a\right)^{\frac{11}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(11/8),x, algorithm="fricas")
```

output
```
integral((b*x^2 - a)^(5/8)/(b^2*x^10 - 2*a*b*x^8 + a^2*x^6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.03

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{11/8}}\,dx=\frac{e^{-\frac{3i\pi}{8}}\,{}_2F_1\left(\begin{matrix}-\frac{5}{2},\frac{11}{8}\\-\frac{3}{2}\end{matrix}\middle|\,\frac{bx^2}{a}\right)}{5a^{\frac{11}{8}}x^5}$$

input
```
integrate(1/x**6/(b*x**2-a)**(11/8),x)
```

output
```
exp(-3*I*pi/8)*hyper((-5/2, 11/8), (-3/2,), b*x**2/a)/(5*a**(11/8)*x**5)
```

Maxima [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(11/8),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^6), x)
```

Giac [F]

$$\int \frac{1}{x^6 (-a+bx^2)^{11/8}}\,dx = \int \frac{1}{(bx^2-a)^{\frac{11}{8}}x^6}\,dx$$

input
```
integrate(1/x^6/(b*x^2-a)^(11/8),x, algorithm="giac")
```

output
```
integrate(1/((b*x^2 - a)^(11/8)*x^6), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{11/8}}\,dx=\int \frac{1}{x^6 \left(b\,x^2-a\right)^{11/8}}\,dx$$

input
```
int(1/(x^6*(b*x^2 - a)^(11/8)),x)
```

output
```
int(1/(x^6*(b*x^2 - a)^(11/8)), x)
```

Reduce [F]

$$\int \frac{1}{x^6 \left(-a+bx^2\right)^{11/8}}\,dx=\frac{432(b\,x^2-a)^{\frac{3}{8}}\,a^2-2812(b\,x^2-a)^{\frac{3}{8}}\,ab\,x^2+1660(b\,x^2-a)^{\frac{3}{8}}\,b^2x^4-10920(b\,x^2-}{}$$

input
```
int(1/x^6/(b*x^2-a)^(11/8),x)
```

output
```
(432*( - a + b*x**2)**(3/8)*a**2 - 2812*( - a + b*x**2)**(3/8)*a*b*x**2 +
1660*( - a + b*x**2)**(3/8)*b**2*x**4 - 10920*( - a + b*x**2)**(3/4)*int((
 - a + b*x**2)**(1/4)/(( - a + b*x**2)**(5/8)*a*x**4 - ( - a + b*x**2)**(5
/8)*b*x**6),x)*a**2*b*x**5 + 12565*( - a + b*x**2)**(3/4)*int(( - a + b*x*
*2)**(1/4)/(( - a + b*x**2)**(5/8)*a*x**2 - ( - a + b*x**2)**(5/8)*b*x**4)
,x)*a*b**2*x**5 - 2905*( - a + b*x**2)**(3/4)*int(( - a + b*x**2)**(1/4)/(
( - a + b*x**2)**(5/8)*a - ( - a + b*x**2)**(5/8)*b*x**2),x)*b**3*x**5)/(2
160*( - a + b*x**2)**(3/4)*a**3*x**5)
```

3.1267 $\int x^7(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int x^7\left(a+bx^2\right)^p\,dx=-\frac{a^3\left(a+bx^2\right)^{1+p}}{2b^4(1+p)}+\frac{3a^2\left(a+bx^2\right)^{2+p}}{2b^4(2+p)}-\frac{3a\left(a+bx^2\right)^{3+p}}{2b^4(3+p)}+\frac{\left(a+bx^2\right)^{4+p}}{2b^4(4+p)}$$

output
```
-1/2*a^3*(b*x^2+a)^(p+1)/b^4/(p+1)+3/2*a^2*(b*x^2+a)^(2+p)/b^4/(2+p)-3/2*a
*(b*x^2+a)^(3+p)/b^4/(3+p)+1/2*(b*x^2+a)^(4+p)/b^4/(4+p)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int x^7\left(a+bx^2\right)^p\,dx=\frac{1}{2}\left(-\frac{a^3\left(a+bx^2\right)^{1+p}}{b^4(1+p)}+\frac{3a^2\left(a+bx^2\right)^{2+p}}{b^4(2+p)}-\frac{3a\left(a+bx^2\right)^{3+p}}{b^4(3+p)}+\frac{\left(a+bx^2\right)^{4+p}}{b^4(4+p)}\right)$$

input
```
Integrate[x^7*(a + b*x^2)^p,x]
```

output
```
(-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/
(b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 +
 p)/(b^4*(4 + p)))/2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^6 \left(bx^2+a\right)^p dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(-\frac{a^3 \left(bx^2+a\right)^p}{b^3}+\frac{3a^2 \left(bx^2+a\right)^{p+1}}{b^3}-\frac{3a \left(bx^2+a\right)^{p+2}}{b^3}+\frac{\left(bx^2+a\right)^{p+3}}{b^3}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(-\frac{a^3 \left(a+bx^2\right)^{p+1}}{b^4 (p+1)}+\frac{3a^2 \left(a+bx^2\right)^{p+2}}{b^4 (p+2)}-\frac{3a \left(a+bx^2\right)^{p+3}}{b^4 (p+3)}+\frac{\left(a+bx^2\right)^{p+4}}{b^4 (p+4)}\right)$$

input
```
Int[x^7*(a + b*x^2)^p,x]
```

output

```
(-((a^3*(a + b*x^2)^(1 + p))/(b^4*(1 + p))) + (3*a^2*(a + b*x^2)^(2 + p))/
(b^4*(2 + p)) - (3*a*(a + b*x^2)^(3 + p))/(b^4*(3 + p)) + (a + b*x^2)^(4 +
 p)/(b^4*(4 + p)))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

method	result
gosper	$-\frac{\left(b\,x^{2}+a\right)^{p+1}\left(-b^{3}p^{3}x^{6}-6b^{3}p^{2}x^{6}-11b^{3}p\,x^{6}+3a\,b^{2}p^{2}x^{4}-6b^{3}x^{6}+9a\,b^{2}p\,x^{4}+6a\,b^{2}x^{4}-6a^{2}bp\,x^{2}-6a^{2}b\,x^{2}+6a^{3}\right)}{2b^{4}\left(p^{4}+10p^{3}+35p^{2}+50p+24\right)}$
orering	$-\frac{\left(b\,x^{2}+a\right)^{p}\left(-b^{3}p^{3}x^{6}-6b^{3}p^{2}x^{6}-11b^{3}p\,x^{6}+3a\,b^{2}p^{2}x^{4}-6b^{3}x^{6}+9a\,b^{2}p\,x^{4}+6a\,b^{2}x^{4}-6a^{2}bp\,x^{2}-6a^{2}b\,x^{2}+6a^{3}\right)\left(b\,x^{2}+a\right)}{2b^{4}\left(p^{4}+10p^{3}+35p^{2}+50p+24\right)}$
risch	$-\frac{\left(-b^{4}p^{3}x^{8}-6b^{4}p^{2}x^{8}-a\,b^{3}p^{3}x^{6}-11b^{4}p\,x^{8}-3a\,b^{3}p^{2}x^{6}-6b^{4}x^{8}-2ap\,x^{6}b^{3}+3a^{2}b^{2}p^{2}x^{4}+3a^{2}p\,x^{4}b^{2}-6a^{3}p\,x^{2}b+6a^{4}\right)\left(b\,x^{2}+a\right)^{}}{2(3+p)(4+p)(2+p)(p+1)b^{4}}$
norman	$\frac{x^{8}{\mathrm e}^{p\ln\left(b\,x^{2}+a\right)}}{2p+8}-\frac{3a^{4}{\mathrm e}^{p\ln\left(b\,x^{2}+a\right)}}{b^{4}\left(p^{4}+10p^{3}+35p^{2}+50p+24\right)}+\frac{ap\,x^{6}{\mathrm e}^{p\ln\left(b\,x^{2}+a\right)}}{2b\left(p^{2}+7p+12\right)}-\frac{3a^{2}p\,x^{4}{\mathrm e}^{p\ln\left(b\,x^{2}+a\right)}}{2b^{2}\left(p^{3}+9p^{2}+26p+24\right)}+\frac{3p\,a^{3}x^{2}{\mathrm e}^{p\ln\left(b\,x^{}\right.}}{b^{3}\left(p^{4}+10p^{3}+35p^{2}+\right.}$
parallelrisch	$\frac{x^{8}\left(b\,x^{2}+a\right)^{p}a\,b^{4}p^{3}+6x^{8}\left(b\,x^{2}+a\right)^{p}a\,b^{4}p^{2}+11x^{8}\left(b\,x^{2}+a\right)^{p}a\,b^{4}p+x^{6}\left(b\,x^{2}+a\right)^{p}a^{2}b^{3}p^{3}+6x^{8}\left(b\,x^{2}+a\right)^{p}a\,b^{4}+3x^{6}\left(b\,x^{2}+a\right)^{p}a^{2}b^{}}{2(4+p)\left(p^{2}+5p+6\right)a(p+}$

input
```
int(x^7*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^4*(b*x^2+a)^(p+1)/(p^4+10*p^3+35*p^2+50*p+24)*(-b^3*p^3*x^6-6*b^3*p
^2*x^6-11*b^3*p*x^6+3*a*b^2*p^2*x^4-6*b^3*x^6+9*a*b^2*p*x^4+6*a*b^2*x^4-6*
a^2*b*p*x^2-6*a^2*b*x^2+6*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int x^7 (a+bx^2)^p \, dx$$
$$= \frac{((b^4p^3+6\,b^4p^2+11\,b^4p+6\,b^4)x^8+6\,a^3bpx^2+(ab^3p^3+3\,ab^3p^2+2\,ab^3p)x^6-3\,(a^2b^2p^2+a^2b^2p)x^4-6\,a}{2\,(b^4p^4+10\,b^4p^3+35\,b^4p^2+50\,b^4p+24\,b^4)}$$

input
```
integrate(x^7*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*((b^4*p^3 + 6*b^4*p^2 + 11*b^4*p + 6*b^4)*x^8 + 6*a^3*b*p*x^2 + (a*b^3
*p^3 + 3*a*b^3*p^2 + 2*a*b^3*p)*x^6 - 3*(a^2*b^2*p^2 + a^2*b^2*p)*x^4 - 6*
a^4)*(b*x^2 + a)^p/(b^4*p^4 + 10*b^4*p^3 + 35*b^4*p^2 + 50*b^4*p + 24*b^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1923 vs. 2(85) = 170.

Time = 2.76 (sec) , antiderivative size = 1923, normalized size of antiderivative = 19.23

$$\int x^7 (a+bx^2)^p \, dx = \text{Too large to display}$$

input
```
integrate(x**7*(b*x**2+a)**p,x)
```

output
```
Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(x - sqrt(-a/b))/(12*a**3*b*
*4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(x + s
qrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x*
*6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7
*x**6) + 18*a**2*b*x**2*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x
**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(x + sqrt(-a/b))/
(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a*
*2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**
6) + 18*a*b**2*x**4*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2
+ 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(x + sqrt(-a/b))/(12*
a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2
*x**4/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) +
 6*b**3*x**6*log(x - sqrt(-a/b))/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*
b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(x + sqrt(-a/b))/(12*a**3*b**4
+ 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3
*log(x - sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*
log(x + sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(
4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x - sqrt(-
a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(x +
 sqrt(-a/b))/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int x^7 (a + bx^2)^p \, dx = \frac{((p^3 + 6\,p^2 + 11\,p + 6)b^4x^8 + (p^3 + 3\,p^2 + 2\,p)ab^3x^6 - 3\,(p^2 + p)a^2b^2x^4 + 6\,a^3bpx^2 - 6\,a^4)(bx^2 + a)^p}{2\,(p^4 + 10\,p^3 + 35\,p^2 + 50\,p + 24)b^4}$$

input
```
integrate(x^7*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*((p^3 + 6*p^2 + 11*p + 6)*b^4*x^8 + (p^3 + 3*p^2 + 2*p)*a*b^3*x^6 - 3*
(p^2 + p)*a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 6*a^4)*(b*x^2 + a)^p/((p^4 + 10*p^
3 + 35*p^2 + 50*p + 24)*b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(92) = 184$.

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.60

$$\int x^7(a+bx^2)^p\,dx$$
$$= \frac{(bx^2+a)^4(bx^2+a)^p p^2 - 3(bx^2+a)^3(bx^2+a)^p ap^2 + 3(bx^2+a)^2(bx^2+a)^p a^2p^2 + 5(bx^2+a)^4(bx^2+a}{}$$
$$-\frac{(bx^2+a)^{p+1}a^3}{2b^4(p+1)}$$

input
```
integrate(x^7*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*((b*x^2 + a)^4*(b*x^2 + a)^p*p^2 - 3*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p^2
 + 3*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2*p^2 + 5*(b*x^2 + a)^4*(b*x^2 + a)^p*p
 - 18*(b*x^2 + a)^3*(b*x^2 + a)^p*a*p + 21*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2
*p + 6*(b*x^2 + a)^4*(b*x^2 + a)^p - 24*(b*x^2 + a)^3*(b*x^2 + a)^p*a + 36
*(b*x^2 + a)^2*(b*x^2 + a)^p*a^2)/(b^4*p^3 + 9*b^4*p^2 + 26*b^4*p + 24*b^4
) - 1/2*(b*x^2 + a)^(p + 1)*a^3/(b^4*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.83

$$\int x^7(a+bx^2)^p\,dx = (b\,x^2+a)^p\left(\frac{x^8\,(p^3+6\,p^2+11\,p+6)}{2\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{3\,a^4}{b^4\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{3\,a^3\,p\,x^2}{b^3\,(p^4+10\,p^3+35\,p^2+50\,p+24)} + \frac{a\,p\,x^6\,(p^2+3\,p+2)}{2\,b\,(p^4+10\,p^3+35\,p^2+50\,p+24)} - \frac{3\,a^2\,p\,x^4\,(p+1)}{2\,b^2\,(p^4+10\,p^3+35\,p^2+50\,p+24)}\right)$$

input
```
int(x^7*(a + b*x^2)^p,x)
```

output
```
(a + b*x^2)^p*((x^8*(11*p + 6*p^2 + p^3 + 6))/(2*(50*p + 35*p^2 + 10*p^3 +
 p^4 + 24)) - (3*a^4)/(b^4*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (3*a^3*p
*x^2)/(b^3*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) + (a*p*x^6*(3*p + p^2 + 2)
)/(2*b*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)) - (3*a^2*p*x^4*(p + 1))/(2*b^2
*(50*p + 35*p^2 + 10*p^3 + p^4 + 24)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int x^7(a+bx^2)^p\,dx$$
$$=\frac{(b\,x^2+a)^p\,(b^4p^3x^8+6b^4p^2x^8+a\,b^3p^3x^6+11b^4p\,x^8+3a\,b^3p^2x^6+6b^4x^8+2a\,b^3p\,x^6-3a^2b^2p^2x^4-3a^2b^2}{2b^4\,(p^4+10p^3+35p^2+50p+24)}$$

input
```
int(x^7*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*( - 6*a**4 + 6*a**3*b*p*x**2 - 3*a**2*b**2*p**2*x**4 - 3*
a**2*b**2*p*x**4 + a*b**3*p**3*x**6 + 3*a*b**3*p**2*x**6 + 2*a*b**3*p*x**6
 + b**4*p**3*x**8 + 6*b**4*p**2*x**8 + 11*b**4*p*x**8 + 6*b**4*x**8))/(2*b
**4*(p**4 + 10*p**3 + 35*p**2 + 50*p + 24))
```

3.1268 $\int x^5(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 72

$$\int x^5\left(a+bx^2\right)^p\,dx=\frac{a^2(a+bx^2)^{1+p}}{2b^3(1+p)}-\frac{a(a+bx^2)^{2+p}}{b^3(2+p)}+\frac{(a+bx^2)^{3+p}}{2b^3(3+p)}$$

output
```
1/2*a^2*(b*x^2+a)^(p+1)/b^3/(p+1)-a*(b*x^2+a)^(2+p)/b^3/(2+p)+1/2*(b*x^2+a
)^(3+p)/b^3/(3+p)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int x^5\left(a+bx^2\right)^p\,dx=\frac{(a+bx^2)^{1+p}\left(2a^2-2ab(1+p)x^2+b^2(2+3p+p^2)\,x^4\right)}{2b^3(1+p)(2+p)(3+p)}$$

input
```
Integrate[x^5*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x^2 + b^2*(2 + 3*p + p^2)*x^4)
)/(2*b^3*(1 + p)*(2 + p)*(3 + p))
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b x^2\right)^p \, dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2} \int x^4 \left(b x^2+a\right)^p \, dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{2} \int \left(\frac{a^2 \left(b x^2+a\right)^p}{b^2}-\frac{2 a \left(b x^2+a\right)^{p+1}}{b^2}+\frac{\left(b x^2+a\right)^{p+2}}{b^2}\right) dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{a^2 \left(a+b x^2\right)^{p+1}}{b^3 (p+1)}-\frac{2 a \left(a+b x^2\right)^{p+2}}{b^3 (p+2)}+\frac{\left(a+b x^2\right)^{p+3}}{b^3 (p+3)}\right)$$

input
```
Int[x^5*(a + b*x^2)^p,x]
```

output
```
((a^2*(a + b*x^2)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x^2)^(2 + p))/(b^3*
(2 + p)) + (a + b*x^2)^(3 + p)/(b^3*(3 + p)))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
gosper	$\frac{\left(b\,x^{2}+a\right)^{p+1}\left(b^{2}p^{2}x^{4}+3b^{2}p\,x^{4}+2b^{2}x^{4}-2abp\,x^{2}-2ab\,x^{2}+2a^{2}\right)}{2b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$
orering	$\frac{\left(b\,x^{2}+a\right)\left(b^{2}p^{2}x^{4}+3b^{2}p\,x^{4}+2b^{2}x^{4}-2abp\,x^{2}-2ab\,x^{2}+2a^{2}\right)\left(b\,x^{2}+a\right)^{p}}{2b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$
risch	$\frac{\left(b^{3}p^{2}x^{6}+3b^{3}p\,x^{6}+a\,b^{2}p^{2}x^{4}+2b^{3}x^{6}+a\,b^{2}p\,x^{4}-2a^{2}bp\,x^{2}+2a^{3}\right)\left(b\,x^{2}+a\right)^{p}}{2(2+p)(3+p)(p+1)b^{3}}$
norman	$\frac{a^{3}e^{p\ln\left(b\,x^{2}+a\right)}}{b^{3}\left(p^{3}+6p^{2}+11p+6\right)}+\frac{x^{6}e^{p\ln\left(b\,x^{2}+a\right)}}{6+2p}+\frac{ap\,x^{4}e^{p\ln\left(b\,x^{2}+a\right)}}{2b\left(p^{2}+5p+6\right)}-\frac{p\,a^{2}x^{2}e^{p\ln\left(b\,x^{2}+a\right)}}{b^{2}\left(p^{3}+6p^{2}+11p+6\right)}$
parallelrisch	$\frac{x^{6}\left(b\,x^{2}+a\right)^{p}b^{3}p^{2}+3x^{6}\left(b\,x^{2}+a\right)^{p}b^{3}p+2x^{6}\left(b\,x^{2}+a\right)^{p}b^{3}+x^{4}\left(b\,x^{2}+a\right)^{p}a\,b^{2}p^{2}+x^{4}\left(b\,x^{2}+a\right)^{p}a\,b^{2}p-2x^{2}\left(b\,x^{2}+a\right)^{p}a^{2}bp+2\left(b\,x^{2}\right.}{2b^{3}\left(p^{3}+6p^{2}+11p+6\right)}$

input
```
int(x^5*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2/b^3*(b*x^2+a)^(p+1)/(p^3+6*p^2+11*p+6)*(b^2*p^2*x^4+3*b^2*p*x^4+2*b^2*
x^4-2*a*b*p*x^2-2*a*b*x^2+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int x^5(a+bx^2)^p\,dx = \frac{((b^3p^2+3\,b^3p+2\,b^3)x^6-2\,a^2bpx^2+(ab^2p^2+ab^2p)x^4+2\,a^3)(bx^2+a)^p}{2\,(b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3)}$$

input
```
integrate(x^5*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*((b^3*p^2 + 3*b^3*p + 2*b^3)*x^6 - 2*a^2*b*p*x^2 + (a*b^2*p^2 + a*b^2*
p)*x^4 + 2*a^3)*(b*x^2 + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(58) = 116$.

Time = 1.30 (sec) , antiderivative size = 920, normalized size of antiderivative = 12.78

$$\int x^5 (a + bx^2)^p \, dx = \text{Too large to display}$$

input
```
integrate(x**5*(b*x**2+a)**p,x)
```

output
```
Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a**2*b**
3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2*b**3
 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 +
 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**
2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
+ 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**
5*x**4), Eq(p, -3)), (-2*a**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2)
 - 2*a**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3
+ 2*b**4*x**2) - 2*a*b*x**2*log(x - sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) -
 2*a*b*x**2*log(x + sqrt(-a/b))/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*
b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2
*log(x + sqrt(-a/b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)),
(2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3)
 - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p
 + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2
 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*
b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*
p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int x^5 (a+bx^2)^p \, dx = \frac{((p^2+3p+2)b^3x^6+(p^2+p)ab^2x^4-2a^2bpx^2+2a^3)(bx^2+a)^p}{2(p^3+6p^2+11p+6)b^3}$$

input
```
integrate(x^5*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*((p^2 + 3*p + 2)*b^3*x^6 + (p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + 2*a^3
)*(b*x^2 + a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int x^5 (a+bx^2)^p \, dx = \frac{(bx^2+a)^3(bx^2+a)^p p - 2(bx^2+a)^2(bx^2+a)^p ap + 2(bx^2+a)^3(bx^2+a)^p - 6(bx^2+a)^2(bx^2+a)^p a}{2(b^3p^2+5b^3p+6b^3)} + \frac{(bx^2+a)^{p+1}a^2}{2b^3(p+1)}$$

input
```
integrate(x^5*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*((b*x^2 + a)^3*(b*x^2 + a)^p*p - 2*(b*x^2 + a)^2*(b*x^2 + a)^p*a*p + 2
*(b*x^2 + a)^3*(b*x^2 + a)^p - 6*(b*x^2 + a)^2*(b*x^2 + a)^p*a)/(b^3*p^2 +
 5*b^3*p + 6*b^3) + 1/2*(b*x^2 + a)^(p + 1)*a^2/(b^3*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^5(a+bx^2)^p\,dx = (b\,x^2+a)^p\left(\frac{a^3}{b^3\,(p^3+6\,p^2+11\,p+6)}+\frac{x^6\,(p^2+3\,p+2)}{2\,(p^3+6\,p^2+11\,p+6)}-\frac{a^2\,p\,x^2}{b^2\,(p^3+6\,p^2+11\,p+6)}+\frac{a\,p\,x^4\,(p+1)}{2\,b\,(p^3+6\,p^2+11\,p+6)}\right)$$

input
```
int(x^5*(a + b*x^2)^p,x)
```

output
```
(a + b*x^2)^p*(a^3/(b^3*(11*p + 6*p^2 + p^3 + 6)) + (x^6*(3*p + p^2 + 2))/
(2*(11*p + 6*p^2 + p^3 + 6)) - (a^2*p*x^2)/(b^2*(11*p + 6*p^2 + p^3 + 6))
+ (a*p*x^4*(p + 1))/(2*b*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int x^5(a+bx^2)^p\,dx = \frac{(b\,x^2+a)^p\,(b^3p^2x^6+3b^3p\,x^6+a\,b^2p^2x^4+2b^3x^6+a\,b^2p\,x^4-2a^2bp\,x^2+2a^3)}{2b^3\,(p^3+6p^2+11p+6)}$$

input
```
int(x^5*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*(2*a**3 - 2*a**2*b*p*x**2 + a*b**2*p**2*x**4 + a*b**2*p*x
**4 + b**3*p**2*x**6 + 3*b**3*p*x**6 + 2*b**3*x**6))/(2*b**3*(p**3 + 6*p**
2 + 11*p + 6))
```

3.1269 $\int x^3(a+bx^2)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int x^3(a+bx^2)^p \, dx = -\frac{a(a+bx^2)^{1+p}}{2b^2(1+p)} + \frac{(a+bx^2)^{2+p}}{2b^2(2+p)}$$

output
```
-1/2*a*(b*x^2+a)^(p+1)/b^2/(p+1)+1/2*(b*x^2+a)^(2+p)/b^2/(2+p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int x^3(a+bx^2)^p \, dx = \frac{(a+bx^2)^{1+p}(-a+b(1+p)x^2)}{2b^2(1+p)(2+p)}$$

input
```
Integrate[x^3*(a + b*x^2)^p,x]
```

output
```
((a + b*x^2)^(1 + p)*(-a + b*(1 + p)*x^2))/(2*b^2*(1 + p)*(2 + p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+bx^2)^p \, dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int x^2 (bx^2+a)^p \, dx^2$$

$$\Big\downarrow \; 53$$

$$\frac{1}{2}\int \left(\frac{(bx^2+a)^{p+1}}{b}-\frac{a(bx^2+a)^p}{b}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{(a+bx^2)^{p+2}}{b^2(p+2)}-\frac{a(a+bx^2)^{p+1}}{b^2(p+1)}\right)$$

input
```
Int[x^3*(a + b*x^2)^p,x]
```

output
```
(-((a*(a + b*x^2)^(1 + p))/(b^2*(1 + p))) + (a + b*x^2)^(2 + p)/(b^2*(2 +
p)))/2
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{(b\,x^2+a)^{p+1}(-x^2pb-b\,x^2+a)}{2b^2(p^2+3p+2)}$	42
orering	$-\frac{(b\,x^2+a)^{p}(-x^2pb-b\,x^2+a)(b\,x^2+a)}{2b^2(p^2+3p+2)}$	47
risch	$-\frac{(-b^2p\,x^4-b^2x^4-abp\,x^2+a^2)(b\,x^2+a)^p}{2b^2(2+p)(p+1)}$	54
norman	$\frac{x^4e^{p\ln(b\,x^2+a)}}{4+2p}-\frac{a^2e^{p\ln(b\,x^2+a)}}{2b^2(p^2+3p+2)}+\frac{pa\,x^2e^{p\ln(b\,x^2+a)}}{2b(p^2+3p+2)}$	83
parallelrisch	$\frac{x^4(b\,x^2+a)^pa\,b^2p+x^4(b\,x^2+a)^pa\,b^2+x^2(b\,x^2+a)^pa^2bp-(b\,x^2+a)^pa^3}{2(2+p)(p+1)a\,b^2}$	87

input
```
int(x^3*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(b*x^2+a)^(p+1)/(p^2+3*p+2)*(-b*p*x^2-b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x^3(a+bx^2)^p\,dx=\frac{(abpx^2+(b^2p+b^2)x^4-a^2)(bx^2+a)^p}{2\,(b^2p^2+3\,b^2p+2\,b^2)}$$

input
```
integrate(x^3*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(a*b*p*x^2 + (b^2*p + b^2)*x^4 - a^2)*(b*x^2 + a)^p/(b^2*p^2 + 3*b^2*p
 + 2*b^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(37) = 74$.

Time = 0.53 (sec) , antiderivative size = 333, normalized size of antiderivative = 6.94

$$\int x^3(a+bx^2)^p\,dx = \begin{cases} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} + \frac{bx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2} - \frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)**p,x)
```

output
```
Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b*
*3*x**2) + a*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 +
2*b**3*x**2) + b*x**2*log(x - sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2) + b*x**
2*log(x + sqrt(-a/b))/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(x - sq
rt(-a/b))/(2*b**2) - a*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1
)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**
2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**
2*p**2 + 6*b**2*p + 4*b**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98

$$\int x^3 (a+bx^2)^p \, dx = \frac{(b^2(p+1)x^4 + abpx^2 - a^2)(bx^2+a)^p}{2(p^2+3p+2)b^2}$$

input
```
integrate(x^3*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*(p + 1)*x^4 + a*b*p*x^2 - a^2)*(b*x^2 + a)^p/((p^2 + 3*p + 2)*b^2
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

$$\int x^3 (a+bx^2)^p \, dx = \frac{(bx^2+a)^2(bx^2+a)^p}{2b^2(p+2)} - \frac{(bx^2+a)^{p+1}a}{2b^2(p+1)}$$

input
```
integrate(x^3*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + a)^2*(b*x^2 + a)^p/(b^2*(p + 2)) - 1/2*(b*x^2 + a)^(p + 1)*a/
(b^2*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int x^3 (a+bx^2)^p \, dx = (b\,x^2+a)^p \left(\frac{x^4\,(p+1)}{2\,(p^2+3\,p+2)} - \frac{a^2}{2\,b^2\,(p^2+3\,p+2)} + \frac{a\,p\,x^2}{2\,b\,(p^2+3\,p+2)}\right)$$

input
```
int(x^3*(a + b*x^2)^p,x)
```

output
```
(a + b*x^2)^p*((x^4*(p + 1))/(2*(3*p + p^2 + 2)) - a^2/(2*b^2*(3*p + p^2 +
 2)) + (a*p*x^2)/(2*b*(3*p + p^2 + 2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int x^3(a+bx^2)^p\,dx = \frac{(b\,x^2+a)^p\,(b^2p\,x^4+b^2x^4+abp\,x^2-a^2)}{2b^2\,(p^2+3p+2)}$$

input
```
int(x^3*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*( - a**2 + a*b*p*x**2 + b**2*p*x**4 + b**2*x**4))/(2*b**2
*(p**2 + 3*p + 2))
```

3.1270 $\int x(a+bx^2)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^{1+p}}{2b(1+p)}$$

output
```
1/2*(b*x^2+a)^(p+1)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\left(a+bx^2\right)^p\,dx=\frac{\left(a+bx^2\right)^{1+p}}{2b+2bp}$$

input
```
Integrate[x*(a + b*x^2)^p,x]
```

output
```
(a + b*x^2)^(1 + p)/(2*b + 2*b*p)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^p \, dx$$

$$\Big\downarrow \text{ 241}$$

$$\frac{(a+bx^2)^{p+1}}{2b(p+1)}$$

input
```
Int[x*(a + b*x^2)^p,x]
```

output
```
(a + b*x^2)^(1 + p)/(2*b*(1 + p))
```

Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
gosper	$\frac{(b\,x^2+a)^{p+1}}{2b(p+1)}$	22
derivativedivides	$\frac{(b\,x^2+a)^{p+1}}{2b(p+1)}$	22
default	$\frac{(b\,x^2+a)^{p+1}}{2b(p+1)}$	22
risch	$\frac{(b\,x^2+a)\,(b\,x^2+a)^{p}}{2b(p+1)}$	27
orering	$\frac{(b\,x^2+a)\,(b\,x^2+a)^{p}}{2b(p+1)}$	27
parallelrisch	$\frac{(b\,x^2+a)^{p}b\,x^2+(b\,x^2+a)^{p}a}{2b(p+1)}$	37
norman	$\frac{x^2 e^{p\ln\left(b\,x^2+a\right)}}{2p+2}+\frac{a\,e^{p\ln\left(b\,x^2+a\right)}}{2b(p+1)}$	45

input
```
int(x*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^2+a)^(p+1)/b/(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(a+bx^2\right)^p\,dx=\frac{(bx^2+a)(bx^2+a)^p}{2\,(bp+b)}$$

input
```
integrate(x*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(b*x^2 + a)*(b*x^2 + a)^p/(b*p + b)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int x(a+bx^2)^p\,dx = \begin{cases} \frac{x^2}{2a} & \text{for } b=0 \wedge p=-1 \\ \frac{a^p x^2}{2} & \text{for } b=0 \\ \frac{\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b} + \frac{\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } p=-1 \\ \frac{a(a+bx^2)^p}{2bp+2b} + \frac{bx^2(a+bx^2)^p}{2bp+2b} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)**p,x)
```

output
```
Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a**p*x**2/2, Eq(b, 0)), (lo
g(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(p, -1)), (a*(a + b
*x**2)**p/(2*b*p + 2*b) + b*x**2*(a + b*x**2)**p/(2*b*p + 2*b), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)^p\,dx = \frac{(bx^2+a)^{p+1}}{2\,b(p+1)}$$

input
```
integrate(x*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)^p\,dx=\frac{(bx^2+a)^{p+1}}{2\,b(p+1)}$$

input
```
integrate(x*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
1/2*(b*x^2 + a)^(p + 1)/(b*(p + 1))
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x(a+bx^2)^p\,dx=\frac{(b\,x^2+a)^{p+1}}{2\,b\,(p+1)}$$

input
```
int(x*(a + b*x^2)^p,x)
```

output
```
(a + b*x^2)^(p + 1)/(2*b*(p + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x(a+bx^2)^p\,dx=\frac{(b\,x^2+a)^p\,(b\,x^2+a)}{2b\,(p+1)}$$

input
```
int(x*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*(a + b*x**2))/(2*b*(p + 1))
```

3.1271 $\int \frac{(a+bx^2)^p}{x} dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{(a+bx^2)^p}{x} dx = -\frac{(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

output
```
-1/2*(b*x^2+a)^(p+1)*hypergeom([1, p+1],[2+p],1+b*x^2/a)/a/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x} dx = -\frac{(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(1,1+p,2+p,1+\frac{bx^2}{a}\right)}{2a(1+p)}$$

input
```
Integrate[(a + b*x^2)^p/x,x]
```

output
```
-1/2*((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a
])/(a*(1 + p))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x}\,dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p}{x^2}dx^2$$

$$\Big\downarrow \; 75$$

$$-\frac{(a+bx^2)^{p+1}\,\text{Hypergeometric2F1}\left(1,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a(p+1)}$$

input
```
Int[(a + b*x^2)^p/x,x]
```

output
```
-1/2*((a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a
])/(a*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x}dx$$

input
```
int((b*x^2+a)^p/x,x)
```

output
```
int((b*x^2+a)^p/x,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x}\,dx = \int \frac{(bx^2+a)^p}{x}\,dx$$

input
```
integrate((b*x^2+a)^p/x,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)^p}{x}\,dx = -\frac{b^p x^{2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix} -p, -p \\ 1-p \end{matrix}\middle| \frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(1-p)}$$

input
```
integrate((b*x**2+a)**p/x,x)
```

output
```
-b**p*x**(2*p)*gamma(-p)*hyper((-p, -p), (1 - p,), a*exp_polar(I*pi)/(b*x*
*2))/(2*gamma(1 - p))
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x}\,dx = \int \frac{(bx^2+a)^p}{x}\,dx$$

input
```
integrate((b*x^2+a)^p/x,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x}\,dx = \int \frac{(bx^2+a)^p}{x}\,dx$$

input
```
integrate((b*x^2+a)^p/x,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x}\,dx = \int \frac{(b\,x^2+a)^p}{x}\,dx$$

input
```
int((a + b*x^2)^p/x,x)
```

output
```
int((a + b*x^2)^p/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x}\,dx = \frac{(b\,x^2+a)^p + 2\left(\int \frac{(b\,x^2+a)^p}{b\,x^3+ax}dx\right)ap}{2p}$$

input
```
int((b*x^2+a)^p/x,x)
```

output
```
((a + b*x**2)**p + 2*int((a + b*x**2)**p/(a*x + b*x**3),x)*a*p)/(2*p)
```

3.1272 $\int \frac{(a+bx^2)^p}{x^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \frac{(a+bx^2)^p}{x^3} dx = \frac{b(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(2, 1+p, 2+p, 1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

output
```
1/2*b*(b*x^2+a)^(p+1)*hypergeom([2, p+1],[2+p],1+b*x^2/a)/a^2/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x^3} dx = \frac{b(a+bx^2)^{1+p} \text{Hypergeometric2F1}\left(2, 1+p, 2+p, 1+\frac{bx^2}{a}\right)}{2a^2(1+p)}$$

input
```
Integrate[(a + b*x^2)^p/x^3,x]
```

output
```
(b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/
(2*a^2*(1 + p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {243, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^3} \, dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{(bx^2+a)^p}{x^4}dx^2$$

$$\Big\downarrow \text{ 75}$$

$$\frac{b(a+bx^2)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{bx^2}{a}+1\right)}{2a^2(p+1)}$$

input
```
Int[(a + b*x^2)^p/x^3,x]
```

output
```
(b*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p, 1 + (b*x^2)/a])/
(2*a^2*(1 + p))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^3}dx$$

input
```
int((b*x^2+a)^p/x^3,x)
```

output
```
int((b*x^2+a)^p/x^3,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = \int \frac{(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^3, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = -\frac{b^p x^{2p-2}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)}{2\Gamma(2-p)}$$

input
```
integrate((b*x**2+a)**p/x**3,x)
```

output
```
-b**p*x**(2*p - 2)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), a*exp_polar(I
*pi)/(b*x**2))/(2*gamma(2 - p))
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = \int \frac{(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^3, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = \int \frac{(bx^2+a)^p}{x^3}\,dx$$

input
```
integrate((b*x^2+a)^p/x^3,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = \int \frac{(b\,x^2+a)^p}{x^3}\,dx$$

input
```
int((a + b*x^2)^p/x^3,x)
```

output
```
int((a + b*x^2)^p/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x^3}\,dx = \frac{-(b\,x^2+a)^p+2\left(\int \frac{(b\,x^2+a)^p}{b\,x^3+ax}dx\right)bp\,x^2}{2x^2}$$

input
```
int((b*x^2+a)^p/x^3,x)
```

output
```
( - (a + b*x**2)**p + 2*int((a + b*x**2)**p/(a*x + b*x**3),x)*b*p*x**2)/(2
*x**2)
```

3.1273 $\int x^6(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int x^6\left(a+bx^2\right)^p\,dx=\frac{1}{7}x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

output
```
1/7*x^7*(b*x^2+a)^p*hypergeom([7/2, -p],[9/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^6\left(a+bx^2\right)^p\,dx=\frac{1}{7}x^7\left(a+bx^2\right)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^6*(a + b*x^2)^p,x]
```

output
```
(x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 +
(b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 (a+bx^2)^p \, dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^6 \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{7}x^7 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{2},-p,\frac{9}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^6*(a + b*x^2)^p,x]
```

output
```
(x^7*(a + b*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(7*(1 +
(b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^6 \left(b\,x^2+a\right)^p dx$$

input
```
int(x^6*(b*x^2+a)^p,x)
```

output
```
int(x^6*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^6 \left(a+bx^2\right)^p \,dx = \int \left(bx^2+a\right)^p x^6 \,dx$$

input
```
integrate(x^6*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^6, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int x^6 \left(a+bx^2\right)^p \,dx = \frac{a^p x^7 {}_2F_1\left(\begin{matrix} \frac{7}{2}, -p \\ \frac{9}{2} \end{matrix}\middle| \frac{bx^2 e^{i\pi}}{a}\right)}{7}$$

input
```
integrate(x**6*(b*x**2+a)**p,x)
```

output
```
a**p*x**7*hyper((7/2, -p), (9/2,), b*x**2*exp_polar(I*pi)/a)/7
```

Maxima [F]

$$\int x^6 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^6, x)
```

Giac [F]

$$\int x^6 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^6 \, dx$$

input
```
integrate(x^6*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^6 (a + bx^2)^p \, dx = \int x^6 \left(b x^2 + a\right)^p \, dx$$

input
```
int(x^6*(a + b*x^2)^p,x)
```

output
```
int(x^6*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^6(a+bx^2)^p\,dx$$
$$=\frac{30(b\,x^2+a)^p\,a^3px-20(b\,x^2+a)^p\,a^2b\,p^2x^3-10(b\,x^2+a)^p\,a^2bp\,x^3+8(b\,x^2+a)^p\,a\,b^2p^3x^5+16(b\,x^2+a)}{}$$

input
```
int(x^6*(b*x^2+a)^p,x)
```

output
```
(30*(a + b*x**2)**p*a**3*p*x - 20*(a + b*x**2)**p*a**2*b*p**2*x**3 - 10*(a
 + b*x**2)**p*a**2*b*p*x**3 + 8*(a + b*x**2)**p*a*b**2*p**3*x**5 + 16*(a +
 b*x**2)**p*a*b**2*p**2*x**5 + 6*(a + b*x**2)**p*a*b**2*p*x**5 + 8*(a + b*
x**2)**p*b**3*p**3*x**7 + 36*(a + b*x**2)**p*b**3*p**2*x**7 + 46*(a + b*x*
*2)**p*b**3*p*x**7 + 15*(a + b*x**2)**p*b**3*x**7 - 480*int((a + b*x**2)**
p/(16*a*p**4 + 128*a*p**3 + 344*a*p**2 + 352*a*p + 105*a + 16*b*p**4*x**2
+ 128*b*p**3*x**2 + 344*b*p**2*x**2 + 352*b*p*x**2 + 105*b*x**2),x)*a**4*p
**5 - 3840*int((a + b*x**2)**p/(16*a*p**4 + 128*a*p**3 + 344*a*p**2 + 352*
a*p + 105*a + 16*b*p**4*x**2 + 128*b*p**3*x**2 + 344*b*p**2*x**2 + 352*b*p
*x**2 + 105*b*x**2),x)*a**4*p**4 - 10320*int((a + b*x**2)**p/(16*a*p**4 +
128*a*p**3 + 344*a*p**2 + 352*a*p + 105*a + 16*b*p**4*x**2 + 128*b*p**3*x*
*2 + 344*b*p**2*x**2 + 352*b*p*x**2 + 105*b*x**2),x)*a**4*p**3 - 10560*int
((a + b*x**2)**p/(16*a*p**4 + 128*a*p**3 + 344*a*p**2 + 352*a*p + 105*a +
16*b*p**4*x**2 + 128*b*p**3*x**2 + 344*b*p**2*x**2 + 352*b*p*x**2 + 105*b*
x**2),x)*a**4*p**2 - 3150*int((a + b*x**2)**p/(16*a*p**4 + 128*a*p**3 + 34
4*a*p**2 + 352*a*p + 105*a + 16*b*p**4*x**2 + 128*b*p**3*x**2 + 344*b*p**2
*x**2 + 352*b*p*x**2 + 105*b*x**2),x)*a**4*p)/(b**3*(16*p**4 + 128*p**3 +
344*p**2 + 352*p + 105))
```

3.1274 $\int x^4(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int x^4(a+bx^2)^p\,dx=\frac{1}{5}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

output
```
1/5*x^5*(b*x^2+a)^p*hypergeom([5/2, -p],[7/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^4(a+bx^2)^p\,dx=\frac{1}{5}x^5(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^4*(a + b*x^2)^p,x]
```

output
```
(x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 +
(b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a+bx^2)^p \, dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^4 \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{5} x^5 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{5}{2},-p,\frac{7}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^4*(a + b*x^2)^p,x]
```

output
```
(x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(5*(1 +
(b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{4}\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^4*(b*x^2+a)^p,x)
```

output
```
int(x^4*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^4 \left(a+bx^2\right)^p \,dx = \int \left(bx^2+a\right)^p x^4 \,dx$$

input
```
integrate(x^4*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^4, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int x^4 \left(a+bx^2\right)^p \,dx = \frac{a^p x^5 {}_2F_1\left(\left.\begin{matrix}\frac{5}{2}, -p \\ \frac{7}{2}\end{matrix}\right| \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

input
```
integrate(x**4*(b*x**2+a)**p,x)
```

output
```
a**p*x**5*hyper((5/2, -p), (7/2,), b*x**2*exp_polar(I*pi)/a)/5
```

Maxima [F]

$$\int x^4 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^4, x)
```

Giac [F]

$$\int x^4 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^4 \, dx$$

input
```
integrate(x^4*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4 (a + bx^2)^p \, dx = \int x^4 \left(b x^2 + a\right)^p \, dx$$

input
```
int(x^4*(a + b*x^2)^p,x)
```

output
```
int(x^4*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^4 (a+bx^2)^p \, dx$$
$$= \frac{-6(b\,x^2+a)^p\,a^2px+4(b\,x^2+a)^p\,ab\,p^2x^3+2(b\,x^2+a)^p\,abp\,x^3+4(b\,x^2+a)^p\,b^2p^2x^5+8(b\,x^2+a)^p\,b^2p\,x}{}$$

input
```
int(x^4*(b*x^2+a)^p,x)
```

output
```
( - 6*(a + b*x**2)**p*a**2*p*x + 4*(a + b*x**2)**p*a*b*p**2*x**3 + 2*(a +
b*x**2)**p*a*b*p*x**3 + 4*(a + b*x**2)**p*b**2*p**2*x**5 + 8*(a + b*x**2)*
*p*b**2*p*x**5 + 3*(a + b*x**2)**p*b**2*x**5 + 48*int((a + b*x**2)**p/(8*a
*p**3 + 36*a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*
p*x**2 + 15*b*x**2),x)*a**3*p**4 + 216*int((a + b*x**2)**p/(8*a*p**3 + 36*
a*p**2 + 46*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15
*b*x**2),x)*a**3*p**3 + 276*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46
*a*p + 15*a + 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)
*a**3*p**2 + 90*int((a + b*x**2)**p/(8*a*p**3 + 36*a*p**2 + 46*a*p + 15*a
+ 8*b*p**3*x**2 + 36*b*p**2*x**2 + 46*b*p*x**2 + 15*b*x**2),x)*a**3*p)/(b*
*2*(8*p**3 + 36*p**2 + 46*p + 15))
```

3.1275 $\int x^2(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 49

$$\int x^2(a+bx^2)^p\,dx=\frac{1}{3}x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

output
```
1/3*x^3*(b*x^2+a)^p*hypergeom([3/2, -p],[5/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^2(a+bx^2)^p\,dx=\frac{1}{3}x^3(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^2*(a + b*x^2)^p,x]
```

output
```
(x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 +
(b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2)^p \, dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^2 \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{1}{3} x^3 (a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{2},-p,\frac{5}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[x^2*(a + b*x^2)^p,x]
```

output
```
(x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -((b*x^2)/a)])/(3*(1 +
(b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2} \left(b \,x^{2}+a \right)^{p}d x$$

input
```
int(x^2*(b*x^2+a)^p,x)
```

output
```
int(x^2*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^2 \left(a + b x^2\right)^p \, dx = \int \left(b x^2 + a\right)^p x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.53

$$\int x^2 \left(a + b x^2\right)^p \, dx = \frac{a^p x^3 {}_{2}F_{1}\left(\begin{matrix} \frac{3}{2}, -p \\ \frac{5}{2} \end{matrix}\middle| {\frac{b x^{2} e^{i \pi}}{a}} \right)}{3}$$

input
```
integrate(x**2*(b*x**2+a)**p,x)
```

output
```
a**p*x**3*hyper((3/2, -p), (5/2,), b*x**2*exp_polar(I*pi)/a)/3
```

Maxima [F]

$$\int x^2 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^2, x)
```

Giac [F]

$$\int x^2 (a + bx^2)^p \, dx = \int (bx^2 + a)^p x^2 \, dx$$

input
```
integrate(x^2*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 (a + bx^2)^p \, dx = \int x^2 \left(b\,x^2 + a\right)^p \, dx$$

input
```
int(x^2*(a + b*x^2)^p,x)
```

output
```
int(x^2*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^2(a+bx^2)^p\,dx$$
$$= \frac{2(b\,x^2+a)^p\,apx+2(b\,x^2+a)^p\,bp\,x^3+(b\,x^2+a)^p\,b\,x^3-8\left(\int \frac{(b\,x^2+a)^p}{4b\,p^2x^2+8bp\,x^2+4a\,p^2+3b\,x^2+8ap+3a}dx\right)a^2p^3-16}{b\,(4p^2+8p+3)}$$

input
```
int(x^2*(b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2)**p*a*p*x + 2*(a + b*x**2)**p*b*p*x**3 + (a + b*x**2)**p*b*
x**3 - 8*int((a + b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b
*p*x**2 + 3*b*x**2),x)*a**2*p**3 - 16*int((a + b*x**2)**p/(4*a*p**2 + 8*a*
p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**2),x)*a**2*p**2 - 6*int((a +
 b*x**2)**p/(4*a*p**2 + 8*a*p + 3*a + 4*b*p**2*x**2 + 8*b*p*x**2 + 3*b*x**
2),x)*a**2*p)/(b*(4*p**2 + 8*p + 3))
```

3.1276 $\int (a+bx^2)^p \, dx$

Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

output
```
x*(b*x^2+a)^p*hypergeom([1/2, -p],[3/2],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (a+bx^2)^p \, dx = x(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Integrate[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {238, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^p\,dx$$

$$\Big\downarrow \ 238$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \ 237$$

$$x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},-\frac{bx^2}{a}\right)$$

input
```
Int[(a + b*x^2)^p,x]
```

output
```
(x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(1 + (b*x^
2)/a)^p
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 238
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^2)
^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(1 + b*(x^2/a))^p, x], x] /
; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p] &&  !GtQ[a, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int((b*x^2+a)^p,x)
```

output
```
int((b*x^2+a)^p,x)
```

Fricas [F]

$$\int \left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p\,dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \left(a+bx^2\right)^p\,dx=a^px\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},-p\\ \frac{3}{2}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)$$

input
```
integrate((b*x**2+a)**p,x)
```

output
```
a**p*x*hyper((1/2, -p), (3/2,), b*x**2*exp_polar(I*pi)/a)
```

Maxima [F]

$$\int \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p \, dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p, x)
```

Giac [F]

$$\int \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p \, dx$$

input
```
integrate((b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p, x)
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)^p \, dx = \frac{x\left(b\,x^2+a\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^p}$$

input
```
int((a + b*x^2)^p,x)
```

output
```
(x*(a + b*x^2)^p*hypergeom([1/2, -p], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^p
```

Reduce [F]

$$\int \left(a+bx^2\right)^p\,dx$$
$$=\frac{\left(b\,x^2+a\right)^p x+4\left(\int \frac{\left(b\,x^2+a\right)^p}{2bp\,x^2+b\,x^2+2ap+a}dx\right)a\,p^2+2\left(\int \frac{\left(b\,x^2+a\right)^p}{2bp\,x^2+b\,x^2+2ap+a}dx\right)ap}{2p+1}$$

input
```
int((b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*x + 4*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**
2),x)*a*p**2 + 2*int((a + b*x**2)**p/(2*a*p + a + 2*b*p*x**2 + b*x**2),x)*
a*p)/(2*p + 1)
```

3.1277 $\int \frac{(a+bx^2)^p}{x^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 47

$$\int \frac{(a+bx^2)^p}{x^2} dx = -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

output
```
-(b*x^2+a)^p*hypergeom([-1/2, -p],[1/2],-b*x^2/a)/x/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x^2} dx = -\frac{(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Integrate[(a + b*x^2)^p/x^2,x]
```

output
```
-(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (
b*x^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^2}dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^2}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{2},-p,\frac{1}{2},-\frac{bx^2}{a}\right)}{x}$$

input
```
Int[(a + b*x^2)^p/x^2,x]
```

output
```
-(((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)/a)])/(x*(1 + (
b*x^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^2}dx$$

input
```
int((b*x^2+a)^p/x^2,x)
```

output
```
int((b*x^2+a)^p/x^2,x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^2}\,dx = \int \frac{(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^2, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \frac{(a+bx^2)^p}{x^2}\,dx = -\frac{a^p {}_2F_1\left(\begin{matrix} -\frac{1}{2}, -p \\ \frac{1}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{x}$$

input
```
integrate((b*x**2+a)**p/x**2,x)
```

output
```
-a**p*hyper((-1/2, -p), (1/2,), b*x**2*exp_polar(I*pi)/a)/x
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^2}\,dx=\int \frac{(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^2, x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^2}\,dx=\int \frac{(bx^2+a)^p}{x^2}\,dx$$

input
```
integrate((b*x^2+a)^p/x^2,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.23

$$\int \frac{(a+bx^2)^p}{x^2}\,dx=\frac{(b\,x^2+a)^p\,{}_2\mathrm{F}_1\left(\frac{1}{2}-p,-p;\ \frac{3}{2}-p;\ -\frac{a}{b\,x^2}\right)}{x\,\left(2\,p-1\right)\,{\left(\frac{a}{b\,x^2}+1\right)}^p}$$

input
```
int((a + b*x^2)^p/x^2,x)
```

output
```
((a + b*x^2)^p*hypergeom([1/2 - p, -p], 3/2 - p, -a/(b*x^2)))/(x*(2*p - 1)
*(a/(b*x^2) + 1)^p)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x^2}\,dx = \frac{(b\,x^2+a)^p + 4\left(\int \frac{(b\,x^2+a)^p}{2bp\,x^4-b\,x^4+2ap\,x^2-a\,x^2}dx\right) a\,p^2 x - 2\left(\int \frac{(b\,x^2+a)^p}{2bp\,x^4-b\,x^4+2ap\,x^2-a\,x^2}dx\right) apx}{x\,(2p-1)}$$

input
```
int((b*x^2+a)^p/x^2,x)
```

output
```
((a + b*x**2)**p + 4*int((a + b*x**2)**p/(2*a*p*x**2 - a*x**2 + 2*b*p*x**4
 - b*x**4),x)*a*p**2*x - 2*int((a + b*x**2)**p/(2*a*p*x**2 - a*x**2 + 2*b*
p*x**4 - b*x**4),x)*a*p*x)/(x*(2*p - 1))
```

3.1278 $\int x^{7/2}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{7/2}(a+bx^2)^p\,dx=\frac{2}{9}x^{9/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^2}{a}\right)$$

output
```
2/9*x^(9/2)*(b*x^2+a)^p*hypergeom([9/4, -p],[13/4],-b*x^2/a)/((1+b*x^2/a)^
p)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^{7/2}(a+bx^2)^p\,dx=\frac{2}{9}x^{9/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^(7/2)*(a + b*x^2)^p,x]
```

output
```
(2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(
9*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{7/2}\left(\frac{bx^2}{a}+1\right)^p\,dx$$

$$\Big\downarrow \text{278}$$

$$\frac{2}{9}x^{9/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{9}{4},-p,\frac{13}{4},-\frac{bx^2}{a}\right)$$

input
```
Int[x^(7/2)*(a + b*x^2)^p,x]
```

output

```
(2*x^(9/2)*(a + b*x^2)^p*Hypergeometric2F1[9/4, -p, 13/4, -((b*x^2)/a)])/(
9*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{\frac{7}{2}} \left(b\,x^{2}+a\right)^{p}d x$$

input
```
int(x^(7/2)*(b*x^2+a)^p,x)
```

output
```
int(x^(7/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{7/2} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p x^{\frac{7}{2}} \, dx$$

input
```
integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{7/2}(a+bx^2)^p\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{7/2}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(7/2), x)
```

Giac [F]

$$\int x^{7/2}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{\frac{7}{2}}\,dx$$

input
```
integrate(x^(7/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(a+bx^2)^p\,dx=\int x^{7/2}\,(b\,x^2+a)^p\,dx$$

input
```
int(x^(7/2)*(a + b*x^2)^p,x)
```

output
```
int(x^(7/2)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{7/2}(a+bx^2)^p\,dx=\frac{-40\sqrt{x}\,(b\,x^2+a)^p\,a^2p+32\sqrt{x}\,(b\,x^2+a)^p\,ab\,p^2x^2+8\sqrt{x}\,(b\,x^2+a)^p\,abp\,x^2+32\sqrt{x}\,(b\,x^2+a)}{}$$

input
```
int(x^(7/2)*(b*x^2+a)^p,x)
```

output
```
(2*( - 20*sqrt(x)*(a + b*x**2)**p*a**2*p + 16*sqrt(x)*(a + b*x**2)**p*a*b*
p**2*x**2 + 4*sqrt(x)*(a + b*x**2)**p*a*b*p*x**2 + 16*sqrt(x)*(a + b*x**2)
**p*b**2*p**2*x**4 + 24*sqrt(x)*(a + b*x**2)**p*b**2*p*x**4 + 5*sqrt(x)*(a
 + b*x**2)**p*b**2*x**4 + 640*int((sqrt(x)*(a + b*x**2)**p)/(64*a*p**3*x +
 240*a*p**2*x + 236*a*p*x + 45*a*x + 64*b*p**3*x**3 + 240*b*p**2*x**3 + 23
6*b*p*x**3 + 45*b*x**3),x)*a**3*p**4 + 2400*int((sqrt(x)*(a + b*x**2)**p)/
(64*a*p**3*x + 240*a*p**2*x + 236*a*p*x + 45*a*x + 64*b*p**3*x**3 + 240*b*
p**2*x**3 + 236*b*p*x**3 + 45*b*x**3),x)*a**3*p**3 + 2360*int((sqrt(x)*(a
+ b*x**2)**p)/(64*a*p**3*x + 240*a*p**2*x + 236*a*p*x + 45*a*x + 64*b*p**3
*x**3 + 240*b*p**2*x**3 + 236*b*p*x**3 + 45*b*x**3),x)*a**3*p**2 + 450*int
((sqrt(x)*(a + b*x**2)**p)/(64*a*p**3*x + 240*a*p**2*x + 236*a*p*x + 45*a*
x + 64*b*p**3*x**3 + 240*b*p**2*x**3 + 236*b*p*x**3 + 45*b*x**3),x)*a**3*p
))/(b**2*(64*p**3 + 240*p**2 + 236*p + 45))
```

3.1279 $\int x^{5/2}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{5/2}(a+bx^2)^p\,dx=\frac{2}{7}x^{7/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^2}{a}\right)$$

output
```
2/7*x^(7/2)*(b*x^2+a)^p*hypergeom([7/4, -p],[11/4],-b*x^2/a)/((1+b*x^2/a)^
p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^{5/2}(a+bx^2)^p\,dx=\frac{2}{7}x^{7/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^(5/2)*(a + b*x^2)^p,x]
```

output
```
(2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(
7*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2} \left(a+b x^2\right)^p \, dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+b x^2\right)^p \left(\frac{b x^2}{a}+1\right)^{-p} \int x^{5/2} \left(\frac{b x^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{2}{7} x^{7/2} \left(a+b x^2\right)^p \left(\frac{b x^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{7}{4},-p,\frac{11}{4},-\frac{b x^2}{a}\right)$$

input
```
Int[x^(5/2)*(a + b*x^2)^p,x]
```

output

```
(2*x^(7/2)*(a + b*x^2)^p*Hypergeometric2F1[7/4, -p, 11/4, -((b*x^2)/a)])/(
7*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{\frac{5}{2}} \left(b\,x^{2}+a\right)^{p}d x$$

input
```
int(x^(5/2)*(b*x^2+a)^p,x)
```

output
```
int(x^(5/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{5/2} \left(a+bx^2\right)^p \, dx = \int { \left(bx^{2} + a\right)}^{p} x^{\frac{5}{2}} \, dx$$

input
```
integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int x^{5/2}\left(a+bx^2\right)^p\,dx=\text{Timed out}$$

input
```
integrate(x**(5/2)*(b*x**2+a)**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^{5/2}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^p x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(5/2), x)
```

Giac [F]

$$\int x^{5/2}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^p x^{\frac{5}{2}}\,dx$$

input
```
integrate(x^(5/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(a+bx^2)^p\,dx=\int x^{5/2}\,(b\,x^2+a)^p\,dx$$

input
```
int(x^(5/2)*(a + b*x^2)^p,x)
```

output
```
int(x^(5/2)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{5/2}(a+bx^2)^p\,dx=\frac{8\sqrt{x}\,(b\,x^2+a)^p\,apx+8\sqrt{x}\,(b\,x^2+a)^p\,bp\,x^3+6\sqrt{x}\,(b\,x^2+a)^p\,b\,x^3-192\left(\int\frac{\sqrt{x}\,(b\,x}{16b\,p^2x^2+40bp\,x^2+16a}\right.}{}$$

input
```
int(x^(5/2)*(b*x^2+a)^p,x)
```

output
```
(2*(4*sqrt(x)*(a + b*x**2)**p*a*p*x + 4*sqrt(x)*(a + b*x**2)**p*b*p*x**3 +
 3*sqrt(x)*(a + b*x**2)**p*b*x**3 - 96*int((sqrt(x)*(a + b*x**2)**p)/(16*a
*p**2 + 40*a*p + 21*a + 16*b*p**2*x**2 + 40*b*p*x**2 + 21*b*x**2),x)*a**2*
p**3 - 240*int((sqrt(x)*(a + b*x**2)**p)/(16*a*p**2 + 40*a*p + 21*a + 16*b
*p**2*x**2 + 40*b*p*x**2 + 21*b*x**2),x)*a**2*p**2 - 126*int((sqrt(x)*(a +
 b*x**2)**p)/(16*a*p**2 + 40*a*p + 21*a + 16*b*p**2*x**2 + 40*b*p*x**2 + 2
1*b*x**2),x)*a**2*p))/(b*(16*p**2 + 40*p + 21))
```

3.1280 $\int x^{3/2}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int x^{3/2}(a+bx^2)^p\,dx=\frac{2}{5}x^{5/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^2}{a}\right)$$

output
```
2/5*x^(5/2)*(b*x^2+a)^p*hypergeom([5/4, -p],[9/4],-b*x^2/a)/((1+b*x^2/a)^p
)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int x^{3/2}(a+bx^2)^p\,dx=\frac{2}{5}x^{5/2}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[x^(3/2)*(a + b*x^2)^p,x]
```

output
```
(2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{279}$$
$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{3/2}\left(\frac{bx^2}{a}+1\right)^p dx$$
$$\Big\downarrow \text{278}$$
$$\frac{2}{5}x^{5/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{5}{4},-p,\frac{9}{4},-\frac{bx^2}{a}\right)$$

input
```
Int[x^(3/2)*(a + b*x^2)^p,x]
```

output

```
(2*x^(5/2)*(a + b*x^2)^p*Hypergeometric2F1[5/4, -p, 9/4, -((b*x^2)/a)])/(5
*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{\frac{3}{2}} \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^(3/2)*(b*x^2+a)^p,x)
```

output
```
int(x^(3/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{3/2} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p x^{\frac{3}{2}} \, dx$$

input
```
integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 162.50 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int x^{3/2}(a+bx^2)^p\,dx=\frac{a^px^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},-p\\\frac{9}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(3/2)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(5/2)*gamma(5/4)*hyper((5/4, -p), (9/4,), b*x**2*exp_polar(I*pi)/a
)/(2*gamma(9/4))
```

Maxima [F]

$$\int x^{3/2}(a+bx^2)^p\,dx=\int (bx^2+a)^px^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(3/2), x)
```

Giac [F]

$$\int x^{3/2}(a+bx^2)^p\,dx=\int (bx^2+a)^px^{\frac{3}{2}}\,dx$$

input
```
integrate(x^(3/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+bx^2\right)^p\,dx=\int x^{3/2}\,\left(b\,x^2+a\right)^p\,dx$$

input
```
int(x^(3/2)*(a + b*x^2)^p,x)
```

output
```
int(x^(3/2)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{3/2}(a+bx^2)^p\,dx=\frac{8\sqrt{x}\,(b\,x^2+a)^p\,ap+8\sqrt{x}\,(b\,x^2+a)^p\,bp\,x^2+2\sqrt{x}\,(b\,x^2+a)^p\,b\,x^2-64\left(\int\frac{\sqrt{x}\,(b\,x^2+}{16b\,p^2x^3+24bp\,x^3+16a\,p^2}\right.}{}$$

input
```
int(x^(3/2)*(b*x^2+a)^p,x)
```

output
```
(2*(4*sqrt(x)*(a + b*x**2)**p*a*p + 4*sqrt(x)*(a + b*x**2)**p*b*p*x**2 + s
qrt(x)*(a + b*x**2)**p*b*x**2 - 32*int((sqrt(x)*(a + b*x**2)**p)/(16*a*p**
2*x + 24*a*p*x + 5*a*x + 16*b*p**2*x**3 + 24*b*p*x**3 + 5*b*x**3),x)*a**2*
p**3 - 48*int((sqrt(x)*(a + b*x**2)**p)/(16*a*p**2*x + 24*a*p*x + 5*a*x +
16*b*p**2*x**3 + 24*b*p*x**3 + 5*b*x**3),x)*a**2*p**2 - 10*int((sqrt(x)*(a
 + b*x**2)**p)/(16*a*p**2*x + 24*a*p*x + 5*a*x + 16*b*p**2*x**3 + 24*b*p*x
**3 + 5*b*x**3),x)*a**2*p))/(b*(16*p**2 + 24*p + 5))
```

3.1281 $\int \sqrt{x}(a+bx^2)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \sqrt{x}(a+bx^2)^p \, dx = \frac{2}{3}x^{3/2}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^2}{a}\right)$$

output
```
2/3*x^(3/2)*(b*x^2+a)^p*hypergeom([3/4, -p],[7/4],-b*x^2/a)/((1+b*x^2/a)^p
)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \sqrt{x}(a+bx^2)^p \, dx = \frac{2}{3}x^{3/2}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[Sqrt[x]*(a + b*x^2)^p,x]
```

output
```
(2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \sqrt{x}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{2}{3}x^{3/2}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{4},-p,\frac{7}{4},-\frac{bx^2}{a}\right)$$

input
```
Int[Sqrt[x]*(a + b*x^2)^p,x]
```

output
```
(2*x^(3/2)*(a + b*x^2)^p*Hypergeometric2F1[3/4, -p, 7/4, -((b*x^2)/a)])/(3
*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \sqrt{x}\, \left(b\, x^{2}+a \right)^{p}d x$$

input
```
int(x^(1/2)*(b*x^2+a)^p,x)
```

output
```
int(x^(1/2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int \sqrt{x}(a+bx^2)^p\,dx = \int (bx^2+a)^p\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 23.45 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \sqrt{x}(a+bx^2)^p\,dx = \frac{a^p x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},-p\\ \frac{7}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(x**(1/2)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(3/2)*gamma(3/4)*hyper((3/4, -p), (7/4,), b*x**2*exp_polar(I*pi)/a
)/(2*gamma(7/4))
```

Maxima [F]

$$\int \sqrt{x}(a+bx^2)^p\,dx = \int (bx^2+a)^p\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*sqrt(x), x)
```

Giac [F]

$$\int \sqrt{x}(a+bx^2)^p\,dx = \int (bx^2+a)^p\sqrt{x}\,dx$$

input
```
integrate(x^(1/2)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+bx^2)^p \, dx = \int \sqrt{x}\,(b\,x^2+a)^p \, dx$$

input
```
int(x^(1/2)*(a + b*x^2)^p,x)
```

output
```
int(x^(1/2)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int \sqrt{x}(a+bx^2)^p \, dx = \frac{2\sqrt{x}\,(b\,x^2+a)^p\,x+16\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^2+3b\,x^2+4ap+3a}dx\right)a\,p^2+12\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^2+3b\,x^2+4ap+3a}dx\right)ap}{4p+3}$$

input
```
int(x^(1/2)*(b*x^2+a)^p,x)
```

output
```
(2*(sqrt(x)*(a + b*x**2)**p*x + 8*int((sqrt(x)*(a + b*x**2)**p)/(4*a*p + 3
*a + 4*b*p*x**2 + 3*b*x**2),x)*a*p**2 + 6*int((sqrt(x)*(a + b*x**2)**p)/(4
*a*p + 3*a + 4*b*p*x**2 + 3*b*x**2),x)*a*p))/(4*p + 3)
```

3.1282 $\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = 2\sqrt{x}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^2}{a}\right)$$

output
```
2*x^(1/2)*(b*x^2+a)^p*hypergeom([1/4, -p],[5/4],-b*x^2/a)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = 2\sqrt{x}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^2}{a}\right)$$

input
```
Integrate[(a + b*x^2)^p/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^p
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int \frac{\left(\frac{bx^2}{a}+1\right)^p}{\sqrt{x}}dx$$

$$\Big\downarrow \text{278}$$

$$2\sqrt{x}(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{4},-p,\frac{5}{4},-\frac{bx^2}{a}\right)$$

input
```
Int[(a + b*x^2)^p/Sqrt[x],x]
```

output
```
(2*Sqrt[x]*(a + b*x^2)^p*Hypergeometric2F1[1/4, -p, 5/4, -((b*x^2)/a)])/(1
 + (b*x^2)/a)^p
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{\sqrt{x}}dx$$

input
```
int((b*x^2+a)^p/x^(1/2),x)
```

output
```
int((b*x^2+a)^p/x^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = \int \frac{(bx^2+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/sqrt(x), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 14.99 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = \frac{a^p\sqrt{x}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},-p\\ \frac{5}{4}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((b*x**2+a)**p/x**(1/2),x)
```

output
```
a**p*sqrt(x)*gamma(1/4)*hyper((1/4, -p), (5/4,), b*x**2*exp_polar(I*pi)/a)
/(2*gamma(5/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = \int \frac{(bx^2+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/sqrt(x), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = \int \frac{(bx^2+a)^p}{\sqrt{x}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx = \int \frac{(b\,x^2+a)^p}{\sqrt{x}}\,dx$$

input
```
int((a + b*x^2)^p/x^(1/2),x)
```

output
```
int((a + b*x^2)^p/x^(1/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{\sqrt{x}}\,dx$$
$$= \frac{2\sqrt{x}\,(b\,x^2+a)^p + 16\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^3+b\,x^3+4apx+ax}dx\right)a\,p^2 + 4\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^3+b\,x^3+4apx+ax}dx\right)ap}{4p+1}$$

input
```
int((b*x^2+a)^p/x^(1/2),x)
```

output
```
(2*(sqrt(x)*(a + b*x**2)**p + 8*int((sqrt(x)*(a + b*x**2)**p)/(4*a*p*x + a
*x + 4*b*p*x**3 + b*x**3),x)*a*p**2 + 2*int((sqrt(x)*(a + b*x**2)**p)/(4*a
*p*x + a*x + 4*b*p*x**3 + b*x**3),x)*a*p))/(4*p + 1)
```

3.1283 $\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

output
```
-2*(b*x^2+a)^p*hypergeom([-1/4, -p],[3/4],-b*x^2/a)/x^(1/2)/((1+b*x^2/a)^p
)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

input
```
Integrate[(a + b*x^2)^p/x^(3/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]
*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^{3/2}}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{1}{4},-p,\frac{3}{4},-\frac{bx^2}{a}\right)}{\sqrt{x}}$$

input
```
Int[(a + b*x^2)^p/x^(3/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-1/4, -p, 3/4, -((b*x^2)/a)])/(Sqrt[x]
*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{\left(b\,x^{2}+a\right)^{p}}{x^{\frac{3}{2}}}dx$$

input
```
int((b*x^2+a)^p/x^(3/2),x)
```

output
```
int((b*x^2+a)^p/x^(3/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^(3/2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 79.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx = \frac{a^p\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-p\\ \frac{3}{4}\end{matrix}\right|\frac{bx^2e^{i\pi}}{a}\right)}{2\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate((b*x**2+a)**p/x**(3/2),x)
```

output
```
a**p*gamma(-1/4)*hyper((-1/4, -p), (3/4,), b*x**2*exp_polar(I*pi)/a)/(2*sq
rt(x)*gamma(3/4))
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx=\int \frac{(bx^2+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx=\int \frac{(bx^2+a)^p}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx=\int \frac{(b\,x^2+a)^p}{x^{3/2}}\,dx$$

input
```
int((a + b*x^2)^p/x^(3/2),x)
```

output
```
int((a + b*x^2)^p/x^(3/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x^{3/2}}\,dx=\frac{2(b\,x^2+a)^p+16\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^4-b\,x^4+4ap\,x^2-a\,x^2}dx\right)a\,p^2-4\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^4-b\,x^4+4ap\,x^2-a\,x^2}dx\right)}{\sqrt{x}\,(4p-1)}$$

input
```
int((b*x^2+a)^p/x^(3/2),x)
```

output
```
(2*((a + b*x**2)**p + 8*sqrt(x)*int((sqrt(x)*(a + b*x**2)**p)/(4*a*p*x**2
- a*x**2 + 4*b*p*x**4 - b*x**4),x)*a*p**2 - 2*sqrt(x)*int((sqrt(x)*(a + b*
x**2)**p)/(4*a*p*x**2 - a*x**2 + 4*b*p*x**4 - b*x**4),x)*a*p))/(sqrt(x)*(4
*p - 1))
```

3.1284 $\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{4},-p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

output
```
-2/3*(b*x^2+a)^p*hypergeom([-3/4, -p],[1/4],-b*x^2/a)/x^(3/2)/((1+b*x^2/a)
^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{4},-p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

input
```
Integrate[(a + b*x^2)^p/x^(5/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/
2)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^{5/2}}dx$$
$$\Big\downarrow \text{279}$$
$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^{5/2}}dx$$
$$\Big\downarrow \text{278}$$
$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{4},-p,\frac{1}{4},-\frac{bx^2}{a}\right)}{3x^{3/2}}$$

input
```
Int[(a + b*x^2)^p/x^(5/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-3/4, -p, 1/4, -((b*x^2)/a)])/(3*x^(3/
2)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{5}{2}}}dx$$

input
```
int((b*x^2+a)^p/x^(5/2),x)
```

output
```
int((b*x^2+a)^p/x^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx=\int \frac{(bx^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^(5/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = \int \frac{(b\,x^2+a)^p}{x^{5/2}}\,dx$$

input
```
int((a + b*x^2)^p/x^(5/2),x)
```

output
```
int((a + b*x^2)^p/x^(5/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x^{5/2}}\,dx = \frac{2(b\,x^2+a)^p+16\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^5-3b\,x^5+4ap\,x^3-3a\,x^3}dx\right)a\,p^2x-12\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^5-3b\,x^5+4ap\,x^3-3a}\right.}{\sqrt{x}\,x\,(4p-3)}$$

input
```
int((b*x^2+a)^p/x^(5/2),x)
```

output
```
(2*((a + b*x**2)**p + 8*sqrt(x)*int((sqrt(x)*(a + b*x**2)**p)/(4*a*p*x**3
- 3*a*x**3 + 4*b*p*x**5 - 3*b*x**5),x)*a*p**2*x - 6*sqrt(x)*int((sqrt(x)*(
a + b*x**2)**p)/(4*a*p*x**3 - 3*a*x**3 + 4*b*p*x**5 - 3*b*x**5),x)*a*p*x))
/(sqrt(x)*x*(4*p - 3))
```

3.1285 $\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{5}{4},-p,-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

output
```
-2/5*(b*x^2+a)^p*hypergeom([-5/4, -p],[-1/4],-b*x^2/a)/x^(5/2)/((1+b*x^2/a
)^p)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = -\frac{2(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{5}{4},-p,-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

input
```
Integrate[(a + b*x^2)^p/x^(7/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5
/2)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{279}$$

$$(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int \frac{\left(\frac{bx^2}{a}+1\right)^p}{x^{7/2}}dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{2(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{5}{4},-p,-\frac{1}{4},-\frac{bx^2}{a}\right)}{5x^{5/2}}$$

input
```
Int[(a + b*x^2)^p/x^(7/2),x]
```

output
```
(-2*(a + b*x^2)^p*Hypergeometric2F1[-5/4, -p, -1/4, -((b*x^2)/a)])/(5*x^(5
/2)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \frac{(b\,x^2+a)^p}{x^{\frac{7}{2}}}dx$$

input
```
int((b*x^2+a)^p/x^(7/2),x)
```

output
```
int((b*x^2+a)^p/x^(7/2),x)
```

Fricas [F]

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(7/2),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^(7/2), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate((b*x**2+a)**p/x**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \int \frac{(bx^2+a)^p}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((b*x^2+a)^p/x^(7/2),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \int \frac{(b\,x^2+a)^p}{x^{7/2}}\,dx$$

input
```
int((a + b*x^2)^p/x^(7/2),x)
```

output
```
int((a + b*x^2)^p/x^(7/2), x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^p}{x^{7/2}}\,dx = \frac{2(b\,x^2+a)^p + 16\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^6-5b\,x^6+4ap\,x^4-5a\,x^4}dx\right)a\,p^2x^2 - 20\sqrt{x}\left(\int \frac{\sqrt{x}\,(b\,x^2+a)^p}{4bp\,x^6-5b\,x^6+4ap\,x^4-5a}\right.}{\sqrt{x}\,x^2\,(4p-5)}$$

input
```
int((b*x^2+a)^p/x^(7/2),x)
```

output
```
(2*((a + b*x**2)**p + 8*sqrt(x)*int((sqrt(x)*(a + b*x**2)**p)/(4*a*p*x**4
- 5*a*x**4 + 4*b*p*x**6 - 5*b*x**6),x)*a*p**2*x**2 - 10*sqrt(x)*int((sqrt(
x)*(a + b*x**2)**p)/(4*a*p*x**4 - 5*a*x**4 + 4*b*p*x**6 - 5*b*x**6),x)*a*p
*x**2))/(sqrt(x)*x**2*(4*p - 5))
```

3.1286 $\int x^m(a+bx^2)^p\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int x^m(a+bx^2)^p\,dx = \frac{x^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{1+m}$$

output
```
x^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/(1+m)/
((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int x^m(a+bx^2)^p\,dx = \frac{x^{1+m}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{1+m}$$

input
```
Integrate[x^m*(a + b*x^2)^p,x]
```

output
```
(x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -
((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int x^m \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{m+1} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)}{m+1}$$

input
```
Int[x^m*(a + b*x^2)^p,x]
```

output
```
(x^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((b*
x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^m (b\,x^2+a)^p\,dx$$

input
```
int(x^m*(b*x^2+a)^p,x)
```

output
```
int(x^m*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^m \left(a+bx^2\right)^p\,dx = \int \left(bx^2+a\right)^p x^m\,dx$$

input
```
integrate(x^m*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int x^m (a+bx^2)^p \, dx = \frac{a^p x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate(x**m*(b*x**2+a)**p,x)
```

output
```
a**p*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int x^m (a+bx^2)^p \, dx = \int (bx^2+a)^p x^m \, dx$$

input
```
integrate(x^m*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^m, x)
```

Giac [F]

$$\int x^m (a+bx^2)^p \, dx = \int (bx^2+a)^p x^m \, dx$$

input
```
integrate(x^m*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m (a+bx^2)^p \, dx = \int x^m \left(b x^2 + a\right)^p dx$$

input
```
int(x^m*(a + b*x^2)^p,x)
```

output
```
int(x^m*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^m (a+bx^2)^p \, dx$$
$$= \frac{x^m (b x^2 + a)^p x + 2\left(\int \frac{x^m (b x^2+a)^p}{bm x^2+2bp x^2+b x^2+am+2ap+a} dx\right) amp + 4\left(\int \frac{x^m (b x^2+a)^p}{bm x^2+2bp x^2+b x^2+am+2ap+a} dx\right) a p^2 + 2\left(\int\right.}{m+2p+1}$$

input
```
int(x^m*(b*x^2+a)^p,x)
```

output
```
(x**m*(a + b*x**2)**p*x + 2*int((x**m*(a + b*x**2)**p)/(a*m + 2*a*p + a +
b*m*x**2 + 2*b*p*x**2 + b*x**2),x)*a*m*p + 4*int((x**m*(a + b*x**2)**p)/(a
*m + 2*a*p + a + b*m*x**2 + 2*b*p*x**2 + b*x**2),x)*a*p**2 + 2*int((x**m*(
a + b*x**2)**p)/(a*m + 2*a*p + a + b*m*x**2 + 2*b*p*x**2 + b*x**2),x)*a*p)
/(m + 2*p + 1)
```

3.1287 $\int (cx)^m \left(a+bx^2\right)^p \, dx$

Optimal result

Integrand size = 15, antiderivative size = 66

$$\int (cx)^m \left(a+bx^2\right)^p \, dx = \frac{(cx)^{1+m}\left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{bx^2}{a}\right)}{c(1+m)}$$

output
```
(c*x)^(1+m)*(b*x^2+a)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/c/
(1+m)/((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int (cx)^m \left(a+bx^2\right)^p \, dx = \frac{x(cx)^m \left(a+bx^2\right)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{1+m}$$

input
```
Integrate[(c*x)^m*(a + b*x^2)^p,x]
```

output
```
(x*(c*x)^m*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, 1 + (1 + m)/2, -
((b*x^2)/a)])/((1 + m)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx)^m \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 279}$$

$$\left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \int (cx)^m \left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{(cx)^{m+1} \left(a+bx^2\right)^p \left(\frac{bx^2}{a}+1\right)^{-p} \text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{bx^2}{a}\right)}{c(m+1)}$$

input
```
Int[(c*x)^m*(a + b*x^2)^p,x]
```

output
```
((c*x)^(1 + m)*(a + b*x^2)^p*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -
((b*x^2)/a)])/(c*(1 + m)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int (cx)^m \left(b\,x^2+a\right)^p dx$$

input
```
int((c*x)^m*(b*x^2+a)^p,x)
```

output
```
int((c*x)^m*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int (cx)^m \left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p(cx)^m\,dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*(c*x)^m, x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (cx)^m \left(a+bx^2\right)^p \, dx = \frac{a^p c^m x^{m+1} \Gamma\left(\frac{m}{2}+\frac{1}{2}\right) {}_2F_1\left(\begin{matrix} -p, \frac{m}{2}+\frac{1}{2} \\ \frac{m}{2}+\frac{3}{2} \end{matrix} \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}$$

input
```
integrate((c*x)**m*(b*x**2+a)**p,x)
```

output
```
a**p*c**m*x**(m + 1)*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,),
 b*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))
```

Maxima [F]

$$\int (cx)^m \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*(c*x)^m, x)
```

Giac [F]

$$\int (cx)^m \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p (cx)^m \, dx$$

input
```
integrate((c*x)^m*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*(c*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (cx)^m \left(a+bx^2\right)^p \,dx=\int (c\,x)^m \,\left(b\,x^2+a\right)^p \,dx$$

input
```
int((c*x)^m*(a + b*x^2)^p,x)
```

output
```
int((c*x)^m*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int (cx)^m \left(a+bx^2\right)^p \,dx$$
$$=\frac{c^m\left(x^m(b\,x^2+a)^p\,x+2\left(\int \frac{x^m\,(b\,x^2+a)^p}{bm\,x^2+2bp\,x^2+b\,x^2+am+2ap+a}dx\right)amp+4\left(\int \frac{x^m\,(b\,x^2+a)^p}{bm\,x^2+2bp\,x^2+b\,x^2+am+2ap+a}dx\right)a\,p^2+\right.}{m+2p+1}$$

input
```
int((c*x)^m*(b*x^2+a)^p,x)
```

output
```
(c**m*(x**m*(a + b*x**2)**p*x + 2*int((x**m*(a + b*x**2)**p)/(a*m + 2*a*p
+ a + b*m*x**2 + 2*b*p*x**2 + b*x**2),x)*a*m*p + 4*int((x**m*(a + b*x**2)*
*p)/(a*m + 2*a*p + a + b*m*x**2 + 2*b*p*x**2 + b*x**2),x)*a*p**2 + 2*int((
x**m*(a + b*x**2)**p)/(a*m + 2*a*p + a + b*m*x**2 + 2*b*p*x**2 + b*x**2),x
)*a*p))/(m + 2*p + 1)
```

3.1288 $\int x^{-7-2p}(a+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 105

$$\int x^{-7-2p}(a+bx^2)^p \, dx = -\frac{b^2x^{-2(1+p)}(a+bx^2)^{1+p}}{a^3(1+p)(2+p)(3+p)} + \frac{bx^{-2(2+p)}(a+bx^2)^{1+p}}{a^2(2+p)(3+p)} - \frac{x^{-2(3+p)}(a+bx^2)^{1+p}}{2a(3+p)}$$

output
```
-b^2*(b*x^2+a)^(p+1)/a^3/(p+1)/(2+p)/(3+p)/(x^(2*p+2))+b*(b*x^2+a)^(p+1)/a
^2/(2+p)/(3+p)/(x^(4+2*p))-1/2*(b*x^2+a)^(p+1)/a/(3+p)/(x^(6+2*p))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int x^{-7-2p}(a+bx^2)^p \, dx = -\frac{x^{-2(3+p)}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-3-p,-p,-2-p,-\frac{bx^2}{a}\right)}{2(3+p)}$$

input
```
Integrate[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*Hypergeometric2F1[-3 - p, -p, -2 - p, -((b*x^2)/a)])/(
(3 + p)*x^(2*(3 + p))*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-7}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{2b\int x^{-2p-5}\left(bx^2+a\right)^p\,dx}{a(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

$$\Big\downarrow \text{245}$$

$$-\frac{2b\left(-\frac{b\int x^{-2p-3}\left(bx^2+a\right)^p dx}{a(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}\right)}{a(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

$$\Big\downarrow \text{242}$$

$$-\frac{2b\left(\frac{bx^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(a+bx^2\right)^{p+1}}{2a(p+2)}\right)}{a(p+3)}-\frac{x^{-2(p+3)}\left(a+bx^2\right)^{p+1}}{2a(p+3)}$$

input
```
Int[x^(-7 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*(a + b*x^2)^(1 + p)/(a*(3 + p)*x^(2*(3 + p))) - (2*b*((b*(a + b*x^2)^
(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2)^(1 + p)/(2*a*
(2 + p)*x^(2*(2 + p)))))/(a*(3 + p))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{x^{-6-2p}\left(b\,x^{2}+a\right)^{p+1}\left(2b^{2}x^{4}-2abp\,x^{2}+a^{2}p^{2}-2ab\,x^{2}+3a^{2}p+2a^{2}\right)}{2a^{3}\left(2+p\right)\left(3+p\right)\left(p+1\right)}$	81
orering	$-\frac{\left(b\,x^{2}+a\right)x\left(2b^{2}x^{4}-2abp\,x^{2}+a^{2}p^{2}-2ab\,x^{2}+3a^{2}p+2a^{2}\right)x^{-7-2p}\left(b\,x^{2}+a\right)^{p}}{2\left(3+p\right)\left(2+p\right)\left(p+1\right)a^{3}}$	87

input
```
int(x^(-7-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(-6-2*p)/a^3/(2+p)/(3+p)/(p+1)*(b*x^2+a)^(p+1)*(2*b^2*x^4-2*a*b*p*x
^2+a^2*p^2-2*a*b*x^2+3*a^2*p+2*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int x^{-7-2p}\left(a+bx^{2}\right)^{p}\,dx = -\frac{\left(2\,b^{3}x^{7}-2\,ab^{2}px^{5}+\left(a^{2}bp^{2}+a^{2}bp\right)x^{3}+\left(a^{3}p^{2}+3\,a^{3}p+2\,a^{3}\right)x\right)\left(bx^{2}+a\right)^{p}x^{-2\,p-7}}{2\left(a^{3}p^{3}+6\,a^{3}p^{2}+11\,a^{3}p+6\,a^{3}\right)}$$

input
```
integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*x^7 - 2*a*b^2*p*x^5 + (a^2*b*p^2 + a^2*b*p)*x^3 + (a^3*p^2 + 3
*a^3*p + 2*a^3)*x)*(b*x^2 + a)^p*x^(-2*p - 7)/(a^3*p^3 + 6*a^3*p^2 + 11*a^
3*p + 6*a^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. 2(90) = 180.

Time = 9.04 (sec) , antiderivative size = 452, normalized size of antiderivative = 4.30

$$\int x^{-7-2p}(a+bx^2)^p\,dx = \frac{a^2a^pp^2x^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)} + \frac{3a^2a^ppx^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)} + \frac{2a^2a^px^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)} - \frac{2aa^pbpx^2x^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)} - \frac{2aa^pbx^2x^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)} + \frac{2a^pb^2x^4x^{-2p-6}\left(1+\frac{bx^2}{a}\right)^{p+3}\Gamma(-p-3)}{2a^2\Gamma(-p)+4abx^2\Gamma(-p)+2b^2x^4\Gamma(-p)}$$

input
```
integrate(x**(-7-2*p)*(b*x**2+a)**p,x)
```

output
```
a**2*a**p*p**2*x**(-2*p - 6)*(1 + b*x**2/a)**(p + 3)*gamma(-p - 3)/(2*a**2
*gamma(-p) + 4*a*b*x**2*gamma(-p) + 2*b**2*x**4*gamma(-p)) + 3*a**2*a**p*p
*x**(-2*p - 6)*(1 + b*x**2/a)**(p + 3)*gamma(-p - 3)/(2*a**2*gamma(-p) + 4
*a*b*x**2*gamma(-p) + 2*b**2*x**4*gamma(-p)) + 2*a**2*a**p*x**(-2*p - 6)*(
1 + b*x**2/a)**(p + 3)*gamma(-p - 3)/(2*a**2*gamma(-p) + 4*a*b*x**2*gamma(
-p) + 2*b**2*x**4*gamma(-p)) - 2*a*a**p*b*p*x**2*x**(-2*p - 6)*(1 + b*x**2
/a)**(p + 3)*gamma(-p - 3)/(2*a**2*gamma(-p) + 4*a*b*x**2*gamma(-p) + 2*b*
*2*x**4*gamma(-p)) - 2*a*a**p*b*x**2*x**(-2*p - 6)*(1 + b*x**2/a)**(p + 3)
*gamma(-p - 3)/(2*a**2*gamma(-p) + 4*a*b*x**2*gamma(-p) + 2*b**2*x**4*gamm
a(-p)) + 2*a**p*b**2*x**4*x**(-2*p - 6)*(1 + b*x**2/a)**(p + 3)*gamma(-p -
 3)/(2*a**2*gamma(-p) + 4*a*b*x**2*gamma(-p) + 2*b**2*x**4*gamma(-p))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.80

$$\int x^{-7-2p}(a+bx^2)^p\,dx = -\frac{(2\,b^3x^6-2\,ab^2px^4+(p^2+p)a^2bx^2+(p^2+3\,p+2)a^3)e^{(p\log(bx^2+a)-2\,p\log(x))}}{2\,(p^3+6\,p^2+11\,p+6)a^3x^6}$$

input
```
integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
-1/2*(2*b^3*x^6 - 2*a*b^2*p*x^4 + (p^2 + p)*a^2*b*x^2 + (p^2 + 3*p + 2)*a^
3)*e^(p*log(b*x^2 + a) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*a^3*x^6)
```

Giac [F]

$$\int x^{-7-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2\,p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 7), x)
```

Mupad [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.47

$$\int x^{-7-2p}(a+bx^2)^p\,dx = -(b\,x^2+a)^p\left(\frac{x\,(p^2+3\,p+2)}{2\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)} + \frac{b^3\,x^7}{a^3\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)} - \frac{b^2\,p\,x^5}{a^2\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)} + \frac{b\,p\,x^3\,(p+1)}{2\,a\,x^{2\,p+7}\,(p^3+6\,p^2+11\,p+6)}\right)$$

input
```
int((a + b*x^2)^p/x^(2*p + 7),x)
```

output
```
-(a + b*x^2)^p*((x*(3*p + p^2 + 2))/(2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6
)) + (b^3*x^7)/(a^3*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) - (b^2*p*x^5)/(a
^2*x^(2*p + 7)*(11*p + 6*p^2 + p^3 + 6)) + (b*p*x^3*(p + 1))/(2*a*x^(2*p +
 7)*(11*p + 6*p^2 + p^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^{-7-2p}(a+bx^2)^p\,dx = \frac{(b\,x^2+a)^p\,(-2b^3x^6+2a\,b^2p\,x^4-a^2b\,p^2x^2-a^2bp\,x^2-a^3p^2-3a^3p-2a^3)}{2x^{2p}a^3x^6\,(p^3+6p^2+11p+6)}$$

input
```
int(x^(-7-2*p)*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*( - a**3*p**2 - 3*a**3*p - 2*a**3 - a**2*b*p**2*x**2 - a*
*2*b*p*x**2 + 2*a*b**2*p*x**4 - 2*b**3*x**6))/(2*x**(2*p)*a**3*x**6*(p**3
 + 6*p**2 + 11*p + 6))
```

3.1289 $\int x^{-5-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 67

$$\int x^{-5-2p}(a+bx^2)^p\,dx=\frac{bx^{-2(1+p)}(a+bx^2)^{1+p}}{2a^2(1+p)(2+p)}-\frac{x^{-2(2+p)}(a+bx^2)^{1+p}}{2a(2+p)}$$

output
```
1/2*b*(b*x^2+a)^(p+1)/a^2/(p+1)/(2+p)/(x^(2*p+2))-1/2*(b*x^2+a)^(p+1)/a/(2
+p)/(x^(4+2*p))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int x^{-5-2p}(a+bx^2)^p\,dx = -\frac{x^{-2(2+p)}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-2-p,-p,-1-p,-\frac{bx^2}{a}\right)}{2(2+p)}$$

input
```
Integrate[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*Hypergeometric2F1[-2 - p, -p, -1 - p, -((b*x^2)/a)])/(
(2 + p)*x^(2*(2 + p))*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-5} \left(a+bx^2\right)^p \, dx$$

$$\Big\downarrow \text{245}$$

$$-\frac{b \int x^{-2p-3} \left(bx^2+a\right)^p dx}{a(p+2)}-\frac{x^{-2(p+2)} \left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

$$\Big\downarrow \text{242}$$

$$\frac{bx^{-2(p+1)} \left(a+bx^2\right)^{p+1}}{2a^2(p+1)(p+2)}-\frac{x^{-2(p+2)} \left(a+bx^2\right)^{p+1}}{2a(p+2)}$$

input
```
Int[x^(-5 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(b*(a + b*x^2)^(1 + p))/(2*a^2*(1 + p)*(2 + p)*x^(2*(1 + p))) - (a + b*x^2
)^(1 + p)/(2*a*(2 + p)*x^(2*(2 + p)))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{x^{-4-2p}\left(b\,x^{2}+a\right)^{p+1}\left(-b\,x^{2}+ap+a\right)}{2a^{2}\left(2+p\right)\left(p+1\right)}$	45
orering	$-\frac{\left(b\,x^{2}+a\right)^{p}x^{-5-2p}\left(-b\,x^{2}+ap+a\right)x\left(b\,x^{2}+a\right)}{2\left(2+p\right)\left(p+1\right)a^{2}}$	51

input
```
int(x^(-5-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(-4-2*p)/a^2/(2+p)/(p+1)*(b*x^2+a)^(p+1)*(-b*x^2+a*p+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x^{-5-2p}\left(a+bx^2\right)^p\,dx=\frac{(b^2x^5-abpx^3-(a^2p+a^2)x)(bx^2+a)^px^{-2\,p-5}}{2\,(a^2p^2+3\,a^2p+2\,a^2)}$$

input
```
integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
1/2*(b^2*x^5 - a*b*p*x^3 - (a^2*p + a^2)*x)*(b*x^2 + a)^p*x^(-2*p - 5)/(a^
2*p^2 + 3*a^2*p + 2*a^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(56) = 112.

Time = 8.75 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.42

$$\int x^{-5-2p}(a+bx^2)^p\,dx = -\frac{aa^p px^{-2p-4}\left(1+\frac{bx^2}{a}\right)^{p+2}\Gamma(-p-2)}{2a\Gamma(-p)+2bx^2\Gamma(-p)} - \frac{aa^p x^{-2p-4}\left(1+\frac{bx^2}{a}\right)^{p+2}\Gamma(-p-2)}{2a\Gamma(-p)+2bx^2\Gamma(-p)} + \frac{a^p bx^2x^{-2p-4}\left(1+\frac{bx^2}{a}\right)^{p+2}\Gamma(-p-2)}{2a\Gamma(-p)+2bx^2\Gamma(-p)}$$

input
```
integrate(x**(-5-2*p)*(b*x**2+a)**p,x)
```

output
```
-a*a**p*p*x**(-2*p - 4)*(1 + b*x**2/a)**(p + 2)*gamma(-p - 2)/(2*a*gamma(-
p) + 2*b*x**2*gamma(-p)) - a*a**p*x**(-2*p - 4)*(1 + b*x**2/a)**(p + 2)*ga
mma(-p - 2)/(2*a*gamma(-p) + 2*b*x**2*gamma(-p)) + a**p*b*x**2*x**(-2*p -
4)*(1 + b*x**2/a)**(p + 2)*gamma(-p - 2)/(2*a*gamma(-p) + 2*b*x**2*gamma(-
p))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int x^{-5-2p}(a+bx^2)^p\,dx = \frac{(b^2x^4 - abpx^2 - a^2(p+1))e^{(p\log(bx^2+a)-2p\log(x))}}{2(p^2+3p+2)a^2x^4}$$

input
```
integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
1/2*(b^2*x^4 - a*b*p*x^2 - a^2*(p + 1))*e^(p*log(b*x^2 + a) - 2*p*log(x))/
((p^2 + 3*p + 2)*a^2*x^4)
```

Giac [F]

$$\int x^{-5-2p}(a+bx^2)^p \, dx = \int (bx^2+a)^p x^{-2p-5} \, dx$$

input
```
integrate(x^(-5-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 5), x)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.43

$$\int x^{-5-2p}(a+bx^2)^p \, dx = -\left(b\,x^2+a\right)^p \left(\frac{x\,(p+1)}{2\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)} - \frac{b^2\,x^5}{2\,a^2\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)} + \frac{b\,p\,x^3}{2\,a\,x^{2\,p+5}\,\left(p^2+3\,p+2\right)}\right)$$

input
```
int((a + b*x^2)^p/x^(2*p + 5),x)
```

output
```
-(a + b*x^2)^p*((x*(p + 1))/(2*x^(2*p + 5)*(3*p + p^2 + 2)) - (b^2*x^5)/(2
*a^2*x^(2*p + 5)*(3*p + p^2 + 2)) + (b*p*x^3)/(2*a*x^(2*p + 5)*(3*p + p^2
+ 2)))
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int x^{-5-2p}\left(a+bx^2\right)^p\,dx=\frac{(b\,x^2+a)^p\,(b^2x^4-abp\,x^2-a^2p-a^2)}{2x^{2p}a^2x^4\,(p^2+3p+2)}$$

input
```
int(x^(-5-2*p)*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*( - a**2*p - a**2 - a*b*p*x**2 + b**2*x**4))/(2*x**(2*p)*
a**2*x**4*(p**2 + 3*p + 2))
```

3.1290 $\int x^{-3-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{x^{-2(1+p)}\left(a+bx^2\right)^{1+p}}{2a(1+p)}$$

output
```
-1/2*(b*x^2+a)^(p+1)/a/(p+1)/(x^(2*p+2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=\frac{x^{-2-2p}\left(a+bx^2\right)^{1+p}}{a(-2-2p)}$$

input
```
Integrate[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(-2 - 2*p)*(a + b*x^2)^(1 + p))/(a*(-2 - 2*p))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-3}\left(a+bx^2\right)^p\,dx$$
$$\Big\downarrow \text{242}$$
$$-\frac{x^{-2(p+1)}\left(a+bx^2\right)^{p+1}}{2a(p+1)}$$

input
```
Int[x^(-3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*(a + b*x^2)^(1 + p)/(a*(1 + p)*x^(2*(1 + p)))
```

Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
gosper	$-\frac{x^{-2p-2}\left(b\,x^{2}+a\right)^{p+1}}{2a(p+1)}$	29
orering	$-\frac{x\left(b\,x^{2}+a\right)x^{-3-2p}\left(b\,x^{2}+a\right)^{p}}{2a(p+1)}$	35

input
```
int(x^(-3-2*p)*(b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^(-2*p-2)/a/(p+1)*(b*x^2+a)^(p+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.13

$$\int x^{-3-2p}(a+bx^2)^p\,dx = -\frac{(bx^3+ax)(bx^2+a)^p x^{-2p-3}}{2(ap+a)}$$

input
```
integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
-1/2*(b*x^3 + a*x)*(b*x^2 + a)^p*x^(-2*p - 3)/(a*p + a)
```

Sympy [A] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^{-3-2p}(a+bx^2)^p\,dx = \frac{a^p x^{-2p-2}\left(1+\frac{bx^2}{a}\right)^{p+1}\Gamma(-p-1)}{2\Gamma(-p)}$$

input
```
integrate(x**(-3-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(-2*p - 2)*(1 + b*x**2/a)**(p + 1)*gamma(-p - 1)/(2*gamma(-p))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{(bx^2+a)e^{\left(p\log\left(bx^2+a\right)-2\,p\log(x)\right)}}{2\,a(p+1)x^2}$$

input
```
integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
-1/2*(b*x^2 + a)*e^(p*log(b*x^2 + a) - 2*p*log(x))/(a*(p + 1)*x^2)
```

Giac [F]

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^p x^{-2\,p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 3), x)
```

Mupad [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\left(b\,x^2+a\right)^p\left(\frac{x}{2\,x^{2\,p+3}\,(p+1)}+\frac{b\,x^3}{2\,a\,x^{2\,p+3}\,(p+1)}\right)$$

input
```
int((a + b*x^2)^p/x^(2*p + 3),x)
```

output
```
-(a + b*x^2)^p*(x/(2*x^(2*p + 3)*(p + 1)) + (b*x^3)/(2*a*x^(2*p + 3)*(p + 
1)))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int x^{-3-2p}\left(a+bx^2\right)^p\,dx=-\frac{\left(b\,x^2+a\right)^p\left(b\,x^2+a\right)}{2x^{2p}a\,x^2\,(p+1)}$$

input
```
int(x^(-3-2*p)*(b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2)**p*(a + b*x**2))/(2*x**(2*p)*a*x**2*(p + 1))
```

3.1291 $\int x^{-1-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int x^{-1-2p}(a+bx^2)^p\,dx = -\frac{x^{-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^2}{a}\right)}{2p}$$

output
```
-1/2*(b*x^2+a)^p*hypergeom([-p, -p],[1-p],-b*x^2/a)/p/(x^(2*p))/((1+b*x^2/
a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int x^{-1-2p}(a+bx^2)^p\,dx = -\frac{x^{-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^2}{a}\right)}{2p}$$

input
```
Integrate[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(p*x^(
2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-1}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{-2p-1}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,-p,1-p,-\frac{bx^2}{a}\right)}{2p}$$

input
```
Int[x^(-1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-1/2*((a + b*x^2)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^2)/a)])/(p*x^(
2*p)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-2p} \left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{-1-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^{2}+a\right)^{p} x^{-2\,p-1} \,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.76 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int x^{-1-2p}\left(a+bx^2\right)^p\,dx=\frac{a^px^{-2p}\Gamma(-p)\,{}_2F_1\left(\begin{matrix}-p,-p\\1-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(1-p\right)}$$

input
```
integrate(x**(-1-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*gamma(-p)*hyper((-p, -p), (1 - p,), b*x**2*exp_polar(I*pi)/a)/(2*x**(
2*p)*gamma(1 - p))
```

Maxima [F]

$$\int x^{-1-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^px^{-2\,p-1}\,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Giac [F]

$$\int x^{-1-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^px^{-2\,p-1}\,dx$$

input
```
integrate(x^(-1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-2p}\left(a+bx^2\right)^p\,dx=\int\frac{\left(b\,x^2+a\right)^p}{x^{2\,p+1}}\,dx$$

input
```
int((a + b*x^2)^p/x^(2*p + 1),x)
```

output
```
int((a + b*x^2)^p/x^(2*p + 1), x)
```

Reduce [F]

$$\int x^{-1-2p}\left(a+bx^2\right)^p\,dx=\int\frac{\left(b\,x^2+a\right)^p}{x^{2p}x}dx$$

input
```
int(x^(-1-2*p)*(b*x^2+a)^p,x)
```

output
```
int((a + b*x**2)**p/(x**(2*p)*x),x)
```

3.1292 $\int x^{1-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int x^{1-2p}(a+bx^2)^p\,dx = \frac{x^{2-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^2}{a}\right)}{2(1-p)}$$

output
```
1/2*x^(2-2*p)*(b*x^2+a)^p*hypergeom([-p, 1-p],[2-p],-b*x^2/a)/(1-p)/((1+b*
x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^{1-2p}(a+bx^2)^p\,dx = \frac{x^{2-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^2}{a}\right)}{2-2p}$$

input
```
Integrate[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a
)])/((2 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{1-2p}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{1-2p}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{2-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(1-p,-p,2-p,-\frac{bx^2}{a}\right)}{2(1-p)}$$

input
```
Int[x^(1 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(2 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b*x^2)/a
)])/(2*(1 - p)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{1-2p} \left(b \,x^{2}+a \right)^{p}d x$$

input
```
int(x^(1-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(1-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{1-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^{2} + a\right)^{p} x^{-2 \, p + 1} \,d x$$

input
```
integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.52 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^{1-2p}(a+bx^2)^p\,dx=\frac{a^px^{2-2p}\Gamma(1-p)\,{}_2F_1\left(\begin{matrix}-p,1-p\\2-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma(2-p)}$$

input
```
integrate(x**(1-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(2 - 2*p)*gamma(1 - p)*hyper((-p, 1 - p), (2 - p,), b*x**2*exp_pol
ar(I*pi)/a)/(2*gamma(2 - p))
```

Maxima [F]

$$\int x^{1-2p}(a+bx^2)^p\,dx=\int(bx^2+a)^px^{-2p+1}\,dx$$

input
```
integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Giac [F]

$$\int x^{1-2p}(a+bx^2)^p\,dx=\int(bx^2+a)^px^{-2p+1}\,dx$$

input
```
integrate(x^(1-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1-2p}\left(a+bx^2\right)^p\,dx=\int x^{1-2\,p}\,\left(b\,x^2+a\right)^p\,dx$$

input
```
int(x^(1 - 2*p)*(a + b*x^2)^p,x)
```

output
```
int(x^(1 - 2*p)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{1-2p}\left(a+bx^2\right)^p\,dx=\frac{\left(b\,x^2+a\right)^p x^2+2x^{2p}\left(\int \frac{\left(b\,x^2+a\right)^p x}{x^{2p}a+x^{2p}b\,x^2}dx\right)ap}{2x^{2p}}$$

input
```
int(x^(1-2*p)*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*x**2 + 2*x**(2*p)*int(((a + b*x**2)**p*x)/(x**(2*p)*a + x
**(2*p)*b*x**2),x)*a*p)/(2*x**(2*p))
```

3.1293 $\int x^{3-2p}(a+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 64

$$\int x^{3-2p}\left(a+bx^2\right)^p \, dx = \frac{x^{4-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^2}{a}\right)}{2(2-p)}$$

output
```
1/2*x^(4-2*p)*(b*x^2+a)^p*hypergeom([-p, 2-p],[3-p],-b*x^2/a)/(2-p)/((1+b*
x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x^{3-2p}\left(a+bx^2\right)^p \, dx = \frac{x^{4-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^2}{a}\right)}{4-2p}$$

input
```
Integrate[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a
)])/((4 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3-2p}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{3-2p}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 278}$$

$$\frac{x^{4-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(2-p,-p,3-p,-\frac{bx^2}{a}\right)}{2(2-p)}$$

input
```
Int[x^(3 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(4 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[2 - p, -p, 3 - p, -((b*x^2)/a
)])/(2*(2 - p)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{3-2p} \left(b \,x^{2}+a \right)^{p}d x$$

input
```
int(x^(3-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(3-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{3-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^{2} + a\right)^{p} x^{-2\,p + 3}\,{d x}$$

input
```
integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.61

$$\int x^{3-2p}\left(a+bx^2\right)^p\,dx=\frac{a^px^{4-2p}\Gamma(2-p)\,{}_2F_1\left(\begin{matrix}-p,2-p\\3-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(3-p\right)}$$

input
```
integrate(x**(3-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(4 - 2*p)*gamma(2 - p)*hyper((-p, 2 - p), (3 - p,), b*x**2*exp_pol
ar(I*pi)/a)/(2*gamma(3 - p))
```

Maxima [F]

$$\int x^{3-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^px^{-2p+3}\,dx$$

input
```
integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Giac [F]

$$\int x^{3-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^px^{-2p+3}\,dx$$

input
```
integrate(x^(3-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3-2p}(a+bx^2)^p\,dx=\int x^{3-2\,p}\,\left(b\,x^2+a\right)^p\,dx$$

input
```
int(x^(3 - 2*p)*(a + b*x^2)^p,x)
```

output
```
int(x^(3 - 2*p)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{3-2p}(a+bx^2)^p\,dx = \frac{(b\,x^2+a)^p\,ap\,x^2+(b\,x^2+a)^p\,b\,x^4+2x^{2p}\left(\int\frac{(b\,x^2+a)^p\,x}{x^{2p}a+x^{2p}b\,x^2}dx\right)a^2p^2-2x^{2p}\left(\int\frac{(b\,x^2+a)^p\,x}{x^{2p}a+x^{2p}b\,x^2}dx\right)a^2p}{4x^{2p}b}$$

input
```
int(x^(3-2*p)*(b*x^2+a)^p,x)
```

output
```
((a + b*x**2)**p*a*p*x**2 + (a + b*x**2)**p*b*x**4 + 2*x**(2*p)*int(((a +
b*x**2)**p*x)/(x**(2*p)*a + x**(2*p)*b*x**2),x)*a**2*p**2 - 2*x**(2*p)*int
(((a + b*x**2)**p*x)/(x**(2*p)*a + x**(2*p)*b*x**2),x)*a**2*p)/(4*x**(2*p)
*b)
```

3.1294 $\int x^{-6-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int x^{-6-2p}(a+bx^2)^p\,dx = -\frac{x^{-5-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-2p),-p,\frac{1}{2}(-3-2p),-\frac{bx^2}{a}\right)}{5+2p}$$

output
```
-x^(-5-2*p)*(b*x^2+a)^p*hypergeom([-p, -5/2-p],[-3/2-p],-b*x^2/a)/(5+2*p)/
((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^{-6-2p}(a+bx^2)^p\,dx = -\frac{x^{-5-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{5}{2}-p,-p,-\frac{3}{2}-p,-\frac{bx^2}{a}\right)}{5+2p}$$

input
```
Integrate[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-5/2 - p, -p, -3/2 - p, -(
(b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-6}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{-2(p+3)}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-2p-5}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p,\frac{1}{2}(-2p-3),-\frac{bx^2}{a}\right)}{2p+5}$$

input
```
Int[x^(-6 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-5 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-5 - 2*p)/2, -p, (-3 - 2*
p)/2, -((b*x^2)/a)])/((5 + 2*p)*(1 + (b*x^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-6-2p} \left(b\,x^2+a\right)^p dx$$

input
```
int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{-6-2p} \left(a+bx^2\right)^p \, dx = \int \left(bx^2+a\right)^p x^{-2\,p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int x^{-6-2p}(a+bx^2)^p\,dx = \frac{a^p x^{-2p-5}\Gamma\left(-p-\frac{5}{2}\right) {}_2F_1\left(\begin{matrix} -p, -p-\frac{5}{2} \\ -p-\frac{3}{2} \end{matrix}\middle| \frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(-p-\frac{3}{2}\right)}$$

input
```
integrate(x**(-6-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(-2*p - 5)*gamma(-p - 5/2)*hyper((-p, -p - 5/2), (-p - 3/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(-p - 3/2))
```

Maxima [F]

$$\int x^{-6-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Giac [F]

$$\int x^{-6-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 6), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-6-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+6}}\,dx$$

input
```
int((a + b*x^2)^p/x^(2*p + 6),x)
```

output
```
int((a + b*x^2)^p/x^(2*p + 6), x)
```

Reduce [F]

$$\int x^{-6-2p}\left(a+bx^2\right)^p\,dx=\frac{-\left(b\,x^2+a\right)^p-2x^{2p}\left(\int \frac{\left(b\,x^2+a\right)^p}{x^{2p}a\,x^6+x^{2p}b\,x^8}dx\right)ap\,x^5}{5x^{2p}x^5}$$

input
```
int(x^(-6-2*p)*(b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2)**p - 2*x**(2*p)*int((a + b*x**2)**p/(x**(2*p)*a*x**6 + x*
*(2*p)*b*x**8),x)*a*p*x**5)/(5*x**(2*p)*x**5)
```

3.1295 $\int x^{-4-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int x^{-4-2p}\left(a+bx^2\right)^p\,dx = -\frac{x^{-3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-p,\frac{1}{2}(-1-2p),-\frac{bx^2}{a}\right)}{3+2p}$$

output
```
-x^(-3-2*p)*(b*x^2+a)^p*hypergeom([-p, -3/2-p],[-1/2-p],-b*x^2/a)/(3+2*p)/
((1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^{-4-2p}\left(a+bx^2\right)^p\,dx = -\frac{x^{-3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-\frac{3}{2}-p,-p,-\frac{1}{2}-p,-\frac{bx^2}{a}\right)}{3+2p}$$

input
```
Integrate[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-3/2 - p, -p, -1/2 - p, -(
(b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-4}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{-2(p+2)}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-2p-3}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p,\frac{1}{2}(-2p-1),-\frac{bx^2}{a}\right)}{2p+3}$$

input
```
Int[x^(-4 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-3 - 2*p)/2, -p, (-1 - 2*
p)/2, -((b*x^2)/a)])/((3 + 2*p)*(1 + (b*x^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-4-2p} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{-4-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p x^{-2\,p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.59 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.77

$$\int x^{-4-2p}(a+bx^2)^p\,dx = \frac{a^p x^{-2p-3}\Gamma\left(-p-\frac{3}{2}\right)\,{}_2F_1\left(\begin{matrix}-p,-p-\frac{3}{2}\\ -p-\frac{1}{2}\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(-p-\frac{1}{2}\right)}$$

input
```
integrate(x**(-4-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(-2*p - 3)*gamma(-p - 3/2)*hyper((-p, -p - 3/2), (-p - 1/2,), b*x*
*2*exp_polar(I*pi)/a)/(2*gamma(-p - 1/2))
```

Maxima [F]

$$\int x^{-4-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Giac [F]

$$\int x^{-4-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 4), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-4-2p}\left(a+bx^2\right)^p\,dx=\int \frac{\left(b\,x^2+a\right)^p}{x^{2\,p+4}}\,dx$$

input
```
int((a + b*x^2)^p/x^(2*p + 4),x)
```

output
```
int((a + b*x^2)^p/x^(2*p + 4), x)
```

Reduce [F]

$$\int x^{-4-2p}\left(a+bx^2\right)^p\,dx=\frac{-\left(b\,x^2+a\right)^p-2x^{2p}\left(\int \frac{\left(b\,x^2+a\right)^p}{x^{2p}a\,x^4+x^{2p}b\,x^6}dx\right)ap\,x^3}{3x^{2p}x^3}$$

input
```
int(x^(-4-2*p)*(b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2)**p - 2*x**(2*p)*int((a + b*x**2)**p/(x**(2*p)*a*x**4 + x*
*(2*p)*b*x**6),x)*a*p*x**3)/(3*x**(2*p)*x**3)
```

3.1296 $\int x^{-2-2p}(a+bx^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int x^{-2-2p}(a+bx^2)^p \, dx = -\frac{x^{-1-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}(-1-2p),-p,\frac{1}{2}(1-2p),-\frac{bx^2}{a}\right)}{1+2p}$$

output
```
-x^(-1-2*p)*(b*x^2+a)^p*hypergeom([-p, -1/2-p],[1/2-p],-b*x^2/a)/(1+2*p)/(
(1+b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int x^{-2-2p}(a+bx^2)^p \, dx = -\frac{x^{-1-2p}(a+bx^2)^p \left(1+\frac{bx^2}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-\frac{1}{2}-p,-p,\frac{1}{2}-p,-\frac{bx^2}{a}\right)}{1+2p}$$

input
```
Integrate[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((
b*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-2}\left(a+bx^2\right)^p dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{-2(p+1)}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-2p-1}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-1),-p,\frac{1}{2}(1-2p),-\frac{bx^2}{a}\right)}{2p+1}$$

input
```
Int[x^(-2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
-((x^(-1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(-1 - 2*p)/2, -p, (1 - 2*p
)/2, -((b*x^2)/a)])/((1 + 2*p)*(1 + (b*x^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-2p-2}\left(b\,x^{2}+a\right)^{p}dx$$

input
```
int(x^(-2*p-2)*(b*x^2+a)^p,x)
```

output
```
int(x^(-2*p-2)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{-2-2p}\left(a+bx^2\right)^p\,dx=\int\left(bx^2+a\right)^px^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.35 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int x^{-2-2p}(a+bx^2)^p\,dx = \frac{a^p x^{-2p-1}\Gamma\left(-p-\frac{1}{2}\right)\,{}_2F_1\left(\begin{matrix}-p,-p-\frac{1}{2}\\ \frac{1}{2}-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{1}{2}-p\right)}$$

input
```
integrate(x**(-2-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(-2*p - 1)*gamma(-p - 1/2)*hyper((-p, -p - 1/2), (1/2 - p,), b*x**
2*exp_polar(I*pi)/a)/(2*gamma(1/2 - p))
```

Maxima [F]

$$\int x^{-2-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Giac [F]

$$\int x^{-2-2p}(a+bx^2)^p\,dx = \int (bx^2+a)^p x^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p - 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2-2p}(a+bx^2)^p\,dx=\int \frac{(b\,x^2+a)^p}{x^{2\,p+2}}\,dx$$

input
```
int((a + b*x^2)^p/x^(2*p + 2),x)
```

output
```
int((a + b*x^2)^p/x^(2*p + 2), x)
```

Reduce [F]

$$\int x^{-2-2p}(a+bx^2)^p\,dx=\frac{-(b\,x^2+a)^p-2x^{2p}\left(\int \frac{(b\,x^2+a)^p}{x^{2p}a\,x^2+x^{2p}b\,x^4}dx\right)apx}{x^{2p}x}$$

input
```
int(x^(-2-2*p)*(b*x^2+a)^p,x)
```

output
```
( - (a + b*x**2)**p - 2*x**(2*p)*int((a + b*x**2)**p/(x**(2*p)*a*x**2 + x*
*(2*p)*b*x**4),x)*a*p*x)/(x**(2*p)*x)
```

3.1297 $\int x^{-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 69

$$\int x^{-2p}(a+bx^2)^p\,dx = \frac{x^{1-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),-p,\frac{1}{2}(3-2p),-\frac{bx^2}{a}\right)}{1-2p}$$

output
```
x^(1-2*p)*(b*x^2+a)^p*hypergeom([-p, 1/2-p],[3/2-p],-b*x^2/a)/(1-2*p)/((1+
b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^{-2p}(a+bx^2)^p\,dx = \frac{x^{1-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}-p,-p,\frac{3}{2}-p,-\frac{bx^2}{a}\right)}{1-2p}$$

input
```
Integrate[(a + b*x^2)^p/x^(2*p),x]
```

output
```
(x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[1/2 - p, -p, 3/2 - p, -((b*x^
2)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{-2p}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{1-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2p),-p,\frac{1}{2}(3-2p),-\frac{bx^2}{a}\right)}{1-2p}$$

input
```
Int[(a + b*x^2)^p/x^(2*p),x]
```

output
```
(x^(1 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(1 - 2*p)/2, -p, (3 - 2*p)/2,
 -((b*x^2)/a)])/((1 - 2*p)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int \left(b\,x^{2}+a\right)^{p} x^{-2p}dx$$

input
```
int((b*x^2+a)^p/(x^(2*p)),x)
```

output
```
int((b*x^2+a)^p/(x^(2*p)),x)
```

Fricas [F]

$$\int x^{-2p}\left(a+bx^2\right)^p\,dx=\int \frac{(bx^2+a)^p}{x^{2\,p}}\,dx$$

input
```
integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p/x^(2*p), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 6.84 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.46

$$\int x^{-2p}(a+bx^2)^p\,dx = \sqrt{b}b^{p-\frac{1}{2}}x\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-p\\ \frac{1}{2}\end{matrix}\middle|\frac{ae^{i\pi}}{bx^2}\right)$$

input
```
integrate((b*x**2+a)**p/(x**(2*p)),x)
```

output
```
sqrt(b)*b**(p - 1/2)*x*hyper((-1/2, -p), (1/2,), a*exp_polar(I*pi)/(b*x**2
))
```

Maxima [F]

$$\int x^{-2p}(a+bx^2)^p\,dx = \int \frac{(bx^2+a)^p}{x^{2p}}\,dx$$

input
```
integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p/x^(2*p), x)
```

Giac [F]

$$\int x^{-2p}(a+bx^2)^p\,dx = \int \frac{(bx^2+a)^p}{x^{2p}}\,dx$$

input
```
integrate((b*x^2+a)^p/(x^(2*p)),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p/x^(2*p), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2p}(a+bx^2)^p\,dx=\int \frac{(b\,x^2+a)^p}{x^{2\,p}}\,dx$$

input
```
int((a + b*x^2)^p/x^(2*p),x)
```

output
```
int((a + b*x^2)^p/x^(2*p), x)
```

Reduce [F]

$$\int x^{-2p}(a+bx^2)^p\,dx=\frac{(b\,x^2+a)^p\,x+2x^{2p}\left(\int \frac{(b\,x^2+a)^p}{x^{2p}a+x^{2p}b\,x^2}dx\right)ap}{x^{2p}}$$

input
```
int((b*x^2+a)^p/(x^(2*p)),x)
```

output
```
((a + b*x**2)**p*x + 2*x**(2*p)*int((a + b*x**2)**p/(x**(2*p)*a + x**(2*p)
*b*x**2),x)*a*p)/x**(2*p)
```

3.1298 $\int x^{2-2p}(a+bx^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int x^{2-2p}(a+bx^2)^p\,dx = \frac{x^{3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(3-2p),-p,\frac{1}{2}(5-2p),-\frac{bx^2}{a}\right)}{3-2p}$$

output
```
x^(3-2*p)*(b*x^2+a)^p*hypergeom([-p, 3/2-p],[5/2-p],-b*x^2/a)/(3-2*p)/((1+
b*x^2/a)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^{2-2p}(a+bx^2)^p\,dx = \frac{x^{3-2p}(a+bx^2)^p\left(1+\frac{bx^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{3}{2}-p,-p,\frac{5}{2}-p,-\frac{bx^2}{a}\right)}{3-2p}$$

input
```
Integrate[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[3/2 - p, -p, 5/2 - p, -((b*x^
2)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{2-2p}\left(a+bx^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\int x^{2-2p}\left(\frac{bx^2}{a}+1\right)^p dx$$

$$\Big\downarrow \text{278}$$

$$\frac{x^{3-2p}\left(a+bx^2\right)^p\left(\frac{bx^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(3-2p),-p,\frac{1}{2}(5-2p),-\frac{bx^2}{a}\right)}{3-2p}$$

input
```
Int[x^(2 - 2*p)*(a + b*x^2)^p,x]
```

output
```
(x^(3 - 2*p)*(a + b*x^2)^p*Hypergeometric2F1[(3 - 2*p)/2, -p, (5 - 2*p)/2,
 -((b*x^2)/a)])/((3 - 2*p)*(1 + (b*x^2)/a)^p)
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{2-2p} \left(b\,x^{2}+a\right)^{p} dx$$

input
```
int(x^(2-2*p)*(b*x^2+a)^p,x)
```

output
```
int(x^(2-2*p)*(b*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{2-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^{2}+a\right)^{p}x^{-2\,p+2}\,dx$$

input
```
integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.91 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.67

$$\int x^{2-2p}\left(a+bx^2\right)^p\,dx=\frac{a^p x^{3-2p}\Gamma\left(\frac{3}{2}-p\right){}_2F_1\left(\begin{matrix}-p,\frac{3}{2}-p\\ \frac{5}{2}-p\end{matrix}\middle|\frac{bx^2e^{i\pi}}{a}\right)}{2\Gamma\left(\frac{5}{2}-p\right)}$$

input
```
integrate(x**(2-2*p)*(b*x**2+a)**p,x)
```

output
```
a**p*x**(3 - 2*p)*gamma(3/2 - p)*hyper((-p, 3/2 - p), (5/2 - p,), b*x**2*e
xp_polar(I*pi)/a)/(2*gamma(5/2 - p))
```

Maxima [F]

$$\int x^{2-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p x^{-2\,p+2}\,dx$$

input
```
integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Giac [F]

$$\int x^{2-2p}\left(a+bx^2\right)^p\,dx=\int \left(bx^2+a\right)^p x^{-2\,p+2}\,dx$$

input
```
integrate(x^(2-2*p)*(b*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((b*x^2 + a)^p*x^(-2*p + 2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2-2p}(a+bx^2)^p\,dx=\int x^{2-2p}\,\left(b\,x^2+a\right)^p\,dx$$

input
```
int(x^(2 - 2*p)*(a + b*x^2)^p,x)
```

output
```
int(x^(2 - 2*p)*(a + b*x^2)^p, x)
```

Reduce [F]

$$\int x^{2-2p}(a+bx^2)^p\,dx$$
$$=\frac{2(b\,x^2+a)^p\,apx+(b\,x^2+a)^p\,b\,x^3+4x^{2p}\left(\int\frac{(b\,x^2+a)^p}{x^{2p}a+x^{2p}b\,x^2}dx\right)a^2p^2-2x^{2p}\left(\int\frac{(b\,x^2+a)^p}{x^{2p}a+x^{2p}b\,x^2}dx\right)a^2p}{3x^{2p}b}$$

input
```
int(x^(2-2*p)*(b*x^2+a)^p,x)
```

output
```
(2*(a + b*x**2)**p*a*p*x + (a + b*x**2)**p*b*x**3 + 4*x**(2*p)*int((a + b*
x**2)**p/(x**(2*p)*a + x**(2*p)*b*x**2),x)*a**2*p**2 - 2*x**(2*p)*int((a +
 b*x**2)**p/(x**(2*p)*a + x**(2*p)*b*x**2),x)*a**2*p)/(3*x**(2*p)*b)
```

3.1299 $\int x^{-1-p}(2+3x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int x^{-1-p}(2+3x^2)^p \, dx = -\frac{2^p x^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{2}\right)}{p}$$

output
```
-2^p*hypergeom([-p, -1/2*p],[1-1/2*p],-3/2*x^2)/p/(x^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int x^{-1-p}(2+3x^2)^p \, dx = -\frac{2^p x^{-p} \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{2}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(2 + 3*x^2)^p,x]
```

output
```
-((2^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (-3*x^2)/2])/(p*x^p))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}(3x^2+2)^p\,dx$$

$$\Big\downarrow \; 278$$

$$-\frac{2^p x^{-p}\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{2}\right)}{p}$$

input
```
Int[x^(-1 - p)*(2 + 3*x^2)^p,x]
```

output
```
-((2^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (-3*x^2)/2])/(p*x^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
meijerg	$-\frac{2^p x^{-p}\,\text{hypergeom}\left(\left[-p,-\frac{p}{2}\right],\left[1-\frac{p}{2}\right],-\frac{3x^2}{2}\right)}{p}$	33

input
```
int(x^(-1-p)*(3*x^2+2)^p,x,method=_RETURNVERBOSE)
```

output
```
-2^p/p*x^(-p)*hypergeom([-p,-1/2*p],[1-1/2*p],-3/2*x^2)
```

Fricas [F]

$$\int x^{-1-p}(2+3x^2)^p\,dx = \int (3\,x^2+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2+2)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.64 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^{-1-p}(2+3x^2)^p\,dx = \frac{2^p x^{-p}\Gamma\left(-\frac{p}{2}\right){}_2F_1\left(\begin{matrix}-p,-\frac{p}{2}\\1-\frac{p}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{2}\right)}{2\Gamma\left(1-\frac{p}{2}\right)}$$

input
```
integrate(x**(-1-p)*(3*x**2+2)**p,x)
```

output
```
2**p*gamma(-p/2)*hyper((-p, -p/2), (1 - p/2,), 3*x**2*exp_polar(I*pi)/2)/(
2*x**p*gamma(1 - p/2))
```

Maxima [F]

$$\int x^{-1-p}(2+3x^2)^p\,dx=\int (3\,x^2+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2+2)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(2+3x^2)^p\,dx=\int (3\,x^2+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2+2)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}(2+3x^2)^p\,dx=\int \frac{(3\,x^2+2)^p}{x^{p+1}}\,dx$$

input
```
int((3*x^2 + 2)^p/x^(p + 1),x)
```

output
```
int((3*x^2 + 2)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}(2+3x^2)^p\,dx = \frac{(3x^2+2)^p + 4x^p\left(\int \frac{(3x^2+2)^p}{3x^p x^3+2x^p x}dx\right)p}{x^p p}$$

input
```
int(x^(-1-p)*(3*x^2+2)^p,x)
```

output
```
((3*x**2 + 2)**p + 4*x**p*int((3*x**2 + 2)**p/(3*x**p*x**3 + 2*x**p*x),x)*
p)/(x**p*p)
```

3.1300 $\int x^{-1-p}(-2+3x^2)^p \, dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int x^{-1-p}\left(-2+3x^2\right)^p \, dx = -\frac{x^{-p}\left(1-\frac{3x^2}{2}\right)^{-p}\left(-2+3x^2\right)^p \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},\frac{3x^2}{2}\right)}{p}$$

output
```
-(3*x^2-2)^p*hypergeom([-p, -1/2*p],[1-1/2*p],3/2*x^2)/p/(x^p)/((1-3/2*x^2
)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{-1-p}\left(-2+3x^2\right)^p \, dx = -\frac{x^{-p}\left(1-\frac{3x^2}{2}\right)^{-p}\left(-2+3x^2\right)^p \text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},\frac{3x^2}{2}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(-2 + 3*x^2)^p,x]
```

output
```
-(((-2 + 3*x^2)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (3*x^2)/2])/(p*x^
p*(1 - (3*x^2)/2)^p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}\left(3x^2-2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(1-\frac{3x^2}{2}\right)^{-p}\left(3x^2-2\right)^p\int x^{-p-1}\left(1-\frac{3x^2}{2}\right)^p\,dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-p}\left(1-\frac{3x^2}{2}\right)^{-p}\left(3x^2-2\right)^p\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},\frac{3x^2}{2}\right)}{p}$$

input
```
Int[x^(-1 - p)*(-2 + 3*x^2)^p,x]
```

output
```
-(((-2 + 3*x^2)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (3*x^2)/2])/(p*x^
p*(1 - (3*x^2)/2)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

method	result	size
meijerg	$-\frac{2^{p} \operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)^{p} \left(-\operatorname{signum}\left(-1+\frac{3 x^{2}}{2}\right)\right)^{-p} x^{-p} \operatorname{hypergeom}\left(\left[-p,-\frac{p}{2}\right],\left[1-\frac{p}{2}\right],\frac{3 x^{2}}{2}\right)}{p}$	57

input
```
int(x^(-1-p)*(3*x^2-2)^p,x,method=_RETURNVERBOSE)
```

output
```
-2^p*signum(-1+3/2*x^2)^p*(-signum(-1+3/2*x^2))^(-p)/p*x^(-p)*hypergeom([-
p,-1/2*p],[1-1/2*p],3/2*x^2)
```

Fricas [F]

$$\int x^{-1-p}\left(-2+3x^2\right)^p\,dx=\int \left(3\,x^{2}-2\right)^{p} x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2-2)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.73 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int x^{-1-p}(-2+3x^2)^p\,dx = \frac{2^p x^{-p} e^{i\pi p}\Gamma\left(-\frac{p}{2}\right) {}_2F_1\left(\begin{matrix} -p, -\frac{p}{2} \\ 1-\frac{p}{2} \end{matrix}\middle| \frac{3x^2}{2}\right)}{2\Gamma\left(1-\frac{p}{2}\right)}$$

input
```
integrate(x**(-1-p)*(3*x**2-2)**p,x)
```

output
```
2**p*exp(I*pi*p)*gamma(-p/2)*hyper((-p, -p/2), (1 - p/2,), 3*x**2/2)/(2*x*
*p*gamma(1 - p/2))
```

Maxima [F]

$$\int x^{-1-p}(-2+3x^2)^p\,dx = \int (3\,x^2-2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2-2)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(-2+3x^2)^p \, dx = \int (3\,x^2-2)^p x^{-p-1} \, dx$$

input
```
integrate(x^(-1-p)*(3*x^2-2)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}(-2+3x^2)^p \, dx = \int \frac{(3\,x^2-2)^p}{x^{p+1}} \, dx$$

input
```
int((3*x^2 - 2)^p/x^(p + 1),x)
```

output
```
int((3*x^2 - 2)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}(-2+3x^2)^p \, dx = \frac{(3x^2-2)^p - 4x^p\left(\int \frac{(3x^2-2)^p}{3x^p x^3-2x^p x}dx\right)p}{x^p p}$$

input
```
int(x^(-1-p)*(3*x^2-2)^p,x)
```

output
```
((3*x**2 - 2)**p - 4*x**p*int((3*x**2 - 2)**p/(3*x**p*x**3 - 2*x**p*x),x)*
p)/(x**p*p)
```

3.1301 $\int x^{-1-p}(a+3x^2)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 57

$$\int x^{-1-p}(a+3x^2)^p\,dx = -\frac{x^{-p}(a+3x^2)^p\left(1+\frac{3x^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{a}\right)}{p}$$

output
```
-(3*x^2+a)^p*hypergeom([-p, -1/2*p],[1-1/2*p],-3*x^2/a)/p/(x^p)/((1+3*x^2/
a)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int x^{-1-p}(a+3x^2)^p\,dx = -\frac{x^{-p}(a+3x^2)^p\left(1+\frac{3x^2}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{a}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(a + 3*x^2)^p,x]
```

output
```
-(((a + 3*x^2)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (-3*x^2)/a])/(p*x^
p*(1 + (3*x^2)/a)^p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {279, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}\left(a+3x^2\right)^p\,dx$$

$$\Big\downarrow \text{279}$$

$$\left(a+3x^2\right)^p\left(\frac{3x^2}{a}+1\right)^{-p}\int x^{-p-1}\left(\frac{3x^2}{a}+1\right)^p\,dx$$

$$\Big\downarrow \text{278}$$

$$-\frac{x^{-p}\left(a+3x^2\right)^p\left(\frac{3x^2}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{2},1-\frac{p}{2},-\frac{3x^2}{a}\right)}{p}$$

input
```
Int[x^(-1 - p)*(a + 3*x^2)^p,x]
```

output
```
-(((a + 3*x^2)^p*Hypergeometric2F1[-p, -1/2*p, 1 - p/2, (-3*x^2)/a])/(p*x^
p*(1 + (3*x^2)/a)^p))
```

Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 279
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^IntP
art[p]*((a + b*x^2)^FracPart[p]/(1 + b*(x^2/a))^FracPart[p])   Int[(c*x)^m*
(1 + b*(x^2/a))^p, x], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] &&
!(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-p} \left(3x^2 + a\right)^p dx$$

input
```
int(x^(-1-p)*(3*x^2+a)^p,x)
```

output
```
int(x^(-1-p)*(3*x^2+a)^p,x)
```

Fricas [F]

$$\int x^{-1-p} \left(a + 3x^2\right)^p \, dx = \int \left(3\,x^2 + a\right)^p x^{-p-1} \, dx$$

input
```
integrate(x^(-1-p)*(3*x^2+a)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + a)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.78 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int x^{-1-p}(a+3x^2)^p\,dx = \frac{a^p x^{-p}\Gamma\left(-\frac{p}{2}\right) {}_2F_1\left(\begin{matrix}-p,-\frac{p}{2}\\1-\frac{p}{2}\end{matrix}\middle|\frac{3x^2e^{i\pi}}{a}\right)}{2\Gamma\left(1-\frac{p}{2}\right)}$$

input
```
integrate(x**(-1-p)*(3*x**2+a)**p,x)
```

output
```
a**p*gamma(-p/2)*hyper((-p, -p/2), (1 - p/2,), 3*x**2*exp_polar(I*pi)/a)/(
2*x**p*gamma(1 - p/2))
```

Maxima [F]

$$\int x^{-1-p}(a+3x^2)^p\,dx = \int (3\,x^2+a)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2+a)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + a)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(a+3x^2)^p\,dx = \int (3\,x^2+a)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(3*x^2+a)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + a)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}\left(a+3x^2\right)^p\,dx=\int \frac{\left(3\,x^2+a\right)^p}{x^{p+1}}\,dx$$

input
```
int((a + 3*x^2)^p/x^(p + 1),x)
```

output
```
int((a + 3*x^2)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}\left(a+3x^2\right)^p\,dx=\frac{-\left(3x^2+a\right)^p+6x^p\left(\int \frac{\left(3x^2+a\right)^p x}{x^p a+3x^p x^2}dx\right)p}{x^p p}$$

input
```
int(x^(-1-p)*(3*x^2+a)^p,x)
```

output
```
( - (a + 3*x**2)**p + 6*x**p*int(((a + 3*x**2)**p*x)/(x**p*a + 3*x**p*x**2
),x)*p)/(x**p*p)
```

3.1302 $\int x^{-1-p}(2+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int x^{-1-p}(2+bx^n)^p\,dx=-\frac{2^px^{-p}\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{2}\right)}{p}$$

output
```
-2^p*hypergeom([-p, -p/n],[1-p/n],-1/2*b*x^n)/p/(x^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x^{-1-p}(2+bx^n)^p\,dx=-\frac{2^px^{-p}\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{2}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(2 + b*x^n)^p,x]
```

output
```
-((2^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, -1/2*(b*x^n)])/(p*x^p))
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}(bx^n+2)^p\,dx$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{2^p x^{-p}\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{2}\right)}{p}$$

input
```
Int[x^(-1 - p)*(2 + b*x^n)^p,x]
```

output
```
-((2^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, -1/2*(b*x^n)])/(p*x^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.88 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.23

method	result	size
meijerg	$-\frac{2^{p} x^{-p} \operatorname{hypergeom}\left(\left[-p, \frac{-1-p}{n}+\frac{1}{n}\right], \left[1+\frac{-1-p}{n}+\frac{1}{n}\right], \frac{i x^{n} b (-1)^{\frac{\operatorname{csgn}(i b)}{2}+\frac{\operatorname{csgn}\left(i x^{n}\right)}{2}-\frac{\operatorname{csgn}\left(i x^{n}\right) \operatorname{csgn}(i b)}{2}}}{2}\right)}{p}$	87

input
```
int(x^(-1-p)*(2+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-2^p/p*x^(-p)*hypergeom([-p,(-1-p)/n+1/n],[1+(-1-p)/n+1/n],1/2*I*x^n*b*(-1
)^(1/2*csgn(I*b)+1/2*csgn(I*x^n)-1/2*csgn(I*x^n)*csgn(I*b)))
```

Fricas [F]

$$\int x^{-1-p}(2+bx^n)^p\,dx=\int(bx^n+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(2+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + 2)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int x^{-1-p}(2+bx^n)^p\,dx=\frac{2^p x^{-p}\Gamma\left(-\frac{p}{n}\right){}_2F_1\left(\left.\begin{matrix}-p,-\frac{p}{n}\\1-\frac{p}{n}\end{matrix}\right|\frac{bx^ne^{i\pi}}{2}\right)}{n\Gamma\left(1-\frac{p}{n}\right)}$$

input
```
integrate(x**(-1-p)*(2+b*x**n)**p,x)
```

output
```
2**p*gamma(-p/n)*hyper((-p, -p/n), (1 - p/n,), b*x**n*exp_polar(I*pi)/2)/(
n*x**p*gamma(1 - p/n))
```

Maxima [F]

$$\int x^{-1-p}(2+bx^n)^p\,dx = \int (bx^n+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(2+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + 2)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(2+bx^n)^p\,dx = \int (bx^n+2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(2+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + 2)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}(2+bx^n)^p\,dx = \int \frac{(b\,x^n+2)^p}{x^{p+1}}\,dx$$

input
```
int((b*x^n + 2)^p/x^(p + 1),x)
```

output
```
int((b*x^n + 2)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}(2+bx^n)^p\,dx$$
$$=\frac{(x^nb+2)^p+2x^p\left(\int\frac{(x^nb+2)^p}{x^{n+p}bnx-x^{n+p}bx+2x^pnx-2x^px}dx\right)n^2p-2x^p\left(\int\frac{(x^nb+2)^p}{x^{n+p}bnx-x^{n+p}bx+2x^pnx-2x^px}dx\right)np}{x^pp\,(n-1)}$$

input
```
int(x^(-1-p)*(2+b*x^n)^p,x)
```

output
```
((x**n*b + 2)**p + 2*x**p*int((x**n*b + 2)**p/(x**(n + p)*b*n*x - x**(n +
p)*b*x + 2*x**p*n*x - 2*x**p*x),x)*n**2*p - 2*x**p*int((x**n*b + 2)**p/(x*
*(n + p)*b*n*x - x**(n + p)*b*x + 2*x**p*n*x - 2*x**p*x),x)*n*p)/(x**p*p*(
n - 1))
```

3.1303 $\int x^{-1-p}(-2+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 59

$$\int x^{-1-p}(-2+bx^n)^p\,dx = -\frac{x^{-p}\left(1-\frac{bx^n}{2}\right)^{-p}(-2+bx^n)^p\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},\frac{bx^n}{2}\right)}{p}$$

output
```
-(-2+b*x^n)^p*hypergeom([-p, -p/n],[1-p/n],1/2*b*x^n)/p/(x^p)/((1-1/2*b*x^
n)^p)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int x^{-1-p}(-2+bx^n)^p\,dx = -\frac{x^{-p}\left(1-\frac{bx^n}{2}\right)^{-p}(-2+bx^n)^p\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},\frac{bx^n}{2}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(-2 + b*x^n)^p,x]
```

output
```
-(((-2 + b*x^n)^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, (b*x^n)/2])/(p*x^
p*(1 - (b*x^n)/2)^p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}(bx^n-2)^p\,dx$$

$$\Big\downarrow \text{889}$$

$$\left(1-\frac{bx^n}{2}\right)^{-p}(bx^n-2)^p\int x^{-p-1}\left(1-\frac{bx^n}{2}\right)^p dx$$

$$\Big\downarrow \text{888}$$

$$-\frac{x^{-p}\left(1-\frac{bx^n}{2}\right)^{-p}(bx^n-2)^p\,\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},\frac{bx^n}{2}\right)}{p}$$

input
```
Int[x^(-1 - p)*(-2 + b*x^n)^p,x]
```

output
```
-(((-2 + b*x^n)^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, (b*x^n)/2])/(p*x^
p*(1 - (b*x^n)/2)^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.81 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.92

method	result
meijerg	$-\frac{2^{p} \operatorname{signum}\left(-1+\frac{b x^{n}}{2}\right)^{p} \left(-\operatorname{signum}\left(-1+\frac{b x^{n}}{2}\right)\right)^{-p} x^{-p} \operatorname{hypergeom}\left(\left[-p, \frac{-1-p}{n}+\frac{1}{n}\right], \left[1+\frac{-1-p}{n}+\frac{1}{n}\right], -\frac{i x^{n} b (-1)^{-\frac{\operatorname{csgn}(ib)}{2}+\frac{\operatorname{csgn}(}{2}}}{2}\right.}{p}$

input
```
int(x^(-1-p)*(-2+b*x^n)^p,x,method=_RETURNVERBOSE)
```

output
```
-2^p*signum(-1+1/2*b*x^n)^p*(-signum(-1+1/2*b*x^n))^(-p)/p*x^(-p)*hypergeo
m([-p,(-1-p)/n+1/n],[1+(-1-p)/n+1/n],-1/2*I*x^n*b*(-1)^(-1/2*csgn(I*b)+1/2
*csgn(I*x^n)+1/2*csgn(I*x^n)*csgn(I*b)))
```

Fricas [F]

$$\int x^{-1-p}(-2+bx^n)^p\,dx = \int (bx^n-2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(-2+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n - 2)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int x^{-1-p}(-2+bx^n)^p\,dx = \frac{2^p x^{-p} e^{i\pi p}\Gamma\left(-\frac{p}{n}\right) {}_2F_1\left(\begin{matrix}-p,-\frac{p}{n}\\1-\frac{p}{n}\end{matrix}\middle|\frac{bx^n}{2}\right)}{n\Gamma\left(1-\frac{p}{n}\right)}$$

input
```
integrate(x**(-1-p)*(-2+b*x**n)**p,x)
```

output
```
2**p*exp(I*pi*p)*gamma(-p/n)*hyper((-p, -p/n), (1 - p/n,), b*x**n/2)/(n*x*
*p*gamma(1 - p/n))
```

Maxima [F]

$$\int x^{-1-p}(-2+bx^n)^p\,dx = \int (bx^n-2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(-2+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n - 2)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(-2+bx^n)^p\,dx=\int (bx^n-2)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(-2+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n - 2)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}(-2+bx^n)^p\,dx=\int \frac{(b\,x^n-2)^p}{x^{p+1}}\,dx$$

input
```
int((b*x^n - 2)^p/x^(p + 1),x)
```

output
```
int((b*x^n - 2)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}(-2+bx^n)^p\,dx$$
$$=\frac{(x^nb-2)^p-2x^p\left(\int \frac{(x^nb-2)^p}{x^{n+p}bnx-x^{n+p}bx-2x^pnx+2x^px}dx\right)n^2p+2x^p\left(\int \frac{(x^nb-2)^p}{x^{n+p}bnx-x^{n+p}bx-2x^pnx+2x^px}dx\right)np}{x^pp\,(n-1)}$$

input
```
int(x^(-1-p)*(-2+b*x^n)^p,x)
```

output
```
((x**n*b - 2)**p - 2*x**p*int((x**n*b - 2)**p/(x**(n + p)*b*n*x - x**(n +
p)*b*x - 2*x**p*n*x + 2*x**p*x),x)*n**2*p + 2*x**p*int((x**n*b - 2)**p/(x*
*(n + p)*b*n*x - x**(n + p)*b*x - 2*x**p*n*x + 2*x**p*x),x)*n*p)/(x**p*p*(
n - 1))
```

3.1304 $\int x^{-1-p}(a+bx^n)^p\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int x^{-1-p}(a+bx^n)^p\,dx = -\frac{x^{-p}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{a}\right)}{p}$$

output
```
-(a+b*x^n)^p*hypergeom([-p, -p/n],[1-p/n],-b*x^n/a)/p/(x^p)/((1+b*x^n/a)^p
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x^{-1-p}(a+bx^n)^p\,dx = -\frac{x^{-p}(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{a}\right)}{p}$$

input
```
Integrate[x^(-1 - p)*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, -((b*x^n)/a)])/(p*
x^p*(1 + (b*x^n)/a)^p))
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {889, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-p-1}(a+bx^n)^p\,dx$$

$$\Big\downarrow \text{ 889}$$

$$(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\int x^{-p-1}\left(\frac{bx^n}{a}+1\right)^p dx$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{x^{-p}(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,-\frac{p}{n},1-\frac{p}{n},-\frac{bx^n}{a}\right)}{p}$$

input
```
Int[x^(-1 - p)*(a + b*x^n)^p,x]
```

output
```
-(((a + b*x^n)^p*Hypergeometric2F1[-p, -(p/n), 1 - p/n, -((b*x^n)/a)])/(p*
x^p*(1 + (b*x^n)/a)^p))
```

Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 889
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p])   Int[(c*x)
^m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0
] &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Maple [F]

$$\int x^{-1-p}(a+b\,x^{n})^{p}\,dx$$

input
```
int(x^(-1-p)*(a+b*x^n)^p,x)
```

output
```
int(x^(-1-p)*(a+b*x^n)^p,x)
```

Fricas [F]

$$\int x^{-1-p}(a+bx^n)^p\ dx=\int (bx^n+a)^p x^{-p-1}\,dx$$

input
```
integrate(x^(-1-p)*(a+b*x^n)^p,x, algorithm="fricas")
```

output
```
integral((b*x^n + a)^p*x^(-p - 1), x)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.16 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x^{-1-p}(a+bx^n)^p \, dx = \frac{a^{-\frac{p}{n}} a^{p+\frac{p}{n}} x^{-p} \Gamma\left(-\frac{p}{n}\right) {}_2F_1\left(\begin{matrix} -p, -\frac{p}{n} \\ 1-\frac{p}{n} \end{matrix} \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1-\frac{p}{n}\right)}$$

input
```
integrate(x**(-1-p)*(a+b*x**n)**p,x)
```

output
```
a**(p + p/n)*gamma(-p/n)*hyper((-p, -p/n), (1 - p/n,), b*x**n*exp_polar(I*
pi)/a)/(a**(p/n)*n*x**p*gamma(1 - p/n))
```

Maxima [F]

$$\int x^{-1-p}(a+bx^n)^p \, dx = \int (bx^n+a)^p x^{-p-1} \, dx$$

input
```
integrate(x^(-1-p)*(a+b*x^n)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^n + a)^p*x^(-p - 1), x)
```

Giac [F]

$$\int x^{-1-p}(a+bx^n)^p \, dx = \int (bx^n+a)^p x^{-p-1} \, dx$$

input
```
integrate(x^(-1-p)*(a+b*x^n)^p,x, algorithm="giac")
```

output
```
integrate((b*x^n + a)^p*x^(-p - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1-p}(a+bx^n)^p\,dx=\int\frac{(a+b\,x^n)^p}{x^{p+1}}\,dx$$

input
```
int((a + b*x^n)^p/x^(p + 1),x)
```

output
```
int((a + b*x^n)^p/x^(p + 1), x)
```

Reduce [F]

$$\int x^{-1-p}(a+bx^n)^p\,dx = \frac{(x^nb+a)^p+x^p\left(\int\frac{(x^nb+a)^p}{x^{n+p}bnx-x^{n+p}bx+x^panx-x^pax}dx\right)a\,n^2p-x^p\left(\int\frac{(x^nb+a)^p}{x^{n+p}bnx-x^{n+p}bx+x^panx-x^pax}dx\right)anp}{x^pp\,(n-1)}$$

input
```
int(x^(-1-p)*(a+b*x^n)^p,x)
```

output
```
((x**n*b + a)**p + x**p*int((x**n*b + a)**p/(x**(n + p)*b*n*x - x**(n + p)
*b*x + x**p*a*n*x - x**p*a*x),x)*a*n**2*p - x**p*int((x**n*b + a)**p/(x**(
n + p)*b*n*x - x**(n + p)*b*x + x**p*a*n*x - x**p*a*x),x)*a*n*p)/(x**p*p*(
n - 1))
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
           return 1
        else:
           return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
           return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
           if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
              return 1
           else:
              return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
           return max(3,expnType(expn.args[0]),expnType(expn.args[1]))  #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file